

Consideration of human resources in the Mixed-model Sequencing Problem with Work Overload Minimization: Legal provisions and productivity improvement

Joaquín Bautista, Rocío Alfaro-Pozo, Cristina Batalla-García

ETSEIB/UPC, Universitat Politècnica de Catalunya. Avda. Diagonal 647, 08028 Barcelona, Spain

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Abstract: Beginning with a variation of the sequencing problem in a mixed-products line (*MMSP-W*: Mixed-Model Sequencing Problem with Workload Minimization), we propose two new models that incorporate a set of working conditions in regard with human resources of workstations on the line. These conditions come from collective agreements and therefore must be respected by both company and labor unions. The first model takes into account the saturation limit of the workstations, and the second model also includes the activation of the operators throughout the working day. Two computational experiments were carried out using a case study of the Nissan motor plant in Barcelona with two main objectives: (1) to study the repercussions of the saturation limit on the decrease in productivity on the line and (2) to evaluate the recovery of productivity on the line via both activation of operators while maintaining the same quality in working conditions achieved by limiting the saturation and auxiliary processors. The obtained results show that the saturation limitation leads to suppose an important increase of work overload, which means average economic losses of 28,731.8 Euros/day. However, the productivity reduction may be counteracted by the work pace factor increase at certain moments of workday and by incorporation of auxiliary processors into the line.

Keywords: Sequencing; Mixed-product line; Work overload, Saturation, Activity factor.

1. Preliminaries

Currently, many production systems exist in which the manufacture or assembly of an entire product (or a subcomponent of the product) is carried out on the production line. At the same time, the increasing market requirements demand that companies offer a wide range of products with different options. This situation is commonly found in the automotive industry in which different products are manufactured and although these products belong to the same family, they have variable characteristics that require different component consumption and resource use, such as different processing times of operations. Obviously, not all vehicles share the same type of motor, and not all are equipped with the same components.

Assembly lines in the automotive industry are a clear example of this type of mixed-product lines, which are known as Mixed-Model Assembly Line (*MMAL*). In this type of lines, different components (seats, steering wheels, pedals, etc.) are incorporated into the vehicle body depending on the type of vehicle that is assembled at each moment. Therefore, these production and assembly lines must be flexible and able to adapt to each type of product assembled in them without incurring excessive costs.

In this way, to increase flexibility and reduce costs in terms of both workforce and storage, the assembly lines for mixed products face two basic problems: (1) the balancing of the line, known in the literature as the Assembly Line Balancing Problem (*ALBP*) (Salveson, 1955), and about which many variants exist (Becker and Scholl, 2006; Battaia and Dolgui, 2013); and (2) the sequencing of mixed products in production lines and workshops.

The latter issue can be classified according to the variability of processing times of the operations required to produce or assemble the products. Consequently, we find the following mixed-model sequencing problems:

- i. Permutation problems, i.e., Flow-Shop. This type of problem corresponds to heterogeneous processing times of the units, in the stages of the production process in a workshop (Bautista, Cano, Companys et al., 2012; Pan and Ruíz 2013).
- ii. Economic Lot Scheduling Problems (*ELSP*). When the processing time of any operation depends on the number of units it is convenient to sequence the units in batches of pieces. Thus, these problems involve determining the size of the lots according to the balance between the launching costs of the line and the

holding costs for the stock of parts (Elmaghraby ,1978; Raza and Akgunduz, 2008).

- iii. Mixed-Model Sequencing Problems (*MMSP*). In this type of problem, the processing times of the mixed products are homogeneous during the stages of the production process. The aim is to establish a manufacturing order for the products (and this order must be maintained as much as possible). These problems appear in the supply chain of production systems governed by the Just In Time (*JIT*, Toyota) and Douki-Seisan (*DS*, Nissan) ideologies. The literature contains various studies dedicated to these problems (Solnon et al., 2008; Boysen et al., 2009a,b; Bautista and Cano, 2011).

In turn, the *MMSP* can be classified according to the optimization criterion that affects one or more elements of the production system:

- a. Minimization of the stock level of products and components. This category contains the Product Rate Variation Problem (*PRVP*) proposed by Miltenburg (1989) and whose purpose is to minimize the variation of production rates; and the Output Rate Variation Problem (*ORVP*) proposed by Monden (1983) and whose aim is to minimize the variation in the component consumption rates.
- b. Minimization of the work overload. The variation of the processing times of operations based on the type of product can cause that sometimes the time assigned to a workstation is less than the processing time of the operation of a product. When this happens, the processor does not have sufficient time to complete the work on the assigned product and then work overload appears. Without extra effort, this situation ends up generating backlog. In this case, the objective is to minimize the uncompleted work, which is also known as work overload. One example of this type of problems is the Mixed-Model Sequencing Problem with Workload Minimization (*MMSP-W*) both the original version, Yano and Rachamadugu (1991), and its variants Bautista et al. (2012a, b).
- c. Minimization of the number of subsequences with special options. These problems are focused on avoiding blockages caused by products that require additional work by offering special options. This problem is known as the Car Sequencing Problem (*CSP*) and was first described by Parrello et al. (1986).

Obviously, the above problems has been extended or combined in many papers in the literature. For instance, Lin and Chu (2013, 2014) among others, minimize the manufacturing total cost of a mixed-product assembly line considering labor, warehouse

capacity and order fulfillment rates. Giard and Jeunet (2010) consider a cost function that involves two elements: the cost associated with the utility workers and the cost of setup. Thus, they simultaneously try: (1) to minimize the number of auxiliary operators necessary to complete the work required and therefore they minimize the work overload (*MMSP-W* objective); and (2) to minimize the setup times between product models (*CSP* objective).

Following the idea of extending the models in the literature, in this article, we focus on the (iii)-(b) category of problems. Therefore, we address the sequencing problem of mixed products in production lines with the objective of minimizing the work overload (i.e., *MMSP-W*: Mixed-Model Sequencing Problem with Workload Minimization).

Although this type of problem has been widely treated in the literature, there are few papers that consider specific aspects of the human resources involved in the production system. Among these few works, Celano et al. (2004) introduce the human resource into the sequencing problem of a mixed-model U-assembly line in order to evaluate human factor policies impact on the optimal solution of the problem.

However, the processors of line workstations usually contain automated systems and operators and it must be taken into account that the operators are subjected to working conditions defined according to laws, rules, and contracts and also negotiations between the company and the workers' representative. Indeed, these conditions affect certain job characteristics, such as the length of working days, the saturation and occupancy rates of the processors, and the normal activity level of the operators, the acceptable performance level, among others.

As a result of works by Alfaro (2015) and Bautista et al. (2012b) and the scarcity of works on the sequencing problem considering the human factor of the processors of the line and the effect of working conditions on the productivity, in this work we propose an extension of the *MMSP-W*. This extension incorporates constraints on working conditions that must be guaranteed to the workers of an assembly line in the automotive Sector.

Specifically, we incorporate through new constraints the fulfillment of the maximum saturation of an operator throughout his working day. These new constraints limit the relation between the time used by the operator to carry out his workload and the available time to work. For this reason, we also incorporate two possible measures to counteract the negative effect of saturation conditions on productivity. First, we incorporate into the model the activity concept according the work by Bautista, Alfaro

and Batalla (2015a). In this way the activation of workers at certain times of their workday will reduce productivity losses due to the limits of saturation. Obviously, the activation level of operators will also fulfill the limits established by collective agreements. Secondly, we reinforce the line with auxiliary processors in order to complete the required work.

This work is structured as follows. In Section 2, the *MMSP-W* problem and some of its variants, such as the reference model used in this paper, are described. Besides, an example is shown in order to understand the problem. In Section 3, working conditions agreed at Collective Agreements are analyzed. Specifically, the focus is on the Work Schedule, the Workday and the processors' saturation. Section 4 is dedicated to incorporate the saturation conditions into the *MMSP-W*. For this purpose, it is necessary previously to distinguish between static and dynamic saturation. This section ends with the formulation of the proposed model, the $M4 \cup 3_{\eta}$ model. In Section 5, the $M4 \cup 3_{\eta}$ model is evaluated through a case study linked to an assembly line of engines of Nissan motor plant in Barcelona. This computational experience measures the impact of the limitation of dynamic saturation in the stations over increasing the global work overload. In Section 6, we propose a series of corrective measures designed to reduce the global uncompleted work. Two of these corrective measures, i.e., the activation of the processors and the incorporation of auxiliary processors, are described in Sections 7 and 8. As a result of the natural extension of the $M4 \cup 3_{\eta}$ model, we consider the activation measure to formulate the $M4 \cup 3_{\dot{\alpha}I}_{\eta}$ model. In Section 9, we evaluate the $M4 \cup 3_{\dot{\alpha}I}_{\eta}$ model and the profits and losses obtained by the proposed models and the measures proposed to increase productivity are analyzed. For this, we use the same data as in Section 5 with the objective of achieving a balance between productivity and ideal working conditions. Finally, we dedicate Section 10 to the conclusions and proposals for future work.

2. The *MMSP-W*. Reference models

The *MMSP-W* consists of establishing the manufacturing order of T units of a set I of product types in an assembly line composed of a set K of workstations arranged in series. Each unit of product type i ($i = 1, \dots, |I|$) requires from each homogeneous processor (operator, robot...) of a workstation k ($k = 1, \dots, |K|$) a processing time $p_{i,k}$

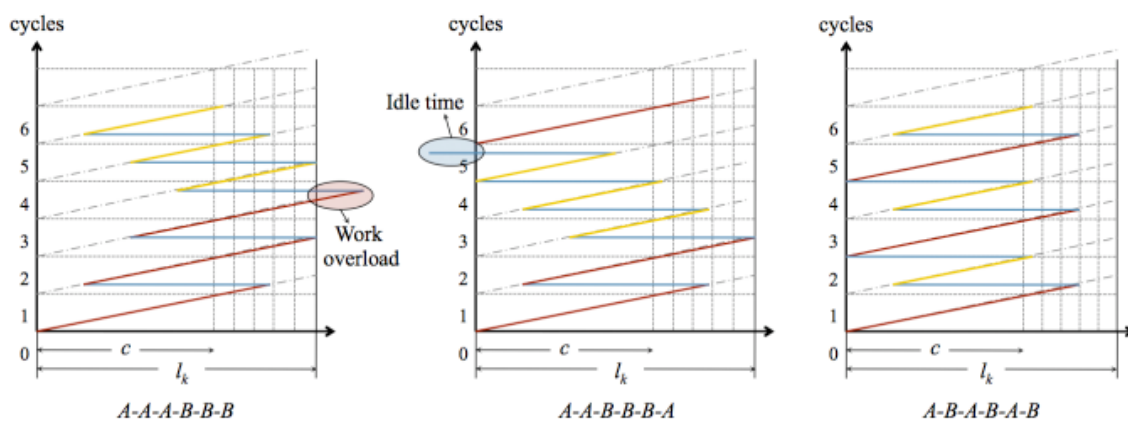
measured at normal activity (α^N). Each processor has a normal working time known as cycle time (c). This time is the standard time in which each processor is available to work on a product unit. Occasionally, to complete the work on a unit, the processor of workstation k ($k = 1, \dots, |K|$) can retain that unit for a time longer than the cycle time. This time is referred to as the time window or temporal window (l_k) and it is fulfilled $l_k - c > 0$. However using this time reduces the time available to work on the next unit of the sequence; when the temporal window is not sufficient to complete the entire work required, the work overload is generated.

