

# System Reliability Assessment Using Covariate Theory

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## *SUMMARY & CONCLUSIONS*

A method is demonstrated that utilizes covariate theory to generate a multi-response component failure distribution as a function of pertinent operational parameters. Where traditional covariate theory uses actual measured life data, a modified approach is used herein to utilize life values generated by computer simulation models. The result is a simulation-based component life distribution function in terms of time and covariate parameters for each failure response. A multivariate joint probability covariate model is proposed by combining the covariate marginal failure distributions with the Nataf transformation approach. Evaluation of the joint probability model produced significant improvement in joint probability predictions as compared to the independent series event approach. The proposed methods are executed for a nominal aircraft engine system to demonstrate the assessment of multi-response system reliability driven by a dual mode turbine blade component failure scenario as a function of operational parameters.

## *1. INTRODUCTION*

Traditional system reliability is highly concerned with the failure structure rather than the actual individual reliability values of the system constituents. Typically, due to difficulties and limitations in running full system reliability tests for large multi-component systems the individual component reliability values are either specified as a constant, an uncertain probability value modeled via an assumed probability distribution [Ref. 1], or explicitly known for a specified operating condition. Often, the individual component reliability values are generated from test data in a way that is off-line from that actual system conditions. In each of these cases, component reliability values are assumed to be independent of their position as well as the system operating conditions and component design parameters.

Although accelerated life testing methods using covariate theory have been successfully executed to model component reliability as a function of explanatory variables, these models are still limited in that non-normality and statistical dependence of local input variables are not appropriately modeled. The approach demonstrated within this study is to create a covariate model of a component's failure distribution(s) using a modeling and simulation

environment of the entire system. In doing so, statistical properties of the local input variables are inherently accounted for and a reliability model as a direct function of system operating conditions is produced. Modeling the component reliability vector as a function of system and component parameters is the focus of this work.

## *2. APPROACH*

### *2.1 Covariate Models*

Covariate models are used to represent the effect of suspect treatments in a lifetime model. A covariate is defined as a treatment or explanatory variable that influences the failure time of the component. A vector of covariates,  $z$ , is chosen where each entry of the vector represents a unique explanatory variable. Typical covariates include those that represent mechanical forces, material properties, and environmental factors. There are two rather popular approaches for linking these covariates to the probability function. The first method, known as Accelerated Life Testing (ALT), is based on modifying the time axis of the survivor function. The original application of ALT was to reduce the time to test of production components by increasing, or accelerating, the primary explanatory factors and using the resulting model to predict component lifetimes under standard in-service conditions. The premise of the second approach, called Proportional Hazard Model (PHM), is to modify the hazard rate function to include the covariates.

In either method, the covariates, represented by the vector  $z$ , are modeled by augmenting the random variable with a 'link' function,  $\psi(z)$ , which is a function of the covariates. A popular link function found in the literature [Ref 2] is the log-linear link function given as

$$\psi(z) = e^{\beta z} \quad (1)$$

where  $\beta$  is an  $n$  by 1 vector of regression coefficients corresponding to each of the  $n$  covariates in the vector,  $z$ . The appealing characteristic of this link function compared to others that have been proposed is that the exponential form is highly compatible with most traditional parametric distributions. Also, the log-linear function is asymptotically stable across large ranges of the regression coefficients.

The resulting survivor function in the ALT approach becomes

$$S(t) = S_o(t\psi(z)) \quad (2)$$

where  $S_o$  is the baseline survivor function<sup>1</sup> determined at the nominal values of each of the potential covariates (i.e.  $z_i=0$ ). This formulation imposes certain requirements on the link function. Namely, the function must be equal to unity at the nominal covariate setting,  $z=0$ , and must be positive for any and all values of  $z$ . The form of the log-linear link function used in both of the two approaches allows for the use of the familiar applied statistical model theory to determine the respective covariate regression coefficients,  $\beta_i$ . Rather than use least squares regression, as is typical for statistical linear models and response surface equations, maximum likelihood estimation (MLE) is often used to estimate the covariate coefficients. The coefficients are treated as parameters of a multivariate distribution to be estimated using MLE. Consequently, statistical significance tests of each coefficient in addition to whole model tests are readily computed resulting in a rapid importance ranking and error estimation of each coefficient and hence its corresponding covariate variable.

