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TOPOLOGY OF SUPERBANANA ORBITS IN TOKAMAKS WITH TF RIPPLES

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The topology of banana guiding center orbits of fast ions in tokamaks with toroidal field (TF) ripples is considered. Analytical expressions determining the stagnation orbits and boundaries of regions with the closed orbits in the phase space are derived. Thoroughly studied is the modification of the topology of superbanana orbits due to the variation of the TF ripple magnitude δ. Contour plots of the adiabatic invariant of banana guiding center motion are presented for different values of δ. A comparison of the results of semi-analytical consideration and graphical interpretation of the adiabatic invariant is provided.
KEY WORDS: banana, superbanana, locally trapped particles, toroidal field ripples, adiabatic invariant of motion, tokamak

ТОПОЛОГИЯ СУПЕРБАНАНОВЫХ ОРБИТ В ТОКАМАХ С ГОФРИРОВКОЙ ТОРОИДАЛЬНОГО ПОЛЯ

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Рассматривается топология орбит ведущих центров бананов в токамаках при наличии гофрировки тороидального поля (ТП). Получены аналитические выражения, определяющие координаты особых точек орбит и границы областей с замкнутыми орбитами в координатах фазового пространства. Проанализировано изменение топологии супербанановых орбит, вследствие варьирования амплитуды гофров ТП. Построены линии равного уровня адиабатического инварианта движения ведущего центра банана для различных значений амплитуды гофрировки ТП. Проведено сравнение результатов квазианалитического рассмотрения и графического представления адиабатического инварианта.
КЛЮЧЕВЫЕ СЛОВА: банан, супербанан, локально запертые частицы, гофрировка тороидального магнитного поля, адиабатический инвариант движения, токамак

ТОПОЛОГІЯ СУПЕРБАНАНОВИХ ОРБІТ В ТОКАМАХ З ГОФРУВАННЯМ ТОРОЇДАЛЬНОГО ПОЛЯ

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Досліджується топологія орбіт ведучих центрів бананів у токамаках з гофруванням тороїдального поля (ТП). Отримано аналітичні вирази для координат особливих точок орбіт та границі областей із замкненими орбітами у координатах фазового простору. Проаналізовано зміну топології супербаннновых орбіт внаслідок варіювання амплітуди гофрування ТП. Побудовано лінії рівного значення адіабатичного інваріанту руху ведучого центру банана для різних значень амплітуди гофрування ТП. Проведено порівняння результатів квазіаналітичного розгляду та графічного представлення адіабатичного інваріанта.
КЛЮЧОВІ СЛОВА: банан, супербанан, локально заперті частинки, гофрування тороїдального магнітного поля, адіабатичний інваріант руху, токамак

In axisymmetric approximation the tokamak magnetic field is treated as a toroidally uniform one. However in reality, the toroidal field in tokamaks is corrugated due to the finite number of toroidal field coils. These toroidal field ripples are generally strongest and consequential for particle transport in the low-B side of a tokamak while being of less significance at the high-B side. Field perturbations of 1% are typical in the outer edge of a tokamak plasma and appear a few orders of magnitude smaller at the magnetic axis.

Toroidal field ripples are known to create secondary magnetic wells at the outer plasma edge, most dominantly in the vicinity of the mid-plane [1]. Particles trapped in these wells are subject to enhanced radial transport and hence poorly confined in the plasma. The criterion for the existence of secondary ripple wells in a circular tokamak with large aspect ratio is $\alpha \equiv \varepsilon|\sin\vartheta|/(Nq\delta) < 1$, where ε denotes the local inverse aspect ratio, ϑ the poloidal angle, N the number of toroidal field coils, q the safety factor and $\delta \equiv (B_{t\max} - B_{t\min})/(B_{t\max} + B_{t\min})$ the ripple amplitude. Note that at the plasma periphery, where δ exceeds the critical value δ_{GWB} given by the Goldstone-White-Boozer stochastisity threshold, toroidally trapped particles are nearly promptly lost from the plasma during a time small in comparison with Coulomb collision times [1,2]. Here we examine the ripple impact on toroidally trapped fast ions orbits with the banana tips in the core plasma region where there are no ripple wells and the ripple magnitude is below the stochasticity threshold, i.e. where $\alpha > 1$ and $\delta < \delta_{GWB}$. The most significant effect of TF ripples occurs for toroidally trapped fast ions which are in resonance with the ripple perturbations [6], i.e. for ions satisfying the resonance condition $l\omega_b - N\omega_d = 0, \quad l = 0, \pm 1, \pm 2, \ldots$, where ω_b and ω_d are the particle's bounce and toroidal precession frequencies. Such resonant toroidally trapped particles, tracing out so-called superbananas, are seen to undergo an even more increased radial diffusion and thus are responsible for a substantial share in TF ripple losses of energetic ions [6-7, 10-13].

