Suppression of Absolute Instabilities by Appropriate Choice of Rheological Parameters of Anisotropic Viscoelastic Tube Conveying Fluid

M. Hamadiche[1], N. Kizilova[2], M. Gad-el-Hak[3]

[1] LMFA, ECL, 36 Av. Guy de Collongue, UMR5509, F 69134 Ecully, France
[2]Department of Theoretical Mechanics, Kharkov National University, Svobody Sq., 4, 61195, Kharkov, Ukraine
[3]Department of Mechanical Engineering, Virginia Commonwealth University, Richmond, VA 23284-3015,

Abstract. The stability of the steady flow of a viscous liquid through a thick-wall, three-layer, viscoelastic tube with different rheological parameters for each layer is studied. It is shown that the system can be in both absolute and convective unstable states. It is shown that the absolute instability of the system can be converted into a convective instability, and in some cases the system can even be stabilized with an appropriate choice of the rheological parameters. It is found that an anisotropic tube composed of layers possessing distinct rheological values can completely eliminate all absolute instability modes. The present model can be applied to blood vessels that are composed of three viscoelastic layers with distinct rheological properties and to distensible tubes conveying fluids in different technical devices.

Introduction

Different phenomena due to fluid-structure interaction can be observed when fluid moves over a deformable surface in heat and mass exchangers or through the tubes and channels of different man-made devices and natural beds. For example, a laminar flow that is ordinarily stable at given a low Reynolds number if the surface is rigid can become unstable with respect to small disturbances if the surface is compliant, as it has been shown by Hamadiche and Gad-el-Hak (2002). Hamadiche and Gad-el-Hak (2004) carried out an extensive analysis to classify the nature of the instability in soft duct flow where they found the instability to be absolute in nature. Hamadiche, M., Kizilova, N. and Gad-el-Hak, M. (2008) considered the flow of a viscous liquid through a thick-wall, three-layer viscoelastic tube with different rheological parameters for each layer and analysed the fluid structure system stability. Influence of the material parameters of the layers (thickness, density, viscosity, Young modules and Poisson ratio) and the Reynolds number on the spatial and temporal amplification rate of the most unstable mode has been investigated. The importance of transition between absolute and convective instabilities and the possibility of eliminating the absolute instability are elucidated.

Spatio--Temporal Instability

The flow is said to be absolutely unstable, if there is at least one growing mode having zero group velocity. This means that the local system response to an initial impulse grows in time. Absolute instabilities can occur when a mechanism exists for upstream disturbance propagation, as for example in the separated flow over a backward-facing step where the flow recirculation provides such mechanism. In this case, some of the growing disturbances may travel upstream and continually disrupt the flow even after the initial disturbance is neutralized. Flow oscillations in this case are self-excited. Therefore, absolute instabilities are generally more dangerous and more difficult to control, consequently, it has to be suppressed as much as possible. In some flows, for example two-dimensional blunt-body wakes, certain regions are absolutely unstable while others are convectively unstable. The upstream addition of acoustic or electric feedback can change a convectively unstable flow to an absolutely unstable one and self-excited flow oscillations can thus be generated. In any case, identifying the character of flow instability facilitates its effective control, i.e. suppressing or amplifying the perturbation as needed.

The instability is considered to be absolute if there is a pinch point in the Fourier contours that prevents the temporal amplification rate from being reduced down to zero. If there are no pinch points, the instability is convective. In other words, observing a pinch point in the unstable zone of the Fourier plane is a necessary and sufficient condition for the presence of an absolute instability.

In the case of an absolute instability, the mode propagates upstream as well as downstream and often has a very small (or even zero) group velocity in comparison with the velocity of the mean flow. The coalescence of two modes coming from two halves of the wave number plane, forming the pinch point of Fourier contour, may be detected by inspection of the dispersion relation. The existence of a pinch point is equivalent to the existence of a saddle point of the dispersion relation in the complex wave number plane formed by two modes coming from two halves of that plane. Kupfer et al. 1987, have shown that such a saddle point is equivalent to a cusp point in the complex frequency plane, i.e. in the Laplace contours. Kupfer et al. 1987 provide fuller account of the procedure outlined here.

