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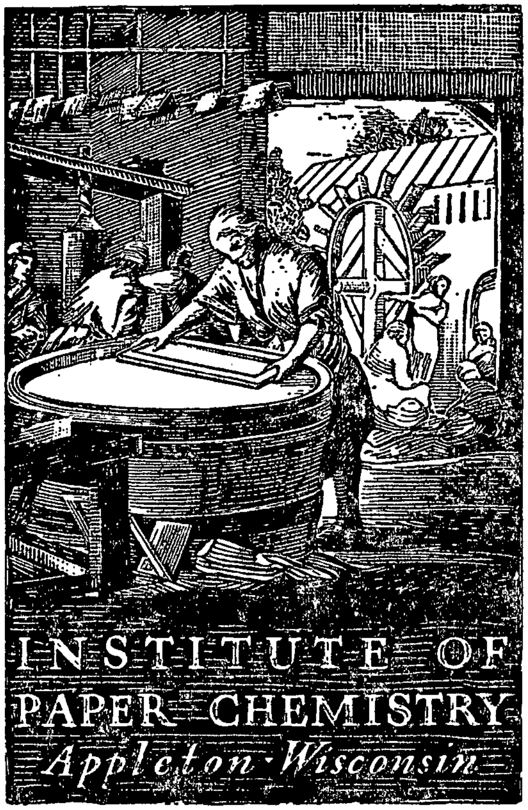

DETERMINATION OF THE REINFORCEMENT AFFORDED SACK PAPER BY AN ELECTRICAL RESISTANCE STRAIN GAGE

Project 2033

Progress Report Eight

to

MULTIWALL SHIPPING SACK
PAPER MANUFACTURERS

September 8, 1959

THE INSTITUTE OF PAPER CHEMISTRY

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SUMMARY

The bonding of an electrical resistance strain gage to a specimen of sack paper affects the apparent stress-strain characteristics of the specimen because of the reinforcement afforded by the gage. In order to estimate the strain in the paper from an electrical resistance strain gage measurement, it becomes necessary to determine by auxiliary means the effect of the reinforcement. This report describes the determination of the gage stiffening effect so that a gage measurement of the strain in a sack at the time of impact may be converted to an estimate of strain in the ungaged paper at that location of the sack.

It is shown analytically that the gage reinforcement effect is dependent upon (a) the relative stiffness of the sack paper and the electric resistance strain gage including the adhesive; (b) the magnitude of the strain, and (c) for biaxial gages, the ratio of strains in the two directions. A calculation is described for evaluating reinforcement factors from tensile test data involving the over-all elongation of a specimen and the strain indicated by the gage.

Specimens of 50-lb. multiwall sack paper with biaxial strain gages adhered to them were subjected to biaxial tensions of various intensities

and with various ratios of strain in the machine and across-machine directions. The results of the study are a series of curves of paper strain vs. gage strain, which may be used to convert measurements of gage strain to ungaged paper strain.

An alternate presentation of the results is given in terms of a quantity termed reinforcement factor, which is defined as the ratio of paper strain to gage strain. The product of strain in the gage and reinforcement factor is an estimate of the strain which would have occurred at the gage location if the gage had not been there. It was found that the reinforcement factors ranged from about 1.5 to 2.8 for the in-machine direction of the sack paper and from 1.1 to 5.0 for the across-machine direction, under conditions of static biaxial tensile testing.

It is reasoned that under high rates of strain the reinforcement factors are less than those presented in this report. This result stems from the consideration of the stiffness of the sack paper at high and low rates of strain.

INTRODUCTION

One of the widely used methods of measuring the strain in a structure is by means of a bonded electrical resistance strain gage. This method has been employed in the investigation of the strains in a multiwall sack at the time of impact (1, 2).

The strain gage consists of a continuous length of fine wire formed into a flat grid which is bonded onto a thin sheet of base paper and protected by a covering of felt. The entire gage is cemented onto the sack paper at a location on the multiwall sack for which the strain behavior under impact conditions is to be determined.

The operation of the gage is based on the principle that the electrical resistance of the wire grid increases when the grid elongates and decreases when the grid contracts. When the paper is stressed, strains in the sack paper are transmitted through the bonding cement to the strain gage grid, causing the latter to strain. Measurement of the change in resistance of the grid, therefore, provides a measurement of the strain in the gage and in the sack paper. The gage may be calibrated so that the change in its resistance can be read directly as unit strain.

The bonding of the electrical resistance strain gage to a material having a low modulus of elasticity affects the stress-strain characteristics of the specimen. When the gage is applied to a sack paper specimen, the gage and cement reinforces the sack paper immediately under the gage. This stiffening action confounds the strain pattern of the specimen and results in an indication of strain at the gaged position of the specimen of somewhat lesser magnitude than would be present in that same portion of the specimen if the gage had not been there. This effect may be shown to be severe when the stiffness of the gage and cement approaches or exceeds the stiffness of the paper. When the absolute strain in the sack paper is desired from an electrical resistance strain gage measurement, it is necessary to determine by auxiliary methods what the paper strain would have been in the paper in the area of the gage if the gage had not been there.

A program for the determination of the reinforcement afforded sack paper by an electrical resistance strain gage was undertaken in conjunction with a study of the strains in a multiwall sack at the time of impact. This report covers the determination of the gage stiffening effect so that a gage measurement of the strain in a sack may be converted to an estimate of strain in the ungaged paper at that location of the sack.

THEORETICAL CONSIDERATIONS

This section of the report is devoted to an analytical description of the reinforcement effect of an electric resistance strain gage on sack paper. Consideration is given first to uniaxial tension behavior, followed by an extension of the reasoning for the case of biaxial tension.

Consider a uniaxial tension specimen with an electric resistance strain gage adhered at mid-length, as pictured in Fig. 1. It is convenient, though not necessary, to consider the specimen as having the same width, b, as the width of the gage. As long as the strains remain elastic, the tensile force per unit width, F, in the paper specimen at section A-A is given by Hooke's law, namely,

$$F = S_p e_p \qquad (1)$$

where F = force per unit width, lb./in.

S_p = tensile stiffness of paper specimen per unit width, lb./in.

e_p = unit tensile strain, in./in.

The tensile stiffness, S_p, is the product of the elastic modulus, E, of the paper and its caliper, t. In particular, S is the slope of the linear portion of the curve of force per unit width *vs.* unit strain of the paper.

At section B-B, the tensile force and unit strain are related by the composite stiffness of the specimen and the gage, namely,

$$F = (S_p + S_g)\, e_g \qquad (2)$$

where S_g = tensile stiffness of the gage per unit width, lb./in.

e_g = unit tensile strain underneath the gage, in./in.

