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ELECTROMAGNETIC DIFFRACTION BY METAL CYLINDER COATED WITH INHOMOGENEOUS MAGNETOACTIVE PLASMA SHEATH.I

Azarenkov N.A., Galaydych V.K.
Physical–Technical Department , V.Karazin Kharkiv National University, 61077, Svobody sq.4, Kharkiv, Ukraine

This paper is devoted to the problem of diffraction of low frequency electromagnetic waves incident from a homogeneous magnetoactive plasma by a metal circular cylinder surrounded by radially inhomogeneous plasma sheath. The external magnetic field is parallel to a cylinder axis; the wavevector of incident plane wave is perpendicular to this axis. It is assumed that plasma is cold and perturbations in it are governed by Maxwell's and two-liquid hydrodynamic equations. The exact solutions for fields in both linear and power plasma sheath profile are obtained. The cross-sections for backward and forward scattering as a function of incident wave vector are derived for various parameters of task. The dependences of these cross-sections upon these parameters are analyzed. The inhomogeneous plasma sheath enhances the oscillation amplitude of the back scattering cross-section and changes in several times the value of the forward scattering cross-section.
KEYWORDS: electromagnetic wave, diffraction, magnetoactive plasma, non-uniform plasma, cross-section.

Problem of electromagnetic wave diffraction by a metal cylinder embedded into magnetoactive plasma is of interest for solving a number of problems such as antenna radiation in a plasma, probe measurements, remote sensing and generally speaking a metal object influence on the electromagnetic wave propagation in various plasma-like media from modern metamaterials to space. Diffraction by metal bodies of various shape in vacuum has been investigated carefully enough in [1]. Electromagnetic scattering by a metal cylinder immersed into homogeneous plasma or/and coated with homogeneous sheaths has been studied in many works ([2] and Refs. in [3]). But within adjacent sheath the plasma density varies with radius under real experiment conditions.

STATEMENT OF THE PROBLEM

In present paper, we have solved the problem of diffraction of low frequency $(\omega < \omega_i)$ magnetohydrodynamic (MHD) waves propagating in a homogeneous magnetoactive plasma on a metal circular cylinder of radius R_c, surrounded by radially inhomogeneous plasma sheath $(1 < r < \rho)$; here and further the radius r is normalized by R_c. The homogeneous plasma occupies region $r > \rho$. The external magnetic field $\vec{H}_0$ is parallel with a cylinder axis, the wavevector of incident plane wave is perpendicular to this axis. It is assumed that plasma is cold and disturbances in it are governed by Maxwell's and two-liquid hydrodynamic equations.

An incident plane low frequency $(\omega < \omega_i)$ wave propagates in homogeneous magnetoactive plasma and has such components (TM -polarization)

$$H_z^{(i)} = E_0 \exp(-ik_T x - i\omega t)\,;\; E_x^{(i)} = -(\omega_i/\omega)(k/k_T)H_z^{(i)}\,;\; E_y^{(i)} = -(k/k_T)H_z^{(i)}\,. \tag{1}$$

Here E_0 is amplitude, $\omega_i = eH_0/m_i c$ is the ion gyrofrequency, e, m_i - are ion charge and mass, respectively, H_0 - value of external magnetic field, c - speed of light in vacuum. The frequency ω and the wavevector k_T of incident plane wave are connected by linear dispersion relation

$$k_T = (\omega/c)(\Omega_{i0}/\omega_i)\,, \tag{2}$$

where $\Omega_{i0} = \left(4\pi e^2 n_0/m_i\right)^{1/2}$ - ion Langmuir frequency, n_0 - undisturbed density of the homogeneous plasma.

Because in the incident wave $E_z = 0$ and other components are independent on z and the inhomogeneity is unlimited on z (i.e. $\partial/\partial z = 0$), therefore these properties remain the same in the scattered wave, i.e. there is no depolarization of wave. We will assume that normally incident plane wave (1) as a result of diffraction on the given structure generates a divergent cylindrical wave. It is possible to consider this wave as the locally plane wave of *TM*- polarization far from a cylinder.

Set of equations that governs the disturbances which propagates perpendicularly to the external magnetic field $\vec{H}_0$ is reduced to two uncoupled subsystems that describe *TE* - and *TM* - waves.

