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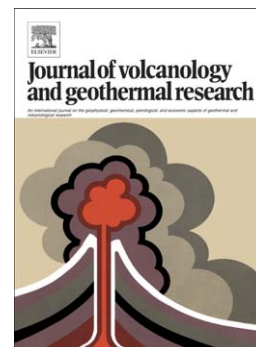
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Uncertainties in volcanic plume modeling: a parametric study using FPLUME

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Abstract

We carry out a parametric study in order to identify and quantify the effects of uncertainties on pivotal parameters controlling the dynamics of volcanic plumes. The study builds upon numerical simulations using FPLUME, an integral steady-state model based on the Buoyant Plume Theory generalized in order to account for volcanic processes (particle fallout and re-entrainment, water phase changes, effects of wind, etc). As reference cases for strong and weak plumes, we consider the cases defined during the IAVCEI Commission on tephra hazard modeling inter-comparison study (Costa et al., 2016). The parametric study quantifies the effect of typical uncertainties on total mass eruption rate, column height, mixture exit velocity, temperature and water content, and particle size. Moreover, a sensitivity study investigates the role of wind entrainment and intensity, atmospheric humidity, water phase changes, and particle fallout and re-entrainment. Results show that the leading-order parameters that control plume height are the mass eruption rate and the air entrainment coefficient, especially for weak plumes.

Keywords: Volcanic plumes, Buoyant Plume Theory, FPLUME, Uncertainty

1. Introduction

Tephra Transport and Dispersal Models (TTDMs; Folch, 2012) are commonly used for volcanic hazard assessment and tephra dispersal (ash cloud) forecasts. The proper quantification of the parameters defining the source term in TTDMs, and in particular the estimation of the Mass Eruption Rate (MER), plume height, and particle vertical mass distribution, is of paramount importance for obtaining reliable results in terms of particle mass concentration in the atmosphere and loading on the ground. Several TTDMs (*e.g.* FALL3D; Costa et al. (2006); Folch et al. (2009); ASH3D; Schwaiger et al. (2012)) can

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10 obtain the source term through coupling with 1D integral plume models, which
11 describe the plume dynamics depending on vent and atmospheric conditions.
12 As a result, uncertainties in plume modeling (*e.g.* in vent conditions, state of
13 the atmosphere, or implicit in the plume model parameterizations) result in un-
14 certain Eruption Source Parameters (ESPs) and propagate to TTDMs strongly
15 affecting its accuracy.

16 We perform a parametric and a sensitivity study to quantify how typical
17 uncertainties in vent conditions and plume model parameterizations affect the
18 ESPs, and in particular the plume height. To this purpose, we use FPLUME
19 (Folch et al., 2016), a steady-state 1-D cross-section averaged eruption column
20 (plume) model based on the Buoyant Plume Theory (BPT) firstly developed
21 by Morton et al. (1956) and later adapted for volcanic plumes (*e.g.* Woods,
22 1988, 1993; Ernst et al., 1996; Bursik, 2001). FPLUME accounts for plume
23 bent over by wind, entrainment of ambient moisture, effects of water phase
24 changes, particle fallout and re-entrainment, a parameterization for the wind
25 entrainment coefficients based on the local Richardson number and a model for
26 wet aggregation of ash particles in the presence of liquid water or ice. Our study
27 focuses on the two reference cases (strong and weak plumes) defined during
28 the volcanic plume model inter-comparison study promoted by the IAVCEI
29 Commission of tephra hazard modeling (Costa et al., 2016). Because of the
30 large number of parameters that can affect plume dynamics, our studies fix
31 the particle grain size distributions and wind profiles for both strong and weak
32 plume.

33 2. Physical Model

34 This section summarizes the governing equations and parameterization of
35 the FPLUME model (for a more detailed description see Folch et al., 2016).
36 FPLUME is a 1D steady-state volcanic plume model based on the Buoyant
37 Plume Theory of Morton et al. (1956) that accounts for different options for es-
38 timating air entrainment (Carazzo et al., 2006, 2008b; Tate & Middleton, 2000),
39 plume bending due to wind effects (Bursik, 2001), fallout of particles from the
40 plume (Bursik, 2001), particle re-entrainment (Ernst et al., 1996), water phase
41 changes (Woods, 1988, 1993), particle wet aggregation (Costa et al., 2010; Brown
42 et al., 2012), and column collapse. The model considers the volcanic plume as
43 a multiphase mixture of volatiles, suspended particles (tephra), and entrained
44 ambient air. For simplicity, water (in vapor, liquid or ice phase) is assumed
45 to be the only volatile species, being either of magmatic or phreatic origin, or
46 incorporated through the ingestion of moist ambient air. Since the governing
47 equations based upon the BPT are not adequate above Neutral Buoyancy Level
48 (NBL), the model uses a semi-empirical approach above this region (see Folch
49 et al. (2016)).

50 2.1. Governing Equations

51 The equations solved by FPLUME up to the NBL are obtained assuming
52 steady-state cross-section averaged equations for axisymmetric plume motion in

53 a turbulent wind (Folch et al., 2016):

$$\frac{d\hat{M}}{ds} = 2\pi r \rho_a u_e + \sum_{i=1}^n \frac{d\hat{M}_i}{ds} \quad (1a)$$

$$\frac{d\hat{P}}{ds} = \pi r^2 (\rho_a - \hat{\rho}) g \sin \theta + u_a \cos \theta (2\pi r \rho_a u_e) + \hat{u} \sum_{i=1}^n \frac{d\hat{M}_i}{ds} \quad (1b)$$

$$\hat{P} \frac{d\theta}{ds} = \pi r^2 (\rho_a - \hat{\rho}) g \cos \theta - u_a \sin \theta (2\pi r \rho_a u_e) \quad (1c)$$

$$\frac{d\hat{E}}{ds} = 2\pi r \rho_a u_e \left((1 - w_a) c_a T_a + w_a h_{wa}(T_a) + gz + \frac{1}{2} u_e^2 \right) + c_p \hat{T} \sum_{i=1}^n \frac{d\hat{M}_i}{ds} \quad (1d)$$

$$\frac{d\hat{M}_a}{ds} = 2\pi r \rho_a u_e (1 - w_a) \quad (1e)$$

$$\frac{d\hat{M}_w}{ds} = 2\pi r \rho_a u_e w_a \quad (1f)$$

$$\frac{d\hat{M}_i}{ds} = -\frac{\chi u_{si}}{r \hat{u}} \left(1 + \frac{f u_e}{u_{si} dr/ds} \right)^{-1} \hat{M}_i + A_i^+ - A_i^- \quad (1g)$$

$$\frac{dx}{ds} = \cos \theta \cos \Phi_a \quad (1h)$$

$$\frac{dy}{ds} = \cos \theta \sin \Phi_a \quad (1i)$$

$$\frac{dz}{ds} = \sin \theta \quad (1j)$$

63 where $\hat{M} = \pi r^2 \hat{\rho} \hat{u}$ is the total mass flow rate, $\hat{P} = \hat{M} \hat{u}$ is the total axial
 64 (stream-wise) momentum flow rate, θ is the plume bent over angle with respect
 65 to the horizontal (*i.e.* $\theta = 90^\circ$ for a plume raising vertically), $\hat{E} = \hat{H} + \hat{M}(gz +$
 66 $\frac{1}{2} \hat{u}^2)$ is the total energy flow rate, \hat{H} is the enthalpy flow rate of the mixture,
 67 $\hat{T} = \hat{T}(\hat{H})$ is the mixture temperature, \hat{M}_a is the mass flow rate of dry air, $\hat{M}_w =$
 68 $\hat{M} x_w$ is the mass flow rate of water, x_w is the mass fraction of water (including
 69 water vapor, liquid and ice, *i.e.* $x_w = x_v + x_l + x_s$), $\hat{M}_i = \hat{M} x_p f_i$ is the mass flow
 70 rate of particles of class i ($i = 1:n$ where n is the number of particle classes),
 71 x and y are the horizontal coordinates, z is height, s is the distance along the
 72 plume axis and Φ_a is the horizontal wind direction (azimuth). The complete
 73 list of symbols and variables is reported in Tables 1 and 2. The equations above
 74 express the conservation of total mass (1a), stream-wise (1b) and radial (1c)
 75 momentum, energy (1d), mass of dry air (1e), mass of water (1f), and mass of
 76 particles (1g). Finally, eqs. (1h) to (1j) determine the 3D plume trajectory as
 77 a function of the length parameter s . The hat above a variable denotes “bulk”
 78 quantities, that is, a variable integrated over a plume cross-section using a top-
 79 hat profile in which a generic quantity ϕ has a constant value $\hat{\phi}(s)$ at a given
 80 plume cross-section and vanishes outside. These equations constitute a set of

81 $9 + n$ first order ordinary differential equations in s for $9 + n$ unknowns: \hat{M} , \hat{P} ,
 82 θ , \hat{E} , \hat{M}_a , \hat{M}_w , \hat{M}_i (for each particle class), x , y and z . Please, note that the last
 83 term in eq. (1b) represents the change in stream-wise momentum due to loss or
 84 re-entrainment of the particles. However, while particles leave the column with
 85 velocity \hat{u} , re-entrained particles enter with the velocity of the environment air.
 86 For simplicity, the difference between outgoing and ingoing particle velocity is
 87 not taken into account and we assume that re-entrained particles enter the plume
 88 with velocity \hat{u} . However, such an assumption introduces in the momentum
 89 balance equation a negligible error (less than a few percent in the investigated
 90 cases).

