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THE FUNDAMENTALS OF FLOTATION DEINKING

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ABSTRACT

Flotation deinking is a separation process in which swarms of air bubbles are injected into a relatively low consistency pulp slurry so that hydrophobic contaminant particles attach to the hydrophobic bubble surface. As the bubbles rise, they carry the contaminants to the surface where they are removed from the system. Understanding the fundamentals of this complex process will help in developing flotation models that can be used in the mill to determine if a given process change will help or hinder flotation performance before an expensive mill trial is attempted. In addition, knowledge of these fundamentals can also be used to improve equipment performance.

The purpose of this paper is to review the fundamentals of flotation separation. Since flotation use in paper recycling was adopted from the mineral processing industry, flotation deinking will be compared to mineral flotation. The fundamentals of mineral flotation, when applied to flotation deinking, are assumed to be similar, and the fundamental microprocesses related to flotation will be reviewed. These microprocesses, including capture (or interception), attachment by sliding, three-phase contact, and stability, are typically utilized to model flotation separation, and selected models will be summarized. Finally, the bridge between the fundamentals of flotation deinking and the current technology used in the pulp and paper industry will be briefly reviewed.

KEY WORDS:
flotation deinking, flotation microprocesses, flotation modeling

INTRODUCTION

Although Ortner [1] indicates that flotation was utilized in the paper processing industry to remove ink particles from secondary fibers well before the 1950s, it was not until then that this practice, which was adopted from mineral processing, became well accepted. The mineral processing industry provides much useful information regarding flotation separation in terms of fundamentals and overviews (see, for example, [2, 3]). The use of flotation in paper recycling has been summarized in [1, 4-6]. These reviews focus primarily on applications and equipment. The purpose of this paper is to review the current state of flotation fundamentals and bridge the theory with some of the newest equipment available for flotation deinking.

In flotation deinking, air bubbles rise through agitated liquid tanks containing suspended cellulose pulp and contaminant particles and preferentially attach to hydrophobized contaminant and/or ink particles that typically fall within the size range of approximately 20 to 300 μm in diameter. These particles are then transported to a froth layer where they may be easily removed. Although flotation cell designs may vary with respect to their geometry, flow configurations, and operating parameters, they all operate on similar principles and incorporate the fundamental processes of pulp aeration, mixing to maximize bubble/particle interaction, and separation of bubble/particle aggregates from the bulk mixture.

The focus of this paper will be in the bubble/particle interaction and the formation of a stable bubble/particle aggregate that can be transported to the froth layer for removal. Before these issues are addressed in detail, the commonalities and differences between mineral flotation and flotation deinking will be briefly summarized. The flotation microprocesses, including capture, attachment by sliding, three-phase contact, and stability will then be presented. The current status of flotation modeling will also be presented. This review will conclude by making a connection between the fundamental processes involved in flotation and how they are utilized in some of the newer flotation technology available for use in deinking mills.

MINERAL FLOTATION AND FLOTATION DEINKING

Much of the theoretical and experimental work related to the fundamental flotation microprocesses has been developed explicitly for mineral flotation. However, many of the fundamentals are assumed to be applicable to flotation deinking if one is aware of the main similarities and differences between these two processes. Some of the similarities and differences are presented in Table 1 [7]. The most significant differences between flotation deinking and mineral processing are that in deinking: the particle density is relatively low and may be comparable to that of water; there may be a wide distribution of particle size and character (i.e., surface energy, geometry, etc.); and the pulp suspension is composed of cellulose fibers that can form a network [7, 8]. In addition, both the contaminants to be removed and the pulp slurry may be highly heterogeneous.

Table 1: A comparison between flotation deinking and mineral flotation (adapted from [7]).

Parameter	Flotation Deinking	Mineral Flotation
Particle surface energy	very complex - low energetic, hydrophobic adhesives - medium energetic, hydrophobic inks - high energetic, hydrophilic filler and fibers	fairly uniform - high energetic, hydrophilic
Particle size	broad distribution	broad distribution
Particle density	very low, in some cases lower than water	broad distribution, but higher than water
Particle liberation from stock	by repulping in the presence of chemicals and surfactants	by grinding without added chemicals
Pulp properties	- very heterogeneous - fiber networks - high temperature (40-60°C) - possible redeposition of particles	- homogeneous - temperature is low - redeposition of particles is unimportant
Final product characterization	by means of - paper sheet brightness - dirt speck analysis - adhesive content	by means of - chemical analysis
Impact of efficiency on additional processing steps	contaminants that are not removed can - damage downstream equipment - reduce product quality	not sensitive

These differences result in separation efficiencies being much lower in flotation deinking than those observed in mineral flotation [9].

Despite these differences, it has been common to assume the fundamentals of each flotation separation process are similar, with the exception of inertia effects, which are typically neglected in flotation deinking. Therefore, the various microprocesses applicable to mineral processing are utilized to describe the relevant events taking place in a flotation deinking cell.

THE FLOTATION MICROPROCESSES

The prevalent viewpoint that has been taken in modeling the overall flotation separation process is that it is a multistage probability process consisting of a sequence of microprocesses with associated probability measures. This sequence includes the approach of a particle to an air bubble, the subsequent interception of that particle by the bubble, the sliding of the particle along the surface of the thin liquid film that separates the particle from the bubble, film rupture, the subsequent formation of a three-phase contact between the bubble, particle, and film, and the stabilization of the bubble/particle aggregate (with its subsequent transport to the froth layer for removal from the flotation cell). These microprocesses take place in all flotation equipment.