At low frequencies $\omega < \omega_i$ in a dense $\left(\Omega_{i0}^2 >> \omega_i^2\right)$ plasma, it is possible to obtain the following set of equations for TM-wave components in the form $A(r,\varphi,t) = A(r)\cdot\exp\left[i(m\varphi - \omega t)\right]$ [4]:

$$\frac{d^2H_z}{dr^2} + \left[\frac{1}{r} - \frac{d}{dr}\ln N(r)\right]\frac{dH_z}{dr} + \left[k_T^2 N(r) - \left(\frac{m}{r}\right)^2 - \frac{m}{r}\frac{\omega}{\omega_i}\frac{d}{dr}\ln N(r)\right]\cdot H_z = 0\,, \tag{3}$$

$$E_r = -\frac{1}{k_T\sqrt{N(r)}}\left(\frac{m}{r}H_z + \frac{\omega}{\omega_i}\frac{dH_z}{dr}\right); \qquad E_\varphi = -\frac{i}{k_T\sqrt{N(r)}}\left(\frac{m}{r}\frac{\omega}{\omega_i}H_z + \frac{dH_z}{dr}\right), \tag{4}$$

where $n(r)$ - the unperturbed plasma densities in the inhomogeneous regions, $N(r) = n(r)/n_0;$, m - azimuthal wavenumber.

A continuous plane MHD wave propagating in an homogeneous region $(r > \rho)$ may be represent in the form

$$H_z^{(i)} = E_0 \exp(-ik_T x) = E_0 \cdot \sum_{m=-\infty}^{\infty}(-i)^m J_m(k_T r)e^{im\varphi}, \; E_y^{(i)} = -\frac{(\omega/c)}{k_T}\cdot H_z^{(i)} \; ; E_x^{(i)} = -(\omega_i/\omega)(k/k_T)H_z^{(i)}. \tag{5}$$

Taking into account the conditions for all components (A) at infinity in the cylindrical coordinate system [5] :

$$\sqrt{r}\cdot(\partial A/\partial r + ik_T A) \to 0 \tag{6}$$

the diffraction field components in the homogeneous plasma region may be chosen in the following form :

$$H_z^{(s)} = E_0 \sum_{m=-\infty}^{\infty}(-i)^m a_m H_m^{(1)}(k_T r)e^{im\varphi},$$

$$E_r^{(s)} = \frac{-1}{k_T\sqrt{N(r)}}\left(\frac{m}{r}H_z^{(s)} + k_T\frac{\omega}{\omega_i}\frac{dH_z^{(s)}}{dr}\right), \tag{7}$$

$$E_\varphi^{(s)} = \frac{-i}{k_T\sqrt{N(r)}}\left(\frac{m}{r}\frac{\omega}{\omega_i}H_z^{(s)} + k_T\frac{dH_z^{(s)}}{dr}\right).$$

It is possible to express the fields inside the inhomogeneous sheath $(1 < r < \rho)$ via two linearly-independent solutions of equation (3):

$$H_z^{(t)} = E_0\cdot\sum_{m=-\infty}^{\infty}(-i)^m\cdot e^{im\varphi}\cdot\left[b_m\cdot F_{1m}(r) + c_m\cdot F_{2m}(r)\right],$$

$$E_r^{(t)} = \frac{-E_0}{k_T\sqrt{N(r)}}\sum_{m=-\infty}^{\infty}(-i)^m e^{im\varphi}\left\{b_m\left[\frac{m}{r}F_{1m}(r) + \frac{\omega}{\omega_i}F'_{1m}(r)\right] + c_m\left[\frac{m}{r}F_{2m}(r) + \frac{\omega}{\omega_i}F'_{2m}(r)\right]\right\}, \tag{8}$$

$$E_\varphi^{(t)} = \frac{-iE_0}{k_T\sqrt{N(r)}}\sum_{m=-\infty}^{\infty}(-i)^m e^{im\varphi}\left\{b_m\left[\frac{m}{r}\frac{\omega}{\omega_i}F_{1m}(r) + F'_{1m}(r)\right] + c_m\left[\frac{m}{r}\frac{\omega}{\omega_i}F_{2m}(r) + F'_{2m}(r)\right]\right\}.$$