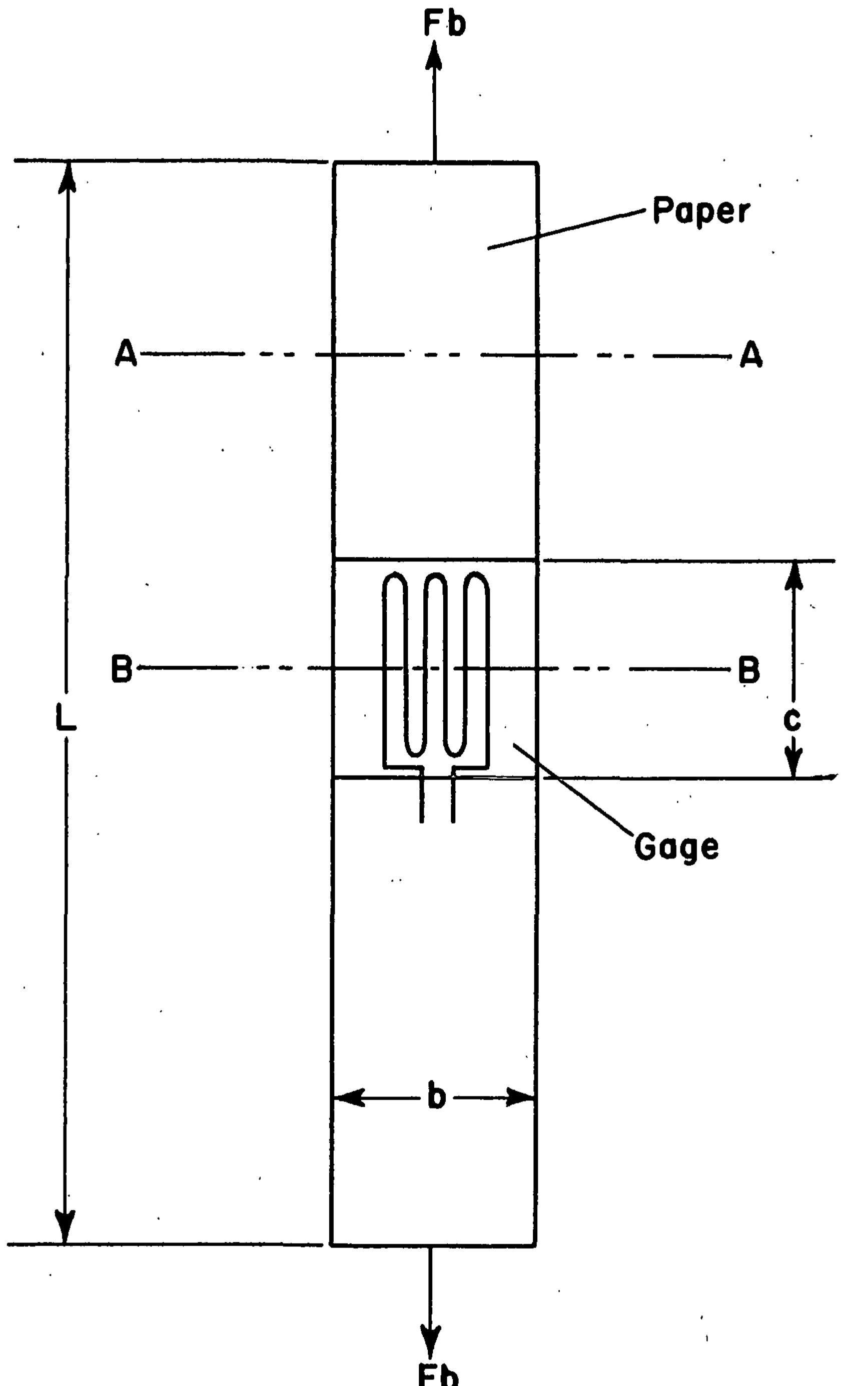

Figure 1

Uniaxial Tension Specimen with an Electrical Resistance Strain Gage

The force F is necessarily the same at sections A-A and B-B. Equations (1) and (2) may be combined, therefore, to yield an expression for e_p/e_g, that is, the ratio of strain in the ungaged portion of the specimen to the strain underneath the gage. This ratio is defined as the reinforcement factor of the gage.

$$R = \frac{e_p}{e_g} = \frac{S_g}{S_p} + 1 \qquad (3)$$

Reference (3) presents an alternate derivation of this equation.

Equation (3) reveals that the reinforcement factor is dependent on the relative stiffness of the gage and of the paper specimen. If the specimen stiffness is vastly greater than the gage stiffness, the reinforcement factor is near unity. In this instance, the strain at the gaged section is essentially the same as the strain outside the gaged region. Accordingly, the strain determined by the electric resistance strain gage may be interpreted as the strain throughout the specimen; that is, no stiffening correction is required. This condition generally exists with strain measurements on metal structures.

If, on the other hand, the gage and specimen are of equal stiffness, Equation (3) reveals that the reinforcement factor is 2.0, meaning that strains in the specimen outside the gaged area may be expected to be twice the strain indicated by the gage. In this instance, all indicated gage strains would be multiplied by the reinforcement factor 2.0 to correct for gage reinforcement.

In the general case, the stiffness of gage and specimen will be in a ratio other than unity, requiring a specific evaluation of R for the purpose of converting gage strain to estimated strain in the paper.

While Equation (3) has been derived for a linear load-deformation characteristic of the specimen, the form of the equation is nonetheless appropriate for any other portion of the load-elongation curve, provided an appropriate modulus is employed in the evaluation of the stiffness. Specifically, the secant modulus (sometimes termed "chord modulus") is required. This modulus is the slope of a straight line from the origin to the point of interest on the load-elongation curve. Accordingly, the reinforcement factor may be expected to vary with strain (or load) intensity beyond the proportional limit.

Equation (3) indicates that the reinforcement factor R may be estimated from the stiffness of the paper and the gage. An alternate method of evaluating the stiffening effect is to determine the intensity of strain e_p in the ungaged portion of the specimen when a given strain e_g is indicated by the gage. The ungaged paper strain e_p may be calculated by subtracting the indicated elongation under the gage from the total elongation of the specimen, and then dividing by the ungaged length of the specimen. The elongation under the gage is the product of the length of the gage (the adhered length) and the unit strain e_g indicated by the gage. Symbolically, the calculation of the strain in the ungaged portion of the specimen is as follows:

$$e_p = \frac{d - c\,e_g}{L - c} \qquad (4)$$

where e_p = average unit strain in ungaged portion of specimen, in./in.

d = total elongation of specimen, in.

e_g = unit strain indicated by electric resistance strain gage, in./in.

L = over-all length of specimen, in.

c = length of strain gage, in.

Finally, the reinforcement factor R is the quotient of e_p by e_g. This method was employed in determining the reinforcement factors of this report.

Returning to consideration of Equation (3), an interesting special case presents itself when the same type of strain gage is used on two paper specimens having differing stiffness, S_{p_1} and S_{p_2}. For each specimen, by Equation (3),

$$R_1 = \frac{S_g}{S_{p_1}} + 1$$

$$R_2 = \frac{S_g}{S_{p_2}} + 1$$

Since the gage stiffness, S_g, is identical for each specimen, these equations may be combined as

$$\frac{R_2 - 1}{R_1 - 1} = \frac{S_{p_1}}{S_{p_2}} \qquad (5)$$

revealing an inverse relationship between the reinforcement factors and the tensile stiffnesses of the two specimens.

Equation (5) has utility in that it permits an estimate to be made of the reinforcement factor of a given type of gage on one specimen of known stiffness in terms of the reinforcement factor for another specimen of known stiffness. For this purpose, Equation (5) may be written more conveniently as

$$R_2 = \frac{S_{p_1}}{S_{p_2}}(R_1 - 1) + 1 \quad (6)$$

In the foregoing, attention has been restricted to a gaged specimen subjected to uniaxial tension. Of primary interest to strain measurements on sacks, however, is the gage reinforcement effect when the sack paper is simultaneously subjected to tension in two perpendicular directions, i.e., biaxial tension, as may be expected when a filled sack is impacted in a laboratory drop test.

