Spectrum Sharing Optimization and Analysis in Cellular Networks under Target Performance and Budget Restriction

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***Abstract*—Dynamic Spectrum Sharing (DSS) aims to provide secondary access to under-utilised spectrum in cellular networks. The main aim of the paper is twofold. Firstly, secondary operator aims to borrow spectrum bandwidths under the assumption that more spectrum resources exist considering a *merchant mode*. Two optimization models are proposed using stochastic and optimization models in which the secondary operator (i) spends the minimal cost to achieve the target GoS assuming unrestricted budget or (ii) gains the maximal profit to achieve the target GoS assuming restricted budget. Results obtained from each model are then compared with results derived from algorithms in which spectrum borrowings were random. Comparisons showed that the gain in the results obtained from our proposed stochastic-optimization framework is significantly higher than heuristic counterparts. Secondly, post-optimization performance analysis of the operators in the form of blocking probability in various scenarios is investigated to determine the probable performance gain and degradation of the secondary and primary operators respectively. We mathematically model the sharing agreement scenario and derive the closed form solution of blocking probabilities for each operator. Results show how the secondary operator perform in terms of blocking probability under various offered loads and sharing capacity.**

***Index Terms*—Spectrum sharing, spectrum allocation, merchant mode, spectrum pricing, mathematical programming, aggregated channel allocation algorithms.**

I. Introduction

A. Background and motivation

The static partitioning of spectrum in cellular networks has significant operational implications, (e.g., *pseudo* scarcity of the available radio spectrum) which have been identified by extensive spectrum utilisation measurements [1, 2]. These measurements show that a large part of the radio spectrum, which is allocated to cellular and Personal Communications Service (PCS) use, are quite well utilised, but the utilisation varies dramatically over time and space. Such variation of spectrum utilisation causes the so-called *spectrum holes* [3, 4].

The current static spectrum management must give way to a new approach that breaks down artificial spectrum access barriers and enables networks and their subscribers to dynamically access the spectrum [5–7]. As a response, for example, in the UK there are plans for spectrum liberalisation between operators with different spectrum holdings [8]. Liberalisation of spectrum of the incumbent holders and mandatory spectrum release may lead to some spectrum being under the control of a third party for secondary use. It is also possible that spectrum might be redistributed not only because of such a mandate and realisation but also as a result of secondary market trading [9–12]. Secondary trading of spectrum enhances the overall spectrum utilisation. As a result, network operators would be allowed to release their under-utilised commodities to potential operators [13, 14].

With the large number of service providers in the mobile cellular network industry, each with their own policy and strategy, a variety of spectrum opportunities could be available for secondary use. To this end, in order to distinguish between options of different bandwidth opportunities, incumbent holders of spectrum licenses may broadcast information in relation to these available bandwidths for possible leasing to secondary operators [15]. Part of the information broadcasted by the spectrum holders are in the form of available spectrum size, location boundaries, maximum allowable transmit power, duration of the lease, type of band and admission cost [16].

Operators aim to provide a stable grade of service (GoS) to their end users with their limited allocated spectrum. However, in high demand periods, operators would require additional spectrum. A solution to increase the spectrum by means of sharing has been addressed in the research domain [17–19]. Spectrum sharing between operators often results in a significant improvement of GoS, although it would incur additional costs to the operators [20]. Since network operators often operate with a limited budget, the borrowing decisions of a network operator would be affected. Consequently, the operators would need to make dynamic, on-demand and correct choices of borrowing additional bandwidths from other operators.

Given a market scenario with several operators, rules and conditions of spectrum access, spectrum requirement and their prices, and other parameters, our main idea is to optimize the resource sharing under a target GoS and budget restriction. We propose two algorithms: the first is to optimize the amount of savings that secondary operator could achieve when they engage in spectrum trading with primary operators (incumbent holders of spectrum licenses) to gain a certain threshold of GoS. Second is to optimize the profit of secondary operator under budget restrictions. However, due to the mutual spectrum sharing agreement between the operators,

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the targeted GoS can not be always guaranteed. Therefore, a post-optimization analysis is needed to calculate the actual GoS in terms of blocking probability. Hence, we derive the blocking probability formulae under a mutual agreement to share spectrum where the leased spectrum bandwidth can be deviated according to the operators internal demand. We allow operators to dynamically access or handover part of the shared spectrum according to their internal demand state.

Major contributions of this paper are summarized as follows:

- a novel purchase approach for dynamic spectrum sharing (DSS) network is proposed in the presence of multiple primary service operators. We introduce two optimization problems in *merchant mode* DSS.
- the robustness of the proposed algorithms are investigated in the presence of large number of cells and various types of spectrum bandwidths and the proposed algorithms are compared with heuristic borrowing algorithm. Comparisons show a substantial gain over the heuristic borrowing algorithms and
- a post-optimization analysis technique of the operators' performance (secondary and primary) in the form of blocking probability is derived, which gives the actual GoS of the operators.

B. Related work

In the literature, a great number of studies has appeared in recent years on the design of dynamic spectrum sharing within cellular networks [21–27]. Interests in this context include secondary leasing and pricing strategies among incumbent spectrum license holders, secondary operators and secondary users. These prior studies mainly focused on approaches using *auction mode* and game theory to implement the spectrum pricing and allocation schemes by taking into account the variation of the networks demands and constraints such as power, price and interference [21–26, 28].

In [28], the authors proposes a multiple-dimension auctioning mechanism through a broker to facilitate an efficient secondary spectrum market. In [27] a knapsack based *auction mode* that dynamically allocates spectrum to the wireless service providers such that revenue and spectrum usage are maximised. A dynamic pricing strategy for the service providers is also proposed. Auction schemes where a central clearing authority auctions spectrum to bidders, while explicitly accounting for communication constraints is proposed in [16]. The used techniques are related to the posterior matching scheme, which is used in systems with channel output feedback. While in [29], spectrum auctions in a dynamic setting where secondary users can change their valuations based on their experiences with the channel quality was studied. The authors in [23] investigate price-based resource allocation strategies for two-tier femtocell networks, in which a central macrocell is underlaid with distributed femtocells, all operating over the same frequency bandwidths. A Stackelberg game is formulated to study the joint utility maximisation of the macrocell and femtocells subject to a maximum tolerable interference power constraint at the macrocell base station. Price-based DSS has also been investigated from the business perspective [9, 30]. For example, in [31] An extensive business portfolio for heterogeneous networks is presented to analyse the benefits due to multi-operator cooperation for spectrum sharing. High resolution pricing models are developed to dynamically facilitate price adaptation to the system State. In [32], a quality-aware dynamic pricing algorithm (QADP) which maximises the overall network revenue while maintaining the stability of the network was studied.

The vast majority of the aforementioned studies consider competitive market scenarios and therefore auction and game theory have been discussed to develop DSS strategies. By using the same assumption, pricing in the context of DSS has mainly been considered from the spectrum owners perspective to maximise their revenues [25, 30, 33]. However, when the number of available bandwidths from multiple license owners is higher than SNO's demand, then *auction mode* is not always the best strategy. This is because the number of bidders might be too small and the best selling price can not be achieved for the license owners by using *auction mode*. A more realistic and pragmatic model in this case is a *merchant mode*, which to the best of the authors' knowledge, has not been investigated in the context of DSS. Moreover, spectrum borrowing when considering budget restrictions has not been addressed. Also, there is currently no published work, which attempted to study the admission cost minimisation in the *merchant mode* with target performance. Thus, the problems that we formulate and solve substantially differ from those available in the literature.

The analysis of blocking probability and dynamic aggregated channel assignment has been extensively considered in the context of cellular networks [34, 35]. However, there are significant differences between *auction mode* and the focus of our work. For example, in *auction mode* network operators are not assumed to claim back the leased spectrum within a single trading window during busy intervals [12]; whereas in our approach, the leased capacity is dynamic in size. To the best of our knowledge, our post-optimization analysis is the first to study the blocking probability behaviour during a trading window with the presence of multiple operators. It also addresses the issue of primary operators' change in state during a single trading window.

The paper is organised as follows: the proposed dynamic spectrum management model is described in Section II. Section III addresses the problem of spectrum allocation in cellular networks and describes our mathematical programming formulations to the problem. Section III-G, presents blocking probability analysis under resource sharing with multiple PNOs. In Section IV, we present our findings. Finally, Section V summarises our conclusions.

II. Dynamic spectrum management model

We consider a cellular network to consists of one secondary network operator (SNO) and $\mathcal{N}$, with size $|\mathcal{N}| = N$, denote the set of primary network operators (PNOs) serving a region $\mathcal{R}$, see Figure 1. Let $\mathcal{L}$, with size $|\mathcal{L}| = L$, be the set of cells in the region. Existence of multi-SNOs in a common area results in competition between operators. Such competition analysis and optimization among SNOs is outside the scope of this paper.

Each operator in the network is licensed with an incumbent bandwidth consisting of a set of component carriers, each of which can be allocated to support the operators' subscribers. The antenna towers/masts at the centre of each cell $i \in \mathcal{L}$ is shared among the operators. In the context of this cellular networks arrangement, we only consider cells with an almost identical radio environment, which is visible to all providers in each cell. An example of this setup is when a town or city requires operators to use common towers for their antennas, due to economy of scale property of telecommunication industry.

