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An Evaluation of Credit Default Swaps and Default Risk Using Barrier Options

by

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May 2007

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An Evaluation of Credit Default Swaps and Default Risk Using Barrier Options

Kevin Lam
Finance 495-Spring 2007
Advisor: Dr. John Teall

ABSTRACT

Credit default swaps, a traded financial instrument that provides credit protection in exchange for a periodic premium, is at the forefront of the exponential growth in the credit derivatives market, which has revolutionized the way credit risk is managed in recent years. This project offers a review into the application of option pricing theory in the valuation of default risk under a plain vanilla analysis and introduces a theoretical model that uses barrier options as a potential and perhaps more accurate tool for assessing default risk and its implications for valuing credit default swaps.

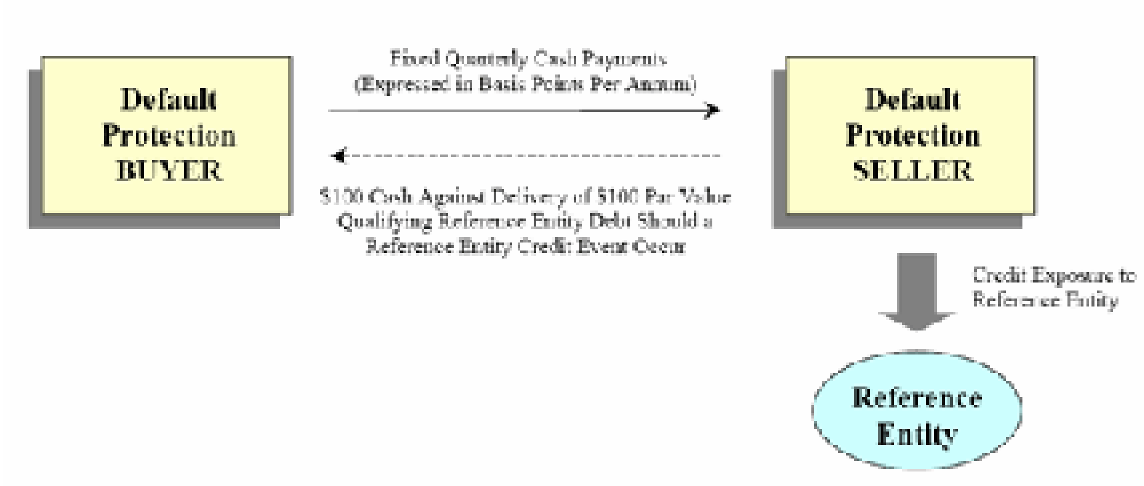
CHAPTER 1

INTRODUCTION

1. A. WHAT IS A CREDIT DEFAULT SWAP?

A Credit Default Swap (CDS) is a traded financial instrument that provides protection against credit risk in exchange for periodic premium payments. It is analogous to an insurance contract between two parties, in which the insurance buyer pays a premium in exchange for loss payments. The company or entity referenced in the CDS is referred to as the reference entity, and a default by the reference entity is known as a credit event. In the case of a CDS, the protection buyer makes periodic payment to the protection seller in exchange for the right to sell a particular bond issued by the reference entity for face value upon the occurrence of a credit event. The bond is referred to as the reference obligation and the total face value of the bond that is specified in the CDS is known as the swap's notional principal.

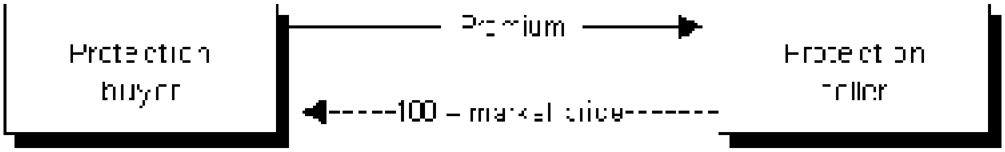
An example will help illustrate the mechanics of a CDS. Suppose the protection buyer enters into a 5-year CDS paying an annual premium of 300 basis points. The swap's notional principal on which protection is purchased is $100 million. Thus, assuming there is no default, the protection buyer will then make annual payments of $3 million (0.03 x $100 million) for the next 5 years. However, in the scenario that a credit event is triggered due to a default by the referenced entity and that the recovery rate is 30%, the protection seller will have to make a loss payment to the protection buyer in the amount of $70 million ((1-30%) x $100 million). Thus, the protection buyer is part of the fixed leg of the swap and the protection seller is part of the floating or contingent leg of the swap. The diagram below is an example of a typical CDS cash flow structure.



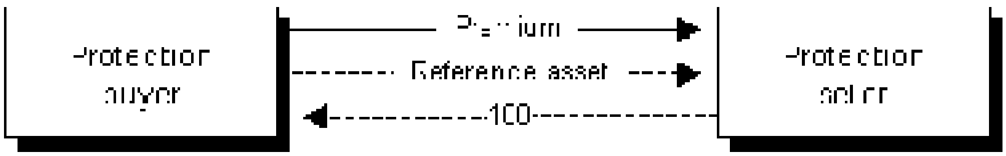
1. B. SETTLMENT PROCEDURES FOR CDS

After the triggering of a credit event, there are two widely used settlement procedures to determine the loss payments on the floating or contingent leg of the swap. The first option is physical settlement and the second option is called cash settlement. In the case of a physical settlement, the protection buyer delivers the defaulted bond or any deliverable obligations of the reference entity as specified in the CDS in exchange for the total face value of the bond. In a cash settlement, a dealer poll is usually conducted to determine the recovery rate or market value of the asset immediately after default, the loss payment by the protection seller is then determined by subtracting the market value of the bond from the total face value of the bond. This value is known as the loss given default (LGD) and is calculated as total face value times (1-R), where R is the recovery rate.

Example of Cash Settlement



Example of Physical Settlement



1. C. ISDA AND ITS ROLE IN THE CREDIT DERIVATIVES MARKET

Most CDS transaction are governed and executed based on a legal framework provided by the International Swap and Derivative Association (ISDA). Documentations evidencing CDS transactions are based on confirmation document and legal definitions set forth by ISDA. In May 2003, the 2003 ISDA Credit Derivatives Definitions took effect, expanding and revising the 1999 Definitions and Supplements. The Definitions provided a basic framework for documentation, but precise documentation remains the responsibility of the parties involved, as a CDS is a bilateral contract.[1]

These Definitions were developed based on case histories of contractual disputes and defaults in the past. Standard Credit Event languages are provided as part of the ISDA Definitions. Credit event languages are critical in the CDS contract because it determines the event or condition under which a loss payment will be due from the protection seller. In the US corporate market, where the reference entity is a corporation, the most common credit events are Bankruptcy, Failure to Pay, and Restructuring

[1] Glen Tasker, Credit Default Swap Primer, 2nd Edition (Bank of America, 2006) 7-8

