COLLABORATIVE PROCUREMENT AND DUE DATE MANAGEMENT IN SUPPLY CHAINS

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COLLABORATIVE PROCUREMENT AND DUE DATE MANAGEMENT IN SUPPLY CHAINS

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SUMMARY

In this thesis we analyze the procurement process of buyers and supply decisions of manufacturers. Companies are looking for ways to decrease their procurement costs, which account for a large percentage of the supply chain costs. We study the effects of demand aggregation and collaborative procurement on buyers' profitability. First, we make a high-level analysis and consider a market with multiple buyers and suppliers where multi-unit transactions for multiple items take place. The procurement costs are effected by economies of scale in the suppliers' production costs and by economies of scope in transportation. We design buyer strategies that model different collaboration levels and assess the role of collaboration under varying market conditions. Next, we analyze the procurement process on a lower level and identify benefits of inter-firm collaboration among buyers who are potential competitors in the end market. We adopt a game-theoretic approach to explore the economics of the basic mechanism underlying collaborative procurement, and determine the conditions that makes it an attractive proposition for the participants.

Besides low procurement costs, important considerations in supplier selection are responsiveness and the reliability of the suppliers in meeting demand. Hence, manufacturers face the pressure for quoting short and reliable lead-times. We cover several aspects of the manufacturer's problem, such as quoting reliable due dates based on the current workload in the system, maximizing profit considering the lateness cost incurred due to late deliveries, and deciding on the level of inventory to increase responsiveness. We employ a model where demand arrival and manufacturing processes are stochastic, and obtain insights on the optimal due date policy and on the optimal inventory level.

CHAPTER I

INTRODUCTION

In this thesis we consider the collaborative efforts in the procurement process among supply chain participants and the challenges that are faced by suppliers in fulfilling demand. Procurement costs account for a high percentage of supply chain costs. According to an article published in Harvard Business Review (Degraeve and Roofhooft 2001), "Purchased products and services account for more than 60% of the average company's total costs. For steel companies, it may go up to 75%; it's 90% in the petrochemical industry... Bringing down procurement costs can have a dramatic effect on the bottom line–a 5% cut can translate into a 30% jump in profits".

In the first part of this thesis, we analyze the potential benefits of collaboration in reducing procurement costs (Chapters 2 and 3). Collaborative procurement is one of the initiatives that helps supply chain partners gain more value-added pricing, service, and technology from their suppliers than could be obtained individually (Hendrick 1997).

In Chapter 2, we determine the conditions under which buyers benefit intra-firm and inter-firm collaboration, whereas in Chapter 3 we analyze the dynamics behind group purchasing, which is a prevailing practice of inter-firm collaboration.

Intra-firm collaboration takes place among the internal units of an enterprise. In many large-scale companies, purchasing is done locally by purchasing units of regional branches that either act independently or have minimal interaction with each other. For example, prior to March 2003, the divisions in Sun Chemical Corp. had independent purchasing procedures. As a result, the company could not realize the potential purchasing power and was not getting the best prices from its suppliers. By enabling collaboration among purchasing units through a centralized purchasing organization, Sun Chemical Corp. aims to reduce the company's total cost of ownership in the supply chain by 10% (Graff 2004). Until recently, Dial Corp.'s purchasing was also decentralized. It was typical for buyers at different Dial plants to buy the same raw material from different suppliers at different prices. This approach was ineffective in taking advantage of Dial's volume and corporate-wide buying power (Reilly 2002). Similarly, until 1997, purchasing at Siemens Medical Systems was done locally, where buyers at Siemens' ultrasound, electromedical, computer tomography, magnetic resonance imaging and angiography divisions independently bought the components and material that their individual plants needed and rarely communicated with each other (Carbone 2001).

Inter-firm collaboration occurs when independent companies work together, synchronize and modify their business practices for mutual benefit. Examples include Covisint (founded by DaimlerChrysler, Ford, General Motors and Renault-Nissan), and Chemconnect, an online exchange in the chemical industry that enables collaboration in purchasing and sales processes.

Motivated by current of intra- and inter-firm collaboration practices, in Chapter 2 we consider a market where buyers have multiple functional divisions responsible for purchasing. We assess the role of third party intermediaries (or e-markets) in enabling collaboration among buyers. We address the following research questions: (i) under which market conditions is collaboration most beneficial?, and (ii) how do the benefits differ in intra-firm and inter-firm collaborations? We analyze and compare the following models: (I) *No collaboration*: Buyer divisions and suppliers trade through traditional sales channels, via one-to-one transactions. No information flow or collaboration exists among the functional divisions of a buyer or among multiple buyers. (II) *Internal collaboration*: Functional divisions of a buyer collaborate internally. (III) *Full collaboration*: A third party intermediary enables collaboration among different buyers, and allows the participants to achieve benefits from both economies of scale and scope due to reduced fixed production and transportation costs.

Chapter 2 ignores competition among the buyers in the end market whereas Chapter 3 considers the case where the buyers compete in the same end market. In this setting, we study under which conditions collaborative procurement benefits the buyers.

Collaborative procurement is practiced often in the health care industry through group purchasing organizations (GPOs), and has been increasingly used in other industries as well.

For example, a Texas based GPO, VHA, saved its members \$813.5 million on purchases of medical supplies and services in 2003 (Vha.com). Other industries where joint purchasing exists include manufacturing (Mfrmall.com, Purchasing Consortia of Manufacturer's Association of Central New York), automotive (Covisint.com), plastics (Polysort.com), and logistics (Transplace.com, Nistevo.com). In the area of logistics, collaborative purchasing takes the form of shippers sharing lanes. For example, the collaborative logistics network, Nistevo, helps buyers to consolidate shipments and share the fleet capacity or arrange backhauls. General Mills is saving approximately $\$800,000$ per year by collaborating with another company on a single tour (Lynch 2000). Transplace has a fuel program that allows member carriers to procure fuel at lower prices at designated fuel stops across the country. In these industries, although the buyers are collaborative partners in purchasing, they could be competing for the same customer base.

The extent of savings may lead potential members of group purchasing programs to question whether it is actually worth joining such a program. The supplier also has parallel concerns: does the potential increase in sales volumes justify offering lower prices? We address the following research questions: (i) what factors are important for group purchasing to be successful?, (ii) is group purchasing always profitable for the buyers and the supplier?, and if not, under what conditions is it profitable for the participants? We adopt a game-theoretic approach to explore the economics of group purchasing, and identify the conditions that makes it an attractive proposition for the participants.

In a recent survey conducted among 500 manufacturers, 83% of the respondents ranked the ability to meet delivery schedules as the most important criterion for selecting a vendor (Keeping 2002). After studying the buyers' collaborative efforts to lower the procurement costs, in the second part of the thesis we consider the supplier's problem of reliability and responsiveness in meeting demand (Chapter 4). Suppliers are increasingly aware that being responsive in fulfilling demand while keeping delivery time promises is an important means by which to differentiate themselves from their competitors.

As a guarantee for meeting delivery promises, firms are offering all or part of the revenue back to the customer should a late delivery occur. For example, in September 2003, less-than-truckload carriers FedEx Freight, Con-Way and USF announced their new service guarantee where the shipper is not charged if the shipment is not on-time (Boyce 2003). Real World, a distributor of products such as CPUs, memory chips, and semiconductors, offers a 5% rebate for late deliveries (Cohodas 1999). During the 2000 Christmas season, online retailer Roxy.com gave $100 compensation to customers for which it failed to deliver the orders on-time to (Sandlund 2000).

In the examples cited above, when there is a late delivery the supplier incurs a cost, which is independent of the duration of the delay. Some companies, however, set a penalty proportional to the length of the delay. Carload services of CSXT and Union Pacific railroad, for example, both guarantee delivery with a lateness penalty of $200 per day (Blanchard 2001). This type of penalty is usually a part of the purchase contract in manufacturing and the amount of the penalty may change with respect to the buyer's production schedule. For example, the cost of a late delivery in FMC Wellhead Equipment Division, a producer of wellhead drilling and completion equipment, may rise up to $250,000 per day (Blanchard 1998). In the aircraft industry the lateness penalties start from $10,000-15,000 and can go as high as $1,000,000 per day (George 2001; Slotnick and Sobel 2004). As these figures imply, late deliveries may have catastrophic consequences. In 1997, Boeing faced more than $200 million in late-delivery penalties (Holmes and France 2002). Delivery promises are also watched by government agencies. The Federal Trade Commission (FTC) charged seven major e-tailers more than $1,5 million in total when they failed to comply with their delivery promises during the 1999 holiday season (Sedlak 2001).

Late deliveries and potential loss of goodwill are important incentives for quoting reliable due dates. Firms may have the tendency to quote slightly longer lead times as a buffer to ensure reliability. For example, to keep the percentage of on-time arrivals high, airlines have been announcing longer scheduled travel times for certain trips. In some cases the scheduled times have increased by as much as 25% over what they were 10 years ago (Peters 2000). However, a delayed due date may cause customers to switch to other suppliers. When quoting due dates (or lead times), companies need to consider the sensitivity of the demand to lead-times. One possible solution for being responsive while not incurring lateness penalties is to keep stock available. For example, in a repair facility or after-sales service department, part availability is crucial for customer satisfaction and loyalty (*Unlocking Hidden Value* 2003).

In Chapter 4, we examine several facets of a due date quotation problem, including quoting reliable due dates based on the current workload in the system, maximizing profit considering the lateness cost incurred due to late deliveries, and deciding on the level of inventory. We ask the following research questions: (i) when is it more profitable to operate in a pure make-to-order environment, and when is it more profitable to keep inventory?, (ii) how much inventory should be kept?, and (iii) how do utilization levels affect the profitability? We develop a model where demand arrival and manufacturing processes are stochastic, and obtain insights on the optimal due date policy and on the optimal inventory level.

For each of the three topics in the thesis, we present a review of the literature in the corresponding chapter, and describe how our work contributes to the literature. Conclusions and future work are discussed in Chapter 5.

CHAPTER II

IMPROVING SUPPLY CHAIN PERFORMANCE THROUGH BUYER COLLABORATION

2.1 Introduction

In this chapter we consider a market with multiple buyers and suppliers where multi-unit transactions for multiple items take place. Buyers have multiple functional divisions where each buyer division is responsible for the procurement of different items. These functional divisions may or may not collaborate in the procurement process to pool their purchasing power. We design alternative procurement strategies that model three different levels of buyer collaboration (see Figure 1). Our objective is to identify the conditions where collaboration is most beneficial to the buyers by testing those strategies under different markets.

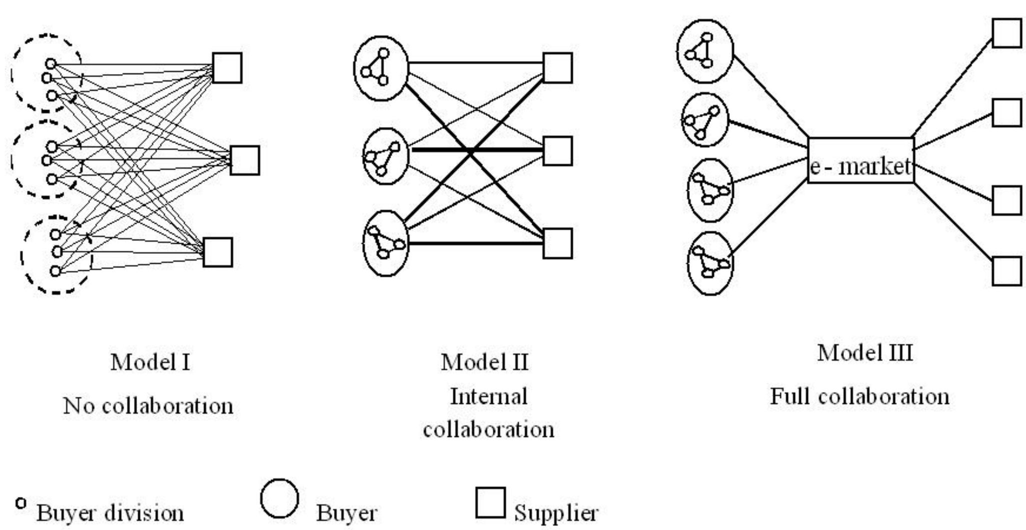


Figure 1: Three models of collaboration among buyers and buyer divisions

2.2 Literature Review

Interactions among participants of a supply chain can be analyzed along several directions. One line of related research focuses on decentralized versus shared information. When the information is decentralized, studies are primarily on constructing different mechanisms to enable coordination in a two-stage setting and to eliminate inefficiencies stemming from double marginalization. Cachon and Zipkin (1999) and Lee et al. (2000) analyze coordination mechanisms in the form of rebates or transfer payments. Weng (1995) considers a system where coordination is established through quantity discounts and franchise fees. Jin and Wu (2001) study supply chain coordination via transfer payments in the presence of e-market intermediaries. Also see Cachon (2003) for an analysis of coordinating contracts under different supply chain settings.

Another line of research focuses on collaboration. Internet and technology have made information sharing possible at every stage, and this leads to different collaborative efforts in supply chains. Several examples include vendor managed inventory (VMI), just-in-time distribution (JITD), and collaborative planning, forecasting and replenishment (CPFR), in which trading partners such as vendors and retailers collaborate vertically. In VMI systems the vendor is given autonomy in replenishing the retailer's orders and in turn manages the retailer's inventory with fewer stockouts and at low levels. In the pilot effort on CPRF in 1997, Wal-Mart and vendors Lucent and Sara Lee were sharing event and point of sales information to jointly forecast sales (http://www.cpfr.org). There is a growing interest in supply chain literature on analyzing the benefits of VMI in supply chains, for example see Cetinkaya and Lee (2000) and Cheung and Lee (2002). Relatively little research exists on collaboration through forecasting. Aviv (2001) studies the benefits of collaborative forecasting with respect to decentralized forecasting.

Horizontal collaboration differs from vertical in the sense that it considers collaboration among those only on the buyer or supplier side. Horizontal coordination and collaboration in the supply chain enabled by quantity discounts is studied by Gurnari (2001) in a single supplier two buyer setting. Some existing research considers the interaction of buyers and suppliers from a resource allocation perspective. Ledyard, Banks and Porter (1989) test allocation mechanisms with uncertain resources and indivisible demand. The results indicate that high efficiency could be obtained if collaboration is enabled among buyers. To the best of our knowledge there has not been much research on the loss of efficiency due to lack of horizontal collaboration. In this chapter, we study the interaction between multiple buyers and multiple suppliers, where horizontal collaboration is enabled by a central mechanism (or intermediary).

A similar problem is studied by Kalagnanam, Davenport and Lee (2000), where the motivation came from electronic markets in the paper and steel industries. The authors consider an e-market in which buyers and suppliers submit bid and ask prices for multiple units of a single product. They show that the problem of determining the clearing price and quantity under different assignment constraints can be solved in polynomial time when demand is divisible, but is NP-hard when demand is not divisible.

We extend the work of Kalagnanam, Davenport and Lee (2000) in several directions. We consider multiple products, rather than a single product, where each supplier needs to decide how to allocate its limited capacity among these multiple products. Furthermore, we consider fixed costs of production and transportation which lead to economies of scale and scope. Finally, in addition to the "centralized" e-market (where all the buyers and suppliers are available in the market at the same time and the buyer-supplier assignments are done centrally) we study two other scenarios where buyers arrive to the market sequentially and select suppliers on a first come first served basis.

2.3 Model

To model buyers' behavior in the market, we assume that buyers or buyer divisions arrive with requests for quotes (RFQ) for each item they want to buy. We assume buyers initiate the trades by submitting RFQs to the suppliers. A buyer requests that her entire demand for an item is satisfied from a single supplier. Hence, a supplier would respond to a buyer's RFQ only if he has enough production capacity to satisfy the buyer's entire demand for that item. Such all-or-nothing buyer behavior is observed in several industries for various reasons.

Splitting an order across multiple suppliers complicates order tracking and transportation arrangements. In addition, order splitting might also lead to inconsistency in quality. For example, in the paper industry the quality of the paper produced by different machines is slightly different, which may create problems in printing. Similarly, in carpet manufacturing, carpet produced at different times or locations has slight color variations which can be noticed when the carpet is installed.

Each buyer has a reservation price for each item, which is the maximum price for the purchase of the set of good (not per unit of the item). A buyer's *surplus* for an item is defined as the buyer's reservation price for that item minus the final contracting price for the entire demand of the buyer for that item. With the goal of maximizing her surplus, if there are no quotes (bids) that are acceptable to a buyer at a given time, she may leave the market and come back later with the hope of getting a better quote.

By evaluating the quotes offered by the suppliers (if any) buyers decide which supplier to choose for each item. A supplier might produce multiple types of items and has limited production capacity to be shared among these items. A supplier's cost for an item consists of four components:

- The manufacturing setup cost for that item (fixed production cost). A supplier initiates production and incurs a setup cost for an item only if a buyer places an order for that item.
- Variable production cost per unit.
- Fixed cost of transportation.
- Variable transportation cost per unit.

In responding to the buyer RFQs, suppliers use a cost plus pricing scheme, i.e., set prices to cover the fixed and variable costs and leave enough *profit margin* for profits. Despite its limitations, cost-plus pricing is commonly used in various industries. For example, in the logistics industry, 33% of third-party logistics companies (3PL) in North America used cost-plus pricing in 2000 (Smyrlis 2000).

To keep the exposition simple, we ignore the *profit margin* component and focus only on the cost component, i.e., the *bid price* quoted by a supplier is obtained by adding up the cost components. However, as we will explain in the following sections, what the buyer pays in the end (*contract price*) might be lower than what the supplier quotes originally.

We assume that the buyers select suppliers based on price alone. Although price is an important criterion in supplier selection, most buyers also consider other factors, such as quality and delivery time reliability, while selecting a supplier. However, we focus on commodity procurement, where multiple vendors with similar quality and delivery performance exist.

In the remainder of this chapter we assume that each buyer division is responsible for the procurement of one item (hence, the index for items and buyer divisions is the same, see Table 1). We also assume that the different divisions of the same buyer are located in the same region, which implies that the locations of the divisions are close enough to allow for consolidation of orders for transportation and thus for price discounts from the carriers; e.g., the "region" can be a state, or the south-east region of the United States. The pricing structure enables buyers to obtain economies of scale and scope. As more buyers place orders with the same supplier for the same item, the associated fixed production cost for each buyer decreases (economies of scale). As a single buyer places orders at the same supplier for multiple items, the associated fixed transportation cost per unit decreases (economies of scope). This type of cost (or price) structure can also be interpreted as a volume discount.

The demand quantities of the buyers and initial capacities available at the suppliers represent the total demand and total supply in the market. Initially there is no production setup at the suppliers. As buyers accept supplier bids and make contracts for the items, suppliers initiate production.

2.3.1 No Collaboration

In this market structure, we model traditional marketplaces, where neither the functional divisions of a buyer nor different buyers in the market collaborate with each other. As

Table 1: A glossary of notation for Chapter 2

i:	index for buyers, $i \in I$
j:	index for items (or buyer divisions), $j \in J$
k:	index for suppliers, $k \in K$
K_j:	set of suppliers that produce item j
Div_{ij}:	buyer i, division j
d_{ij}:	demand of buyer i for item j
D_j:	total demand in the market for item j, $\sum_i d_{ij}$
D:	total demand in the market, $\sum_j D_j$
q_{ijk}:	quantity of item j produced at supplier k (upon receiving an RFQ from buyer i)
t_{jk}:	total quantity of item j produced at supplier k
res_{ij}:	reservation price of buyer i for the total demanded quantity of item j
fpc_{jk}:	fixed production cost for item j at supplier k
vpc_{jk} :	variable production cost per unit for item j at supplier k
ftc_{ik} :	fixed cost for transportation between buyer i and supplier k (might also include the fixed transaction costs)
vtc_{ik} :	variable cost for transportation per unit between buyer i and supplier k
c_{jk}:	capacity required to produce one unit of item j at supplier k
tc_k:	total capacity at supplier k
cap_k:	available capacity at supplier k, upon receiving an RFQ
ave(c_{jk}):	capacity required to produce one unit of item j averaged out over all suppliers that produce item j, $\frac{\sum_k c_{jk}}{\|K_j\|}$
Q:	estimated total supply in the market, $\frac{\sum_k tc_k}{\text{ave}(c_{jk})}$
1^p_{jk}:	indicator variable equal to 1, if supplier k started production for item j
1^t_{ik} :	indicator variable equal to 1, if buyer i and supplier k have contracted before

discussed earlier, many firms have uncoordinated purchasing divisions. For example, until recently Chevron's procurement structure was fragmented and decentralized where "people at many different locations were buying many materials, (often the same materials) from their favorite suppliers, or on an as-needed basis" (Reilly 2001).

We assume that the functional divisions arrive sequentially and independently to the market. Therefore the marketplace can be thought of as a queue of buyer divisions each with an RFQ for a specific item. When a buyer division makes a contracting decision, she is unaware of the other buyer divisions' demands or procurement decisions (including the ones both from the same and different companies), or about the current order status at the suppliers. After submitting an RFQ, a buyer division makes the contracting decision only based on the unit prices quoted by the suppliers. Once the contracting decision is made and the purchase order is submitted by a buyer division, another buyer division arrives to the market and submits an RFQ.

Buyer divisions' decisions in contracting will depend on the kind of information they receive from the suppliers. A supplier can provide either a *pessimistic* or an *optimistic* quote to a buyer. When providing a pessimistic quote, the supplier regards that buyer as if she will be the last one to contract for that item. When providing an optimistic quote (OPT), the supplier assumes that he will supply all the demand in the market for that item. The optimistic quote is a lower bound on the final contract price, whereas the pessimistic quote is an upper bound.

Given an RFQ by buyer division j of company i for d_{ij} units of item j, the supplier computes the pessimistic quote (bid price per unit) as follows:

$$bid_{ijk} = \frac{fpc_{jk}}{q_{ijk} + d_{ij}} + vpc_{jk} + \frac{ftc_{ik}}{\sum_{m \in S} d_{im}} + vtc_{ik} \tag{1}$$

The first two terms in equation (1) correspond to the unit production cost, $P_k(d_{ij})$. The fixed production cost for an item is shared among multiple buyer divisions (from different companies) who placed orders for that item with the supplier. q_{ijk} is the total quantity for item j already contracted at supplier k upon receiving the RFQ of company i division j.

The last two terms of equation (1) correspond to the unit transportation cost, $T_k(d_{ij})$.

The fixed transportation cost is shared among multiple buyer divisions from the same company who placed orders (for different items) with the same supplier. The set S contains the current buyer division and the other buyer divisions of the same company that have already contracted with supplier k.

Note that if a buyer division is the first one to place an order for an item at a supplier, then she is quoted all the fpc. Similarly, when a supplier receives an RFQ for the first time from a buyer division of a particular company, he incorporates all the ftc in the bid.

To compute the optimistic quote, supplier k needs to first compute (an upper bound on) the maximum total quantity of orders for item j he could produce upon receiving an RFQ, which is:

$$Q_{ijk} = \min\{D_j, q_{ijk} + \frac{cap_k}{c_{jk}}\}$$

The maximum total quantity is bounded by the minimum of two terms. The first is the total demand for item j in the market. The second is the quantity of item j already produced by supplier k plus the maximum additional quantity of item j that can be produced by supplier k.

Therefore the following term is a lower bound on the fixed production cost per unit:

$$\max\{\frac{fpc_{jk}}{D_j}, \frac{fpc_{jk}}{q_{ijk} + \frac{cap_k}{c_{jk}}}\}.$$

Upon receiving an RFQ, supplier k computes the following optimistic quote for buyer i per unit of item j:

$$\text{OPT}_{ijk} = \frac{fpc_{jk}}{Q_{ijk}} + \frac{ftc_{ik}}{\sum_{m \in S} d_{im}} + vpc_{jk} + vtc_{ik} \quad (2)$$

In the remainder of the chapter, we assume that the supplier provides a pessimistic quote is provided by the supplier unless otherwise stated.

If buyer i contracts with supplier k, the final contract price she pays per unit of item j is:

$$price_{ijk} = \frac{fpc_{jk}}{t_{jk}} + \frac{ftc_{ik}}{\sum_{m \in S} d_{im}} + vpc_{jk} + vtc_{ik}, \quad (3)$$

where t_{jk} is the total quantity of item j produced at supplier k in the final matching. The contract price of an item has a similar structure to the bid price for that item. In the contract price, the fpc portion to be paid by each contracting buyer is calculated as $\frac{fpc_{jk}}{t_{jk}}$. Hence, as the number of buyers contracting with a supplier for the same item increases, the unit price to be paid by a buyer decreases. Therefore, in the end the price paid by a buyer might be lower than the quoted price.

The practice of lower final prices paid by the buyers compared to the initial bids offered by the suppliers is commonly observed in group purchasing programs. For example, the price of a product goes down as more buyers join the group and agree to buy that product at the current posted price. Although some buyers may have joined the group (and committed to purchasing) while the price was higher, in the end all the buyers pay the final, lowest price.

In the final matching, the surplus of buyer i contracted with supplier k for item j is:

$$surplus_{ij} = (res_{ij} - price_{ijk} \cdot d_{ij}) \tag{4}$$

Note that when considering the supplier bids, the buyer division multiplies the bid price with the total demand for the item to evaluate her surplus.

Buyer Strategies

A buyer division submits RFQs to the suppliers for the item she demands and chooses a supplier with the goal of maximizing her surplus for the item she demands. We consider the following buyer strategies for accepting or rejecting a bid.

1. If some of the supplier bids are lower than her reservation price, she accepts the minimum bid.

2. If all the bids are higher than the buyer division's reservation price, she accepts the minimum offer with probability α.

3. If the buyer division rejects all the quotes (with probability 1-α), then with probability β, she leaves the market permanently. With probability $1-\beta$ she returns to the market (i.e., joins the end of the queue) since there is a possibility that the minimum bid the

buyer receives later is lower than the current minimum bid. The buyer stays in the market until her surplus becomes positive or the bid prices stop decreasing, whichever happens first.

Having defined a general scheme, we now describe alternative buyer strategies. A buyer division makes a contract with the supplier that offers the minimum bid, if that bid is below her reservation price. Otherwise, the buyer division:

Myopic Strategy, MYOPIC ($\alpha = 0$, $\beta = 1$) Leaves the market.

Accept the minimum bid, MIN ($\alpha = 1$, $\beta = 0$) Makes a contract with the supplier that offers the minimum bid.

Leave and possibly return later, LOQ ($\alpha = 0$, $0 < \beta < 1$) Leaves the market with probability $0 < \beta < 1$, returns to the market and joins the queue with probability $1 - \beta$.

Leave and return later, Q ($\alpha = 0$, $\beta = 0$) Returns to the market and joins the end of the queue.

Accept the lowest bid or leave and return later, AOQ ($0 < \alpha < 1$, $\beta = 0$) Accepts the minimum bid with probability α. With probability 1-α she rejects all the bids and joins the end of the queue.

Minimum optimistic bid, MOB Accepts the minimum optimistic bid, $\min_k\{\text{OPT}_{ijk}\}$.

Example 1. Consider a marketplace where there are four buyer divisions, Div_{ij}, $i = 1, 2$, $j = 1, 2$ (two companies with two divisions each), three suppliers, S_k, $k = 1, \ldots, 3$ and two items, I_1 and I_2. A buyer division j is responsible for item I_j, $j = 1, 2$. The buyer divisions arrive to the market in the following order: (1) company 1 division 1, (2) company 2 division 1, (3) company 1 division 2, (4) company 2 division 2. Buyer divisions use the *accept the minimum bid* strategy for contracting decisions. For simplicity we assume that the suppliers are uncapacitated. The information regarding buyer divisions (Div), suppliers (S) and items (I) is listed in the tables below.

Company 1 division 1 places an RFQ for item 1. The quoted bids for per unit of item 1 by supplier 1, supplier 2 and supplier 3 are 18.75, 131.37 and 19 respectively (refer to equation 1). Supplier 1 wins the contract for item 1.

Next, company 2 division 1 arrives to the market and submits an RFQ for item 1.

Table 2: Demand and reservation prices

	Div_{11}	Div_{12}	Div_{21}	Div_{22}
Demand for I_1	40	-	10	-
Demand for I_2	-	25	-	75
Res. price for I_1	2000	-	500	-
Res. price for I_2	-	3750	-	7500

Supplier 1 has already initiated production for item 1. Therefore supplier 1 reflects only some portion of the fixed production cost in the quote, whereas supplier 2 and supplier 3 reflect all the fixed production cost in their quotes. The quotes per unit of item 1 by suppliers 1, 2 and 3 are 24, 486 and 30 respectively. Company 2 division 1 contracts with supplier 1 for item 1.

Table 3: Fixed and variable production costs

	S_1	S_2	S_3
I_1	$fpc_{11} = 100$ $vpc_{11} = 10$	$fpc_{12} = 140$ $vpc_{12} = 10$	$fpc_{13} = 100$ $vpc_{13} = 10$
I_2	$fpc_{21} = 9125$ $vpc_{21} = 10$	$fpc_{22} = 100$ $vpc_{22} = 10$	$fpc_{23} = 4605$ $vpc_{23} = 10$

Table 4: Fixed and variable transportation costs

	S_1	S_2	S_3
B_1	$ftc_{11} = 50$ $vtc_{11} = 5$	$ftc_{12} = 4515$ $vtc_{12} = 5$	$ftc_{13} = 60$ $vtc_{13} = 5$
B_2	$ftc_{21} = 50$ $vtc_{21} = 7$	$ftc_{22} = 4570$ $vtc_{22} = 5$	$ftc_{23} = 50$ $vtc_{23} = 5$

Next, company 1 division 2 places an RFQ for item 2. While quoting the bids, supplier 1 incorporates only some portion of the fixed transportation cost, since supplier 1 has already contracted with division 1 of the same company for item 1. The bids quoted by suppliers 1, 2 and 3 are 380.77, 199.60, 201.60, respectively. For item 2, company 1 division 2 contracts with supplier 2 and is charged for two different fixed transportation costs by both supplier 1 and supplier 2. The surplus she obtains for item 2 is $res_{12} - bid_{122} \cdot d_{12} = 3750 - 199.60 \cdot 25 = -1240$. Although the surplus value is negative, since the strategy under consideration is *accept the minimum bid*, this does not impose any restriction on contracting.

Finally company 2, division 2 submits an RFQ for item 2. Supplier 2 has initiated production for item 2. The quotes for item 2 by suppliers 1, 2 and 3 are 139.25, 76.93, 77.07, respectively. For item 2, company 2 division 2 contracts with supplier 2. She is also charged for two different fixed transportation costs. Bid prices and contracted suppliers are shown in Table 5.

Table 5: Bid prices under no collaboration

	S_1	S_2	S_3
Div_{11}	$\mathbf{bid_{111} = 18.75}$	$bid_{112} = 131.75$	$bid_{113} = 19$
Div_{12}	$bid_{121} = 380.77$	$\mathbf{bid_{122} = 199.60}$	$bid_{123} = 201.60$
Div_{21}	$\mathbf{bid_{211} = 24}$	$bid_{212} = 486$	$bid_{213} = 30$
Div_{22}	$bid_{221} = 139.25$	$\mathbf{bid_{222} = 76.93}$	$bid_{223} = 77.07$

The contract prices paid by the buyers are shown in Table 6 and the total surplus is 2115. The matchings for the no collaboration example are shown in Figure 2(a). In the following sections we analyze the same example under different collaboration models.

Table 6: Contract prices and surplus under no collaboration

	$price_{ijk}$	$surplus_{ij}$	$surplus_i$
Div_{11}	$\frac{100}{40+10} + 10 + \frac{50}{40} + 5 = 18.25$	$2000 - 18.25 \cdot 40 = 1270$	105
Div_{12}	$\frac{100}{25+75} + 10 + \frac{4515}{25} + 5 = 196.6$	$3750 - 196.6 \cdot 25 = -1165$	
Div_{21}	$\frac{100}{10+40} + 10 + \frac{50}{10} + 5 = 22$	$500 - 22 \cdot 10 = 280$	2010
Div_{22}	$\frac{100}{25+75} + 10 + \frac{4570}{75} + 5 = 76.93$	$7500 - 76.93 \cdot 75 = 1730$	

2.3.2 Internal Collaboration

Advances in information technology and enterprise systems have increased the availability of real-time data. This, in turn, has led to increased levels of information sharing and collaboration among the divisions (or business units) of a company. Before 1997, each division of Siemens Medical Systems had its own supplier and this significantly deteriorated buying power. Centralization of purchasing has saved 25% on material costs (Carbone 2001). Similarly, Chevron Corp. is aiming to cut 5% to 15% from annual expenditures by centralizing the procurement system and by leveraging volume buys (Reilly 2001).

In this section we assume that the procurement function is centralized within each company. Equivalently, multiple divisions within a company collaborate for procurement. We also assume that buyers know the structure of the transportation cost (ftc and vtc) for each supplier, possibly specified with a long-term contract. Examples of such practices are commonly found in the procurement of transportation services, such as trucking or sea cargo. Shippers contract with multiple carriers where each contract specifies a volume-based price and capacity availability. However, the buyers usually do not have to commit to a shipment volume in these contracts; even if they do, such minimum volume commitments are typically not enforced by the carriers. In our model, the price structure is defined by fixed and variable costs, i.e., suppliers offer volume discounts. We assume that a company i can request and receive information about the available total capacity, cap_k, and the volume-based price quote $P_k(d_{ij})$ (first two terms of equation (1)) for any item j from a supplier k for her entire demand d_{ij}. This implies that a company can determine the total cost for an order using the information on the transportation cost component and the quote on the production cost component for any item-supplier combination.

Under internal collaboration each company decides how much of each product to procure from each supplier using a centralized mechanism. The procurement decision of a company, say buyer i, can be modelled by the following linear integer program.

x_{ijk}: 1, if buyer i contracts with supplier k for item j.

y_{ik}: 1, if buyer i contracts with supplier k.

$$\max \sum_j \sum_k res_{ij} \cdot x_{ijk} - \sum_j \sum_k (P_k(d_{ij}) + vtc_{ik}) \cdot d_{ij} \cdot x_{ijk} - \sum_k ftc_{ik} \cdot y_{ik} \qquad (5)$$

subject to

$$
\text{(IP-I)} \qquad
\begin{aligned}
\sum_j c_{jk} \cdot d_{ij} \cdot x_{ijk} &\le cap_k & \forall k & \qquad (6)\\
x_{ijk} &\le y_{ik} & \forall j,k & \qquad (7)\\
\sum_k x_{ijk} &= 1 & \forall j & \qquad (8)
\end{aligned}
$$

Constraint (6) ensures that the amount of demand satisfied by a supplier does not exceed the current available capacity of the supplier. Constraint (7) ensures that when a contract is made with a supplier the corresponding fixed transportation cost is charged to the buyer. Constraint (8) ensures that the buyer contracts with a single supplier per item.

