2001-01-3033

A Bayesian Approach to Non-Deterministic Hypersonic Vehicle Design

George C. Mantis *, **Dimitri N. Mavris** †
Aerospace Systems Design Laboratory
Georgia Institute of Technology
Atlanta, GA 30332

ABSTRACT

Affordable, reliable endo- and exoatmospheric transportation, for both the military and commercial sectors, grows in importance as the world grows smaller and space exploration and exploitation increasingly impact our daily lives. However, the impact of disciplinary, operational, and technological uncertainties inhibit the design of the requisite hypersonic vehicles, an inherently multidisciplinary and non-deterministic process. Without investigation, these components of design uncertainty undermine the designers' decision-making confidence.

In this paper, the authors propose a new probabilistic design method, using Bayesian Statistics techniques, which allows assessment of the impact of disciplinary uncertainty on the confidence in the design solution. The proposed development of a two-stage reusable launch vehicle configuration highlights the means to first quantify the fidelity of the disciplinary analysis tools utilized, then propagate such to the vehicle system level.

BACKGROUND

For the second time since its inception, the United States Air Force (USAF) Scientific Advisory Board (SAB) researched the future needs and present shortcomings in the USAF's overall mission effectiveness. This assessment includes definitions of and requirements for future mission areas. The SAB compiled their findings in a series of volumes, collectively known as the New World Vistas, describing aerospace systems concepts with the potential to provide the greatest mission capabilities to the future Air Force [1]. For many cases, the United States needs to develop and field hypersonic systems to best equip its Air Force for its missions. In fact, the SAB considers hypersonic vehicles one of seven top-level system categories to answer the challenges facing tomorrow's Air Force. Specifically, its members envision hypersonics taking operational form as tactical and strategic missiles, global-reach spaceplanes, and reusable launch vehicles. These systems could fulfill the strike, bombing, mobility, reconaissance, and space access roles more effectively than other candidates, owing to their higher speeds and lethality.

Market forecasts reveal strong growth in worldwide passenger travel. This growing demand, and the desire for shorter trip times, point to a hypersonic commercial transport to satisfy passenger's needs. A correspondingly stronger world air cargo market, and the desire for faster route travel, indicate a possible freighter role for the hypersonic transport as well.

Growing opportunities exist for commercial vehicles for access to space as well. Taking the form of reusable launch vehicles (RLVs), commercial hypersonic systems would fulfill the missions of rapid intercontinental transport, satellite delivery and on-orbit maintenance, and civil space missions including manned spaceflight and the International Space Station [2].

Cost drives the desire to shift the world's space launch burden off of its fleet of expendable launchers. The expense of operating the Space Shuttle, and expendable launchers such as the Titan IV, constrains commercial and government efforts in space, especially with today's declining budgets [3]. Reusable launch vehicles (RLV's) potentially offer more reliable and affordable access to space. However, NASA cannot retire the veteran Shuttle until commercial RLVs not only become available, but have demonstrated safe and reliable operation [2].

MOTIVATION

DISCIPLINARY UNCERTAINTY The lack of a credible multidisciplinary simulation capability for hypersonic vehicles stands between those vehicles in service today and those needed tomorrow. Any aircraft design process re-

* Ph.D. Candidate, ASDL
† Director, ASDL; Professor and Boeing Chair in Advanced Aerospace Systems Analysis

quires a multidisciplinary approach to vehicle synthesis and sizing to best evaluate the compromises between the various disciplines. Hypersonic vehicle design takes this “necessary evil” to the extreme, for it exhibits an exceptionally tight coupling of disciplines [4], as shown in Figure 1. The more stringent requirements and constraints of the design force this close coupling, with greater sensitivity to changes.

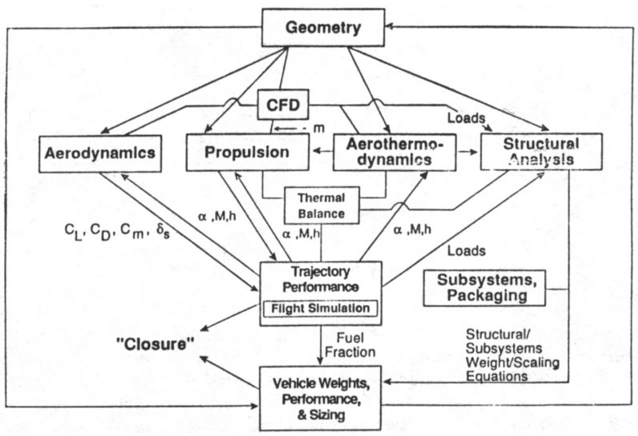

Figure 1: Coupling of disciplines in hypersonic vehicle design [4]

A synthesis and sizing capability takes form as an automated computational environment of integrated disciplinary analysis tools [5]. The term “disciplines” implies both those in the classic engineering sense, such as aerodynamics and propulsion, and the product life-cycle sense, including manufacturing and operations. Analysis tools may run the gamut from simple regression equations, to physics- and process-based analyses (e.g., computational fluid dynamics), to experimental databases.

The fidelity of the synthesis and sizing environment’s constituent analyses comes into question [6], as do their algorithmic accuracy. The use of lower-fidelity tools results from trading accuracy for computational speed, or from the utter lack of tools of higher analytical fidelity. Lower-fidelity tools implement first-order analyses in the form of regression of historical data or oversimplifications of the physics involved. As such, they utilize only a minimal input vehicle configuration, and in turn, require a minimum of time for problem setup and execution. These tools thus run quickly, yet provide dubious results. The inability to analytically predict the exact value of parameters defines **disciplinary uncertainty**, the uncertainty ultimately manifest in the use of lower-fidelity tools.

Other than accounting for the impact of uncertainty, only the use of higher-fidelity tools mitigates this problem. However, their physics-based analyses require detailed input and long setup and execution times, rendering their use at each design iteration impractical. Also, higher-fidelity codes don’t completely mitigate the problem of design uncertainty. For example, such codes include calibration factors to finely adjust the values of the outputs; these factors exist to circumvent the uncertainty in calculations of the given values.

Also, particularly in the hypersonic flight regime, high-fidelity disciplinary analysis tools elude use by vehicle designers. The hypersonic flight literature documents a number of efforts to develop such analyses, including those for aerodynamics (References [7, 8]) and propulsion (References [9, 10]). Meanwhile, several more efforts seek experimental solutions, as described in References [11–14]. Yet Blankson et al. discuss the “inability of ground facilities to generate hypersonic test data at real flight conditions for validating design tools” [15]. The tendency of flight test data “to raise more questions than answers” compounds the problem [16].

Until recent efforts come to fruition, disciplinary uncertainty will hamper conceptual and preliminary hypersonic vehicle design. The inadequacies of test facilities contribute to the lack of knowledge of the hypersonic aerothermodynamic environment, resulting in extreme limitations of current multidisciplinary simulation capability [15].

ACCOUNTING FOR UNCERTAINTY NASA Administrator Daniel Goldin states that [17]

> . . . the hypersonic and space environments are filled with uncertainty, so traditional numerical approaches will not work. . . In order to account for the uncertainty and to quantify the risk level, we need to move from the traditional deterministic methods to non-deterministic methods. . .

Deterministically derived, single-value solutions fail to capture the effects of uncertainty, a random phenomenon by definition. Thus, probability distributions must serve to represent uncertain quantities, for “probabilities are the language of uncertainty; probability laws are the grammar of that language” [18]. Past research by the Aerospace Systems Design Laboratory asserts and demonstrates this point in modeling disciplinary, operational, and technological uncertainty [19–21].