(defined in table below). Additionally, if the reference entity is a sovereign, credit events might include Repudiation/Moratorium or even Obligation Acceleration if the reference entity is an emerging market. In the instance that no pre-specified credit event occurs during the life of the transaction, the protection seller receives the periodic premium payment in compensation for assuming the credit risk on the reference entity. Conversely, if a credit event occurs during the life of the transaction, the protection buyer receives a form of loss payment depending on the settlement procedure of the contract (cash/physical delivery). The protection seller receives only the accrued periodic payment up to and including the Event Determination Date (effectively the date a credit event occurs).[2]

Standard Credit Events and its definitions are listed in the table below:[3]

Bankruptcy	A corporation's insolvency or inability to pay its debts. Not relevant to sovereign issuers. 2003 Definitions explicitly require the inability to pay debts to be part of a judicial, regulatory or administrative proceeding.
Failure to Pay	A reference entity's failure to make due payments in the form of interest or principal. Failure to Pay takes into account any applicable grace period and usually sets a minimum dollar threshold of $1 million.
Restructuring	A change in the debt obligation's terms that is adverse to creditors, such as reduction in interest rate or loan principal, or extension of final maturity. The US investment grade market uses Modified Restructuring. The US high yield market sometimes uses No Restructuring (i.e., Restructuring does not constitute a credit event).
Obligation Acceleration	When an obligation has become due and payable earlier than normal because of a reference entity's default or similar condition. Obligation Acceleration is subject to a minimum dollar threshold amount.
Repudiation/Moratorium	A reference entity's rejection or challenge of the validity of its obligations.

[2] Glen Tasker, Credit Default Swap Primer, 2nd Edition (Bank of America, 2006) 7-8)
[3] Ibid. 13-14

1. D. MARKET DEVELOPMENT OF CDS

Standard documentation produced by the International Swaps and Derivatives Association (ISDA) for trading CDS in 1998 preceded the rapid growth of the CDS market and facilitated heavy trading volume. The participants in the CDS market include hedge funds, insurance companies, and financial institution acting as both protection sellers and protection buyers. Aside from managing credit exposure, the main motivation for participating in the CDS market is to the opportunity to employ significantly higher leverage and thus achieve higher yield relative to other markets. CDS are customized products with any maturity but are mainly traded in the in the Over the Counter market with 5-year maturity. According to ISDA, the total notional value of debt referenced by CDS grew from $630 million in 2001 to over $12 billion in 2005. The most liquid CDS contracts are usually written on reference entities with credit ratings that are low investment grade.[4]

Most of the growth in the market has been concentrated in the single-name CDS, but other areas, such as the CDX credit index market, and other structured credit products, such as synthetic collateralized debt obligations (SCDO) have grown tremendously. The size of the single-name CDS market is estimated to be at $7.7 trillion for 2006, compared with $1.8 trillion in 2003. The SCDO market grew to $3.9 trillion in 2006, from $639 billion in 2003, and the index market grew to $2.2 trillion in 2006, from $319 billion in 2003.[5]

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[4] Glen Tasker, Credit Default Swap Primer, 2nd Edition (Bank of America, 2006) 7-8
[5] Ibid.

1. E. OBJECTIVES OF THE STUDY

Despite the exponential growth of the CDS market, there is no standardized approach in its valuation. A key parameter in the valuation of CDS is the default risk or default probability associated with the reference entity on the contract. Thus, the lack of a universal model to estimate default risk gives rise to the difficulties in valuing credit default swaps.

The purpose of this research paper is to provide a brief overview of the theoretical literature in credit risk, with particular emphasis on the structural model of credit risk introduced by Robert Merton in the 1970's that utilizes option pricing theory to evaluate default risk under a "plain vanilla" or simplified analysis. Building from Merton's framework, I then introduce a theoretical model that relaxes some of the assumptions or limitations inherent in the Merton model. The organization of this paper is divided into two main components. The first part of the paper offers a review into the plain vanilla analysis of default risk and its implications in valuing CDS. The second part of the paper introduces the theoretical model and its applications in assessing default risk and valuing CDS.

CHAPTER 2

LITERATURE REVIEW

The purpose of this section of the paper is to offer a review into the literature of default risk and credit default swaps. Before the introduction of credit default swaps, the study of default risk and credit risky instruments, such as corporate securities have received tremendous attention in the literature. The development of the theoretical literature in credit risk has indicated that the modeling of credit risk is divided into many forms. Among them was the structural form, which was introduced by Robert Merton in the 1970's. Structural models are based on a theory of option pricing to derive a mathematical probability that a firm will default on its debt. The concept of the model is based on the view that the capital structure of the firm can be viewed as option positions on the assets of the firm. For example, this approach recognizes that the limited liability equity of a firm can be modeled as a long position on a call option on the firm's assets. The call option will be exercised if the assets of the firm exceed the exercise price, which is the face value of the firm's debt. Similarly, the firm experiences a default when the assets of the firm drop below the exercise price, in which case, the call option will expire worthless and the bondholders of the firm are entitled to the remaining assets of the firm. Thus, we can estimate the default probability as the probability that the call option will expire out of the money.

Furthermore, the model became a major focus of future studies. In light of the Basel II accord, Jacob and Gupta (2004) presented a comparison between the standardized approach and Internal Rating Based (IRB) approach in estimating risk capital requirements. Under the standardized approach, Jacob and Gupta noted that default probability estimation and, thus risk capital requirements were based on credit rating assigned by external rating agency. On the other hand, IRB approach focused mainly on using some form of credit risk models to estimate default probability, which is then used to calculate the risk requirement of banks. Using a sample of Indian firms, Jacob and Gupta presented an application of the IRB approach using option-pricing theory introduced by Black and Scholes (1973) and the Merton (1974) framework to estimate default probabilities.

An alternative to the structural approach using option-based information is the traditional approach, which uses historical accounting information to estimate default risk. In 2006, Papanastasopoulos suggested a hybrid approach to default risk modeling that combines accounting based information used in the traditional approach with option based information used in the structural approach. The purpose of this hybrid approach, as illustrated by Papanastasopoulos in a binary probit regression model, will serve to overcome some of the shortcomings evident in the traditional and structural approach. The results of this study indicated that although market information as used in an options approach can be extremely useful in estimating default probability of listed firms, its estimation and predictive power will be greatly enhanced when combined with accounting based measures in a hybrid model.

Although there are many different forms of credit risk models, most of them can arguably be divided into either the structural form (uses application of option pricing theory) or the reduced form. Under the reduced form approach, default probabilities are estimated using the market prices of risky bonds. This approach is based on the assumption that a risky bond can be broken down into a risky component and a risk-free component. The price of a risk-free bond is the present value of the bond's certain or risk-free future cash flows. On the other hand, the price of a risky bond is the present value of its uncertain future cash flows, which reflects default probability of the issuer and an assumed recovery rate in the event of default. Thus, a reduced form model derives an "implied" default probability from the price difference between a risky bond and an equivalent risk-free bond. In practice, the reduced form model is a lot easier to implement than the structural model, default probabilities can be readily estimated using either bond prices or spread data gathered from the growing CDS market. Despite the advantages of the reduced form model, there are also disadvantages. It is important to recognize that default probabilities from reduced form models are highly dependent upon the assumed recovery rate, which is also difficult to assess in practice. Additionally, the model might run into a problem when market prices of debt are not readily available, such as in the case of distressed credits.

