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THE QUANTUM TUNNEL EFFECT FROM THE POINT OF VIEW OF QUANTUM MECHANICS AND CLASSICAL PHYSICS

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Parallel analyses of matter penetrating a barrier are analyzed, one using electrodynamics, the other using quantum mechanics. Mathematical operations are performed to show the identical nature of the phenomena of reflection above the barrier and of penetration of the barrier in quantum mechanics and in classical electrodynamics. It is shown that, in reality, the tunneling effect is not a “purely quantum phenomenon”.
KEY WORDS: quantum mechanics, barrier, penetration, reflection, tunnel effect.

ТУННЕЛЬНЫЙ ЭФФЕКТ С ТОЧКИ ЗРЕНИЯ КВАНТОВОЙ МЕХАНИКИ И КЛАССИЧЕСКОЙ ФИЗИКИ
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Проведено параллельное рассмотрение прохождения материи через барьер методами электродинамики и квантовой механики. Математически строго получена абсолютная идентичность надбарьерного отражения и подбарьерного прохождения, как в квантовой механике, так и в классической электродинамике. Показано, что определение туннельного эффекта как «чисто квантового явления» – не соответствует действительности.
КЛЮЧЕВЫЕ СЛОВА: квантовая механика, барьер, прохождение, отражение, туннельный эффект.

ТУНЕЛЬНИЙ ЕФЕКТ З ТОЧКИ ЗОРУ КВАНТОВОЇ МЕХАНІКИ ТА КЛАСИЧНОЇ ФІЗИКИ
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Проведено паралельне дослідження проходження матерії через бар’єр методами електродинаміки та квантової механіки. Математично виключно отримана абсолютна ідентичність надбар’єрного відбиття та підбар’єрного проходження як у квантовій механіці так і в класичній електродинаміці. Показано, що визначення тунельного ефекту як «чисто квантового явища» – не відповідає дійсності.
КЛЮЧОВІ СЛОВА: квантова механіка, бар'єр, проходження, відбиття, тунельний ефект.

“In questions of science the authority of a thousand is not worth the humble reasoning of a single individual” (Galileo Galilee).

Over 450 years before the beginning of the Common Era, Empedocles said that everything on this planet was made up of earth, water, air, and fire. A century later, Aristotle supported and built upon this theory. Aristotle’s authority was so great that the validity of the statement was not doubted for over a millennium, until Robert Boyle did not put into motion the modern chemistry movement. As of today, an analogous scenario is playing out regarding the “purely quantum nature of the tunnel effect”. When quantum mechanics was only beginning to blossom as a science, it was said that the tunnel effect could not be encountered anywhere in classical mechanics. This correct, but narrow statement went from textbook to textbook, article to article, until it became known that the tunnel effect was a purely quantum phenomenon, a statement which has undermined the potential of the science. Lev Okun once said that “First of all, perverted information will inevitably lead to a mistake in some unusual situation. Second of all, a clear understanding of the basics of science is more important than mindless work with formulas and numbers” [1].

