Bypassing the Natural Visual-Motor Pathway to Execute Complex Movement Related Tasks Using Interval Type-2 Fuzzy Sets

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Abstract—In visual-motor coordination, the human brain processes visual stimuli representative of complex motion-related tasks at the occipital lobe to generate the necessary neuronal signals for the parietal and pre-frontal lobes, which in turn generates movement related plans to excite the motor cortex to execute the actual tasks. The paper introduces a novel approach to provide rehabilitative support to patients suffering from neurological damage in their pre-frontal, parietal and/or motor cortex regions. An attempt to bypass the natural visual-motor pathway is undertaken using interval type-2 fuzzy sets to generate the approximate EEG response of the damaged prefrontal/parietal/motor cortex from the occipital EEG signals. The approximate EEG response is used to trigger a pre-trained joint coordinate generator to obtain desired joint coordinates of the link endpoints of a robot imitating the human subject. The robot arm is here employed as a rehabilitative aid in order to move each link end-points to the desired locations in the reference coordinate system by appropriately activating its links using the well-known inverse kinematics approach. The meansquare positional errors obtained for each link end-points is found within acceptable limits for all experimental subjects including subjects with partial parietal damage, indicating a possible impact of the proposed approach in rehabilitative robotics. Subjective variation in EEG features over different sessions of experimental trials is modelled here using interval type-2 fuzzy sets for its inherent power to handle uncertainty. Experiments undertaken confirm that interval type-2 fuzzy realization outperforms its classical type-1 counterpart and back-propagation neural approaches in all experimental cases, considering link positional error as a metric. The proposed research offers a new opening for the development of possible rehabilitative aids for people with partial impairment in visual-motor coordination.

Index Terms—Interval type-2 fuzzy sets, Fuzzy mapping, Bypassing natural visual-motor pathways, Prediction of positional body joint coordinates and Inverse kinematics in Robotics, Rehabilitative aids for visual-motor impairment

I. INTRODUCTION

Brain-computer interfacing (BCI), which is currently passing its infancy, has gained immense popularity over the last decade for its increasing applications in rehabilitative robotics [1]-[5]. Patients suffering from paraplegia [6], paraparesis [6], cerebral palsy [97], Optic ataxias [8], [9], Balint's Syndrome [10] and other brain-related diseases [11] including post stroke patients [7] usually have reduced functioning in prefrontal, parietal lobe and/or motor cortex, prohibiting them to correctly control their limb and body movements due to impairments in sensory-motor coordination. The conventional BCI techniques utilize the signals acquired directly from the motor cortex during motor imagery tasks to decode the motor executions/imaginations. [12]-[14], [82]. In case of partial damages of the brain modules on the visualmotor coordination pathways, the subjects are unable to perform the coordination tasks. The brain signals acquired from motor cortex of those people are compromised since the pathways for motor coordination and execution are disrupted in these cases. Consequentially, decoding of BCI motor imagery directly from motor cortical signals in these cases is not beneficial. One approach to solve this problem is to bypass the pathways containing the damaged brain modules. This paper attempts to rehabilitate patients with such visual-motor coordination impairment by arranging an alternative (artificial) pathway from the occipital lobe to the motor cortex (via the pre-frontal and parietal lobe) through a two-step non-linear mapping process.

The problem addressed in the paper is briefly outlined as follows. Let \vec{x} and \vec{y} be two distinct feature vectors obtained from EEG signals acquired from two brain regions/lobes. Let $M: \vec{x} \to \vec{Y}$ be a mapping function. If we can recover M from successful instances of \vec{x} and \vec{y} , then for an unknown $\vec{x'}$ close enough to \vec{x} , we can retrieve $\vec{Y'}$. Nonlinear regression [90], [91] and function approximation by supervised neural learning [92], [93] are widely used techniques to develop the mapping function M. These mapping techniques work well when the feature vectors are free from noise.

Measurements in real world problems are often found to be contaminated with various forms of noise [81]. EEG signals acquired from a subject during his/her experimental trials to perform a cognitive task often are found to be contaminated with noise for the following reasons. A few common sources of noise that influence the acquired EEG signals include lack of subjective concentration, parallel undesirable cognitive thoughts while performing the main cognitive tasks, undesirable head/body movements/eye blinking, and noisy ambience. Naturally, the features extracted from the EEG signals in presence of the above noisy sources are affected with noise.

Traditional mapping policies that attempt to generalize $M: \vec{X} \to \vec{Y}$ from several X_i s and corresponding Y_i s for i = 1 to *n* trials cannot correctly capture the non-linearity in the function Y = M(X) for non-uniform fluctuation in noise over the different trials. The problem in the present context is to design a suitable mapping M, which would not be significantly influenced in presence of random fluctuation of noise over the EEG trials.

The logic of fuzzy sets has proved itself an interesting tool for decision-making under uncertainty and noise [15-25], [Fill me in]. Type-2 fuzzy sets, which has been introduced by Zadeh in 1975 [26], and has been popularized by Mendel [27] - [34] over the last decade, have an inherent representational characteristics to model measurement uncertainty by membership functions [35-39], [98]. An interval type-2 fuzzy set (IT2FS) often is characterized by two membership functions (MFs), called upper membership function (UMF) and lower membership function (LMF). The interval between the UMF and the LMF

embodies an infinitely large number of (embedded) type-1 fuzzy sets [40-42], and is referred to as *footprint* of uncertainty (FOU). Thus, for example, for a fuzzy concept: height is MEDIUM, we have n different type-1 MFs obtained from n sources. The UMF (LMF) of IT2FS at a given value of linguistic variable x is obtained here by taking the maximum (minimum) of the type-1 fuzzy MFs at the same x obtained from n sources. Thus at a given height (x=6 feet, say), the FOU bounded by the UMF and the LMF has a wide space of uncertainty in the membership of height is MEDIUM.

The EEG features acquired during task-planning having wider variance, IT2FS seems to be an efficient tool for EEG feature encoding (and also mapping). Given two brain lobes L_1 and L_2 , suppose we have n sets of features for both the lobes for *n* sets of experimental trials, aimed at planning/performing a given cognitive task. We construct one FOU for each feature of lobe L_1 using the numeric values of the same feature over *n*-successful-trials for a given subject *i*. Similarly, we also obtain one FOU for each feature of lobe L_2 using the numeric values of the same feature over *n* successful trials for the same subject *i*. Now, given an unknown experimental trial, for which we obtain all the features from the EEG of lobe L_1 , we use these to instantiate the IT2FS MFs of lobe L₁, and using standard IT2 inferential procedure attempt to infer the IT2 MFs for each unknown feature of lobe L₂. Now, by type-2 defuzzification (fuzzy decoding) of each FOU, dedicated for each feature, we obtain the predicted features of lobe L_2 . This process is used to determine features from successful instances of occipital EEG to parietal and prefrontal EEG and then by using the resulting IT2MFs of the parietal and prefrontal regions we, ultimately obtain the EEG features of the motor cortex region. Experiments undertaken confirm that for normal subjects, the predicted motor cortex features obtained from the measured occipital features by the proposed IT2FS technique is very accurate with *mean square error* less than or equal to 1.25.

It is thus apparent that to predict EEG response for movement related task from the EEG response to visual stimuli, we need to have prior EEG data from successful instances of occipital lobe, parietal lobe, prefrontal lobe and motor cortex regions, where these *a priori* data are used to develop the brain model for mapping occipital to motor cortex via the parietal lobe. However, unfortunately, for patients diagnosed to

have Balint's Syndrome, optic ataxia, optic apraxia, Parkinsson's diseases, paraplegia, paraparesis, cerebral palsy, or other brain ailments cannot perform complex planning due to partial non-functionality of the parietal, pre-frontal and/or motor cortex [6], [7], [10]. Since the visual-motor pathways are affected for these types of patients, the proper signals cannot be acquired directly from the motor cortex. Thus conventional BCI systems [2], [4], [5], [80], [82], [83], aiming at generation of control commands for rehabilitative aids (such as brain-commanded artificial limbs [Fill me: Saugat Journal paper-P300 based limb]) from the acquired EEG signals captured directly from the motor cortex are unsuitable for the above types of patients. Thus, focusing on the patients with impaired visual-motor coordination due to damaged prefrontal, parietal and/or motor cortex, this paper attempts to derive the mapping of occipital to parietal and prefrontal lobe to motor cortical EEG features from successful instances of visual-motor coordination task and use this mapping in future to offer rehabilitative aids to these patients.

There exists a lot many works on EEG driven motor planning/control [2], [4], [5], [80], [82], [83]. A few works that require special mention in this regard include EEG driven mind controlled wheelchair [1], [84], [85], brain-actuated asynchronous control of humanoid robots [86], BCI based unmanned car control [88], virtual gaming [89] and other applications [82]. Unfortunately, none of these works consider bypassing visual-motor pathways by EEG-BCI. The present work thus seems to be a promising research in the BCI literature.



Fig. 1. Neural pathways (redline) inside the brain that transforms input visual stimulation to complex movement plans (at parietal), and movement execution (in motor cortex) (a) Active pathway from occipital to motor cortex in visual-motor coordination, (b) Schematic view of the architectural connection in (a)

The EEG features derived for motor cortex from such mapping now can be used to predict target joint coordinates of subjects' limb movement associated with complex sensory motor coordination task. The target joint coordinates of subjects' shoulder, elbow and wrist are captured from his/her successful movement of these joints in a visual-motor coordination task. The capturing of this coordinates is done using a Kinect sensor system [43-49] to determine the artificial mapping of occipital features to parietal and prefrontal features to motor cortical features to joint coordinates of shoulder, elbow and wrist of the right-handed subjects. The mapping are later used to test the feasibility of artificial mapping introduced above in visual-motor coordination impairments. The mapping of motor cortex features to joint coordinates is then performed by the IT2FS technique mentioned above. The joint coordinates of subjects thus predicted from his/her occipital EEG data are then used to subsequently input to a robotic arm with multiple links, each having correspondence to specific limbs of a human subject. During resetting, the robot

aligns its links similar to start-up positions of the subjects' (fixed) natural limb positions (hanging down). Next the robot determines the angular shifts/displacement required for each link to reach the desired goals for individual joint by an inverse kinematic approach. In our simple system, we attempted to imitate only three joints of the upper arm (shoulder, elbow and wrist).

