Journal of Mechanical Engineering. 2005. N5(56)

MODELLING OF LAMINATED GROWING BIOLOGICAL MATERIALS

Kizilova N.N., Egorova E.S.

The continued model of the multilayered plate from a growing biological material is presented. Introducing the Airy's stress function the problem is reduced to differential equation of the fourth order. Solution of the problem is obtained for the two-layered plate with a rectangle cross-section. For some relations between the rheological coefficients of the model the governing equations are simplified and the corresponding theoretical estimations are obtained. Numerical calculations of the stress and velocity fields for the model parameters that correspond to the growing plant leaves are presented. Any difference in own growth rates of the adjacent layers leads to stretching of the tempered layer and compressing of the rapidly growing one that results in growth acceleration of the former and growth retardation of the latter. Thus the mechanical stress field can regulate coordinated growth of the laminated biological materials.

Key words: continuous media, growing biological materials, laminated composites.

INTRODUCTION

Growing material is a new sort of composite materials whose mass increases with the time due to absorption of new substances on the distributed system of inner surfaces and on the outer surfaces of the body. The models of growing composites resemble plant and animal tissues and new substances are delivered by liquid flow which is provided by special long-distance conducting systems. During individual growth and development, natural materials obtain optimal mechanical properties [1,2] which can be thoroughly investigated and embedded then into artificial composite materials to improve their mechanical strength and durability. The growing materials can absorb new mass in accordance with principals of the stress tensor that leads to strengthening of the growing body in the line of the external loading and formation of lightweight materials. Engineering applications of the new sort of adaptive materials might very useful.

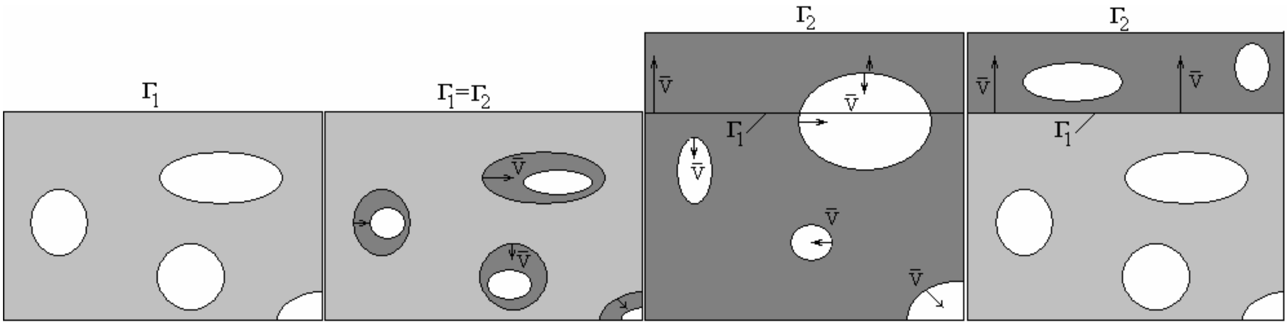
,

Mechanical factors play an important role in the growth of plant and animal tissues. Stimulating role of the stretching loadings and oppressing role of the compressed ones have been confirmed in numerous experiments [3,4]. Applied to the bones the phenomenon has been used in clinical practice in Ilisarov-apparatus for ostheosynthesis. The method can control the growth and shape of the bone. For plant tissues the role of the mechanical factors in control over the growth processes has not been sufficiently investigated. In contemporary mechanics the growing materials are considered as two-phased viscoelastic continuous media which consist of a deformable porous skeleton (totality of the cell walls) filled with viscous liquids (intracellular and extracellular fluids). The extracellular fluid that is transported through the conducting elements of the plant provides continuous flux of new substances which is absorbed by the porous skeleton (onto the distributed system of surfaces of the solid phase) due to biochemical reactions.

Three different types of growth can be distinguished in biological materials [4]:

1. Inner growth, when mass increase is observed on the inner surfaces of the pores only. The specimen is at zero-stress state ($\sigma = 0$) in this case and its volume does not change. The porosity θ decreases due to mass deposition on the surface of the pores (fig.1 a,b). During inner rebuilding the substances can be removed from one pore to others at $\theta = \text{const}$.
2. Volumetric growth, when mass increase of each infinitesimal element leads to deformation of the adjoining elements and the whole specimen (fig.1 ,c). The inner structure of the body can be deformed due to stress field $\sigma \neq 0$. The shape of pores can be changed by growth deformation at $\theta = \text{const}$.
3. Surface growth, when mass increase takes place on the outer surface of the specimen only. Deformation of the sample is absent, $\sigma = 0$.

In the nature different types of growth are usually mixed. For instance the surface growth can be accompanied by inner re-building of body structure as well as the volumetric growth can be combined with mass deposition on the outer surface and distributed inner surfaces of the porous body. The same classification is valid for the opposite case of mass decrease.



a b c d

Fig. 1. Main types of growth of biological media: inner growth (b), volumetric growth (c), surface growth (d); (a) is initial cross-section of the specimen, $_1$ and $_2$ are outer surfaces at different periods of time, $\vec{V}$ is growth velocity. Initial material and new-grown one are highlighted by light-grey and dark-grey respectively.