Probability measures, which are associated with some of the elementary microprocesses referenced above, have appeared in many places in the literature (e.g., [2, 9-12]), while mathematical descriptions of one or more of these microprocesses have appeared in these references as well as in [13-25]. The majority of these models assume that the bubble and particle are spherical in nature. In addition, most of the microprocess analyses address one particle interacting with one bubble. The effects of multiple particles interacting with a single bubble [26, 27] and bubble swarms [26, 28], and their influence on the flotation microprocesses complicate the analysis considerably.

In general, the process of collecting contaminant particles by bubbles in a pulp slurry takes place in a complex and highly turbulent environment. However, as the distance separating the particle from the bubble decreases, the flow conditions relative to the bubble/particle pair are typically idealized and simplified to correspond to unperturbed flow [17]. This allows for considerable simplifications that are universally employed in modeling the flotation microprocesses. These microprocesses, involving capture, attachment by sliding, three-phase contact, and stability, will now be reviewed.

Capture

A particle must travel close enough to a given bubble to be captured by the bubble. Some authors refer to this process as collision or interception (e.g., [17]). When capture (or collision) occurs, it does not imply that a bubble/particle aggregate has formed; rather, it only means that short-range forces and thin film dynamics become significant (to be discussed in the next section). A critical parameter governing the particle approach to the bubble and subsequent capture is R_c (Fig. 1), the radius of the streaming tube within which the particle must move so as to be intercepted by the bubble.

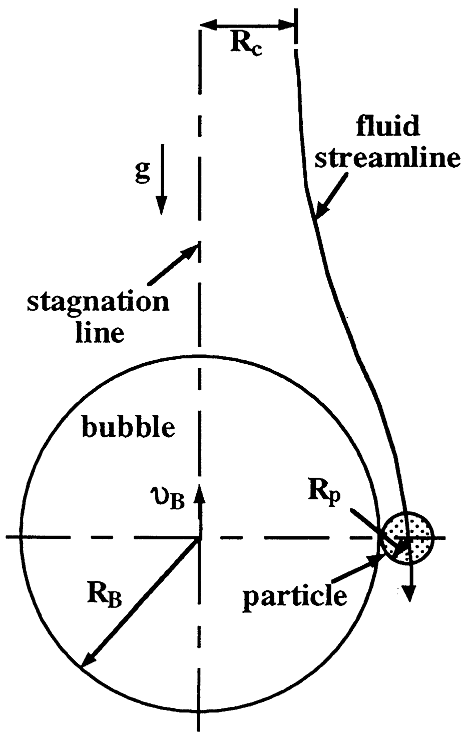

Fig. 1: All particles within the streaming tube of radius R_c will be captured by (or collide with) the air bubble.

The corresponding probability of capture (or collision), P_c, depends on the particle and bubble radii, R_p and R_B, respectively, as well as on the assumptions one makes about the flow field in which the particle moves. Particles the size of typical ink particles in a flotation cell experience negligible inertial forces and tend to follow the streamlines in the flow field around the bubble. The probability of capture is then defined as

$$P_c = \left(\frac{R_c}{R_B}\right)^2 \tag{1}$$

which represents the ratio of the number of particles (with R_p assumed to be less than R_B) within the streaming tube of cross-sectional area πR_c^2 that encounter a bubble per unit time, to the number of all particles that approach a bubble in a streaming tube with cross-sectional area πR_B^2. The Stokes number (St) and the bubble Reynolds number (Re_B) are critical in defining the flow parameters that characterize P_c. The Stokes number represents a ratio of inertia to drag forces and is defined as

$$St \equiv \frac{Re_B\,\rho_p d_p^2}{9\rho_\ell d_B^2} = \frac{\rho_p d_p^2 \upsilon_B}{9\mu_\ell d_B} \tag{2}$$

where d_B is the bubble diameter, d_p is the particle diameter, ρ_p is the particle density, ρ_ℓ is the liquid density, μ_ℓ is the fluid dynamic viscosity, and υ_B is the bubble rise velocity. For typical ink particles, St « 1 and inertia forces have a negligible influence on particle motion. Hence, there are basically no impact collisions (i.e., such as two billiard balls) between the particles and bubbles, and the particles follow the streamlines in the flow around the bubble (Fig. 1). In Eq. (2), the bubble Reynolds number is defined by

$$Re_B = \frac{\upsilon_B d_B \rho_\ell}{\mu_\ell} \tag{3}$$

The bubble surface mobility may also influence P_c [10], but it can be assumed that in flotation deinking, the bubble is coated with surfactants and/or other surface contaminants and the rising bubble may be approximated as a rigid sphere. By considering only the long-range hydrodynamic forces which act on a particle as it approaches a bubble (i.e., the drag, gravitational, and buoyancy forces), a system of equations (see [2, 28, 29]) with associated initial conditions may be written down for the velocity components of the particle whose structure depends in a crucial manner on the nature of the velocity field of the liquid in a neighborhood of the bubble. For flow conditions in which Re_B « 1, the fluid flow is described by Stokes flow, and P_c can be determined by [17]

$$P_c = \frac{3}{2}\left(\frac{R_p}{R_B}\right)^2 \tag{4}$$

When the bubble Reynolds number is very large, potential flow conditions exist around the bubble and the probability of capture may be expressed as [17, 30]

$$P_c = 3\left(\frac{R_p}{R_B}\right) \quad (5)$$

In typical flotation cells, Re_B is thought to be in the range of $1 < Re_B < 100$. For this bubble Reynolds number range (termed intermediate flow), a Stokes number St « 0.1, and the assumption of a rigid bubble surface, Yoon and Luttrell [17] have interpolated between Stokes flow and potential flow, and then correlated the resulting streamlines with experimental data obtained at various Reynolds numbers, to yield

$$P_c = \left[\frac{3}{2} + \frac{4\,Re_B^{0.72}}{15}\right]\left(\frac{R_p}{R_B}\right)^2 \quad (6)$$

More complex correlations for P_c are available in the literature [31-33].