91 The enthalpy flow rate of the mixture is a non-decreasing function of the
 92 temperature \hat{T} , given by:

$$\hat{H} = \hat{M}[x_a c_a \hat{T} + x_p c_p \hat{T} + x_v h_v(\hat{T}) + x_l h_l(\hat{T}) + x_s h_s(\hat{T})] \quad (2)$$

93 where h_v , h_l and h_s are, respectively, the enthalpy per unit mass of water vapor,
 94 liquid and ice:

$$h_s(\hat{T}) = c_s \hat{T} \quad (3a)$$

$$h_l(\hat{T}) = h_{l0} + c_l(\hat{T} - T_0) \quad (3b)$$

$$h_v(\hat{T}) = h_{v0} + c_v(\hat{T} - T_0) \quad (3c)$$

97 where $c_s = 2108 \text{ J K}^{-1} \text{ kg}^{-1}$ is the specific heat of ice, T_0 is a reference tem-
 98 perature, $h_{l0} = 3.337 \times 10^5 \text{ J kg}^{-1}$ is the enthalpy of the liquid water at the
 99 reference temperature, $c_l = 4187 \text{ J K}^{-1} \text{ kg}^{-1}$ is the specific heat of liquid wa-
 100 ter, $h_{v0} = 2.501 \times 10^6 \text{ J kg}^{-1}$ is the enthalpy of vapor water at the reference
 101 temperature and $c_v = 1996 \text{ J K}^{-1} \text{ kg}^{-1}$ is the specific heat of vapor water. For
 102 convenience, the reference temperature T_0 is taken equal to the temperature of
 103 triple point of the water ($T_0 = 273.15 \text{ K}$). The energy and the enthalpy flow rate
 104 are related by:

$$\hat{E} = \hat{H} + \hat{M}(gz + \frac{1}{2}\hat{u}^2) \quad (4)$$

105 For the integration of eq. (1d) and for evaluating the aggregation rate terms in
 106 eq. (1g), the temperature \hat{T} and the mass fractions of ice (x_s), liquid water (x_l)
 107 and vapour (x_v) need to be evaluated. These quantities are obtained by the
 108 direct inversion of eq. (2), with the use of eqs. (1d) and (4) and by assuming
 109 that the pressure inside the plume P is equal to the atmospheric pressure at the
 110 same altitude (z).

111 The model uses a pseudo-gas assumption considering that the mixture of air
 112 and water vapour behaves as an ideal gas:

$$P = P_v + P_a ; \quad P_v = n_v P ; \quad P_a = n_a P \quad (5a)$$

$$n_v = \frac{x_v/m_v}{x_v/m_v + x_a/m_a} ; \quad n_a = \frac{x_a/m_a}{x_v/m_v + x_a/m_a} \quad (5b)$$

114 where P_v and P_a are, respectively the partial pressures of the water vapour and
 115 of the air in the plume, n_v and n_a are the molar fractions of vapour and air in

116 the gas phase ($n_v + n_a = 1$) and $m_v = 0.018$ kg/mole and $m_a = 0.029$ kg/mole
 117 are the molar weights of vapour and air.

118 For the particle re-entrainment parameter f in eq. (1g) we adopt the fit
 119 proposed by Bursik (2001) on the basis of the experimental results of Ernst
 120 et al. (1996) for plumes not affected by wind:

$$f = 0.43 \left(1 + \left[\frac{0.78 u_s P_o^{1/4}}{F_o^{1/2}} \right]^6 \right)^{-1} \quad (6)$$

121 where $P_o = r_o^2 \hat{u}_o^2$ and $F_o = r_o^2 \hat{u}_o \hat{c}_o \hat{T}_o$ are the specific momentum and thermal
 122 fluxes at the vent ($s = 0$) (see Folch et al., 2016, for more details).

123 Particle terminal settling velocity u_{si} is parameterized as Costa et al. (2006);
 124 Folch et al. (2009):

$$u_{si} = \sqrt{\frac{4g(\rho_{pi} - \hat{\rho})d_i}{3C_d \hat{\rho}}} \quad (7)$$

125 where d_i is the class particle diameter and C_d is a drag coefficient that depends
 126 on the Reynolds number $Re = d_i u_{si} \hat{\rho} / \hat{\mu}$. Here we use the parameterisation
 127 proposed by Ganser (1993), which considers the effects of particles sphericity
 128 Ψ .

129 2.2. Solving Strategies

130 Given a closure equation for the turbulent air entrainment velocity (u_e) and
 131 an aggregation model (defining the mass aggregation coefficients A_i^+ and A_i^-),
 132 eqs. (1a) to (1j) can be integrated along the plume axis from the inlet (volcanic
 133 vent) up to the NBL. Inflow (boundary) conditions are required at the vent
 134 ($s = 0$) for total MER \hat{M}_o (*i.e.* the total mass flow rate at the vent), bent over
 135 angle $\theta_o = 90^\circ$, temperature \hat{T}_o , exit velocity \hat{u}_o , fraction of water x_{wo} , null
 136 air mass flow rate $\hat{M}_a = 0$, vent coordinates (x_o, y_o and z_o), and MER for each
 137 particle class \hat{M}_{i_o} . The latter is obtained from the total MER given the particle
 138 grain size distribution at the vent:

$$\hat{M}_{i_o} = f_{i_o} \hat{M}_o (1 - x_{wo}) \quad (8)$$

139 where f_{i_o} is the mass fraction of class i at the vent.

140 Alternatively, equations can also be solved given the plume height rather
 141 than the total MER at the vent \hat{M}_o . The inverse problem of finding \hat{M}_o from an
 142 assigned height is solved by changing \hat{M}_o iteratively until the obtained column
 143 height approximates the required value within a specified tolerance (≈ 10 m).
 144 The search algorithm is based on the bisection method. However, although
 145 the direct method (find height h given \hat{M}_o) always gives a solution, the inverse
 146 problem cannot always find a \hat{M}_o that gives a required column height. The
 147 reason for this is the non-linear relationship between MER and column height
 148 due to air stratification, wind, column collapse conditions, etc.

149 3. Parametric study on the input parameters

150 Firstly, we performed a parametric study of the model inputs to quantify
 151 how uncertainties at the vent (*i.e.* on \hat{M}_o , \hat{u}_o , \hat{T}_o , x_{wo} , and particle size) affect
 152 the Eruption Source Parameters (ESP). Emphasis is given on plume height
 153 because of its pivotal role on atmospheric dispersal. Our study focus on the two
 154 test cases defined in the IAVCEI inter-comparison study (Costa et al., 2016) for
 155 strong and weak plumes considering both windy and windless conditions. For
 156 the strong plume scenario, meteorological data were obtained from the European
 157 Centre for Medium-Range Weather Forecasts (ECMWF) and corrected above
 158 20 km by Costa et al. (2013) for Pinatubo volcano at 13:40 PLT of 15 June
 159 1991 (column height 39 km). For the weak plume scenario, meteorological data
 160 were provided by the Japan Meteorological Agency's Non-Hydrostatic Model
 161 (Hashimoto et al., 2012) for Shinmoe-dake volcano at 00:00 JST on 27 January
 162 2011 (column height 8 km).

163 The parametric study consists on a series of runs varying the model input pa-
 164 rameters one at a time. When possible, the rest of the parameters are kept con-
 165 stant as in the reference case (see Table 3) or are modified for ensuring a physical
 166 consistency. In particular, at the vent, the mass eruption rate \hat{M}_o , the density
 167 of the mixture $\hat{\rho}_o = \hat{\rho}_o(P_0, \hat{T}_o, x_{wo})$, the exit velocity $\hat{u}_o = \hat{u}_o(P_0, \hat{T}_o, x_{wo})$ and
 168 the vent radius r_0 are related by the relationship:

$$\hat{M}_o = \pi r_0^2 \hat{\rho}_o \hat{u}_o \quad (9)$$

169 where P_0 is the pressure at the vent, assumed equal to the atmospheric pressure
 170 at the same quote. In this study, unless otherwise specified, when a single
 171 parameter among \hat{M}_o , \hat{T}_o , \hat{x}_{wo} , \hat{u}_o is varied, then the vent radius r_0 is modified
 172 accordingly, in order to satisfy eq. (9). For the reference case, according to the
 173 values of Table 3), vent radius and column density at the vent are, respectively,
 174 708 m and 3.46 kg/m³ for the strong plumes, and 27 m and 4.85 kg/m³ for the
 175 weak plumes.

176 The response of the model was explored within the following ranges, repre-
 177 sentative of typical uncertainties at the vent:

- 178 1. Total MER \hat{M}_o ranging from 1/5 to 5 times the reference values (1.5×10^9
 179 and 1.5×10^6 kg/s for strong and weak plumes, respectively);
- 180 2. Eruption column heights varying $\pm 20\%$ with respect the reference values;
- 181 3. Mixture exit velocities \hat{u}_o varying $\pm 30\%$ with respect the reference values
 182 (275 and 135 m/s for strong and weak);
- 183 4. Mixture exit temperatures \hat{T}_o varying ± 100 K with respect the reference
 184 values (1053 and 1273 K for strong and weak);
- 185 5. Erupted water mass fraction x_{wo} varying ± 2 wt% with respect the refer-
 186 ence values (5 and 3% for strong and weak).

187 The grain size distributions for both strong and weak plumes were assumed
 188 as in Costa et al. (2016) and are reported in Table 4. However, in order to
 189 explore the role of particle size we also considered additional runs with a single
 190 particle class varying $\pm 6\Phi$ with respect a reference value of $\Phi = 2$ (250 μ m).