Mechanical processes are involved in growth, development and reconstruction of plant tissues. In the pictures of cross-sections of growing plant leaves at different development stages some separate cellular layers can be distinguished (fig.2). The outer layers (1,4 in fig.2) consist of epidermal cells. During their growth the cells elongate in the direction of the surface of the leaf blade. The inner layers (2,3 in fig.2) make the palisade parenchyma (2 in fig.2) and sponge parenchyma (3 in fig.2) which have different porosity of the layers during the latest growth stages at the expense of cell separation and elongation in different directions. Growth rates of adjacent layers differ at certain growth stages. No-slip conditions for the adjacent cells of different layers lead to interaction of the layers by means of stress field [5-6]. At early stages of leaf development the fast division and elongation of the cells of the outer layers promote frequent divisions of the cells of the inner layers without increasing porosity that lead to elongation of the palisade cells in the perpendicular direction of the leaf surface (fig 2a,b). At later stages of leaf development the fast elongation of the cells of the outer layers results in separation of parenchyma cells and increasing porosity of the inner layers (fig.2 b,c) [7-8]. In that way mechanical stress field defines the coordinated growth of adjacent cellular layers and formation

’

of special inner structure of intercellular space which is important for appropriate water evaporation from the outer cell walls and gas exchange during physiological cellular processes. In present paper the continued model of a growing laminated plate which corresponds to growing plant leaves is investigated.

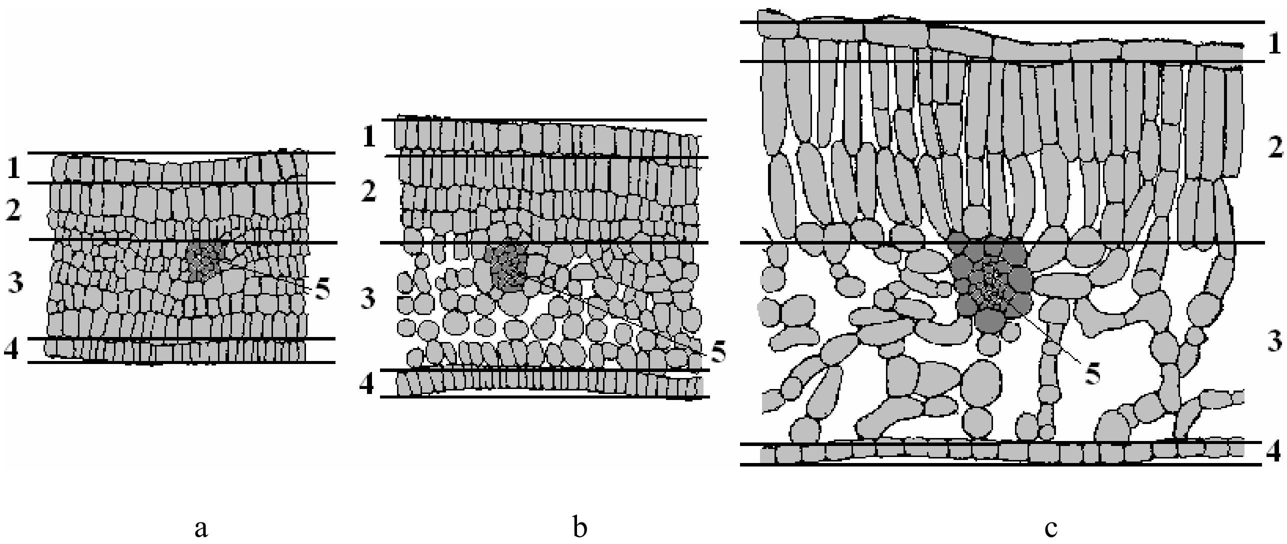


Fig.2. Transverce cross-sections of the growing leaf at different growth stages (a,b,c). Here 1,4 are epidermal layers, 2 is the palisade parenchyma, 3 is the sponge parenchyma, 5 is cross-section of the conducting element.

GOVERNING EQUATIONS

The strain tensor is considered as an additive function of elastic ε_{ik}^{e} and growth ε_{ik}^{g} strain tensors. The elastic strain obeys Gook’s law $\varepsilon_{ik}^{e} = E_{iklm}^{-1}\sigma_{lm}$, where E_{iklm} is elastic module tensor, σ_{lm} is stress tensor. According to various experimental data [4] the growth strains obey linear strain rate - stress dependence $\frac{D}{Dt}\varepsilon_{ik}^{g} = A_{ik} + B_{iklm}\sigma_{lm}$, where A_{ik} is tensor of own growth rates (growth rates at zero-stress conditions $\sigma_{lm} \equiv 0$), B_{iklm} is matrix of reverse viscosity coefficients, $[B_{iklm}] = (Pa \cdot s)^{-1}$, D/Dt is time derivative. Rheological parameters A_{ik}, B_{iklm} for a given growing material have to be defined by solving the corresponding inverse problem.

Rheology of growing material corresponds to Maxwell model with additional term A_{ik} which defines zero-stressed growth. The model has been used for investigation of growth of the plant root as a thin long cylinder [9-10], a monolayer of cells [11-12] and multilayered round plate as a model of plant leaf [13]. Finally we obtain the constitutive relation for the growing material in the form:

$$v_{ik} = A_{ik} + B_{iklm}\sigma_{lm} + \frac{D}{Dt}\left(E^{-1}_{iklm}\sigma_{lm}\right) \tag{1}$$

where $v_{ik} = (\partial V_i/\partial x_k + \partial V_k/\partial x_i)/2$ is strain rate tensor, $\vec{V}$ is velocity of the growth deformations.

