Integrated approach to assignment, scheduling and routing problems

Laura Hervert-Escobar, Francisco López-Ramos, Oscar A. Esquivel-Flores

Instituto Tecnológico de Estudios Superiores de Monterrey

laura.hervert@itesm.mx

Abstract-*This research considers a real life case study that determines the minimum number of sellers required to attend a set of customers located in a certain region taking into account the weekly schedule plan of the visits, as well as the optimal route. The problem is formulated as a combination of assignment, scheduling and routing problems. In the new formulation, case studies of small size subset of customers of the above type can be solved optimally. However, this subset of customers is not representative within the business plan of the company. To overcome this limitation, the problem is divided into three phases. A greedy algorithm is used in Phase I in order to identify a set of cost-effective feasible clusters of customers assigned to a seller. Phase II and III are then used to solve the problem of a weekly program for visiting the customers as well as to determine the route plan using MILP formulation. Several real life instances of different sizes have been solved demonstrating the efficiency of the proposed approach.*

I. INTRODUCTION

Network models and integer programs are applicable to an enormous known variety of decision problems. In a real life, the cost efficient management decision is defined by a combination of different models.

Consider a set of customers ***C={1,2,...,i,...,j,...N}*** dispersed in a given region where their locations are given by coordinates ***(gx_j, gy_j)***. It is required to design a business plan that includes the minimum number of sellers ***Y={1,2,...,s,...Y}*** to attend these customers, in days ***D={1,2,3,4,5,6}*** denoted by index *t* in the scheduling plan per week, providing the optimal daily routing. The decision should consider the demand ***(Dem)*** and the service time ***(T_i)*** of the customers. Also, the daily capacity *(Cap)* and available time of the sellers ***(T_s)***. Decision variables of the model are as follows:

Y_i^s Binary variable denoting whether customer *i* is assigned to seller *s*

V_{it}^s Binary variable denoting whether seller *s* visits a customer *i* at day *t*

X_{ij}^t Binary variable denoting whether customer *i* is visited

e_i Continuous variable denoting the order in which customer *i* is visited in the route plan of seller *s* during day *t*.

The formulation of the matematical model is as follows:

$$\min \sum_{s\in S}\sum_{i\in C} 2^{s-1} Y_i^s + \sum_i\sum_j\sum_s\sum_t d_{i,j}\cdot X_{i,j}^{s,t} \qquad (1)$$

Subject to:

$$\sum_{s\in S} Y_i^s = 1, \quad \forall i \qquad (2)$$

$$V_{i,t}^s \le Y_i^s, \quad \forall i,t,s \qquad (3)$$

$$X_{i,j}^{s,t} \le V_{i,t}^s, \quad \forall i,j,t,s,\ i\ne j\in N \qquad (4)$$

$$X_{i,j}^{s,t} \le V_{j,t}^s, \quad \forall i,j,t,s,\ i\ne j\in N \qquad (5)$$

$$\sum_i X_{i,j}^{s,t} = \sum_j X_{i,j}^{s,t}, \quad \forall i,j,t,s,\ i\ne j\in N \qquad (6)$$

$$\sum_i X_{i,j}^{s,t} + \sum_j X_{i,j}^{s,t} = 2V_{i,t}^s, \quad \forall i,j,t,s,\ i\ne j\in N \qquad (7)$$

$$\sum_i T_i\cdot V_{i,t}^s \le T^s, \quad \forall t,s \qquad (8)$$

$$\sum_t\sum_s V_{i,t}^s = \left\lceil \frac{Dem_i}{6*Cap} \right\rceil, \quad \forall i \qquad (9)$$

$$V_{i,t}^s + V_{i,t+1}^s \le 1, \quad \forall i,s,\ t\le 5,\ Freq_i \le 3 \qquad (10)$$

$$e_i^{s,t} - e_j^{s,t} + MX_{i,j}^{s,t} \le M-1, \quad \forall i,j,t,s,\ i\ne j\in N \qquad (11)$$

$$e_i^{s,t} \le \sum_j V_{j,t}^s, \quad \forall i,j,t,s,\ i\ne j\in N \qquad (12)$$

The objectivefunction (1) represents the sum of two goals, the minimization of the number of sellers required to service the customers and the minimization of the traveling distance to visit each customer for each routing plan.