## 2.2 Covariate Model Generation

Traditional data used to generate covariate models is usually provided by actual physical experiments or even extracted from operational databases. However, with the advent of more sophisticated modeling capabilities, failure data can be generated using models of the system and components operating within such system. Yet, for computational efficiency an intelligent choice of the variable value combinations and replications is still required. Planning the simulation-based covariate model generation is now discussed.

As with any exercise of determining an appropriate model, a sample set which is indicative of the space to be modeling should be pursued. Determining the appropriate sample set or test matrix, however, can be challenging. Meeker and Escobar recommend several considerations when designing the test matrix [Ref. 4]. The considerations relevant to this problem are now discussed. First, one can estimate the ALT regression coefficients and even baseline distribution parameters using expert knowledge and/or existing data. An appropriate test matrix could then be devised quickly by assessing the quality of the MLE fit under variations of the test parameters. Such an initial estimate can be used to evaluate different test schemes such as the spacing of covariate vector values as well as the number of simulations allocated to each of the the various cases. Equal spacing and allocation of test points is the typical starting point. The

<sup>1</sup> The baseline survivor function is assumed to be known. If the survivor function is not known then existing or generated data could be used with EDF based statistical hypothesis methods such as Kolomogorov-Smirnoff and Anderson-Darling to statistically justify the most appropriate distribution (see Stephens 1974 for a complete comparison between the primary EDF methods used [Ref. 3])

authors of this paper also feel that under a limit of the number of experiments to run, one should allocate the number of experiments appropriately to each design of experiments case so that the variation of the maximum likelihood parameter estimates are minimized. For instance, if certain combinations of the covariates produce a lower probability level then one should allocate more experiments to these cases so that the accuracy at this probability level is improved. Alternatively, one could use the classical power of test method to statistically determine the required number of total experiments (see Sethuraman 1982 [Ref. 5]).

The example of covariate modeling given in the subsequent chapter utilizes the design of experiments method to sample the available failure space. Should there be a small number of candidate variables and variable levels, a full-factorial design could be utilized. However, if a larger number of variables and levels are to be pursued, a fractional factorial DOE, such as the popular Box-Benkin or Central Composite Design, is recommended. The levels selected should be the smallest possible that would be reflective of the linearity of the problem. For instance, a known linear problem would only require two levels for each variable while a quadratic response would require at least three levels.

## 2.3 Joint Probability Model

Often, independent statistical behavior is assumed for components within a system failure structure. This greatly simplifies the mathematics involved with computing the system reliability but at the expense of significant inaccuracy should joint randomness exist. Fortunately, there are several methods available to model such a condition. One such method, often found in structural reliability, is that of Nataf [Ref. 6]. The proposition made by Nataf is that joint randomness can be approximated by multiplying the products of the individual marginal distributions with a separate weight function that captures joint randomness through a transformation of the joint, non-normal space into an approximate joint, normal space. The joint randomness term is generated using a function of standard normal variates,  $\mathbf{Z} = (z_1, z_2, \dots, z_n)$ , which are determined through a marginal transformation of the random variables,  $\mathbf{X} = (x_1, x_2, \dots, x_n)$ , given by  $Z_i = \Phi^{-1}[F_{X_i}(x_i)]$  where  $\Phi$  is the standard cumulative normal function. Assuming that  $\mathbf{Z}$  is standard normal, then by the principle of inverse probability transformation the joint probability density function is given by

$$f(\mathbf{x}) = \prod_{i=1}^n f_{X_i}(x_i) \cdot \frac{\phi(\mathbf{z}, \mathbf{C}')}{\phi(z_1)\phi(z_2)\dots\phi(z_n)} \quad (3)$$

where  $\phi(\mathbf{z}, \mathbf{C}')$  is the  $n$ -dimensional standard normal PDF of the standard normal variates,  $\mathbf{Z}$ , and  $\mathbf{C}'$  is the correlation coefficient matrix of the transformed space with elements  $\rho'_{ij}$ . The general solution of  $\rho'_{ij}$  must be found iteratively through numerical integration. Fortunately, Der Kiureghian and Liu [Ref. 7] have determined several empirical formulas that provide  $\rho'_{ij}$  as a function of  $\rho_{ij}$  for several combinations of

random variable distribution pairs. Finally, the integration of the joint normal space produced by the transformation in the second term of equation (3) can be found using existing approximation techniques for the multi-normal integral [Ref. 8].