Figure 2, based on Reference [22] and professional experience, depicts a representative design space exploration process for probabilistic system design. From a given mission, and a “baseline” vehicle sized to that mission, the process starts with the definition of a design space. The design variables of interest, and ranges of values for those variables, define the design space. Inclusion of operational uncertainty requires further definition by way of relevant noise factors, and ranges of possible values for them. Probability distributions further define each variable. Control factors receive uniform distributions, for they lie within the designer’s control and thus may take any value in the range with equal likelihood. Noise factors receive such probability distributions as Normal or Beta to represent their randomness. Commencement of the pro-

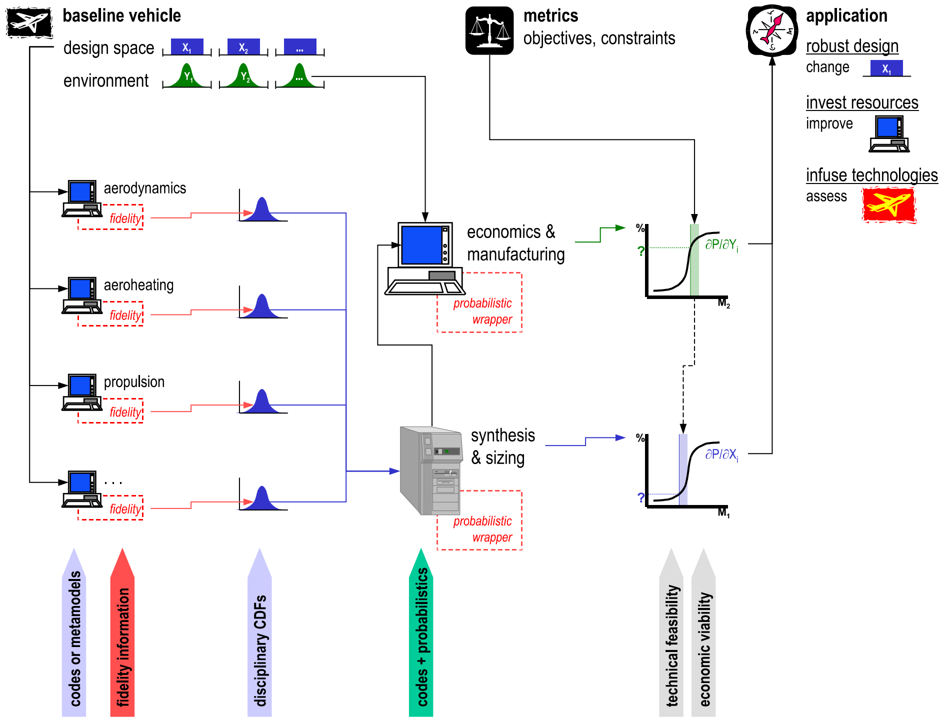

Figure 2: Generic probabilistic design environment

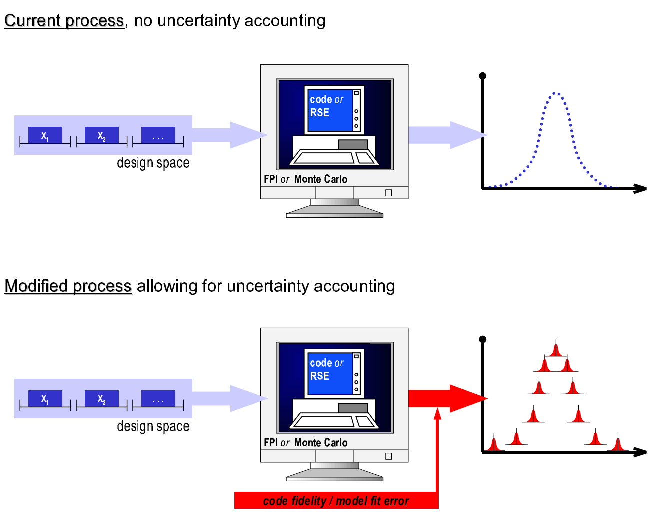

Figure 3: Modification for design uncertainty quantification

cess includes metric identification, by defining objectives and constraints for the system.

The vehicle simulation capability comprises a series of disciplinary analysis codes, linked to a synthesis and sizing tool (e.g., a trajectory analysis code), and further linked to an economics (with manufacturing) code for life-cycle cost analysis. The linking of codes in this manner creates an automated multidisciplinary simulation environment, an integral part of any probabilistic design effort [5, 6]. The disciplinary and economic analysis tools in Figure 2 need not be actual computer programs or experimental databases. These may take the form of analysis and cost modules internal to a monolithic design code, integrated with synthesis and sizing routines. Or, they may be metamodels, like Response Surface Equations (RSE's), linked with a synthesis and sizing tool, a procedure well-documented in the ASDL literature (e.g. References [23–25]).

Notice the interim step between the disciplinary analyses and the synthesis and sizing tool. The probability distributions following each analysis in Figure 2 represent the effects of design uncertainty. Instead of "point" values for disciplinary outputs, one set per set of input values, there exist distributions borne of fidelity and model fit error accounting. These distributions then proceed to the synthesis and sizing code. *This detailed process of quantifying and propagating disciplinary uncertainty provide the foundation of the proposed work.*

A probabilistic tool (e.g., the Southwest Research Institute's Fast Probability Integrator, or FPI) integrates disciplinary uncertainty information, obtained *a priori*, into the disciplinary outputs. Likewise, the probabilistic tool inputs this information to the synthesis and sizing, and economics codes, ultimately providing the distributions on the system-level responses, i.e., the previously defined objectives and constraints. "Technical feasibility" assessment entails comparison of performance objectives with imposed constraints [26]. For a launch vehicle, one such objective might be gross lift-off weight (GLOW), subject to the constraint of a maximum of 5 million pounds. "Economic viability" assessment likewise compares life-cycle cost objectives to constraints. Again for the launch vehicle, an example is cost per pound of payload to low Earth orbit, constrained to under $1000/lb. The CDF's generated for each assessment indicate the probability of "success," or the percentage of the design space defined in the design variable ranges that, subject to uncertainty, satisfies the imposed constraints on the objectives considered. From another perspective, the designer may view the system-level CDF's as a measure of design confidence; for the point on the CDF curve intersected by the constraint, the corresponding probability value equals the level of confidence in the design achieving that value. In the $/lb example, if the constraint, say $1000/lb, meets the curve at the 30% probability level, then the designer may state he/she believes "the design can meet the target of $1000/lb to LEO with 30% confidence."

PROBLEM STATEMENT "Zoom in" on any one of the disciplinary analysis codes (and its probabilistic output) from Figure 2, and Figure 3 results. The upper half depicts the treatment of disciplinary analyses in existing design methods. The probabilistic tool employed accepts the uniformly distributed control factors of the design space, executes the code (or evaluates the metamodel) accordingly, and outputs the resulting distribution for the subsystem metric. Existing methods, without accounting for the uncertainty in the analysis, treat the output canonically; the point values defining the probability density function (PDF) curve are truly discrete points.

The lower half of Figure 3 shows the effect of design uncertainty accounting. Application of the knowledge of the nature of the design uncertainty in question, obtained beforehand, alters the output PDF of the analysis. Each point on the PDF, accounting for uncertainty, becomes a distribution of its own. The output PDF, now a collection of distributions instead of discrete points, takes on a new form.