Attachment by Sliding

Not all particles that are captured by a bubble become attached. In general, only those particles that are hydrophobic enough attach themselves to the bubble through the formation of a three-phase contact with a finite contact angle [17]. Before this three-phase contact appears, the liquid film between the bubble and particle that forms as soon as the particle is captured by a bubble must thin sufficiently to rupture. Upon liquid film formation, the particle begins to slide over the surface of the bubble and resides on it for a finite period of time, which is generally referred to as the sliding time. This sliding process subjects the film to a weak surface deformation, which tends to thin the film out and may lead to film rupture. For attachment by sliding to occur, the contact time of the particle with the liquid film must be greater than the induction (drainage) time of the film up to the point of rupture.

To study this problem, one must model the motion of a particle, as it moves over the surface of the liquid film between the bubble and the particle, so as to be able to predict the film thickness h as a function of the position angle ϕ of the particle. Note that both h and ϕ vary with the time t and that $\phi(t{=}0) = \phi_T$, which is referred to as the touching angle. It has been common in the literature to make the following assumptions in modeling the particle motion during the sliding process [10, 11]: (i) the particles move in a quasistationary manner on an almost circular path along the bubble surface; (ii) the sliding path L » h and $dL/dt > dh/dt$; (iii) for $0 < \phi < \pi/2$, the influence of the flow boundary layer is negligible; and (iv) as in the computation of P_c, the tangential component of the fluid velocity field may be modeled by the intermediate flow of Yoon and Luttrell [17].

Particle attachment by sliding over the bubble surface is probably the single most important microprocess in the overall flotation macroprocess, and it is also possibly the most complicated. The sliding motion of a particle is governed by a force balance as the particle slides over the bubble. Depending on the assumptions made and the magnitude of the forces involved, the following forces may be included in this force balance [2, 10, 34-36]: the resistive force generated during the drainage of the liquid film surrounding the bubble surface; the (apparent) weight of the particle; the centrifugal force acting on the sliding particle; the flow force acting on the sliding particle close to the bubble wall; the lift force acting on the particle; and the drag force acting on the particle (which is strongly dependent on the flow field about the bubble). From the force balance, a system of ordinary differential equations can be written in terms of (ϕ,h) which govern the variation of the disjoining film thickness with respect to the position of the (sliding) particle with the associated initial condition $h(\phi_T) = h_0$. If h_{crit} denotes the critical thickness that the film must thin down to in order for rupture to occur, then the critical position angle ϕ^*_{crit} is defined to be the largest touching angle ϕ_T (<90°) for which $h = h_{crit}$ will be achieved at a position angle ϕ_{crit} such that $\phi_T < \phi_{crit} < \pi/2$. Figure 2 depicts the relationship that exists among ϕ_T, h_{crit}, and ϕ^*_{crit}. Note that the process of the particle sliding over a bubble surface does not guarantee that a bubble/particle aggregate will form.

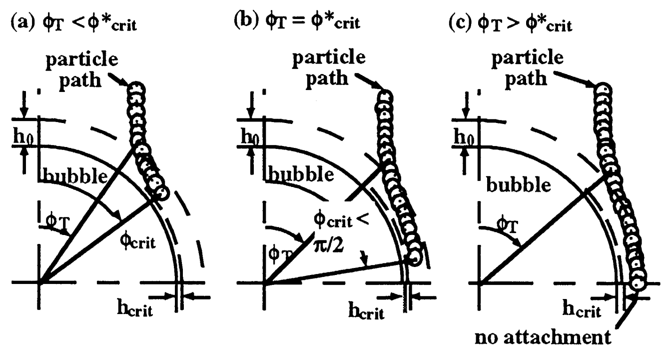

Fig. 2: Schematic representation of the relationship among ϕ_T, h_{crit}, and ϕ^*_{crit}.

Knowledge of film drainage and rupture is important in this microprocess to estimate the particle induction time, τ_i, or the critical film thickness at which rupture occurs, h_{crit}. Thin film dynamics has been addressed in the literature [19, 21, 22, 37-39], and there is continued interest in this subject because questions still remain regarding which forces are important to film rupture [37].

