Updating: A Psychologically Basic Situation of Probability Revision

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Abstract

The Bayesian model has been used in psychology as the standard reference for the study of probability revision. In the first part of this paper we show that this traditional choice restricts the scope of the experimental investigation of revision to a stable universe. This is the case of a situation that, technically, is known as *focusing*. We argue that it is essential for a better understanding of human probability revision to consider another situation called *updating* (Katsuno & Mendelzon, 1992), in which the universe is evolving. In that case the structure of the universe has definitely been transformed and the revision message conveys information on the resulting universe. The second part of the paper presents four experiments based on the Monty Hall puzzle that aim to show that updating is a natural frame for individuals to revise their beliefs.

Keywords: Probability revision, Bayes' rule, updating, focusing

Updating: A psychologically Basic Situation of Probability Revision

Imagine your host has just proposed a fruit to you and she goes to the kitchen to find the fruit basket. Talking from the kitchen she informs you that the basket contains two bananas, two apples, and one pear. Even though you would prefer the pear you tell her to choose any fruit randomly. From your mental representation of the basket you know that there is one chance out of five for her to bring back the pear. Then she tells you that the fruit she has chosen (at random) is not a banana. You now focus on the two apples and the pear and change your initial degree of belief to get a pear at one chance out of three, in accordance with the new partition elicited by the message, that is, a subset of three fruit resulting from a partition which you could have envisaged initially among others (see Fig. 1 left column). In this example, you have an initial knowledge that refers to a well defined *class of reference (*universe of possibilities), namely the composition of the fruit basket (more generally, a population). You learn a precise piece of information (*factual evidence*) and you change the reference class in order to focus on just those fruits that share with the chosen fruit the same property (not to be a banana). You have operated a change of reference class which results in a change from a prior probability (1/5) to a posterior probability (1/3). This is not a revision process proper because the posterior probability is virtually available (among others) and the information "the fruit chosen (at random) is not a banana" just prompts the selection of a particular posterior probability (among others). There is no true alteration of your initial belief about the content of the basket but a stricter *focus* on your initial state of knowledge specified by new factual evidence. In other words, the message brings in a refinement of your initial knowledge. This situation of revision is *atemporal*, meaning that your initial and final degrees of belief are just two possible values that can theoretically be defined in advance. It concerns a "belief revision" in a *stable universe,* that is to say a situation where the problem's class of reference (known to you)[1] is not modified by the message (indeed your knowledge of the composition of the basket has not been modified by the message saying that the chosen fruit is not a banana). This traditional situation of revision where you learn a message that concerns an object drawn at random from a population of objects which constitutes a certain stable universe is called *focusing* on a reference class (Dubois & Prade, 1992, 1997) and the appropriate way to modify your belief (focusing and normalisation) is the well known conditioning by Bayes' rule[2] (de Finetti, 1974).

Updating and the Minimal rule

Updating

Imagine that when your friend informs you that the fruit chosen at random is not a banana, you do not focus on the two kinds of fruit that remain possible; instead you mentally represent the basket as if the bananas did not exist any more; that is, to compute your chances of eating the pear you represent the basket *without bananas*. You estimate your chances of eating your favorite fruit to be equal to one third. Now, *in so doing, you have acted as if you were in another situation of revision*. You have inferred from the message "the fruit chosen is not a banana" the same information as you would have, had you heard the message "the bananas have been removed from the basket", and you have conceived a new probability distribution consistent with your representation of a new basket without bananas. Clearly this situation of revision is different from focusing. You learn that the universe that consists of the fruit basket has been modified (a basket with bananas at time t_0 versus a basket without bananas at time t_1). You turn your representation into a new basket *without bananas* (see Fig. 1, right column). Your beliefs, while correct at one time, may become obsolete due to this change. Thus, the situation of revision here is *temporal* and your final degree of belief corresponds truly to a new degree of belief inferred from the new composition of the basket. This situation where individuals have to revise their degrees of belief when the universe —which now is considered as susceptible of evolving— has changed according to the indication of the new message is called *updating*. A word of caution about the terminology is in order. It is common to use the term *updating* to refer to probability revision in general (e.g., Hogarth, 1992). Here we are using this term in a technical sense that differs from this generic sense. In the present paper we follow the usage in the AI (Katsuno & Mendelzon, 1992; Kern-Isberner, 2001; Winslett, 1990), philosophy (Hansson, 1999) and cognitive economics (Bourgine & Nadal, 2004; Walliser, 2007) communities where "updating" refers specifically to a situation of revision that takes place in an evolving universe. The updating operation takes into account the fact that the universe referred to in the knowledge base has evolved, and so the set of beliefs must evolve accordingly. Formally, it differs subtly, but also sharply from the focusing situation. It has a clear definition backed up by a set of axioms for belief revision (Katsuno & Mendelzon, 1992) and for probability revision (Walliser & Zwirn, 2002).

The Minimal rule

However, the new probability distribution entailed by the temporal modification of the universe (suppressing the bananas) cannot be calculated by Bayes' rule (for a technical analysis see Gärdenfors, 1988; Walliser & Zwirn, 2002)[3]. Instead, it is specified by a two-stage process, called the *Minimal rule* (Walliser & Zwirn, in press). It consists of a very simple dynamic process: (i) representing the new universe/class of reference at t_1 that results from the modification of the initial one at t_0 (like in our example where the message of suppression of the bananas entails the existence of a basket without bananas), and (ii) inferring the new probability distribution for the universe/class of reference at t_1. In our example, point (ii) is equivalent to conditioning by Bayes' rule since the new probability to choose the pear from the new basket without bananas is equal to the initial probability to choose the pear modified after normalisation (to get a sum equal to 1). The Minimal rule generalises Bayes' rule by incorporating the temporal aspect. It should not be understood as an alternative to Bayes' rule, but rather as a complementary rule applicable to the situation in question where Bayes' rule just does not apply.