#### 2.4 Joint Covariate Model

An interesting modification to Nataf's proposition is to replace the marginal distribution functions of equation (3) with covariate lifetime models. Assuming that the correlation coefficient matrix remains constant over the space of covariate values under consideration, the modified model would therefore provide a unique and straightforward means of modeling both the parametric lifetime distribution behavior of the components as well as joint randomness between their failure distributions. Furthermore, since system simulation data already would exist if a set of component lifetime covariate models have been generated (section 2.2) a correlation matrix between the component failure distributions could easily be quantified using such data.

The resulting generalized covariate joint probability function is found by combining equations (2) and (3) given as

$$f(\mathbf{x}, \mathbf{z}) = \prod_{i=1}^n f_{X_i}(x_i, \mathbf{z}) \cdot \frac{\varphi(\mathbf{z}, \mathbf{C}')}{\varphi(z_1)\varphi(z_2)\cdots\varphi(z_n)} \quad (5)$$

where  $f_{X_i}(x_i, \mathbf{z}) = -S'(x_i, \mathbf{z})$  which is the marginal covariate probability density function of the  $x_i^{\text{th}}$  variable. Then for a series event system of dependent component lifetimes the covariate system reliability function using probability event theory is given as

$$S_X(\mathbf{x}, \mathbf{z}) = \sum_i S_{X_i}(x_i, \mathbf{z}) - \sum_{i < j} S_{X_i X_j}(x_i, x_j, \mathbf{z}) + \sum_{i < j < k} S_{X_i X_j X_k}(x_i, x_j, x_k, \mathbf{z}) - \cdots + (-1)^{n+1} S_X(\mathbf{x}, \mathbf{z}) \quad (6)$$

where  $S_{X_i}(x_i, \mathbf{z})$  is the marginal covariate survivor function of the  $x_i^{\text{th}}$  variable and  $S_{X_i X_j}(x_i, x_j, \mathbf{z})$  is the bivariate covariate survivor function of the  $x_i^{\text{th}}$  and  $x_j^{\text{th}}$  variables and so on. For a parallel event system the system reliability function is given as

$$S(\mathbf{x}, \mathbf{z}) = \int_x^\infty \int_x^\infty \cdots \int_x^\infty f(\mathbf{x}, \mathbf{z}) dx_1 dx_2 \cdots dx_n \quad (7)$$

which is the overlapping event space (intersection) between the  $\mathbf{x}$  variables. The multivariate joint probability terms in either equation (6) or (7) can then be solved for the limiting values of the random variables, or failure modes, using multi-normal integration techniques such as that of Gollwitzer and Rackwitz [Ref. 8].

### 3. APPLICATION: TURBINE BLADE RELIABILITY

A nominal aircraft engine system was chosen as an example of the reliability assessment approach discussed in this study. The propulsion system selected is the CFM56 separate flow turbofan engine which powers the B737-400

medium size transport aircraft. The vehicle mission used in this study is typical of a B737 aircraft; although, it has been simplified to facilitate the demonstration of the proposed method. Only the cruise condition is considered for this study. The take-off and landing segments are modeled as discontinuous jumps in engine rotor speed between the shut-down and cruise segments of the operating profile. The cruise segment occurs at an altitude of 35,000 feet and a speed of Mach 0.745 as is typical for the B737-400. Mission parameters, or system operating conditions, identified for this study are the cruise Altitude and Mach number.

Safe, reliable operation of the entire aircraft is highly dependent on the propulsion system utilized. One of the most critical components affecting the safety and reliability of turbine engines is the turbine blade. Failure of a turbine airfoil could cause a high-energy part to be released from the rotor system destroying the entire engine through a cascade of subsequent events. Such a situation could cause a catastrophic condition for the entire vehicle. Therefore, the propulsion system reliability is determined, within this study, by conducting a reliability assessment of a single turbine airfoil. The airfoil is assumed to be limited by two failure modes, fatigue and overstress.

The blade fatigue and overstress behavior is analyzed using a first-principle, integrated turbine blade multi-physics environment. The physical analysis structure begins with system operational and ambient conditions providing input to a thermo-dynamic cycle model. This cycle model is used to solve for the engine station thermodynamic state conditions as well as the mechanical speed of the rotors. A thermo-mechanical analysis is then conducted at the part level to determine the thermal and mechanical state of the part as required for the failure analysis.