191 *3.1. Effect of MER variations on height*

192 Figure 1 shows the variation of the column height h as function of the total
 193 MER at the vent \hat{M}_o . The mass flow rate is changed by varying the vent radius,
 194 keeping constant the exit velocity, the temperature and the water mass fraction
 195 at the vent (see Table 3). In the investigated range of MER, the vent radius
 196 varies from 317 to 1583 m for the strong plumes and from 12 to 61 m for the
 197 weak plumes. Results are given as absolute and relative variations, the latter
 198 showing the column height variation factor h/h^{ref} given a relative MER variation
 199 $100 \times (\hat{M}_o - \hat{M}_o^{\text{ref}})/\hat{M}_o^{\text{ref}}$ ranging from -80 to 400. This range corresponds to a
 200 variation between 1/5 and 5 of the ratio $\hat{M}_o/\hat{M}_o^{\text{ref}}$. As expected (see eg. Wilson
 201 et al., 1978; Bursik & Woods, 1991; Bursik, 2001; Degruyter & Bonadonna,
 202 2012), the column height increases with \hat{M}_o following approximately a power
 203 law. Note how, for windless conditions, the strong plume collapses at MERs
 204 larger than about 4.9×10^9 kg/s whereas, in the presence of wind, the collapse is
 205 not observed because the increased entrainment of air. This result is consistent
 206 with the work of Degruyter & Bonadonna (2013) who find that wind increases
 207 air entrainment and prevents column collapse.

208 *3.2. Effect of vent radius variations on height*

209 The effect of the variation of the vent radius on the column height is implicit-
 210 itly contained in the results obtained by varying the MER. In fact, according to
 211 eq. (9), keeping constant \hat{T}_0 (i.e. ρ_0 , under the assumption that the exit pressure
 212 equals the atmospheric pressure) and \hat{u}_0 , a variation of \hat{M}_0 is equivalent to a
 213 variation of r_0 . Referring to Figure 1, for the strong plumes, the vent radius
 214 varies from 317 m for MER 3×10^8 kg/s to 1583 m for MER 7.5×10^9 kg/s.
 215 For strong plumes, in the windless condition, column collapse occurs for MER
 216 4.9×10^9 kg/s, corresponding to a vent radius of 1293 m, For the weak plumes, the
 217 vent radius varies from 12 m for MER 3×10^5 kg/s to 61 m for MER 7.5×10^6 kg/s.

218 *3.3. Effect of height variations on MER*

219 In practice, it is more convenient to quantify the variations on \hat{M}_o result-
 220 ing from column height uncertainties because column height is much easier to
 221 observe (or, at least, to constrain). Results are shown in Figure 2 for relative
 222 variations $100 \times (h - h^{\text{ref}})/h^{\text{ref}}$ in the range $\pm 20\%$ with respect the reference
 223 value h^{ref} . As observed, to produce an increase of only 10% in the column height
 224 requires of an increase in the MER by 50 and 25% for strong and weak plumes
 225 respectively. In other words, small errors (uncertainties) in column height mea-
 226 surements will result on much larger (relative) errors in the estimation of MER
 227 and, consequently, in the concentration downstream.

228 For the strong plumes, in the investigated range of column height (or mass
 229 eruption rate), Figures 1 and 2 show the presence of small bumps. These are
 230 related to the release of heat in the plume, due to the water phase change. The
 231 effect disappears when the latent heat for condensation or freezing of water are
 232 set equal to zero.

233 *3.4. Effect of exit velocity variations on height*

234 The effect of a variation in the mixture exit velocity \hat{u}_o on column height
 235 is reported in Figure 3. The variation of the velocity is performed at constant
 236 MER, by adjusting the vent radius accordingly. In the investigated range of
 237 exit velocity, the vent radius varies from 621 to 846 m for the strong plumes
 238 and from 24 to 32 m for the weak plumes. The temperature and the water
 239 mass fraction at the vent was kept constant (see Table 3). As observed, column
 240 heights are almost insensitive to variations on exit velocities within the explored
 241 uncertainty range ($\pm 30\%$ relative variations). However, for the strong plume
 242 case in windless conditions, columns collapse for velocities lower than about
 243 220 m/s. This reflects the existence of a minimum value of \hat{u}_o (or of \hat{M}_o) to
 244 sustain the plume buoyantly.

245 Moreover, the exit velocity was varied in the same ranges shown in Figure 3,
 246 but keeping fixed vent radius, exit temperature and water fraction (same as the
 247 reference case); MER is changed accordingly, in order to satisfy eq. (9). Results
 248 are not reported, since they do not differ significantly from Figure 3. In this
 249 case, for the strong plumes, in the windless condition, column collapse occurs
 250 for an exit velocity of 207 m/s, corresponding to a MER of 1.13×10^9 kg/s.

251 The effect of exit velocity on column collapse was previously described by
 252 Sparks & Wilson (1976), Wilson (1976), Wilson et al. (1978) and Wilson et al.
 253 (1980) who found that the conditions leading to collapse involve large vent
 254 radii, low gas velocities, and low gas contents. Similar results were also ob-
 255 tained by Valentine & Wohletz (1989); Kaminski & Jaupart (2001); Degruyter
 256 & Bonadonna (2013) and Dellino et al. (2014). Our findings are consistent with
 257 these previous works.

258 *3.5. Effect of exit temperature variations on height*

259 Most of the height of a volcanic eruption column is dominated by buoyancy
 260 effects (Sparks, 1986) and, to a first approximation, the height of the plume
 261 is related to the thermal flux at the vent (Wilson et al., 1978; Settle, 1978;
 262 Sparks, 1986). In the parametric study we varied the exit temperature keeping
 263 fixed the external (atmospheric) pressure, the mass eruption rate (MER) and
 264 the exit velocity. This implies that the density of the mixture at the vent varies
 265 as a consequence of the variation of the density of the gas phase (vapor). We
 266 assume that the gas density follows the equation of state of the ideal gas. In the
 267 investigated range of exit temperature, the vent radius varies from 673 to 741 m
 268 for the strong plumes and from 26 to 28 m for the weak plumes, whereas the
 269 density of the mixture at the vent varies from 3.1 to 3.7 kg/m³ for the strong
 270 plumes and from 4.5 and 5.2 kg/m³ for the weak plumes.

271 Results are reported in Figure 4 which shows the variation of column height h
 272 on mixture exit temperature \hat{T}_o for variations in the range ± 100 K the reference
 273 value. The effect of \hat{T}_o is noticeable for strong plumes (*e.g.* an increase of 5%
 274 in \hat{T}_o results on an increase of about 2.5% in column height) but negligible for
 275 weak plumes (as reflected by the flat lines in Fig. 4c and 4d).

276 *3.6. Effect of the erupted water content variations on height*

277 Figure 5 shows the effect of the erupted water mass fraction x_{wo} on the
 278 column height for relative variations (uncertainties) in the range ± 2 wt% the
 279 reference value. The water content, affecting the mixture density, was varied
 280 keeping constant the MER and the exit velocity. The vent radius was adjusted
 281 accordingly. In the investigated range of initial water content, the vent radius
 282 varies from 548 to 838 m for the strong plumes and from 16 to 35 m for the weak
 283 plumes. Column height slightly increases as the erupted water content increases.
 284 This effect is clear for strong plumes (*e.g.* an increase of 2 wt% results on an
 285 increase of about 2.5% in column height) but, as occurs with the mixture exit
 286 temperature, it is almost negligible for weak plumes.

287 *3.7. Effect of particle size variations on height*

288 In order to investigate the effect of particle size variations on column height
 289 we performed additional runs with a single granulometric class ranging $\pm 6\Phi$
 290 with respect to a reference value. The densities of each particle class were set
 291 as in Costa et al. (2016) and are reported in Table 4. As shown in Figure 6,
 292 the effect of particle size is visible only for windless conditions and particles
 293 in the millimetric range. This result is consistent with the works of Woods &
 294 Bursik (1991); de' Michieli Vitturi et al. (2015); Pouget et al. (2016), who found
 295 negligible variations of the column height with mean grain size in the range
 296 ($-6 \leq \Phi \leq 0$). In contrast, because of a different assumption on the grain size
 297 distribution (*i.e.* a power-law number distribution) and a larger particle size
 298 range, results of Girault et al. (2014, 2016) indicate that column height can be
 299 significantly affected by particle size distribution.

300 **4. Sensitivity study on model parameterizations**

301 A sensitivity study was also performed on the FPLUME model parameters
 302 related to wind entrainment, wind intensity, water phase change, air humidity
 303 (moisture), and particle fallout and re-entrainment. The effect of these processes
 304 was investigated by turning on and off the corresponding term in the model
 305 equations or by varying the parameters controlling the process (*e.g.* for studying
 306 the effect of air entrainment in the column).

307 *4.1. Effect of entrainment coefficients on column height*

308 The amount of entrained air in the column is described by the entraining
 309 velocity u_e , usually parameterized as a function of the rising velocity of the
 310 column and the wind velocity (eg. Hewett et al., 1971; Bursik, 2001; Suzuki &
 311 Koyaguchi, 2015; Woodhouse et al., 2015; Folch et al., 2016):

$$u_e = \alpha|\hat{u} - u_a \cos \theta| + \beta|u_a \sin \theta| \quad (10)$$

312 where \hat{u} and u_a are, respectively, the velocity of the plume along the centerline
 313 and the velocity of the wind. In the FPLUME model α and β can be set as

314 constants or calculated at each point depending on the local Richardson number
 315 (Tate, 2002; Carazzo et al., 2006, 2008b,a; Folch et al., 2016). For the test cases,
 316 we adopted the formulation based on the local Richardson number (Folch et al.,
 317 2016), predicting entrainment coefficients varying from 0.07 to 0.17 for α and
 318 from 0.43 to 1.00 for β . However, for the sensitivity study these parameters were
 319 assumed as constants varying in the ranges $\alpha = 0.05 - 0.15$ and $\beta = 0.1 - 1.0$,
 320 as dictated by Costa et al. (2016).

