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Strength Development of Concrete Made With Recycled Glass Aggregates Subjected to Frost Curing Conditions

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ABSTRACT

An experimental investigation was undertaken to study whether the strength behavior of concrete made with glass aggregate differed significantly from that made with natural aggregates when concretes cured in low temperatures. The aim of the research work presented is to examine the strength behavior of glass concrete when cured under freezing conditions at -15°C and -10°C. The results showed that when glass concrete is cured at low curing temperature, the 28 day compressive strength is higher than control concrete. Glass concrete that had been cured at low temperatures and subsequently allowed normal curing recovered 100% of its strength while the recovery for control concrete was of about 50%. These findings suggest that concrete made with recycled glass could have an important application, on cold temperature concreting.

Keywords: Glass aggregates, compressive strength, low curing temperature, strength development.

1. INTRODUCTION

The beneficial effects of low temperature curing on the long term strength development of concrete has been confirmed by the research findings of other researchers that stated that when concrete is cured at a low temperature the long – term compressive strength is higher than these of concretes cured at higher temperatures, Sangha and Dhir, (1975).

However there is a limit in curing temperature beyond which fresh concrete is permanently damaged and strength may never recover. It has been stated in literature that when the internal temperature of concrete falls to -2°C the free water in the pores begins to crystallise as ice. Freezing of water causes an increase in volume of 9% and generates stresses that will incorporate defects within the concrete, Korhonen, (2006).

When fresh concrete is subjected to sub–zero temperatures, the temperature inside the concrete mass starts to drop. The ability of concrete to withstand damage caused by the expanded ice depends on the strength of cement paste. However, strength development of cement paste is a time dependent phenomenon and therefore it is of vital importance to identify the exact time when ice starts to form in cement paste**. *Freezing point*** is defined as the temperature at which water in mass of concrete starts to freeze.

Korhonen, (1992) stated that when the temperature within concrete drops below -3°C, 90% of the water will freeze. However, there is a relationship between the temperature at which water in pores of concrete freezes and the size of the pores. According to Mindess and Young, (1981) water in pores of 10nm diameter will not freeze until -5°C and in pores of 3.5nm diameter water will not freeze until -20°C. The relationship between the size of capillary pores and the temperature that water in pores freezes has also been investigated by other researchers. Powers and Helmuth, (1953) stated that gel pore water will not freeze and ice can only form in some of the capillary pores and aggregates pores.

Earlier research findings have shown that a maximum temperature rise of 6.8°C was achieved by control concrete 12 hours after casting while the rise of temperature during hydration of glass aggregate concrete was about 13.7°C. Thus the rise in temperature for the glass concrete was approximately twice that of control concrete, Poutos *et al.*, (2007). Poutos *et al.*, (2007), explained that glass aggregate absorbs less heat than control aggregate due to its low specific heat and as a result the water absorbs a higher amount of the heat produced during cement hydration. In this way the concrete becomes hotter and the higher temperature accelerates the hydration of cement. The accelerating influence of higher temperatures was investigated by Soroka, (1993) who showed for example that when the temperature raises from 20^{o}C to 40^{o}C the rate of hydration of cement increases by a factor of 2.45.

Thus an investigation has been carried out in order to evaluate the ability of glass concrete to accelerate strength when cured under freezing conditions. On this purpose glass concrete has been tasted for compressive strength and has been compared with control concrete with the same w/c ratio of 0.50. The freezing curing temperatures used were *-15°C and -10°C.*

2. MATERIALS

2.1 Aggregates

The control aggregate was land based flint from Ridge Quarry, near Romsey, Hampshire, England. Soda lime silica recycled glass cullet was produced by the Krysteline implosion technique process. Sieve analysis was performed on the natural aggregates in accordance with BS 812 – 103.1 (British Standards Institution, 1985). The grading of the natural aggregates complied with BS 882 requirements for 10 mm all - in aggregate (British Standards Institution, 1992). The glass aggregate was sieved and separated into batches according to particle size. The different sizes were mixed in appropriate proportioning in order to produce aggregate which was identical to the grading of the natural aggregate.
The 24 hour absorption test was conducted in accordance with BS EN 1097 – 6, test method for mechanical and physical properties of aggregates, determination of particle density and water absorption (British Standards Institution, 2000). The 24 hour water absorption for both glass and natural aggregates are presented in Table 1.

Table 1: Percentage by mass of 24 hours water absorption for glass and natural aggregates

Aggregates	Water absorption by mass (%)
Natural Aggregates	3.0
Glass Aggregates	0.0

2.2 Cement

Ordinary Portland Cement PC – RM CEM I supplied by Lafarge Cement and produced at the Westbury Works, UK, was used throughout the test program. Details on the chemical composition of the cement used in this research as well as supplementary properties of it were supplied by the Research Department of Lafarge Cement.

