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Financial and Sovereign Debt Crises in Spain: Fiscal Limits and Spillovers

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Abstract

Sovereign debt crises have four consistent features: 1) financial crises tend to coincide with them; 2) they are followed by credit crunches; 3) the domestic costs of default are higher where financial institutions hold large portions of their sovereign's debt; and 4) sovereign risk premiums are countercyclical and exhibit nonlinear dynamics with respect to debt levels. These facts indicate that spillover between financial and debt crises are important means of amplifying economic downturns. Current models cannot replicate all four of these facts because they either lack investment, an endogenous fiscal limit on the accumulation of sovereign debt, or a nonlinear solution. I create a dynamic stochastic general equilibrium (DSGE) model with collateralized sovereign bonds used by entrepreneurs to obtain investment funds and an endogenous fiscal limit that instigates default. These two components are essential in explaining facts 1-3. I solve my model globally through the monotone map method so that it accurately matches the nonlinear behavior of sovereign risk premiums and accounts for fact 4. I calibrate my model to Spanish data from 1999-2012 to test if it captures these four facts as well as the cyclical behavior of macroeconomic aggregates.

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1 Introduction

Concurrent financial and sovereign debt crises in Spain have led to tumbling assets prices in private financial markets and the market for Spanish sovereign debt since the onset of the Global Financial Crisis. The Spanish economy is one of the most devastated in Europe in recent years; this is particularly troubling given its large size. Spain has experienced declines in GDP and virtually no growth since 2008 and great uncertainty persists as to how and when economic recovery will begin.

Coinciding sovereign debt and financial crises have occurred across countries for many years and I propose an economic framework to explain why we may observe this phenomenon and what influences the behavior of macroeconomic aggregates during these crises. Historically, financial and sovereign debt crises regularly occur simultaneously and each type of crisis has preceded the other. Many times during these episodes, private credit severely contracts (a credit crunch).

Sovereign debt literature has long focused on the international costs of default such as lost access to international goods and capital markets. However, costs to sovereign default also originate domestically when residents are holders of sovereign debt. In particular, domestic financial institutions in many countries (including Spain) hold large levels of public debt for collateral purposes as regulatory institutions consider them to be extremely safe. The collateral role of government bonds means that a negative feedback loop may arise. Falling government bond prices harm the balance sheets of domestic financial institutions and the resulting declines in private financial activity reduce the tax-base used by the sovereign to pay for its debt.

Nonlinear solution methods are needed to replicate the empirically observed nonlinear relationship between sovereign bond prices and levels of sovereign debt. Additionally, when modeling crises, nonlinear solutions better capture the behavior of macroeconomic aggregates. During a crisis, variables can make extreme and sudden changes; a local linear approximation quickly loses its accuracy under these circumstances.

The primary contribution of this research is a novel characterization of the role sovereign bonds play in domestic markets, capable of explaining these four empirical regularities surrounding debt and financial crises. When a sovereign's debt is held by domestic banks, it raises the costs of default and makes default less likely *ceteris paribus*. At higher debt levels, the cost of continuing to service the debt is greater and bondholders are aware that the probability of continuing to be repaid falls; the bondholders then require greater yields for the additional risk. When sovereign bonds help extend private credit, the fall in prices can instigate a credit crunch and reduce real investment. Financial crises brought about by exogenous factors or debt crises can further worsen the economic environment by reducing the tax-base available to the sovereign for servicing its debt.

In what follows, I present my model, its solution method, and apply it to Spanish data from 1999-2012. In section 2 I discuss the literature related to four stylized facts about sovereign default and illustrate the presence of these facts in Spanish macroeconomic data. Section 3 details the components of the model and section 4 discusses the solution method and calibration of model parameters. I analyze the effect of shocks to total factor productivity (TFP) and fiscal policy in section 5 where I evaluate the ability of my model to replicate the stylized facts described above as well as Spanish macroeconomic aggregates. Section 6 concludes.

2 Sovereign Default Facts and Literature

2.1 The Coincidence of Financial and Debt Crises

Financial and sovereign debt crises historically coincide, but there is no definitive direction of causality; both crises can propagate each other. In a panel study covering 81 countries from 1980-2005, 74 of 110 default episodes were accompanied by banking crises (Gennaioli et al., 2012). In 30 of these coincident crises, the financial crisis had begun prior to the debt crisis while the opposite is true for the remaining cases. Both types of crises can influence the other and catalyze a negative feedback loop. Spain's financial sector was negatively impacted by the global financial crisis by as early as 2007 (see figure 1). After the Eurozone debt crisis began to accelerate in late 2009, Spanish sovereign and financial sector stress indicators began to comove contemporaneously.

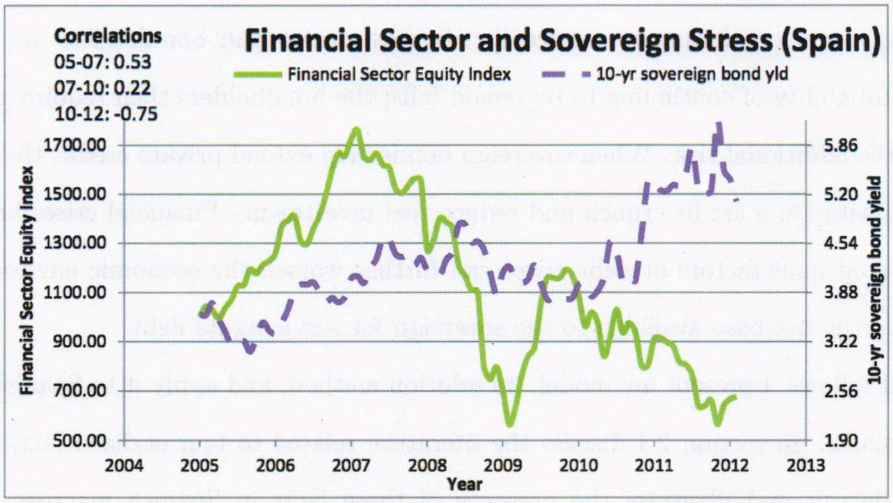

Figure 1: Measures of sovereign and financial stress at a monthly frequency. Bond yields are from Eurostat and the financial sector equity index was provided by the Chicago Fed.

Many existing models do not reflect this coincidence of events because they do not explicitly model capital and a financial sector. Eaton and Gersovitz (1981) inspired many contemporary sovereign default models such as Arellano (2008) where output/income is exogenous. In these models, random shocks to output can trigger default which further entails ad hoc costs to output. Since output is exogenous in these models, they do not provide a framework for explaining why rising sovereign bond yields and outright default have an effect on any of the components of output (investment included).

Bi (2011) presents a nuanced model of default with an endogenous fiscal limit similar to the one in this paper where sovereign default endogenously reduces output. Default poses a trade-off: default can be desirable as fewer taxes are necessary to service the debt after default, but households holding sovereign bonds lose resources when their government does not repay them. Unlike Bi, I model a financial sector where entrepreneurs hold public debt for collateral purposes and rent capital to firms. The collateral role of sovereign bonds and their role in augmenting investment and subsequently output directly links the financial sector and the sovereign and makes both debt and financial crises capable of amplifying each other.

2.2 Ensuing Credit Crunches

Private sector credit crunches often follow sovereign debt crises; the collateral value of sovereign debt plays an important capital role on bank balance sheets and facilitates private sector lending. Gennaioli et al. (2012) find that within a year after a sovereign defaults on a portion of their debt, private credit falls on average by 2.5 percent of GDP[1]. Debt crises can bring about credit crunches through the use of sovereign bonds as collateral. Binding borrowing constraints, as in Bianchi (2011), trigger deleveraging when capital declines in value. After the onset of the financial crises in 2007, loan supply shocks played a central role in shrinking real loan volume and curbing real GDP growth (Hristov et al., 2011). Sovereign debt crises significantly affect domestic private credit markets which in turn play a significant role in the deepening of debt crises.

Gennaioli et al. (2012) and Bolton and Jeanne (2011) model a channel through which sovereign stress spills over to domestic banks. They endogenize the costs of default, abstracting from the ad hoc costs to default constructed in Eaton and Gersovitz (1981) and Arellano (2008). Gennaioli et al. (2012) and Bolton and Jeanne (2011) both have financial sectors where banks hold sovereign bonds for their liquidity value and use them to make loans to other economic sectors.

Financial institutions in the Eurozone hold public debt because it can be used as collateral for cheap lending through central bank discount windows and regulations such as the Basel accords require that they maintain certain capital ratios and collateral levels[2]. When sovereigns default or the price of their debt falls, bank capital declines. This limits the ability of banks to extend credit to the economy and thus instigates a credit crunch.[3] Bolton and Jeanne (2011) specifically design the loans to resemble repurchase agreements

[1]This estimated effect is both statistically and economically significant.

[2]Public bonds have been especially desirable for collateral and capital maintenance in the Eurozone since banks may assign them a zero risk-weighting (Pisani-Ferry and Merler, 2012)—treating Greek, German, and US bonds identically in terms of risk.

[3]See Adrian and Shin (2010); Iacoviello (2010); Bernanke et al. (1999); Gilchrist et al. (2009); Kiyotaki and Moore (1997) for examples of how declines in bank capital can amplify economic downturns.

(repo loans[4]).

Mendoza and Yue (2012) were the first to endogenize the decision to default as the outcome of declines in the costs of default. In their model, sovereign default leads international agents to exclude the delinquent government and domestic private firms from foreign capital markets. Producers are forced to substitute foreign capital with less efficient domestic capital. The sovereign is a benevolent social planner who solves an infinite horizon problem to maximize the utility of domestic households. Thus default occurs when not making public debt payments frees up more resources for consumption than are lost when excluded from international markets.

2.3 Domestic Costs of Default

Trade and capital sanctions are often short-lived and rarely observed in modern times (Gennaioli et al., 2012), thus the frequency with which we observe sovereign default only makes sense if there are significant domestic costs facing the sovereign. Domestic costs manifest themselves as output lost as a result of a the sovereign's decision to default. Similarly to Gennaioli et al. (2012), I posit that losses in real investment resulting from a credit crunch are the domestic costs born by the defaulting nation.

Gennaioli et al. construct a three-period, small open economy model where banks hold a substantial share of government bonds. Default destroys the capital of financial institutions and reduces their ability to extend credit to the rest of the economy. An important insight of their model is that there exists a complementarity between public and private borrowing when bonds have a liquidity value. "Strong" financial institutions, meaning those integrated with much of the domestic economy, have a larger negative effect on credit markets if the bonds that they hold go into default. Weaker financial institutions lead to lower costs of default and therefore make it more likely for a government to default (all else the same).