Due to spectrum liberalisation, the PNOs $|\mathcal{N}|$ will have the freedom to lease their spectrum bandwidths to the SNO. Leasing spectrum bandwidths would mean that the secondary operator will have to pay a certain compensation to the primary operator for using the spectrum bandwidths, and naturally the amount of compensation is expected to be proportional to the amount of allowed spectrum leasing by the primary system. We assume the compensation paid to the PNO is in form of monetary value. The PNOs broadcast specific information about their available bands for leasing and admission cost (per unit bandwidth) at each cell $i \in \mathcal{L}$ at fixed identical intervals (e.g., every 2 hours). The lease conditions may specify additional parameters such as the extent of spatial region for spectrum use and maximum power. The compliant use of leased spectrum requires that the SNO returns the spectrum to the PNO at the end of the lease interval. The duration of each lease could be decided by the network providers under a mutual agreement, and/or any other regulatory bodies' conditions (e.g., minutes, hours, days).

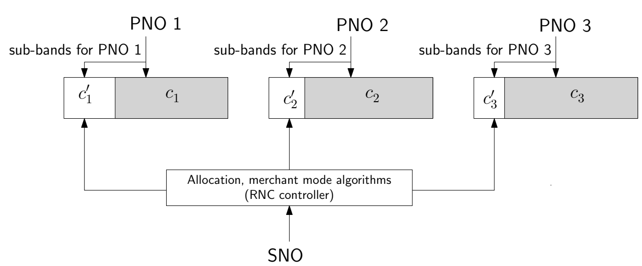


Fig. 1: Network model for cellular network with 3 PNO and 1 SNO

III. Problem formulation

Considering the system model described in the previous section, the problem now becomes how the SNO acquires additional spectrum from PNOs. The spectrum borrowing for an SNO can be performed by considering one of the following objectives:

- to minimise borrowing cost in each time slot by selecting the lowest cost combinations of available spectrum from the primary networks to achieve a specified grade of service (GoS) and
- to maximise profit in each time slot by borrowing the highest profit combinations of available spectrum from the primary networks under restricted budget to achieve a specified grade of service.

In principle, the SNO's objective is to minimise overall operating cost or to maximise revenue as well as to maximise utility to the end users. Next, we formulate each problem as a finite horizon nonlinear stochastic program whose computation time is polynomial in the input size.

A. *Modelling assumptions*

We identify the part of network information which is assumed to be known to the SNO:

- arrival rate of the SNO at ith cell for jth type of spectrum band $\lambda_{i_j},\ \forall i, j$,
- service rate of the SNO at ith cell for jth type of spectrum band $\mu_{i_j},\ \forall i, j$,
- available bandwidth of the SNO at ith cell for jth type of spectrum band $w_{i_j},\ \forall i, j$,
- borrowing cost of the SNO for unit bandwidth from the PNOs at ith cell for jth type of spectrum band $c_{i_j k},\ \forall i, j, k$ (which are assumed to be announced periodically by the PNOs),
- allocated budget for borrowing bandwidths to the SNO at ith cell for jth type of spectrum bandfrom the PNOs $b_{i_j},\ \forall i, j$,
- available bandwidth of the kth PNO at ith cell for jth type of spectrum band $a_{i_j k},\ \forall i, j, k$, (which are assumed to be announced periodically by the PNOs), and
- expected profit of the SNO at ith cell for jth type of spectrum band for borrowing unit bandwidth from kth PNO $\gamma_{i_j k},\ \forall i, j, k$.

Time is divided into equal-length slots $\mathcal{T} = \{0, 1, 2, \ldots\}$. At each time slot $t \in \mathcal{T}$ the process of aggregated channel borrowing is repeated. We use the time indicator (t) to emphasise the vectors dependancy in time. Trading of bandwidth is done between primary and secondary providers separately in each of successive time windows of a particular duration. Henceforth, we focus on the the process of channel borrowing and optimization in a single window.

B. *Notations used in Problem 1 and Problem 2:*

Let us define the following quantities which are used later in mathematical programming problems (Problem 1 and Problem 2):

$c_{i_j k}(t)$:= cost of unit bandwidth to be borrowed from kth PNO for j type resource at ith cell during time interval t, where $c_{i_j k}(t) \in \mathbb{R}_{\geq 0}^{L \times N_{i_j}}$.

$x_{i_j k}(t)$:= unit of spectrum bandwidths (or sub-bands) to be borrowed from kth PNO for j type resource at ith cell during time interval t, where $x_{i_j k}(t) \in \mathbb{R}_{\geq 0}^{L \times N_{i_j}}$.

$\theta_{i_j k}(t)$:= PNOs intrinsic quality (e.g., the extent of the coverage area and/or maximum allowable transmit power), where $\{\theta_{i_j 1}, \theta_{i_j 2}, \ldots, \theta_{i_j k}, \ldots, \theta_{L \times N}\}$.

$p_{i_j}(t)$:= target blocking probability for j type resource at ith cell during time interval t for the secondary network operator.

$a_{i_jk}(t)$:= unit bandwidth available from kth PNO to be leased to SNO for jth type resource at the ith cell during time interval t, where $a_{i_jk}(t) \in \mathbb{R}_{\geq 0}^{L\times N_{i_j}}$.

$r_{i_j}(t)$:= unit bandwidth required to satisfy the target blocking probability $p_{i_j}(t)$ for the SNO's for jth type resource at ith cell during time interval t, where $r_{i_j}(t) \in \mathbb{R}_{\geq 0}^{L}$.

$\gamma_{i_jk}(t)$:= the expected profit for borrowing unit bandwidth from kth PNO for jth type resource at ith cell during time interval t, where $\gamma_{i_jk}(t) \in \mathbb{R}^{L\times N_{i_j}}$.

C. Spectrum allocation by minimising borrowing cost

We now formulate the spectrum allocation problem, that is, how much spectrum bandwidths to be borrowed from each PNO to keep the blocking probability in a specific level, for instance, at 1%. Given a set of possible available spectrum resources $\{a_{i_jk}(t)\}$ and their associated prices $\{c_{i_jk}(t)\}$, the problem is to find the feasible set of spectrum bandwidths $\{x_{i_jk}(t)\}$ by minimising the total borrowing cost. The PNOs set their prices according to the maximum allowed transmit power ϖ_{i_jk} and the pricing coefficient φ_{i_jk}, which can be written as [20]

$$c_{i_jk} = \frac{\sum\limits_{k\,\in a_{i_jk}} \left[\log\left(1+\frac{h\,\varpi_{i_jk}}{\varrho_i}\right) - (\varpi_{i_jk}\cdot\varphi_{i_jk})\right]}{a_{i_jk}} \tag{1}$$

where h is the average aggregated channel gain and ϱ_i is the additive noise received by SNO users at cell i. Resource acquisition in this case is obtained by solving the following optimization problem:

Problem 1:

$$\text{minimise}\left[\sum_{i_j=1}^{L}\sum_{k=1}^{N_{i_j}} c_{i_jk}(t)\cdot x_{i_jk}(t)\cdot\theta_{i_jk}(t)\right], \tag{2}$$

subject to

$$\begin{aligned}
\underset{x_{i_jk}\forall i,j,k}{\arg\min}\ \Pr\left(\lambda(t),\mu(t),r_{i_jk}(t)+w_{i_j}\right) &\leq p_{i_j}(t), \quad \forall i_j,k \quad (3)\\
x_{i_jk}(t) &\leq a_{i_jk}(t), \quad \forall i_j,k \quad (4)\\
\sum_{k=1}^{N_{i_j}} x_{i_jk}(t) &\leq r_{i_j}(t), \quad \forall i_j,k \quad (5)
\end{aligned}$$

While borrowing cost for each cell i can be calculated as

$$\sum_{k=1}^{N_{i_j}} c_{i_jk}(t)\cdot x_{i_jk}(t)\cdot\theta_{i_jk}(t). \tag{6}$$

The parameter $\theta_{i_jk}(t)$ $(0 \leq \theta_{i_jk}(t) \leq 1)$ defines the intrinsic quality by weighing the cost of borrowing spectrum bandwidths. The intrinsic quality represents the quality of the available heterogeneous aggregated channels to carry the data for transmission. Therefore, the price per unit bandwidth in each PNO can vary, i.e., $c_{i_jk}(t) \lesseqgtr c_{i_jl}(t)$, $\forall i_j$ and $\forall k,l$ with $k \neq l$. We thus refer to this pricing scheme as *non-uniform pricing*.

The blocking probability in constraint (3) is a non-linear function of spectrum bandwidth for each cell. Therefore, the above optimization problem is considered as a non-linear optimization problem which can be solved in two phases: the SNO set the target blocking probability for each cell (e.g., $p_{i_j} = 0.01$, $\forall i_j$). Then it calculates the bandwidth $r_{i_j}(t)$ required to achieve the target blocking probability $p_{i_j}(t)$ for each cell i. Next the SNO finds the amount of bandwidth required to borrow from primary networks. Blocking probability at the ith cell of SNO can be defined as

$$P_{(b)}(t) = \frac{1}{\nu!}\left(\frac{\lambda(t)}{\mu(t)}\right)^{\nu}\left[\sum_{n=0}^{\nu}\frac{1}{n!}\left(\frac{\lambda(t)}{\mu(t)}\right)^{n}\right]^{-1}. \tag{7}$$

Now with the initial fixed bandwidth w_{i_j}, we first calculate the total required bandwidth $\tau_{i_j}(t)$ to achieve the target blocking probability for the ith cell of the SNO

$$\tau_{i_j}(t) = f^{-1}\left(\Pr\left(\lambda_{i_j}(t),\mu_{i_j}(t),w_{i_j}\right)\right). \tag{8}$$

where $f^{-1}(\cdot)$ is the inverse function of $P_{(b)}(t)$ (equation 7) used to derive the required capacity over the existing capacity.