The rest of the paper is structured into six sections. Section II provides a system overview along with the proposed T1FS and IT2FS based mapping. Experimental details and corresponding results are given in section III. System validation is undertaken in section IV with an overall discussion of the proposed system in section V. Conclusions are summarized in section VI.

II. PRINCIPLES AND METHODOLOGY

This section provides a thorough discussion on the proposed feature mapping technique using Type-1 fuzzy sets (T1FS) and IT2FS. It also gives an overview of the complete scheme employed for occipital to parietal, and parietal/pre-frontal to motor cortex feature mapping.

Let,

 ${}^{j}f_{i,r}(t)$ be the j^{th} instance of the i^{th} feature of an EEG signal acquired on day t from the r^{th} cortical region of the scalp, where j, i, and t lie in [1, l], [1, n], and [1,k] respectively.

 ${}^{j}F_{i,R}(t)$ be the j^{th} instance of the i^{th} feature of an EEG signal acquired on day t from the R^{th} cortical region of the scalp, where $j \in [1, l]$, $i \in [1, m]$ and $t \in [1, k]$.

 $f_{i,r}(t)$ be a random variable with mean $m_i(t) = \frac{1}{l} \sum_{j=1}^{l} {}^j f_{i,r}(t)$ and variance

$$s_i^2(t) = \frac{1}{l} \sum_{j=1}^{l} \left({}^j f_{i,r}(t) - m_i(t) \right)^2.$$

 $F_{i,R}(t)$ be a random variable with mean $M_i(t) = \frac{1}{l} \sum_{j=1}^{l} {}^{j} F_{i,R}(t)$ and variance

$$S_i^{2}(t) = \frac{1}{l} \sum_{j=1}^{l} \left({}^{j} M_{i,R}(t) - M_i(t) \right)^2$$

We here propose a mapping scheme from feature set $f = \{f_{i,r}(t), i = 1 \text{ to } n\}$ to feature set $F = \{F_{i,R}(t), i = 1 \text{ to } n\}$, where the parameters involved in the sets are defined above. The randomness in $f_{i,r}(t)$ is captured by a Gaussian type membership function (MF) $\mu_{CLOSE-TO-MEAN_i}(f_{i,r}(t))$ or hereafter, $\mu_{C_i}(f_{i,r}(t))$ for brevity, where the MF indicates the degree of closeness of $f_{i,r}(t)$ with the mean value $m_i(t)$ of the random variable $f_{i,r}(t)$. Similarly, the randomness in $F_{i,R}(t)$ is captured by a Gaussian type MF $\mu_{CLOSE-TO-MEAN_i}(F_{i,R}(t))$ or $\mu_{D_i}(F_{i,R}(t))$. Choice of Gaussian type MF here is induced by the experimental observation that the random variable $f_{i,r}(t)$ always lies in the interval $[m_i - 3s_i, m_i + 3s_i]$. Similarly, the random variable $F_{i,R}(t)$ always lies in the interval $[M_i - 3S_i, M_i + 3S_i]$.

To keep the proposed mapping free from the effect of diurnal variation, t, we use random variables $f_{i,r}$ and $F_{i,R}$ with respective mean and variance obtained by *central limit theorem* [79]. The mean and

variance of $f_{i,r}$ are obtained as $\overline{f_{i,r}} = \sum_{t=1}^{k} m_i(t)$ and $v_i^2 = \sum_{t=1}^{k} s_i^2(t)$, whereas the mean and variance of

 $F_{i,R}$ are given by $\overline{F_{i,R}} = \sum_{t=1}^{k} M_i(t)$ and $V_i^2 = \sum_{t=1}^{k} S_i^2(t)$ respectively. The Gaussian MFs $\mu_{C_i}(f_{i,r}(t))$ and

 $\mu_{D_i}(F_{i,R}(t))$ with mean and variance as introduced above are given by

$$\mu_{C_i}(f_{i,r}) = e^{-\frac{\left(f_i^r - \overline{f_i^r}\right)^2}{2v_i^2}},$$
(1)

and
$$\mu_{D_i}(F_{i,R}) = e^{-\frac{\left(F_i^R - \overline{F_i^R}\right)^2}{2V_i^2}}$$



Fig. 2. Feature mapping using proposed IT2FS

A. Type-2 Fuzzy Feature Mapping

Type-1 fuzzy (T1FS) technique, introduced above, attempts to model the variations of a feature across experimental trials by a type-1 MF. However, for simplicity in representation, the measurements containing diurnal variation in a feature are represented by a single fuzzy (Gaussian type) MF. Such MF, however, fails to include fluctuation over days. This section overcomes the above limitation by combining the type-1 MFs describing diurnal variation with the help of an interval type-2 (IT2) representation. The uncertainty involved within and across diurnal variations of features thus can be better modeled by IT2FS (See Fig. A.1 in the Appendix). The fuzzy mapping induced by the following interval type-2 rule thus is expected to yield more realistic inferences than its type-1 counterpart (See Fig. A.2 in the Appendix), indicating parietal and motor cortex EEG features from the measured occipital features.

Rule \mathbf{R}_{i} : If $(f_{1,r} \text{ is } \tilde{C}_{1})$ and $(f_{2,r} \text{ is } \tilde{C}_{2})$ and and $(f_{n,r} \text{ is } \tilde{C}_{n})$ Then $(F_{i,R} \text{ is } \tilde{D}_{i})$

(2)

where \tilde{C}_{j} for j=1 to n and \tilde{D}_{i} are IT2FS. We have n such rules with the same antecedent as for IT2 Rule R_{i} but varied consequent $F_{i,R}$ for i = 1 to n.

The randomness of a feature over different instances on a day is modeled here by a Gaussian MF $\mu_{C_i}(f_{i,r}(t))$ with mean $m_i(t)$ and variance $s_i^2(t)$ as defined earlier. The MF is given by

$$\mu_{\tilde{C}_{i}}(f_{i,r}(\mathbf{t})) = e^{-\frac{(f_{i,r}(t) - m_{i}(t))^{2}}{2s_{i}^{2}(t)}}.$$
(6)

We now construct a IT2 MF with upper Membership Function (UMF) and Lower Membership Function (LMF) for feature $f_{i,r}$ given by

$$UMF(f_{i,r}) = \underset{t=1}{\overset{k}{\max}} \left[\mu_{\tilde{C}_i}(f_{i,r}(t)) \right]$$
(7)

and
$$LMF(f_{i,r}) = \underset{t=1}{\overset{k}{\min}} \left[\underset{C_i}{\mu_{c_i}}(f_{i,r}(t)) \right]$$
 (8)

The region between the UMF and the LMF is called the footprint of uncertainty (FOU). Similarly, we define $UMF(F_{i,R})$ and $LMF(F_{i,R})$. Once the computation of $UMF(f_{i,r})$, $LMF(f_{i,r})$, $UMF(F_{i,R})$ and $LMF(F_{i,R})$ for i=1 to n is over, we employ the following four steps for predicting the EEG features of region R from the measured features $f_{i,r}$ at region r.

Step 1: Instantiate UMF $(f_{i,r})$ and LMF $(f_{i,r})$ by the measurements $f'_{i,r}$ for i=1 to n to determine the lower and upper firing strengths $LFS_{i,r}$ and $UFS_{i,r}$ given by

$$LFS_{i,r} = \left[LFM(f_{i,r}) \right]_{f_{i,r} = f_{i,r}'} \qquad \text{Replace LFM by LMF}$$
(9)
and $UFS_{i,r} = \left[UFM(f_{i,r}) \right]_{f_{i,r} = f_{i,r}'} \qquad \text{Replace UFM by UMF}$ (10)

Step 2: The composite lower and upper firing strengths LFS_r and UFS_r are now obtained by taking fuzzy aggregation, here Min, of the $LFS_{i,r}s$ for i=1 to n.

$$LFS_r = M_{i=1}^n \left(LFS_{i,r} \right) \tag{11}$$

and
$$UFS_r = \underset{i=1}{\overset{n}{\min}} \left(UFS_{i,r} \right)$$
 (12)

Step 3: We next determine the FOU q of the consequent membership space by performing fuzzy t-norm *(min)* over the *LFS_r* (*UFS_r*) and the q^{th} consequent *LMF*($F_{q,R}$) (*UMF*($F_{q,R}$)) for q = 1 to n. This is given by

$$LMF(F'_{q,R}) = Min(LFS_r, LMF(F_{q,R}))$$
(13)

and
$$UMF(F'_{q,R}) = Min(UFS_r, UMF(F_{q,R}))$$
 (14)

Step 4: The feature $F'_{q,R}$ is now evaluated using the following two sub-steps. First, we compute the lower and the upper end point centroids ($C_{l,q}$ and $C_{u,q}$) of the resulting FOU q by the following expressions [50-55]:

$$C_{l,q} = \frac{\int_{-\infty}^{C_{l,q}} UMF_q \cdot x \, dx + \int_{C_{l,q}}^{\infty} LMF_q \cdot x \, dx}{\int_{-\infty}^{C_{l,q}} UMF_q dx + \int_{C_{l,q}}^{\infty} LMF_q dx}$$
(15)

$$C_{u,q} = \frac{\int_{-\infty}^{C_{u,q}} LMF_q. x \, dx + \int_{C_{u,q}}^{\infty} UMF_q. x \, dx}{\int_{-\infty}^{C_{u,q}} LMF_q dx + \int_{C_{u,q}}^{\infty} UMF_q dx}$$
(16)

Here we use the well-known Karnik-Mendel [50-55] iterative algorithm to compute $C_{l,q}$ and $C_{u,q}$ using the above two equations. In the next step, we evaluate feature $F'_{q,R}$ by taking average of $C_{l,q}$ and $C_{u,q}$, i.e.,

$$F'_{q,R} = \frac{1}{2} \Big(C_{l,q} + C_{u,q} \Big) \tag{17}$$

for q = 1 to n. The outline of the IT2FS algorithm is graphically explained in Fig. 2. For prediction of

parietal features from occipital features, prefrontal features from occipital features and joint coordinates prediction from motor cortical features, the IT2FS scheme presented in Fig. 2 is executed.