When an orthotropic material is simultaneously under tension in two directions (denoted by x and y), the elastic force-elongation relationship in the x-direction is (from Reference (4)):

$$\frac{F_x}{e_x} = \frac{1 + v_x \frac{e_y}{e_x}}{1 - v_x v_y} S_x \quad (7)$$

F_x = force per unit width in x-direction, lb./in.
e_x = unit strain in x-direction, in./in.
e_y = unit strain in y-direction, in./in.
S_x = uniaxial tension stiffness in x-direction per unit width, lb./in.
v_x = Poisson ratio associated with contraction in x-direction
v_y = Poisson ratio associated with contraction in y-direction.

An analogous equation holds for the y-direction of the material and is obtained by interchanging the roles of x and y.

The factor by which S_x is multiplied in the right-hand side of Equation (7) is necessarily greater than unity. Thus, the ratio of force-to-strain, which may be termed the effective stiffness, S'_x, in biaxial tension, is larger than the uniaxial stiffness S_x. Symbolically, from Equation (7),

$$S'_x = \frac{F_x}{e_x} = K\,S_x,\; K > 1 \tag{8}$$

The effect of biaxial tension on the effective stiffness, S'_x, presumably occurs with both the sack paper and the electric resistance strain gage (including the adhesive). Thus, the reinforcement factor, R, defined by Equation (3) for uniaxial tension, may be expected to be of the following form for biaxial tension (for either direction of the sack paper):

$$R = \frac{K_g\,S_g}{K_p\,S_p} + 1 \tag{9}$$

In general, it may be anticipated that K_g is not equal to K_p. Thus, the reinforcement factor, R, in either direction of the sack paper under biaxial tension will be different from the uniaxial reinforcement factor. Furthermore, in view of the dependence of K on e_y and e_x (see Equations 7 and 8), it may be expected that the reinforcement factor, R, will vary with the ratio of the strains in the two directions of the sack paper.

In summary, it has been shown analytically that the gage reinforcement effect is dependent on the relative stiffness of the sack paper and the electric resistance strain gage including the bonding cement. The stiffening effect of a given type of gage employed with a given sack paper may be expressed as the ratio of strain in the paper outside the gaged region to the strain underneath the gage; this ratio is termed the reinforcement factor. Multiplication of a strain indicated by the gage and the reinforcement factor yields an estimate of the strain which would have existed at the gage location had the gage not been there. The reinforcement factor may be expected to vary with strain intensity when the latter is greater than the proportional limit strain.

A calculation was described for evaluating the reinforcement factor from experimental data involving the over-all elongation of a specimen and the strain indicated by the gage. Furthermore, it was shown how the reinforcement factor of a given type of gage may be estimated for one sample of sack paper in terms of its factor for another sample of paper, knowing the relative stiffness of the two samples.

Finally, it was reasoned that the reinforcement factor of a biaxial strain gage, employed with sack paper subjected to biaxial tension, varies according to the ratio of strains in the two directions as well as the strain intensity.

In view of these analytical results, a test program was performed wherein specimens of sack paper with biaxial strain gages adhered to them were subjected to biaxial tension of various intensities and with various ratios of strain in the two directions to determine the stiffening effect of an electrical resistance strain gage on sack paper.

MATERIALS

The reinforcement effect of a strain gage on sack paper was determined for Type AX-5-1 rosette strain gages (Baldwin-Lima-Hamilton Corporation). These gages were of the same type selected for use in the impact strain studies. Sack paper specimens were taken from Roll A-3 of the recent fabrication study. Paper from this roll constituted the outer ply of the sacks used in the impact strain investigation. The paper specimens were subjected to standard conditioning prior to test.

TEST PROCEDURE

In order to simulate the biaxial nature of strains in a sack at the time of impact, the specimens employed in this study of the stiffening effect of electric resistance strain gages on sack paper were strained in a biaxial stress-strain tester (5). The sack paper specimens were six inches square. An AX-5-1 rosette strain gage, approximately one-inch square, was cemented to each specimen at its center with Baldwin SR-4 strain gage cement. The two strain-sensitive axes of the gage were aligned with the machine and across-machine directions of the paper, with the slightly longer dimension of the gage parallel to the machine direction. The cement was allowed to cure under the pressure of a one-pound weight for twenty-four hours before the specimen was tested.

The specimen was clamped in the four jaws of the tester and the gage circuit connected to the strain gage. Each gage grid was placed into a circuit as one arm of a Wheatstone bridge, which was completed in an Ellis bridge-and-amplifier, model BA-2 or BA-12. The DC outputs of the bridges were connected alternately through a double-pole, double-throw switch into a Du Mont oscilloscope, type 403. The scale on the oscilloscope screen was calibrated in units of strain with the calibration provision of the Ellis equipment.

The specimen was then deformed in tension in both the machine and across-machine directions of the paper so that prescribed amounts of strain in each direction of the paper were indicated by the gages. Two dial indicators, which spanned the distance between opposite clamps, were read at each of the several levels of gage strain to determine the over-all specimen deformation. The process was continued until the specimen ruptured. Strain data was obtained at about ten levels of strain intensity throughout the test life of the specimen.

As shown in Theoretical Considerations, the stiffening effect of electric strain gages in biaxial tension may be expected to depend on the ratio of the biaxial strains as well as their magnitude. Accordingly, various ratios of across-machine to in-machine gage strain were applied to the several specimens, namely: 4.0, 3.0, 2.0, 1.0, 0.5, 0.33, and 0.25. At least two specimens were tested for each of these ratios.

DISCUSSION OF RESULTS

Electric resistance strain gages have been employed to measure the magnitude and rate of strain in the outer-ply sack paper during impact of a filled sack. Application of the strain gage to the sack paper, however, causes a local reinforcement of the paper in the area of the gage. Consequently, the impact strain in this region of the sack paper (and hence in the gage) is less than if the gage had not been there. To properly interpret the measured impact strains it is necessary to correct for the stiffening effect of the gage.

GAGE REINFORCEMENT UNDER STATIC TEST CONDITIONS

An experiment was performed for the purpose of estimating the reinforcement afforded sack paper by a strain gage rosette of the type employed in impact strain measurements on sacks. Gages were affixed to six-inch square specimens of sack paper corresponding to the outer ply of the sacks used in the impact strain tests. The gaged specimens were tested under static conditions in a biaxial tensile stress-strain tester at various intensities of strain and various ratios of strain in the in- and across-machine directions of the paper. The deformation data obtained from these tests enabled computation of the reinforcement characteristic of the gage as described below.

An example of the data analysis will be given with the data from the in-machine direction of specimens tested in an applied strain ratio of 1.0. The first step of the data reduction process was to plot the total elongation of the specimen vs. the gage signal displacement (in grid divisions on the oscilloscope screen) as shown in Fig. 2. These values constituted the primary data obtained from three tests. The curves for Specimens 1 and 3 appear to be coincident for a portion of the plot, whereas the curve for Specimen 2 is displaced somewhat from the others.