321 Figures 7a and 7c show the sensitivity of column height to variations in α for
 322 strong and weak plumes without wind (note that, for the windless case, $u_a = 0$
 323 and β plays no role). Note how variations in α within the considered range imply
 324 variations of up to $\pm 20\%$ and $\pm 30\%$ for weak and strong plumes respectively.
 325 This effect is largely magnified when considering the combined effect of α and
 326 β . In the case of a weak plume, with stronger wind, the column height can
 327 decrease up to a factor 2.5 with respect to the reference value if $\beta \geq 0.5$ (see
 328 Fig. 7d).

329 4.2. Effect of wind velocity on height

330 In order to investigate the influence of wind velocity on column height, the
 331 reference wind profiles (from Costa et al. (2016)) were multiplied by a factor
 332 f_w ranging between 0 and 2 (a value of $f_w = 1$ indicates the reference wind
 333 used in this work whereas a value of $f_w = 0$ corresponds to plumes in windless
 334 conditions). The resulting sensitivity of column height is shown in Figure 8. As
 335 expected, plume bending increases with wind, resulting on a decrease of plume
 336 height (see e.g. Bursik, 2001; Folch et al., 2012; Devenish, 2013; Woodhouse
 337 et al., 2013; Mastin, 2014). Because of the stronger intensity characterizing the
 338 reference wind profile, the effect is more pronounced for the weak plumes, with
 339 differences of up to 80% between windless and reference windy conditions.

340 4.3. Effects of various physical processes on height

341 In addition to the parameters described above, the height of the volcanic
 342 column is affected by various processes such as water phase change, entrainment
 343 of moisture, particle fallout and re-entrainment. The effect of the moisture and
 344 water phase change in the plume was previously investigated by Woods (1993)
 345 who found that in Plinian eruptions ($MER > 10^7$ kg/s) the latent heat released
 346 by condensation of vapor is relatively small in comparison with the thermal
 347 energy provided by the hot clasts and therefore moisture has no significant
 348 effect upon the eruption column dynamics. The largest influence of the phase
 349 change of water may occur for small or moderately sized eruptions where the
 350 energy released on condensation contributes significantly to the energy of the
 351 plume (Woods, 1993; Sparks et al., 1997; Woodhouse et al., 2013).

352 Due to gravity, particles tend to escape from the plume. This process was
 353 initially modeled by Woods & Bursik (1991), who assume that when a clast
 354 reaches a height at which the drag force equals its weight, the clast escapes
 355 from the plume. Moreover, due to the vortexes at the boundary of the plume, a
 356 fraction of the escaped particles may be re-entrained into the plume. The com-
 357 bined effect of fallout and re-entrainment was modeled by Bursik (2001) and is

358 represented by eq. (1g) and (6). In this work, the effect of these processes (phase
 359 changes, entrainment of moisture, and particle fallout and re-entrainment) was
 360 investigated by turning off one process at a time and comparing with the refer-
 361 ence runs. As observed in Figure 9 these effects are negligible for both strong
 362 and weak plumes. However, it should be stressed that the effect of moisture
 363 entrainment has been investigated only for the meteorological conditions of the
 364 reference tests. Previous works (eg. Woods, 1993; Bursik, 2001; Degruyter &
 365 Bonadonna, 2012) found that atmospheric humidity may have a significant effect
 366 on volcanic plumes. We expect that under other conditions (plumes in moist
 367 environment) the role of ambient moisture can become much more important.

368 5. Summary and discussion

369 We have performed a parametric and a sensitivity study to quantify how
 370 uncertainties in vent conditions and FPLUME plume model parameterizations
 371 affect the ESPs, in particular, the eruption column height. Uncertainties were
 372 explored within typical ranges for the two test cases (strong and weak) defined
 373 during the IAVCEI Commission on tephra hazard modeling inter-comparison
 374 study. The goal was to explore the leading order role of each parameter in order
 375 to assess which should be better constrained to better quantify ESPs for later
 376 use by TTDMs.

377 Results, summarized in Table 5, show that uncertainties in total MER at the
 378 vent \hat{M}_o are the ones that most affect column height for both weak and strong
 379 plume cases. Conversely, uncertainties in plume height determination strongly
 380 impact on the source strength quantification (*e.g.* uncertainties of $\pm 20\%$ in h
 381 result on MER variations of roughly $\pm 50\%$). Uncertainties (variations) in wind
 382 entrainment coefficients and wind intensity are also of first order (consistent
 383 with results of Woodhouse et al., 2015), especially for the weak plume case.
 384 The combined effect of variations in α and β has a dramatic effect on the model
 385 results (see Fig. 7). In contrast, mixture exit velocity \hat{u}_o and erupted water
 386 mass fraction \hat{x}_{wo} have a second order effect for the considered range. Finally,
 387 the effect of mixture exit temperature \hat{T}_o and particle size variations are almost
 388 negligible. Other physical phenomena such as water phase change, air humidity
 389 (moisture), and particle fallout and re-entrainment have been found to have
 390 little influence on model results for the test cases. However, it should be noted
 391 that atmospheric conditions have been not varied in our study. Other conditions
 392 different from those of the inter-comparison study (*e.g.* moist atmosphere) could
 393 result in notably different results.

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- 401 Brown, R. J., Bonadonna, C., & Durant, A. J. (2012). A review of volcanic ash
402 aggregation. *Phys. Chem. Earth*, 45-46, 65–78. doi:10.1016/j.pce.2011.11.001.
- 403 Bursik, M. I. (2001). Effect of wind on the rise height of volcanic plumes.
404 *Geophys. Res. Lett.*, 18, 3621–3624.
- 405 Bursik, M. I., & Woods, A. W. (1991). Buoyant, superbuoyant and col-
406 lapsing eruption columns. *J. Volcanol. Geotherm. Res.*, 45, 347–350.
407 doi:10.1016/0377-0273(91)90069-C.
- 408 Carazzo, G., Kaminski, E., & Tait, S. (2006). The route to self-
409 similarity in turbulent jets and plumes. *J. Fluid Mech.*, 547, 137–148.
410 doi:10.1017/S002211200500683X.
- 411 Carazzo, G., Kaminski, E., & Tait, S. (2008a). On the dynamics
412 of volcanic columns: A comparison of field data with new model of
413 negatively buoyant jets. *J. Volcanol. Geotherm. Res.*, 178, 94–103.
414 doi:10.1016/j.jvolgeores.2008.01.002.
- 415 Carazzo, G., Kaminski, E., & Tait, S. (2008b). On the rise of turbulent
416 plumes: Quantitative effects of variable entrainment for submarine hydrother-
417 mal vents, terrestrial and extra terrestrial explosive volcanism. *J. Geophys.*
418 *Res.*, 113. doi:10.1029/2007JB005458.
- 419 Costa, A., Folch, A., & Macedonio, G. (2010). A model for wet aggregation of
420 ash particles in volcanic plumes and clouds: I. Theoretical formulation. *J.*
421 *Geophys. Res.*, 115. doi:10.1029/2009JB007175.
- 422 Costa, A., Folch, A., & Macedonio, G. (2013). Density-driven transport in the
423 umbrella region of volcanic clouds: Implications for tephra dispersion models.
424 *Geophys. Res. Lett.*, 40, 1–5. doi:10.1002/grl.50942.
- 425 Costa, A., Macedonio, G., & Folch, A. (2006). A three-dimensional Eulerian
426 model for transport and deposition of volcanic ashes. *Earth Planet. Sci. Lett.*,
427 241, 634–647.
- 428 Costa, A., Suzuki, Y. J., Cerminara, M., Devenish, B. J., Esposti Ongaro, T.,
429 Herzog, M., Van Eaton, A. R., Denby, L. C., Bursik, M., de' Michieli Vitturi,
430 M., Engwell, S., Neri, A., Barsotti, S., Folch, A., Macedonio, G., Girault,
431 F., Carazzo, G., Tait, S., Kaminski, E., Mastin, L. G., Woodhouse, M. J.,
432 Phillips, J. C., Hogg, A. J., Degruyter, W., & Bonadonna, C. (2016). Results
433 of the eruption column model inter-comparison study. *J. Volcanol. Geotherm.*
434 *Res.*, . doi:10.1016/j.jvolgeores.2016.01.017.
- 435 Degruyter, W., & Bonadonna, C. (2012). Improving on mass flow
436 rate estimates of volcanic eruptions. *Geophys. Res. Lett.*, 39, L16308.
437 doi:10.1029/2012GL052566.