B. Proposed System Architecture

The principle of type-1(T1FS) and IT2 fuzzy (IT2FS) approach for EEG feature mapping from finally region r to region R on the human scalp has been extended here for rehabilitative application of subjects with damaged pre-frontal, parietal and/or motor cortex regions. We here attempt to utilize the proposed fuzzy mapping policy to map the EEG features extracted from occipital region to predict the EEG features of the same subject for the parietal/prefrontal and motor cortex regions. Fig. 3 explains the fuzzy mapping principles involved to predict the EEG features of parietal, prefrontal and motor cortex regions. Here, for known measurements $f_{i,o}$ for i=1 to n obtained from the occipital region "O" of subject-1 and known $F_{j,P}$ for j=1 to n obtained from the parietal region "P" and $F_{j,pF}$ for j=1 to n obtained from the prefrontal region "pF", we construct type-1 or IT2 fuzzy sets describing $f_{i,o}$ is C_i and $F_{j,P}$ is D_j and $F_{j,pF}$ is E_j for i= 1 to n and j=1 to n. Then for known observations about $f_{i,o}$ i =1 to n, we attempt to predict $F_{j,P}$ for j=1 to n and $F'_{j,pF}$ (Do correct formatting for $F'_{j,pF}$) for j=1 to n using the principle discussed above. In the second phase, we similarly construct type-1 or IT2 MFs for $f_{j,P}$ is C_j , $F_{j,PF}D_j$ and $F_{k,MC}$ is E_k and then for *j*, k = 1 to *n*, we instantiate $f_{j,P}$ is C_j by predicted $F'_{j,P}$, (i.e., by setting $f_{j,P} = F'_{j,P}$) and $f_{j,pF}$ is D_j by $F'_{j, pF}$ for j=1 to n, to predict $F'_{k,MC}$ for k=1 to n using the geometric principles introduced in the Appendix (See Fig. A.3).



Fig. 3. IT2FS fuzzy inference generation system



Fig.4. The complete scheme during task execution session



Fig. 5. Inverse kinematics solution for joint coordinates

The motor cortex EEG features thus obtained is used to predict three distinct joint-coordinates: J1, J2, and J3 (Fig. 4), resembling shoulder, elbow and wrist joints respectively of the subject (Fig. 5(a)), while J0 resembling the subject's waist is being used as the reference joint (see Fig.5(a)), for the subject with damaged parietal, prefrontal and/or motor cortex. The prediction involves again functional mapping, which has been performed by both T1 and IT2FS techniques introduced above. The predicted joint coordinates are now used to determine the angular movements required for different links of the robot, which is performed here by inverse kinematic technique used in robotics [56], [78]. A brief outline to the proposed inverse kinematic approach is given below for convenience of the non-specialist readers. The overall schematic is presented in Fig. 4.

Inverse Kinematics: Let the initial coordinate of joint J_i be $\overset{\rightarrow}{X_i} = (x_i, y_i, z_i)^T$ and its final coordinate after

rotation be $X'_i = (x'_i, y'_i, z'_i)^T$. Let ${}^{j-1}T_j$ be the transformation matrix for link j (= 1 to 3 here). Then

$$\begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix} = \begin{pmatrix} 0 T_1 \cdot T_2 \dots ^{j-1} T_i \end{pmatrix} \cdot \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix}$$

$$= {}^o A_j \cdot \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix}, \text{ say}$$

$$(18)$$

where, ${}^{o}A_{j} = {}^{0}T_{1} {}^{1}T_{2} {}^{...}{}^{j-1}T_{j}$.

Thus given $\begin{bmatrix} x_i, y_i, z_i \end{bmatrix}^T$, $\begin{bmatrix} x_i, y_i, z_i \end{bmatrix}^T$, and ${}^{0}T_1 \cdot {}^{1}T_2 \dots {}^{j-2}T_{j-1}$, we can find ${}^{j-1}T_j$ from the following

expression:

$$\begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix} = \begin{pmatrix} {}^{o}T_1 \cdot {}^{1}T_2 \dots {}^{j-2}T_{j-1} \end{pmatrix} \cdot \begin{pmatrix} {}^{j-1}T_j \end{pmatrix} \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix}$$

$$= {}^{o}A_{j-1} \cdot \begin{pmatrix} {}^{j-1}T_j \end{pmatrix} \cdot \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix}, \text{ say}$$

$$(19)$$

where ${}^{o}A_{j-1}$ is known. Computing ${}^{j-1}T_{j}$ and hence the angle of rotation θ_{i} around a given axis, which is expressed in the form of *Sin* θ_{i} and/or *Cos* θ_{i} in ${}^{j-1}T_{j}$, is the inverse kinematic problem [101]. The inverse kinematic problem is solved here stepwise starting from link 1. The general structure of solving the problem in the present context is given in Fig. 5(b).

III. EXPERIMENTS AND RESULTS

Principles of feature prediction of the parietal and motor cortex EEG from the measured EEG features of the occipital region, introduced earlier, are experimentally tested in this section. We here briefly outline the experimental set-up, followed by experimental steps and main results.

A. Experimental set-up

An EEG headset with 24 electrodes, manufactured by Nihon Kohden [57]-[58], Japan, has been employed

to extract EEG signals. Experiments are conducted on 10 right handed normal healthy individuals (4 female and 6 male of age group 40-55 years) as well as on 30 right handed disabled subjects. 30 diseased subjects included people suffering from six types of diseases/disability, namely isolated optic ataxia (OA) (with damaged in one side of the post parietal cortex), paraplegia (PG) (loss in motor control), Balint's syndrome (BS) (suffering from visual-motor coordination), paraparesis (PR) (partial loss in motor control), post stroke (PS) patients with damaged parietal/prefrontal or motor cortex and cerebral palsy (CP) (affects motor movement and muscle coordination). These diseased subjects are mainly suffering from impairments of frontal/ parietal and/motor cortex regions leading to visual, motor and visual-motor coordination. Five subjects belonging to each of these six disease/disability groups are chosen, thus comprising thirty subjects, including 17 male and 13 female in the age group 40-55years.

Subjects are engaged in a movement-related task-planning through visually inspired stimuli. EEG signals are acquired from the occipital (channels O1 and O2), parietal (channels P3 and P4), pre-frontal (FP1 and FP2) and motor cortex regions (channels C3 and C4) of subjects using the standard 10/20 EEG configuration [59]-[60]. A Kinect sensor system, manufactured by Microsoft Corporation, USA, is used in conjunction with the EEG system to measure the joint-coordinates in the right arm of the subjects throughout the experiments (Fig. 6(a)). The Kinect system includes two cameras, one in the visual wavelength and the other in the infra-red (IR) wavelength. The camera in the visual wavelength gives image information and the one in the IR wavelength gives depth information. Both the visual and depth information are jointly used to construct a skeleton of the human subject with positions of 20 joint coordinates of the subject within its field of view. We, however, use the 3D coordinates of wrist, elbow and shoulder of the subject only. The sampling rate of Nihon Kohden EEG machine is set at 500 Hz and that of Kinect sensor is 30Hz. A humanoid (JACO) robot arm [61]-[62] (manufactured by Kinova, USA) capable of mimicking one complete arm of a normal human being, is used to test/validate the predicted movement of subjects from his/her predicted motor cortex features.

Here, we used Event Related De-synchronization (ERD)/Event Related synchronization (ERS) [80], [82],

[95] modality of EEG. In our initial experiments, we considered power spectral density (PSD) [13], [63]-[65] Adaptive Autoregressive (AAR) parameters [66]-[69] and Daubechies-4 wavelet coefficient [70]-[73] features, but later discovered that only PSD features are a good choice, as AAR and wavelet coefficients features do not add any improvements in the results of final joint coordinate prediction.



Fig. 6 (a) The skeleton obtained by processing the Kinect output showing the corresponding joints (J1-J4), with the waist joint J0 is taken as reference (b) The JACO Robot Arm showing the corresponding joints (J1-J4), with the robot arm base J0 is taken as reference and the respective links (L1 - L3)

B. MF Construction

The experiment includes throwing a ping-pong ball toward a subject from a distance of 20 feet at a speed of 2feet/sec approximately, where the subject recognizes the stimulus, plans and executes the movement-related task to hit the ball with a bat held at his/her right arm. Only the successful instances, where hit occurs are considered here repeatedly over 10 epochs/day and over 10 days on each of the subjects to design the Type-1/IT2 membership functions of the acquired occipital, parietal, pre-frontal, and motor cortex EEG features of each subject.

C. Testing Phase

During the testing phase, the subject observes the ball movement and attempts to plan the movement of his/her arm to hit the ball. The principles employed for prediction of Type-1/IT2 pre-frontal, parietal and motor cortex features from the measured occipital features of each subject are used to determine their

motor cortex features. The overall system introduced in Fig. 4 is invoked to determine the joint coordinates

of shoulder, elbow, and wrist of subject from the predicted features of motor cortex of the same subject.