Three boundary relations were used for determining the coefficients a_m, b_m, c_m in (8). The two of them consist of the continuity of the tangential component of the electric field E_φ and of the magnetic field H_z at the boundary $r = \rho$:

$$E_\varphi^{(i)} + E_\varphi^{(s)} = E_\varphi^{(t)} \quad ; \quad H_z^{(i)} + H_z^{(s)} = H_z^{(t)}. \tag{9}$$

The third is zero relation for tangential component of the electric field E_φ at the metal surface at $r = 1$:

$$E_\varphi^{(t)}(r = 1) = 0. \tag{10}$$

From obtained set of equations we have

$a_m = D_{ma}\cdot D_m^{-1}, \quad D_m = -H_m^{(1)}(x_T)\cdot\Delta_{1m} + G_m\cdot\Delta_{2m}; D_{ma} = J_m(x_T)\cdot\Delta_{1m} - Q_m\cdot\Delta_{2m};$

$\Delta_{1m} = P_{2m}(1)\cdot P_{1m}(\rho) - P_{1m}(1)\cdot P_{2m}(\rho);\ \Delta_{2m} = P_{2m}(1)\cdot F_{1m}(\rho) - P_{1m}(1)\cdot F_{2m}(\rho);\ P_{jm}(r) = \mu F_{jm}(r) + r\cdot F'_{jm}(r), j = 1,2;$

$P_{jm}(r) = \mu F_{jm}(r) + r\cdot F'_{jm}(r), j = 1,2; G_m = q_m\cdot H_m^{(1)}(x_T); Q_m = q_m\cdot J_m(x_T);\ q_m = x_T + \mu\ ; x_T = \rho k_T R_C\ ;\ \mu = m(\omega/\omega_i).$

As a characteristic of scattering, the differential scattering cross-section per unit length $\sigma(\varphi)$ may be introduced. It is equal to the ratio of the power diffracted at an angle φ to the magnitude of the Poynting vector [5]

$$\sigma(\varphi) = \lim_{r\to\infty}\frac{\pi r}{2\rho}\left(\left|H_z^{(s)}\right|^2 \Big/ \left|E_0\right|^2\right). \tag{11}$$

At far field zone $(k_T r >> 1)$ it is possible to use the well known asymptotic of Hankel's function and the expression for $\sigma(\varphi)$ can be written as

$$\sigma(\varphi) = \frac{4}{k_T R_c}\left|\sum(-1)^m a_m e^{im\varphi}\right|^2. \tag{12}$$

Graphs in Fig.1 show the cross-sections for backward $\sigma(0)$ and forward scattering $\sigma(\pi)$ as a function of $k_T R_c$ for a metal cylinder in homogeneous magnetoactive plasma without any surrounding sheaths. In the diffraction theory observed oscillations of the back scattering cross-section are qualitatively explained by the effect of the superposition of the diffractional field (‘leakage wave’) and the field reflected from a metal surface [8].

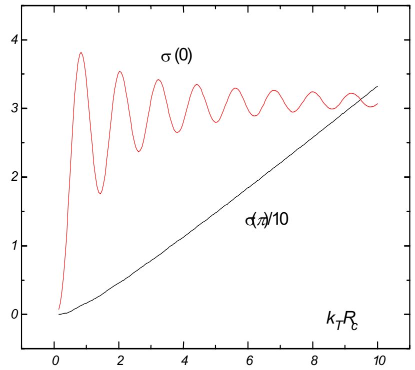

Fig.1. Differential cross-sections (18) of the metal cylinder in a *homogeneous* magnetoactive plasma

Let us consider the differential cross-section (12) of the metal cylinder surrounded by a rarefied inhomogeneous plasma sheath. For modeling the rarefied adjacent plasma sheath, we have chosen the plasma density distribution inside inhomogeneous region in a such form:

$$N(r) = (r/\rho)^S \ , \ 1 \le r \le \rho \ , \ s \text{ is real.} \tag{13}$$

This type of dependence allows us to use the well-known solutions of equations (3) and, on the other hand, under varying values of ρ and s, to take into account quite a wide class of density profiles. For such profiles (13), we can write down two linearly-independent solutions (3) (see [6])

$$F_{1m}(r) = r^{(s/2)} J_\eta(\kappa); F_{2m}(r) = r^{(s/2)} Y_\eta(\kappa), \text{ where } \eta = \frac{1}{s+2}\sqrt{s^2 + 4(m^2 + s\mu)}; \kappa = \frac{2}{s+2} k_T \sqrt{p} \cdot r^{(s+2)/2}, \tag{14}$$

and J_η, Y_η are the η th-order Bessel's and Neumann's functions; $p = N(r=1) = \rho^{-S}$.