Based on the above, the objective of the problem is to minimize the work overload or maximize the completed work by taking into consideration the variation in the processing times of the operations according to the product types; and assuming that the cycle time of the processors is determined from the average processing times of each type of product in regard with its presence in the demand plan.

This objective can be achieved in two ways, first, by minimizing the work that has not been completed (work overload) by the processors, which ensures that most of the work required is achieved; and second, by minimizing the downtime (idle time) of the processors.

Fig. 1 shows the effect produced in a workstation by three sequences of six units of two types of products (A with high load and B with low load). The first sequence ($AAABBB$) generates work overload, and the second sequence ($AABBBA$) generates idle time, whereas the third sequence ($ABABAB$) produces neither of these effects.

Fig. 1. Work overload, idle time, and completed work as a function of the sequence.



From the model proposed by Yano and Rachamadugu (1991), $M1$, and the model proposed by Scholl et al. (1998), $M2$, many variants of the $MMSP-W$ have been studied. Indeed, from these models in which no links between stations are contemplated,

Bautista and Suárez (2009) formulated two new models, i.e., $M3$ and $M4$, with links between stations and the assignment of a maximum time equal to l_k for all units of products. Next, Bautista et al. (2011) extended the $M4$ model to obtaining the $M4'$ model, which besides to minimizing the work overload, uses relative start instants and considers the temporal window in all of the workstations for all products and links between workstations; this model takes into account that a workstation can be viewed as more than a homogenous processor. Finally, Bautista et al. (2012b) formulated two new equivalent models, the $M3 \cup 4$ model and the $M4 \cup 3$ model.

To summarize, Table 1 lists the differences between the models mentioned above. To differentiate each model, we use the following characteristics: (1) the objective function with the minimization of the work overload (W) or the maximization of the completed work (V); (2) the variables associated with the start instants of the operations on absolute ($s_{k,t}$) and relative ($\hat{s}_{k,t}$) scales; (3) variables associated with the completed processing times, which are: the applied processing time at normal activity ($v_{k,t}$) and the generated work overload ($w_{k,t}$); (4) the maximum time that the processors of workstations can work on the last product unit ($t = T$); (5) the range of the number of processors (b_k) by workstation; (6) the consideration of links between serial stations.

Table 1. Comparative study of the literature models.

	$M1$	$M2$	$M3$	$M4$	$M4'$	$M3 \cup 4$	$M4 \cup 3$
Objective function	$\max V$	$\min W$	$\max V$	$\min W$	$\min W$	$\max V / \min W$	$\max V / \min W$
Start instants	$s_{k,t}$	$\hat{s}_{k,t}$	$s_{k,t}$	$\hat{s}_{k,t}$	$\hat{s}_{k,t}$	$s_{k,t}$	$\hat{s}_{k,t}$
Process variables	$v_{k,t}$	$w_{k,t}$	$v_{k,t}$	$w_{k,t}$	$w_{k,t}$	$v_{k,t}, w_{k,t}$	$v_{k,t}, w_{k,t}$
Temporal Window $t = T$	$l_k \forall k$	$c \forall k$	$l_k \forall k$	$l_k \forall k$	$l_k \forall k$	$l_k \forall k$	$l_k \forall k$
Range for b_k	$b_k \geq 1$	$b_k = 1$	$b_k \geq 1$	$b_k = 1$	$b_k \geq 1$	$b_k \geq 1$	$b_k \geq 1$
Seasonable links	No	No	Yes	Yes	Yes	Yes	Yes

Note that $M3$ and $M4$ models are an extension from the models proposed by Yano and Rachamadugu (1991) and Scholl et al. (1998). Likewise, the $M3 \cup 4$ and $M4 \cup 3$ models are an extension from $M3$ and $M4$, respectively. Thus, for this paper we use the $M3 \cup 4$ and $M4 \cup 3$ models (Bautista et al., 2012b) as reference for the new

models which will contemplate working conditions such as the level of occupation or saturation of the processors and the workers' activity.

3. Working conditions established by law

When a production system involves human resources, there are a set of legal provisions that must be met. These conditions imposed by laws, regulations or collective agreements refer to many labor issues, which affect the workers, such as ergonomics in the workplace, workers' rights, etc (Llovera et al., 2014).

In our case, we will analyze the legal provisions relating to the work schedule and the daily work organization in order to adapt the reference model for the *MMSP-W* to real situations where human resources are present, by means of the extension of the work presented by Bautista et al. (2012a,b).

Particularly, we will focus our research on the automotive industry. Indeed, based on our case study that is focused on the Nissan plant in Barcelona (Nissan Motor Ibérica, *NMISA*), we begin with the study of the work schedule and the length of the workday with the aim of determining the productive working time available within the legal limits.

Once the effective work hours in a workday have been determined, we will discuss the saturation of workers and the limits set by collective agreements in this regard. Finally, we will formulate the new mathematical model considering these limitations.

3.1. The Work Schedule and the Workday

The working time must be understood according to Article 2 of the 2003/88/CE Directive of the European Parliament and the Council of 4 November 2003 concerning certain aspects of the organization of working time. This Article defines the working time as "Any period during which the worker is working, at the employer's disposal and carrying out his activity or duties, in accordance with national legislation and / or national practice".

The duration of working day is decided by means of collective agreements or work contracts, according to the Article 34 of the Statute of the Workers' Rights. In general terms, after the analysis of various collective agreements in the automotive industry and especially from Nissan, we can establish that the work schedule with an annual horizon is segmented into numbered weeks (52 or 53), and the number days considered as

working days is approximately 227. The daily working time might be eight hours from Monday to Friday with three work shifts organized as follows: from 6:00 to 14:00, from 14:00 to 22:00, and from 22:00 to 6:00 (not including the special shifts that can be set for weekends).

Once determined the hours of presence of a work shift, it is necessary to analyze non-productive time in order to establish the useful work time within a working day.

- A total of 18 minutes for snack break.

Both Article 4 of the 2003/88/CE Directive of the European Parliament and the Council of 4 November 2003 and Article 34 of the Statute of the Workers Rights recognize that, if the duration of the working day exceeds 6 hours, a rest period should be established. This rest period is known as the snack break, which does not count as effective work time if it has not been established in this manner in the collective agreements or work contracts, according to Article 34.4 of the Statute of the Workers Rights.

- A total of 10 minutes for a scheduled pause after the snack break.
This pause, which is added to the 18 minutes of the snack break, increases the rest period to a total of 28 minutes. This pause is applied to cover the fatigue that derives from the type of work that the worker performs and/or a supplement for personal needs.
- A pause of 0 to 14 minutes that workers choose according to their own criteria to cover fatigue and personal needs.
- Three scheduled pauses of 10 minutes, each of which is distributed throughout the working day for a total of 30 minutes.
- Three minutes intended for preparation of materials and tools.
- From 0 to 30 minutes of downtime.

Within the downtime, we distinguish between stoppages that are payable, i.e., lack of work, forced waits, allowed movements, and breakdowns, and non-payable stoppages. Within the non-payable stoppages, we find cases that are directly attributable to the worker or cases of force majeure.

Therefore, considering the analyzed minimum and maximum limits for the non-productive time, it is established a non-productive time between 1 hour and 1 minute as minimum and 1 hour and 45 minutes as maximum. These limits translate into a

minimum limit for useful time of work of 6 hours and 15 minutes and a maximum limit of 6 hours and 59 minutes within a working day of 8 hours.

Finally, when the effective hours of the workday (H) and the number of units to manufacture in a day (T) are known, the time given to any workstation to process any operation, i.e., the cycle time (c), is determined. Obviously, $c = H/T$.

3.2. Daily work organization. Saturation of the processors

In the work organization scope, three basic aspects are considered: work simplification and improvement of methods and industrial or administrative proceedings; analysis and determination of the work performance; and the selection of the correct workforce.

These aspects, in addition to meeting the general principle of adaptation of work for the person, must respect the conditions of the collective agreements in this regard. Thus, once established the operative methods and the permissible levels of quality in the manufacturing processes, the correct workloads and the performance levels are determined.

The establishment of operator workloads throughout the workday must respect the limits of saturation established by the collective agreement. The saturation is the ratio between the actual time used by an operator to carry out his workload and the available time that the operator has for performing that work.

The saturation to which processors (operators) are subjected throughout the workday in a production line is one of the conditions that the major automotive companies negotiate with the workers' representatives. Indeed, European companies usually set limits for the maximum saturation (η_{\max}^{∞}) and the average saturation ($\eta_{\text{med}}^{\infty}$).

The maximum saturation is defined as the proportion of time used by the operation (workload) with the longest processing time regarding the cycle time available to perform that operation at a certain time of the workday. On the other hand, the average saturation is the proportion between all work required (in time units) with respect to all time available to complete it. In Nissan's case, the usual limits for the maximum and average saturation are: $\eta_{\max}^{\infty} = 1.2$ and $\eta_{\text{med}}^{\infty} = 0.95$, respectively.

Obviously, these conditions, regarding saturation of operators, will limit the amount of work completed by the operator. Indeed, a maximum average saturation equal to 0.95

means that the amount of work that an operator can perform in an effective workday of 6 hours and 15 min is equal to 5 hours and 56 minutes whereas normal activity. Therefore, it is important to take into account the saturation limits when the daily production will be scheduled.

4. Incorporating Saturation limits into the *MMSP-W*

Before incorporating the saturation conditions into the mixed-model sequencing problem, it is necessary to distinguish between static and dynamic saturation.

4.1. Previous Definitions

The static saturation is determined by a cycle time (c), a matrix of the processing times (at normal activity) of the operations ($\mathbf{P} := (p_{i,k}) : i \in I, k \in K$), and a demand plan ($\vec{d} = (d_1, \dots, d_{|I|})$) composed of T units of mixed products.

