MULTITONE TRACKING WITH COUPLED EKFs AND HIGH ORDER LEARNING

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ABSTRACT[1]

A multitone tracker is described using two basic principles in optimum frequency estimation, namely: Processing bandwidth depending on the distance from the estimate to the actual frequency values; and, parallel estimates with inhibitory paths to ensure orthogonality between the enhanced tones. The first feature is provided by Extended Kalman Filters (EKF), and the second one is achieved by high order rule for the learning of the inhibitory cells. It is shown that the independence between signals is linked to the high order function of the learning process. The resulting multitone tracker seems to be a potential alternative to adaptive high resolution methods or time-frequency tools.

I. INTRODUCTION

This paper deals with the problem of simultaneous acquisition and tracking of complex sinusoids either in white and colored noise. Regardless the motivations of this problem can be found almost in any signal processing application, the objective of the authors was to solve the problem of collisions produced when two simultaneous users ask for a dedicated beam in an adaptive array beamforming with temporal reference. Being more specific, the access to the satellite link is provided to any user which issues a pure (bandwidth less than 5% of the global frequency slot) sinusoid in a pre-assigned frequency band. The received frequency is regenerated by an EKF at the array output. This regenerated signal, being compared with the array output, in the same frequency band, produces the residual signal which enables the adaptive algorithm to steer the desired source yet preserving adequate nulling to interferences (see /1/ for more details in Time Reference Beamforming (TRB)).

The problem of collision, as mentioned above, appears when two users send two different unmodulated references at the same time. In order to increase the throughput of the communication link, it is necessary to form two beams, time or frequency multiplexed for the corresponding DOAs of the two users. To do this we require

that, when the users send the two pure tones, two EKFs acquire and track independently the two references.
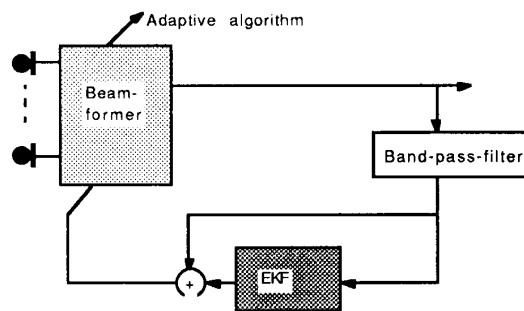

Figure1. EKF in the regeneration of unmodulated references in TRB systems.

In summary, the problem to be faced is a multitone tracking in white or colored noise as it corresponds to the described application. Naming x(t) the signal, which contains the two pure tones at the array output, its formulation will be (1).

$$x(t)=e_1(t).exp(jF_1(t))+$$
$$e_2(t).exp(jF_2(t))+w(t) \qquad (1)$$

Being $e_1(t)$ and $e_2(t)$ the envelope of each tone. These magnitudes, as indicated, may fluctuate in time +/-3 dB during tracking. In consequence all the signal to noise ratios provided will be given from their average.

$$SNR_i=ave(e_i^2(t))/s^2 \; ; s^2=1; \text{ref level} \qquad (2)$$

The instantaneous phase obeys to a given nominal frequency assigned to the user f_i (i=1,2). At the base-band level, this frequency will be in the range of 1 up to 3KHz, for a sampling rate of 8 KHz. The main difficulties come from the existing doppler that will be in the range of +/- 1.5 KHz, with a doppler rate of 4KHz per second. The scheme depicted in figure 2, shows the frequency range where the operating frequencies may vary.
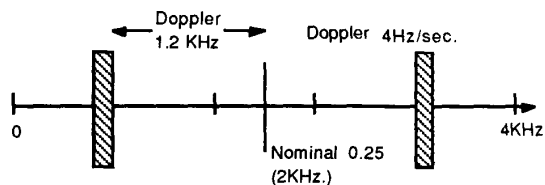
Figure 2.- Frequency range and bounds

It is worthwhile to note that it is just the doppler rate the effect which increases the inherent difficulty of the problem. In summary, two instantaneous frequencies together with their slow time varying magnitudes have to be acquired and tracked.

2. SPECTRAL METHODS AND EXTENDED KALMAN FILTERING.

There are many alternatives which may be used in trying to solve the problem of multitone tracking.

