This is a repository copy of *Income Tax Avoidance and Evasion: A Narrow Bracketing Approach*.

White Rose Research Online URL for this paper:
http://eprints.whiterose.ac.uk/104895/

Version: Accepted Version

eprints@whiterose.ac.uk
https://eprints.whiterose.ac.uk/

Income Tax Avoidance and Evasion: A Narrow Bracketing Approach*

Duccio Gamannossi degl'Innocenti[†]

duccio.gamannossi@imtlucca.it

Matthew D. Rablen[‡]

m.rablen@sheffield.ac.uk

September 19, 2016

Abstract

We characterize optimal individual tax evasion and avoidance when taxpayers "narrow bracket" the joint avoidance/evasion decision by exhausting all gainful methods for legal avoidance before choosing whether or not also to evade illegally. We find that (i) evasion is an increasing function of the audit probability when the latter is low enough, yet tax avoidance is always decreasing in the probability of audit; (ii) an analogous finding to the so-called Yitzhaki puzzle for evasion also holds for tax avoidance – an increase in the tax rate decreases the level of avoided income and the level of avoided tax; and (iii) that, holding constant the expected return to evasion, it is not always the case that the combined loss of reported income due to avoidance and evasion can be stemmed by increasing the fine rate and decreasing the audit probability.

JEL Classification: H26, K42, D82, H21.
Keywords: Tax avoidance, Tax evasion, Narrow bracketing, Financial intermediaries.

1 Introduction

Individuals take a variety of actions to reduce their tax liabilities. The UK tax authority, for instance, distinguishes three distinct types of action (HM Treasury and HM Revenue and Customs 2011): those that breach tax law (tax evasion); those that "use the tax law to get a tax advantage that Parliament never intended" (tax avoidance); and those that "use tax allowances for the purposes intended by Parliament" (tax planning). By these definitions, both tax evasion and tax avoidance are responsible for significant losses in public revenue: estimates provided by the UK tax authority put the value of tax avoidance at £2.7 bn. and the value of tax evasion at £4.4 bn. (HM Revenue and Customs 2015). Given the first order significance of tax avoidance, it is of note that the first economic studies relating to tax compliance (e.g., Allingham and Sandmo 1972; Srinivasan 1973; Yitzhaki 1974; Christiansen 1980) neglect the possibility of tax avoidance altogether, and the economic literature that followed has largely retained this bias.

In this paper we introduce tax avoidance into the portfolio model of tax evasion (Yitzhaki 1974). To model the joint tax avoidance/evasion decision we build on insights developed in psychology and behavioral economics. In particular, we allow for a pervasive propensity among human decision makers facing multiple-dimension problems – that of *narrow bracketing.* In our context, a decision maker who narrow brackets would decompose sequentially the joint decision {avoidance,evasion} into narrow brackets, e.g., {avoidance} followed by {evasion}. A key feature of narrow bracketing is that the decision maker tends to choose an option in each stage without full regard to the other decisions and circumstances that he or she faces (Rabin and Weizsäcker 2009). In the present context, therefore, the choice made first is made without the decision maker being assumed to have pre-meditated on how the second choice will be made. Important in this context is whether the taxpayer is more likely to make the avoidance or evasion choice first. This question – as to the order in which a complex decision is mentally staged – is thought to depend heavily on mentally focal qualitative features of the choice set. We argue that a focal feature of the choice set is that avoidance is ostensibly legal whereas evasion is illegal. Indeed, judiciaries on both sides of the Atlantic have long upheld the right of a citizen to challenge the proper interpretation of tax law and to pay only the tax they owe in law. Thus, while tax avoidance can be seen as the rightful exercise of a basic right by some lights, tax evasion lacks an equivalent interpretation. Accordingly, if taxpayers qualitatively prefer avoidance to evasion, we suppose they would focus on exhausting opportunities for legal tax avoidance before subsequently

focusing on opportunities for illegal evasion.

We are by no means the first to propose that taxpayers distinguish qualitatively between avoidance and evasion, however. This distinction has previously been represented by supposing that a cost owing to social stigma and/or personal guilt is attached to the illegal act of tax evasion. This cost can be financial (e.g., Chetty 2009; Lee 2001) or psychic (e.g., Benjamini and Maital 1985; Gordon 1989; Myles and Naylor 1996; Kim 2003; Dell'Anno 2009). The concept of narrow bracketing offers an alternative perspective on this distinction: in our model, illegal evasion is not considered until all gainful avenues for legal avoidance have been exhausted.[1] In this way our paper relates to a literature on two-stage decision making (e.g., Gorman 1959; Strotz 1959; Blackorby et al. 1970). Unlike this literature, however, we do not seek a sequential approach to the joint avoidance/evasion decision that coincides with the outcome that would obtain were the joint decision assumed to be made simultaneously.