3. EXPERIMENTAL PROCEDURES

3.1. Concrete mixes

Concrete cubes 100 mm in size were prepared using natural aggregate and glass aggregates. In total, 6 concrete mixes were cast using natural and green glass aggregates. The w/c ratios used was 0.50. Glass concrete was cast with 100% Brown glass cullet as aggregate.

3.2 Curing and testing Method

Throughout this research various curing conditions and methods were followed in order to study their effect on glass concrete strength. Concrete specimens were tested after being cured at normal curing temperature of 20°C, freezing curing temperature of -10°C and -15°C and combined curing at freezing and normal temperature. The following Sections details the methods used.

3.2.1 Normal Curing

Normal curing of concrete was carried out in accordance with BS 1881 – Part 111 code, for methods of a normal curing of test specimens (20°C method), (British Standards Institution, 1983). All the test specimens were demoulded 24 hours after casting and were stored in the water tank. The water inside the water tank was thermostatically controller at 20 ± 2°C.

3.2.2 Combined Curing

A specific research program was established to investigate the effect of low temperature curing and the influence of the duration of the exposure at these conditions on the compressive strength of concrete.
Concrete specimens were subjected to combined curing conditions as illustrated in Figure 1. The bar colored in blue indicates the duration of normal curing at 20°C while the red bar indicates the duration of curing at freezing temperatures. The curing method illustrated at the figure below was performed for two different freezing temperatures, namely -15°C and -10°C.

More specifically, -10ºC curing were allowed for 1, 3, 7, 21 and 28 days. Following freezing, the specimens were transferred in a water tank at room temperature (20 ± 2ºC) for an extra 28 days curing. Then, these specimens were tested for compressive strength.

When the non – normal curing temperature was -10°C, 30 concrete cubes were used, 6 of them were undergo freezing for 1 day after casting, after which 3 were tested for compressive strength and 3 more were placed in the water tank at 20°C for a further 28 days before testing for compressive strength. Similarly, the same numbers of cubes were frozen for 3, 7, 21 and 28 days and either tested or placed in the water tank for a further 28 days of normal curing, Figure 1.

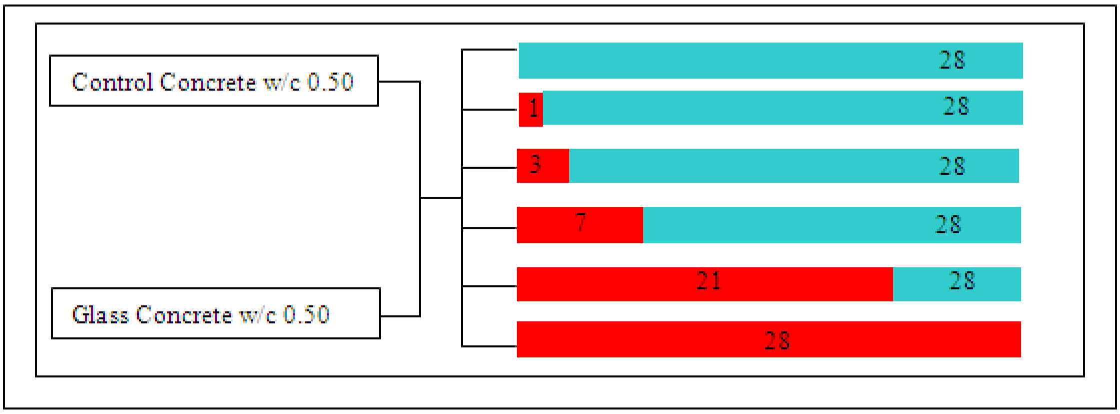

Figure 1 Combined curing method of glass and concrete specimens made with different w/c ratio.

4. RESULTS AND DISCUSSION

4.1 Strength behaviour of Concretes cured at 20°C

Figure 2 illustrate the 1 day, 3 day, 7 day 21 day and 28 day compressive strength results for the control concrete and glass concrete cured at 20°C.