We note that direct measurements of fast ions lost in ripple wells were performed on TFTR [4-7], JET [8, 5] and TORE SUPRA [9, 5]. Relevant theoretical treatment of fast ion ripple transport based on Fokker-Planck calculations (3D in constants-of-motion (COM) space) is found in some of our previous studies [6, 7, 10-13]. Other approaches were recently developed [18-19]. It should be mentioned that this problem of TF ripple induced transport stays actual nowadays, especially for ITER [14-17]. Other approaches are developed in [18-19] recently.

In this paper, we analyze the toroidally trapped particle motion using the adiabatic invariant on superbanana orbits [11]. This invariant can be represented as $p^2/2 - M\cos(\xi + p)\cos\psi = h = const$, where (p, ψ) denote the guiding center coordinates, ξ can be assumed as a constant parameter and $M \cong \delta(Nq)^{3/2}/\varepsilon$.

Our aim here is to derive analytical expressions describing the separatrices between the domains of closed orbits and of orbits with infinite motion in ψ [20]. Those expressions are necessary to provide an algorithm for the superbanana averaging procedure required for the derivation of the 3D Fokker-Planck equation in COM variables. Because of singularities of the thus averaged transport coefficients at the boundary between these domains [11] it is important to analytically treat these separatrices as modified by the value of M.

The paper is organized as follows. In the sections THE COORDINATES OF THE STAGNATION POINTS, THE TYPE OF THE STAGNATION POINTS the coordinates and the type of the stagnation points of the Hamiltonian h are determined. Further an effect of the ripple magnitude on the separatrices between the domains of closed orbits and of orbits with infinite motion in angular coordinate is analyzed for different values of parameter M in section SEPARATRICES BETWEEN THE DOMAINS OF CLOSED ORBITS AND OF ORBITS WITH INFINITE MOTION IN ψ. The conclusions are given in section SUMMARY AND DISCUSSION.

THE COORDINATES OF THE STAGNATION POINTS

As it was mentioned in Introduction the analysis of superbanana orbits can be reduced to consideration of 1D problem with effective Hamiltonian h given by following equation

$$h = p^2/2 - M\cos(\xi + p)\cos\psi, \tag{1}$$

where (p, ψ) is the banana guiding center coordinates in the phase space, M and ξ can be assumed as the constant parameter near the chosen resonance. Rather complex dependence $h(p, \psi)$ does not allow to find the explicit analytical expressions describing the stagnation points for arbitrary values of parameter M. However, required analysis can be carried out numerically.

To investigate the stagnation points of Hamiltonian h we calculate the following derivatives

$$\frac{\partial h}{\partial p} = p + M\sin(\xi + p)\cos(\psi), \tag{2}$$

$$\frac{\partial h}{\partial \psi} = M\cos(\xi + p)\sin(\psi), \tag{3}$$

$$\frac{\partial^2 h}{\partial p^2} = 1 + M\cos(\xi + p)\cos(\psi), \tag{4}$$

$$\frac{\partial^2 h}{\partial \psi^2} = M\cos(\xi + p)\cos(\psi), \tag{5}$$

$$\frac{\partial^2 h}{\partial p \partial \psi} = M\sin(\xi + p)\sin(\psi). \tag{6}$$