We also distinguish between average static saturation and maximum static saturation. The average static saturation, $\eta_{med}^{\circ}(k, c, \vec{d}, \mathbf{P})$, is associated with the workstation $k \in K$, the cycle time (c), the demand plan (\vec{d}), and the set of processing times (\mathbf{P}). In this way, knowing that the average static saturation is the proportion of time required by each processor to complete the required work with respect to the time available, we have:

$$\eta_{med}^{\circ}(k, c, \vec{d}, \mathbf{P}) = \frac{1}{c \cdot T} \cdot \sum_{i=1}^{|I|} p_{i,k} \cdot d_i \quad (k = 1, \dots, |K|) \quad (1)$$

Similarly, the maximum static saturation, $\eta_{max}^{\circ}(k, c, \mathbf{P})$, is associated with the workstation $k \in K$, the cycle time (c), and the set of processing times (\mathbf{P}). Thus, considering that the maximum static saturation is the proportion of time that each processor requires to complete the most laborious operation with respect to the cycle time, we define:

$$\eta_{max}^{\circ}(k, c, \mathbf{P}) = \frac{1}{c} \cdot \max_{i \in I} \{ p_{i,k} \} \quad (k = 1, \dots, |K|) \quad (2)$$

Consequently, if we take into consideration the conditions established between the company and the workers' representative, the following equations should be satisfied:

$$\eta_{med}^{\circ}(k, c, \vec{d}, P) \leq \eta_{med}^{\infty} \quad (k = 1, \dots, |K|) \quad (3)$$

$$\eta_{max}^{\circ}(k, c, P) \leq \eta_{max}^{\infty} \quad (k = 1, \dots, |K|) \quad (4)$$

On the other hand, the dynamic saturation of the workstations depends on the manufacturing sequence of the products ($\pi(T) = \{\pi_1, \dots, \pi_T\}$), the cycle time (c), and a matrix of times (at normal activity) corresponding to the work completed ($V := (v_{k,t}) : k \in K, t = 1, \dots, T$) at each workstation and position in the sequence. The average and maximum dynamic saturation gives the following equations:

$$\eta_{med}^{\circ}(k, c, \pi(T), V) = \frac{1}{c \cdot T} \cdot \sum_{t=1}^{|T|} v_{k,t} \quad (k = 1, \dots, |K|) \quad (5)$$

$$\eta_{max}^{\circ}(k, c, \pi(T), V) = \frac{1}{c} \cdot \max_{1 \leq t \leq T} \{v_{k,t}\} \quad (k = 1, \dots, |K|) \quad (6)$$

Therefore, to limit the average and maximum dynamic saturation in the line workstations, we add to the *MMSP-W* models the following restrictions:

$$\sum_{t=1}^{|T|} v_{k,t} \leq \eta_{med}^{\infty} \cdot c \cdot T \quad (k = 1, \dots, |K|) \quad (7)$$

$$v_{k,t} \leq \eta_{max}^{\infty} \cdot c \quad (k = 1, \dots, |K|; t = 1, \dots, T) \quad (8)$$

As shown, the static saturation depends on the required work time and the dynamic saturation depends on the completed work time. Therefore, it is possible that saturation levels are above the limit values set by collective agreements, in the case of a demand plan with a highly demanding load or an inappropriate fabrication sequence. If this occurs should be noted that the violation of the maximum saturation is considered unacceptable and requires review by the department of times and measures to search for alternatives in the assembly process that will reduce processing times.

However, this situation does not occur with average saturation. Indeed, when the average saturation, either static or dynamic, exceeds the admissible η_{med}^{∞} limit value in certain workstations, the processors do not have sufficient time to complete the required work. Therefore, a work overload will be generated with a value that will grow against the reduction of the η_{med}^{∞} value.

Hence, the incorporation of the average saturation limits into the *MMSP-W* could cause an increase of work overload due to both the static and dynamic over-saturation. Indeed, it is possible to determine the static work overload associated with each processor of the workstation $k \in K$ as follows:

$$\omega_0(k, c, \vec{d}, P, \eta_{med}^\infty) = c \cdot T \cdot \max\{0, \eta_{med}^\circ(k, c, \vec{d}, P) - \eta_{med}^\infty\} \quad (k = 1, \dots, |K|) \quad (9)$$

And considering all the set of workstations of the line, it is possible to estimate the static work overload of the line by means of the adding of static work overloads of workstations multiplied by their processors $\vec{b} = (b_1, \dots, b_{|K|})$:

$$W_0(c, \vec{d}, \vec{b}, P, \eta_{med}^\infty) = \sum_{k=1}^{|K|} b_k \cdot \omega_0(k, c, \vec{d}, P, \eta_{med}^\infty) \quad (10)$$

On the other hand, the dynamic work overloads associated with the workstations and the line are determined in the following manner:

$$\omega(k, c, \pi(T), V, \eta_{med}^\infty) = c \cdot T \cdot \max\{0, \eta_{med}(k, c, \pi(T), V) - \eta_{med}^\infty\} \quad (k = 1, \dots, |K|) \quad (11)$$

$$W(c, \vec{b}, \pi(T), V, \eta_{med}^\infty) = \sum_{k=1}^{|K|} b_k \cdot \omega(k, c, \pi(T), V, \eta_{med}^\infty) \quad (12)$$

Obviously, the work overload value that derives from the dynamic saturation (W) will always be equal or greater than the work overload derived from the static saturation (W_0). This situation occurs because the effects produced by the variation of processing times in combination with the sequence must be added in the dynamic situation.

4.2. *MMSP-W* Models with Saturation Constraints

Taking into account the above definitions of dynamic saturation and the $M3 \cup 4$ and $M4 \cup 3$ models for the *MMSP-W* presented by Bautista et al. (2012b), we propose below two new equivalents models for the *MMSP-W*, whose difference is the time scale used to calculate the start instants of operations at workstations. The new models, $M3 \cup 4_ \eta$ and $M4 \cup 3_ \eta$, consider the saturation conditions imposed by the collective agreements and its parameters and variables are the following:

Parameters:

K Set of workstations, arranged in series, that makes up the line ($k = 1, \dots, |K|$).

b_k	Number of homogeneous processors at workstation k ($k = 1, \dots, K $).
I	Set of product types that must be manufactured in the line ($i = 1, \dots, I $).
d_i	Programmed demand of the product type i ($i = 1, \dots, I $).
$p_{i,k}$	Processing time (normal activity) required by one unit of a product type i ($i = 1, \dots, I $) at workstation k ($k = 1, \dots, K $) for each homogeneous processor.
T	Total demand. Obviously: $\sum_{i=1}^{ I } d_i = T$.
t	Position index in the sequence ($t = 1, \dots, T$).
c	Cycle time. Standard time assigned to each homogeneous processor in the workstations ($k = 1, \dots, K $) to process any product unit.
l_k	Temporal window. Maximum time that each homogeneous processor of workstation k ($k = 1, \dots, K $) is allowed to work on any unit of product; once the cycle has been completed, the maximum time that a unit of product can be retained in station k is $l_k - c > 0$.
η_{med}^{∞}	Allowable average saturation by the processors of each workstation ($k = 1, \dots, K $).
η_{max}^{∞}	Allowable maximum saturation by the processors of each workstation ($k = 1, \dots, K $).

Variables:

$x_{i,t}$	Binary variable equal to 1 if the product unit i ($i = 1, \dots, I $) is assigned to the position t ($t = 1, \dots, T$) of the sequence and 0 otherwise.
$s_{k,t}$	Start instant for the t^{th} unit of the sequence of products at workstation k ($k = 1, \dots, K $).
$\hat{s}_{k,t}$	Relative start instant. Positive difference between the start instant and the earliest start instant of the t^{th} operation in the workstation k ($k = 1, \dots, K $). It is fulfilled $\hat{s}_{k,t} = [s_{k,t} - (t + k - 2)c]^+$.
$v_{k,t}$	Processing time applied by each homogeneous processor (at normal activity) to the t^{th} product unit sequenced in the workstation k ($k = 1, \dots, K $).
$w_{k,t}$	Work overload measured in units of time (at normal activity) generated by the

t^{th} product unit sequenced in each homogeneous processor of the workstation
 k ($k = 1, \dots, |K|$).

And the new mathematical models are the following:

$M3 \cup 4_{-\eta}$ model:

$$\max V = \sum_{k=1}^{|K|} \left(b_k \sum_{t=1}^T v_{k,t} \right) \Leftrightarrow \min W = \sum_{k=1}^{|K|} \left(b_k \sum_{t=1}^T w_{k,t} \right) \quad (13)$$

Subject to:

$$\sum_{t=1}^T x_{i,t} = d_i \quad i = 1, \dots, |I| \quad (14)$$

$$\sum_{i=1}^{|I|} x_{i,t} = 1 \quad t = 1, \dots, T \quad (15)$$

$$v_{k,t} + w_{k,t} = \sum_{i=1}^{|I|} p_{i,k} \cdot x_{i,t} \quad k = 1, \dots, |K|; t = 1, \dots, T \quad (16)$$

$$\sum_{t=1}^T v_{k,t} \leq \eta_{med}^{\infty} \cdot c \cdot T \quad k = 1, \dots, |K| \quad (17)$$

$$v_{k,t} \leq \eta_{max}^{\infty} \cdot c \quad k = 1, \dots, |K|; t = 1, \dots, T \quad (18)$$

$$s_{k,t} \geq (t + k - 2)c \quad k = 1, \dots, |K|; t = 1, \dots, T \quad (19)$$

$$s_{k,t} \geq s_{k,t-1} + v_{k,t-1} \quad k = 1, \dots, |K|; t = 2, \dots, T \quad (20)$$

$$s_{k,t} \geq s_{k-1,t} + v_{k-1,t} \quad k = 2, \dots, |K|; t = 1, \dots, T \quad (21)$$

$$s_{k,t} + v_{k,t} \leq (t + k - 2)c + l_k \quad k = 1, \dots, |K|; t = 1, \dots, T \quad (22)$$

$$s_{k,t}, v_{k,t}, w_{k,t} \geq 0 \quad k = 1, \dots, |K|; t = 1, \dots, T \quad (23)$$

$$x_{i,t} \in \{0,1\} \quad i = 1, \dots, |I|; t = 1, \dots, T \quad (24)$$

$M4 \cup 3_{-\eta}$ model:

$$\min W = \sum_{k=1}^{|K|} \left(b_k \sum_{t=1}^T w_{k,t} \right) \Leftrightarrow \max V = \sum_{k=1}^{|K|} \left(b_k \sum_{t=1}^T v_{k,t} \right) \quad (25)$$

Subject to:

$$\sum_{t=1}^T x_{i,t} = d_i \quad i = 1, \dots, |I| \quad (26)$$

$$\sum_{i=1}^{|I|} x_{i,t} = 1 \quad t = 1, \dots, T \quad (27)$$

$$v_{k,t} + w_{k,t} = \sum_{i=1}^{|I|} p_{i,k} \cdot x_{i,t} \quad k = 1, \dots, |K|; t = 1, \dots, T \quad (28)$$

$$\sum_{t=1}^T v_{k,t} \leq \eta_{med}^{\infty} \cdot c \cdot T \quad k = 1, \dots, |K| \quad (29)$$

$$v_{k,t} \leq \eta_{max}^{\infty} \cdot c \quad k = 1, \dots, |K|; t = 1, \dots, T \quad (30)$$

$$\hat{s}_{k,t} \geq \hat{s}_{k,t-1} + v_{k,t-1} - c \quad k = 1, \dots, |K|; t = 2, \dots, T \quad (31)$$

$$\hat{s}_{k,t} \geq \hat{s}_{k-1,t} + v_{k-1,t} - c \quad k = 2, \dots, |K|; t = 1, \dots, T \quad (32)$$

$$\hat{s}_{k,t} + v_{k,t} \leq l_k \quad k = 1, \dots, |K|; t = 1, \dots, T \quad (33)$$

$$\hat{s}_{k,t}, v_{k,t}, w_{k,t} \geq 0 \quad k = 1, \dots, |K|; t = 1, \dots, T \quad (34)$$

$$x_{i,t} \in \{0,1\} \quad i = 1, \dots, |I|; t = 1, \dots, T \quad (35)$$

$$\hat{s}_{1,1} = 0 \quad (36)$$

Although the proposed models are extensions of the original models proposed by Yano and Rachamadugu (1991) and Scholl et al. (1998) and each one uses a different time scale to determine the start instants of operations, both models are equivalent, as was the case with the reference models, $M3 \cup 4$ and $M4 \cup 3$ (Bautista et al., 2012a,b). Indeed the objective functions (13) and (25) are the same and express the equivalence between the minimization of the total dynamic work overload (W) and the maximization of the total completed work (V). Constraints (14) and (26) represent the satisfaction of the programmed demand. Constraints (15) and (27) force that each product unit can only be assigned to one position of the sequence. The set of constraints (16) and (28) fix the relationships among the required processing time, the completed work, and the work overload in each workstation and in each moment. Constraints (17) and (29) restrict the average saturation, and constraints (18) and (30) restrict the maximum saturation at workstations. The sets (19)-(22) determine the absolute start instants of the operations at the workstations; and the sets (31)-(33) denote the relative start instants of the t ($t = 1, \dots, T$) unit sequenced at workstation k ($k = 1, \dots, |K|$). Constraints (23) and (34) establish the non-negativity of the variables. Constraints (24) and (35) establish as binary the assignment variables of units to the sequence. Finally, the equality (36) fixes the start instant of the operations.

