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A Probabilistic Approach to Multivariate Constrained Robust Design Simulation

Dimitri N. Mavris and Oliver Bandte
Georgia Institute of Technology

ABSTRACT

Several approaches to robust design have been proposed in the past. Only few acknowledged the paradigm shift from performance based design to design for cost. The incorporation of economics in the design process, however, makes a probabilistic approach to design necessary, due to the inherent ambiguity of assumptions and requirements as well as the operating environment of future aircraft. The approach previously proposed by the authors, linking Response Surface Methodology with Monte Carlo Simulations, has revealed itself to be cumbersome and at times impractical for multi-constraint, multi-objective problems. In addition, prediction accuracy problems were observed for certain scenarios that could not easily be resolved. Hence, this paper proposes an alternate approach to probabilistic design, which is based on a Fast Probability Integration technique. The paper critically reviews the combined Response Surface Equation/ Monte Carlo Simulation methodology and compares it against the Advanced Mean Value (AMV) method, one of several Fast Probability Integration techniques. Both methods are used to generate cumulative distribution functions, which are being compared in an example case study, employing a High Speed Civil Transport concept. Based on the outcome of this study, an assessment and comparison of the analysis effort and time necessary for both methods is performed. The Advanced Mean Value method shows significant time savings over the Response Surface Equation/Monte Carlo Simulation method, and generally yields more accurate CDF distributions. The paper also illustrates how by using the AMV method for distribution generation, robust design solutions to multivariate constrained problems may be obtained. These robust solutions are optimizing the objective function for a given level of risk the decision maker is willing to take.

INTRODUCTION

Systems design, in particular as applied to aerospace vehicles, has experienced a paradigm shift from emphasizing performance to maximizing affordability.[1, 2] The resulting new 'design for affordability' requires the addition of cost estimation as a new discipline to systems design. But since most of the economic assumptions and ground rules, such as number of paying passengers, fluctuations in fuel price, travel distance, etc.[1, 3], are inherently uncertain, more emphasis has been put on replacing "point" by probabilistic estimates that quantify the uncertainty of the predicted outcome. This new way of thinking has shifted the design focus from optimizing to 'compromising', where compromising describes a decision process that yields a robust solution[2, 4], i.e. a design that is insensitive to the variation of those economic parameters that are difficult or impossible to control. Such a design might be preferable to a true optimum which exhibits low confidence of achieving that optimum consistently. In order to quantify and minimize the uncertainty of a design outcome, a methodology called Robust Design Simulation (RDS) has been introduced.[5, 6, 7, 8] It is based on a Concurrent Engineering (CE)/Integrated Product and Process Development (IPPD) approach and opens up the traditional deterministic to a probabilistic approach to systems design. The methodology treats the cost parameters as random variables and models their variation with probability distributions. The paper critically reviews two approaches to probabilistic robust systems design, that allow for random changes in the assumptions made in the design process and aircraft operating environment.

ROBUST DESIGN SIMULATION

An aircraft synthesis and sizing process, utilizing appropriate analytical tools, evaluates the system value to the customer for each aircraft configuration through selected objectives such as performance, cost, profit, quality, or reliability. Regardless of the defined objective, customer satisfaction can be achieved only if all system design and environmental constraints are met. This algorithm is displayed in Figure 1, depicting the dependence of the objective on economic and discipline uncertainties as well as technological and schedule risk.

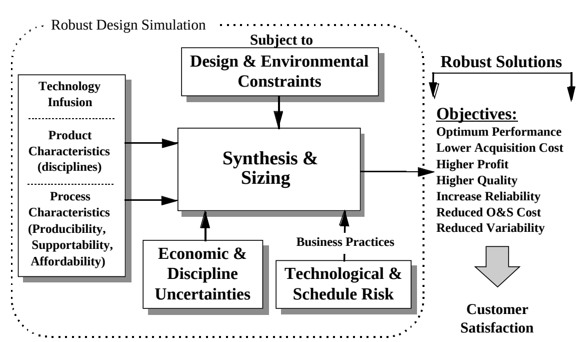

Figure 1: Robust Design Simulation[6]

The uncertainties in the design assumptions are usually accounted for in the form of variability distributions for system inherent random variables. These random variables introduce a variability in the objective that can be modeled as a probability distribution. So far, most robust design methodologies[4, 8, 9] strived to reduce the variability of the objective, assuming that one can reach a higher customer satisfaction with such reduction. In contrast, an alternate approach is proposed here where customer satisfaction is achieved by enabling him or her to chose the level of risk at which decisions are made. The proposed approach transforms the objective function from being dependent on design/control and noise/random variables into being dependent on design/control variables and level of risk. Given the level of risk the objective function can then be optimized by varying the controllable parameters, while concurrently satisfying all imposed design and environmental constraints.

PROBABILISTIC METHODS FOR ROBUST DESIGN

One of the major obstacles in applying probabilistic design methods is accommodating the large variety of existing deterministic computer codes used in modern systems design. It is impractical for all of them to be modified to enable a probabilistic problem formulation. Hence, a more generic methodology is proposed, which calls on some kind of ‘wrapper’ that, when linked to the selected analyses codes, drives the program and yields the desired results. Based on this formulation, probability functions can be assigned to each of those input variables which are considered to be uncertain and a cumulative probability distribution function (CDF) for each of the desired objectives may subsequently be obtained. Most probabilistic analyses, e.g. Monte Carlo Simulation [10], estimate their probability distribution functions based on a large number of samples generated over the design space, defined by the random variable ranges. While the usage of computer models allows for an easy perturbation of input values, as design problems increase in complexity, computerized analysis tools increase in complexity accordingly, often dramatically increasing run-time. Fox[11] lists three methods that incorporate such complex computer programs in a probabilistic systems design approach. Method #1 directly links a time consuming, due to a large number of repetitions needed, thus inefficient probabilistic method, such as the Monte Carlo Simulation, to the traditional systems design codes used by the traditional deterministic approach. Although computer speed has significantly increased in recent years, the extreme complexity of some design codes yields computation times that may prohibit a large number of program evaluations within the allotted time frame for the design process. Thus, Method #1 may not be a feasible option for a probabilistic design approach.