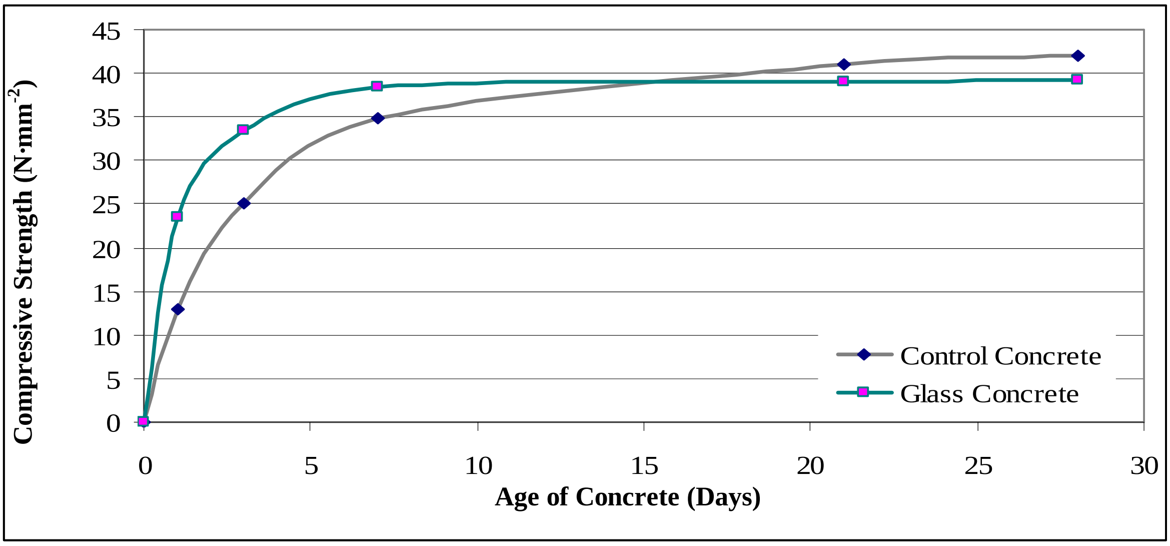

Figure 2 Influence of age on the compressive strength of control and glass concrete.

Glass concrete produces lower 28 days compressive strength when compared with control concrete. However, glass concrete results in significantly higher rates of strength development during the first 7 days after casting. As a result, the 1 day, 3 day and 7 day compressive strengths of glass concrete were higher than control concrete.

The high rate of strength development for the first 7 days after casting for glass concrete, compared with control concrete, can be partially attributed to the significantly higher temperatures developed during the hydration of glass concrete. This consequently accelerates the hydration of cement at an early age. Poutos *et al.*, (2007), found that a maximum temperature rise of 6.8^{o}C was achieved by control concrete 12 hours after casting. When 2% by mass of cement of calcium chloride was used as an accelerator the highest temperature rise occurred about 8 hours after casting and was 10.1°C. However for the green glass cullet concrete, the highest rise of temperature also occurred about 9 hours after casting but was 13.7^{o}C.

Thus the rise in temperature for the glass concrete was approximately twice that of control concrete. Poutos *et al.*, (2007), explained that glass aggregate absorbs less heat than control aggregate due to its low specific heat, (Tipler, 1999) and as a result the water absorbs a higher amount of the heat produced during cement hydration. In this way the concrete becomes hotter and the higher temperature accelerates the hydration of cement. The accelerating influence of higher temperatures was investigated by Soroka, (1993) who showed for example that when the temperature raises from 20°C to 40°C the rate of hydration of cement increases by a factor of 2.45.

Figure 3 illustrates the relative strength development of glass concretes as a percentage of the strength of control concretes.