To clarify the influence of inhomogeneity properly, we have calculated the differential cross-sections $\sigma(0), \sigma(\pi)$ using formula (12) for "averaged" model. The latter means the homogeneous sheath of the average density <*N*>, which has a thickness equal to that of the inhomogeneous one. The dotted line in Fig.2,3 corresponds to this model.

$$\langle N \rangle = \frac{1}{\rho - 1}\int_1^\rho N(r)dr \ . \tag{15}$$

The influence of the inhomogeneous region on the wave diffraction varies depending on the model parameters. The analysis of the obtained results allows interpreting some regularity.

The influence of increasing (deep into plasma) density region on the backscattering cross-section $\sigma(0)$ depends upon rarefiedness of the coating plasma sheath. While a rarefiedness parameter value is $1 > p > 0.5$, graphs for both inhomogeneous and averaged models are very similar to corresponding ones in Fig.1. There are quantitative (not qualitative) differences that consist on the faster decreasing of oscillations of $\sigma(0)$. The considerable rarefiedness strongly enhances the oscillations of the back-scattering cross-section $\sigma(0)$ and *increase* $\sigma(\pi)$ in several times (Fig. 2).

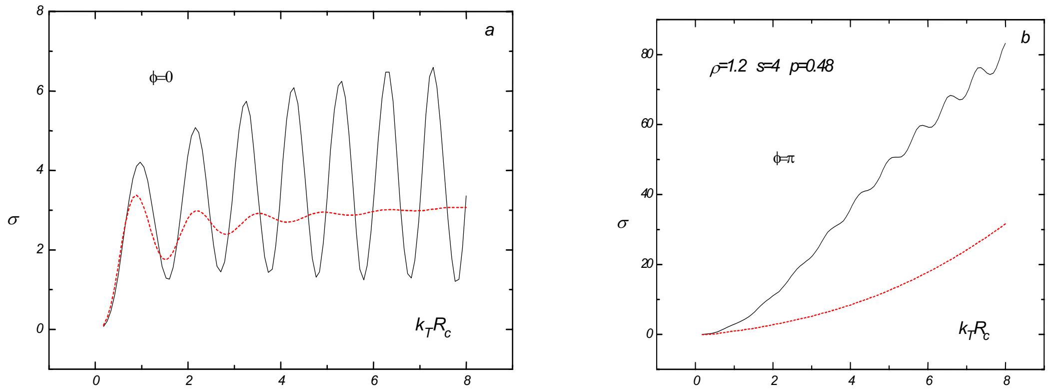

Fig. 2. The cross-section for back scattering $\sigma(0)$ (a), and forward scattering (b) as a function of $k_T R_c$ for the power sheath profile $N(r) = (r/\rho)^S$.

Consider the influence of the sheath of decreased (deep into plasma) plasma density on the wave diffraction. Another words, cylinder is surrounded by more dense plasma then background. Let choose the $N(r)$ in the form

$$N(r) = A + B \cdot r \ . \tag{16}$$

Two linearly independent solutions of equations (3) will be obtained by Frobenius' method [7]:

$$F_{1m}(r) = \sum_{l=0}^{\infty} \alpha_l r^l, F_{2m}(r) = a \cdot \ln r \cdot F_{1m}(r) + \sum_{k=0}^{\infty} \beta_k r^{-m+k} \ , \tag{17}$$

where all the coefficients must be found with the help of the recurrence relationships (see [4]).

The power series (17) converges under the condition $A + B > 0$ [7]. The inhomogeneity region of small $(p \le 1.1)$ density gradient does not change considerably the cross-section similarly to the previous case. Under the increasing of modulus of the gradient we have the enhancement of the oscillation amplitude $\sigma(0)$ versus $k_T R_C$ under $k_T R_c > 2$ (see

Fig. 3). As for the forward scattering $\sigma(\pi)$ we can establish a fact of decreasing of value and appearance of oscillation on $k_T R_C$ under $k_T R_c > 5$. With the increasing of the gradient modulus oscillation amplitude $\sigma(0)$ (as a function of $k_T R_C$) increases when $k_T R_c > 2$ (see Fig. 3). For the forward scattering cross-section $\sigma(\pi)$ we have fixed the *decreasing* of its value and appearance of oscillation when $k_T R_c > 5$.