5. Effect of the saturation conditions in the *MMSP-W*. Case Study.

Next, we evaluate the effect of the saturation limitation on the increase in work overload using a case study linked to the assembly line of engines at Nissan in Barcelona.

Specifically, we will run the $M3 \cup 4_{-\eta}$ and $M4 \cup 3_{-\eta}$ models for a set E of 23 different demand plans (see Block I of Table 7 in Bautista et al., 2012b) in order to obtaining sequences with minimum work overload. All data instances consist of a workday divided into two shifts of 8 hours which means a total of 13.125 effective hours of work per day, after discount the statutory breaks and rest periods, i.e. 6 hours with 33 minutes per shift.

Furthermore, all plans must satisfy the same total demand of engines, $T = 270$, which is divided into nine types of engines, $|I| = 9$, grouped into three families according to the vehicle types: (1) 4x4 vehicles (p_1, p_2 y p_3), (2) vans (p_4 y p_5), and (3) average tonnage trucks (p_6, \dots, p_9). It should be noted that each engine type has different processing time of operations at workstations, $p_{i,k}$ ($i = 1, \dots, |I|$; $k = 1, \dots, |K|$).

All engines of the demand plans must be assembled in a line with 21 workstations arranged in series, $|K| = 21$. All these workstations have one homogeneous processor ($b_k = 1, \forall k \in K$) that is equivalent to a team of two workers with identical skills and tools and the same requirements for auxiliary equipment. Moreover, the processors of all workstations get the same cycle time, $c = 175s$, to work, and an identical time window that is equal to $l_k = 195s$. This time window supposes a slack over the 10% of cycle time to work on a product unit in the sequence at any workstation.

Finally, regarding the saturation limits, we use the values established by the collective agreement of Nissan, that is, $\eta_{\max}^{\infty} = 1.2$ for the maximum saturation and $\eta_{\text{med}}^{\infty} = 0.95$ for the average saturation.

Considering the above data set, we run the models through the LP Solver of the Gurobi Optimizer 4.5.0 in an Apple Macintosh iMac computer with an Intel Core i7 2.93-GHz processor, 8 GB of RAM memory, and a MAC OS X 10.6,7 operating system with a CPU time limit for each production plan to 7200 s.

Before running the models, the values for the maximum and average static saturation are calculated for each demand plan, according to equations (1) and (2). Thus, we can check whether all production plans fulfill the maximum value of static saturation permitted per workstation and therefore all of them are technically feasible on the line. Furthermore, we can also calculate, from equation (10), the inevitable overload generated at stations that present an average static saturation higher than the allowed by Nissan ($\eta_{med}^{\infty} = 0.95$).

Table 2 shows the calculated results and these obtained after running the $M3 \cup 4_{\eta}$ and $M4 \cup 3_{\eta}$ models. Indeed, the first column of the table represents the number of the production plan ($\varepsilon \in E$). The second column shows the set of workstations that contain oversaturated processors when only the static saturation is considered. The third column shows the values of the global work overload ($W_{3 \cup 4}, W_{4 \cup 3}$) achieved by the reference models, $M3 \cup 4$ and $M4 \cup 3$, from Bautista et al. (2012a). The fourth column shows the inevitable overload of the line due to the limit of the average static saturation ($W_{0_{\eta}}$) calculated from equation (10). The fifth column presents the values of the dynamic work overload of the line obtained by the both models ($W_{4 \cup 3_{\eta}} = W_{3 \cup 4_{\eta}}$). Finally, the sixth column shows the work overload increase because of the incorporation of the saturation constraints in both models (RPD_1, RPD_2); these values are calculated by the following equations:

$$RPD_1(\varepsilon) = \frac{W_{3 \cup 4_{\eta}}(\varepsilon) - W_{3 \cup 4}(\varepsilon)}{W_{3 \cup 4}(\varepsilon)} \cdot 100 \quad (\varepsilon = \#1, \dots, \#|E|) \quad (37)$$

$$RPD_2(\varepsilon) = \frac{W_{4 \cup 3_{\eta}}(\varepsilon) - W_{4 \cup 3}(\varepsilon)}{W_{4 \cup 3}(\varepsilon)} \cdot 100 \quad (\varepsilon = \#1, \dots, \#|E|) \quad (38)$$

Table 2. Oversaturated workstations ($k \in K$), work overload ($W_{4 \cup 3}, W_{0_{\eta}}, W_{4 \cup 3_{\eta}}$), and percentage increase of the dynamic work overload obtained by limiting saturation for 23 demand plans ($\varepsilon \in E$).

ε	$k \in K : \eta_{med}^{\infty}(k) > \eta_{med}^{\infty}$	$W_{3 \cup 4}$	$W_{4 \cup 3}$	$W_{0_{\eta}}$	$W_{3 \cup 4_{\eta}} = W_{4 \cup 3_{\eta}}$	RPD_1	RPD_2
#1	4, 9, 10, 16, 17, 18	251.0	187.0	12315.0	12315.0*	4806.4	6485.6
#2	4, 9, 10, 16, 17, 18	444.0	341.0	12458.0	12458.0*	2705.9	3553.4
#3	4, 9, 10, 11, 16, 17, 18, 21	477.0	427.0	12210.0	12210.0*	2459.7	2759.5
#4	4, 9, 10, 16, 17, 18	402.0	310.0	12470.0	12470.0*	3002.0	3922.6
#5	4, 9, 10, 16, 17, 18, 21	754.0	633.0	13012.5	13012.5*	1625.8	1955.7
#6	4, 9, 10, 16, 17, 18	525.0	413.0	12910.0	12910.0*	2359.0	3025.9
#7	4, 9, 10, 16, 17, 18, 21	818.0	742.0	12722.5	12722.5*	1455.3	1614.6

#8	4, 9, 10, 16, 17, 18	228.0	139.0	12018.0	12018.0*	5171.1	8546.0
#9	4, 9, 10, 16, 17, 18	824.0	732.0	13363.0	13363.0*	1521.7	1725.5
#10	4, 9, 10, 11, 16, 17, 18, 21	1208.0*	1208.0*	13122.0	13122.0*	986.3	986.3
#11	4, 9, 10, 11, 16, 17, 18	165.0	78.0	11792.5	11792.5*	7047.0	15018.6
#12	4, 9, 10, 16, 17, 18	406.0	284.0	12246.0	12246.0*	2916.3	4212.0
#13	4, 9, 10, 16, 17, 18	383.0	286.0	12551.0	12551.0*	3177.0	4288.5
#14	4, 9, 10, 16, 17, 18	500.0	420.0	12646.0	12646.0*	2429.2	2911.0
#15	4, 9, 10, 16, 17, 18, 21	506.0	433.0	12393.5	12393.5*	2349.3	2762.2
#16	4, 9, 10, 16, 17, 18	321.0	227.0	12363.0	12363.0*	3751.4	5346.3
#17	4, 9, 10, 16, 17, 18, 21	550.0	478.0	12597.5	12597.5*	2190.5	2535.5
#18	4, 9, 10, 16, 17, 18	673.0	605.0	13208.0	13208.0*	1862.6	2083.1
#19	4, 9, 10, 11, 16, 17, 18, 21	949.0	945.0*	12810.0	12810.0*	1249.8	1255.6
#20	4, 9, 10, 16, 17, 18	233.0	139.0	11875.0	11875.0*	4996.6	8443.2
#21	4, 9, 10, 16, 17, 18	652.0	560.0	13065.0	13065.0*	1903.8	2233.0
#22	4, 9, 10, 16, 17, 18, 21	1006.0	987.0	13062.5	13062.5*	1198.5	1223.5
#23	4, 9, 10, 11, 16, 17, 18	188.0	140.0	11902.5	11902.5*	6231.1	8401.8
Average values		541.9	465.8	12570.2	12570.2	2930.3	4143.0

Based on obtained results (Table 2), we observe the following:

1. None of the 23 demand plans violates the limitation of both static and dynamic maximum saturation ($\eta_{\max}^{\infty} = 1.2$) imposed on all of the workstations by collective agreement. Therefore, both models can find a solution.
2. Workstations 4, 9, 10, 16, 17 and 18 present an average static saturation higher than the admissible in the 23 production plans. However, workstations 1, 2, 3, 5, 6, 7, 8, 12, 13, 14, 15, 19, and 20 are never oversaturated, which means that none of them fails to the limit imposed on the medium static saturation ($\eta_{\text{med}}^{\infty} = 0.95$).
3. The incorporation of the saturation constraints supposes a considerable increase of work overload. Indeed, taking only the static situation into account (W_{0_n}), the work overload values get worse, on average, until to reach 12570.2 seconds. This value turns into a number of uncompleted engines of 72 engines per day, considering the cycle time, $c = 175s$.
4. Both models, $M3 \cup 4_n$ and $M4 \cup 3_n$, offer the same solutions for all instances. In addition, in all plans, the static work overload of the line (W_{0_n}) coincides with the values of dynamic work overload obtained with the models ($W_{3 \cup 4_n} = W_{4 \cup 3_n}$). This indicates the following three observations: (1) the optimal values (maximums) for the completed work are reached in the 23 plans; (2) none of the 23 sequences of manufacture generates idle time; and (3) the effect of saturation

on the work overload generated in the workstations causes the sequence does not affect the results.

5. The increase in work overload values (RPD_1, RPD_2) obtained with the model with absolute time scale, $M3 \cup 4$, by limiting the average saturation of stations ranges from 986.3% ($\varepsilon = \#10$) to 7047.0% ($\varepsilon = \#11$), with an average value of 2930.3%. Meanwhile, the $M4 \cup 3_{\eta}$ model, with relative start instants, reaches increases of 986.3% ($\varepsilon = \#10$), 15018.6% ($\varepsilon = \#11$), and 4143.0%, on average.
6. The improvement in working conditions by the average saturation constraints at workstations can involve a daily work loss whose average value is equivalent to 72 engines with a range of 9 engines because of the quantity of uncompleted work. Without this limitation, the daily work loss is equivalent to an average of 3 engines and rank of 7 engines.
7. Taking into account that production line of engines supposes a consolidated operation profit of 10% over the total value of a motor (4,000 €), i.e., the loss of an engine means a cost for the line of 400 €, we can calculate the daily economic losses. Indeed, the economic losses due to the limitation of the average saturation are equivalent to 28,731.8 Euros/day, on average, with a range of 3,589.7 Euros/day; without the limitation, the average economic loss because of work overload is 1,064.7 Euros/day, with a range of 2,582.9 Euros/day.

Obviously, the limitation of the occupation level of workers gets worse the productivity of the assembly line. However, thanks to these saturation constraints, the legal conditions imposed by collective agreements are fulfilled and therefore the working conditions are improved.