Three questions result from the above consideration; *it is these questions that ultimately motivate this work.*

1. **How does a designer systematically quantify design uncertainty for a disciplinary analysis?**
2. **By what process does one measure the uncertainty, and then recompute the analysis results in light of the uncertainty?**
3. **Upon quantifying design uncertainty, how does the designer propagate the results to the vehicle/system level?**

Through this paper, the authors present the future framework for systematically answering these questions.

THEORY: BAYESIAN STATISTICS

OVERVIEW Bernardo and Smith state that "Bayesian Statistics offers a rationalist theory of personalistic beliefs in contexts of uncertainty. . . The goal, in effect, is to establish rules and procedures for disciplined uncertainty accounting" [27]. From this scholarly assurance, the authors confidently assert Bayesian Statistics' ability to answer the research questions posed.

Over its long history, the statistics community derived several well-known probability distribution function (PDF) types. This work concerns itself only with the Uniform, Normal, and Beta distributions. Past work in aerospace systems design indicates that the Normal and Beta distributions adequately model uncertainty parameters, whereas the Uniform distribution represents control factors [24–26]. Also, Phillips reveals that in general practice, the Uniform, Normal, and Beta PDF's suffice to solve *any* problem in Bayesian Statistics [18].

Equation 1 states Bayes' Theorem in terms of discrete events, e.g., rolling dice or drawing cards. A_i and A_j represent *n* discrete events partitioning a given sample space. These events are *partitions* because they are assumed to be mutually exclusive, i.e., they have no outcomes in common. B represents a further event, and $P(B|A_i)$ equals the probability that event B occurs *when it is known that event A_i has occurred*, also known as the **conditional probability** of B, conditional on the occurrence of A_i.

$$P(A_i|B) = \frac{P(A_i) \cdot P(B|A_i)}{\sum_{j=1}^{n} P(A_j) \cdot P(B|A_j)} \quad (1)$$

The above formulation of Bayes' Theorem represents but one specific perspective on the theory. In general, "opinions are expressed in probabilities, data are collected, and these data change the prior probabilities, through the operation of Bayes' Theorem, to yield posterior probabilities" [18]. In these more general terms, $P(A_i)$, the **prior probability**, represents one's hypothesis about the outcome of event A_i. $P(B|A_i)$ represents the data collected or observations made when testing the hypothesis. Finally, $P(A_i|B)$, the **posterior probability**, equals the new hypothesis, or the original hypothesis now "corrected" in light of the new information provided by $P(B|A_i)$.

QUESTIONS 1 & 2 - QUANTIFYING UNCERTAINTY Past work [24, 25] indicate that consideration of disciplinary uncertainty typically rests with so-called expert opinion of that uncertainty. Expert opinion regarding a code's fidelity typically stems from observation of a few cases, and thus omits the effects of dissimilar vehicle types and geometries, and flight conditions. In other words, the fidelity uncertainty varies with the specifics of the problem, so "blanket statements" about the fidelity become suspect. Therefore, the belief regarding the nature of the uncertainty requires revision based on data, collected for the purpose of testing the belief. *This task is the raison d'être of Bayesian Statistics.* In addition, proper accounting for fidelity uncertainty requires observations on multiple data sources which, together with expert opinion, will derive a more representative probability distribution to model disciplinary uncertainty.

Theoretical Background Lee presents Bayes' Theorem reformulated in a form more suited to engineering and design, i.e., in terms of probability density functions (PDF's) rather than discrete events [28]:

$$p(\theta|x) = \frac{p(\theta) \cdot p(x|\theta)}{\int_\theta p(\theta) \cdot p(x|\theta) d\theta} = \frac{p(\theta) \cdot p(x|\theta)}{p(x)} \quad (2)$$

or more simply,

$$p(\theta|x) \propto p(\theta) \cdot p(x|\theta) \quad (3)$$

In this formulation, $p(\theta)$ equals the prior density, $p(x|\theta)$ represents the data observed when testing the prior, and $p(\theta|x)$ is the resulting posterior density.

Bayes' Theorem applies with equal validity for single or multiple observations. That is, **x** may be a single set of observed data x, or a vector of multiple, independent observations $\vec{x}$. The theorem conveniently allows for sequential application of the method in the latter case, as shown in Figure 4. Taking *n* as the number of observations such that $\vec{x} = \{x_1, x_2, \ldots, x_n\}$, one applies Bayes' Theorem using the prior density, $p(\theta)$, and the first data set, $p(x_1|\theta)$. The resulting posterior density, $p(\theta|x_1)$, becomes the *prior* density in a subsequent calculation, using the data set $p(x_2|\theta)$, and so on. Furthermore, the same final posterior density results regardless of the order of introduction of the data sets; "shuffling" the $p(x_i)$ values won't affect $p(\theta|\vec{x})$.

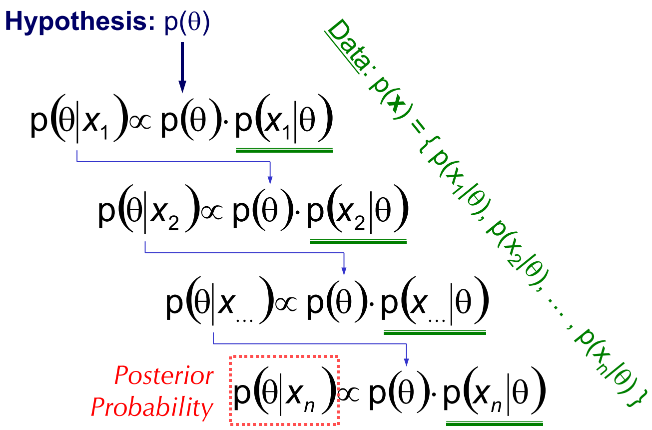

Figure 4: Sequential use of Bayes' Theorem

Implementation: Prior Density For a new design problem, the designer's initial estimation of disciplinary uncertainty stems from "expert" opinion. Intuitively, then, the prior density of Bayes' Theorem represents this estimation.

The developer(s) and/or expert user(s) of a given disciplinary analysis code typically refer to the code's output(s) as "good to within plus or minus θ percent," with θ stated as some small integer value. Implicit in such a statement is the belief that the code's fidelity errors are Gaussian (defined by a Normal PDF), with mean μ = the output value, and standard deviation $\sigma = \frac{1}{6} \times [upper\ limit - lower\ limit]$ [24]. For example, $\pm 3\%$ translates to a standard deviation of $\sigma = \frac{1}{6} \times [3 - -3] = 1$. (99.7% of the Normal PDF curve lies within $\pm 3\sigma$ of the mean [29]; for all practical purposes, this "6σ" spectrum captures the *entire* range of values under the PDF curve.) Note that this description follows for the case of the standard Normal distribution, with $\mu = 0$; for other values, μ scales σ. In the Bayesian sense, the stated fidelity uncertainty of a code forms the prior density, normally distributed with parameters μ (mean) and σ^2 (variance, the square of the standard deviation); symbolically,

$$\theta \sim N(\mu, \sigma^2), \ \sigma = \frac{(UL\% \times \mu) - (LL\% \times \mu)}{6.0} \quad (4)$$

Note that this calculation follows for *each* disciplinary metric output from *each* analysis code.

Implementation: Observations As for the Bayesian prior density and initial uncertainty estimation, "observations" and "data" translate to test cases for the code in question, i.e., comparisons of the code's outputs to physical data of the same disciplinary metrics. To truly test a disciplinary analysis requires modeling existing vehicles in the code, vehicles similar to the design in question and for which physical test data exist. For example, one could test the disciplinary analyses in the design of a re-entry vehicle using the Space Shuttle and the Apollo capsules as validation cases; for a spaceplane design, NASA's X-15 and XB-70 research vehicles (see Reference [30]) could provide these cases.