- 438 Degruyter, W., & Bonadonna, C. (2013). Impact of wind on the condition
439 for column collapse of volcanic plumes. *Earth Planet. Sci. Lett.*, *377-378*,
440 218–226. doi:10.1016/j.epsl.2013.06.041.
- 441 Dellino, P., Dioguardi, F., Mele, D., D’Addabbo, M., Zimanowski, B., Büttner,
442 R., Doronzo, D. M., Sonder, I., Sulpizio, R., Dürig, T., & La Volpe, L. (2014).
443 Volcanic jets, plumes, and collapsing fountains: evidence from large-scale ex-
444 periments, with particular emphasis on the entrainment rate. *Bull. Volcanol.*,
445 *76*, 834. doi:10.1007/s00445-014-0834-6.
- 446 Devenish, B. J. (2013). Using simple plume models to refine the source mass
447 flux of volcanic eruptions according to atmospheric conditions. *J. Volcanol.*
448 *Geotherm. Res.*, *256*, 118–127. doi:10.1016/j.jvolgeores.2013.02.015.
- 449 Ernst, G. J., Sparks, R. S. J., Carey, S. N., & Bursik, M. I. (1996). Sedimen-
450 tation from turbulent jets and plumes. *J. Geophys. Res.*, *101*, 5575–5589.
451 doi:10.1029/95JB01900.
- 452 Folch, A. (2012). A review of tephra transport and dispersal models: Evolution,
453 current status, and future perspectives. *J. Volcanol. Geotherm. Res.*, *235-236*,
454 96–115. doi:10.1016/j.jvolgeores.2012.05.020.
- 455 Folch, A., Costa, A., & Basart, S. (2012). Validation of the FALL3D ash disper-
456 sion model using observations of the 2010 Eyjafjallajökull volcanic ash cloud.
457 *Atmos. Environ.*, *48*, 165–183. doi:10.1016/j.atmosenv.2011.06.072.
- 458 Folch, A., Costa, A., & Macedonio, G. (2009). FALL3D: A computational model
459 for transport and deposition of volcanic ash. *Comput. Geosci.*, *35*, 1334–1342.
460 doi:10.1016/j.cageo.2008.08.008.
- 461 Folch, A., Costa, A., & Macedonio, G. (2016). FPLUME-1.0: An integral
462 volcanic plume model accounting for ash aggregation. *Geosci. Model Dev.*, *9*,
463 431–450. doi:10.5194/gmd-9-431-2016.
- 464 Ganser, G. H. (1993). A rational approach to drag prediction of spherical
465 and nonspherical particles. *Powder Technol.*, *77*, 143–152. doi:10.1016/0032-
466 5910(93)80051-B.
- 467 Girault, F., Carazzo, G., Ferrucci, F., & Kaminski, E. (2014). The effect of total
468 grain-size distribution on the dynamics of turbulent volcanic plumes. *Earth*
469 *Planet. Sci. Lett.*, *394*, 124–134. doi:10.1016/j.epsl.2014.03.021.
- 470 Girault, F., Carazzo, G., Tait, S., & Kaminski, E. (2016). Combined effects
471 of total grain-size distribution and crosswind on the rise of eruptive volcanic
472 columns. *J. Volcanol. Geotherm. Res.*, . doi:10.1016/j.jvolgeores.2015.11.007.
- 473 Hashimoto, A., Shimbori, T., & Fukui, K. (2012). Tephra fall simulation for the
474 eruptions at Mt. Shinmoe-dake during 26-27 January 2011 with JMANHM.
475 *SOLA*, *8*, 37–40. doi:10.2151/sola.2012-010.

- 476 Hewett, T. A., Fay, J. A., & Hault, D. P. (1971). Laboratory experiments of
 477 smokestack plumes in a stable atmosphere. *Atmos. Environ.*, *5*, 767–789.
 478 doi:10.1016/0004-6981(71)90028-X.
- 479 Kaminski, E., & Jaupart, C. (2001). Marginal stability of atmospheric eruption
 480 columns and pyroclastic flow generation. *J. Geophys. Res.*, *106*, 21,785–
 481 21,798. doi:10.1029/2001JB000215.
- 482 Mastin, L. G. (2014). Testing the accuracy of a 1-D volcanic plume model in
 483 estimating mass eruption rate. *J. Geophys. Res. Atmos.*, *119*, 2474–2495.
 484 doi:10.1002/2013JD020604.
- 485 de' Michieli Vitturi, M., Neri, A., & Barsotti, S. (2015). PLUME-MoM 1.0:
 486 A new integral model of volcanic plumes based on the method of moments.
 487 *Geosci. Model Dev.*, *8*, 2447–2463. doi:10.5194/gmd-8-2447-2015.
- 488 Morton, B. R., Taylor, G., & Turner, J. S. (1956). Turbulent gravitational con-
 489 vection from maintained and instantaneous sources. *Proc. Roy. Soc. London,*
 490 *Ser. A*, *234*, 1–23.
- 491 Pouget, S., Bursik, M., Singla, P., & Singh, T. (2016). Sensitivity analysis of a
 492 one-dimensional model of a volcanic plume with particle fallout and collapse
 493 behavior. *J. Volcanol. Geotherm. Res.*, . doi:10.1016/j.jvolgeores.2016.02.018.
- 494 Schwaiger, H. F., Denlinger, R. P., & Mastin, L. G. (2012). Ash3d: A finite-
 495 volume, conservative numerical model for ash transport and tephra deposi-
 496 tion. *J. Geophys. Res.*, *117*, B04204. doi:10.1029/2011JB008968.
- 497 Settle, M. (1978). Volcanic eruption clouds and the thermal power output of
 498 explosive eruptions. *J. Volcanol. Geotherm. Res.*, *3*, 309–324.
- 499 Sparks, R. S. J. (1986). The dimensions and dynamics of volcanic eruption
 500 columns. *Bull. Volcanol.*, *48*, 3–15. doi:10.1007/BF01073509.
- 501 Sparks, R. S. J., Bursik, M. I., Carey, S. N., Gilbert, J. S., Glaze, L. S., Sigurd-
 502 son, H., & Woods, A. W. (1997). *Volcanic Plumes*. Chichester, U.K.: John
 503 Wiley & Sons Ltd.
- 504 Sparks, R. S. J., & Wilson, L. (1976). A model for the formation of ign-
 505 imbrite by gravitational column collapse. *J. Geol. Soc. London*, *132*, 441–451.
 506 doi:10.1144/gsjgs.132.4.0441.
- 507 Suzuki, Y. J., & Koyaguchi, T. (2015). Effects of wind on entrain-
 508 ment efficiency in volcanic plumes. *J. Geophys. Res.*, *110*, 6122–6140.
 509 doi:10.1002/2015JB012208.
- 510 Tate, P. M. (2002). *The rise and dilution of buoyant jets and their behaviour*
 511 *in an internal wave field*. Phd thesis University of New South Wales, School
 512 of Mathematics. URL: <http://trove.nla.gov.au/version/19798635> last
 513 access: 16 September 2015.

- 514 Tate, P. M., & Middleton, J. H. (2000). Unification of non-dimensional solu-
515 tions to asymptotic equations for plumes of different shape. *Boundary-Layer*
516 *Meteorol.*, *94*, 225–251.
- 517 Valentine, G. A., & Wohletz, K. H. (1989). Numerical models of Plinian
518 eruption columns and pyroclastic flows. *J. Geophys. Res.*, *94*, 1867–1887.
519 doi:10.1029/JB094iB02p01867.
- 520 Wilson, L. (1976). Explosive volcanic eruptions III. Plinian eruption columns.
521 *Geophys. J. R. Astron. Soc.*, *45*, 543–556.
- 522 Wilson, L., Sparks, R. S. J., Huang, T. C., & Watkins, N. D. (1978). The control
523 of volcanic column heights by eruption energetics and dynamics. *J. Geophys.*
524 *Res.*, *83*, 1829–1836. doi:10.1029/JB083iB04p01829.
- 525 Wilson, L., Sparks, R. S. J., & Walker, G. P. L. (1980). Explosive volcanic
526 eruptions IV. The control of magma properties and conduit geometry on
527 eruption column behavior. *Geophys. J. R. Astron. Soc.*, *63*, 117–148.
- 528 Woodhouse, M. J., Hogg, A. J., Phillips, J. C., & Rougier, J. C. (2015). Un-
529 certainty analysis of a model of wind-blown volcanic plumes. *Bull. Volcanol.*,
530 *77*, 83. doi:10.1007/s00445-015-0959-2.
- 531 Woodhouse, M. J., Hogg, A. J., Phillips, J. C., & Sparks, R. S. J. (2013).
532 Interaction between volcanic plumes and wind during the 2010 Eyjafjal-
533 lajökull eruption, Iceland. *J. Geophys. Res. Solid Earth*, *118*, 92–109.
534 doi:10.1029/2012JB009592.
- 535 Woods, A. W. (1988). The fluid dynamics and thermodynamics of eruption
536 columns. *Bull. Volcanol.*, *50*, 169–193.
- 537 Woods, A. W. (1993). Moist convection and the injection of volcanic ash into
538 the atmosphere. *J. Geophys. Res.*, *98*, 17,627–17,636.
- 539 Woods, A. W., & Bursik, M. I. (1991). Particle fallout, thermal disequilibrium
540 and volcanic plumes. *Bull. Volcanol.*, *53*, 559–570. doi:10.1007/BF00298156.

Table 1: List of latin symbols. Quantities with a hat denote bulk (top-hat averaged) quantities. Throughout the text, the subindex o (*e.g.* \hat{M}_o , \hat{u}_o , etc.) indicates values of quantities at the vent ($s = 0$).