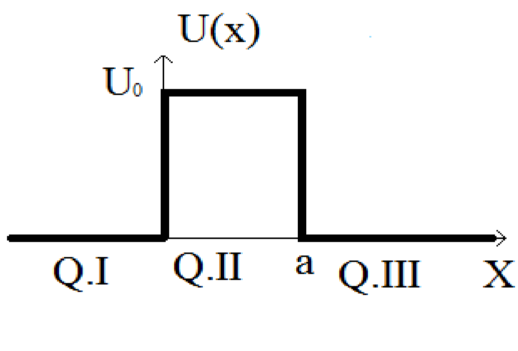
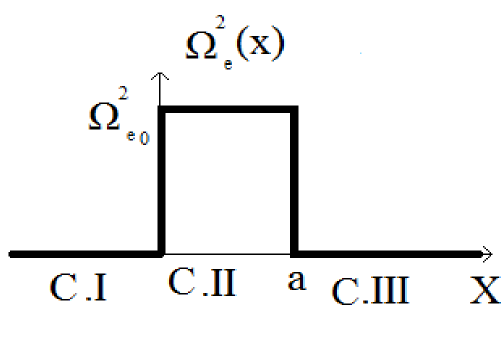
One of the most interesting applications of the De Broglie hypothesis, as well as the Schrodinger equation is that there is a certain probability that, when a particle adheres to quantum laws, it will occupy a space which it could not occupy according to the laws of classical mechanics. To illustrate this, let’s analyze a particle tunneling through a potential barrier. From the point of view of classical mechanics, a particle cannot pass through the barrier if its kinetic energy was lower than the height of the potential barrier. From the point of view of quantum mechanics, however, a particle can have any kinetic energy and still be able to pass through the barrier, albeit the lower the energy of the particle – the lower the probability of the particle succeeding. Starting from these interesting applications, the physicist George Gamow proposed the theory of alpha decay, in which he proposed that an alpha particle has a small chance of tunneling out of the potential well of the nucleus. From the point of view of classical physics, alpha-radiation was an unsolved riddle, since the kinetic energy of the alpha particles, according to calculations, was not enough to traverse outside of the nucleus' potential well. But the detail that stumped physicists of that era most was the process' complete unpredictability. The alpha particle is like a prisoner held in a deep hole, constantly trying to get out. From the point of view of classical mechanics, the prisoner's fate is sealed. But in quantum mechanics (and, as experimentally proven, in real life) the prisoner has a small chance of climbing outside of the hole. The problem, therefore, lies not in the ability of the prisoner to get out, but knowing which of his attempts will succeed. This is a general overview of the unpredictable behavior of particles adhering to the laws of quantum mechanics. It is also evident that this contradicts modern philosophical ideas, which state that every action has a clearly predefined reaction.

The study of determinism combined with the continuous nature of our surroundings has been a pillar of modern philosophy since the time of Spinoza. Albert Einstein, a big fan of Spinoza's work, was surprised by the random nature of quantum physics, and his unwillingness to accept them, put him at odds with Niels Bohr. In this situation, however, nature itself supported Bohr and the other founders of quantum mechanics in their conflict with Einstein. The tunnel effect seen in many experiments followed Schrodinger's equation precisely. Later, It was determined that we owe our very existence to this wonderful effect, as it plays a key role in the nuclear reactions that take place on the Sun. Einstein, however, was unmoved and kept with his opinions, while the scientific community moved on.

QUANTUM AND CLASSICAL MODELS AND THEIR RESPECTIVE SOLUTIONS

So, what is the truth behind the "purely quantum" nature of the tunnel effect? To answer this question, let's analyze the following situation: we have a particle cannon at $x \to -\infty$ which sends out matter (in the most general, philosophical meaning of the word, a meaning that is even more general than the relativistic definition of the word, which already encompasses substance and radiation) with the energy K.

The matter flies along the X axis, away from the cannon, and comes across a barrier, located on $0 \le x \le a$. Matter is neither destroyed nor created on the borders of the barrier. Let's analyze this well-known problem from the point of view of quantum mechanics (left) and classical physics (right).

Quantum point of view	Classical point of view
Let's assume that the matter shot out is of the quantum nature – electrons. Then, the potential barrier looks like this: $U(x)=\begin{cases}0, x\in]-\infty;0[\cup]a;+\infty[\\ U_0, x\in[0,a]\end{cases}$ (Q.1)	Let's assume that the matter shot out is of the classical nature – an electromagnetic wave, and the barrier is a layer of plasma with the frequency $\Omega_e^2=\frac{4\pi e^2 n_0}{m_e}$, where n_0 is the density of the electron component of the plasma, m_e is the mass of the electron. The plasma barrier will look like $\Omega_e^2(x)=\begin{cases}0, x\in]-\infty;0[\cup]a;+\infty[\\ \frac{4\pi e^2 n_0}{m_e}, x\in[0,a]\end{cases}$ (C.1)
 Fig. QP. 1. Q.I – space before the barrier, Q.II – the barrier's space, Q.III – space after the barrier	 Fig. CP. 1. C.I – space before the barrier, C.II – the barrier's space, C.III – the barrier's space
Since the potential energy only depends on one	The wave propagates along the X axis, and during it's