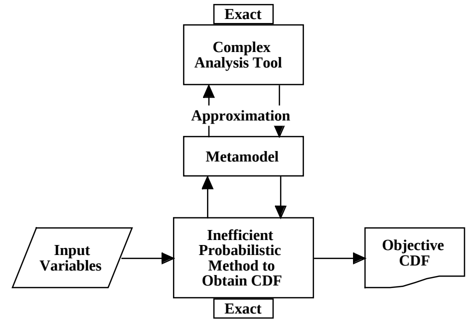

Figure 2: Probabilistic Method #2[11]

Method #2, displayed in Figure 2, proposes the use of a metamodel which approximates the design codes. The advantage of creating such a metamodel is a significantly reduced execution time, allowing a Monte Carlo Simulation to be applied to the metamodel rather than on the actual computer code. Several different metamodels have been proposed and applied. Some of the more common regression models are based on experimental designs [12], artificial neural networks [13], or Fuzzy Graph based metamodeling.[14]

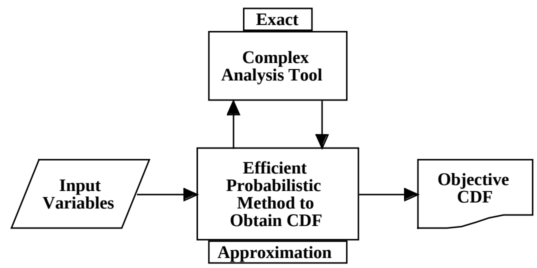

Figure 3: Probabilistic Method #3[11]

Method #3, displayed in Figure 3, takes a different approach, approximating the probability distribution function rather than the design code. This is based on the notion that in order to obtain the cumulative distribution function (CDF) not all probability levels need to be identified. The method selects several percentile levels and calculates the according objective value. Note that this calculation is based on the exact computer code, not on an approximating metamodel. These objective values and their probabilities can than be used to fit the typical S-shape of a CDF. The details of this method are described in [15] and in later sections of this paper.

Method #2 has found the widest application and has also been used in the past by the authors.[6, 8, 16] In particular, the use of statistical regression models, based on Taylor series expansions, in combination with experimental designs is very popular.[5, 11, 17, 18, 19, 20, 21] The two main reasons for its popularity are its easy application to numerous computer simulation problems, e.g. aircraft synthesis, and the large number of statistical analysis tools commercially available. Nonetheless, there are two major problems in metamodeling of complex computer codes with a high number of inputs. First, the number of input variables handled by this approach is typically limited to eight or nine. This problem can often be solved through a screening process

[22] that identifies the major contributors to variation in the model output. However, the metamodel created based on the screened parameters can never capture the variation of any of the other, 'less important' input parameters.

The second problem with Method #2 is in the mathematical background of such regression methods as Response Surface Methodology (RSM) and Design Of Experiments (DOE), which are based on random rather than deterministic variables (see [10], [12], [22], [23]). Therefore, the authors would like to stress here the use of caution in the straightforward application of these methods. Fundamental statistical knowledge is critical for obtaining reasonable approximations of the computer model. Many of the statistical analysis results the commercial packages offer are based on random error (ε) estimation and do not reflect the accuracy of the metamodel, since random error does not exist in deterministic computer simulation. A discussion on accuracy and behavior of statistical regression metamodels in computer simulations can be found in [12], [24], [25], and [26]. In general, the best validation of the accuracy of the metamodel is an extensive test at randomly distributed points over the design space to compare predicted values with the exact computer simulation values. Unfortunately, this test increases the computational effort put into the generation and validation of the metamodel. As shown in a previous paper[27], variation of only a subset of the variables in the metamodel can cause an additional prediction error not accounted for by testing the whole model.

COMBINED RESPONSE SURFACE/MONTE CARLO SIMULATION APPROACH

Despite the aforementioned problems of the Response Surface Methodology (RSM), it can, if applied correctly, provide some valuable insight into the systems design code behavior. Hence, it has been used by the authors as a metamodel generator to facilitate probabilistic aerospace systems design methods.[8] In order to compare that approach (Method #2) with the one introduced in this paper (Method #3), a very brief overview of the combined Response Surface Equation/Monte Carlo Simulation (RSE/MCS) method is provided here. For more detailed information refer to [6], [8], [22], and [23].

RSM is based on a statistical approach to build and rapidly assess empirical metamodels.[22, 23] By carefully designing and analyzing experiments or simulations, the methodology seeks to relate and identify the relative contributions of various input variables to the system response. However, aerospace systems are extremely complex, and the responses of interest could be a function of hundreds of design variables. The first step in constructing a Response Surface Equation (RSE) as a metamodel is to conduct a screening test to identify the variables which make the greatest contribution to the response of the system. The screening test is a two level fractional factorial Design of Experiments (DOE) that accounts for main effects of variables only (i.e. no interactions).[23] It allows the rapid investigation of many variables to gain a first understanding of the problem.