6. Measures used to Reduce the Work Overload

We have shown that consideration of specific aspects of real production systems, such as the saturation limitation of workers may result in an increase in work overload and therefore an increase in economic losses because of the uncompleted work.

Consequently, keeping in mind the main objective of *MMSP-W* and the obligation to respect the saturation limits established in the collective agreements, the following alternatives are proposed to counter the violation of the saturation limits, and thus, to improve the working conditions without damaging the productivity of the assembly line.

1. Resort to methods and times department to search for alternatives in the assembly process in order to reduce the processing times of operations. This action is not immediate because it requires the intervention of the product and process engineering.
2. Increase the activation level of the processors (based on activity factor) like the authors Bautista, Alfaro and Batalla made in their work (Bautista et al., 2015a). In this way, the work assigned to each processor will be carried out in less time, and thus, the ratio between the total available time and the real work time will be reduced. In others words, this action would reduce the average saturation. Indeed, given a demand plan, the average saturation of a processor will be higher or lower depending on whether the activity factor of this processor is lower or higher, respectively. Obviously, this activation may not exceed the limits established by the collective agreements at any time of the workday.
3. Strengthen the production line to increase its capacity by incorporating auxiliary processors, either multi-skilled or not in the system. Thus, the auxiliary processors could assist the oversaturated workstations.
4. Resort to rotation between consecutive stations. Using this approach, the oversaturated workstations may be offset, in the medium term, with lower saturations of other workstations with higher ergonomic quality. It should be noted that this measure is not permitted in companies that belong to the Organization for Economic Cooperation and Development (*OCDE*) because they do not allow exceeding the average saturation limit at any time of day and at any workstation. However, this measure could be considered for those countries where no such limit exists.

In this work, we focus on the second and third measures proposed to reduce the overload. Then, we study the impact produced by the increasing of activity of processors on the line productivity in order to reduce the economic losses derived from the saturation limits; and the auxiliary processors necessary to comply the saturation conditions and the totality of required work.

7. Incorporating the Processors' Activation

In order to reduce production losses generated by limiting saturation processors and based on the work published by Bautista et al., (2015a,b), a series of previous concepts

regarding the variation of the activity or work pace of the operators are described. Similarly, a new mathematical model that incorporates the possibility of increasing the work pace of processors to reduce the workload while respecting saturation limits established by the collective agreement is formulated.

7.1. Processing times and work pace factor

Normally the Automotive Sector uses timing techniques, tables /estimates and *MTM* (Methods Time Measurement) systems as time measuring instruments. These techniques allow for assigning a predetermined time to any operation or manual method through the decomposition of the operation into basic movements. After these standard processing times of operations have been predetermined, the required level of activity or normal work pace is determined by means of the collective agreements.

This required activity level is equivalent to the normal activity accepted by the company and it corresponds to the work pace with which workers must perform their workload, in normal conditions.

In this regard, once the standard processing times are established, according to the *MTM* system, the normal processing times $(p_{i,k} : i \in I, k \in K)$ must be established, considering the normal activity set by the company. Therefore, considering the case of Nissan, the Collective Agreement establishes the normal processing times as the resultant of the direct application of the *MTM* _100 times (standard times with centesimal scale) with an activity of 110 (normal work pace set by Nissan). This makes necessary changing the scale of the standard times that correspond with *MTM* _100 system, to the scale of normal times established from the normal activity (*MTM* _110 in the case of Nissan):

$$p_{i,k}(MTM_110) = p_{i,k}(MTM_100) \cdot \frac{100}{110} \quad \forall i \in I, \forall k \in K \quad (39)$$

Where $p_{i,k} = p_{i,k}(MTM_110) : i \in I, k \in K$ are the processing times (pre-set) required by products to the processors of workstations, when processors work at normal activity or work pace.

In this way, defining the work pace factor by product and workstation,

$\alpha_{i,k} (i \in I, k \in K)$, as follows (40) it is possible to change the processing times with standard activity scale, $p_{i,k}$, to another one, $\hat{p}_{i,k}$.

$$\alpha_{i,k} \cdot \hat{p}_{i,k} = \alpha^N \cdot p_{i,k} \quad \text{if } \alpha^N = 1 : i \in I, k \in K \quad \alpha_{i,k} = p_{i,k} / \hat{p}_{i,k} \quad \forall i \in I, \forall k \in K \quad (40)$$

Obviously, if activity increases above de normal (α^N), the processing times of operations will be reduced. In contrast, if the work pace of an operator is lower than the normal, the processing time of operations will be longer. In this way, if an operator can work with a greater work pace, he will take less time to perform his workload and therefore his saturation level will be less and the work overload will be reduced.

However, it must be noted that automotive industries and labor unions also agree a maximum or optimal activity level over the the activity established as normal through collective agreements. This optimal activity corresponds to the 120% of the normal activity, in the case of Nissan, and is the maximum work pace which an operator can withstand without harming its health.

Therefore, taking into account the work pace levels defined in the Nissan's Collective Agreement, we can associate to the activities denominated standard, normal and optimal, the scale times MTM_{100} , MTM_{110} and MTM_{132} respectively, which correspond to activity factors $\alpha_{i,k}^{\circ} = 0.90$, $\alpha_{i,k}^N = 1.0$ and $\alpha_{i,k}^* = 1.2$.

7.2. The work pace throughout the workday

In 1908, the researchers Robert Yerkes and John Dodson demonstrated that when stress increases up to a certain optimum point of stress, the operators' performance improves too. However, when the optimum point is reached the efficiency decreases drastically. To reflect this idea the authors established the Yerkes-Dodson curve, also called inverted U-shaped curve, where it is shown that optimum stress lies between the excess and deficiency of activation.

Furthermore, Bautista et al., (2015a) established a direct correlation between the activity factor and the stress level over time, taking into consideration firstly, the above idea and, secondly, the idea that worker's performance varies throughout the working day, and therefore, the relationship between the operators' performance and their level of "activation", which is reflected by their level of stress, follows a concave function (Muse et al., 2003). This correlation, allowed, to the authors, set, first, a stepped function for the work pace factor throughout time (Bautista et al., 2014) and, afterwards, three additional functions, the triangular, the trapezoidal and the parabolic one (Bautista et al., 2015a).

Thanks to these functions the authors were able to reduce the work overload by reducing the processing times of operations by means of the increase of dynamic activity factor at specific instants of the workday in accordance with each of the defined functions. Indeed, with only an average increase of 5% of normal factor (triangular function), the authors completely eliminated the work overload (Bautista et al., 2015a). For this reason, this paper is based on the activation functions defined by Bautista et al. (2015a) to reduce the great work overload generated by the saturation conditions. Specifically, we incorporate the dynamic activity factor into the $M3 \cup 4_{-\eta}$ and $M4 \cup 3_{-\eta}$ models, following the procedure used, by Bautista et al., (2015a), to formulate the models $M3 \cup 4_{-\dot{\alpha}I}$ and $M4 \cup 3_{-\dot{\alpha}I}$.

7.3. The *MMSP-W* with saturation constraints and activity factor variation

From the $M3 \cup 4_{-\eta}$ and $M4 \cup 3_{-\eta}$ models and considering the variation of work pace factor of operators throughout their workday according the functions defined by Bautista et al., (2015a), we propose the $M3 \cup 4_{-\dot{\alpha}I_{-\eta}}$ and $M4 \cup 3_{-\dot{\alpha}I_{-\eta}}$ models in order to reduce the work overload generated by the saturation limitations.

The parameters and the additional variables of the new models are:

Parameters:

-
- $\dot{\alpha}_{k,t}$ Dynamic factor of the work pace or activity associated with the t^{th} operation of the product sequence ($t = 1, \dots, T$) at the workstation k ($k = 1, \dots, |K|$). Note that $\alpha_{i,k}$ is the activity factor by product and workstation and it not depends on the sequence.
 - $\dot{\alpha}_t$ Dynamic factor of the work pace or activity associated with the period t ($t = 1, \dots, T + |K| - 1$) of the extended workday. This extended workday includes T manufacturing cycles (total demand) and $|K| - 1$ additional cycles, which are required to complete the required work by the production units in all the workstations. Note that if we associate the same dynamic factor with each moment of the workday in all of the workstations, we will have: $\dot{\alpha}_{k,t} = \dot{\alpha}_{t+k-1}$ ($k = 1, \dots, |K|$; $t = 1, \dots, T$).
-

Variables:

$\hat{v}_{k,t}$ Processing time reduced by the dynamic activity factor $\dot{\alpha}_{k,t}$. It is established that $v_{k,t} = \dot{\alpha}_{k,t} \cdot \hat{v}_{k,t}$ ($k = 1, \dots, |K|$; $t = 1, \dots, T$).

It should be noted that if $\dot{\alpha}_{k,t} = 1$ ($\forall k, \forall t$) (normal work pace factor), the processing times will correspond to the *MTM* _110 system. Similarly, the *MTM* _121 and *MTM* _132 time scales are equivalent to the dynamic work pace factor $\dot{\alpha}_{k,t} = 1.1$ and $\dot{\alpha}_{k,t} = 1.2$, respectively.

Hence, considering the equality $\dot{\alpha}_{k,t} = \dot{\alpha}_{t+k-1}$ ($k = 1, \dots, |K|$; $t = 1, \dots, T$) and the parameters and variables defined above, the new models for the *MMSP-W* are:

M3 \cup *4* _ $\dot{\alpha}$ *I* _ η model:

$$\min W = \sum_{k=1}^{|K|} \left(b_k \sum_{t=1}^T w_{k,t} \right) \Leftrightarrow \max V = \sum_{k=1}^{|K|} \left(b_k \sum_{t=1}^T v_{k,t} \right) \quad (44)$$

Subject to: (14) – (16) and (24) from *M3* \cup *4* _ η model

$$\dot{\alpha}_{t+k-1} \cdot \hat{v}_{k,t} - v_{k,t} = 0 \quad k = 1, \dots, |K|; t = 1, \dots, T \quad (45)$$

$$\sum_{t=1}^{|T|} \hat{v}_{k,t} \leq \eta_{med}^{\infty} \cdot c \cdot T \quad k = 1, \dots, |K| \quad (46)$$

$$\hat{v}_{k,t} \leq \eta_{max}^{\infty} \cdot c \quad k = 1, \dots, |K|; t = 1, \dots, T \quad (47)$$

$$s_{k,t} \geq (t + k - 2)c \quad k = 1, \dots, |K|; t = 1, \dots, T \quad (48)$$

$$s_{k,t} \geq s_{k,t-1} + \hat{v}_{k,t-1} \quad k = 1, \dots, |K|; t = 2, \dots, T \quad (49)$$

$$s_{k,t} \geq s_{k-1,t} + \hat{v}_{k-1,t} \quad k = 2, \dots, |K|; t = 1, \dots, T \quad (50)$$

$$s_{k,t} + \hat{v}_{k,t} \leq (t + k - 2)c + l_k \quad k = 1, \dots, |K|; t = 1, \dots, T \quad (51)$$

$$s_{k,t}, v_{k,t}, w_{k,t}, \hat{v}_{k,t} \geq 0 \quad k = 1, \dots, |K|; t = 1, \dots, T \quad (52)$$

M4 \cup *3* _ $\dot{\alpha}$ *I* _ η model:

$$\min W = \sum_{k=1}^{|K|} \left(b_k \sum_{t=1}^T w_{k,t} \right) \Leftrightarrow \max V = \sum_{k=1}^{|K|} \left(b_k \sum_{t=1}^T v_{k,t} \right) \quad (53)$$