In addition to the legal distinction between avoidance and evasion, we further assume that avoidance is costly whereas evasion is costless. Devising avoidance schemes that reduce a tax liability without ostensibly violating tax law invariably requires a detailed understanding of tax law, coupled with a degree of ingenuity. A classical form of avoidance scheme, for instance, involves the implementation of a circular sequence of self-cancelling option agreements that return the seller to his or her original position, but in the process create an allowable loss. As, however, few taxpayers are equipped to conceive of and implement independently such avoidance schemes, it is necessary to purchase them on the open market.[2] Satisfying this demand for tax avoidance is a substantial industry dedicated to the development and marketing of avoidance schemes (see, e.g., Sikka 2012; Committee of Public Accounts 2013; Addison and Mueller 2015). By contrast, many forms of tax evasion require no technical or legal expertise. Intentionally understating income on the tax return, for instance, may readily be performed independently.

We find that, in two respects, allowing for tax avoidance importantly changes the characteristics of optimal tax evasion. First, under plausible conditions, evasion is an *increasing* function of the probability of audit. Second, we re-examine the finding of Christiansen (1980,

---

[1]Strictly speaking, the stigma cost approach and ours and not mutually exclusive. To present our approach in the simplest possible light, however, we do not allow for a stigma cost.

[2]People not only have difficulties in understanding tax law, but also show poor knowledge about tax rates and basic concepts of taxation. For evidence that people often significantly under-estimate marginal tax rates see, e.g., Lewis (1978) and Gideon (in press).

391) that "if the fine is increased, but the efforts to detect tax evaders are adjusted so as to keep the expected gain from tax evasion unaltered, risk averters will always reduce their tax evasion." We are again able to prove this result, yet in our model it is the total amount of lost tax (through both avoidance and evasion) that is economically pertinent. When we consider both avoidance and evasion, however, it is possible that taxpayers declare more income if the probability of audit is increased, and the fine decreased, holding the expected gain from tax evasion constant.

The paper adds to the small, but growing, economic literature on tax avoidance. The two closest analyses to ours are Alm and McCallin (1990) and Alm (1988). The former describes avoidance and evasion as risky assets – each asset has a return characterized by a mean and variance, and the interdependence between the two returns is characterized by a covariance – while the latter characterizes avoidance as a riskless, albeit costly, asset. Whereas both of these analyses consider the simultaneous determination of evasion and avoidance, in our framework we argue that these are chosen sequentially. Different from Alm and McCallin, we model the mean, variance and covariance of evasion and avoidance, rather than taking these quantities as exogenous. Unlike in Alm (1988), we take avoidance to be risky, owing to the possibility of effective anti-avoidance measures by the tax authority.

Much of the remaining literature on tax avoidance is, however, concerned with whether income tax has "real" effects upon labor supply, or simply leads to changes in the "form" of compensation. Accordingly, in these studies the term "tax avoidance" typically refers to all form-changing actions that reduce a tax liability.[3] This definition overlaps with ours, but is broader in the sense that it also includes actions that fall in to our notion of tax planning. In the context of this broader definition of tax avoidance, Slemrod and Kopczuk (2002), Piketty, Saez, and Stantcheva (2014) and Uribe-Teran (2015) analyze theoretically the elasticity of taxable income in the presence of avoidance. In the empirical literature, Slemrod (1995, 1996) finds pronounced tax avoidance effects in the response of high-earners to tax changes, while Feldstein (1999) finds that accounting for tax avoidance significantly increases estimates of the implied deadweight loss of income taxation. Lang, Nöhrbaß, and Stahl (1997) estimate that tax avoidance costs the German exchequer an amount equal to around 34 percent of income taxes paid. Fack and Landais (2010) show that the response of charitable deductions to tax rates is concentrated primarily along the avoidance margin

---

[3]For a detailed discussion of these "form-changing" actions see, e.g., Stiglitz (1985) and Slemrod and Yitzhaki (2002).

(rather than the real contribution margin), while Gruber and Saez (2002) show that the elasticity of a broad measure of income is notably smaller than the equivalent elasticity for taxable income, suggesting that much of the response of taxable income comes through deductions, exemptions, and exclusions.

