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A Cardinal Sin: The Infinite in Spinoza's Philosophy

Samuel H. Eklund

Macalester College, sam.eklund@gmail.com

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A Cardinal Sin:

The Infinite in Spinoza's Philosophy

By: Samuel Eklund

Macalester College Philosophy Department

Honors Advisor: Geoffrey Gorham

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Chapter I. A Look at the Letter

i. Introduction

To many readers, the most striking feature of Spinoza's *Ethics* is its geometric order: the step-by-step progression from definitions and axioms to numbered propositions and scholia. Thus, it is tempting to read the *Ethics* strictly as a deductive system, with the justification of the later arguments coming from the groundwork established by earlier propositions. Despite this temptation, I believe a richer understanding will come from both reading between the lines and looking outside of the system. Once we examine the technical terms and ideas explicated in Spinoza's other writings, we can gain a clearer understanding of the *Ethics* as a philosophical system that draws upon theories external to the book. One concept which can be saved from ambiguity through this methodology is the infinite, which is crucial to many aspects of Spinoza's philosophy, including Substance, Attributes, and certain kinds of Modes.¹ Although it plays a role in several key places in the *Ethics*, there are no explicit distinctions among the divergent ways Spinoza conceives of infinity; nor is any detailed definition of the infinite present in the *Ethics*.

However, Spinoza's earlier "Letter on the Infinite," (Letter 12) explains the meaning of the term "infinite" in considerable detail. This chapter reviews three types of infinity identified in Letter 12. First there are things that are (1) "infinite by their own nature," next are things (2) "infinite by virtue of the cause in which they inhere," and

¹ Capitalization in original.

finally (3) “things that can be called infinite, or if you prefer, indefinite, because they cannot be expressed by any number, while yet being conceivable as greater or less.”

(106).² After explaining these three categories and their places in Letter 12, I propose a way to understand these categories and use them as a lens to gain a clearer understanding of key arguments in the *Ethics*.

Figure 1 below represents Spinoza’s taxonomy of the infinite. Corresponding to each of the kinds of infinity are distinct ontological, temporal, and spatial categories. For example, things infinite by cause are the Modes, which possess Duration and Quantity.

Kind of Infinity	Ontological Level	Temporal Properties	Spatial Properties
Infinite by Nature	Substance	Eternity	Extension
Infinite by Cause	Modes	Duration	Quantity
Indefinite	Aids of the Imagination	Time	Measure

Figure 1.

Spinoza’s Letter 12 was written in 1663 to Lodewijk Meyer, Spinoza’s editor and personal friend. The letter aimed to explain “the causes of the errors and confusion that have arisen regarding the question of the infinite” (106). For Spinoza, these errors arise “through [a] failure to distinguish” between different types of infinity and their corresponding properties (101). Because Spinoza believed that confusion about the

² All references to page numbers refer to Shirley’s 1995 edition of *The Letters*. Additionally, citations from the *Ethics* come from Curley’s 1985 *The Collected Works of Spinoza*.

infinite typically arises through conflating distinct concepts, he thought it necessary to fully explain the different types of infinity. Having done so, he shows how their spatial and temporal properties follow from their breed of infinity.

Spinoza's discussion of the infinite draws on several sources, including Descartes, who distinguished between infinities comprehended by the understanding and those conceived in the imagination. The former cannot be divided, while the latter can. Descartes often referred to this latter type as the "indefinite."³ However, unlike Descartes and previous philosophers, Spinoza believes that puzzles stemming from the infinite are not a consequence of the mind's limitations, but are the result of a failure to adequately distinguish between the different kinds of infinity. Spinoza summarizes these three kinds of infinity near the end of Letter 12:

“[1] certain things are infinite by their own nature and cannot in any way be conceived as finite, [2] while other things are infinite by virtue of the cause in which they inhere; and when the latter are conceived *in* abstraction, they can be divided into parts and be regarded as finite. [3] Finally, there are things that can be called infinite, or if you prefer, indefinite, because they cannot be accurately expressed by any number, while yet being conceivable as greater or less” (106, numbering added).

In the sections that follow, I take up the three levels of infinity in turn.

ii. Substance: Eternity and Extension

In Letter 12, Spinoza makes three important observations about Substance. First, Substance necessarily exists by definition, meaning that its essence and existence are one

³ Descartes, *Letter to Mersenne* 27 May 1630. cf Ariew 1990.

and the same. Second, “Substance is not manifold.” Spinoza explains that by this, he means that for any essence, there can only be one substance which possesses that essential feature.⁴ And third, Substance is necessarily infinite, conceptually as well as actually (102). For Spinoza, it is as self-contradictory to attribute finitude to substance as it is to attribute quadri-linearity to circle. These features of Substance entail that it is metaphysically prior to all other parts of Spinoza’s world. Only Substance can exist without depending on anything else. In the *Ethics*, Spinoza refers to the one existing Substance as God.⁵ Although his view of God is highly unorthodox in many respects, Spinoza’s concept of Substance as necessary and infinite is similar to traditional theological accounts of God, that don’t assign a creator to God.

Thus, Spinoza’s Substance requires an account of what it means for something to exist by definition. He provides this account through his explication of Eternity. Eternal existence is a temporal property for Spinoza in only a very broad sense, for it designates beings which exist outside of any possible alteration or limitation. Platonic Forms, or mathematical truths, like “ $2+2=4$,” would normally be considered Eternal in this sense (even though Spinoza does not think such objects exist).

Spinoza maintains that the necessity and immutability of Eternal existence require it to be indivisible. Consequently, Eternity has no parts and cannot be evaluated with comparative terms like more or less, or before and after (103). Because they are

⁴ This property remains in the *Ethics*, but rather than talking about Substance not being “manifold,” it is described in terms of Substances being unable to share Attributes. *cf* EIP5.

⁵ EIP14.

absolutely necessary, Eternal being never begin or cease to exist; Spinoza calls “an infinite enjoyment of existence or - pardon the Latin - of being (*essendi*)” (102).

The spatial or Extended Attributes of Substance likewise do not allow parts or division. This way of conceiving Extended Substance obviously diverges heavily from common conceptions of space. As Alison Peterman argues, when Spinoza refers to the Extension of Substance, this is not the type of extension possessed by three-dimensional objects. Instead, she argues that the type of Extension used by Spinoza is fundamentally indivisible in reality.⁶ If, contrary to Spinoza, we follow the usual way of conceiving Extension, as an unbounded three-dimensional Euclidean space with no inherent divisions, we end up with a space that can still be divided in the imagination. For instance, we could arbitrarily divide this space into different parts by using a coordinate system. But Spinoza would object that this merely conceptual division is contrary to the true nature of Extended Substance, for we are discussing something which has a necessary unity. This is why Spinoza compares the idea that Substance is an infinite aggregation of parts with the contradictory notion of a square created by adding circles together (103). What we get from adding parts together endlessly is not the full-blown infinity of Extended Substance. Beings “infinite by their own nature” are necessarily indivisible, and not merely unbounded.

⁶ Peterman, draft. pp. 7&11.

iii. Modes: Duration and Quantity

Modes are on the next rung of Spinoza's ontological ladder. He defines Modes as "the affections of Substance" (102).⁷ As such, they are ontologically posterior to Substance; Modes cannot exist without Substance, although Substance does not depend on the existence of Modes.⁸ For Spinoza, this level comprises all the familiar entities of the universe, which are not individual Substances, as other Modern philosophers claimed. The level of Modes includes human beings, animals, and atoms. But how exactly do the Modes relate to Substance, considering that modes depend on substance, but not vice-versa? In particular, how do finite material things relate to Infinite Extension, as understood by Spinoza? According to Bennett's influential "field-metaphysic" interpretation, the Modes are just different parts of Extension possessing various properties, such as redness or softness, at different times and locations.⁹ Despite its influence, I would like to argue this reading is untenable, given the clarifications provided by Letter 12. But first, I should explain how Modes are understood in this letter.

Spinoza's use of Modes in Letter 12 contains terminology that parallels the discussion of Substance. Whereas Substance falls under the category of "Eternity," Modes have the property of "Duration." Duration, like Eternity, is a way of understanding an object's existence. Consider a particular Mode, such as a single cup.

⁷ There are different ways of interpreting the ontology of modes, such as whether they are *bona fide* objects in the usual sense of the term, or just properties predicated on Substance (Newlands). For the purposes of this paper, I will be neutral about their ultimate nature, and describe how they are examples of that which is infinite by "the force of the cause in which they inhere," comparing this infinity with the type possessed by Substance.

⁸ Melamed, 2000. p. 207.

⁹ Bennett, 1984. pp. 91-3.

Considered apart from the whole order of nature, there is nothing intrinsic to the idea of that cup which determines the beginning or end of its existence. However, considered in relation to other objects in nature, it will begin and end through the effects of other Modes. Thus, we can speak of other events which happened before its existence, during its Duration, and after its end (102-3). We can imagine cases where the Mode began or ended in a different way, or even failed to exist. The cup exists not by definition, but because of the causal ordering of nature. For this reason, its existence is described by Duration, rather than Eternity.

Spinoza elaborates upon this line of thought in the *Ethics*. In Part I, he postulates a world consisting of only 20 people (IP8schol2). The reason why there are only 20 people in this world cannot be derived from the concept of human nature; rather, the explanation must invoke the order and connection of other finite entities. This collection of 20 people is a Mode, and endures with a Duration, for unlike Eternal Substance, its existence is not entailed by its essence alone. In fact, nothing in the Mode considered in itself determines what its Duration will be. In Spinoza's words, "we can arbitrarily delimit the existence and duration of Modes without thereby impairing to any extent our conception of them" (103). So, while the concept of a Substance necessarily entailed its existence, additional facts are needed to explain the Duration of Modes.

So far, I have discussed the Modes with finite Duration, but despite their lack of intrinsically necessary existence, Modes can still be described as infinite. Suppose a particle, despite its place in the order of nature, exists without end. It simply happens to have always existed, and through the causes present in nature, it always will exist. Despite its continual existence, we can imagine alternate facts about the world that could

cause the particle to be destroyed at some point in time. This is not possible for Substance, Spinoza argues, since Substance has an “infinite enjoyment of existence” by its very essence (102). This is why even unlimited Duration is separate from Eternity, for Substance could have never possessed anything but Eternity, and we cannot conceive of how Substance could fail to be Eternal. We can entertain counterfactual situations of Modes with infinite Duration coming into and out of being by changing other facts of the world, yet this is not so with Substance. Therefore, we can say that even though Modes can have an infinite Duration, they never rise to the level of infinity possessed by Substance.

The distinction between Eternal things and Durational beings has not been fully appreciated by many writing on Spinoza’s metaphysics. Some commentators, such as Jonathan Bennett have argued against the strict distinction between Eternity and Duration claiming that Eternal objects also participate in an endless Duration. Bennett claims that “Eternity is a species of Duration, marked off by the differentia ‘necessary.’”¹⁰ On his view, Eternal Substance exists temporally alongside the Modes. Bennett acknowledges that although Modes can exist without end, they are not Eternal, since they do not exist by necessity. However, I believe that Bennett’s arguments ignore the points present in Letter 12. While Bennett correctly holds that Eternity and unlimited Duration (also known as sempiternity)¹¹ are two distinct concepts, he incorrectly states that the Eternity of an object implies its sempiternity. The distinction between Eternity and unlimited Duration is not just that Eternity implies absolute certainty about existence. Rather,

¹⁰ Bennett, p. 205.

¹¹ Kneale, 1979.

Eternity is entirely separate from Duration, for in Spinoza's words: "it is to the existence of Modes alone that we can apply the term Duration" (102), and this is true even of the infinite Duration of sempiternity.

The reason why Bennett believes Eternity entails sempiternity is because he thinks that if Extension is divisible, then eternity must be divisible as well.¹² I agree with the conditional, yet infer that because Eternity is indivisible, then Extension cannot be divided. In other words, his *modus ponens* is my *modus tollens*. I will now explain how Modes relate to the infinite Extension of Substance, and what this means for interpretations about the infinite nature of Extension.

Just as Eternity and Duration represent the kinds of temporal infinity which correspond to Substance and Mode, there is a pair of properties that express two distinct spatial infinities, namely Extension and Quantity. Quantity is a property of the Modes, and Extension belongs to Substance. Although Spinoza does not give a detailed account of Quantity in Letter 12, he clearly considers it as analogous to Duration.¹³ Spinoza says that we can conceive of Quantity in two ways, either abstractly, or in itself (103). Using a clear understanding, "if we have regard to it as it is in the intellect and we apprehend the thing as it is in itself (and this is very difficult), then it is found to be infinite, indivisible, and one alone." (103). However, conceived in the abstract sense, we imagine the Quantity of individual Modes, which can be divided by the mind.

We have already seen how a never-ending progression of Duration is distinct from Eternity, and Spinoza would likewise be committed to saying that a Quantity

¹² Bennett p. 206.

¹³ Peterman, p. 9

comprising the collection of every Mode in the universe is distinct from Extended Substance. This Quantity is posterior to the existence of Modes, but no individual Mode is involved in the bare essence of Substance. Furthermore, this supposed collection of modes, since it consists of parts, cannot be attributed to the indivisible infinity of Substance. Instead, it is infinite only because it is composed by infinite Modes.