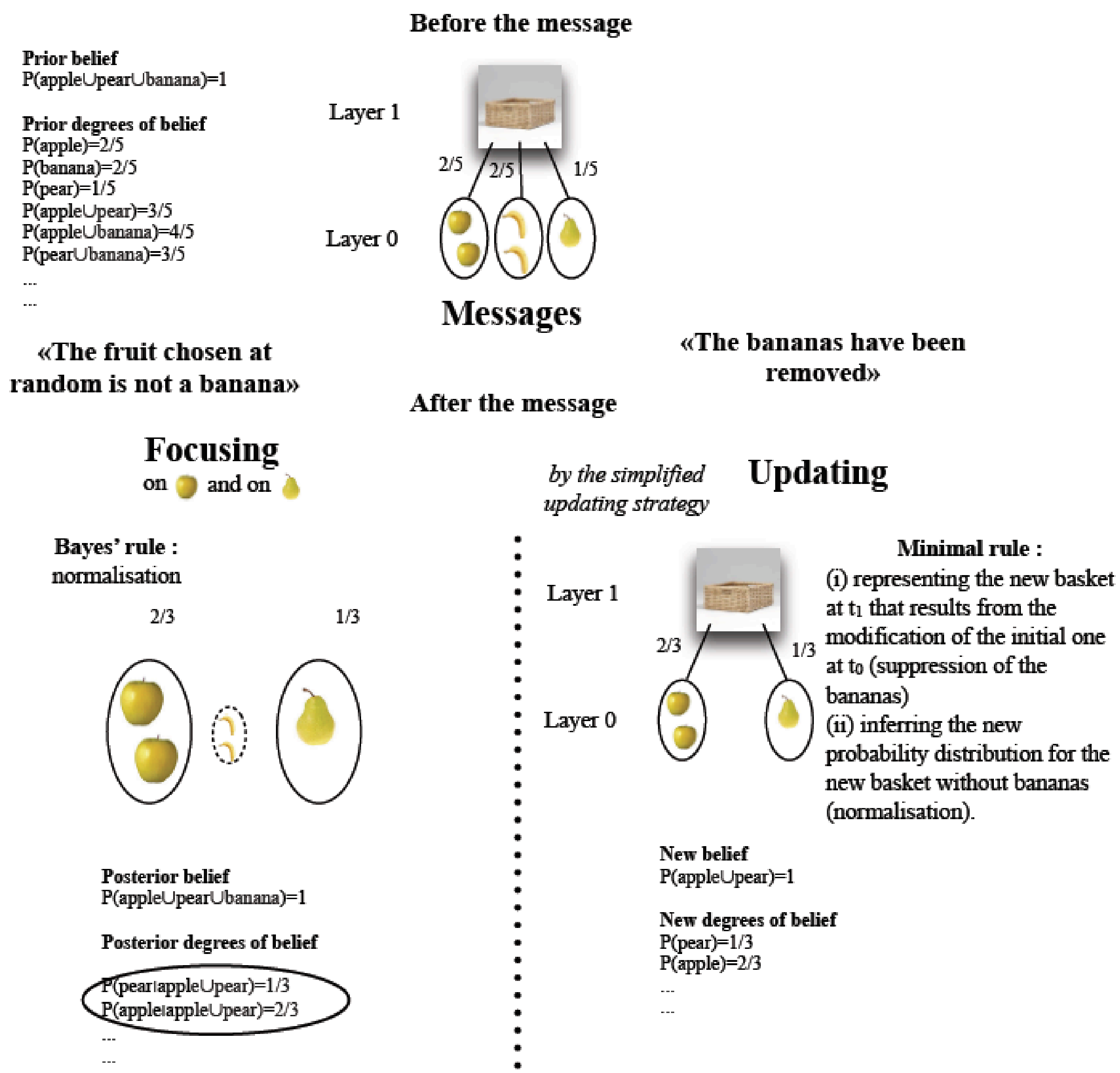


Figure 1. The fruit basket in the focusing and updating situations

The simplified updating strategy

The updating situation is easy to illustrate when its source is an action that modifies the initial knowledge of the actual universe as in our fruit basket example. But it is noteworthy that updating may occur without an explicit action. This would be the case with the outbreak of a new disease that transforms the knowledge base of a physician, or with new statistical data that alter radically a demographic model. At this stage, it is likely that readers who have worked out for themselves the solution of the fruit basket example fail to grasp any computational difference between the two situations. Indeed, the essential claim of the present paper concerns the hypothesis that reasoners in the focusing situation fail to represent part of the possibilities (the bananas, even though they still exist), and so handle a new problem which puts them in an updating situation. We believe that to infer probabilities individuals tend to set themselves spontaneously in an updating situation even when the situation does not coincide with this type of revision. Initially individuals envisage a probability distribution of possible occurrences with respect to their knowledge of a given environment. Upon receiving new information, they infer a new probability distribution, thinking (or imagining) that the environment has been modified by the message. This process of updating leads one to the normatively correct probability distribution when it concerns a single level of belief (which involves a single distribution as opposed to nested distributions as will be explained shortly), like in our example of the fruit basket. You have inferred the probability distribution that is correct with regard to the focusing context by means of the interpretation of the context as one of updating. Because of the simple structure of the fruit basket example the two situations yield the same result. However, as we shall see shortly, with revisions that involve several levels of belief, this *simplified updating strategy* of revision may lead to an incorrect probability distribution, and consequently an incorrect response, even though the problem seems computationally very simple, so that a paradox arises. This has given rise to well-known puzzles that we are going to exploit.

The primary aim of this paper is to present, possibly for the first time, an experimental investigation of focusing and updating, which distinguishes a stable universe and a changing universe. The second aim is to show and illustrate that this distinction is indispensable for a better understanding of human probability revision, *even when we restrict ourselves to the study of revision in a certain stable universe.* This is because, even if the experimenter assumes or instructs participants that the universe is fixed, it may be the case that they nevertheless consider a universe in evolution. So, when assessing the coherence of their judgment, the experimenter must be aware of this possibility and be ready to use the appropriate formalism (Baratgin & Politzer, 2006). This paper will be organised as follows. In the next section, the two different situations of probability revision, namely focusing and updating are illustrated by means of the well known “Monty Hall puzzle” (henceforth MHP). This is an emblematic example of people's inconsistency in making probability judgment (Granberg & Brown, 1995) that reveals the structural complexity of many classic tasks and paradigms used in the psychological literature. This complexity stems from the existence of two levels of belief, which involves nested probability distributions. This is expounded in the Appendix (section 1) to which we defer the technical points linked to the MHP. In the main part of the paper we take up the theoretical analysis developed in Baratgin (2009) which claims that (i) the situation of the MHP as conceived by the experimenter is a focusing situation; and (ii) participants' modal response stems from an updating interpretation of the situation, that is, the MHP is difficult to solve because, for reasons essentially pragmatic, the specific characteristics of the puzzle prompt a biased interpretation of the revision message which is processed within the framework of updating. This will be followed by four experiments that test our approach. The first two use a novel experimental paradigm aiming to reveal these crucial characteristics of the puzzle. The last two experiments show that it is possible to avoid such a biased interpretation, in which case participants give the experimenter's expected (normative) answer. In the general discussion, after producing evidence that backs up the pragmatic claims, we consider the implications of our results for the psychological research on probability revision.

The Two Situations of Revision (Focusing and Updating) in The Monty Hall Puzzle

A Typical Example of a Biprobabilistic Structure of Belief

All the experimental paradigms that have been used in the psychological literature share a common characteristic, namely the existence of nested probability distributions. In other words, these paradigms are built on a distribution of distributions. Participants must have beliefs about elementary properties of a given object and the source of these properties (for example, the various colours of balls extracted from different urns, the various symptoms caused by different kinds of disease, etc.) In the statement of the problem, the experimenter gives, explicitly or implicitly, (i) the probability distribution of these properties for each source (e. g., the content of the urns, or the probability distribution of the symptoms for each disease): This is the nested distribution (or level 1 distribution); and (ii) the probability distribution of these sources (e. g., the number of urns, or the probability distribution of the diseases, etc.): This is the overarching distribution (or level 2 distribution). To build such a two-level hierarchy of distributions, one needs three hierarchical layers of worlds, such that a first overarching distribution relates the layer 2 world to the layer 1 world, and the nested distribution relates the layer 1 world to the layer 0 world.

A standard version of the MHP presented in a probabilistic format[4], is the following:

> A TV host shows you three doors D1, D2, and D3 all equally likely, one hiding a car and the other two hiding goats. (i) You get to pick a door, winning whatever is behind it. You choose door D1, say. The host asks you to estimate the probability that the car is behind D1. (ii) The host, who knows where the car is, tells you: "I will show you one door (out of the other two) behind which there is a goat". (iii) Then he opens D3 to reveal a goat. (iv) Finally he asks you what is the probability that the car is behind D1.

The experimenter's solution is deeply counterintuitive: The probability that the car is behind D1 equals 1/3. Indeed, following Bayes' rule, the correct answer after the host's opening D3 to reveal a goat, is that there are twice as many chances that the car is behind D2 as behind D1 (the explanation is given in section 2 of the Appendix). Yet the crushing majority of people (whatever their level of instruction) think that there are equal chances for the car to be behind D1 and D2 and accordingly answer 1/2 (for a review see Krauss & Wang, 2003). From now on, we will refer to the solution 1/3 as (the experimenter's) "Bayesian Solution".

We will argue that the crux of the MHP lies in this: In a genuine situation of focusing like the MHP, the universe of reference is fixed, and the correct response is 1/3 as calculated by the experimenter. However, the course of events in the MHP renders the focusing hard to perceive and induces an updating interpretation within which it is rational to give the modal answer (1/2)[5] (see section 3 of the Appendix). In other words, the latter response is coherent within an updating interpretation, although it is of course incorrect from the point of view of the experimenter's focusing conception of the task.

Based on the hypothesis that participants' modal answer is induced by a particular interpretation of the situation of probability revision we are in a position to propose a unified explanation which is not contradictory to the explanations found in the experimental literature (for more detail see Baratgin, 2009). It will also become clear that the MHP refers to a fairly complex situation, both theoretically and cognitively. The fact that this puzzle exists in various guises such as the classic "three drawers problem" of Bertrand (1889) or the famous "three prisoners problem" (Shimojo & Ichikawa, 1989) points to the existence of a cognitive difficulty to represent their common isomorphic structure, which requires an explanation.

The Focusing and Updating Situations From a Cognitive Point of View

In most studies of probability revision, participants are assumed to understand the problem exactly as it has been conceived by the experimenter; in particular, a focusing problem built by the experimenter is assumed to be understood as such by participants. In the frequent cases where the participants' answers do not coincide with the result calculated by Bayes' rule participants are deemed incoherent. However, if the participants have not interpreted the revision situation as a focusing situation, there is an alternative diagnostic: They have a different interpretation of the problem that may be suggested by the statement and the context of the problem, while remaining fully coherent within their own interpretation. In that case, participants cannot be considered as incoherent (from a normative point of view) but only as victims of an error of interpretation of the revision situation. We concur with Oaksford & Chater's (1996) idea that participants' apparent errors of judgment and reasoning may result from their rationally solving a task determined by their interpretation of it instead of solving the task that was initially conceived by the experimenter.

An examination of the literature on the MHP shows that when experimenters envisage the different answers they generally fail to consider that participants' interpretation of the host’s message might not conform to the only situation that they envisage and expect, namely a focusing situation of revision. On the contrary, the participants' interpretation of the MHP might be that of an updating situation which induces the solution 1/2. The strategy leading to this solution is detailed in the Appendix.

There are strong arguments supporting the hypothesis that participants routinely identify the revision message as an updating message. These will be considered in some detail in the general discussion. Suffice it to say now that across the different stages of the MHP the pieces of information concerning the door opened by the host have a presumption of relevance (Sperber & Wilson, 1995) and pragmatically suggest a modification of the class of reference (from three doors to two doors)[6].

Taking into Account the Participants' Interpretation of the Situation of Revision

We present four experiments. The first two test the pragmatic claim that has just been made and the last two aim to show that participants give the Bayesian Solution as a modal answer when the hierarchical structure of the puzzle and the focusing character of the message are both made explicit.