The first approach may come from time-varying or adaptive versions of high resolution frequency detectors. Most of the cases these procedures are based in SVD analysis of data matrix and the algorithms to update such decompositions, recently reported, should be used /2/. In reference /3/, and for the case of DOA estimation with a filled linear array, some examples of crossing trajectories (i.e. crossing frequencies evolution in this paper). Regardless that the computational load and the sequential character of these algorithms is very high, the resulting performance does not achieves the same levels of quality that we are going to observe hereafter. Furthermore, the degree of parallelism of the scheme to be proposed hereafter is high and it supports analog and optical implementation.

The second approach is the use of time-frequency methods which have been shown excellent performance in revealing the structure of frequency modulated signals. Still, we do believe that the optimum procedure cannot be achieved by these techniques for the multitone tracking, because they work, somehow, under a broadband basis. To be more precise, an optimum frequency processor should be in such a way that, by the time new data arrives, it concentrates more and more its performance in a narrowband around the actual frequency position. In other words, the processing bandwidth must be dependent on the vicinity of the estimated frequency versus the actual frequency. This is the main guide-line to anticipate the degree of quality that can be achieved from a given frequency estimate. Of course, the asymptotic bandwidth, theoretically cero, has to be set above some threshold depending on the desired persecution error in time varying scenarios.

From the above paragraphs it is clear that only PLLs or FLLs follows the basic guide-lines in order to achieve optimum performance. This justifies why they last almost during fifty years without competition in single tone acquisition and tracking in communications receivers. In other words, whenever a pure tone can be isolated in a given frequency band, no other system may compete with a PLL. In

this sense, The Extended Kalman Filter (EKF) has been proved to be a steep ahead to classical PLL, being the main contribution of digital technology in this field. We will review hereafter the basic equations for an EKF.

In an EKF a signal model is required. For a single tone in noise the two equations of this model are:

$$A_n^T = (e(n),f(n),f(n)); A_{n+1}^T = F \cdot A_n^T + \underline{v}(n)$$
$$x(n) = e(n) \cdot \exp(jf(n)) + w(n) \qquad (3a\text{-}3b)$$

The so-called state vector contains the actual envelope, instantaneous phase and instantaneous frequency; F is the transition matrix; and $\underline{v}(n)$ is a random vector with independent components representing the uncertainty in the time evolution of the state parameters.

$$F = \begin{pmatrix} 1\ 0\ 0 \\ 0\ 1\ 2p \\ 0\ 0\ 1 \end{pmatrix} \quad E(\underline{v}(n) \cdot \underline{v}^H(n)) = Q \text{ (diagonal)} \quad (3.c)$$

The corresponding EKF equations are:

$$\underline{a}(n+1) = F \cdot \underline{a}(n) + K_n \cdot \underline{x}_n \qquad (4.a)$$
$$z(n) = e(n) \cdot \exp(jQ(n)) \text{ or}$$
$$\underline{z}^T_n = (e(n) \cdot \cos(Q(n)), -e(n) \sin(Q(n))) \qquad (4.b)$$

where K_n is the so-called gain matrix, a(n) the state estimate, vector $\underline{x}_n$ is the signal error between the measured signal x(n) and z(n) in vector form with the in-phase and quadrature components. The crucial point in getting the EKF equations is to set the waveform error $\underline{x}_n$ in a linear dependence of the state error $\underline{x}_{an}$ (i.e. the difference between the actual state A_n and the EKF estimate $\underline{a}(n)$. This can be done assuming that the tracking error in the state vector is small enough to approximate the waveform error by (5).

$$\underline{x}_n = H_n \cdot \underline{x}_{an} + \underline{w}(n) \qquad (5)$$

being

$$H = \begin{pmatrix} \cos Q(n)\ -\sin Q(n)\ 0 \\ -\sin Q(n)\ -\cos Q(n)\ 0 \end{pmatrix} \qquad (6)$$

The design equations follow straight forward from the necessary orthogonality between the state error estimate $\underline{x}_{an+1}$ and the waveform error $\underline{x}_n$. The resulting gain matrix is:

$$K_n = F \cdot S_n \cdot E^{-1} \qquad (7.a)$$
$$E = E(\underline{x}_n \cdot \underline{x}_n^H) = H_n \cdot S_n \cdot H_n^H + I \qquad (7.b)$$

where it has been assumed the uncorrelated character of the noise vector $\underline{w}(n)$, with power normalised to one. The state covariance S_n is updated from the difference between the state model error (3.a) and the EKF update (4.a).

$$S_{n+1}=F.S_n.F^H-K_n.E.K_n^H-Q \qquad (8)$$

It is important to note, concerning the involved complexity, that all the above matrix are of a maximum dimension of 3x3. Also, we recall that the above scheme is nearly optimum to track a single tone in colored gaussian noise.