For each disciplinary metric utilized from a given code's output, cross-plotting the code's results with the physical data for the system illustrates the distribution (Normal or Beta) derived to model the uncertainty for the given case [25], as in Figures 5 and 6. Figure 5 provides a Stability and Controls example of the difference between analytical results and test data for a particular disciplinary metric - here, moment coefficient versus angle of attack. Figure 6 shows these differences on the vertical axis, versus the analytical results on the horizontal axis.

In fact, Figure 6 is a histogram, a key statistical tool indicating the nature of the PDF governing an "experiment's" behavior [29]. Observation of this histogram leads to identification of the PDF type and parameters. In this particular example, the histogram shape clearly indicates a Beta distribution. The parameters "a" and "b," the lower (-0.0525) and upper (0.0025) bounds, are obvious directly from the histogram. The shaping parameters "q" and "r" require study of Beta PDF shapes, such as the gallery thereof in Reference [28]; in this case, one concludes $q = 3.0$ and $r = 1.5$.

The above process leads to a PDF modeling fidelity of *one* disciplinary metric from *one* disciplinary analysis code, for *one* test case. A fair and accurate uncertainty assessment, again for *each* metric, requires repetition of the process for *each* vehicle data set. The result is a series of

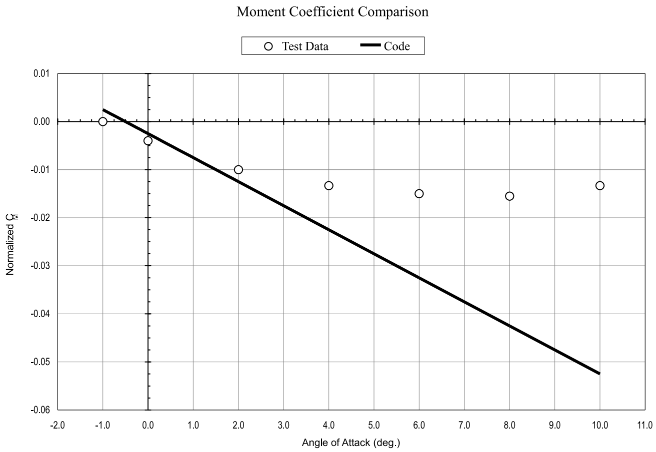

Figure 5: Comparison of analytical results and physical data

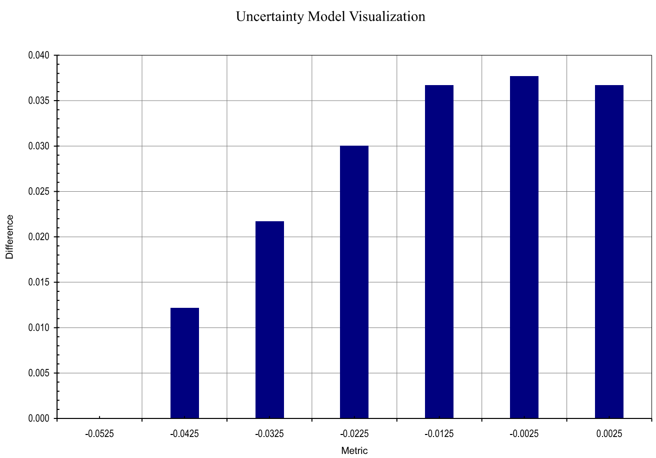

Figure 6: Visualization of uncertainty model

PDF's representing the "observations" in Bayes' Theorem.

Integration: Constructing the Uncertainty Model The PDF's computed for Bayesian prior and observed densities, combined via Bayes' Theorem, result in the posterior density - the probabilistic model of disciplinary uncertainty, as Figure 4 illustrates. The initial hypothesis $p(\theta)$ at the upper-left of the figure represents the prior distribution, the disciplinary uncertainty model assumed initially. Modeling existing vehicles in analysis codes, and comparing the results for a given metric to the data for the vehicles, result in PDF's representing observations in support of the hypothesis - the data, $p(x_i|\theta)$, down the right side of Figure 4. Sequential application of Bayes' Theorem with these elements yields the PDF quantifying disciplinary uncertainty.

The literature unanimously concludes the necessity of representing prior and observed distributions with specifically-paired PDF types. Failure to adhere to this conclusion in applying Bayes' Theorem results in uncategorizable posterior distributions. Such cases greatly complicate the propagation of computed disciplinary uncertainty information. For the two distributions considered in this work, the combination of a Normal prior distribution and Normal observations yields a Normal posterior distribution [28], while a Beta prior yields a Beta posterior *if the observed data take the form of the Binomial distribution* [18]. The Binomial, one of several PDF's for modeling *discrete* random variables, works well for problems in counting the number of "successes" resulting from a series of discrete events [29]. Such a PDF therefore finds no use in this work, given that continuous functions, not discrete events, describe disciplinary metrics in aerospace engineering problems. Of course, the counterpoint regarding the nature of observed data immediately arises: References [19, 25] show this data to be distributed either as Normal or Beta.

Nevertheless, the literature points out several means to circumvent this difficulty, and still provide satisfactory results. Phillips describes the most viable option, and recommends its use in applications of Bayesian Statistics. Since the Beta PDF often resembles that of the Normal distribution, one may use the Normal PDF to approximate a Beta distribution. By equating the expectation and variance of the two distributions, calculations on the parameters of the Beta PDF yield the parameters of the approximate Normal PDF. Thus,

$$E(X) = \mu|_N = \left[a + \frac{q}{q+r} \cdot (b-a)\right]_\beta \quad (5)$$

$$Var(X) = \sigma^2|_N = \left[\frac{q \cdot r}{(q+r)^2 \cdot (q+r+1)} \cdot (b-a)^2\right]_\beta \quad (6)$$

As a result, the authors propose this method for all calculations in this work. *The task of working with arbitrary PDF's, without resorting to transformations, thus remains as one item of future work. Another is the introduction of the error due to this approximation as a source of model fit error.*

The literature provides several calculations of Bayes' Theorem for each prior-observed pair of PDF's, such as in the case of a Normal prior and Normal observation(s). Consider the first calculation step in Figure 4. Suppose prior opinion defines a given output from a given code as $\theta \sim N(\theta_0, \phi_0)$, that is, Gaussian with output value $\mu = \theta_0$ and error bounds resulting in $\sigma^2 = \phi_0$. Further suppose that for the first test case, comparison of the output and physical data yield a distribution of $x \sim N(\theta_x, \phi_x)$. Lee shows that the posterior distribution, $\theta|x \sim N(\theta_1, \phi_1)$ [28], where

$$\phi_1 = \frac{1}{\phi_0^{-1} + \phi_x^{-1}} \quad (7)$$

$$\theta_1 = \phi_1 \cdot \left(\frac{\theta_0}{\phi_0} + \frac{\theta_x}{\phi_x}\right) \quad (8)$$

If the observed distribution represents the sole test case, then *the disciplinary uncertainty is quantified, as defined* *by the distribution,* $\theta|x \sim N(\theta_1, \phi_1)$. If other test cases exist, then this distribution represents only the first step in the Bayesian calculation, the posterior density of which becomes the prior density for the next test case and calculation step.