The buyers contract with the suppliers sequentially as in the no collaboration model. The model does not include any reservation price constraint. This implies that if the buyer surplus is negative after solving the IP-I, the buyer still makes the contract. Each buyer makes the contracting decision based on maximizing her current surplus in equation (5). In the final matching, the contract price and surplus of buyer i for item j is obtained as in equations (3) and (4).

Example 1. (cont.) We analyze our previous example assuming that each company makes its procurement decisions centrally. Buyer 1 (both divisions of company 1) arrives to the market and makes the contracting decisions for both items by solving the IP model above. Based on the outcome of the model, buyer 1 contracts with supplier 2 for both items and is charged the fixed transportation cost only once.

When buyer 1 makes her contract, supplier 2 initiates production for both items. Therefore, if buyer 2 also contracts with supplier 2, the associated fixed production cost for both items will be shared among the two buyers. Buyer 2 solves the same model with updated $P_2(d_{21})$ and $P_2(d_{22})$ values. The solution suggests that buyer 2 also contracts with supplier 2 for both items. Therefore the buyers benefit from both economies of scale and scope.

For buyer 1 the final contract price per unit of item 1 is 87.26 and the final price per unit of item 2 is 85.46. Therefore the total surplus of buyer 1 is 123. For buyer 2, the contract prices per unit of item 1 and 2 are, 71.57 and 69.76. The total surplus of buyer 2 is 2052. As compared to the traditional market, both buyers have increased their total surplus. The overall buyer surplus obtained in the market is 2175.

Table 7: Contract prices and surplus under internal collaboration

	I_1	I_2	$surplus_i$
B_1	$price_{112} = 87.26$	$price_{122} = 85.46$	$(2000 - 87.26 \cdot 40)+$ $(3750 - 85.46 \cdot 25) = 123$
B_2	$price_{212} = 71.57$	$price_{222} = 69.76$	$(500 - 71.57 \cdot 10)+$ $(7500 - 69.76 \cdot 75) = 2052$

2.3.3 Full Collaboration

In the full collaboration model, we assume that a third party intermediary enables collaboration among multiple buyers. We use the terms *e-market* and *full collaboration* interchangeably. However, in practice not all e-markets enable full collaboration. Some e-markets provide only catalog services where suppliers and buyers post supply and demand quantities. 3PL providers such as Transplace or C.H. Robinson, where shippers and carriers do not contract directly with each other but through the 3PL intermediary, may enable full collaboration.

In this model, each buyer submits her demand and reservation price for each item, and each supplier submits cost and capacity information to the intermediary. The intermediary in turn matches supply and demand in the market with the objective of maximizing the total buyer surplus.

The matching problem faced by the intermediary can be modelled by the following linear integer problem, which is a slight modification of (IP-I):

z_{jk}: 1, if supplier k initiates production for item j.

$$\max \sum_i \sum_j \sum_k res_{ij} \cdot x_{ijk} - \sum_j \sum_k fpc_{jk} \cdot z_{jk} - \sum_i \sum_k ftc_{ik} \cdot y_{ik} - \sum_i \sum_j \sum_k (vpc_{jk} + vtc_{ik}) \cdot d_{ij} \cdot x_{ijk} \tag{9}$$

subject to

$$(7) \text{ and } (8) \qquad \forall i$$

$$\sum_i \sum_j c_{jk} \cdot d_{ij} \cdot x_{ijk} \leq tc_k \qquad \forall k \tag{10}$$

$$x_{ijk} \leq z_{jk} \qquad \forall i, j, k \tag{11}$$

Constraint (10) ensures that the amount of demand satisfied by a supplier does not exceed the total capacity of the supplier. Constraint (11) ensures that when production is initiated at a supplier for an item, a fixed production cost is incurred for that item.

In the final matching, the contract price and surplus of buyer i for item j is obtained as in equations (3) and (4). While matching supply and demand, it is possible that some buyers have a negative surplus.

Example 1. (cont.) We illustrate the full collaboration model using our previous example where the intermediary simultaneously matches buyers and suppliers. Under this model both companies contract with supplier 3 on both items. The contract prices and the surplus are given in Table 8. The total buyer surplus under this model is 6685.

Table 8: Contract prices and surplus under full collaboration

	I_1	I_2	$surplus_i$
B_1	$price_{113} = 17.92$	$price_{123} = 61.97$	$(2000 - 17.92 \cdot 40)+$ $(3750 - 61.97 \cdot 25) = 3483.75$
B_2	$price_{213} = 17.58$	$price_{223} = 61.63$	$(500 - 17.58 \cdot 10)+$ $(7500 - 61.63 \cdot 75) = 3201.25$

As seen in Figure 2, increasing collaboration levels among buyers leads to different matchings.

2.4 *Experimental Design*

In this section we test how the three collaboration models perform under different market conditions. We consider a market with 25 buyers, 6 suppliers and 3 items. Each buyer places RFQs for all 3 items and suppliers have the capability to produce any of the items. We assume that the capacity required to produce one unit of any item is 1.

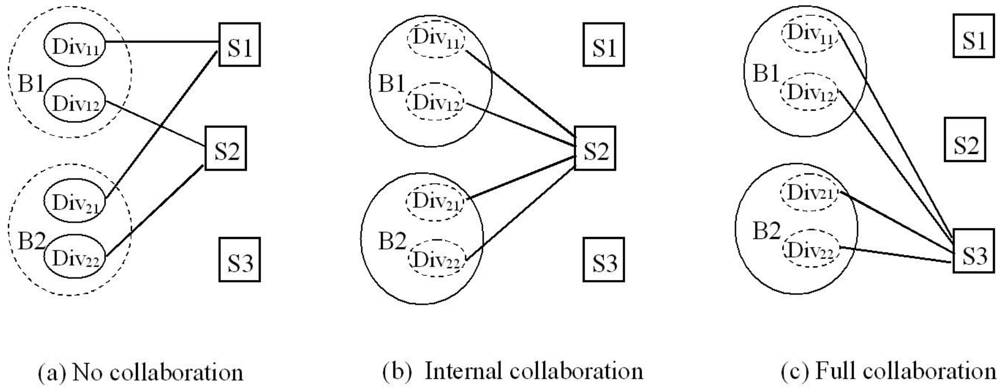


Figure 2: Matchings under different models of buyer collaboration

Table 9: Surplus obtained by each buyer under different collaboration levels

	B_1	B_2	Total
no collaboration	105	2010	2115
internal collaboration	123	2052	2175
full collaboration	3483.75	3201.25	6685

The parameters that define the marketplace are listed in Table 10. We selected the following three factors for controlling the market structure.

1. market supply (total capacity).

2. manufacturing set-up cost (fpc) versus variable production cost (vpc).

3. fixed transportation cost (ftc) versus variable transportation cost(vtc).

We define two levels (low and high) for each of the three factors and obtain 8 different market settings.

2.4.1 Design Parameters

Market Supply vs. Market Demand

The demand of each buyer for each item is generated from a uniform distribution U[$\underline{d}$,$\overline{d}$] (see Table 10). The market supply is defined as the total production capacity of the market, which can be either "low" or "high" compared to the expected total demand in the market.

We model low (high) market supply by setting the supply equal to the lower (upper) bound of the total demand.

Average total capacity required to satisfy low market demand =

TC_{low} = (# of buyers)·(# of items per buyer)·($\underline{d}$ per item)·(avg. capacity required per unit)

Average total capacity required to satisfy high market demand =

TC_{high} = (# of buyers)·(# of items per buyer)·($\bar{d}$ per item)·(avg. capacity required per unit)

In our experiments we assume that the capacities of the suppliers are equal and hence the capacity per supplier is the total market capacity divided by the number of suppliers.

Fixed Costs of Manufacturing and Transportation

The average setup cost of production, i.e., the mean of fpc, is set at either "low" or "high" levels as compared to the average variable production cost vpc, where μ_{fpc}=50 or 1000 and μ_{vpc}=10. fpc values are generated randomly from the uniform distributions U[46,54] and U[980,1020], for the low and high fpc levels, respectively.

The fixed cost of transportation, ftc, is set either at "low" or "high" levels as compared to vtc, where μ_{ftc}=20 or 500 and μ_{vtc}=5. ftc values are generated randomly from the uniform distributions U[18,22] and U[485,515], for the low and high cases, respectively.

Note that rather than the individual values of fpc, ftc, vpc and vtc, we consider the ratios $\frac{fpc}{vpc}$ and $\frac{ftc}{vtc}$ as the design factors in our experiments.

Reservation Price

We generate the reservation prices of the buyers randomly from the uniform distribution $U \sim [(vpc + vtc) \cdot \overline{d}, fpc + ftc + (vpc + vtc) \cdot \overline{d}]$. The lower bound corresponds to the case where (in the limit) the buyer only pays the variable costs. The upper bound corresponds to the worst case where a buyer incurs the entire fixed production and transportation cost, in addition to the variable costs.

The design factors and factor levels are shown in Table 12.

2.4.2 Buyer strategies

In our experiments, we consider the following buyer strategies defined previously:

Table 10: The parameters of the market

Demand per item	$U(10, 20)$
Total capacity per supplier	125 for tight, 250 for relaxed
fpc	$U(46, 54)$ $U(980, 1020)$
vpc	$U(8, 12)$
Reservation price per item	$U(300, 370)$ $U(300, 1320)$ $U(300, 850)$ $U(300, 1800)$
ftc	$U(18, 22)$ $U(485, 515)$
vtc	$U(4, 6)$

Table 11: Upper and lower bound for reservation price in four different market types

market type	μ_{fpc}, μ_{ftc}	reservation price
1	(50,20)	U(300,370)
2	(1000,20)	U(300,1320)
3	(50,500)	U(300,850)
4	(1000,500)	U(300,1800)

Table 12: Design factors and levels

	Levels	low	high
factor 1	Supply/Demand	.66	1.33
factor 2	fpc/vpc	5	100
factor 3	ftc/vtc	4	10

1. No collaboration

   (a) Myopic strategy (MYOPIC)

   (b) Accept the minimum bid (MIN)

   (c) Leave and possibly return later (LOQ) with $\beta = \frac{D}{Q}$

   (d) Leave and return later (Q)

   (e) Accept the lowest bid or leave and return later (AOQ) with $\alpha = 0.25$

   (f) Minimum optimistic bid (MOB)

2. Internal collaboration (INT)

3. Full collaboration (e-MARKET)

2.4.3 Performance Measures

To evaluate the effectiveness of different bidding strategies under various market conditions, we consider several performance measures. One important measure is the amount of market liquidity (i.e., the number of participants in the market or the number of trades taking place) generated by a strategy. The liquidity in the marketplace increases both with the increase in buyer satisfaction (i.e., the amount of satisfied demand) and with the amount of total surplus obtained in the market. Buyers also benefit by the quality of the surplus, i.e., the amount of surplus obtained per item.

One other evaluation criteria is whether a strategy helps to decrease the total setup effort in the market, which can be measured by the total cost incurred. However, since a low total cost may imply a low satisfied demand, we also consider the cost per unit.

As a result we defined four performance measures:

1. % of satisfied demand $= \frac{\text{quantity of satisfied demand}}{\text{total market demand}} \cdot 100$

2. total surplus = total reservation price - total cost

3. average surplus per unit$= \frac{\text{total surplus}}{\text{quantity of satisfied demand}}$

4. average cost per unit$= \frac{\text{total cost}}{\text{quantity of satisfied demand}}$

Current e-marketplaces focus on different performance measures. For example, Transplace uses the capacity of several carriers to satisfy demand, which makes it an attractive choice for shippers in terms of demand satisfaction. Similarly, consortia e-marketplaces such as Covisint.com help buyers to achieve economies of scale and therefore reduce average cost per unit. It is important to note that when comparing different bidding strategies, the most appropriate one does not necessarily need to perform well across all measures, since buyers may have different preferences.

2.5 Experimental Results

In this section we compare the strategies with respect to the performance measures. Each market consists of 25 buyers 6 suppliers and 3 items. The results are obtained and compared using a t-test for 8 market types and 8 strategies, with 15 runs for each market type; strategy combination.

The market types are indicated by a 3 digit code *abc*, where $\{a,b,c\} \in \{0,1\}$. 0 corresponds to *low* level setting for that factor and 1 corresponds to *high* level. a indicates capacity level in the market, b indicates $\frac{fpc}{vpc}$ level and c indicates $\frac{ftc}{vtc}$ level. For example, the 101 market corresponds to high capacity, low fixed cost of production and high fixed cost of transportation.

The results are presented in Tables 13-19. In Tables 14, 16, 18 and 20 a ">" sign indicates that a strategy has resulted in a higher value at the indicated significance level (SL). If two strategies are in the same set, then their performances are not significantly different from each other.

2.5.1 Results for % of satisfied demand

Observation 1 *When capacity is high*

i. AOQ, MIN, MOB *and* INT *perform best in satisfying the demand, followed by e-*MARKET*, followed by* Q*, followed by* LOQ *and* MYOPIC*.*

ii. If there are no economies of scale and scope (market 100), then LOQ *and* MYOPIC *have similar performance; otherwise,* LOQ *outperforms* MYOPIC*.*

It is not surprising that MOB, MIN and INT perform well since they do not consider reservation prices. Since AOQ accepts offers with some probability even if they are above the reservation price, it also performs well in satisfying the demand.

Observation 2 *When capacity is low*

i. e-MARKET is at least as good as any other strategy in satisfying the demand.

ii. When benefits from economies of scale or scope are high MYOPIC and LOQ are always the worst, followed by Q.

iii. MIN, INT, AOQ and MOB have similar performance when there are economies of scope. When there are no economies of scope, e-MARKET dominates other strategies.

Note that MYOPIC and LOQ are always the worst in satisfying the demand except for 000, followed by Q, regardless of the capacity level in the market.

Table 13: Satisfied demand in different markets

Market type	MYOPIC	MIN	Q	AOQ	LOQ	MOB	INT	e-MARKET
000	64.96%	65.03%	64.96%	64.82%	64.96%	64.33%	64.78%	65.49%
001	21.15%	64.52%	27.20%	64.77%	21.15%	64.46%	64.78%	64.91%
010	29.80%	64.93%	35.88%	65.02%	29.80%	63.90%	64.55%	65.41%
011	19.91%	64.53%	26.55%	64.49%	19.91%	64.52%	64.60%	65.06%
100	95.08%	100.00%	97.38%	100.00%	95.53%	100.00%	100.00%	99.37%
101	22.19%	100.00%	35.38%	99.82%	25.02%	100.00%	99.70%	94.70%
110	40.93%	100.00%	57.12%	100.00%	46.65%	100.00%	100.00%	96.33%
111	26.00%	100.00%	42.68%	99.82%	30.19%	100.00%	99.71%	84.67%

2.5.2 Results for total surplus

Observation 3 *e-MARKET outperforms all other strategies in terms of total surplus under any market structure. The benefit of e-MARKET compared to the next best strategy is highest when the capacity is low and there are economies of scale and scope (Figure 3).*

Observation 4 *In maximizing the total surplus*

i. INT outperforms MOB when there are economies of scope (markets 001, 011, 101 and 111).

ii. MIN outperforms INT when there are economies of scale but not scope (markets 010 and 110).

Table 14: Comparison of strategies with respect to the percentage of satisfied demand

Market type	Performance	SL
000	e-MARKET > {MIN, MYOPIC=LOQ=Q, AOQ, INT, MOB}	97%
001	{e-MARKET, INT, AOQ, MIN, MOB} > Q > {MYOPIC = LOQ}	99%
010	e-MARKET > {AOQ, MIN } > INT > MOB > Q > {MYOPIC=LOQ}	96%
011	{ e-MARKET, INT, MIN, MOB, AOQ} > Q > {MYOPIC=LOQ}	99%
100	{AOQ=MIN=MOB=INT} > e-MARKET > Q > {LOQ, MYOPIC}	99%
101	{MIN=MOB, AOQ, INT}> e-MARKET > Q > LOQ > MYOPIC	99%
110	{AOQ=MIN=MOB=INT}> e-MARKET > Q > LOQ > MYOPIC	99%
111	{MIN=MOB, AOQ, INT}> e-MARKET > Q > LOQ > MYOPIC	99%

iii. Q *outperforms* LOQ *when there are either economies of scale or scope or when the capacity is high. In market 000,* Q *and* LOQ *have the same performance.*

iv. MOB *outperforms or does as well as* MIN *in all markets. The difference between* MOB *and* MIN *is greatest when the capacity is high and there are only economies of scope (market 101).*

v. AOQ *outperforms* LOQ *when there are economies of scale (markets 010, 011, 110, 111); otherwise* LOQ *either outperforms or does as well as* AOQ*.*

Observation 4.i is in line with the fact that INT tries to consolidate orders of the same buyer for different products and therefore saves on transportation cost, whereas MOB focuses on lowering the production cost by consolidating orders from different buyers for the same product.

Observation 4.ii implies that when the benefits from economies of scale are high, the MIN strategy leads to fewer production initiations as compared to the INT strategy. In the INT strategy, due to collaboration, the buyer divisions arrive to the system at the same time and contract with a set of suppliers to maximize their overall surplus. Although the fixed cost of transportation is low, experimental results indicate that it is still more beneficial for a buyer to contract with fewer suppliers than the number of items she demands. Please note that this may not be the case if the variance of *variable transportation cost* is sufficiently high. In that case a buyer could select a different supplier for each of her items. In the MIN strategy, buyer divisions arrive to the market independently. For most of the cases two divisions of the same buyer are not assigned to the same supplier because they arrive at different times and the same supplier is no longer available for the division that arrives later. Therefore in most of the instances a supplier uses all of his capacity to produce a single item, whereas in the INT strategy a supplier uses his capacity to produce two or more items. In the INT strategy, each buyer contracting with a supplier on two or more items causes more suppliers to initiate production as compared to the MIN strategy. As a result, while losses from economies of scale are high, gains from economies of scope are insignificant in the INT strategy.

Observation 4.iv indicates that the "lookahead" policy employed by the MOB strategy helps to increase the surplus compared to the MIN strategy, which does not consider potential future arrivals. Observations 4.iii and 4.v show that the total surplus can increase if the buyers either accept an offer even if the best quote is above their reservation price, or return to the market with some probability.

We would like to note that the observations may partly depend on the experimental settings. For instance, observation 4.ii might change if the cost values assigned to each supplier had much smaller variance. In that case, the INT strategy would not necessarily lead to more production initiations and buyer-supplier assignments would have a similar structure to the MIN strategy.

In our experimental design we limited the number of items that each buyer is willing to buy to three, due to computational difficulties. However we conjecture that as the number of items increases, INT strategy will achieve a higher total surplus in the presence of economies of scope. When only economies of scale exist, INT strategy might perform worse due to observation 4.ii.

2.5.3 Results for average surplus per unit

Observation 5 *When the capacity is low*

*i. e-*MARKET *outperforms or does at least as well as any other strategy in maximizing the average surplus.*

ii. AOQ *outperforms* MOB *and* MIN*.*

Table 15: Total surplus in different markets

Market type	MYOPIC	MIN	Q	AOQ	LOQ	MOB	INT	e-MARKET
000	5031.71	4629.24	5031.71	4939.71	45031.71	4530.15	4587.78	6702.80
001	3220.01	−3494.72	4613.46	−314.21	3220.01	−1849.48	6046.07	10706.19
010	13403.87	16743.97	16175.58	21443.45	13403.87	18245.39	13680.84	29635.13
011	8417.88	5063.56	11216.48	12826.18	8417.88	6653.07	10107.94	24931.93
100	7606.56	7617.25	7647.31	7625.48	7610.80	7611.89	7750.16	8318.25
101	3557.05	700.15	6671.03	3692.67	4342.45	5970.44	12524.31	13078.07
110	18201.16	33262.55	24332.07	33654.85	20330.07	34265.37	31780.89	35950.08
111	11723.80	16596.69	18110.15	16768.02	13366.80	20299.64	30367.68	33501.98

Table 16: Comparison of the strategies with respect to total surplus in different markets

Market type	Performance	SL
000	e-MARKET > {MYOPIC=LOQ=Q} > AOQ > {MIN, INT, MOB}	99%
001	e-MARKET > INT > Q > {MYOPIC=LOQ} > AOQ > MOB > MIN	99%
010	e-MARKET > AOQ > MOB > {MIN, Q} > {INT, MYOPIC=LOQ}	95%
011	e-MARKET > AOQ > {Q, INT} > {MYOPIC=LOQ} > MOB > MIN	90%
100	e-MARKET > INT > Q > {AOQ, MIN, MOB, LOQ , MYOPIC}	90%
101	e-MARKET > INT > {Q, MOB} > {LOQ, AOQ, MYOPIC} > MIN	98%
110	e-MARKET > {MOB, AOQ, MIN} > INT > Q > LOQ > MYOPIC	99%
111	e-MARKET> INT > {MOB, AOQ, Q, MIN} > {LOQ, MYOPIC}	99%

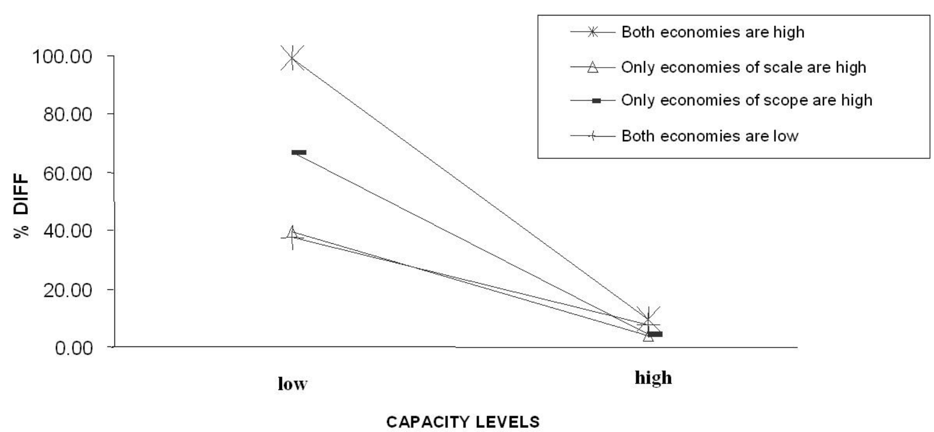


Figure 3: % difference in total surplus between e-MARKET and next best strategy under different market types

Observation 6 *In maximizing the average surplus,*

i. MYOPIC*,* Q *and* LOQ *perform at least as well as or better than all strategies except* e-MARKET*.*

ii. MIN *has the worst performance except when there are only economies of scale (markets 010 and 110), in which case* INT *performs worst.*

Note that the strategies which consider reservation prices (MYOPIC, LOQ or Q) result in higher average surplus compared to the strategies which do not. Recall that these strategies do not perform well in satisfying the demand (Observation 1). On the other hand, strategies which do not consider reservation prices (MIN and MOB) result in lower average surplus levels, but higher percentages of satisfied demand. In these strategies it is possible that some buyers obtain a negative surplus. These results imply a significant trade-off between high satisfied demand and the average surplus.

In addition, under tight capacity Observation 4.i also holds for maximizing the average surplus, i.e., MOB does better when there is economies of scale and INT does better when there are economies of scope.

Table 17: Average surplus per unit in different markets

Market type	MYOPIC	MIN	Q	AOQ	LOQ	MOB	INT	e-MARKET
000	6.89	6.33	6.89	6.77	6.89	6.26	6.30	9.10
001	13.52	−4.82	15.26	−0.44	13.52	−2.54	8.30	14.67
010	40.05	22.93	40.27	29.34	40.05	25.40	18.85	40.28
011	37.32	6.96	37.47	17.70	37.32	9.17	13.92	34.09
100	7.12	6.78	6.99	6.79	7.09	6.78	6.90	7.45
101	14.12	0.63	16.79	3029	15.40	5.32	11.18	12.30
110	39.54	29.60	38.25	29.94	38.74	30.49	28.28	33.28
111	40.22	14.78	38.45	14.95	39.53	18.06	27.10	35.44

Table 18: Comparison of the strategies with respect to average surplus in different markets

Market type	Performance	SL
000	e-MARKET > {MYOPIC=LOQ=Q} > AOQ > {MIN, INT, MOB}	99%
001	{Q, e-MARKET} > {MYOPIC=LOQ} > INT >AOQ > MOB > MIN	88%
010	{e-MARKET, Q, MYOPIC=LOQ} > AOQ > MOB > MIN > INT	99%
011	{Q, MYOPIC=LOQ, e-MARKET} > AOQ > INT > MOB > MIN	99%
100	e-MARKET > MYOPIC > LOQ > {Q, INT} > {AOQ, MIN, MOB}	90%
101	Q > LOQ > MYOPIC > e-MARKET > INT > MOB > AOQ > MIN	95%
110	MYOPIC > {LOQ, Q} > e-MARKET > {MOB, AOQ, MIN} > INT	95%
111	{MYOPIC, LOQ, Q} > e-MARKET > INT > MOB > {AOQ, MIN}	93%

2.5.4 Results for average cost per unit

Observation 7 e-MARKET *always has the lowest average cost, except when the capacity is high and there are only economies of scale (market 110).*

Observation 8 MIN *has the highest average cost except when there are only economies of scale (markets 010 and 110), in which case* INT *has the highest average cost.*

It is interesting to note that when INT is not the worst performer in average cost, it is the second best. This leads us to conclude that collaboration helps to decrease the average cost per unit, especially when economies of scale and scope are high. In comparing MOB and INT, Observation 4.i also holds for minimizing the average cost, i.e., MOB does better when there are economies of scale and INT does better when there are economies of scope.

2.6 Concluding Remarks

We analyzed markets where multi-unit transactions over multiple items take place. We considered three different trading models with increasing levels of collaboration among buyers. The "no collaboration" model considers traditional markets where there is no collaboration among buyers or buyer divisions. In the "internal collaboration" model, purchasing divisions of a buyer collaborate for procurement. In the "full collaboration" model an intermediary enables collaboration among different buyers.

Table 19: Average cost per unit in different markets

Market type	MYOPIC	MIN	Q	AOQ	LOQ	MOB	INT	e-MARKET
000	16.03	16.16	16.03	16.05	16.03	16.20	16..	15.42
001	40.83	42.61	36.45	41.25	40.83	40.49	29.44	28.31
010	26.81	29.83	24.68	28.72	26.81	27.51	33.96	23.73
011	62.70	61.55	58.84	60.75	62.70	59.58	54.44	50.44
100	15.40	15.47	15.41	15.44	15.40	15.43	15.31	14.82
101	39.81	36.95	32.32	34.31	37.14	32.26	26.41	25.02
110	22.49	22.99	20.08	22.65	21.23	22.10	24.31	21.14
111	57.04	53.14	52.07	53.00	55.18	49.86	40.76	39.59

Table 20: Comparison of the strategies with respect to average cost in different markets

Market type	Performance	SL
000	{MOB, MIN} > {AOQ, Q=MYOPIC=LOQ, INT} > e-MARKET	99%
001	MIN > {AOQ, MYOPIC=LOQ, MOB} > Q > INT > e-MARKET	99%
010	INT > {MIN, AOQ} > {MOB, MYOPIC=LOQ} > Q > e-MARKET	95%
011	{MYOPIC=LOQ, MIN, AOQ} > {MOB, Q} > INT > e-MARKET	95%
100	{MOB, MIN, AOQ, Q, LOQ, MYOPIC, INT} > e-MARKET	99%
101	MYOPIC> {LOQ, MIN} > AOQ > {Q, MOB} > {INT, e-MARKET}	98%
110	INT > {MIN, AOQ, MYOPIC, MOB} > {LOQ, e-MARKET} > Q	80%
111	MYOPIC > LOQ > {MIN, AOQ, Q} > MOB > INT > e-MARKET	97%

We developed six different buyer strategies for the no collaboration model, and one for the internal collaboration model. These strategies are tested against the centralized buyer-seller matching mechanism employed by the intermediary in the “full collaboration” model.

The experimental results show that when there is tight capacity in the market and when potential benefits from economies of scope are high (i.e., when the fixed cost of transportation is high), the “full collaboration” model performs significantly better than other strategies in terms of total surplus obtained. These benefits are much more significant when benefits from economies of scale are also high (i.e., when the fixed costs of both manufacturing and transportation are high). The extra benefits obtained by full collaboration are relatively low when the capacity is high and the fixed cost factors are low.

We also observe that internal collaboration performs very well, provided that the potential benefits from economies of scope are high. On the other hand, when the potential benefits from economies of scale are high, buyer strategies with a “look-ahead” perform well. These are the strategies which consider potential future trades in the market by other buyers while contracting with a supplier.

It is clear from the analysis that intermediaries will be most beneficial in capacitated markets with a high fixed production cost and/or high fixed transportation cost. Process industries such as rubber, plastic, steel and paper are typically characterized by high fixed production costs. High fixed transportation costs can arise in industries that have to either manage their own distribution fleet or establish contracts with carriers that guarantee a minimum capacity in order to ensure adequate service levels (e.g., Ford Motor Company guarantees deliveries to their dealers every three days and hence has a high fixed transportation cost).

CHAPTER III

COLLABORATIVE PROCUREMENT BETWEEN COMPETING BUYERS

3.1 Introduction

Motivated by the applications of group purchasing in practice, in this chapter we study a collaborative procurement mechanism where the supplier offers a volume discount scheme allowing multiple buyers to pool their purchasing power and get lower prices. After procurement, the buyers engage in a single-period Cournot competition and make decisions about the quantity to sell in a market characterized by price-sensitive demand (Figure 4).

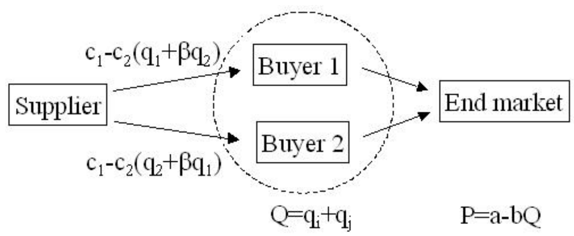


Figure 4: Representation of the collaborative procurement problem

Our objective is to characterize the Nash equilibria (purchasing quantities of the buyers) both under independent and joint procurement and investigate under which conditions joint procurement benefits the buyers. Having characterized the buyers' response to a given supply price function, we then consider the supplier's decision of choosing β, and the conditions under which the supplier is better off due to joint procurement.

In Section 3.2, we discuss the literature relevant to the problem, and state how our study differs from the existing ones. The mathematical model in Section 3.3 is followed by Section 3.4 where we consider uncapacitated buyers. In Section 3.5, we consider buyers with different capacity levels (sizes) and see how the benefits differ from the uncapacitated case. Insights and concluding remarks constitute the final section of this chapter.

3.2 Literature Review

The collaborative efforts among supply chain participants have been analyzed along several lines. In supply chain literature mainly vertical relationships between buyers and suppliers have been considered. There is a growing body of work on supply chain coordination which discusses contracting mechanisms (Lariviere 1999; Anupindi and Bassok 1999; Cachon 1999) such as return policies (Emmons and Gilbert 1998; Pasternack 1985), rebates, revenue sharing (Cachon and Lariviere 2002; Taylor 2002), and price protection (Lee et al. 2000).

Most studies analyze horizontal collaboration in the research and development (R&D) and research joint ventures context (D'Aspremont and Jacquemin 1988; Kamien, Muller and Zang 1992; Cabral 2000). These models consider the interaction among the firms that are partners in R&D investment but competitors in the end market. Essig (2000) shows through an empirical study of consortium sourcing among 13 small and medium-sized German companies that consortium sourcing can be beneficial to all involved members. Zentes and Swoboda (2000) discuss the IT network structures of groups of small to medium-sized retailers in Central Europe. They conclude that modern information and communication technologies allow these allied groups to set up complex networks, which help members to safeguard their competitive positions in the market.

For the buyers, the main motivation behind the formation of buyer groups or group purchasing organizations is the quantity discounts provided by the suppliers. In a quantity discount scheme, the supplier sets price break(s) (Lam and Wong 1996) or uses continuous pricing (Ladany and Sternlieb 1974; Lal and Staelin 1984; Dave, Fitzpatrick and Baker 1996). Dolan (1987) provides a thorough analysis and categorization of the studies on quantity discounts. The main motivations for the suppliers according to Buchanan (1953) are, (i) price discrimination against buyer(s), (ii) to pass part of the supply chain costs on to the buyer and/or to increase overall system efficiency, by changing the buyer's ordering pattern.

A significant portion of the supply chain literature on quantity discounts focuses on the second motivation: achieving cost minimization from supplier's (or buyer's) perspective and/or achieving overall cost minimization through channel coordination. Introducing quantity discounts changes the ordering quantity of the buyer and this reduces the supply chain related costs such as inventory holding, order replenishment or purchasing (see Lal and Staelin 1984; Rosenblatt and Lee 1985; Weng 1995). If the demand is price-sensitive, discounting would attract more demand, benefiting the supplier. Among the studies in supply chain literature that consider price-sensitive demand are Abad (1994), Parlar and Wang (1994) and Yang (2004). On the other hand, most of the studies on quantity discounts in the economics literature focus on the first motivation and try to design an optimal discount scheme that extracts all or some of the consumer surplus (see Oi 1971; Katz 1984; Armstrong 1996).

Research on collaborative relations and group-buying include Mathewson and Winter (1996), Spiegel (1993) and Anand and Aron (2003). Mathewson and Winter (1996) study a problem where a group of buyers negotiates and makes a contract with a group of suppliers to get lower prices. In turn, the buyer group gets supplies only from the contracting group which implies a trade-off between low price and low product availability. They conclude that as the number of suppliers increases, the buyer group is more likely to benefit from contracts and buyer groups might be welfare increasing or decreasing depending on the model's parameters. Spiegel (1993) addresses production sub-contracting between two rival firms operating at the same horizontal stage in the supply chain. He shows that this arrangement, if it occurs at all, always increases production efficiency. Anand and Aron (2003) study the optimal design of an online business-to-customer group-buying scheme under demand uncertainty. In their model, the buyers arrive and demand single units, and as the number of units demanded increases the price drops. The demand function is not known to the supplier before he decides on price-quantity tuples. Under this setting, the supplier's benefit from group-buying increases as demand heterogeneity (the difference in the slopes of the demand curves of the buyers) increases. Furthermore, group-buying outperforms single pricing when the goods are produced after total demand is realized under scale economies.

We consider collaborative procurement in a business-to-business setting, where the buyers collaborate for procurement and then compete in the end market. Being potential competitors affects the purchasing quantities of the buyers. Furthermore, we consider the case where buyers have limited capacities, which affects their purchasing power. We consider the purchasing strategies of the buyers and the selling strategy of the supplier and identify the conditions under which all parties benefit from group purchasing.