Solving the system of equations resulting from Eqs. (2,3)

$$p + M\sin(\xi + p)\cos(\psi) = 0 \tag{7}$$

$$M\cos(\xi+p)\sin(\psi)=0 \tag{8}$$

one can find the two groups of stationary points:
i -group

$$\psi_i=\pi i\,,\ (-1)^{i+1}\frac{p_i}{M}=\sin(\xi+p_i)\,, \tag{9}$$

and j -group

$$\cos\psi_j=\frac{(-1)^{j+1}}{M}\left(\frac{\pi}{2}-\xi+\pi j\right),\ p_j=\frac{\pi}{2}-\xi+\pi j\,, \tag{10}$$

where i and $j\in\mathbb{Z}$.

It follows from Eqs. (9), (10) that position of stagnation points in the phase space is essentially dependent on the values of M and ξ. Furthermore, j-points can exist only if $M>|\pi/2-\xi+\pi j|$. For qualitative analysis we use the graphic representation of Eqs. (9), (10) in Figs.1 and 2.

As it seen from Fig.1 the value of parameter M defines the inclination angel of the line given by equation $y(p_i)=\pm p_i/M$ and thus it determines the quantity of the intersections between the line and the sine graphs, i.e. the quantity of roots. The same conclusion for the j-group can be done from Fig.2 because only the lines with

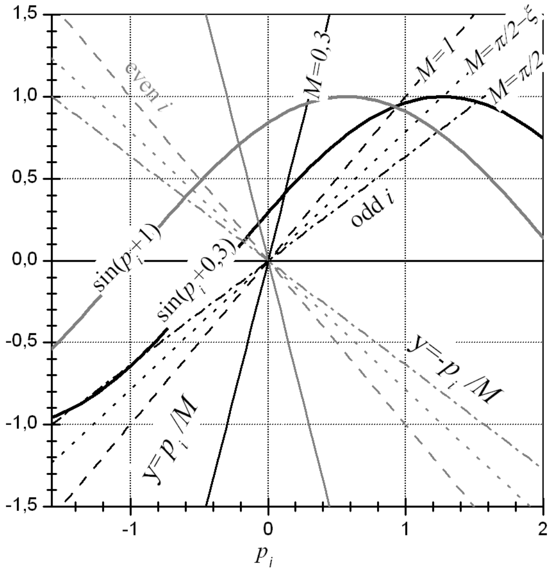

Fig. 1. Graphical representation of Eq.(9) determining *i*-points.

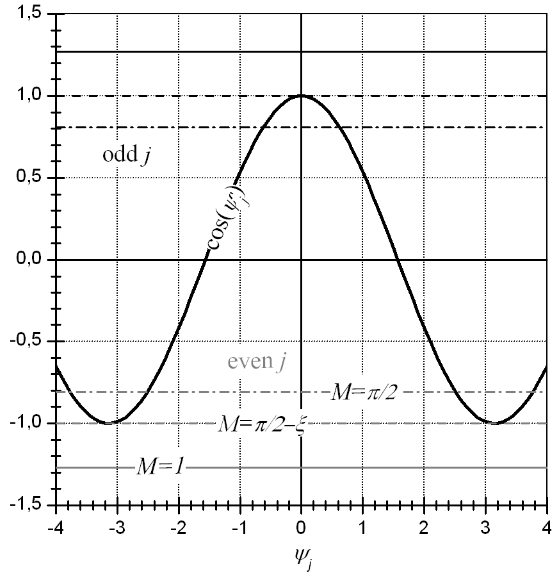

Fig. 2. Graphical representation of Eq.(10) determining *j*-points.

$M>(\pi/2-\xi+\pi j)$ can intersects the graph of the cosine.

It should be noted that under the certain values M the type of the i-th stagnation point changes. The particular feature of j-th stagnation points is that these points are placed by pairs around the stagnation point from i -group. The location symmetry of this pair resulting from the cosine parity is seen from Fig.2.