Subtracting the fixed bandwidth w_{i_j} from the total required $\tau_{i_j}(t)$, we obtain the required bandwidth $r_{i_j}(t)$ at the ith cell of the SNO during time interval t

$$r_{i_j}(t) = \tau_{i_j}(t) - w_{i_j}. \tag{9}$$

Now the problem is to find the feasible set of bandwidth $x_{i_jk}(t)$ from the PNOs which minimises the borrowing cost. This is done in the next mathematical programming phase.

In this phase, we set up the borrowing cost $c_{i_jk}(t)$ and the maximum possible bandwidth available $a_{i_jk}(t)$. The borrowing decisions of the SNO are made subject to the lowest price from the set $\{a_{i_jk}(t)\}$. The decision variable $x_{i_jk}(t)$ in this context can be a combination of a number of acquisitions, e.g., SNO selects the lowest price from the available set of bandwidths from the PNOs. If the acquired resources $a_{i_jk}(t)$ are insufficient to reach the target blocking probability $p_{i_j}(t)$ (i.e., $r_{i_jk}(t) - a_{i_jk}(t) > 0$), then the SNO borrows from the remaining bandwidths from the set $\{a_{i_j1}(t), a_{i_j2}(t), \ldots, a_{i_jN}(t)\} \not\ni a_{i_jk}(t)$ for which the cost is minimum. If the required blocking probability $p_{i_j}(t)$ is reached, then the SNO stops acquiring new spectrum bandwidths until the next time interval $(t+1)$.

Once we solve the problem, the new blocking probability can be calculated as

$$\begin{aligned}
P_{(b_{i_j}^{new})}(t) &= \Pr\left(\lambda_{i_j}(t),\mu_{i_j}(t),\left(w_{i_j}+\sum_{k=1}^{N_{i_j}} x_{i_jk}(t)\right)\right)\\
&= \frac{1}{\nu_{i_j}!}\left(\frac{\lambda_{i_j}(t)}{\mu_{i_j}(t)}\right)^{\nu_{i_j}}\left[\sum_{n=0}^{\nu_{i_j}}\frac{1}{n!}\left(\frac{\lambda_{i_j}(t)}{\mu_{i_j}(t)}\right)^{n}\right]^{-1}. \quad (10)
\end{aligned}$$

where

$$\nu_{i_j} = w_{i_j} + \sum_{k=1}^{N_{i_j}} x_{i_jk}(t).$$

Consequently, the SNO will achieve the blocking probability with the required amount of bandwidths satisfying the target blocking probability $p_{i_j}(t)$ or with the highest possible borrowed bandwidths which is mathematically expressed as

$$P_{(b^{new}_{i_j})}(t) = \begin{cases} p_{i_j}(t), & \sum_{k=1}^{N_{i_j}} a_{i_j k}(t) \geq r_{i_j}(t) \\ P_{(b^{new}_{i_j})}(t), & \text{otherwise.} \end{cases} \tag{11}$$

Algorithm 1 Optimal spectrum borrowing

Initialisation: Number of cells in the network = L, number of operators in the network = N and number of types of spectrum bands = M.
Calculate r_{i_j} $\forall i, j$ which satisfies p_{i_j}, and get $c_{i_j k}$ and $a_{i_j k}$ $\forall i, j, k$.
for every time slot (t) **do**
    for all cells $i \leftarrow 1 : L$ **do**
        for all PNOs $k = 1 : N$ **do**
            Solve the nonlinear stochastic *Problem 1* s.t. constraints (3), (4) and (5)
        end for
    end for
end for
return

D. Spectrum allocation using heuristic algorithm

In this approach, spectrum acquisition is performed randomly as illustrated in Algorithm 2. The optimal borrowing cost using this algorithm can only be found randomly from the set of capacity values $a_{i_j k}$ by satisfying the constraints in equation (4) and (5).

Algorithm 2 Heuristic spectrum borrowing

Initialisation: Number of cells in the network = L, number of operators in the network = N and number of types of spectrum bands = M.
Calculate r_{i_j} $\forall i, j$ which satisfies p_{i_j}, and get $c_{i_j k}$ and $a_{i_j k}$ $\forall i, j, k$.
for every time slot (t) **do**
    for all cells $i \leftarrow 1 : L$ **do**
        Set $x \leftarrow \{0_N\}$.
        Set counter $\leftarrow \sum x$.
        Choose a random integer $n \in \{1, 2, \ldots, N\}$.
        for all PNOs $k = n : N \quad 1 : (n-1)$ **do**
            if $0 < a_{i_j k} > (r_{i_j} - \text{counter})$ **then**
                $x_{i_j k} \leftarrow (r_{i_j} - \text{counter})$.
                BREAK
            else if $a_{i_j k} > 0$ & counter $< r_{i_j}$ **then**
                $x_{i_j k} \leftarrow a_{i_j k}$.
                counter $\leftarrow$ counter $+ x_{i_j k}$.
            else
                $x_{i_j k} \leftarrow 0$.
            end if
        end for
    end for
end for
return

For all i, j and k, equation (5) ensures that the SNO does not borrow more than the network's bandwidths demand by controlling the borrowed spectrum bandwidth size in each iteration, which can be expressed mathematically as

$$x_{i_j k}(t) = \begin{cases} a_{i_j k}(t), & r_{i_j}(t) \geq a_{i_j k}(t) \\ r_{i_j}(t), & \text{otherwise.} \end{cases} \tag{12}$$

This scenario can also be regarded as round-robin scheduling algorithm, where SNOs randomly gain access to the PNOs' available spectrum, and the PNOs serve one SNO in each turn. The resource allocation in algorithm 2 evolves in two main discrete steps:

- compute the spectrum demand in each cell r_{i_j}, $\forall i, j$ from equation (9)
- randomly obtain $x_{i_j k}$ subject to equations (4) and (5) from the vector $a_{i_j k}$

The only difference between the two formulations is that in the heuristic formulation, the cost of spectrum access is not considered, where spectrum acquisition is performed randomly from the set $\{a_{i_j k}\}$. Note that when $\sum a_{i_j k} \leq r_{i_j}$ the feasible set $\{x_{i_j k}\}$ is equal for both formulations. We also note that when $\sum_{k=1}^{N_{i_j}} a_{i_j k}(t) > r_{i_j}(t)$, the optimal and heuristic algorithm may achieve the same outcome in terms of total borrowing cost, however, this is a result of randomness in the selection process with probability

$$P(\text{selecting optimal bandwidths}) = \begin{cases} \dfrac{1}{N} & a_{i_j k} \geq r_{i_j}, \forall i_j \\ \dfrac{1}{\left|\{\bar{a}_{i_j}..\}\right|} & \sum_m \{\bar{a}_{i_j lm},\ \forall l, m\} \geq r_{i_j}, \forall i_j \\ 1 & \sum_{k=1}^{N_{i_j}} a_{i_j k} \leq r_{i_j}, \forall i_j \end{cases} \tag{13}$$

where $\{\bar{a}_{i_j lm},\ \forall l, m\} \subset \{a_{i_j k,\ \forall i_j,\ k}\}$, and $\left|\{\bar{a}_{i_j}..\}\right|$ is the number of subsets in the set $\{\bar{a}_{i_j}..\}$ which satisfy the bandwidth requirement for the ith cell with jth type of spectrum band.

E. Expected profit maximisation under restricted budget

In this section, we formulate the second spectrum allocation problem that illustrates how much spectrum bandwidths to be borrowed from each PNO to keep the blocking probability in a specific level. Given a set of possible available spectrum resources $\{a_{i_j k}(t)\}$, their associated prices $\{c_{i_j k}(t)\}$ and expected profit $\{\gamma_{i_j k}(t)\}$, the problem is to find the feasible set of spectrum bandwidths $\{x_{i_j k}(t)\}$ by maximising the total profit of the SNO, under allocated budget and performing the selection according to the highest possible profit combination. Resource acquisition in this case is obtained by solving the following optimization problem:

Problem 2:

$$\text{maximise} \left[\sum_{i=1}^{L} \sum_{j=1}^{M} \sum_{k=1}^{N_j} \gamma_{i_j k}(t) \cdot x_{i_j k}(t) \right] \tag{14}$$

subject to

$$
\begin{aligned}
\underset{x_{i_jk} \forall i,j,k}{\arg\min} \; \Pr\left(\lambda(t), \mu(t), r_{i_jk}(t) + w_{i_j}\right) &\leq p_{i_j}(t),\ \forall i_j, k \quad (15)\\
x_{i_jk}(t) &\leq a_{i_jk}(t), \forall i_j, k \quad (16)\\
\sum_{k=1}^{N_{i_j}} x_{i_jk}(t) &\leq r_{i_j}(t),\ \forall i_j, k \quad (17)\\
\sum_{k=1}^{N_j} c_{i_jk}(t) \cdot x_{i_jk}(t) &\leq b_{i_j},\ \forall i_j, k, \quad (18)
\end{aligned}
$$

where $\gamma_{i_jk}(t)$ consists of two parts: the expected revenue $v_{i_j}(t)$ and cost $c_{i_jk}(t)$, which can be obtained as

$$\gamma_{i_jk}(t) = v_{i_jk}(t) - c_{i_jk}(t), \quad (19)$$

Here

$$v_{i_jk}(t) = f\left(\beta_{i_j}(t), \theta_{i_jk}(t)\right). \quad (20)$$

where $\beta_{i_j}(t)$ is the selling price per unit bandwidth for the ith cell and jth type service during time period t. In equation (20), the expected revenue $v_{i_jk}(t)$ is the function $f(\cdot)$ of the selling price $\beta_{i_jk}(t)$ and the intrinsic quality ($\theta_{i_jk}(t)$) which may take, in general, a non-linear form. In the simplest case, the function can be defined as

$$v_{i_jk}(t) = \beta_{i_jk}(t)\left[-e^{-\theta_{i_jk}(t)}\right]. \quad (21)$$

We consider the the intrinsic quality per unit bandwidth ($\theta_{i_jk}(t)$) for each PNO, which can vary, i.e., $\theta_{i_jk}(t) \lesseqgtr \theta_{i_jl}(t)$, $\forall i_j$ and $\forall k, l$ with $k \neq l$ according to spatial structure of the base stations, allowed transmission power, bandwith types, etc. In this problem formulation, the parameter $\theta_{i_jk}(t)$ influences the optimal spectrum borrowing decisions.