We consider a two-dimensional problem for the growing plate which consists of two rectangular layeres $S_1 = \{x \in [0, L(t)], y \in [0, h(t)]\}$ and $S_2 = \{x \in [0, L(t)], y \in [-h(t), 0]\}$ (fig.3) with different rheological properties. The axis 0x is chosen as a border between the layers. Due to large charachteristic growth time the growth acceleration in the momentum equation and momentary elastic deformations in (1) are omitted and in the absence of outer forces the governing equations have been taken to be the following:

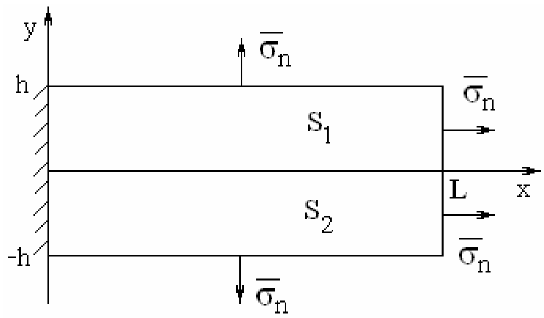


Fig.3. A two-dimensional model of a two-layered growing plate.

$$\frac{\partial \sigma^j_x}{\partial x} + \frac{\partial \sigma^j_{xy}}{\partial y} = 0 \tag{2}$$

$$\frac{\partial \sigma^j_{xy}}{\partial x} + \frac{\partial \sigma^j_y}{\partial y} = 0 \tag{3}$$

$$\frac{\partial V^j_x}{\partial x} = A^j_{11} + B^j_{11}\sigma^j_x + B^j_{12}\sigma^j_y \tag{4}$$

,

$$\frac{\partial V_y^j}{\partial y} = A_{22}^j + B_{21}^j \sigma_x^j + B_{22}^j \sigma_y^j \tag{5}$$

$$\frac{\partial V_x^j}{\partial y} + \frac{\partial V_y^j}{\partial x} = 2(A_{12}^j + B_{33}^j \sigma_{xy}^j) \tag{6}$$

where $j = 1,2$ are numbers of the layers, V_x^j, V_y^j are velocity components, σ_{ik}^j are stress tensors, A_{ik}^j are own growth rates, B_{ik}^j are inverse viscosity tensors of the layers. Stretching leads to growth retardation down to growth cessation but not to mass decrease that means $B_{ik}^j = 0$ when $\left|\sigma_{ik}^j\right| < 0$. The boundary conditions are presented by no-slip conditions between the layers, zero-loading on free surfaces of the body and fastening on the surface $x = 0$ which correspond to the place of attaching of the leaf blade to the petiole:

$$\sigma_x^j\Big|_{x=L} = 0 \qquad \sigma_y^j\Big|_{y=\pm h} = 0 \qquad \sigma_{x,y}^1\Big|_{y=0} = \sigma_{x,y}^2\Big|_{y=0} \tag{7}$$

$$V_y^j\Big|_{y=0} = 0 \qquad V_x^1\Big|_{y=0} = V_x^2\Big|_{y=0} \qquad V_x^j\Big|_{x=0} = 0 \tag{8}$$

ANALYSIS OF ZERO-STRESS GROWTH

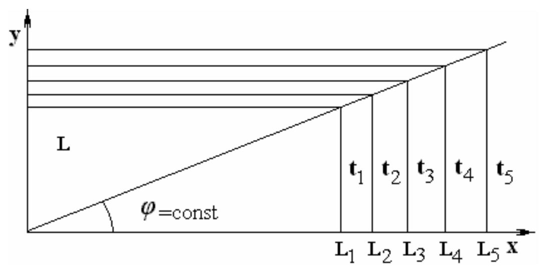
For zero-stress growth when $\sigma_{ik}^j = 0$ and (2)-(3) is identically valid, the relations (4)-(6) can be considered as a system of equations for the velocity components V_x^j, V_y^j:

$$\frac{\partial V_x^j}{\partial x} = A_{11}^j, \qquad \frac{\partial V_y^j}{\partial y} = A_{22}^j, \qquad \frac{\partial V_x^j}{\partial y} + \frac{\partial V_y^j}{\partial x} = 2A_{12}^j \tag{9}$$

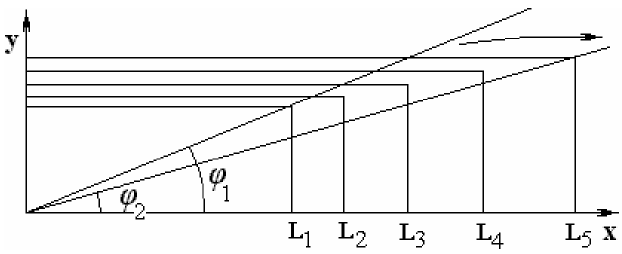
Three relations (9) for two unknown values V_x^j, V_y^j give strain rate compatibility condition which is the equation for components A_{ik}^j:

$$\frac{\partial^2 A_{11}^j}{\partial y^2} + \frac{\partial^2 A_{22}^j}{\partial x^2} - 2\frac{\partial^2 A_{12}^j}{\partial x \partial y} = 0 \tag{10}$$