The first two experiments investigate the *pragmatic* claims. Due to the expectations resulting from ostensive communication, the MHP is interpreted as a temporal problem of revision, that is, as a problem in which the initial structure is modified by the new information. Our hypothesis is that the stronger the participants' expectations about new information, the greater their tendency to consider any new message released by the experimenter to be relevant. Therefore even if the message may actually be uninformative, participants will tend to infer information from it. In Experiment 1 (which corresponds to an isomorphic version of the MHP) the experimenter delivers an uninformative message like, for example, an already known message, and so we predict that a sizeable number of participants will nevertheless modify their initial degrees of belief. This experiment is a close replication of an experiment carried out in Baratgin (2007) that shows a redundancy effect, that is, a revision performed after a non informative message in an isomorphic version of the MHP (the three prisoners problem). Experiment 1 tests the same hypothesis in a novel isomorphic version of the MHP: Whereas in the three prisoners paradigm participants read the statement of a scenario and the puzzle is virtual, here we use a task in which participants really act in the game with concrete objects, like in the MHP. Experiment 2 studies explicitly participants' interpretation of the situation of revision with this novel material that we now describe. Three hollow tubes are used (corresponding to the three doors of the MHP), one of which contains a ball (corresponding to the car) while the other two tubes are empty (the MHP goats). Participants experience the same four stages as in the MHP, namely: (i) choosing a tube and estimating the probability that the ball is in the chosen tube; (ii) listening to the experimenter's notice that he will open one of the two non-chosen tubes and show explicitly that this tube is empty; (iii) observing the physical opening by the experimenter of one empty tube selected among the two tubes that the participant has not chosen. Finally, (iv) the request to give a new estimate of the probability that the ball is in the tube initially chosen.

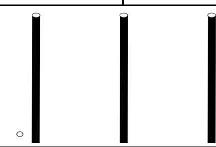
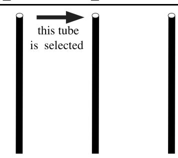
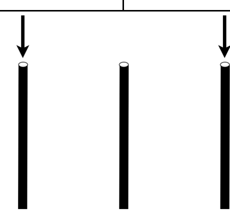
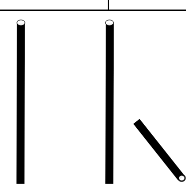
Experiment 1: Redundancy Effect in a Novel Isomorphic Version of the MHP

We claim that the misinterpretation of the situation as one of updating is pragmatically triggered: In.this experiment we aim to show that if participants expect a piece of information to be delivered, the content of which they assume to be relevant to their belief, then they will be inclined to revise, even if the message is uninformative.

Method

Participants and design. Forty-seven students from the University of the Mediterranean (France) participated in this experiment. They were ignorant of the MHP. They were tested in groups of eight to ten and allocated to one of two conditions. They were required to justify their answers in writing at the end of the experiment.

Figure 2. Conditions and results in percent for Experiment 1.

Condition **N**	**Standard** **23**	**Non-informative** **24**
Materials		
Stage 1 "Please choose a tube"		
Initial judgment	"What is the probability that the ball is in the tube initially chosen?"	
Answer 1/3	100%	
Stage 2	"I will show you one tube (out of these two tubes) that is empty"	"at least one tube (out of these two tubes) is empty"
		
Intermediate judgment		"What is the probability that the ball is in the tube initially chosen?"
Answer 1/3		54.2%
Answer 1/2		45.8%
Stage 3 Final judgment		
	"What is the probability that the ball is in the tube initially chosen?"	
Answer 1/3[a]	4.3%	0%
Answer 1/2[b]	95.7%	100%

a Corresponds to the Bayesian Solution

b Corresponds to the solution by the Minimal rule

Materials and procedure. Three tubes of different colours made of cardboard that could be closed with stoppers, and one cotton ball were used. There were two experimental conditions (see Fig. 2). The first condition (the *Standard condition)* was isomorphic to the standard version of the MHP. It fulfilled two objectives: One was to replicate the classic results of the MHP, and the other was to provide a control for the second condition (the *Non-informative* condition).

In the Standard condition the three tubes and the ball were used instead of the three doors and the car, respectively. First, the three tubes and the ball were displayed on a table. The experimenter showed that these three tubes were empty and said that he was going to introduce the ball into one of the tubes (without a preference for any tube). Participants were invited to manipulate these tubes and the ball (to see for themselves that the material was not rigged). The experimenter invited participants to leave the room and introduced the ball into a tube. When participants were back, the experimenter asked one of them (chosen randomly) to designate a tube. The experimenter placed this tube slightly aside while the other two remained close to him. Then the experimenter asked an initial judgment: "*What is the probability that the ball is in the chosen tube?* The answer to this question (and similarly for the other quantitative questions of this and the other experiments) was produced by the participant on a sheet of paper as a percentage or a fraction. Then the experimenter showed the other two tubes and added: "*I know where the ball is*; *I will show you one tube --out of these two tubes-- that is empty"*. The experimenter also explained explicitly his strategy to select the empty tube. In particular, he stated that if the ball was in the tube initially chosen, he would choose randomly the tube to open, and that if this was not the case he would necessarily show the empty tube. He then opened one of these two tubes to reveal that it was empty and put it lying on the table. Then he asked a final judgment "*What is the probability that the ball is in the tube initially chosen?"***.**

The Non-informative condition involved three judgments of probability. The first (initial judgment) took place after a participant had chosen a tube, as in the Standard condition; an intermediate judgment took place after the experimenter uttered the following non-informative message "*I know where the ball is*; *at least one tube --out of these two tubes-- is empty*"; and a final judgment took place after the experimenter had opened one of the two other tubes to reveal that it was empty.

Results and discussion

Fig. 2 shows the percentages of response in each condition. Every participant in the two conditions gave 1/3 for their initial probability estimate. In the Standard condition all but one participant gave 1/2 as their final estimate, so replicating the result of the classic MHP. The majority gave explanations supporting the Minimal rule, a typical instance of which is: "the problem has been transformed, there are now only two tubes, hence their probabilities are equal". In the Non-informative condition 46% of the participants revised their probability to 1/2, a proportion whose 95% confidence interval is [.33; .61]. This is a remarkable result considering that participants had received the non-informative message "at least one tube --out of these two tubes-- is empty". For the final estimate (after the experimenter had revealed an empty tube), every participant answered 1/2. A majority of participants who had revised their probability after the non-informative message explained that they did not revise their probability on the third estimation because when the experimenter showed the empty tube this was no news. In brief, participants expect a message from the experimenter to be relevant and are eager to revise their first judgment of probability.

This study confirms our hypothesis that participants' expectation of a relevant message from the experimenter has an effect on the interpretation of the three-tube problem, and in all likelihood on the isomorphic problems (the MHP and the three prisoners problem). Participants have a tendency to modify their prior probability as if the redundant message was an updating message. Participants' *most frequent* verbal justification for the response 1/2, which supports the hypothesis of an updating interpretation, is that the initial problem (find one tube among three tubes) has been modified by the experimenter's action into an equivalent problem with just two tubes (as is the case for the MHP, see the Appendix, section 3). It is important to note that participants understood the experimenter's explanations regarding his strategy to select the empty tube in the standard condition (viz., a random choice if the tube initially chosen was not empty and a constrained choice if the tube initially chosen was empty). Evidence of this was obtained during debriefing. However, this knowledge was not exploited and seems to have been useless. So, interpreting the experimenter's action as a structural modification of the initial problem into a new problem seems to be the essential reason why in the end participants considered only two possibilities (one empty tube, one non empty tube). The next step is to get direct experimental confirmation of this updating interpretation.

Experiment 2: An Updating Interpretation of the Focusing Message in an Isomorphic Version of the MHP

We use the Standard condition of Experiment 1 and a simplified version of the three-tube problem (the *Naive*[7] condition) in which the experimenter simply removes a tube. As explained earlier, the main consequence of an updating interpretation is that in the participants' representation of the problem one tube is missing. If participants use the simplified updating strategy, they will behave similarly on these two conditions, that is, they will tend to give the same answers and to be unaware of the difference.

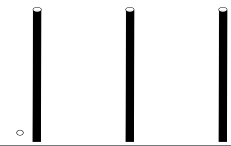
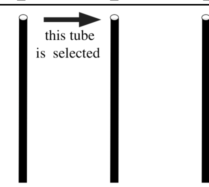
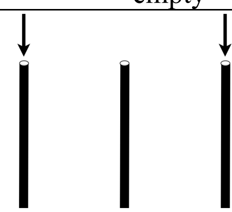
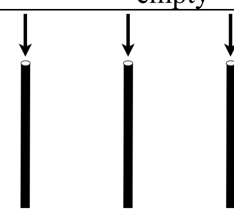
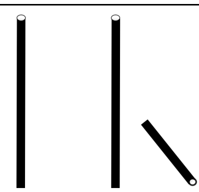
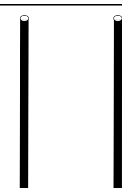
Method

Participants and materials. One hundred students from the University of the Mediterranean participated in this experiment. They were ignorant of the MHP. The same materials as in Experiment 1 were used.

Design and procedure. Participants were tested in groups of eight to twelve. To control for order effects one half of the participants received the Standard condition first and the Naive condition second; this order was reversed for the other half. Finally, participants were asked to indicate if, in their opinion, the two problems were "the same" or "different", and to explain their answers in writing. This type of questioning rendered a within-participant design necessary (and similarly for the next two experiments).

The Standard condition was identical to that of Experiment 1. The Naive condition differed as follows: At stage (ii), instead of saying, like in the Standard condition, "I know where the ball is; I will show you one tube --out of these **two** tubes-- that is empty", the experimenter said: "I know where the ball is; I will show you one tube --out of these **three** tubes-- that is empty". He added that he would choose the tube to open randomly from any of the two tubes that he knew were empty. Then the experimenter opened one of the two tubes not initially chosen to reveal that it was empty and put it lying on the table. The procedure is summarised in Fig. 3.