Subject to: (26) – (28) and (35) – (36) from *M4* \cup *3* _ η

$$\dot{\alpha}_{t+k-1} \cdot \hat{v}_{k,t} - v_{k,t} = 0 \quad k = 1, \dots, |K|; t = 1, \dots, T \quad (54)$$

$$\sum_{t=1}^{|T|} \hat{v}_{k,t} \leq \eta_{med}^{\infty} \cdot c \cdot T \quad k = 1, \dots, |K| \quad (55)$$

$$\hat{v}_{k,t} \leq \eta_{max}^{\infty} \cdot c \quad k = 1, \dots, |K|; t = 1, \dots, T \quad (56)$$

$$\hat{s}_{k,t} \geq \hat{s}_{k,t-1} + \hat{v}_{k,t-1} - c \quad k = 1, \dots, |K|; t = 2, \dots, T \quad (57)$$

$$\hat{s}_{k,t} \geq \hat{s}_{k-1,t} + \hat{v}_{k-1,t} - c \quad k = 2, \dots, |K|; t = 1, \dots, T \quad (58)$$

$$\hat{s}_{k,t} + \hat{v}_{k,t} \leq l_k \quad k = 1, \dots, |K|; t = 1, \dots, T \quad (59)$$

$$\hat{s}_{k,t}, v_{k,t}, \hat{v}_{k,t}, w_{k,t} \geq 0 \quad k = 1, \dots, |K|; t = 1, \dots, T \quad (60)$$

The new set of constraints (45) and (55) serves to reduce or lengthen the applied processing times depending on the work pace factor. The remainder of the constraints coincide with the $M3 \cup 4_{-\eta}$ and $M4 \cup 3_{-\eta}$ models, leaving out both the constraints that limit the saturation of the processors, i.e., (46) – (47) and (56) – (57), and the ones that determine the start instants of the operations, i.e., (48) – (51) and (58) – (60), that now consider the reduced processing time applied, $\hat{v}_{k,t}$. Obviously, if we consider $\alpha_i = 1, \forall t$, the $M3 \cup 4_{-\alpha I_{-\eta}}$ and $M4 \cup 3_{-\alpha I_{-\eta}}$ models are equivalent to the $M3 \cup 4_{-\eta}$ and $M4 \cup 3_{-\eta}$ models, respectively.

On the other hand, in terms of average and maximum static saturations, it is now necessary to consider the matrix of the static activity factors ($A := (\alpha_{i,k}) : i \in I, k \in K$):

$$\eta_{med}^{\circ}(k, c, \vec{d}, P, A) = \frac{1}{c \cdot T} \cdot \sum_{i=1}^{|I|} \frac{p_{i,k}}{\alpha_{i,k}} \cdot d_i \quad (k = 1, \dots, |K|) \quad (61)$$

$$\eta_{max}^{\circ}(k, c, P, A) = \frac{1}{c} \cdot \max_{i \in I} \left\{ \frac{p_{i,k}}{\alpha_{i,k}} \right\} \quad (k = 1, \dots, |K|) \quad (62)$$

Therefore, the static work overload with activity factors that currently support each one of the processors of the workstation $k \in K$ has the following form:

$$\omega_0(k, c, \vec{d}, P, A, \eta_{med}^{\infty}) = c \cdot T \cdot \max \left\{ 0, \eta_{med}^{\circ}(k, c, \vec{d}, P, A) - \eta_{med}^{\infty} \right\} \quad (k = 1, \dots, |K|) \quad (63)$$

And the static work overload of the line is calculated in the following manner:

$$W_0(c, \vec{d}, \vec{b}, P, A, \eta_{med}^{\infty}) = \sum_{k=1}^{|K|} b_k \cdot \omega_0(k, c, \vec{d}, P, A, \eta_{med}^{\infty}) \quad (k = 1, \dots, |K|) \quad (64)$$

Obviously, for practical purposes, for determining both the maximum and average static saturation of the processors of workstations, given a production plan (\vec{d}) and the

processing times of operations (P), the set of activity factor values (A) will be independent of the workstations ($k \in K$) and products ($i \in I$). Therefore, the average activity factor, determined from the corresponding values at every moment of the working day, will be used.

On the other hand, if we take into consideration the dynamic activity factors ($\dot{A} := (\dot{\alpha}_{k,t}) : k \in K, t = 1, \dots, T$) and the manufacturing sequence $\pi(T) = \{\pi_1, \dots, \pi_T\}$, the equations that, respectively, determine the dynamic saturation (medium and maximum) and the dynamic work overload (elemental and global) are the following:

$$\eta_{med}(k, c, \pi(T), V, \dot{A}) = \frac{1}{c \cdot T} \cdot \sum_{t=1}^{|T|} \frac{v_{k,t}}{\dot{\alpha}_{k,t}} \quad (k = 1, \dots, |K|) \quad (65)$$

$$\eta_{max}(k, c, \pi(T), V, \dot{A}) = \frac{1}{c} \cdot \max_{1 \leq t \leq T} \left\{ \frac{v_{k,t}}{\dot{\alpha}_{k,t}} \right\} \quad (k = 1, \dots, |K|) \quad (66)$$

$$\omega(k, c, \pi(T), V, \dot{A}, \eta_{med}^{\infty}) = c \cdot T \cdot \max \left\{ 0, \eta_{med}(k, c, \pi(T), V, \dot{A}) - \eta_{med}^{\infty} \right\} \quad (k = 1, \dots, |K|) \quad (67)$$

$$W(c, \bar{b}, \pi(T), V, \dot{A}, \eta_{med}^{\infty}) = \sum_{k=1}^{|K|} b_k \cdot \omega(k, c, \pi(T), V, \dot{A}, \eta_{med}^{\infty}) \quad (68)$$

Note that equations represent the general case. In our particular case study, we have considered that operators perform their workload with the same work pace regardless of the workstation in which they find themselves; that is:

$$\dot{\alpha}_{k,t} = \dot{\alpha}_{t+k-1} \quad (k = 1, \dots, |K|; t = 1, \dots, T).$$

8. Increased Line Capacity by Auxiliary Processors

An alternative measure to deal with the increased work overload produced by saturation conditions is to incorporate reinforcement operators into the line. Really, once known the inevitable work overload per station, both static and dynamic, it is possible to determine the number of auxiliary workers needed to complete this unfinished work.

This calculation can be done by two ways:

- I. **Auxiliary processors required by the line:** From the obtained overall work overload, both static and dynamic, and the total available time by processor throughout the effective shift work, it is possible to determine a lower bound of the

number of auxiliary processors necessary to eliminate the work overload. For that matter, the following points should be noted:

- a. The auxiliary processors are multi-skilled and therefore they can perform suitably any task assigned to any workstation.
- b. The movement of processors between stations is considered negligible.
- c. There are not more workstations overloaded simultaneously than the number of auxiliary processors.

In such conditions, the number of auxiliary multi-skilled processors is calculated as follows:

$$\Delta b^\circ = \sum_{k=1}^{|K|} b_k \cdot \max \left\{ 0, \eta_{med}^\circ(k, c, \vec{d}, P, A) - \eta_{med}^\infty \right\} \quad (69)$$

$$\Delta b = \sum_{k=1}^{|K|} b_k \cdot \max \left\{ 0, \eta_{med}(k, c, \vec{d}, V, \dot{A}) - \eta_{med}^\infty \right\} \quad (70)$$

Obviously, this number of multi-skilled processors can be determinate for a specific workstation:

$$\Delta b_k^\circ = \max \left\{ 0, \eta_{med}^\circ(k, c, \vec{d}, P, A) - \eta_{med}^\infty \right\} \quad (k = 1, \dots, |K|) \quad (71)$$

$$\Delta b_k = \max \left\{ 0, \eta_{med}(k, c, \vec{d}, V, \dot{A}) - \eta_{med}^\infty \right\} \quad (k = 1, \dots, |K|) \quad (72)$$

Fulfilling the following:

$$\Delta b^\circ = \sum_{k=1}^{|K|} b_k \cdot \Delta b_k^\circ \quad (73)$$

$$\Delta b = \sum_{k=1}^{|K|} b_k \cdot \Delta b_k \quad (74)$$

II. Auxiliary processors by workstation: from the work overload generated at each station and the total available time by processor throughout the effective shift work, an upper bound, of the number of processors necessary to complete the required work by workstation, can be obtained. In this way, it must be noted:

- a. The auxiliary processors are specialized in operations assigned to a single station and, therefore, they are assigned exclusively to a workstation. That is, each workstation overloaded receives individualized assistance of a specialized processor.

Accordingly, the number of specialized processors necessary to eliminate the work overload generated by the saturation conditions, both static and dynamic, is determined as follows:

$$\hat{\Delta}b_k^\circ = \sum_{k=1}^{|K|} [b_k \cdot \Delta b_k^\circ] \quad (75)$$

$$\hat{\Delta}b = \sum_{k=1}^{|K|} [b_k \cdot \Delta b_k] \quad (76)$$

Along these lines, we consider two opposite ways to counteract the negative effect of limiting the processors' saturation. First, we propose multi-skilled processors, that can provide assistance to any workstations and, secondly, we suggest specialized processors that are assigned to one specific station.

It should be noted that, so far, we have only consider the calculation of auxiliary processors. However, in our case study, each processor consists of a team of two workers. For this reason, we denote as H the number of operators that compose one processor and therefore we define the auxiliary operator functions as follows:

$$\Delta H^\circ = [H \cdot \Delta b^\circ] \quad (77)$$

$$\Delta H = [H \cdot \Delta b] \quad (78)$$

$$\hat{\Delta}H^\circ = \sum_{k=1}^{|K|} [H \cdot b_k \cdot \Delta b_k^\circ] \quad (79)$$

$$\hat{\Delta}H = \sum_{k=1}^{|K|} [H \cdot b_k \cdot \Delta b_k] \quad (80)$$

9. Effect of Activation and Saturation Conditions. Case Study

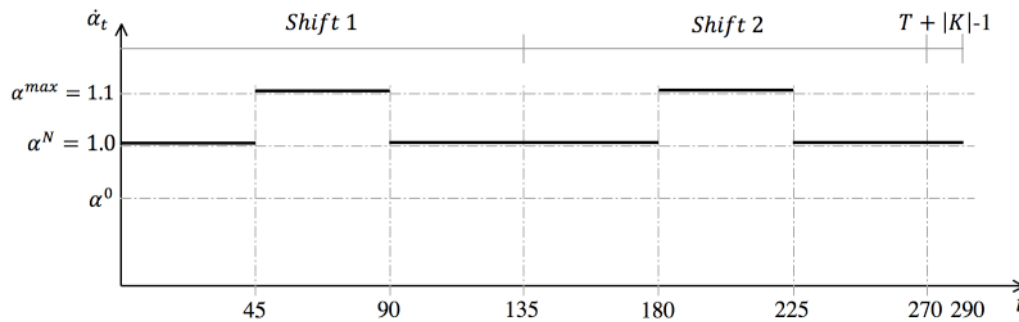
Next, we determine the inevitable static work overload of the line and the dynamic work overload according to the $M3 \cup 4_{\dot{\alpha}I}_{\eta}$ and $M4 \cup 3_{\dot{\alpha}I}_{\eta}$ models. We achieve this goal by working from the same case corresponding to the Nissan engine plant in Barcelona that was used in the exploitation of $M3 \cup 4_{\eta}$ and $M4 \cup 3_{\eta}$ models but considering simultaneously the limitation of saturation and the activation of the processors.