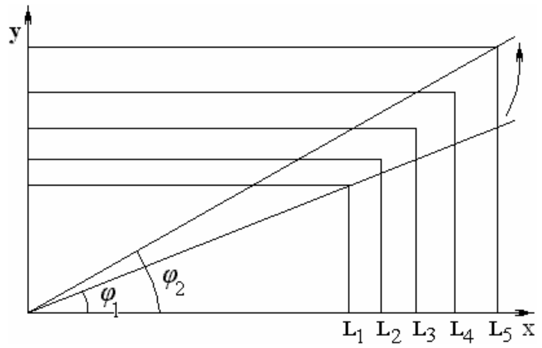
Relation (10) is valid for uniform isotropic growth when $A^j_{11} = A^j_{22}$, $A^j_{12} = 0$ (fig.4a) and for uniform anisotropic growth when $A^j_{11,22} = \text{const}$, $A^j_{11} \neq A^j_{22}$, $A^j_{12} = 0$ (fig.4 b,c) that correspond to growth of different types of leaves (fig.5a,b). Consequent shapes in fig.4 have been obtained at a given time step $\Delta t = \text{const}$. Growth displacement (u_x, u_y) of separate points of the consequent shapes in fig.4 have been obtained by integrating (4)-(5):

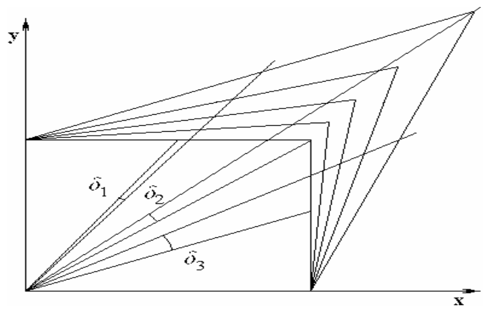


a



b





d

Fig.4. Different types of uniform isotropic growth $A_{11} = A_{22} = A = \text{const}$ (a), anisotropic growth $A_{11} > A_{22}$ (b) and $A_{11} < A_{22}$ (c), unreal type of growth at $A_{11}=y^2$, $A_{22}=x^2$, $A_{12}=0$ (d).

$$u_x(x,y) = \Delta t \int A_{11}(x,y)dx, \qquad u_y(x,y) = \Delta t \int A_{22}(x,y)dy \tag{11}$$

where $A_{11,22}$ are growth rates of the plant material in x and y directions. The first type of the anisotropic growth ($A_{11} > A_{22}$) describes gradual elongation of the leaf (fig.5a) whereas the second type ($A_{11} < A_{22}$) corresponds to growth unfolding of the basal part of the leaf blade (fig.5b). In these three cases growth deformations are similarity transformations (homothety) that is

,

inherent to many real natural bodies and tissues [1,2]. When the components A_{ik} do not satisfy (10) one can obtain non-realistic growth deformations (11) of the initial leaf-like shape (fig.4d) that look similar to growth of leaves of genetically modified plants [14-15]. In the last case the angular displacements of different points are different in both values and directions (angles δ_{1-3} in fig.4d).

A CASE OF GROWTH WHEN THE SHAPE OF THE GROWING BODY REMAINS RECRTANGULAR

We consider now the growth of the two-layered sample (fig.3) when the rectangular shape is maintained by cell divisions in the direction of the system axes 0x, 0y only. In that case the newly-grown cellular walls are always parallel to the borders of the layers. That kind or growth is possible when

$$\sigma_x^j = \sigma_x^j(x), \ \sigma_y^j = \sigma_y^j(y) \tag{12}$$

Applying the derivative $\partial/\partial x$ to (1) and $\partial/\partial y$ to (2) and subtracting the two equations we obtain the condition of existence of the solution in the form (12):

$$\frac{\partial^2 \sigma_x^j}{\partial x^2} = \frac{\partial^2 \sigma_y^j}{\partial y^2} = a^j = \text{const}$$

Taking into account (6) we obtain the solution:

$$\sigma_x^j = a^j(L^2 - x^2) + b^j(L - x)$$

$$\sigma_y^j = a^j(h^2 - y^2) - c^j(y \mp h) \tag{13}$$

$$\sigma_{xy}^j = 2a^j xy + a^j x + b^j y + d^j$$

Substituting (13) in (4)-(6) and integrating the first relation by x and the second one by y we have:

$$V_x^j = \int_0^x A_{11}^j dx + B_{11}^j a^j L^2 x - \frac{1}{3} B_{11}^j a^j x^3 + B_{11}^j b^j Lx - \frac{1}{2} B_{11}^j b^j x^2 + B_{12}^j a^j (h^2 - y^2) x -$$
$$- B_{12}^j c^j x (y \mp h) + f_1^j(y) \tag{14}$$
$$V_y^j = \int_0^y A_{22}^j dy + B_{21}^j a^j y (L^2 - x^2) + B_{21}^j b^j y (L - x) + B_{22}^j a^j h^2 y - \frac{1}{3} B_{22}^j a^j y^3 -$$
$$- B_{22}^j c^j \frac{y^2}{2} \pm B_{22}^j c^j hy + f_2^j(x)$$