Arora, Bohn, and Zhu (2005) presented an analysis between the structural form and the reduced form models of credit risk. They offered a brief literature review into the original structural model based on the basic Merton framework, and the various extensions provided to the model. Their research is based on an empirical analysis of two structural models, (basic Merton and Vasicek-Kealhofer (VK) models) and one reduced form model (Hull and White (HW)) of credit risk. The VK model is a more sophisticated extension of Merton's model. Their research highlighted the relative value of the structural models versus the reduced form models based on their ability to accurately predict CDS spread. Their study also explored the ability of the models to discriminate defaulter and non-defaulters. The conclusion of their study indicated that the HW model (reduced form) was more effective in explaining CDS spread than the basic Merton model when a given firm issues a large number of bonds. However, the VK model (sophisticated structural model), for the most part, outperformed both the HW model and the basic Merton model in discriminating defaulters from non-defaulters and in explaining CDS spreads. Overall, the conclusion of the study suggested that a simple structural model is not enough, appropriate modifications to the framework can produce significant value. On the other hand, the effectiveness of reduced form models depends to a great extent on the quality and quantity of data available, which means many issuers, will not be modeled well unless they have enough traded debt outstanding.

The exponential growth of the credit derivatives market in recent years, particularly in credit default swaps have greatly expanded the amount of literature on this subject area. For example, the increasing liquidity in the CDS market has allowed empirical studies into the relationship between CDS spreads and credit risk. Das and Hanouna (2006) suggested that CDS spreads are better indicator of credit risk than bond spreads . In 2004, Hull, Predescu, and White examined the effects of credit rating announcements on CDS spreads, and the ability of CDS spreads to anticipate credit rating announcements. Firstly, they concluded that credit rating announcement, such as review for downgrades contained significant information that produced changes in CDS spreads but announcements, such as negative outlook and downgrades did not. Secondly, they concluded that there is anticipation of all three types of credit announcement by the credit default swap market.

The purpose of the model presented on this paper builds on the theoretical literature conducted on default risk, particularly on addressing the shortcomings of the basic structural model and offers a theoretical argument on valuing credit default swaps. As a motivation for the theoretical model presented in this paper, prior research in the literature directly relating to its applicability was introduced by Ericsson and Reneby (1998) and Skinner and Townend (2002). By relating to the Black and Scholes (1973) and Merton (1974) framework, Townsend and Skinner (2002) showed how credit default swaps appeals to put-call parity and how it can be viewed as a put option. Thus, using linear regression containing five variables essential to option pricing theory, Townsend and Skinner concluded that at least 3 or 4 of these variables are also important in pricing credit default swaps. Namely, the statistically significant variables are: risk-free rate, yield on the reference bond, maturity and volatility.

Furthermore, Ericsson and Reneby (1998) illustrated how certain ideas of barrier contracts can be applied in the valuation of corporate securities. Following certain assumptions, Ericssion and Reneby showed how corporate securities can be valued as a portfolio of a barrier options by replicating payoffs of corporate securities with combinations of the three building blocks. According to Ericsson and Reneby, the firm can default if the asset value of the firm drops below a certain level (reorganization barrier) or if the value of the firm is below the face value of debt at debt maturity. Ericsson and Reneby suggested different economic and judicial ways to determine and interpret the reorganization barrier, usually related to the value of outstanding debt, and liquidity of the firm. Furthermore, Ericsson and Reneby suggested various payoffs for both creditors and equity holders in the event of default based on the remaining assets of the firm. Thus, the theoretical model presented in this paper combines the ideas introduced by Skinner and Townend (2002) and Ericssion and Reneby (1998) and offers a different perspective into the valuation of default risk and credit default swaps.

CHAPTER 3

PLAIN VANILLA VALUATION MODEL APPLICATIONS

3. A. EUROPEAN PUT AND CALL OPTIONS

A European call option is a financial instrument that gives the holder the right but not the obligation to purchase a particular asset at a predetermined price (exercise price) at a predetermined time in the future (maturity). For example, imagine a European call option on a company's stock with a strike price of $30 and a maturity date of 5 years. Therefore, if you own or long the call option, you have the right but not the obligation to purchase that company's stock 5 years from now for a price of $30. Similarly, a European put option gives the holder the right but not the obligation to sell a particular asset at a predetermined price (exercise price) at a predetermined time in the future (maturity).

3. B. BLACK-SCHOLES OPTIONS PRICING MODEL

One of the most widely used option pricing model is the Black-Scholes Options Pricing Model, developed in the 1970's. The model prices options as a function of five key inputs:

1. Stock price (S_0)
2. Risk-free return rate (r_f)
3. Variance of the stock return
4. Time to maturity of the contract (T)
5. The exercise price of the option: (X)

The derivation of the model is based on the following assumptions:

1. There exist no restrictions on short sales of stock or writing of call options.
2. There are no taxes or transactions costs.
3. There exists continuous trading of stocks and options.
4. There exists a constant riskless interest rate that applies for both borrowing and lending.
5. Stock prices are continuous.
6. The underlying stock will pay no dividends during the life of the option.
7. The option can is a European Option that can only be exercise on its maturity date.
8. Shares of stock and option contracts are infinitely divisible.
9. Stock prices are assumed to be normally distributed and can take on any positive value at any time.[6]

Thus, if the assumptions hold, the model states the price of an option can be determined using the following equation and the five inputs described earlier.

$$C_0 = S_0 N(d_1) - Xe^{-rt} N(d_2)$$

$$P_0 = Xe^{-rt} N(-d_2) - S_0 N(-d_1)$$

$$d_1 = \frac{\ln(S_0 / X) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S_0 / X) + (r - \sigma^2 / 2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

3. C. PUT-CALL PARITY

The put-call parity is a simple relationship between the call and put values that must hold to avoid the occurrence of arbitrage opportunities. This relationship allows us to determine the price of a call given the price of a put written on the same underlying with the exact same terms, and vice versa. If we assume that there exist a call option (with a current price of C_0) and a put option (with a current price of P_0) written on the

[6] John L. Teall, FIN 396Q Coursepack (Unpublished:), 32-33

same underlying with the same exercise price (X) and maturity date of (T), the put-call parity formula states that the following equation must hold in order to prevent arbitrage:

$$C_0 + Xe^{-rT} = P_0 + S_0$$

In other words, the relationship states that a portfolio consisting of one call with an exercise price of X and a risk-free zero-coupon note with a face value equal to the exercise price (X) must be equal to the second portfolio consisting of a put written with the same exercise price and a share of the stock underlying both the call and put options. Therefore, by manipulating the formula, we can calculate the price of a call if we are given the price of a put with the exact same terms:[7]

$$C_0 = P_0 + S_0 - Xe^{-rT}$$

Similarly, we can infer the price of a put if we are given the price of a call with the exact same terms:

$$P_0 = C_0 - S_0 + Xe^{-rT}$$

3. D. APPLICATION OF OPTION PRICING THEORY ON CORPORATE SECURITIES UNDER A PLAIN VANILLA B-S ANALYSIS

In the early 1970's, Fischer Black, Myron Scholes, and Robert Merton were the pioneers of option pricing and they showed that options can be used to characterize the capital structure of the firm (known as the Merton framework). By using options to characterize the capital structure of the company, the Merton framework showed that option pricing theory can be used to value corporate securities, such as limited liability

