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GLOBAL STABILITY IN A REGULATED LOGISTIC GROWTH MODEL

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Abstract. We investigate global stability of the regulated logistic growth model (RLG) $n'(t) = rn(t)(1-n(t-h)/K-cu(t)), u'(t) = -au(t)+bn(t-h)$. It was proposed by Gopalsamy and Weng [1, 2] and studied recently in [4, 5, 6, 9]. Compared with the previous results, our stability condition is of different kind and has the asymptotical form. Namely, we prove that for the fixed parameters K and $\mu = bcK/a$ (which determine the levels of steady states in the delayed logistic equation $n'(t) = rn(t)(1 - n(t - h)/K)$ and in RLG) and for every $hr < \sqrt{2}$ the regulated logistic growth model is globally stable if we take the dissipation parameter a sufficiently large. On the other hand, studying the local stability of the positive steady state, we observe the improvement of stability for the small values of a: in this case, the inequality $rh < \pi(1 + \mu)/2$ guaranties such a stability.

1. **Introduction.** This paper is inspired by the recent work [9], where an analog of the 3/2-stability criterion was established for the regulated logistic growth model

$$\begin{cases} n'(t) = & rn(t)\,(1 - n(t-h)/K - cu(t))\,, \\ u'(t) = & -au(t) + bn(t-h). \end{cases} \tag{1}$$

Here $(n, u) \in \mathbb{R}^2_+$ and all the parameters r, K, h, c, a, b are positive. In [9], proving the global stability of the positive equilibrium of (1) for $rh \le 3/2(1 - bcK/a)$, the authors have found the sharpest global stability condition for (1) ever reported before (see Table 1 below). In the limit case when $bc = 0$, the above inequality takes the form $rh \le 3/2$ coinciding with the well-known result by Wright for the delayed logistic growth equation

$$n'(t) = rn(t)(1 - n(t - h)/K). \tag{2}$$

Comparing (2) and (1), it is natural to suppose that the values of r, h, K are fixed and that the positive numbers a, b, c are regulating parameters. Actually, for the first time system (1) was proposed in [1, 2] to control the equilibrium level of a population modelled by Eq. (2): using the parameter $\mu = bcK/a$ one can move

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