QUESTION 3 - PROPAGATING UNCERTAINTY The implementation of Bayesian Statistics for disciplinary uncertainty propagation spans a subset of the implementation for uncertainty quantification. The difference lies with the use of the quantified disciplinary uncertainty, previously a Bayesian posterior density, now as the "observed" density.

Consider again Equation 3, Lee's formulation of Bayes' Theorem [28]. Canonical treatment of the output distribution from a disciplinary code, for an input design space (the upper half of Figure 3), forms the designer's initial "hypothesis." The term $p(\theta)$ in Bayes' Theorem represents this hypothesis, expressed as a PDF. Quantification of disciplinary uncertainty, as described above, leads to the PDF modeling "observations" in support of the hypothesis, as represented by $p(x|\theta)$. The posterior distribution, $p(\theta|x)$, thus equals the PDF for the disciplinary metric, resulting from the probabilistic treatment of the design space and the inclusion of uncertainty effects.

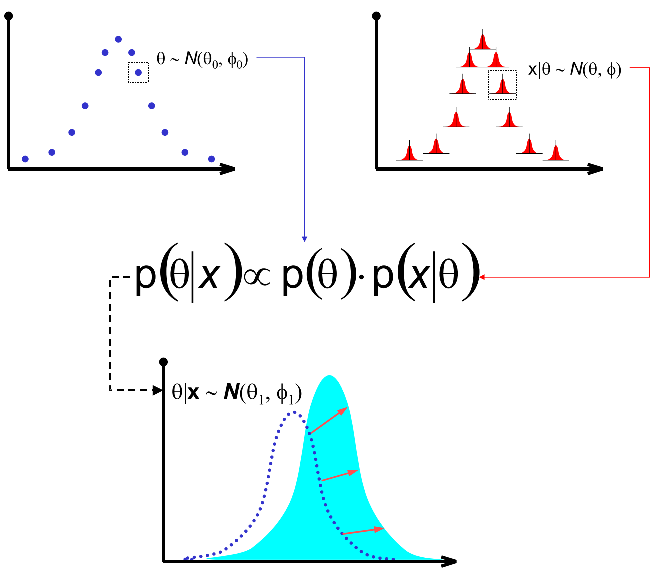

Figure 7: Application of Bayes' Theorem to uncertainty propagation

The mathematical formulation for this approach follows Lee's derivation, as summarized in Figure 7. With no uncertainty quantification, probabilistic execution of a design code with an input design space results in a PDF for the output disciplinary metric; by the Central Limit Theorem [29], the PDF is Normal, with some mean θ_0 and variance ϕ_0. That is, the output value $\theta \sim N(\theta_0, \phi_0)$; but the above quantification approach reveals that each point on the PDF curve, $x \sim N(\theta, \phi)$. This likewise embodies the spirit of Bayesian Statistics: *measurement of a random variable, the parameters of which are themselves random variables.* Knowledge of disciplinary uncertainty, in effect, provides error bounds on analytical results, as modeled by the PDF defining x. This provides the similar result as for uncertainty quantification: $\theta|x \sim N(\theta_1, \phi_1)$, with θ_1 and ϕ_1 as given by Equations 7, 8. This PDF thus models the output from a disciplinary analysis code, executed probabilistically, with full accounting of the disciplinary uncertainty it introduces.

APPROACH AND EXAMPLE

OBJECTIVE Much like aircraft design, spaceplane and launch vehicle design entails "closure" of the design concept on fuel weight and volume. The designer calculates the weight and volume of fuel required for the vehicle to fly the given mission, then contrasts this with the fuel available aboard the current iteration of the vehicle [31]. Closure, by matching required and available fuel, requires modifications to the design through changes in geometry and assumed technology level, to effect needed improvements in key disciplinary metrics.

The difference between air and space vehicles lies in the additional consideration of the hypersonic or launch vehicle, the **weight growth margin**. Given the less advanced state of the art in spaceplane / launch vehicle design (relative to aircraft design), a great deal of uncertainty in all its forms arises. Thus, conceptual design for these types follows a safety factor approach by adding additional "payload" capacity beyond that required for the mission [32]. The quotation marks denote how this added capacity is not intended for true "payload." Rather, designing for the additional capacity masks the impact of uncertainty, the realization of which (through procession of the design to the hardware development phase) leads to added vehicle empty weight and propellant weight. Note that weight growth margin, through long-standing use in launch vehicle design, becomes one of the system-level metrics of any such design, examples of which appear in References [33, 34].

The proposed application of Bayesian Statistics seeks to quantify disciplinary uncertainty and forecast its effects on system-level metrics. Successfully doing so endeavors to replace the traditional method of desensitizing the vehicle to the impact of disciplinary uncertainties, the weight growth margin approach. However, differences between initial estimates of disciplinary parameters and "real" values determined later yield an overdesigned vehicle; the classic examples of the Concorde and Space Shuttle show that this form of design robustness reduces both performance and profitability. Furthermore, gross inaccuracies in the estimates, such that the vehicle weight "grows" beyond the allowed margin, force performance trade-offs or even a complete redesign.

VALIDATION A meaningful validation effort entails an "apples to apples" comparison of the Bayesian and traditional approaches. To that end, part of this research will seek to design the same vehicle to the same mission, using the same design tool(s). However, even under these conditions, the comparison must be largely qualitative. Different design approaches entail different assumptions and solution methods, leading to differences in the values for design metrics. This precludes meaningful quantitative comparisons.

Qualitative comparisons call for assessments of the results of each method. Rather than weigh the specific, numeric values resulting from one design against the other, it is important to ascertain the means by which the values were obtained, and thus, the basis for and validity of the values. Obviously, the results of the Bayesian design are grounded in the theory of Bayesian Statistics. Those for the traditional design may result from engineering "guesstimates" and other expert opinion. The latter point, of course, remains to be seen in the course of this work.

PRELIMINARIES

Computer Tools Two programs form the computational design environment; FPI, the probabilistic "wrapper" previously described (Reference [35] provides more detail), links with HAVOC, the Hypersonic Aircraft Vehicle Optimization Code. HAVOC, a monolithic design code developed and provided by the Systems Analysis Branch of the NASA-Ames Research Center, aids in conceptual-level synthesis and sizing of hypersonic vehicles [36]. With its internal aerothermodynamics, propulsion, mass properties, and trajectory modules, HAVOC handles several vehicle types, including transatmospheric aircraft, SSTO launch vehicles, conventional rockets, and hybrids thereof. A third program, commercially-available Microsoft Excel, aids in the quantification of disciplinary uncertainty as described previously. Shell scripts, written and executed as part of the research, handle Bayesian calculations and data manipulation. An integration environment, such as Engineous Software's iSIGHT or Phoenix Integration's Model Center, links the tools together to automate computational tasks.

Validation Case The information required includes vehicle data and metrics. Definition of key disciplinary and system-level metrics leads the information needed for the design process. The disciplinary metrics are the instrument by which to quantify and propagate disciplinary uncertainty. Additionally, the quantification of disciplinary uncertainty requires ascertaining expert opinion regarding HAVOC's fidelity and accuracy in computing these values. System-level metrics, measurements of the performance and effectiveness of the vehicle system, provide the means by which to compare design methods.