In practice, a criterion that allows the segregation between absolute and convective instabilities may be expressed in terms of the complex wave number, $k= k_r + I k_i$, and the complex frequency number, $s=s_r + I s_i$, $I = (-1)^{1/2}$, through the examination of the dispersion relation $D(k,s)=0$. First, the system is unstable if $s_r(k) > 0$ for at least one real k and stable otherwise. In order to distinguish between the two kinds of instabilities, we consider Laplace contour L with very large s_r, then the Fourier contour are computed via the dispersion equation. Thus, the set of branches $k^{+(s)}$ in the upper half of the complex k-plane as well as the set of branches $k^{-(s)}$ in the lower half of the same plane are obtained by varying s_i for large and fixed s_r, which, of course, requires computing the spatial eigenvalues for all s belonging to L. Decreasing s_r leads to the displacement of those set of branches $k^{+(s)}$ and $k^{-(s)}$ in the complex k-plane. The existence of absolute instability is revealed by a saddle point (a pinch point in the Fourier contour) in the complex k-plane formed by two branches, one belongs to the set $k^{+(s)}$ and the other belongs to the set $k^{-(s)}$. Let k_0 and s_0 be the complex wave number and the complex frequency for which a pinch point occurs. At this point the dispersion relation has double roots. Therefore,

$D(\{k\},\{s\})= 0,\ dD(k,s)/dk=0$ at $k=k_0$, and $s = s_0$

As long as the preceding equations are satisfied for $s_{0r} >$

0 , the instability is absolute. The boundary of absolute instability is the locus of the roots of the precedent equation when $s_{0r} = 0$. Namely,

D ({k},{s})= 0, dD(k,s)/dk=0, and $k=k_0$, $s = s_0$, $s_{0r} = 0$

When the double roots cross the boundary of absolute instability while the equation D $(k_0,s_0)= 0$ alone is satisfied for $s_{0r} > 0$, the system becomes convectively unstable, otherwise it becomes stable.

Absolute Instability in Flows Over Compliant Walls

In boundary layer flows Gad-el-Hak et al. (1984) and Gad-el-Hak (1986) experimentally investigated the different instabilities occurring over a compliant surface interacting with a boundary layer flow. The conditions for the onset of these instabilities were documented and their wave amplitude, length and phase speed were measured using a novel non-intrusive probe.

These authors observed that an elastic or a viscoelastic solid subjected to the forces of a turbulent boundary layer is susceptible to forming two types of waves distinguished by their wave speeds. One of those two waves, static divergence, has a slow propagation speed and the other, flutter, has a fast propagation one. In retrospect, it has been found that those two modes form a common future of the mechanism involving a fluid-structure interaction. A first step toward the comprehension of the dynamic of the modes observed experimentally has been offered by Sen and Arora (1988) where they found that a powerful instability termed transitional could be formed by the coalescence of a solid-based unstable mode and a fluid-based unstable mode. However, the absolute or convective nature of any of those instabilities were not clarified at this point of time.

The existence of an absolute instability over a compliant flat-plate in laminar flow has been demonstrated analytically by Yeo et al. (1996; 1999). In these studies, Blasius boundary layer is used as a base flow. As it is well known, a Blasius boundary layer developing over a rigid wall is subjected only to convective instability, it has been understood that the absolute nature of the instability found by Yeo et al. (1999) is due to the interaction between the flow and the compliant wall. Later, Yeo et al. (2001) have shown that the slow waves, termed static-divergence waves, observed when a turbulent boundary layer flows over a compliant wall represents an absolute instability, and that the fast waves, termed flutter, represent a convective instability. This analysis offers a more precise physical picture of the growth and development of the fast and slow modes developing over a compliant surface.

A model-based on three-dimensional linear elasticity equations to describe the wall motion and perturbation of the exact Navier--Stokes equations for the fluid motion has been suggested by Hamadiche (2002). He has shown that both absolute and convective instabilities in the form of axisymmetric waves may occur in a viscoelastic tube conveying fluid flow. In his model, the tube is tethered to an exterior rigid wall, therefore it is prohibited from collapsing by the no-displacement condition imposed at the tethering surface. When the outer surface of the compliant tube is free from constraint, the tube may collapse under the action of the transmural pressure and the system becomes much more vulnerable to absolute and convective instabilities, as it has been shown by Hamadiche and Gad-el-Hak (2004). These authors found that the absolute unstable modes are non-axisymmetric. In all cases, when the wall is non-rigid and is allowed to interact with the flow, the absolute instability exists for certain values of the parameters of the system. It appear as if the flexibility of the structure conveying the flow increases the number of degrees of freedom of the system, thus permitting certain unstable modes to interact and form a more powerful instability, i.e. absolute instability. However, those degrees of freedom offered by the system may be chosen in order to force the same system to be in a state far from absolute instability. In other words, careful choice of the system's parameters may prevent the onset of absolute instability, as will be shown in the following section.