[7] John L. Teall, FIN 396Q Coursepack (Unpublished), 32-33

equity and risky debt. Today, this model is widely used by financial institutions to evaluate a company's credit risk.[8]

In order to understand how option pricing can be applicable, we have to understand the relationship between a firm's capital structure and options. The key to the analysis is that creditors and shareholders of a corporation can be viewed as having positions on options written on the firm's assets. To illustrate, consider a company that has assets that are financed with zero-coupon bonds and equity. Suppose that the bonds mature in five years at which time a principal payment of X is required. Furthermore, assume the company pays no dividends. In the case that the firm performs well and the assets are worth more than X in five years, the equity holders will choose to repay the bondholders and keep the remaining assets of the firm.

On the other hand, if the firm performs poorly and the assets are worth less than X, the equity holders will choose not to repay bondholders and declare bankruptcy, leaving the bondholders with ownership of the firm. The value or the payoff for the equity holders in five years is $\max(A_T\text{-}X, 0)$, where A_T is the asset value of the firm in five years or debt's maturity. This payoff is the same as the payoff of a European call option. Thus, the equity holders of a corporation can be viewed as having a long position in a European call option written on the firm's assets with an exercise price of X or the face value of debt. The total debt value of the firm can then be referred to as the exercise price. On the other hand, the payoffs for the bondholders must then be $\min(A_T, K)$ in five years, which is the same as $X - \max(X\text{-}A_T, 0)$. The second part of the payoff is the same as a European put option. Thus, the bondholders of a corporation are viewed as having a combination of riskless debt and a short position on a put option written on the firm's

[8] John C. Hull, Fundamentals of Futures and Options Markets, 5th Edition (New Jersey: Pearson), 215

assets. This is true because the shareholders, in a sense, have the right to put the firm's assets on to the creditors when total debt level exceeds assets (bankruptcy).[9]

To summarize, if C and P are the values of the call and put options, respectively then:

$$\text{Value of equity} = C$$
$$\text{Value of debt} = PV(X) - P$$

The value of the company's assets must be equal to equity plus debt or Assets = C + PV(X) – P. By rearranging the equation, we have C + PV(X) = Assets + P, which is identical to the put-call parity relationship, where "Assets" and PV(X) is analogous to S_0 and Xe^{-rT} in the original put-call parity formula, respectively. Thus, the option positions described suggest that corporate securities, such as limited liability equity and risky debt can be valued using option pricing methodology, such as the put-call parity formula or Black-Scholes Model.

3. E. ESTIMATING DEFAULT PROBABILITIES USING OPTION PRICING THEORY

As discussed earlier, corporate securities such as limited liability equity and risky debt can be viewed as options on the firm's assets, and can therefore be valued using option pricing models. Based on the Black and Scholes (1973) and Merton (1974) framework, equity of a firm can be valued in the form of a call option on the firm's assets. Default probabilities can then be derived from the option price or equity value of the firm. The model assumes that a company has a certain amount of zero-coupon debt that will become due at a future time T. The company defaults if the value of its assets is less than the promised debt repayment at time T. The equity of the company is a

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[9] John C. Hull, Fundamentals of Futures and Options Markets, 5th Edition (New Jersey: Pearson), 215

European call option on the assets of the company with a maturity T and a strike price equal to the face value of the debt. In the Merton framework the payment to the shareholders at time T, is given by E_T = max (A_T – D, 0). This shows that the equity is a call option on the assets of the firm with strike price equal to the promised debt payment. Using Black-Scholes Options Pricing Model, the current equity price is therefore:[10]

$E_0 = A_0 N(d_1) - De^{-rT} N(d_2)$, *which is equivalent to* $C_0 = S_0 N(d_1) - Xe^{-rt} N(d_2)$

$$d_1 = \frac{\ln\left(\frac{A_0}{D_T}\right) + \left(r + \frac{\sigma_A^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma_A \sqrt{T}$$

Where:

E_0 - Current market value of equity
A_0 - Implied value of assets of the firm
D_T = Debt payable at the end of time horizon T
T = Time horizon
$N(d_1)$- Standard Normal Distribution value corresponding to d1
$N(d_2)$ - Standard Normal Distribution value corresponding to d2
σ_A^2 = Implied variance of assets

Under the Black-Scholes Options Pricing Model, the probability of the call option being in-the-money at maturity (producing a positive payoff) is equivalent to $N(d_2)$, which means the shareholder will decide to exercise the option. Therefore, $1-N(d_2)$ must be the probability that the option will expire out-of-the-money or worthless at maturity, which means the shareholders will decide not to exercise their option. As discussed earlier, the only situation, in which the shareholders will not exercise their option, is

[10] Joshy Jacob and Piyush Gupta, Estimation of Probability of Default Using Merton's Option Pricing Approach: An Empirical Analysis, (Indian Institute of Mangement, 2002)

when the asset value of the firm drops below the face value of debt, forcing the firm into immediate default. Thus, 1 – N (d2) also represents the probability of default.

3. F. VALUING CDS POSITIONS BASED ON A PLAIN VANILLA B-S ANALYSIS OF CDS TERMS

Imagine a very simple setting where the payoff of the CDS contract is contingent on the default of the referenced entity, which can only occur at debt's maturity when asset value drops below the face value of debt. Under this framework, the CDS contract can be valued using a plain vanilla B-S analysis. As a bondholder of a leveraged firm under the Merton framework, he/she is holding to a risk-free component and a risky component of debt. If the bondholder enters into a CDS contract as a protection buyer on the referenced bond, he/she is insured against default of the debt. In other words, by entering into the CDS contract, the bondholder has essentially removed or offset the risky component of the bond. Thus, the value of the CDS contract is analogous to the value of the risky component of the referenced debt. Recall that debt can be broken down into two components, a risk-free component and a risky component (Debt = Risk-free component + Risky Component), where the risk-free component is the face value of debt and the risky component of debt can be viewed as a form of put option on the firm's assets. In other words, the equation can be rewritten: Debt = Par – Put Option. We illustrated earlier that under a plain-vanilla analysis, corporate securities represent option positions on the firm's assets, which conforms to put-call parity and can be calculated by applying the Black-Scholes Model.

C + PV(Par) = Assets + P is analogous to $C_0 + Xe^{-rT} = P_0 + S_0$

Where,

$C = C_0$ = Equity
$PV(Par) = X^{e-rT}$ present value of the risk-free component of debt
Assets = S_0
$P = P_0$ = Risky component of debt
We can therefore assess the value of a CDS position by determining the value of the put option under a B-S analysis.