coordinate, Schrodinger's equation has the following form [1, 2]

$$\frac{\partial^2\psi(x)}{\partial x^2}+\frac{2m_e}{\hbar^2}[K-U(x)]\psi(x)=0 \qquad (Q.2)$$

where $\psi(x)$ is the wave function, characterized by the following formula:

$$\psi(x)=\begin{cases} e^{ikx}+Ae^{-ikx} & on \quad Q.I \\ Be^{ik_1x}+B'e^{-ik_1x} & on \quad Q.II \\ Ce^{ikx} & on \quad Q.III \end{cases}, \qquad (Q.3)$$

where $k^2=\frac{2m_e}{\hbar^2}K$, $k_1^2=\frac{2m_e}{\hbar^2}(K-U_0)$.

collision with the plasma layer Maxwell's electric field equation has the following form [3]

$$\frac{d^2E_y(x)}{dx^2}+\frac{1}{c^2}[\omega^2-\Omega_e^2(x)]E_y(x)=0 \qquad (C.2)$$

where $E_y(x)$ is the wave component perpendicular to the electrical wave component with the frequency ω, characterized by the following formula:

$$E_y(x)=\begin{cases} E_0e^{ikx}+Ae^{-ikx} & on \quad C.I \\ Be^{ik_1x}+B'e^{-ik_1x} & on \quad C.II \\ TE_0e^{ikx} & on \quad C.III \end{cases}, \qquad (C.3)$$

where $k^2=\frac{\omega^2}{c^2}$, $k_1^2=\frac{1}{c^2}(\omega^2-\Omega_{e_0}^2)$

Since matter is not created on and between $x=0$ and $x=a$ the probability density function and the flux density are both continuous. [1, 2]

$$\begin{cases} \psi_I(0)=\psi_{II}(0), & \frac{d\psi_I(0)}{dx}=\frac{d\psi_{II}(0)}{dx} \\ \psi_{II}(a)=\psi_{III}(a), & \frac{d\psi_{II}(a)}{dx}=\frac{d\psi_{III}(a)}{dx} \end{cases} \qquad (Q.4)$$

Since matter is not created on and between $x=0$ and $x=a$, the tangential component of the electric and magnetic field obtains a continuous nature [3]

$$\begin{cases} E_{yI}(0)=E_{yII}(0), & \frac{dE_{yI}(0)}{dx}=\frac{dE_{yII}(0)}{dx} \\ E_{yII}(a)=E_{yIII}(a), & \frac{dE_{yII}(a)}{dx}=\frac{dE_{yIII}(a)}{dx} \end{cases}, \qquad (C.4)$$

where $H_{z_j}(0,a)=-\frac{i}{k}\frac{dE_{y_j}(0,a)}{dx}$, $j=I,II,III$.

If one carefully analyzes the corresponding pairs of formulas (C.1) – (Q.1); (C.2) – (Q.2); (C.3) – (Q.3); (C.4) – (Q.4), then, from the point of view of mathematical formalism, they are absolutely identical, and therefore all future analyses also must be identical. All that remains is to test the validity of the approximations in which the analysis will be performed.

Performing simple mathematical calculations yields, along with [1, 2], the following formula for the calculation of the transmission coefficient

$$D_Q=\frac{1}{1+\left[\frac{k^2-k_1^2}{2kk_1}\sin k_1a\right]} \qquad (Q.5)$$

Performing several substitutions and transformations on [3] yields the transmission coefficient

$$D_c=|T|^2=\left|\frac{i2\sqrt{\varepsilon}}{(\varepsilon+1)\sin k_1a+i2\sqrt{\varepsilon}\cos k_1a}\right|^2=$$

$$=\frac{1}{1+\left[\frac{k^2-k_1^2}{2kk_1}\sin k_1a\right]^2}, \qquad (C.5)$$

where $\varepsilon=1-\frac{\Omega_{e_0}^2}{\omega^2}$.