This point is confirmed in the *Ethics*, where Spinoza states that as an Attribute of God, the Extension of Substance cannot be composed by aggregating finite pieces. He states that the absurdities which follow from imagining that God is corporeal “do not at all follow from the fact that an infinite quantity is supposed, but from the fact that they suppose an infinite quantity to be measurable and composed of finite parts” (EIP15). However, there appears to be a contradiction, coming from the scholium to the 7th Lemma of the “physical interlude” of Book II, where Spinoza states: “But if we should further conceive a third kind of Individual, composed [NS: of many individuals] of this second kind, we shall find that it can be affected in many other ways, without any change to its form. And if we proceed in this way to infinity, we shall easily conceive that the whole of nature is one Individual, whose parts, i.e., all bodies vary in infinite ways, without any change of the whole Individual” (EII7Schol). It seems that he contradicts himself by talking about “the whole of nature” possessing parts, while at the same time denying parthood to quantity as conceived through Substance in IP15.

Lee Rice attempts to solve this inconsistency by arguing that the second passage starts with the infinite collection as a given whole, rather than trying to build the concept up from finite pieces. Under this interpretation, the infinity of Substance can be divided

into finite parts, but it cannot be constructed from adding finite pieces.¹⁴ To go back to Spinoza's analogy, this is not like adding circles together to try and get a square, which would be trying to fit incompatible pieces together. Instead, it is like trying to add geometric points together to form a continuous line. We cannot construct a line by adding points together piece by piece in our minds, but once given the line, we can identify points within it. Rice resolves the contradiction by saying that EIP15 means that Extension cannot be iteratively composed of adding pieces together, while the Scholium to the 7th Lemma is analogous to identifying points on a previously given line. However, there is a superior way of resolving this contradiction. This comes from recognizing that Spinoza is employing two separate concepts in both passages: Extension in the case of IP15 and Quantity in the case the 7th Lemma. Even though he refers to it as a "quantity" in the quote provided, he goes on to describe that the quantity can be conceived as divisible through the distorted lens of the imagination, or as an "infinite, unique, and indivisible"¹⁵ property of Substance, meaning that he is referring to Extension in the technical sense of the term. Conceived properly, Extension cannot be divided into parts, but collections of Quantity can. This resolution involves recognizing that these really are differences of kind, not degree.¹⁶

Bennett takes the Eternity and Extension of Substance to be equally divisible, whereas Peterman and I take them both to be indivisible, based on the evidence in Letter

¹⁴ Rice, 1996. p. 35.

¹⁵ This line, and the full sentence in which it occurs, is an almost verbatim repetition of the quote from Letter 12 from two paragraphs ago, showing how committed Spinoza was to the arguments of Letter 12 while writing *The Ethics*.

¹⁶ This reading may be amenable to Rice's overall interpretation of Spinoza, as he acknowledges that Spinoza conceives of infinities as differing by kind, not merely degree.

12. Given the framework provided in this letter, we can see that it is a category error to attribute infinite Duration or Quantity to Substance. Substance cannot have infinite Duration (or Quantity), as Spinoza says that “it is to the existence of Modes alone that we can apply the term Duration.” (102). In the very next paragraph, Spinoza goes on to argue that attending to the right kinds of infinity shows us that “it is nonsense, bordering on madness, to hold that extended Substance is composed of parts or bodies really distinct from one another.” (103). Given what Spinoza says about Eternity and Duration, the best way to read this passage is that the “nonsense” involves predicating the wrong type of infinity to the wrong category of objects; it involves ascribing to Substance the infinity that corresponds to causes, rather than its proper type: that which is infinite by nature. The former can be conceived of as divisible, while the latter cannot.

If Spinoza were offering some sort of set-theoretical account of infinity, it might make sense to say that infinite Duration follows from the greater infinity of Eternity, just as the integers form a subset of the real numbers. Anachronistically, a solution in this vein commits the error of using mathematical concepts developed well after Spinoza’s death to interpret his work.¹⁷ Even more problematic is the fact that it ignores the crucial difference between the levels of infinity presented by Spinoza, which is neither cardinal nor ordinal, but metaphysical.

iv. Aids of the Imagination: Time, Measure, and Number

Aids of the imagination constitute the final level in the metaphysical system of Letter 12. He lists these as Time, Measure, and Number. When we observe modes

¹⁷ Ariew, 1990. p. 16.

coming into and going out of existence, we use Time in order to quantify and easily compare different Durations. Similarly, to compare the Quantity of Modes, we apply Measure (103-4). Time and Measure are not concepts which inhere in nature, but are “aids of the imagination” or “mental constructs.”(104). The process of abstraction is also the source of Number, which is used to convey information about classes constructed by the mind (104). For example, in Letter 50, Spinoza says “he who holds in his hand a penny and a dollar will not think of the number two unless he can apply a common name to this penny and dollar, that is, pieces of money or coins” (259).

Interestingly, Spinoza’s account of number seems to anticipate a view later deployed by Frege. Indeed, Frege cites this letter when explaining his concept of number as an extension of a concept.¹⁸ It is only once we have divided objects into categories that we can apply a number to them. I may say that there is one book on the table, or 404 pages, depending on which concept I apply. Spinoza considers such concepts to be human impositions on reality. We say that there are 404 pages because we ignore the different words on each page, and the different molecules which compose each individual page, for we focus on the similarities relevant to practical interest instead. Melamed notes that Spinoza acknowledges that numbers can possess a rigorous order, but Melamed explains that for Spinoza, these “are orders which do not reflect the real order of modes within substance.”¹⁹ Due to their abstraction from the true order of nature, Spinoza classifies Numbers as an aid of the imagination. Unlike Frege, who is a realist about concepts and numbers, Spinoza’s nominalism about concepts leads him to reject

¹⁸ Frege 1959, p. 62.

¹⁹ Melamed, 2000. p. 11

the idea that number can apply to matters of deep metaphysics. Number represents a type of double abstraction from reality for Spinoza; we first use our imagination to create classes of objects, and then extend that concept to produce Number.

As aids of the imagination, all three of Time, Measure, and Number fail to capture key aspects of reality. They may be useful on pragmatic grounds, but they cannot provide us with information about Substance or Eternity.²⁰ All three concepts contain properties that are distorted reflections of the ways things really are. Spinoza's epistemology draws a line between imagining a concept and understanding it; "there are many things that can in no way be apprehended by the imagination but only by the intellect, such as Substance, Eternity, and other things." (104). Because our reasoning with Number, Time, and Measure all involve ignoring the relations the Modes have within Substance, the resulting relations among the aids of imagination have left the realm of understanding.²¹

Spinoza thinks that Zeno-style paradoxes arise from a misapplication of the aids of the imagination to reality. He considers a person who wonders how an hour of Time could pass, for first half an hour must pass, and then a quarter, and so on, *ad infinitum*. He rejects one way of solving this problem, which is to suppose that Duration is composed of discrete moments, a proposal Spinoza compares to "say[ing] that Number is made up of simply adding noughts together" (104). Spinoza's alternative diagnosis of the paradox is that it conflates Duration with Time, the aid of our imaginations. The person supposes that the Duration of the hour has the structure of Time, that is, composed of

²⁰ Schliesser, draft. p. 28

²¹ Melamed. p. 11

infinitely divisible parts. But this is to confuse the structure of our mental aids the underlying reality.

For an additional example illustrating the misapplication of imaginary constructs to reality consider that we use names to designate the colors of the spectrum: red, blue, yellow, green, white, and so on. When we confuse these categories with the spectrum itself, we are led to puzzles like when one category of colors ends, and another begins, such as “what is the exact frequency of light where yellow become orange?” These puzzles only arise from confusing mental categories with objective reality, just like Spinoza’s diagnosis of the Zeno paradox. Once we realize that our color concepts are a matter of convention, these puzzles dissolve, for color concepts do not carve nature at its joints.

There remains a difficulty in Spinoza’s account of the aids of the imagination, He writes: “it is obvious from the above [an example of infinitely dividing time] that neither Number, Measure, nor Time, being merely aids to the imagination can be infinite, for in that case Number would not be number, nor Measure measure, nor Time time.” (p. 104). His use of capitalization indicates, I believe, that he wants to underscore the particular function of these aids. In particular, the function of Measure requires a conventional metric that allows us to navigate the world in the proper ways. Time requires a unit like hours, Measure needs something like meters, and Number is represented by integers. He is not saying that mental constructs *must* be finite by logical necessity; instead, he believes that in order to aid us in the standard ways, they need to possess a finite structure. We could think of a ruler with neither ends nor markings, but it wouldn’t fill the role that normal finite rulers play in our society. If we use this ruler as the basis for

our system of something we call “Measure,” we wouldn’t be in line with our typical usage of the term “measure.” It would be like constructing a morality that permits killing without needing to provide a stronger justification than “I felt like it.” It may be an “Ethical” system under a formal definition that states that all we need for a system of Ethics is any set of action-guiding principles, but it would by no means be “ethical” under standard uses of the word.

Understanding why Spinoza believes that aids of the imagination must be finite informs us about his views on the way metrics relate to nature. Spinoza is not saying that the parts of nature that we measure must be finite; his point focuses on the tools that we use to do this. To show why this is the case, we must begin with the purposes of Time, Measure, and Number. As aids of the imagination, by definition, they must aid us. By using these aids as standard metrics, I can say: “I’ll see you in an hour,” rather than describing each event which will occur between now and when I see you as specific, unique concepts. As metrics, these concepts give us terms we can use to navigate the world and communicate with one another by abstracting to obtain the relevant properties.

One standard metric of Measure could be a specific bar of metal; I abstract from the exact causal history and specific molecules which compose the bar’s Quantity, and arrive at a unit of Measure, say a meter. Using this unit of Measure, I could discuss what it means for object to be two meters or half a meter; I simply double or halve the initial unit of Measure. If I am 5 meters away from something and take a 1-meter step back, I would be 6 meters away. But what if my starting unit were infinite? If the universe has no furthest distance, I could define my unit of Measure as the distance from my position as I write this paper to the infinite distance directly in front of me; let’s call this Measure an

infinimeter. What would it mean to say that something is twice as long as an infinimeter, or half as long, since we would still have an infinite length no matter how we divide or multiply it in this way? If I am an infinimeter away from the edge of the universe, and take one step back, I will still be an infinimeter away, despite having moved.

I believe that this is what Spinoza means when he says that Measure cannot be infinite. Spinoza is not talking about Quantity itself, but rather the units we use to measure it. Infinite “Measure” is not “measure” because of its radical difference from the finite metrics which can be associated with addition, division, and multiplication in familiar ways. The problem with the infinimeter is that dividing it in half or multiplying it by two does not give us the same results as multiplying and dividing a meter, a finite unit of Measure. And clearly a similar point could be made about Time, as an abstraction from Duration. What if rather than measuring the world in seconds, minutes, or hours, we started with an infinite metric, such as the “infiniyear,” which starts now, and proceeds without end? We run into the same problems as we did with units of Measure based on an infinite Quantity. What sense can we make of comparing events when applying infinite Time, since one infiniyear starting at a given moment would mark off just as much time as using 1/100th of an infiniyear proceeding from the same starting point? As concepts which help us navigate the world, Measure and Time must be divisible in ways that produce numerable units when split in half or added together in finite iterations, rather than being infinite all the way down.

Given that in order to be useful, our units of Time and Measure must be finite, explaining why Number cannot be infinite is the next step. This claim may be confusing to the modern reader. After all, given any number, we can always find one which is

bigger. Isn't this enough to say that the series of numbers are infinite?²² While this may allow us to think of the series as infinite, the best reading of the claim that Number cannot be infinite is that no individual Number can be infinite, just as no single unit of Measure or Time can be infinite. What if instead of giving a number composed of numerals, we used ∞ , ω , \aleph , or another symbol to signify an infinite number and its properties? I think that to Spinoza, such signs are not Numbers, as they are not the result of combining other integers together in a finite process.

the categories of Modes we create. In order to show that an infinite "Number" would not have the standard properties of numbers, we can think about the distorted relations between traditional part/whole relations which occur in infinite collections, which are show by Galileo's Paradox:

Positive Integers:	1	2	3	4	5	6	7	...
Perfect Squares:	1	4	9	16	25	36	49	...

For every integer, there is a corresponding square. Given that both series are infinite, we will never "run out" of integers or perfect squares. However, the perfect squares are a proper subset of the positive integers, but we have just established a 1-1 correspondence between the two series. For those unfamiliar with the properties of transfinite math, this would be as shocking as claiming that I have the same number of fingers on my left hand as I do on both hands. Our standard intuitions about numerical relationships are subverted when considering infinite series because a proper part can be equal to its whole. This may be why Spinoza did not think that it was proper to call any type of infinity a Number.

²² In this case, the degree of infinity associated with this series is known as the "indefinite," a description of which occurs below.

Spinoza was not alone in this practice; equating “Number” with “finite Number” was a practice shared by most Early Modern philosophers and mathematicians.²³

Since Cantor, and the development of transfinite set theory, this difficulty has been resolved. We now accept that in the case of infinite collections, a proper subset can have the same cardinality as the whole. Indeed, this is one way of defining what it means for a set to be infinite. However, for Spinoza, these aids cannot serve their function when they are finite, as then there will be ambiguities about how exactly the infinimeter divides Quantity, to name one example.