After identifying the variables which will form the RSE, an experimental Design of Experiments has to be selected. For the purposes of this study, a face-centered central composite design was used as a scheme for the input variable levels to be tested. This experimental design is a three level composite design formed by combining a two-level full factorial with a star design.[22] Typically, a second order model in k-variables is assumed to exist. This second order polynomial for a response, R, can be written as:

$$R = b_o + \sum_{i=1}^{k} b_i x_i + \sum_{i=1}^{k} b_{ii} x_i^2 + \sum \sum_{i<j}^{k} b_{ij} x_i x_j \qquad (1)$$

where: b_i are regression coefficients for linear terms

b_{ii} are coefficients for pure quadratic terms

b_{ij} are coefficients for cross-product terms

x_i, x_j are the design variables of interest

Refer to [22] and [23] for a detailed description of a response surface generation. After the RSE is developed, the effect of uncertain variables can be incorporated into a systems level design through the use of a Monte Carlo Simulation (MCS). A Monte Carlo Simulation is effectively a random number generator that selects values for each random variable with a frequency proportional to the shape of the corresponding probability distribution. Usually 5,000 to 10,000 trials are needed for a good representation of the response probability distribution. Without the aid of the RSE, this task would be computationally excessive and in many cases impractical considering that MCS would have to execute the design simulation code each time (Method #1, Figure 1).

FAST PROBABILITY INTEGRATION APPROACH

To avoid the often difficult generation of a metamodel (Method #2), this paper suggest, the use of a fast probability integration technique as an approach to Method #3. This technique is reviewed here in greater detail. The Fast Probability Integration (FPI) computer program[15], developed by researchers at the Southwest Research Institute (SwRI) for the NASA Lewis Research Center, is a probability analysis code based on the Most Probable Point (MPP) analysis frequently used in structural reliability analysis. The MPP analysis utilizes a response function Z(X) that depends on several random variables X_i (see Figure 4 for a 2-D example). Each point in the design space spanned by the X's has a specific probability of occurrence according to their joint probability distribution function (see Figure 5). However, each point in the design space also corresponds to one specific response value Z(X). Hence, each response value has the same probability of occurrence as the corresponding point in the design space.

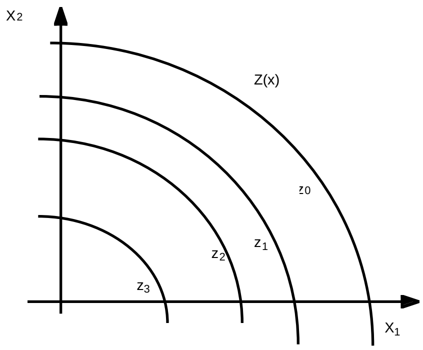

Figure 4 : Objective Function Contours

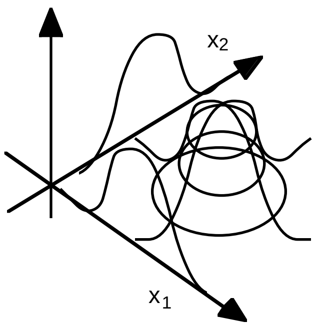

Figure 5 : Joint Probability Distribution

In cost analysis and other disciplines involving random variables, it is often desired to find the probability of achieving response values below a critical value of interest z_0. This critical value can be used to form a limit-state function (LSF):

$$g(X) = Z(X) - z_0 \quad (2)$$

where values of $g(X) \geq 0$ are undesirable. The MPP analysis calculates the cumulative probability of all points that yield $g(X) \leq 0$ for the given z_0 (see Figure 6). Since the LSF 'cuts off' a section of the joint probability distribution (see Figure 7) a point with maximal probability of occurrence can be identified on that LSF. This point is called the Most Probable Point. It is found most conveniently in a transformed space (see Figure 7), in which all random variables are normally distributed. Once the MPP and the cumulative probability are identified, the process can be repeated for several values of z_0, mapping each probability to the corresponding value of z_0. The resulting cumulative probability distribution for Z(X) can then be differentiated to obtain the probability density function of the response.

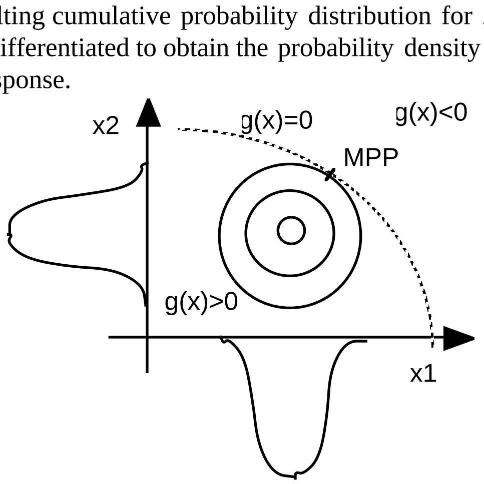

Figure 6: Most Probable Point (MPP) Location

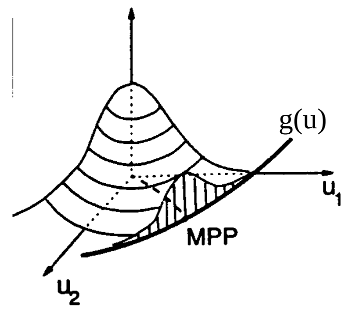

Figure 7: Visualization of MPP [9]

The FPI code offers several techniques to find the MPP and the probability of a given LSF value z_0 for the response function. Some of these techniques are very efficient and eliminate the need for an expensive Monte Carlo Simulation. An additional advantage of FPI is the fact that it is directly linked to the analysis code, eliminating the need for a metamodel and its limit in the number of variables. However, all Fast Probability Integration techniques approximate the LSF locally at the Most Probable Point.