Figure 3. *Conditions and results in percent for Experiment 2.*

Condition **N**	**Standard** **100**	**Updating** **100**
Materials		
Stage 1 "Please choose a tube"		
First judgment	"What is the probability that the ball is in the tube initially chosen?"	
Answer 1/3	100%	
Stage 2	"I will show you one tube (out of these two tubes) that is empty"	"I will show you one tube (out of these three tubes) that is empty"
		
Stage 3		
Second judgment	"What is the probability that the ball is in the tube initially chosen?"	
Answer 1/3[a]	2%	0%
Answer 1/2[b]	98%	100%

a Corresponds to the Bayesian Solution

b Corresponds to the solution by the Minimal rule

Results and discussion

We analyse first the quantitative answers. If participants view the two conditions as equivalent, the answers will tend to be the same. The results overwhelmingly support this prediction as the answers 1/3 (first estimate) and 1/2 (second estimate) were given by 98 participants in the Standard condition and by every participant in the Naive condition. However, one might object that identical answers could stem from some uncontrolled factor. This is why we asked participants to justify their judgment of sameness/difference. If participants tend to offer the same justifications in the two conditions, then this clearly supports the hypothesis that their revision stems from similar representations of the problem structure rather than from some extraneous factor. This is what an analysis of the qualitative answers shows. In both conditions *the majority of the justifications* for their judgment of sameness/difference were of the type, *"there are now only two tubes",* which is in line with the Minimal rule. Seventy-six participants judged the two conditions identical, a proportion whose 95% confidence interval is [.67; .83]. The remaining 24 participants declared the two conditions different but generally did not offer clear justifications for this. Thirteen wrote that the experimenter showed one empty tube out of the two non-chosen tubes in the Standard condition and one out of the three tubes in the Naive condition, with no further elaboration. Out of these 13 participants only five expressed that the two conditions asked for different responses but did not explain why.

This study supports our main hypothesis that participants understand the structure of the MHP as a problem where one possibility is cancelled by the experimenter. However the result could be analysed differently under the hypothesis that participants are not able to represent the biprobabilistic structure of the MHP (even though they seem to understand the experimenter's strategy for choosing the tube which is made verbally explicit). In such a case, the answer would stem from a revision by Bayes' rule applied to a one-level naive probabilistic structure (for this hypothesis see Fox & Levay, 2004; Johnson-Laird, Legrenzi, Girotto, & Sonino-Legrenzi, 1999). We are going to address this question in the next two experiments where we present a version of the three-tube problem in which the biprobabilistic structure that corresponds to Fig. 6a1 is made explicit. In one condition the message is unambiguously a focusing message whereas in another one it is unambiguously an updating message. It was hypothesised that participants would no more interpret the focusing situation as an updating, resulting in a shift from the modal response to the Experimenter's solution and that participants should even be able to tell the difference between *focusing* and *updating* messages (while failing to differentiate the updating situation from a standard condition used as a control).

Experiment 3: Focusing and Updating Conditions in the Standard Three-Tube Problem

Method

Participants. Twenty-four participants3[8] (doctorate and post-doc students, researchers, University staff at the University of the Mediterranean) participated in this experiment. They had some knowledge of elementary probability calculus but we had reason to expect that they would not perform differently than more naive participants. This was based on the fact that the counter-intuitive character of the MHP and of its isomorphic variants seem to be universal.[9]

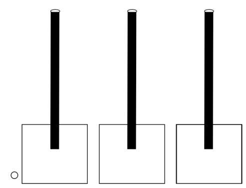
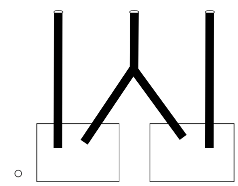
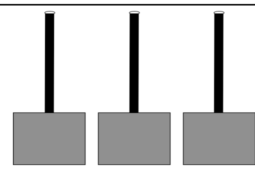
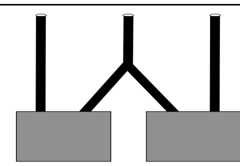
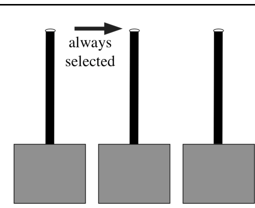
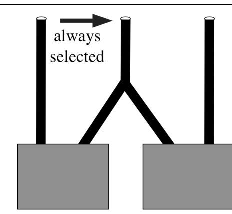
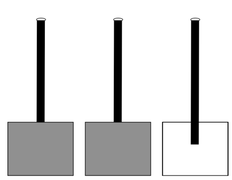
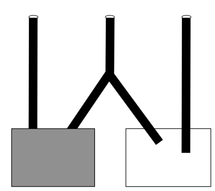
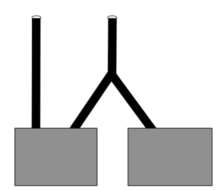
Materials. Three tubes that ended in different boxes and two screens were displayed in front of the participants (see Fig. 4). Depending on the condition, one of the tubes could be Y-shaped (an inverted Y).

There were three conditions (to be described below). In the Standard condition, which served as a control, the materials consisted of the same three straight tubes as in Experiments 1 and 2, and a cotton ball. In the other two conditions, there were two straight tubes, one inverted Y-shaped tube, two transparent boxes, two screens, and a cotton ball.

Procedure and design. Participants were tested in six groups of four. The material was displayed on a table. Each group was submitted to the three conditions in one of the six possible orders. After completion of the three problems, participants had to indicate in writing if, in their opinion, these problems were “equivalent” or “different” and to justify this judgment.

The Standard condition was equivalent to the Standard condition of Experiments 1 and 2. This time the participants had to estimate *the probability that the ball had fallen through one of the three vertical tubes*, each of which ended in a different opaque box. As in the standard condition of Experiments 1 and 2 the steps were identical to the MHP (that is, the participant's

Figure 4. Conditions and results in percent for Experiment 3.

Condition **N**	**Standard** **24**	**Focusing** **24**	**Updating** **24**
Materials			
The boxes are masked			
Stage 1 “Please choose a tube”			
First judgment: “What is the probability that the ball fell through the tube that you have selected?”			
Answer 1/3	100%		
Stage 2 “I will show you, among the two other tubes, one tube through which the ball did not fall”			
Second judgment: “What is the probability that the ball fell through the tube that you have selected?”			
Answer 1/3[a]	4.2%	79.2%	0%
Answer 1/2[b]	95.8 %	8.3%	100%
Answer a, with a < 1/3	0%	12.5%	0%

a Corresponds to the Bayesian Solution

b Corresponds to the solution by the Minimal rule

choice of a tube, the experimenter's notice that he would indicate one tube among the two non chosen tubes through which the ball had not passed, with the same explicit explanations and designation of the tube).

Participants could initially manipulate the tubes to see for themselves that the material was not rigged. In particular, they could observe that when the ball was introduced into a straight tube it fell (with certitude) straight into the box underneath. The experimenter asked participants to leave the room. When the participants were back, the three boxes had been concealed by the screens so that participants could not know through which tube the ball had fallen. The experimenter asked a pseudo-participant to choose one tube. In fact, this person acted as a confederate who always chose the middle tube to match the two conditions below (and was of course discounted from the genuine participants). The experimenter asked: "*What is the probability that the ball fell through the selected tube?*" The experimenter then pointed to the two other tubes and said: "*I will show you one tube --out of these two tubes-- through which the ball did not fall*" and he removed a screen to reveal one of the empty boxes, following which he asked: "*What is the probability that the ball fell through the tube that you have selected?"*

In the Focusing and Updating conditions there were two boxes and three tubes were also displayed in front of the participant. As in the Standard condition the two straight tubes had their extremity ending in a box; the Y-shaped tube was inverted and had each of its branches ending in one of two boxes (see Fig. 4). This display allows a visual representation of the biprobabilistic structure of the MHP. In particular, if the ball was placed in the middle tube (the inverted Y-shaped tube), it could fall into either the box that was common with the left tube or the box that was common with the right tube. If the question concerns the probability that the ball has fallen through the middle tube (whose end is Y-shaped), one gets back to the MHP structure. As in the

Standard condition, participants initially manipulated the tubes to see that the material was not rigged. In particular, they could observe that when the ball was introduced into the inverted Y-shaped tube it could fall (in a symmetric manner) either into the left box or into the right box. In addition, the experimenter specified that when the ball was introduced into the inverted Y-shaped tube, it had the same probability (1/2) to fall into either box.

In the Focusing condition the experimenter asked participants to leave the room and masked the two boxes with the screens**.** One participant was asked to choose a tube (this participant was a confederate who always chose the Y-shaped middle tube). The experimenter asked participants "*What is the probability that the ball fell through the tube that you have selected?"* Then the experimenter designated the other two tubes and said: "*I will show you one tube--out of these two tubes--through which the ball did not fall"*. Following this, the experimenter removed one screen (see Fig. 4) and revealed that one box was empty.