In this experiment, a number of features of the assembly line have been taken into account to define the stepped ($\dot{\alpha}^S$), triangular ($\dot{\alpha}^T$), trapezoidal ($\dot{\alpha}^Z$) and parabolic ($\dot{\alpha}^P$) functions of the dynamic factor of activity. We state the following:

- Factor of normal activity and maximum allowed: $\alpha^N = 1.0$, $\alpha^{\max} = 1.1$. It should be noted that the considered maximum value is lower than the optimal value established by the collective agreement ($\alpha^* = 1.2$).
- Shift 1: $t_0 = 45$, $t_{med} = 67$, $t_\infty = 90$.
- Shift 2: $t_0 = 180$, $t_{med} = 202$, $t_\infty = 225$.
- Average of the activity factor: $\overline{\alpha^S} = 1.0\hat{3}$; $\overline{\alpha^T} = 1.05$; $\overline{\alpha^Z} = \overline{\alpha^P} 1.0\hat{6}$.

In this way, all workers will synchronously perform their work, period by period, throughout the workday and will follow the work pace set by the activity factor imposed by the functions defined in Bautista et al. (2015a) and their respective functions with average value. For illustration see the Figure 3 (Fig. 3) that corresponds to the stepped function ($\dot{\alpha}^S$).

Fig. 3. Stepped function for the work pace factor in the NISSAN case.



The work overload results achieved in this experiment are shown in Table 3. The first column enumerates the 23 plans of production ($\varepsilon = \#1, \dots, \#23$). The following four columns determine the inevitable work overload (W_0) obtained by each instance when the static saturation limitations are considered. Specifically, there are shown the values obtained without considering activation of processors (second column) and the values obtained when an average activation is considering according to the average values of the four functions for work pace of workers.

Furthermore, the last six columns represent the values obtained with the $M3 \cup 4_{\dot{\alpha}I}_{\eta}$ and $M4 \cup 3_{\dot{\alpha}I}_{\eta}$ models when all the cases for the workers'

activation are considered. Indeed, the sixth column shows the work overload given by $M3U4_{\eta}$ and $M4U3_{\eta}$ models, that is equal to consider a work pace function equal to the normal activity factor all along the workday.

Table 3. Inevitable work overload values due to the production mixes (ε), the limitation of the static average saturation ($\eta_{med}^{\infty} = 0.95$) and the activation level of workers ($\alpha_{i,k}$); and work overload given by $M3U4_{\eta}$, $M4U3_{\eta}$, $M3U4_{\dot{\alpha}}_{\eta}$ and $M4U3_{\dot{\alpha}}_{\eta}$ models, with the work pace functions ($\dot{\alpha}_i$).

ε	$W_0(\alpha_{i,k}, d_i)$				$W_{3U4_{\eta}} = W_{4U3_{\eta}}$	$W_{3U4_{\dot{\alpha}}_{\eta}} = W_{4U3_{\dot{\alpha}}_{\eta}}$				
	-	$\overline{\alpha^S}$	$\overline{\alpha^T}$	$\overline{\alpha^Z} = \overline{\alpha^P}$		$\dot{\alpha}^S$	$\dot{\alpha}^T$	$\overline{\alpha^S}$	$\overline{\alpha^T}$	$\dot{\alpha}^Z(1)$
#1	12,315.0	4,220.6	591.1	0.0	12,315.0	4,601.8	796.0	4361.3	620.6	0.0
#2	12,458.0	4,312.6	725.2	0.0	12,458.0	4,692.7	884.9	4456.3	761.5	0.0
#3	12,210.0	4,123.9	794.6	0.0	12,210.0	4,509.2	895.4	4261.3	834.4	0.0
#4	12,470.0	4,269.0	638.7	0.0	12,470.0	4,649.5	843.2	4411.3	670.6	0.0
#5	13,012.5	4,549.7	1,053.8	0.0	13,012.5	4,929.8	1,202.9	4701.3	1,106.5	0.0
#6	12,910.0	4,506.1	872.0	0.0	12,910.0	4,882.2	1,072.5	4656.3	915.6	0.0
#7	12,722.5	4,361.0	1,080.4	0.0	12,722.5	4,743.9	1,212.2	4506.3	1,134.4	0.0
#8	12,018.0	4,075.5	448.2	0.0	12,018.0	4,459.6	657.1	4211.3	470.6	0.0
#9	13,363.0	4,694.8	1,071.9	0.0	13,363.0	5,066.8	1,258.2	4851.3	1,125.5	0.0
#10	13,122.0	4,404.5	1,545.1	0.0	13,122.0	4,795.6	1,660.4	4551.3	1,622.4	0.0
#11	11,792.5	3,838.4	355.7	0.0	11,792.5	4,231.1	520.8	3966.3	373.5	0.0
#12	12,246.0	4,209.0	649.0	0.0	12,246.0	4,590.8	801.1	4349.3	681.5	0.0
#13	12,551.0	4,335.8	704.4	0.0	12,551.0	4,715.1	907.0	4480.3	739.6	0.0
#14	12,646.0	4,398.7	789.0	0.0	12,646.0	4,777.3	968.3	4545.3	828.5	0.0
#15	12,393.5	4,271.9	796.7	0.0	12,393.5	4,653.1	946.6	4414.3	836.5	0.0
#16	12,363.0	4,249.7	619.6	0.0	12,363.0	4,630.4	823.9	4391.3	650.6	0.0
#17	12,597.5	4,375.5	872.9	0.0	12,597.5	4,755.0	1,022.3	4521.3	916.5	0.0
#18	13,208.0	4,646.4	1,010.1	0.0	13,208.0	5,020.7	1,210.2	4801.3	1,060.6	0.0
#19	12,810.0	4,312.6	1,294.6	0.0	12,810.0	4,700.6	1,403.5	4456.3	1,359.4	0.0
#20	11,875.0	3,983.5	440.5	0.0	11,875.0	4,370.0	599.7	4116.3	462.5	0.0
#21	13,065.0	4,554.5	919.6	0.0	13,065.0	4,929.1	1,120.4	4706.3	965.6	0.0
#22	13,062.5	4,452.9	1,330.8	0.0	13,062.5	4,838.9	1,440.5	4601.3	1,397.4	0.0
#23	11,902.5	3,935.2	318.5	0.0	11,902.5	4,323.0	524.0	4066.3	334.4	0.0
	12,570.2	4,307.9	822.7	0.0	12,570.2	4,689.8	990.0	4,451.5	863.9	0.0

As can be noted, the inevitable work overload is considerably reduced when the work pace of operators increases at some period of workday. Specifically, if the activity factor is increased through a stepped function, which only represents an average increase of 3.33%, the inevitable work overload is 4,307.9 seconds on average. Similarly, whether the average activation is equal to 5%, the work overload is 822.7 seconds. However when the activation corresponds to the 6.66%, the inevitable work overload is completely eliminated.

On the other hand, the models $M3 \cup 4_{\dot{\alpha}I}_{\eta}$ and $M4 \cup 3_{\dot{\alpha}I}_{\eta}$ also have reached all the optimal solutions, such as the case of the first computational experience. Nevertheless, now, these solutions do not coincide with the inevitable values. Indeed the $M3 \cup 4_{\dot{\alpha}I}_{\eta}$ and $M4 \cup 3_{\dot{\alpha}I}_{\eta}$ worsen the work overload value by 8.9% and 24.8%, on average, when the stepped and the triangular functions are considered. Even so the incorporation of a largest activity factor at certain moments of the workday allows a reduction of the overall overload values. Specifically, the stepped function reduces the non-completed work by 62.7%, on average, in regard with the solutions given by $M3 \cup 4_{\eta}$ and $M4 \cup 3_{\eta}$ models; the triangular function decreases these values by 92.1%; and the trapezoidal and parabolic functions and their corresponding functions with a mean linear activation are able to eliminate the 100% of the non-completed work.

9.1. Result analysis

Obviously, the activation of workers produces a productivity improvement even considering the saturation conditions established by the collective agreements. This improvement can be economically translated (€/year) whether the consolidated operation profit of the line (10%), the total value of a engine (4,000€/engine), the production time per engine (175s) and the working calendar (225 days/year) are taken into account.

However these improvements are achieved by means of an excess effort by operators, since they must increase their work pace at specific moments of their workday. Because of this required extra-effort and although the demanded activity factor does not overtake the maximum value established by law ($\alpha_{i,k}^* = 1.2$) at no moment of the day, we calculate the extra cost resulting from the effort demanded to the workers. For that purpose and taking into account that our case study is located in Spain, we consider the current hourly cost per operator in the Spanish automotive sector (i.e. [20€/hour ; 25€/hour]). Therefore, according the used function and the 21 workstations of the line and the two operators by processor, we will have the following costs:

- Stepped function: [81,860.63 €/year-line ; 102,235.78 €/year-line].
- Triangular function: [124,031.25 €/year-line; 155,039.06 €/year-line].
- Trapezoidal / Parabolic function: [165,358.46 €/year-line; 206,718.54 €/year-line].

Based on the above, we can calculate the net profits obtained by means of the activation of operators, regarding the number of uncompleted engines, in contrast with the situation where the saturation conditions are fulfilled but the work pace factor always is equal to the normal factor ($\alpha^N = 1.0$). Thus, considering the rank of the effort costs, we have the following net profits (Table 4):

Table 4. Annual net profits (in millions of Euros) obtained by the reduction of work overload given by $M3\cup4_c\dot{l}_ \eta$ and $M4\cup3_c\dot{l}_ \eta$ models and taking into account the excess effort costs.

Net profit (M€/year-line)	$M3\cup4_c\dot{l}_ \eta = M4\cup3_c\dot{l}_ \eta$				
	$\dot{\alpha}^s$	$\dot{\alpha}^t$	$\overline{\alpha}^s$	$\overline{\alpha}^t$	$\dot{\alpha}^z, \dot{\alpha}^z, \overline{\alpha}^s, \overline{\alpha}^s$
Min.	[3.81 ; 3.79]	[5.67 ; 5.64]	[3.94 ; 3.92]	[5.77 ; 5.74]	[5.90 ; 5.86]
Max.	[4.18 ; 4.16]	[5.89 ; 5.86]	[4.30 ; 4.28]	[5.91 ; 5.88]	[6.71 ; 6.67]
Average	[3.97 ; 3.95]	[5.83 ; 5.80]	[4.09 ; 4.07]	[5.90 ; 5.87]	[6.30 ; 6.26]

After confirming that the increase factor of work pace brings benefits when average dynamic saturation of processors is limited, including the compensation of operators for that excess effort, we analyze whether also the reinforcement of the line by auxiliary processors to reduce overload is feasible or not.

Indeed, considering the number of both multi-skilled and specialized auxiliary processors (equations 78 and 80) needed to eliminate the work overload in each studied scenario (Table 5), we can determine the cost of this approach that is addressed to counteract the negative effect of saturation limitations.

Table 5. Number of multi-skilled auxiliary operator (ΔH) and specialized auxiliary operators ($\hat{\Delta H}$) needed to eliminate the workload obtained with $M3\cup4_ \eta$, $M4\cup3_ \eta$, $M3\cup4_c\dot{l}_ \eta$ and $M4\cup3_c\dot{l}_ \eta$ models.