To test their model's predictions, they analyze 81 emerging and developed economies

[4]A repurchase agreement or *repo loan* is an agreement where the issuer purchases a security, in this case government bonds, which the seller/borrower agrees to repurchase for a higher price at a specified later date.

from 1980 to 2005. Gennaioli et al. find that declines in private credit volume are more severe (rising from 2.5 to 2.9 percent of GDP) for banks holding a percentage of their sovereign's bonds above the sample median. This means that default is more costly when banks hold a large amount of their sovereign's debt. When making the decision to default, a sovereign considers what they must forgo in order to service their debt (e.g., higher taxes and lower economic activity) versus the domestic costs incurred through default. Since the onset of the global financial crises, Spain and Greece, whose banks hold increasingly large shares of their respective sovereign's total debt, have already faced the largest losses of output in the Eurozone.

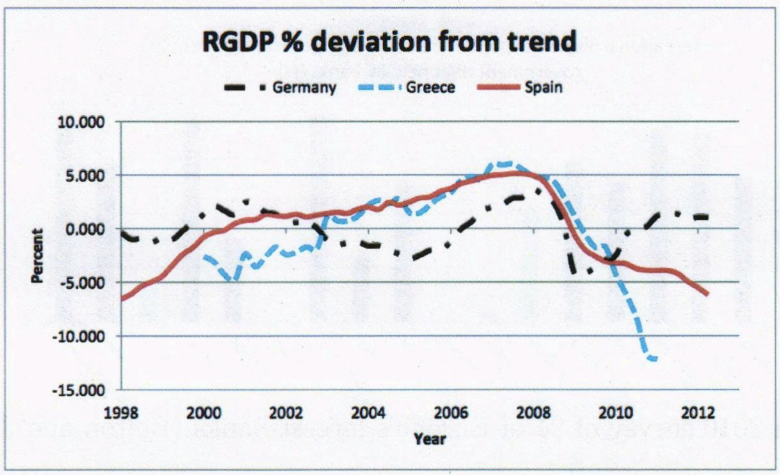

Figure 2: Real GDP percent deviations from trend. Data are quarterly and from Eurostat.

Domestic banks throughout the EU tend to hold an unusually large amount of sovereign debt as collateral compared to the rest of the world (Pisani-Ferry and Merler, 2012). This is especially true for Spain, Greece, and Germany as seen in figure 2. This pattern has declined for Germany while it has risen for Spain (Pisani-Ferry and Merler, 2012). Moreover, figure 3 shows that Germany differs from Greece and Spain in that its overall sovereign bond portfolio is significantly more diversified by country. Spanish and Greek banks are the least diversified in terms of national origin of public bonds. These holding patterns make Spanish banks exceptionally vulnerable to rising sovereign bonds yields which may degrade their collateral[5].

[5]30 percent of EU banks attributed poor performance to collateral degradation (ECB Bank Lending Survey, 2012)

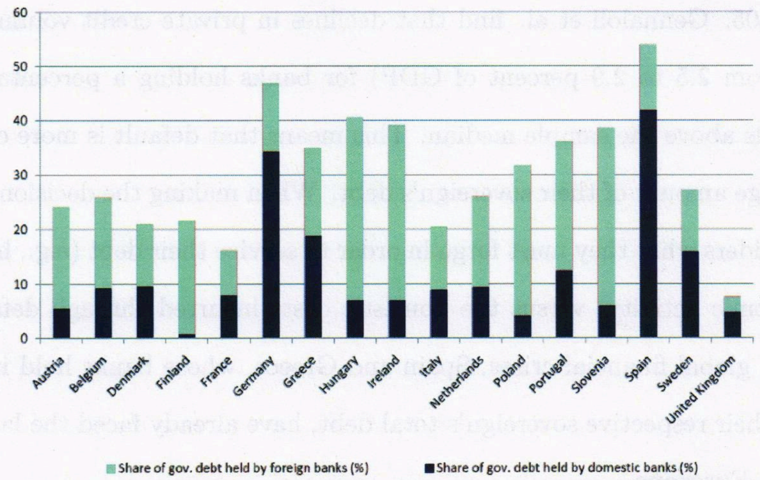

Figure 3: From a 2010 survey of 91 of Europe's largest banks (Bolton and Jeanne, 2011).

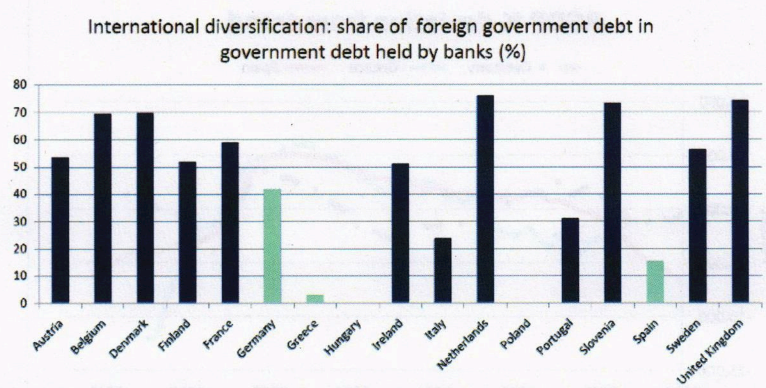

Figure 4: From a 2010 survey of 91 of Europe's largest banks (Bolton and Jeanne, 2011).

2.4 Nonlinear Risk Premiums

Sovereign risk premiums rise more rapidly at higher debt levels. Baldacci and Kumar (2010) analyze 31 developed and emerging economies from 1980-2008 to find that higher deficits and public debt cause statistically significant and nontrivial increases in the long-term sovereign bond yields. The magnitude of these increases depends on initial debt levels and the quality of domestic institutions. On average, for OECD countries, increasing debt-to-GDP above 60 percent puts pressure on yields to rise more rapidly.

Ghosh et al. (2011) also find a similar nonlinear relationship between long-term sovereign bond yields and debt levels using a dataset of 23 developed economies from 1970-2007. At a low debt-to-GDP ratio, there is little or a slightly negative relationship between lagged debt and the primary balance[6]. At higher levels the rate of change in bond yields rises until a very high threshold when the rate of change begins to slow. These studies imply that the relationship between yields and debt levels is nonlinear and that bond yields are positively related to debt at higher debt levels. Intuitively, this can arise from higher yields making subsequent borrowing more costly (requiring further debt). This suggests there may exist a threshold for borrowing at which greater debt levels and high bond yields become mutually reinforcing and suddenly, strongly and positively correlated.

The relationships of Spanish and Greek sovereign yields with their respective public debt levels are best approximated, in terms of R^2, with a 2$^{\text{nd}}$ degree polynomial function as shown below:

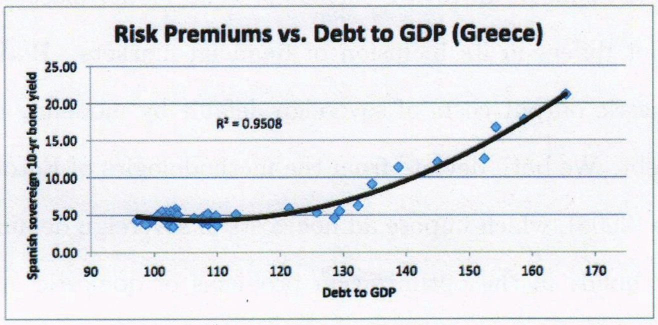

Figure 5: Greek 10-year sovereign bond yield versus gross consolidated government debt to GDP. Data are from 1999:1-2012:1.

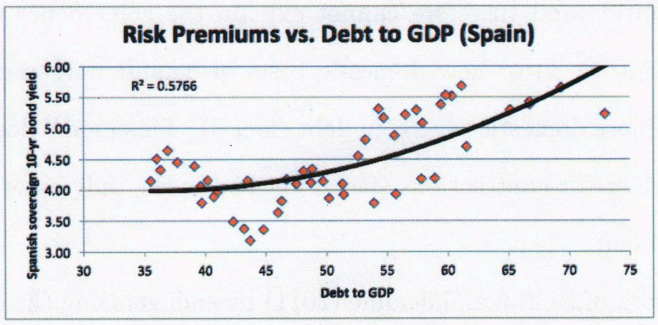

Figure 6: Spanish 10-year sovereign bond yield versus gross consolidated government debt to GDP. Data are from 1999:1-2012:1.

DSGE models typically have highly nonlinear characterizations of their equilibrium and their policy functions are often approximated linearly. In order to allow sovereign yields to exhibit the nonlinear behavior of bond yields that has taken place in Spain

[6]The primary balance is revenue less expenditures excluding gross interest payments.

and many other countries across time, it is necessary to solve the model globally and describe bond prices with a nonlinear policy function. In general, economic crises bring out nonlinear behavior as variables tend to travel far from their steady states during the notable downturns that become labeled as crises. When this is the case, a local linear solution will quickly lose accuracy.

2.5 Features of My Model

This paper models domestic financial markets where sovereign bonds are used as collateral to augment investment. The sovereign defaults on its debt if it exceeds an endogenous fiscal limit—discounted sum of all future surpluses. The model most closely resembles that of Bi (2011) but differs in its inclusion of financial markets. Both of our papers endogenize the domestic output costs of sovereign default by modeling domestic agents who purchase the debt. We both deviate from the methodologies of Eaton and Gersovitz (1981) and Arellano (2008), which impose ad hoc costs of sovereign default, by specifying a role for sovereign bonds in the optimization problems of domestic agents. I aim to explain several stylized facts about the relationship between sovereign debt and financial crises therefore must incorporate capital markets. Her model does not possess a concept of capital or investment, and therefore cannot explain the coincidence of financial and sovereign debt crises (fact 1) or how domestic costs of default may rise when financial institutions possess more domestic sovereign debt (fact 3). This model does not lend itself to simulating private credit contractions (fact 2) either as the only asset available is the domestic sovereign bond.

I augment the work of Bolton and Jeanne (2011) by endogenizing the consequences on tax policy of sovereign debt crises. In their model, the endogenous output loss of default follows from a lump-sum"tax" increase. Implicit in their model, this "tax" on the banks is the portion of the debt held by banks repudiated by the sovereign. I explicitly model a distortionary labor income tax that rises when debt repayments to entrepreneurs increase. This enables me to better model a debt crisis and not only the actual event of default because households may experience taxes rising to help pay off debt without the sovereign having to default. I construct a Laffer curve to find the maximum possible tax revenue. Agents are aware of this curve and form expectations about the government's ability to service its current debt payments. Specifically, they consider whether or not the discounted total future government surpluses resulting from taxing at the most revenue-maximizing rate are enough to pay back bondholders. Moreover, I construct an economy with an infinite time horizon whereas both Bolton and Jeanne (2011) and Gennaioli et al. (2012) feature three-period economies. Both of these models make profound contributions by illustrating how rising sovereign yields during a debt crisis may degrade capital. However, modeling this phenomenon with an infinite time horizon and a distortionary tax augments their work by illustrating the build up to, and fallout from, a debt crisis and not only the immediate consequences following the event of sovereign default.