The Updating condition was the same as the Focusing condition except that after uttering "*I will show you one tube--out of these two tubes--through which the ball did not fall"* the experimenter selected one of the two straight tubes (see Fig. 4) and removed it; then he asked, "*What is the probability that the ball fell through the tube that you have selected?"*

Results and discussion

If participants have the same representation in the Standard and the Updating situations their answers will be the same. So we expect each participant to give a revised probability of 1/2, in agreement with the Minimal rule. This is what was observed in the crushing majority of the cases as 23 our of 24 participants revised their estimate from 1/3 to 1/2 in both conditions together. Similarly, if participants have alternative representations in the Standard and the Focusing situations they will tend to give the specific answers determined by their representation. So we expect each participant to give a revised probability of 1/3 in the Focusing condition in accordance with the Bayesian solution. This too was observed in the great majority of the cases as 19 out of 23 participants (79%) maintained the probability of 1/3 in the Focusing condition (while updating to 1/2 in the Standard condition). Both results are highly significant (sign test, $p< 10^{-3}$). The distributions in percent given in Fig. 4 shows the near identity of the Standard and Updating conditions and the strong reversal between the Standard and Focusing conditions. These results are born out by participants' justifications of their similarity judgments: For the Standard and Updating conditions most participants (79%) judged the two conditions equivalent, typically justifying this by saying that in both cases the problem is transformed into a new problem in which one just considers the possibility that the ball comes from one of the two remaining tubes. So, for these participants the three conditions are equivalent *before* the experimenter's action. This means that (i) the representation of the problem in the Standard condition is viewed as equivalent to the explicit biprobabilistic representation in the other two conditions. This is another confirmation that the participants have correctly understood and taken into account the host's strategy (represented explicitly here by the biprobabilistic structure) at the beginning of the experiment.; and (ii) what creates the judgment of equivalence is whether there has been a physical transformation of the initial problem. So, *after* the experimenter's action, the focusing condition according to which participants were aware that no physical transformation of the initial problem had taken place was perceived as non equivalent to the Standard and Updating conditions. These results bring further evidence that for participants in the Standard condition interpret it from an updating point of view.

Finally, in this experiment administered to participants who were not ignorant of probability theory almost everybody failed to give the Bayesian solution in the Standard condition, exactly like in the previous experiment in which participants were ignorant of probability theory. This shows that there was nothing peculiar that would distinguish the former population from the latter. It might be objected, though, that the high rate of production of the Bayesian solution in the Focusing condition was favoured by the educational level of the participants. This does not seem to hold given the limited cognitive ability and the very elementary understanding of probability that are required to answer correctly in the Focusing condition. After the screen has been removed by the experimenter to reveal an empty box and participants have inferred that the ball has fallen into the other box, which is still screened, they must (i) understand that there are twice as many chances that the ball has been introduced into the straight tube as into the inverted Y-shaped tube (as participants have been familiarised during the initial phase of manipulation), and (ii) then normalise to one, that is, reckon that one chance for an event against two chances for the other event amount to a probability of one third for the first event. Technically, normalisation requires an application of the constraint of additivity of probability and in principle it could be a source of difficulty for novice participants if the probability distribution becomes complicated. However the literature shows that very few participants violate this constraint when they are confronted with two complementary hypotheses made explicit at the same time (for example see Baratgin & Noveck, 2000). Nevertheless, to ascertain that there was no effect of educational background we have carried out a replication of Experiment 3 with a population that had not received training in probability at the university level. Nineteen students attending the second year of Computer Science at the University of the Mediterranean received the three conditions (Standard, Focusing and Updating). The results were very close to those found in Experiment 3. Every participant gave the response 1/2 for their second estimate in the Standard and Updating conditions whereas sixteen of them (84%) gave 1/3 in the Focusing condition.

The Bayesian Solution was given by a great majority of participants (79% in Experiment 3 and 84% in its replication). To the best of our knowledge such high percentages have never been observed. This indicates that if participants are helped to form the correct focusing representation of the problem (the same as the experimenters' representation), they tend to be Bayesians. In Experiment 4 we aim to generalise this result to a more counterintuitive version in presenting a problem with unequal prior probabilities. Indeed, the result of some experiments in which the problem is formulated with explicitly unequal prior probabilities is that participants give the Bayesian Solution even less often than they do with the standard MHP or its relevant isomorphic versions (Granberg, 1996; Ichikawa, 1989; Ichikawa & Takeichi, 1990; Yamagishi, 2003).

Experiment 4: Focusing and Updating Conditions in the Three-Tube Problem with Unequal Prior Probabilities

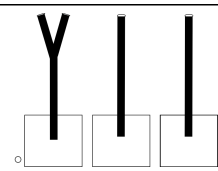
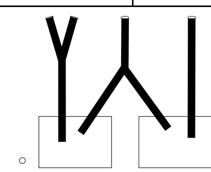
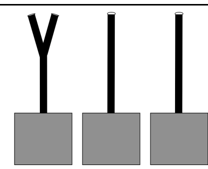
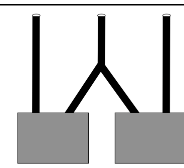
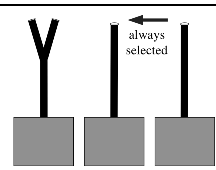
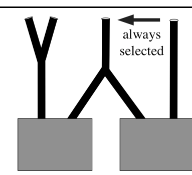
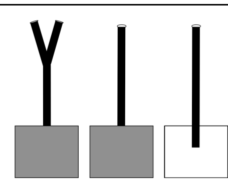
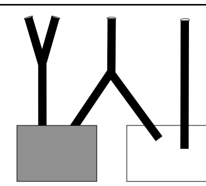
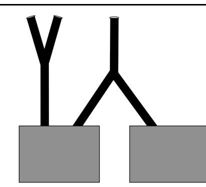
The method differed slightly from Experiment 3: The material was modified to make the prior probabilities equal to {2/4; 1/4; 1/4} for the three tubes, respectively. In this case the Bayesian Solution is {4/5; 1/5; 0} and the Minimal rule solution is {2/3; 1/3; 0}.[10]

Method

Participants (N = 12) were selected from the same population (with basic knowledge of probability theory) as for Experiment 3 but none of them participated in both experiments. They were tested by groups of two. The three conditions of Experiment 3 were modified as follows. One of the two tubes previously straight was now Y-shaped *at the top,* so that there were twice as many chances for the ball to fall through that tube as there were for it to fall through each of the two other tubes (see Fig. 5). Thus the prior probabilities were explicitly 2/4, 1/4 and 1/4. The experimenter always revealed the box corresponding to the last tube on his right so that for the three problems it was the tube with a prior of 1/4 that was the locus of a transformation. Consequently, the result was 1/5 in a focusing interpretation and 1/3 in an updating interpretation. Again all possible orders of presentation were used.

Results and Discussion

Figure 5. Conditions and results in percent for Experiment 4

Condition **N**	**Standard** **12**	**Focusing** **12**	**Updating** **12**
Materials			
The boxes are masked			
Stage 1 "Please choose a tube"			
First judgment: "What is the probability that the ball fell through the tube that you have selected?"			
Answer 1/4	100%		
Stage 2 "I will show you, among the two other tubes, one tube through which the ball did not fall"			
Second judgment: "What is the probability that the ball fell through the tube that you have selected?"			
Answer 1/5[a]	8%	67%	0%
Answer 1/3[b]	92 %	0%	83%
Answer a, with a < 1/3	0%	33%	17%

a Corresponds to the Bayesian Solution

b Corresponds to the solution by the Minimal rule

The distributions of the answers for the three conditions are given in Fig. 5. Results are similar to those of Experiment 3. All the participants in the three conditions gave 1/4 as the prior probability for the tube they chose. For the second estimate, 9 out of 12 participants gave the Minimal rule solution of 1/3 in both the Standard and Updating conditions. The Focusing condition gave rise to a reversal as 10 out of 12 participants gave both the Bayesian solution 1/5 in the Focusing condition and the Minimal rule solution 1/3 in the Standard condition. Sign tests applied to these frequencies indicate that the latter result is significant ($p<.02$) but the former is only close to significance ($p<.073$), presumably for lack of statistical power due to the small numbers. However, any doubt regarding the reliability of the results can be eliminated by considering participant's justifications for their comparison judgments, like for the previous experiment. The justifications routinely supported the Minimal rule. For the Standard as well as the Updating condition they expressed that there was one possibility left for a tube and two for the other. Eight participants judged these two conditions equivalent. By contrast, in the Focusing condition eight participants gave the Bayesian Solution (1/5) accompanied by a justification that agrees with Bayes' rule to the effect that there was one possibility for a tube and four for the second; three participants explicitly expressed that the probability had decreased but they had difficulty giving a quantitative value, which suggests that they may have experienced difficulty in normalising the probabilities; the last answer was 1/4 justified by the notion that nothing has changed with the tube.

Burns & Wieth (2004) showed an improvement in participants' performance with the MHP versions where the biprobabilistic structure is explicit. Experiments 3 and 4 generalise this hypothesis in underscoring that when this is the case, participants also distinguish between focusing and updating interpretations of the message: In these two conditions, the relations between layers are revealed by the experimenter's message. When the screen is removed by the experimenter, there is no change in the structure of the problem and participants focus on the subset of different possibilities for the ball to fall into the other box. But when a straight tube is removed, the structure of the problem is modified and participants give their new probability distribution corresponding to this new problem. In contrast, participants clearly identify the Standard condition as equivalent to the Updating condition.

In brief, the result of the last two experiments is that participants answer rationally as a function of the perceived situation of revision. As we have already argued, this proceeds from the individual's spontaneous adoption of the naive set with one degree of belief composed of the three tubes. Participants carry on a simplified updating strategy by considering the set composed of the two tubes that are not invalidated by the message. However, as illustrated by the example of the fruit basket, with a one-level of belief set the situations of focusing and updating are indistinguishable: In this case the Minimal rule is equivalent to Bayes' rule.