The revenue earned through the sale of the borrowed bandwidth can be equal, higher or lower than the cost. However, for simplicity, we model the revenue $v_{i_jk}(t)$ earned through the sale of the borrowed bandwidth to exceed the borrowing cost, i.e., $v_{i_jk}(t) > c_{i_jk}(t)$ due to the assumption that profit of the SNO for borrowing a unit bandwidth is always positive ($\gamma_{i_jk}(t)$).

The inequality constraint in equation (18) implies that the SNO maximises its profit by taking into account the limitations imposed by cost of the utility and the maximum allowable expenditure which the SNO can spend for borrowing spectrum demand in each cell. Next, we solve the the above non-linear optimization problem in two phases:

In the first phase, the same steps are performed using equation (15) as for solving Problem 1. The SNO calculates the spectrum demand to meet its time varying target blocking probability over time and location. The spectrum demand is adjusted dynamically based on the network information provided by the expected cell demand, service rate and existing spectrum bandwidth.

In the second phase, we set up the vectors $\{c_{i_jk}(t)\}$, $\{a_{i_jk}(t)\}$ and $\{\gamma_{i_jk}(t)\}$. The borrowing decisions of the SNO are made subject to achieving the maximum profit for each acquisition from the PNOs. In Problem 1, the budget restriction is not considered, where the SNO is allowed to make spectrum bandwidth borrowing until it meets the spectrum demand, i.e.,

$$\sum_{k=1}^{N_j} x_{i_jk}(t) = r_{i_j}(t), \text{ assuming } \sum_{k=1}^{N_j} a_{i_jk}(t) \geq r_{i_j}(t). \quad (22)$$

However, in this formulation, the borrowing capacity of the SNO is restricted to budget allocation b_{i_j}. Note that in the case where the SNO's budget is too small to provide the required GoS, then Problem 2 is infeasible. The SNO only achieves a best effort service to minimise the blocking probability so far as the budget permits.

Next, we list the detailed procedure in Algorithm 3.

Algorithm 3 Optimal spectrum borrowing under restricted budget

```
Initialisation: Number of cells in the network = L, number of operators in the network = N and number of types of spectrum bands = M.
Calculate r_{i_j} ∀i, j which satisfies p_{i_j}, and get c_{i_jk}, a_{i_jk}, γ_{i_jk} and θ_{i_jk}(t) ∀i, j, k.
Set maximum allowed budget expenditure for every cell b_{i_j}.
for every time slot (t) do
    for all cells i ← 1 : L do
        for all PNOs k = 1 : N do
            Solve the nonlinear stochastic Problem 2 s.t. (15), (16), (17) and (18)
        end for
    end for
end for
return
```

F. Spectrum allocation using heuristic algorithm under budget constraint

In this subsection, we solve the problem of spectrum allocation under budget constraint by a heuristic bandwidth selection algorithm (Algorithm 4). The algorithm performs all the steps as in Algorithm 3. However, Algorithm 4 does not perform spectrum selection according to the highest possible profit combination from the set $\{a_{i_jk}\}$, rather runs on randomly selected combination from the set $\{a_{i_jk}\}$ to satisfy the spectrum demand r_{i_j}. The optimal profit using Algorithm 4 can only be found from the set of capacity values $\{a_{i_jk}\}$ satisfying the constraints in equation (16), (17) and (18) with probability given in equation (13). To satisfy the constraints in equation (16), (17) and (18) we use

$$x_{i_jk}(t) = \begin{cases} a_{i_jk}(t), & r_{i_j}(t) \geq a_{i_jk}(t),\ b_{i_j} \geq c_{i_jk}, \\ r_{i_j}(t), & r_{i_j}(t) < a_{i_jk}(t),\ b_{i_j} \geq c_{i_jk}, \\ 0, & b_{i_j} < c_{i_jk} \text{ or } r_{i_j}(t) = 0. \end{cases} \quad (23)$$

Algorithm 4 Heuristic spectrum borrowing under restricted budget

Initialisation: Number of cells in the network = L, number of operators in the network = N and number of types of spectrum bands = M.
Calculate r_{i_j} $\forall i, j$ which satisfies p_{i_j}, and get c_{i_jk}, a_{i_jk}, γ_{i_jk} and $\theta_{i_jk}(t)$ $\forall i, j, k$.
Set maximum allowed budget expenditure for every cell b_{i_j}.
for every time slot (t) **do**
  for all cells $i \leftarrow 1 : L$ **do**
    Set $x \leftarrow \{0_N\}$.
    Set counter $\leftarrow \sum x$.
    Choose a random integer $n \in \{1, 2, \ldots, N\}$.
    for all PNOs $k = n : N$ and $1 : (n-1)$ **do**
      if $(0 < a_{i_jk}) \leq (r_{i_j} - \text{counter})$ & $(c_{i_jk} * a_{i_jk}) \leq b_{i_j}$ **then**
        $x_{i_jk} \leftarrow a_{i_jk}$.
        counter $\leftarrow$ counter + $\sum x_{i_jk}$.
        $b_{i_j} \leftarrow b_{i_j} - \sum(x_{i_jk} * c_{i_jk})$.
      else if $(a_{i_jk} > 0)$ & $c_{i_jk} \leq (b_{i_j} - \text{counter})$ & $(a_{i_jk} * c_{i_jk}) \geq b_{i_j}$ **then**
        $x_{i_jk} \leftarrow \left\lfloor \frac{b_{i_j}}{c_{i_jk}} \right\rfloor$ where $\lfloor x \rfloor$ means the floor of x.
        counter $\leftarrow$ counter + $\sum x_{i_jk}$.
        $b_{i_j} \leftarrow b_{i_j} - \sum x_{i_jk} .* c_{i_jk}$.
      else if counter $\leq r_{i_j}$ & $a_{i_jk} > 0$ & $a_{i_jk} \geq (r_{i_j} - \text{counter})$ & $(a_{i_jk} * c_{i_jk}) \leq b_{i_j}$ **then**
        $x_{i_jk} \leftarrow r_{i_j} - \text{counter}$.
        counter $\leftarrow$ counter + $\sum x_{i_jk}$.
        $b_{i_j} \leftarrow b_{i_j} - \sum x_{i_jk} .* c_{i_jk}$.
        break
      else if counter $\leq r_{i_j}$ & $a_{i_jk} > 0$ & $a_{i_jk} \geq (r_{i_j} - \text{counter})$ & $(a_{i_jk} * c_{i_jk}) \geq b_{i_j}$ **then**
        $x_{i_jk} \leftarrow min\left\{\left\lfloor \frac{b_{i_j}}{c_{i_jk}} \right\rfloor\right\}$.
        counter $\leftarrow$ counter + $\sum x_{i_jk}$.
        $b_{i_j} \leftarrow b_{i_j} - \sum x_{i_jk} * c_{i_jk}$.
      else if
then
        $x_{i_jk} \leftarrow 0$.
      end if
    end for
  end for
end for
return

G. Performance analysis under resource sharing between the SNO and PNOs

In the optimization problems above, the PNOs lease part of their spectrum resources to the SNO for monetary benefits. The leasing and borrowing was based on expected demand and available spectrum resources. However, the demand in the PNOs may change during trading window causing one or more of PNOs' state to change from the underloaded to overloaded and their blocking probability would increase. As a consequence, a PNO may react by deviating part or all of its leased spectrum resources under mutual agreement, which results in reducing the size of the shared spectrum resources. This dynamic mechanism affects the performance of all operators involved in the trading. The complexity of the problem depends primarily on the number of PNOs involved and the level of interactions between them. In this paper we will consider three cases as follows:

1) case 1: SNO is sharing with three PNOs: Consider a cell consisting of an SNO with capacity c_0 and three PNOs with capacity c_1, c_2 and c_3. Under a sharing agreement all three PNO share part of their resources c'_1, c'_2 and c'_3, respectively with the SNO determined using the optimization approach discussed in the previous sections. These resources may also be used by the corresponding PNO under mutual agreement. A state of this network is a vector

$$\mathbf{n} = (n_0, n_1, n_2, n_3, n_{01}, n_{02}, n_{03}, n_{11}, n_{22}, n_{33})$$

where n_i are the number of channel requests in progress in the secondary operator and primary operators $1, 2, 3$ respectively, $n_{0i}, i = 1, 2, 3$ are the number of channel requests in the shared resources of the ith primary operator from the secondary operator and $n_{ii}, i = 1, 2, 3$ are the number of calls of the ith primary operator on its own shared resources. The states $\mathbf{n}$ are restricted due to resource sharing constraints. The set of feasible states can be written as

$$\Omega_s = \{\mathbf{n} : A\mathbf{n} \leq \mathbf{s}\} \tag{24}$$

where A is a $d \times 10$ matrix, and $\mathbf{s}$ is a d-vector, where d is the number of constraints. The network topology is reflected in the matrix A, and the vector $\mathbf{s}$.