ADVANCED MEAN VALUE METHOD

The Advanced Mean Value (AMV) method is one of the twelve analysis methods included in the FPI code. It combines a simple Mean Value method with the MPP analysis and determines the CDF for the response function Z(X). The Mean Value (MV) method is based on a simple Taylor series expansion of the response function Z(X) (Equation 3), assuming Z(X) to be smooth and the expansion to exist at the mean:

$$\begin{aligned} Z(X) &= Z(\mu) + \sum_{i=1}^{n} \left(\frac{\partial Z}{\partial X_i} \right) \cdot \left(X_i - \mu_i \right) + H(X) \\ &= a_o + \sum_{i=1}^{n} a_i X_i + H(X) \quad (3) \\ &= Z_{MV}(X) + H(X) \end{aligned}$$

The derivatives are evaluated at the mean values and $Z_{MV}(X)$ represents the sum of the first order terms and H(X) represent higher order terms. For n random variables, the a_i's can be estimated with n+1 function evaluations and a numerical differentiation method. Based on this linear approximation the CDF for $Z_{MV}(X)$ can be obtained directly, since the distributions for the random variables X_i are fully defined and $Z_{MV}(X)$ is explicit. For nonlinear Z-functions the MV solution for the CDF is not sufficiently accurate. One possible means for improving accuracy is increasing the order of the Taylor series expansion, which becomes difficult and inefficient for implicit response functions and a large number of random variables (n).

A more efficient approach is proposed through the AMV method:

$$Z_{AMV} = Z_{MV} + H(Z_{MV}) \quad (4)$$

$H(Z_{MV})$ is defined as the difference between Z and Z_{MV} at the Most Probable Point Locus (MPPL) of Z_{MV}, where the

MPPL combines the MPP's for several values of z_0.[28] In other words, $H(Z_{MV})$ in Equation 4 approximates $H(X)$ in Equation 3. Z_{AMV} would be exact if the MPPL was known and exact, i.e. $MPPL(Z_{MV}(X)) = MPPL(Z(X))$. Since the MPPL is not known, the AMV method approximates the locus based on Z_{MV}, which is for smooth response functions a good approximation.[15] Again, to avoid confusion with Method #2, the AMV method does not approximate the response function to obtain the CDF but rather the MPP (MV method). This approximation, however, is corrected by the 'move' in the AMV method, as depicted in Figure 8. The steps for a CDF generation with the AMV method are summarized in Figure 8.

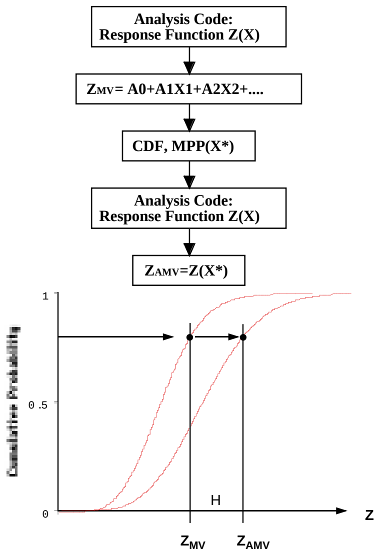

Figure 8: AMV Method [9]

One of the dominant advantages of the AMV method is the small number of function calls necessary, since n+1 analysis code executions are sufficient for the linear approximation of the response function Z_{MV} and ten additional program evaluations are needed to obtain the updated Z_{AMV} for ten selected levels of z_0.[15] This translates into significant time savings over the RSE/MCS method which usually requires several hundred function evaluations for the generation of the RSE.[22] Additionally, the AMV is principally not limited to a small number of variables. The current limit within the FPI code of 100 variables is due to vector formatting and not the fast probability integration technique itself. Nonetheless, there is an additional gain associated with the extended effort in the RSE generation. It can serve as a valuable tool to gain understanding of the behavior of the underlying model. The AMV method, on the other hand, will only return a probability distribution without providing any further insight into the analysis code.

APPLICATION EXAMPLE: A HIGH SPEED CIVIL TRANSPORT

To further compare the two approaches to probabilistic robust design, an aerospace systems design example is examined in some detail here. The aircraft baseline used for this example is a High Speed Civil Transport (HSCT) depicted for review in Figure 9. The vehicle has an area-ruled fuselage (maximum diameter of 12 ft.), a double delta planform, and four nacelles below the wing, housing mixed flow turbofan (MFTF) power plants. The values for some of the important design parameters are given in Table I. The mission profile for this aircraft is depicted in Figure 10, where the length of the subsonic cruise segment is assumed to be 15% of the design range. This split subsonic/supersonic mission is a result of the restriction for supersonic flight over land.

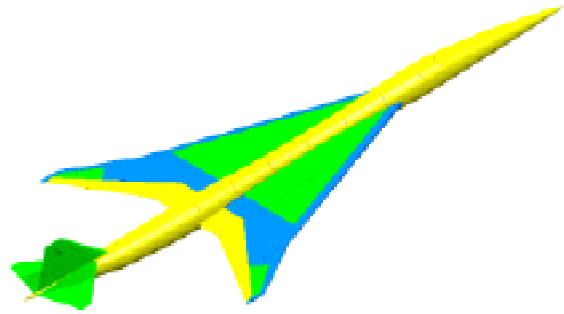

Figure 9: HSCT Example

Table I: Description of the Baseline HSCT

Parameter	Baseline
Range	5000 nm
Payload	300 Passengers
Fuselage length	310 ft.
Span	77.5 ft.
Inboard Sweep	74 deg.
Outboard Sweep	45 deg.
Reference Area	9,000 ft^2
Mach Number	2.4
Cruise Altitude	~63,000 ft.
Sustained Load	2.5 g