Substituting (14) into (6) we get the condition of existence of (12):

$$2B_{33}^j + B_{12}^j + B_{21}^j = 0 \tag{15}$$

Then from (14) and the boundary conditions (8) we obtain

$$\sigma_x^j = a^j (L^2 - x^2), \quad \sigma_y^j = a^j (h^2 - y^2), \quad \sigma_{xy}^j = 2a^j xy$$

$$V_x^j = \int_0^x A_{11}^j dx + B_{11}^j a^j L^2 x \left(1 - \frac{x^2}{3}\right) + B_{12}^j a^j x (h^2 - y^2) \tag{16}$$

$$V_y^j = \int_0^y A_{22}^j dy + B_{21}^j a^j y (L^2 - x^2) + B_{22}^j a^j y \left(h^2 - \frac{y^2}{3}\right)$$

When $A_{11}^j = A_{22}^j = const$, then continuity conditions (8) for the velocities V_x^j at $y = 0$ gives $a^j \neq 0$ when $B_{11}^1 / B_{11}^2 = B_{12}^1 / B_{12}^2$ only. In the case stress tensor components are continuous $< \sigma_{x,y} > \equiv (\sigma_{x,y}^1 - \sigma_{x,y}^2)\big|_{y=0} = 0$ at the border between the layers. Otherwise $< \sigma_y > = a^1 h^2 (1 - B_{11}^1 / B_{11}^2) \neq 0$. When $A_{11}^j \neq A_{22}^j$, then the component $\sigma_x(x,y)$ is not continuous and $< \sigma_x > = (A_{11}^1 - A_{11}^2) x \neq 0$. Experimental observations of kinematics of growth of different types of leaves [2,6,12] revealed three types of growth (fig.4a-c) when $A_{11,22}^j$ are constant values. As a theoretical case any other expressions $A_{11,22}^j(x, y)$ may be substituted in (16) and the values for a^j can be obtained from the continuity conditions (8).

’

GENERAL SOLUTION OF THE PROBLEM

When (15) is not valid the general solution of (2)-(8) can be obtained. Introducing Airy's stress function Φ^j for the layers and expressions for the components of stress tensor

$$\sigma_x^j=\frac{\partial^2\Phi^j}{\partial y^2}\quad \sigma_y^i=\frac{\partial^2\Phi^j}{\partial x^2}\quad \sigma_{xy}^j=-\frac{\partial^2\Phi^j}{\partial x\partial y}$$

we obtain from (4)-(6) the equation for Φ as:

$$B_{11}^j\frac{\partial^4\Phi^j}{\partial y^4}+(B_{12}^j+B_{21}^j+2B_{33}^j)\frac{\partial^4\Phi^j}{\partial x^2\partial y^2}+B_{22}^j\frac{\partial^4\Phi^j}{\partial x^4}+$$
$$+2\left(\frac{\partial B_{11}^j}{\partial y}\frac{\partial^3\Phi^j}{\partial y^3}+\frac{\partial B_{22}^j}{\partial x}\frac{\partial^3\Phi^j}{\partial x^3}+\frac{\partial(B_{12}^j+B_{33}^j)}{\partial y}\frac{\partial^3\Phi^j}{\partial x^2\partial y}+\frac{\partial(B_{21}^j+B_{33}^j)}{\partial x}\frac{\partial^3\Phi^j}{\partial x\partial y^2}\right)+ \tag{17}$$
$$\left(\frac{\partial^2 B_{22}^j}{\partial x^2}+\frac{\partial B_{12}^j}{\partial y^2}\right)\frac{\partial^2\Phi^j}{\partial x^2}+\left(\frac{\partial^2 B_{21}^j}{\partial x^2}+\frac{\partial^2 B_{11}^j}{\partial y^2}\right)\frac{\partial^2\Phi^j}{\partial y^2}+2\frac{\partial^2 B_{33}^j}{\partial x\partial y}\frac{\partial^2\Phi^j}{\partial x\partial y}=0$$

After integrating (17) the unknown coefficients have to be defined from the boundary conditions that can be obtained from (7)-(8) as:

$$x=0:\qquad \int\left(A_{11}^j+B_{11}^j\frac{\partial^2\Phi^j}{\partial y^2}+B_{12}^j\frac{\partial^2\Phi^j}{\partial x^2}\right)dx=0$$

$$x=L:\qquad \frac{\partial^2\Phi}{\partial y^2}=0$$

$$y=\pm h:\qquad \frac{\partial^2\Phi}{\partial x^2}=0 \tag{18}$$

$$y=0:\qquad \int\left(A_{22}^j+B_{21}^j\frac{\partial^2\Phi^j}{\partial y^2}+B_{22}^j\frac{\partial^2\Phi^j}{\partial x^2}\right)dy=0$$

$$\int\left(A_{11}^1+B_{11}^1\frac{\partial^2\Phi^1}{\partial y^2}+B_{12}^1\frac{\partial^2\Phi^1}{\partial x^2}\right)dx=\int\left(A_{11}^2+B_{11}^2\frac{\partial^2\Phi^2}{\partial y^2}+B_{12}^2\frac{\partial^2\Phi^2}{\partial x^2}\right)dx$$