Let calls arrive to the secondary and ith primary operators according to a non-homogeneous Poisson process, with rates $\lambda(t)$ and $\lambda_i(t)$ at time t. These calls are admitted if $\mathbf{n} + \mathbf{e}_i \in \Omega_s$, where $\mathbf{e}_i$ is the ith unit vector with 1 in place i and 0 elsewhere. When all c_0 resources of the secondary operator is full then calls are diverted and admitted to the ith primary operator if $\mathbf{n} + \mathbf{e}_{0i} \in \Omega_s$, where $\mathbf{e}_{0i}$ is the ith unit vector. Similarly, being all c_i resources occupied calls are diverted to its shared resources c'_i for the ith primary network if $\mathbf{n} + \mathbf{e}_{ii} \in \Omega_s$, where $\mathbf{e}_{ii}$ is the ith unit vector. Assume that admitted calls in secondary and primary operators i have exponential holding times with rates $\mu(t)$ and $\mu_i(t)$ respectively at time t. Under these assumptions, the network can be modeled as a non-homogeneous continuous time Markov chain $\mathbf{X}(t) = (X_i(t), X_{0i}(t), X_{ii}(t);\ i = 1, 2, 3,\ t \geq 0)$ that records the number of channel requests in progress from all operators. The state space of the Markov chain is specified in (24), and its transition rates $\mathbf{Q}(t) = (q(\mathbf{n},\mathbf{n}',t), \mathbf{n},\mathbf{n}' \in \Omega_s)$ are given by

$$q(\mathbf{n},\mathbf{n}',t) = \begin{cases} \lambda(t) & \mathbf{n}' = \mathbf{n} + \mathbf{e}_1 \text{ or } \mathbf{n}' = \mathbf{n} + \mathbf{e}_{0i}, \\ & \quad \text{if } n_0 = c_0,\ i = 1,2,3 \\ \lambda_i(t) & \mathbf{n}' = \mathbf{n} + \mathbf{e}_i \text{ or } \mathbf{n}' = \mathbf{n} + \mathbf{e}_{ii}, \\ & \quad \text{if } n_i = c_i,\ i = 1,2,3 \\ n\mu(t) & \mathbf{n}' = \mathbf{n} - \mathbf{e}_1 \\ n_i\mu_i(t) & \mathbf{n}' = \mathbf{n} - \mathbf{e}_i,\ i = 1,2,3 \\ n_{0i}\mu_i(t) & \mathbf{n}' = \mathbf{n} - \mathbf{e}_{0i},\ i = 1,2,3 \\ n_{ii}\mu_i(t) & \mathbf{n}' = \mathbf{n} - \mathbf{e}_{ii},\ i = 1,2,3 \\ 0 & \text{otherwise.} \end{cases} \tag{25}$$

Theorem 1. *The closed-form solution of* $\mathbf{n}$ *channel requests in progress at time* t *is given by equation (26).*

Proof. Define the state probabilities

$$P(\mathbf{n},t) := Pr(X(t) = \mathbf{n}), \quad \mathbf{n} \in \Omega_s,\ t \geq 0 \tag{28}$$

with initial condition $P_0(\mathbf{n}) = Pr(X(0) = \mathbf{n})$. Since the network has a finite state space, these probabilities are the unique solution of the Kolmogorov forward equations given in (29) for $\mathbf{n} \in \Omega_s, t > 0$.

The Kolmogorov forward equations can be defined as

$$\begin{aligned} \frac{\mathrm{d}P(\mathbf{n},t)}{\mathrm{d}t} = & \left[\lambda(t) \cdot (\mathbb{1}(n_0 < c_0) + \mathbb{1}(n_0 = c_0 \cap_{i\in\{1,2,3\}} \mathbf{n} + \mathbf{e}_{0i})) + \sum_{i=1}^{3} \lambda_i(t) \cdot (\mathbb{1}(n_i < c_i) + \mathbb{1}(n_i = c_i \cap \mathbf{n} - \mathbf{e}_{ii}))\right] P((\mathbf{n} - \mathbf{e}_i), t) \\ & + (n+1)\mu(t) P((\mathbf{n} + \mathbf{e}_i), t) + \sum_{i=1}^{3} (n_{0i} + 1)\mu_i(t) \cdot P(\mathbf{n} + \mathbf{e}_{0i}) \\ & - \left[\lambda(t) \cdot (\mathbb{1}(n_0 < c_0) + \mathbb{1}(n_0 = c_0 \cap_{i\in\{1,2,3\}} \mathbf{n} + \mathbf{e}_{0i})) + \sum_{i=1}^{3} \lambda_i(t) \cdot (\mathbb{1}(n_i < c_i) + \mathbb{1}(n_i = c_i \cap \mathbf{n} - \mathbf{e}_{ii}) \right. \\ & \left. + n\mu(t) + \sum_{i=1}^{3} n_{0i}\mu_i(t))\right] P(\mathbf{n},t) \end{aligned} \tag{29}$$

where $\mathbb{1}(A)$ is the indicator function for an event A.

Equations in (29) can be written in the operator form as given by

$$\frac{\mathrm{d}\mathbf{P}(t)}{\mathrm{d}t} = \mathbf{P}(t)\,\mathbf{Q}(t), \quad \mathbf{P}(0) = \mathbf{P}_0,\ t > 0 \tag{30}$$

where $\mathbf{P}(t)$ is the vector of probabilities $\mathbf{P}(\mathbf{n},t)$. The formal solution of the equation (30) is given by

$$\mathbf{P}(t) = \mathbf{P}_0\, E_Q(t), \quad t \geq 0 \tag{31}$$

where $E_Q(t)$ is the time-ordered exponential of the generator $\mathbf{Q}(t)$, that is the unique operator solution to the equation

$$\frac{\mathrm{d}E_Q(t)}{\mathrm{d}t} = E_Q(0)\,\mathbf{Q}(t), \quad t \geq 0 \tag{32}$$

where $E_Q(0) = \mathbf{I}$, the identity operator. The unique solution of the Kolmogorov forward equations (29) is then given by the equation (26). □

The blocking probability formula can then be derived from the closed-form solution (26). The blocking probability for an operator i (ith operator could be the SNO or a PNO), is then given by

$$\begin{aligned} P_{b_i}(t) &= \sum_{\mathbf{n}\in S_R} P(\mathbf{n},t) \\ &= \frac{\displaystyle\sum_{\mathbf{n}\in S_R} \frac{\left[\rho^{(n+n_{01}+n_{02}+n_{03})}(t)\right] \cdot \left[\rho_1^{(n_1+n_{11})}(t)\right] \cdot \left[\rho_2^{(n_2+n_{22})}(t)\right] \cdot \left[\rho_3^{(n_3+n_{33})}(t)\right]}{(n_0+n_{01}+n_{02}+n_{03})!\,(n_1+n_{11})!\,(n_2+n_{22})!\,(n_3+n_{33})!}}{\displaystyle\sum_{\mathbf{n}\in \Omega_s} \frac{\left[\rho^{(n+n_{01}+n_{02}+n_{03})}(t)\right] \cdot \left[\rho_1^{(n_1+n_{11})}(t)\right] \cdot \left[\rho_2^{(n_2+n_{22})}(t)\right] \cdot \left[\rho_3^{(n_3+n_{33})}(t)\right]}{(n_0+n_{01}+n_{02}+n_{03})!\,(n_1+n_{11})!\,(n_2+n_{22})!\,(n_3+n_{33})!}} \quad \forall\ \mathbf{n} \in \Omega_s \end{aligned} \tag{33}$$

where the set S_R is the restricted state space, and varies for the SNO and PNOs. For the SNO, it is defined as

$$S_R = \{\mathbf{n} \in \Omega_s \mid (n_0 = c_0 \cap n_{01} + n_{11} = c_1' \cap n_{02} + n_{22} = c_2' \cap n_{03} + n_{33} = c_3')\}, \tag{34}$$

and for the ith PNO, S_R can be replaced by S_i and defined as

$$S_i = \{\mathbf{n} \in \Omega_s \mid (n_i = c_i \cap n_{0i} + n_{ii} = c_i')\}, \quad i = 1,2,3. \tag{35}$$

$$P(\mathbf{n},t) = \mathcal{K}^{-1} \frac{\left[\rho^{(n+n_{01}+n_{02}+n_{03})}(t)\right] \cdot \left[\rho_1^{(n_1+n_{11})}(t)\right] \cdot \left[\rho_2^{(n_2+n_{22})}(t)\right] \cdot \left[\rho_3^{(n_3+n_{33})}(t)\right]}{(n_0+n_{01}+n_{02}+n_{03})!\,(n_1+n_{11})!\,(n_2+n_{22})!\,(n_3+n_{33})!} \quad \forall \;\; \mathbf{n} \in \Omega_s \tag{26}$$

where $\mathcal{K}$ is the normalizing constant and given by

$$\mathcal{K} = \sum_{\mathbf{n}\in\Omega_s} \frac{\left[\rho^{(n+n_{01}+n_{02}+n_{03})}(t)\right] \cdot \left[\rho_1^{(n_1+n_{11})}(t)\right] \cdot \left[\rho_2^{(n_2+n_{22})}(t)\right] \cdot \left[\rho_3^{(n_3+n_{33})}(t)\right]}{(n_0+n_{01}+n_{02}+n_{03})!\,(n_1+n_{11})!\,(n_2+n_{22})!\,(n_3+n_{33})!}. \tag{27}$$