I include entrepreneurs similar to those in Bolton and Jeanne (2011) in my model, but also add government recapitalization of entrepreneurs. Entrepreneurs leverage their sovereign bonds and countercyclical government capital transfers to obtain repo loan financing on private investment markets. Recapitalization is costly and increases the government's debt burden; a larger debt burden with no increases in government revenue moves debt levels closer to their limit and the probability of default grows. The increasing chances of default lower the expected payoff of sovereign bonds and yields rise. Recapitalization is an important addition since it may further worsen a credit crunch when it is so costly to the sovereign that on net it actually reduces bank capital through falling prices. These declines in economic activity shrink the tax-base on which sovereigns rely to service their debt and thus further grow the public debt burden. My model replicates fact 1, the coincidence of debt and financial crises. Since banks and sovereigns may transmit stress to each other, either type of crisis can catalyze the other. This is because investment is funded by loans for which sovereign bonds are used as collateral. A rise in default risk reduces the value of collateral via falling prices of sovereign bonds. This reduces real investment and the tax-base at the disposal of the sovereign for servicing their debt. If a sovereign responds with recapitalization to restore balance sheets, this helps the bank less than one would initially suspect since this further raises the default risk and has an offsetting negative impact on bank capital.

Sosa-Padilla (2012) extends the research of Mendoza and Yue and, similarly to my own work, models capital losses as a result of a credit crunch following default. Sosa-Padilla and I differ in our focus on developed economies (Spain, in my case) and emerging economies (Argentina). Additionally, in his model, default is the optimal decision of a benevolent government while in mine, investors make optimal portfolio decisions by assessing the probability of a sovereign approaching a dynamic and stochastic fiscal limit and defaulting. My specification lends itself to calculating a hypothetical fiscal limit which may be used to improve our understanding of how close Spain is to default. Moreover, the sovereign in my model only repudiates a portion of its debt while Sosa-Padilla models 100 percent repudiation. A positive recovery rate slows the rise of risk premiums and allows a country to reach higher and more realistic levels of public debt before default is triggered Wright (2011). The starkest contrast between my model and those of Sosa-Padilla and Mendoza and Yue is that I will globally solve my model and thus capture the nonlinear behavior of bond yields with respect to debt levels. This is also beneficial for modeling economic crises since during such extreme events variables are prone to travel far from their steady states; a nonlinear solution provides an improved approximation of the policy functions.

3 Model Description

The model economy is populated by four types of infinitely-lived agents. Households earn wages from labor to spend on consumption and pay a distortionary labor income tax; they smooth consumption through making saving internationally. An entrepreneur chooses a portfolio of assets an aims to maximize their own utility. The entrepreneur has an initial capital stock which they can rent or invest and can further augment their capital stock with international loans[7]. In order to receive loans from abroad, the entrepreneur must purchase sovereign bonds to use as collateral. The entrepreneur combines these

[7]We can think of the entrepreneur as being an investment bank and the firm as being a non-financial corporation that borrows capital from the investment bank.

funds with a capital endowment and recapitalization from the sovereign to rent capital to the firm. Combining labor and capital, the firm employs a Cobb-Douglas production technology to create consumption goods that it sells to households and the government. The government raises a distortionary tax and sells bonds to finance recapitalization, transfers to households, an exogenous expenditure, and previous debt obligations.

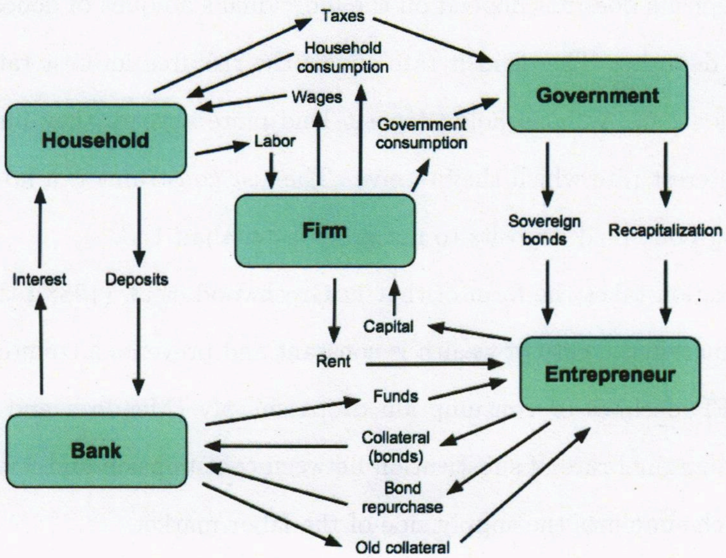

Figure 7: Interactions between economic agents

3.1 Households

A continuum of households with mass $\eta \in (0,1)$, indexed by $i \in (0,\eta)$, choose consumption c_t, labor h_t, and financial deposits d_t to solve:

$$
\begin{aligned}
\max_{\{c_t^i, h_t^i, d_t^i\}_{t=0}^{\infty}} \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[c_t^i - \chi (h_t^i)^{\omega}/\omega\right]^{1-\phi} - 1}{1-\phi} \\
\text{s.t.} \quad & c_t^i + d_t^i = w_t(1-\tau_t)h_t^i + (1+r_{t-1}^d)d_{t-1}^i + \frac{T}{\eta} \\
& r_t^d = r^f - \varrho^d D_t \\
& \lim_{t\to\infty} \frac{d_t^i}{\prod_{t=0}^{\infty}(1+r_t^d)} = \lim_{t\to\infty} \beta^t d_t^i = 0 \\
& c_t \in \mathbb{R}_+, h_t \in [0,1], d_t \in \mathbb{R}
\end{aligned}
$$

with $\beta \in (0,1)$ and $\chi, \phi > 0, \varrho^d > 0$, denoting the discount factor, the relative disutility of labor, coefficient of relative risk aversion, and the elasticity of the deposit rate relative to total deposits, respectively. The expression $1/(\omega-1)$ is the Frisch elasticity of labor. The budget constraint implies that household consumption and deposits equal after-tax income $w_t(1-\tau_t)h_t$, returns on deposits $(1+r_t^d)d_{t-1}$, and government transfers $\frac{T}{\eta}$. Note that the interest rate on deposits does not depend on the individual's amount of deposits, but the *aggregate* level of deposits. The deposit rate equals the risk-free interest rate, r^f, when aggregate deposits are 0. As households deposit/lend more abroad, they put downward pressure on the interest rate which they receive. The last constraint is a no-Ponzi game condition requiring household deposits to not grow faster than $1/\beta$.

The utility function takes the form of that in Greenwood et al. (1988); this is preferable because the marginal utility of wealth is constant and prevents a counterfactual rise in labor when TFP declines or consumption drops sharply (Mendoza and Yue, 2012). Additionally, the marginal rate of substitution between consumption and leisure depends only on labor which simplifies the supply side of the labor market.

The first-order conditions for the households are:

$$\left[c_t^i - \chi (h_t^i)^\omega/\omega\right]^{-\phi} = \lambda_t \tag{1}$$

$$h_t^i = \left[\frac{w_t(1-\tau_t)}{\chi}\right]^{1/(\omega-1)} \tag{2}$$

$$\lambda_t = \beta(1+r_t^d)\mathbb{E}_t\lambda_{t+1} \tag{3}$$

$$r_t^d = r^f - \varrho^d D_t \tag{4}$$

where λ_t is the Lagrange multiplier associated with the budget constraint. Since each member of the unit mass of households is identical, aggregate values are

$$C_t^h = \eta c_t^i = \int_0^\eta c_t^i \, di, \quad H_t = \eta h_t^i = \int_0^\eta h_t^i \, di, \quad D_t = \eta d_t^i = \int_0^\eta d_t^i \, di \,.$$

3.2 Firm

A single perfectly competitive firm hires labor, rents capital and is endowed with a Cobb-Douglas production technology. The firm maximizes profits

$$\pi_t = Y_t - w_t H_t - r_t^k K_t,$$

where production exhibits constant returns to scale and has the form:

$$Y_t = z_t K_t^{\alpha} H_t^{1-\alpha} \tag{5}$$

and TFP follows an AR(1) process

$$z_t = \rho_z z_{t-1} + 1 - \rho_z + \epsilon_t^z, \quad \epsilon_t^z \sim N(0, \sigma_z^2) \tag{6}$$

with $\rho_z \in (0,1)$. Both wages and capital are determined competitively, thus in equilibrium:

$$w_t = z_t(1-\alpha)(K_t/H_t)^{\alpha} \tag{7}$$

$$r_t^k = z_t \alpha (H_t/K_t)^{1-\alpha} \tag{8}$$

$$Y_t = z_t K_t^{\alpha} H_t^{1-\alpha}. \tag{9}$$

3.3 Government

The government employs a distortionary labor tax τ_t to raise revenue $\tau_t w_t H_t$ and chooses to borrow B_t at price q_t. These resources finance spending G_t, financial sector recapitalization $\mathcal{R}_t$, and transfers to households T. Every period the government pays back the undefaulted portion of their previous period borrowing $B_t^d = (1 - \Delta_t) B_{t-1}$ where Δ_t is the percent of the debt repudiated by the government. Thus the government's budget constraint in each period is:

$$\tau_t w_t H_t + q_t B_t = B_t^d + G_t + \mathcal{R}_t + T. \tag{10}$$

Recapitalization and taxes respond to TFP and debt repayment, respectively:

$$\ln\left(\frac{\mathcal{R}_t}{\mathcal{R}}\right) = \nu \ln\left(\frac{z_t}{z}\right) \tag{11}$$

$$\tau_t - \tau = \theta(B_t^d - B). \tag{12}$$

and government expenditure is characterized by an AR(1) process

$$G_t = \rho_G G_{t-1} + (1-\rho_G)G + \epsilon_t^G, \quad \epsilon_t^G \sim N(0, \sigma_G^2). \tag{13}$$

3.4 Entrepreneurs (The Financial Sector)

Risk-neutral entrepreneurs with a capital endowment, a_t, aim to maximize their consumption, c_t^e. They may augment their investment by obtaining loans abroad. Each period the entrepreneurs purchase government bonds b_t at price q_t to use as collateral to borrow loans l_t. As in Kiyotaki and Moore (1997), I assume when entering this financial contract that the entrepreneur pledges the market value of the asset. At the same time, they are repaid by the government the non-defaulted portion of their sovereign bond $b_{t-1}(1-\Delta_t)$ used as collateral in the previous period. They must also pay interest and return the principal $l_{t-1}(1+r_{t-1}^l)$ of the loan which they borrowed. Loans are desirable because entrepreneurs are able to transform them at a one-for-one rate into usable capital which may be combined with their initial endowment, a_t, and sold to the firm as $k_t = a_t + l_t$. The entrepreneur is constrained in their ability to use sovereign bonds as collateral and may only do so at a ratio of $\gamma < 1$. This implies a leverage ratio of $\frac{\gamma}{1-\gamma}$.