	PERCENTAGE OF NORMALIZED POSITIONAL LINK ERROR FOR NORMAL AND DISEASED SUBJECTS								
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	Subject ID	Percentage of	f Normalized Positional	Link error for	Time taken for				
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$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	D-Diseased)		(%)	(sec)					
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	Link No.	L1	L2	L3	L1	L2	L3		
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	1-N	6.1	3.4	5.01	2.5	2.0	3.2		
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$		(7.2)	(3.8)	(6.2)	(4.0)	(3.4)	(5.4)		
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	2-N	7.0	3.25	3.40	1.8	2.2	4.3		
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$		(8.4)	(5.2)	(5.6)	(2.6)	(3.1)	(5.1)		
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	3-N	7.80	3.16	3.60	2.2	1.9	3.8		
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$		(9.4)	(6.4)	(6.3)	(2.4)	(2.7)	(3.1)		
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	4-N	7.20	3.45	3.75	2.0	2.8	3.1		
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$		(9.1)	(4.9)	(4.2)	(2.5)	(3.2)	(2.5)		
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	5-N	7.45	3.84	4.02	1.7	2.0	2.7		
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$		(8.6)	(4.8)	(5.7)	(2.1)	(2.6)	(2.9)		
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	6-N	3.4	4.7	3.84	2.4	2.6	3.3		
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$		(5.6)	(7.5)	(6.8)	(3.0)	(2.9)	(3.9)		
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	7-N	6.35	5.27	4.78	2.2	2.0	34		
8-N 7.58 5.2 5.8 2.7 2.9 3.3 9-N 7.2 3.5 4.3 2.1 2.8 3.3 (7.7) (4.6) (6.8) (2.5) (3.4) (3.8) 10-N 7.7 (4.6) (6.8) (2.5) (3.4) (3.8) 10-N 7.7 (4.8) (4.5) 1.9 2.6 3.2 (6.2) (7.7) (4.6) (6.2) (7.9) (2.3) (3.1) (3.5) 1.0 173.17.6.16.8, 14.5,12.6,13.5, 13.9,13.4,14.6, 4.6,4.9 4.7,5.1 6.1,6.4 (0A) (15.9,17.1) 14.7,14.4 14.2,15.2 4.6,4.9 4.7,5.1 6.1,6.4 (0A) (17.4),(17.9) (15.1),(14.9) (14.8),(15.9) (5.3),(5.7) (5.3),(6.1) (6.6),(6.8) (17.4),(17.9) (15.1),(17.1),(17.8),(23.4,(25.6),(21.5),(7.1),(6.3),(6.9,(7.4),(8.3),(8.6),(8.0),(2.8),(24.2),(23.8),(27.4),(18.2),(17.7),(17.8),(23.4),(25.6),(21.5),(7.1),(6.5),(5.8),(6.5),(7.4),(7.4),(8.3),(8.6),(8.0),(2.8),(2.4),(2.3),(2.5),(2.5),(2.5),(7.1),(6.3),(5.8),(6.5),(7.2),(7.4),(8.8),(8.7),(8.8) <tr< td=""><td>, 11</td><td>(8.1)</td><td>(10.4)</td><td>(7.5)</td><td>(2.8)</td><td>(2.7)</td><td>(4.7)</td></tr<>	, 11	(8.1)	(10.4)	(7.5)	(2.8)	(2.7)	(4.7)		
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	8-N	7.58	5.2	5.8	2.7	2.9	3.3		
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	011	(8.1)	(6.8)	(7.2)	(3.2)	(3.6)	(4.1)		
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	9-N	7.2	3.5	43	21	2.8	33		
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$		(7.7)	(4.6)	(6.8)	(2.5)	(3.4)	(3.8)		
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	10-N	77	4.8	4 5	1.9	26	3.2		
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	10 10	(8.3)	(6.2)	(7.9)	(2 3)	(3.1)	(3.5)		
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$		173 176 168	14 5 12 6 13 5	139 134 146	464138	524854	61 63 56		
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	1-D	16.9.17.1	14.5, 12.0, 15.5, 14.7, 14.7, 14.4	14.2 15.2	46.49	4751	61 64		
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	(OA)	(185) (182) (177)	(15.7) (13.3) (14.5)	(14.6) (13.7) (15.1)	(52)(47)(43)	(5.9) (5.3) (5.8)	(64) (67) (58)		
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$		(174), (17.9)	(15.1), (14.9)	(14.8), (15.9)	(5.2), (117), (115), (115), (5.3), (5.7)	(5.3), (6.1)	(6.6), (6.8)		
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$		22.5.21.0.23.6	167 164 171	20.0.20.2.18.3	64 56 67	736669	798377		
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	2-D	23.8.21.6	17.8.16.9	21.4, 19.6	69.51	7.1.74	8.2.8.5		
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	(PG)	(24.2), (23.8), (27.4).	(18.2), (17.9), (17.8)	(234) (256) (215)	(7.1) (6.3) (6.9)	(8.5) (6.9) (7.4)	(8.3) (8.6) (8.0)		
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	(10)	(28.1), (23.7)	(18.4), (17.3)	(26.3), (23.5)	(7.4), (5.7)	(8.2), (8.8)	(8.7), (8.8)		
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$		12.7.12.3.13.4	9.5.9.7.10.2	96.91.98	49.48.5.3	65.69.71	86.88.7.7		
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	3-D	12.8.13.1	8.9. 9.4	8.7. 10.1	5.7. 4.7	6.1. 6.8	7.4. 8.2		
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	(BS)	(14.0), (13.6), (14.2).	(11.3), (10.4), (11.1),	(11.2) (9.7) (10.4)	(5.2) (5.3) (5.9)	(7.0) (7.4) (8.2)	(9.1) (9.6) (8.5)		
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	(22)	(13.5), (13.9)	(9.7), (9.9)	(9.3) (10.8)	(6.5), (5.8)	(6.5), (7.2)	(7.8) (8.8)		
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		27.9.24.5.26.1	20.5.21.8.19.5	23.6.24.5.22.7	79.66.74	76.72.78	80.83.7.9		
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	4-D	28.4. 28.0	17.8.18.4	27.7.25.6	81.67	81.74	81.86		
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	(PR)	(29.1), (29.5), (30.2),	(24.7), (23.3), (21.6),	(28.6), (29.4), (25.4),	(8.6), (6.8), (7.9),	(8.3), (7.0), (8.3),	(8.9), (9.1), (8.4),		
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	(/	(33.8), (24.7)	(19.4), (20.7)	(30.5), (27.9)	(8.7), (7.2)	(8.6), (7.9)	(8.7), (8.9)		
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		26.0. 23.4. 27.8.	25.3. 26.8. 23.6.	24.7. 26.1. 27.3.	6.5, 6.3, 6.1,	6.9. 7.2. 7.6.	7.2, 7.4, 7.1,		
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	5-D	26.0, 25.5	22.5, 26.1	22.4, 25.6	6.8, 6.6	6.4. 6.2	6.9. 7.5		
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	(PS)	(29,3), (25.6), (30.3)	(28.3), (29.0), (28.1)	(29.3), (28.2), (31.4)	(7,4), (6,7), (6,4)	(7.6), (7.7), (7.9)	(7.9), (7.7), (7.6)		
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	(- ~)	(27.3), (29.3)	(25.2), (29.7)	(26.7), (27.1)	(7.4), (7.0)	(6.6), (6.5)	(7.3), (8.1)		
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$		18.8. 19.2. 21. 5	20.4. 22.9. 18.3	16.7. 17.2. 15.4	7.7.8.3.7.4	7.9. 7.2. 8.5	8.2. 8.6. 8.9		
(CP) (20.2), (23.6), (24.7), (22.2), (24.7), (20.5), (18.5), (19.2), (17.4), (8.1), (8.7), (7.6), (8.8), (7.3), (8.9), (9.1), (8.9), (9.6), (18.2), (23.6), (22.3), (26.3) (16.3), (15.1) (7.2), (7.8) (8.4), (8.1) (8.6), (8.9)	6-D	17.6. 20.4	19.4, 22.6	15.7, 14.8	6.9, 7.5	8.2. 7.7	7.9. 8.4		
(18.2), (23.6) (22.3), (26.3) (16.3), (15.1) (7.2), (7.8) (8.4), (8.1) (8.6), (8.9)	(CP)	(20.2), (23.6), (24.7).	(22.2), (24.7), (20.5).	(18.5), (19.2), (17.4).	(8.1), (8.7), (7.6).	(8.8), (7.3), (8.9).	(9.1), (8.9), (9.6).		
	()	(18.2), (23.6)	(22.3), (26.3)	(16.3), (15.1)	(7.2), (7.8)	(8.4), (8.1)	(8.6), (8.9)		

TABLE I

TABLE IIA (ANWESHA: CITE THIS IN TEXT)

COMPARISON OF PERFORMANCE IN TERMS OF HIT RATE FOR FOUR CASES: (A) WITHOUT AID, B) WHILE MAPPING IS PERFORMED FROM PREFRONTAL FEATURES TO JOINT COORDINATES, (C) PLANNING DIRECTLY FROM MOTOR CORTEX AND, (D) WHILE MAPPING IS PERFORMED FROM OCCIPITAL TO PREFRONTAL/PARIETAL FEATURES TO MOTOR CORTEX FEATURES TO JOINT COORDINATES

	Hit rate (%)							
Subject	While patients are directly involved to hit the ball	While mapping is performed from prefrontal features to joint coordinates	While mapping is performed from motor cortex features to joint coordinates	While mapping is performed from occipital to prefrontal/parietal features to motor cortex features to joint coordinates	While mapping joint coordinates directly from compromised parietal and motor cortex features			
Normal	85.4	82.3	84.3	80.2	NA			
1-D (OA)	21.2	26.3	40.2	53.8	24.1			
2-D (PG)	18.4	22.1	38.1	51.4	21.8			
3-D (BS)	11.3	17.4	32.4	48.1	15.2			
4-D (PR)	35.4	29.2	54.4	70.4	26.2			
5-D (PS)	20.2	26.4	39.2	52.3	23.8			
6-D (CP)	24.4	32.1	43.1	57.2	29.3			

TABLE IIB (ANWESHA: CITE THIS IN TEXT) RUN TIME COMPLEXITY OF EXISTING AND PROPOSED TECHNIQUES

	Run time complexity					
Mapping Algorithms	While mapping is performed from prefrontal features directly to joint coordinates (in Sec)	While mapping is performed from motor cortex features to joint coordinates (in Sec)	While mapping is performed from occipital to prefrontal/parietal features to motor cortex features to joint coordinates (in Sec)	While mapping joint coordinates directly from compromised parietal and motor cortex features (in Sec)		
T1FS	4.2	2.4	7.6	6.2		
IT2FS	6.4	3.6	9.2	7.3		
BPNN	5.6	4.4	8.4	8.2		



Fig. Hit rate (%) variations over the increases number of sessions for all the six groups of disabled subjects i.e. 1-D (OA), 2-D(PG), 3-D(BS), 4-D(PR), 5-D(PS) and 6-D(CP) depicted for the various mapping algorithms. Blue, green and red lines represents BPNN, T1FS and IT2FS respectively.