Given that there cannot be an infinite Number, in what sense can an aid of the imagination be called infinite? He tells us that there are “things that can be called infinite, or if you prefer, indefinite, because they cannot be expressed by any number, while yet being conceivable as greater or less” (106). To illustrate this he provides an instructive example in Letter 12. Consider two inscribed non-concentric circles:

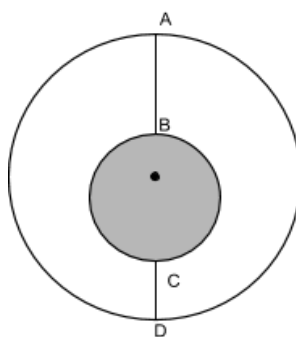


Figure 2: Spinoza’s circles.

Spinoza points out that in this example, we can know the distance of segments AB and CD, which are the maximum and minimum distance between the two circles, respectively

²³ Rice. p. 34.

(105). However, we can create a never-ending number of additional segments between this maximum and minimum²⁴:

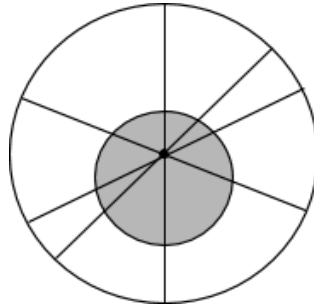


Figure 3: More distances.

In other words, the question of how many differences in distances there are between the two circles cannot be associated with a finite number. And, since we already saw that Spinoza regards all numbers as finite, Spinoza would say that we cannot assign any number to this collection, but it is still infinite. However, Spinoza says that we can conceive of these infinities as greater or less, since the amount of distances in a part of this space can be conceived of as lesser than the amount of distances in the whole of the image. For example, if we cut the figure into quarter circles, there will still be an infinite number of lines we can draw, and for any one quarter there will be lines that are in the whole, but not this part. However, we still call both call the part and the whole infinite.²⁵ Therefore, even though infinities of the third kind are such that “number is inapplicable to it without manifest contradiction,” we can still conceive of them as “greater or less,” since we can talk about part/whole relations despite (105-106). This is the most moderate

²⁴ Gueroult p. 204

²⁵ Gueroult 204.

kind of infinity, for it only comes about as a result of the mind's ability to imagine infinite division.

v. Beyond the Letter

In this chapter, I have attempted to clarify Spinoza's provocative claim that there are three radically different infinities, corresponding to Substance, Modes, and the aids of the imagination. By clarifying these concepts, I have shown how to solve standard problems in interpreting Spinoza's thought, which assume that Spinoza's concept of infinity is akin to our more unified contemporary notion. By attending to the sharp lines Spinoza places between the types of infinity, we can come to definitive interpretations of what Spinoza meant by "Extension" and "Quantity," to name just one pair of terms which have puzzled commentators. In the following chapters, I use the type of infinity which applies to substance to answer questions about how many attributes there are within substance, as well as how to reconcile the indivisibility of substance with the separation between the attributes. From there, I address the second degree of infinity, that which is infinite by nature, to explain Spinoza's concept of an infinite mode. The concept of infinite by cause can also be used to understand infinite collections of finite modes, which can provide insights into Spinoza's famed parallelism doctrine. I also explore Spinoza's legacy when it comes to the infinite, showing the influence he may have had on Leibniz and Cantor.

Chapter 2: A God of Infinite Attributes.

Now that I have set down the groundwork for understanding Spinoza's concept of infinity, it is time to turn to Spinoza's magnum opus, *The Ethics*. This chapter will focus on the Attributes, which Spinoza defines in ID4 as "That which the intellect perceives of a substance, as constituting its essence," and God, who is defined in ID6 as "a being absolutely infinite, that is, a substance consisting of an infinitely many attributes, of which each one expresses an eternal and infinite essence." Despite the promising structure of the *ordo geometrico*, there are numerous puzzles as to exactly what Spinoza means by these definitions. For instance, are the attributes really distinct from one another in God, or is their division merely conceived in the human mind? Is God ontologically prior to the attributes, or does God emerge from taking the attributes as a collected whole? Additionally, Spinoza says there are an infinity of attributes, yet only lists two in *The Ethics*, thought and extension. Spinoza was aware that claiming God had infinite attributes and then only listing two would leave many unsatisfied; Spinoza addresses this concern in "Letter 64," written to G.H. Schuller in 1675, albeit in an equally unsatisfying way. With an eye towards Letter 12, it is time to visit the scholarship around these questions.

But before entering into the interpretive questions, it is important first to have a sense of what the text asserts. First of all, the attributes are connected to substance, with substance defined as "what is in itself and is conceived through itself, that is, that whose concept does not require the concept of another thing from which it must be formed" (ID3). As mentioned above, attributes are defined as "That which the intellect perceives

of a substance, as constituting its essence” (IP4). Considered in themselves, the attributes are prior to any individual finite being. So, the attribute of thought has an ontological priority over any specific mind, and the attribute of extension is required for there to be any specific bodies. This is what Spinoza means when he claims in IP1 that “a substance is prior in nature to its affections.” Spinoza also claims no two substances can share the same attribute, in a very puzzling proposition which will be discussed later. He goes on to say that God is the only substance in nature, and “whatever is, is in God” (IP14-15). Therefore, all the objects of our everyday world are just minor properties of substance, rather than beings which have an existence independent of, or “outside of,” God. Does this mean that humans are the union of a thinking thing and a separate extended thing, where the mind tells the body to act, and the body gives information to the mind? For Spinoza, the answer is an emphatic “no.” Since they are infinite in kind, the attributes possess autonomy over their domains, so to speak. Nothing in the attribute of thought can affect any mode whatsoever in the attribute of extension, and vice versa. This is argued through the fact that the attributes of mind and extension have no essential commonalities, and IP3 states “if things have nothing in common with one another, one of them cannot be the cause of the other.” Despite being causally isolated from one another, the structures of the modes in both attributes are isomorphic, as explained by Spinoza’s famous IIP7: “The order and connections of ideas is the same as the order and connection of things.” So the order and connection of the physical world involves bodies related to each other in causal ways, such as there being six chairs around the dining table. And in the attribute of the intellect, there will be a relationship isomorphic to this physical arrangement.

Combined with the isolation across attributes, this means that we can construct different causal stories for the same event, but under different attributes. I could talk about the world in purely physical terms, saying that light of a certain wavelength struck my eyes, caused the rods and cones to trigger activity in the nerves leading to my brain, which caused complex processes involving signals being sent to my arm causing me to reach forward, and so on. Or, the same order of events could be described under the attribute of thought, by saying that I had a perception of seeing a slice of pizza, and then I felt an urge to eat the pizza, followed by an experience of my arm extending forward, and so on. For Spinoza, these two stories describe the same structural sequence of events, but considered under different attributes. Although the causal sequence in thought has a counterpart in extension, neither requires us to call in other attributes during the course of our explanation. However, in Spinoza's account, many questions are left unanswered. I will now try to address some of these problems.

i. The Substance with All Attributes

Proposition 5 of Book I states "In nature, there cannot be two or more substances of the same nature or attribute." While this may not seem like an important matter at first glance, it is one of the foundations of his whole system. Later on, in IP14, Spinoza uses this proposition to argue that there is just one substance, i.e. God. Spinoza's strategy to prove this substance monism is to first show that no attribute can be possessed by two substances. From there, he shows that a substance outside of God would entail a contradiction. Due to this foundational role, a lot rides on the proof that no two substances can have the same attribute.

The proof of this crucial proposition starts with two possible criteria for distinguishing substances from one another: by a difference in attributes or in affections. He says if we distinguish them by their attributes “then it will be conceded that there is only one of the same attribute,” which means that if we’re using the fact that they possess different attributes to distinguish the substances, then there will be no attributes that are shared (more on this later). The other option to distinguish two substances is to look at their modes. If we distinguish them by a difference in affections or modes, then we’re committing a category error, since particular affections/modes aren’t a part of the essential nature of a substance. Instead, we need to consider the nature of the attributes themselves. And once we do this, we would have two of the same attributes, so there would be no way to distinguish between the supposedly different attributes.²⁶ A substance which has extension and an ordering of nature which differs from our own, where Earth is the 4th planet from the sun, for instance, could not be considered as a possible counterexample, since neither ordering of the planets is contained in the very definition of any attribute. But even if we concede this point to Spinoza in the case of substances with only one attribute, we could still raise objections in the cases of substances which have multiple attributes. Why can’t the substance consisting of thought and extension be distinguished from the substance consisting only of thought? Or even worse, what about a substance with attributes A and B and another one with attributes B

²⁶ Note that this argument is based on the fact that there would be no way to distinguish between the two substances, there could not be two such substances. For a further discussion of how this principle of sufficient reason functions in Spinoza’s work, see Della Rocca, 2003.

and C? Even though they share B in common, we could distinguish them by virtue of their disjoint attributes: A and C.²⁷

The proper response to this line of argument is that the non-shared attributes are irrelevant when considering the supposedly shared attributes. After all, Spinoza says in IP10 that “each attribute of a substance must be conceived through itself.” Like the causal stories about modes, explained above, identification of substance needs to be restricted to considering just one attribute at a time. Here, Spinoza refers to Definitions 4 and 3: that an attribute is what an intellect ascribes to the essence of a substance, and the definition of a substance entails that we can grasp the substance with its essence alone, which makes the objection which ended the last paragraph invalid. For if we think about the attribute B from the previous paragraph, it’s implied that conceiving of it alone will mean we’re conceiving of it as constituting the essence of a substance. Let’s call this Substance_B. However, we supposed that there is a substance with A, as well as a substance with C. So, now we actually have two substances: Substance_{B+A} and Substance_{B+C}. Unfortunately for this solution, this involves introducing another attribute to properly conceive of a substance, contrary to the definition of substance provided. So, B cannot properly function as the essence of a substance, which means it cannot be an attribute.²⁸ Or, to use more contemporary terminology, an attribute is a definite description, so it must denote only one object. Instead of thinking of God as “*an* extended substance,” the concept of an attribute means we must think of God as “*the* extended substance;” Spinoza’s concept of essence and substance entail this conclusion. Note that

²⁷ Bennett, pp. 69-70.

²⁸ *cf.* Della Rocca, 2008. pp. 48-9.

this doesn't mean that each substance can only have one attribute. It just means that one attribute is sufficient to pick out a substance.

Have we reached the end of objections about Spinoza allocating the attributes to a single substance? Unfortunately, nothing in Spinoza scholarship is that simple. As witnessed in the previous chapter, I am fond of the concept of one person's *modus ponens* being another's *modus tollens*. In this case, the conditional would be "if there is a substance with all attributes, then there cannot exist a substance which consists of a finite number of attributes." Spinoza thinks the antecedent is true, since God contains all attributes, meaning there are no substances outside of God. But why couldn't we simply say that the converse is true: there could exist substances which consist of only one attribute, so therefore a substance containing any of these attributes in a larger collection cannot exist. This would be especially problematic for a substance that supposedly contains every possible attribute, which would Spinoza's proof of God's existence false. And although Descartes would likely be appalled by Spinoza's definition of God, he would say that if we had to choose between many singular substances and one substance with many attributes, he'd go with the former option. This comes from his view in the *Meditations* that conceptual distinction entails actual distinction. Thus, the conceptual distinction between thought and extension would entail that they belong to fundamentally separate beings. Spinoza has an argument for establishing that God needs to exist, but it is very weak as it stands in the *Ethics*. However, with the addition of Letter 12, we can construct an argument that's a bit more convincing.

Spinoza's argument for the existence of God in the ethics occurs at IP11. It's relatively straightforward in its demonstration. God is a substance, and therefore we

cannot deny its existence, since substance has a self-caused existence. But why not take the following argument: the substance consisting of only one attribute must exist, since this thing is defined as a substance and therefore it pertains to its nature to exist. In the early propositions of Book 1, Spinoza has only proven that it is the nature of substance to exist, but not that a substance of infinite attributes must take precedence over an infinite amount of mono-attribute substances. In order to produce the result that no substance can exist outside of God, Spinoza needs to show that the antecedent, that God exists, is necessarily true in a way that rules out the possibility of there being an any number of substances with different attributes, a situation that when combined with the no shared attribute doctrine would rule out the existence of God as defined by *The Ethics*. Only after this fact has been established can the no shared attribute doctrine be used to show that there is nothing outside of God.