General Discussion

Many psychologists, in the wake of Edwards, Lindman, & Savage (1963), hold the Bayesian model as the behavioural norm for human probability judgment (Chater & Oaksford, 2008). This choice implies that probabilities are interpreted as subjective degrees of belief and that Bayes' rule is adopted as the revision rule (de Finetti, 1964). The adoption of a Bayesian standpoint for the study of the revision of probability judgment has two consequences. The first consequence is to impose a theoretical limitation on the scope of experimental studies of revision: The use of Bayes' rule restricts one (by definition) to studying revision in a *stable* universe of reference. Thus, psychologists working in the field of probability revision have never considered the conceptual differences between the situations of revision that have been defined in AI, philosophy and cognitive economics. The quasi-totality of the experimental paradigms addresses focusing (for a review, see Baratgin & Politzer, 2007), whereas updating has not been explicitly considered.[11] The other consequence is that if theorists do endorse the subjective nature of degrees of belief as implied by the Bayesian model, then they must take into account participants' expectations and attributions of mental states to the experimenter such as intentions and epistemic states. This is why a pragmatic analysis of all the components of the method (design, content of the task, materials, administration, response format) is crucial (Baratgin & Politzer, 2006).

In the present investigation we have taken a serious view on these two consequences: considering an evolving universe and integrating the pragmatic constraints. We take up these two issues in turn. Regarding the former we will show, in the next sub-section, the interest of our approach at a meta-theoretic level as the various attempts made in the literature to explain the MHP paradoxical results can all be recast within the framework of updating. Regarding the latter, it was mentioned earlier that they are crucial to characterise the MHP, a claim that we are going to vindicate in the second sub-section.

Viewing the universe as evolving

The different explanations of participants' response found in the literature can be reanalysed as evidence for an updating interpretation of the host's message. We present four arguments in support of this view (for technical points see Baratgin, 2009).

1. The explanations based on the use of "simple heuristics" (Ichikawa, 1989; Ichikawa & Takeichi, 1990; Shimojo & Ichikawa, 1989; Yamagishi, 2003) can be reinterpreted, from a mathematical point of view, as the application of the Minimal rule to different interpretations.

2. The explanations based on the mental models theory according to which people would not be able to construct a complete set of mental models of the puzzle due to limitation in working memory (Girotto & Gonzalez, 2000; Johnson-Laird, et al., 1999) or on the partition-edit-count strategy according to which people would evaluate conditional probabilities by *subjectively partitioning* the sample space into a set of interchangeable events (Fox & Levay, 2004) can also be viewed as descriptions of a process of revision using the Minimal rule (see section 3 of the Appendix). In other words, the participants who have initially taken into account and understood the host's strategy will not take them into consideration to evaluate the new probability distribution in an updating interpretation. In particular when the experimenter explicitly specifies the strategy of the TV host as well as the rules of MHP, participants still give the modal response (Baratgin & Politzer, 2007; Burns & Wieth, 2004; Krauss & Wang, 2003; Shimojo & Ichikawa, 1989).

3. According to Glymour (2001) people have difficulty comprehending the causal structure underlying the MHP and subsequently fail to recognise *the collider principle*: The choice of the host to place the car behind a door and participants' initial selection (here D1) determines which door the host will then open to show a goat. This can be reinterpreted by noticing that in updating there cannot be a collider principle because the problem becomes a simple problem (symmetrical) of two doors. It follows that participants neglect the collider principle (Burns & Wieth, 2004), and also have the general belief described in Shimojo (1989) that *the posterior probabilities of the remaining alternatives should never be less than their prior probabilities when one alternative is removed.*

4. The cases of improvement reported in the literature can be reinterpreted and explained as cases where the manipulation encourages the interpretation of the host's choice to show a goat behind

door D3 (out of the two doors D2 and D3) as a focusing message rather than an updating message. Various experimental studies show that probability judgments tend to conform better to Bayes' rule when the experimental paradigm refers to natural frequencies (as opposed to single-event probabilities) (Brase, Cosmides, & Tooby, 1998; Cosmides & Tooby, 1996; Gigerenzer & Hoffrage, 1995; Hoffrage, Gigerenzer, Krauss, & Martignon, 2002), when the distributions of probabilities are formulated in term of chance (Girotto & Gonzalez, 2001) and more generally when the "nested sets", that is, subsets relative to larger sets in the task structure, are explicit (Sloman, Over, Slovak, & Stibel, 2003). This is the case with the isomorphic problems of the MHP written in a natural frequency format (Yamagishi, 2003) and also with the standard statement when the experimenter gives a diagram that illustrates the biprobabilistic structure and the focusing character of the revision message (Cheng & Pitt, 2003; Girotto & Gonzalez, 2000; Ichikawa, 1989; Yamagishi, 2003). However these specific formats which facilitate the Bayesian solution actually both clarify the biprobabilistic structure and enhance the focusing understanding of the situation of revision.

Pragmatics constraints

It was assumed earlier that the experimenter and the participants may not have the same representation of the revision process (focusing vs. updating). In support of this assumption, we develop three points, the last two being based on relevance theory (Sperber & Wilson, 1995).

1. As explained earlier, the focusing situation is an atemporal situation of probability revision. After a *focusing* message individuals modify their frame of reference in considering a smaller set, but they stick to their initial probability distribution that encompasses the whole initial set. The situation of focusing is easier to comprehend when one can have a statistical interpretation of the problem, for example when imagining a sufficiently large number of games (Dubois &

Prade, 1992). However, the MHP is proposed as a “one-shot” game where a situation of temporal change is suggested. At time t_0 participants are invited to work out their prior distribution; at time t_1 they learn a new message implied by the host’s action that seems to modify the initial problem structure (shifting from three to two closed doors); then the experimenter asks a judgment of probability that can be interpreted as a request for a brand new probability distribution. In more intuitive terms, participants may think that because the problem has changed, so has the probability.

2. The Bayesian Solution proves in contradiction with pragmatic principles of communication. In the present case, participants make their initial choice of a door (here D1) without any special information related to this object. They base their choice on the sole information, “The car is hidden behind one of the three doors”. The uncertainty leads them to the attribution of a probability of 1/3 to door D1. Within the framework of relevance theory, the communicative principle posits that any utterance conveys a presumption of its own (optimal) relevance. In other words, participants assume that if the experimenter sends a message, it deserves to be treated. Thus for participants any new information given by the experimenter carries a presumption of relevance in relation with the aim of the problem (to know if the car is behind D1). In fact, the new information can be expected to be all the more important (relevant) as participants have initially chosen the door randomly, and this knowledge is shared with the experimenter. Consequently, they are strongly invited to modify their prior probability regarding the door that is the most relevant, namely D1. But in the focusing situation the learning of the new information (the host’s choice to show a goat behind door D3, out of the two doors D2 and D3 not chosen by the participant) does not change participants’ initial degree of belief that the car is behind door D1, so that the Bayesian Solution (1/3) is countermanded by the principle of relevance. It only

modifies participants' initial degree of belief that the car is behind one of the doors D2 and D3. The experimental situation is akin to the well-known experiment by McGarrigle and Donaldson (1975) on the conservation of number in children which demonstrates the effect of repeating a question after the experimenter's intervention: This results in the child's presuming that the response must change.

3. Interpreting the host's choice to show a goat behind door D3 (out of the two doors D2 and D3) as an the updating message, "the possibility that the host shows that there is a goat behind D2 has been suppressed", achieves greater relevance than interpreting it as a focusing message. According to relevance theory, participants are more inclined to pay attention to messages with an important contextual effect and with an expected treatment that requires little cognitive effort. The host's choice to show a goat behind door D3 arrives after the host's notice that one of the two doors conceals a goat, whose implied meaning is "one of the two doors (D2 or D3) will be excluded". This information conveys a strong contextual effect with little cognitive effort (world D2 or D3 will be eliminated by the host). It actually comprises two compacted pieces of information. The first piece, as we have just mentioned, concerns the host's choice of one door among D2 and D3. The second piece concerns what is behind D3, namely a goat. The first element constitutes the key to construct the representation of the puzzle in a *focusing* framework but the comprehension of its consequences requires a considerable cognitive effort because participants must work out the alternative possibility that the host shows a goat behind D2, and then keep this in memory (even after the host's action and the perception that D2 is open). This element has no important contextual effect because it is difficult to apprehend its relevance with respect to the aim of the puzzle if participants have not constructed the full representation of the puzzle. On the contrary, the second element of the message (its ostensive part, whatever it may

be, to show a goat behind door D 2 or to show a goat behind door D3) is expected by participants as it follows the participants' interpretation of the verbal part of the host's message "one of the two doors (D2 or D3) will be excluded". Consequently, the second element of the message benefits from an important contextual effect in concretely eliminating the door D3 from the set of possible doors. Participants are then left with two doors (D1 and D2).

Conclusion: A more optimistic view on participants' consistency in focusing situations

Our experiments strongly support the hypothesis of an updating process in human probability revision. Experiment 1 indicates that updating seems a situation that can be routinely induced by the context. Experiment 2 indicates that updating seems to be the usual interpretation in our MHP isomorphic problem. In this updating interpretation, participants answer in agreement with the Minimal rule. Experiments 3 and 4 show that people can distinguish the situations of focusing and updating when the biprobabilistic structure of the problem is explicit and that they produce their answer accordingly, that is, the Bayesian answer in the case of focusing and the answer based on the Minimal rule in the case of updating.