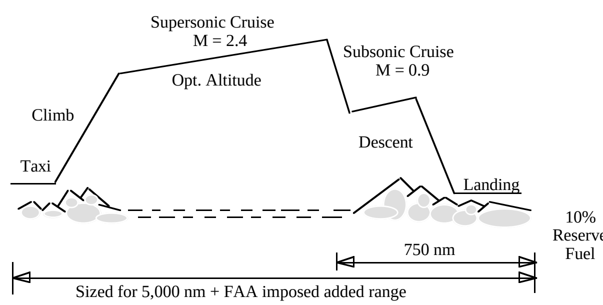

Figure 10: Baseline Mission Profile

Finding an optimal configuration for a supersonic transport vehicle is a multidisciplinary and very difficult task. Choosing a wing planform shape, for example, is driven by the need for efficient performance at both sub- and supersonic cruise conditions, a conflicting design objective in itself.[5, 29] Furthermore, the trades involved in planform selection are being complicated by different discipline considerations for aerodynamics, structures, propulsion, etc., and the presence of design and performance constraints at the system level which are directly related to the wing. The limit on approach speed, for example, is mostly a function of wing loading. Fuel volume requirements impact the wing size and shape. Both become sizing criteria and are treated as constraints that tend to increase the wing in size. On the other hand, increased wing area yields higher induced and skin friction drag, thus increasing fuel consumption. Based on the way the sizing code models an aircraft and its mission, a fuel requirement, Rf, can be constructed in the form of a ratio of available fuel to required fuel for completing the mission. Hence, Rf has to have a value greater than one to satisfy the fuel requirement. Additional design challenges are presented by takeoff and landing field length limitations (less than 10,000 ft) that are also modeled as design constraints for this study. Table II summarizes the design objective, required average yield per revenue passenger mile, ¢/RPM, and all constraints considered for this study that need to be satisfied during an optimization. ¢/RPM was chosen as an objective function for this study, because it captures implicitly the interests of both passengers and airline, reflecting the average ticket price given a fixed return on investment for the airline.

Table II: Summary of Objective and Constraints

Response	Requirement
¢/RPM	minimize
Fuel Requirement Rf	> 1
Approach Speed	< 154 kts
Takeoff Field Length	< 10,000 ft
Landing Field Length	< 10,000 ft

The Flight Optimization System (FLOPS) [30] code was selected for this study as the design simulation tool, while the Aircraft Life Cycle Cost Analysis (ALCCA) [31] program was selected as the economics model. Based on previous screening tests performed by the authors [6, 8] an inclusive list of design and economic variables were identified and are listed in Table III as the main contributors to the response average required yield per revenue passenger mile (¢/RPM).

Note the distinction made between the Economic and the Design Range. The former represents the average distance an airplane will fly from one airport to another during its life, while the latter depicts the maximum distance the aircraft is designed to fly. All nine parameters have been characterized as either design or uncertainty variables, where the uncertainty variables are associated with normal probability distributions. The design variables are not random but rather assumed to be under the control of the designer. Note that all randomness in this study is inherent to the economic uncertainty.

Table III: Control and Noise Variable Descriptions and Ranges

Variable	Type	Name	Range	Mean	Std.Dev.
Thrust to Weight Ratio	Control	TWR	0.28 - 0.32		
Wing Area	Control	WingArea	8.5 - 9.5 $*$ 10^3 ft^2		
Longitudinal Kink Location	Control	x1	1.54 - 1.62		
Spanwise Kink Location	Control	y1	0.5 - 0.58		
Turbine Inlet Temperature	Control	TIT	3 - 3.25 $*$ 10^3 °F		
Fan Pressure Ratio	Control	FPR	3.5 - 4.5		
Fuel Cost	Noise	$-Fuel	0.55 - 1.1 $/gal	0.75	0.07
Load Factor	Noise	LF	0.55 - 0.75	0.65	0.04
Economic Range	Noise	Ec-Range	3 - 5 $*$ 10^3 nm	4000	350

As part of the Response Surface Methodology, a face centered Central Composite Design (CCF) [22] is identified for the nine variables in Table III. The typically assumed second order regression model, Equation 1, is used to estimate the relationship between design and economic variables and the responses. Using the obtained RSEs, prediction profiles, depicted in Figure 11, can show the individual dependency or sensitivity of the response to the design and economic variables. All sensitivities are displayed for the baseline aircraft as the variable settings indicate. The random variables are set at their mean values. It should also be mentioned that throughout the study all actual values for ¢/RPM have been reduced by a constant to protect any sensitive data.

R^2 values for the regression 0.991 to 0.999 indicate a successful fit of the data generated by the CCF design to the model in Equation 1. These values, however, do not reflect