It may be noted that none of the curves tend to go through the origin of the graph. The curve for Specimen 2 intersects the vertical axis at about 0.0005 inch of specimen elongation. The other curves intersect the horizontal axis at about 1.8 divisions of signal displacement. Both of these observations tend to refute a fundamental concept that if the strain gage indicated a strain, the entire specimen must be strained and, conversely, if the specimen is strained, the gage must indicate a strain (assuming adequate sensitivity).

It is believed that the nonzero intercepts of the curves of Fig. 2 are attributable to experimental error. Consider the effect of slack in the specimen after insertion in the clamps of the tester, i.e., slack not detected and removed by the operator prior to the test. This slack would be taken up by the movement of the clamps at the start of the test and would be interpreted as elongation of the specimen even though strain was not actually induced in either the specimen or the strain gage. This behavior would lead to a nonzero intercept of the type exhibited by Specimen 2 in Fig. 2.

On the other hand, it was observed that the drying of the strain gage cement frequently induced wrinkles or "cupping" of the specimen at the gage location, with the result that the gage itself was warped out of plane

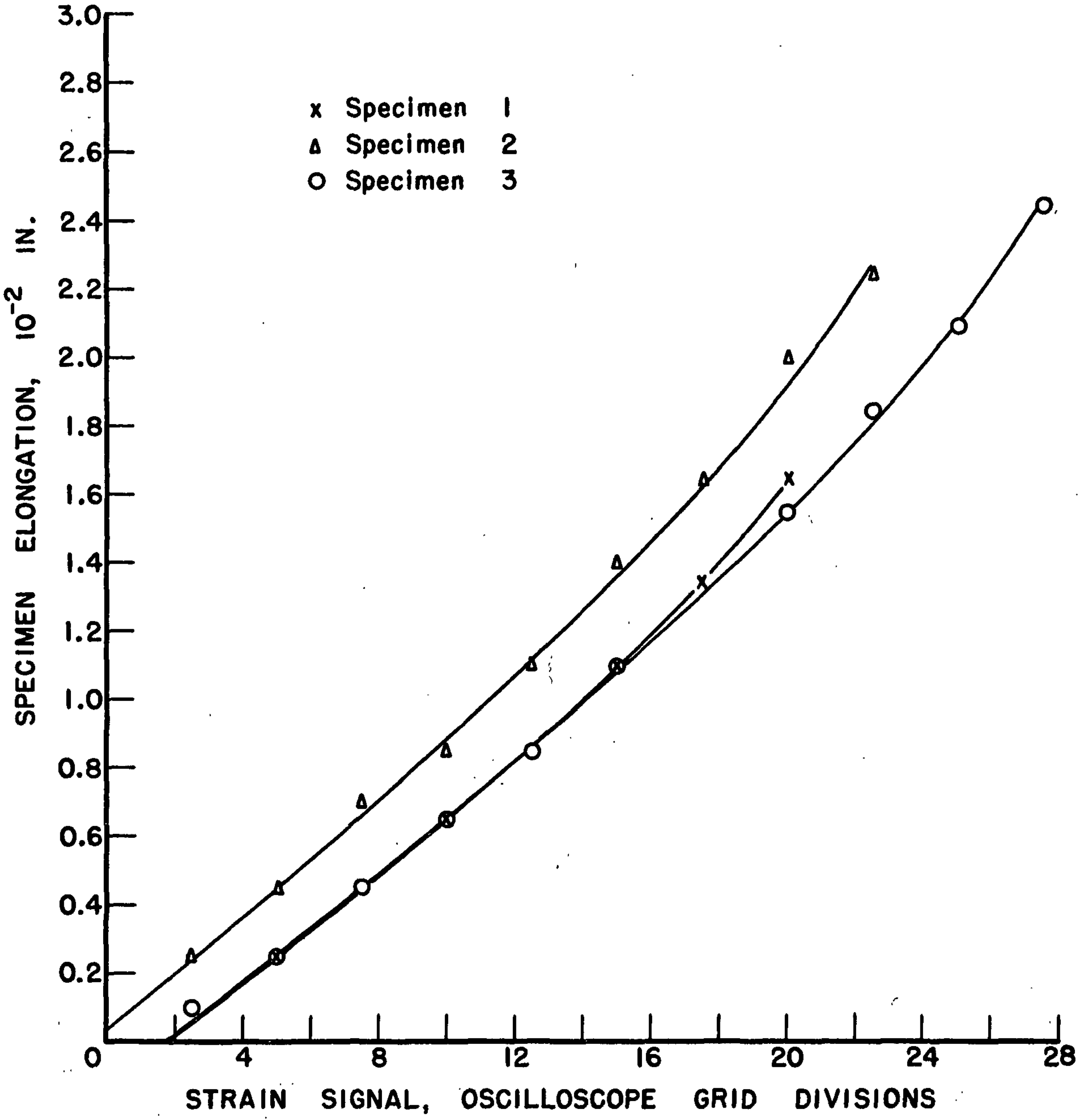

Figure 2

Example of Experimental Data Relating Specimen In-Machine Direction Elongation and Electric Resistance Strain Gage Signal

(Biaxial Strain Ratio = 1.0)

at the start of the test. It may be expected that initial deformation of the specimen produced some flexing of the gage in bringing it back to planeness at the start of the test, yielding a strain gage signal which was not proportional to the average tension strain across the caliper of the specimen. This behavior may lead to the nonzero intercept of the type displayed by Specimens 1 and 3.

It is usual in physical testing that the precision of the experiment be least at the outset of the test, for reasons analogous to those suggested above. In such instances the data may be rendered more understandable and reliable if the resulting curve is extrapolated backward into the initially nonlinear portion of the graph which corresponds to the start of the test. An example is the determination of zero elongation in a conventional tensile test by means of the intercept of the tangent line to the initial portion of the curve and the elongation axis.

In the belief that the gage reinforcement data would be more meaningful if the results were adjusted to compensate for possible experimental errors at the start of the test, each curve of Fig. 2 and all analogous curves of the study were extrapolated to what appeared to be a reasonable intercept on the horizontal or vertical axis. Thereafter, each intercept was taken as zero for the subject curve.

After this adjustment of the origins of the curves was accomplished, the total specimen elongation at arbitrarily selected magnitudes of gage signal were read from the curves of Fig. 2 and tabulated as shown in Table I for the example treated in this discussion. The trace displacements shown in the

TABLE I

EXAMPLE CALCULATION OF REINFORCEMENT FACTOR OF ELECTRICAL RESISTANCE STRAIN FOR THE IN-MACHINE DIRECTION OF THE PAPER AND A BIAXIAL STRAIN RATIO OF 1.0

Specimen	Gage-Cement Length, c, inch	Trace Displacement, divisions	Specimen Elongation, d, inch	Gage Strain, e_g, inch/inch	Ungaged Paper Strain, e_p, inch/inch	Reinforcement Factor, R
1	1.00	8.2	0.0065	0.000596	0.00118	1.98
		14.2	0.0119	0.001033	0.00217	2.10
		18.2	0.0165	0.001324	0.00304	2.30
2	1.10	10.0	0.0084	0.000727	0.00155	2.13
		16.0	0.0143	0.001164	0.00266	2.29
		22.5	0.0225	0.001636	0.00422	2.58
3	0.85	10.0	0.0078	0.000774	0.00139	1.80
		20.0	0.0171	0.001548	0.00306	1.98
		26.0	0.0249	0.002013	0.00450	2.24