3.3 Model

Our model has the following characteristics. There is one supplier and two buying firms, both of whom procure input from the supplier and then produce/sell a homogeneous product. We assume that the buyers are identical in terms of their production costs which we characterize by constant returns to scale. Without loss of generality, we assume that the unit production cost is zero, i.e., the only cost for the buyers is the cost of procurement of the input materials from the supplier. We assume that the market price per unit is given by $P = a - bQ$ where Q is the demand (or total quantity sold by the buying firms) in the end market. Such linear downward sloping price/demand functions are commonly used in the economics and operations management literature (see for instance, Albaek 1990; Corbett and Karmarkar 2001).

The unit supply prices under independent (I) and joint (J) procurement are given by:

$$S_i^I = c_1 - c_2 q_i^I, \ i = 1, 2 \tag{12}$$

$$S_i^J = c_1 - c_2(q_i^J + \beta q_j^J), \quad \beta \in [0, 1], \ i, j = 1, 2 \text{ and } i \neq j \tag{13}$$

Note that uniform price is a special case of the above price structure with $c_2 = 0$. In case of joint procurement, buyers consolidate their order quantities in order to get a better price deal from the supplier. β is the *spillover factor*, which determines the additional discount that a buyer gets due to the quantity purchased by the other buyer. The extreme cases are $\beta = 0$, which corresponds to independent procurement; and $\beta = 1$, which implies complete spillover, i.e., the two buyers can act as a single buyer and thus each buyer could achieve the maximum benefit from the reduction in the supply price.

If buyers i and j produce/procure q_i and q_j, respectively, then $Rev_i(q_i, q_j) = (a - b(q_i + q_j))q_i$ denotes the revenue and $C_i(q_i, q_j) = (c_1 - c_2(q_i + \beta q_j))q_i$ denotes the cost of buyer i. Hence, the marginal revenue and marginal cost of buyer i are given by $MR_i(q_i, q_j) = a - b(2q_i + q_j)$ and $MC_i(q_i, q_j) = c_1 - c_2(2q_i + \beta q_j)$. As q_i increases, both MR_i and MC_i decreases, at the rates $2b$ and $2c_2$, respectively. We make the following assumptions:

A1. $b > c_2$: Marginal revenue falls faster than the marginal cost as the quantity procured increases, i.e., the quantities are bounded.

A2. $a > c_1$: Market price is greater than the supply price for the first unit, i.e., it is profitable to procure a positive quantity.

A3. $a < c_1 \frac{b}{c_2}$: Total procurement cost for buyer i is an increasing function of the quantity purchased, q_i (see Appendix A.1).

We assume that all the parameters of the model are common knowledge and all parties are profit maximizers. As a building block we first analyze the single supplier-single buyer (monopoly) model, where the supply price is given by $S = c_1 - c_2 q$. The buyer solves the following optimization problem to find the order quantity that maximizes his profits:

$$\max_{q \geq 0} \ \pi_M = (a - bq - (c_1 - c_2 q))q.$$

The objective function is concave in q, hence, from first order conditions, the optimal monopoly quantity is

$$q_M = \frac{a - c_1}{2(b - c_2)}. \tag{14}$$

By assumptions A1 and A2, $q_M > 0$.

3.4 Uncapacitated Buyers

In this section we consider the case of uncapacitated buyers. We analyze the procurement decisions of two independent buyers under joint and independent procurement given a supply price function. After characterizing the buyers' behavior, we then study the supplier's choice of β.

3.4.1 Buyers' Procurement Decisions

Consider the procurement decisions of two independent buyers. Given buyer j's procurement/production decision q_j, buyer i's profit maximization problem is:

$$\begin{aligned}\max_{q_i \geq 0} \ \pi_i &= Rev_i(q_i, q_j) - C_i(q_i, q_j) \\ &= \Big(a - b(q_i + q_j)\Big)q_i - \Big(c_1 - c_2(q_i + \beta q_j)\Big)q_i, \ \ i, j = 1, 2 \text{ and } i \neq j\end{aligned} \tag{15}$$

Note that when $\beta = 0$ the expression above models independent procurement and otherwise it corresponds to joint procurement. From second order conditions ($\frac{\partial^2 \pi_i}{\partial q_i^2} = -2(b - c_2) < 0$) the profit function is concave. Hence, from first order conditions

$$\frac{\partial \pi_i}{\partial q_i} = (a - 2bq_i - bq_j) - (c_1 - 2c_2 q_i - \beta c_2 q_j) \tag{16}$$

The best response of buyer i, $q_i = R_i(q_j)$, which maximizes π_i for a given q_j is given by

$$q_i = R_i(q_j) = r_i(q_j)^+ \tag{17}$$

where $x^+ = \max\{0, x\}$, and

$$r_i(q_j) = \frac{(a - c_1)}{2(b - c_2)} - \frac{b - \beta c_2}{2(b - c_2)} q_j, \quad i, j = 1, 2 \text{ and } i \neq j \tag{18}$$

Let us consider the conditions under which $r_i(q_j)$ attains its maximum value or becomes 0. Since q_j is non-negative, $r_i(q_j)$ is bounded above by $r_i(0) = q_M$, the monopoly quantity.

From (17) and (18), $r_i(q_j) = 0$ if $q_j \geq \bar{q}$, where,

$$\bar{q} = \frac{a - c_1}{b - \beta c_2}. \tag{19}$$

That is, if $q_j > \bar{q}$, the marginal revenue of procuring any positive amount is less than the marginal cost for buyer firm i. Furthermore, if firm j procures q_j, the market price is at most $P(0 + q_j)$ and the supply price of purchasing the first unit for firm i is at least $S_i(0, q_j)$. When $q_j \geq \bar{q}$, we have $P(0 + q_j) - S_i(0, q_j) \leq 0$, i.e., firm i's profit of procuring the first unit is negative. Since by Assumption A1, the market price falls faster than the supply price, the marginal profit of procuring additional units would always remain negative for firm i. Therefore, when $q_j \geq \bar{q}$, firm i's best response is not to procure at all. Hence, $\bar{q}$ denotes the minimum quantity a firm needs to procure to leave the other firm completely out of the market. Note that $\bar{q}$ increases in β. When $\beta > \beta^c$, where $\beta^c = 2 - \frac{b}{c_2}$, $\bar{q}$ exceeds q_M and, at equilibrium, it is not possible for a firm to leave the other firm out of market.

$\bar{q}$ and q_M are the points where r_1 and r_2, respectively, intercept the q_2 axis in Figure 5. We have to consider the following three cases depending on the relationship between $\bar{q}$ and q_M:

Case A. $\bar{q} > q_M$, equivalently, $\beta > \beta^c$ (Figure 5.(A)).

Case B. $\bar{q} < q_M$, equivalently, $\beta < \beta^c$ (Figure 5.(B)).

Case C. $\bar{q} = q_M$, equivalently, $\beta = \beta^c$ (Figure 5.(C)).

At β^c, the system switches from Case A to Case B. Under independent procurement ($\beta = 0$) we obtain the conditions for Case A as $b > 2c_2$ and for Case B as $b < 2c_2$.

It is interesting to note that if we had a constant unit supply price, c_1, instead of a quantity discount schedule, we would have $\bar{q} > q_M$. This implies at $q_i = q_M$, $MR_j - MC_j > 0$ and it is not possible for a firm to leave the other firm completely out of market. However, under the quantity discount schedule, the marginal cost of procuring the initial unit may be greater than the marginal revenue (depending on the other firm's current procurement quantity) and a firm may choose not to procure/sell.

In Case A, we have a unique equilibrium (q_U, q_U) where

$$q_U = \frac{a - c_1}{3b - (2 + \beta)c_2} \tag{20}$$

By inserting $\beta = 0$, the equilibrium under independent procurement is (q_U^I, q_U^I) where

$$q_U^I = \frac{a - c_1}{3b - 2c_2} \tag{21}$$

The profits of the buyers are

$$\pi_i(q_U, q_U) = (b - c_2)(q_U)^2, \ i = 1, 2 \tag{22}$$

In Case B, there exist two additional (pure strategy) equilibria, $(0, q_M)$ and $(q_M, 0)$.

In Case C, any (q_1, q_2) pair is an equilibrium as long as it satisfies Equation (18) (see Figure 5.(C)).

We focus on Cases A and B.

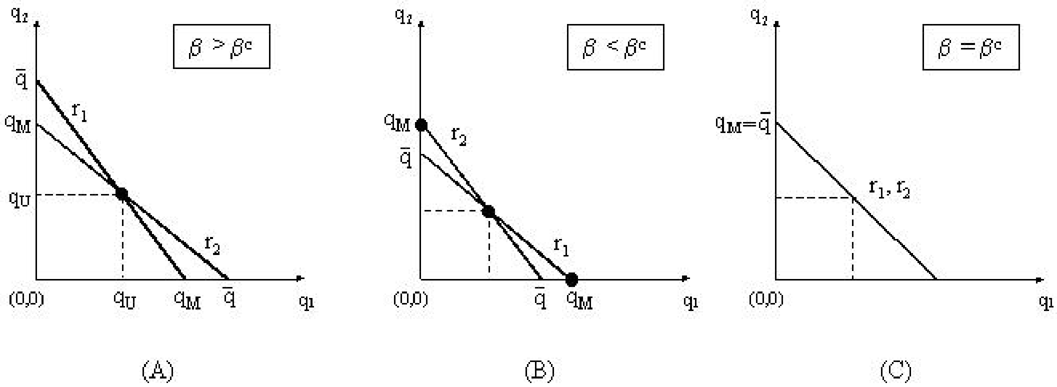


Figure 5: Reaction functions under Case A, Case B and Case C

Observation 9 *In equilibrium, the total quantity procured by the two buying firms increases as* $\frac{a}{c_1}$ *increases and/or* $\frac{b}{c_2}$ *decreases.*

Observation 10 *Under equilibrium* (q_U, q_U)*, the total quantity procured by the two buying firms increases as* β *increases.*

These observations follow directly from Equations (14) and (20). They can also be explained intuitively. All other parameters being fixed, the larger the value of $\frac{a}{c_1}$, i.e., the larger the difference between the base supply price and the base market price, the more the buyers can procure before the marginal cost becomes higher than the marginal revenue. Similarly, the smaller the value of $\frac{b}{c_2}$, i.e., the smaller the difference between the rate of decline of the supply price and the market price, the more the buyers can procure before the marginal cost becomes higher than the marginal revenue. Finally, a larger value of β makes the supply price fall faster and it effectively makes the value of $\frac{b}{c_2}$ smaller.

Observation 11 *Under equilibrium* (q_U, q_U)*, uncapacitated buyer firms and end consumers are better off under joint procurement as compared to independent procurement.*

If the equilibrium is (q_U, q_U), q_U increases in β (Equation (20)) and the profits of the buying firms increase in q_U (Equation (22)), making the buying firms better off under joint procurement. Total quantity produced under joint procurement is higher compared to independent procurement, and since the market price $P = a - bQ$ decreases in the total quantity, the end consumers are also better off under joint procurement.

3.4.2 Supplier's Choice of β when the Buyers are Uncapacitated

We analyzed uncapacitated buyers' procurement decisions given the supply price function in Equation (13). We now consider how the supplier's choice of β impacts the profits. In choosing β, the supplier needs to trade-off the decrease in the revenue due to quantity discounts and the increase in revenue due to the increase in the quantity sold.

Recall from Figure 5 that if the supplier sets $\beta > \beta^c$, then the unique equilibrium is (q_U, q_U). If the supplier sets $\beta < \beta^c$, then there exist two additional pure strategy equilibria, $(0, q_M)$ and $(q_M, 0)$.

The decision mechanism of the supplier is characterized as below:

1. The supplier determines the β that maximizes his profits under equilibrium (q_U, q_U); call this β_U^*, where $\beta_U^* \in [0, 1]$.

2.a. If $b > 2c_2$, then the unique equilibrium is (q_U^*, q_U^*), where $q_U^* = \frac{a-c_1}{3b-(2+\beta_U^*)c_2}$. The supplier sets $\beta = \beta_U^*$.

b. If $b < 2c_2$, then the supplier may choose to set $\beta < \beta^c$ and sell $(q_M, 0)$. Note $\pi_S(q_M, 0)$ is independent of β. The supplier compares $\pi_S(q_U^*, q_U^*, \beta_U^*)$ and $\pi_S(q_M, 0)$.

Let us first identify the β that maximizes the supplier's profit under (q_U, q_U). The supplier's profit function is:

$\pi_S^J(\beta) = 2(c_1 - c_2 q_U(1 + \beta))q_U$

Under $\beta \leq 1$ and assumption A1, there exists a single β that satisfies the first order conditions:

$$\beta^* = \frac{3\frac{b}{c_2}(2 - \frac{a}{c_1}) - 2}{\frac{a}{c_1}} \tag{23}$$

For $\beta \in [0, 1]$, $\frac{\partial \pi_S^J}{\partial \beta}$ is positive for $\beta < \beta^*$ and negative for $\beta > \beta^*$. This implies that the

β that maximizes the supplier's profit (π_S^J) is given by,

$$\beta_U^* = \begin{cases} 0, \text{ if } \frac{a}{c_1} > 2 - \frac{2c_2}{3b} \quad (\beta^* < 0) \\ 1, \text{ if } \frac{a}{c_1} < 2 - \frac{4}{3\frac{b}{c_2}+1} \quad (\beta^* > 1) \\ \beta^*, \text{otherwise} \end{cases} \tag{24}$$

From Equation (24), if $\beta_U^* > 0$ (equivalently, $\beta^* > 0$), then the supplier is better off under joint procurement. This is more likely to be the case for low values of $\frac{a}{c_1}$ and high values of $\frac{b}{c_2}$, i.e., if the initial market price is low relative to the base supply price or if the market price decreases rapidly as compared to the supply price. Conversely, if $\beta_U^* = 0$ (equivalently, $\beta^* \leq 0$), the supplier is more likely to be better off under independent procurement. This happens under high values of $\frac{a}{c_1}$ and low values of $\frac{b}{c_2}$, where the quantities procured by the buyers are already high, therefore it is not beneficial for the supplier to give additional discounts.

If $\beta^* < 0$ ($\beta^* > 1$), then this is an indication that the supplier is offering higher (lower) discounts than she should. In this case, decreasing (increasing) c_2 would limit (increase) the discount level and increase the supplier's profits.

The following proposition shows the supplier's optimal decision when the buyers are uncapacitated. For expositional clarity we present the proofs in Appendix A.

Proposition 1 *When the buyers are uncapacitated, the supplier maximizes her profit by setting* $\beta = \beta_U^*$ *and by selling* q_U^* *to each buyer.*

Proposition 1 implies that, both under independent and joint procurement, rather than selling a large quantity to a single buyer, the supplier is always better off by selling smaller and equal quantities to both buyers. She can achieve this by limiting the quantity she sells to any buyer to q_U^*.

3.5 Capacitated Buyers

In this section, we assume that the buying firms have limited production capacities K_1 and K_2, where without loss of generality $K_1 > K_2$. Having capacity limitations restricts the buyers' procurement/production quantities and impacts how much they (or the supplier) can benefit from joint procurement.

Intuitively, one would think that a "large" buyer (with higher capacity, buyer 1 in this case) would have less incentive to collaborate with a smaller buyer on procurement, since the large buyer already has enough volume to obtain a good price from the supplier. The "small" buyer, on the other hand, might prefer to collaborate with a large buyer, since it will obtain additional price breaks due to the volume of the large buyer. Given these conflicting incentives, we might expect that joint procurement would occur mainly among roughly equal size buyers (in terms of capacity and purchase volume).

However, we find that depending on the market characteristics, collaboration may occur among different size buyers. Furthermore, depending on the capacity of the large buyer, the small buyer may not always be willing to collaborate.

3.5.1 Buyers' Procurement Decisions

First, we analyze the buyers' procurement decisions in equilibrium, given the supply price function in Equation (13). From (15)-(18), the best response of buyer i is,

$$R_i(q_j) = \min\{K_i, r_i(q_j)^+\} \tag{25}$$

We say that buyer i has "tight" capacity, if $K_i < q_U$, has "medium" capacity, if $q_U \leq K_i < q_M$, and has "ample" capacity, if $q_M \leq K_i$ (the capacity ranges are defined similarly for independent procurement by replacing q_U with q_U^I). Note that q_U is a function of β. This implies that whether the buyer's capacity is tight or medium depends on the relationship between K_i and β. For instance, a buyer may have medium capacity under independent procurement and tight capacity under joint procurement.

From Equation (25), if both buyers have $K_i \geq q_M$, this is equivalent to the uncapacitated case. Therefore without loss of generality, we assume $K_2 < K_1 \leq q_M$ (Note that even if $K_1 > q_M$, buyer 1 will never procure more than q_M, hence without loss of generality we can assume $K_1 \leq q_M$).

We obtain the equilibrium (equilibria) for a given β, under the cases where buyer 2 has tight capacity and medium capacity. The analysis holds for all $\beta \in [0,1]$. Note that for $\beta = 0$, the analysis corresponds to independent procurement.

Theorem 1 *If buyer 2 has tight capacity ($K_2 < q_U$), the equilibrium procurement quantities are characterized as follows:*

$\beta > \beta^c$: $(q_1, q_2) = (min\{K_1, r_1(K_2)\}, K_2)$.

$\beta < \beta^c$: $(q_1, q_2) = (K_1, min\{K_2, r_2(K_1)^+\})$.

Theorem 2 *If buyer 2 has medium capacity ($q_U \leq K_2 < q_M$), the equilibrium procurement quantities are characterized as follows:*

$\beta > \beta^c$: $(q_1, q_2) = (q_U, q_U)$.

$\beta < \beta^c$: $(q_1, q_2) = (q_U, q_U)$ *or* $(K_1, r_2(K_1)^+)$ *or* $(r_1(K_2)^+, K_2)$.

A list of all possible equilibria with respect to K_1, K_2 and β values is presented in Table 21. Figure 6 shows how the equilibria (equilibrium) change(s) with respect to K_1 and K_2 for a given β value.

Table 21: Total characterization of equilibria

		$\beta > \beta^c$	$\beta < \beta^c$
E1	(K_1, K_2)	$K_1 \leq r_1(K_2)$	$K_2 \leq r_2(K_1)$
E2	(q_U, q_U)	$K_1, K_2 \geq q_U$	$K_1, K_2 \geq q_U$
E3	$(K_1, r_2(K_1))$	-	$K_1 \leq \bar{q}$, $K_2 \geq r_2(K_1)$
E4	$(K_1, 0)$	-	$K_1 \geq \bar{q}$
E5	$(r_1(K_2), K_2)$	$K_2 \leq q_U$, $K_1 \geq r_1(K_2)$	$K_2 \leq \bar{q}$, $K_1 \geq r_1(K_2)$
E6	$(0, K_2)$	-	$K_2 \geq \bar{q}$

Next we analyze, for a given (K_1, K_2), how the equilibrium changes with respect to β. We take $\beta = 0$ (independent procurement) as a starting point. There exist 10 (mutually exclusive) cases when buyer 2's capacity is tight under independent procurement ($K_2 < q_U^I$), and 9 cases when buyer 2's capacity is medium under independent procurement ($q_U^I \leq K_2 \leq q_M$). We present a summary of these cases in Figures 7 and 8, respectively. See Appendix A.4 for the analysis of why each condition leads to a particular series of equilibria.

In Table 22, we list and explain the β values of Figures 7 and 8, at which the equilibrium switches from one to another. For example at β_{51} the equilibrium switches from E5 to E1,

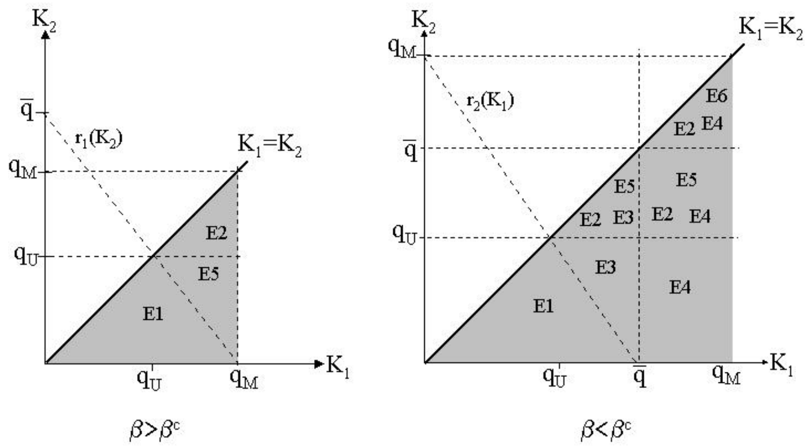


Figure 6: All possible equilibria of Theorem 1 and 2 for a given β, as a function of K_1 and K_2

i.e., $r_1(K_2)$ becomes equal to K_1. Similarly, β_2 indicates the value at which buyer 2's capacity switches from medium to tight (note, E2 corresponds to the case where buyer 2 has medium capacity).

In Figure 8 we assume that there exists a single equilibrium, E2, for $\beta < \beta_2$ (although Theorem 2 states that there exist three equilibria when $\beta < \beta^c$ and buyer 2 has medium capacity). However, in Proposition 3 of Section 3.5.2 we show that the supplier's optimal decision is to sell equal quantities to each buyer, rather than $(K_1, r_2(K_1)^+)$ or $(r_1(K_2)^+, K_2)$, whenever the optimal β for the supplier is less than β_2. Therefore in the remainder of our analysis, we only consider E2 when $\beta < \beta_2$.

Table 22: β values at switching points

Equilibria	Condition	β
E5 to E1	$r_1(K_2) = K_1$	$\beta_{51} = \frac{bK_2-(a-c_1)+2(b-c_2)K_1}{c_2K_2}$
E3 to E1	$r_2(K_1) = K_2$	$\beta_{31} = \frac{bK_1-(a-c_1)+2(b-c_2)K_2}{c_2K_1}$
E3 to E5	$b = (2-\beta)c_2$	$\beta^c = 2 - \frac{b}{c_2}$
E4 to E3	$r_2(K_1) = 0$	$\beta_{43} = \frac{bK_1-(a-c_1)}{c_2K_1}$
E2 to E3, E4 or E5	$q_U^J = K_2$	$\beta_2 = \frac{(3b-2c_2)K_2-(a-c_1)}{c_2K_2}$

In the following examples, we show how different market conditions, buyer capacities and discount levels affect buyers' willingness to collaborate.

Example 2.a. Suppose $a = 100$, $c_1 = 88$, $b = 1$, $c_2 = 0.7$, $K_1 = 10$ and $K_2 = 6$.

System conditions				Initial equilibrium (β=0)	All possible equilibria under joint procurement	β values where equilibrium changes	Case
$K_2 < q_U^I$	$b>2c_2$	$K_1 \le q_M - \frac{b}{2(b-c_2)}K_2$		E1	E1		1
		$q_M - \frac{b}{2(b-c_2)}K_2 \le K_1 \le q_M - \frac{K_2}{2}$		E5	E5 → E1	β_{51}	2
		$K_1 > q_M - \frac{K_2}{2}$		E5	E5		3
	$b<2c_2$	$K_1 < \bar{q}$	$K_2 < q_M - \frac{b}{2(b-c_2)}K_1$	E1	E1		4
			$q_M - \frac{b}{2(b-c_2)}K_1 \le K_2 < q_M - K_1$	E3	E3 → E1	β_{31}	5
			$q_M - K_1 \le K_2 \le 2(q_M - K_1)$	E3	E3 → E5 → E1	β^c, β_{51}	6
			$K_2 > 2(q_M - K_1)$	E3	E3 → E5	β^c	7
		$K_1 \ge \bar{q}$	$q_M - \frac{b}{2(b-c_2)}K_1 < K_2 < q_M - K_1$	E4	E4 → E3 → E1	β_{43}, β_{31}	8
			$q_M - K_1 \le K_2 \le 2(q_M - K_1)$	E4	E4 → E3 → E5 → E1	$\beta_{43}, \beta^c, \beta_{51}$	9
			$K_2 > 2(q_M - K_1)$	E4	E4 → E3 → E5	β_{43}, β^c	10

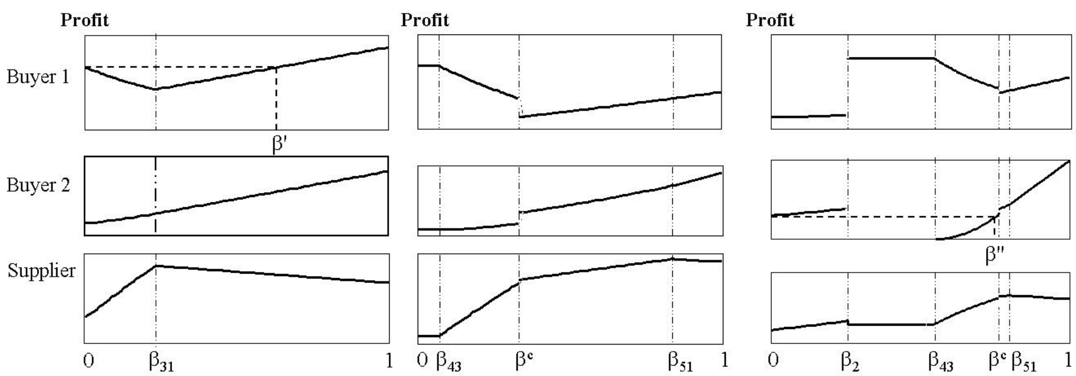
Figure 7: All possible equilibria of Theorem 1 and Theorem 2 as a function of β for a given (K_1, K_2), when buyer 2 has tight capacity under independent procurement

We have $q_U^I = 7.5$, $\bar{q} = 12$, $q_M = 20$, and the system conditions are such that, $K_2 < q_U^I$, $b < 2c_2$, $K_1 \ge \bar{q}$, and $q_M - \frac{b}{2(b-c_2)}K_1 \le K_2 < q_M - K_1$. Hence the system is in Case 5. The equilibrium is E3 at $\beta = 0$, and switches to E1 at $\beta_{31} = 0.228$. The profits of the two buyers and the supplier for $\beta \in [0, 1]$ are shown in Figure 9.a.

From Figure 9.a, buyer 1 prefers collaboration if β is sufficiently high ($\beta > \beta'$). Under independent procurement, buyer 1 procures at capacity, whereas buyer 2 procures $r_2(K_1)$. As β increases, buyer 1 still procures at capacity, whereas procurement quantity of buyer 2 increases. This results in a decrease in buyer 1's profit. In other words, for buyer 1, the decrease in the supply price due to β does not outweigh the decrease in the market price. However, once β reaches β_{31}, both buyers procure at capacity and the market price does not decrease as β increases, whereas the supply price continues to decrease. When the decrease in the supply price is sufficiently high, buyer 1 prefers joint procurement over independent procurement. As we will see in Proposition 2, for this example $\beta' = 0.635$. Buyer 2 prefers collaboration for all $\beta \in (0, 1]$, since the decrease in the supply price outweighs the decrease in the market price for $\beta \in (0, 1]$. The supplier's profit is maximized at β_{31}, however, she is better off if buyers collaborate as compared to independent procurement, for all $\beta \in (0, 1]$.

System conditions				Initial equilibrium ($\beta=0$)	All possible equilibria under joint procurement	β values where equilibrium changes	Case
$K_2 \geq q_U^I$	$K_2 \geq \frac{a-c_1}{3(b-c_2)}$			E2	E2		11
	$\frac{a-c_1}{4(b-c_2)} < K_2 < \frac{a-c_1}{3(b-c_2)}$	$K_2 < 2(q_M-K_1)$		E2	E2→E5→E1	β_2, β_{51}	12
		$K_2 \geq 2(q_M-K_1)$		E2	E2→E5	β_2	13
	$K_2 \leq \frac{a-c_1}{4(b-c_2)}$	$\frac{1}{K_2} < \frac{1}{K_1} + \frac{1}{q_M}$	$K_2 < q_M - K_1$	E2	E2→E3→E1	β_2, β_{31}	14
			$q_M - K_1 \leq K_2 \leq 2(q_M-K_1)$	E2	E2→E3→E5→E1	$\beta_2, \beta^c, \beta_{51}$	15
			$K_2 > 2(q_M-K_1)$	E2	E2→E3→E5	β_2, β^c	16
		$\frac{1}{K_2} \geq \frac{1}{K_1} + \frac{1}{q_M}$	$K_2 < q_M - K_1$	E2	E2→E4→E3→E1	$\beta_2, \beta_{43}, \beta_{31}$	17
			$q_M - K_1 \leq K_2 \leq 2(q_M-K_1)$	E2	E2→E4→E3→E5→E1	$\beta_2, \beta_{43}, \beta^c, \beta_{51}$	18
			$K_2 > 2(q_M-K_1)$	E2	E2→E4→E3→E5	$\beta_2, \beta_{43}, \beta^c$	19

Figure 8: All possible equilibria of Theorem 1 and Theorem 2 as a function of β for a given (K_1, K_2), when buyer 2 has medium capacity under independent procurement



(a) Example 2.a: Case 5 (b) Example 2.b: Case 9 (c) Example 2.c: Case 18

Figure 9: Profits of the buyers and the supplier in Cases 5, 9 and 18 for $\beta \in [0, 1]$.

Example 2.b. Suppose $a = 100$, $c_1 = 88$, $b = 1$, $c_2 = 0.6$, $K_1 = 12.5$ and $K_2 = 4$. We have $q_U^I = 6.67$, $\bar{q} = 12$, $q_M = 15$, and the system is in Case 9. Buyer 1 does not prefer collaboration for $\beta \in (0, 1]$ (actually he prefers collaboration if $\beta \geq \beta' = 1.92$), whereas buyer 2 prefers collaboration for all $\beta \in (0, 1]$. The supplier's profit is maximized at $\beta_{51} = 0.83$. However, due to buyer 1, collaboration does not take place in this example (Figure 9.b).

Example 2.c. Suppose $a = 100$, $c_1 = 88$, $b = 1$, $c_2 = 0.8$, $K_1 = 21$ and $K_2 = 10$. We have $q_U^I = 8.57$, $\bar{q} = 12$, $q_M = 30$, and the system is in Case 18. Buyer 1 prefers collaboration for all $\beta \in (0,1]$, whereas buyer 2 prefers collaboration if β is sufficiently small or sufficiently high (if $\beta < \beta_2 = 0.25$ or $\beta > \beta'' = 0.74$). The supplier's profit is maximized at $\beta_{51} = 0.8$. At 0.8, both buyers are willing to collaborate (Figure 9.c).

Knowing how the equilibrium changes as β changes helps the supplier to choose the *optimal* or the *best feasible* β values. By *optimal* β, we refer to the β that maximizes the supplier's profit, whereas, by *best feasible* β, we refer to the β that maximizes the supplier's profit at which both buyers are willing to collaborate. Recall that, when the buyers are uncapacitated, the optimal β is also the best feasible β, since for all $\beta \in (0,1]$, the buyers prefer joint procurement. However when they are capacitated, a buyer may or may not prefer joint procurement, depending on β.

In Example 2.a, while β_{31} is optimal for the supplier, buyer 1 would not be willing to collaborate at $\beta = \beta_{31}$. Hence the best feasible β is β', at which both buyers are willing to collaborate. As in Example 2.a, there might exist a β that maximizes the supplier's profit, however, if at least one buyer is worse off at that value (compared to independent procurement), the supplier would have to compromise. Therefore, in Proposition 2 we determine the conditions at which the buyers prefer joint procurement, and then based on those conditions, in Proposition 4 we analyze the supplier's choice of β.

Proposition 2 *(1) If buyer 2 has tight capacity under independent procurement ($K_2 < q_U^I$), then he is better off under joint procurement. Buyer 1 is better off if and only if $b > 2c_2$ or $\beta > \beta'$, where $\beta' = \frac{b}{c_2}(1 - \frac{r_2(K_1)}{K_2})$.*

(2) If buyer 2 has medium capacity under independent procurement ($K_2 \geq q_U^I$), then buyer 1 is better off under joint procurement. Buyer 2 is better off if and only if one of the following holds:

(i) $K_2 \geq \frac{a-c_1}{4(b-c_2)}$,

(ii) $\beta \leq \beta_2$,

(iii) $\beta > min\{\beta^c, \beta''\}$, where $\beta'' = \frac{b}{c_2} - \frac{a-c_1-2(b-c_2)q_U^I}{c_2 K_1}$.

Proposition 2 shows that, for each buyer there exists a threshold β (possibly $= 0$ or > 1), such that when the supplier sets β above the threshold value of the buyer, the buyer prefers joint procurement. These threshold values may be different for each buyer, i.e., it is possible to have β values where one buyer is willing to collaborate whereas the other buyer is not. Furthermore, the threshold values change as market conditions and the buyers' capacities change (see also Examples 2.a, 2.b and 2.c).

The existence of threshold values can be explained intuitively. The decrease in the supply price due to β is an incentive for the buyers to collaborate. Note that, there exists a β at which the equilibrium quantities are (K_1, K_2). For higher values of β, the market price stays constant at $a-b(K_1+K_2)$, whereas the supply price decreases. Therefore there should be a β (which is an upper bound on the threshold values) that makes the buyers better off under joint procurement. This means, if the supplier gives enough quantity discounts, then buyers should be willing to collaborate.

Under joint procurement, at least one of the buyers increases his procurement quantity at equilibrium, unless there are capacity restrictions. That is, as β increases MC decreases for both buyers, and this is an incentive for the buyers to procure more. The proof of Proposition 2 indicates that, if the procurement quantity of a buyer increases under joint procurement, the buyer prefers joint procurement and if the quantity decreases under joint procurement, the buyer prefers independent procurement. If the quantity is the same under joint procurement, the supplier should give sufficient discount for the buyer to collaborate.

Buyer 1 does not collaborate in Cases 6-10 (since either the buyer's procurement quantity decreases under joint procurement or his threshold β value exceeds 1). In Case 5, collaboration may or may not take place depending on β. For all other cases, buyer 1 prefers collaboration. If buyer 1 is procuring at capacity under independent procurement, K_1 and K_2 being "low" ($K_1 < \frac{a-c_1}{b}$ and $K_1 + K_2 < q_M$) is a necessary condition for him to collaborate. The reason is, if K_1 or K_2 is "low", although the procurement quantity of buyer 2 increases under joint procurement, the amount of increase is limited.

The procurement quantity of buyer 2 possibly decreases under joint procurement, only in Cases 14-19 (depending on β). If K_1 is "low" with respect to K_2, then buyer 2 collaborates under a wide range of parameter settings, since under "low" K_1, buyer 2's procurement quantity under joint procurement increases (for instance, if $\frac{K_1}{K_2} < 1 - q_U^I/q_M$, buyer 2 collaborates for any β). See Figure 10 for a summary of the cases and the conditions under which each buyer prefers collaboration.