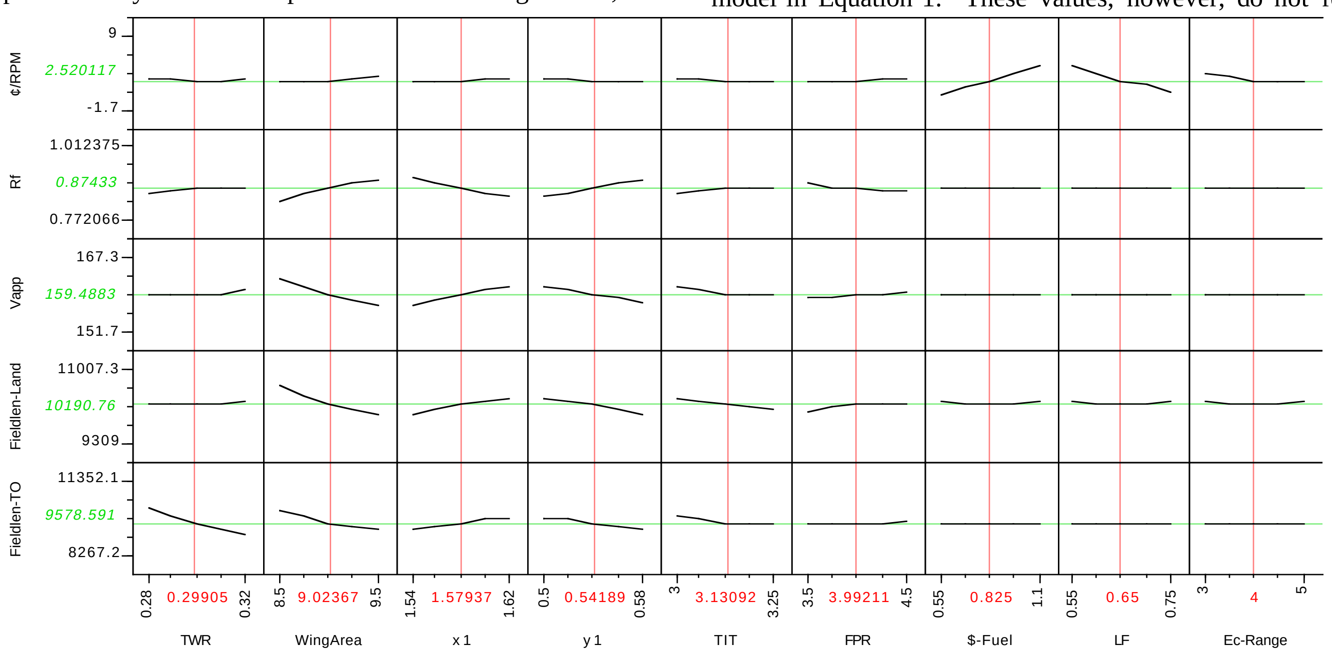

Figure 11: Response Surface Prediction Profiles

the prediction performance of the equations at ‘off design’ points, i.e. points not selected in the DOE. To verify the prediction accuracy of the RSEs, 500 data points randomly distributed over the design space were generated, both for the actual design code and the RSEs. With these data points correlation plots can be generated that compare the synthesis code results with the values predicted by the RSE. These plots, Figures 12 to 16, and correlation values, R^2, ranging between 0.9965 for ¢/RPM and 0.864 for Approach Speed indicate a good prediction performance of the RSEs. A perfect correspondence of both data sets would be indicated by a 45 degree line through all data points from the bottom left to the top right corner and by a correlation value of 100%. The differences in prediction performance between the responses can be explained by the way they are simulated. Economics calculations are usually comprised of regression equations that are well behaved (e.g. no discontinuities), while the estimation of gross weight embodies several nonmonotonic equations and table look up procedures that are not being approximated well by second order polynomials. The prediction ellipse, also depicted in the graphs, represents a confidence interval that encloses 95% of the data generated.

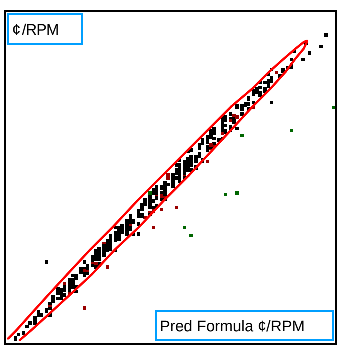

Figure 12: Correlation of Actual with Predicted Values for ¢/RPM

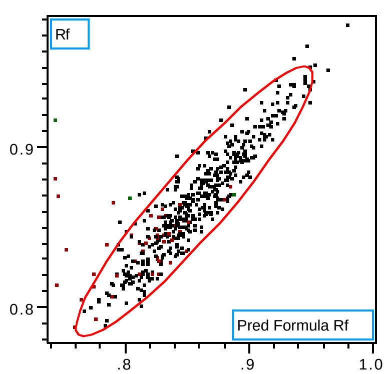

Figure 13: Correlation of Actual with Predicted Values for Rf

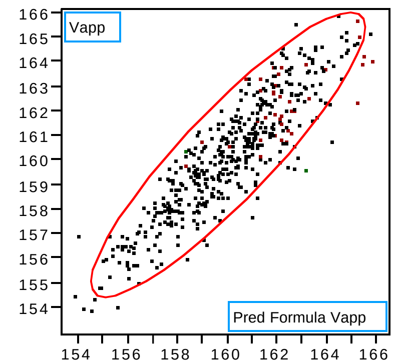

Figure 14: Correlation of Actual with Predicted Values for Approach Speed

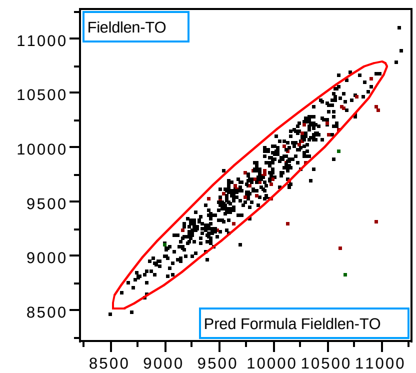

Figure 15: Correlation of Actual with Predicted Values for Takeoff Field Length

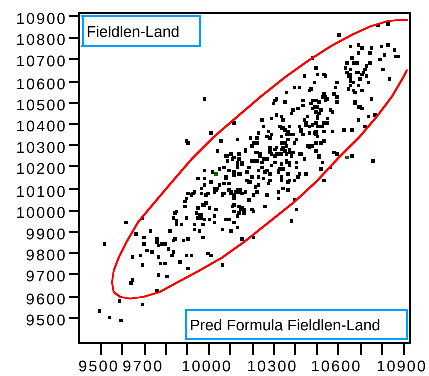

Figure 16: Correlation of Actual with Predicted Values for Landing Field Length

COMPARISON OF CDFS FOR RSE/MCS AND AMV APPROACH

It has been demonstrated in a previous study performed by the authors[27] that the AMV method clearly yields a better estimate of the CDF. However, to confirm this preference for the present case study, a Monte Carlo Simulation is performed with the actual design code at a randomly selected design point. The CDF of this simulation can be compared in Figure 17 against the CDFs from both methods, AMV and RSE/MCS. Inspection of this figure indicates that the CDF from the AMV method and the actual Monte Carlo Simulation are almost identical, while the CDF from the RSE/MCS method shows a clear distinction.