Results

The system under consideration herein consists of a thick-wall circular tube of inner radius R filled with a Newtonian fluid. The tube wall is composed of three co-axial, viscoelastic layers of different densities, thicknesses, and rheological coefficients. Generally each layer is anisotropic and rheological coefficients of the layers are different. We assume that the tube and the steady flow inside it are in dynamic equilibrium. The tube is assumed to be sufficiently long to allow a uniform and steady velocity field parallel to the axis of the tube. The temporal and the eigenvalues of the system are computed. Figure 1 shows the trajectory of the two coalescing modes when the Laplace contour is lowered. The figure shows that the two modes coalesce when the amplification rate approaches zero but before reaching zero indication by the way that the instability is absolute. Figure 2 shows the amplification rate versus the viscosities of the one of the layer while the viscosity of the others are maintained constant. The figure show too the amplification rate when the viscosities of all the layers are changed by the same amount.

Effect of the viscosities of the layers on the instability and on the absolute instability of the system is shown in figure 2; for the following system parameters. The inner radius of the tube R is the unity of distance, Reynolds number equal 10, ratio of elastic forces to hydrodynamics forces equal 10, the thickness of the three layers respectively from inside to the outside of the duct read $h_1 = 0.08$, $h_2 = 0.14$, $h_3= 0.1$, Poisson coefficient equal 0.4, wave number k=2.5. We note the viscosities of the three layers from inside of the tube toward the outside, respectively, ν_1, ν_2, ν_3 . We note the viscosity of an arbitrary layer ν_r . The label of the curves are as follow: curve labelled +, $(\nu_1, \nu_2, \nu_3)=(\nu_r, \nu_r, \nu_r)$, curve labelled x, $(\nu_1, \nu_2, \nu_3)=(\nu_r, 0, 0)$, curve labelled *, $(\nu_1,\nu_2,\nu_3)=(0,\nu_r ,0)$, and curve labelled “box”, $(\nu_1, \nu_2, \nu_3)=(0, 0, \nu_r)$. (b) Amplification rate at the cusp point versus the viscosities of the layers, all other conditions are the same as part (a). It is found that the system could be stabilised and the absolute instability could be eliminated by increasing the viscosity of the second layer, curve labelled *, while increasing the viscosities of the three layers together by the same amount do not stabilise the system.

References

Gad-el-Hak, M., (1986a) ``Boundary Layer Interactions with Compliant Coatings:\ An Overview," *Appl. Mech. Rev.* **39**, pp. 511--524.

Gad-el-Hak, M., (1986b) ``The Response of Elastic and Viscoelastic Surfaces to a Turbulent Boundary Layer,"

J. Appl. Mech. **53**, pp. 206--212.
Hamadiche, M. (2002) ``Absolue et convective instabilités de l'écoulement de Poiseuille en conduite viscoélastique,'' *Comptes Rendus Mécanique*, **330**, pp. 769-775.
Hamadiche, M., Kizilova, N. and Gad-el-Hak, M. (2008) "Suppression of Absolute Instabilities in the Flow Inside a Compliant Tube." Admitted in *journal Communications in Numerical Methods in Engineering.*
Hamadiche, M., and Gad-el-Hak, M. (2002) ``Temporal Stability of Flow Through Viscoelastic Tubes,'' *J. Fluids Structures*, **16**, no. 3, pp. 331--359.
Hamadiche, M., and Gad-el-Hak, M. (2004) ``Spatio-temporal stability of flow through collapsible, viscoelastic tubes,'' *AIAA Journal*, v.42, pp. 772--786.
Hamadiche, M., and Kizilova, N. (2005) ``Temporal and Spatial Instabilities of the Flow in the Blood Vessels as Multi-Layered Compliant Tubes,'' *International Journal of Dynamics of Fluids*, **1**, no.\ 1, pp. 1--25.
Hamadiche, M., and Abu Shadi, H. (2006) ``Flow induced vibration during the optic fiber coating process,'' *Journal of Fluids and Structures*, **22**, no. 5, pp. 599--615.
Kupfer, K., Bers, A., and Ram, A.K. (1987) ``The Cusp Map in the Complex-Frequency Plane for Absolute Instabilities,'' *Physics of Fluids*, **30**, no. 10, pp. 3075--3082.
Sen, P.K., and Arora, D. S. (1988) ``On the Stability of Laminar Boundary-Layer Flow over a Flat Plate with a Compliant Surface,'' *J. Fluid Mech*., **197**, pp. 201--240.
Yeo, K. S., Zhao, H. Z., and Khoo, B. C. (2001) ``Turbulent Boundary Layer over a Compliant Surface:\ Absolute and Cconvective Instabilities,'' *Journal of Fluid Mechanics*, **449**, pp. 141-168.