The value of the CDS position depends on the value of the put option, which is linked to the asset value of the firm. For example, as the value of the firm declines, or its credit deteriorate; the value of the put option will increase in value, which will increase the position value of the protection buyer. Thus, the value or cost to neutralize (offset) a CDS position can be determined based on the value of the put option at a point in time. It is important to note that under market convention, CDS prices are quoted as spreads on a per annum basis (i.e. 200 basis points on a reference notional amount of $100 million over 5 years, which is similar to an annuity of $2 million dollar per year for the next 5 years). However, in contrast to market convention, our analysis illustrates the value of the put option as the value of the CDS contract in present value terms (present value of the comparable annuity), which is equivalent to the price quoted under market convention but in different form. Bear in mind that the value of the CDS contract will depend on the credit events terminology on the contract, which is usually much more complicated than this simple setting where our single credit event is restricted to default occurring only at maturity. In reality, default can occur at any time prior to maturity, and in different ways, such as a breach of financial covenants or technical insolvency. In order to accommodate and incorporate the various forms and timing of default, we must extend this plain vanilla analysis to the use of more exotic options to value corporate securities and obtain a more "realistic" value for the risky component of debt. The following is a list of key assumptions in this plain vanilla analysis of CDS positions:

Key assumptions in our modeling of CDS positions

Under plain vanilla analysis:

1. ***Credit Event is limited to default occurring at debt's maturity, where default is defined as asset value dropping below face value at debt's maturity.***
2. ***Recovery for the reference obligation is equal to the asset value of the firm upon default***
3. ***Default risk or risky component of debt is equal to European put option.***
4. ***A CDS position is analogous to the risky component of debt.***
5. ***The payoff of the CDS is thus equivalent to the payoff of a European put option*** $Max\ (X - A_T, 0)$

CHAPTER 4

INTRODUCTION TO THEORICAL MODEL

4. A. INTRODUCTION TO BARRIER OPTIONS

Barrier options are a type of exotic options that are categorized as either knock-out options or knock-in options. Knock-out options ceases to exist when the price of the underlying asset reaches a certain level. On the other hand, knock-in options come into existence when the underlying asset price reaches a particular level. The four common types of knock-out options are down-and-out call options, up-and-out call options, down-and-out put options, and up-and-out put options. An up-and-out call option can be defined as a European call option that ceases to exist once the price of the underlying reaches a certain level that is set above the asset price at inception. A down-and-out call option can be defined similarly but with the barrier level set below the asset price at inception. Down-and-out put options and up-and-out put options are defined similarly with barrier level set below and above asset price at initiation, respectively. The prices of barrier options are related to regular options. For example, the price of an up-and-out call option plus the price of an up-and-in call option must be equal to the price of a regular European option.[11] The following is the payoff structures for the various forms of barrier options:

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[11] John C. Hull, Fundamentals of Futures and Options Markets, 5th Edition (New Jersey: Pearson), 434

Types of barrier options:	**$t<T$** **S_T = asset's value at maturity** **S_t = asset's value prior to maturity** **S_d = lower barrier level** **S_u = upper barrier level**
Down-and-out Call	**(If $S_t > S_d$, then max(S_T -X, 0), 0)**
Up-and-out Call	**(If $S_t < S_u$, then max (S_T -X, 0), 0)**
Down-and-out Put	**(If $S_t > S_d$, then max (X- S_T, 0), 0)**
Up-and-out Put	**(If $S_t < S_u$, then max (X- S_T, 0), 0)**
Down-and-in Call	**(If $S_t > S_d$, max (S_T -X, 0), 0)**
Up-and-in Call	**(If $S_t < S_u$, then 0, max (S_T –X, 0))**
Down-and-in Put	**(If $S_t > S_d$, then 0, max (X- S_T, 0))**
Up-and-in Put	**(If $S_t < S_u$, then 0, max (X- S_T, 0))**

4. B. INTRODUCTION TO MODEL

As a motivation, the purpose of the model is to capture the various timing of default associated with corporate securities, which is ignored in the plain vanilla analysis under the Merton's framework that assumes default can only occur at debt's maturity. Instead of modeling default risk using European put options, the model uses barrier options as a proxy to model the risk of default occurring on and prior to debt's maturity. In the plain vanilla analysis, default can only occur at debt's maturity when asset value at maturity (A_T) is below the face value of debt (X). However, default can also occur prior to debt's maturity, such as technical insolvency, which can occur at any point in time when asset value drops below face value of debt. Furthermore, bondholders of leveraged firm are usually protected through financial covenants that place certain restrictions on the financial condition of the firm. Violations of such covenants will force the firm into immediate default. For example, bond indentures usually include a financial covenant on the firm's debt ratio. A firm's debt ratio, usually calculated as total debt/total assets, is a measure of the firm's leverage. A common financial covenant in most bond indentures is to prevent the debt ratio from going beyond a certain number. A table of possible defaults captured by the model is provided below:

Technical Insolvency	A firm is considered to be technical insolvent at any point in time when its asset value drops below the total face value of debt.
Breach of financial covenants	A firm is forced into immediate default when a financial covenant, such as debt ratio (total debt/total assets) is breached.

Recall that under the Merton framework and the plain vanilla analysis, the risk of default at debt's maturity is captured with the use of European put options. Under such an analysis, the stockholders always have the right to "put" the assets of the firm onto the bondholders, which is usually exercised when the firm defaults or when asset value at maturity is below face value of debt. Thus, bondholder's payoff is characterized as X – max(X- A_T, 0), where X is the face value of debt and A_T is the asset value at debt's maturity. The second component of the payoff, max (X- A_T, 0), represents the default risk of the firm, which is modeled using a European put option. In reality, as discussed earlier, default can occur at any point prior to maturity when asset value drops below a certain value, or in the case of technical insolvency, when it drops below face value. This value or threshold is analogous to a barrier level (A_B), which triggers a default when it is hit. Thus, bondholder's payoff can now be redefined as X if there is no default and X – max (X- A_T, 0) if default occurs when At < A_B, where A_t is the asset value of the firm at any point in time prior to maturity and A_B is the barrier level. This payoff structure is analogous to the payoff structure of a European down-and-in put option (If $S_t > S_d$, then 0, otherwise, max(X-ST, 0)), where as S_t, S_T, and S_d is defined similarly as A_t, A_T, and A_B, respectively. Thus, the model assumes that if the payoff structures are the same then the value of the risky component of debt must equal to the value of a down-and-in put option. The following is table of bondholder's payoff structure under scenario #1, where default can occur only at maturity and scenario #2, where default can occur on and prior to maturity.