The probability of attachment by sliding, P_{asl}, can now be defined as the fraction of particles in the path of the bubble that actually adheres, compared to the maximum possible. Referring to Fig. 3, this should be the ratio of the area inscribed by the limiting radius R_{crit}, the radius from the stagnation line to the line corresponding to the touching angle associated with ϕ^*_{crit}, to the area inscribed by the sum of the bubble, particle, and critical film thickness ($R_B + R_p + h_{crit}$). Since we can expect $h_{crit} \ll R_p$, it is common to write P_{asl} as [10, 11, 17]

$$P_{asl} = \frac{R^2_{crit}}{\left(R_B + R_p\right)^2} \tag{7}$$

Relating R_{crit} to ϕ^*_{crit}, Eq. (7) can be written as

$$P_{asl} = \sin^2 \phi^*_{crit} \tag{8}$$

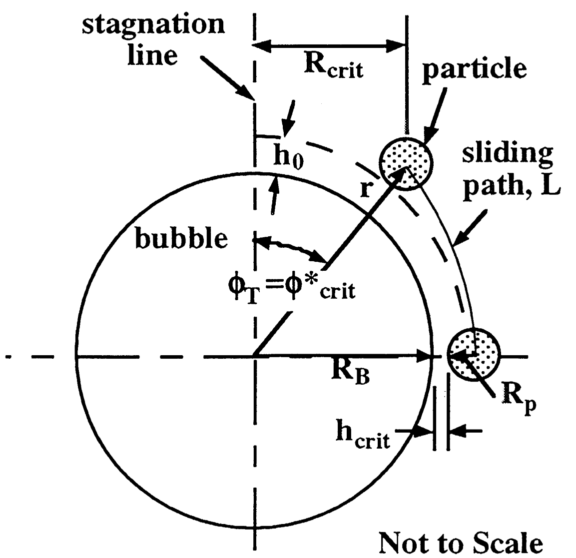

Fig. 3: Schematic representation of P_{asl}.

The critical position angle ϕ^*_{crit} is thought to be a complex function of the system parameters (i.e., the bubble and particle radius, turbulent energy density, etc.) that may be determined only by numerically solving the system of ordinary differential equations which govern the adhesion by sliding process [10]. Some authors have proposed expressions for ϕ^*_{crit} and/or expressions for the particle induction time, τ_i [17, 33, 40-42]. For example, Yoon and Luttrell [17] propose

$$P_{asl} = \sin^2\left[2 \arctan \exp\left\{\frac{-(45 + 8\,Re_B^{0.72})\upsilon_B \tau_i}{30 R_B (R_B / R_p + 1)}\right\}\right] \tag{9}$$

for intermediate flow conditions. However, the bubble rise velocity, υ_B, and the particle induction time, τ_i, must be known before P_{asl} can be determined.

Three-Phase Contact

Once the thin film surrounding a bubble has ruptured, a relatively large three-phase contact (TPC) between the liquid film, particle, and bubble must form in a sufficiently short time span τ_{tpc} to provide a strong enough force of attachment to prevent the bubble/particle aggregate from immediately separating. This aggregate formation is schematically represented in Fig. 4, where θ represents the contact angle measured in the liquid.

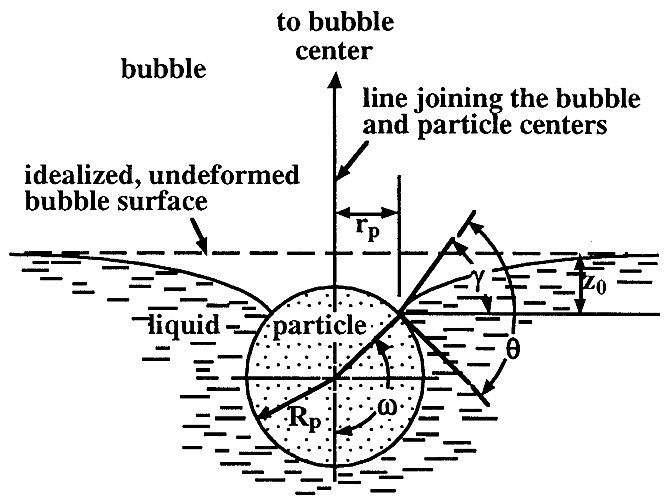

Fig. 4: The three-phase contact between the bubble, particle, and liquid regions.

If τ_υ denotes the average lifetime for turbulent vortices within the flotation cell, then the time required to form a three-phase contact to create a bubble/particle aggregate must satisfy $\tau_{tpc} < \tau_\upsilon$. Schulze [10] has used an exponential distribution of the approximate form

$$P_{tpc} \cong 1 - \exp\left(-\frac{\tau_\upsilon}{\tau_{tpc}}\right) \tag{10}$$

for the probability of extension of the three-phase contact, P_{tpc}. However, it has been shown that to within (about) 1%, $P_{tpc} \approx 1$ over a wide range of particle sizes [10]. Many authors imply that $P_{tpc} = 1$ and omit this term from the overall probability of flotation [7, 14, 17, 25, 34, 35, 38], which seems justified.

Stability

Once a bubble/particle aggregate forms, it must remain stable on its journey to the froth layer to be removed from the system. It has generally been accepted [2, 10, 24, 43] that bubble/particle stability can be determined by performing a force balance on the particle attached to the bubble. Figure 5 summarizes these forces.