Fig. . Hit rate (%) variations over the number of sessions for the three different mapping algorithms (a) BPNN, (b) T1FS and (c) IT2FS.

TABLE III

COMPARISON OF TIME TAKEN FOR HIT FOR FOUR CASES: (A) WITHOUT AID, (B) WHILE MAPPING IS PERFORMED FROM PREFRONTAL FEATURES TO JOINT COORDINATES, (C) PLANNING DIRECTLY FROM MOTOR CORTEX AND, (D) WHILE MAPPING IS PERFORMED FROM OCCIPITAL TO PREFRONTAL/PARIETAL FEATURES TO MOTOR CORTEX FEATURES TO JOINT COORDINATES

	Time taken to hit the ball from the onset of throw for successful hits (in sec)					
Subject	While patients are directly involved to hit/miss the ball	While mapping is performed from prefrontal features to joint coordinates	While mapping is performed from motor cortex features to joint coordinates	While mapping is performed from occipital to prefrontal/parietal features to motor cortex features to joint coordinates		
Normal	40.2 (hit)	40.3	40.8	41.6		
1-D	42.2(hit)	42.12 (hit)	42.12 (hit)	42.12 (hit)		
(OA)	Miss	42.12(hit)	42.12(hit)	42.13 (hit)		
2-D	42.4 (hit)	42.21(hit)	42.21(hit)	42.21 (hit)		

(PG)	miss	42.21 (hit)	42.21 (hit)	42.21 (hit)
3-D	43.4(hit)	43.13(hit)	43.13(hit)	43.13(hit)
(BS)	Miss	43.13(hit)	43.13(hit)	43.13(hit)
4-D	41.3(hit)	41.02 (Hit)	41.02 (Hit)	41.02 (Hit)
(PR)	Miss	41.02 (Hit)	41.02 (Hit)	41.02 (Hit)
5-D	42.3 (hit)	42.14(hit)	42.14(hit)	42.14(hit)
(PS)	Miss	42.14(hit)	42.14(hit)	42.14(hit)
6-D	42.1 (hit)	41.73 (hit)	41.73 (hit)	41.73 (hit)
(CP)	Miss	41.73 (hit)	41.73 (hit)	41.73 (hit)

The JACO robot arm is then commanded to turn its links accordingly to reach the desired position of the ball to hit it. The desired coordinate of the end-points of link 1, link 2 and link 3 of JACO robot arm (Fig. 7) are determined by inverse kinematics [100]. The coordinates of the actual end points of the robot are also determined through measurements. The normalized positional error (NPE), defined by actual positional error (Euclidean distance between desired and actual link end point coordinates) committed, divided by the corresponding link length, is measured. Table-I provides the results of percentage of normalized positional errors for the end-points of three links (Link-1: upper arm, Link-2 lower arm, and Link-3: palm) of 10 normal subjects and six diseased groups including five subjects of each group are given in sequence and the time taken for each link to align itself from initial to the goal position. The percentage calculation is done by multiplying the normalized positional error by 100. It is clear from the Table that IT2FS NPEs are relatively smaller than type-1 fuzzy logic based systems, in all cases. It is also observed that the positional errors and time taken by diseased subjects are more than the normal subjects and IT2FS yielded better results than its Type-1 counterpart. Table II and III provides the hit rate (i.e., the number of hits to the ball by the bat divided by a sum of the number of hits and misses) and time taken to hit/miss the ball respectively for four situations i.e., while patients are directly involved to hit the ball, while mapping is performed directly from prefrontal features to joint coordinates, while mapping is performed directly from motor cortex features to joint coordinates, and while mapping is performed from motor cortex features to joint coordinates. It is observed that the hit rates for disabled subjects are maximum (increased by around 35% on an average with respect to direct hitting of ball by individuals without aid) when the robot arm is controlled by the joint coordinated predicted from occipital to parietal/prefrontal to motor cortex features. The time taken is also minimized in the proposed prediction technique of mapping joint coordination from

IV. PERFORMANCE ANALYSIS

The performance of the proposed system is analyzed with respect to two viewpoints. First, we compare the *mean-square error* in the predicted features for type-1 fuzzy, IT2FS and neural [94], [96] (back-propagation algorithm) [74]-[77] realization (see Fig. 7 and Fig. 8), where the mean square error (MSE) is defined as

$$MSE = \frac{1}{n} \sum_{i=1}^{n} (t_i - c_i)^2.$$
 (20)

Here, $t_i = i^{th}$ target parietal (or motor cortex) EEG feature of a given subject *s*, and $c_i = i^{th}$ computed parietal (or motor cortex) EEG feature of the same subject *s*. It is observed from TABLE-IV that the mean square error for parietal, prefrontal and motor cortex regions by the IT2FS realization outperforms its competitors for all subjects. The average time taken for feature prediction by T1FS, IT2FS and BPNN is 0.06ms, 0.078 ms and 0.085 ms respectively.

Second, we compare the performance of the three mapping techniques in presence of noise. Since EEG signals are often contaminated with noise due to involuntary eye/head movements, poor signal ambience, cognitively induced noise (such as, parallel undesirable thoughts by the subjects), examining the performance of the algorithm in presence of noise is very important.



Subject, P2



Fig. 7.Training phase of feature mapping using neural nets (NN)

Fig. 8. Experimental testing phase of feature mapping using Fuzzy/neural technique

PERFORMANCE OF IT2FS (T1FS) IN PREDICTING THE FEATURES OF NORMAL SUBJECTS							
Subject ID (N-Normal, D-Diseased)	Mapping Technique Used	Parietal Feature Error (MSE)	Prefrontal Feature Error (MES)	Motor Cortex Feature Error (MSE)			
	T1FS	0.3287	0.2957	0.5536			
Normal	IT2FS	0.0522	0.0496	0.0587			
	BPNN	0.4671	0.3701	1.2513			
1 D	T1FS	0.8534	0.9234	1.0102			
1-D (OA)	IT2FS	0.0831	0.0910	0.0923			
(OA)	BPNN	0.8923	0.8843	0.9774			
2.0	T1FS	0.6572	0.6874	0.7123			
2-D (DC)	IT2FS	0.0781	0.0710	0.7881			
(PG)	BPNN	0.7124	0.7754	0.9512			
2.D	T1FS	0.6682	0.7122	0.8874			
3-D (DS)	IT2FS	0.0643	0.0711	0.0831			
(D3)	BPNN	0.7681	0.7982	1.0233			
4.D	T1FS	0.7183	0.7451	0.8921			
4-D (DD)	IT2FS	0.0832	0.0883	0.8921			
(PK)	BPNN	0.8674	0.9163	1.1280			
5 D	T1FS	0.7013	0.7364	0.7524			
5-D (DS)	IT2FS	0.0683	0.0712	0.0823			
(13)	BPNN	0.7714	0.7983	1.0702			
6 D	T1FS	0.6912	0.7284	0.7731			
0-D	IT2FS	0.0834	0.0881	0.0953			
(CP)	BPNN	0.7274	0.0894	1.0241			

TABLE IV PERFORMANCE OF IT2FS (T1FS) IN PREDICTING THE FEATURES OF NORMAL SUBJECTS

TABLE V

 PERFORMANCE OF T1FS, IT2FS AND BPNN IN NOISY ENVIRONMENT FOR PREDICTING THE FEATURES OF NORMAL/DISEASED INDIVIDUAL

Subjects	Additive Gaussian noise with mean=0 and variance=	Mapping Technique	Positional Link error (%)			Time taken for IT2FS (sec)		
			L1	L2	L2	L1	L2	L3
		IT2FS	18.14	22.00	25.28	2.81	2.98	3.31
	0.5	T1FS	27.25	24.00	29.35	3.09	2.82	3.42
		BPNN	30.51	28.70	32.40	2.65	2.83	3.20
Normal	2	IT2FS	16.08	20.60	19.05	3.44	3.67	3.81
		T1FS	22.45	21.76	23.31	3.23	3.32	3.72
		BPNN	20.87	23.97	26.48	3.02	3.18	3.91
	4	IT2FS	8.23	8.55	8.88	2.71	2.95	2.90
		T1FS	10.56	11.72	10.05	2.98	3.03	3.43
		BPNN	13.44	12.00	11.28	3.28	3.83	3.04
		IT2FS	32.21	37.52	43.40	6.10	7.31	7.52
	0.5	T1FS	37.61	38.22	47.91	6.51	6.72	7.81
		BPNN	43.20	42.51	47.52	6.52	7.61	7.52
Diseased		IT2FS	25.12	27.32	32.51	7.31	7.91	8.32
	2	T1FS	28.62	29.01	30.02	7.52	7.82	7.81
		BPNN	29.51	31.00	34.31	7.61	7.61	8.41
		IT2FS	18.22	21.22	25.11	8.52	8.82	9.32
	4	T1FS	26.21	28.61	29.12	8.61	8.71	9.51
		BPNN	28.42	30.12	33.70	8.90	9.82	9.82

We here examined the effect of noise on the performance of the three realizations (algorithms) by measuring percentage of positional link errors, when the measured occipital features are induced with Gaussian noise of zero mean and varying standard deviation. TABLE-V provides the results in positional link errors when noise variance is set to 0.5, 2.0, and 4.0 which are averaged over all 10 normal and 30 diseases subjects respectively. It is observed from the TABLE-V that in all situations IT2FS outperforms T1 FS and back-propagation algorithm in positional link errors for all the three links. The results given in TABLE-V thus confirm that the proposed IT2FS realization carries a significant merit in bypassing the neural pathways for diseased subjects, and thus has immense scope for practical realization as the next generation rehabilitative aid. Unfortunately, the nicety of IT2FS in EEG feature prediction has not been explored in the current literature.