The process also requires vehicle data, starting with a baseline launch vehicle. The Space Launch Initiative (SLI), part of NASA's Integrated Space Transportation Plan, seeks to develop a second generation RLV capable of a 10-fold reduction in costs, and a 100-fold increase in reliability, in the 2010 timeframe [37]. The Marshall Space Flight Center, leading the SLI effort, envisions this RLV as a part airbreathing, part rocket-based two-stage-to-orbit (TSTO) system. The authors thus propose validation via a NASA-developed reusable TSTO system concept, available in the public domain. This requires the collection of data on the system's mission and performance, in terms of both disciplinary and system metrics, as ascertained by traditional design methods. Furthermore, the end steps of the validation process require knowledge of the basis for the data, e.g., physics-based analyses, approximations, expert opinion.

"Test Data" Cases The disciplinary uncertainty quantification task, as described previously, utilizes Bayesian observations originating in existing vehicles that each share some traits with the design under consideration. For a reusable TSTO, the NASA X-15 and XB-70 research vehicles, and the Space Shuttle, provide excellent test cases (see Figure 8) for implementing the design method. The X-15 provides the database for high dynamic pressure, sustained atmospheric hypersonic flight. The XB-70 indicates the design drivers for low-drag, lifting high-speed flight utilizing airbreathing propulsion. The Shuttle, with its long operational history as the world's sole reusable Space Transportation System, offers volumes of data regarding the disciplinary and operations issues associated with access to space and reentry from orbit. Data related to the disciplinary metrics of interest for these vehicles are essential to the uncertainty quantification process.

Figure 8: Counterclockwise from upper left: X-15, XB-70, Space Shuttle [30]

CONCEPTUAL EXAMPLE The example considers, for a notional TSTO, the disciplinary metrics of lift-to-drag ratio (L/D) for aerodynamics, thrust specific impulse (I_{sp}) for propulsion, and wing weight (W_{wing}) for mass properties. The system-level metrics include gross lift-off weight ($GLOW$), representing technical feasibility, and cost per pound of payload to low-Earth orbit ($\$/lb$) for economic viability.

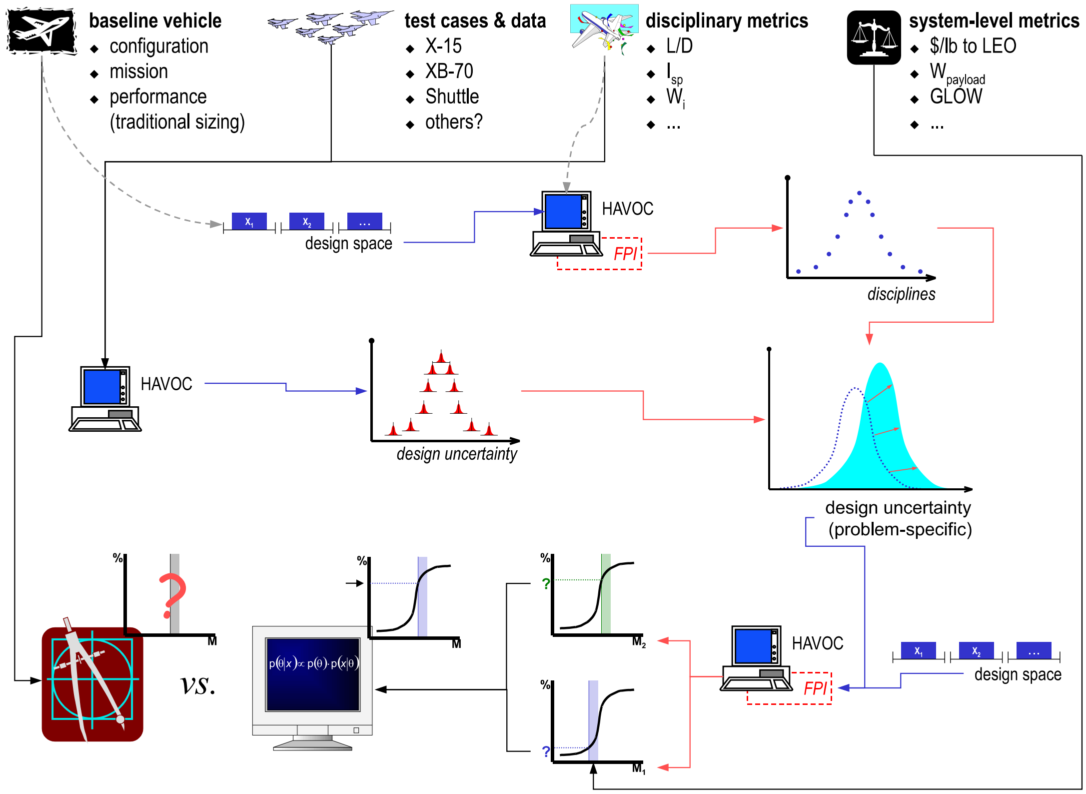

Figure 9: Proposed design method implementing Bayesian Statistics

Quantifying Uncertainty   In the Bayesian approach, the combination of expert opinion and observations yield the PDF which quantifies disciplinary uncertainty for a particular disciplinary metric. In this specific case, quantifying uncertainty begins with modeling each of the test vehicles in HAVOC and tabulating each of the disciplinary metrics of interest for each case. Following the previously outlined procedure, HAVOC outputs for the vehicles are each cross-plotted with the data obtained for the vehicles. "Observations" in the Bayesian sense are concluded from the resulting plot, in the form of PDF's. Equations 3, 7, and 8 then calculate, from these PDF's and the PDF defining expert opinion of HAVOC, the probability distribution quantifying uncertainty in each discipline.

Upon modeling the supplied TSTO concept (after creation and execution of an input file for the baseline vehicle as given), suppose HAVOC's developers and expert users agree that the accuracy of the disciplinary analyses follows that shown in Table 1. With that information, and after modeling the TSTO concept in HAVOC (as provided), the last column of Table 1 reveals the resulting Bayesian prior distribution established for each metric, calculated from Equation 4.

After establishing prior PDF's, Bayesian observations take place by modeling the X-15, XB-70, and Shuttle in HAVOC and tabulating the L/D vs. Mach number, I_{sp} vs. altitude, and W_{wing} versus wing geometry outputs for each vehicle. These results, cross-plotted with the physical data for the actual systems, lead to graphs like those of Figures 5 and 6. Following this process, the Bayesian "observed" PDF's listed in Table 2 result. Notice the presence of two Beta distributions; Equations 5 and 6 easily provide the transformations of these PDF's to the Normal type. Table 3 shows these results, and thus lists the distributions representing the Bayesian "observations."

With prior and observed distributions now defined, the sequential use of Bayes' Theorem shown in Figure 4 computes the quantified disciplinary uncertainty, given in Equation 9 for this example.

$$\begin{aligned} L/D &\sim N(2.589, 0.0971^2) \\ I_{sp} &\sim N(874.5, 5.923^2) \\ W_{wing} &\sim N(19007., 120.97^2) \end{aligned} \qquad (9)$$

Propagating Uncertainty   With HAVOC's disciplinary uncertainty quantified, propagation of the uncertainty begins with definition of a design space: selecting design variables of interest, and assigning a range of values to each. Through FPI, this design space is input as a series of Uniform PDF's to HAVOC for probabilistic execution, from which only PDF's for the disciplinary metrics are recorded. The Bayesian approach, via Equations 3, 7, and 8, as well as the uncertainty PDF's obtained in the prior step, reshape the outputs to produce new disciplinary PDF's. FPI/HAVOC is then re-executed, this time not only with the design space input as before, but also with the new disciplinary PDF's just obtained, now as inputs for disci-