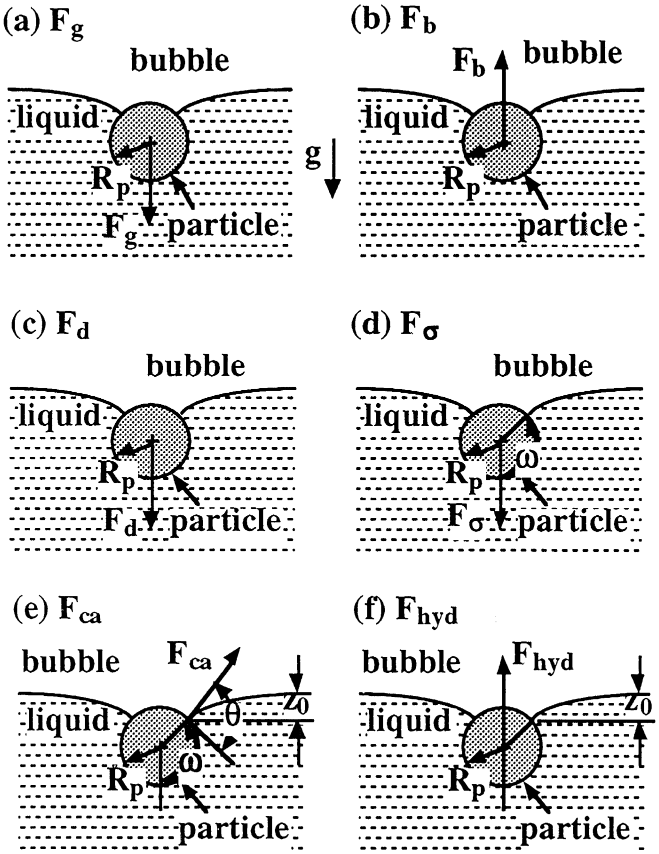

Fig. 5: Stabilizing and destabilizing forces that act on a bubble/particle aggregate.

Assuming spherical particles, the gravitational force (Fig. 5a) is specified by

$$F_g = \frac{4}{3}\pi R_p^3 \rho_p g \qquad (11)$$

where ρ_p is the particle density, and g is the acceleration due to gravity. The static buoyancy force that acts on the particle (Fig. 5b), assuming that the entire particle is immersed in the liquid [24], is

$$F_b = \frac{4}{3}\pi R_p^3 \rho_\ell g \qquad (12)$$

with ρ_ℓ the liquid density. The buoyant and gravitational force can be combined to obtain an expression for the apparent particle weight

$$F_{wt} = \frac{4}{3}\pi R_p^3 (\rho_p - \rho_\ell) g \qquad (13)$$

For the detaching force due to fluid drag (Fig. 5c),

$$F_d = \frac{4}{3}\pi R_p^3 \rho_p a_c \qquad (14)$$

where expressions for the fluid acceleration, a_c, depend on both the structure and intensity of the turbulence within the flotation cell. For aggregates where the particle size is smaller than the bubble size, it has been determined that the fluid acceleration can be related to the energy dissipation in the tank by [10]

$$a_c \approx \frac{1.9\varepsilon^{2/3}}{(R_B + R_p)^{1/3}} \qquad (15)$$

where ε is the turbulent energy density. The force generated by the capillary pressure in the gas bubble that acts on the contact area of the attached particle (Fig. 5d) is given by

$$F_\sigma \approx \pi R_p^2 \left(\frac{2\sigma}{R_B} - 2R_B \rho_\ell g \right) \sin^2 \omega \qquad (16)$$

where σ is the surface tension, and ω is the angle specified in Fig. 4. The capillary force exerted on the three-phase contact in the z-direction (Fig. 5e) is

$$F_{ca} = -2\pi R_p \sigma \sin\omega \sin(\omega + \theta) \qquad (17)$$

where θ is the contact angle. Finally, the hydrostatic pressure force (Fig. 5f) of the liquid of height z_o above the contact area of radius r_p (= $R_p \sin\omega$ in Fig. 4) is

$$F_{hyd} = \pi R_p^2 \rho_\ell g z_o \sin^2 \omega \qquad (18)$$

Therefore, the net detachment force which acts on a bubble/particle aggregate is

$$F_{det} = F_{wt} + F_d + F_\sigma \quad (19)$$

and the net attachment force is given by

$$F_{ad} = F_{ca} + F_{hyd} \quad (20)$$

The stability of bubble/particle aggregates is then characterized by comparing the net detachment force to the net attachment force by the following dimensionless parameter, with $F_{hyd} \approx 0$ and F_{ca} replaced by the maximum capillary force F_{cam} [10],

$$Bo' = \frac{F_{detachment}}{F_{attachment}} \quad (21)$$

where

$$F_{detachment} = 4R_p^2\left(\Delta\rho_p g + \frac{1.9\rho_p \varepsilon^{2/3}}{\left(R_p + R_B\right)^{1/3}}\right) + 3R_p\left(\frac{2\sigma}{R_B} - 2R_B\rho_\ell g\right)\sin^2\left(\pi - \frac{\theta}{2}\right) \quad (22)$$

and

$$F_{attachment} = \left|6\sigma \sin\left(\pi - \frac{\theta}{2}\right)\sin\left(\pi + \frac{\theta}{2}\right)\right| \quad (23)$$

In Eq. (22), $\Delta\rho_p = \rho_p - \rho_\ell$. As cited in Schulze [10], taking into account the experimental results of Plate [44], a reasonable form for P_{stab} is

$$P_{stab} = 1 - \exp\left(1 - \frac{1}{Bo'}\right) \quad (24)$$

Recent studies suggest that once a bubble/particle aggregate forms by attachment by sliding, the bubble has a strong grip on the particle, which tends to resist breakup [45]. However, these results contradict observations made by equipment manufacturers concerning turbulence [46]. Additionally, flotation cell product literature describes regions in most flotation cells where turbulence is reduced to promote the formation of stable bubble/particle aggregates [47-52]. The influence turbulence has on bubble/particle aggregate destruction has also recently been stressed by Julien Saint Amand [53].