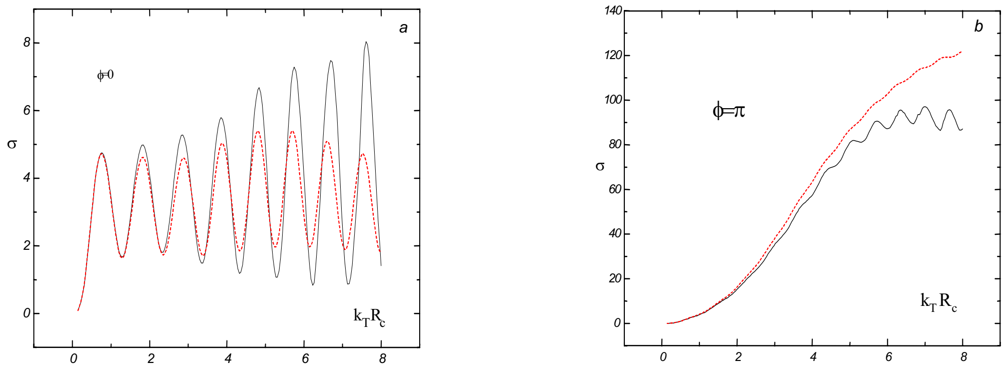

Fig. 3. The cross-section for back scattering $\sigma(0)$ (a), and forward scattering $\sigma(\pi)$ (b), as a function of $k_T R_c$ for the linear sheath profile $N(r) = A + B \cdot r$; $A = 4$, $B = -2$, $\rho = 1.5$, $p = 2$.

To perform the calculations, the typical plasma parameters of the Earth ionosphere were chosen: $\left(m_i / m_p\right) \approx 30; \left(NO^+, O_2^+\right)$; $R_C = 5\,\text{m}$; $\omega_i R_C / c = 2.4 \cdot 10^{-3}$.

So in present paper, it has been shown that a radially inhomogeneous plasma sheath surrounding an ideal conducting cylinder which is immersed into homogeneous magnetoactive plasma can change significantly scattering of low frequency plane wave. The inhomogeneous sheath leads to the enhancement of the oscillation amplitude of back scattering cross-section and to the considerable (in several times) change of forward scattering cross-section. Sign of that change depends on the sign of sheath density gradient.

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РАССЕЯНИЕ ЭЛЕКТРОМАГНИТНЫХ ВОЛН НА МЕТАЛЛИЧЕСКОМ ЦИЛИНДРЕ, ОКРУЖЕННОМ СЛОЕМ НЕОДНОРОДНОДНОЙ МАГНИТОАКТИВНОЙ ПЛАЗМЫ

Н.А. Азаренков, В.К. Галайдыч
ХНУ им. В.Н. Каразина,
61077, пл. Свободы 4, Харьков, Украина

В данной работе решена задача дифракции низкочастотной электромагнитной волны, падающей из однородной магнитоактивной плазмы на металлической цилиндр, окруженный радиально неоднородным слоем плазмы. Внешнее магнитное поле параллельно оси цилиндра; волновой вектор падающей плоской волны перпендикулярен к этой оси. Плазма предполагается холодной и возмущения в ней описываются уравнениями Максвелла и двухжидкостными гидродинамическими уравнениями. Получены точные решения для полей для линейного и степенного профилей неоднородности плотности плазменного слоя. Сечения рассеяния вперед и назад как функция волнового вектора падающей волны получены для различных параметров задачи. Проанализированы зависимости этих сечений рассеяния от параметров задачи. Слой неоднородной плазмы увеличивает амплитуду колебаний сечения обратного рассеяния и изменяет в несколько раз величину сечения рассеяния вперед.

КЛЮЧЕВЫЕ СЛОВА: электромагнитная волна, дифракция, магнитоактивная плазма, неоднородная плазма, сечение рассеяния.