The entrepreneur purchases a loan which is, in essence, the same as a repo loan. The timing in this model is such that the entrepreneur has already repurchased the sovereign debt prior to the sovereign's decision to default or not—so they bear the loss, not the international lender, when the sovereign defaults. Therefore entrepreneurs may be thought of as banks with investment opportunities that assets leverage via interbank markets. The borrowing constraint has analogous implications to a capital-to-assets ratio requirement, similar to that which Spanish banks maintain to abide by the Basel accords. Another resource at the disposal of the entrepreneurs is recapitalization from the government $\mathcal{R}_t$. Recall that this variable depends on the current period total factor productivity realization and therefore acts as a capital injection (for $z_t < z$) when a bad shock is realized.

Each period the entrepreneurs purchase bonds, obtain interbank/repo loans which are transformed into capital, their total capital stock is rented to the firm at price r_t^k and they also invest, x_t, in order to accumulate capital. Used capital depreciates at rate δ. The continuum of entrepreneurs have mass $1 - \eta$ and are indexed by $h \in (0,1)$ and each solves:

$$\begin{aligned}
\max_{\{c_t^h, b_t^h, l_t^h, a_{t+1}^h, x_t^h\}_{t=0}^{\infty}} \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_e^t c_t^{e,h} \\
\text{s.t.} \quad & c_t^{e,h} + x_t^h + l_{t-1}^h(1 + r_{t-1}^l) + q_t b_t^h \\
& = r_t^k(a_t^h + l_t^h) + (1-\delta) l_t + b_{t-1}^h(1 - \Delta_t) + \frac{\mathcal{R}_t}{1-\eta} \\
& x_t^h = a_{t+1}^h - (1-\delta) a_t^h \\
& k_t^h = a_t^h + l_t^h \\
& l_t^h \leq \gamma \left(q_t b_t^h + \frac{\mathcal{R}_t}{1-\eta} \right) \\
& r_t^l = r^f + \varrho^l L_t \\
& b_t^h, l_t^h \in \mathbb{R}, \quad c_t^h, k_{t+1}^h \geq 0.
\end{aligned}$$

In order to prevent the entrepreneur reaching the point where they accumulate enough capital to be self-financing, they discount at a higher rate than all other agents: $\beta_e < \beta$ (Carlstrom and Fuerst, 1997). Letting μ_t denote the Lagrange multiplier associated with the leverage constraint, the first-order conditions for the entrepreneur are

$$q_t = \frac{\beta_e \mathbb{E}_t(1-\Delta_{t+1})}{1-\gamma\mu_t} \tag{14}$$

$$1 = \beta_e \mathbb{E}_t(r^k_{t+1} + 1 - \delta) \tag{15}$$

$$\mu_t = r^k_t + (1-\delta) - \beta_e(1+r^l_t) \tag{16}$$

$$l^h_t = \gamma(q_t b^h_t + \mathcal{R}_t) \tag{17}$$

$$k^h_t = a^h_t + l^h_t \tag{18}$$

$$r^l = r^f + \varrho^l L_t \tag{19}$$

$$c^{e,h}_t = -x^h_t - l_{t-1}(1+r^l_{t-1}) - q_t b^h_t \tag{20}$$

$$+r^k_t(a^h_t + l^h_t) + (1-\delta)l^h_t + b^h_{t-1}(1-\Delta_t) + \frac{\mathcal{R}_t}{1-\eta}. \tag{21}$$

Note that the borrowing constraint binds in the steady state as the multiplier μ is nonzero. Variables associated with the entrepreneurs are aggregated as follows:

$$C^e_t = (1-\eta)c^{e,h}_t = \int_0^1 c^{e,h}_t \, dh, \quad B_t = (1-\eta)b^h_t = \int_0^1 b^h_t \, dh, \quad L_t = (1-\eta)l^h_t = \int_0^1 l^h_t \, dh,$$
$$A_t = (1-\eta)a^h_t = \int_0^1 a^h_t \, dh, \quad K_t = (1-\eta)k^h_t = \int_0^1 k^h_t \, dh, \quad X_t = (1-\eta)x^h_t = \int_0^1 x^h_t \, dh \, .$$

Consumption per capita is the sum of household and entrepreneur consumption: $C_t = C^h_t + C^e_t$.

3.5 Laffer Curve, Fiscal Limit and Default

Since the labor tax is distortionary and labor is supplied elastically, a Laffer curve can be constructed to determine the revenue-maximizing rate of taxation. From the household's intratemporal condition (equation 2), tax revenues are:

$$\mathcal{T}_t = \tau_t w_t H_t = \tau_t w_t [w_t(1-\tau_t)/\chi]^{1/(\omega-1)}.$$

The greatest possible tax revenue, $\mathcal{T}_t^{\max}$, is obtained for a tax rate $\tau_t^{\max}$ that satisfies:

$$\frac{\partial \mathcal{T}_t}{\partial \tau_t} = w_t[w_t(1-\tau_t)/\chi)]^{1/(\omega-1)} + \tau_t w_t \frac{1}{\omega-1}[w_t(1-\tau_t)/\chi)]^{1/(\omega-1)-1}\left(-\frac{w_t}{\chi}\right) = 0.$$

The tax rate that maximizes revenue is $\tau_t^{\max} = \frac{\omega-1}{\omega}$; note that this is a function of the Frisch elasticity $\frac{1}{\omega-1}$. As in Sargent (2012), forward-looking agents forecast whether or not the government will be able to service its current debt payments with the expected discounted sum of all future surpluses. I model agents who assume the best-case scenario regarding tax revenues; the public forecasts whether or not the government will have enough resources to service its debt if they taxed a revenue-maximizing tax rate. Formally, the public is aware that the sovereign will default when the gross debt payment for the current period (B_{t-1}) exceeds the discounted sum of all future surpluses:

$$\begin{aligned} \mathcal{S}_t(z_t, G_t, K_t) &= \mathbb{E}_t \sum_{j=0}^{\infty} \beta^{t+j}(\mathcal{T}_{t+j}^{\max} - G_{t+j} - \mathcal{R}_{t+j} - T) \\ &= \mathbb{E}_t \sum_{j=0}^{\infty} \beta^{t+j}(\tau^{\max} w_{t+j} H_{t+j} - G_{t+j} - \mathcal{R}_{t+j} - T). \end{aligned}$$

If the current repayment exceeds this *fiscal limit*, then the government defaults on a fixed portion ζ of its debt. Since $\mathcal{S}_t$ depends on the stochastic realization of the state, this limit is both dynamic and stochastic and therefore has a distribution conditional on the state variables. In my simulations, I randomly draw a fiscal limit $\mathcal{S}_t^*$ from the distribution of $\mathcal{S}_t$ as in Bi (2011). The amount of debt repudiated is therefore:

$$\Delta_t = \begin{cases} 0 & : B_{t-1} \leq \mathcal{S}_t^* \\ \zeta & : B_{t-1} > \mathcal{S}_t^* \end{cases}.$$

Randomly drawing the fiscal limit serves to approximate the inherent uncertainty in the political process that leads up to a government's decision to default or not. Pressure on policy makers from various interest groups can make this outcome quite uncertain in reality and this modeling method reflects the conditional probabilities rational agents form when anticipating how the sovereign will proceed.

The distribution of the fiscal limit is calculated via Markov chain Monte Carlo (MCMC) simulations of the two Markov processes (G_t and z_t) over a discretized state space $\{z_t, G_t, K_t\}$. For $j = 0$, the surplus depends only on the current state. Future periods make use of the law of iterated expectations to substitute in the entrepreneur's first order condition with respect to capital (see section A of the appendix for details) so that the entire sum only depends on the current state and not directly on future states. Since the tax rate is assumed to be fixed at $\tau^{\max}$, it is possible for all $j > 0$ to eliminate hours and other endogenous variables.

The simulations for Spain[8] suggest that the mean of the fiscal limit relative to output is highly sensitive to the level of government expenditure (see figure 8). Increases in government expenditure also reduce the variance in the distribution of the fiscal limit relative to GDP.

Simulations show that rises in TFP slightly increase the mean of the fiscal limit relative to GDP while a larger capital stock reduces the mean of the fiscal limit relative to GDP. The effect of capital on the fiscal limit may appear counterintuitive, but increases in the capital stock do in fact increase the mean of the fiscal limit (not relative to GDP). However, a larger capital stock increases GDP more than it increases the mean of the fiscal limit. In essence, this suggests that, all things the same, a larger capital stock raises the borrowing limit in absolute terms but lowers it relative to national income/output.

[8]The histograms are based on 10,000 simulations of 600 time periods over gridded variables with 15 nodes. In the debt-to-GDP ratio, debt is measured as a stock and GDP as a flow. Therefore, quarterly debt-to-GDP ratios should be approximately four times greater than annual measures of this ratio. I simulated quarterly data, but I approximated a corresponding annual limit by dividing the debt-to-GDP ratio by four to make the numbers more directly comparable to the more often-cited annual debt-to-GDP ratios.

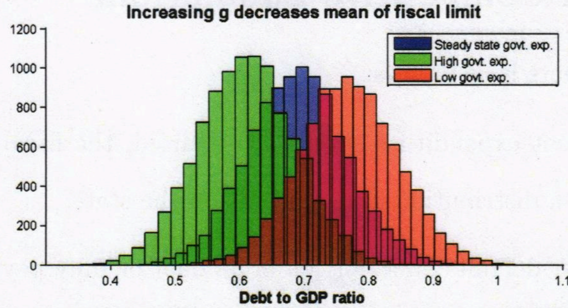

Figure 8: Histograms of annual fiscal limits relative to output (debt-to-GDP) for steady states and one standard deviations above and below of government expenditure. Increased government expenditure lowers the fiscal limit.

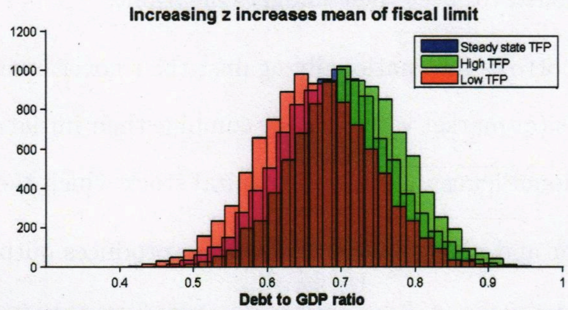

Figure 9: Histograms of annual fiscal limits relative to output (debt-to-GDP) for steady states and one standard deviations above and below of TFP. Increasing TFP slightly raises the fiscal limit.