V. DISCUSSION

The objective of the present work is to bypass the damaged/partially damaged regions/lobes in the human brain by artificial means with a motivation to develop rehabilitative aids to people suffering from visualmotor coordination problems. An IT2FS based mapping strategy has been incorporated to indirectly utilize the lost coordination between any two successive brain modules in the signaling pathways [99] used for visual-motor coordination. Any traditional mapping techniques, including regression, neuro-computational and the like could have been used to solve the present problem. However, the choice of IT2FS is induced by the additional merits of fuzzy sets in general and IT2FS in special to eliminate the effect of noise that may enter into the acquired EEG signals (due to thoughts other than the targeted task) from selected channels.

Experiments are performed with 30 diseased individuals with partial damage in parietal and/or motor cortex regions, where these patients are asked to hit a ball thrown from a distance of 20 feet at low speed (2 feet/second approximately). The motivation of the experiment is to study the normal coordination in their brain between each two modules lying on the signaling pathways used to perform visual-motor coordination. It is observed that success rate in hit is only 35% on an average when experimented with subjects with partial parietal and or motor cortex failures. The experiment, however, gave a success rate over 84% when performed with 10 healthy subjects. The high failure rate in hit by the patients with partial parietal, prefrontal and/or motor cortex inspired us to generate motor cortex features from the occipital features of these subjects by an IT2FS based mapping from occipital features. The mapping is developed from successful trials of the subjects, i.e., when they could hit the ball properly.

The last part of the experimental set-up is developed to engage a robot to hit the ball for a patient. While the patient engages himself in watching the throw and trajectory of the ball, the occipital EEG signals are acquired and relevant features are extracted and then using the mapping policy introduced above the motor cortex features are extracted. Finally, one more mapping is required to determine the joint coordinates of the robot holding a bat to orient itself properly to hit the flying ball. The mapping is developed with the measured parietal features and joint coordinates of the right hand of the subject, obtained from a Kinect sensor. We here used only three joint coordinates (shoulder, elbow and wrist) to orient the bat properly to hit the ball. Although a better arrangement could be the orientation of the palm (to control the in- and outswings of the palm), the present Kinect based scheme, however, cannot serve the problem.

It is interesting to note that the artificial mapping used to generate joint coordinates directly from the occipital region yields less link error and execution time in comparison to the link errors obtained while mapping from prefrontal region, motor cortex to joint coordinates or direct playing by the diseased subjects. Consequently, the mapping of joint coordinates from occipital to prefrontal and parietal to motor cortex features increases the hit rate by additional 35% with respect to the hit rate when subjects are directly involved in hitting the ball. On the other hand, the hit rate merely increases by 5 % and 20 % when mapping is performed directly from prefrontal and motor cortex to joint coordinates respectively with respect to subjects' direct hitting of the ball without aid. [Anwesha: Please add comparisons with some References, indicating them in [] box. This is very important.]

VI. CONCLUSION

Sensory-motor coordination remained an open area of active research for the next generation BCI applications. This paper introduces a novel approach to visual-motor coordination with a possible emphasis to rehabilitate patients with partial failure in such coordination. Considering the well-known functional architecture of the brain, this paper attempts to develop an artificial mapping between the features generated from two active brain regions/lobes during the execution of visual-motor coordination phase. The conventional functionality of occipital, parietal, prefrontal and motor-cortex regions in visual signal processing, planning and decision making and motor execution respectively is presumed, and a mapping of the responses from occipital to parietal and prefrontal, and next prefrontal/parietal to motor cortex is developed using Type-1 fuzzy, IT2FS and neural techniques.

IT2FS being more robust to noise, in comparison to its Type-1 counterpart and neural networks, has been selected to perform the mapping. Experiments undertaken reveal that the IT2FS based mapping yields relatively small positional link error and faster speed to its competitors (Type1 fuzzy sets and BPNN).

Neural back-propagation scheme, which has wider application in function approximation has been used here as a reference model to supplement the proposed fuzzy techniques. However, experiments undertaken confirm that although the neural approach has comparable performance in occipital to parietal/motor cortex feature mapping, it performs poorly in presence of noise in the EEG features. As infiltration of noise in EEG cannot be prevented, the proposed Type-2 fuzzy mapping technique seems to have immense importance with respect to neural and type-1 fuzzy mapping techniques. Experiments undertaken also confirm the above results.

Experimental results further reveal that direct mapping of prefrontal and motor cortex features to joint coordinates results in a miss for a maximum of 83% and 68% respectively for diseased subjects, which, however, can be reduced to 48% by an automatic mapping from occipital features to parietal/prefrontal features to motor cortex to joint coordinates. Here lies the importance of the proposed technique. The principles adopted in the paper can be used in the next generation rehabilitative aids for people with partial visual-motor coordination impairment.

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References

- [1] H. Dandan, Q. Kai, F. Ding-Yu, J. Wenchuan, C. Xuedong and B. Ou, "Electroencephalography (EEG)-Based Brain Computer Interface (BCI): A 2-D Virtual Wheelchair Control Based on Event-Related Desynchronization/Synchronization and State Control," *IEEE Trans. Neural Systems and Rehabilitation Engineering*, vol. 20, pp. 379-388, 2012.
- [2] A. Frisoli, C. Loconsole, D. Leonardis, F. Banno, M. Barsotti, C. Chisari and M. Bergamasco, "A New Gaze-BCI-Driven Control of an Upper Limb Exoskeleton for Rehabilitation in Real-World Tasks," *IEEE Trans. Systems, Man, and Cybernetics, Part C: Applications and Reviews*, vol. 42, pp. 1169-1179, 2012.

- [3] C. Escolano, J.M. Antelis and J. Minguez, "A Telepresence Mobile Robot Controlled With a Noninvasive Brain Computer Interface," *IEEE Trans. Systems, Man, and Cybernetics, Part B: Cybernetics*, vol. 42, pp. 793-804, 2012
- [4] Y. Takata, M. Saeki, J. Izawa, K. Takeda, Y. Otaka, K. Ito and T. Kondo, "Analysis of key factors on ERD production for BCI neuro-robotic rehabilitation," in 4th IEEE RAS & EMBS International Conference on Biomedical Robotics and Biomechatronics (BioRob), pp. 240-245, 2012.
- [5] M. Sarac, E. Koyas, A. Erdogan, M. Cetin and V. Patoglu, "Brain Computer Interface based robotic rehabilitation with online modification of task speed," in 2013 IEEE International Conference on Rehabilitation Robotics (ICORR), pp. 1-7, 2013.
- [6] J.N. Kornegay, "Alzheimer's Paraparesis (paraplegia), tetraparesis (tetraplegia), urinary/fecal incontinence. Spinal cord diseases," in *Probl Vet. Med.*, vol. 3 (3), pp. 363-377, 1991.
- [7] B.B. Johansson, "Brain plasticity and stroke rehabilitation," in Stroke, vol. 31, pp. 2230-230, 2000.
- [8] M.T. Perenin and A. Vighetto, "Optic ataxia: a specific disruption in visual-motor mechanisms. Different aspects of the deficit in reaching for objects," in *Brain*, vol. 111(3), pp. 643-674, 1988.
- [9] E.J. Hwang, M. Hauschild, M. Wilkie and R.A. Andersen, "Inactivation of parietal reach region causes optic ataxia, impairing reaches but not saccades," in *Neuron*, Elsevier, vol. 76, pp. 1021-1029, 2012.
- [10] M. Rizzo, "Balint's syndrome and associated visual-spatial disorders," in *Baillieres Clinical Neurology*, vol. 2(2), pp. 415-437, 1993.
- [11] A. Hasan, T. Bergener, M.A. Nitsche, W. Strube, T. Bunse, P. Falkai and T. Wobrock, "Impairment of motor cortex responses to unilateral and bilateral direct current stimulation in schizophrenia," in *Frontiers in Psychiatry*, vol. 4, 2013.
- [12] S. Bhattacharyya, A. Sengupta, T. Chakraborti, A. Konar and D.N. Tibarewala, "Automatic feature selection of motor imagery EEG signals using differential evolution and learning automata," *Medical & Biological Engineering & Computing, Springer*, pp. 1-9, 2013.
- [13] A. Khasnobish, S. Bhattacharyya, A. Konar, D.N. Tibarewala and A.K. Nagar, "A Two-fold classification for composite decision about localized arm movement from EEG by SVM and QDA techniques," in *The 2011 International Joint Conference on Neural Networks (IJCNN)*, pp. 1344-1351, 2011.
- [14] T. M. Rutkowski, A. Cichocki, T. Tanaka, D.P. Mandic, C. Jianting and A.L. Ralescu, "Multichannel spectral pattern separation - An EEG processing application," in *IEEE International Conference on Acoustics, Speech and Signal Processing, 2009 (ICASSP)*, pp. 373-376, 2009.
- [15] N. Hiroshi and A. L. Ralescu, "Implementation of fuzzy systems using multilayered neural network," in *IEEE International Conference on Neural Networks*, vol.1, pp. 317-322, 1993.
- [16] S. Visa and A. Ralescu, "A fuzzy classifier for imbalanced and noisy data," in *IEEE International Conference on Fuzzy Systems*, vol.3, pp. 1727-1732, 2004.
- [17] S. Visa and A. Ralescu, "Towards online learning of a fuzzy classifier," in Annual Meeting of the North American Fuzzy Information Processing Society, NAFIPS 2005, pp. 557-561, 2005.