Metric	Error Distribution *Type*	*Accuracy*	Bayesian Prior
L/D	Gaussian	$\pm 5\%$	$\theta_{L/D} \sim N(5.000, 0.0500^2)$
I_{sp}	Gaussian	$\pm 2\%$	$\theta_{Isp} \sim N(1250., 8.333^2)$
W_{wing}	Gaussian	$\pm 1\%$	$\theta_W \sim N(20,000., 133.33^2)$

Table 1: Code fidelity based on expert opinion (example)

Metric	Vehicle		
	X-15	*XB-70*	*Shuttle*
L/D	$x_{X-15}\|\theta_{L/D} \sim N(2.500, 0.1000^2)$	$x_{XB-70}\|\theta_{L/D} \sim \beta(1.750, 2.250, 1.500, 3.000)$	$x_{STS}\|\theta_{L/D} \sim N(2.500, 0.1000^2)$
I_{sp}	$x_{X-15}\|\theta_{Isp} \sim N(200.0, 20.00^2)$	$x_{XB-70}\|\theta_{Isp} \sim N(1200., 25.00^2)$	$x_{STS}\|\theta_{Isp} \sim N(400.0, 10.00^2)$
W_{wing}	$x_{X-15}\|\theta_W \sim \beta(3450., 5100., 2.500, 1.500)$	$x_{XB-70}\|\theta_W \sim N(75,000., 2000.^2)$	$x_{STS}\|\theta_W \sim N(30,000., 500.0^2)$

Table 2: Code fidelity for test-case vehicles (example)

Metric	Vehicle		
	X-15	*XB-70*	*Shuttle*
L/D	$x_{X-15}\|\theta_{L/D} \sim N(2.500, 0.1000^2)$	$x_{XB-70}\|\theta_{L/D} \sim N(1.917, 0.1005^2)$	$x_{STS}\|\theta_{L/D} \sim N(2.500, 0.1000^2)$
I_{sp}	$x_{X-15}\|\theta_{Isp} \sim N(200.0, 20.00^2)$	$x_{XB-70}\|\theta_{Isp} \sim N(1200., 25.00^2)$	$x_{STS}\|\theta_{Isp} \sim N(400.0, 10.00^2)$
W_{wing}	$x_{X-15}\|\theta_W \sim N(4481.3, 357.24^2)$	$x_{XB-70}\|\theta_W \sim N(75,000., 2000.^2)$	$x_{STS}\|\theta_W \sim N(30,000., 500.0^2)$

Table 3: Transformed code fidelity for test-case vehicles (example)

Metric	*Prior*	*Observed*	*Posterior*
L/D	$\sim N(4.575, 0.0510^2)$	$\sim N(2.589, 0.0971^2)$	$\sim N(4.146, 0.0452^2)$
I_{sp}	$\sim N(904.2, 7.463^2)$	$\sim N(874.5, 5.923^2)$	$\sim N(884.4, 4.317^2)$
W_{wing}	$\sim N(21,050., 99.576^2)$	$\sim N(19,007., 120.97^2)$	$\sim N(20,225., 76.880^2)$

Table 4: Propagated disciplinary uncertainty for TSTO (example)

plinary data. CDF's for the system metrics are output as the final result of the Bayesian approach.

In this example, the design space takes the simple form of varying wing area (S_W) between 4500 and 7500 square feet, and nose radius (R) between 30 and 45 inches. Definition of this design space precedes formation of the link between FPI and HAVOC. HAVOC is then executed probabilistically, treating the two design variables as Uniform random variables bound by the defined limits: $S_W \sim U(4500., 7500.)$ and $R \sim U(30.0, 45.0)$. This first run of HAVOC focuses on disciplinary metrics, and with this probabilistic input, FPI/HAVOC outputs Normal PDF's for L/D, I_{sp}, and W_{wing}, as given in the first column of Table 4. The single-step use of Bayes' Theorem takes the PDF's in Equation 9, the Bayesian prior distributions (the second column of Table 4), and combines them with the Bayesian observations. Table 4 presents these calculation elements and the results (the third column), the posterior distributions instrumental to propagation of disciplinary uncertainty.

The next step again executes FPI/HAVOC, with an expanded set of inputs. After disabling HAVOC's internal analysis routines for calculating L/D, I_{sp}, and W_{wing}, the PDF's from Table 4 are input via FPI, along with the design space defined earlier ($S_W \sim U(4500., 7500.)$, $R \sim U(30.0, 45.0)$). The results of interest now are the system-level CDF's for $TOGW$ and $\$/lb$.

Validation If the TSTO utilizes existing Shuttle launch facilities, then its $GLOW$ must be less than or equal to 4.5 million pounds [38]. Furthermore, in order to compete with existing launch vehicles and meet NASA targets, $\$/lb$ must be less than or equal to $1000/lb. Plotting these constraint lines on the CDF plots for $GLOW$ and $\$/lb$ allows measurement of the probability of "success," i.e., the portion of the design space satisfying the imposed constraints, with inclusion of disciplinary uncertainty.

Comparison of these results with those for the "traditionally" sized TSTO measures the worth of this new design approach, in relation to the traditional approach. The comparison spans less the values for all metrics in each case, and more the confidence computed in those values. Of particular interest is the source of the confidence estimates resulting from the traditional design approach.

SUMMARY Figure 9 illustrates the overall process described in the aforementioned example. The authors propose this method, embodying Bayesian Statistics principles, for quantifying and propagating disciplinary uncertainty.

CONCLUSION

The authors confidently propose the utilization of Bayesian Statistics theory to quantify and propagate disciplinary uncertainty. Of the three sources of uncertainty defined by the ASDL literature, disciplinary uncertainty receives the least attention, yet most undermines the decision-making confidence of the hypersonic vehicle designer. The proposed design methods, relying on probabilistic techniques, shall provide the means to ascertain the impact of disciplinary uncertainty on this confidence. The specified tools and data will merge to create the environment and example to seek to prove the method's utility to hypersonic vehicle design. In so doing, the authors endeavor to demonstrate the method's superiority over traditional uncertainty accounting methods.

ACKNOWLEDGEMENTS

The authors wish to acknowledge the NASA Langley Research Center, for their support via Grant NAG-1-2235, and the inspiration and support of Dr. Kevin Bowcutt and Dr. Marty Bradley of The Boeing Company.

REFERENCES

[1] McCall, G.H. and Corder, J.A., editor. *New World Vistas: Air and Space Power for the 21st Century*. United States Air Force, Wright-Patterson Air Force Base, Dayton, Ohio, December 1995.

[2] Eisele, A. Multiple RLVs to Follow Shuttle. *Space News*, July–August 1997.

[3] Benson, J. Conversations with Gary Payton. *Aerospace America*, June 1996.

[4] Hunt, J.L. Hypersonic Airbreathing Vehicle Design. In *Hypersonics, Volume I*.

[5] Mavris, D.N., DeLaurentis, D.A., Hale, M.A., Tai, J.C.M. Elements of an Emerging Virtual Stochastic Life Cycle Design Environment. Number AIAA-1999-01-5638 in 1999 World Aviation Conference, San Francisco, California, October 1999. SAE International and the American Institute for Aeronautics and Astronautics.

[6] Mavris, D.N., DeLaurentis, D.A., Bandte, O., Hale, M.A. A Stochastic Approach to Multi-disciplinary Aircraft Analysis and Design. Number AIAA-98-0912 in 36th Annual Aerospace Sciences Meeting and Exhibit, Reno, Nevada, January 1998. American Institute for Aeronautics and Astronautics.