FLOTATION MODELING

It has been stated in the mineral flotation literature that the simplest studies addressing the flotability of a particular ore involves at least 25 clearly identifiable variables, and a full-scale study may require the consideration of at least 100 variables. In addition, the general knowledge of the interactions between these variables is sparse [3]. Considering flotation deinking involves a more complex fibrous suspension, additional variables should be considered in a full-scale study of flotation deinking. Since experimental studies and system trails can be very expensive, there has been considerable effort to mathematically model the flotation process to predict separation efficiencies and general trends before full-scale experiments are performed. These models typically utilize the various microprocess models to formulate an overall flotation model.

The complexity of the flotation microprocesses has prevented the development of a flotation model based on first principles. However, in what has become a common assumption, Schuhmann [54] assumed that the individual microprocess probabilities for a given particle are uncorrelated. Therefore, the overall probability that a stable bubble/particle aggregate will form and be lifted to the froth layer may be written as

$$P_{overall} = P_c P_{asl} P_{tpc} P_{stab} \approx P_c P_{asl} P_{stab} \quad (25)$$

Additionally, based on much mineral flotation experimental evidence, the flotation process is thought to be analogous to a chemical reactor, where the overall rate of particle removal is expressed as a first-order reaction in the form [55]:

$$X(\text{particles}) + Y(\text{bubbles}) \rightarrow Z(\text{floated aggregates}) \quad (26)$$

If the bubble concentration is constant, and assuming that the removed particles represent a small volume, then the rate of change in solids is proportional to the concentration. Therefore, flotation is assumed to be governed by [2, 10, 12, 13, 30, 53, 55-59]

$$\frac{dc}{dt} = -kc \quad (27)$$

where c is the particle concentration, and k is the rate constant involving the physical, chemical, and surface properties of the system. The form of this equation suggests that given a long enough flotation time, all particles will eventually be removed, which is typically not observed in practice.

Many expressions for the rate constant are available in the literature (see [17, 25, 35, 53, 55, 60] for examples) and these may depend on the specific system being studied. Schulze [2, 9, 10] has employed relations where the rate constant is specified as

$$k = ZP_{overall} \tag{28}$$

where Z is the number of bubble/particle collisions per unit time (collision frequency). In the absence of bubble/particle collision frequencies specific to flotation deinking, a reasonable first estimate may be represented by [23]

$$Z = 2^{7/9}\frac{5}{3}n_p n_B\left(\frac{\varepsilon^{4/9}}{\nu_\ell^{1/3}\rho_\ell^{2/3}}\right)\left(R_p + R_B\right)^2 \times\left(R_p^{14/9}\Delta\rho_p^{4/3} + R_B^{14/9}\Delta\rho_B^{4/3}\right)^{1/2} \tag{29}$$

where ν_ℓ is the (fluid) kinematic viscosity, $\Delta\rho_p = \rho_p - \rho_\ell$, $\Delta\rho_B = \rho_B - \rho_\ell$, and n_p and n_B are the total number of particles and bubbles in the unit volume (system), respectively. Other forms of the collision frequency are also available (e.g., [13, 61-63]).

The idea of a kinetic or population balance type of equation was extended by Heindel and Bloom [43, 64] to construct a kinetic equation of the form

$$\frac{dn_p^f}{dt} = -k_1 n_p^f n_B^f + k_2 n_B^a \tag{30}$$

where n_p^f corresponds to the number of free particles in a unit volume available to attach to bubbles, n_B^f is the number of bubbles in the unit volume that are available for particle attachment, and n_B^a represents the number of bubbles with attached particles in the unit volume. The first term on the right-hand side represents the overall probability that a free particle will successfully attach to a bubble that is initially free of particles. The second term is a measure of the probability that a bubble/particle aggregate will become unstable and split to yield a "new" free particle. This term has not been explicitly included in previous flotation models (i.e., [2, 10, 12, 30, 53, 56, 60]). The kinetic constants k_1 and k_2 are positive numbers described by the various microprocess probabilities and the collision frequency:

$$k_1 = ZP_c P_{asl} P_{tpc} P_{stab} \tag{31}$$

$$k_2 = P_{destab} = 1 - P_{stab} \tag{32}$$

A more detailed model of flotation has been proposed by Schulze [2, 12] that includes a system of partial differential equations that govern free and attached particles in the system and accounts for particle advection and diffusion, as well as particle source and sink terms within the system. However, no attempt has been made at solving this system of partial differential equations.

Although models of the flotation separation process can be very simplistic or very complicated, they all have the goal of predicting the appropriate trends observed in practice. If this is accomplished, the models can be utilized to predict changes in system performance when it is desired to change one or more of the system parameters. However, current operating conditions, and changes to these conditions resulting from a change in the desired parameter, must be known. Values of selected parameters (i.e., turbulent energy density, surface tension, etc.) can be estimated, but predictions may depend on the accuracy of these estimates.

FLOTATION FUNDAMENTALS APPLIED IN INDUSTRY

Understanding flotation fundamentals is important to developing a realistic model of the overall flotation process. Knowledge of flotation fundamentals can also assist in flotation deinking equipment design and proper utilization because the flotation microprocesses take place in all flotation cells. For example, Pfalzer [46] noticed that efficiency is reduced if the turbulence in a flotation cell is too high. Turbulence plays a roll in the collision frequency (i.e., getting the particles and bubbles together) as well as stability (i.e., a highly turbulent fibrous flow may separate bubble/particle aggregates). These competing effects can have opposing results on the flotation cell performance.