The mechanical analysis for determining the bulk radial stress can be accomplished using Newton's second law applied to a rotating body. The centrifugal force produced anywhere in the airfoil can be determined by calculating the product of the mass of the supported material at the radius of interest and the square of the component rotational speed. Dividing this quantity by the airfoil cross-sectional area gives the bulk centrifugal stress at this radius. The rotor speed is calculated using the cycle model described earlier. With the thermo-mechanical conditions solved for, a failure analysis can then be conducted.

The overstress failure mode is simply determined by defining a limit state which is the limit at which the yield strength of the material, modeled as a function of temperature, is exceeded by the centrifugal stress of the component. Here the material temperature is provided as an input to the temperature dependent strength of the material. The material temperature is computed using the cooling flow temperature and core gas flow temperature which are output from the thermodynamic cycle model.

The fatigue failure mode is modeled as follows. Assuming that linear-elastic conditions exist, the straightforward stress-life, S-N, approach can be utilized to estimate the number of cycles to failure given completely reversed, constant stress cycle amplitude. Morrow [Ref. 9]

suggests a variation to the S-N approach to compensate for non-zero mean stress, which is given as

$$N_{fatigue} = 0.5 \left[ \frac{\sigma_a}{\sigma_f' \left( 1 - \frac{\sigma_m}{\sigma_f'} \right)} \right]^{\frac{1}{b}} \quad (9)$$

where  $\sigma_a$ ,  $\sigma_m$ ,  $\sigma_f'$ , and  $b$ , are the stress amplitude, mean stress, fatigue strength coefficient, and fatigue exponent of the blade material. The Morrow fatigue function assumes that the stress cycle amplitude is constant. The stress cycle generated within the context of this problem is defined as the centrifugal stress cycling between the zero stress condition before starting the engine and the stress state at the cruise condition.

#### 4. RESULTS

##### 4.1 Identification of Baseline Distribution

Data from 10,000 simulations was used to determine appropriate parametric distributions for the overstress and fatigue life failure modes. The Anderson-Darling Test statistic is used to compare how well each distribution fits the given data [Ref. 3]. The baseline distribution appropriate to the fatigue failure data is the lognormal distribution with location and scale equal to 14.49 and 1.71, respectively. A similar exercise showed that the overstress failure condition follows a normal distribution with a location and scale equal to 22.32 and 12.75, respectively. These results were validated using the A-D goodness-of-fit test provided earlier. The critical A-D value for both failure mode distributions can be found using Tables published by Stephens (1974) for a normal distribution.

##### 4.2 Covariate Model of Component Failure Distributions: Fatigue and Overstress

As discussed earlier, one can use either the PHM or ALT methods to parameterize the failure distribution(s) to include multiple explanatory (covariate) variables. The PHM method is advantageous because it can be used in a semi-parametric sense to prevent erroneously specifying an incorrect baseline distribution. However, our baseline analysis and subsequent A-D goodness-of-fit hypothesis tests revealed that the two failure modes follow standard parametric distributions. Therefore, the ALT method was selected.

The log-quadratic link function is used to account for the covariate parameters in the ALT model. A quadratic, polynomial function is assumed for the exponential component of the link function given as

$$\psi(\mathbf{z}) = e^{g(\beta\mathbf{z})} = e^{\beta_1 z_1 + \beta_2 z_2 + \beta_3 z_1 z_2 + \beta_4 z_1^2 + \beta_5 z_2^2} \quad (10)$$

The ALT survivor function then becomes

$$S_f(x, \theta_f, \beta_f, \mathbf{z}) = 1 - \Phi \left( \frac{\ln(x\psi(\mathbf{z})) - \theta_{f_1}}{\theta_{f_2}} \right) \quad (11)$$

for the lognormal fatigue failure distribution and

$$S_o(x, \theta_o, \beta_o, \mathbf{z}) = 1 - \Phi \left( \frac{x\psi(\mathbf{z}) - \theta_{o_1}}{\theta_{o_2}} \right) \quad (12)$$

for the overstress failure mode. The objective then is to acquire samples of the life at various values of the covariate vector,  $\mathbf{z}$ , and use the maximum likelihood method to estimate the coefficient vectors  $\beta_f$  and  $\beta_o$  as well as the baseline distribution parameter vectors,  $\theta_f$  and  $\theta_o$ .