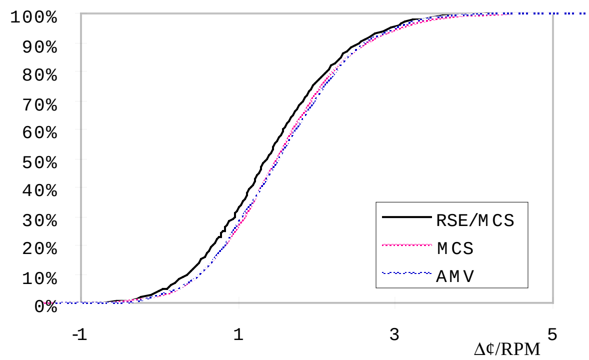

Figure 17: CDF Comparison for a Random Point

It is not necessarily intuitive to contribute the noticeable difference of the CDFs in Figure 17 to the prediction error of the RSE, which is rather small as demonstrated before. However, the design variables are deemed to be responsible for the prediction error, since for the same random variable distributions the CDF differs with the setting of the design variables. This phenomenon was thoroughly discussed in [27], and is briefly examined here in Figures 18 to 20 again. The two CDFs in each figure are generated for the same design point by the two methods, RSE/MCS and AMV, while the design point varies from figure to figure. It can be concluded from these figures alone that the CDF prediction error depends on the design variables setting, since the error changes with varying design variables.

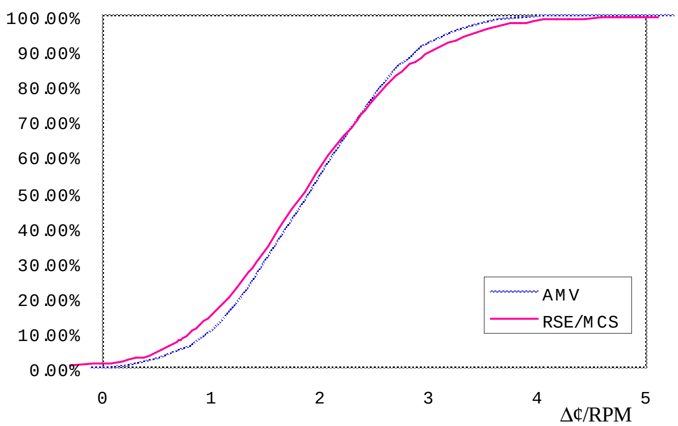

Figure 18: CDF Comparison for Design Point 1

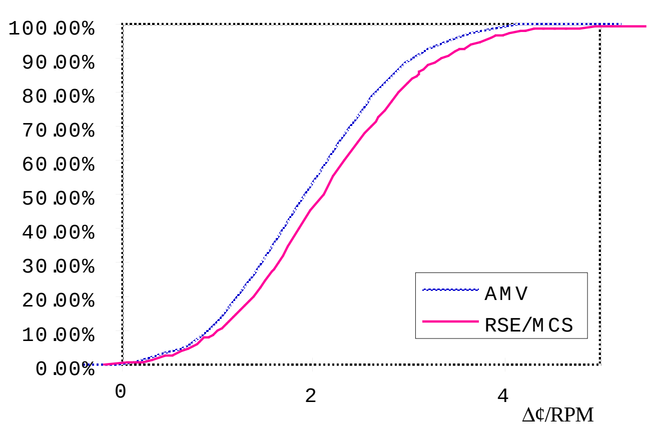

Figure 19: CDF Comparison for Design Point 2

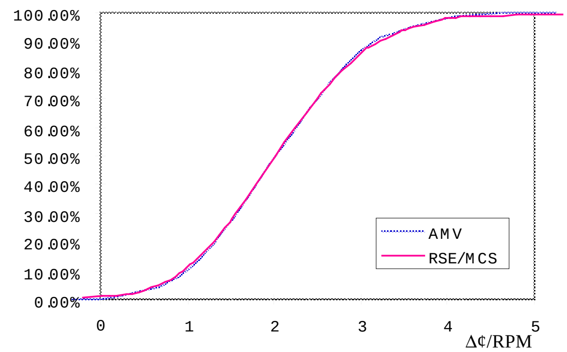

Figure 20: CDF Comparison for Design Point 3

CONSTRAINED ROBUST DESIGN OF AN HSCT

After having established AMV as the more reliable CDF prediction method, a constrained robust design methodology can be demonstrated using the HSCT example case. The first step in a probabilistic robust design is to identify the different probability levels, or risk, at which the objective function is being investigated. For each of these levels an RSE is created employing a CCF design for only the design variables. The noise variables are not part of this DOE, since it is them that generate the variability that is under investigation. Thus, each of the cases of the DOE generates one CDF for ¢/RPM (see also [8] and [16]). From this CDF the probability level of interest and its corresponding ¢/RPM value are determined. These data comprise the sample for the regression problem. Since the distributions of the noise variables are held constant, all variation in the ¢/RPM value at a particular probability level has to be contributed to the variation of the design variables from case to case. Hence, an RSE for ¢/RPM dependent on design variables can be generated for each probability level. For this study three levels, 82%, 50%, and 14%, as well as the mean were selected to be displayed in Figure 21. The constraints, on the other