Scenario #1	$X - \max(X - A_T, 0)$
Scenario #2	X if there is no default and $X - \max(X - A_T, 0)$ if default occurs when At < AB

The barrier level or default threshold can be manipulated to model the different default possibilities. The barrier level is usually a multiple of X or the face value of debt. For example, default under technical insolvency can be modeled by setting the barrier level equal to the face value of debt. Similarly, default under a breach of financial covenant, such as a violation of the debt ratio can be modeled by setting the barrier level as a multiple of X that is usually greater than 1. For simplicity, assume that bond indentures for all leveraged firms contain a financial covenant that restricts them from having a debt ratio of more than 0.50, which means total asset value must be at least 2x total debt level. The firm will be forced into immediate default at any point if asset value falls below 2x debt level. By definition, default under technical insolvency occurs when asset value drops below total debt. However, in reality there is usually a lag or time interval before bondholders realize that asset value has dropped. Thus, default will occur when asset value falls below total debt level by a certain amount. In this special or "extreme" case of technical insolvency, the barrier level is set at a multiple of less than 1. The following table summarizes the various possibilities of default and trigger levels.

Default possibilities:	**Trigger levels:**
Technical Insolvency	$A_B = X$
Breach of financial covenants	A_B = multiple of X, where multiple is > 1
Special case ("extreme") of technical insolvency	A_B = multiple of X, where multiple is < 1

In summary, the model assumes that default can occur at any point in time prior to maturity where events of default include technical insolvency (including special case) and breach of financial covenants. Under this framework when default can occur at any point prior to maturity, bondholders' payoff is transformed from:

$X - \max(X - A_T, 0)$ ----------------→ X, or $X - \max(X - A_T, 0)$ if default occurs.

The risky component of the new payoff structure is analogous to a European down-and-in put option. Thus, under this argument, the model assumes that the value of the risky component of debt must be equal to the value of a down-and-in put option. The following is a table that summarizes the various events of default and bondholders' payoff structure under triggering (default) and non-triggering scenarios (non-default). An illustration of the model is provided in the appendix attached to this report.

Events of Default:	**Trigger Levels**	**Payoff structures under non-triggering scenario (no default, $A_t > A_B$)**	**Payoff structure under trigger scenario (default, $A_t < A_B$)**
Default can only occur at maturity (plain vanilla analysis)	N/A	$X - \max(X - A_T, 0)$	$X - \max(X - A_T, 0)$
Technical Insolvency	$A_B = X$	X	$X - \max(X - A_T, 0)$
Breach of financial covenants	A_B = multiple of X, where multiple is > 1	X	$X - \max(X - A_T, 0)$
Special case of technical insolvency	A_B = multiple of X, where multiple is < 1	X	$X - \max(X - A_T, 0)$

4. C. VALUING CDS POSITIONS UNDER BARRIER OPTIONS MODEL/FRAMEWORK

Under this analysis, the barrier option now represents a more "realistic" value of the risky component of debt that accommodates default occurring prior to maturity compared to the plain vanilla analysis. This value is also equivalent to the value of a CDS referencing the leveraged firm. Recall that under the plain vanilla analysis, the credit event language is limited to default occurring only at maturity when asset value is below face value of debt. Under the barrier model, key assumption #2 is relaxed and credit event languages can now be extended to include technical insolvency and breach of financial covenants.

CDS under analysis type:	Credit Events:
Plain vanilla analysis	1. Asset value < Total debt level at maturity
Barrier Model analysis	1. Technical Insolvency 2. Breach of financial covenants

Key assumptions in our modeling of CDS positions

Under plain vanilla analysis:

1. ***Credit Event is limited to default occurring at debt's maturity, where default is defined as asset value dropping below face value at debt's maturity.***
2. ***Recovery for the reference obligation is equal to the asset value of the firm upon default***
3. ***Default risk or risky component of debt is equal to European put option.***
4. ***A CDS position is analogous to the risky component of debt.***
5. ***The payoff of the CDS is thus equivalent to the payoff of a European put option Max $(X-A_T, 0)$***

Under Barrier Options Model analysis:

1. ***Credit Events are limited to default occurring on and prior to debt's maturity, where default is defined as technical insolvency or breach of financial covenants.***
2. ***Recovery for the reference obligation is equal to the asset value of the firm upon default***
3. ***Default risk or risky component of debt is equal to a European down-and-in put option***
4. ***CDS is equal to risky component of debt.***
5. ***CDS must be equal to value of down-and-in put options***

Thus, Barrier model extended credit event languages.

4. D. LIMITATIONS OF THE MODEL

There are two main shortcomings with the current model. First, although the current model is able to capture the various timing of default, it fails to account for the immediate transfer of firm's assets from equity holders to bondholders upon default. Under the current model, bondholders payoff under a default (triggering) scenario is X – max(X-A_T, 0), where "A_T" essentially assumes assets are transferred at debt's maturity. In reality, it is more realistic to assume that assets are transferred immediately from equity holders to bondholders upon default. Secondly, the model assumptions with regard to valuing CDS positions are not truly reflective of reality. In particular, the credit event languages assumed in the model do not necessarily reflect market convention for corporate CDS as illustrated in the table under section 1.C.

4. E. EXTENSION TO MODEL

The purpose of this section is to introduce an extension to the current model in order to accommodate the immediate transfer of assets upon default. In order to account for the immediate transfer of firm's assets upon default, bondholders payoff must be adjusted to $X - \max(X-A_t, 0)$, where "A_t" captures asset's value at the time of default. This payoff structure is identical to the payoff of an American down-and-in put option, which is $\max(X-A_t, 0)$ if $A_t<A_B$. The use of American-style barrier options incorporates early exercise, which was prohibited in the original model that uses European-style barrier options as a proxy for valuing default risk and CDS positions. Thus, this extended model is able to capture both the various timing of default and the immediate transfer of assets upon default. A mathematical illustration of the model is provided in the appendix attached to this report.

CHAPTER 5

CONCLUSION OF THE STUDY

The study presented in this paper examined the Merton model of credit risk and introduced its applicability in valuing CDS positions under a plain vanilla setting. Limitations inherent in the Merton model, particularly its failure to capture the various forms of default timing were noted. As an alternative, a theoretical model using barrier options as a proxy for valuing default risk is introduced to address the limitations in the Merton model. Finally, the study concluded that the use barrier of options can possibly offer a more accurate measure of default risk. The implications of this study suggest that other exotic options, such as compound options or Asian-style option should be explored in future studies of credit risk and credit risk modeling.

SELECTED BIBLIOGRAPHY

Arora, Navneet, Jeffrey R. Bohn and Falin Zhu, "Reduced Form vs. Structural Models of Credit Risk: A Case Study of Three Models" Moody's KMV, February 17, 2005.

Carr, Peter and Liuren Wu, "Stock Options and Credit Default Swaps: A Joint Framework for Valuation and Estimation", Working paper, New York University and Baruch College, September 26, 2006.

Das, Sanjiv R. and Paul Hanouna, "Credit Default Swap Spreads", Working paper, Santa Clara University and Villanova University, June 21, 2006.

Ericsson, Jan and Joel Reneby, "A Framework for Valuing Corporate Securities", Working paper, Catholic University of Louvain and Stockholm School of Economics, October 1998.