More generally, these experiments illustrate the importance of analysing the interpretation of the revision message prior to the experimental study. We believe that a complete experimental program designed to study probability revision should take into account the different situations of revision and adapt its methodology accordingly. In the present paper we have concerned ourselves with focusing and updating. Indeed, psychologists have not distinguished these situations of revision, with the possible exception of causal reasoning in which the notion of intervention reveals that updating appears natural to lay people. Moreover, such a complete experimental program might have to go beyond the restricted framework of additive probabilities.

The overwhelming majority of the studies have concerned themselves with focusing situations. In these studies experimenters often interpret the term "Bayesianism" as a simple synonym of Bayes' rule techniques (for a review see Baratgin & Politzer, 2006). In most cases the question addressed is whether participants perform in agreement with Bayes' rule. The participants' probabilities (prior and likelihoods) are assumed to correspond to the fixed values provided by the experimenter in the instructions. The discrepancies found with the "true result" defined by experimenters are analysed as evidence for human irrationality (Piattelli-Palmerini, 1994) or as evidence for the inadequacy of the Bayesian model (Lopes, 1991). A number of cognitive explanations of participants' responses have been proposed and debated: Heuristics (Kahneman, Slovic, & Tversky, 1982), frequentist format (Gigerenzer & Hoffrage, 1995), nested-set relations (Sloman, et al., 2003) or mental models (Johnson-Laird, et al., 1999). The experimental results that have been exploited in the debate on participants' rationality and the robustness of the Bayesian model can be reinterpreted in the light of the classification of the three situations of revision: The statement of the problem used in each experimental paradigm is open to an interpretation in terms of one of these situations of revision. As these involve different rules, it is clear that a response incorrect under one interpretation may turn out to be correct under another interpretation.

Our investigation started from the observation that a large majority of paradigms used in the experimental literature corresponds to a situation of focusing, but none *explicitly* corresponds to a situation of updating. Thus, there exists a significant imbalance in the various studies of probability revision. The lack of consideration for the situation of updating can also invite one to question the very results of these studies because an answer considered as erroneous in a situation of focusing may well be viewed as coherent in a situation that is interpreted as updating. In the present study, we have identified the pragmatic origin of the updating interpretation of a focusing situation. It remains for future work to determine what factors other than pragmatic (such as individual, social, etc.) may induce people to favour either interpretation.

Finally, two points at a metatheoretical level are in order. One, the conceptual approach that consists of distinguishing several situations of revision together with the restriction of the application of Bayes' rule is in no way a critique of Bayesianism and of the importance of Bayes' rule. The fundamental question which we consider is 'how to revise when the world/problem has undergone a transformation?' This situation is, classically, kept aside from the realm of Bayesian revision (see de Finetti, 1974). The new approach offers an enrichment to standard Bayesianism, and what is called the *Minimal rule* should not be understood as an alternative to Bayes' rule, but rather as a complementary rule applicable to the situation in question where Bayes' rule just does not apply. Two, as suggested above, the distinction between focusing and updating enables one to reinterpret the various explanations of performance on the MHP found in the psychological literature. Our approach helps reveal their implicit hypotheses or missing justifications. It does not constitute a challenge to these theories but rather a parsimonious way of analysing and characterising them.

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Footnotes

[1.] There is also another revision situation in a stable universe (not considered in this paper) that is called in the literature *revising* (Alchourron, Gärdenfors, & Makinson, 1985). When revising you do not know precisely the class of reference (your beliefs are erroneous or incomplete) and the message you learn specifies or disconfirms your initial belief.

[2.] There are several specifications of "Bayes' rule". Although these specifications are equivalent, in this paper it appears important to distinguish two major forms. Let H and $\neg H$ be two alternative hypotheses, let M be one event such that $M \neq \varnothing$, P(H) the prior probability of H, P(H|M) the posterior probability, P(M|H) and $P(M|\neg H)$ the likelihoods and finally P(M) the probability of message M.

Form 1, "conditional probability": $P(H \mid M) = \dfrac{P(H \wedge M)}{P(M)}$,

Form 2, "Bayes' identity": $P(H \mid M) = \dfrac{P(H)\,P(M \mid H)}{P(M)}$.

Bayes's rule is a revision rule that is defined only for a stable universe. It is the only possible rule in a focusing situation. It operates a change of reference class, namely it leads from a prior to a posterior probability (de Finetti, 1974). Bayes' rule also proves an adequate rule (among others) to change degrees of belief in a situation of revising (Walliser & Zwirn, 2002).

[3.] From a theoretical point of view, this rule is a particular instantiation of *Lewis's General Imaging rule* (Cross, 2000; Dubois & Prade, 1993; Gärdenfors, 1988; Lepage, 1997; Lewis, 1976; Perea, 2007).

[4.] The MHP is usually presented in two versions. One standard version originates from a popular television game show, *Let's Make A Deal,* programmed in 1963 in America. It is set up as a problem of choice where participants decide to switch or keep the door initially chosen (vos Savant, 1990). An older version called the Three Prisoners Problem is cast in a probabilistic format (Gardner, 1959). In this paper we analyse the puzzle in a probabilistic format.

5. This solution of the MHP was first suggested by Dubois & Prade (1992).

6. Numerous studies of probability judgment have supported the hypothesis that participants infer a different representation of the task than experimenters do, based on various pragmatic considerations (Dulany & Hilton, 1988; Krosnick, Li, & Lehman, 1990; Macchi, 1995; Politzer & Macchi, 2005; Politzer & Noveck, 1991).

7. We borrow here the term *naive* from the theorists who differentiate a Bayesian conditioning on a *naive set* and a Bayesian conditioning on a more *sophisticated* set in the MHP (Grünwald & Halpern, 2003; Jeffrey, 1988). Independently, it was also used by some psychologists in a similar sense (Fox & Levay, 2004; Johnson-Laird, et al., 1999).

8. Unlike the first two experiments, in this and the next experiment the numbers were relatively small. The first two experiments were run with larger numbers because we used novel materials, so that the base rate of incorrect answers and the order of magnitude of the effects to be demonstrated were unknown.

9. A number of academic people, including a world famous mathematician, wrote to Vos Savant (1991) to express their disagreement that 1/3 was the correct answer to the MHP (see also Vazsonyi, 1999).

10. The solutions 1/5 in the focusing situation and 1/3 in the updating situation are obvious by changing the prior probabilities (1/3, 1/3, 1/3) to (2/4, 1/4, 1/4) in Fig. 6.

[11.] A very small minority of studies deals with the situation of *revising* (see note 1) (Baratgin & Politzer, 2007). Updating has already been considered explicitly in a *deductive* framework, in connection with the problem of belief revision for knowledge bases (Elio & Pelletier, 1997; Politzer & Carles, 2001), and implicitly in the field of counterfactual reasoning: In effect, a counterfactual statement does express a modification of the universe, albeit virtual. Finally, the situation called *intervention* in the field of causal reasoning (Pearl, 2000) can be considered as a special case of updating. A few experimental studies on causal reasoning have shown that the situation of intervention seems natural for participants (Lagnado & Sloman, 2004, 2006; Sloman & Lagnado, 2005; Steyvers, Tenenbaum, Wagenmakers, & Blum, 2003).

Appendix: The focusing and updating solutions of the MHP

1. A two-level probabilistic structure

The MHP (after the participant has chosen door D1) is a problem characterised by a hierarchical belief structure. Participants must have beliefs about elementary properties of doors (to be opened by the host and to have the car or a goat behind them). The existence of different layers in the MHP can be made explicit and its analysis in terms of possible worlds can easily be visualised by a "*two-level probabilistic structure* " of belief (Walliser & Zwirn, in press) as