3.MULTITONE TRACKING AND HIGH ORDER LEARNING

In this section we deal with the extension of the previous system to the case where multiple tones (two) are present in the measured signal.

A classical approach is to extend the state vector with the addition of three new variables which correspond to the parameters of the second tone. The performance of this procedure for a different case of pseudoperiodic signal tracking may be viewed in /4/. No matter the performance in a more difficult and general case is going to be better with the procedure to be reported hereafter, note that the mentioned approach extends the dimensions up to 6x6 in the corresponding algorithm, avoiding the natural parallelism of the problem. This point becomes crucial when dealing with other technologies.

The alternative we propose is to use two EKFs in parallel with two inhibitory cells. These inhibitory cells consist in an adder with a weight (synapsis) subject to some learning rule. The scheme is depicted in figure 3.
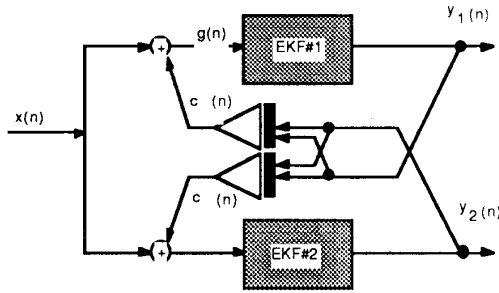

Figure 3. Multitone tracker with two inhibitory cells and two EKFs.

This scheme avoids many inherent difficulties and it has been used with fixed weights in frequency acquisition with no doppler and fixed magnitude by some authors /5/ with success under the mentioned conditions. Also it is worth to mention the use of coupled PLLs in spectral estimation and array processing, following the above scheme /6/. In any case, the presence of hard doppler rates, and slowly fluctuating envelopes in severe noise conditions, hardly degrades the reported performance of the mentioned procedures.

Here, we propose to use an adaptive learning rule to ensure that the pure tones, generated at the EKFs outputs,

remain orthogonal in very short time periods. In other words, the learning rule for the weights $c_1(n)$ and $c_2(n)$ should be in such a way that it tends to minimise the magnitude of the scalar products between the two outputs in a given interval of time T.

$$F=\left(\int_0^T y_1(t).y_2^*(t).dt\right)^2_{minimum} \qquad (9)$$

The learning rule is obtained from the derivative of this objective with respect the corresponding weight (let us concentrate in c_1). This derivative is:

$$\frac{dF}{dc_1}=\iint y_1^*.y_2.y_2^*.\frac{dy_1}{dc_1}dt\,dt \qquad (10)$$

At this moment, note that $y_1(t)$ is a non-linear function of $g(t)$ (equal to $x(t)-c_1.y_2(t)$) being, in consequence, rather difficult to get some closed form for this derivative. An approximation could be done from the following expression:

$$\frac{dy_1}{dt}\cdot\frac{dg}{dc_1}=\frac{dy_1}{dg}\cdot(-y_2(t)) \qquad (11)$$

by assuming the first derivative almost constant and neglecting its contribution to the global gradient learning rule. After this, the corresponding learning rule can be set as indicated in (12).

$$c_1(n+1)=c_1(n)+m.y_1^*.y_2./y_2/^2 \qquad (12)$$

It is very important to note two basic points in deriving the above learning rule: First note the mentioned assumption concerning the derivative of the non-linearity; second, the instantaneous value of the gradient has been taken. Also we would like to give emphasis in two important issues concerning this learning rule. First, any scheme with two identical adaptive systems, regardless the inhibitory cells, will require unsymmetrical learning in order to separate the two signals. In other words, with symmetrical learning, the two EKFs will track the same signal. Furthermore, even with different learning parameter m controlling a symmetric (i.e. order two moment $y_1.y_2^*$), the scheme will have not the same power, in achieving the objective, when compared with the unsymmetrical m and the four order moment involved in the herein proposed learning rule. Second, and in the same sense, the order four of our objective precludes the use of lower order moments in the corresponding learning. This sentence supports the success of the above rule in problems like source separation and blind equalizers /7/,/8/.

In order to asses the above claims, the following figure shows the performance of the order two rule and the proposed order four rule. The signal contains two tones at 0 dB of signal to noise ratio. The signal bandwidth was 8KHz and

the doppler rate was sinusoidal with maximum at +/- 1KHz and rate, on average, of 4KHz per second.