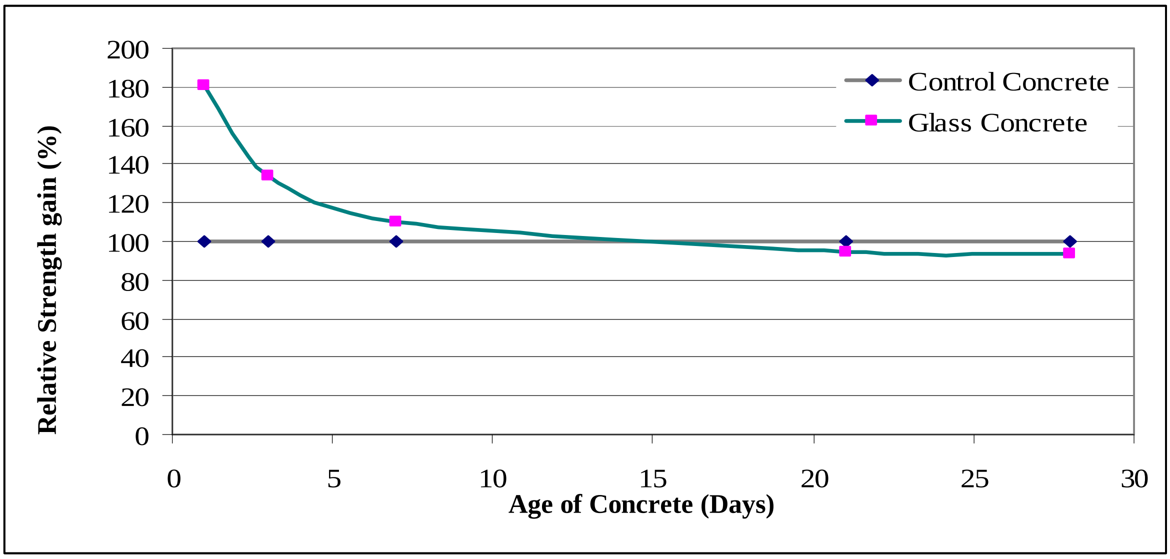

Figure 3 Relative strength of glass concrete as a percentage of the strength of control concrete.

The relative strength of glass concrete for the 1st, 3rd and 7th day after casting is 81%, 34% and 10% higher than the control concrete. These results again highlight the very significant accelerating effect of glass during the first 24 hours.

The increased compressive strength obtained by the glass concretes is evidence of the accelerating activity of the increased temperatures experienced during the hydration of glass concrete. Swamy *et al.*, (2006) stated that the early evolution of the heat of hydration is beneficial for the early age strength of concrete as the retained heat from hydration produces concrete with markedly increased compressive strength.

BS EN 934 – 2 specifies the requirements for admixtures that increase the rate of development of early strength of concrete, (British Standards Institution, 2001). The specific requirements for hardening accelerating admixtures are:

i. When concrete is cured at 20°C, the 24 hours compressive strength should be not less than 120% of the control mix.
ii. When concrete is cured at 5°C, the 24 hours compressive strength should be not less than 130% of the control mix.

Glass concrete achieves the first requirement of the code confidently. Thus it can be concluded that glass concretes having a w/c ratio less than or equal to 0.50 will satisfy requirement (i). Although no work was done at 5°C, results reported in the next section 4.2 relating to performance at - 10°C and -15°C suggest that requirement (ii) would very easily be satisfied.

It is interesting to note that glass concrete also achieves the ASTM C 928-89 specifications that requires a very rapid – hardening material to have a compressive strength of 3,000 psi (20.68 $N \cdot mm^{-2}$) in 24 hours (American Society for Testing Materials, 1989). The glass concrete achieved a compressive strength of 23.5 $N \cdot mm^{-2}$, Figure 2.

The reduced compressive strength of glass concretes after the 7th day is the consequence of the relatively high temperature attained during the hydration of the glass concrete. It is well documented in literature that high temperatures during hydration speeds up the chemical reaction with beneficial effects on early strength development but possible adverse affects on strength after 7 days. Kanda *et al.*, (1992), Verberck and Helmuth, (1968).

4.2 Strength behaviour of Concretes cured at -10°C

Figure 4 illustrates the 1 day, 3 day, 7 day 21 day and 28 day compressive strength results for the control and glass concretes with w/c ratios 0.40 and 0.60 cured continuously in a freezer at -10°C.