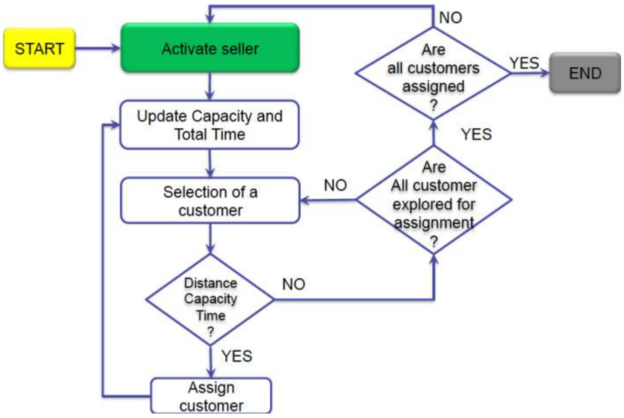
As for constraints, (2) ensures that a customer is attended by only one seller. Equation (3) guaranties that a customer is assigned to the seller that actually visits that customer. Equations (4) and (5) link the scheduling variables to the routing ones. Equations (6) and (7) are used for connectivity purposes. Next equation (8) ensures that the available time of the seller is not compromised during the scheduling of visits to the customers assigned per day. In this way, equation (9) establishes the number of visits to carried out per customer according to the given frequency. Equation (10) avoids consecutive visits to those customers whose

er week. Finally, equations (11) and (12) allow to assign the proper order of visits to customers during the routing plan to avoid sub-tours.

II. SOLUTION METHOD

The solution method for the problem is divided into three phases. The Phase I find a cluster of customers using the nearest neighbor approach. This objective is known as the tightest cluster of *m* points. This is similar to the one facility version of the max-cover problem [1], for planar models [2], for one facility [3], and for several facilities [4], where we wish to find the location of several facilities which cover the maximum number of points within a given distance. The procedure is illustrated in Fig. 1.

Fig. 1. Greedy algorithm for phase I, the assignment of customers to sellers.



After defining the clusters of customers, the sequence of visits for each seller (cluster) is determined by solving a scheduling problem (phase II). Finally, the routing is solved per day and seller in phase III.

Caceres et al. [5] present a survey on VRPs apply to real life problems. A classification that applies for this case study is Multi-Period/Periodic VRP with Multiple Visits/Split deliveries. In this classification, the clients are visited several times as vehicles may deliver a fraction of the customer's demand. Moreover, optimization is made over a set of days, considering a different frequency of visits to each client.

III. Results

To test the performance of the proposed models, several instances were tested. The data for each instance correspond to a real life case consisting of a soft-drinks manufacturer. The solving time is an important issue for the company due to the deadline to generate the business plan each week. Therefore, the results are given in terms of both objective functions as well as solving times. The greedy algorithm, which determines the total number of sellers needed to satisfy the customers demand, was implemented in C++ 9.0.21. The scheduling and routing models were implemented using AMPL to call the optimizer CPLEX v.12.6.0. A time limit of 3600 sec is used as a stopping criteria when scheduling and routing are jointly solved. The cover area criteria for the greedy algorithm was set to 10 km.

The results are given in table 1. For each combination of territory and seller, the table provides the total number of customers per territory, the total number of required sellers, the optimal solution provided by the scheduling solution from the three-phase approach (OF(Scheduling)) and the computational times of both approaches.

TABLE I

RESULTS OF SOLUTION METHODS

Territory	TypeSeller	#Cusm	Sellers	OF(Scheduling)	T_{CPU} (2-p/3-p)
T1	D	15	5	1207	0.03/0.06
T4	D	17	6	1446	0.04/0.06
T3	AS	26	2	8472	0.19/0.4
T2	AS	33	5	5426	0.1/0.51
T1	B	37	1	13957	0.28/2.46
T4	A	59	7	6467	0.11/0.22
T3	F	112	3	28118	0.49/14.25
T1	C	163	5	26784	0.47/11.59
T4	F	243	6	28985	0.36/16.75
T3	C	405	11	38512	0.57/14.5
T2	E	1645	6	103299	4.26/611.46

The results shows that the total number of required sellers is not related to the size of the instance but to a combination of distance and demand of the customers. On the other hand, the objective function of the scheduling increases with the customers per territory. Concerning the computational time, it should be noted that the three-phase approach is faster than the two-phase one. Moreover, the three-phase approach achieves the same quality of the solution or even better. The computational performance improves with tight time windows and high node geographical density. Due to the use of the greedy algorithm, the critical size of the cluster-based MILP formulation significantly decreases and the hybrid approach becomes much more efficient.

IV. Conclusions

The described approach allows tackling the uncertainties stemming from practical problems such as different sizes of territory and particular features of the demand such as the distance and the service time. Future research lines include the development of a metaheuristic for further improving the solution provided by the three-phase approach, as well as the addition of stochastic data to represent a raise/fall in the clients demand and the appearance/loss of clients. Moreover, this approach is better suited for parallel implementation for larger problems.

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