##### 4.3 Model Generation

As with any exercise of determining an appropriate model, a sample set which is indicative of the space to be modeling should be pursued. Determining the appropriate sample set, however, can be challenging. Consequently, the design of experiments method is employed here. To be able to model quadratic behavior, a 3-level full-factorial design is created using the two covariate variables, Altitude ( $z_1$ ) and Mach number ( $z_2$ ). The tri-level design is chosen here to be able to capture quadratic behavior, should it exist. Also, the small number of candidate variables permits a full-factorial design to be utilized. However, should a larger number of variables and levels be pursued a fractional factorial DOE, such as the popular Box-Benkin or Central Composite Design, is recommended.

The ranges of the two covariates are given in Table 1. Nine different DOE cases each with a unique and orthogonal value of Altitude and Mach number are required for the full-factorial design. The number of simulations to be executed for each combination of covariate variable values is somewhat subjective. Within this study the number of simulations for each case was 10,000.

The results of the MLE fit of the covariate coefficients to 90,000 Monte Carlo simulations, 10,000 per case, are given in Table 2. The whole model tests of these two failure mode data sets verify that the covariates considered provide an exceptional model for representing the failure distribution while the standard error for each of the covariate coefficient estimates is minimal. However, the likelihood ratio tests for each vector of coefficients suggests that the interaction and quadratic terms can be neglected in the final ALT model. Based on the model statistics produced by the maximum likelihood estimation approach, the number of case simulations was deemed to be sufficient.

Table 1: Covariate Variable Ranges

Covariate	Minimum (-1)	Centerpoint (0)	Maximum (+1)
Altitude	34000	35000	36000
Mach	0.725	0.745	0.765

Table 2: Covariate Coefficient Vectors

	Intercept ( $\beta_0$ )	Scale	$\beta_1$	$\beta_2$
Fatigue	$e^{14.49}$	$e^{1.71}$	0.803	-0.333
Overstress	22.32	12.76	3.332	-2.071

#### 4.4 Validation of Covariate Model

As with any model, one should determine whether it is an adequate representation of the actual physical phenomena. However, with a probabilistic response this quickly becomes prohibitive for physical validation and thus ideal using computational simulation. Large simulation Monte Carlo analyses or alternative probabilistic methods such as variance reduction techniques and sensitivity-based approaches can be used to validate the model accuracy by verifying the probability or life prediction at various values of Altitude and Mach number. The choice of covariate value cases from which to evaluate the model or even how many cases to pursue is somewhat arbitrary. In the interest of computational efficiency the minimal number of cases necessary to assess the accuracy of the model is ideal. For this study a random sample of validation cases was generated numbering roughly around half of the number of original cases used to create the model. The results of this validation step for the fatigue life ALT model are given in Table 3. The metric to be used to assess the accuracy of the model is the predicted life at three probability levels; 0.01, 0.1, and 0.50. Four cases of large sample Monte Carlo probability calculations were randomly generated to validate the two covariate models. As shown in Table 3, the predicted fatigue life using the covariate model agrees very well with the actual life. The fourth case shows a few percent error between the actual and predicted values. However, considering the lognormal behavior of the fatigue life this is considered acceptable. Several techniques suggested by Meeker and Escobar [Ref 4] can be applied to reduce this error even further. Also, future work is anticipated to explore the use of the semi-parametric proportional hazards model as an improved approach.

Table 3: Fatigue Model Validation Results

Altitude	Mach	P=	Actual Life			Predicted Life		
			0.01	0.10	0.50	0.01	0.10	0.50
0.208	0.494		37,725	223,681	1,985,131	37,495	222,647	1,979,569
-0.456	-0.110		26,404	158,532	1,428,802	26,911	159,801	1,420,801
-0.602	0.864		16,337	99,497	912,551	16,987	100,868	896,820
-0.970	-0.068		16,670	101,517	930,941	17,459	103,671	921,748

#### 4.5 Joint Randomness Evaluation

As stated earlier, the strong assumption of independence of reliability responses is commonly made. However, whether or not the responses are truly independent is seldom verified. For this study, the correlation coefficient, a measure of linear statistical dependence, was computed between the fatigue and overstress random variables for each DOE case. For the DOE cases explored, moderate correlation existed between the two variables as the correlation coefficient fluctuated around a mean of 0.272 with a maximum and minimum value of 0.287 and 0.240, respectively. Further, the error in computing the probability of fatigue failure ( $P[\text{Life} < 5E4]$ ) using the independence

assumption varied between 20% and 30% across the nine DOE cases. Thus, even the moderate joint randomness should be accounted for in the final bivariate covariate reliability model.