THE TYPE OF THE STAGNATION POINTS

To find out the type of stagnation points we investigate the sign of the determinant D given as

$$D=\frac{\partial^2 h}{\partial\psi^2}\frac{\partial^2 h}{\partial p^2}-\left(\frac{\partial^2 h}{\partial p\partial\psi}\right)^2. \tag{11}$$

Using the explicit expressions for derivatives (Eqs.(4)-(6)) and for stationary coordinates (Eqs. (9)-(10)) we represent this determinant as

$$D_i=M^2\cos^2(\xi+p_i)+M\cos(\xi+p_i)(-1)^i,\ \text{for } i \text{ -group} \tag{12}$$

and

$$D_j = -M^2 \sin^2(\psi_j)\text{, for } j\text{-group.} \tag{13}$$

Taking into account that $D > 0$ for the stable stagnation points (O-points) and correspondingly $D < 0$ for the unstable stagnation points (X-points) we concluded that j-group consists of X-points only, while i -group includes both O-points and X-points depending on the values of p_i. For example, for $M = 0,3$ even i corresponds to O-point and odd i to X-point.

To determine the maximums and minimums of h one should estimate the sign of the $\partial^2 h/\partial\psi^2$ or $\partial^2 h/\partial p^2$. In our case it is convenient to use $\partial^2 h/\partial\psi^2$:

$$\left.\frac{\partial^2 h}{\partial\psi^2}\right|_{(\psi_i, p_i)} = M\cos(\xi + p_i)(-1)^i. \tag{14}$$

In the mentioned above case $M = 0,3$ for $\psi_i = 0$ and $-\xi < p_i < 0$ ($i = 0$ is even) and therefore $\partial^2 h/\partial\psi^2 > 0$, i.e. this stagnation corresponds to minimum of h. In another case $M = \pi/2$ for $\psi_i = \pi$ and $\pi/2 - \xi < p_i < \pi - \xi$ (see Fig.1.) ($i = 1$ is odd) and therefore $\partial^2 h/\partial\psi^2 < 0$, i.e. this stagnation point corresponds to maximum of h.

It should be pointed out that the direction of rotation in domain with the maximum of h is opposite to the domains with the minimum of h. It can be easily seen from analyzing the sign of the $\dot{p} = -\partial h/\partial\psi$.

SEPARATRICES BETWEEN THE DOMAINS OF CLOSED ORBITS AND OF ORBITS WITH INFINITE MOTION IN ψ

Figure 3 displays the contour plots of $h(p,\psi)$ for the different M values.

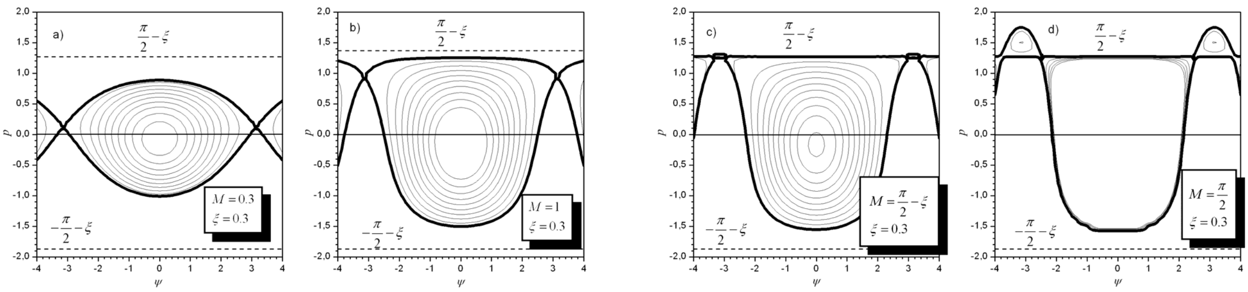

Fig. 3. Phase portraits of superbanana orbits determined by the Hamiltonian of Eq. ((1)) for the case of ξ=0.3 for different TF ripple parameters: (a) M=0.3, (b) M=1, (c) M=π/2-ξ and (d) M=π/2.

These graphs were obtained calculating h at the mesh points with the equal step on p and ψ with the magnitude 0,01 and following 2D splinting.