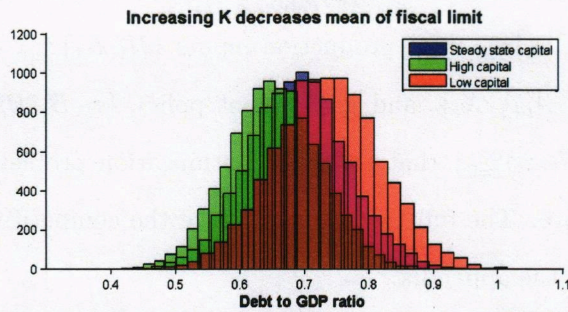

Figure 10: Histograms of annual fiscal limits relative to output (debt-to-GDP) for steady states and shocks above and below the steady state of capital of 10 percent. Increasing the capital stock increases GDP relatively more than it increases the limit on the amount of debt, reducing the limiting debt-to-GDP ratio.

3.6 Timing and Competitive Equilibrium

The timing of model events is as follows:

1. TFP and government expenditure shocks are realized, the fiscal limit is then randomly drawn from a distribution that depends on the state.
2. The sovereign either defaults on a portion of its debt or fully pays its previous obligation based on the fiscal limit. The sovereign then chooses taxes and recapitalizes the entrepreneur while also making transfer payments to households. The sovereign borrows what is needed to meet their budget constraint.
3. The entrepreneur borrows internationally against their government capital injection and sovereign bonds (at market value). They combine their initial capital endowment with their international loans into a single capital stock which they rent to the firm.
4. The firm hires labor and, using the rented capital, produces output.
5. The household works, consumes, saves through deposits, and enjoys leisure.

A competitive equilibrium for this economy is a set of prices $\{w_t, r_t^k, r_t^d, r_t^l, q_t\}_{t=0}^{\infty}$, household allocations $\{C_t, H_t, D_t\}_{t=0}^{\infty}$, firm production inputs $\{H_t, K_t\}_{t=0}^{\infty}$, entrepreneur portfolio choices $\{C_t^h, B_t, L_t, A_{t+1}, X_t\}$, and government policy $\{\tau_t, B_t, B_t^d, \Delta_t, \mathcal{R}_t\}_{t=0}^{\infty}$ given exogenous $\{A_0, r^f, T, \{G_t, z_t\}_{t=0}^{\infty}\}$ that solve the optimization problems and satisfy the constraints outlined above. The full characterization of the competitive equilibrium can be found in section B of the appendix.

4 Solution Method and Calibration

4.1 Description of Solution Method

I solve the model globally in order to capture the nonlinear behavior of the interest rates and maintain the accuracy of my policy functions when values deviate far from their steady states[9]. The mathematical object that constitutes a solution to this dynamic, stochastic

[9]Deviating far from steady state values is much more likely to occur when modeling features like default which may cause a sudden and large change in the allocation of resources. Thus, local linear methods lose

system is a set of policy functions that map exogenous and predetermined endogenous (state) variables to the remaining endogenous choice variables.

I employ the monotone map method which was first developed by Coleman (1991) and since utilized by, e.g., Davig (2004) and Bi (2011) to find policy functions for suboptimal economies. Coleman (1991) identifies conditions, met in by my paper[10], under which a generated sequence of approximating functions converge to the true policy function for suboptimal economies. The method relies on the ability to construct a monotone self-map of a nonempty, partially-ordered, compact set. This means *mapping the policy function* to itself via the Euler equations. The fixed "point" of this monotone map is the desired policy function.

The monotone map method works by making an initial guess for the policy function and solving for the value of the function applied to a fixed set of arguments. This updated function becomes the new guess for the next iteration and the procedure is repeated until the function converges. Let $S_t = \{z_t, G_t, K_t, B_{t-1}, D_{t-1}\}$ denote the set of state variables (state space) at time t. The system of 23 equations that characterize the equilibrium can be reduced to two Euler equations in $\{S_t, S_{t+1}, D_{t+1}\}$. I iterate to find the policy functions $D_t = h^d(S_t)$ and $B_t = h^B(S_t)$:

$$
\begin{aligned}
\frac{B_t^d + G_t + \mathcal{R}_t + T - \tau_t w_t H_t}{B_t} &= \frac{\beta_e \mathbb{E}_t(1 - \Delta_{t+1})}{1 - \gamma[r_t^k + (1-\delta) - \beta_e(1 + r_t^l)]} \\
\left(\eta^{\omega-1} C_t^h - \chi H_t^\omega / \omega\right)^{-\phi} &= \beta \mathbb{E}_t(1 + r_t^d)\left(\eta^{\omega-1} C_{t+1}^h - \chi H_{t+1}^\omega / \omega\right)^{-\phi}
\end{aligned}
$$

After obtaining the policy functions h^d and h^b, it is possible to solve for the remaining policy functions.

The procedure differs from that employed by Bi (2011) who uses projection methods to iterate a single Euler equation and solve for the state space given a guess for her policy

significant accuracy when applied to models replicating these phenomena

[10]For the specifics of the necessary conditions, see Coleman (1991). They are mainly restrictions on the functional forms for the production technology and utility (such as continuity, continuous differentiability, $f(0) = 0$, monotonicity, etc.). The Cobb-Douglas and AK production functions both satisfy this; most utility functions commonly utilized in DSGE literature do as well including Greenwood et al. (1988) preferences and Cobb-Douglas utility.

function. I simultaneously iterate two Euler equations and solve for two policy functions. The method of Bi (2011) is comparable to fixed point iteration as discussed in Rendahl (2012) and the endogenous grid method (EGM) innovated by Carroll (2006).

Fixed point iteration finds a policy function $x_{t+1} = g^n(x_t)$ by iterating $g^{n+1}(x_t) = f(x_t, g^n(x_t), g^n(g^n(x_t)))$. Rendahl (2012) shows that this method has inferior convergence properties compared to time iteration. Fixed point iteration can diverge and explode while time iteration is guaranteed to converge if a solution exists as the policy function is a contraction mapping. Rendahl (2012) further generalizes the convergence proof of Coleman (1991) to show that time iteration converges to the policy function(s) for an arbitrarily large state and choice space even in the presence of occasionally binding constraints.

Rendahl also notes that these convergence properties hold when time iteration is synthesized with EGM. EGM solves for particular state variables given a set potential policy function values instead of finding the optimal values of policy function given the state space. It is called EGM because the endogenous choice variable that one usually approximates is now discretized as a grid and what was previously a discretized state variable becomes "endogenous" since it changes with each iterative step.

In Barillas and Fernández-Villaverde (2006) the authors generalize the EGM procedure to solve systems with multiple state and choice variables, record the superior accuracy and speed associated with EGM over conventional value function iteration, and note that this makes it an attractive method especially for highly nonlinear models. I cannot utilize EGM exactly as in Carroll (2006) since my model has more than one Euler equation. I first apply fixed point iteration to solve for current period bonds in the first-order condition with respect to bonds and time iteration and EGM to solve for deposits in the previous period from the first-order condition with respect to deposits.

The steps of my variant of these algorithms are:

1. Discretize a 5-dimensional space into a grid where D_t replaces D_{t-1}, in the original state space $\mathcal{G}_t = \{z_t, G_t, K_t, B_{t-1}, D_t\}$.
2. Make an initial guess for the functions[11] $D_t = h^d(S_t)$ and $B_t = h^b(S_t)$. Denote these

[11]Recall that the policy functions are mappings from the state space to choice variables; this is why the policy

initial guess functions as h_0^d and h_0^b.

3. Plug in guesses $D_{t+1} = h_0^d(S_{t+1})$ and $B_t = h_0^b(S_t)$ and discrete grid values. There is only one K_{t+1} in the system, so this term must be the K_{t+1} evaluated as the guess. The variable B_t appears twice in the Euler equations and it is most convenient to make the value used to compute the fiscal limit the initial guess and solve for the B_t is the first-order condition with respect to bonds. Time iteration is applied to deposits while fixed point iteration is used to solve for bonds.

4. Use Gauss-Hermite quadrature to numerically approximate the expectation terms. The expectation of a function of one stochastic process z_t is approximated as

$$\mathbb{E}_t[F(D_{t+1}, z_{t+1})|z_t] \approx \frac{1}{\sqrt{\pi}} \sum_{j=1}^{n} \mathcal{W}_j F(D_{t+1}, \sqrt{2}\sigma_z \hat{z}_j + \mu_z)$$

Where σ_z is the standard deviation of the stochastic process, μ_z is the mean of the process, $\hat{z}_j$ is the j^{th} root of the n^{th} Hermite polynomial, and $\mathcal{W}_j$ is the j^{th} is the corresponding weight (Tauchen and Hussey, 1991). In the double integral case where one is approximating the expectation of a function depending on two stochastic processes z_t and g_t, one evaluates

$$\mathbb{E}_t[F(D_{t+1}, z_{t+1}, g_{t+1})|(z_t, g_t)] \approx \frac{1}{\pi} \sum_{i=1}^{m} \sum_{j=1}^{n} \mathcal{W}_j \mathcal{V}_i F(D_{t+1}, \sqrt{2}\sigma_z \hat{z}_j + \mu_z, \sqrt{2}\sigma_g \hat{g}_i + \mu_g)$$

Where $\mathcal{W}_j$ and $\mathcal{V}_i$ are the j^{th} and i^{th} weights of the n^{th} and m^{th} Hermite polynomial, corresponding to $\hat{z}_j$ and $\hat{g}_i$ (the j^{th} and i^{th} roots), respectively (Vladislav, 2004).

5. Plug in the approximated expectation(s) and solve the Euler equations for D_{t-1} and B_t.

6. Interpolate to find an updated policy function $D_t = h_1^d(S_t)$ where S_t contains the updated value of D_{t-1}. Evaluate the function at S_{t+1} to determine the updated $D_{t+1} = h_1^d(S_{t+1})$. Update the guess for as $B_t = h_1^b(S_t)$.

functions shown in these steps are defined on S_t. They are not functions on $\mathcal{G}_t$; $\mathcal{G}_t$ is the grid that must be predefined in the code used to implement this procedure.

7. Return to step 4 and repeat until $|h_{n+1}^d - h_n^d| < \epsilon$ and $|h_{n+1}^b - h_n^b| < \epsilon$ (where ϵ is the desired precision).

4.2 Parameter Calibration

Model parameters are calibrated to match several important moments from Spanish macroeconomic data. The Spanish data are quarterly (unless otherwise noted) and span 1999:1-2012:2.