Most of the new flotation equipment available for paper recycling mills attempt to increase bubble/particle interaction in one region of a cell to promote bubble/particle aggregate formation. Once an aggregate is formed, the flotation cell may have another region to enhance aggregate removal from the bulk stock flow. McCool [5] and Ferguson [6] provide reviews of flotation deinking equipment. Selected equipment is also shown in Figs. 6-11. These figures do not represent all of the equipment available to a recycling facility for flotation deinking, and new developments are continually being introduced [58].

Ahlstrom Kamyr Inc. has recently modified the Gas Sparged Cyclone (GSC) to provide a high shear and turbulence zone to increase bubble/particle interaction (Fig. 6). In this version, the GSC is the aeration unit where bubble/particle collision is the primary focus [49]. The air vent allows for excess air to be sparged through the feedstock. The air/stock mixture is then injected into a separation tower with a primary purpose of promoting stable formation of bubble/particle aggregates and separating them from the system.

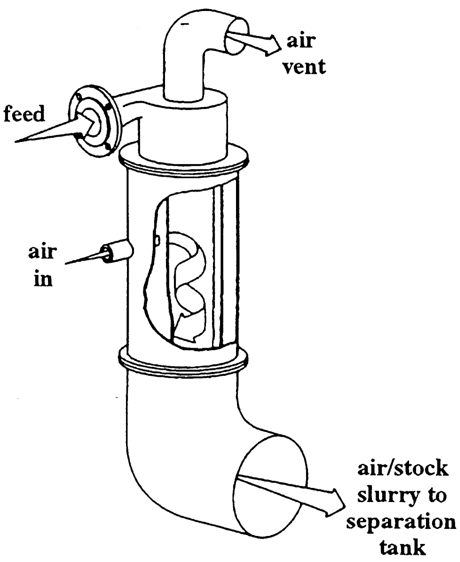

Fig. 6: The Gas Sparged Cyclone used as an aeration unit [49].

When Beloit's Pressurized Deinking Module (PDM) was designed (Fig. 7), the emphasis was to independently optimize the collision, attachment, and separation (i.e., stability) microprocesses [47]. The aeration zone is where the air is introduced into the flotation cell. The mixing zone is designed to have turbulent flow, which increases the number of bubble/particle collisions and subsequent attachments. The separation zone reduces the turbulence level and allows stable bubble/particle aggregates to rise to the surface where they are rejected.

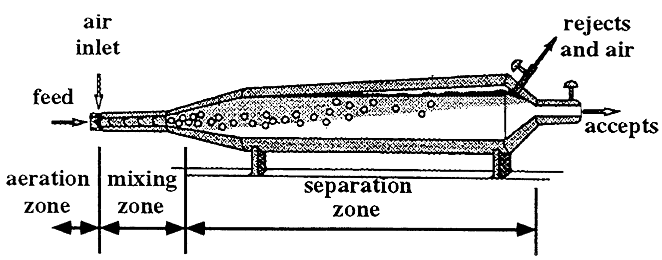

Fig. 7: Beloit Pressurized Deinking Module [47].

The Black Clawson/IIM Flotator (Fig. 8) utilizes a high air-to-stock ratio, a high shear mixing area, and a long retention time to promote bubble/particle collision and attachment [48]. The high air-to-stock ratio, promoted by multiple passes of the stock past the air injector, ensures that a large number of bubbles in the system are available for particles to attach to. High shear mixing results in increased turbulence and promotes bubble/particle collisions as well as provides for a wide range of bubble sizes. The long stock retention times also increase the probability that a particle will collide with, and subsequently attach to, a bubble. The mixing and air injection take place near the bottom of the flotation cell, and the top of the cell provides less mixing for contaminant removal.

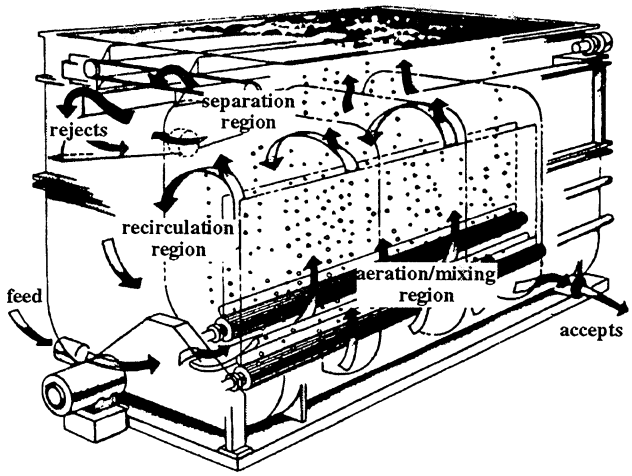

Fig. 8: The Black Clawson/IIM Flotator [48].

The Fiberprep/Lamort MacCell (Fig. 9) is a stacked cell where the air/stock mixture enters at E1 and these accepts (A1) are recirculated to E2 for additional aeration. This continues for each stage where the final accepts are discharged at A5 and the rejects are removed at R [50]. The multiple aeration steps enhance bubble/particle collision, and each stage has a separation region to promote bubble/particle attachment and stability. Additionally, each stage allows for individual control of the turbulence level. According to the product literature, this allows for removal of relatively large particles because excess turbulence has a negative impact on these particles. This turbulence effect was also discussed by Julien Saint Amand [53].