Direct solution of the problem (17)-(18) as applied to real animal and plant tissues is difficult because of the absence of the reliable values B_{ik}. It might be reasonable that $B_{12}/B_{11} \le 1$, $B_{21}/B_{22} \le 1$ which means that any stretching accelerates growth in the same direction. We can estimate the order of values B_{ik} assuming that all terms in (4)-(6) have the same order so as $A_{ik} \sim B_{ik}\sigma^*$ where σ^* is the threshold stress which influences growth processes at cellular level. For plant leaves the characteristic values $A^* \sim 0.1\ \text{day}^{-1}$ [2,5,6] and $\sigma^* \sim 0.03–0.05$ MPa [16] that give $B^* \sim (2-4)\cdot 10^{-11}\ (\text{Pa}\cdot\text{s})^{-1}$. Different forms of dependencies $B_{ik}(x,y)$ in (17)-(18) can be considered as theoretical cases only because the real dependencies are connected with the inner structure of the body and not investigated yet. When the materials of the layers are kept uniform during the growth then B^j_{ik} are constant values and (17) can be reduced to the equation

$$B^j_{11}\frac{\partial^4\Phi^j}{\partial y^4} + B^j\frac{\partial^4\Phi^j}{\partial x^2\partial y^2} + B^j_{22}\frac{\partial^4\Phi^j}{\partial x^4} = 0 \tag{19}$$

where $B^j = B^j_{12} + B^j_{21} + 2B^j_{33}$. We consider here the self-similar growth when the initial rectangular cross-section of the plate (fig.3) remains rectangular with $L = L(t), H = 2h(t)$ and $\sigma^j_{x,y} = \sigma^j_{x,y}(x,y)$, $\tau^j = \tau^j(x,y)$. The solution of (19)-(18) can be obtained as expansion

$$\Phi(x,y) = \Phi_0(y) + \sum_{k=1}^{\infty}\Phi_k(y)\cos\beta_k x \tag{20}$$

where $\beta_k = (\pi/2 + \pi k)/L$. Then the second condition (18) is satisfied when $\Phi_0^{//} = 0$. Substituting (20) into (19) we get the equations for unknown functions $\Phi^j_k(y)$ in the form:

$$B^j_{11}(\Phi^j_k)^{////} - B^j\beta_k^2(\Phi^j_k)^{//} + B^j_{22}\beta_k^4 = 0 \tag{21}$$

The solution of (21) is

’

$$\Phi_k^j = \sum_{r=1}^{4} C_{rk}^j \exp(\lambda_{rk}^j y), \qquad \lambda_{rk}^j = \pm\beta_k \left(\frac{1}{2B_{11}^j} \left(B \pm \sqrt{B - 4B_{11}^j B_{22}^j} \right) \right)^{0.5} \tag{22}$$

Substituting (22) into (18) we obtain for the unknown parameters C_{rk}^j the next relations:

$$\begin{aligned} P_k^j = {} & (U_{1k}^j e^{(\lambda_{1k}^j - \lambda_{4k}^j)h} - U_{4k}^j)(e^{(\lambda_{1k}^j - \lambda_{2k}^j)h} - e^{(\lambda_{2k}^j - \lambda_{1k}^j)h}) - (e^{(\lambda_{4k}^j - \lambda_{1k}^j)h} - \\ & - e^{(\lambda_{1k}^j - \lambda_{4k}^j)h})(U_{2k}^j - U_{1k}^j e^{(\lambda_{1k}^j - \lambda_{2k}^j)h}) \end{aligned}$$

$$\begin{aligned} Q_k^j = {} & (e^{(\lambda_{3k}^j - \lambda_{1k}^j)h} - e^{(\lambda_{1k}^j - \lambda_{3k}^j)h})(U_{2k}^j - U_{1k}^j e^{(\lambda_{1k}^j - \lambda_{2k}^j)h}) - \\ & - (U_{1k}^j e^{(\lambda_{1k}^j - \lambda_{3k}^j)h} - U_{3k}^j)(e^{(\lambda_{1k}^j - \lambda_{2k}^j)h} - e^{(\lambda_{2k}^j - \lambda_{1k}^j)h}) \end{aligned} \tag{23}$$

$$\begin{aligned} C_{2k}^j = {} & (C_{3k}^j (U_{1k}^j e^{(\lambda_{1k}^j - \lambda_{3k}^j)h} - U_{3k}^j) + C_{4k}^j (U_{1k}^j e^{(\lambda_{1k}^j - \lambda_{4k}^j)h} - U_{4k}^j))(U_{2k}^j - \\ & U_{1k}^j e^{(\lambda_{1k}^j - \lambda_{2k}^j)h})^{-1} \end{aligned}$$

$$C_{1k}^j = -\left(C_{2k}^j U_{2k}^j + C_{3k}^j U_{3k}^j + C_{4k}^j U_{4k}^j \right) / U_{1k}^j, \qquad C_{3k}^j = P_k^j C_{4k}^j / Q_k^j$$

$$U_{rk}^j = B_{21}^j \lambda_{rk}^j - \beta_k^2 B_{22}^j / \lambda_{rk}^j$$

From the continuity conditions at $y = 0$ one can get the equations:

$$\sum_{r=1}^{4} W_{rk}^1 C_{rk}^1 - W_{rk}^2 C_{rk}^2 = \zeta_k(0), \qquad \sum_{r=1}^{4} C_{rk}^1 - C_{rk}^2 = 0 \tag{24}$$

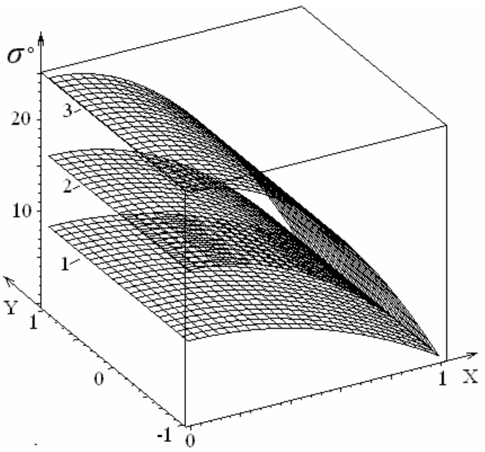
where $W_{rk}^j = B_{11}^j (\lambda_{rk}^j)^2 / \beta_k - \beta_k B_{12}^j$, $\zeta_k(y) = \frac{2}{L} \int_0^L F(x,y) \sin \beta_k x dx$, $F = \int_0^x (A_{11}^2 - A_{11}^1) dx$.