Stochastic Parameters TFP is set equal to 1 in its steady state in order to keep the process stationary. Using GDP, average weekly hours worked, and a capital stock series[12], I estimate TFP as the Solow residual. I take its natural logarithm and apply the HP-filter to the series. Afterwards, I regress this series on its lags to calculate the persistence parameter as the regression coefficient and estimate the standard deviation of the noise in the TFP process to be the standard deviation of the regression residuals. The process parameters for government expenditure are found similarly, however the steady state value of government expenditure is chosen so that in the steady state the government expenditure-to-GDP ratio in the model data matches the empirical long-run average of this ratio for Spain.

Parameter	Economic Meaning	Value	Source
ρ_z	TFP persistence	0.928	Own estimate
σ_z	TFP noise standard deviation	0.0055	Own estimate
g	Steady state G/Y	0.19	Own estimate
ρ_g	G persistence	0.84	Own estimate
σ_g	G noise standard deviation	0.0074	Own estimate

Table 1: Model parameter estimates; calibrations marked with "TBD" are yet to be finalized.

Household Parameters The disutility of labor parameter is chosen so that the steady state value of hours worked matches the percent of weekly hours worked out of

[12] I estimated this series via the perpetual inventory method and used Spanish data on gross fixed capital formation and consumption of fixed capital.

the total number of hours per week[13]. The parameter governing the Frisch elasticity of labor is 3, implying a Frisch elasticity of 0.5. This value and the elasticity of substitution between consumption and leisure come from values often utilized in Real Business Cycle literature (exact sources are noted in the table below) and the discount factor is chosen so that the quarterly deposit rate is 0.0526.

Parameter	Economic Meaning	Value	Source
χ	Disutility of labor	6.95	Own estimate
$\frac{1}{\omega-1}$	Frisch elasticity	0.5	Boscá et al. (2009)
ϕ	Consupmption—leisure elasticity of subs.	2	Schmitt-Grohé and Uribe (2003)
β	Discount factor	0.9959	Quarterly deposit rate 0.0129
η	Household mass	0.75	TBD

Table 2: Model parameter estimates; calibrations marked with "TBD" are yet to be finalized.

Production Parameters The capital share of income is chosen so that $\alpha \in [0.2, 0.36]$ as recommended in Gollin (2002). The depreciation rate is found along with a quarterly capital stock series by employing the perpetual inventory method. The entrepreneurs and the bank are constrained in their ability to leverage their equity to borrow.

Parameter	Economic Meaning	Value	Source
α	Capital income share	0.3	Gollin (2002)
δ	Depreciation rate	0.0054	Own estimate
β_e	Entrepreneur discount factor	0.9479	TBD
γ	Entrepreneur borrowing constraint	0.9	TBD
r^f	Risk-free interest rate	0.0296 percent	Own estimate
ϱ^d	Elasticity of deposit rate to deposits	0.0018	Own estimate
ϱ^l	Elasticity of loan rate to loans	0.0538	Own estimate

Table 3: Model parameter estimates; calibrations marked with "TBD" are yet to be finalized.

Financial Sector Parameters Entrepreneurs discount at a higher rate than households, presently I employ a value approximately 0.01 lower than the discount factor of households. The leverage ratio associated with the calibrated borrowing constraints is 10.

[13]For the time period of interest, the steady state value of hours is 0.245

I will perform robustness checks to explore the sensitivity of the model to changes in the parameters which I have not yet calibrated before proceeding with further calibration. For the time being I have chosen parameters that are consistent with model theory (e.g., bankers and entrepreneurs leverage their equity at a rate greater than one). The first order conditions imply that increasing the leverage ratio will only raise the price that entrepreneurs would be willing to pay for risky debt but will not otherwise affect the default dynamics of the model.

I calculate the risk-free interest rate as the average quarterly interest of long-term German public debt from 2000:Q1 to 2012:Q3. The data I use to calculate the elasticity of deposits and loans are quarterly Spanish deposit and loan rates from 2003:Q1 to 2012:Q1.[14] From the steady state equations of the interest rate rules (see section C), I select elasticities to impose that in the steady state, the spread relative to the risk-free rate for the long-run average of deposit and loan rates is consistent with Spanish data.

Government Parameters The steady state values of transfers (T) made to the household, recapitalization ($\mathcal{R}$), and debt are chosen so that in the steady state the ratios to GDP match the long-run average of these ratios for Spain. Both datasets used for calibrating these measures are of annual frequency.[15]

Recapitalization data has only recently been cataloged for Eurozone countries by the European Commission for Spain since 2008. I combine this with European Commission data on government transfers (non-crisis-related) to the financial sector; these data are available only as early as 2005. While less than ideal, these are good proxies for recapitalization. The elasticity of recapitalization is currently chosen so that theoretical notion that recapitalization is countercyclical holds.

Similarly, the elasticity of the labor income tax with respect to GDP is chosen so that the distortionary tax increases when the government has to make a larger debt payment as compared to the steady level of debt repayment. I will perform sensitivity analysis with

[14] It would be ideal use have this data from 2000:Q1 to 2012:Q3, however it is not available.

[15] Since all of these series are flows and not stocks, each year's value approximates the average for its four quarters.

respect to both of these elasticities before investigating alternative calibration schemes. I The fiscal limit moments were generated from the MCMC simulations described in section 3.5 and the default percentage is a normalization of the of the variability of fiscal limit's distribution with respect to its mean.

Parameter	Economic Meaning	Value	Source
t	Steady state T/Y	0.126	Own estimate
$\bar{\mathcal{R}}$	Steady state $\mathcal{R}/Y$	0.0139	Own estimate
b	Steady state B/Y	0.512	Match B/Y
τ	Steady state taxes	0.512	Match B/Y
ν	Elasticity of recapitalization	-0.3	TBD
θ	Elasticity of taxes	0.5	TBD
$\mathbb{E}(\mathcal{S})$	Mean of steady state fiscal limit	0.8036	MCMC
$\sigma_{\mathcal{S}}$	SD of steady state fiscal limit	0.1236	MCMC
ζ	Default percentage	0.3077	$2\sigma_{\mathcal{S}}/\mathbb{E}(\mathcal{S})$

Table 4: Model parameter estimates; calibrations marked with "TBD" are yet to be finalized.

5 Analysis of Model Data

5.1 Impulse Response Functions

My modeling of the IRFs still has some numerical flaws on which I am currently working to correct[16]. However, if we cautiously interpret the IRFs they may suggest that the economic framework that I have developed is appropriate for understanding the Spanish debt and financial crises. If my model is close to being numerically correct as is, it appears that it does not replicate fact 4 (countercyclical interest rates and nonlinearity) completely, and does not replicate fact 2 (credit crunches) at all. My model successfully replicates fact 1 (coincidence of crises) and fact 3 (domestic costs).

To see if my model replicates the coincidence of crises (fact 1), one looks for if rising sovereign borrowing is associated with lower investment. It appears the sovereign borrowing is countercyclical and that investment is procyclical. When government borrowing

[16]In what follows, I mistakingly iterate the Euler equations while evaluating Δ_t as an expectation and not randomly drawing it. I am currently experimenting with ways to iterate the Euler equations while randomly sampling Δ_t from its distribution without sacrificing the convergence properties of time iteration.

initially falls in figure 14, investment rises. But when the sovereign begins to take advantage of low interest rates and increase its borrowing, though investment has already returned to the steady state it makes a small downward deviation. The model relationship between these variables matches what we empirically observe about the coincidence of financial and debt crises. My model suggests that reduced sovereign borrowing and default risk make it easier for the entrepreneur to amass capital and thus the investment is more profitable.

Fact 3 is also present in my model; we can see in figure 11 and 14 that though initially sovereign borrowing falls during good economic times (a positive TFP shock), it picks up soon as the price of borrowing falls. The sovereign becomes less likely to default, but as borrowing gets sufficiently cheap they find it in their interest to purchase more debt. This increases the probability of default about 10-20 quarters after the TFP shock and borrowing again begins to fall in an attempt to maintain the government budget constraint (see figure 14). When entrepreneurs possess additional bonds, from the temporary increase in the sovereign's borrowing, default probabilities rise. The subsequent wind-down in government borrowing leads to a drop in GDP just before 20 quarters after the shock (see figure 11). GDP growth doesn't resume until around the twentieth period when government borrowing declines.

Fact 2, credit crunches ensue during debt crises, does not appear in my simulation. In figure 13 it appears that lending contracts during a boom—the opposite of what we regularly witness in reality. However, I believe that this results from a numerical error in the current version of the model. It appears that loans converge to a lower steady and I believe that what we are seeing is in fact a boom in lending during positive TFP shocks that returns to its steady state. Rerunning of the model is necessary to confirm this. If this is the case, my model does in fact show that private borrowing contracts during periods when default is likely or takes place.

For fact 4, we can see that a nonlinear relationship appears between government bonds and their price, but we do not see countercyclical bond prices. A higher bond price is indicative of investors perceiving a bond as less risky, thus we observe and expect that during periods of lower default risk a sovereign's bond price will be higher. This is not the case according to figure 14. I suspect also in this case that the steady state value for bonds is not correct and that it is slightly lower. Thus it may be that bond prices are procyclical. We do see the negative correlation between debt levels and bond prices as well as a nonlinear relationship. Note that in figure 14 that sovereign borrowing and bond prices move in opposite directions in near perfect unison. Additionally, as debt levels fall, the rate at which bond prices change is lower.

The impulse response function for the expected amount of repudiated debt (figure 14)suggests a counterintuitive relationship, but theoretically consistent, relationship between TFP shocks and probabilities of default. Immediately after a positive TFP shock, the probability of default falls as the distribution of the fiscal limit shifts right while for the government is still paying back the steady state level of debt. But quickly investors bid up the price of sovereign debt. As sovereign debt is the entrepreneur's only choice variable that can act as collateral, they aim to increase in order to obtain more loans and take advantage of increased returns to renting capital following the TFP shock. Since entrepreneurs have no other asset to which they may turn, this puts substantial upward pressure on bond prices. This, in effect, makes borrowing *too easy* for the sovereign and they quickly increase their borrowing. While positive TFP shocks raise the mean of the fiscal limit, lowering the default risk, this can be offset by increased borrowing and it is possible for the risk of default to increase following a TFP shock.

6 Conclusion

In this paper I establish four empirical regularities about sovereign debt crises and their interaction with financial crises. These crises frequently coincide and both are able to propagate each other. Credit crunches often follow the onset of a debt crisis and, in countries where banks hold large amounts of domestic sovereign bonds, the damages to bank capital resulting from falling bond prices can worsen the domestic output costs of default. Sovereign bond yields also empirically display a nonlinear relationship with respect to debt levels.