Observation 12 *When buyers are capacitated, end consumers are better off under joint procurement.*

Proposition 2 implies that, if joint procurement takes place, then each buyer's procurement quantity either increases or stays the same, whereas at least one of the buyers increases his procurement quantity. As a result, in the end market the total production quantity increases, the price decreases and the end consumers are better off.

3.5.2 Supplier's Choice of β when the Buyers are Capacitated

In this section we analyze the supplier's choice of β. Let $\beta_C^* \in [0, 1]$ denote the β that maximizes the supplier's profit function when the buyers are capacitated. As β increases from 0 to 1, the system may pass through several equilibria as indicated in Cases 1-19 of Figures 7 and 8. The supplier finds the β values that maximizes her profit function under each equilibrium, and compares those profits to find the β that achieves the maximum profit. Since the system passes through different equilibria, the supplier's profit function may have discontinuities at the switching β values.

We characterize β_C^* for each of the 19 cases in Theorem 3. When there exist multiple equilibria, each equilibrium leads to a different profit, therefore how much to sell to each buyer is also a decision the supplier has to make when determining β_C^*. In Proposition 3 we show that whenever $\beta_C^* \leq \beta_2$, the optimal decision for the supplier is to sell (q_C^*, q_C^*), where $q_C^* = \frac{a-c_1}{3b-(2+\beta_C^*)c_2}$. This enables us to assume under $0 \leq \beta \leq \beta_2$, E2 is the only equilibrium.

Proposition 3 *When the buyers are capacitated, for $\beta_C^* \leq \beta_2$ the supplier maximizes her profit by selling q_C^* to each buyer.*

Proposition 3 implies if optimal decision of the supplier is to sell $(K_1, r_2(K_1)^+)$, then $\beta_C^* > \beta_2$.

Theorem 3 *β_C^* is characterized as in Figure 10.*

Analysis in Theorem 3 indicates that β_C^* is never equal to β_{43}, i.e., it is never optimal for the supplier to sell $(K_1, 0)$. This, together with Proposition 3 imply, rather than selling a large quantity to a single buyer, the supplier prefers selling to both buyers in smaller quantities. This result is similar to Proposition 1 of Section 3.4.2.

Case	Condition		β_C^*	Buyer 1	Buyer 2
1, 4			0	Collaborates	Collaborates
2, 3			$\min\{\beta_5, \beta_{51}, 1\}$		
5, 8			$\min\{\beta_3, \beta_{31}\}^+$	Collaborates for $\beta > \beta'$, Case 5 Does not collaborate for Case 8	
6, 7, 9, 10	$\beta_3 < \beta^c$		$\mathrm{argmax}\{\pi_S(\beta_3), \pi_S(\beta^c)\}$	Does not collaborate	
	$\beta^c < \beta_3$	$\beta_5 < \beta^c$	β^c		
		$\beta^c < \beta_5$	$\min\{\beta_5, \beta_{51}, 1\}$		
11, 12, 13	$\beta_U^* < \beta_2$		β_U^*	Collaborates	Collaborates
	$\beta_2 < \beta_U^*$	$\beta_2 < \beta_5$	$\min\{\beta_5, \beta_{51}, 1\}$		
		$\beta_5 < \beta_2$	β_2		
14, 17	$\beta_3 < \beta_2$		β_U^*		Collaborates
	$\beta_2 < \beta_3$		$\mathrm{argmax}\{\pi_S(\min\{\beta_2, \beta_U^*\}), \pi_S(\min\{\beta_3, \beta_{31}\})\}$		Collaborates for $\beta > \beta''$
15, 16, 18, 19	$\beta_3 < \beta_2$		$\mathrm{argmax}\{\pi_S(\beta_U^*), \pi_S(\beta^c)\}$		Collaborates
	$\beta_2 < \beta_3 < \beta^c$		$\mathrm{argmax}\{\pi_S(\min\{\beta_2, \beta_U^*\}), \pi_S(\beta_3), \pi_S(\beta^c)\}$		Collaborates for $\beta > \beta''$
	$\beta^c < \beta_3$	$\beta_5 < \beta^c$	$\mathrm{argmax}\{\pi_S(\min\{\beta_2, \beta_U^*\}), \pi_S(\beta^c)\}$		Collaborates
		$\beta^c < \beta_5$	$\mathrm{argmax}\{\pi_S(\min\{\beta_2, \beta_U^*\}), \pi_S(\min\{\beta_5, \beta_{51}, 1\})\}$		

Figure 10: The $\beta_C^* \in [0, 1]$ values that maximizes the supplier's profit function for each of the 19 Cases.

If $\beta_C^* = 0$, the supplier does not offer a discount. If $\beta_C^* > 0$, the supplier checks whether the buyers are willing to collaborate. If either buyer is not willing to collaborate at β_C^*, the supplier may change β by considering the β values given in Proposition 2. In Figure 10 we also present for each of the 19 cases, whether each buyer is willing to collaborate. Proposition 4 characterizes the supplier's profit maximizing strategy under capacitated buyers.

Proposition 4 *The profit maximizing strategy of the supplier when the buyers are capacitated is as follows:*

(1) In Cases 1-4 or 11-13, $\beta = \beta_C^*$.

(2) In Case 5, if $1 - \frac{r_2(K_1)}{K_2} < \frac{c_2}{b}$ *and* $c_1 > c_2(K_2 + r_2(K_1)) + b(2K_1)$, *then* $\beta = \beta'$, *otherwise* $\beta = 0$.

(3) In Cases 6-10, collaboration does not take place.

(4) In Cases 14-19,

(a) *If* $\beta_C^* \neq \beta_3$ *or* $\beta_C^* = \beta_3 \geq \beta''$, $\beta = \beta_C^*$.

(b) *If* $\beta_C^* = \beta_3 < \beta''$, *buyer 2 does not collaborate at* β_C^*. *For Cases 14 and 17,* $\beta =$*argmax*$\{\pi_S(min\{\beta_2, \beta_U^*\}), \pi_S(\beta'')\}$, *and for Cases 15,16, 18 and 19,* $\beta =$*argmax*$\{\pi_S(min\{\beta_2, \beta_U^*\}), \pi_S(\beta''), \pi_S(\beta^c)\}$.

Note that, if collaboration does not take place it is either due to buyer 1 or due to the supplier. Buyer 2 and the supplier could always compromise on a β that both would agree (unless $\beta_U^* = 0$). This implies, although willing to collaborate with the small buyer in most cases, the large buyer has more impact on the collaboration process.

Below, we present an example where buyer 2 is not willing to collaborate at β_C^*.

Example 3. Suppose $a = 124$, $c_1 = 88$, $b = 1$, $c_2 = 0.7$, $K_1 = 37.49$ and $K_2 = 22.51$. We have $q_U^I = 22.5$, $\bar{q} = 36$, $q_M = 60$, and the system falls under Case 18. The β values are, $\beta_2 = 0.001$, $\beta_U^* = 0.378$, $\beta_3 = 0.568$, $\beta^c = 0.5714$, and $\beta_5 = 0.296$. $\beta_2 < \beta_3 < \beta^c$, therefore:

$$\beta_C^* = \text{argmax}\{\pi_S(\min\{\beta_2, \beta_U^*\}), \pi_S(\beta_3), \pi_S(\beta^c)\}.$$

$$= \text{argmax}\{\pi_S(\beta_2), \pi_S(\beta_3), \pi_S(\beta^c)\}$$

$$= \text{argmax}\{3251.66, 3266.38, 3266.34\}$$

$$\beta_C^* = \beta_3$$

Since buyer 2 has medium capacity, from Proposition 2 buyer 1 is willing to collaborate for all $\beta \in (0, 1]$ and buyer 2 is willing to collaborate if $\beta < \beta_2$ or if $\beta >$min$\{\beta^c, \beta''\}$. $\beta'' = 0.5712 > \beta_3$. In this example supplier compromises by setting $\beta_C^* = \beta''$. Table 23 shows the profits of the two buyers and the supplier under independent procurement, in comparison with the profits under β_C^* and β''.

Table 23: Example 3: A comparison of the profits of the buyers and the supplier

	q_1	q_2	π_1	π_2	π_S
Independent	22.5	22.5	151.88	151.88	3251.25
Joint (β_C^*)	37.49	22.37	422.89	150.17	3266.38
Joint (β'')	37.49	22.51	421.74	151.88	3266.34

3.6 Concluding Remarks

We have shown that if the market conditions are appropriate, cooperation between competing firms provides benefits for all participants. We analyzed two cases with respect to buyers' capacities. When the buyers are uncapacitated, their procurement quantities increase under joint procurement, and as a result they are willing to collaborate for any discount given by the supplier. When the buyers are capacitated, each buyer is willing to collaborate for certain discount levels, provided that he could increase his procurement quantity under joint procurement. Intuitively, one would think that a small buyer would always prefer collaborating with a large buyer. However, our study indicates that depending on large buyer's capacity and the discount level, the small buyer may not find it beneficial to procure jointly. For both the uncapacitated and the capacitated cases, total quantity in the end market increases under joint procurement, and end users are better off.

CHAPTER IV

DYNAMIC DUE-DATE QUOTATION FOR BASE-STOCK INVENTORY SYSTEMS

4.1 Introduction

In this chapter, we consider a due date quotation and inventory keeping problem of a retailer/manufacturer. The demand occurs in the form of customer arrivals. If there are items in stock then demand is met immediately, while an order is placed for a new item. If there are no available items in stock, then the customer has to wait until an item arrives from the depot, in which case he is quoted a due date. If the item arrives later than the time quoted to the customer, a lateness cost is incurred per part, per unit time. Furthermore a holding cost is incurred per item, per unit time for any unassigned item that waits in the inventory. This type of problem can also arise in a repair facility of a manufacturer. In that case instead of ordering an item from the depot, the part is placed into repair.

In our problem the customers are sensitive to the quoted due dates; as the quoted due date increases, the customer is less likely to place his order. If the customer places his order, he brings a revenue. Given these settings, our objective is to find the optimal inventory level and the optimal due date policy.

General Electric (GE) Medical Systems provides a relevant industry practice for our problem definition. In GE Medical Systems, service parts are categorized as "consumable" and "exchange" items. The exchange items are the parts than can be repaired and are priced below the full price. The customer places an order, receives a replacement part and sends the defective part within 30 days of the receipt.[1]

In Section 4.2 we review the literature on due date quotation. In Section 4.3, we make a structural analysis of the optimal due date quotation policy under a given base-stock level

[1] http://www.gehealthcare.com/services/repl_parts/faq.html

and obtain insights on the optimal base-stock level. In Section 4.4 we conduct experiments to identify the conditions under which keeping inventory is beneficial. We present our concluding remarks in Section 4.5.

4.2 Literature Review

There exists voluminous amount of work on due date quotation. Researchers have been studying different aspects of the problem such as developing scheduling/sequencing algorithms to meet due dates, quoting lead-times to maximize profit, or to minimize cost, subject to some service level constraint. In many cases, the revenue obtained per job, holding cost and earliness/lateness penalties are the main elements of the models. The production environments can be deterministic versus stochastic with lead-time sensitive versus insensitive customers. If customers are lead-time sensitive, the due date policy implicitly controls the customer arrivals to the system. Several papers take into consideration the long-run effects of the due date quotation policy. In this section, we present a representative subset of the due date quotation literature.

A line of research focus on scheduling/sequencing customer orders to meet an objective such as minimizing average tardiness, maximum lateness or an aggregate cost function. Keskinocak and Tayur (2003) provides a thorough survey on coordinated scheduling and due date quotation. In some of the studies the lead-times are customer-specified, all arriving orders are accepted (ElHafsi 2000), and the processing times are deterministic (Chhajed and Chand 1992), but there exist exceptions. Keskinocak, Ravi and Tayur (2001) consider both scheduling and lead-time quotation decisions under lead-time sensitive revenue. Duenyas and Hopp (1995) models a due date quotation problem with stochastic arrival and processing times and find an optimal sequencing rule to maximize profit with lead-time sensitive customers.

In our model the due dates are set by the manufacturer. There exist a body of work that considers endogenous due date quotation. In some studies, the due-date decisions are based on a service constraint. Hopp and Sturgis (2001) construct a dynamic due date quotation policy to minimize average lead-times subject to various service level constraints.

Spearman and Zhang (1999) compare two types of service level constraints; fraction of tardy jobs versus average tardiness, where first type leads to unethical policies. So and Song (1998) consider a manufacturer's price, lead-time and capacity decisions under service level constraints. On the other hand, some papers consider a profit or cost function with lateness cost component instead of a service level constraint. Weng (1999) studies a problem with holding, earliness and tardiness costs, and conclude that the optimal lead-time is the manufacturing flow time adjusted by earliness and tardiness costs. Dellaert (1991) considers a model with earliness/lateness penalties. Dellaert (1991) and So and Song (1998) also consider the affect of customer sensitivity on due date decisions. Similarly Palaka, Erlebacher and Kropp (1992) study a model where price and lead-time are the two decision variables which affect demand. Several papers incorporate the affect of accepting a customer on system congestion and future due date decisions (Duenyas and Hopp 1995; Duenyas 1995), whereas others do not (Chatterjee, Slotnick and Sobel 2002; Weng 1999). Our work differs from the previous literature in the sense that we consider an initial inventory as a means to increase responsiveness. The due date decisions are set endogenously and depend on the workload status in the system. Furthermore, future effects of accepting an order on system congestion is considered. We do not consider a service level constraint, rather maximize a profit function that includes a lateness cost component. See Figure 11 for a classification of previous work that are relevant to our problem and how our study differs from the existing ones.

Determining the type of the manufacturing environment is another objective in our study. Li (1992) studies a manufacturer's production policy where price, production cost, holding cost and quoted lead-times are exogenous, and finds that the optimal policy is to operate under a base-stock inventory system. The conditions for a make-to-order (MTO) system is determined and the change in the likelihood of the firms to keep inventory is analyzed with respect to competition. Rajagopalan (2002) studies a production system where items could be make-to-order or make-to-stock. He shows that it may not be always true that low demand items should be MTO and high demand items should be make-to-stock. Veatch and Wein (1994) model the production system as tandem stations with queues

	Due-dates endogenous?	Inventory	Lead-time sensitivity	Long-run effects	Workload status	Service constraints
Dellaert 1992	✓	✓	✓	✓	✓	
Li 1992		✓	✓			
Duenyas-Hopp 1995	✓		✓	✓	✓	
Palaka et.al.1998	✓		✓	✓	✓	✓
So-Song 1998	✓		✓			✓
Weng 1999	✓	✓		✓		
Spearman-Zhang 1999	✓			✓	✓	✓
ElHafsi 2000		✓			✓	✓
Hopp Sturgis 2001	✓			✓	✓	✓
Keskinocak et.al.2001	✓		✓			✓
Chatterjee et.al.2002	✓		✓		✓	
Our study	✓	✓	✓	✓	✓	

Figure 11: A classification of previous studies on due date quotation and a comparison with our study

where production rates can be controlled under the presence of inventory and backordering costs. Their objective is to identify the conditions that different production policies such as Kanban or base-stock are preferred. In our work we control the arrival rates (by quoting due dates) instead of production rates, and determine when to operate under MTO versus under base-stock systems.

4.3 Model

The customers arrive according to a Poisson process. We assume all customers and parts are identical, and customers place orders one at a time. We further assume that the fixed cost of sending an item to depot is negligible. For this reason, one-for-one replenishment is appropriate.

If a customer is quoted lead-time d, then he places the order with probability $f(d)$, where $f(d)$ is a decreasing function of d. We assume $f(0) = 1$ and that there exists a maximum lead-time, d_{max}, where $f(d_{max}) = 0$. When an order is placed, it brings a revenue of R. The service discipline is FCFS (first come first served). Customer interarrival and part repair times are exponential with rate λ and μ, respectively. The repair service is capacitated, with a single server. Unit time is defined as the expected service time, $1/\mu$, which without loss of generality we assume 1. The objective is to find the optimal base-stock level and the optimal due date quoting policy. For this we consider a two stage solution methodology. In the first stage we find the optimal due date quotation policy for a given base-stock level. In the second stage we obtain insights on the optimal base-stock level.

4.3.1 The optimal due date quotation policy, given a base-stock level

The service center makes a decision each time a customer arrives to the system, i.e., stages of the problem (or decision epochs) are determined by the customer arrivals, and the model is formulated as a Semi-Markov Decision Process (SMDP).

The state and action spaces, reward function, and transition probabilities are defined in Table 24. The state space is defined as $I = \{-s, -s+1, .., 0, 1, ..\infty\}$, where s is the base-stock level. Negative states correspond to the number of parts waiting in the inventory, and positive states correspond to the number of customers waiting in the queue. Note that between two customer arrivals the state of the system may change several times, i.e., with the arrival of parts from repair, the state may decrease. The action space is $[0, d_{max}]$.

The reward function, $r_i(d)$, is the expected reward obtained from an arriving customer, given the state is i upon arrival and the quoted due date is d. Reward consists of revenue, lateness cost and inventory cost:

$$r_i(d) = f(d)\Big(R - L_i(d) - I_{i+1}\Big) + (1 - f(d))(-I_i) \tag{26}$$

The expected lateness cost given the state is i upon arrival and the quoted due date is d, is defined by:

$$L_i(d) = \begin{cases} l\int_d^{\infty}(x-d)E_{i+1}dx, & \text{if } i \geq 0 \\ 0 & \text{, otherwise} \end{cases} \tag{27}$$

where $E_j = \frac{\mu(\mu x)^{j-1}}{(j-1)!}e^{-\mu x}$ (Erlang(μ, j)) corresponds to the probability distribution function of the sum of j exponentially distributed variables, each with parameter μ. The lateness cost per part per unit time is l. When $i \geq 0$, if an arriving customer places an order, he waits for the i parts in repair plus his own service time. When $i < 0$, a lateness cost is not

Table 24: A glossary of notation for Chapter 4

s	: base-stock level
i	: index for the state, $i \epsilon I$
d	: due date quoted to a customer
d_{max}	: upper bound on the due date quoted
$\mathbf{d}$	: due date policy, a vector of d values
$f(d)$	: probability of placing an order if quoted due date is d
$r_i(d)$	: expected reward obtained when the due date quoted to the arriving customer is d, given the state is i upon arrival
R	: the revenue obtained when customer places an order
$L_i(d)$	: expected lateness cost when the due date quoted to the arriving customer is d, given the state is i upon arrival
l	: lateness cost per unit per unit time
I_i	: expected inventory cost incurred until next customer arrival, given the state is i upon arrival
h	: holding cost per unit per unit time
$P(j\|i,d)$	: the probability that the next state is j, given the current state is i and the arriving customer is quoted d
$g^{\mathbf{d}}$	: the long-run average return **per customer** under due date policy $\mathbf{d}$
g^*	: the long-run optimal average return **per customer**
$g^{\mathbf{d}}(s)$	: the long-run return **per unit time** under due date policy $\mathbf{d}$ and base-stock level s $g^*(s)$ corresponds to the return **per unit time** under the optimal due date policy $g^0(s)$ corresponds to the return **per unit time** under the 'accept-all' policy
$v(i)$	: relative value of starting in state i
$\pi_i(\mathbf{d}, s)$	: the steady-state probability of being in state i under due date policy $\mathbf{d}$ and base-stock level s

incurred since the demand is satisfied from inventory.

The expected inventory cost incurred until next customer arrival given the state is i, is defined by:

$$I_i = h\Big(\max\{-\frac{i}{\lambda}, 0\} + \sum_{j=\max\{i,0\}+1}^{s+i} \int_0^\infty \int_0^t (t-x)E_j dx \lambda e^{-\lambda t} dt\Big), \tag{28}$$

where h is the holding cost per part per unit time. If $i < 0$, then there are -i parts in inventory and $s+i$ parts in repair. Until the next customer arrival, parts already in inventory incur an expected inventory cost of $\frac{h\cdot(-i)}{\lambda}$. The parts in repair incur inventory cost after they arrive. The expected waiting time of the j^{th} part arriving from repair given the next customer arrives at time t is $\int_0^t (t-x)E_j dx$. Therefore the j^{th} part in repair incurs an expected inventory cost of $h\int_0^\infty \int_0^t (t-x)E_j dx \lambda e^{-\lambda t} dt$ until the next customer arrival. For $i \geq 0$, no parts are waiting in inventory, but $s+i$ parts are in repair. Therefore for $i \geq 0$ the first term is 0, and cost is possibly incurred for up to s parts until the next arrival.

The transaction probability function, $P(j|i,d)$, is defined as the probability that the next state is j, given the current state is i and the arriving customer is quoted d.

$$P(j|i,d) = \begin{cases} 0 & \text{, if } j > i+1 \\ \frac{\lambda}{\lambda+\mu}\Big(f(d)(\frac{\mu}{\mu+\lambda})^{i-j+1} + (1-f(d))(\frac{\mu}{\lambda+\mu})^{i-j}\Big) & \text{, if } i+1 \geq j > -s \\ f(d)(\frac{\mu}{\mu+\lambda})^{i+s+1} + (1-f(d))(\frac{\mu}{\lambda+\mu})^{i+s} & \text{, if } j = -s \end{cases} \tag{29}$$

Given state i, if the customer places an order, then there must be exactly $i-j+1$ part arrivals before the next customer arrival, for the next state to reach $j > -s$. We indicate this probability with $(\frac{\mu}{\mu+\lambda})^{i-j+1}\frac{\lambda}{\mu+\lambda}$. If the customer does not place an order, then there must be $i-j$ part arrivals before the next customer arrival, for the state to reach j. On the other hand, for $j = -s$, if the customer places an order, then there should be $i+s+1$ part arrivals before the next customer arrival. Since $i+s+1$ is the maximum number of parts that can arrive, this probability is equivalent to the probability that there are more than $s+i$ part arrivals, as expressed below (say, for $d = 0$):

$$
\begin{aligned}
P(j|i,0) &= 1 - P(\text{exactly 0 part}) - P(\text{exactly 1 part}) - \cdots - P(\text{exactly s+i parts})\\
&= 1 - \frac{\lambda}{\lambda+\mu} - \frac{\lambda}{\lambda+\mu}(\frac{\mu}{\lambda+\mu}) - \cdots - \frac{\lambda}{\lambda+\mu}(\frac{\mu}{\lambda+\mu})^{s+i}\\
&= 1 - \frac{\lambda}{\lambda+\mu}(1 + \frac{\mu}{\lambda+\mu} + \cdots + (\frac{\mu}{\lambda+\mu})^{s+i})\\
&= (\frac{\mu}{\mu+\lambda})^{i+s+1}
\end{aligned}
$$

The analysis is similar for $j = -s$ when the customer does not place the order.

Choosing the optimal due dates upon customer arrivals is equivalent to choosing the arrival rates at each state. Note that the problem under consideration is an $M/M/1$ queue with the initial state -s. Proposition 5 shows that at optimality the system has finite number of states (Proofs are presented in Appendix B).

Proposition 5 *The problem under consideration has finite number of states at optimality.*

In the remainder of this chapter, we make the analysis under finite state space, and we simply indicate the final state as N. Next, we show that there exists a stationary optimal policy.

Corollary 1 *(Ross 1970, Corollary 6.20, p.149) If the state space is finite and every stationary policy gives rise to an irreducible Markov chain, then there exists an optimal stationary policy. Or if there exists a state, reachable from every other state under every stationary policy, then there exists an optimal stationary policy.*

Observation 13 *In our problem, since the server is busy whenever there are parts in repair, -s is reachable from every other state under every policy. Therefore there exists a single communicating class and the optimal policy is a stationary policy.*

Let $g^{\mathbf{d}}$ be the long-run average return **per customer** under a stationary policy **d**. We introduce the following definition.

Definition. (Tijms 1986, p.166) *The relative value $v(i)$ indicates the transient effect of the initial state i on the total expected return under a given policy. Let $V_n^{\mathbf{d}}(i)$ be the total* *expected return after n stages (decision epochs) under policy $\mathbf{d}$, with initial state i. Then $lim_{n\to\infty} V_n^{\mathbf{d}}(i)/n = g^{\mathbf{d}}$. For n large, $V_n^{\mathbf{d}}(i) \approx ng^{\mathbf{d}} + v^{\mathbf{d}}(i)$.*

After defining the components, the SMDP is:

$$g^* + v^*(i) = max_d\{r_i(d) + \sum_j P(j|i,d)v^*(j)\}, \tag{30}$$

where, g^* is the long-run optimal average return **per customer**, d is the due date quoted to a customer (a scalar value), and $v(i)$ is the relative value of starting in state i.

We use the notation g^* and $g^{\mathbf{d}}$ in Section 4.3.1, where we determine the optimal due date quotation policy under an exogenous base-stock level. On the other hand, in Section 4.3.2, where we study the optimal base-stock level, we use the notation $g^{\mathbf{d}}(s)$, which corresponds to the long-run return **per unit time** (see also Table 24). Note, $g^{\mathbf{d}}(s) = \lambda g^{\mathbf{d}}$. Since λ is a constant, the due date policy that results in optimal average return per customer is the same as the policy that results in optimal return per unit time.

Next we characterize the optimal policy. Let $\mathbf{d}^*(\mathbf{s})$ be the optimal due date policy under base-stock level s and $\mathbf{d}_i^*(s)$ be the optimal due date quoted in state i (a scalar value). For simplicity, we use $\mathbf{d}^*$ instead of $\mathbf{d}^*(\mathbf{s})$.

We show in Proposition 6 below that whenever there are parts available in inventory, the optimal policy is to quote 0.

Proposition 6 $\mathbf{d}_i^* = 0$ *for* $i =$-$s, \cdots, -1$.

Proof. First we show the optimal decision in state $i < 0$ is either to quote $d = 0$ (i.e, accept the order with probability 1) or to quote $d = d_{max}$ (i.e., reject). Suppose d is the optimal decision in state i, where $0 < d < d_{max}$. Then from (30),

$$v^*(i) = r_i(d) - g^* + \textstyle\sum_j P(j|i,d)v^*(j) = max_d\{r_i(d) - g^* + \sum_j P(j|i,d)v^*(j)\},$$

This implies,

$$r_i(d) + \sum_j P(j|i,d)v^*(j) > r_i(0) + \sum_j P(j|i,0)v^*(j). \tag{31}$$

Note that, for $i < 0$, $r_i(d)$ and $P(j|i,d)$ are convex combinations of $r_i(0)$ and $r_i(d_{max})$, and $P(j|i,0)$ and $P(j|i,d_{max})$, respectively. From Equation (31),

$$\Big(f(d)r_i(0) + (1-f(d))r_i(d_{max})\Big) +$$

$$\sum_j \Big(f(d)P(j|i,0) + (1-f(d))P(j|i,d_{max})\Big)v^*(j) > r_i(0) + \sum_j P(j|i,0)v^*(j)$$

$$r_i(d_{max}) + \sum_j P(j|i,d_{max})v^*(j) > r_i(0) + \sum_j P(j|i,0)v^*(j)$$

which implies,

$$r_i(d_{max}) + \sum_j P(j|i,d_{max})v^*(j) > r_i(d) + \sum_j P(j|i,d)v^*(j)$$

Since d is the optimal decision at state i, this is a contradiction. Therefore the optimal decision in state i is either to accept or reject the order. Suppose $-s \leq k < 0$ is the state in which an arriving customer is rejected, i.e., for all $i < k$ the customers are accepted. Let us compare this policy (policy A) with a policy where the customers are accepted in k and rejected in state $k+1$ (policy B). Let π_i and $\pi_i{}'$ be the steady-state probabilities of being in state i under policy A and policy B, respectively. An inventory cost of $-i \cdot h$ is incurred per unit time whenever the system is in state i. Since,

$\sum_{i=-s}^{k+1} \pi_i{}' = \sum_{i=-s}^{k} \pi_i = 1$,

$\sum_{i=-s}^{k} \pi_i{}' < \sum_{i=-s}^{k} \pi_i$,

i.e., the system spends less time in states $i \leq k$ under policy A than it does under policy B. This implies under policy B, a lower long-run inventory cost is incurred per unit time. Furthermore, since $\pi_{k+1}{}' < \pi_k$, in steady-state a lower fraction of customers are rejected under policy B, i.e., the long-run revenue per unit time is higher. As a result, under the optimal policy, $\mathbf{d}_i^* = 0$ for $i = -s, \cdots, -1$. □

Next, we show that for non-negative states, the optimal policy has a monotone structure, i.e., as the number of customers in the system increases the quoted due date increases.

Let $E^{\mathbf{d}}(i,j)$ be the total expected profit and $t^{\mathbf{d}}(i,j)$ be the expected number of customers that arrive to the system (but not necessarily place an order) between one visit from state i to state j, under the stationary policy $\mathbf{d}$.

Theorem 4 *(Ross 1970, Theorem 7.5, p. 160) Suppose* $\boldsymbol{d}$ *is a stationary policy of the*

SMDP in (30). If $t^{\mathbf{d}}(j,j) < \infty$*, then*

$$g^{\mathbf{d}} = \frac{E^{\mathbf{d}}(j,j)}{t^{\mathbf{d}}(j,j)} \qquad \textit{for all } j \tag{32}$$

If $t^{\mathbf{d}}(j,j) < \infty$, then j is a positive recurrent state. In our problem, states $-s, \cdots, -1, 0$ are positive recurrent under optimality, since optimal decision is to quote 0 under states $-s, \cdots, -1$. In general we can write the following for any policy $\mathbf{d}$, a given positive recurrent state j, and a starting state i (Tijms 1986):

$$v^{\mathbf{d}}(i) = E^{\mathbf{d}}(i,j) - g^{\mathbf{d}} \cdot t^{\mathbf{d}}(i,j) \tag{33}$$

Note that (32) and (33) imply $v^{\mathbf{d}}(j) = 0$. At optimality (under the optimal policy):

$$v^*(i) = E^*(i,j) - g^* \cdot t^*(i,j) \quad \forall\ i$$

where $E^*(i,j)$ corresponds to the total expected return until first visit to j starting from i under the optimal policy, $t^*(i,j)$ corresponds to the expected number of customer arrivals until first visit to j starting from i under the optimal policy, and $v^*(i)$ corresponds to the relative value of starting in state i under the optimal policy.

For any stationary policy $\mathbf{d}$, $g^* > g^{\mathbf{d}}$,

$E^{\mathbf{d}}(i,j) - g^* \cdot t^{\mathbf{d}}(i,j) \le E^{\mathbf{d}}(i,j) - g^{\mathbf{d}} \cdot t^{\mathbf{d}}(i,j) = v^{\mathbf{d}}(i) \le v^*(i)$

From Equation (30), $v(i)$ takes its maximum value under the optimal policy. Therefore,

$E^{\mathbf{d}}(i,j) - g^* \cdot t^{\mathbf{d}}(i,j) \le v^{\mathbf{d}}(i) \le v^*(i)$

and equality is obtained if $E^{\mathbf{d}}(i,j) - g^* \cdot t^{\mathbf{d}}(i,j)$ is maximized over all $\mathbf{d}$, $\forall\ i$.

Now consider another SMDP problem. In this problem, the objective is to maximize the total return until the system reaches state j starting from state i, where a constant c is subtracted from the return at each customer arrival. Call this the modified problem. The modified SMDP problem is different from the original SMDP problem in (30) in two aspects:

1) The reward function of the original problem is $r_i(d)$, whereas the reward function of the modified problem is $r_i(d) - c$.

2) (Under a stationary policy) In the original problem, state j is positive recurrent, whereas in the modified problem, state j is absorbent. Therefore we can say that the original problem is infinite-horizon, whereas the modified problem is finite-horizon.

If g^* is used in place of the constant c in the modified problem, then the original and the modified problem are equivalent in the sense that the optimal policies of the two problems are the same. Maximizing $E_{\mathbf{d}}(i,j) - g^* \cdot t_{\mathbf{d}}(i,j)$ over all $\mathbf{d}$, gives the optimal policy for the original problem, as we mentioned above. Also $E_{\mathbf{d}}(i,j) - g^* \cdot t_{\mathbf{d}}(i,j)$ is the expression for the total expected return until reaching j starting from state i, with constant g^* is subtracted at each customer arrival. Maximizing this quantity over all $\mathbf{d}$ gives the optimal policy for the modified problem (for a similar definition see Stidham and Weber 1989; Duenyas and Hopp 1995).

We will determine whether the optimal policy of the modified problem is monotone for non-negative states. Let $E^*_m(i,j)$ be the maximum total expected return starting from state i until reaching j, of the modified problem. We set the constant c to g^*. $E^*_m(i,j)$ satisfies the following optimality equation:

$$E^*_m(i,j) = \max_d \Big\{ \frac{max\{-i,0\} \cdot h}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu} f(d)\Big(- g^* + R - L_i(d) + E^*_m(i+1,j)\Big) + \frac{\lambda}{\lambda+\mu}(1-f(d))\Big(- g^* + E^*_m(i,j)\Big) + \frac{\mu}{\lambda+\mu} E^*_m(i-1,j)\Big\} \text{ for } i \geq -(s-1) \tag{34}$$

For $i = -s$, we obtain $E^*_m(-s,j)$ as:

$E^*_m(-s,j) = \max_d\{\frac{(-s)\cdot h}{\lambda} + f(d)(-g^* + R + E^*_m(-s+1,j)) + (1-f(d))(-g^* + E^*_m(-s,j))\}$

In (34), first observe that, $\frac{1}{\lambda+\mu}$ is the expected time for an event (a part or a customer arrival) to occur. The system spends $\frac{1}{\lambda+\mu}$ amount of time in state i on expectation, and during that period, the cost of being in state i is incurred. $\frac{\lambda}{\lambda+\mu}$ indicates the probability that a customer arrival occurs before a part arrival. Since this is the modified problem, whenever a customer arrives, g^* is subtracted regardless of the customer's decision. If a customer places an order (with $f(d)$), the next state is $(i+1)$. If the customer does not place an order (with $1 - f(d)$), the next state is i. Finally, $\frac{\mu}{\lambda+\mu}$ indicates the probability a part arrival occurs before a customer arrival. When a part arrives, the next state is $(i-1)$.

Before showing the monotonicity of the optimal policy in Proposition 7, we introduce the following lemmas.

Lemma 1 *$\lambda(d_1)L_i(d_1) - \lambda(d_2)L_i(d_2)$ is increasing in i, for $d_1 < d_2$, $i \geq 0$.*

Lemma 2 *(Puterman 1994, Lemma 4.7.1) Suppose ε is a superadditive function on $I \times D$ and for each $i \in I$, $max_{\{d \in D\}}\ \varepsilon(i,d)$ exists. Then*

$f(i) = max\{argmax_{\{d \in D\}}\ \varepsilon(i,d)\}$

is monotone non-decreasing in i.