In *The Ethics*, one of the arguments which most clearly relates to this point is also one of the more obscure. IP10, the proposition that each attribute must be conceived through itself was mentioned above, but the Scholium to this proposition explains that a substance can still have more than one attribute despite the need to conceive of each attribute independently. Here he states that “the more reality, or being it [substance] has, the more it has attributes which express necessity, or eternity, and infinity.” Huenemann refers to Spinoza’s *Short Treatise* to explain this claim.²⁹ In the *Short Treatise*, an analogy is made between modes and the attributes. Just as the extended modes of my body, my house, the planet Earth, and all other beings are all united in the attribute of

²⁹ 2013. pp. 52-54.

extension, the attribute of extension and thought are like “modes” (loosely speaking) of God. That is, just as an infinite number of things are bound under the attribute of extension, an infinite number of attributes are similarly bound under the infinite substance: which is God. Melamed takes this approach as well, though he is agnostic regarding the exact nature of the relationship between attributes/substance. What he does stress is that it’s important to acknowledge that the relationship between the attributes and substance is similar in structure to the one between the attributes and modes.³⁰ Just as extended modes emerge from and are dependent upon the attribute of extension, the infinite attributes all stem from God as the infinite substance consisting of all possible attributes.

Once again, why must we say that we ought to think of these attributes as all united under substance instead of as separate entities? The modes need to be united under an attribute since they cannot exist through their own power, but attributes are not subject to this requirement. To imagine an attribute is to imagine a substance with that attribute, as mentioned above. However, a reason as to why the attributes should be conceived of as united can be found in Letter 12.

Imagine that there are an infinite number of separate substances with different attributes. What kind of infinity would we use to describe this situation? Using the taxonomy of Letter 12, this would be the third type of infinity, since it could be conceived of greater or less in a metaphysical sense (e.g. all the attributes without exception, and all the attributes except for thought). But for Spinoza this type of infinity

³⁰ Melamed, 2013. p. 85

is extremely weak, not at all suitable for an account of the fundamental ontology of the universe. On the other hand, if all the attributes were united in a single substance, there would be no collection of independent entities of which to speak, since they are all connected under one substance, which is infinite in the highest degree: as a necessary unified whole. This is why one substance with all the attributes would be more fundamental than any collection of independent substances. Thus substance monism is a result of Spinoza's belief that a unified infinity which exists absolutely is metaphysically stronger than any amalgamation of lesser beings.

Identifying God as presented in *The Ethics* with the infinite by nature of Letter 12 is also a move Nadler presents, arguing that there is enough of an overlap between the ways that God is described in *The Ethics* and the properties of the first kind of infinity identified in Letter 12. Nadler points out that the first type of infinity is "what medieval philosophers referred to as an absolutely infinite being," by the nature of "infinity" meaning "all" in this context.³¹ Since the first kind of infinity in Letter 12 possesses the most perfection of all the types of infinity, for Spinoza a substance with all attributes is a more rational account than a world where there is no being to call absolutely infinite.

Although this may solve the problem of whether a single being with infinite attributes can be posited in opposition to a world containing a plurality of substances with separate attributes, one problem remains. In Letter 12, that which is infinite by nature can admit to no separation without resulting in a loss of coherence. But God contains all attributes, which are separate essences. How could God retain this degree of essential

³¹ Nadler, p. 68.

infinity while still containing these distinct attributes? To show that this is not a problem for Spinoza, I now turn to the nature of the distinctions among substance, and what is meant by the term “indivisible” in this context.

ii. Substance/Attribute Relationship

So we face a problem: if God is the infinite collection of attributes, why can't we divide God into different subsets of attributes, thus creating a division (contrary to both *The Ethics* and Letter 12)? One solution is to say that the differences of attributes are merely conceptual, being non-actual differences that stem only from our finite intellect. In this reading, thought and extension are not really different essences, but are the result of the mind seeing the same thing in different ways, and mistakenly concludes that this means that there really are differences between extension and thought independently of our minds. In reality, according to this view, there is no natural separation between the different attributes. One possible defense for this view is that when Spinoza defines attributes, he says that they are what “the intellect perceives of a substance, as constituting its essence” (IP4). This is different than simply saying that attributes constitute the essence of substance, without making any reference to the intellect. However, while this may be a possible reading of IP4 itself, other passages in *The Ethics* make this interpretation overwhelmingly implausible.

First of all, Spinoza argues repeatedly that two substances cannot have the same attribute. Although he argues for this based on our inability to mentally distinguish two substances via a common attribute, the conclusion he draws is ontological, not epistemological. In general, Spinoza tends to argue for features of God based on

supposed *a priori* concepts.³² And if our ideas of division in the attributes don't reflect actual divisions in nature, then why does Spinoza use our other ideas of substance as a pathway to describing how things truly are? IP2, for instance, uses the fact that if there are no conceptual connections between different substances, then they have nothing in common. This is a metaphysical conclusion drawn from epistemic premises. And in IP19, Spinoza says that all of God's attributes are eternal. Eternal beings must exist by their own necessity, as shown in Letter 12. If attributes are created by the mind, then something is required to create these concepts. But if the attributes are just subjective concepts, then the mind has to create them at some point, meaning they would not have the self-contained existence of eternal objects. For these reasons, among others, the best overall interpretation is that the different attributes are the result of objective divisions among the attributes.

We can now return to the central question: if God is infinite in the first sense, how is it that God can have real distinctions, given the way the first kind of infinity is presented in Letter 12? The converse is equally problematic: if God is infinite in the first sense, how could God possess real divisions? This is especially pressing given IP13, which states that "A substance which is absolutely infinite [i.e. God] is indivisible." After all, if thought and extension are separate, with nothing in common, then doesn't that create a division within God, contrary to IP13, and so many of Spinoza's central claims?

The solution I propose depends on the definitional distinction between God and substance. Once we reach the midpoint of Book I, it is clear that God and Substance are

³² As mentioned above, see Della Rocca 2003 for an explanation of how Spinoza makes use of epistemology to explain ontology.

one and the same in nature (essence and existence) but prior to that, substance is talked about in general. Using contemporary terminology, substance and God have the same extension, but different intensions. For instance, water and H₂O refer to the same thing, but are different concepts with respect to their intensions. And in *The Ethics*, a similar situation occurs. God and substance are defined separately, and Spinoza has to invoke other concepts and arguments to show that the two are identical. Thus, to show how division can occur within God, we must distinguish between dividing God and retaining the defined nature of substance, and dividing God and retaining the defined properties of God.

As mentioned previously, God is defined as “a substance consisting of infinite attributes,” and substance is defined as “what is in itself and conceived through itself” (ID6 and ID3). Through later developments in *The Ethics*, these two definitions are shown to necessarily correspond to the same being, but substance is actually a “wider” concept when considered in itself. All that is required for a substance is a necessary (self-caused) existence, which is manifested through some attribute. Thus to conceive of God through attribute A is to conceive of a substance consisting of attribute A. Have we divided our concept of God? We have taken one of God’s attributes and then conceived of a new being with no intrinsic contradictions. Obviously, this new substance cannot exist independently, given Spinoza’s arguments for substance monism. However, when considered in itself, there is nothing incoherent about this concept. Dividing God’s attributes into separate substances therefore does not lead to an internal inconsistency, like a square circle, but it is a possibility which is ruled out by other necessary and *a priori* facts about the world. Once we have done this division, we have annihilated our

concept of God, as we are now considering a substance which does not possess every attribute but only one, which contradicts the defined nature of God. So, if by division, we simply mean that we can take one of God's attributes and conceive the properties of that attribute and its modes in isolation, then God is divisible, as Spinoza himself suggests in the early parts of Book II. But, if we mean that God can actually be divided into two separately existing parts and retain the nature of God (like how macroscopic matter can be actually divided and both parts will still retain the nature of matter), then God cannot be divided.

Thus, my reading of IP13, which states that "A substance which is absolutely infinite [i.e. God] is indivisible," is that we can perceive the distinct attributes of God, but we cannot state that we have found new substances which consist of those different attributes. To use an imperfect analogy, we can recognize the distinct blocks in a Jenga tower with many blocks removed, while being unable to remove any of the blocks and form a separate tower without the whole structure collapsing. Or to use another example, the type of division which is forbidden isn't imagining the top half and lower half of a piece of paper geometrically, it's actually ripping them apart. Thus, to say that God is indivisible doesn't mean that distinct attributes cannot be identified. It means that once those attributes are considered, it does not follow that these attributes can be separated into new substances. In this sense the relationship between substance and attributes is that attributes do form distinct properties within God, but cannot be given a separate existence independent of God, as proven in Book I of the *Ethics*.

iii. How Many Attributes are There?

So far, we have seen there is one substance, with infinite attributes, and it cannot be divided into separate substances, through an actual separation. But what exactly does “infinite attributes” mean? Does it mean that there are a finite number of attributes, each of which is infinite, or does it mean that there are an infinite number of them? So far the second answer has been assumed, but there are compelling arguments for the former which deserve attention. Additionally, why is it that of the infinite collection of attributes, we are only capable of conceiving of two of them, as Spinoza admits in IIA5? This is an axiom that states “we neither feel nor perceive any singular things, except bodies and modes of thinking.”

One way to resolve this issue is to consider, once again, at the definition of God. Spinoza adds a note to this definition explaining what he means by absolutely infinite: “if something is absolutely infinite, whatever expresses essence and involves no negation pertains to its essence.” To use a mathematical analogy, the set of all integers may be infinite in kind, since there are no integers which fall outside of its scope. However, this set is not absolutely infinite, since there are numbers in the mathematical universe which are not contained within this set.³³ This still doesn’t answer the question of how many attributes there are, since without any clue as to what the other attributes could be, it’s entirely possible that there are no attributes other than thought and extension. If every attempt to construct properties other than the two which are always given result in a

³³ Of course, even the mathematical universe is not absolutely infinite in the Spinozistic sense, since there are physical objects which are not a part of the mathematical universe.

contradiction, then two is the maximum amount of attributes there can be. That would mean that it is possible for “absolutely infinite” to mean two, but only if those two are the only things which “expresses essence and involve no negation,” the criteria which make God an absolutely infinite being. The best solution to this problem would be to give an example of an additional attribute, or, better yet, a set of guidelines for identifying new attributes.

But Spinoza doesn't even hint towards any of these solutions, although he repeats his insistence that we can only know two of God's attributes in Letter 64. He says in response to G.H. Schuller that “the human mind – i.e., the idea of the human body – involves and expresses no other attributes of God except these two [thought and extension].”³⁴ Therefore, the best evidence we have to support an answer to the question of whether or not there are more than just two attributes is plausibility. If the two mentioned attributes were the only existing attributes, we lose potential diversity within God, but now we have an understanding of God that's stronger than “a being of infinite attributes,” since we can list the attributes under consideration. On the other hand, a God of with an infinite amount of attributes would have more essences united under it than an account where God has just two attributes, but we cannot come close to comprehending, or even naming, the basic essences of God. If we can explanation the gap that emerges between the epistemology and the metaphysics of this latter account, it gives a more tenable alternative to the scenario where God has just two attributes. Recently, Melamed

³⁴ In Shirley, p.298.

has offered a convincing account, and I will expand upon it in the remainder of the chapter.

One way to think about the attributes is that the essence they correspond to constitute that attribute's basic "job." So the job of extension as an attribute is to possess geometric distance relations, and the job of the intellect as an attribute is to represent modes in the other attributes. This leads to an imbalance in the importance of intellect compared with the other attributes. In this account, the strict separation between the attributes remains, since none require the fundamental natures of any other attribute in their essences. And since the intellect has the job of representation, it reflects the features of all of the other modes. As mentioned before, the mind and body are parallels of each other, and our experience of our mind, just is a representation of those modes (IIP13). Now let's introduce a different mode, a correlate of my body, but under attribute X, whatever that may be. Since intellect's essence seems to involve representing other modes, there will be a reflection of this mode of Attribute X in the intellect, just as my body as an extended being is represented as such under the intellect.³⁵ But there are also ideas representing all the other attributes, so why doesn't my mind have ideas of those? Melamed's answer is that each mode in the intellect has a complex internal structure, like a multifaceted gem (but with infinite sides). In Melamed's own words, "each idea has infinitely many aspects, so that each idea-aspect uniquely represents a mode of God

³⁵ For an account of why the intellect is representational in nature, consider IP30: "An actual intellect, whether finite or infinite, must comprehend God's attributes and God's affections, and nothing else." This proposition is not phrased as extension extending all of God's attributes and affections. If structural parallelism is true, then these two phrases might be equivalent, but the necessity of structural parallelism will be debated in the next chapter.

under a particular attribute.”³⁶ And each of these facets are just as isolated from each other as the differing attributes they reflect. Put more concretely (or at least as concrete as one can be in the realm of abstract metaphysics), my mind is the representation of my body and all the other extended things which interact with it. And my body is only affected by other extended modes, neither X modes, Y modes, nor any the other modes which belong to the attributes other than thought and extension. The reason why we can’t conceive of any of these mysterious attributes is because the intellect reflects the causal and relational orders in nature, and without any causal bridges between the attributes, there is no mapping between the attributes in the intellect. And thus, we cannot conceive of them. A textual justification for this comes from IIP26, which states that “The mind does not perceive any external body as actually existing, except through the ideas of the affections of its own body.” In other words, our ideas are just parallels of things acting on our bodies. And with no modes from other attributes acting on our bodies, *our* minds cannot form perceptions of them (although there will necessarily be minds in the intellect that do form perceptions of them, but not extension or any other attribute). Since ID4 states that the attributes are what “the intellect perceives of a substance, as constituting its essence,” without anything from another attribute to affect our bodies, “the mind does not perceive” these attributes.