As it seen from carried out analysis for the small values of $M < \pi/2 - \xi$ there is only one domain with the closed orbits. These orbits correspond to the trapped bananas (superbananas). The later ones correspond to the passing bananas.

These different kinds of orbits are separated in the phase space by separatrix given by $h_{sep} = h(\pi, p_{i=1})$. In the general case the separatrix of the chosen domain is given by the following expression

$$h(\psi, p) = h_{sep}, \tag{15}$$

where $h_{sep} = h(\psi_X, p_X) - const$, (ψ_X, p_X) are the coordinates of the X-point of the chosen domain.

If $M > \pi/2 - \xi$ and therefore upper boundary of the separatrix exceeds value $\pi/2 - \xi$, another domain with the closed orbits is formed above line $p = \pi/2 - \xi$. It should be noted that the direction of rotation in this domain is opposite to the mentioned above domain. Continuing the increasing the value of M the new domains are formed. The direction of rotation in these new domains depends on kind of O-point: minimum h – clockwise rotation, maximum h – counter rotation.

SUMMARY AND DISCUSSIONS

The topology of banana guiding center orbits in phase space is considered. The domains with the different behavior of superbanana orbits are found (see Fig.3. d). The effect of the toroidal field ripple magnitude on the domain with the closed orbits formation, location and size is analyzed.

The coordinates of the stagnation points of these orbits are derived. It is shown that the quantity of stagnation points depends on the toroidal field ripple magnitude. The two qualitatively different groups of stagnation points are obtained. Eqs.(9) and (10). It is marked that stagnation points located on the horizontal lines $p=\pi/2-\xi+\pi j, j\in\mathbb{Z}$ have the threshold of existence, that is $M>\pi/2-\xi$.

The kinds of the stagnation points are determined. It is shown that the stagnation points located on the horizontal lines $p=\pi/2-\xi+\pi j, j\in\mathbb{Z}$ are X-points (Eq.(13)). And the stagnation points located on the vertical lines $\psi=\pi i, i\in\mathbb{Z}$ can be O-points and X-points depending on the values of M and ξ parameters (Eq.(12)). Besides that it is pointed out that the O-points corresponds to both the minimums and maximums of adiabatic invariant h and the direction of the rotation along the closed orbits are opposite in these two cases (Eq.(14)).

The analytical expressions for boundary of regions with the closed particles are derived (Eq.(15)).

The contour plots of the adiabatic invariant of motion for banana guiding center equation are presented for the set of the toroidal field ripple values. The comparison between the results of semianalytical consideration and graphical interpretation of the adiabatic invariant is provided. It is shown that the semianalytical analysis is in good agreement with the behavior of orbits in the phase space.