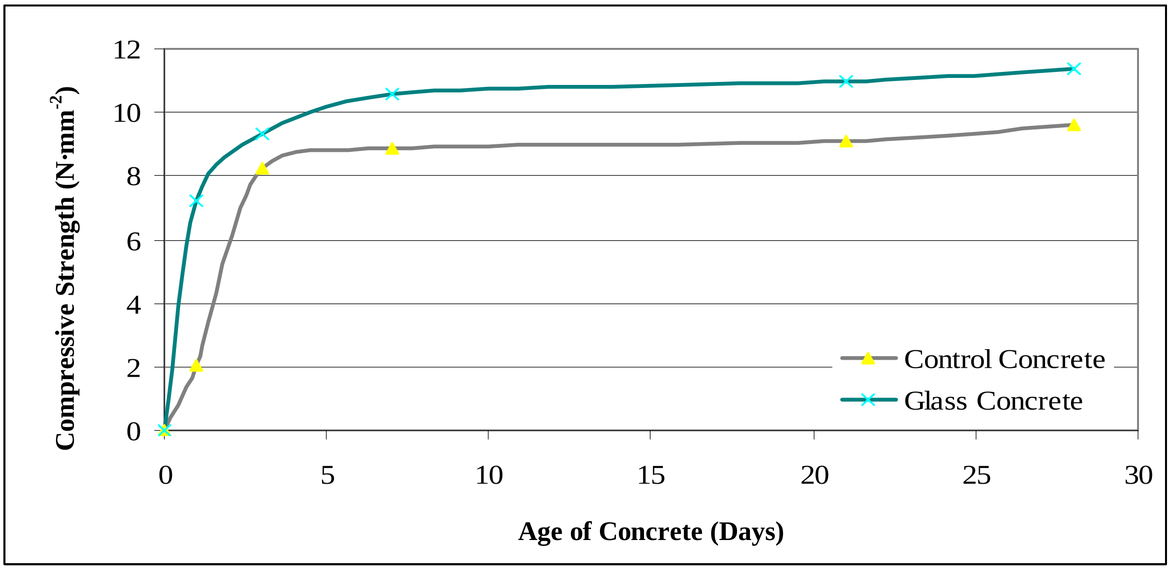

Figure 4 1 day, 3 day, 7 day and 28 day compressive strength of concrete cured and tested at -10°C.

When concrete is cured at low temperatures, setting time and rate of strength gain is significantly delayed since the rate of hydration is significantly reduced. The setting time of concrete increases by approximately 33% for every 6°C drop in temperature, down to 4°C, (American Concrete Institute, 1992).

Poutos et *al* (2007), showed that a significantly higher temperature is developed during hydration of glass concrete when compared to control concrete. Findings presented earlier in this paper highlighted the ability of glass concrete to have an accelerating effect on the strength development of glass concrete. Findings presented in this section showed that this early temperature rise in glass concrete benefits strength development when glass concrete is cured at -10°C. The glass concrete developed a significantly higher 1 day compressive strength. The 1 day compressive strength of glass concrete was three times that of control concrete. It is interesting to note that after 3 days there is very little difference in strength of glass concrete. The relatively better performance of glass concrete can be attributed to the excellent bonding of glass aggregate (Sangha et *al*, (2004)) even when exposed to severe freezing. On the other hand, control concrete did not develop the required strength to withstand damage from ice formation. The 24 hour strength of these concretes was below 3.5 N·mm^2.

The greater similarity in strength for all concretes after the age of 3 days is probably due to the influence of the strength of ice that is produced inside the concrete mass. Masterson *et al.*, (1997) performed compressive strength tests on natural ice cores. The ice temperature was varied from -6°C to -15°C and they found that the compressive strength varied between 6 N·mm^2 and 11 N·mm^2 and these results were confirmed by other researchers (Cox *et al.*, 1983). It is interesting to note that the strength of all the concretes tested was approximately 11 N·mm^{-2} at 28 days.

4.3 Curing at -10°C and 20°C

In the previous section 4.2 it was stated that the ability of glass concrete to develop sufficient strength during the first 24 hours enables it to resist damage from ice formation. Therefore it is important to consider whether glass concrete which has been subjected to severe freezing conditions when then exposed to normal curing conditions will develop strength without any significant effect on the ultimate strength. In order for this statement to be tested, concrete cubes were cast and cured following the curing procedure detailed in Figure 1. Figure 5 illustrate the compressive strength results for control and glass concretes cured for different duration under freezing conditions at -10°C followed by 28 day of standard curing at 20°C.

It is worth noting that the initial ability of glass concrete to develop strength is of vital importance for later strength development. When the internal temperature of concrete falls to -2°C, free water in the pores begins to crystallize as ice. Powers and Helmuth, (1953) stated that when water in concrete freezes, it expands, and its volume increases by 9% and the increased volume of ice generate stresses that incorporate defects within the concrete.