Melamed’s interpretation is that there are actually two parts of IIP7. First, there is the claim that all ideas reflect things in the universe. This is the main proof of IIP7, which states “the order and connection of ideas is the same as the order and connection of

³⁶ Melamed, p. 156.

things.” Melamed’s reading tells us that all we can infer from this statement is that for each thing, there is a mirror idea of that thing in the intellect. But this statement is not a two-way street. There could be two separate orders of things in attributes X and Y, and there would be a parallel structure of the ideas of those things in the intellect. But this does not entail that there has to be a universal structure across all of the attributes. In other words, for any attribute, there exists some part of the intellect which has the same structure of that attribute. But it’s still possible that there are structures in the intellect which only have a counterpart in one attribute, but not another. For instance, it could be that my extended body is mirrored in the intellect, but not any other attribute. IIP7 only states that for any attribute, something exists in the intellect whose structure is isomorphic to that attribute’s modes. But this does not entail that everything must share a similar structure across the attributes. This is why the claim that there is a universal order and connection across every attribute requires an additional premise.

Melamed points out that this second premise comes from Spinoza’s scholium to IIP7, which contains the phrase: “whether we conceive of nature under the attribute of extension, or under the attribute of thought, or under another attribute, we shall find one and the same order.” According to Melamed, “E2P7 asserts a parallelism between *two* domains of entities (ideas and things), while E2P7S asserts a parallelism between *infinitely* many domains (the infinitely many attributes).”³⁷ The scholium claims that in the Spinozistic universe every set of causes and effects under one attribute has an isomorphism in every other attribute. While this second doctrine presents a view of a

³⁷ Melamed, p. 142.

universe which involves beautifully complex parallels between all of the attributes, it stretches too far beyond the propositions and axioms Spinoza presents. The only thing we can infer from IIP7 itself, (which only relies on IA4) is that modes outside of the intellect have a reflection within the intellect, not that there is a similar order and connection among the non-intellect attributes. But this leads us to the modes, and not attributes, which are the focus of this chapter. What's important here is that the existence of other attributes can be accounted for by the lack of causal interaction between any of the modes of different attributes.

In this chapter, I took the concept of infinite by nature present in Letter 12 and showed how its properties can be applied to God and its attributes in the context of *The Ethics*. Despite being distinct entities, the attributes are not separable from God in the ways that produce independently existing objects. Additionally, a possible answer was given to why God's other attributes are both unknown and unknowable to us as mind which are ideas of bodies. The next step is to address the next rung on the ladder of infinite categories, that which is infinite by cause.

Chapter 3: Infinite by Cause: Big, But not Everything

The next level of infinity to examine is “infinite by virtue of the cause in which they inhere; and when the latter [this kind of infinity] are conceived in abstraction, they can be divided into parts and regarded as finite.”³⁸ This is the type of infinity which corresponds to the world of modes. As mentioned in the first chapter, modes describe the objects of everyday experiences. In this level, nothing is intrinsically necessary, which means that everything depends on the existence of other things, whether substance itself or other modes. Once the whole causal chain is put forth, everything becomes necessary, but in themselves, the essences of modes don’t involve existence. It is only once we consider the order of nature that we can determine whether or not a mode exists. This is what Spinoza proves in propositions 24-29 of Book I. IP29 is “In nature there is nothing contingent, but all things have been determined from the very necessity of the divine nature to exist and produce an effect in a certain way.” The proof of that is the following:

Whatever is, is in God (by P15); but God cannot be called a contingent thing. For (by P11) he exists necessarily, not contingently. Next, the modes of the divine nature have also followed from it necessarily and not contingently (by P16) - either insofar as the divine nature is considered to act in a certain way (by P28). Further, God is the cause of these modes not only insofar as they simply exist (by P24C), but also (by P26) insofar as it is impossible, not contingent, that they should determine themselves. Conversely (by P27) if they have been determined by God, it is not contingent, but impossible, that they should render themselves undetermined. So all things have been determined from the necessity of the divine nature, not only to exist, but to exist in a certain way, and to produce effects in a certain way. There is nothing contingent. q.e.d.

³⁸ Letter 12. p. 106.

Besides giving an insight into how confusing Spinoza's writing style can be at times, this passage shows that even though the existence of a specific thing, like my chair, may not be a self-caused and intrinsically necessary being, it is a part of a causal ordering whose necessity follows from God when conceived as a substance. This chapter will focus on the relationship between the infinitely many modes and the one substance, and relevant to this paper is the fact that duration goes backwards infinitely for Spinoza, making this causal chain infinite. Specifically, the two major topics will be how the infinite sequence of modes are entailed by timeless and unchanging infinite substance, and whether or not it is necessary that every attribute have the same order and connection with respect to their modes.

i. From the Infinite Universe to the Closed World

The first kinds of things that directly follow from the nature of an attribute are the infinite modes. These strange types of mode are introduced in IP21, where Spinoza describes "things which follow from the absolute nature of any of God's attributes." Spinoza argues through a proof by contradiction that these modes must be infinite. For if they were finite, then they would have to be limited by something (which is what ID2 asserts about the nature of finitude). But we're talking about the first mode to be entailed by an attribute, so there are no modes out there to limit it yet. And if we say that it has a finite duration, that too would have to be limited by some other mode destroying it. Since these modes follow directly from the eternal substance, there is nothing which could possibly limit it. Thus, they are infinite, but in the sense which belongs to modes, not substances, since they are not self-caused. Unlike the attributes, the infinite modes do not

constitute the essence of a substance, and since they cannot be conceived as constituting the essence of a substance, they are not an essential feature of the attributes.

Additionally, we can classify the infinite modes into two separate categories, those which follow directly from an attribute, and those which follow from another infinite mode. In Letter 64, Spinoza gives two examples of the first kind of infinite mode, also known as an immediate infinite mode. In the case of thought, we have an infinite intellect (which is called “The idea of God” in *The Ethics*), and in the case of extension, we have “motion and rest”. And an example of an infinite mode which follows from another infinite mode, which are sometimes called mediate infinite modes, is “the face of the whole universe.”³⁹ This statement can be seen as the first test of trans-attribute parallelism, a test that I could not find addressed in the literature. For if the order and connection of thought and extension really are the same, then the immediate infinite modes of both attributes must serve the same function in the overall structure of their respective modes. What this means is that the causal role that an infinite intellect plays in the attribute of thought must be isomorphic to the causal role that motion and rest plays in extension. In order for Spinoza’s structural parallelism to hold, there must then be some commonality between an infinite intellect and motion and rest.

The best answer to the question of how motion and rest can serve the same the same function as an infinite intellect comes from Spinoza’s definition of an individual in the “physical interlude” of Book II. Here, an individual thing is defined as the continuation of certain proportions of motion and rest (IIP13S). Using this principle of

³⁹ Spinoza. Letter 64. In Shirley 1995. p.299

individuation, we can apply a similarity between the seemingly separate concepts of motion and rest and an infinite intellect, since every idea of a physical individual will be an idea of a certain proportions of motion and rest. Therefore, bare motion and rest has the ability to produce all of the things which can fall under the infinite intellect, allowing us to say that they are parallel concepts. Despite the success in explaining how these two concepts can parallel each other, we still need to say how the infinite modes follow from substance, as well as how finite modes follow from infinite modes.

The main problem with understanding how infinite modes lead to finite modes comes straight from Spinoza's text. IP22 asserts that what follows from an infinite mode necessarily must be infinite. This seems to imply that there can be no step at which an infinite mode entails that a finite mode follows. In Friedman's analysis of this issue, he argues that Spinoza has two separate accounts of necessary entailment. The first is a definitional type of necessity and can be done in a finite number of steps. For instance, it is necessary that, in Euclidean space, a triangle's angles will add to 180 degrees.⁴⁰ And it is necessary that given my essence, I am a human and not a goat. But if we were to ask why I exist, we need to look at the order and cause of nature, a point which has been made repeatedly throughout this thesis. And though it is necessary that given the state of the world, I came into being, the analysis of this necessity is different than reasoning with essences and definitions. This is because definitional reasoning about things other than God can only get us "if-then" reasoning about necessary properties.

⁴⁰ Friedman, "How the Finite Follows from the Infinite in Spinoza's System." pp. 384-6

If I exist, we can conclude that it will be necessary that I am a human being. But to see why it is necessary that I exist, an infinite regress into the order and connection of causes and effects must be performed, an impossible task for finite minds. So in one sense, necessity can describe why something is the way it is (its essence), and in another it answers why something is (its existence).⁴¹

For Spinoza, the essences of finite things give us truths about what is analytically necessary, but causal necessity comes from external facts. The distinction between these two types of necessity leads Friedman to conclude that Spinoza might not be as strict of a necessitarian as is often assumed, since he allows the possibility of the world being ordered differently, by ignoring other facts about how modes follow from other modes/substance. A world with different events would violate the causal structure of this world, but a world with a different causal structure would be a coherent idea that doesn't cause contradictions, according to Friedman. This analysis may seem tangential to the question of how the finite follows from the infinite, but for Friedman, it is the bedrock of his analysis. If we look for some definitional aspect of substance which leads to the intrinsic necessity of the world of finite things, we'll find no answer. Instead, the type of necessity with which the finite follows from the infinite is causal in nature, meaning that the question of how the world of our experience follows from the unchanging eternal essence of substance has to be analytical, not causal. When explaining this point,

⁴¹ And of course, in God these two concepts are united, which allows for *a priori* proofs of God's existence.

Friedman even goes as far as to say that Leibniz's doctrine of possible worlds might be compatible with a Spinozistic system.⁴²

One virtue of this reading is that it allows us to distinguish the differences between an infinite intellect, and the face of the universe, the first two infinite modes of thought. An infinite intellect would comprehend all of the formal essences of things. Formal essences are mentioned in IIP5, which states that the formal essences of things have God as their cause, as well as IIP8, which deals with the nature of non-existent things. For Spinoza, when considered formally, the essential properties of finite things which actually exist and those which don't are the same; it's the order and connection of nature which determine whether or not any finite thing is actualized. Therefore, the absolutely infinite intellect (as well as motion and rest for extension) involves every formal essence and all the ways that they could relate to one another in possible configurations of the world. And the first mediate infinite mode, the "face of the whole universe," is the infinite mode which corresponds to the arrangement of all of the finite modes in the actual world.

The problem that this picture faces compared to Leibniz's is that unlike Leibniz, Spinoza doesn't believe in a God who chooses which possibility to actualize. For Leibniz, God could create any possible world, but chooses to create the best, due to God's beneficence. But Spinoza sees the practice of assigning teleological purposes to God's actions as an unnecessary and unproductive error, as he repeatedly asserts in the Appendix to Book I of *The Ethics*. Unlike Leibniz, this cosmological question of why any

⁴² p. 396

one particular universe is chosen from a group of equally logically consistent universes remains a mystery. In other words, if we admit a plurality of logically possible orderings, what is the process which gets us the actual ordering of the finite modes?

By breaking Spinoza's account of necessity into two categories (logical and causal), Friedman has inadvertently introduced a new level of complexity into the problem of how finite modes follow from infinite modes. His interpretation may fall within the text of Spinoza's work, but by the strict necessity of definition from the specific ordering and connection of this world, he has opened a gap between the unlimited logical possibilities thought of in the infinite intellect, and the reality of the face of the whole universe. However, both the immediate and the mediate modes are infinite by cause, despite the fact that the "face of the universe" doesn't have the indivisible infinity of substance, since we could imagine this part of the universe, or just one of the formal essences comprehended by the infinite intellect. Because we can conceive of parts of these modes, they are infinite in a way that permits divisibility, which makes them distinct from the infinity which applies to substance.

The division between definitional necessity and causal necessity can be grounded by the propositions of *The Ethics*. In IP11 and IP17S Spinoza uses the example of a triangle and its necessary properties to show the difference between something which is intrinsically necessary, and something which necessary as a result of external forces:

This reason, *or* cause, [for a thing's existence] must either be contained in the nature of the thing, or outside it. For example, the very nature of a square circle indicates the reason why it does not exist, namely, because it involves a contradiction. On the other hand, the reason why a substance exists also follows from its nature alone, but it involves existence (see P7). But the reason why a

circle or triangle exists, or why it does not exist, does not follow from the nature of these things, but from the order of the whole of corporeal Nature. For from this [order] it must follow either that the triangle necessarily exists now or that it is impossible for it to exist now. (IP11).

Continuing with the triangle theme, Spinoza asserts in a discussion of God's lack of free will that:

Others think that God is a free cause because he can (so they think) bring it about that the things which we have said follow from his nature (i.e., which are in his power) do not happen or are not produced by him. But this is the same as if they were to say that God can bring it about that it would not follow from the nature of a triangle that its three angles are equal to two right angles; *or* that from a given cause the effect would not follow - which is absurd. (IP17S)

What Spinoza is saying here is that if a triangle were to exist, the interior sum theorem would necessarily hold.⁴³ This is different than saying that it is necessary that a triangle with 180 degrees exists. It just says that if one did exist, there is nothing that can stop it from following that a geometric analysis will show that its interior angles add up to 180 degrees. And for Spinoza, only substance's existence is a necessary consequence of its definition.