REFERENCES

1. Yushmanov P.N. // Reviews of Plasma Physics (Edited by M. A. Leontovich, Published by Consultants Bureau New York, London). – 1987. – Vol 16. – P. 117-191.
2. Goldston R.J., Towner H.H. Effects of toroidal field ripple on suprathermal ions in tokamak plasmas // Journal of Plasma Physics. – 1981. – Vol.26, № 02. – P. 283-307.
3. Goldston R.J., White R.B., Boozer A.H. Confinement of High-Energy Trapped Particles in Tokamaks // Physical Review Letters. – 1981. – Vol.47, № 9. – P. 647-649.
4. Zweben S.J., Budny R.V., Darrow D.S., Medley S.S., Nazikian R. et.al. Alpha particle physics experiments in the Tokamak Fusion Test Reactor // Nuclear Fusion. – 2000. – Vol.40, № 1. – P. 91-148.
5. Heidbrink W.W., Sadler G.J. The behaviour of fast ions in tokamak experiments // Nuclear Fusion. – 1994. – Vol.34, № 4. – P. 535-615.
6. Goloborod'ko V.Ya, Reznik S.N., Yavorskij V.A., Zweben S.Z. Fokker-Planck 3-D modeling of axisymmetric collisional of fusion products in TFTR // Nuclear Fusion. – 1995. – Vol.35, № 12. – P. 1523-1535.
7. Yavorskij V.A., Edenstrasser J.W, Goloborod'ko V.Ya. Reznik S.N., Zweben S.Z. Fokker-Planck modelling of delayed loss of charged fusion products in TFTR // Nuclear Fusion. – 1998. – Vol.38, № 10. – P. 1565-1576.
8. Saibene G., Sartori R., Mcdonald Dc, Beurskens M., Salmi A et. al. Results of the variable toroidal field ripple experiments in JET // 22th IAEA Fusion Energy Conference, Geneva, October 13-18. – 2008. – IAEA-CN-165/EX/2-1.
9. Assas S., Eriksson L.-G., Nguyen F., Basiuk V. On the asymmetry of ripple induced losses in ICRF heated Tore Supra discharges // Plasma Physics and Controlled Fusion. – 2003. – Vol.45, № 2. – P. 145-158.
10. Goloborod'ko V.Ya., Yavorskij V.A. Kinetic equation for high-energy particles of axisymmetric tokamaks in terms of constants of motion // Fusion Technology. – 1990. – Vol.18, № 11. – P. 414-424.
11. Yavorskij V.A., Andrushchenko Zh.N., Edenstrasser J.W, Goloborod'ko V.Ya. Three-dimensional Fokker-Planck equation for trapped fast ions in a Tokamak with weak toroidal field ripples // Physics of Plasmas. – 1999. – Vol.6, № 10. – P. 3853-3867.
12. Goloborod'ko V.Ya., Yavorskij V.A. On formulation of 3D boundary-value problem for the fast particle distribution function in axisymmetric tokamak plasmas // Ukrainian Journal of Physics. – 1997. – Vol. 42, № 2. – P. 143-152.
13. Yavorskij V.A., Schoepf K., Edenstrasser J.W, Goloborod'ko V.Ya. 3D Fokker Planck Model for High and Moderate Energy Ions in Weakly Ripple Tokamaks // 18th IAEA Fusion Energy Conference, Sorrento (Italy), October 4-10. – 2000. – IAEA-THP2/30.
14. Fasoli A., Gormenzano C., Berk H.L., Breizman B., Briguglio S. et al Chapter 5: Physics of Energetic Ions // Nuclear Fusion. – 2007. – Vol.47, № 6. – P. S264-S284.
15. Putvinski S.V. Physics of Energetic Particles in ITER // Nuclear Fusion. – 1998. – Vol.38, № 9. – P. 1275-1281.
16. Kurki-Suonio T., Asunta O., Hynönen V., Johnson T., Koskela T. Fast Particle Losses in ITER // 22th IAEA Fusion Energy Conference, Geneva, October 13-18. – 2008. – IAEA-CN-165/IT/P6-2.
17. Kramer G.J., White R.B., Nazikian R., Berk H.L. Fusion-born Alpha Particle Ripple Loss Studies in ITER // Princeton Plasma Physics Laboratory Preprint. – 2009. – March. – PPPL-4363.
18. Hideyuki M., Tsutsui H., Tsuji-iio S., Ryuichi S., Tani K. A Theoretical Model of Ripple Resonance Diffusion of Alpha Particles in Tokamaks(P1-35) // 18th International Toki Conference, Ceratopia Toki, Japan, December 9-12. – 2008. – ITC(18)-P1-35. – P. 261-264.
19. Hideyuki M., Tani K., Tsutsui H., Tobita K., Tsuji-iio S. et. al. Numerical Study of the Ripple Resonance Diffusion of Alpha Particles in Tokamaks // Plasma and Fusion Research. . – 2009. – Vol.4. – P. 008-1-008-8.
20. Yavorskij V.A., Moskvitin A.A., Moskvitina Yu.K., Goloborod'ko V., Schoepf K. Fokker-Planck description of TF ripple induced collisional transport of fast ions in tokamaks // 11th IAEA Technical Meeting on "Energetic Particles in Magnetic Confinement Systems" Kyiv, Ukraine, September 21-23. – 2009. – IAEA-P23, Book of Abstracts, p.73.