ϵ	$\Delta H(M3\cup4_c\dot{l}_ \eta = M4\cup3_c\dot{l}_ \eta)$			$\hat{\Delta H}(M3\cup4_c\dot{l}_ \eta = M4\cup3_c\dot{l}_ \eta)$					
	-	$\dot{\alpha}^s, \dot{\alpha}^t, \overline{\alpha}^s, \overline{\alpha}^t$	$\dot{\alpha}^z, \dot{\alpha}^z, \overline{\alpha}^z, \overline{\alpha}^z$	-	$\dot{\alpha}^s$	$\dot{\alpha}^t$	$\overline{\alpha}^s$	$\overline{\alpha}^t$	$\dot{\alpha}^z^{(1)}$
#1	4	2	0	24	20	20	20	20	0
#2	4	2	0	24	20	20	20	16	0
#3	4	2	0	32	20	12	20	12	0
#4	4	2	0	24	20	20	20	20	0
#5	4	2	0	28	20	16	20	16	0
#6	4	2	0	24	20	20	20	20	0
#7	4	2	0	28	20	16	20	12	0
#8	4	2	0	24	20	20	20	20	0
#9	4	2	0	24	20	20	20	16	0
#10	4	2	0	32	20	12	20	12	0
#11	4	2	0	28	20	16	20	16	0
#12	4	2	0	24	20	16	20	16	0
#13	4	2	0	24	20	20	20	20	0
#14	4	2	0	24	20	20	20	16	0
#15	4	2	0	28	20	16	20	16	0
#16	4	2	0	24	20	20	20	20	0

#17	4	2	0	28	20	16	20	16	0
#18	4	2	0	24	20	20	20	20	0
#19	4	2	0	32	20	12	20	12	0
#20	4	2	0	24	20	16	20	16	0
#21	4	2	0	24	20	20	20	20	0
#22	4	2	0	28	20	16	20	12	0
#23	4	2	0	28	20	20	20	12	0

Table 6 shows the sum of the auxiliary operator costs and the excess effort costs for all cases, multi-skilled and specialized operators and all the work pace functions. The costs have been calculated in order to eliminate completely the overall work overload.

Table 6. Average costs (in thousands of Euros) from eliminating the work overload by means of hiring multi-skilled (ΔH) and specialized ($\hat{\Delta H}$) auxiliary workers and compensation of over-effort required to increase the work pace factor, considering the range of hourly cost per worker in Spain [20 ; 25€/hour].

	$M3 \cup 4_ \dot{\alpha} _ \eta$	$M3 \cup 4_ \dot{\alpha} _ \eta = M4 \cup 3_ \dot{\alpha} _ \eta$				
	$M4 \cup 3_ \dot{\alpha} _ \eta$	$\dot{\alpha}^s$	$\dot{\alpha}^T$	$\overline{\alpha}^s$	$\overline{\alpha}^T$	$\dot{\alpha}^z$
ΔH	[115.6 ; 144.4]	[140.9 ; 176.1]	[183.1 ; 228.9]	[140.9 ; 176.1]	[183.1 ; 228.9]	[165.4 ; 206.7]
$\hat{\Delta H}$	[775.5 ; 969.4]	[672.5 ; 840.5]	[714.6 ; 803.4]	[600.6 ; 750.6]	[606.8 ; 758.5]	[165.4 ; 206.7]

According to the costs listed in Table 6 and taking into account the economic losses from the excess of work overload when saturation conditions agreed by collective agreement are incorporated in sequencing models we denote that both measures, work pace activation and auxiliary operators, are economically feasible. Indeed, based on the average work overload given by $M3 \cup 4_ \eta$ and $M4 \cup 3_ \eta$ models (12,570.2s) we have an average annual loss of 6,464.6 thousands of Euros, but if we activate the operators through a trapezoidal function we will only have a maximum excess effort cost of 206.7 thousands of Euros. This situation where the costs of measures to eliminate the work overload are less than the costs associated to the uncompleted engines, is given in all studied scenarios. Therefore, if we want to fulfill the conditions set in collective agreements it is necessary to activate the workers at certain moments of the workday and/or to incorporate auxiliary processors.

Obviously, it should be noted that the best situation, where the work overload is eliminated only by means of multi-skilled operators, may not be technically feasible because the workers must move along the line and an only worker cannot work at two stations simultaneously.

9. Conclusions

After establishing a set of legal bases on the working conditions in the automotive industry, such as the workday duration and the saturation of the workstations, we have proposed two equivalent mathematical models for the *MMSP-W*. These new models are focused on measuring the impact produced by such labor characteristics on the work overload generated in a mixed-model assembly line.

The proposed models, $M3 \cup 4_{-\eta}$ and $M4 \cup 3_{-\eta}$, were applied to a case study of the Nissan engine plant in Barcelona that consist of 23 different demand plans. The computational experience determined that the improvement in working conditions designed to limit the average saturation of the workstations causes a drastic drop in production. This drop is estimated in terms of 75 daily engines in a line with a capacity of 270 engines in two shifts.

In order to relieve the production drop, we have proposed the activation of the processors and the incorporation of auxiliary processors at the assembly line.

The first proposal consists on increasing the work pace of operators (while always respecting the agreed working conditions) under the guidance of temporary functions of the activity factor that unfolds throughout the workday. This action have been concretized in two new models, the $M3 \cup 4_{-\dot{\alpha}l_{-\eta}}$ and $M4 \cup 3_{-\dot{\alpha}l_{-\eta}}$, that incorporate the activity factor in the execution times of the operations.

After running the models with saturation and activation conditions, we have observed how the economic losses derived from the number of uncompleted engines are reduced whenever the average level activity of workers increases. Indeed, from the losses due to the work overload given by $M3 \cup 4_{-\eta}$ and $M4 \cup 3_{-\eta}$ models, we have calculated the net profits that would be obtained by require workers more effort, considering the extra-cost that supposes the compensation of this over-effort at certain moments of workday. This has allowed us to observe how, whatever the increased activity of operators, benefits are obtained.

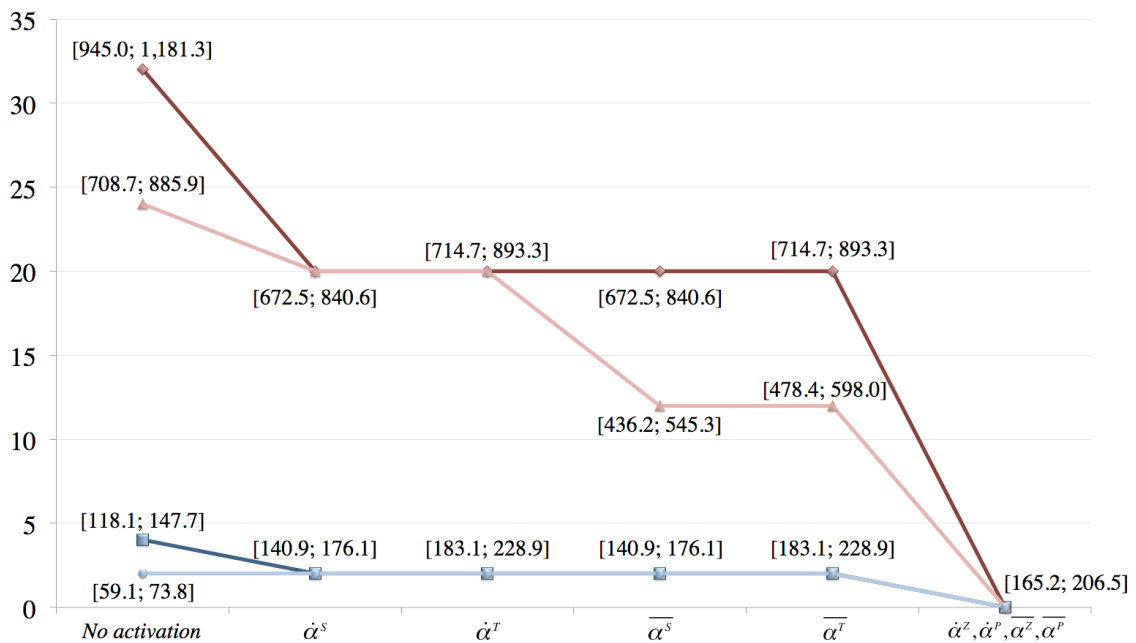
However, the work pace rise does not clear the 100% of work overload for cases in which the increase is 3.33% or 5% (stepped and triangular function). For this reason and in order to assess the feasibility of incorporating auxiliary operators, we have also calculated the minimum and maximum reinforcement operators needed to achieve null work overload in all cases.

Thus, if we consider multi-skilled workers able to move about stations in a negligible time and able to develop any task, four workers will be needed to eliminate overloads obtained with the $M3 \cup 4_{\eta}$ and $M4 \cup 3_{\eta}$ models; 2 operators for overloads given by $M3 \cup 4_{\dot{\alpha}I_{\eta}}$ and $M4 \cup 3_{\dot{\alpha}I_{\eta}}$ models, when the activity factor follows a stepped or triangular function; and none to the other cases.

On the other hand, considering the extreme situation where the auxiliary workers are assigned to a single station because they are specialized, a maximum number of 32 operators are necessary when working at normal activity, 20 when the activity factor is increased by 3.33 % (Stepped function) and 5% (triangular function) on average and none when the average increase is 6.66%.

Thus, given that the hourly cost per worker in the automotive sector in Spain ranges between 20 and 25 €; engine line represents a cost of 10% from value of a engine and therefore, a lost engine supposes a loss of 400 €; and the work schedule is 225 working days; we have calculated the costs in regard with both the level activation of workers and the number of auxiliary processors.

Figure 4. Maximum and minimum number of multi-skilled operators (blue lines); the maximum and minimum specialized operators (red lines) needed to completely eliminate the work overload associated to one workday composed by two shifts; and the cost range due to the reinforcement (considering that auxiliary operators work at normal activity) and the excess effort required to operators.



The results show that all evaluated scenarios reported profits because of none supposes a costs greater than the losses derived from the incorporation of saturation condition

(6,464.6 thousands of Euros on average). Indeed, except in the case of trapezoidal or parabolic functions in which no auxiliary operators are required, net profits after combination of both measures (activation and reinforcement operators) are higher than those obtained with a single measure to reduce the effect of saturation and also to complete 100% of the required work.

In short, the models proposed in this paper allow fulfilling the legal conditions imposed by collective agreements and, therefore, approach the problem from literature to real environments, all this without losing sight of the objective of completing all the required work and maximizing productivity. In other words, this study has able to establish

In future works, the incorporation of auxiliary processors will be studied more thoroughly. Specifically, an intermediate situation where there are specialized operators in the operations assigned to a set of consecutive workstations will be analyzed. Thus, a single processor may support a specific set of consecutive workstations overloaded, but not all at once, so that operators can move from one to another and do not require such demanding training degree. We also study an alternative to improve the average saturation by means of rotational programs among the workstations with the purpose of reducing the medium-and long-term saturation. Finally, in order to bring closer the *MMSP-W* to real and current situations of productive environments, we will incorporate, into the models proposed in this paper, *JIT* characteristics, with the purpose of improving the production regularity and thus favoring both a regular consumption of components and a regular use of resources.

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