Substituting (23) in (24) gives the algebraic system for C_{4k}^1, C_{4k}^2. Solution of the system can be substituted then in (23). After determining all the coefficients C_{rk}^j in (22) we finally obtain:

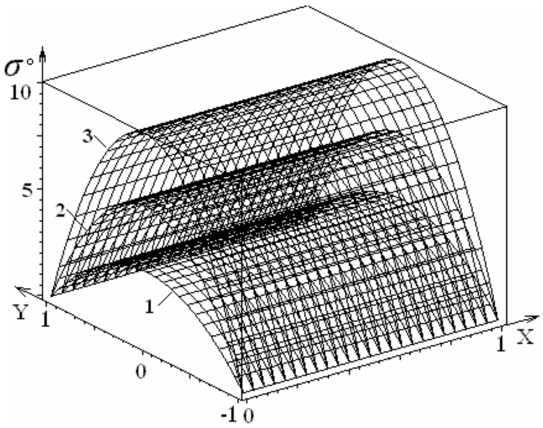
$$\sigma_x^j = \sum_{k=1}^{\infty}\sum_{r=1}^{4} C_{rk}^j (\lambda_{rk}^j)^2 \exp(\lambda_{rk}^j y)\cos\beta_k x, \quad V_x^j = \int_0^x A_{11}^j dx + \sum_{k=1}^{\infty}\sum_{r=1}^{4} W_{rk}^j C_{rk}^j \exp(\lambda_{rk}^j y)\sin\beta_k x$$

$$\sigma_y^j = -\sum_{k=1}^{\infty}\sum_{r=1}^{4} C_{rk}^j \exp(\lambda_{rk}^j y)\beta_k^2 \cos\beta_k x, \quad V_y^j = \int_0^y A_{22}^j dy + \sum_{k=1}^{\infty}\sum_{r=1}^{4} Z_{rk}^j C_{rk}^j \exp(\lambda_{rk}^j y)\cos\beta_k x \qquad (25)$$

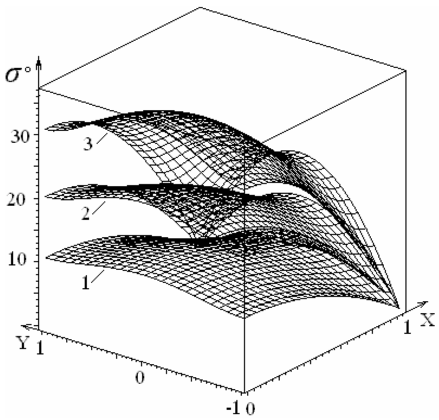
where $Z_{rk}^j = B_{21}^j \lambda_{rk}^j - B_{22}^j \beta_k^2 / \lambda_{rk}^j$. Some results of numerical calculation on (23)-(25) of stress and velocity fields are presented in fig.5. For the sake of definiteness simple relations $B_{11}^j : B_{22}^j = \{1:1; 1:2; 2:1\}$, $B_{11}^j : B_{12}^j = \{1:1; 1:0.5\}$, $B_{12}^j = B_{21}^j = B^*$, $B_{33}^j = 0$ have been used.