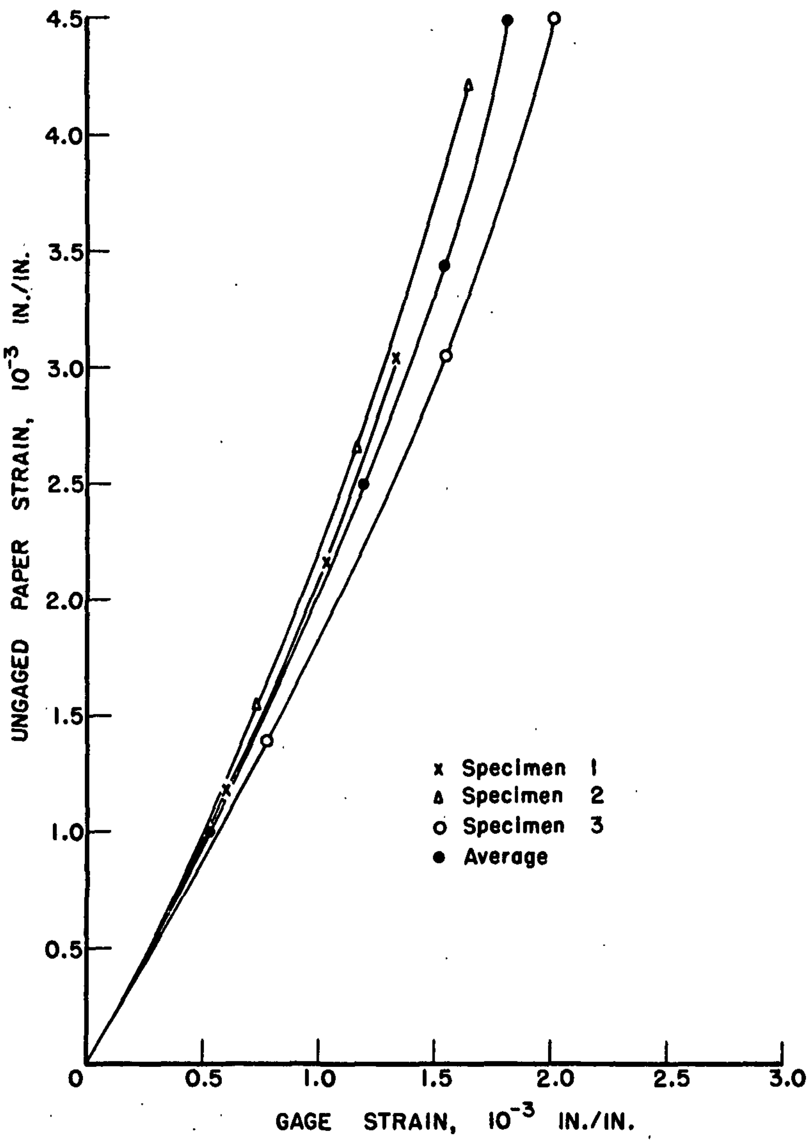

Figure 3

Example of Processed Data Relating Specimen In-Machine Strain and Electric Resistance Gage Strain

(Biaxial Strain Ratio = 1.0)

table were changed to unit strain values by multiplying by the calibration factor determined for each gage. Then Equations (4) and (3) of Theoretical Considerations were used to calculate ungaged paper strain, e_p, and reinforcement factor, R. The calculated ungaged paper strains for each specimen shown in Table I were then plotted against the strain indicated by the gage, as shown in Fig. 3.

An average curve was constructed from the three specimen curves and is also shown in Fig. 3. This average curve is the primary result of the data analysis for in-machine gage reinforcement at a biaxial strain ratio of 1.0. It may be noted in Fig. 3 that for any value of strain indicated by the gage, the strain in the ungaged sack paper was somewhat larger, indicating that the strain gage provided reinforcement to the paper beneath it. The curves of Fig. 3 are concave upward, revealing that the gage reinforcement effect becomes more severe as the intensity of strain increases.

The average curves for in-machine gage reinforcement for all biaxial strain ratios investigated are shown in Fig. 4. It may be noted that the curve for a strain ratio of unity, which was discussed by way of example of the data analysis, is the median curve of the family of curves shown in Fig. 4. The effect of biaxial strain ratio on gage reinforcement may be examined in terms of the curves of Fig. 4. For a given gage strain in the machine direction, an increase in the across-machine gage strain (increasing strain ratio) requires an increase in the ungaged paper strain to maintain the same gage signal. This behavior is associated with the Poisson effect in the paper and the gage under biaxial tension stresses, as demonstrated analytically in Theoretical Considerations.