Lemma 3 *$E^*_m(i, i-1)$ is decreasing in i, for $i > 0$.*

Proposition 7 *The optimal policy of the modified problem is monotone increasing in i, for $i = 0, 1, \cdots$.*

Proof. There are two steps that must be shown:

1) The due date policy $\mathbf{d}^*$ that attains $E^*_m(i,j)$ is composed of $\mathbf{d}^*_i$, where $\mathbf{d}^*_i$ maximizes $E_m(i, i-1)$.

2) $E^*_m(i, i-1)$ is superadditive.

1) In $E^*_m(i,j)$, let $j = 0$ (which is positive recurrent under optimal policy). We can restrict our analysis to $i \geq 0$, since we are interested in monotonicity of those states. $E^*_m(i,0)$ is as in (34). Let $\lambda(d) = \lambda f(d)$,

$$\begin{aligned}
E^*_m(i,0) = \max_d\{ & \frac{-\lambda \cdot g^*}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu} f(d)(R - L_i(d) + E^*_m(i+1,0)) \\
& + \frac{\lambda}{\lambda+\mu}(1-f(d))E^*_m(i,0) + \frac{\mu}{\lambda+\mu}E^*_m(i-1,0)\} \\
E^*_m(i,0) = \max_d\{ & \frac{-\lambda g^* + \lambda(d)(R - L_i(d) + E^*_m(i+1,0))}{\lambda+\mu} \\
& + \frac{(\lambda - \lambda(d))E^*_m(i,0) + \mu E^*_m(i-1,0)}{\lambda+\mu}\}
\end{aligned}$$

that is,

$$\begin{aligned}
E_m^*(i,0)(\lambda+\mu) \geq & -\lambda g^* + \lambda(d)(R - L_i(d) + E_m^*(i+1,0)) + (\lambda - \lambda(d))E_m^*(i,0) \\
& + \mu E_m^*(i-1,0) \ \ \forall\, d \\
E_m^*(i,0) \geq & \frac{-\lambda g^* + \lambda(d)(R - L_i(d) + E_m^*(i+1,0)) - \lambda(d)E_m^*(i,0)}{\mu} \\
& + E_m^*(i-1,0) \quad \forall\, d,
\end{aligned}$$

$$E_m^*(i,0) = \max_d\{\frac{-\lambda g^* + \lambda(d)(R - L_i(d) + E_m^*(i+1,0)) - \lambda(d)E_m^*(i,0)}{\mu}\} + E_m^*(i-1,0) \tag{35}$$

The system under consideration is skip-free in the negative (positive) direction, i.e, the system cannot pass from state i to a lower (upper) state $j < i$ ($j > i$) without passing through all the intervening states (Keilson 1965). We use the skip-free negative property to write:

$$E_m^*(i,0) = E_m^*(i,i-1) + E_m^*(i-1,0).$$

(For a similar usage of skip-free property, see Wijngaard and Stidham 1986.)

Plugging $E_m^*(i+1,i) + E_m^*(i,0)$ in place of $E_m^*(i+1,0)$ in (35),

$$E_m^*(i,i-1) = \max_d\{\frac{-\lambda g^* + \lambda(d)(R - L_i(d)) + \lambda(d)E_m^*(i+1,i)}{\mu}\}. \tag{36}$$

Therefore $\mathbf{d}^*$ is composed of $\mathbf{d}_i^*$, where $\mathbf{d}_i^*$ maximizes $E_m(i,i-1)$.

2) We show $\mathbf{d}_i^*$ is increasing in i. For this, it is sufficient to show that the term inside parenthesis in (36), say $\varepsilon(i,d)$, $\varepsilon(i,d) = \frac{-\lambda g^* + \lambda(d)(R-L_i(d)) + \lambda(d)E_m^*(i+1,i)}{\mu}$, is superadditive (see Lemma 2). In other words, it is sufficient to show for any two scalars d_1 and d_2, such that, $d_1 < d_2$, $\varepsilon(i,d_2) - \varepsilon(i,d_1)$ is increasing in i, $i > 0$.

$$\varepsilon(i,d_2) - \varepsilon(i,d_1) = \frac{\lambda(d_2)(R-L_i(d_2)) - \lambda(d_1)(R-L_i(d_1))}{\mu} + \frac{(\lambda(d_2)-\lambda(d_1))E_m^*(i+1,i)}{\mu}$$

Note, the second term is increasing in i, since $E_m^*(i+1,i)$ is decreasing in i (Lemma 3), and $\lambda(d_2) - \lambda(d_1)$ is negative. The first term is equal to,

$$\frac{R(\lambda(d_2) - \lambda(d_1)) + \lambda(d_1)L_i(d_1) - \lambda(d_2)L_i(d_2)}{\mu}$$

and $\lambda(d_1)L_i(d_1) - \lambda(d_2)L_i(d_2)$ is increasing in i (Lemma 1). As a result $\varepsilon(i,d)$ is super-additive for $i \geq 0$.

We conclude that, the optimal policy for the original and the modified problem is monotone increasing for states $i \geq 0$. $\square$

Theorem 5 *The optimal policy* $\mathbf{d}^*$ *of the problem in (30) is characterized as,* $d_i^* = 0$ *for* $i < 0$ *and* d_i^* *increasing in i for* $i \geq 0$.

Proof. From Propositions 6 and 7.

4.3.2 The optimal base-stock level, s^*

In this section we obtain insights on the optimal base-stock level, s^*. Let us first write the return **per unit time** under due date policy $\mathbf{d}$ and base-stock level s:

$$g^{\mathbf{d}}(s) = revenue(g^{\mathbf{d}}(s)) - cost(g^{\mathbf{d}}(s))$$
$$= \lambda R \sum_{i=-s}^{N} \pi_i(\mathbf{d},s)\cdot f(\mathbf{d}_i) - h\sum_{i=-s}^{-1} \pi_i(\mathbf{d},s)\cdot(-i) - \lambda l \sum_{i=0}^{N} \pi_i(\mathbf{d},s)\cdot f(\mathbf{d}_i)\cdot L_i(\mathbf{d}_i)$$

Consider a newsboy problem where the overage cost is h, underage cost is l, the order quantity is s, and the demand has a discrete nature with probability distribution $P(D = i) = \pi_{i-s}(\mathbf{d},s)$. We denote the cost of this newsboy problem as below:

$$N^{\mathbf{d}}(s) = h\sum_{i=0}^{s-1}(s-i)\cdot \pi_{i-s}(\mathbf{d},s) + l\sum_{i=s+1}^{\infty}(i-s)\cdot \pi_{i-s}(\mathbf{d},s). \tag{37}$$

Now, consider a system where the due date policy is to quote 0 at every state, which we call as *accept-all* policy. This system has infinite number of states. Whenever $i < 0$, the cost incurred is $i \cdot h$ per unit time, and whenever $i > 0$, the cost incurred is $i \cdot l$ per unit time [2]. We denote the return per unit time under the accept-all policy by $g^0(s)$:

$$g^0(s) = \lambda R - h\sum_{i=-s}^{-1}(-i)\cdot \pi_i(0,s) - l\sum_{i=1}^{\infty} i\cdot \pi_i(0,s) \tag{38}$$

where $\pi_i(0,s) = (1-\lambda)\lambda^{i+s}$.

[2] The reason is, when the due date quoted is 0 at every state, existence of a customer implies a lateness cost is incurred (similar to existence of a part implies an inventory cost is incurred). On the other hand, if a customer is quoted a due date $d > 0$, then the service center does not incur a lateness cost due to this customer for the first d time units after the arrival. In that case even if there are i customers in the system, lateness cost per unit time would be less than $i \cdot l$.

Let us consider the corresponding newsboy problem with demand probability distribution equivalent to the steady-state probability obtained under the accept-all policy. We indicate the cost incurred as $N^0(s)$:

$$N^0(s) = h\sum_{i=0}^{s-1}(s-i)\cdot\pi_{i-s}(0,s) + l\sum_{i=s+1}^{\infty}(i-s)\cdot\pi_{i-s}(0,s). \tag{39}$$

From (38) and (39), $cost(g^0(s))$ is identical to the cost incurred in the newsboy problem, $N^0(s)$. Therefore the optimal base-stock quantity of the due date problem under accept-all policy, say s_1, is equal to the optimal order quantity of the newsboy problem. Note that, under accept-all policy the revenue in the due date problem, λR, is a constant and hence does not have any affect on the optimal base-stock level.

Example 4. Consider a system where $h = 2$/part/unit time, $l = 7$/customer/unit time, $\lambda = 0.7$/unit time and $\mu = 1$/unit time. $\rho = \frac{\lambda}{\mu} = 0.7$.

$P(x) = \pi_{(-s+x)} = (1-\rho)\rho^x$

Define the cumulative distribution function of the demand as $F(x) = P(D \leq x)$. Then $F(s_1) = \frac{7}{7+2} = 0.778$

Table 25: Cumulative distribution function for Example 4

x	$P(x)$	$F(x)$
0	0.3	0.300
1	0.21	0.510
2	0.147	0.657
3	0.103	0.760
4	0.072	0.832
.	.	.

We take the smallest x value where $F(x) \geq \frac{l}{l+h}$. In this example s_1 is 4, and we obtain, $N^{\mathbf{0}}(4) = 8.375$.

Next, we show that s_1 is an upper bound on the optimal base-stock level of the due date problem, s^*. Let $\mathbf{d}^*(s^*)$ be the optimal due date policy with base-stock level s^*. For simplicity we indicate this only with $\mathbf{d}^*$. We show $s^* \leq s_1$ in two steps: (i) given a parameter set, $N^{\mathbf{d}^*}(s)$ has an optimal order quantity $\bar{s} \geq s^*$, (ii) $s_1 \geq \bar{s}$.

Proposition 8 *$\bar{s} \geq s^*$.*

Proof. Consider $N^{\mathbf{d}^*}(s)$. We show that ordering a quantity of s^* results in lower cost than ordering $s^* - 1$, i.e., $N^{\mathbf{d}^*}(s^*) < N^{\mathbf{d}^*}(s^* - 1)$. Since for a given policy $\mathbf{d}$, $N^{\mathbf{d}}(s)$ is convex in s, this implies $\bar{s} \geq s^*$ (for convexity of the cost with respect to s under the newsboy problem, see Nahmias 1997).

First we show that the following inequality,

$$N^{\mathbf{d}}(s) - cost(g^{\mathbf{d}}(s)) < N^{\mathbf{d}}(s-1) - cost(g^{\mathbf{d}}(s-1)) \tag{40}$$

holds under a base-stock s and a due date policy $\mathbf{d}$, where $\mathbf{d}$ corresponds to a policy with first s states quoted 0. That is, if policy $\mathbf{d}$ is employed under base-stock s, then due date quoted is 0 for $i = -s, -(s-1), \cdots, -1$, and if policy $\mathbf{d}$ is employed under base-stock $s-1$, then due date quoted is 0 for $i = -(s+1), \cdots, -1, 0$. Furthermore $\mathbf{d}_i$ is quoted in state i under base-stock s and in state $i+1$ under base-stock $s-1$, $i = -s, -(s-1), \cdots$.

Plugging s^* in place of s, and $\mathbf{d}^*$ in place of $\mathbf{d}$ in (40), we show $N^{\mathbf{d}^*}(s^*) < N^{\mathbf{d}^*}(s^* - 1)$.

$$\begin{aligned} N^{\mathbf{d}}(s) - cost(g^{\mathbf{d}}(s)) = & h \sum_{i=-s}^{-1} \pi_i(\mathbf{d}, s) \cdot (-i) + l \sum_{i=0}^{N} \pi_i(\mathbf{d}, s) \cdot (i) \\ & - h \sum_{i=-s}^{-1} \pi_i(\mathbf{d}, s) \cdot (-i) - \lambda l \sum_{i=0}^{N} \pi_i(\mathbf{d}, s) \cdot f(\mathbf{d}_i) \cdot L_i(\mathbf{d}_i) \end{aligned}$$

Note, the inventory cost incurred per unit time is the same for both problems. Only the lateness cost differs.

$$\begin{aligned} N^{\mathbf{d}}(s) - cost(g^{\mathbf{d}}(s)) &= \lambda l \sum_{i=0}^{N} \pi_i(\mathbf{d}, s) \cdot f(0) \cdot L_i(0) - \lambda l \sum_{i=0}^{N} \pi_i(\mathbf{d}, s) \cdot f(\mathbf{d}_i) \cdot L_i(\mathbf{d}_i) \\ &= \lambda l \sum_{i=0}^{N} \pi_i(\mathbf{d}, s) \cdot [f(0) \cdot L_i(0) - f(\mathbf{d}_i) \cdot L_i(\mathbf{d}_i)] \end{aligned} \tag{41}$$

$N^{\mathbf{d}}(s-1)$ and $cost(g^{\mathbf{d}}(s-1))$ are the costs obtained under due date policy $\mathbf{d}$ and base-stock $s-1$. Since $\mathbf{d}_i$ is quoted in state $i+1$, $\pi_i(\mathbf{d}, s) = \pi_{i+1}(\mathbf{d}, s-1)$, for $i = -s, \cdots, N$.

$$
\begin{aligned}
N^{\mathbf{d}}(s-1) - cost(g^{\mathbf{d}}(s-1)) =& \lambda l \sum_{i=0}^{N+1} \pi_i(\mathbf{d}, s-1) \cdot f(0) \cdot L_i(0) \\
& - \lambda l \sum_{i=0}^{N+1} \pi_i(\mathbf{d}, s-1) \cdot f(\mathbf{d}_{i-1}) \cdot L_i(\mathbf{d}_{i-1}) \\
=& \lambda l \pi_0(\mathbf{d}, s-1) \cdot [f(0) \cdot L_0(0) - f(0) \cdot L_0(0)] \\
& + \lambda l \sum_{i=1}^{N+1} \pi_i(\mathbf{d}, s-1) \cdot [f(0) \cdot L_i(0) - f(\mathbf{d}_{i-1}) \cdot L_i(\mathbf{d}_{i-1})] \\
=& 0 + \lambda l \sum_{i=1}^{N+1} \pi_i(\mathbf{d}, s-1) \cdot [f(0) \cdot L_i(0) - f(\mathbf{d}_{i-1}) \cdot L_i(\mathbf{d}_{i-1})] \\
=& \lambda l \sum_{i=0}^{N} \pi_i(\mathbf{d}, s) \cdot [f(0) \cdot L_{i+1}(0) - f(\mathbf{d}_i) \cdot L_{i+1}(\mathbf{d}_i)]
\end{aligned} \tag{42}
$$

Note the relation between (41) and (42). Since $f(0) \cdot L_i(0) - f(\mathbf{d}_i) \cdot L_i(\mathbf{d}_i) < f(0) \cdot L_{i+1}(0) - f(\mathbf{d}_i) \cdot L_{i+1}(\mathbf{d}_i)$ (by Lemma 1), we conclude that,

$$
\begin{aligned}
N^{\mathbf{d}}(s) - cost(g^{\mathbf{d}}(s)) &< N^{\mathbf{d}}(s-1) - cost(g^{\mathbf{d}}(s-1)) \\
cost(g^{\mathbf{d}}(s-1)) - cost(g^{\mathbf{d}}(s)) &< N^{\mathbf{d}}(s-1) - N^{\mathbf{d}}(s)
\end{aligned}
$$

Next, we need to show, $0 <$cost$(g^{\mathbf{d}}(s-1))-$cost$(g^{\mathbf{d}}(s))$, for $s = s^*$ and $\mathbf{d} = \mathbf{d}^*$. Note that $0 <$cost$(g^{\mathbf{d}}(s-1))-$cost$(g^{\mathbf{d}}(s))$ is equivalent to $0 < g^{\mathbf{d}}(s) - g^{\mathbf{d}}(s-1)$ since $revenue(g^{\mathbf{d}}(s)) = revenue(g^{\mathbf{d}}(s-1))$.

$$
0 < g^{\mathbf{d}^*}(s^*) - g^{\mathbf{d}^*}(s^*-1) \tag{43}
$$

since $g^{\mathbf{d}^*}(s^*)$ corresponds to the optimal profit. (40) and (43) implies $N^{\mathbf{d}^*}(s^*) < N^{\mathbf{d}^*}(s^*-1)$. Therefore $\bar{s}$ can not be less than s^*. □

To show $s_1 \geq \bar{s}$, we introduce the following lemma.

Lemma 4 *Consider two birth-death processes with the same state space $S = \{0, 1, \cdots\}$. Let the transition intensity from state $i-1$ to i be $\lambda_i \geq 0$, and the transition intensity from* *state i to $i-1$ be $\mu_i > 0$, for $i = 1, 2, \cdots$. Define ρ_i^j $(= \frac{\lambda_i^j}{\mu_i^j})$, for $i = 1, 2, \cdots$ and $j = 1, 2$. If $\rho_i^1 \geq \rho_i^2$ for all i and $\rho_i^1 > \rho_i^2$ for at least one i, $i = 0, 1, \cdots < \infty$, then,*

$$\sum_{i=0}^{n} \pi_i^1 < \sum_{i=0}^{n} \pi_i^2 \qquad n = 0, 1, \cdots < \infty,$$

where π_i^j denotes the steady-state probability of state i of process j.

Proposition 9 $s_1 \geq \bar{s}$

Proof. Consider the two newsboy problems $N^0(s_1)$ and $N^{\mathbf{d}^*}(\bar{s})$. Let π^1 and π^2 be the demand probability functions of $N^0(s_1)$ and $N^{\mathbf{d}^*}(\bar{s})$, respectively.

By the definition of s_1,

$$\sum_{i=0}^{s_1} \pi_i^1 \geq \frac{l}{l+h} > \sum_{i=0}^{s_1-1} \pi_i^1, \tag{44}$$

and by definition of $\bar{s}$,

$$\sum_{i=0}^{\bar{s}} \pi_i^2 \geq \frac{l}{l+h} > \sum_{i=0}^{\bar{s}-1} \pi_i^2. \tag{45}$$

Combining (44) and (45),

$$\sum_{i=0}^{s_1} \pi_i^1 \geq \frac{l}{l+h} > \sum_{i=0}^{\bar{s}-1} \pi_i^2$$

This holds only if $s_1 \geq \bar{s}$, since by Lemma 4 $\sum_{i=0}^{\bar{s}-1} \pi_i^1 < \sum_{i=0}^{\bar{s}-1} \pi_i^2$. $\square$

4.4 Comparison of Hybrid and Make-to-Order Systems

An interesting question that arises is whether keeping inventory is more beneficial than operating in a pure make-to-order environment. In this section we conduct experiments to make a comparison of two systems: (i) a *hybrid* system where (optimal) inventory is kept, and (ii) a *make-to-order (MTO)* system where all orders are back-ordered. We observe the conditions under which keeping inventory is preferable, and analyze how the benefits change with respect to the system parameters.

Intuitively, one might think that it would be profitable to keep inventory when revenue per customer, R, is high. This would allow the manufacturer to meet as many customers as possible without losing any portion of the revenue. However this is may not always be the case. Specifically, if $\lambda \leq \frac{h}{h+l}$, then operating in a MTO environment is more beneficial regardless of the R value. Note, when $\lambda \leq \frac{h}{h+l}$, the upper bound on the optimal inventory level is zero, $s_1 = 0$. This implies, if the customer arrival rate is low (that is, if inventory is to be kept over long periods), or if holding cost, h, is high with respect to lateness cost, l, then keeping inventory is more likely to lower the profits.

Before starting the experimental analysis, we present the following proposition on operating under the MTO system versus under a hybrid system.

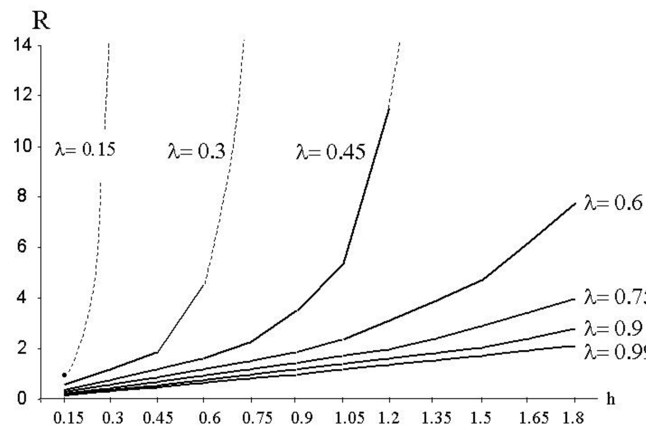


Figure 12: R^* value under varying conditions ($l = 1.5$)

Proposition 10 *Keeping inventory is more profitable than operating under MTO system, if* $\lambda > \frac{h}{h+l}$ *and revenue per customer is greater than* R^**, where* R^* *is the threshold value at which the retailer is indifferent between keeping an inventory level of 1 and keeping no inventory.*

We derive R^* in Appendix B.5. Figure 12 shows the threshold values for different levels of holding cost and customer arrival rates (where lateness cost is constant).

For the experimental analysis, we consider 3 factors that affect the system behavior: (i) revenue per customer, R, (ii) holding cost per unit per unit time, h, and (iii) arrival rate, λ. In the experiments lateness cost per unit per unit time is hold constant.

We perform the experiments for different levels of each factor. For R, h, and λ we test 7 levels each, $R = \{5, 7.5, 10, 15, 25, 50, 100\}$, $h = \{0.15, 0.3, 0.6, 0.9, 1.2, 1.5, 1.8\}$ and $\lambda = \{0.15, 0.3, 0.45, 0.6, 0.75, 0.9, 0.99\}$. We assume $d_{max} = 4$, and the probability of placing an order is $f(d) = 1 - (\frac{d}{d_{max}})^{\frac{1}{4}}$, which is a convex function of d. $f(d)$ being convex implies customers are more sensitive to the changes in due dates when the duration of due date quoted is short. We discretize the action space as $0,\ 0.5,\ 1, \cdots,\ 4$. In practice firms quote discrete due dates rather than continuous ones. Our comparison of this 'incremental' due date policy with an accept-reject policy (where the action set is $\{0, 4\}$) indicates the profit loss due to narrowing the action space, is very small. The highest relative decrease is when $R = 5$ and the % difference is at most 5% this case, with a scalar value of 0.118 (see Figure 13).

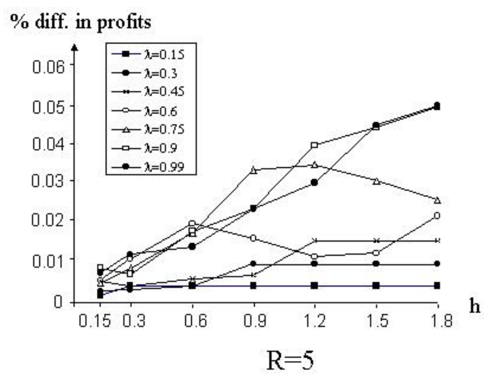


Figure 13: % difference in profits under incremental versus accept-reject policies

We assume $l = 1.5$. Note that if R, h and l are decreased or increased in the same proportion, profit also changes in the same proportion, whereas the optimal base-stock level and due date policy do not change. Therefore by setting $l = 1.5$ we consider the following ratios; $R/l = \{3.33, 5, 6.67, 10, 16.67, 33.33, 66.67\}$, and $h/l = \{0.1, 0.2, 0.4, 0.6, 0.8, 1, 1.2\}$.

The "hybrid system" has optimal base-stock level, s^*, and the "MTO system" has base-stock level 0. We always consider the optimal due date policy unless otherwise stated. Note the profit under the hybrid system is always greater than or equal to the profit under the MTO system. We could not obtain the optimal base-stock level (for a given parameter set) in closed-form. Therefore we make an exhaustive enumerative search of possible base-stock levels. In section 4.3.2 we determined s_1 as an upper bound on the optimal base-stock level. Proposition 11 states that a tighter upper bound may exist.

Proposition 11 *The optimal base-stock level under the accept-reject policy is an upper bound on the optimal base-stock level under the incremental policy.*

The optimal base-stock levels under the incremental and accept-reject policy types when $R = 7.5$ is given in Table 26. Note that the two policy types result in the same optimal base-stock level for all settings under $R = 7.5$, except when $h = 0.3$ and $\lambda = 0.6$. This implies under the current experimental settings, accept-reject policy provides a pretty tight upper bound.

Table 26: Optimal base-stock levels under the incremental policy (left column) and the accept-reject policy (right column) for $R = 7.5$

$\lambda \setminus h$	0.15		0.3		0.6		0.9		1.2		1.5		1.8	
0.15	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0.3	1	1	1	1	1	1	0	0	0	0	0	0	0	0
0.45	2	2	2	2	1	1	1	1	0	0	0	0	0	0
0.6	4	4	**2**	**3**	2	2	1	1	1	1	1	1	1	1
0.75	5	5	4	4	2	2	2	2	1	1	1	1	1	1
0.9	7	7	5	5	3	3	2	2	2	2	1	1	1	1
0.99	9	9	6	6	4	4	3	3	2	2	2	2	1	1

To find the optimal base-stock level under the accept-reject policy an exhaustive search between 0 and s_1 is conducted.

The following performance measures are used in the analysis of experimental results:

1. Long-run average profit per customer:

   average revenue $-$ average inventory cost $-$ average lateness cost

   $= R\sum_{-s}^{N} \pi_i(\mathbf{d})f(\mathbf{d}_i) - h/\lambda \sum_{-s}^{-1} \pi_i(\mathbf{d})(-i) - l\sum_{0}^{N} \pi_i(\mathbf{d})f(\mathbf{d}_i)L_i(\mathbf{d}_i)$

   $=$ (equivalently) $\sum_{-s}^{N} r_i(\mathbf{d}_i)\pi_i(\mathbf{d})$

2. Long-run profit per unit time:

   revenue per unit time $-$ inventory cost per unit time $-$ lateness cost per unit time

$= \lambda R \sum_{-s}^{N} \pi_i(\mathbf{d}) f(\mathbf{d}_i) - h \sum_{-s}^{-1} \pi_i(\mathbf{d})(-i) - \lambda l \sum_{0}^{N} \pi_i(\mathbf{d}) f(\mathbf{d}_i) L_i(\mathbf{d}_i)$

3. Expected number of customers in the system: $\sum_1^N \pi_i(\mathbf{d}) \cdot i$

Based on the experimental results, we make the following observations and provide some intuition to the dynamics behind each one.

Observation 14 *(a) In a MTO environment the average profit per customer decreases as λ increases.*

(b) For a given R, as λ increases and h decreases, the % difference in average profit per customer between an hybrid system and a MTO system increases. (Figure 15.a)

(c) For a given λ, as R and h decreases, the % difference in average profit between a hybrid system and a MTO system increases. (Figure 15.b)

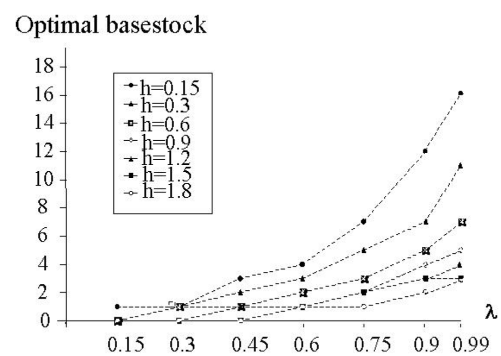


Figure 14: Optimal base-stock level with respect to h and λ

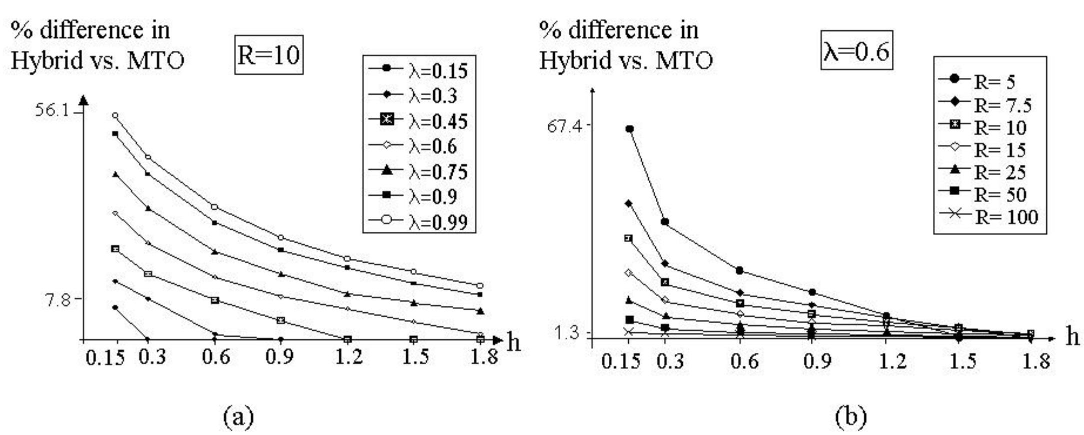


Figure 15: Change in average profit as R or λ changes

(a) Under MTO, since there does not exist any inventory, profit consists of revenue and lateness cost. As arrival rate increases, expected waiting time per customer increases. This causes either an increase in lateness cost per customer or (with an effort to avoid lateness cost) a decrease revenue per customer. This implies average profit per customer decreases as the customer arrival rate increases.

(b) The optimal basestock level increases as λ increases and h decreases (Figure 14). In other words, the hybrid system can make use of high customer arrival rates and low holding cost by increasing the base-stock level accordingly. As a result, the % difference in average profit between an hybrid system and a MTO system increases as λ increases and h decreases (Figure 15.a).

(c) The scalar difference in average profits obtained under the hybrid and MTO systems does not change very much with R. For instance for $\lambda = 0.3$ and $h = 0.15$, the hybrid system increases the profits by 1.13 under $R = 7.5$, and by 1.15 under $R = 50$. Similarly, when $\lambda = 0.45$ and $h = 0.6$, the hybrid system increases the profits by 0.7579 under $R = 15$ and by 0.7667 under $R = 100$. This implies the impact of keeping inventory on average profits is more under low R (Figure 15.b).

Observation 15 *The profit per unit time under the hybrid system increases with λ (Figure 16). When λ is sufficiently large, the increase has diminishing returns.*

An increase in customer arrival rate increases the profit per unit time, since at optimality it is always possible to reject a customer if accepting the customer decrease the profits. On the other hand, for instance under an accept-all policy, the profit per unit time is likely to decrease with increasing λ (Consider the case where, $h = 2$, $l = 7$, and $R = 50$. Under $\lambda = 0.95$, the profit per unit time is 18.89, and under $\lambda = 0.99$, -94.79). Under the optimal policy, the benefits from an increase in customer arrival rate decreases under high arrival rates (Figure 16).

Under high customer arrival rates, intuitively, it would be harder to meet customer demand on-time and an increasing number of customers would be rejected. This implies the average profit per unit customer decreases under high customer arrival rate.

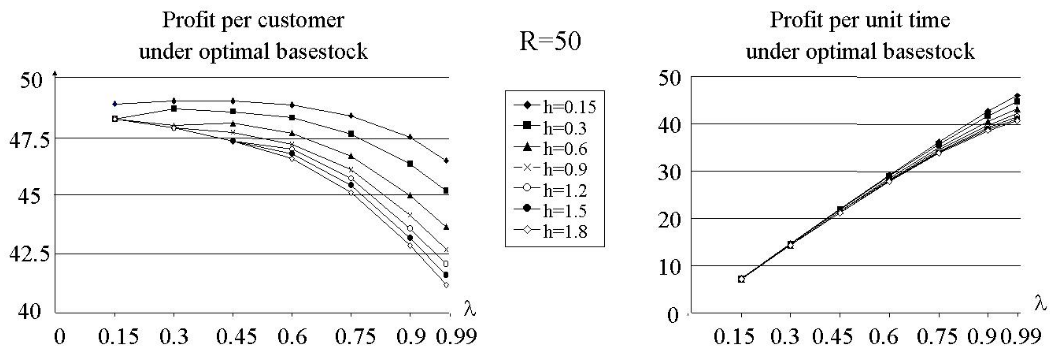


Figure 16: Average profit per customer and profit per unit time under the optimal base-stock level, as a function of λ

Observation 16 *The expected number of customers under the MTO system is greater than or equal to the expected number of customers under the hybrid system.*

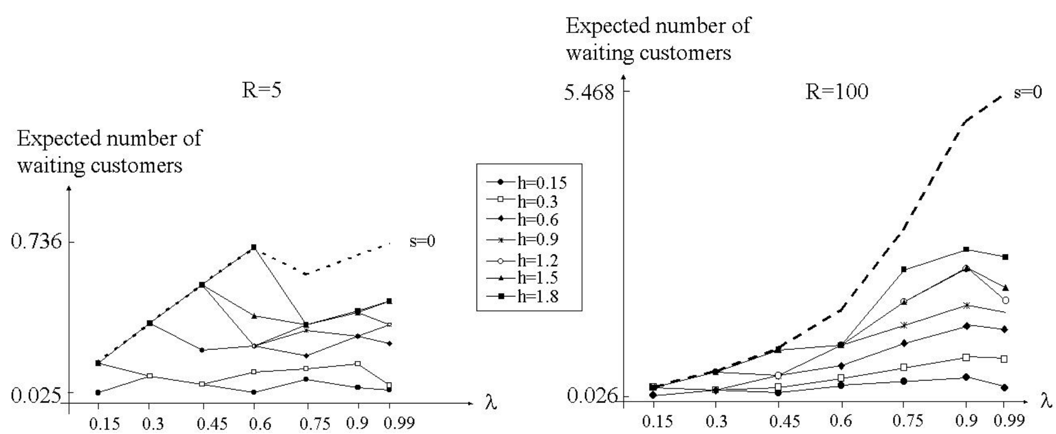


Figure 17: Expected number of customers as a function of λ

Since no inventory is kept under the MTO system, the expected number of customers is always high as compared to the hybrid system. We observe from Figure 17 that the difference in expected number of customers under the two systems is high especially under high R, high λ and low h.

4.5 Concluding Remarks

We considered a manufacturer's dynamic due date quotation problem with an option of keeping base-stock. We found that when there are parts in inventory, the optimal decision is to meet the demand immediately. If there are no parts available, then the optimal due date quoted has a monotone increasing structure with respect to the number of customers in the system. To find the optimal base-stock level, first we determined a general upper bound that applies for any policy type (such as accept-reject, incremental or continuous). We further obtained that the optimal base-stock level under the accept-reject policy is an upper bound on the optimal due date policy under the incremental policy.

In this problem setting we studied whether it is more beneficial to operate under a MTO environment or under a hybrid environment where initial inventory is kept. Analysis indicated that although high R is an incentive to keep initial inventory, if the customer arrival rate is less than a threshold level, then operating under MTO is more beneficial than keeping initial inventory regardless of how high R is. Our comparison of the two systems indicates that the relative benefits of keeping inventory is high if holding cost is low, customer arrival rate is high, and revenue per customer is low. The profit per unit time increases as utilization increases and the increase has diminishing returns when customer arrival rate is high. High utilization levels lead to higher number of customers in the system under the MTO environment especially when the revenue is high.