- [18] J. M. Garibaldi and T. Ozen, "Uncertain Fuzzy Reasoning: A Case Study in Modelling Expert Decision Making," *IEEE Trans. Fuzzy Systems*, vol. 15, pp. 16-30, 2007.
- [19] A. B. Cara, C. Wagner, H. Hagras, H. Pomares and I. Rojas, "Multiobjective Optimization and Comparison of Nonsingleton Type-1 and Singleton Interval Type-2 Fuzzy Logic Systems," *IEEE Trans. Fuzzy Systems*, vol. 21, pp. 459-476, 2013.
- [20] R. A. Francelin and F. A. C. Gomide, "A neural network for fuzzy decision making problems," in Second IEEE International Conference on Fuzzy Systems, vol. 1, pp. 655-660, 1993.
- [21] F. J. Lin, S. Y. Chen, P. H. Chou, and P. H. Shieh, "Interval Type-2 Fuzzy Neural Network Control for X-Y-Theta Motion Control Stage Using Linear Ultrasonic Motors," *Neurocomputing*, vol. 72, pp. 1138-1151, 2008.
- [22] A. Lemos, W. Caminhas and F. Gomide, "Evolving fuzzy linear regression trees," in IEEE International Conference on Fuzzy Systems (FUZZ), pp. 1-8, 2010.
- [23] L. Faa-Jeng, L. Chih-Hong and S. Po-Hung, "Self-constructing fuzzy neural network speed controller for permanent-magnet synchronous motor drive," *IEEE Trans. Fuzzy Systems*, vol. 9, pp. 751-759, 2001.
- [24] G. Nakamiti, R. Freitas, J. Prado and F. Gomide, "Fuzzy distributed artificial intelligence systems," in Fuzzy Systems, *IEEE World Congress on Computational Intelligence.*, vol. 1, pp. 462-467, 1994.
- [25] B. Garg, M.M.S. Beg and A.Q. Ansari, "Fuzzy time series model to forecast rice production," in *IEEE International Conference on Fuzzy Systems (FUZZ)*, pp. 1-8, 2013.
- [26] L. A. Zadeh, "The concept of a linguistic variable and its application to approximate reasoning-I," *Inform. Sci.*, vol. 8, pp. 199–249, 1975.
- [27] J. M. Mendel and R. B. John, "Type-2 fuzzy sets made simple," IEEE Trans. Fuzzy Systems, vol. 10, pp. 117-127, 2002.
- [28] N. N. Karnik and J. M. Mendel, "Introduction to type-2 fuzzy logic systems," in *IEEE World Congress on Computational Intelligence.*, pp. 915-920, 1998.
- [29] L. Qilian and J. M. Mendel, "Interval type-2 fuzzy logic systems: theory and design," *IEEE Trans. Fuzzy Systems*, vol. 8, pp. 535-550, 2000.
- [30] W. Hongwei and J. M. Mendel, "Uncertainty bounds and their use in the design of interval type-2 fuzzy logic systems," *IEEE Trans. Fuzzy Systems*, vol. 10, pp. 622-639, 2002.
- [31] J. M. Mendel and W. Hongwei, "Type-2 Fuzzistics for Symmetric Interval Type-2 Fuzzy Sets: Part 1, Forward Problems," *IEEE Trans. Fuzzy Systems*, vol. 14, pp. 781-792, 2006.
- [32] J. M. Mendel and W. Hongwei, "Type-2 Fuzzistics for Symmetric Interval Type-2 Fuzzy Sets: Part 2, Inverse Problems," *IEEE Trans. Fuzzy Systems*, vol. 15, pp. 301-308, 2007.
- [33] J. M. Mendel and R. I. John, "Type-2 fuzzy sets made simple," *IEEE Trans. on Fuzzy Systems*, vol. 10, pp. 117–127, Apr. 2002.
- [34] J. M. Mendel, "Type-2 Fuzzy Sets and Systems: An Overview," *Computational Intelligence Magazine*, *IEEE*, vol. 2, pp. 20-29, 2007.

- [35] Q. Liang, N. N. Karnik, and J. M. Mendel, "Connection admission control in ATM networks using survey-based type-2 fuzzy logic systems," *IEEE Trans. Syst., Man, Cybern. C*, vol. 30, pp. 329–339, Aug. 2000.
- [36] L. Qilian and J. M. Mendel, "Equalization of nonlinear time-varying channels using type-2 fuzzy adaptive filters," *IEEE Trans. Fuzzy Systems, on,* vol. 8, pp. 551-563, 2000.
- [37] J. M. Mendel, "Fuzzy sets for words: A new beginning," in Proc. Of IEEE Int'l. Conf. on Fuzzy Systems, St. Louis, MO, pp. 37–42, May 2003.
- [38] H. Hagras, "A hierarchical type-2 fuzzy logic control architecture for autonomous mobile robots," *IEEE Trans. Fuzzy Syst.*, vol. 12, no. 4, pp. 524–539, Aug. 2004.
- [39] H. Hagras, "Type-2 FLCs: A new generation of fuzzy controllers," *IEEE Comput. Intell. Magazine*, vol. 2, no. 1, pp. 30–43, Feb. 2007.
- [40] D. Wu and J. M. Mendel, "A vector similarity measure for linguistic approximation: Interval type-2 and type-1 fuzzy sets," *Information Sciences*, vol. 178, pp. 381-402, 2008.
- [41] W. Dongrui, "On the Fundamental Differences between Interval Type-2 and Type-1 Fuzzy Logic Controllers," *IEEE Trans. Fuzzy Systems*, vol. 20, pp. 832-848, 2012.
- [42] T. Bilgic and I. B. Turksen, "Measurement of membership functions: Theoretical and empirical work," in *Handbook of Fuzzy Systems: Foundations*, D. Dubois and H. Prade, Eds. Boston, MA: Kluwer, vol. 1, pp. 195–228, 2000.
- [43] A. Doumanoglou, S. Asteriadis, D.S. Alexiadis, D. S., D. Zarpalas and P. Daras, "A dataset of Kinect-based 3D scans," in IEEE 11th IVMSP Workshop, 2013, pp. 1-4, 2013.
- [44] M. Draelos, N. Deshpande and E. Grant, "The Kinect up close: Adaptations for short-range imaging," in IEEE Conference on *Multisensor Fusion and Integration for Intelligent Systems (MFI)*, pp. 251-256, 2012.
- [45] H. Jungong, S. Ling, X. Dong and J. Shotton, "Enhanced Computer Vision with Microsoft Kinect Sensor: A Review," IEEE Trans. Cybernetics, vol. 43, pp. 1318-1334, 2013.
- [46] E. Machida, C. Meifen, T. Murao and H. Hashimoto, "Human motion tracking of mobile robot with Kinect 3D sensor," in SICE Annual Conference (SICE), pp. 2207-2211, 2012.
- [47] A. K. Roy, Y. Soni and S. Dube, "Enhancing effectiveness of motor rehabilitation using kinect motion sensing technology," in *IEEE Global Humanitarian Technology Conference: South Asia Satellite (GHTC-SAS)*, pp. 298-304, 2013.
- [48] G. Tao, P.S. Archambault and M.F. Levin, "Evaluation of Kinect skeletal tracking in a virtual reality rehabilitation system for upper limb hemiparesis," in *Virtual Rehabilitation (ICVR), 2013 International Conference on*, pp. 164-165, 2013.
- [49] R. Zhou, Y. Junsong, M. Jingjing and Z. Zhengyou, "Robust Part-Based Hand Gesture Recognition Using Kinect Sensor," *IEEE Trans. Multimedia*, vol. 15, pp. 1110-1120, 2013.
- [50] N. N. Karnik and J. M. Mendel, "Centroid of a type-2 fuzzy set," Inf. Sci., vol. 132, pp. 195–220, 2001.
- [51]J. M. Mendel, Rule-Based Fuzzy Logic Systems: Introduction and New Directions. Upper Saddle River, NJ: Prentice–Hall, 2001.