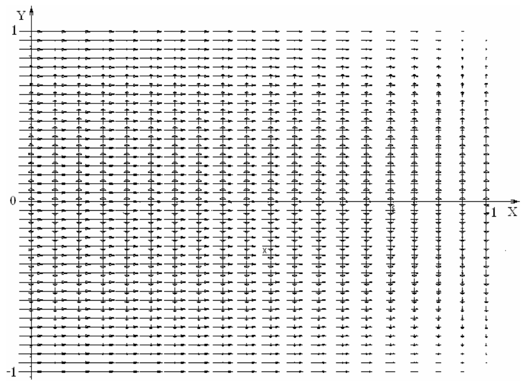
a



b

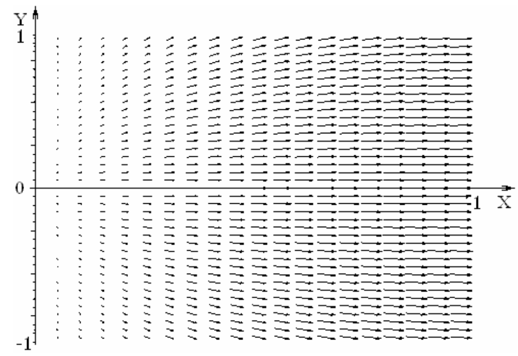


c

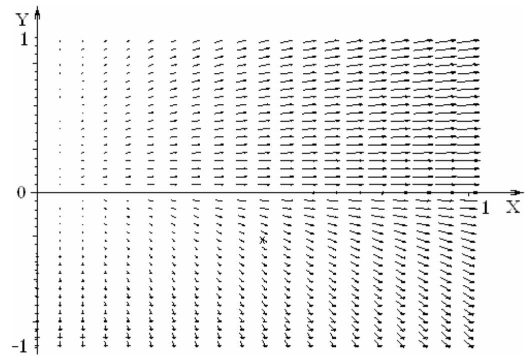


d

,



e



f

Fig.5. Components of stress tensor $\sigma_x^{1,2}(X,Y)$ (a), $\sigma_y^{1,2}(X,Y)$ (b), intensities $\sigma^{1,2}(X,Y)$ (c) and principals of stress tensor (d), velocity field $\vec{V}^{1,2}(x,y)$ at $\sigma_{ik}^j \neq 0$ (e) and $\sigma_{ik}^j = 0$ (f). Here $\sigma^\circ = \sigma/\sigma^*$ is dimensionless stress, x/L, $Y = y/h$, L=0.008 m, h=0.002 m, $B_{11}^j : B_{12}^j = 1:0.5$. Numbers 1-3 correspond to $B_{11}^j : B_{22}^j = \{1:1; 1:2; 2:1\}$ respectively.

CONCLUSIONS

When growth rates inherent to the layers are coordinated and $A_{11}^1 = A_{11}^2$, a continuous stress field accelerates the uniform growth of the body. The x-component σ_x^j which predominantly accelerates the growth along the x-axe (fig.5a), decreases with x whereas the y-component σ_y^j which accelerates mainly the growth along the y-axe, has a dome shape (fig.5b). When $\sigma_x^j = \sigma_x^j(x)$, $\sigma_y^j = \sigma_y^j(y)$ the corresponding dependencies have the parabolic shape (16). Intensity of the mechanical stress $\sigma^j = \sqrt{(\sigma_x^j)^2 + (\sigma_y^j)^2}$ decreases toward the edges $x = L$, $y = \pm h$ of the cross-section (fig.5c). When $B_{33}^j = 0$ the principals of the stress tensor practically coincide the coordinate axes (fig.5d) and newly-grown cellular walls appear in the directions which are parallel to the system axes. Most likely the natural growth does not cause rotation of the cellular walls until external shear force is applied to the growing body. When $B_{ik} \sim A_{ik}/\sigma^*$ the stress field plays a very important role in the coordinated growth of the layers. When $A_{11}^1 \neq A_{11}^2$, $A_{22}^1 \neq A_{22}^2$ and $\sigma_{ik}^j = 0$ their own growth rates cause quite different velocity fields in the adjacent layers (fig.5f). When $\sigma_{ik}^j \neq 0$ the stress field leads to smoothing of the growth differences in the attached layers and makes the balanced growth in x- and y- directions (fig.5e).

Computer simulation at wide variation of the parameters of the model reveal that when the adjacent layers grow at different growth rates inherent to them $A_{ik}^1 \neq A_{ik}^2$, the stretching stresses appear in the tempered layer and the compressing stresses in the rapidly growing one due to the attachment of the layers and no-slip condition at the boundary y=0. In accordance with (3)-(5) the stress field will stimulate the growth of the tempered layer and impede the growth of the rapidly growing one that promotes smoothing of the growth velocity field (fig.5e-f). In such a way the mechanical stress can be a long-distance regulator of a synchronized growth of different layers of the body that can growth with quite different own growth rates. The mechanism can be imitated in the artificial composite materials when the chemical reactions which lead to mass absorption are accelerated by stretching and decelerated by compression. The materials can be used in clinical practice as adaptable prosthesis and in technology as strengthened adaptive constructions.

REFERENCES

1. BEJAN, A.: Shape and Structure, from Engineering to Nature. Cambridge Univ. Press. 2000.
2. KIZILOVA, N.N. : Lecture Notes in Computer Science, 3044, 2004. p.476.
3. SKALAK, R. : Journal of Theoretical Biology. *94*, 1982, p.555.
4. REGIRER, S.A., STEIN, A.A. : In: Modern Problems of Biomechanics. Riga, Zinatne. 1985, p.3.
5. TABER, L.A. : Applied Mechanical Reviews. *48*, 1995, p.490.
6. KIZILOVA, N.N., KRAVCHENKO, E.P. : Mechanics of Solid Body. *33*, 2003, p.158.
7. TAIZ, L., ZEIGER, E. : Plant physiology. Sinauer Associates, Sunderland. 2002.
8. ESAU . : Anatomy of seed plants. Moscow, Mir, 1980.

9. STEIN, A.A. : Applied Mathematics and Mechanics. *59*, 1995, p.149.

10. STEIN, AA. : Biofizika. 41, 1996, p.1097.

11. STEIN, A.A., LOGVENKOV, S.A. : Doklady Academii Nauk SSSR. *328*, 1993, p.443.

12. KANTOR, B.Ya., KIZILOVA, N.N. : Theoretical and Applied Mechanics. *37*, 2003, p.130.

13. KIZILOVA, N.N. : In: Euromech Colloquium 'Advanced Methods in Validation and Identification of Nonlinear Constitutive Equations in Solid Mechanics'. Moscow, Moscow University Press. 2004, p.47.

14. DOLAN, L., POETHIG, R.S. : American Journal of Botany. *85*, 1998, p.315.

15. DOLAN, L., POETHIG, R.S. : American Journal of Botany. *85*, 1998, p.322.

16. COSGROVE, D.J.: Plant Physiology. 84, 1987, p.561.