[7] Moore, F.G, McInville, R.M., Hymer, T.C. The 1998 Version of the Aeroprediction Code (AP98). Number AIAA-99-0762 in 37th AIAA Aerospace Sciences Meeting and Exhibit, Reno, Nevada, January 1999. American Institute for Aeronautics and Astronautics.

[8] Morris, R.A. et al. Characterization and Prediction of Hypersonic Plasma Effects. Number AIAA-99-0630 in 37th AIAA Aerospace Sciences Meeting and Exhibit, Reno, Nevada, January 1999. American Institute for Aeronautics and Astronautics.

[9] Kanda, T. Simulation of an Airframe-Integrated Scramjet Engine. Number AIAA-98-3427 in 34th AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit, Cleveland, Ohio, July 1998. American Institute for Aeronautics and Astronautics.

[10] Kanda, T. 3-D Hypersonic Inlet Optimization Using a Genetic Algorithm. Number AIAA-98-3582 in 34th AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit, Cleveland, Ohio, July 1998. American Institute for Aeronautics and Astronautics.

[11] Reijasse, P., Bur, R., Chanetz, B. The French Program PREPHA: Experimental Analysis of Aerodynamical Interactions Occurring on a Hypersonic Spacecraft. Number AIAA-99-0610 in 37th AIAA Aerospace Sciences Meeting and Exhibit, Reno, Nevada, January 1999. American Institute for Aeronautics and Astronautics.

[12] Mishin, G.I. Experimental Investigation of the Flight of a Sphere in Weakly Ionized Air. Number AIAA-97-2298 in 15th Applied Aerodynamics Conference, Atlanta, Georgia, June 1997. American Institute for Aeronautics and Astronautics.

[13] Roudakov, A.S., Semenov, V.L, Hicks, J.W. Recent Flight Test Results of the Joint CIAM-NASA Mach 6.5 Scramjet Flight Program. Technical Memorandum NASA/TM-1998-206548, NASA Dryden Flight Research Center, Edwards, California, April 1998.

[14] Kanda, T. et al. Mach 8 Testing of a Scramjet Engine Model. Number AIAA-99-0617 in 37th AIAA Aerospace Sciences Meeting and Exhibit, Reno, Nevada, January 1999. American Institute for Aeronautics and Astronautics.

[15] Blankson, I.M., Schindel, L., Snyder, C. Propulsion-Airframe Integration of a Small Mach 4-8 Hydrocarbon-Fueled Airbreathing Cruise Research Vehicle. Number ISABE-97-7198 in XIII International Symposium on Airbreathing Engines, Chattanooga, Tennessee, September 1997. International Society for Airbreathing Engines.

[16] Cockrell, C.E., Huebner, L.D. Generic Hypersonic Inlet Module Analysis. Number AIAA-91-3209 in 9th Applied Aerodynamics Conference, Baltimore, Maryland, September 1991. American Institute for Aeronautics and Astronautics.

[17] Goldin, Daniel S. Tools of the Future. Presentation remarks from the WWW at ftp://ftp.hq.nasa.gov/pub/pao/Goldin/1998/ise.txt, January 1998.

[18] Phillips, L.D. *Bayesian Statistics for Social Scientists*. Thomas Y. Crowell Company, New York, New York, 1973.

[19] DeLaurentis, D.A., Mavris, D.N. Uncertainty Modeling and Management in Multidisciplinary Analysis and Synthesis. Number AIAA-2000-0422 in 38th Aerospace Sciences Meeting and Exhibit, Reno, Nevada, January 2000. American Institute for Aeronautics and Astronautics.

[20] Mavris, D.N., Bandte, O. Economic Uncertainty Assessment Using a Combined Design of Experiments/Monte Carlo Simulation Approach With Application to an HSCT. 17th Annual Conference, San Diego, California, May 1995. International Society of Parametric Analysts.

[21] Mavris, D.N., Soban, D.S., Largent, M.C. An Application of a Technology Impact Forecasting (TIF) Method to an Uninhabited Combat Aerial Vehicle. Number AIAA-1999-01-5633 in 1999 World Aviation Conference, San Francisco, California, October 1999. SAE International and the American Institute for Aeronautics and Astronautics.

[22] Mavris, D.N., DeLaurentis, D.A. A probabilistic approach for examining aircraft concept feasibility and viability. *Aircraft Design - An International Journal: Theory, Technology, Applications*, 3:79–101, 2000.

[23] Mavris, D.N., Hayden, W.T. Formulation of an IPPD Methodology for the Design of a Supersonic Business Jet. Number AIAA-96-5591 in 1st World Aviation Congress, Los Angeles, California, October 1996. American Institute of Aeronautics and Astronautics.

[24] Hayden, W.T., Mavris, D.N. Probabilistic Assessment of the Impact of New Technologies on an HSCT. Number AIAA-1999-01-5633 in First Internet Conference on Approximations and Fast Reanalysis in Engineering Optimization, Conducted entirely on the Internet, June 1998. The International Society for Structural and Multidisciplinary Optimization (ISSMO), the NASA Langley Research Center, and the American Institute of Aeronautics and Astronautics (AIAA).

[25] DeLaurentis, D.A. *A Probabilistic Approach to Aircraft Design Emphasizing Stability and Control Uncertainties*. PhD thesis, Georgia Institute of Technology, November 1998.

[26] Mavris, D.N., Mantis, G.C., Kirby, M.R. Demonstration of a Probabilistic Technique for the Determination of Economic Viability. Number SAE-97-5585 in 2nd World Aviation Congress and Exposition, Anaheim, California, October 1997. Society of Automotive Engineers, Inc.

[27] Bernardo, J.M., Smith, A.F.M. *Bayesian Theory*. John Wiley and Sons, New York, New York, 1994.

[28] Lee, P.M. *Bayesian Statistics: An Introduction*. Oxford University Press, New York, New York, 1989.

[29] Hayter, A.J. *Probability and Statistics for Engineers and Scientists*. PWS Publishing Company, Boston, Massachusetts, 1996.

[30] Research Airplane Program Fact Sheet. NASA Dryden Flight Research Center. On the WWW at http://www.dfrc.nasa.gov/PAO/PAIS/HTML/FS-031-DFRC.html.

[31] Raymer, Daniel P. *Aircraft Design: A Conceptual Approach*. American Institute of Aeronautics and Astronautics, Washington, D.C., Second edition, 1992.

[32] Olds, J.R. AE8133: Introduction to Space Vehicles. Course Notes, Georgia Institute of Technology, 1996.

[33] Cohen, A. et al. Design of a Single-Stage-to-Orbit Launch Vehicle.

[34] Olds, J.R., Bellini, P.X. Argus, a Highly Reusable SSTO Rocket-Based Combined Cycle Launch Vehicle with Maglifter Launch Assist. Number AIAA-98-1557 in Eighth International Space Planes and Hypersonic Systems and Technologies Conference, Norfolk, Virginia, April 1998. American Institute of Aeronautics and Astronautics (AIAA).

[35] Southwest Research Institute, San Antonio, Texas. *FPI User's and Theoretical Manual*, 1995.

[36] Bowles, J. et al. *Hypersonic Aircraft Vehicle Optimization Code*. NASA Ames Research Center, Moffett Field, California.

[37] The Space Launch Inititative: Technology to pioneer the space frontier. Technical Report FS-2001-06-122-MSFC, NASA Marshall Space Flight Center, June 2001. On the WWW at http://std.msfc.nasa.gov/index.html.

[38] Shuttle Statistics. NASA Johnson Space Center.