- [52] J. M. Mendel and L. Feilong, "Super-Exponential Convergence of the Karnik-Mendel Algorithms for Computing the Centroid of an Interval Type-2 Fuzzy Set," *IEEE Trans. Fuzzy Systems*, vol. 15, pp. 309-320, 2007.
- [53] N. N. Karnik and J. M. Mendel, An Introduction to Type-2 Fuzzy Logic Systems, USC Signal Image Process. Rep., 1998
- [54] W. Dongrui and J. M. Mendel, "Enhanced Karnik--Mendel Algorithms," *IEEE Trans. Fuzzy Systems*, vol. 17, pp. 923-934, 2009.
- [55] J. M. Mendel, "On KM Algorithms for Solving Type-2 Fuzzy Set Problems," *IEEE Trans. Fuzzy Systems*, vol. 21, pp. 426-446, 2013.
- [56] K. S. Fu, R. Gonzalez and C. S. G. Lee, Robotics: Control Sensing. Vis, McGraw Hill, 1987.
- [57] http://www.nihonkohden.de/products/neurology/eeg
- [58] K. Serizawa, et al., "Comparison of quantitative EEGs between Parkinson disease and age-adjusted normal controls," *Journal of Clinical Neurophysiology*, vol. 25, pp. 361-366, 2008.
- [59] Teplan M., "Fundamentals of EEG Measurement", J. Measurement Sc. Review., vol. 2(2), 2002.
- [60] Geddes L.A. and Baker L.E., Principles of applied biomedical instrumentation, Wiley Interscience Pub., 2nd Ed. 1975.
- [61] http://kinovarobotics.com/products/jaco-research-edition/
- [62] V. Maheu, J. Frappier, P.S. Archambault and F. Routhier, "Evaluation of the JACO robotic arm: Clinico-economic study for powered wheelchair users with upper-extremity disabilities," in *IEEE International Conference on Rehabilitation Robotics* (*ICORR*), pp. 1-5, 2011.
- [63] S. Bhattacharyya, A. Khasnobish, S. Chatterjee, A. Konar, and D.N. Tibarewala, "Performance analysis of LDA, QDA and KNN algorithms in left-right limb movement classification from EEG data," in *International Conference on Systems in Medicine and Biology (ICSMB)*, pp. 126-131, 2010.
- [64] P. Herman, T. Prasad, T.N. McGinnity and D. Coyle, "Comparative Analysis of Spectral Approaches to Feature Extraction for EEG-Based Motor Imagery Classification," *IEEE Trans. Neural Syst. and Rehab. Engg.*, vol. 16, pp. 317-326, 2008.
- [65] H. Meng, L. Jiaojie, L. Guang, T. Xiaowei and D. Quiping, "Classification of Normal and Hypoxia EEG Based on Approximate Entropy and Welch Power-Spectral-Density," in *International Joint Conference Neural Networks (IJCNN '06)*, pp. 3218-3222, 2006.
- [66] G. S. Dharwarkar and O. Basir, "Enhancing Temporal Classification of AAR Parameters in EEG single-trial analysis for Brain-Computer Interfacing," in 27th Annual International Conference of the Engineering in Medicine and Biology Society (IEEE-EMBS), pp. 5358-5361, 2005.
- [67] N.J. Huan and R. Palaniappan, "Classification of mental tasks using fixed and adaptive autoregressive models of EEG signals," in 26th Annual International Conference of the IEEE Engineering in Medicine and Biology Society, pp. 507-510, 2004.
- [68] G. Pfurtscheller, C. Neuper, A. Schlogl and K. Lugger, "Separability of EEG signals recorded during right and left motor imagery using adaptive autoregressive parameters," *IEEE Trans. Rehab. Engg.*, vol. 6, pp. 316-325, 1998.

- [69] M. Xin, "The research of brain-computer interface based on AAR parameters and neural networks classifier," in 2011 International Conference on Computer Science and Network Technology (ICCSNT), pp. 2561-2564, 2011.
- [70] M. Vetterli and C. Herley, "Wavelets and filter banks: Theory and design," *IEEE Trans. Signal Process.*, 40(9), pp.2207–2232, 1992.
- [71] S. Qin and Z. Ji, "Multi-Resolution Time-Frequency Analysis for Detection of Rhythms of EEG Signals," *IEEE Proceedings on Signal Processing*, 11'th International DSP Workshop, vol. 4, pp, 338-341, 2004.
- [72] S.G. Mallat, "A theory for multiresolution signal decomposition: the wavelet representation," *IEEE Trans. Pattern Anal. & Mach. Intelligence*, Vol. 11(7), pp, 674–693, Jul. 1989.
- [73] I. Daubechies, "Orthogonal bases of compactly supported wavelets," Commun, Pure Math, 41, pp. 909-996, 1988.
- [74] R. Hecht-Nielsen, "Theory of the backpropagation neural network," in *International Joint Conference on Neural Networks*, vol.1, pp. 593-605, 1989.
- [75] B. Widrow and M. A. Lehr, "30 years of adaptive neural networks: perceptron, Madaline, and backpropagation," *Proceedings of the IEEE*, vol. 78, pp. 1415-1442, 1990.
- [76] A. Turnip, H. Keum-Shik and S.S. Ge, "Backpropagation neural networks training for single trial EEG classification," in 29th Chinese Control Conference (CCC), pp. 2462-2467, 2010.
- [77] R. Roy, A. Konar, D.N. Tibarewala and R. Janarthanan, "EEG driven model predictive position control of an artificial limb using neural net," in *Third International Conference on Computing Communication & Networking Technologies (ICCCNT)*, pp. 1-9, 2012.
- [78] H. Hexmoor, Essential Principles for Autonomous Robotics, Morgan and Claypool Pub, 2013.
- [79] H. Taub and D.L. Schilling, Principles of Communication Systems, McGraw-Hill, pp.70-71, 1971.
- [80] A. Konar, D.N. Tibarewala, S. Bhattacharyya and A. Khasnobish, *Human-Computer Interface for Rehabilitative Applications*, Springer, 2014 (to appear).
- [81] J. P. Pijn, J. Van Neerven, A. Noest and F.H. Lopes da Silva, "Chaos or noise in EEG signals; dependence on state and brain site," *Electroencephalography Clinical Neurophysiology*, vol. 79(5), pp. 371-381, 1991.
- [82] G. Dornhege, Towards Brain-Computer Interfacing, MIT Press, 2007.
- [83] A. Nijholt, and D. Tan, "Brain-computer interfacing for intelligent systems," *IEEE Intel. Sys.*, vol. 23, no. 3, pp.72-79, May-June 2008.
- [84] Y. Long, Y. Li, H. Wang, T. Yu, J. Pan, and F. Li, "A hybrid brain computer interface to control the direction and speed of a simulated or real wheelchair," *IEEE Trans. Neural Sys. Rehab. Eng.*, vol. 20, no. 5, pp. 720-729, Sept. 2012.
- [85] F. Galan, M. Nuttin, E. Lew, P.W. Ferrez, G. Vanacker, J. Philips, and J. del R. Millan, "A brain-actuated wheelchair: asynchronous and non-invasive brain-computer interfaces for continuous control of robots," *Clin. Neurophy.*, vol. 119, pp. 2159-2169, 2008.
- [86] Y. Chae, J. Jeong, and S. Jo, "Toward brain-actuated humanoid robots: Asynchronous direct control using an EEG-based BCI," *IEEE Trans. Robotics*, vol. 28, no. 5, pp. 1131-1144, 2012.

- [87] M.D. Serruya, N.G. Hatsopoulos, L. Paninski, M.R. Fellows, and J.P. Donoghue, "Brain-machine interface: Instant neural control of a movement signal," *Nature*, vol. 416, pp. 141-142, 2002.
- [88] E. Curran, P. Sykacek, M. Stokes, S.J. Roberts, W. Penny, I. ohnsrude, and A.M. Owen, "Cognitive tasks for driving a brain-computer interfacing system: a pilot study *IEEE Trans. Neural Sys. Rehab. Eng.*, vol. 12(1), pp. 48-54, March 2004.
- [89] C.C. Cheng and H. Hexmoor, "Improve path planning performance by monitoring human decisions," Int. Conf. Art. Intell., Las Vegas, pp. 453-460, 2013.
- [90] C. Notarnicola, L. Bruzzone and L. Pasolli, "Multiobjective model selection for non-linear regression techniques," in *IEEE International Geoscience and Remote Sensing Symposium (IGARSS)*, pp. 268-271, 2010.
- [91]O. Rudovic, I. Patras and M. Pantic, "Regression-Based Multi-view Facial Expression Recognition," in *International Conference on Pattern Recognition (ICPR)*, pp. 4121-4124, 2010.
- [92] Z. Zhiwei, J. Qiang and K.P. Bennet, "Nonlinear Eye Gaze Mapping Function Estimation via Support Vector Regression," in 18th International Conference on Pattern Recognition, pp. 1132-1135, 2006.
- [93] H. Hongmei and Q. Zengchang, "A k-hyperplane-based neural network for non-linear regression," in *9th IEEE International Conference on Cognitive Informatics (ICCI)*, pp. 783-787, 2010.
- [94] E. Vonk, L.C. Jain and L.P.J. Veelenturf, "Neural network applications," in *Proceedings Electronic Technology Directions* to the Year 2000, pp. 63-67, 1995.
- [95] B. Sen Bhattacharya, D. Coyle and L.P. Maguire, "Assessing alpha band event-related synchronization/desynchronization using a bio-inspired computational model," *J. Uni. Comp. Sc.*, vol. 18, no. 13, pp. 1888-1904, 2012.
- [96] E. Vonk, L.P.J. Veelenturf and L.C. Jain, "Neural networks: implementations and applications," *IEEE Aerospace and Electronic Systems Magazine*, vol. 11, pp. 11-16, 1996.
- [97] M. W. Jones, E. Morgan, J. E. Shelton, C. Thorogood, "Cerebral Palsy: Introduction and Diagnosis (Part I)", J Pediatric Health Care, vol.21(3), pp. 146-152, 2007
- [98] A. Halder, A. Konar, R. Mandal, A. Chakraborty, P. Bhowmik, N.R. Pal and A.K. Nagar, "General and Interval Type-2 Fuzzy Face-Space Approach to Emotion Recognition," *IEEE Trans. on Systems, Man, and Cybernetics: Systems*, vol. 43, pp. 587-605, 2013.
- [99] R. Kar, A. Konar, A. Chakraborty, A.K. Nagar, "Detection of signaling pathways in human brain during arousal of specific emotion," in *Intl. Joint Conf. Neural Networks (IJCNN)*, pp.3950-3957, 6-11 July 2014.
- [100] D. Tolani, A. Goswami, N. I. Badler, "Real-Time Inverse Kinematics Techniques for Anthropomorphic Limbs," *Graphical Models*, vol. 62(5), Pages 353-388, 2000.
- [101] K. S. Fu, R. Gonzalez, C.G.C. Lee, Robotics: Control, Sensing, Vision and Intelligence, McGraw-Hill Education, New York, 1987.
- [102] A. Halder, A. Konar, R. Mandal, A. Chakraborty, P. Bhowmik, N.R. Pal, A.K. Nagar, "General and Interval Type-2 Fuzzy Face-Space Approach to Emotion Recognition," *IEEE Trans. Sys., Man, and Cyber.: Sys.*, vol.43(3), pp.587-605, 2013.

[103] S. Chakraborty, A. Konar, A. Ralescu, N.R. Pal, "A Fast Algorithm to Compute Precise Type-2 Centroids for Real-Time Control Applications," *IEEE Trans. Cyber.*, vol.45(2), pp. 340-353, 2015.