Geske, Teri, "Back to Basics: The Basics of Credit Risk Modeling", CMS BondEdge-Fixed Income Portfolio and Credit Risk Analytics, December 2002, <http://www.interactivedata-fia.com/b2b/bb_cred_risk.html>, Accessed April 1, 2007.

Hull, John C. Fundamentals of Futures and Options Markets, 5th Edition (New Jersey: Pearson), 2004.

Hull, John, Mirela Predescu and Alan White, "The Relationship Between Credit Defaut Swap Spreads, Bond Yields, and Credit Rating Announcements", Working paper, University of Toronto, January, 2004.

Hull, John and Alan White, "Valuing Credit Default Swps I: No Counterparty Default Risk", Working paper, University of Toronto, April 2000.

Ishizaka, Motokazu and Koichiro Takaoka, "On the Pricing of Defaultable bonds Using the Framework of Barrier Options", Asia-Pacific Financial Markets, Vol 10, No. 2, 2003, 151-162.

Jacob, Joshy and Piyush Gupta, "Estimation of Probability of Default Using Merton's Option Pricing Approach: An Empirical Analysis", Working paper, Indian Institute of Management, Lucknow, December 2005.

Papanastasopoulos, George, "Using Option Theory and Fundamentals to Assessing Default Risk of Listed Firms", Working paper, University of Peloponnese, Tripolis Campus, June 2006.

Saikat, Nandi, "Valuation Models for Default-Risky Securities: An Overview", Federal Reserve Bank of Atlanta Economic Review, Fourth Quarter 1998.

Skinner, Frank S. and Timothy G. Townend, “An Empirical Analysis of Credit Default Swaps”, International Review of Financial Analysis, Vol 11, No. 3, 2002, 297-309.

John Teall, L., “FIN 396Q Coursepack”, (Unpublished)
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APPENDIX A

MENU OF DEFAULT POSSIBILITIES

APPENDIX A

MENU OF DEFAULT POSSIBILITIES

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Menu of default possibilities:

Technical Insolvency [illegible]

Downandin put option with barrier and exercise set at face value of debt.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Breach of financial covenants

[illegible]

[illegible]

[illegible]

Example:

[illegible] Total Debt = 100

Total Assets = 2[illegible]

[illegible]

Downandin put option with barrier set at Asset value of 200 and exercise set at face value of debt.

Special case of Technical Insolvency

[illegible]

[illegible]

Downandin put option with barrier set at a multiple of debt, where the multiple is less than 1 and the exercise is still face value of debt.

[illegible]

[illegible]

A_{t} = 7[illegible]

A_{t} = 20[illegible]

[illegible]

APPENDIX B

ILLUSTRATION OF MODEL

APPENDIX B

ILLUSTRATION OF MODEL

B.1. TECHNICAL INSOLVECY

Under technical insolvency, the barrier level is set equal to the strike rate (face value of debt)

AB = Multiple of X, where multiple is = 1

AB = X

Example

Please enter the current asset value (A_0)	100
Please enter the risk free rate (r)	10%
Please enter the barrier level (A_B)	80
Please enter the exercise price (X)	80
Please enter the volatility	40%
Please enter the maturity (T)	5

λ =	[illegible]	N(x) =	[illegible]	N(y) =	[illegible]
y =	[illegible]	N(y) =	[illegible]		
x_1 =	[illegible]	N(y_1) =	[illegible]		
y_1	[illegible]				

When A₀ is less than strike price:

Value of down and in put =

[illegible]

Value of Down and In put = 6.955700

Therefore,

Value of Risky Component of debt	[illegible]
Value of CDS position =	[illegible]

B.2. SPECIAL CASE ("EXTREME") TECHNICAL INSOLVENCY

Under technical insolvency (special), the barrier level is set as a multiple of the strike rate (face value of debt) that is less than 1.

AB = Multiple of X, where multiple is < 1

AB < X

Example:

Please enter the current asset value (A_0):	100
Please enter the risk-free rate (r):	10%
Please enter the the barrier level (A_B):	80
Please enter the exercise price (X):	110
Please enter the volatility:	40%
Please enter the maturity (T):	5

λ =	[illegible]	N(x₁) =	[illegible]	N(y₁) =	[illegible]
x	0.470705	N(x)	0.666652		
y₁	[illegible]	N(y)	[illegible]		
y	0.756743				

When A_B is less than strike price:

Value of Down-and-in put

$$-A_0 N(-x_1) - Xe^{-rT} N(-x_1 - \sigma\sqrt{T}) + A_0 (A_B/A_0)^{2\lambda} [N(y) - N(y_1)] - Xe^{-rT} (A_B/A_0)^{2\lambda-2} [N(y - \sigma\sqrt{T}) - N(y_1 - \sigma\sqrt{T})]$$

Value of Down-and-in put = **14.72731**

Therefore:

Value of Risky Component of debt =	**14.72731**
Value of CDS position =	**14.72731**

B.3. BREACH OF FINANCIAL COVENANTS

Under breach of financial covenants, the barrier level is set as a multiple of the strike rate (face value of debt) that is greater than 1.

AB = Multiple of X, where multiple is > 1

AB > X

Example

Please enter the current asset value (A_0)	100
Please enter the risk free rate (r)	10%
Please enter the the barrier level (A_B)	120
Please enter the strike price (X)	90
Please enter the volatility:	10%
Please enter the maturity (T):	5

λ =	[illegible]	$N(x_1)$ =	[illegible]	$N(y_1)$ =	[illegible]
y =	[illegible]	$N(y)$ =	[illegible]		
x_1 =	[illegible]	$N(y_1)$ =	[illegible]		
y_1 =	1.210072				

When A_T is less than strike price:

Value of down-and-in put =

$$-A_0 N(-x_1) + Xe^{-rT} N(-x_1 + \sigma\sqrt{T}) + A_0 (A_B/A_0)^{2\lambda} [N(y) - N(y_1)] - Xe^{-rT} (A_B/A_0)^{2\lambda - 2} [N(y - \sigma\sqrt{T}) - N(y_1 - \sigma\sqrt{T})]$$

Value of Down-and-in put = 9.211619

Therefore,

Value of Risky Component of debt =	9.211619
Value of CDS position =	9.241619

APPENDIX C

EXTENSION OF MODEL

APPENDIX C
EXTENSION OF MODEL

C.1. Technical Insolvency

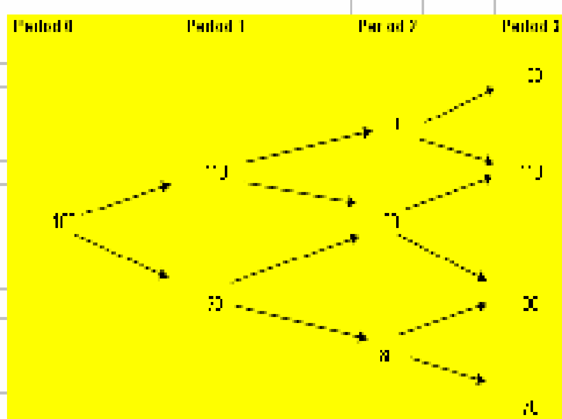
Under technical insolvency, the barrier level is set equal to the strike rate (face value of debt)
AB = Multiple of X, where multiple is = 1
AB = X