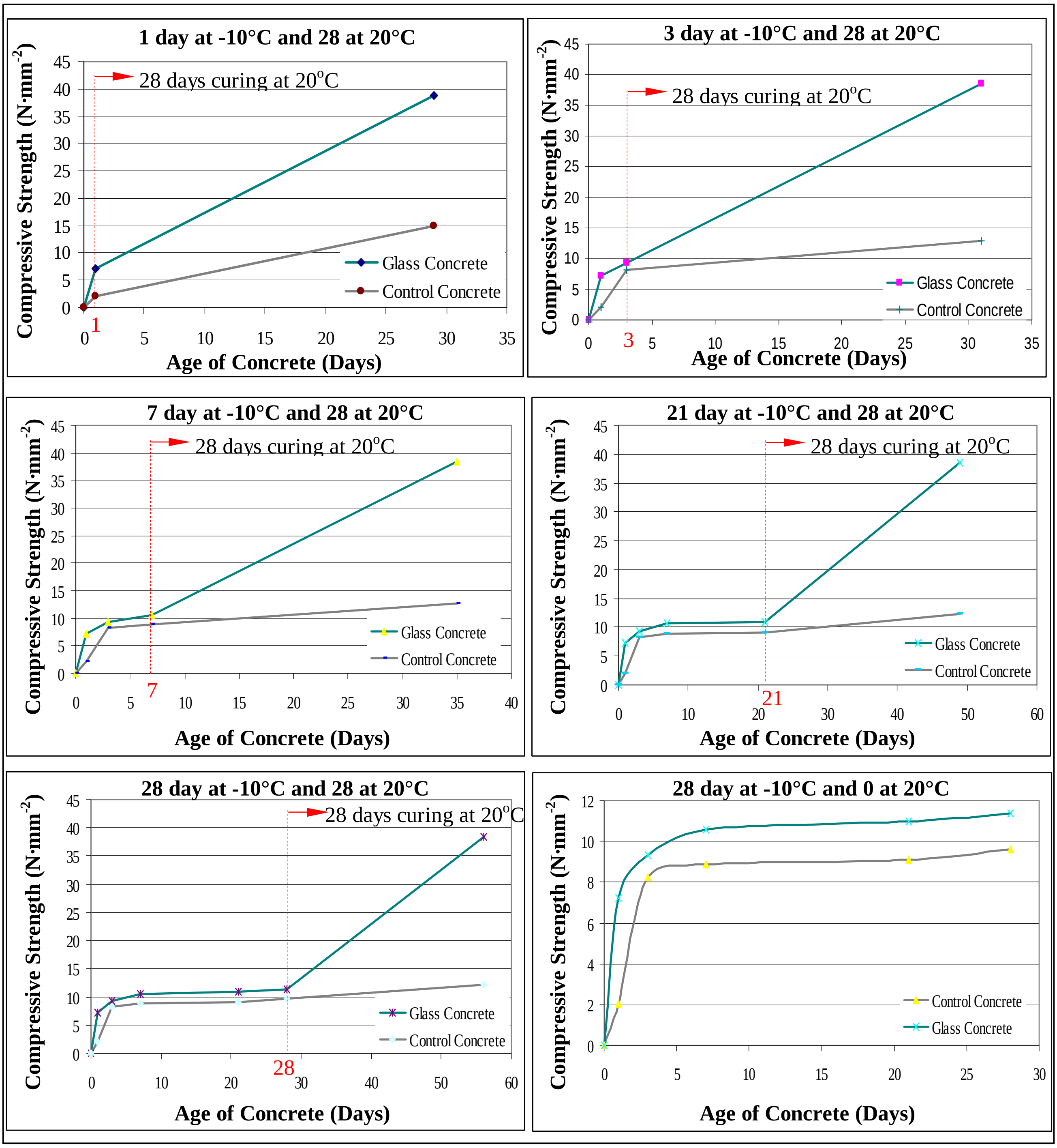

Figure 5 Compressive strength of control and glass concrete cured for different duration at -10°C followed by 28 day of standard curing at 20°C

The compressive strength of control concrete is significantly affected when cured at -10^{o}C immediately after casting. The most critical period for the strength development of control concrete is the first 24 hours after casting. A large reduction in strength for control concrete occurred for the concrete cured at -10^{o}C for the first 24 hours. Thereafter there was a gradual but much smaller reduction in strength for increased duration of exposure at -10^{o}C. The strength of glass concrete exposed to different durations at -10^{o}C was very much better. The effect on strength of glass concrete was very limited. Glass concrete cured at -10^{o}C for 1 day, 3, days 7 days 21 days and 28 days followed by curing at 20^{o}C for an extra 28th day resulted in strengths of about and 39 N·mm^{2}. This strength was just 2% less than the strength of glass concrete cured at 20^{o}C. Thus it can be stated that glass concrete exposed to -10^{o}C curing can fully recover strength when subsequently cured under normal curing regardless to the duration of exposure under freezing conditions.

In spite of the fact that under normal curing conditions control concretes achieved higher 28 days strength than glass concretes, exposure to sever freezing conditions results in stronger concrete when glass is used. This trend was noted for curing at -10°C for all durations. As discussed in the previous section, concrete cured at -10°C for 28 days and then tested after immediate removal from the freezer is probably influenced by the strength of ice and the potential for strength development is not indicated by the results.
The results show that when glass aggregate is used to produce concrete, the strength of glass concrete cured for 1 day, 3 days, 7 days and 21 days at -10°C is 251%, 12% 20% and 20% higher than the strength of control concrete. Thus as noted earlier, the strength benefits of glass are greatest for concretes made with lower w/c ratios.

Figure 6 illustrates the percentage loss of the ultimate 28 days compressive strength of control and glass concrete cured for different durations at -10°C and allowed for an extra 28 day of normal curing. The percentage reduction is calculated with respect to the strength obtained for 28 days standard curing.