2) case 2: SNO is sharing with two PNOs: When two primary operators (1 and 2) is sharing with the SNO, the product form solution (26) takes the following form

$$P(\mathbf{n},t) = \mathcal{K}^{-1} \frac{\left[\rho^{(n+n_{01}+n_{02})}(t)\right] \cdot \left[\rho_1^{(n_1+n_{11})}(t)\right] \cdot \left[\rho_2^{(n_2+n_{22})}(t)\right]}{(n_0+n_{01}+n_{02})!\,(n_1+n_{11})!\,(n_2+n_{22})!} \quad \forall \;\; \mathbf{n} \in \Omega_s \tag{36}$$

where

$$\mathcal{K} = \sum_{\mathbf{n}\in\Omega_s} \frac{\left[\rho^{(n+n_{01}+n_{02})}(t)\right] \cdot \left[\rho_1^{(n_1+n_{11})}(t)\right] \cdot \left[\rho_2^{(n_2+n_{22})}(t)\right]}{(n_0+n_{01}+n_{02})!\,(n_1+n_{11})!\,(n_2+n_{22})!}. \tag{37}$$

The blocking probability formula for the ith operator can be given by

$$P_{b_i}(t) = \sum_{\mathbf{n}\in S_R} P(\mathbf{n},t) = \frac{\sum_{\mathbf{n}\in S_R} \frac{\left[\rho^{(n+n_{01}+n_{02})}(t)\right] \cdot \left[\rho_1^{(n_1+n_{11})}(t)\right] \cdot \left[\rho_2^{(n_2+n_{22})}(t)\right]}{(n_0+n_{01}+n_{02})!\,(n_1+n_{11})!\,(n_2+n_{22})!}}{\sum_{\mathbf{n}\in\Omega_s} \frac{\left[\rho^{(n+n_{01}+n_{02})}(t)\right] \cdot \left[\rho_1^{(n_1+n_{11})}(t)\right] \cdot \left[\rho_2^{(n_2+n_{22})}(t)\right]}{(n_0+n_{01}+n_{02})!\,(n_1+n_{11})!\,(n_2+n_{22})!}} \quad \forall \;\; \mathbf{n} \in \Omega_s \tag{38}$$

where the set S_R is again the restricted state space, and varies for the SNO and PNOs. For the SNO, it is defined as

$$S_R = \{\mathbf{n} \in \Omega_s \mid (n_0 = c_0 \cap n_{01} + n_{11} = c_1' \cap n_{02} + n_{22} = c_2')\}, \tag{39}$$

and for the ith PNO, S_R can be replaced by S_i and defined as

$$S_i = \{\mathbf{n} \in \Omega_s \mid (n_i = c_i \cap n_{0i} + n_{ii} = c_i')\}, \quad i = 1,2. \tag{40}$$

3) case 3: SNO is sharing with one PNO: Under the sharing agreement when only the primary operator 1 is sharing with the secondary operator the equation (26) takes the following form

$$P(\mathbf{n},t) = \mathcal{K}^{-1} \frac{\left[\rho^{(n_0+n_{01})}(t)\right] \cdot \left[\rho_1^{(n_1+n_{11})}(t)\right]}{(n_0+n_{01})!\,(n_1+n_{11})!} \quad \forall \;\; \mathbf{n} \in \Omega_s \tag{41}$$

where

$$\mathcal{K} = \sum_{\mathbf{n}\in\Omega_s} \frac{\left[\rho^{(n_0+n_{01})}(t)\right] \cdot \left[\rho_1^{(n_1+n_{11})}(t)\right]}{(n_0+n_{01})!\,(n_1+n_{11})!}. \tag{42}$$

The blocking probability formula for the ith operator is given by

$$P_{b_i}(t) = \sum_{\mathbf{n}\in S_R} P(\mathbf{n},t) = \frac{\sum_{\mathbf{n}\in S_R} \frac{\left[\rho^{(n+n_{01})}(t)\right] \cdot \left[\rho_1^{(n_1+n_{11})}(t)\right]}{(n_0+n_{01})!\,(n_1+n_{11})!}}{\sum_{\mathbf{n}\in\Omega_s} \frac{\left[\rho^{(n+n_{01})}(t)\right] \cdot \left[\rho_1^{(n_1+n_{11})}(t)\right]}{(n_0+n_{01})!\,(n_1+n_{11})!}} \quad \forall \;\; \mathbf{n} \in \Omega_s \tag{43}$$

where the set S_R is again the restricted state space, and varies for the SNO and PNOs. For the SNO, it is defined as

$$S_R = \{\mathbf{n} \in \Omega_s \mid (n_0 = c_0 \cap n_{01} + n_{11} = c_1')\}, \tag{44}$$

and for the ith PNO, S_R can be replaced by S_1 and defined as

$$S_1 = \{\mathbf{n} \in \Omega_s \mid (n_1 = c_1 \cap n_{01} + n_{11} = c_1')\}. \tag{45}$$

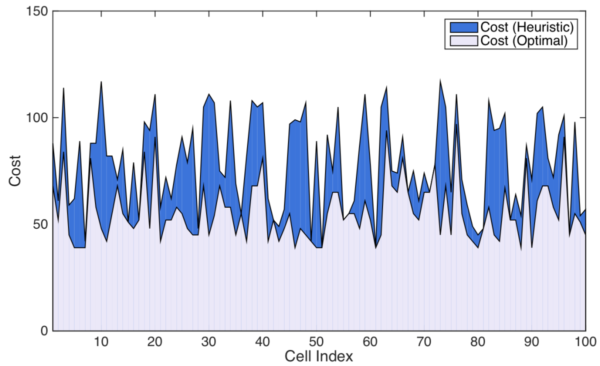
IV. Results and analysis

In this section, we show the analysis of optimal borrowing solutions by Algorithms 1 and 3 corresponding to the cost minimization and profit maximization with restricted budget scenarios, respectively. To explore the advantages of the proposed formulations, we compare the results from Algorithm 1 and 3 with a heuristic spectrum selection formulation by Algorithm 2 and 4, respectively.

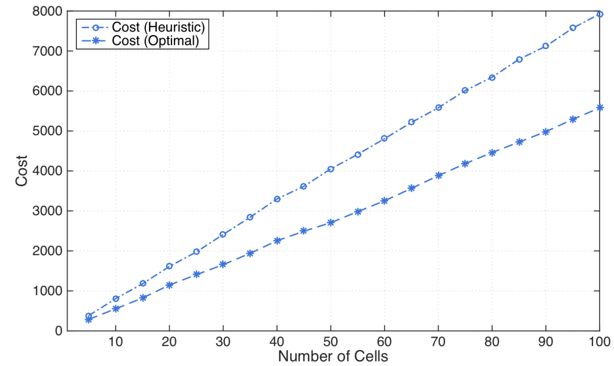
We simulate the functionalities of the network management, which are necessary to generate the optimal solution and to compare with the heuristic spectrum selection algorithms. We consider one SNO and four PNOs ($N = 4$) to simulate the dynamics of the *merchant mode* resource sharing mechanism. Some parameters are determined randomly by the algorithms with specific distribution (e.g., λ_i, μ_i, w_i) and other parameters are preset (e.g., L, p_{i_j}). The algorithms are tested for different scenarios subject to those network parameters.

A. Cost analysis under target performance (Problem 1)

In the simulation, we consider the PNOs spectral usage for all cells L, where four base stations of primary network operators in each cell are deployed (collocated topology), e.g., the case in densely populated cities. The demand of service for each provider (primary or secondary) vary over time and location. We assume the spectral utilisation of secondary provider at time interval t is high whereas the primary operators are underloaded in the same time interval and at the same location. The number of idle spectrum resources of PNOs and the level of spectrum demand of the SNO vary over time and location.



(a)



(b)

Fig. 2: Cost with optimal and heuristic algorithms for (a) per cell and (b) for varying number of cells.

The PNOs charge the SNO at variable rates. The charges may be assessed by the market on the basis of the current supply-demand balance for each individual operator at each cell and possibly other factors [36]. However, we set limits to the price of unit bandwidth as maximum $X_{(max)}$ and minimum price $X_{(min)}$ to structure the problem space. For the purpose of analysis, we parametrise the borrowing cost as

$$c_{i_j k}(t) = \left\{ c_{i_j k}(t) \mid X_{(min)} \leq c_{i_j k}(t) \leq X_{(max)} \right\}, \qquad (46)$$

where $c_{i_j k}(t)$ follows a uniform distribution from $[X_{(min)} = 3, X_{(max)} = 9]$. We keep the difference between $X_{(max)}$ and $X_{(min)}$ relatively small at all cells. This assumption captures the highly competitive nature of the market economic environment. We determine the admission cost per unit bandwidth based on a discrete uniform random variables. In our mathematical model all possible variations of the available bandwidth values $a_{i_j k}(t)$ to provide the SNO demand are considered. This assumption provides realistic scenarios where PNOs could have different values of leasable spectrum resources. More details about the simulation parameters are given in Table I.