Example (American-Style barrier):

Assumptions:
1) zero time-value of money
2) 50% probability in each node
3) Equity holders have an incentive to hold on to the assets of the firm for as long as possible, which means they will only exercise (down-and-out call option) at maturity
4) Assets are transferred immediately upon default (put is exercised immediately when barrier is triggered and the option is "in the money")

[illegible] = [illegible]
[illegible] = [illegible]

Path of asset values



Calculations:

Payoff table and its probabilities
C = Call, X = Strike Price, P = Put, S = Asset value

C	X	P	S	Probability
[illegible]	81	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	81		[illegible]	[illegible]

[illegible] = [illegible] 19.25

P_0 [illegible] 0.25

X = 81

S_0 = [illegible] = 100

[illegible]
[illegible]
S_0 = Value of the [illegible]

CORPORATE SECURITIES AND CDS VALUATIONS:

Value of Equity = Value of down-and-out call option =	$ 19.25
Value of [illegible]	
[illegible]	$ 81.00
[illegible]	$ 0.25
Value of Risky [illegible]	$ 80.75
100 = Value of assets = Value of equity + Value of risky debt = [illegible]	$ 100.00

Put-Call parity = [illegible]

[illegible]

C.2. Special Case of Technical Insolvency

Under technical insolvency (special), the barrier level is set as a multiple of the strike rate (face value of debt) that is less than 1.

AB = Multiple of X, where multiple is < 1

AB < X

Example (American-Style barrier):

Assumptions:

1) zero time-value of money

2) 50% probability in each node

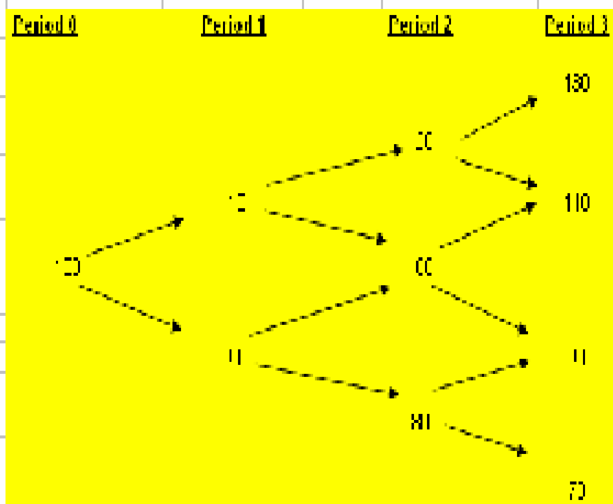
3) Equity holders have an incentive to hold on to the assets of the firm for as long as possible, which means they will only exercise (down-and-out call option) at maturity

4) Assets are transferred immediately upon default (put is exercised immediately when barrier is triggered and the option is "in the money")

Strike price (X) = 90

Barrier level (A_B) = 80

Path of asset values



Calculations:

C = Call, X = Strike Price, P = Put, S = Asset value

C	X	P	S	Probability
40	90		130	0.125
20	90	0	110	0.375
0	90	0	90	0.25
0	90	10	80	0.25

C_0 = 0.125 40 + 0.375 20 + 0.25 0 + 0.25 0 = 12.5

P_0 = 0.25 10 = 2.5

X = 90

S_0 = 12.5 + 90 − 2.5 = 100

C_0 = Price of down-and-out call

P_0 = Price of down-and-in put

S_0 = Value of the firm (asset value)

CORPORATE SECURITIES AND CDS VALUATIONS:

Put-Call parity = C_0 + X − P_0 = S_0

Value of Equity = Value of down-and-out call option =	$ 12.50	[illegible]
Value of Debt = Risk-free component − Risky component Risk-free component = face value of debt = strike price =	$ 90.00	
Risky component = down-and-in put = Value of CDS contract	$ 2.50	
Value of Risky Debt = Face Value of debt − Down-and-in Put =	$ 87.50	
100 = Value of assets = Value of equity + Value of risky debt = $12.50 + $87.50 =	$ 100.00	

C.3. Breach of Financial Covenants

Under breach of financial covenants, the barrier level is set as a multiple of the strike rate (face value of debt) that is greater than 1.

AB = Multiple of X, where multiple is > 1

AB > X

Example (American-Style barrier):

Assumptions:

1) zero time-value of money

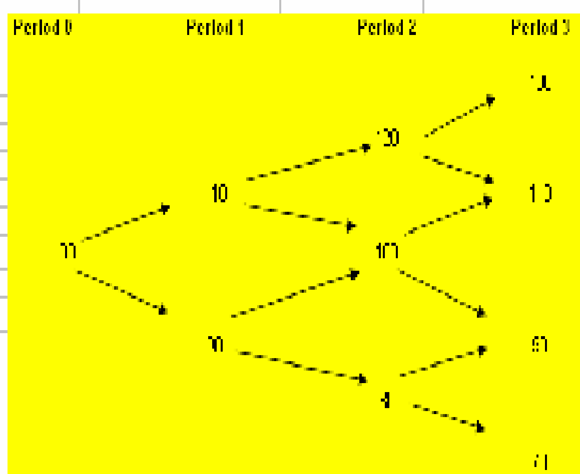
2) 50% probability in each node

3) Equity holders have an incentive to hold on to the assets of the firm for as long as possible, which means they will only exercise (down-and-out call option) at maturity

4) Assets are transferred immediately upon default (put is exercised immediately when barrier is triggered and the option is "in the money")

Strike price (X) =	87
Default barrier (AB) =	90

Path of asset values



Calculations:

C = Call, X = Strike Price, P = Put, S = Asset value

C	X	P	S	Probability
43	87	0	130	0.125
23	87	0	110	0.375
3	87	0	90	0.25
0	87	7	80	0.25

$C_0 = (0.125)(43) + (0.375)(23) + (0.25)(3) =$ 14.75

$P_0 = (0.25)(7) =$ 1.75

$X =$ 87

$S_0 = 14.75 + 87 - 1.75 =$ 100

C_0 = Value of down-and-out call

P_0 = Value of down-and-in put

S_0 = Value of the firm (asset value)

Put-Call parity: $C_0 = S_0 - X + P_0$

14.75 + 87 − 1.75 = 100 *Thus, parity always holds*

CORPORATE SECURITIES AND CDS VALUATIONS:

Value of Equity = Value of down-and-out call option =	$ 14.75
Value of Debt = Riskless component + Risky component	
Riskless component = Face value of debt = strike price =	$ 87.00
Risky Component = Down-and-in put = Value of CDS premium	$ 1.75
Value of Risky Debt = Face Value of debt + Short down-and-in put =	$ 85.25
100 = Value of assets = Value of equity + value of risky debt = $12.50 + $87.50 =	$ 100.00