The formula (Q.5) states that $D_Q\le 1$. When $k_1a=\pi n$, where $n=1;2;3...$, we have $D_Q=1$, meaning the barrier is completely transparent to the material with the discrete values $K_n>U_o$:

$$K_n=\frac{\pi^2n^2\hbar^2}{2m_ea^2}+U_0 \qquad (Q.6)$$

where $n=1;2;3...$

The formula (C.5) states that $D_c\le 1$. When $k_1a=\pi n$, where $n=1;2;3...$, we have $D_c=1$, meaning the barrier is completely transparent to the electromagnetic wave with the frequency $\omega_n^2>\Omega_{e_0}^2$

$$\omega_n^2=\frac{\pi^2c^2n^2}{a^2}+\Omega_{e_0}^2 \qquad (C.6)$$

where $n=1;2;3...$

RESULTS AND DISCUSSION THEREOF

All quantum mechanics textbooks are absolutely right in their statement that when $K < U_0$ we get $D_Q < 1$, but it exists and is positive nonetheless:

$$D_Q = \frac{1}{1 + \left[\frac{k^2 + \chi^2}{2k\chi} \sinh \chi a \right]^2} \quad , \tag{Q.7}$$

where $k_1 = i\chi = i\sqrt{\frac{2m_e}{\hbar^2}(U_0 - K)}$, meaning that there exists a possibility of the penetration of the "barrier" of potential energy by the "matter", whose energy is lower than that of the barrier.

This phenomenon is called:

- «In classical mechanics, the potential barrier is impregnable by particles; in quantum mechanics, however, a particle can, with a non-zero probability, move "through the barrier" (this phenomenon is called quantum tunneling). … The penetration of a potential barrier is an example of a process that **cannot occur in classical mechanics.**» [1].
- «The wave in the first region corresponds to a particle trying to get into the second region, but the amplitude there falls off rapidly. There is some chance that it will be observed in the second region—where **it could never get classically**…This effect is called the **quantum mechanical "penetration of a barrier."**» [4].
- «The fact that D does not vanish for $K < U_0$ is a **purely quantum mechanical result**. This phenomenon of particles passing through barriers higher than their own incident energy is known as *tunneling.*» [5].

All of this poses an obvious question: «Why is the existence of a CLASSICAL TUNNEL EFFECT deemed impossible? ». After all, if $\omega^2 < \Omega_{e_0}^2$ then $k_1 = i\chi = i\sqrt{\frac{1}{c^2}(\Omega_{e_0}^2 - \omega^2)}$, which, when taken into account, transforms (C.5) into

$$D_c = \frac{1}{1 + \left[\frac{k^2 + \chi^2}{2k\chi} \sinh \chi a \right]^2} \quad , \tag{C.7}$$

which is identical to (Q.7). Likewise, there is an exponential decay of the amount of «matter» (the electromagnetic wave) through the «barrier» with an effective potential energy that is larger than the energy (frequency) of the «matter» itself.

For a clearer understanding of the situation, let's analyze, from both points of view, the situation of a wide barrier $\chi a \gg 1$, where $\sinh \chi a \approx \frac{1}{2} e^{\chi a}$, and the transmission coefficients transform into

$D_Q = \frac{16K}{U_0}\left(1 - \frac{K}{U_0}\right) e^{-\frac{2a}{\hbar}\sqrt{2m(U_0 - K)}}$ (Q.8)	$D_c = 16\frac{\omega^2}{\Omega_{e_0}^2}\left(1 - \frac{\omega^2}{\Omega_{e_0}^2}\right) e^{-\frac{2a}{c}\sqrt{\Omega_{e0}^2 - \omega^2}}$ (C.8)