CHAPTER V

CONCLUSIONS

In this thesis we analyzed horizontal collaborative efforts among supply chain participants in the procurement process and the challenges that are faced by the suppliers in meeting demand. We analyzed horizontal collaboration among buyers in two different settings. In the first setting we considered a market where buyers and suppliers make transactions over multiple items in multiple units (Chapter 2). Buyers are composed of buyer divisions each responsible for a single item. The divisions are located in the same region which enables them to aggregate their demand and obtain savings on the fixed cost of transportation. Similarly, buyer divisions of different buyers that procure the same item may aggregate their demand and share the fixed cost of production charged by the supplier. We compared three models with increasing levels of collaboration: (i) no collaboration; (ii) internal collaboration; and (iii) full collaboration. We tested these models under different market conditions determined by the supply level in the system, fixed cost of transportation and fixed cost of production. The experimental results indicate that when the fixed cost of transportation are high, internal collaboration, which takes place only among the buyer divisions of a single buyer, benefits the buyers. On the other hand, when also fixed cost of production are high and there exists low supply in the system as compared to demand, full collaboration provides the highest benefits.

We would like to briefly discuss how our assumptions of “the buyer divisions being located in the same region” and “each buyer division being responsible for the procurement of one item” affect the results obtained. If these two assumptions are relaxed, the structure of the model and the form of collaboration would change. In this case divisions that belong to different buyers located in the same region could achieve savings from transportation (economies of scope), whereas buyer divisions of the same buyer located at different regions could achieve savings from production by leveraging their purchasing power economies of scale). This could lead to a conclusion that internal collaboration is beneficial when benefits from economies of scale are high. However, full collaboration would still be most beneficial when savings both from the fixed costs of production and transportation are high.

In these collaborative environments, the central mechanism which enables full collaboration (in our study, an e-market intermediary) should understand the implications of antitrust laws. Collaboration in the e-market might give incentives to the participants to collude in the upstream or downstream marketplaces. Therefore intermediaries should keep sensitive information such as output levels, reservation prices, costs or capacity levels, confidential. The total profit obtained under full collaboration provides an upper bound on the profit that would be obtained under less collaborative environments. Thus even if full collaboration is not possible due to antitrust laws or other reasons, the upper bound would provide value information for the intermediary and the participants.

Our results from Chapter 2 suggest that when buyers have roughly equal procurement quantities and compete in different end markets, collaboration is always beneficial for the buyers. To understand the impact of competition on the benefits of collaboration, in Chapter 3 we consider competing buyers, who may procure different quantities due to their own capacity restrictions.

In Chapter 3 we model a supply chain with two buyers and one supplier where buyers collaborate and obtain quantity discounts from the supplier and then engage in a Cournot competition in a secondary (production) market. The buyers decide how many units to procure/sell, and given a discount schedule, whether or not to participate in collaborative procurement. The supplier chooses the amount of the quantity discount. We consider two cases depending on the buyers' procurement capacities. In the first case buyers do not have any limitations on procurement quantities. We find that buyers are always willing to collaborate in this case. In the second case we consider capacitated buyers. We find that a buyer is willing to collaborate if he can increase his procurement quantity under collaboration. Furthermore, we observe that buyers of different size may be willing to collaborate, whereas roughly equally sized buyers may not always benefit from collaboration. When buyers have different sizes, the buyer with higher buying power has more impact on the collaboration process, in the sense that, when collaboration does not take place, it is either due to this buyer or due to the supplier. Finally, since the buyers' procurement quantities increase under collaboration, the price in the secondary market decreases and end consumers are better off.

The model we studied in Chapter 3 is a single-period model. Although such a model has limitations compared to a multi-period model which considers collaboration among buyers over multiple periods, it provides useful insights and is relatively easier to analyze. In our model, buyers do not have to be engaged in a long-term contract with each other or with suppliers to obtain discounts. This model would be applicable, for example, if collaborative purchasing is achieved through a group purchasing organization (GPO), where each buyer signs a contract with the GPO but not necessarily with each other. For example, in Nistevo shippers form groups to share space provided by the carriers and such environments may be represented by a single-period model (although in some cases replication might exist).

Another limitation of our model is the assumption that all parameters such as buyer capacities are known to the participants. In our model, the "buyer" could be a manufacturer. In practice manufacturers have capacity restrictions and it is not difficult to know such physical restrictions. For example, if the buyer is a paper manufacturer, the information on the number of paper machines in a mill and their capacities can be obtained easily. Furthermore, as the buyers join the group purchasing agreement, they usually specify a range for the minimum and maximum amounts they would like to procure, which could be interpreted as capacity.

Finally, many of our results in Chapter 3 depend on the assumption of linear demand curve and the supply cost function. Although both the linear demand and the supply cost function are used in the literature (see Albaek 1990; D'Aspremont and Jacquemin 1988), obtaining insights on more general demand and supply curves would be quite interesting. Analysis of a relatively general demand and supply function is under way.

In a recent survey conducted among 500 manufacturers, 83% of the respondents ranked the ability to meet delivery schedules as the most important criterion for selecting a vendor (Keeping 2002). After studying the buyers' collaborative efforts to lower the procurement costs, in the second part of the thesis we considered the supplier's problem of reliability and responsiveness in meeting demand (Chapter 4).

We considered the supplier's due date quotation decisions based on customers' sensitivity to lead-times, workload status in the system and the long-run affects of accepting a customer on system congestion. As a means to increase responsiveness the supplier considers keeping inventory. In this setting we characterized the supplier's due date decisions and optimal inventory level. Our analysis indicated that meeting demand immediately when there are parts available in stock is the optimal decision. This is intuitive due to the assumption of identical parts. If there are no parts available in stock then the supplier's optimal quoted due date increases as the number of customers in the system increases. To find the optimal base-stock level we conducted enumeration over all possible base-stock levels staring from an upper bound to 0.

After the analysis of the optimal due date policy and base-stock level, we addressed the question whether it is more beneficial to keep inventory or to operate under a make-to-order (MTO) system. The experimental results indicate that when the arrival rate is below or holding cost is above a certain threshold, it is always better to operate under MTO regardless of how high the revenue is. In other cases, benefits of keeping inventory increase under high revenue, high arrival rate and low holding cost.

Our model in Chapter 4 assumes a single part (customer) type and Poisson arrivals and departures. These assumptions can be relaxed in several directions. For instance, the firm may have different customer classes each with different revenues. Furthermore, the firm may have several processing options such as regular processing versus expediting, each with different costs. These variations would affect the optimal base-stock and due date quotation policies; in particular, the firm would benefit from reserving some parts for high-revenue customers. Future work also includes studying the model under general arrivals and departures.

APPENDIX A

ADDENDUM FOR CHAPTER 3

A.1 Assumption A3

We show that for buyer i, given q_j, the total procurement cost is increasing in $q_i \in [0, r_i(q_j)]$ (we assume q_i is bounded above, since from (25), $q_i \leq r_i(q_j)$).

$S_i^J(q_i, q_j)q_i = (c_1 - c_2(q_i + \beta q_j))q_i$

Taking derivative with respect to q_i, we obtain:

$\frac{\partial S_i^J q_i}{\partial q_i} = c_1 - 2c_2 q_i - \beta c_2 q_j$

i) Given $q_j < \bar{q}$, $\frac{\partial S_i^J q_i}{\partial q_i}$ attains its minimum at $r_i(q_j)$, which is the maximum value of q_i (both when buyer i is uncapacitated or capacitated) :

$\frac{\partial S_i^J q_i}{\partial q_i} = c_1 - 2c_2 \frac{a - c_1 - (b - \beta c_2)q_j}{2(b - c_2)} - \beta c_2 q_j$

$= \frac{c_1 b - ac_2 + bc_2 q_j(1-\beta)}{b - c_2}$

Since $q_j > 0$ and $\beta \leq 1$, we obtain $a < c_1 \frac{b}{c_2}$ as a sufficient condition for $\frac{\partial S_i^J q_i}{\partial q_i} > 0$.

ii) Given $q_j \geq \bar{q}$, $q_i = 0$, which implies $S_i^J q_i = 0$.

A.2 Proposition 1

We show that the supplier prefers setting $\beta = \beta_U^*$ and selling (q_U^*, q_U^*) over setting $\beta < \beta^c$ and selling $(0, q_M)$ (or $(q_M, 0)$). For $\beta_U^* = 1$ ($\beta^* \geq 1$) or $\beta^c < 0$ ($b > 2c_2$), there exists a single equilibrium.

1) For $\beta_U^* = \beta^*$, we show that $\pi_S(q_U^*, q_U^*) \geq \pi_S(q_M, 0)$.

$$\pi_S(q_U^*, q_U^*) >^? \pi_S(q_M, 0)$$

$$2(c_1 - c_2 q_U^*(1 + \beta^*))q_U^* >^? (c_1 - c_2 q_M)q_M$$

To simplify the expressions, we use x instead of $\frac{a}{c_1}$ and y instead of $\frac{b}{c_2}$.

$$\frac{2}{3y-(2+\beta^*)}(1-\frac{x-1}{3y-(2+\beta^*)}(1+\beta^*)) >^? \frac{1}{2(y-1)}(1-\frac{x-1}{2(y-1)})$$

Plugging in $\frac{3y(2-x)-2}{x}$ in place of β^* we obtain the following:

$$\frac{2x^2}{(3y-1)(x-1)} >^? \frac{(2y-x-1)}{(y-1)^2}$$

Since by assumption A1, $x > 1$ and $y > 1$, the denominators of both sides are positive and the inequality is equivalent to:

$2x^2(y-1)^2 - (2y-x-1)(3y-1)(x-1) >^? 0$

Rearranging the terms,

$$\Big(2(y-1)^2 + (3y-1)\Big)x^2 - 2y(3y-1)x + (2y-1)(3y-1) >^? 0 \tag{46}$$

Note that since $0 < \beta^c = 2 - \frac{b}{c_2} = 2 - y$, we have $y < 2$. For $1 < y < 2$, the left hand side (LHS) of (46) does not have any real roots, and LHS is positive. Therefore the inequality holds for $1 < y < 2$.

This implies the supplier prefers to setting $\beta = \beta^*_U$ and selling (q^*_U, q^*_U) to setting $\beta < \beta^c$ and selling $(q_M, 0)$.

2) For $\beta^*_U = 0$, we show that $\pi_S(q^I_U, q^I_U) \geq \pi_S(q_M, 0)$. $\beta^*_U = 0$ implies $x > 2 - \frac{2}{3y}$.

Assumption A3 states $x < y$. This implies, for $\beta^* < 0$ to hold, $2 - \frac{2}{3y} < y$, or equivalently $3y^2 - 6y + 2 > 0$ should hold. $3y^2 - 6y + 2$ takes negative values for $y \in [1 - \frac{1}{\sqrt{3}}, 1 + \frac{1}{\sqrt{3}}]$. Since $y > 1$, $y > 1 + \frac{1}{\sqrt{3}}$ is a necessary condition for $\beta^* < 0$. Furthermore since we consider $0 < \beta^c$, we assume $y < 2$.

Now let us check whether $\pi_S(q^I_U, q^I_U) > \pi_S(q_M, 0)$ holds under $1 + \frac{1}{\sqrt{3}} < y < 2$.

$$\pi_S(q^I_U, q^I_U) >^? \pi_S(q_M, 0)$$

$$2q^I_U(c_1 - c_2 q^I_U) >^? q_M(c_1 - c_2 q_M)$$

$$\frac{2(3y-x-1)}{2y-x-1} >^? \Big(\frac{3y-2}{2y-2}\Big)^2$$

which is equivalent to,

$$f(x,y) = 2(3y - x - 1)(2y-2)^2 - (2y - x - 1)(3y-2)^2 >^{?} 0 \tag{47}$$

$f(x,y)$ is linear and increasing in x (since $\frac{\partial f}{\partial x} = y^2 + 4y - 4 > 0$ for $1 + \frac{1}{\sqrt{3}} < y < 2$) and $f(x,y) = 0$ at

$x = -\frac{6y^3 - 23y^2 + 20y - 4}{y^2 - 4y + 4}$

Therefore, if we show that x always takes greater values than this quantity, we show that $f(x,y)$ in (47) is always greater than 0. Since $x > 2 - \frac{2}{3y}$, it is sufficient to show that:

$2 - \frac{2}{3y} > \frac{6y^3 - 23y^2 + 20y - 4}{y^2 - 4y + 4}$

Note that RHS is decreasing in y for $y \in [1 + \frac{1}{\sqrt{3}}, 2]$. Therefore an upper bound on RHS is 1.28 which is attained at $y = 1 + \frac{1}{\sqrt{3}}$. On the other hand, LHS is increasing in y and a lower bound on LHS is $1 + \frac{1}{\sqrt{3}} > 1.28$, which is attained at $y = 1 + \frac{1}{\sqrt{3}}$. We conclude that for $y \in [1 + \frac{1}{\sqrt{3}}, 2]$,

$x > 2 - \frac{2}{3y} > \frac{6y^3 - 23y^2 + 20y - 4}{y^2 - 4y + 4}$

Hence, when $\beta^* < 0$, $\pi_S(q_U^I, q_U^I) > \pi_S(q_M, 0)$, and the supplier prefers to sell (q_U^I, q_U^I) to $(q_M, 0)$.

A.3 Theorems 1 and 2

Proof of Theorem 1

$(K_2 < q_U)$

$\beta > \beta^c$: Note in this case, $q_M < \bar{q}$, $q_j < \bar{q}$, therefore $r_i(q_j) > 0$. From Equation (25), $q_i \leq r_i(q_j),\ i, j = 1, 2, i \neq j$ for all equilibria (q_1, q_2).

For $0 \leq q_2 < q_U$, $q_2 < r_2(r_1(q_2))$. This implies $q_2 < r_2(\min\{r_1(q_2), K_1\})$, i.e., $q_2 < r_2(q_1)$. In that case, since $q_2 =$min$\{K_2, r_2(q_1)\}$ from (25), $q_2 = K_2$. $q_1 =$min$\{r_1(K_2), K_1\}$.

$\beta < \beta^c$: Note in this case, $q_M > \bar{q}$. q_1 can be greater or less than $\bar{q}$. $q_2 \leq r_2(q_1)$ holds only for $q_1 \leq \bar{q}$.

Suppose $q_1 \geq \bar{q}$: The only possible equilibrium quantity for firm 2 is: $q_2 = 0$. Therefore $q_1 = K_1$, $q_2 = 0$ are the equilibrium quantities for the two firms. This case is possible if and only if $K_1 \geq \bar{q}$.

Suppose $q_1 \leq \bar{q}$: For $0 \leq q_2 < q_U$, $q_2 > r_2(r_1(q_2))$ (since $\beta < \beta^c$). We also know from (25) that $q_2 \leq r_2(q_1)$, therefore $q_1 \neq r_1(q_2)$, i.e., $q_1 = K_1$. In this case $q_2 = \min\{K_2, r_2(K_1)\}$. Note, since we consider the case where $K_1 \leq \bar{q}$, $r_2(K_1) \geq 0$.

Therefore the equilibrium under $\beta < \beta^c$ is $(K_1, \min\{K_2, r_2(K_1)^+\})$.

Proof of Theorem 2.

$(q_U \leq K_2 < q_M)$

Since $r_2(r_1(q_U)) = q_U$, and $\min\{q_U, K_i\} = q_U$, Equation (25) implies (q_U, q_U) is an equilibrium. Let us check whether there exist other equilibria.

$\beta > \beta^c$: Note in this case, $q_M < \bar{q}$, $q_j < \bar{q}$, therefore $r_i(q_j) > 0$. From Equation (25), $q_i \leq r_i(q_j)$, $i, j = 1, 2, i \neq j$ for all equilibria (q_1, q_2).

We divide the equilibrium region into two: $0 \leq q_2 < q_U$ and $q_U < q_2$, and check whether there exists any equilibrium in either region.

(i) If $0 \leq q_2 < q_U$, then $q_2 < r_2(r_1(q_2))$. This implies $q_2 < r_2(\min\{r_1(q_2), K_1\})$, i.e., $q_2 < r_2(q_1)$. In that case, since $q_2 = \min\{K_2, r_2(q_1)\}$ from (25), $q_2 = K_2$. This is a contradiction since $q_2 < q_U$.

(ii) If $q_U < q_2$, then $q_2 > r_2(r_1(q_2))$. Since by (25) $q_2 \leq r_2(q_1)$, we conclude that $q_1 \neq r_1(q_2)$, i.e., $q_1 = K_1$. But this is not possible since, if $q_U < q_2$, then $r_1(q_2) < q_U$, i.e., $q_1 < q_U$.

There do not exist any other equilibria under $\beta > \beta^c$ case.

$\beta < \beta^c$: Note in this case, $q_M > \bar{q}$. q_i can be greater or less than $\bar{q}$. $q_j \leq r_j(q_i)$ holds only for $q_i \leq \bar{q}$.

Suppose $q_1 \geq \bar{q}$: The only possible equilibrium quantity for firm 2 is: $q_2 = 0$. Therefore $q_1 = K_1$, $q_2 = 0$ are the equilibrium quantities for the two firms. This case is possible if and only if $K_i \geq \bar{q}$.

Suppose $q_1 \leq \bar{q}$:

(i) If $0 \leq q_2 < q_U$, then $q_2 > r_2(r_1(q_2))$ (since $\beta < \beta^c$). We also know from (25) that $q_2 \leq r_2(q_1)$, therefore $q_1 \neq r_1(q_2)$, i.e., $q_1 = K_1$. In this case $q_2 = \min\{r_2(K_1), K_2\} = r_2(K_1)$. Note, since we consider the case where $K_1 \leq \bar{q}$, $r_2(K_1) \geq 0$.

(ii) If $q_U < q_2$, then $q_2 < r_2(r_1(q_2))$. This implies $q_2 < r_2(\min\{r_1(q_2), K_1\})$, i.e.,

$q_2 < r_2(q_1)$. The only possible equilibrium quantity for firm 2 is K_2. In this case equilibrium quantity for firm 1 is q_1 =min$\{K_1, r_1(K_2)\} = r_1(K_2)$.

Therefore there exist two additional equilibria under $\beta < \beta^c$ case, $(K_1, r_2(K_1)^+)$ and $(r_1(K_2)^+, K_2)$.

A.4 Analysis of Figure 7 and Figure 8

We analyze Case 5 of Figure 7 and Case 18 of Figure 8. Analysis for other cases is similar.

For Case 5, since $b < 2c_2$, the system is in Case B when $\beta = 0$ and in Case A when $\beta = 1$. $K_1 < \frac{a-c_1}{b}$, therefore $r_2(K_1) > 0$, i.e., the quantity procured by buyer 2 is positive for all $\beta \in [0, 1]$. $q_M - \frac{b}{2(b-c_2)}K_1 \leq K_2$ implies, under independent procurement $r_2(K_1) < K_2$, i.e, initially the equilibrium is at E3. $K_2 < q_M - K_1$ implies $\beta_{31} < \beta^c$. Depending on β the system is at E3 or E1.

For Case 18, $K_2 \geq q_U^I$ and under independent procurement the system is in E2. $\frac{1}{K_2} \geq \frac{1}{K_1} + \frac{1}{q_M}$ implies $\beta_2 \leq \beta_{43}$ and the equilibrium switches from E2 to E4 at $\beta = \beta_2$ and from E4 to E3 at β_{43}. Since $K_2 \geq q_M - K_1$, $\beta_{31} > \beta^c$ and at β^c the equilibrium switches from E3 to E5. $K_2 \leq 2(q_M - K_1)$ implies $\beta_{51} < 1$, at which the equilibrium switches from E5 to E1. Depending on β the system is at E2, E4, E3, E5 or E1.

A.5 Proposition 2

Let q_i^I and q_i^J denote the equilibrium quantities under independent and joint procurement, respectively. We introduce the following lemma.

Lemma 5 *If $MR_i^J - MC_i^J \geq MR_i^I - MC_i^I$ and $q_i^J \geq q_i^I$, buyer i is better off or no worse off, and if $MR_i^J - MC_i^J \leq MR_i^I - MC_i^I$ and $q_i^J \leq q_i^I$, buyer i is worse off or no better off under joint procurement as compared to independent procurement.*

Proof (Lemma 5). If $MR_i^J - MC_i^J \geq MR_i^I - MC_i^I$ and $q_i^J \geq q_i^I$, then $\pi_i^J \geq \pi_i^I$:

$$
\begin{aligned}
MR_i^J - MC_i^J &\geq MR_i^I - MC_i^I \\
\Big(a - b(2q_i^J + q_j^J) - c_1 + c_2(2q_i^J + \beta q_j^J)\Big)q_i^J &\geq \Big(a - b(2q_i^I + q_j^I) - c_1 + c_2(2q_i^I)\Big)q_i^I \\
\pi_i^J - q_i^{J2}(b - c_2) &\geq \pi_i^I - q_i^{I2}(b - c_2) \\
\pi_i^J - \pi_i^I &\geq (b - c_2)(q_i^{J2} - q_i^{I2})
\end{aligned} \tag{48}
$$

Last inequality of (48) implies, if $MR_i^J - MC_i^J \geq MR_i^I - MC_i^I$ and $q_i^J \geq q_i^I$, then $\pi_i^J \geq \pi_i^I$. $\square$

We show that for all of the cases in Figures 7 and 8, and for all $\beta \in (0, 1]$, either $MR_i^J - MC_i^J \geq MR_i^I - MC_i^I$ and $q_i^J \geq q_i^I$, or $MR_i^J - MC_i^J \leq MR_i^I - MC_i^I$ and $q_i^J \leq q_i^I$, $i = 1, 2$. We identify the β values (if any), under which $MR_i^J - MC_i^J \geq MR_i^I - MC_i^I$ and $q_i^J \geq q_i^I$.

Note, when β increases, MC decreases, i.e., $MR - MC$ increases for both buyers (assuming the quantities remain the same). Furthermore, if at equilibrium buyer i procures at capacity, $q_i = K_i$, then $MR_i \geq MC_i$; if he procures less than capacity, $q_i = r_i(q_j)$, then $MR_i = MC_i$, and if he does not procure, $q_i = 0$, then $MR_i \leq MC_i$.

(1) When buyer 2 has tight capacity, the system can be in one of the four equilibria (E1, E3, E4, E5) under independent procurement. Depending on the initial equilibrium and β, the system can be in one of the four equilibria (E1, E3, E4, E5) under joint procurement (see Figure 7). For each initial equilibrium, we explain if and when the buyers are better off under joint procurement:

(i) E1: For all $\beta \in (0, 1]$, the equilibrium is E1, i.e., $(q_1^J, q_2^J) = (K_1, K_2)$. $MR_i^J - MC_i^J > MR_i^I - MC_i^I$ and $q_i^I = q_i^J$, $i = 1, 2$. Buyers are better off under joint procurement.

(ii) E5: For buyer 1, under independent procurement $MR_1^I = MC_1^I$. For $\beta \in (0, \beta_{51}]$, the equilibrium is E5, and for $\beta \in [\beta_{51}, 1]$ the equilibrium is $E1$. Under both E1 and E5 (for $\beta \in (0, 1]$), the quantity of buyer 1 increases, $q_1^J > q_1^I$, and $MR_1^J - MC_1^J \geq MR_1^I - MC_1^I = 0$, therefore buyer 1 is better off. For buyer 2, under independent procurement $MR_2^I > MC_2^I$. The equilibrium quantity under joint procurement is the same, $q_2^I = q_2^J = K_2$. If equilibrium under joint procurement is E5,

$(MR_2^J - MC_2^J) - (MR_2^I - MC_2^I) = b \cdot \frac{a - c_1 - bK_2}{2(b - c_2)} - (b - \beta c_2)\frac{a - c_1 - (b - \beta c_2)K_2}{2(b - c_2)} > 0.$

In this case buyer 2 is better off. If equilibrium under joint procurement is E1, $(MR_2^J - MC_2^J) - (MR_2^I - MC_2^I) = b \cdot \frac{a-c_1-bK_2}{2(b-c_2)} - (b - \beta c_2)K_1 > 0$ (since $K_1 < r_1(K_2)$), which implies buyer 2 is better off under joint procurement.

(iii) E3: Under joint procurement the equilibrium is either E3, E5 or E1. This implies for $\beta \in (0, 1]$, $q_2^J > q_2^I$ and $MR_2^J - MC_2^J \geq MR_2^I - MC_2^I = 0$, therefore buyer 2 is better off. Whether buyer 1 is better or worse off depends on the equilibrium under joint procurement. If under joint procurement the equilibrium is:

(a) E3, then $q_1^J = q_1^I$ and $(MR_1^J - MC_1^J) - (MR_1^I - MC_1^I) = \frac{\beta c_2}{2(b-c_2)}(a - c_1 - (2b - \beta c_2)K_1) < 0$. Buyer 1 is worse off.

(b) E5, then $q_1^J < q_1^I$ and $0 = MR_1^J - MC_1^J < MR_1^I - MC_1^I$. Buyer 1 is worse off.

(c) E1, then $q_1^I = q_1^J$. $(MR_1^J - MC_1^J) - (MR_1^I - MC_1^J) = b \cdot r_2(K_1) - (b - \beta c_2) \cdot K_2$. Since $q_1^I = q_1^J$, β determines whether buyer 1 is better or worse off. For $\beta > \beta'$ where $\beta' = \frac{b}{c_2}(1 - \frac{r_2(K_1)}{K_2})$, $MR_1^J - MC_1^J > MR_1^I - MC_1^I$, therefore buyer 1 is better off, whereas for $\beta < \beta'$, buyer 1 is worse off under joint procurement. Note $r_2(K_1)$ corresponds to the procurement quantity of buyer 2 under independent procurement, whereas K_2 is his the procurement quantity under joint procurement.

Note, when $\beta > \beta'$, equilibrium can not be E5 or E3, the system can only be in E1. To see why, suppose $\beta > \beta'$ and the equilibrium is E5. This implies $\beta' < \beta_{51}$. $\beta > \beta'$ implies $(MR_1^J - MC_1^J) - (MR_1^I - MC_1^I) > 0$ when $q_1 = K_1$ and $q_2 = K_2$. However at $\beta_{51} > \beta'$, $q_1 = r_1(K_2) = K_1$ and $(MR_1^J - MC_1^J) - (MR_1^I - MC_1^I) = 0 - (MR_1^I - MC_1^I) < 0$. This is a contradiction, therefore when $\beta > \beta'$, the equilibrium can not be E5. Similar analysis holds for E3.

(iv) E4: $MR_1^I - MC_1^I = (a - c_1) - 2(b - c_2)K_1$, and $MR_2^I - MC_2^I \leq 0$. Under joint procurement the equilibrium is either E1, E3, E4 or E5. If the equilibrium is E4 under joint procurement, buyer 1 and buyer 2 are indifferent. For any of the equilibria E1, E3 or E5 (for $\beta \in (\beta_{43}, 1]$), buyer 1 is worse off since $MR_1^J - MC_1^J < MR_1^I - MC_1^I$ and $q_1^J \leq q_1^I$, whereas buyer 2 is better off since $q_2^J \geq 0$ and $MR_2^J - MC_2 \geq 0$.

We conclude that when buyer 2 has tight capacity, buyer 1 is better off if only if $b > 2c_2$ or $\beta > \beta'$, whereas buyer 2 is always better off.

(2) When buyer 2 has medium capacity under independent procurement, initially the equilibrium is E2, $q_i^I = q_U^I$, and $MR_i^I = MC_i^I,\ i = 1, 2$. The system can be in one of the five equilibria (E1, E2, E3, E4, E5) under joint procurement (see Figure 8). For each possible equilibrium under joint procurement we check if and when the buyers are better off:

(i) If under joint procurement equilibrium is E2 (i.e., if $K_2 \geq \frac{a-c_1}{3(b-c_2)}$ or if $K_2 < \frac{a-c_1}{3(b-c_2)}$ and $\beta \leq \beta_2$), then both buyers are better off, since $MR_i^J = MC_i^J$ and $q_i^J > q_i^I,\ i = 1, 2$.

(ii) If under joint procurement equilibrium is E5 or E1 (i.e, if $\frac{a-c_1}{4(b-c_2)} \leq K_2 < \frac{a-c_1}{3(b-c_2)}$ and $\beta > \beta_2$ or if $K_2 < \frac{a-c_1}{4(b-c_2)}$ and $\beta >$min$\{\beta_{31}, \beta^c\}$), then $MR_i^J - MC_i^J \geq MR_i^I - MC_i^I$ and $q_i^J > q_i^I$, $i = 1, 2$. The buyers are better off under joint procurement.

(iii) If under joint procurement equilibrium is E3, i.e, if,

$K_2 < \frac{a-c_1}{4(b-c_2)}$, $\beta_2 < \beta <$min$\{\beta_{31}, \beta^c\}$,

then $MR_1^J > MC_1^J$ and $q_1^J > q_1^I$ and buyer 1 is better off under joint procurement. Since $MR_2^J = MC_2^J$, buyer 2 is better off under joint procurement if and only if $q_2^J = r_2(K_1) > q_2^I = q_U^I$. The β value at which $r_2(K_1) = q_U^I$ is $\beta'' = \frac{b}{c_2} - \frac{a-c_1-2(b-c_2)q_U^I}{c_2 K_1}$. For $\beta > \beta''$ buyer 2 is better off, whereas for $\beta < \beta'$ buyer 2 is worse off. Note that $\beta'' < \beta_{31}$, since at β_{31}, $r_2(K_1) = K_2 > q_U^I$.

(iv) If under joint procurement equilibrium is E4, then $MR_1^J - MC_1^J > MR_1^I - MC_1^I = 0$, and $q_1^J > q_1^I$, buyer 1 is better off under joint procurement. $MR_2^J - MC_2^J < MR_2^I - MC_2^I$ and $q_2^J < q_2^I$, buyer 2 is worse off under joint procurement.

We conclude that when buyer 2 has medium capacity under independent procurement, buyer 1 is always better off under joint procurement. Buyer 2 is better off if only if $K_2 \geq \frac{a-c_1}{4(b-c_2)}$ or $\beta \leq \beta_2$ or $\beta >$min$\{\beta^c, \beta''\}$. holds.

A.6 Proposition 3

We show that under Cases 11-19, setting $\beta_C^* <$min$\{\beta_2, \beta^c\}$ and selling $(K_1, r_2(K_1)^+)$ is never the optimal decision of the supplier. Whenever $\beta_C^* \leq \beta_2$, optimal decision is to sell q_C^*, where $q_C^* = \frac{a-c_1}{3b-(2+\beta_C^*)c_2}$, to each buyer. When there exist multiple equilibria, we only compare $(K_1, r_2(K_1)^+)$ versus (q_U, q_U), since the analysis is symmetrical for $(r_1(K_2)^+, K_2)$.

We analyze the following cases:

a) $\beta_{43} \leq \beta_2$

b) $\beta_2 < \beta_{43}$

For the proof of Proposition 3, we use the following lemma.

Lemma 6 *For* $\beta < min\{\beta_2, \beta^c\}$*,* $\pi_S^J(q_U, q_U, \beta) > \pi_S^J(K_1, r_2(K_1), \beta)$*.*

Proof (Lemma 6). It is sufficient to show that, for $0 \leq \beta \leq \beta^c$, and $K_1 > q_U$, $\pi_S^J(q_U, q_U, \beta) > \pi_S^J(K_1, r_2(K_1), \beta)$.

For this, we show for $0 < \beta < \beta^c$, $\pi_S^J(K_1, r_2(K_1), \beta)$ is a concave function of K_1, and is maximized at K_1^*, where $K_1^* < q_U = \frac{a-c_1}{3b-(2+\beta)c_2}$. Note, $K_1 > q_U$ for $\beta \in [0, \beta_2]$, since $K_1 > K_2$. Therefore for $0 \leq \beta \leq \beta_2$, $\pi_S^J(q_U, q_U, \beta) > \pi_S^J(K_1, r_2(K_1), \beta)$.

$$
\begin{aligned}
\pi_S^J(K_1, r_2(K_1), \beta) =& \Big(c_1 - c_2(K_1 + \beta r_2(K_1))\Big)K_1 + \Big(c_1 - c_2(\beta K_1 + r_2(K_1))\Big)r_2(K_1) \\
\pi_S^J(K_1, r_2(K_1)) =& c_1(K_1 + r_2(K_1)) - c_2(K_1^2 + r_2(K_1)^2) - 2\beta c_2 K_1 r_2(K_1) \\
\frac{\partial \pi_S^J}{\partial K_1} =& c_1 + c_1\frac{\partial r_2(K_1)}{\partial K_1} - 2c_2K_1 - 2c_2 r_2(K_1)\frac{\partial r_2(K_1)}{\partial K_1} - 2\beta c_2(r_2(K_1) + \frac{\partial r_2(K_1)}{\partial K_1}K_1) \\
=& c_1 + c_1\frac{-(b-\beta c_2)}{2(b-c_2)} - 2c_2K_1 + 2c_2\frac{a-c_1-(b-\beta c_2)K_1}{2(b-c_2)} \cdot \frac{b-\beta c_2}{2(b-c_2)} \\
&- 2\beta c_2\Big(\frac{a-c_1-(b-\beta c_2)K_1}{2(b-c_2)} - K_1\frac{b-\beta c_2}{2(b-c_2)}\Big) \\
=& \frac{c_1(b-(2-\beta)c_2)}{2(b-c_2)} - 2c_2\Big(K_1 - \frac{a-c_1-(b-\beta c_2)K_1}{2(b-c_2)} \cdot \frac{b-\beta c_2}{2(b-c_2)}\Big) \\
&- 2\beta c_2\Big(\frac{a-c_1}{2(b-c_2)} - 2K_1\frac{b-\beta c_2}{2(b-c_2)}\Big) \\
=& \frac{c_1(b-(2-\beta)c_2)}{2(b-c_2)} + \frac{c_2(a-c_1)(b-\beta c_2)}{2(b-c_2)^2} - \frac{c_2\beta(a-c_1)}{(b-c_2)} \\
&- 2c_2K_1\Big(1 + \frac{(b-\beta c_2)^2}{4(b-c_2)^2} - \frac{\beta(b-\beta c_2)}{(b-c_2)}\Big) \\
=& \frac{c_1(b-(2-\beta)c_2)}{2(b-c_2)} + \frac{c_2(a-c_1)(b+\beta c_2-2\beta b)}{2(b-c_2)^2} \\
&- 2c_2\Big(1 + \frac{(b-\beta c_2)^2}{4(b-c_2)^2} - \frac{\beta(b-\beta c_2)}{(b-c_2)}\Big)K_1
\end{aligned}
$$

The coefficient of K_1 is negative:

$$-2c_2\Big(1+\frac{(b-\beta c_2)^2}{4(b-c_2)^2}-\frac{\beta(b-\beta c_2)}{(b-c_2)}\Big)=-c_2\cdot\frac{4(b-c_2)^2+(b-\beta c_2)^2-4\beta(b-\beta c_2)(b-c_2)}{2(b-c_2)^2}$$
$$=-c_2\cdot\frac{(b-(2-\beta)c_2)^2+4(1-\beta)(b-\beta c_2)(b-c_2)}{2(b-c_2)^2}$$

Note, $\frac{\partial^2\pi_S^J}{\partial {K_1}^2}<0$, i.e., $\pi_S(K_1,r_2(K_1),\beta_2)$ is concave in K_1. Let K_1^* be the value at which $\pi_S(K_1,r_2(K_1),\beta)$ is maximized.