In the case of modes (both finite and infinite), essential properties tell us that if they exist, they'll have certain properties, but whether or not they actually exist is an open question to be filled by the order and connection of nature. This is what IA7 tells us about finite things: "If a thing can be conceived as not existing, its essence does not involve existence." Since we can conceive of a different ordering of things in the world,

⁴³ Like everyone else in his time, Spinoza thought that Euclidean geometry was necessarily true.

the ordering of things in this world is not intrinsically necessary, but must be determined by a fact stemming from one of the self-determined parts of nature, i.e. God.

God directly determines the immediate infinite modes of infinite intellect/motion and rest/whatever the infinite modes of the other attributes are, and then they cause a chain reaction of entailments which leads to the existence of finite things. This is what Spinoza claims in IP28 and 29, where he says that no finite mode is determined to exist by intrinsic necessity, but at the same time, nothing in nature is undetermined.

The problem is that it doesn't seem like there's any specific fact in the immediate infinite modes, the ones which are about every possible configuration of finite modes that cause nature to have the structure that it does. For these reasons there don't seem to be any guiding principles that can give a coherent account of why there's a necessary relation between the possible connections of formal essences and the actual connections of instantiated modes. Once the laws of physics and ordering of modes are set, then everything else follows by causal necessity. But there doesn't seem to be any necessary reason why of all the infinite possibilities for physical laws and events, only one specific one occurs. Thus, rather than solving the problem of how the finite follows from the infinite, the nature of the immediate infinite modes actual complicates this question by opening the door to a plethora of unrealized possibilities.

ii. Parallelism: Orders and Connections

Let's review the parallelism doctrine once more. Spinoza states that the order and connection of ideas is the same as the order and connection of things, meaning that the physical world and the idea of that world have one and the same structure, despite the

fact that no causal interactions exist between the attributes. This lack of causality was previously used to explain why we can form no ideas of other attributes. Ideas are representations of something, and without a link between other attributes: there is no physical structure which can form the idea of these other attributes in our minds.

However, this chapter will explore a possible gap in Spinoza's claim that there is a single order that exists across all of the attributes. Although I agree that in Spinoza's world, it follows that there are mappings from the structure of things onto the structure of ideas, I don't think that this is a conclusion he can give without bringing in results that go what can be established by his previous work. Spinoza does claim in the scholium to IIP7 that we will find the same order and connection in each attribute, but I believe that this is an *ad hoc* addition which isn't justified by any other parts of *The Ethics*.

The first place to start with this analysis is to once again look at IIP7. This proposition is simply that "the order and connection of ideas is the same as the order and connection of things." Its demonstration is:

This is clear from IA4. For the idea of each thing caused depends on the knowledge of the cause of which it is the effect. (IIP7)

And IA4 simply states that "the knowledge of an effect depends on, and involves, the knowledge of the cause." How IA4 and the concept of order and connection are related is not as immediately obvious as Spinoza seems to think. The best explanation for how these two passages are related comes from the fact that it is assumed that knowledge of an external world exists. And since knowledge the knowledge of an effect involves knowledge of a cause, knowledge of the world as it currently exists can only come about from world as it once was. And that past can't be known unless its past is also known, and so on. And because of Spinoza's strict necessitarianism about the relation between

causes and effects, the future is derivable from the state of affairs that exists in the present. So this knowledge will have the same structure as the universe, since it involves knowing causes and effects, while the structure of the physical world consists of those same causes and effects. For instance, the book on my table as an extended mode has other extended modes involved in the causes leading to its creation and its placement in its current position. And the knowledge of all of those things in the intellect will be structured in the same way, since to understand an object, Spinoza says we need to understand its causes. The fact that knowledge depends on a combination of both effects and causes leads the ideas of the world to mirror the way that the world is. Of course, full knowledge of all causes and effects is beyond the scope of our finite minds. In this case, trying to form ideas without the full scope is what leads to erroneous judgments, meaning that we make mistakes through a lack of knowledge, not through any positive ideas (IIP33 & IIP35).⁴⁴

In this mirroring relationship, Spinoza gives primacy to thought. The first instance of this is simply that he gives an account of the infinite idea (as a mode, not an attribute) of God in Book II before he gives an account of God's infinite extension. In this unique idea of God, Spinoza claims that an infinite number of things follow in an infinite way (IIP3). In this case, infinite once again means "all possible," so from the idea of God, every possible object and relation between objects follows in some formal way. However, what does Spinoza mean by the idea of God? Is it an idea belonging to God, an idea with God as its focus, or is it God's mind? The answer is technically all three. The idea

⁴⁴ A similar move is made by Descartes when accounting for how error arises during *The Meditation*.

belongs to God, since it is a mode which exists within God. But so is every other idea, a fact which Spinoza proved in IP15, and which has been mentioned previously. Whatever the idea of God is, it, like everything else, is within God. But more importantly, this is an idea of God in the sense that it is about God, since it is “an idea, both of his essence and of everything which necessarily follows from his essence” (IIP3). In other words, the objects of this idea are all of the attributes (God’s essence) and all of the modes which reside within those attributes (everything which necessarily follows from God’s essence). It is also God’s mind, since it contains every essential property of substance, as well as the essences of the modes. This makes it the mind of God, since there is nothing outside of substance, attributes, and modes in the universe, meaning that an intellect which understands all these things will understand everything there is to know in Spinoza’s account of ontology. And as an omniscient being, Spinoza’s God has a mind which thinks all of these things, and nothing more, making the idea of God understood as an absolutely infinite intellect the mind of God as well.

However, the properties of God’s mind make a balanced parallelism untenable. The idea of God involves comprehending *every* one of God’s essences/attributes, as well as an additional infinite hierarchy. For each idea, there is an idea of that idea. And there is an idea of that idea of that idea, all the way to infinity (IIP21). However, is not an exact parallel of this structure in the attribute of extension. The closest we can get is the essence of extension and a formal collection of all extended modes, but we won’t find the essence of the intellect or any other attribute in this infinite mode. Additionally, if we have a body, there won’t be a body of that body, like there is an idea of an idea. And in attribute X, whatever that may be, we can have the essence of X, and a collection of all

the X modes, but no infinite hierarchy like in the intellect. Therefore, the idea of God cannot have a parallel in the “body” of God, since the idea of God involves a concept of extension and ideas about those ideas, but extension doesn’t have an analogue which captures this reasoning. Thus, the intellect plays a privileged role among the attributes, since its modes can relate to any other attribute, as well as other modes within the intellect in a unique hierarchy, while the modes of other attributes do not have this reflexive power.

I called the theory which is untenable in light of this argument one where there is a *balanced* parallelism for a reason. We could say that there is an unbalanced parallelism among the attributes. In this account, the intellect’s essence of representing the other attributes is an essence which means that although both collections are infinite in their own kind, the collection of all of the modes of the intellect is greater than the collection of all the modes of any other attribute. This is because the intellect will contain an idea of all of the modes of the other attribute, plus the ideas of those ideas. But we could say that something similar remains among all of these essences and modes: structural relations.

This is what Spinoza says in the Scholium to IIP7:

A circle existing in Nature and the idea of the existing circle, which is also in God, are one and the same thing, which is explained through different attributes. Therefore, whether we conceive Nature under the attribute of extension, or under the attribute of thought, or under any other attribute, we shall find one and the same order, or one and the same connection of causes, that is, the same things follow from one another.

When I said [NS: before] that God is the cause of the idea, say of a circle, only insofar as he is a thinking thing, and [the cause] of the circle, only insofar as he is an extended thing, this was for no other reason than because the formal being of

the idea of the circle can be perceived only through another mode of thinking, as its proximate cause, and that mode again through another, and so on, to infinity. Hence, so long as things are considered as modes of thinking, we must explain the order of the whole of Nature, or the connection of causes, through the attribute of thought alone. And insofar as they are considered as modes of extension, the order of the whole of Nature must be explained through the attributes of extension alone. I understand the same concerning the other attributes. (IIP7S).

In this account, not only is it true that for any structure in an attribute, there exists a similar structure within the intellect, but it tells us that for any attribute, *every* other attribute shares the exact same order and connection. For instance, the mode of intellect which has my extended body as its subject will relate to the mode of the idea of my shoes in the same way that my actual body relates to my actual shoes. And if structure is the same across all the modes, there is some mode under attribute X which relates to some other mode in that same attribute which relate to one another in the same way that my body and shoes are linked in physical space. And of course, there will be an idea of this relation of these X modes as well. And in this theory, there will be objects which are similarly related to one another under every other attribute.

Although this account is different than how Melamed constructs his infinitely-faceted modes of the intellect, a similar picture emerges when considering the idea of my body as extended and the idea of my body under attribute X as two separate modes. Melamed would say that the parallel of my body would have just one mode in the intellect, but with an infinite number of “idea aspects” constructing the mode of thought,

where every aspect is a representation of a given mode under each different attribute.⁴⁵

Instead of postulating idea aspects to account for the ideas of modes under different attributes, we can combine the idea of my body and all of the ideas of its analogues in other attributes into a collection of modes, and do the same for all of the other sets of related modes. Instead of talking about a single mode with an infinite number of facets which represent all of the different attributes that share similar orders and connections, we say that we're talking sets which have an infinite number of modes as elements, with each set containing every representation of a mode and its parallels as an element (ex. the idea of *a* as an extended thing, the idea of *a* as something under attribute X, the idea of *a* as a mode of attribute Y, and so on). Each set, as well as the collection of all of these sets represents something which is infinite by cause. Each one possesses an infinite number of elements, but we can divide these sets into their constituent modes of the intellect, without a contradiction.

⁴⁵ Melamed, pp. 161-3.

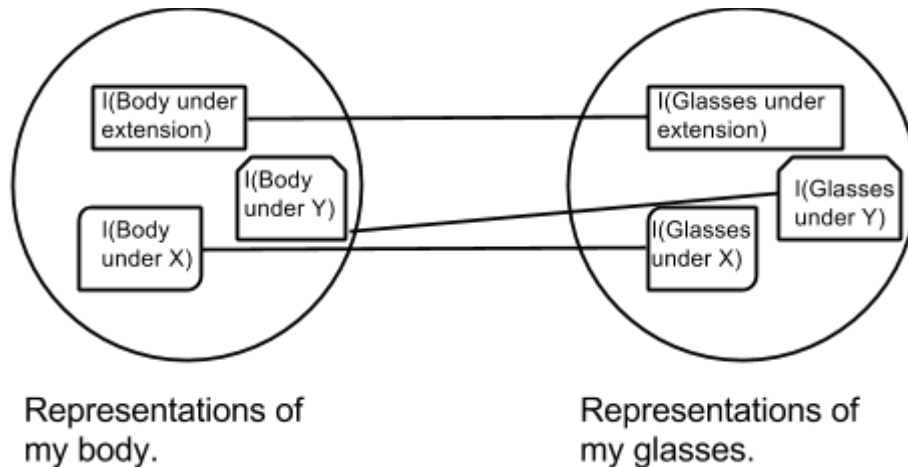


Fig. 4

An example of sets of ideas of things under different attributes, where $I(x)$ simply means the idea of x . According to IIP7, although each collection will contain an infinite number of different modes, the elements of one group will relate to the corresponding elements of another group in a similar way (represented by the lines).

These infinite collections of the modes of intellect fit well within the account Spinoza gives of infinity in Letter 12. None of these collections have the self-caused existence of substance, but they're still larger than any finite number could account for. Additionally, we can divide these collections into finite parts without getting a contradiction, since the finite parts would just be collections of finite modes. And if something was logically incoherent about collections of finite modes, we would be in trouble, since that would jeopardize most of our everyday reasoning about objects. Since I cannot draw an infinite number of things, Figure 4 is not a full drawing of what these infinite collections in the intellect would look like, but it is a finite approximation. All the ideas of my body under the infinitely many attributes could be put into one set, and all of the ideas of my glasses can be placed into separate groups. And the connection between

the ideas of my body and glasses under the extension will be the same as the actual connection between my body and glasses in extension. This connection will also be the same as the one between the idea of my body and glasses under attribute X, which is the same as the actual modes under attribute X. And adding the ideas of ideas, we get something like Figure 5. Here, the ideas of objects are connected to each other, and the ideas of ideas connect both to ideas and other ideas of ideas. And even though they aren't on the graph, there will be an infinite number of ideas of things, an infinite number of ideas of ideas, and an infinite hierarchy of ideas of ideas of ideas...