The formula (Q.8) gives us the value of the transmission coefficient of the quantum object – the electron - through the wide barrier with energy higher than the energy of the electron itself. The quantum object is a wave-particle from the point of view of quantum mechanics and therefore has the energy

$$K = \frac{p^2}{2m} \tag{Q.9}$$

and, correspondingly, the de Broglie wavelength

$$\lambda = \frac{2\pi\hbar}{p} \tag{Q.10}$$

and with the corresponding de Broglie frequency

$$\omega_Q = \frac{2\pi v}{\lambda}. \tag{Q.11}$$

Using the substitutions found in (Q.11) and (Q.10) on (Q.9), we see that

$$K = \frac{\hbar^2 \omega_Q^2}{2mv^2}. \tag{Q.12}$$

Then we can assign an effective barrier frequency Ω_Q , as a result of which

$$U_0 = \frac{\hbar^2 \Omega_Q^2}{2mv^2}. \tag{Q.13}$$

Using the substitutions found in (Q.12) and (Q.13) on (Q.8), we see that

$$D_Q = 16 \frac{\omega_Q^2}{\Omega_Q^2} \left(1 - \frac{\omega_Q^2}{\Omega_Q^2} \right) e^{-\frac{2a}{v}\sqrt{\Omega_Q^2 - \omega_Q^2}}. \tag{Q.14}$$

The only difference between (Q.14) and (C.8) lies in the following: if an electron has the velocity v, then the electromagnetic wave propagates with the speed of light c, while the effective barrier frequency Ω_Q from (Q.14) , in the case of a plasma layer, becomes the plasmon frequency and corresponds to Ω_{e_0} .

Analogously, (C.8) yields (Q.8), in which $m = \frac{\hbar\omega}{c^2}$, and where K and U_0 are (Q.12) and (Q.13) respectively, where $v = c$.

The above statements make it clear that the tunnel effect ((Q.7), (C.7)) and reflection above the barrier ((Q.5), (C.5)) both exist in quantum mechanics, as well as in classical electrodynamics. The tunnel effect that the particles experience is a result of their wave-like nature, which is inseparable from their corpuscular nature. The misunderstanding relating to the “purely quantum nature of the tunnel effect” is most likely linked to the fact that in classical electrodynamics the phenomenon would only be observed much later, when intensive study of plasma began.

The non-zero $\hbar(\hbar \neq 0)$ leads to a uniform description of quantum particles (ones that experience wave-particle duality) in quantum physics, which are separated in classical physics. It is also important to remember that waves behave similarly in quantum mechanics and classical physics. Since there is a phenomenon similar to the tunnel effect in classical physics for waves, the tunnel effect is not purely a quantum one. A visual example of this is the situation where a particle cannon is sending out quantum objects, that tunnel through a barrier when $\hbar \neq 0$, the objects being photons, which have mass, impulse, concentration and so on. Correspondingly, when $\hbar = 0$, tunneling should not take place, a statement which contradicts the conclusions of the classical wave theory seen above.

By the way, the analogy of the tunnel effect for alpha-particles radiating from a nucleus is, in classical wave physics, the radiation of waves with the frequency ω by a generator located in bounded plasma with the plasma frequency $\Omega_{e_0} > \omega$. In this case the wave experiences the skin effect while in the plasma, but a small part of the wave reaches the border of the plasma and radiates.

Therefore, in the quantum realm, particles and waves behave analogously when faced with high energy barriers $\{K < U_0\}$ and low energy barriers $\{K > U_0\}$, as a result of wave-particle duality. The tunnel effect for waves, however, is not bound to the quantum realm – it is also observed when $\hbar = 0$. In reality, the entire misconception about the “purely quantum nature” owes its existence to the wave-particle duality which was used to explain the tunnel effect when it was first observed, and not giving due thought to the possibility of its existence in other realms of science.

Nature has much more unity than is first made apparent by analysis of individual phenomena and the methods of analysis that we are limited to.

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