The Nataf joint probability model, given by equation (3), is employed to account for the joint randomness between the two variables. The transformed space correlation coefficient can be found using an available correction factor [Ref. 7] for the lognormal-normal variable pair case and is given as

$$\rho' = F(\delta)\rho = \frac{\delta}{\sqrt{\ln(1 + \delta^2)}}\rho \quad (13)$$

where  $\delta$  is the coefficient of variation of the lognormal variable. The transformed correlation coefficient would then need to be recomputed each time the covariate model prediction value changes as the coefficient of variation is dependent upon the covariate model.

Using the Nataf covariate model with equation (6) for a series event failure scenario, the predicted failure probability was improved by at least 20% compared to the actual joint probability solution across the four validation cases given in Table 3. Further inspection of the results between the independence calculation and dependence model reveals that the second term of equation (6) is significantly underpredicted when assuming independence between the two failure functions. Therefore, for this bivariate failure scenario the Nataf joint probability model provides a considerable advantage in accuracy over the common independence assumption. The drawback, which is common for most joint probability calculations, is that numerical integration is required to solve for the joint probability of failure.

#### 4.6 Parametric Reliability Assessment

Now that the joint covariate ALT model has been created, several exercises are possible. For instance, the life corresponding to a certain probability of failure can be predicted for any combination of Altitude and Mach number within the specified ranges. Thus, an optimal setting of the two flight condition parameters that would maximize the life for a given acceptable probability of failure can be searched and quantified probabilistically. The probability of fatigue failure as a function of Altitude and Mach number is shown by Figure 1. Interestingly, the covariate model results suggest that for an improved system life the aircraft should be operated at the higher altitude and at the lower flight speed. The thinner air and reduced speed would result in less thrust, a lower rotor speed, and therefore a slower consumption of the fatigue life of the part. Intuitive as this result might be, the usefulness of the method is that a predicted failure probability is permissible across the entire space of interest. The overstress probability contour as a function of Mach and Altitude is shown by Figure 2. As expected, the trend of the overstress failure probability contour is similar to that of the fatigue failure probability, although it is more linear. As fatigue is a function of stress, and both fatigue and stress are



functions of the Altitude and Mach number, a similar behavior is intuitive.

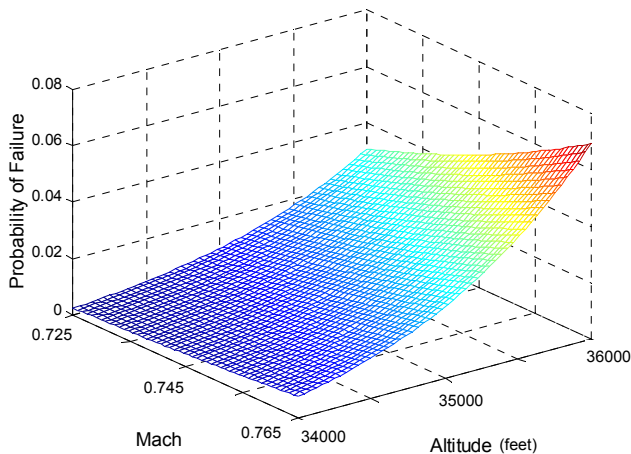


Figure 1: Covariate Model of Fatigue Failure Probability ( $N_{\text{fatigue}}=5,000$  cycles).

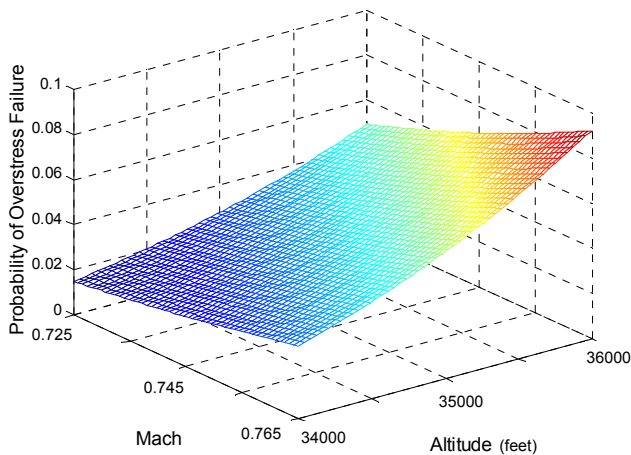


Figure 2: Covariate Model of Overstress Failure Probability ( $P_{\text{failure}}=P[\text{Stress}>\text{Strength}]$ ).

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