I propose a model to explain the first three facts as a result of the collateral role of sovereign bonds. In my model, a TFP shock can lower output and thus the tax-base on which a sovereign relies to service its debt. When the bank gives capital injections to entrepreneurs, they make it even more difficult to service their debt after already losing tax revenue. This pushes the government closer to exceeding its endogenous fiscal limit and to ultimately defaulting on its debt. When default becomes more probable, they must sell their debt at lower prices (higher yields) in order for investors to accept the increased default risk. This degrades the collateral of banks leveraging sovereign bonds to obtain repo loan financing and thus lowers real investment, spurring further output declines. I solve the model globally with the intent to accurately capture the behavior of macroeconomic variables when they travel far from their steady state values and to replicate the nonlinear relationship between bond yields and debt levels.

I use MCMC to simulate a fiscal limit on borrowing for the sovereign with parameters calibrated for Spain. Currently, the results suggest that Spain's fiscal limit is very dispersive. In the steady state the limit ranges from roughly a 50-90 percent debt-to-GDP (annual) ratio and is centered around 70 percent. Negative TFP shocks lower this limit slightly on average, but a one standard deviation increase in government expenditure relocates the mean of the fiscal limit to approximately 60 percent debt-to-GDP. This suggests that if Spain cut its government expenditure, its risk of default would be significantly reduced as its ability to tolerate debt would rise substantially.

Future versions of this paper likely would succeed in replicating the contractions in credit usually following debt crises if financial intermediaries are incorporated and international investors can purchase sovereign bonds. If entrepreneurs have access to another collateralizable asset, such as deposits received by a financial intermediary, then they will diversify their portfolio of collateral. Entrepreneurs will not rely so heavily on sovereign bonds and thus will not be forced to hold all of the sovereign's debt. This makes it more likely that TFP shocks will raise bond prices and not the number of bonds issued. Further, it would be ideal to conduct an analysis of the effects of fiscal policy such as changes in government expenditure, recapitalization, or taxation. Thus this model has the potential to address questions about what policies would have mitigated the crises at their onset and can compare the effects of policy changes on the status quo as well. This makes my model a useful tool for exploring policy counterfactuals, able to lend insight into the current debate surrounding austerity.

Future studies may augment this work by incorporating money and potentially a zero lower bound to explore the consequences of the monetization of debt in countries such as Japan. To apply this model to developed countries, it may be useful to augment this model further with bonds issued in various currencies and allow foreign exchange rates to influence debt and default decisions. Another worthwhile addition would be to study the effects of bailouts, such as those provided through the EU, on stabilizing debt and financial crises.

I solve the full nonlinear model and demonstrate that the risk of default has a non-trivial influence on an economy's domestic financial markets. My model supports that the mechanisms described here may govern the dynamics of macroeconomic aggregates during debt and financial crises in cases such as Spain. Although my model is not perfected yet, it illustrates that debt and financial crises tend to coincide, the domestic costs on an economy in terms of GDP are greater when domestically more bonds are held and there is default risk, and bonds and their prices have a nonlinear relationship. My model does not yet successfully replicate the empirical observation that credit contractions often follow a debt crisis. But I am optimistic that this model, with further revision, is capable of illustrating this.

The most recently updated version of this paper is available at:
https://sites.google.com/a/macalester.edu/aindarte/.

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Appendix

A Fiscal Limit

The dynamic, stochastic fiscal limit is defined as:

$$\begin{aligned}\mathcal{S}_t &= \mathbb{E}_t \sum_{j=0}^{\infty} \beta^{t+j} (\mathcal{T}_{t+j}^{\max} - G_{t+j} - \mathcal{R}_{t+j} - T) \\ &= \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j (\tau^{\max} w_{t+j} H_{t+j} - G_{t+j} - \mathcal{R}_{t+j} - T) \\ &= \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j [Y_{t+j} \tau^{\max}(1-\alpha) - G_{t+j} - \mathcal{R} z_{t+j}^{\nu} - T].\end{aligned}$$

The first order condition for the household can be rearranged as:

$$H_t = \eta^{\frac{\omega-1}{\omega-1+\alpha}} [(z_t(1-\alpha)K_t^{\alpha}(1-\tau^{\max})\chi^{-1}]^{1/(\omega-1+\alpha)}$$

and it follows that

$$\begin{aligned}Y_{t+j} &= z_{t+j} K_{t+j}^{\alpha} H_{t+j}^{1-\alpha} \\ & \quad z_{t+j} K_{t+j}^{\alpha} [\eta^{\omega-1} z_{t+j}(1-\alpha) K_{t+j}^{\alpha} (1-\tau^{\max})\chi^{-1}]^{[(1-\alpha)/(\omega-1+\alpha)]} \\ & \quad z_{t+j}^{\frac{\omega}{\omega-1+\alpha}} K_{t+j}^{\frac{\alpha\omega}{\omega-1+\alpha}} [\eta^{\omega-1}(1-\alpha)\omega^{-1}\chi^{-1}]^{(1-\alpha)/(\omega-1+\alpha)}.\end{aligned}$$

Thus for $j = 0$, the surplus depends only on $\{z_t, K_t, G_t\}$. Before presenting the rewritten sum for $j > 1$, note that the firm's first order condition with respect to capital can be rewritten as

$$K_t = (z_t \alpha / r_t^k)^{1/(1-\alpha)} H_t.$$

This holds for any arbitrary time $t + j$. Additionally, recall the entrepreneur's first order condition with respect to capital implies that today's expectation of next period's rental rate is $\mathbb{E}_t r_{t+1}^k = 1/\beta_e - (1-\delta)$. By the law of iterated expectations, the current expectation of the rental rate in any period $t+j$ is also $\mathbb{E}_t r_{t+j}^k = 1/\beta_e - (1-\delta)$. Therefore

$$\mathbb{E}_t K_{t+j} = \mathbb{E}_t (z_{t+j}\alpha / r_{t+j}^k)^{1/(1-\alpha)} H_{t+j} = \mathbb{E}_t \{z_{t+j}\alpha / [1/\beta_e - (1-\delta)]\}^{1/(1-\alpha)} H_{t+j}.$$

For an arbitrary period $t + j$, the household's first order condition with respect to hours is

$$H_{t+j} = \eta^{\frac{\omega-1}{\omega-1+\alpha}} \{z_{t+j}(1-\alpha) K_{t+j}^{\alpha} [1 - (\omega - 1)/\omega] \chi^{-1}\}^{1/(\omega-1+\alpha)}.$$

Note that above I have substituted $\tau^{\max} = \frac{\omega-1}{\omega}$. Plugging the above expression in for H_{t+j} to that of $\mathbb{E}_t K_{t+j}$ yields

$$\mathbb{E}_t K_{t+j} = \mathbb{E}_t \eta \left[\frac{\alpha z_{t+j}}{1/\beta_e - 1 + \delta}\right]^{\frac{\omega-1+\alpha}{(1-\alpha)(\omega-1)}} \left[\frac{z_{t+j}(1-\alpha)}{\omega\chi}\right]^{\frac{1}{\omega-1}}$$

Then for $j > 1$

$$\begin{aligned}
\mathbb{E}_t Y_{t+j} &= \mathbb{E}_t z_{t+j}^{\frac{\omega}{\omega-1+\alpha}} K_{t+j}^{\frac{\alpha\omega}{\omega-1+\alpha}} [\eta^{\omega-1}(1-\alpha)\omega^{-1}\chi^{-1}]^{[(1-\alpha)/(\omega-1+\alpha)]} \\
&= \mathbb{E}_t z_{t+j}^{\frac{\omega}{\omega-1+\alpha}} \eta^{\frac{\alpha\omega}{\omega-1+\alpha}} \left[\frac{\alpha z_{t+j}}{1/\beta_e - 1 + \delta}\right]^{\frac{\alpha\omega}{(1-\alpha)(\omega-1)}} \left[\frac{z_{t+j}(1-\alpha)}{\omega\chi}\right]^{\frac{\alpha\omega}{(\omega-1)(\omega-1+\alpha)}} \\
&\quad \times [\eta^{\omega-1}(1-\alpha)\omega^{-1}\chi^{-1}]^{[(1-\alpha)/(\omega-1+\alpha)]} \\
&= E_t z_{t+j}^{\frac{\omega}{(1-\alpha)(\omega-1)}} \Omega_z, \text{ where } \Omega_z = \eta^{\frac{\alpha\omega}{\omega-1+\alpha}} \left(\frac{\alpha}{1/\beta_e - 1 + \delta}\right)^{\frac{\alpha\omega}{(1-\alpha)(\omega-1)}} \left(\frac{1-\alpha}{\omega\chi}\right)^{\frac{1}{\omega-1}}
\end{aligned}$$

The discounted sum of fiscal surpluses is a function of solely of state variables $\{z_t, G_t, K_t\}$. It can be written as

$$\mathcal{S}_t = Y_t[(\omega-1)/\omega](1-\alpha) - G_t - \mathcal{R}z_t^{\nu} - T + \mathbb{E}_t \sum_{j=1}^{\infty} \beta_e^j [Y_{t+j}(\omega-1)/\omega(1-\alpha) - G_{t+j} - \mathcal{R}z_{t+j}^{\nu} - T].$$

B Competitive Equilibrium Characterizations

1. The households' intratemporal and intertemporal conditions are satisfied:

$$\begin{aligned}
H_t &= \eta \left[\frac{w_t(1-\tau_t)}{\chi}\right]^{1/(\omega-1)} \\
\left(\eta^{\omega-1} C_t^h - \chi H_t^{\omega}/\omega\right)^{-\phi} &= \beta \mathbb{E}_t (1 + r_t^d) \left(\eta^{\omega-1} C_{t+1}^h - \chi H_{t+1}^{\omega}/\omega\right)^{-\phi}
\end{aligned}$$

2. The firm optimally chooses labor and capital while taking their prices as exogenous:

$$\begin{aligned}
w_t &= z_t(1-\alpha)(K_t/H_t)^{\alpha} \\
r_t^k &= z_t \alpha (H_t/K_t)^{1-\alpha}
\end{aligned}$$

3. Government policy is determined by:

$$\begin{aligned}
\ln\left(\frac{\mathcal{R}_t}{\mathcal{R}}\right) &= \nu \ln\left(\frac{z_t}{z}\right) \\
\tau_t - \tau &= \theta(B_t^d - B^d) \\
B_t^d &= (1 - \Delta_t) B_{t-1} \\
\mathcal{S}_t &= \mathbb{E}_t \sum_{j=0}^{\infty} \beta_e^{t+j} (\tau^{\max} w_{t+j} H_{t+j} - G_{t+j} - \mathcal{R}_{t+j} - T) \\
\Delta_t &= \begin{cases} 0 & : B_{t-1} \leq \mathcal{S}_t^* \\ \zeta & : B_{t-1} > \mathcal{S}_t^* \end{cases}
\end{aligned}$$