We show that $K_1^*<q_U$, for $0<\beta<\beta^c$.

$$K_1^*=\frac{c_1(b-(2-\beta)c_2)(b-c_2)+c_2(a-c_1)(b+\beta c_2-2\beta b)}{c_2\Big((b-(2-\beta)c_2)^2+4(1-\beta)(b-\beta c_2)(b-c_2)\Big)}\leq^?\frac{a-c_1}{3b-(2+\beta)c_2}$$
$$\frac{ac_2(b+\beta c_2-2\beta b)+c_1(b-(2-\beta)c_2)(b-c_2)-c_1c_2(b+\beta c_2-2\beta b)}{c_2\Big((b-(2-\beta)c_2)^2+4(1-\beta)(b-\beta c_2)(b-c_2)\Big)}\leq^?\frac{a-c_1}{3b-(2+\beta)c_2}$$

$$\frac{ac_2(b+\beta c_2-2\beta b)+c_1(b^2-(4-3\beta)bc_2+2(1-\beta)c_2^2)}{c_2\Big((b-(2-\beta)c_2)^2+4(1-\beta)(b-\beta c_2)(b-c_2)\Big)}\leq^?\frac{a-c_1}{3b-(2+\beta)c_2}\tag{49}$$

To simplify the analysis, we assume $c_2=1$ without loss of generality. This implies $1<b<2-\beta$. Let us arrange the terms in the inequality so that,

$$a\Big(\frac{(b+\beta-2\beta b)}{(b-(2-\beta))^2+4(1-\beta)(b-\beta)(b-1)}-\frac{1}{3b-(2+\beta)}\Big)\leq^?\tag{50}$$

$$-c_1\Big(\frac{(b^2-(4-3\beta)b+2(1-\beta))}{(b-(2-\beta))^2+4(1-\beta)(b-\beta)(b-1)}+\frac{1}{3b-(2+\beta)}\Big)$$

The LHS of the inequality is linear in a. Therefore if we check for the lower and upper bounds on a and observe that the inequality holds, then it holds for any a.

From assumption A2, a lower bound on a is c_1. We rewrite (50) as:

$$\frac{(b+\beta-2\beta b)}{(b-(2-\beta))^2+4(1-\beta)(b-\beta)(b-1)}-\frac{1}{3b-(2+\beta)}\leq-\frac{b^2-(4-3\beta)b+2(1-\beta)}{(b-(2-\beta))^2+4(1-\beta)(b-\beta)(b-1)}-\frac{1}{3b-(2+\beta)}$$

Comparing the first terms of LHS and RHS of the inequality:

$$b+\beta-2\beta b<-b^2+(4-3\beta)b-2(1-\beta)$$
$$0<3b-\beta b+\beta-b^2-2$$
$$0<-b^2+(3-\beta)b-(2-\beta)$$
$$0<-(b-(2-\beta))(b-1)$$

Since $1<b<2-\beta$, the inequality holds and this implies (50) holds.

From assumption A3, an upper bound on a is $c_1 b$. Plugging 1 in place of c_2 and $c_1 b$ in place of a in (49):

$$\frac{b(b+\beta-2\beta b)+b^2-(4-3\beta)b+2(1-\beta)}{(b-(2-\beta))^2+4(1-\beta)(b-\beta)(b-1)} \leq \frac{b-1}{3b-(2+\beta)}$$
$$\frac{2(1-\beta)(b-1)^2}{(b-(2-\beta))^2+4(1-\beta)(b-\beta)(b-1)} \leq \frac{b-1}{3b-(2+\beta)}$$
$$2(1-\beta)(b-1)(3b-(2+\beta)) \leq (b-(2-\beta))^2+4(1-\beta)(b-\beta)(b-1)$$
$$2(1-\beta)(b-1)(b+\beta-2) \leq (b+\beta-2)^2$$

Since $b<2-\beta$, LHS is always negative and RHS is always positive, the inequality is satisfied.

Therefore K_1^* is always less than q_U and $\pi_S(q_U,q_U,\beta)>\pi_S(K_1,r_2(K_1),\beta)$, for $0<\beta<\beta_2$ and $K_1>q_U$. $\square$

Now we present the proof of Proposition 3.

Proof (Proposition 3).

a) $\beta_{43}<\beta_2$: (Note $\beta_{43}\leq\beta^c$) For $\beta<\beta_{43}$, one of the equilibria is $(K_1,0)$ and at β_{43}, $(K_1,0)=(K_1,r_2(K_1)$. Since $\beta_3>\beta_{43}$ (see Lemma 7), from Lemma 6, we conclude that for β_C^* <min$\{\beta_2,\beta^c\}$, $\pi_S(q_C^*,q_C^*,\beta_C^*)>\pi_S(K_1,r_2(K_1)^+,\beta_C^*)$ (note, $\beta_{43}\leq\beta^c$).

This analysis corresponds to cases 11-16.

b) $\beta_{43}>\beta_2$: Under this case $\beta^c>\beta_2$ and for $\beta<\beta_2$ one of the equilibria is $(K_1,0)$. For $\beta<\beta_2$, $\pi_S(K_1,0)$ could be greater than $\pi(q_U,q_U,\beta)$. However, since $\beta_3>\beta_{43}$ (Lemma

7), when equilibrium is E3, the profit of the supplier increases. Therefore, again, setting $\beta_C^* <\min\{\beta_2, \beta^c\}$ and selling $(K_1, r_2(K_1)^+)$ is not the optimal decision.

This analysis corresponds to cases 17-19.

A.7 Theorem 3

Before starting the proof, let us first determine the β values that maximize the supplier's profit under each equilibrium:

E1: This is the case, for example, if the system is in Case 1, or if the system is in Case 2 and $\beta \in [\beta_{51}, 1]$. Since under E1 the buyers' procurement quantities do not change with β, the profit function is linearly decreasing in β, and maximized at the minimum β value that is feasible for that case. For example, $\beta = 0$ and $\beta = \beta_{51}$ maximize the supplier's profit under in Case 1 and Case 2, respectively.

E2: This is the case if the system is in Case 11, or if the system is in Cases 12-19 and $\beta \in [0, \beta_2]$. The supplier's profit function under E2 is $\pi_s(q_U, q_U, \beta)$, is maximized at β_U^* (in Equation (24)). Under E2, maximum profit is attained at $\min\{\beta_U^*, \beta_2, 1\}$.

E3: This is the case, for example, if the system is in Case 9 and $\beta \in [\beta_{43}, \beta^c]$. The supplier's profit function under E3, $\pi_s(K_1, r_2(K_1), \beta)$, is concave in β, and from first order conditions, $\pi_s(K_1, r_2(K_1), \beta)$ is maximized at:

$$\beta_3 = \frac{c_1(b-c_2)+(c_2-2b)(a-c_1-bK_1)}{c_2K_1(4b-3c_2)}$$

For example in Case 9, maximum profit is attained at $\min\{\max\{\beta_{43}, \beta_3\}, \beta^c\}$.

E4: Buyer 1 procures at capacity whereas buyer 2 does not procure. The supplier's profit function is constant for all β values.

E5: This is the case, for example, if the system is in Case 15 and $\beta \in [\beta^c, \beta_{51}]$. The supplier's profit function under E5, $\pi_s(r_1(K_2), K_2, \beta)$, is concave in β and from first order conditions, $\pi_s(r_1(K_2), K_2, \beta)$ is maximized at:

$$\beta_5 = \frac{c_1(b-c_2)+(c_2-2b)(a-c_1-bK_2)}{c_2K_2(4b-3c_2)}$$

For example in Case 15, maximum profit is attained a $\min\{\max\{\beta^c, \beta_5\}, \beta_{51}\}$.

(We do not consider E6, since in Proposition 3, we show that $(K_2, 0)$ can never be the optimal quantity tuple that the supplier sells).

We determine β_C^* for all of the Cases 1-19. First, we introduce the following lemma.

Lemma 7 *(1) In Cases 11-13, if* $\beta_U^* < \beta_2$, *then* $\beta_5 < \beta_2$.

(2) In Cases 14-19, if $\beta_U^* > \beta_2$, *then* $\beta_3 > \beta_2$.

(3) a. In Cases 14-16, there exists a discontinuity and the profit function decreases as the equilibrium switches from E2 to E3.

b. In Cases 17-19, there exists a discontinuity and the profit function decreases as the equilibrium switches from E2 to E4, except when $\beta_2 < \beta_U^*$ *and* $\beta_2 < \beta_{43}$.

(4) If $\beta_{43} > 0$, *then* $\beta_{43} < \beta_3$.

(5) In Cases 6, 7, 9, 10, 15, 16, 18, 19, there exists a discontinuity and the profit function increases as the equilibrium switches from E3 to E5.

(6) If $\beta_3 < \beta^c$, *then* $\beta_5 < \beta_3$.

Proof (Lemma 7).

(1) Note in Cases 11-13, $K_2 \geq \frac{a-c_1}{4(b-c_2)}$. The condition that leads to $\beta_U^* < \beta_2$ is:

$$\begin{aligned}
\frac{3b(2c_1-a)-2c_1c_2}{ac_2} &< \frac{(3b-2c_2)K_2-(a-c_1)}{c_2K_2} \\
3b(2c_1-a)-2c_1c_2 &< (3b-2c_2)a - \frac{a(a-c_1)}{K_2} \\
6bc_1-3ab-2c_1c_2-3ab+2ac_2 &< -\frac{a(a-c_1)}{K_2} \\
6b(c_1-a)-2c_2(c_1-a) &< -\frac{a(a-c_1)}{K_2} \\
2(3b-c_2) &> \frac{a}{K_2} \\
K_2 &> \frac{a}{2(3b-2c_2)}
\end{aligned}$$

The condition that leads to $\beta_5 < \beta_2$ is:

$$\frac{c_1(b-c_2)+(c_2-2b)(a-c_1-bK_2)}{c_2K_2(4b-3c_2)} < \frac{(3b-2c_2)K_2-(a-c_1)}{c_2K_2}$$

$$c_1(b-c_2)+(a-c_1)(c_2-2b)-bK_2(c_2-2b) < (3b-2c_2)(4b-3c_2)K_2-(a-c_1)(4b-3c_2)$$

$$c_1(b-c_2)+(a-c_1)(c_2-2b+4b-3c_2) < (10b^2-16bc_2+6c_2^2)K_2$$

$$(2a-2c_1+c_1)(b-c_2) < 2(b-c_2)(5b-3c_2)K_2$$

$$2a-c_1 < 2(5b-3c_2)K_2$$

$$K_2 > \frac{2a-c_1}{2(5b-3c_2)}$$

We simply check whether $K_2 > \frac{a}{2(3b-c_2)}$ and $K_2 \geq \frac{a-c_1}{4(b-c_2)}$ implies $K_2 > \frac{2a-c_1}{2(5b-3c_2)}$, or equivalently, we check $\max\{\frac{a}{2(3b-c_2)}, \frac{a}{4(b-c_2)}\} > \frac{2a-c_1}{2(5b-3c_2)}$:

$$\Big(\frac{a}{2(3b-c_2)}-\frac{2a-c_1}{2(5b-3c_2)}\Big)\Big(\frac{a}{2(2b-2c_2)}-\frac{2a-c_1}{2(5b-3c_2)}\Big) <^? 0$$

$$\Big(a(5b-3c_2)-(2a-c_1)(3b-c_2)\Big)\Big((a-c_1)(5b-3c_2)-(2a-c_1)(2b-2c_2)\Big) <^? 0$$

$$\Big((c_1-a)(3b-c_2)+a(2b-2c_2)\Big)\Big((a-c_1)(3b-c_2)-a(2b-2c_2)\Big) <^? 0$$

$$-\Big((a-c_1)(3b-c_2)-a(2b-2c_2)\Big)^2 <^? 0$$

LHS is always negative, therefore in Cases 11-13, if $\beta_u^* < \beta_2$, $\beta_5 < \beta_2$.

(2) Let us consider β_2 as a function of K_2 and β_3 as a function of K_1. First we introduce the following observation on β_3 and β_2.

Observation 17 *a. β_2 is increasing in K_2.*

b. β_3 may be decreasing or increasing in K_1 depending on the parameters.

Then depending on β_3, there exist two cases: Case(1) The coefficient of $\frac{1}{K_1}$ is positive in β_3, Case(2) The coefficient of $\frac{1}{K_1}$ is negative in β_3.

Case(1). β_3 is decreasing in K_1. We introduce the following lemma.

Lemma 8 *At $K_1 = q_M$, $\beta_3 > \beta^c$.*

Proof.

$$\begin{aligned}
\beta_3 &>^? \beta^c \\
\frac{c_1(b-c_2)+(c_2-2b)(a-c_1-bK_1)}{c_2K_1(4b-3c_2)} &>^? 2-\frac{b}{c_2} \\
c_1(b-c_2)+(c_2-2b)(a-c_1-bK_1) &>^? (2c_2-b)(4b-3c_2)K_1 \\
c_1(b-c_2)+(c_2-2b)(a-c_1) &>^? \Big((2c_2-b)(4b-3c_2)+b(c_2-2b)]K_1 \\
2c_1c_2+2ab-3bc_1-ac_2 &<^? 6(b-c_2)^2K_1
\end{aligned}$$

Plugging q_M in place of K_1:

$$\begin{aligned}
2c_1c_2+2ab-3bc_1-ac_2 &<^? 3(ab-c_1b-ac_2+c_1c_2) \\
2ac_2 &<^? ab+c_1c_2 \\
\frac{a}{c_1} &<^? \frac{c_2}{2c_2-b}
\end{aligned}$$

The inequality holds since $\frac{c_2}{2c_2-b} > \frac{b}{c_2}$, and since from Assumption A3 $\frac{a}{c_1} < \frac{b}{c_2}$. □

Under Case(1), β_3 is decreasing in K_1 and since $K_1 \le q_M$ we conclude that $\beta_3 > \beta^c$ for any given K_1. We consider Cases 14-19 where $\beta_2 < \beta^c$. Therefore $\beta_3 > \beta_2$.

Case(2). Under this case β_3 is increasing, i.e., the coefficient of K_1 is negative. We would like to check whether $\beta_3 > \beta_2$ in this case. For this it is necessary to check whether there exists K_1 and K_2 that satisfy:

$$\begin{aligned}
&\min\ \beta_3-\beta_2 < 0 \\
&\text{st.}\quad q_U^I < K_2 < \text{upper bound} \\
&\qquad\quad K_2 < K_1
\end{aligned}$$

If, for all possible K_1 and K_2, $\beta_3-\beta_2 > 0$ is positive, then we conclude that $\beta_3 > \beta_2$.

$\beta_3-\beta_2 = \frac{c_1(b-c_2)+(c_2-2b)(a-c_1)}{c_2(4b-3c_2)} \cdot \frac{1}{K_1} + \frac{a-c_1}{c_2} \cdot \frac{1}{K_2}$

Lemma 9 below shows that the coefficient of $\frac{1}{K_2}$ is greater than the coefficient of $\frac{1}{K_1}$.

Lemma 9 $\frac{c_1(b-c_2)+(c_2-2b)(a-c_1)}{c_2(4b-3c_2)} + \frac{a-c_1}{c_2} > 0.$

Proof.

$$\begin{aligned}
\frac{a-c_1}{c_2} &> -\frac{c_1(b-c_2)+(c_2-2b)(a-c_1)}{c_2(4b-3c_2)} \\
\frac{a-c_1}{c_2} &> \frac{(2b-c_2)(a-c_1)-c_1(b-c_2)}{c_2(4b-3c_2)} \\
(a-c_1)(4b-3c_2-2b-c_2) &> -c_1(b-c_2) \\
\frac{a}{c_1} &< \frac{3c_2-b}{2(2c_2-b)}
\end{aligned}$$

Since $\frac{3c_2-b}{2(2c_2-b)} > \frac{b}{c_2}$, and since from Assumption A3 $\frac{a}{c_1} < \frac{b}{c_2}$, we conclude that the inequality holds. □

This implies, setting K_2 to its maximum quantity and setting K_1 to K_2 gives the minimum value of $\beta_3 - \beta_2$.

$$\beta_3 >^? \beta_2$$

Plugging K_2 in place of K_1:

$$\begin{aligned}
\frac{c_1(b-c_2)+(c_2-2b)(a-c_1-bK_2)}{4b-3c_2)} &> (3b-2c_2)K_2-(a-c_1) \\
(c_1+2a-2c_1)(b-c_2) &> 2(5b^2-8bc_2+3c_2^2)K_2 \\
(2a-c_1) &> 2(5b-3c_2)K_2
\end{aligned}$$

$$\frac{2a-c_1}{2(5b-3c_2)} > K_2 \tag{51}$$

We check whether this inequality holds for the upper bound of K_2. There exist two upper bounds on K_2: (i) the value at which $\beta_U^* = \beta_2$, $K_2 = \frac{a}{(3b-c_2)}$ (note, for higher values of K_2, $\beta_U^* < \beta_2$), (ii) $K_2 \leq \frac{a-c_1}{4(b-c_2)}$ (since we consider the Cases 14-19). Depending on which upper bound is tighter we obtain two subcases:

(i) $\frac{a}{(3b-c_2)} < \frac{a-c_1}{4(b-c_2)}$, equivalently, $\frac{a}{c_1} > \frac{3b-c_2}{b+c_2}$. Plugging $\frac{a}{(3b-c_2)}$ in (51):

$$\begin{aligned}\frac{2a-c_1}{2(5b-3c_2)} &> \frac{a}{(3b-c_2)}\\ \frac{a}{c_1} &> \frac{3b-c_2}{b+c_2}\end{aligned}$$

(ii) $\frac{a}{(3b-c_2)} > \frac{a-c_1}{4(b-c_2)}$, equivalently, $\frac{a}{c_1} < \frac{3b-c_2}{b+c_2}$. Plugging $\frac{a-c_1}{4(b-c_2)}$ in (51):

$$\begin{aligned}\frac{2a-c_1}{2(5b-3c_2)} &> \frac{a-c_1}{4(b-c_2)}\\ \frac{a}{c_1} &< \frac{3b-c_2}{b+c_2}\end{aligned}$$

We conclude $\beta_3 > \beta_2$. □

(3.a) In Cases 14-16, at β_2, the equilibrium switches from E2 to E3. In Lemma 6, we showed $\pi_S(q_U, q_U, \beta) > \pi_S(K_1, r_1(K_2), \beta)$ for $0 \leq \beta < \beta^c$. Since in Cases 14-16 $\beta_2 < \beta^c$, at β_2 there exists a downward jump in the profit function.

(3.b) In Cases 17-19, at β_2, the equilibrium switches from E2 to E4. Note, $\pi_S(K_1, 0) = \pi_S(K_1, r_1(K_2), \beta_{43}) < \pi_S(q_U, q_U, \beta_{43})$, since $\beta_{43} < \beta^c$. If $\beta_U^* < \beta_2$, then $\pi_S(q_U, q_U, \beta_2) > \pi_S(q_U, q_U, \beta_{43}) > \pi_S(K_1, 0)$. However, if $\beta_2 < \beta_U^*$, whether $\pi_S(q_U, q_U, \beta_2)$ depends on system parameters. Therefore if $\beta_2 < \beta_U^*$ there may be a downward or an upward jump in the profit function when switching from E2.

(4) We check whether $\beta_{43} < \beta_3$, given $\beta_{43} > 0$.

$$\begin{aligned}\frac{bK_1 - (a-c_1)}{c_2K_1} &<^? \frac{c_1(b-c_2) + (c_2-2b)(a-c_1-bK_1)}{c_2K_1(4b-3c_2)}\\ bK_1 - (a-c_1) &<^? \frac{c_1(b-c_2) + (c_2-2b)(a-c_1-bK_1)}{4b-3c_2}\\ -2(a-c_1-bK_1)(b-c_2) &<^? c_1(b-c_2)\end{aligned}$$

$\beta_{43} > 0$ implies $K_1 > \frac{a-c_1}{b}$ and assumption A1 states $b > c_2$. Therefore the inequality holds.

(5) For the proof, we compare $\pi_S(K_1, r_2(K_1), \beta^c)$ and $\pi_S(r_1(K_2), K_2, \beta^c)$. Note, in Cases 6, 7, 9, 10, 15, 16, 18, 19, $K_2 \geq q_M - K_1$.

$$\begin{aligned}
\pi_S(K_1, r_2(K_1), \beta^c) &\leq^? \pi_S(r_1(K_2), K_2, \beta^c)\\
\Big(c_1 - c_2[K_1 + \beta^c(q_M - K_1)]\Big)K_1 &\\
+\Big(c_1 - c_2[q_M - K_1 + \beta^c K_1]\Big)(q_M - K_1) &\leq^?\\
&\Big(c_1 - c_2[K_2 + \beta^c(q_M - K_2)]\Big)K_2\\
&+ \Big(c_1 - c_2[q_M - K_2 + \beta^c K_2])\Big)(q_M - K_2)\\
c_1 q_M - c_2 q_M^2 - 2c_2 K_1^2(1-\beta^c) &\\
+2c_2 K_1 q_M(1-\beta^c) &\leq^? c_1 q_M - c_2 q_M^2 - 2c_2 K_2^2(1-\beta^c)\\
&+ 2c_2 K_2 q_M(1-\beta^c)\\
2(1-\beta^c)c_2(K_1 - K_2)q_M &\leq^? 2(1-\beta^c)c_2(K_1^2 - K_2^2)\\
q_M &\leq K_1 + K_2
\end{aligned}$$

This holds as strict inequality for $K_2 \geq q_M - K_1$, in which case there exists an upward jump in the profit function when switching from E3 to E5. It holds as equality for $K_2 = q_M - K_1$ and in that case, the function is continuous when switching from E3 to E5.

(6) Let us first identify the condition(s) for $\beta_3 = \beta_5$.

$$\begin{aligned}
\frac{c_1(b-c_2) + (c_2 - 2b)(a - c_1 - bK_1)}{c_2 K_1(4b - 3c_2)} &= \frac{c_1(b-c_2) + (c_2 - 2b)(a - c_1 - bK_2)}{c_2 K_2(4b - 3c_2)}\\
c_1(b-c_2)K_2 + (c_2 - 2b)(a - c_1 - bK_1)K_2 &= c_1(b-c_2)K_1 + (c_2 - 2b)(a - c_1 - bK_2)K_1\\
c_1(b-c_2)K_2 + (c_2 - 2b)(a - c_1)K_2 &= c_1(b-c_2)K_1 + (c_2 - 2b)(a - c_1)K_1\\
\Big(c_1(b-c_2) + (c_2 - 2b)(a - c_1)\Big)(K_2 - K_1) &= 0
\end{aligned}$$

For the inequality to hold,

$$c_1(b-c_2)+(c_2-2b)(a-c_1)=0$$
$$a=c_1\frac{3b-2c_2}{2b-c_2}$$

Note, as a decreases both β_5 and β_3 increases, and since $K_2 < K_1$, the amount of increase in β_5 is greater than the amount of increase in β_3.

We find β_3 at $a = c_1\frac{3b-2c_2}{2b-c_2}$:

$$\begin{aligned}
\beta_3 &= \frac{c_1(b-c_2)+(c_2-2b)\Big(c_1\frac{3b-2c_2}{2b-c_2}-c_1-bK_1\Big)}{c_2K_1(4b-3c_2)} \\
&= \frac{c_1(b-c_2)+(c_2-2b)\Big(c_1(\frac{b-c_2}{2b-c_2})-bK_1\Big)}{c_2K_1(4b-3c_2)} \\
&= \frac{(2b-c_2)bK_1}{c_2K_1(4b-3c_2)} \\
&= \frac{(2\frac{b}{c_2}-1)}{(4\frac{b}{c_2}-3)}\cdot\frac{b}{c_2}
\end{aligned}$$

We will show that, β_3 (or β_5) value is always greater than β^c when $a = c_1\frac{3b-2c_2}{c_2-2b}$:

$$\begin{aligned}
\frac{(2\frac{b}{c_2}-1)}{(4\frac{b}{c_2}-3)}\cdot\frac{b}{c_2} &>^? (2-\frac{b}{c_2}) \\
2(\frac{b}{c_2})^2-(\frac{b}{c_2}) &>^? -4(\frac{b}{c_2})^2+11(\frac{b}{c_2})-6 \\
6\Big((\frac{b}{c_2})^2-2(\frac{b}{c_2})+1\Big) &>^? 0 \\
6\Big((\frac{b}{c_2})-1\Big)^2 &>^? 0
\end{aligned}$$

This inequality always holds, i.e., at $a = c_1\frac{3b-2c_2}{2b-c_2}$, $\beta^c < \beta_3 = \beta_5$. For lower values of a, $\beta^c < \beta_3 < \beta_5$, and for higher values of a, $\beta_5 < \beta_3$. Therefore we conclude that if $\beta_3 < \beta_5$, $\beta^c < \beta_3$, or equivalently if $\beta_3 < \beta^c$, $\beta_5 < \beta_3$. □

Now we introduce the proof of Theorem 3.

Proof (Theorem 3). (In accordance with Figure 10)

Cases 1 and 4: The system is in E1. $\beta_C^* = 0$.

Cases 2 and 3: The system passes through E5, and E1 (Case 2). β_C^* =min$\{\beta_5, \beta_{51}, 1\}^+$.

Cases 5 and 8: The system passes through E4 (Case 8), E3 and E1. From Lemma 7.4, $\beta_{43} < \beta_3$ under Case 8. Therefore supplier's profit function is increasing until $\min\{\beta_3, \beta_{31}\}^+$. $\beta_C^* = \min\{\beta_3, \beta_{31}\}^+$.

Cases 6, 7, 9 and 10: The system passes through E4 (Cases 9 and 10), E3, E5, and E1 (Cases 6 and 9). The function is constant under E4. There exist three subcases:

(i) $\beta_3 < \beta^c$: Under E3, the supplier's profit function is maximized at β_3. Lemma 7.5 states there exists an upward jump switching from E3 to E5, and Lemma 7.6 implies under E5 the function is decreasing. The function continues to decrease under E1. $\beta_C^* = \text{argmax}\{\pi_S(\beta_3), \pi_S(\beta^c)\}$.

(ii) $\beta^c < \beta_3$ and $\beta_5 < \beta^c$: Under E3, the profit function is increasing and under E5 the function is decreasing. From Lemma 7.5, there exists an upward jump switching from E3 to E5. $\beta_C^* = \beta^c$.

(iii) $\beta^c < \beta_3$ and $\beta^c < \beta_5$: Under E3, the profit function is increasing and under E5 the function is increasing until $\min\{\beta_5, \beta_{51}\}$. $\beta_C^* = \min\{\beta_5, \beta_{51}, 1\}$.

Cases 11, 12 and 13: The system passes through E2, E5 (Cases 12 and 13) and E1 (Case 12). There exist three subcases:

(i) $\beta_U^* < \beta_2$: Under E2, the supplier's profit function is maximized at β_U^*. Lemma 7.1 implies, under E5 the function is decreasing. Note there does not exist a discontinuity in the profit function as the system switches from E2 to E5, since at β_2, the equilibrium quantities change from $(q_U, q_U) = (K_2, K_2)$ to $(r_1(K_2), K_2) = (K_2, K_2)$. The function keeps decreasing under E1. $\beta_C^* = \beta_U^*$.

(ii) $\beta_U^* > \beta_2$ and $\beta_2 < \beta_5$ (Cases 12 and 13): Under E2, the profit function is increasing. Since $\beta_2 < \beta_5$, under E5 the function keeps increasing until $\min\{\beta_5, \beta_{51}\}$. The function is decreasing under E1. $\beta_C^* = \min\{\beta_5, \beta_{51}, 1\}$.

(iii) $\beta_U^* > \beta_2$ and $\beta_5 < \beta_2$ (Cases 12 and 13): Under E2, the profit function is increasing. Since $\beta_5 < \beta_2$, under E5 the function is decreasing. The function keeps decreasing under E1. $\beta_C^* = \beta_2$.

Cases 14 and 17: The system passes through E2, E4 (Case 17), E3, and E1. There exist two subcases:

(i) $\beta_3 < \beta_2$ (Case 14): From Lemma 7.2, under E2 the system is maximized at β_U^*. As the equilibrium switches from E2 to E3, there is a downward jump (by Lemma 7.3.a). The function decreases under E3 and keeps decreasing under E1. $\beta_C^* = \beta_U^*$.

(ii) $\beta_2 < \beta_3$: Under E2, the system is maximized at $\min\{\beta_U^*, \beta_2\}$. There exists a jump when switching from E2 to E4 or E3. Under E3 the function keeps increasing until $\min\{\beta_3, \beta_{31}\}$. The function decreases under E1.

$\beta_C^* = \text{argmax}\{\pi_S(\min\{\beta_2, \beta_U^*\}), \pi_S(\min\{\beta_3, \beta_{31}\})\}$

Cases 15, 16, 18 and 19: The system passes through E2, E4 (Cases 18 and 19), E3, E5 and E1 (Cases 15 and 18). There exist four subcases:

(i) $\beta_3 < \beta_2$ (Cases 15 and 16): From Lemma 7.2, under E2 the system is maximized at β_U^*. When switching from E2 there exists a downward jump (by Lemma 7.3.a) and the function decreases under E3. At β^c there exists an upward jump (Lemma 7.5). Since $\beta_5 < \beta^c$ by Lemma 7.6, the function decreases in E5 and keeps decreasing in E1. $\beta_C^* = \text{argmax}\{\pi_S(\beta_U^*), \pi_S(\beta^c)\}$.

(ii) $\beta_2 < \beta_3 < \beta^c$: Under E2, the system is maximized at $\min\{\beta_U^*, \beta_2\}$. At β_2 there exists a discontinuity, and the profit function is constant under E4. Under E3, the function increases until β_3. At β^c there exists an upward jump (Lemma 7.5). Since $\beta_5 < \beta^c$ by Lemma 7.6, the function decreases in E5 and keeps decreasing in E1. $\beta_C^* = \text{argmax}\{\pi_S(\min\{\beta_2, \beta_U^*\}), \pi_S(\beta_3), \pi_S(\beta^c)\}$.

(iii) $\beta^c < \beta_3$ and $\beta_5 < \beta^c$: Under E2, the system is maximized at $\min\{\beta_U^*, \beta_2\}$. At β_2 there exists a discontinuity and the profit function is constant under E4. Since $\beta^c < \beta_3$, the function increases under E3. At β^c there exists an upward jump and the profit function decreases under E5 and E1. $\beta_C^* = \text{argmax}\{\pi_S(\min\{\beta_U^*, \beta_2\}), \pi_S(\beta^c)\}$.

(iv) $\beta^c < \beta_3$ and $\beta^c < \beta_5$: Similar to (ii), however when the system is in E5, the function is maximized at $\min\{\beta_5, \beta_{51}\}$. $\beta_C^* = \text{argmax}\{\pi_S(\min\{\beta_U^*, \beta_2\}), \pi_S(\min\{\beta_5, \beta_{51}, 1\}\}$.

A.8 Proposition 4

(1) For Cases 1 and 4, supplier sets $\beta = \beta_C^* = 0$. For cases 2, 3, and 11-13, Proposition 2 indicates buyers are willing to collaborate for any β

(2) In Case 5, buyer 1 collaborates when both buyers procure at capacity (which is realized at β_{31}) and $\beta > \beta'$ (where $\beta' > \beta_{31}$). Note, an upper bound on β_C^* is β_{31}. In this case, if $\beta' > 1$, or equivalently if $1 - \frac{r_2(K_1)}{K_2} > \frac{c_2}{b}$, collaboration does not take place. If $1 - \frac{r_2(K_1)}{K_2} < \frac{c_2}{b}$, then the supplier compares the profit under no collaboration with the profit under $\pi_S(K_1, K_2, \beta')$:

$$\pi_S(K_1, K_2, \beta') >^? \pi_S(K_1, r_1(K_2), 0)$$
$$c_1 >^? c_2(K_2 + r_2(K_1)) + b(2K_1)$$

is the condition that the supplier agrees to compromise and set $\beta = \beta'$.

(3) In Cases 6-10, Proposition 2 indicates buyer 1 does not collaborates.

(4) In Cases 14-19, buyer 1 collaborates, whereas buyer 2 may not collaborate.

(a) If $\beta_C^* \neq \beta_3$, then either $\beta_C^* < \beta_2$, $\beta_C^* = \beta_{31} > \beta''$ or $\beta_C^* > \beta^c$. Proposition 2 indicates buyer 2 is willing to collaborate for those β values. If $\beta_C^* = \beta_3 > \beta''$, then again ,buyer 2 is willing to collaborate.

(b) If $\beta_C^* = \beta_3 < \beta''$, then buyer 2 does not collaborates. Supplier sets β as:

(i) For Cases 14 and 17: $\beta =$argmax$\{\pi_S(\min\{\beta_2, \beta_U^*\}), \pi_S(\beta_{''})\}$

(ii) For Cases 15,16, 18 and 19:

$\beta =$argmax$\{\pi_S(\min\{\beta_2, \beta_U^*\}), \pi_S(\beta''), \pi_S(\beta^c)\}$.

APPENDIX B

ADDENDUM FOR CHAPTER 4

B.1 Proposition 5

First we introduce the following lemma.

Lemma 10 *1) Lateness cost, $L_i(d)$, is convex increasing in i, for $i \geq 0$, for any given $d \in [0, d_{max}]$.*

2) Inventory cost, I_i, is convex decreasing in i, for $i \geq 0$, and bounded by 0 below.

Proof (Lemma 10).

1) $L_i(d)$ is convex increasing in i, for $i \geq 0$:

$$\begin{aligned} L_i(d) =& l \int_d^{\infty} (x-d) E_{i+1} dx \\ =& le^{-d} \sum_{k=0}^{i} (i+1-k) \frac{d^k}{k!} \end{aligned}$$

$L_i(d)$ is increasing in i.