ESTIMATES AT THE LAST ITERATION

————EKF #1————
Magnitude... 0.7952707   Magnitude estimate.. 1.497996
Frequency... 0.3179419   Frequency estimate.. 0.1791485

————EKF #2————
Magnitude... 1.414214   Magnitude estimate.. 0.1818030
Frequency... 0.1820581   Frequency estimate.. 0.1916193

ESTIMATES AT THE LAST ITERATION

————EKF #1————
Magnitude... 0.7952707   Magnitude estimate.. 0.5765772
Frequency... 0.3179419   Frequency estimate.. 0.3227653

————EKF #2————
Magnitude... 1.414214   Magnitude estimate.. 1.457179
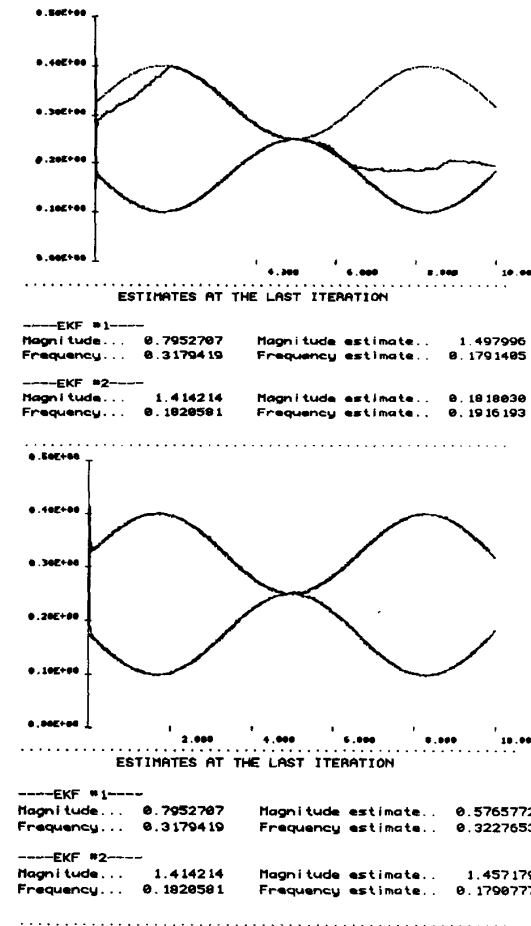Frequency... 0.1820581   Frequency estimate.. 0.1790777

Figure 4. Performance of the order two learning when compared with the proposed order four learning.

When comparing the plot on top of figure 4 (order 2 learning with unsymmetrical μ : 1.2 for inhibitory cell of c_1 and 0.8 for inhibitory cell of c_2) with the bottom graphic, which corresponds to the high order learning rule, it is evident the superiority of this rule in separating the two tones at the EKFs outputs. In ordinates we have normalised frequency versus number of samples divided by 1000 (10000 samples has been used in this experiment). The signal to noise ratio was -5 dB. for the high frequency and 0 dB for the low frequency. The diagonal values of the diagonal matrix Q in the EKFs algorithms were set to 1.e-3,1.e-5 and 1.e-5 respectively. The tables below the graphics show the accuracy in magnitude and frequency estimates at the last iteration.

Finally, the case of 1000 samples data record has been used in figure 5. The SNRs were the same, and the diagonal values of Q have been modifyed to 1.e-2,1.e-3 and 1.e-3 respectively in order to cope with the severe doppler and doppler rate involved in this experiment.
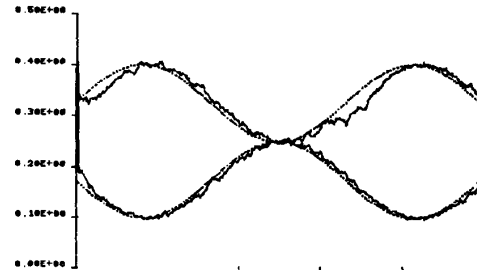

Figure 5. The same experiment that in figure 4 for 1000 samples data record using the high order learning multitone tracker

4. CONCLUSIONS

The multitone tracker described in this paper uses two basic guide-lines in its design namely: High degree of parallelism and high order learning in the inhibitory cells. The principle of perfect frequency estimation, concerning the processing bandwidth of an optimum system, has been preserved because the closeness of the EKF to every tone is granted whenever they are uncoupled. The reported estimate seems to exhibit better performance than currently reported methods using either adaptive minimum norm or time-frequency methods.

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