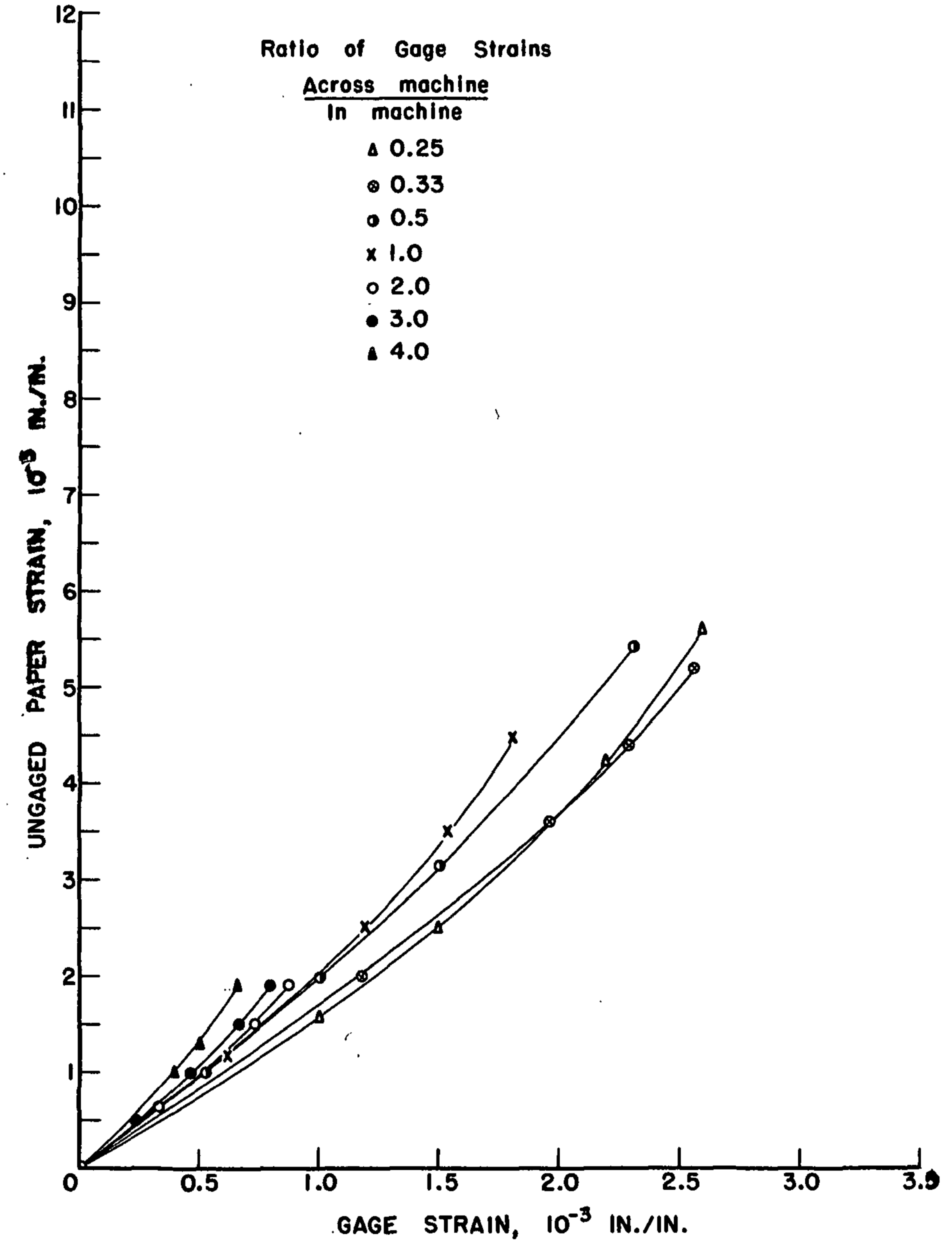

Figure 4

Relationship Between In-Machine Strain in Ungaged 50-lb. Kraft Sack Paper and Strain Indicated by an AX-5-1 Electric Resistance Strain Gage for Various Biaxial Strain Ratios

Curves describing the gage reinforcement effect for the across-machine direction of the sack paper are presented in Fig. 5. Again, for a given level of gage strain in the across-machine direction, an increase in the transverse (in-machine) gage strain (i.e., decrease in strain ratio) requires an increase in ungaged paper strain to maintain a constant gage signal. The dependence of gage reinforcement on strain ratio is even more pronounced for the across-machine direction than for the machine direction of the sack paper.

Figures 4 and 5 provide the means for converting strain gage readings to estimated strains in the sack paper--that is, the strains which would have occurred underneath the gage on a sack if the gage had not been there. This is accomplished by reading into the curve of the appropriate biaxial strain ratio at the values of strain indicated by the gage in either direction of the sack paper. For example, suppose the gage strains recorded during impact of a sack were 0.002 and 0.001 in./in. for the machine and across-machine directions, respectively. The biaxial strain ratio is 0.001/0.002 = 0.5, which defines the curve of interest in each of Fig. 4 and 5. Reading into this curve of Fig. 4 at a gage strain of 0.002 in./in., it is found that the strain in the machine direction of the ungaged paper is about 0.0045 in./in. Similarly, Fig. 5 reveals that the strain in the across-machine direction (corresponding to 0.001 in./in. gage strain) is about 0.0035 in./in.

In general, interpolation between the curves of Fig. 4 and 5 will be required, since the biaxial strain ratios will usually be other than were investigated in this study. For the most part, the curves of both Fig. 4 and 5 are arrayed fairly orderly with respect to strain ratio. Exceptions are

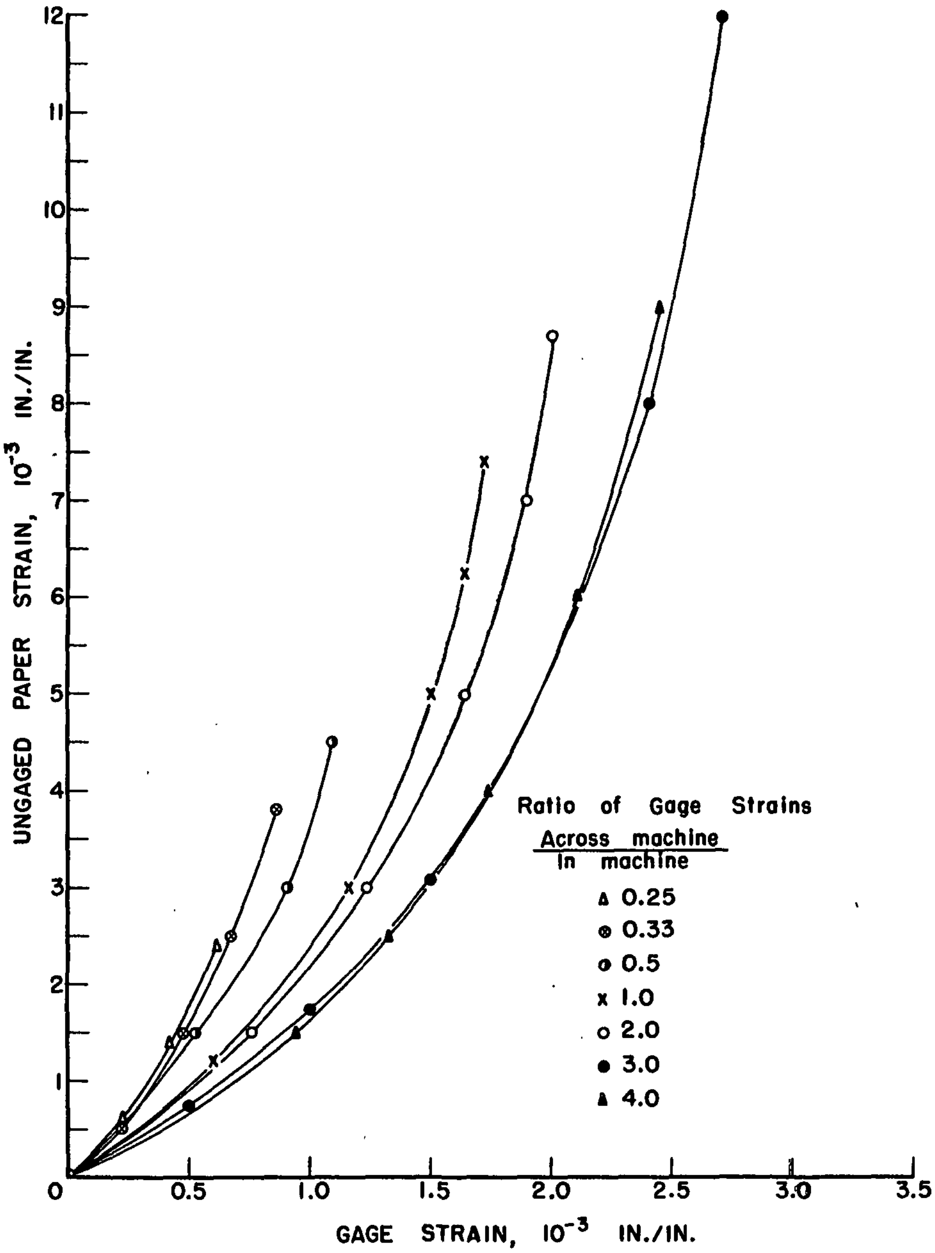

Figure 5

Relationship Between Across-Machine Strain in Ungaged 50-lb. Kraft Sack Paper and Strain Indicated by an AX-5-1 Electric Resistance Strain Gage for Various Biaxial Strain Ratios the two extreme right-hand curves in either Fig. 4 or 5, which exhibit an inversion at the higher strain levels. This observation suggests that the precision of the experiment does not warrant an elaborate interpolation between the curves.