4. The entrepreneurs choose risk-neutral portfolios of loans, capital, and bonds such that:

$$\begin{aligned}
q_t &= \frac{\beta_e \mathbb{E}_t(1-\Delta_{t+1})}{1-\gamma[r_t^k+(1-\delta)-\beta_e(1+r_t^l)]} \\
\mathbb{E}_t r_{t+1}^k &= 1/\beta_e-(1-\delta) \\
L_t &= \gamma(q_t B_t+\mathcal{R}_t) \\
X_t &= A_{t+1}-(1-\delta)A_t \\
K_t &= A_t+L_t
\end{aligned}$$

5. Deposit and loan interest rates follow

$$\begin{aligned}
r_t^d &= r^f-\varrho^d D_t \\
r_t^l &= r^f+\varrho^l L_t
\end{aligned}$$

6. Household, firm, entrepreneur, bank, government and aggregate resource constraints are satisfied:

$$\begin{aligned}
C_t^h &= w_t(1-\tau_t)H_t-D_t+(1+r_{t-1}^d)D_{t-1}+T \\
Y_t &= z_t K_t^\alpha H_t^{1-\alpha} \\
C_t^e &= -X_t-L_{t-1}(1+r_{t-1}^l)-q_t B_t \\
&\quad +r_t^k(A_t+L_t)+(1-\delta)L_t+B_t^d+\mathcal{R}_t \\
B_t^d+G_t+\mathcal{R}_t+T &= \tau_t w_t H_t+q_t B_t \\
Y_t &= C_t+X_t+G_t-(1-\delta)L_t \\
&\quad +D_t-D_{t-1}(1+r_{t-1}^d)-L_t+L_{t-1}+(1+r_{t-1}^l)
\end{aligned}$$

C Steady State Conditions

In the steady state[17] values for the preceding equilibrium conditions are:

$$
\begin{aligned}
H &= \eta \left[\frac{w(1-\tau)}{\chi} \right]^{1/(\omega-1)} && (22)\\
r^d &= 1/\beta - 1 = r^f - \varrho^d D && (23)\\
w &= (1-\alpha)(K/H)^{\alpha} && (24)\\
r^k &= \alpha(H/K)^{1-\alpha} && (25)\\
B &= B^d && (26)\\
\mathcal{S} &= \mathcal{S}(z, g, K) && (27)\\
\Delta &= 0 && (28)\\
q &= \frac{\beta_e}{1-\gamma[r^k + (1-\delta) - \beta_e(1+r^l)]} && (29)\\
r^k &= 1/\beta_e - (1-\delta) && (30)\\
L &= \gamma(qB + \mathcal{R}) && (31)\\
X &= \delta A && (32)\\
K &= A + L && (33)\\
r^l &= 1/\beta_e - 1 = r^f + \varrho^l L && (34)\\
C^h &= w(1-\tau)H + r^d D + T && (35)\\
Y &= K^{\alpha} H^{1-\alpha} && (36)\\
C^e &= -X - r^l L - qB + r^k K + B^d + \mathcal{R} - \delta L && (37)\\
\tau &= \frac{g + t + b(1-q) + \bar{\mathcal{R}}}{1-\alpha} && (38)\\
C &= C^h + C^e && (39)\\
Y &= C + \delta K + G && (40)
\end{aligned}
$$

Closed-Form Steady States

$$r^k = 1/\beta_e - (1-\delta)$$

Note that I choose r^l and r^d to, in the steady state, be consistent with average quarterly Spanish loan and deposit rates. I also select discount parameters such that $r^l = 1/\beta_e - 1$ and $r^d = 1/\beta - 1$ in the steady state. Thus these values are known and I may find the following closed-form representations for other state

[17]Steady state values are the dynamic variables shown no longer with a time subscript (e.g., C is the steady state value of C_t)

variables:

$$
\begin{aligned}
q &= \frac{\beta_e}{1-\gamma[r^k+(1-\delta)-\beta_e(1+r^l)]} \\
\tau &= \frac{g+t+b(1-q)+\bar{\mathcal{R}}}{1-\alpha}
\end{aligned}
$$

Define $\Omega_Y = r^k\alpha$. Then:

$$
\begin{aligned}
K &= \eta\left[\frac{(1-\alpha)(1-\tau)}{\chi}\right]^{\frac{1}{\omega-1}}\Omega_Y^{\frac{1-\omega-\alpha}{(1-\alpha)(\omega-1)}} \\
Y &= \Omega_Y K \\
B &= bY, \quad T = tY, \quad G = gY \quad , \mathcal{R} = \bar{\mathcal{R}}Y \\
H &= \eta^{\frac{\omega-1}{\omega}}\left[\frac{(1-\alpha)(1-\tau)Y}{\chi}\right]^{\frac{1}{\omega}} \\
w &= (1-\alpha)Y/H \\
L &= \gamma(qB+\mathcal{R}) \\
A &= K-L \\
X &= \delta A \\
C^e &= -X - r^lL - qB + r^kK + B + \mathcal{R} - \delta L \\
C^h &= Y - C^e - \delta K - G \\
D &= \frac{w(1-\tau)H + C^h + T}{r^d}
\end{aligned}
$$

The parameters governing the deposit and loan elasticities are implied by the steady state values:

$$
\begin{aligned}
r^d &= r^f - \varrho^d D \\
r^l &= r^f + \varrho^l L
\end{aligned}
$$

D Impulse Response Functions

D.1 TFP Shock

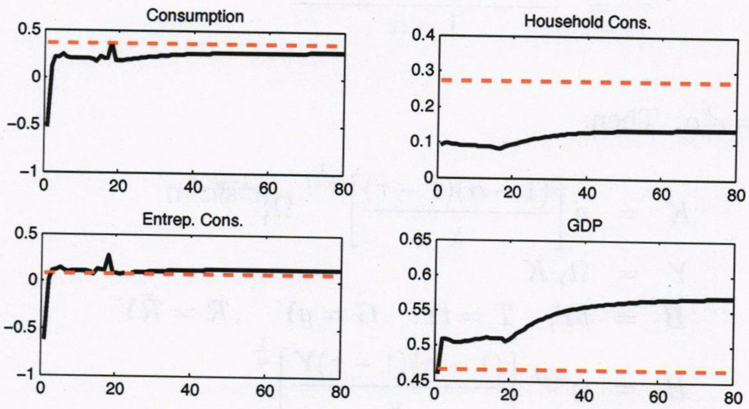

Figure 11: Level deviations in response to a one standard deviation shock to TFP.

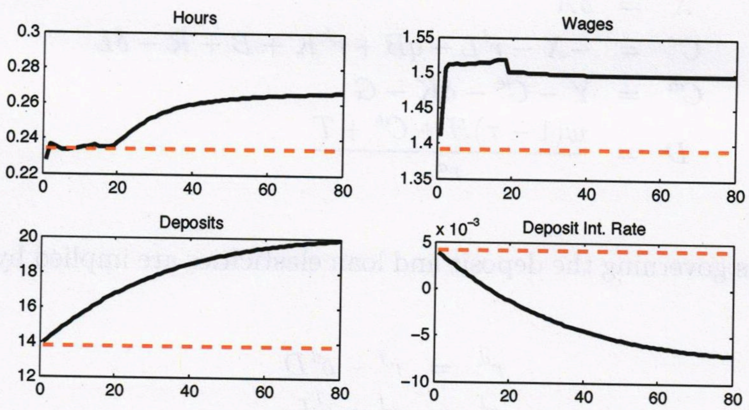

Figure 12: Level deviations in response to a one standard deviation shock to TFP.

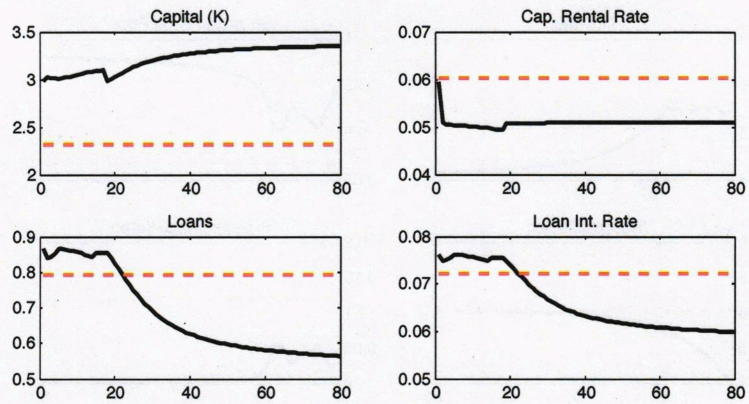

Figure 13: Level deviations in response to a one standard deviation shock to TFP.

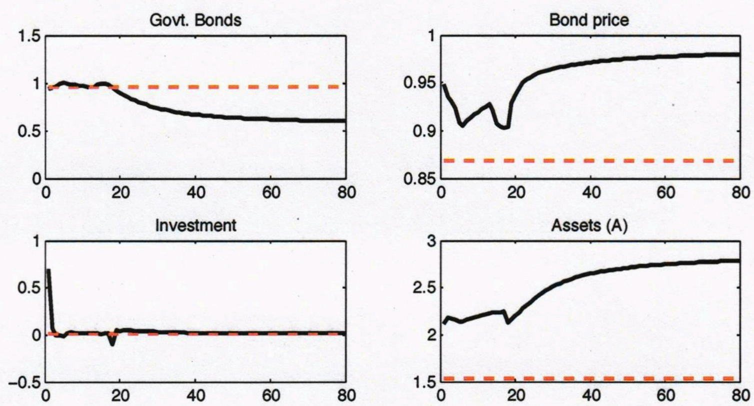

Figure 14: Level deviations in response to a one standard deviation shock to TFP.

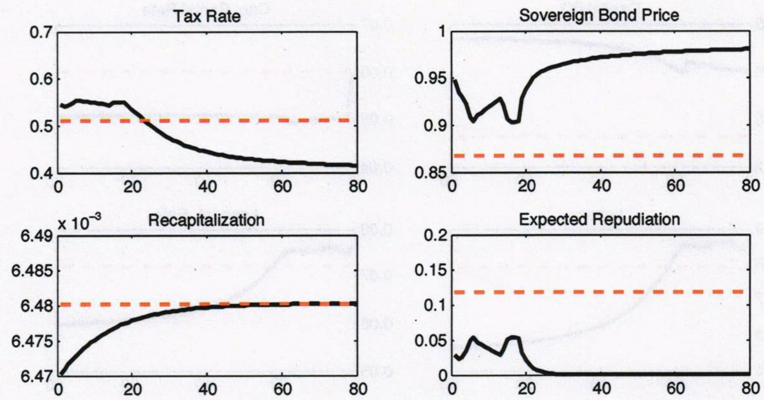

Figure 15: Level deviations in response to a one standard deviation shock to TFP.