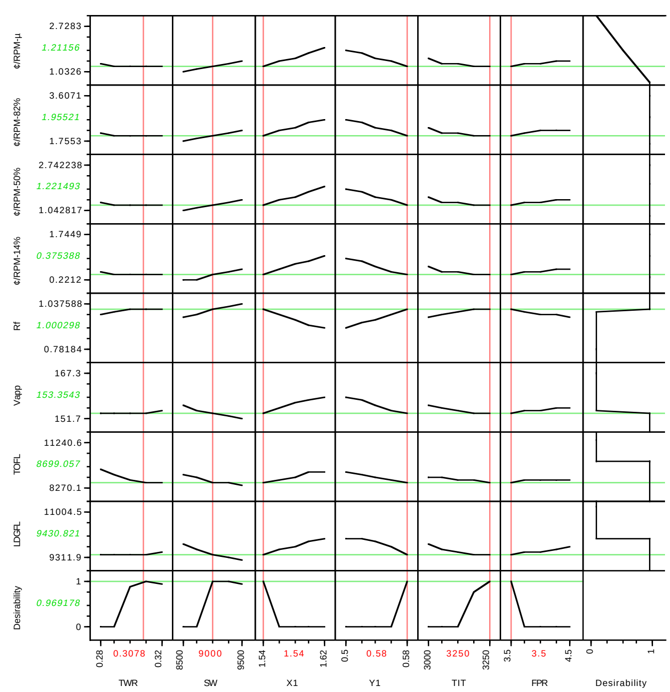

Figure 21: Prediction Profiles for the Robust Design Example

hand, are dependent on the same design variables, as displayed in Figure 21. Hence, a constrained optimization can be executed, minimizing ¢/RPM at a particular probability level without violating any constraints under consideration.

Also depicted in Figure 21 is the robust design solution for the mean of ¢/RPM. This solution was determined with a method called desirability functions, which is a feature of JMP®[32], the statistical package used to generate the DOEs, equations, and all graphs presented in this paper. The user assigns desirability values between zero and one (last column of Figure 21), one being most desirable, to the response function. For example, if a response is supposed to be minimized, like ¢/RPM-μ, low values of that response are assigned high desirability values. By perturbing the variable setting, each outcome of a response yields a desirability value in the bottom row of Figure 21. If more desirability functions are being assigned to different responses, all desirability's are multiplied with each other and displayed in the bottom row of the graph. This method allows to transform a multiple into a single objective optimization problem which maximizes desirability.

Thus, this feature is able to handle constraints by assigning a desirability of zero to all constraint response values that violate their requirement and a desirability of one to those that satisfy it. Hence, all variable settings that violate a constraint will cause a desirability of zero in the bottom row of Figure 21, since the desirability's of all responses are multiplied. On the other hand, if a variable setting satisfies the constraint, the solution will not be influenced since it is multiplied by one. Refer to Rf in Figure 21 as an example, where all values for Rf below one are assigned a desirability of zero, while values greater than one are assigned a desirability of one. If a response, such as ¢/RPM-82%, should not influence the desirability value of the solution, all values are being assigned a desirability of one. This method enables the designer to obtain a solution to an optimization problem quickly and very visibly on the screen without the need for a separate optimization execution. Table IV summarizes the obtained robust design solution and compares it with the baseline.

Table IV: Robust Design Solution Summary

Parameter	Robust Solution	Baseline
T/W Ratio	0.3078	0.3
Wing Area	9000 ft^2	9000 ft^2
x1	1.54 x span	1.58 x span
y1	0.58 x span	0.54 x span
TIT	3250°F	3125°F
FPR	3.5	4.0

Based on the results presented in Table IV the AMV method was employed one more time in order to compare the cumulative distribution for ¢/RPM of this robust design solution to the original one of the baseline, as displayed in Figure 22. The robust design solution always yields a lower ¢/RPM value for the same level of risk. Naturally the probability increases with increasing values for the ¢/RPM. However, it can be seen that for "small" and "very large" values the difference in probability between the robust design solution and the baseline is very small. The difference increases, however, for values around the means of the distributions.

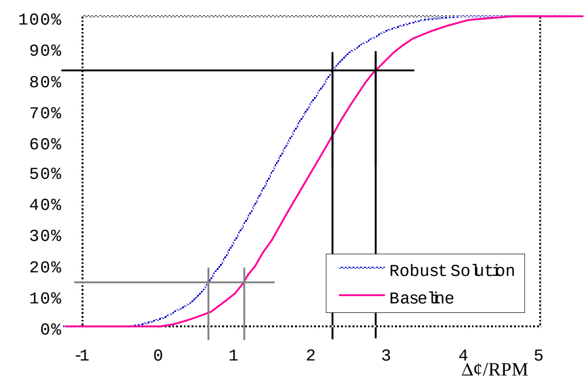

Figure 22: Distribution Comparison Between Baseline and Robust Design Solution

CONCLUSION

The present paper reviewed three different probabilistic approaches to robust design, which were executed and compared to each other. Method #1 applies a Monte Carlo Simulation directly to the analysis codes used in the design process. This approach yields an accurate estimate of the objective probability distribution, but requires a large number of code evaluations, which is usually too time consuming for most modern systems design tools. Method #2 requires the generation of an approximate metamodel to facilitate a Monte Carlo Simulation in a more time efficient manner. Method #3 approximates the probability distribution rather than the design simulation code, extracting the desired probabilistic information based on a significantly reduced number of function calls. Methods #2 and #3 were compared against each other on the basis of a study, using a High Speed Civil Transport concept as an aerospace systems design example. An experimental design was used as an approach to Method #2, generating a Response Surface Equation which was employed in a Monte Carlo Simulation. Method #3 utilized the Advanced Mean Value method, which is based on a fast probability integration technique. The generated cumulative distribution functions from both methods were compared against each other, determining that the Advanced Mean Value method yields the more accurate estimate of the probability distribution. Considering also the reduced number of function calls necessary for the analysis and the ability to accommodate more variables, the authors conclude the Advanced Mean Value method to be the more efficient and effective approach to probabilistic robust design. The Advanced Mean Value Method was finally used to generate the cumulative distribution function for the robust design example study. For given levels of probability, or risk, robust design solutions were identified that yield a minimum objective function value and satisfy all imposed constraints. This solution and the corresponding cumulative distribution function were presented and compared.

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