Symbol	Definition	Units
A_i^+ (A_i^-)	Aggregation source (sink) terms	$\text{kg s}^{-1} \text{m}^{-1}$
c_a	Specific heat capacity of air at constant pressure	$\text{J kg}^{-1} \text{K}^{-1}$
c_l	Specific heat capacity of liquid water	$\text{J kg}^{-1} \text{K}^{-1}$
c_p	Specific heat capacity of particles (pyroclasts)	$\text{J kg}^{-1} \text{K}^{-1}$
c_s	Specific heat capacity of solid water (ice)	$\text{J kg}^{-1} \text{K}^{-1}$
c_v	Specific heat capacity of water vapor	$\text{J kg}^{-1} \text{K}^{-1}$
c_w	Specific heat capacity of water (generic)	$\text{J kg}^{-1} \text{K}^{-1}$
\hat{E}	Energy flow rate	J s^{-1}
f_i	Mass fraction of particle class i	—
g	Gravitational acceleration	m s^{-2}
h	Column height	m
h_l	Enthalpy per unit mass of liquid water	J kg^{-1}
h_s	Enthalpy per unit mass of ice	J kg^{-1}
h_v	Enthalpy per unit mass of vapour	J kg^{-1}
h_{l0}	Enthalpy per unit mass of liquid water at $T = T_0$	J kg^{-1}
h_{s0}	Enthalpy per unit mass of ice at $T = T_0$	J kg^{-1}
h_{v0}	Enthalpy per unit mass of vapour at $T = T_0$	J kg^{-1}
\hat{H}	Enthalpy flow rate	J s^{-1}
m_a	Molar weight of air	kg/mole
m_v	Molar weight of water	kg/mole
\hat{M}	Total mass flow rate	kg s^{-1}
\hat{M}_a	Mass flow rate of dry air	kg s^{-1}
\hat{M}_i	Mass flow rate of particles of class i	kg s^{-1}
\hat{M}_w	Mass flow rate of volatiles (water in any phase)	kg s^{-1}
n_a	Molar fraction of air in the gas phase	—
n_v	Molar fraction of vapour in the gas phase	—
\hat{P}	Axial (stream-wise) momentum flow rate	kg m s^{-2}
P	Pressure	Pa
P_a	Partial pressure of air	Pa
P_v	Partial pressure of water vapor	Pa
s	Distance along the plume axis	m
\hat{T}	Mixture temperature	K
T_a	Ambient air temperature	K
T_0	Reference temperature (273.15 K)	K
\hat{u}	Mixture velocity along the plume axis	m s^{-1}
u_a	Horizontal wind (air) velocity	m s^{-1}
u_e	Air entrainment velocity (by turbulent eddies)	m s^{-1}
w_a	Mass fraction of water in the entrained ambient air	-
x	Horizontal coordinate	m
x_l	Mass fraction of liquid water	—
x_s	Mass fraction of solid water (ice)	—
x_v	Mass fraction of water vapor	—
x_p	Mass fraction of particles (pyroclasts)	—
x_w	Mass fraction of volatiles (water)	—
y	Horizontal coordinate	m
z	Vertical coordinate	m

Table 2: List of greek symbols.

Symbol	Definition	Units
α	stream-wise (shear) air entrainment coefficient	-
β	cross-flow (vortex) air entrainment coefficient	-
$\hat{\rho}$	Mixture density	kg m^{-3}
ρ_a	Ambient air density	kg m^{-3}
Φ_a	Horizontal wind direction (azimuth)	rad
Ψ	Particle sphericity	-
χ	Constant giving the probability of fallout	-

Table 3: Reference values of the parameters for the strong and weak plume cases (Costa et al., 2016).

Parameter	Symbol	Units	Strong	Weak
Mass flow rate	M	kg/s	1.5×10^9	1.5×10^6
Vent height (a.s.l)	h_v	m	1500	1500
Velocity at the vent	u_0	m/s	275	135
Temperature at the vent	T_0	K	1053	1273
Water mass fraction at the vent	w_0	—	5%	3%

Table 4: Total particle grain size distribution at the vent for strong and weak plumes discretized in $n=14$ classes. The Φ units are defined so that the particle diameter (in mm) is $d = 2^{-\Phi}$. The sphericity parameter Ψ is assumed equal to 0.9. The mean particle densities are 2646.3 and 2414.4 kg/m^3 , respectively for the strong and the weak plumes.

(Φ)	Diameter (mm or μm)	Strong Plumes		Weak Plumes	
		Density (kg/m^3)	wt.% (—)	Density (kg/m^3)	wt.% (—)
-6	64	—	—	1700.0	0.01
-5	32	2200.0	0.01	1792.3	0.11
-4	16	2253.8	0.10	1884.6	0.59
-3	8	2307.7	0.59	1976.9	2.24
-2	4	2361.5	2.23	2069.2	5.77
-1	2	2415.4	5.76	2161.5	10.26
0	1	2469.2	10.16	2253.8	12.86
1	500	2523.1	12.37	2346.2	12.39
2	250	2576.9	10.74	2438.5	11.52
3	125	2630.8	7.99	2530.8	12.39
4	62.5	2684.6	7.99	2623.1	12.86
5	31.25	2738.5	10.74	2715.4	10.26
6	15.62	2792.3	12.37	2807.7	5.77
7	7.8	2846.2	10.16	2900.0	2.96
8	3.9	2900.0	8.71	—	—

Table 5: Summary of results. Effect of input uncertainties on column height expressed as h/h^{ref} (i.e. values close to 1 imply little effect.)

Parameter	Case	wind	Increase range	h/h^{ref}	Decrease range	h/h^{ref}
\hat{M}_o	strong	no	$\hat{M}_o \times 3.3^{(*)}$	1.35	$\hat{M}_o \times 1/5$	0.72
		yes	$\hat{M}_o \times 5$	1.48	$\hat{M}_o \times 1/5$	0.72
	weak	no	$\hat{M}_o \times 5$	1.39	$\hat{M}_o \times 1/5$	0.62
		yes	$\hat{M}_o \times 5$	1.78	$\hat{M}_o \times 1/5$	0.64
\hat{u}_o	strong	no	$\hat{u}_o + 30\%$	0.93	$\hat{u}_o - 17\%^{(*)}$	1.05
		yes	$\hat{u}_o + 30\%$	0.93	$\hat{u}_o - 30\%$	1.06
	weak	no	$\hat{u}_o + 30\%$	0.99	$\hat{u}_o - 30\%$	1.02
		yes	$\hat{u}_o + 30\%$	0.99	$\hat{u}_o - 30\%$	1.03
\hat{T}_o	strong	no	$\hat{T}_o + 100 \text{ K}$	1.02	$\hat{T}_o - 100 \text{ K}$	0.97
		yes	$\hat{T}_o + 100 \text{ K}$	1.02	$\hat{T}_o - 100 \text{ K}$	0.97
	weak	no	$\hat{T}_o + 100 \text{ K}$	negligible	$\hat{T}_o - 100 \text{ K}$	negligible
		yes	$\hat{T}_o + 100 \text{ K}$	negligible	$\hat{T}_o - 100 \text{ K}$	negligible
x_{wo}	strong	no	$x_{wo} + 2 \text{ wt}\%$	1.04	$x_{wo} - 2 \text{ wt}\%$	0.97
		yes	$x_{wo} + 2 \text{ wt}\%$	1.04	$x_{wo} - 2 \text{ wt}\%$	0.95
	weak	no	$x_{wo} + 2 \text{ wt}\%$	1.04	$x_{wo} - 2 \text{ wt}\%$	0.96
		yes	$x_{wo} + 2 \text{ wt}\%$	1.03	$x_{wo} - 2 \text{ wt}\%$	0.97
particle size (1 class at $\Phi_o = 2$)	strong	no	$\Phi_o + 4\Phi$	negligible	$\Phi_o - 4\Phi$	0.96
		yes	$\Phi_o + 4\Phi$	negligible	$\Phi_o - 4\Phi$	negligible
	weak	no	$\Phi_o + 4\Phi$	negligible	$\Phi_o - 4\Phi$	0.85
		yes	$\Phi_o + 4\Phi$	negligible	$\Phi_o - 4\Phi$	negligible
Wind entrainment coefficients $\alpha_o = 0.1$ $\beta_o = 0.0$	strong	no	$\alpha_o + 0.05$	0.90	$\alpha_o - 0.05$	1.16
		yes	$\alpha_o + 0.05, \beta_o + 1.0$	0.87	$\alpha_o - 0.05, \beta_o$	1.16
	weak	no	$\alpha_o + 0.05$	0.88	$\alpha_o - 0.05$	1.27
		yes	$\alpha_o + 0.05, \beta_o + 1.0$	0.38	$\alpha_o - 0.05, \beta_o$	1.27
Wind intensity f_w	strong	yes	$f_w \in (1, 2)$	0.95	$f_w \in (0, 1)$	1.08
	weak	yes	$f_w \in (1, 2)$	0.83	$f_w \in (0, 1)$	1.82

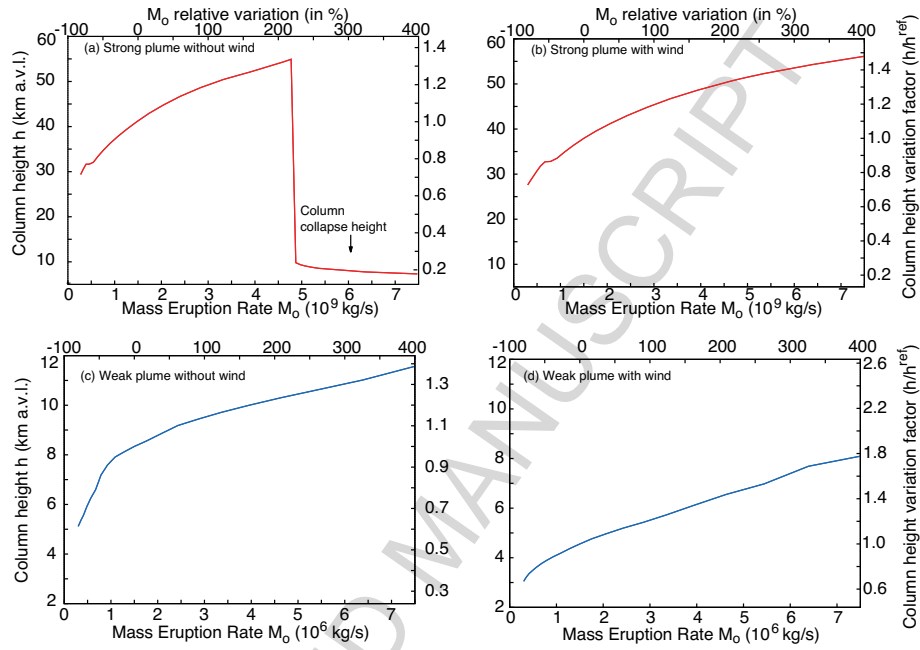


Figure 1: Variation of the column height as function of the mass eruption rate (MER) for the strong (top) and weak (bottom) plumes without (left) and with (right) wind. The top and right axes indicate, respectively, the relative MER variation with respect to the reference value (1.5×10^9 and 1.5×10^6 kg/s for strong and weak plumes respectively) and its effect on the column height. Note that, in absence of wind, the column collapses for MER larger than about 4.9×10^9 kg/s. For the strong plume case (red lines), the small bumps in the left part of the plots are due to the effect of water phase change.