Another way of expressing the data in Fig. 4 and 5 is by means of the reinforcement factor of the gage, which is defined as the ratio of ungaged paper strain-to-gage strain. The larger the reinforcement factor, R, the greater is the stiffening influence of the gage on the sack paper. Referring to the example calculations of Table I, the reinforcement factors tabulated in the last column were plotted as a function of the gage strain, e_g, for each of the three specimens. These three curves were averaged, and this average curve was replotted in Fig. 6 along with the analogous average curves for the remaining biaxial strain ratios. The left-hand side of Fig. 6 pertains to in-machine strains and the right-hand side to across-machine strains.

The reinforcement factors ranged from about 1.5 to 2.8 for the in-machine direction of the sack paper and from 1.1 to 5.0 for the across-machine direction. At any one biaxial strain ratio, the reinforcement factor increases with increasing strain intensity. The across-machine direction factors exhibit the greater increase in this respect, as well as the greater over-all range of the reinforcement factor. These latter observations may be explained in terms of the relative tensile stiffnesses in the two directions of the sack paper. It was shown in Theoretical Considerations that the reinforcement factors of two paper specimens are inversely related to their tensile stiffnesses. Since the tensile stiffness is greater for the in-machine direction of the paper than for the across-machine direction (at any given magnitude of strain), it may be

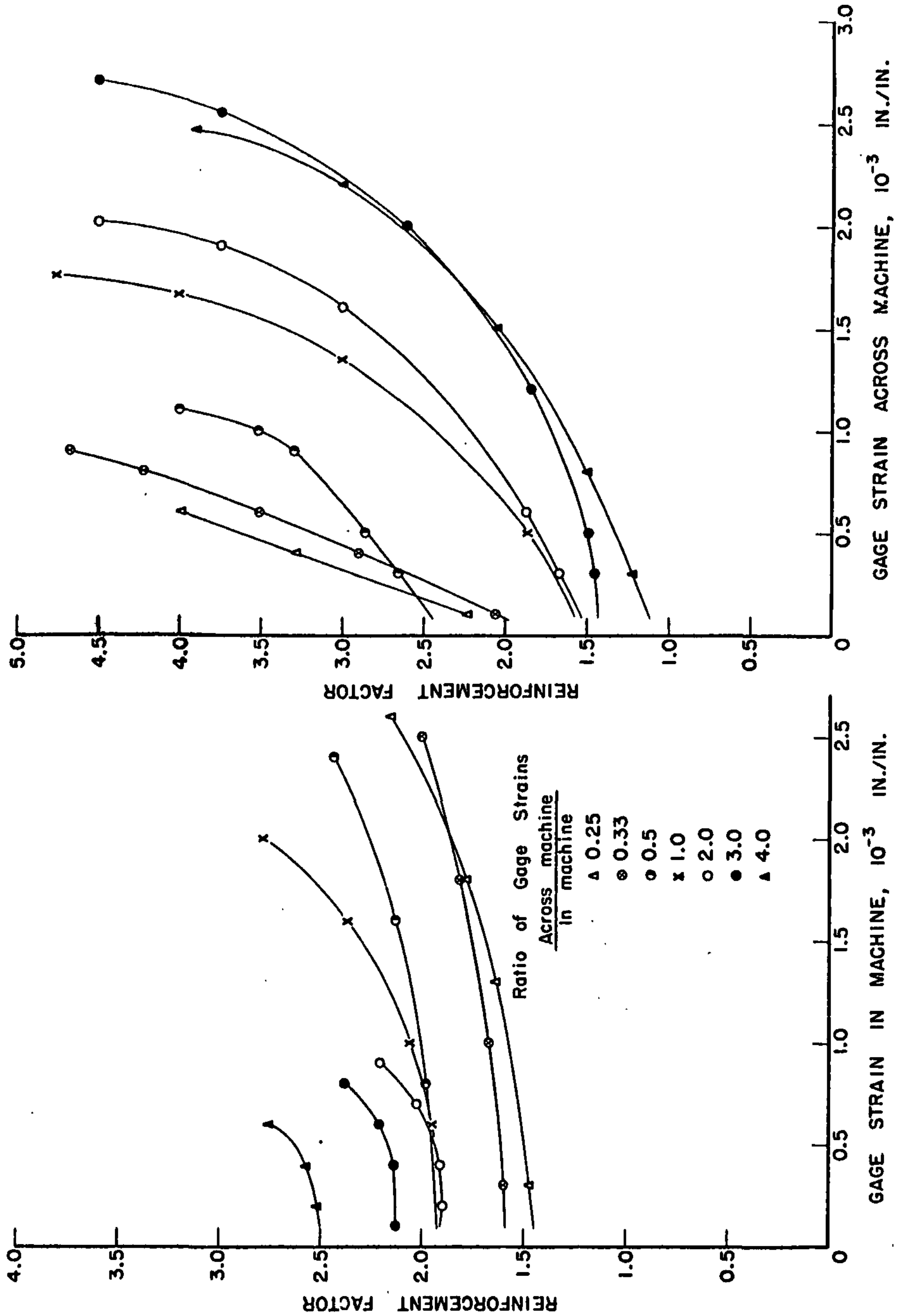

Figure 6

Reinforcement Factors for an AX-5-1 Electric Resistance Strain Gage Affixed to 50-lb. Kraft Sack Paper for Various Ratios of Biaxial Strain expected that the reinforcement factors would be less for the machine direction than for the across-machine direction. Similarly, the high rate of increase of the across-machine factors is probably related to the rapid decrease in tensile stiffness beyond the proportional limit.

The curves of Fig. 6 are less orderly arrayed than the curves of Fig. 4 and 5. Since the reinforcement factor is the slope of a radial line from the origin to a point on a curve of Fig. 4 and 5, it is very sensitive to the shape of those curves and thereby magnifies errors in them.

The reinforcement factor curves of Fig. 6 may be used to convert gage strain to ungaged paper strain in much the same manner as Fig. 4 and 5 were used. One reads into the appropriate strain ratio curve at the measured gage strain and reads out the reinforcement factor. This factoris then multiplied by the gage strain to calculate the ungaged paper strain. It is noted that an additional step is needed to convert the gage signal to ungaged paper strain beyond what is necessary with Fig. 4 and 5.

A shortcoming of the data obtained in this study is the limited range of strains presented in Fig. 4, 5 and 6. The specimens tested in the biaxial tester appeared to fail prematurely, that is, at strain levels lower than might be expected, based on uniaxial stretch properties of the sack paper (5). As shown in Reference (2), impact strains have been measured which exceed the levels of gage strain presented in this report. Extrapolation of the curves of Fig. 4, 5, and 6 must be done cautiously in view of the steep ascent at the higher strain levels--particularly for the across-machine direction.

CONSIDERATION OF THE EFFECT OF THE RATE OF STRAINING ON GAGE REINFORCEMENT

The biaxial test used for determining the reinforcement factor was a static test, whereas the strains in a bag at the time of impact may develop at a high rate. The advisability of interpreting the high speed test with slow speed reinforcement factors, as may be undertaken with the data of this report, is open to question. It is possible, however, to estimate the nature of the effect of the rate of straining on the reinforcement factor.