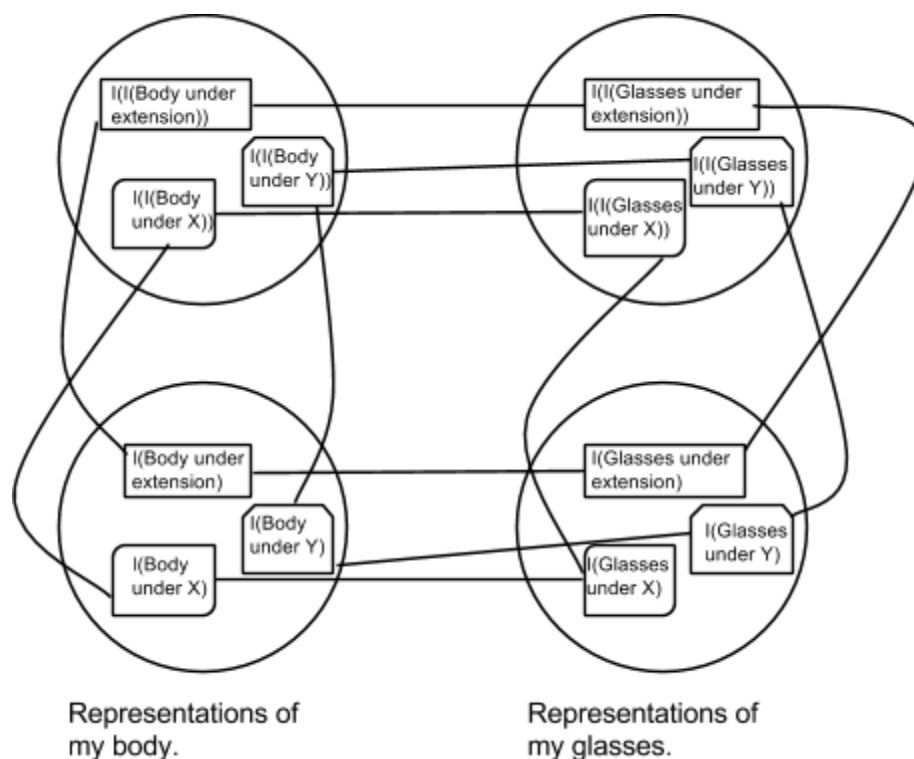


Fig. 5.
The order and connection of ideas, as well as ideas of ideas.

However, this picture does not necessarily emerge if we ignore the scholium to IIP7. If we just look at the proposition itself, all we have is that the order of ideas (of X)

is the same as the order and connection of things (modes of X), where X is the attribute in question. Thus, it would be possible that the idea of my body and glasses under extension relate to one another as the actual extended modes, yet it could still be possible for my body under attribute X to relate to my glasses under attribute X in a completely different way. It is only through the scholium that we get one universal order and connection. However, the scholium which asserts a single structure across the modes of every attributes does not cite any previous propositions, definitions, or axioms. Therefore, we can say that although it isn't what Spinoza intended, if we can show that the negation of a universal structure is consistent with the rest of Spinoza's system, then it will be a feature that's independent of the rest of *The Ethics*.

The reason why order and connection doesn't have to be preserved is rather simple. If we imagine a diagram of the modes of intellect, we can group them by the attributes which contain the modes they represent. So, there would be a cluster of all of the ideas of extended modes, and a cluster of all the modes of attribute X, the cluster of Y-modes, and so on (Fig. 6). Since Spinoza has a strong prohibition against cross-attribute causality, there would be no causal connections between any of these clusters, making them disjoint subgraphs, to use a more modern concept.⁴⁶ The order and connection of any of these components will be identical to the order and connection of one of the attributes. This is because of IA4, mentioned above, which is given for the justification of the parallelism doctrine. Since knowledge of an effect involves (and

⁴⁶ For an account of how to express Spinoza's universe in set theoretical terms, see Friedman, 1974. Although this account is primarily concerned with an account where a similar order and connection occurs across all of the modes, a project in this spirit could be made to model Spinoza's philosophy in contemporary mathematical expressions.

requires) knowledge of its cause, the collections of any ideas will share the same order and connection as the things themselves. And vacuously, if we select a group of things with no connections (i.e. they come from two separate attributes), then there will be no connection between the ideas of each of those objects. This is what we can determine with certainty, given Spinoza's previous definitions and axioms.

If we introduce the negation of the universal parallelism asserted in IIP7S, we get something like Figure 6. Here each mode of the intellect corresponds to a mode under one of the outer attributes. And for any attribute, the order of its modes is represented by one of the clusters within the intellect, meaning there is no contradiction with IP4. In fact, I can see no contradiction with the rest of Spinoza's system when introducing this possibility. Since *The Ethics* is consistent with either the theory that there is a parallelism across the attributes or the interpretation that the modes of each attributes have different structures, this fact is independent of the derivations given about Spinoza's world on the level of modes.⁴⁷

⁴⁷ Except for the parts that already take this as a given. But in that case, they assume what needs to be more explicitly argued for.

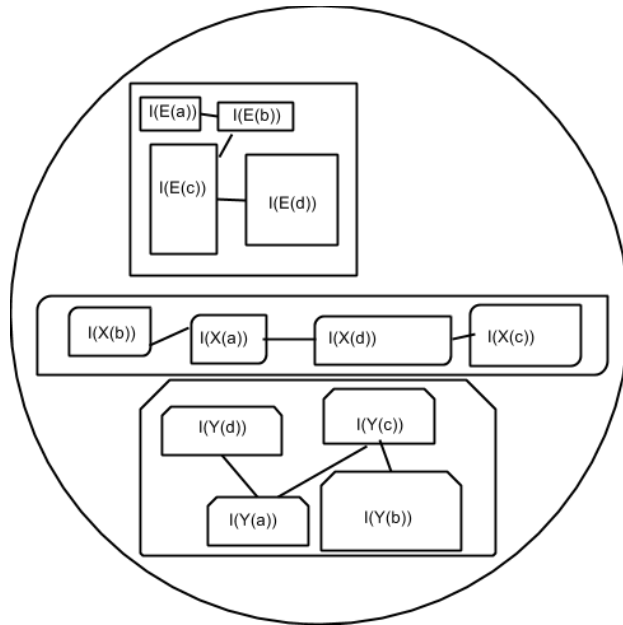


Fig. 6. The circle is the attribute of thought, and $I(E(a))$ represents the idea of mode a under extension. And $I(X(a))$ means the idea of a under attribute X. Here, each mode under every attribute is represented in the intellect, but with a different order and connection.

Chapter IV. Spinoza's Legacy: An Overview

In this final section, I explore Spinoza's intellectual legacy, as it pertains to theories of infinity. Spinoza's views on the infinite were highly complex and deeply rooted in his view of God and nature. When evaluating his thinking on the infinite, we can do so in two ways. One is to ask how well this concept fits with his overall philosophical system. Obviously, this analysis has been conducted in the previous chapters. Another way to show the quality of Spinoza's thought is to show that he influenced later generations of philosophers in a positive way. This means showing that he impacted their thought in ways stronger than simply showing what not to do. In particular, I focus first on one of Spinoza's contemporaries, Gottfried Leibniz, and show how his thinking on infinity was influenced by Spinoza. I then discuss Georg Cantor, an influential mathematician who worked more than 200 years after Spinoza. Even though Cantor revolutionized the way we conceive of infinity inside of mathematics, there is ample evidence that he was influenced by Spinoza's arguments that an actual infinity was present in nature, even though Spinoza did not think these types of infinity could be assigned a proper number.

Historically, interpretations of Spinoza's philosophy have fluctuated between extremes. During his life, critics accused his view of God as collapsing into the natural world, making him an atheist. However, during the 19th Century, readers viewed Spinoza as offering weak criteria which divided the finite from the infinite, causing the natural world to collapse into God, which is why Hegel said that Spinoza's system contained

“too much God.”⁴⁸ In my opinion, those who see Spinoza as replacing God with nature ignore the fact that Spinoza reserved a special kind of infinity just for God, one which cannot be defined by all of the finite objects in nature. On the other hand, interpretations which deny reality to the world of finite modes rest on the mistake that although Spinoza placed God as the foundation of his metaphysical system, modes really do exist, making the diversity in the world more than just an illusion. As is often the case, the best interpretation likely lies somewhere in the middle between these two extremes. Melamed points out, even though Spinoza’s criteria for individuation are relatively weak, there isn’t strong enough evidence to think that he intended to restore a Parmenidean view of a static and unchanging universe. In fact, Melamed reminds us that Spinoza had not one, but two different ways we can differentiate between finite beings in nature. Melamed says the presence of two different accounts of differentiation “might indicate that Spinoza was still experimenting with various ways to cut nature at its joints. Alternatively, it may well be the case that Spinoza intentionally designed the building blocks of his finite world as fuzzy units, in order to stress their inferiority to the self-subsisting, self-explaining, and well-defined substance.”⁴⁹ Regardless of how we want to classify the exact relationship between how the infinite leads to actual finite beings, there is ample evidence that Spinoza intended for some type of criterion to serve this purpose.

I previously mentioned that Leibniz and Cantor will be the focus of this chapter. The reason for choosing these two is that both used philosophical arguments to argue that an actual infinite exists in nature, while resisting the idea that infinity jeopardizes

⁴⁸ Hegel. *History of Philosophy*, 3:282.

⁴⁹ Melamed, 2010. p. 91.

diversity or minimizes the role of God as creator. Thus these two thinkers used Spinoza's philosophy of the infinite to walk the path between atheism and "acosmism" (the denial of the physical world).

i. Leibniz: Infinite Monads

During his life, Leibniz criticized many of Spinoza's ideas. No doubt there was considerable political pressure to distance himself from Spinoza, given Spinoza's label as a dangerous radical. Despite his unwillingness to publicly associate himself with Spinoza's philosophy, there are similarities between Leibniz's view on infinity and Spinoza's. The main theme running throughout Matthew Stewart's *The Courtier and the Heretic* is that despite his public denunciations of Spinozistic philosophy, Leibniz had a secret admiration for the work of Spinoza. One area of overlap between the two thinkers emerges in the realm of the infinite. As explained in the opening chapter, prior to Spinoza, philosophy had classified "infinite" collections as merely being unbounded. Thus, to say that space is infinite wouldn't actually mean that we consider it as an actual system containing every possible point. Instead this "infinity" consists of having no bound, making it a potential infinity, rather than an actual one. Contrary to these potential infinities, the indivisible omnipotence of God is an actual infinite, not merely a potential one. We have already seen how Spinoza rejected the claim that nature doesn't contain any actual infinities, and Leibniz's philosophical work also sought to find a type of infinity within nature.

In Leibniz's later philosophical system, he identified monads as the fundamental ontological units of the world. Admittedly, of all of the philosophical doctrines I've

studied, I find the nature of the monads to be one of the most puzzling. In Leibniz's system, the monads function as the atomic units of reality. They are not spatially related, but their perceptions of one another constitute the spatial structure of the universe. However, we could identify an infinite number of perspectives for any given location, due to the dense nature of mathematical points. Perhaps this is why Leibniz says in the *Monadology* "Just as the same city viewed from different directions appears entirely different and, as it were, multiplied perspectively, in just the same way it happens that, because of the infinite multitude of the simple substances, there are, as it were, just as many universes, which are, nevertheless, only perspectives on a single one, corresponding to the different points of view of each monad."⁵⁰ Since Leibniz is committed to the actual existence of the atomic monads, he needs an account of how an infinite collection can exist in the natural world.

In addition to nature, Leibniz's philosophical system also held that God is infinite. Even though Leibniz does not speak of infinite attributes, and Spinoza refrains from talk about us existing in the best of all possible worlds, both refer to God as infinite. In his "A Tale of Two Thinkers: One Meeting, and Three Degrees of Infinity," Ohad Nachtomy draws the comparison that both philosophers use infinity in a non-numerical and non-quantitative manner when referring to God.⁵¹ What this means for Spinoza has already been explained in detail, so I will focus on what Leibniz meant by the infinity of divine substance.

⁵⁰ Section 57

⁵¹ 2011, p. 938

For Leibniz, statements like “the number of all numbers” or “the greatest possible speed” were contradictory concepts. For instance, in the number of all numbers, it would be possible to have a part which was the size of the whole, as demonstrated by Galileo's Paradox, the conclusion explained in Chapter 1 that shows how a 1-1 correspondence can exist between the integers and perfect squares. Leibniz recognized these apparent contradictions and became worried about the following line of reasoning:

- (1) God contains all perfections.
- (2) Existence is a perfection
- (C) God exists.

After all, if a collection containing all numbers could not exist (under Leibniz’s view of number, at least), what is different about the collection of all perfections? Leibniz found that this argument for the existence of God needed to be augmented to show the internal coherence of the first premise.⁵² Nachtomy argues that Leibniz’s search for an answer to this question is what leads him to seek answers in Spinoza’s work on infinity.

Like Spinoza, Leibniz also had three different categories of infinity: Omnia; Maximum; and Infinitum.⁵³ The Omnia represented an entity which contained everything, while a Maximum contained everything of its kind. Thus, unbounded space could be infinite by being Maximum extension, but it is not Omnia, since it does not contain every time, every thought, and all other possible attributes (in a non-Spinozistic

⁵² Ibid. 993-4

⁵³ Nachtomy, p. 950

sense) that God contains.⁵⁴ And infinitum is “infinite in the lowest degree whose magnitude is greater than we can expound by sensible things, even though there exists something greater than those things.”⁵⁵ Nachtomy points out that there is a correspondence between these three categories of infinity, and the three described by Spinoza. However, he claims only tentatively that the monads can fit into one of these categories, since it is not a question that Leibniz directly addresses. While an entire article could be written on the subject, I offer analogies between Leibnizian monads and the concept of infinite in kind.