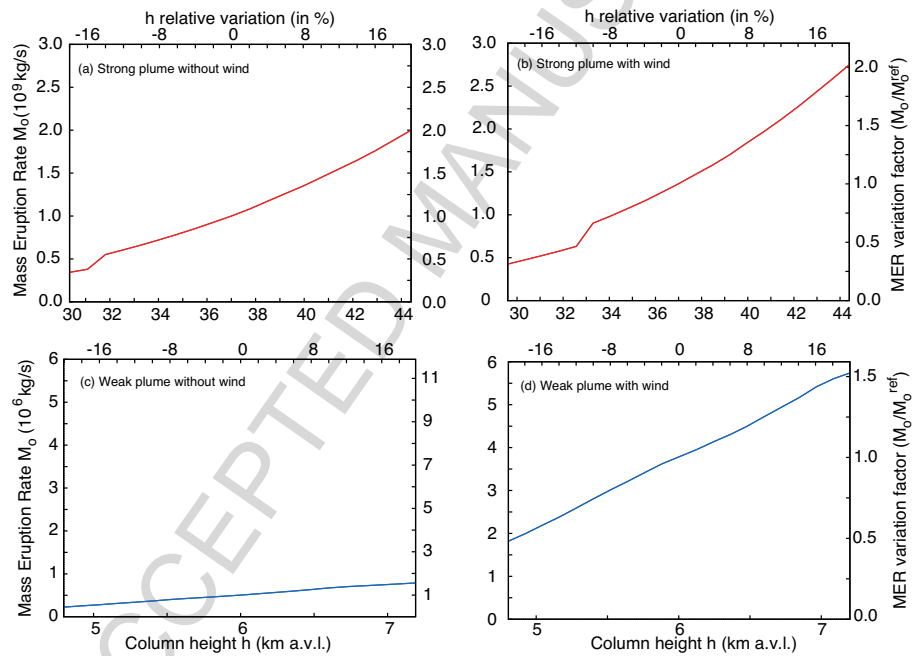


Figure 2: Variation of the mass eruption rate (MER) with column height for the strong (top) and weak (bottom) plumes without (left) and with (right) wind. The top and right axes indicate, respectively, the column height variation with respect to the reference values (37 and 6 km for strong and weak respectively) and its effect on the MER. For the strong plume case (red lines), the small bumps in the left part of the plots are due to the effect of water phase change. Note that plots agree with Figure 1 in the ranges shown.

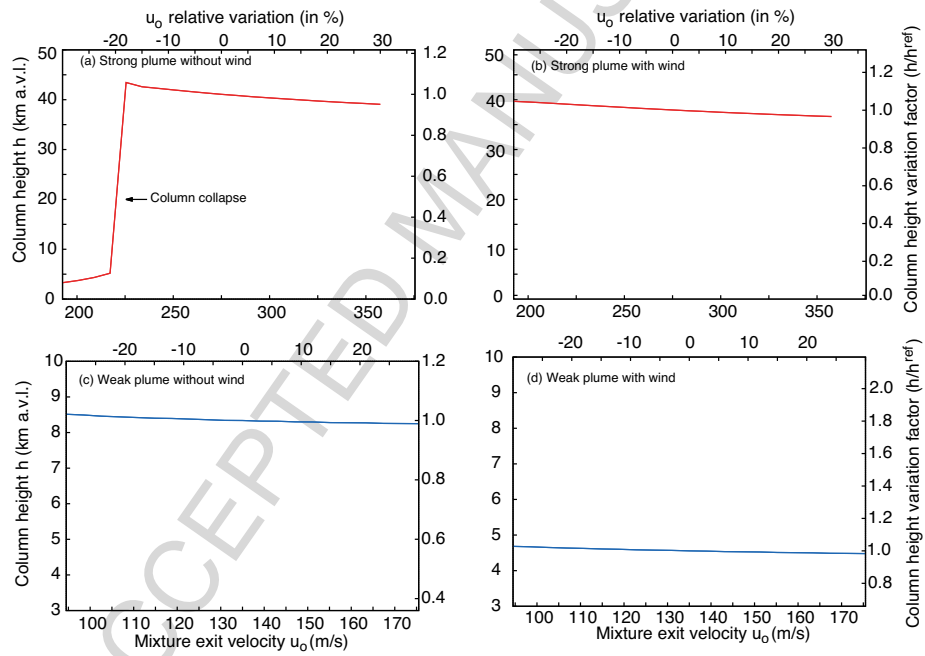


Figure 3: Variation of column height with plume velocity at the vent for the strong (top) and weak (bottom) plumes without (left) and with (right) wind. The top and right axes indicate, respectively, the exit velocity variation with respect to the reference values (275 and 135 m/s for strong and weak respectively) and its effect on the column height. Note that, in absence of wind, the column collapses if velocities at the vent are smaller than about 220 m/s.

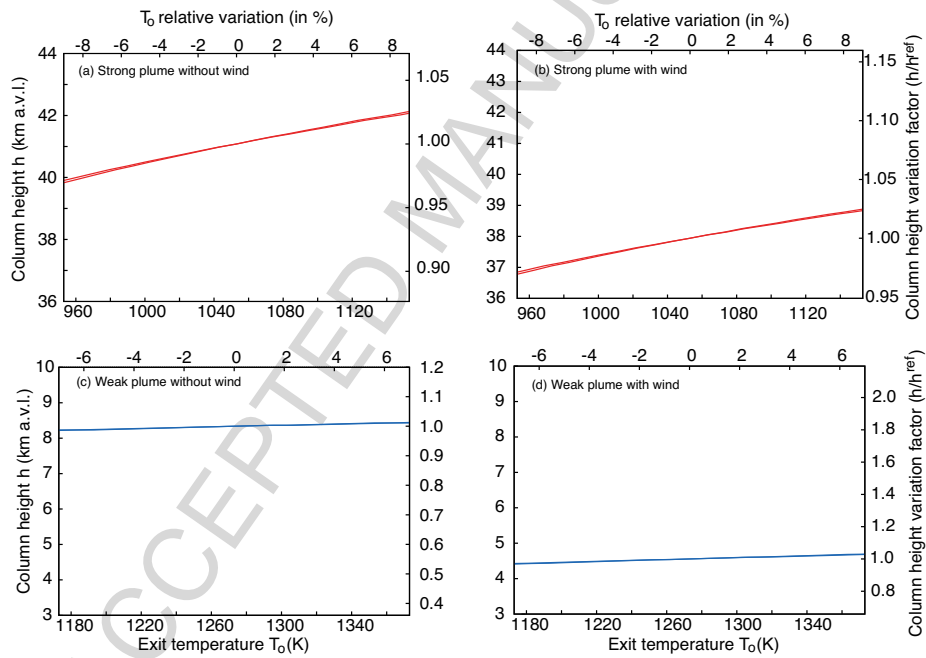


Figure 4: Variation of column height with temperature at the vent for the strong (top) and weak (bottom) plumes without (left) and with (right) wind. The top and right axes indicate, respectively, the temperature variation with respect to the reference values (1053 and 1273 K for strong and weak respectively) and its effect on the column height.

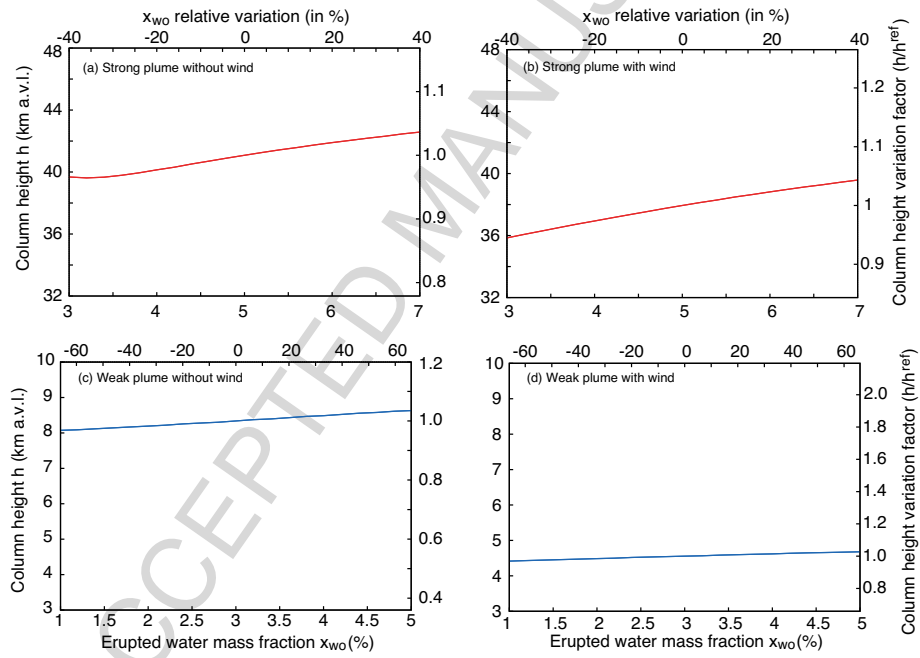


Figure 5: Variation of column height with initial water content for the strong (top) and weak (bottom) plumes without (left) and with (right) wind. The top and right axes indicate, respectively, the variation of water content with respect to the reference values (5 and 3% for strong and weak respectively) and its effect on the column height. Mass eruption rate, exit velocity and temperature are kept fixed, whereas vent radius is allowed to vary.