It was shown analytically in Theoretical Considerations that the electrical resistance strain gage reinforcement effect is dependent on the relative stiffness of the sack paper and the gage. The tensile stiffness of a specimen is the slope of the linear portion of the curve of the force per unit width vs. unit strain for points within the proportional limit of the curve. As the strain (or load) increases beyond the proportional limit, the stiffness of the material is the secant modulus, that is, the slope of a straight line from the origin of the curve to the point of interest on the curve.

Data reported by Steenberg (6) indicate that the uniaxial tension stiffness of sack paper increases with increasing strain rate. By Equation (5), therefore, it follows that the reinforcement factor of a strain gage in uniaxial tension should be less for a high rate of strain than for a low rate of strain provided the stiffness of the gage does not change with rate of strain.

It appears reasonable to assume that a similar difference in sack paper stiffness would manifest itself for slow- and high-speed biaxial strain rates. Accordingly, one may expect that the reinforcement factors of the biaxial strain gages at high rates of strain are less than those presented in this report.

PROPOSAL FOR FUTURE WORK

One obvious shortcoming of the reported determination of gage reinforcement was the inability to obtain gage strains in the biaxial specimen as high as those found in impact strain measurements on sacks. This discrepancy was attributed to premature failure at the corners of the biaxial specimen which prevented it from exhibiting the potential biaxial tensile strength of the sack paper.

On the basis of the biaxial stress-strain studies (5), it appears that the strain level at specimen failure may be elevated by a judicious selection of specimen shape. One suggested specimen shape is pictured in Fig. 7. Even though exploratory studies with this type of specimen did not offer prospect of an appreciable improvement for biaxial strength studies, it may afford a sufficiently large increase in biaxial gage strain to be of value in correcting sack impact strains with regard to gage reinforcement.

When specimens of the type pictured in Fig. 7 were tested in the biaxial tester, they failed in uniaxial tension in one of the legs of the cruciform. Thus, it cannot be said that this specimen shape reveals the biaxial strength of the sack paper, since the centermost region of the

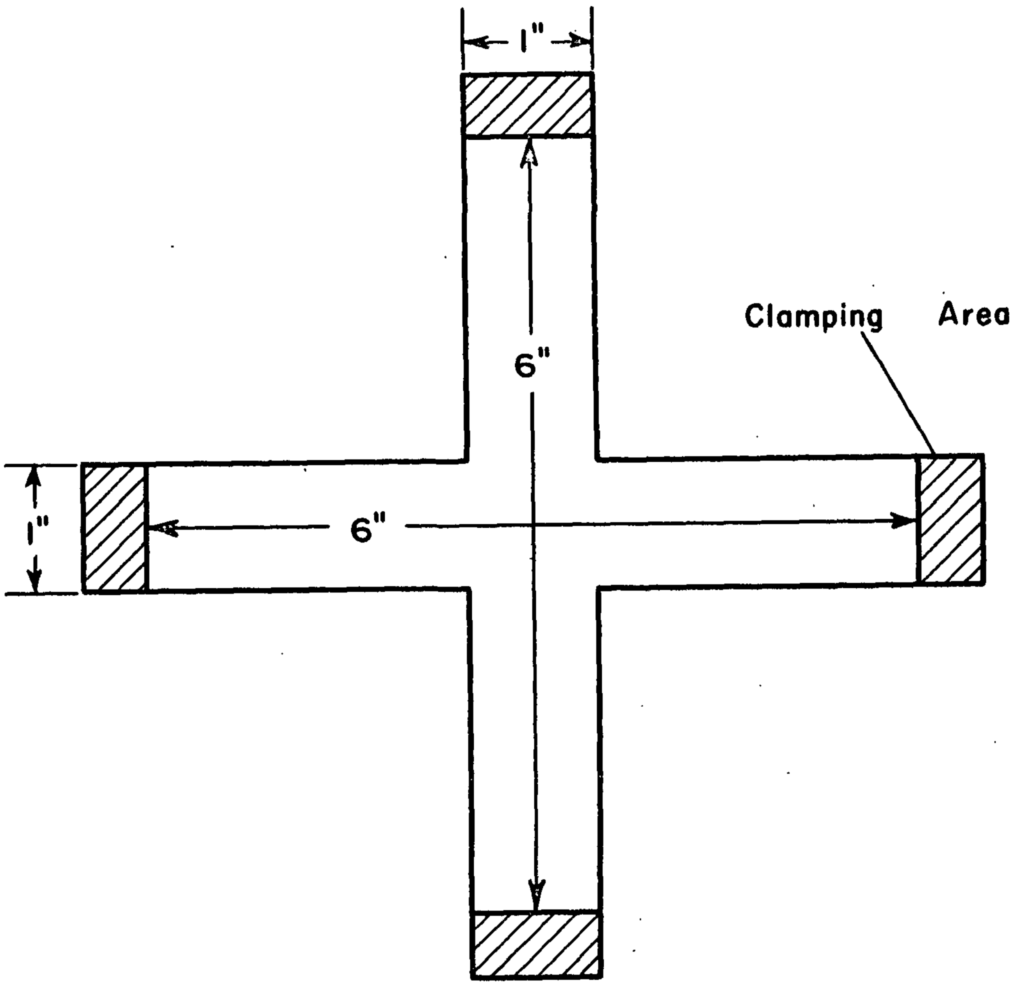

Figure 7

Proposed Specimen Shape for Future Gage Reinforcement Determinations specimen which is subjected to biaxial stresses does not fail during the test life of the specimen. Allowing for some "funneling" of, say, the in-machine direction load into the transverse legs of the specimen, the load intensity in the biaxial region conceivably still may be almost as great as the uniaxial strength of the sack paper. If this assumption is correct, then greater biaxial strains may be achieved with this specimen shape than with the six-inch-square specimens previously employed, in which the loads attained only about 85% of the uniaxial strength.

The contemplated use of this cruciform specimen for gage reinforcement studies will demand additional instrumentation for the measurement of strains in the ungaged paper since the entire specimen does not act in biaxial tension. It is believed that an extensometer with a one-inch gage length would fulfil this requirement for the specimen pictured in Fig. 7.

The following test procedure may be suitable for a specimen of this shape. A biaxial strain gage is affixed to the sack paper at the junction of the legs. While maintaining the in- and across-machine gage strains in a prescribed ratio, the in-machine gage strain is increased to a given level, whereupon the forces in both the machine and across-machine directions are noted. Subsequently, a like specimen without a strain gage, but with an extensometer in its place and oriented in the machine direction is subjected to the same in-and across-machine loads, and the strain in the ungaged, biaxially stressed paper is noted. Thus, both specimens have been subjected to the same biaxial stress condition and the strain in both strain gaged and "un-strain-gaged" sack paper has been determined, whereupon the in-machine reinforcement factor may be calculated. A third specimen with the extensometer oriented in the across-machine direction provides the data for calculation of the across-machine reinforcement factor.

The method described above requires three times as many specimens as the method previously employed. Even greater replication may be required to minimize the effects of variability between the specimens of each set. On the other hand, the requirement of measuring the paper strains locally with an extensometer probably will lead to an improvement compared with measurement of deformation over the total span, since it eliminates possible errors due to slippage or nonuniform strains at the clamps.

The specimen shape of Fig. 7 is suggested for its simplicity. If this specimen type fails to achieve the desired increase in strain level, other more elaborate (reinforced) shapes may be resorted to, as discussed in Reference (5).

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