At time t and in each cell i, the SNO has a particular blocking probability target p_{i_j}. By considering the SNO's expected traffic load λ_{i_j} in the next time interval, the available capacity w_{i_j} and service rate μ_{i_j}, each cell determines its required number of channels $r_{i_j}(t)$.

TABLE I: Simulation parameters.

Parameter	Value(s) (Problem 1)	Value(s) (Problem 2)
L	100	100
M	1	1
N	4	4
p_{i_j}	0.01	0.01
λ_{i_j}	10	(40, 120)
μ_{i_j}	1	(1, 5)
w_{i_j}	1	(1, 5)
$c_{i_j k}$	(3, 9)	(10, 13)
$a_{i_j k}$	(5, 10)	(30, 40)
b_{i_j}	−−	50

For comparisons, we simulate the interactions between the network providers and we solve the resource allocation problem by the optimal and the heuristic allocation as described in Algorithms 1 and 2 , respectively. For the simulation of the heuristic allocation, each cell i makes heuristic selection of aggregated channels for dynamic access from the set $\{a_{i_j k}\}$ which are collocated in the same cell. The selection of aggregated channels is performed regardless of the admission cost associated with the choice of selected channels. Algorithm 2 is allowed to perform spectrum borrowing until the demand is satisfied, assuming $\sum_{k=1}^{N_{i_j}} a_{i_j k}(t) \geq r_{i_j}(t)$. If $\sum_{k=1}^{N_{i_j}} a_{i_j k}(t) < r_{i_j}(t)$ then the algorithm takes all available bandwidths, however, the target blocking probability will not be satisfied, such that, $P_{(b_{new})}(t) < p_{i_j}(t)$.

For the Algorithm 1, the cells of SNO select the combination $\{x_{i_j k}\}$ with the lowest admission cost from the set $\{a_{i_j k}(t)\}$, $\forall k, i$, to achieve the optimal channel borrowing admission costs. It is possible that there may be multiple solutions for the allocation problem which provide the same required bandwidth to the SNO with different costs.

The main observation here is that the optimal model achieves lower costs compared to the heuristic algorithm, except for cells with $\sum_{k=1}^{N_{i_j}} a_{i_j k}(t) < r_{i_j}(t)$, see Figure 2a. It is also observed that the total borrowing cost of both the heuristic and optimal configuration vary in every cell due to the stochastic nature of the costs and number of available channels.

If we consider the admission cost for large number of cells, as we can see from Figure 2b, we notice that as the number of cells increase, the difference in cost between the heuristic and the optimal selection algorithm becomes larger, which implies substantial savings for operators with large territories when the optimal algorithm is used.

We also investigate the effect of target blocking probability on the admission cost. In Figure 3 we show the results for different target blocking probabilities ranging from $0-0.9$ for a single cell. We clearly see that as $p_{i_j} \to 0$, the admission cost increases for both algorithms. However, the optimal algorithm (Algorithm 1) provides lower borrowing cost for most of the points.

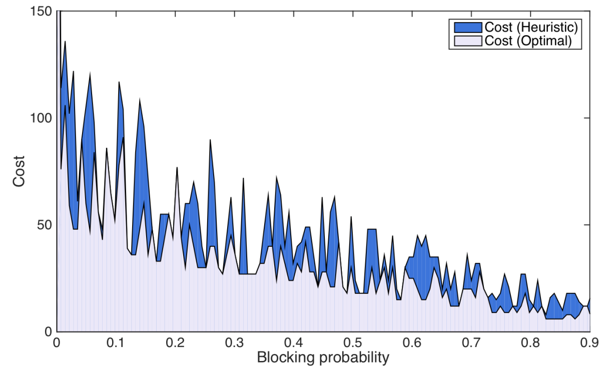


Fig. 3: Effect of varying target blocking probability on cost for optimal and heuristic algorithms.

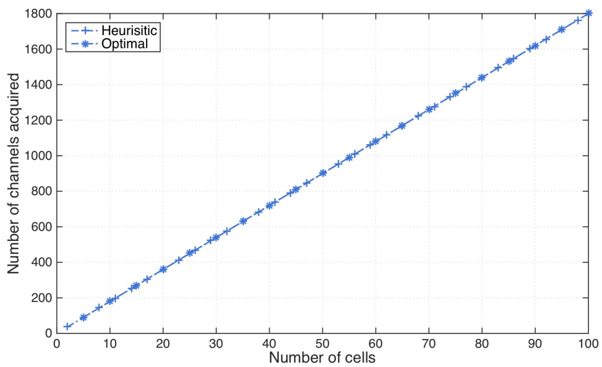


Fig. 4: Effect of borrowing on bandwidth acquisition for the optimal and heuristic algorithms.

The total number of aggregated channels which are acquired through borrowing by using Algorithms 1 and 2 is equal, see Figure 4. This is because both algorithms allow channels to be acquired until a certain grade of service is reached or until all channels from the available bandwidths of primary operators $\{a_{i_jk}\}$ are consumed. This also implies that $P^{rand}_{(b_{new})}(t) = P^{opt}_{(b_{new})}(t)$, where $P^{rand}_{(b_{new})}(t)$ results from using the heuristic algorithm 1 and $P^{rand}_{(b_{new})}(t)$ results from using Algorithm 2.

B. Expected profit under budget constraints analysis (Problem 2)

The objective of the SNO can be described from both economic and system performance perspective. Firstly, the SNO aims to lower the blocking probability for its subscribers. Secondly, the SNO attempts to maximise its profit by leasing additional spectrum from the PNOs in terms of cost and intrinsic quality. However, since network operators often operate with limited budget e.g., SNO can only spend $b_{i_j}(t)$ amount of resources/money at a cell i and time interval t. This is imposed by the government and regulatory bodies to keep the fairness of spectrum leasing among network operators.

To demonstrate the gain of the optimal algorithm, detailed investigation has been made and the results are compared with the heuristic allocation algorithm (see Figure 5a). The figure shows the optimal algorithm achieves a substantial gain in comparison to the heuristic allocation approach. However, both algorithms provide acceptable efficiency in terms of GoS. We also notice that as the number of cells increase the profit of the SNO gets larger, see Figure 5b.

Figure 6 shows the effect of budget and target blocking probability on achievable profit with varying budget expenditure between $0-500$ and target blocking probability between $0-0.8$ for a single cell. It is clear that as we increase the budget further $b_{i_j} \to 500$, the profit increases with respect to the increase of budget and demand. However, as the budget reaches a certain value, the profit does not increase because the budget is larger than required.

We also study how the optimal allocation based on profit maximisation affects the amount of acquired bandwidths. With number of cells between $1-100$, we compare the two algorithms presented in problem 2, see Figure 7. We find that, the optimal algorithm can achieve higher number of aggregated channels due to the higher efficiency in spectrum borrowing under the restricted budget.

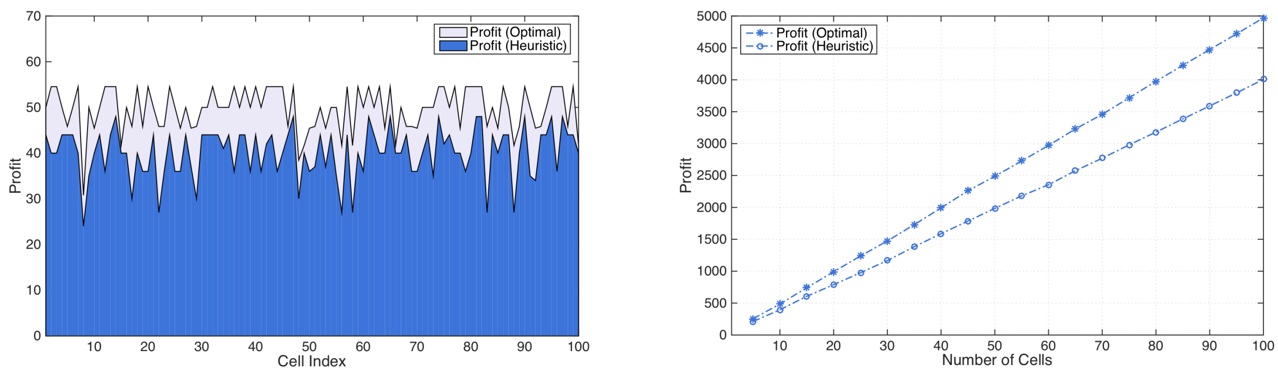
C. Expected profit under budget constraints with multiple types of services (Problem 2)

In the above analysis, we considered only one type of spectrum band ($M = 1$), which is provided to users at all cells (e.g., 900 MHz). In a more general model, different types of bands (e.g., 900 MHz, 2.3-2.4 GHz and 2.40-2.4835 GHz) can be operated by one network operators. Different bands provide different quality in the mobile broadband services [12]. The measures of quality include data rate and coverage. Therefore, they cannot be treated equally. In the proposed algorithms, we added a functionality to allow the trading to be managed more effectively by assigning each cell with a particular band type. In order to quantify the impact of the proposed algorithms we simulated a network which could support three different bands, ($M = 3$). We also tested the algorithms with two different budgets. In the simulation of 10 cells and allocated budget of 50 and 500 for each cell, we observed a markedly increased profit in both cases, see Figure 8. We can also see from the figures (top and bottom figures) that in all types of bands, the optimal algorithm outperforms its heuristic counterpart.