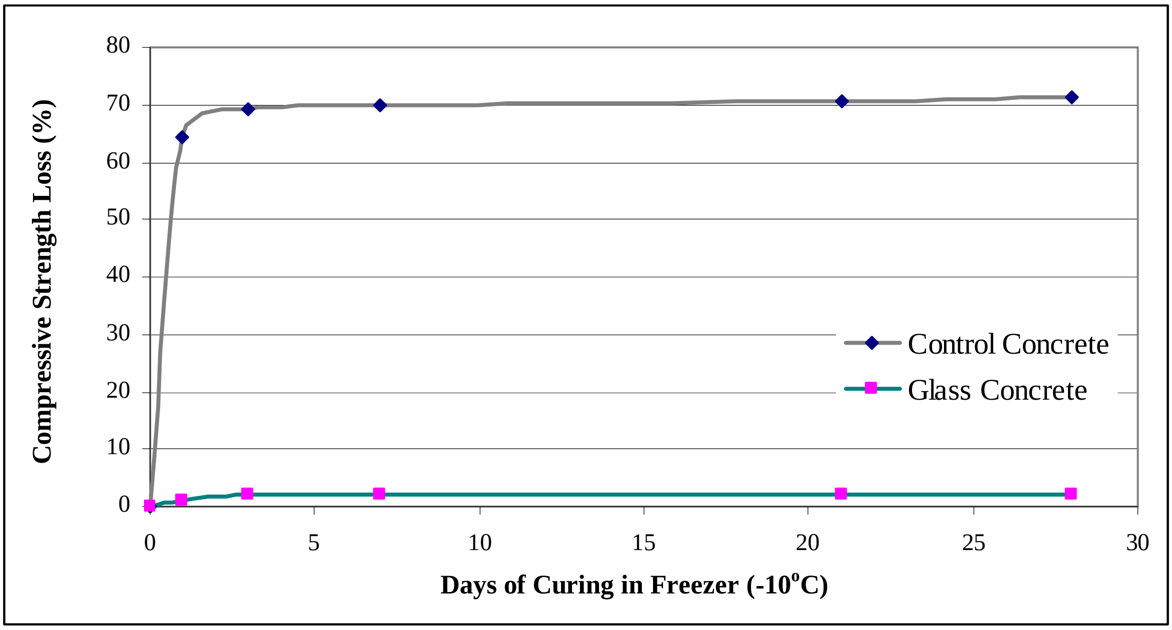

Figure 6 Percentage loss of strength for concretes cured at -10°C for differed durations followed by 28 days of standard curing

The results clearly highlight that when control concrete cured to a temperature of -10°C, loses between 65% and 75% of the ultimate 28 days compressive strength. The percentage loss for the glass concretes can be seen to be very much better, about 2%. The results of this research are consistent with the findings of Korhonen *et al.*, (1997). They performed compressive strengths test on concrete that was cured at -10°C for the first 56 days and then cured at 20°C for 28 more days and found that concrete loses 60% to 70% of its ultimate 28 days strength. Moreover the damage on the compressive strength of concrete due to frost action when subjected to low temperature is irreversible. Otherwise, if concrete subjected freezing conditions before develop sufficient strength to withstand frost damage it will never recover strength regardless of the duration of the subsistent normal curing.

An explanation of the damage to strength when water freezes at early ages was given by Powers and Helmuth, (1953). They suggested that the damage of the cement paste occurs due to the hydrostatic pressure produced by the expanded water that freezes. Moller, (1956) performed tests on the volume stability of concretes subjected to subzero temperature and related the volume change of concrete specimens with the age of concrete when freezing took place and the length of exposure to frost. He found that when freezing takes place at 10 hours after casting and for exposure to frost for more than 5 hours, the increase in concrete volume was approximately 1.5%. An increase in volume by 1.5% has the result of decreasing density by approximately 1.5%. This observation is consistent with the findings of this study. Figure 7 illustrates the density of control concrete and glass concrete for different durations of curing at -10°C.