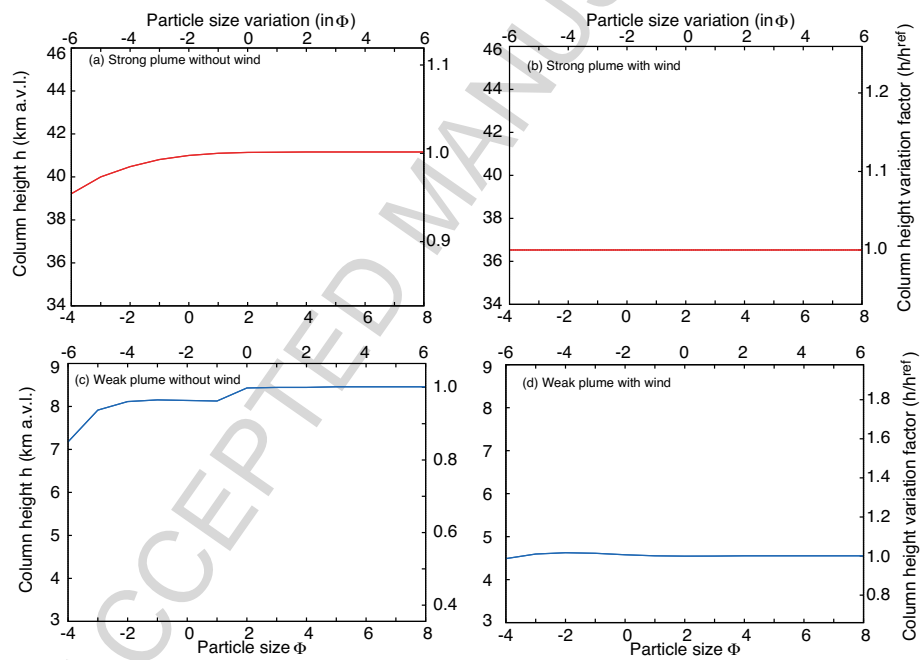


Figure 6: Variation of column height with particle grain size for the strong (top) and weak (bottom) plumes without (left) and with (right) wind. Note that one single class is assumed in these particular runs. The top and right axes indicate, respectively, the variation of class size with respect to a reference value ($\Phi = 2$) and its effect on the column height.

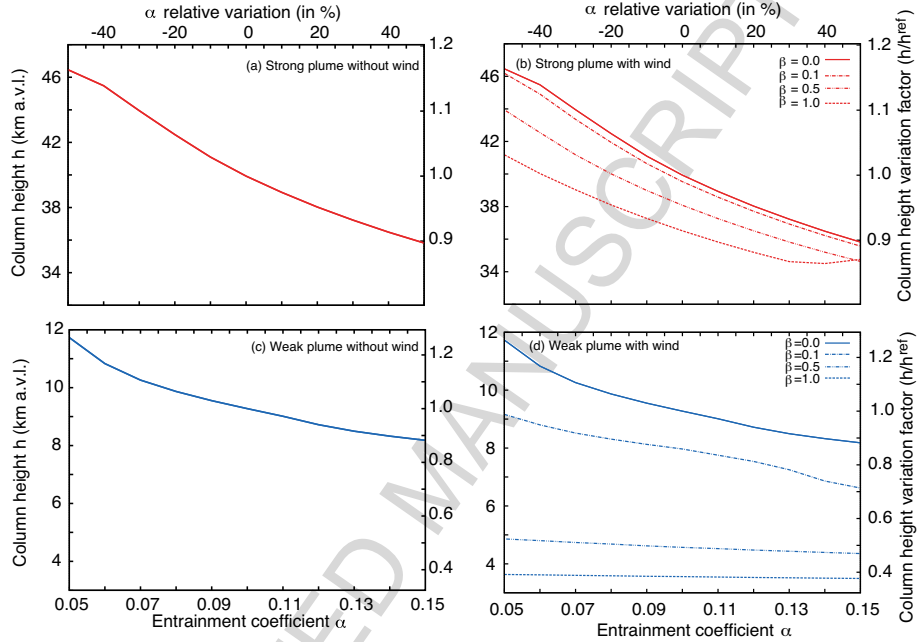


Figure 7: Variation of column height with entrainment coefficient α for the strong (top) and weak (bottom) plumes without (left) and with (right) wind. The top and right axes indicate, respectively, the variation of α with respect to the reference value ($\alpha = 0.1$) and its effect on the column height. In case of wind, results are given for different β values of 0, 0.1, 0.5 and 1.

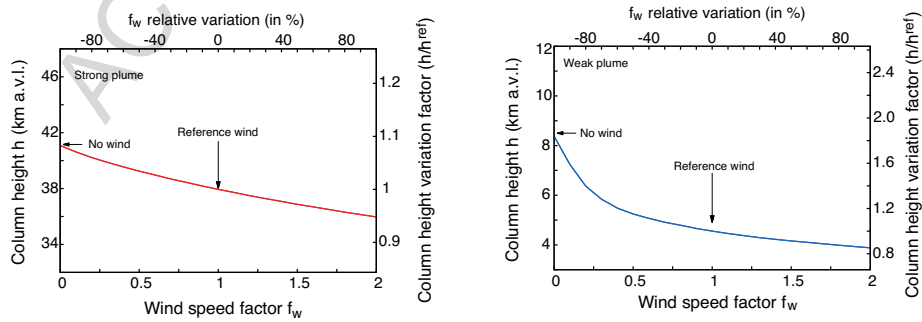


Figure 8: Variation of the column height depending on wind intensity (wind speed factor f_w) for strong (left) and weak (right) plumes. A value of $f_w = 1.0$ corresponds to the reference wind used in this work whereas a value of $f_w = 0.0$ corresponds to plumes in absence of wind. The top and right axes indicate, respectively, the variation of wind speed factor with respect to the reference values and its effect on the column height.

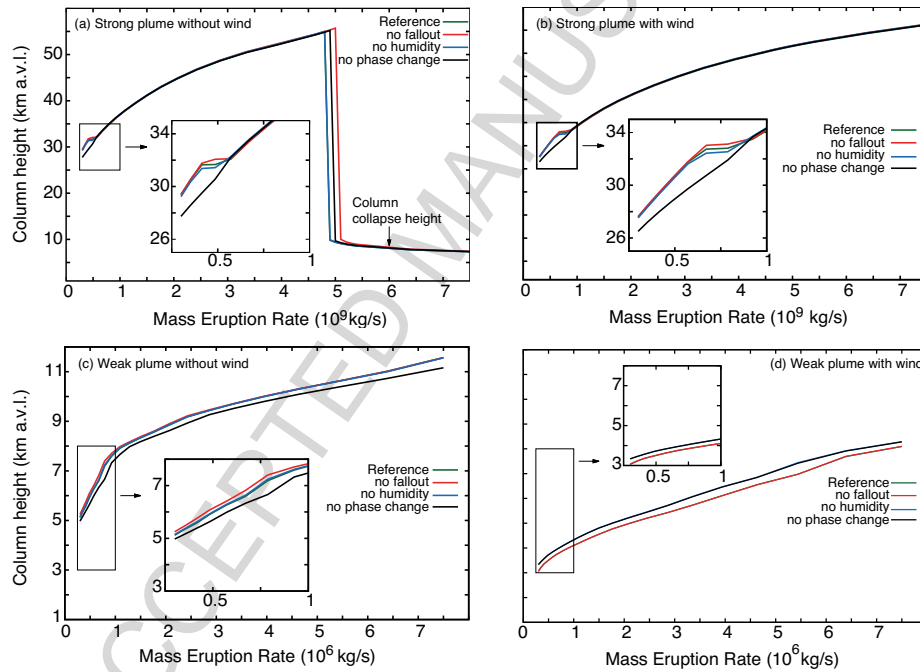


Figure 9: Variation of the column height as function of the mass eruption rate (MER) for the strong (top) and weak (bottom) plumes without (left) and with (right) wind. The reference simulations (green lines) are compared with those obtained by neglecting particle fallout (red lines), atmospheric humidity (blue lines) and the water phase change (black lines). Note that, in absence of wind, the column collapses for MER larger than about 5×10^9 kg/s. The small bumps in the left part of the plots (zoomed areas) are due to the effect of water phase change.

Highlights

- We perform a sensitivity study on input parameters of a volcanic plume model
- Effects of input parameter variation on plume model results were estimated
- Effects on entrainment parameter variation and wind intensity was estimated
- Typical uncertainty on mass flow rate and plume height estimation was assessed