D. Impact on the performance of the operators

To analyze the impact of unilateral deviation strategy of the PNOs, we used the closed form formulae presented in Subsection III-G to compute the blocking probability of operators. The arrival processes involved in all operators are non-homogenous Poisson with rates λ, λ_1, λ_2, and λ_3, respectively. The offered loads are λ/μ and λ_i/μ_i for the secondary and ith PNO, respectively. The number of aggregated channels and traffic intensities in each operator are independent as shown in table II. The results show that the operators could obtain an actual blocking probability values to determine their benefits when they engage in spectrum trading.

In Figure 9a, we observe the performance of the SNO by varying the traffic load at the PNOs. If we fix a particular value of traffic intensity at the SNO ($\rho = 15$) and change it for



(a) (b)

Fig. 5: Profit using the optimal and heuristic algorithms for (a) per cell and (b) for varying number of cells.

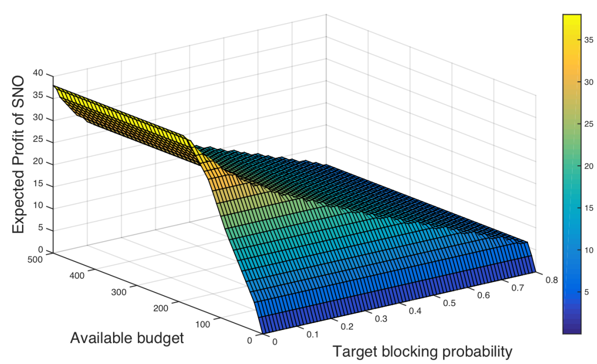


Fig. 6: Expected profit of the SNO for spectrum borrowing with target blocking probability = 0 to 0.8 and budget = 0 to 500.

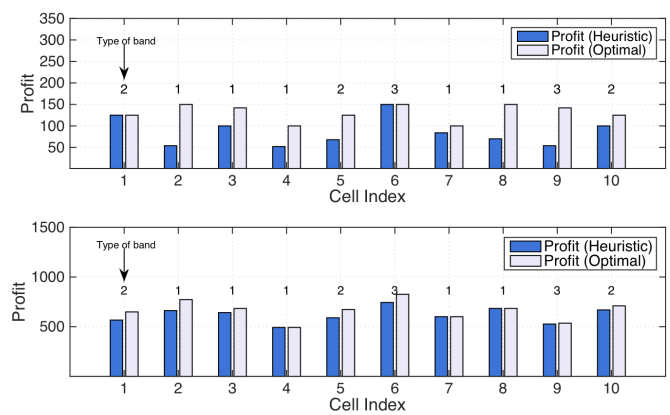


Fig. 8: Effect of spectrum borrowing on profit with budget = 50 (top) and budget= 500 (bottom).

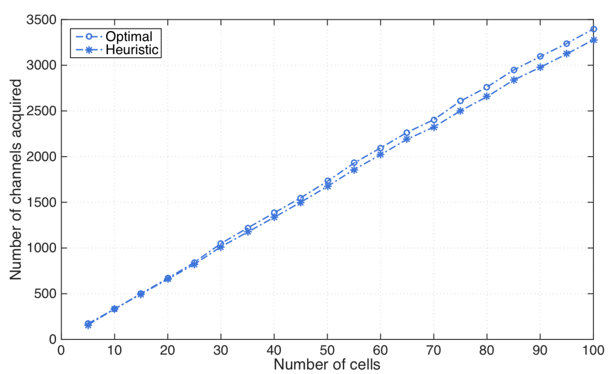


Fig. 7: Bandwidth acquisition of the SNO for spectrum borrowing by the optimal and heuristic algorithms.

the PNOs, then the SNO's blocking probability increases due to the available capacity for sharing (c_1', c_2' and c_3') becomes overloaded by the PNOs' own traffic. We notice that the severity of traffic intensity change in the PNOs affects the performance of the SNO.

To maintain the GoS, SNO should be able to limit the resulting interference, caused by each PNO, by increasing the frequency of trading windows. More specifically, the trading window is repeated more regularly to recompense the lost shared capacity caused by the deviation mechanism of the PNOs.

In Figure 9b, we analyse the impact of change in state of the PNOs from overloaded to underloaded. As the shared capacity becomes ample to meet the SNO's demand, we notice a significant reduction in blocking probability at all operators. The results demonstrate that the derived blocking probabilities can provide a crucial insight to the sharing strategies between operators.

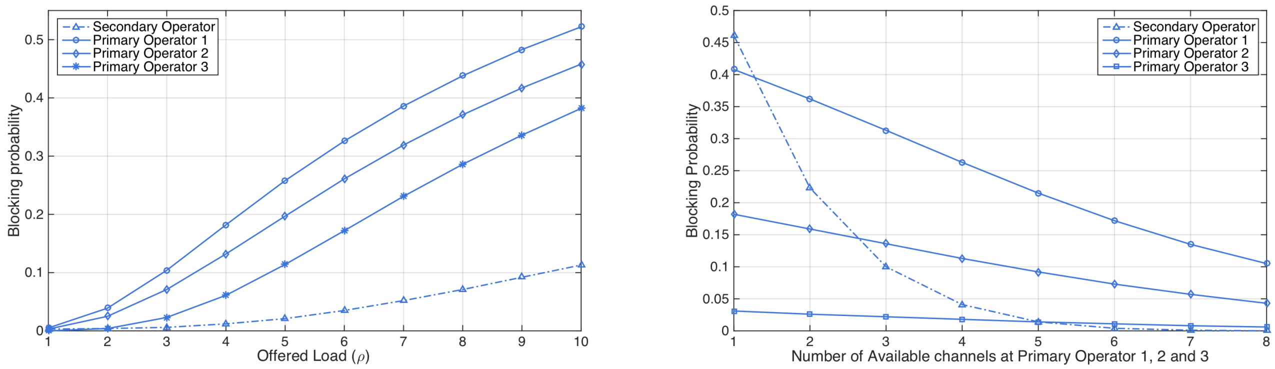
V. Conclusion

In this paper, we presented two finite horizon nonlinear optimization algorithms to solve two optimization problems for dynamic spectrum sharing. The efficiency of the proposed algorithms is compared with their corresponding heuristic algorithms. We also presented the post-optimization performance analysis of the SNO and PNOs through blocking probability, which provides useful details about spectrum sharing strategy.

The optimization problems investigated by considering a comprehensive process of delivering the secondary network

TABLE II: Configurations used in Figure 9a and 9b.

	Number of channels							Load (ρ)			
	SNO	PNO 1		PNO 2		PNO 3		SNO	PNO 1	PNO 2	PNO 3
	c	c_1	c'_1	c_2	c'_2	c_3	c'_3	ρ	ρ_1	ρ_2	ρ_3
Figure 9a	2	4	2	4	3	6	2	4	(0, 10)	(0, 10)	(0, 10)
Figure 9b	2	3	(1, 8)	4	(1, 8)	5	(1, 8)	15	4	3	2



(a) (b)

Fig. 9: Blocking probability for each operator when (a) $\rho = 4$, ρ_1, ρ_2 and ρ_3 are varying from 0 to 10 and (b) c_1, c_2 and c_3 are varying from 1 to 8 (b). See Table II for full configuration details.

operator's (SNO's) bandwidth demand and the solution algorithms ensured that either minimum cost of bandwidth borrowing or maximum profit under budget restrictions are achieved depending on the aim of the SNO. In both cases the SNO aims to achieve a target performance by borrowing spectrum from other network operators (PNOs) on temporal and spatial basis. Results obtained from each model are then compared with results derived from algorithms in which spectrum borrowing were heuristic. Detailed comparisons are presented and they showed that the gain in the results obtained from our proposed stochastic-optimization framework is markedly higher than heuristic borrowing algorithms. Our proposed approaches facilitate a dynamic purchasing (also called *automation of licensing*) scheme for such complex problems, which provide incentives to the network operators wishing to adopt dynamic spectrum sharing as well as substantial benefits for efficient use of spectrum. The proposed algorithms showed significant opportunities to increase spectrum utilisation while keeping GoS at a particular level and ensuring minimum cost. We also shown that our proposed optimization solution not only reduce the total borrowing cost of the SNO, but also finds maximum spectrum access under any allocated budget.

A major challenge with the spectrum sharing optimization models is to guarantee the operational grade of service (GoS) under different sharing protocols. Although a vast amount of literature addressed various spectrum sharing issues very little has discussed the post-optimization results which are crucial for the operators to gain the detailed insight and final GoS. To study these issues and provide the final GoS, we derived the blocking probability behavior using a time-dependent continuous time Markov chain framework under various settings. Results showed that the final GoS is largely affected by the increase of traffic at the PNOs and the amount of shared resources. This post-optimization analysis of spectrum sharing among the operators is an emerging topic that requires further research that would cover other issues, for instance, different sharing strategies and configurations.

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