As you may be tired of reading at this point, Spinoza’s view is that attributes are infinite in kind, meaning that Extension contains all of the extended modes. And as something which is infinite in kind, it cannot be changed by any extended mode. And due to Spinoza’s view of causality, nothing can affect it. Is there an analogue to this type of infinity in Leibniz’s account of nature? Although there is no perfect mapping between the type of infinity in by Spinoza’s attributes and Leibniz’s monads (what he sees as the fundamental building blocks of nature), there are numerous areas of overlap between these two doctrines. In his *Monadology*, Leibniz asserts that the monads have no “doors or windows” (find exact quote), meaning they cannot directly affected by any of the other monads, just as the attributes of Spinoza’s God cannot be affected by any other attribute. Unlike Spinoza’s account of attributes, however, monads are created by God, meaning they can be affected by forces external to themselves. However, the point remains that

⁵⁴ Adapted from an example in Leibniz’s *Labyrinth of the Continuum*, quoted in Nachtomy, p. 957

⁵⁵ Ibid.

each monad (also referred to as a created substance) cannot be destroyed or directly affected by any other mode, giving them an isolated autonomy similar to the attributes of Spinoza's system.

However, as strange as it may sound, monads "perceive" one another in some sense. And it is the collection of these perceptions from different viewpoints which create a relational harmony. This harmony is responsible for the phenomena on the macroscopic level of our daily lives.⁵⁶ However, there is no genuine causal interaction among these modes in the standard sense of the term. Instead of my arm slamming on the desk being the result of mechanical interaction among matter, the monads all shift their perceptions to give rise to the new phenomena. So, instead of the monads in my hand perceiving themselves as being a few feet away from the monads in the desk, they have a new logical relationship which gives rise to a harmony where my hand is touching the desk. I realize how confusing that sounds, but it's what Leibniz's system boils down to. What's important for the comparison with Spinoza is that there is no causal interaction occurring at this level. Instead of the monads in my hand acting on the ones in the table, there is a "pre-established harmony" which causes the monads to mirror themselves in a way that gives rise to this new arrangement once I form the volition to act. In this way, the monads are metaphysically autonomous from one another, just like Spinoza's description of the attributes, yet they all share a similar structure, like the modes that exist within each attribute. Spinoza created a system in which such a being can be called infinite, yet be

⁵⁶ For a further discussion of the monads which uses modern logic and mereology to explicate Leibniz's system, see Roy T Cook. "Monads and Mathematics: The Logic of Leibniz's Mereology."

“lesser” than the absolute infinity of God, and Leibniz used that distinction when giving his account of the world.

ii. Cantor: The Father of Modern Set Theory

In Letter 12, Spinoza concluded that “I do not believe there remains any question regarding the infinite on which I have not touched, or which cannot be readily solved from what I said.”⁵⁷ About two hundred years later, Georg Cantor wrote in the Preface to his 1883 *Foundations of a General Theory of Manifolds*⁵⁸ (hereafter referred to as the *Grundlagen*): “I do not believe that, in such a difficult, complicated, and all-embracing subject as the infinite, I shall have said the last word.”⁵⁹ Despite differing claims in self-importance, there is a compelling reason to believe that Cantor was influenced by Spinoza’s treatment of infinity. In Section 5 of his *Grundlagen*, Cantor lists Spinoza, along with Locke, Descartes, and Leibniz, as important figures who helped cement the tradition of finitism in mathematics. However, he also mentions Spinoza’s Letter 12 in the next paragraph, calling it “the highly important letter, rich in content, of Spinoza to L. Meyer,” and he promised a work which would provide an analysis of this letter, as well as the work of other philosophers.⁶⁰ Unfortunately, this work was never produced; but there are still compelling reasons to hold that not only was Cantor familiar with Spinoza’s work, but that it also influenced Cantor’s own developments.

⁵⁷ Letter 12. p. 106.

⁵⁸ The full German title is *Grundlagen einer allgemeinen Mannigfaltigkeitslehre. Ein mathematisch-philosophischer Versuch in der Lehre des Unendlichen*, which translates to *Foundations of a General Theory of Manifolds: A Mathematico-Philosophical Investigation into the Theory of the Infinite*.

⁵⁹ p. 881. In Ewald, trans and ed.

⁶⁰ Section 5.2

One of the primary developments in Cantor's *Grundlagen* is the concept of the transfinite ordinal. These newly-discovered numbers gave Cantor the ability to hypothesize a well-ordering of infinite sets. The first transfinite ordinal is denoted by ω , and comes from taking the set of all natural numbers in their typical order. Cantor's innovation was to not only call ω a number, but to show that numbers exist which come after ω . The strange thing about these infinite ordinals is that the familiar property of commutativity of addition fails. In finite ordinals, it's obvious to us that $5+7 = 7+5$, which equals 12. However, the number which comes directly after ω is $\omega + 1$, which is different than $1 + \omega$. $1 + \omega$ is simply ω .⁶¹ While this fact may be old news to readers familiar with set theory, one immensely interesting aspect is how Cantor introduces the failure of commutativity: with a discussion on Spinozistic metaphysics. He starts §5.6 with "An especially difficult point in Spinoza's system is the relationship of the finite modes to the infinite one; it remains unexplained how and under what circumstances the finite can maintain its independence with respect to the infinite, or the infinite with respect to still higher infinities." As seen in Chapter 3 of this thesis, Spinoza's theory of infinite modes is still puzzling to scholars and the fact that Cantor alluded to this technical question and offered his own solution is good evidence that he had a strong understanding of Spinoza's works. Cantor's remedy to stopping the infinite modes from absorbing the finite is to focus on the way that infinite quantities are ordered. He says that if the finite is placed before the infinite, "it merges into the infinite and vanishes therein; but if it *contents* itself to take its place *after* the infinite it is preserved and unites with it

⁶¹ Cantor 5.6

to form a new, because modified, infinite.” And with this sentence, Cantor ends §5 of his *Grundlagen*. Although this does not answer puzzles about the properties of infinite modes and their role in nature, the view that the finite modes would disappear if placed in the sequence of the infinite modes (like how finite integers only change the value of an ordinal if added in the right way) is a possible way to conceive of the logical relation between the finite and infinite modes.

Additionally, Cantor disagreed with Spinoza’s view that the infinite cannot be represented as a number due to the paradoxes which arise. Take Spinoza’s example in IP15schol of assigning a number to the infinite measure of the universe in feet. He argues that we could also count that distance in inches, making it 12 times the infinity of the measurement in feet. This would make one infinity be twelve times another infinity, which Spinoza regarded as absurd. However, Cantor’s insight into the nature of infinity shows that regardless of which units we use, we’ll end up with the same number of infinite units if we count with discrete units. Also, even though the interval $[0, 1]$ is longer than $[0, 2]$, if we want to list the sizes of real-number points which lie on both lines, we actually end up with the same infinite measure, \aleph . To return to the example of the two non-concentric circles Spinoza gives in Letter 12, it turns out that the “number of distances” in each section is the same \aleph , regardless of which portion of the diagram we discuss. Cantor’s resolution involves recognizing that Spinoza’s dismissal of infinite numbers was based on the tacit assumption that the properties of finite numbers should hold in the domain of the infinite.⁶²

⁶² Bussotti and Tapp. p. 26.

So far, I have compared and contrasted Cantor and Spinoza's views on infinities which can be divided in the mind. But as I hope this thesis has made clear, there is more to Spinoza's theory of the infinite than these groups of cases. Instead, Spinoza demands that we keep the infinity of Substance in our minds when discussing infinity. Recall that this type of infinity does not involve parthood, and cannot be represented as finite without losing coherence. And even though Cantor doesn't share every aspect of this view, there are close parallels which can be made between Spinoza's first type of infinity, and Cantor's concept of the absolute infinite.

As opposed to the infinitely infinite infinity of transfinite numbers (and even that hyperbole fails to capture just how many different transfinite numbers there are), Cantor believed that there was a type of infinity which could not be captured mathematically, which pertained to God alone.⁶³ In her "Cantor on Infinity in Nature, Number, and the Divine Mind," Anne Newstead gives a compelling account of how Cantor's separation between the absolute infinite and the transfinite allowed him to keep his views consistent with Catholic Theology. In a 1886 letter to Cardinal Johannes Franzelin, Cantor is careful to distinguish his theory from the then-fashionable heresy of pantheism. By showing that it was possible for an actual infinity to exist without contradiction, it would be possible that God could be contained in the world, resulting in pantheism. Or so the German Idealists of the time thought.

⁶³ Newstead, Anne. 2009.

One of the techniques Cantor uses to distinguish himself from pantheism was to put forth three different kinds of infinity.⁶⁴ First, there are the Abstract infinities, which are the numbers subject to mathematical analysis, such as ω , ω^2 , and ϵ_0 . Then there is the Concrete infinite, which can be found in nature, and would be the unbounded nature of space and time. And finally, there is the Absolute infinite, which is possessed by God alone. Cantor says that pantheism arrives by conflating the Concrete infinite with the Absolute infinite, and he even uses the Spinozistic language of “*natura naturata*” to describe the Concrete infinite, and “*natura naturans*” to describe God’s infinity in one of his letters to Cardinal Franzelin.⁶⁵ Therefore, just as Spinoza held that God could not be identified with the modes of the universe, Cantor held that the infinity present in God could not be identified as a collection of transfinite measurements or orderings. And since it is not a proper set, the Absolute infinite and the transfinite are different in kind, not degree. As should be familiar by this point, this is exactly what Spinoza believed.

There is one further concern about Cantor’s Absolute infinity. Was it a genuine belief that Cantor had, or was it his way of protecting himself from accusations of heresy? And if Cantor genuinely believed this theory, is it a theory worth retaining? As for the first question, I think it’s a belief Cantor genuinely held, due to evidence present in one of Cantor’s letters to Dedekind, where he points out that the well-ordered sequence of all numbers (which Cantor denotes with Ω) cannot have a number associated with it. For if it did, then that number, δ , would occur at a fixed location in Ω , since it contains all numbers in a well-ordered manner. But δ is a number, so then $\delta + 1$ would be a number in

⁶⁴ Newstead. 548

⁶⁵ Newstead, pp. 548-9

Ω greater than δ , meaning that δ is not the largest number. Thus, even though Cantor disproved Spinoza's claim that there could be no infinite number by showing that we cannot apply our all of our intuitions about finite collections to the realm of the infinite, it is still true that just as there is no largest finite number, there can be no largest transfinite number. For this reason, Cantor calls Ω "*an inconsistent, absolutely infinite multiplicity*"⁶⁶ Although Cantor's work with infinity showed that the number corresponding to an infinite distance measured in feet would be the same as that same length measured in inches (contrary to Spinoza's discussion in IP15S), he still recognized that not every infinite quantity could be assigned a number, finite or infinite. Whereas Cantor thought both finite and transfinite and transfinite sets could be correlated with a definite number, he was aware that Ω , the set of all ordinals, could not be given the same treatment. Therefore, Cantor appears to have acknowledged that not every collection is a proper set.⁶⁷ This is an approach shared by von-Neuman-Bernays-Gödel (NBG) set theory, which divides collections into two different categories. Sets are collections which can be members of other collections. Therefore, ω is a set under the standard definition of an ordinal, since it is an element of $\omega + 1$, as well as the higher infinite ordinals. NBG set theory adds the concept of proper classes, which are collections that are not elements of any set. One example of a proper class is the set of all sets. Since it is not itself a set, it is not included in this collection, which avoids the paradoxes associated with the set of all sets. The other major system of axiomatic set theory, Zermelo-Frankel, does not include the concept of proper class. Instead, it places limits on constructing sets, which block the

⁶⁶ Cantor. 3 August 1899 Letter to Dedekind. in Ewald, trans. and ed. Emphasis in original.

⁶⁷Bussotti and Tapp. p. 30.

creation of sets using properties like $\{x|x=x\}$, which would be the set of all sets. Even though Cantor's Absolute Infinite may have lost the metaphysical association with God, modern set theory continues to acknowledge that collections such as Cantor's Ω or the universal class V are different kinds of entities than ω or \aleph .

Therefore, even though Cantor expanded the reach of the numbers, he acknowledged the Spinozistic point that some concepts remain outside of their grasp. In fact, Hallett argues that one of the primary lessons of set theory is that even when dealing with a hierarchy of infinite measures, there are still concepts which are too large for set theory to handle in the same way as transfinite sets (Hallett, p. 165).

iii. Conclusion

No matter how we frame it, infinity is weird. Most of our normal intuitions about how objects and relations should interact go straight out the window once we reach an infinite domain. Spinoza recognized this fact, and used it to say that one type of infinity is not adequate to explain all of the intuitions that we have. For instance, the intuition that multiplication and addition yield larger quantities is captured by the third kind of infinity, the one which applies to measure, time, and number. And calling God infinite means something completely different than calling quantity and duration infinite, so Spinoza had different types of infinity for those two concepts. Although we can question whether or not the domain in question really has the properties that Spinoza thinks they should, or whether or not Spinoza was consistent in calling God indivisible despite recognizing the differences among the attributes, we should acknowledge that he had a developed theory of infinity. Furthermore, his theory is not some dusty pre-Cantorian mess which should

be looked on as outdated. Rather, it should be seen as a historical source of many of our contemporary theories.

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