A common flotation column design used by the mineral processing industry has been modified by Kvaerner Hymac and applied to recovered paper processing [51]. This flotation column is shown in Fig. 10 and utilizes countercurrent flow where the stock is introduced at a height about two-thirds above the column base and accepts are removed from the bottom. Air is introduced in the column through a sparger system located near the column base. The countercurrent action created by descending stock and ascending air promotes mixing and bubble/particle interactions. Bubble/particle aggregates that form rise to the collection zone and are subsequently removed in the foam removal zone.

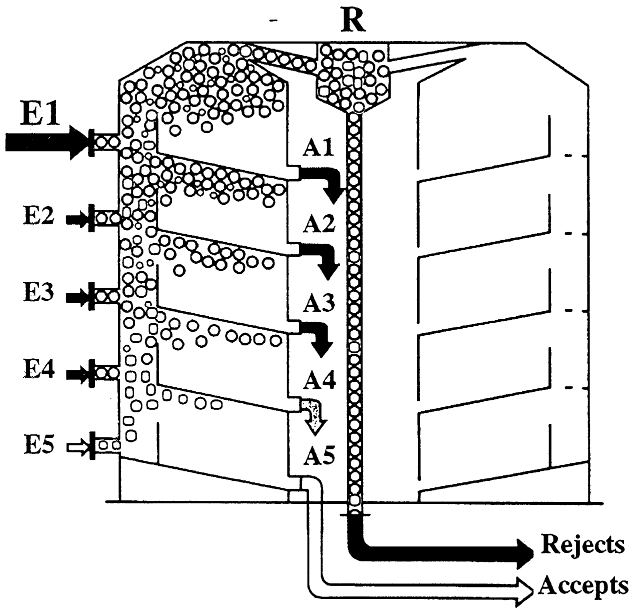

Fig. 9: The Fiberprep/Lamort MacCell [50].

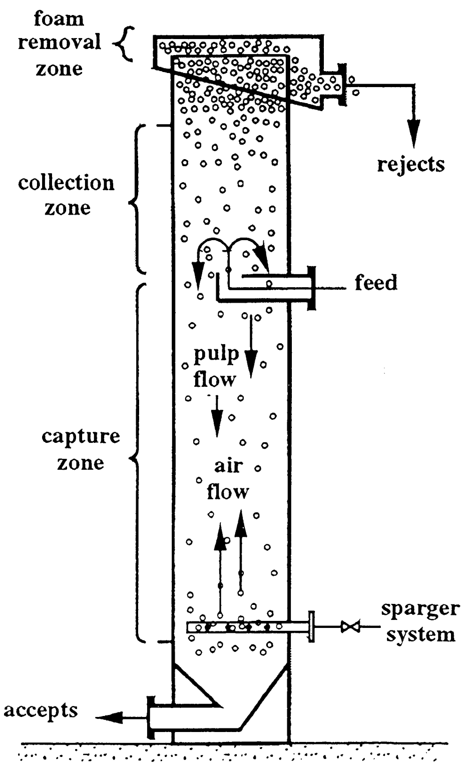

Fig. 10: The Kvaerner Hymac flotation column [51].

The new Voith Sulzer EcoCell (Fig. 11) is a combination of the Compact Flotation Cell (CFC) from Sulzer Escher Wyss and the Type E Flotation Machine from Voith [52]. Air is introduced in the inlet pipe through an aspirator and step diffuser and then passes through an impact mixer to promote air/stock mixing. The primary purpose of this section is to increase bubble/particle collision. The air/stock mixture is then expelled into a large oval tank to reduce stock velocity and turbulence and to promote attachment of the particles to the bubble. Additionally, the low turbulence in this region enhances stable bubble/particle aggregate formation for eventual removal in the froth.

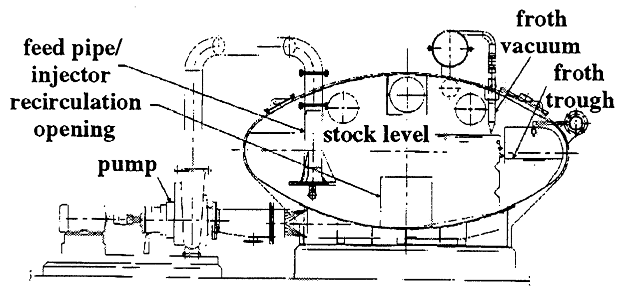

Fig. 11: The Voith Sulzer EcoCell [52].

It is apparent that although the equipment manufacturers have very different flotation cell designs, they all are trying to enhance the probabilities that are associated with bubble/particle collision, attachment, and stability. Knowledge of these microprocesses is also beneficial in a mill setting because proper flotation cell utilization will enhance contaminant removal efficiencies.

CONCLUSIONS

Flotation deinking is such a complicated process that it may seem fortuitous that it actually works in a recycle mill. However, understanding the fundamental microprocesses associated with flotation deinking can actually lead to better flotation cell performance. These microprocesses, including capture, attachment by sliding, three-phase contact, and stability, have been reviewed. Additionally, various models for the overall flotation process have been outlined and typically follow a kinetic-type formulation. Finally, various flotation cells used in fiber recovery systems have been reviewed to indicate that the fundamental flotation microprocesses are relevant to all types of flotation equipment. Understanding the primary goals of each region in a flotation cell, and how it relates to the flotation microprocesses, can lead to better equipment utilization and contaminant removal efficiencies.

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