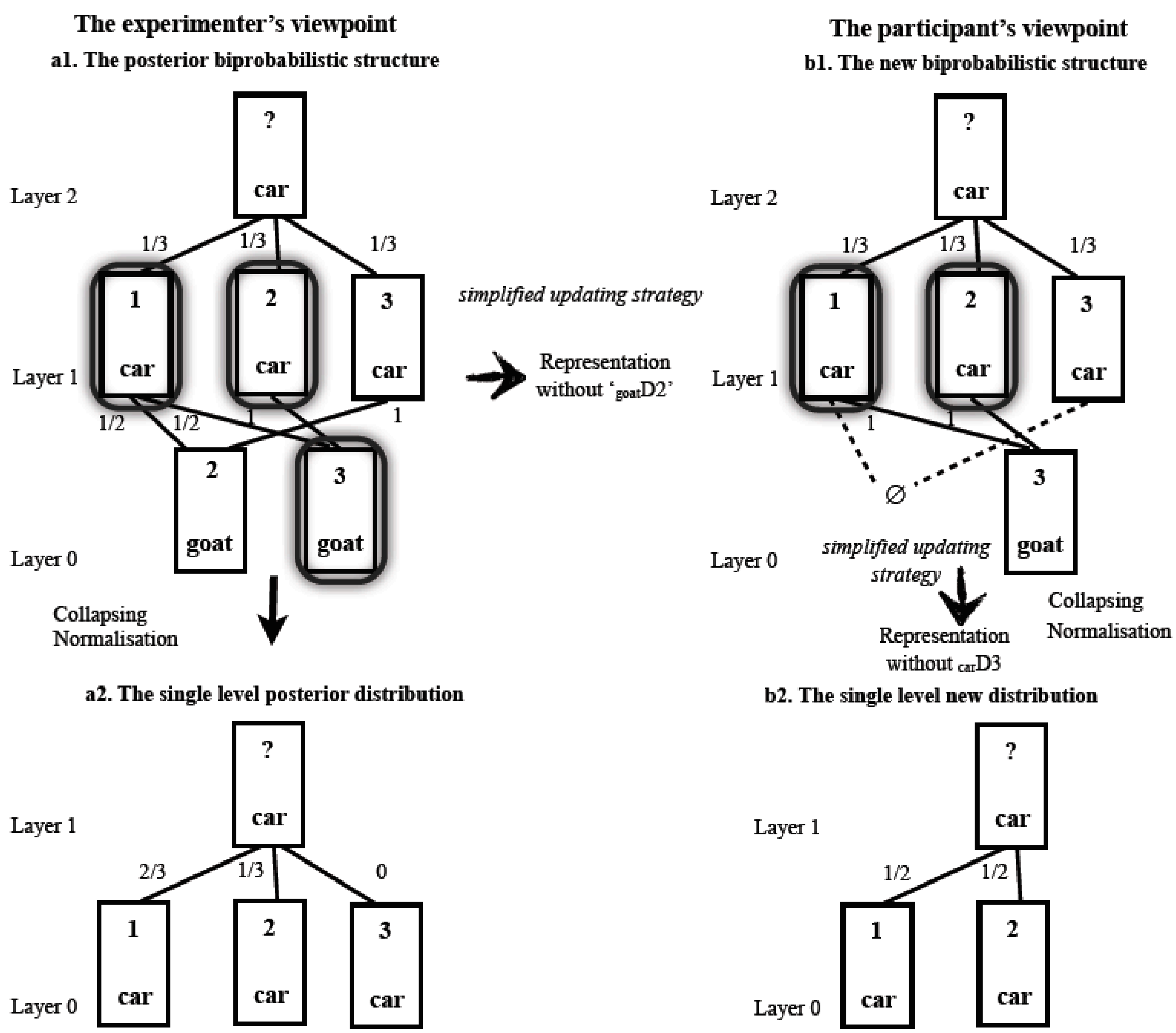

Figure 6. Belief structures of the Monty Hall Puzzle after the focusing message '$_{goat}$D3'.

follows (see Fig. 6(a1)):

- A layer 0 composed of the two possible choices for the host to show a goat behind one door (out of the two doors D2 and D3 not chosen by the participant) and noted respectively with single quotation marks '$_{goat}$D2' and '$_{goat}$D3'.

- A layer 1 composed of the three possibilities that the car is behind each of the three doors. It corresponds to the following worlds: {$_{car}$D1, $_{car}$D2, $_{car}$D3}.

- A layer 2 composed of the initial choice of a door made by the participant (D1).

The two probability distributions from layer 1 to 0 and from layer 2 to 1 are defined by the statement of the MHP. The layer 2 (the actual world) defines a probability distribution on three layer 1 worlds (the three possibilities corresponding to the presence of a car behind a door) and each layer 1 world defines a probability distribution on two layer 0 worlds ('$_{goat}$D2' and '$_{goat}$D3'). The probability distributions can be inferred from the statement of the MHP with implicit and explicit hypotheses about the host's action. In the beginning (corresponding to point (i) of the MHP statement) it is assumed that the host has no preference for a specific door when he initially places the car. The prior probabilities of the three possible worlds $_{car}$D1, $_{car}$D2 and $_{car}$D3 are equal to 1/3 ("all three equally likely"). It is also assumed that the host has no preference between doors D2 and D3 should the car be behind D1 (which is implicit in point (ii) of the statement in the majority of the versions of the puzzle). The probabilities of the two possible worlds '$_{goat}$D2' and '$_{goat}$D3' are 1/2 when world $_{car}$D1 is the actual world. Finally we know that when the car is behind D2, the host is bound to show D3 and when the car is behind D3 the host is bound to show D2 (see Fig. 6a1). We note that this two-level probabilistic structure can be collapsed into an equivalent one-level probabilistic structure with the set of four possible worlds (called "reference class" (Halpern, 2005)) obtained by crossing the layer 1 world and the layer 0 world: { $_{car}$D1∧ '$_{goat}$D2', $_{car}$D1∧ '$_{goat}$D3', $_{car}$D2∧ '$_{goat}$D3' (= $_{car}$D2), $_{car}$D3∧ '$_{goat}$D2' (= $_{car}$D3)} with the prior probabilities {1/6, 1/6, 1/3, 1/3}.

2. The coherent solution 1/3 in the focusing interpretation.

As mentioned in the introduction, focusing *does not really correspond to a temporal process of belief revision*. It is assumed that one object is selected from the universe and that a message releases information about this selected object. Then a reference class different from the initial one is considered by focusing attention on a given subset of the original set, in the present case focusing on the host's choice of D3 as opposed to all the choices (D3 or D2). After the message '$_{goat}$D3', you still consider the same probability distribution but you change the reference class of the whole initial set of possible worlds. You *focus* your attention on the subset of the original system in which world '$_{goat}$D3' is the actual world. This means: (i) executing the collapsing process that has just been mentioned in order to specify the reference class {$_{car}$D1∧ '$_{goat}$D2', $_{car}$D1∧ '$_{goat}$D3', $_{car}$D2∧ '$_{goat}$D3', $_{car}$D3∧ '$_{goat}$D2'}; (ii) focusing on those (crossed) worlds in the reference class that are consistent with the message (in bold) {$_{car}$D1∧ '$_{goat}$D2' (=1/6), **$_{car}$D1∧ '$_{goat}$D3'(=1/6)**, **$_{car}$D2∧ '$_{goat}$D3'(=1/3)**, $_{car}$D3∧ '$_{goat}$D2'(=1/3)} and (iii) normalising the probabilities of the focused worlds (see Fig. 6a2). Thus your probability is still 1/3 because there are twice as many chances for $_{car}$D2 to be the actual world as there are for $_{car}$D1 (the probabilities before normalisation are 1/3 and 1/6, respectively). In brief, to give the Bayesian Solution, participants must (i) build the correct hierarchical structure of Fig. 6(a1) and (ii) interpret correctly the message 'goatD3' in a focusing framework.

3. The coherent solution 1/2 in an updating interpretation.

We now consider the representation which leads participants to the modal answer. This solution is obtained after receiving the message interpreted as "the possibility that the host shows that there is a goat behind D2 has been suppressed” (noted ~~'$_{goat}$D2'~~). This message gives information about the possible transformations of the actual universe. The revision must be obtained by the Minimal rule. As we have seen with the example of the fruit basket, applying the Minimal rule to obtain the new reference class consists of, first excluding the possibility '$_{goat}$D2', and second normalising to obtain the new probability distribution corresponding to the representation of the new universe. So you consider first a new MHP where the host does not open D2 (noted ∅, see Fig. 6(a1) and 6(b1)). Hence the new reference class is {$_{car}$D1∧'$_{goat}$D3', $_{car}$D2∧ '$_{goat}$D3', $_{car}$D3∧∅} associated with the probability distribution {1/3, 1/3, 1/3}. Now if you know that there is a goat behind D3, this message constitutes a *focusing* message on the new universe and you focus on D1 and D2 because you are certain that the car is behind the doors D1 or D2. The solution is 1/2.

We can easily identify the relationship that obtains between the two revision situations and the two revision rules (for a theoretical approach see Walliser & Zwirn, in press). The situation of updating on the first reference class at t_0 {$_{car}$D1∧‘$_{goat}$D2', $_{car}$D1∧‘$_{goat}$D3’, $_{car}$D2∧‘$_{goat}$D3’, $_{car}$D3∧‘$_{goat}$D2’} when it is known that ~~'$_{goat}$D2'~~ actually is equivalent to a focusing situation on a second reference class at t_1 {$_{car}$D1(= $_{car}$D1∧ '$_{goat}$D3'), $_{car}$D2(= $_{car}$D2∧ '$_{goat}$D3'), $_{car}$D3(= $_{car}$D3∧∅)} in which the received information is “the car is not behind D3”. So, *the Minimal rule* applied within the reference class at t_0 *leads one to conditioning by Bayes’ rule after the message* on the reference class at t_1 {$_{car}$D1, $_{car}$D2, $_{car}$D3}. This solution amounts to neglecting the relation (inferred from the knowledge of the host's strategy) from layer 1 to layer 0 and considering only layer 1. This amounts to considering first the set {$_{car}$D1, $_{car}$D2, $_{car}$D3} and then focusing on doors D1 and D2 after the focusing message implying that the car is not behind door D3. Hence we obtain 1/2. However participants might reach this solution by another path: Instead of following this focusing strategy, they could follow the simplified updating strategy once again by representing the set "without the door D3" (because they know that there is no car behind D3) so that in the end they get the new set formed by the two doors D1 and D2, and infer an equiprobability between D1 and D2 (see Fig. 6(b2)).

It is noteworthy that the solution 1/3 for the MHP in an updating framework is theoretically possible (Cross, 2000). However this would require a different hierarchical structure. This would be the case if the two possible worlds "the car is behind door D2" and "the car is behind door D3" were regarded as closer to each other than they are to "the car is behind door D1". But this is not the case in our experiments 3 and 4 in which the experimental paradigm which is isomorphic to the three-level structure, explicitly imposes an equidistance between the three possible worlds. These considerations in terms of possible worlds are somehow technical (see Baratgin, 2009) and we cannot develop them in this paper.