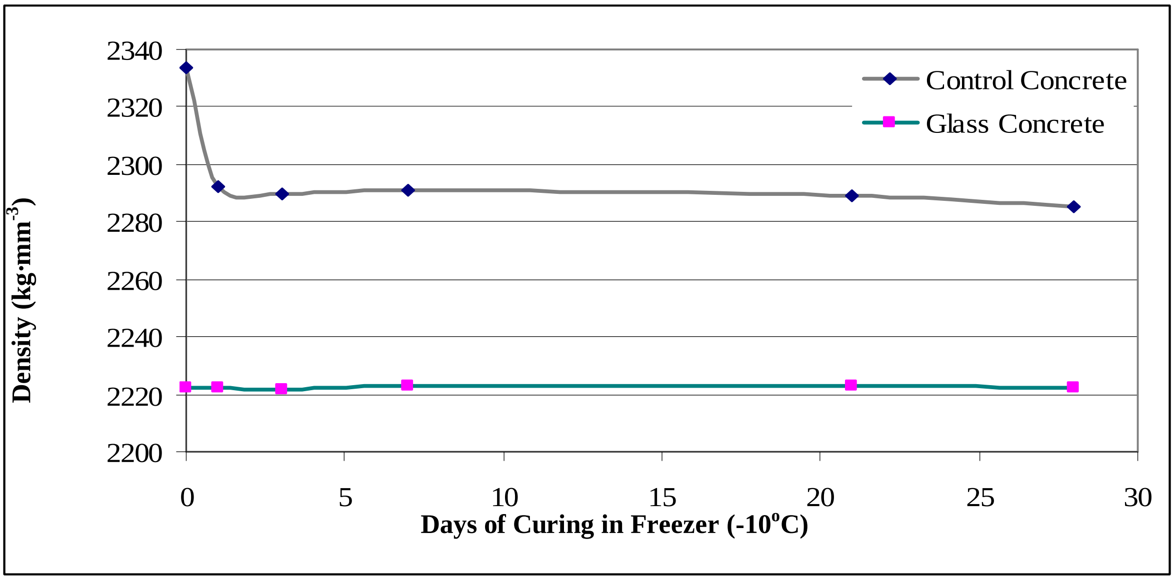

Figure 7 Density changes for control and glass concretes.

The results show that control concrete experiences a significant reduction in its density when cured at -10°C. When control concrete was cured for 1 day at -10°C a reduction of 1.8% of its density was observed. There was no further reduction of density when concrete was cured at -10°C for 3 days, 7 days and 21 days and 28 days. This is one more evidence that the majority of damage occurs on the 1st day of curing at low temperature.

On the other hand glass concrete showed no change in density when cured for different durations at -10°C. This is further evidence that during early age strength development of glass concretes, it is able to withstand the pressure of the expanded water and therefore to achieve volume stability. One of the factors influencing frost damage of concrete is the degree of saturation. Degree of saturation decreases during the hydration process as water is consumed during the hydration process. Therefore as glass concrete produces high rates of hydration, the quantity of water decreases and therefore frost damage is minimized.
The strength loss relationship of glass concretes with duration of low temperature curing follows a different pattern to that produced by control concretes. The main difference results from the fact that glass concrete does not show any significant strength loss when cured for the 1st day at -10°C.

It is the author’s opinion that the strength loss of glass concretes is different to control concretes not only in quantity but in nature as well. The strength loss because of the low temperature curing that occurs for control concrete is mostly because of the damage due to the frost formation. Strength of glass concrete with was only slightly affected by the low temperature curing for the first 7 days. This statement is strongly supported by the fact that glass concretes show an excellent volume stability which means that concrete is not affected by frost formation.

It is the author’s opinion that the ability of glass concrete to recover strength when removed from -15°C then subjected to standard curing without any significant reduction in its ultimate 28 days strength is due to:

1. Higher strength development its early history
2. Practically 0% water absorbs of glass aggregate and
3. Excellent bonding of glass aggregate with the cement matrix which confirmed by microscopic observations

The ability of glass concrete to resist low temperature curing damage can also be explained by the pore structure of glass concrete. Corinaldesi *et al.*, (2005) found that 8% of the pores were less than 10nm diameter where water freezes at less than -5°C and the great majority of pores were in the range of micropores. For control concrete most of the pores are in the range of mesopores and there are no nanopores. Water in smaller pores freezes at much lower temperatures. Similar where the trends when control and glass concrete subjected to - 10°C.

5. CONCLUSIONS

1. The research findings suggests that if concrete made with glass aggregates is used for construction, there may be potential for an important applications, namely, cold weather concreting.

2. Exposure to prolonged curing of concrete at -10°C and -15°C produced dramatic differences between control and glass concretes. Glass concretes recovered virtually all of the strength reductions after subsequent standard curing. However control concretes experienced very large reductions in strength even after subsequent standard curing. Macro and microscopic examination of the structure of the concrete clearly showed that glass concrete was relatively unaffected by freezing conditions, whereas the freezing of water in control concretes severely damaged aggregate bonding. This contrasting behavior can be attributed to:

   i. The smaller pore structure of glass concrete, Corinaldesi *et al.*, (2005)
   ii. The thermal characteristics of glass and its influence on hydration, Poutos et *al (2004)*
   iii. The higher early strength development of glass concrete

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