We check whether $L_{i+1}(d) - L_i(d)$ is increasing in i:

$$\begin{aligned} L_{i+1}(d) - L_i(d) =& le^{-d} \Big(\sum_{k=0}^{i+1} (i+2-k) \frac{d^k}{k!} - \sum_{k=0}^{i} (i+1-k) \frac{d^k}{k!} \Big) \\ =& le^{-d} \Big(\sum_{k=0}^{i+1} (i+2-k) \frac{d^k}{k!} - \sum_{k=0}^{i+1} (i+1-k) \frac{d^k}{k!} \Big) \\ =& le^{-d} \sum_{k=0}^{i+1} (i+2-k-i-1+k) \frac{d^k}{k!} \\ =& le^{-d} \sum_{k=0}^{i+1} \frac{d^k}{k!} \end{aligned}$$

$le^{-d}\sum_{k=0}^{i+1}\frac{d^k}{k!}$ is increasing in i, and therefore $L_i(d)$ is convex.

2) I_i is decreasing in i:

(a) For $i \leq -1$,

$$I_{i+1} - I_i = \frac{-1}{\lambda} + \int_0^{\infty}\int_0^t (t-x)E_{s+i+1}\lambda e^{-\lambda t}dxdt$$

Since $\int_0^t (t-x)E_{s+i+1}dx < t$, $\int_0^{\infty}\int_0^t (t-x)E_{s+i+1}\lambda e^{-\lambda t}dxdt < \frac{1}{\lambda}$, and $I_{i+1} - I_i < 0$ for $i \leq -1$. Furthermore the second term decreases in i, therefore $I_{i+1} - I_i$ decreases in i and I_i is concave decreasing for $i \leq -1$.

(b) For $i \geq 0$,

$$I_{i+1} - I_i = \sum_{j=i+1}^{s+i+1}\int_0^{\infty}\int_0^t (t-x)E_j\lambda e^{-\lambda t}dxdt - \sum_{j=i}^{S+i}\int_0^{\infty}\int_0^t (t-x)E_j\lambda e^{-\lambda t}dxdt$$

$$\begin{aligned} I_{i+1} - I_i &= \int_0^{\infty}\int_0^t (t-x)E_{s+i+1}\lambda e^{-\lambda t}dxdt - \int_0^{\infty}\int_0^t (t-x)E_i\lambda e^{-\lambda t}dxdt \\ &= -\frac{(s+1)\mu^{s+i}}{(\lambda+\mu)^{s+i+1}} - \frac{(s)\mu^{s+i-1}\lambda}{(\lambda+\mu)^{s+i+1}} - \frac{(s-1)\mu^{s+i-2}\lambda}{(\lambda+\mu)^{s+i}} - \cdots - \frac{(1)\mu^{i}\lambda}{(\lambda+\mu)^{i+2}} \end{aligned}$$

This term is always negative, therefore I_i is decreasing for $i \geq 0$. Furthermore for $i \geq 0$ $I_{i+1} - I_i$ is increasing in i. Therefore I_i is convex decreasing for $i \geq 0$. □

Proof (Proposition 5). We assume that at optimality the average return is positive. The proof has two steps:

(1) Show that $\exists$ a state N for which $r_i(d_{max}) > r_i(d)$ for $d \in [0, d_{max})$, $i \geq N$.

(2) Show that N is an upper bound on the state where the customer is rejected at optimality.

(1) We show $\exists$ a state N such that $\frac{\partial r_i(d)}{\partial d} > 0$, for $d \in [0, d_{max}]$, $i \geq N$.

$r_i(d) = f(d)(R - L_i(d) - I_{i+1}) + (1 - f(d))(-I_i)$

$$\begin{aligned} \frac{\partial r_i(d)}{\partial d} &= f'(d)(R - L_i(d) - I_{i+1}) + (-\frac{\partial L_i(d)}{\partial d})f(d) - f'(d)(-I_i) \\ &= f'(d)(R - L_i(d) + I_i - I_{i+1}) - f(d)\frac{\partial L_i(d)}{\partial d} \\ &= f'(d)(R - L_i(d) + I_i - I_{i+1}) + f(d)\int_d^{\infty} E_{i+1}dx \end{aligned}$$

$f'(d) < 0$ and from Lemma 10, $(R - L_i(d) + I_i - I_{i+1}) < 0$ for sufficiently large i. Therefore $\frac{\partial r_i(d)}{\partial d} > 0$ for $d \in [0, d_{max})$ and for sufficiently large i.

(2) For a given due date policy $\mathbf{d}$, average return is expressed as (Puterman 1994):

$g^d(s) = \sum_{i=-s}^{\infty} \pi_i(\mathbf{d}) r_i(\mathbf{d}_i)$

where $\pi_i(\mathbf{d})$ is the steady-state probability of being in state i and $\mathbf{d}_i$ is the due date quoted at state i, under due date policy $\mathbf{d}$.

We show that for any state $k \geq N$, rejecting a customer at k yields a higher average return than quoting a due date $d \in [0, d_{max})$ at k and rejecting at $k+1$. Let us assume $k = N$ wlog. Let policy 1 correspond to rejecting in state N and policy 2 correspond to rejecting in $N+1$, all other states being quoted $\mathbf{d}_i$ under both policies. Let π_i^1 and π_i^2 be the steady-state probabilities of state i, and $g_1^{\mathbf{d}}$ and $g_2^{\mathbf{d}}$ be the average returns for policy 1 and policy 2, respectively.

Let $a_{-s} = 1$, and

$$\begin{aligned} a_i =& a_{i-1} f(\mathbf{d}_{i-1})\lambda \\ =& f(\mathbf{d}_{-s}) f(\mathbf{d}_{-s+1}) \cdots f(\mathbf{d}_{i-1}) \lambda^{i+s} \text{ for } -s < i \leq N+1 \end{aligned}$$

Then,

$\pi_i^1 = \frac{a_i}{\sum_{-s}^{N} a_i}$ for $-s \leq i \leq N$

and

$\pi_i^2 = \frac{a_i}{\sum_{-s}^{N} a_i + a_N f(\mathbf{d}_N)\lambda}$, for $-s \leq i \leq N+1$.

where $d_N \in [0, d_{max})$ and $d_{N+1} = d_{max}$. We check

$$g_1^{\mathbf{d}} >^? g_2^{\mathbf{d}}$$

$$\frac{\sum_{-s}^{N-1} a_i r_i(\mathbf{d}_i) + a_N r_N(d_{max})}{\sum_{-s}^{N} a_i} >^? \frac{\sum_{-s}^{N-1} a_i r_i(\mathbf{d}_i) + a_N r_N(d_N) + a_{N+1} r_{N+1}(d_{max})}{\sum_{-s}^{N+1} a_i}$$

$$\frac{\sum_{-s}^{N+1} a_i}{\sum_{-s}^{N} a_i} \Big(\sum_{-s}^{N-1} a_i r_i(\mathbf{d}_i) + a_N r_N(d_{max}) \Big) >^? \sum_{-s}^{N-1} a_i r_i(\mathbf{d}_i) + a_N r_N(d_N) + a_{N+1} r_{N+1}(d_{max})$$

Note $\frac{\sum_{-s}^{N+1} a_i}{\sum_{-s}^{N} a_i} > 1$. We showed in step (1) that $r_N(d_{max}) > r_N(d)$. Since $r_{N+1}(d_{max}) = -I_{i+1} < 0$,

$\sum_{-s}^{N-1} a_i r_i(\mathbf{d}_i) + a_N r_N(d_{max}) > \sum_{-s}^{N-1} a_i r_i(\mathbf{d}_i) + a_N r_N(d_{max}) + a_{N+1} r_{N+1}(d_{max})$.

There exist two possibilities:

1) $\sum_{-s}^{N-1} a_i r_i(\mathbf{d}_i) + a_N r_N(d_{max}) > 0$. In this case $g_1^{\mathbf{d}} > g_2^{\mathbf{d}}$ and at optimality there exists finite number of states.

2) $\sum_{-s}^{N-1} a_i r_i(\mathbf{d}_i) + a_N r_N(d_{max}) < 0$. In this case since $r_i(d) < 0$ for $i \geq N$ and $d \in [0, d_{max}]$, the system can not both have infinite number of states and a positive average return. Therefore N should be an upper bound on the state where customers are rejected.

$\square$

B.2 Lemma 1

Proof. We check whether $\lambda(d_1)\Big(L_{i+1}(d_1) - L_i(d_1)\Big) - \lambda(d_2)\Big(L_{i+1}(d_2) - L_i(d_2)\Big)$ is positive:

$\lambda(d_1) l e^{-d_1} \sum_{k=0}^{i+1} \frac{{d_1}^k}{k!} - \lambda(d_2) l e^{-d_2} \sum_{k=0}^{i+1} \frac{{d_2}^k}{k!}$

This expression is positive since $\lambda(d)$ and $e^{-d} \sum_{k=0}^{i+1} \frac{d^k}{k!}$ are decreasing in d. $\square$

B.3 Lemma 3

Proof. We use backward induction with the following steps:

Step 1. Show that $E_m^*(i, i-1) = E_m^*(i+1, i)$ for $i \geq N$ (N is the upper limit on the state where optimal decision is to quote $d = d_{max}$).

Step 2. Assume $E_m^*(i, i-1) \geq E_m^*(i+1, i)$ for $i = k, k+1, \cdots, N-1$.

Step 3. Show $E_m^*(i, i-1) \geq E_m^*(i+1, i)$ holds for $i = 0, 1, \cdots, k-1$.

Let d_i^* be the due date that maximizes $E_m^*(i, i-1)$, i.e.,

$$E_m^*(i, i-1) = \frac{-\lambda g^* + \lambda(d_i^*)(R - L_i(d_i^*)) + \lambda(d_i^*) E_m^*(i+1, i)}{\mu}$$

Step 1. For $i \geq N$, since $d_i^* = d_{max}$, $\lambda(d_i^*) = 0$, which implies $E_m^*(N, N-1) = E_m^*(N+1, N) = \cdots = \frac{-g^*}{\mu}$.

Step 2. Assume $E_m^*(i, i-1) \geq E_m^*(i+1, i)$ for $i = k, k+1, \cdots, N-1$.

Step 3. Note, for $i = k-1$,

$$E_m^*(k-1, k-2) = max_d\{\frac{-\lambda g^* + \lambda(d)(R - L_{k-1}(d)) + \lambda(d) E_m^*(k, k-1)}{\mu}\} \geq \frac{-\lambda g^* + \lambda(d_k^*)(R - L_{k-1}(d_k^*)) + \lambda(d_k^*) E_m^*(k, k-1)}{\mu} \tag{52}$$

The rhs of the inequality in (52) is greater than $E^*_m(k, k-1)$. The reason is $\lambda(d^*_i)\Big(R - L_{i-1}(d^*_i)\Big) \geq \lambda(d^*_i)\Big(R - L_i(d^*_i)\Big)$ for all i, and $E^*_m(k, k-1) \geq E^*_m(k+1, k)$ by the assumption in Step 2.

The structure of $E^*_m(i, i-1)$ is the same for all $i \geq 0$ (from (34)), therefore induction analysis holds for all $i \geq 0$. This implies $E^*_m(i, i-1)$ is decreasing in i for $i \geq 0$. $\square$

B.4 Lemma 4

Proof.

$\pi^j_0 = \frac{1}{1+\rho^j_1+\rho^j_1\rho^j_2+\cdots+\rho^j_1\rho^j_2\cdots\rho^j_n+\cdots}$ for $j = 1, 2$.

$\pi^j_i = \frac{\rho^j_1\rho^j_2\cdots\rho^j_i}{1+\rho^j_1+\rho^j_1\rho^j_2+\cdots+\rho^j_1\rho^j_2\cdots\rho^j_n+\cdots}$ for $i = 1, 2, \cdots,\ j = 1, 2$.

Note,

$$1 + \rho^1_1 + \rho^1_1\rho^1_2 + \cdots + \rho^1_1\rho^1_2\cdots\rho^1_n + \cdots > 1 + \rho^2_1 + \rho^2_1\rho^2_2 + \cdots + \rho^2_1\rho^2_2\cdots\rho^2_n + \cdots \tag{53}$$

since $\rho^1_i \geq \rho^2_i$ $for\ all$ i and $\rho^1_i > \rho^2_i$ for at least one i. (53) implies $\pi^1_0 < \pi^2_0$.

For any n,

$$\frac{1}{\rho^1_1\rho^1_2\ldots\rho^1_{n+1}} + \frac{1}{\rho^1_2\ldots\rho^1_{n+1}}\cdots + \frac{1}{\rho^1_{n+1}} \leq \frac{1}{\rho^2_1\rho^2_2\ldots\rho^2_{n+1}} + \frac{1}{\rho^2_2\ldots\rho^2_{n+1}}\cdots + \frac{1}{\rho^2_{n+1}}$$
$$\frac{\rho^1_1\rho^1_2\cdots\rho^1_{n+1}}{1+\rho^1_1+\cdots+\rho^1_1\rho^1_2\cdots\rho_n} \geq \frac{\rho^2_1\rho^2_2\cdots\rho^2_{n+1}}{1+\rho^2_1+\cdots+\rho^2_1\rho^2_2\cdots\rho_n}$$

Multiplying both sides by $(1 + \rho^j_{n+2} + \rho^j_{n+2}\rho^j_{n+3} + \cdots)$ and adding 1,

$$
\begin{aligned}
1+\frac{\rho_1^1\rho_2^1\cdots\rho_{n+1}^1}{1+\rho_1^1+\cdots+\rho_1^1\cdots\rho_n}(1+\rho_{n+2}^1+\rho_{n+2}^1\rho_{n+3}^1+\cdots) \\
\geq 1+\frac{\rho_1^2\rho_2^2\cdots\rho_{n+1}^2}{1+\rho_1^2+\cdots+\rho_1^2\cdots\rho_n}(1+\rho_{n+2}^2+\cdots)
\end{aligned}
$$

$$
\frac{1+\rho_1^1+\rho_1^1\rho_2^1+\cdots}{1+\rho_1^1+\rho_1^1\rho_2^1+\cdots+\rho_1^1\cdots\rho_n^1}\geq\frac{1+\rho_1^2+\rho_1^2\rho_2^2+\cdots}{1+\rho_1^2+\rho_1^2\rho_2^2+\cdots+\rho_1^2\cdots\rho_n^2}
$$

$$
\frac{1+\rho_1^1+\rho_1^1\rho_2^1+\cdots+\rho_1^1\rho_2^1\cdots\rho_n^1}{1+\rho_1^1+\rho_1^1\rho_2^1+\cdots}\leq\frac{1+\rho_1^2+\rho_1^2\rho_2^2+\cdots+\rho_1^2\rho_2^2\cdots\rho_n^2}{1+\rho_1^2+\rho_1^2\rho_2^2+\cdots}
$$

$$
\pi_0^1(1+\rho_1^1+\rho_1^1\rho_2^1+\cdots+\rho_1^1\rho_2^1\cdots\rho_n^1)<\pi_0^2(1+\rho_1^2+\rho_1^2\rho_2^2+\cdots+\rho_1^2\rho_2^2\cdots\rho_n^2)
$$

$$
\sum_{i=0}^{n}\pi_i^1<\sum_{i=0}^{n}\pi_i^2 \quad \square
$$

B.5 Proposition 10

Consider a system with inventory level s. If all states are quoted 0, then total revenue per unit time and total lateness cost per unit time (TLC) reach their highest values. Given a base-stock level s, as R increases, optimal policy will be quoting 0 (i.e, accepting an order) in an increasing number of states, and as R goes to infinity, optimal policy will be an accept-all policy. The reason is, under accept-all policy total lateness cost per unit time is bounded above by a constant:

$$
\begin{aligned}
TLC = & l\sum_{i=1}^{\infty}(1-\lambda)\lambda^{s+i}\cdot\ i \\
= & l(1-\lambda)\lambda^{s+1}\sum_{i=1}^{\infty}\lambda^{(i-1)}\cdot\ i \\
= & l(1-\lambda)\lambda^{s+1}\frac{\partial\sum_i^{\infty}\lambda^i}{\partial\lambda} \\
= & l(1-\lambda)\lambda^{s+1}\frac{\partial\frac{\lambda}{1-\lambda}}{\partial\lambda} \\
= & l\frac{\lambda^{s+1}}{1-\lambda}
\end{aligned}
$$

whereas there is no upper bound on the revenue.

As R increases, $g^*(s, R)$, the optimal return per unit time (a function of s and R), approaches to the return per unit time of the accept-all policy, $g^0(s, R)$, while $g^0(s, R)$, which is a linear increasing function of R, is always a lower bound on $g^*(s, R)$. Note $g^*(s, R)$ is continuous in R.

Assuming $s_1 > 0$, let us consider $s-1$ and $s \leq s_1$. We know that $g^0(s, R) > g^0(s-1, R)$ (since $0 < s \leq s_1$). Also note that $g^*(s-1, 0) > g^*(s, 0)$. Since $g^*(s-1, R)$ approaches to $g^0(s-1, R)$ and $g^*(s, R)$ approaches to $g^0(s, R)$, there should be an R value at which $g^*(s-1, R)$ and $g^*(s, R)$ intersect. Call this value, where the retailer is indifferent between keeping inventory of $s-1$ and s, R_s^*. If $R > R_s^*$, the retailer prefers keeping at least s units of inventory (see Figure 18).

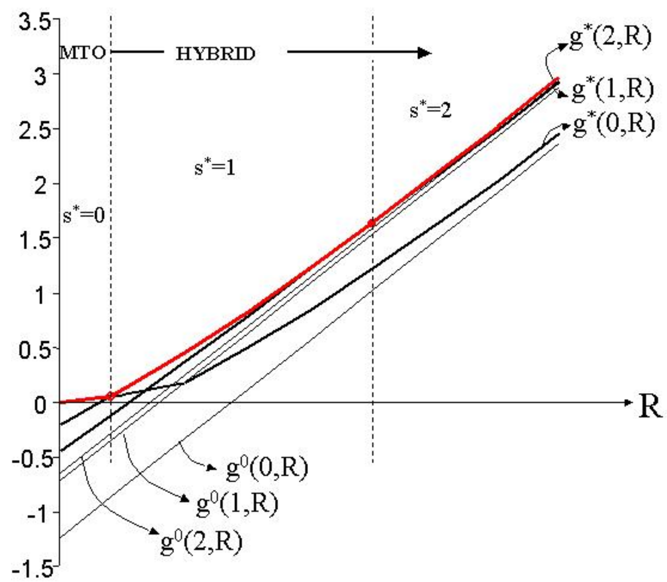


Figure 18: The optimal profit per unit time with respect to R ($h = 0.3$, $l = 1.5$, $\lambda = 0.45$)

The following lemma states, at R_s^* the due date quoted under the optimal policies for the two inventory levels $s-1$ and s, is the same in non-negative states, in other words the two policies are 'equivalent'.

Lemma 11 *Let $\mathbf{d}^{s-1}$ and $\mathbf{d}^s$ be the optimal due date policies at R_s^* when inventory levels are $s-1$ and s respectively. Then $\mathbf{d}^{s-1}$ and $\mathbf{d}^s$ are 'equivalent', i.e., $\mathbf{d}_i^{s-1} = \mathbf{d}_i^s$ for $i = 0, 1, \cdots$.*

Proof. First, we derive an expression for $R_s^{\mathbf{d}}$, where $g^{\mathbf{d}}(s-1, R_s^{\mathbf{d}}) = g^{\mathbf{d}}(s, R_s^{\mathbf{d}})$, given a policy $\mathbf{d}$. For simplicity, $\mathbf{d}$ implies the due dates for states $0, 1, \cdots$, and for negative states the due date quoted is 0. Let us define $\pi_i(\mathbf{d})$ as the steady-state probability of being in state i when base-stock level is $s-1$, and $\pi_i{}'(\mathbf{d})$ as the steady-state probability of being in state i when base-stock level is s.

$$
\begin{aligned}
&g^{\mathbf{d}}(s-1, R_s^{\mathbf{d}}) = g^{\mathbf{d}}(s, R_s^{\mathbf{d}}) \\
&\lambda R_s^{\mathbf{d}} \sum_{-s+1}^{\infty} \pi_i(\mathbf{d}) f(\mathbf{d}_i) - \lambda l \sum_{i=0}^{\infty} \pi_i(\mathbf{d}) f(\mathbf{d}_i) L_i(\mathbf{d}_i) \\
&\qquad -h \sum_{-s+1}^{-1} \pi_i(\mathbf{d})(-i) = \lambda R_s^{\mathbf{d}} \sum_{i=-s}^{\infty} \pi_i{}'(\mathbf{d}) f(\mathbf{d}_i) \\
&\qquad\qquad - \lambda l \sum_{i=0}^{\infty} \pi_i{}'(\mathbf{d}) f(\mathbf{d}_i) L_i(\mathbf{d}_i) - h \sum_{-s}^{-1} \pi_i{}'(\mathbf{d})(-i)
\end{aligned}
$$

$$
\begin{aligned}
&\lambda R_s^{\mathbf{d}} \sum_{-s+1}^{\infty} \Big(\pi_i(\mathbf{d}) - \pi_i{}'(\mathbf{d})\Big) f(\mathbf{d}_i) - \lambda l \sum_{0}^{\infty} \Big(\pi_i(\mathbf{d}) - \pi_i{}'(\mathbf{d})\Big) f(\mathbf{d}_i) L_i(\mathbf{d}_i) \\
&\qquad -h \sum_{-s+1}^{-1} \Big(\pi_i(\mathbf{d}) - \pi_i{}'(\mathbf{d})\Big)(-i) = (\lambda R_s^{\mathbf{d}} - s \cdot h)\pi_{-s}{}'(\mathbf{d})
\end{aligned}
\tag{54}
$$

Note $\pi_i = (\frac{1}{1-\pi_{-s}{}'})\pi_i{}'$. By plugging in $\frac{1}{1-\pi_{-s}{}'}\pi_i{}'$ in place of π_i in (54), we obtain:

$$
\begin{aligned}
&\lambda R_s^{\mathbf{d}} \sum_{-s+1}^{\infty} (\frac{\pi_{-s}{}'}{1-\pi_{-s}{}'})\pi_i{}' f(\mathbf{d}_i) - \lambda l \sum_{0}^{\infty} (\frac{\pi_{-s}{}'}{1-\pi_{-s}{}'})\pi_i{}' f(\mathbf{d}_i) L_i(\mathbf{d}_i) \\
&\qquad -h \sum_{-s+1}^{-1} (\frac{\pi_{-s}{}'}{1-\pi_{-s}{}'})\pi_i{}'(-i) = (\lambda R_s^{\mathbf{d}} - s \cdot h)\pi_{-s}{}'
\end{aligned}
$$

Dividing both sides by $\frac{\pi_{-s}}{1-\pi_{-s}}$ and then adding $(\lambda R_s^{\mathbf{d}} - s \cdot h)\pi_{-s}{}'$,

$$
\lambda R_s^{\mathbf{d}} \sum_{i=-s}^{\infty} \pi_i{}'(\mathbf{d}) f(\mathbf{d}_i) - \lambda l \sum_{i=0}^{\infty} \pi_i{}'(\mathbf{d}) f(\mathbf{d}_i) L_i(\mathbf{d}_i) - h \sum_{-s}^{-1} \pi_i{}'(\mathbf{d})(-i) = \lambda R_s^{\mathbf{d}} - s \cdot h
$$

$$g^{\mathbf{d}}(s, R_s^{\mathbf{d}}) = \lambda R_s^{\mathbf{d}} - s \cdot h \tag{55}$$

Similarly by plugging $(1 - \pi_{-s}{}')\pi_i$ in place of $\pi_i{}'$ in (54),

$$\pi_{-s}{}'\Big(\lambda R_s^{\mathbf{d}} \textstyle\sum_{-s+1}^{\infty} \pi_i(\mathbf{d}) f(\mathbf{d}_i) - \lambda l \sum_{i=0}^{\infty} \pi_i(\mathbf{d}) f(\mathbf{d}_i) L_i(\mathbf{d}_i) - h \sum_{-s+1}^{-1} \pi_i(\mathbf{d})(-i)\Big)$$
$$= (\lambda R_s^{\mathbf{d}} - s \cdot h)\pi_{-s}{}'(\mathbf{d})$$

Dividing both sides by $\pi_{-s}{}'(\mathbf{d})$,

$$R_s^{\mathbf{d}} = \frac{s \cdot h - h \sum_{-s+1}^{\infty} \pi_i(\mathbf{d})(-i) - \lambda l \sum_{0}^{\infty} \pi_i(\mathbf{d}) f(\mathbf{d}_i) L_i(\mathbf{d}_i)}{\lambda\Big(1 - \sum_{-s+1}^{\infty} \pi_i(\mathbf{d}) f(\mathbf{d}_i)\Big)} \tag{56}$$

We use (55) and (56) in showing 'equivalence' of $\mathbf{d}^{s-1}$ and $\mathbf{d}^s$. Suppose $\mathbf{d}_i^{s-1}$ and $\mathbf{d}_i^s$ are not 'equivalent'. Then,

$$g^{\mathbf{d}^s}(s, R_s^*) > g^{\mathbf{d}^{s-1}}(s, R_s^*) \tag{57}$$

since $\mathbf{d}^s$ is the optimal policy for s at R_s^*.

$$g^{\mathbf{d}^s}(s, R_s^*) > g^{\mathbf{d}^{s-1}}(s, R_s^*) = g^{\mathbf{d}^{s-1}}(s, R_s^{\mathbf{d}^{s-1}}) + \lambda(R_s^* - R_s^{\mathbf{d}^{s-1}}) \sum_{-s}^{\infty} \pi_i{}'(\mathbf{d}^{s-1}) f(\mathbf{d}_i^{s-1})$$

From (55),

$$\lambda R_s^* - s \cdot h > \lambda R_s^{\mathbf{d}^{s-1}} - s \cdot h + \lambda(R_s^* - R_s^{\mathbf{d}^{s-1}}) \sum_{-s}^{\infty} \pi_i{}' \mathbf{d}^{s-1} f(\mathbf{d}_i^{s-1})$$
$$0 < \lambda(R_s^* - R_s^{\mathbf{d}^{s-1}})\Big(1 - \sum_{-s}^{\infty} \pi_i{}' \mathbf{d}^{s-1} f(\mathbf{d}_i^{s-1})\Big)$$

This implies $R_s^* > R_s^{\mathbf{d}^{s-1}}$.

On the other hand, $g^{\mathbf{d}^s}(s, R_s^*) = g^{\mathbf{d}^{s-1}}(s-1, R_s^*) > g^{\mathbf{d}^{s-1}}(s, R_s^*)$. $g^{\mathbf{d}^{s-1}}(s-1, R_s^*) > g^{\mathbf{d}^{s-1}}(s, R_s^*)$ implies, at R_s^* base-stock level $s-1$ yields higher profit than base-stock level s under policy $\mathbf{d}^{s-1}$. Therefore the revenue at which the profit functions intersect under policy $\mathbf{d}^{s-1}$, $R_s^{\mathbf{d}^{s-1}}$, is greater than R_s^*. This is a contradiction. Therefore $\mathbf{d}^{s-1}$ is 'equivalent' to $\mathbf{d}^{s-1}$, and this implies:

$R_s^* = R_s^{\mathbf{d}^{s-1}}$, i.e., $g^{\mathbf{d}^{s-1}}(s-1, R_s^*) = g^{\mathbf{d}^{s-1}}(s, R_s^*)$. $\square$

R_s^* is the fixed point of the function in (56), say $k(R)$:

$$k(R) = \frac{s \cdot h - h \sum_{-s+1}^{\infty} \pi_i(\mathbf{d})(-i) - \lambda l \sum_0^{\infty} \pi_i(\mathbf{d}) f(\mathbf{d}_i) L_i(\mathbf{d}_i)}{\lambda \Big(1 - \sum_{-s+1}^{\infty} \pi_i(\mathbf{d}) f(\mathbf{d}_i)\Big)} \tag{58}$$

where $\mathbf{d}$ is the optimal policy under base-stock $s-1$ and revenue R. (Although there does not exist any R term in (58), since $\mathbf{d}$ is a function of R, so is k.)

Note $k(R)$ is a unimodal function and attains its maximum point at R_s^* as in Figure 19.

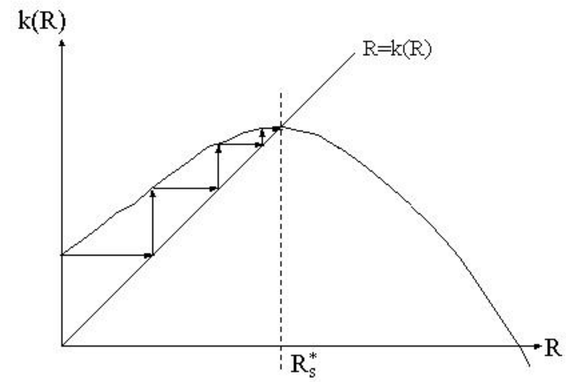


Figure 19: k(R)

Using unimodality of $k(R)$ function we can find R_s^* (with ϵ closeness) as below:

1) Find the optimal policy for $R = 0$ under base-stock $s-1$.

2) Iteratively set $R = k(R)$, find the optimal due date policy and plug in (58), until the difference between two consecutive R values are ϵ.

The threshold R value R^*, that indicates whether operating under the MTO system or a hybrid system is more beneficial, is simply R_1^*.

B.6 Proposition 11

We show that the optimal base-stock level obtained under accept-reject policy with action set $\{0, 4\}$ is an upper bound on the optimal base-stock level obtained under the incremental policy with action set $\{0, 0.5, \cdots, 4\}$.

Let us indicate the optimal profit per unit time under accept-reject policy with $g^{ar}(s)$. Then for any R it holds that, $g^*(s, R) > g^{ar}(s, R) > g^0(s, R)$, and as R increases, $g^*(s, R)$

and $g^{ar}(s,R)$ approach to $g^0(s,R)$. Let the revenue at which $g^{ar}(s-1,R)$ and $g^{ar}(s,R)$ intersect be R_s^{ar}.

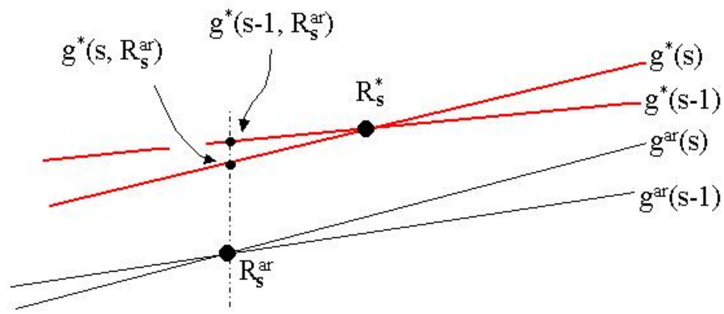


Figure 20: R_s^{ar} and R_s^*

To show that optimal base-stock under the accept-reject policy is an upper bound on the incremental policy, we show $R_s^{ar} < R_s^*$, which is equivalent to the following:

$g^*(s-1,R_s^{ar}) > g^*(s,R_s^{ar})$

(see Figure 20).

First we show $g^{\mathbf{d}}(s-1,R_s^{ar}) > g^{\mathbf{d}}(s,R_s^{ar})(= g^*(s,R_s^{ar}))$, where $\mathbf{d}$ is the optimal incremental due date policy under base-stock level s and revenue R_s^{ar}. For simplicity let us indicate the steady probability in state i under base-stock $s-1$ with π_i, and under base-stock s with $\pi_i{}'$. Again, the relation $\pi_i = (\frac{1}{1-\pi_{-s}{}'})\pi_i{}'$ holds between π_i and $\pi_i{}'$.

$$g^{\mathbf{d}}(s-1,R_s^{ar}) >^? g^{\mathbf{d}}(s,R_s^{ar})$$

$$\lambda R_s^{ar}\sum_{-s+1}^{\infty}\pi_i f(\mathbf{d}_i) - \lambda l\sum_{0}^{\infty}\pi_i f(\mathbf{d}_i)L_i(\mathbf{d}_i) - h\sum_{-s+1}^{-1}\pi_i(-i) >^? \lambda R_s^{ar}\sum_{-s}^{\infty}\pi_i{}' f(\mathbf{d}_i) - \lambda l\sum_{0}^{\infty}\pi_i{}' f(\mathbf{d}_i)L_i(\mathbf{d}_i) - h\sum_{-s}^{-1}\pi_i{}'(-i)$$

Reorganizing the terms,

$$\lambda R_s^{ar}\sum_{-s+1}^{\infty}\Big(\pi_i-\pi_i{}'\Big)f(\mathbf{d}_i) - \lambda l\sum_{0}^{\infty}\Big(\pi_i-\pi_i{}'\Big)f(\mathbf{d}_i)L_i(\mathbf{d}_i) - h\sum_{-s+1}^{-1}\Big(\pi_i-\pi_i{}'\Big)(-i) >^? (\lambda R_s^{ar} - s\cdot h)\pi_{-s}{}'$$

Plugging $(\frac{1}{1-\pi_{-s}{}'})\pi_i{}'$ in place of π_i,

$$\lambda R_s^{ar} \sum_{-s+1}^{\infty} (\tfrac{\pi_{-s}'}{1-\pi_{-s}'}) \pi_i' f(\mathbf{d}_i) - \lambda l \sum_0^{\infty} (\tfrac{\pi_{-s}'}{1-\pi_{-s}'}) \pi_i' f(\mathbf{d}_i) L_i(\mathbf{d}_i)$$
$$-h \sum_{-s+1}^{-1} (\tfrac{\pi_{-s}'}{1-\pi_{-s}'}) \pi_i'(-i) >^? (\lambda R_s^{ar} - s \cdot h) \pi_{-s}'$$

Dividing both sides by $\frac{\pi_{-s}'}{1-\pi_{-s}'}$, and then adding $(\lambda R_s^{ar} - s \cdot h)\pi_{-s}'$,

$$\lambda R_s^{ar} \sum_{i=-s}^{\infty} \pi_i' f(\mathbf{d}_i) - \lambda l \sum_0^{\infty} \pi_i' f(\mathbf{d}_i) L_i(\mathbf{d}_i) - h \sum_{-s}^{-1} \pi_i'(-i) >^? \lambda R_s^{ar} - s \cdot h$$

Note the left-hand-side of the inequality is equal to $g^{\mathbf{d}}(s, R_s^{ar})$. From equation (55), right-hand-side is equal to $g^{ar}(s, R_s^{ar})$. We obtain,

$$g\mathbf{d}(s, R_s^{ar}) >^? g^{ar}(s, R_s^{ar})$$

This inequality holds since for a given R, optimal profit of the accept-reject policy is a lower bound on the optimal return of the incremental policy. Therefore,

$$g^*(s-1, R_s^{ar}) \geq g\mathbf{d}(s-1, R_s^{ar}) > g^{ar}(s, R_s^{ar})$$

which implies $R_s^{ar} < R_s^*$ and optimal base-stock under the accept-reject policy is an upper bound on the optimal base-stock under the incremental policy. $\square$

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