The plan of the article is as follows: in section 2 we motivate the key behavioral assumptions behind our analysis, from which section 3 develops a formal model. Section 4 performs the main analysis and 5 compares our findings to the literature. We extend the model in section 6 to allow for risk in the tax authority's efforts to illegalize avoidance schemes, and section 7 concludes. All proofs are in the Appendix.

2   Deciding to Avoid and/or Evade Tax

Two key features of our modelling of the joint decision to avoid and/or evade are that (i) the taxpayer makes the avoidance and evasion decisions sequentially; and (ii) that the avoidance decision is made first. The first feature – that complex decisions are routinely broken down into smaller ones – is often termed *narrow bracketing* in the behavioral literature. The second feature – the choice of how to stage the sub-decisions within the larger composite decision – is sometimes termed *decision staging* (Johnson et al. 2012). We discuss each of these features in turn.

2.1   Narrow Bracketing

A mass of evidence suggests that people narrowly bracket: a decision maker who faces a multi-dimensional decision tends to break the decision down sequentially, proceeding at each stage to isolate a single dimension of the problem without full regard to the other dimensions of the problem. In the context of monetary risk, Tversky and Kahneman (1981) present an experiment that demonstrates how powerful this propensity is. In their experiment, people narrowly bracket even when faced with only a pair of independent simple binary decisions that are presented on the same sheet of paper. Narrow bracketing lies at the heart of current explanations of phenomena such as the stock market participation puzzle (Barberis, Huang, and Thaler 2006), the equity premium puzzle (Benartzi and Thaler 1995; Gneezy and Potters 1997) and choice among lotteries (Camerer 1989; Battalio, Kagel, and Kiranyakul 1990; Langer and Weber 2001).

Cognitive limitations – in perception, attention, memory, and analytical processing – are

4

thought to be an important reason for narrow bracketing (Read, Loewenstein, and Rabin 1999). Accordingly, in the experiment of Read et al. (2001) subjects who were required to resolve a complex choice problem sequentially actually made better choices than those subjects who were required to proceed simultaneously. Clearly, however, decision making outcomes under narrow bracketing can, in other contexts, appear worse than those arrived at from a wide bracketing perspective. For instance, decision making under narrow bracketing may violate first-order stochastic dominance (Rabin and Weizsäcker 2009) and, in a famous example, the "one day at a time" bracketing observed among New York cab drivers fails to maximize earnings per hour across days (Camerer et al. 1997). Thus, while the desirability of narrow bracketing is still the subject of academic debate (see, e.g., Palacios-Huerta 1999; Kőszegi and Rabin 2009), the pervasiveness of the phenomenon is not in doubt. It is thus of relevance to understand the nature of the joint avoidance and evasion decision under narrow bracketing.

2.2   Decision Staging

Having established that taxpayers may well mentally separate the joint avoidance and evasion decision there remains the question as to how this is done. According to Kahneman (2003), the way individuals will choose to stage or frame a decision is heavily shaped by the features of the situation at hand that come to mind most easily – to use the technical term, by the features that are most "accessible." This notion is supported in the context of tax-related decision making by McCaffery and Baron (2004, 2006). These authors employ a slightly different terminology – the *isolation effect* – which, however, refers to the tendency of respondents in their study to "decide complex matters – and tax raises a host of complex matters – by responding to the most salient or obvious aspect of a choice set or decision problem."

In the context of the joint avoidance and evasion decision we argue that the most accessible feature or aspect of the choice set is that tax avoidance and evasion are qualitatively distinct: one is legal, and the right to practice it has often been defended by the judiciary, whereas the other is a crime. Courts on both sides of the Atlantic have for many years upheld the right of citizens to challenge the interpretation of tax law (Barker 2009; Prebble and Prebble 2010). In 1936 Lord Tomlin, ruling in a case involving the Duke of Westminster, surmised that "[e]very man is entitled, if he can, to order his affairs so that the tax attaching under the appropriate Acts is less than it otherwise would be. If he succeeds in ordering them so

as to secure this result, then, however unappreciative the Commissioners of Inland Revenue or his fellow taxpayers may be of his ingenuity, he cannot be compelled to pay an increased tax." This so-called "Duke of Westminster principle" dominated UK tax avoidance law until the 1980s and remains influential today. Similarly, in the US, Judge Learned Hand stated in 1947 (in Commissioner vs. Newman) that "[o]ver and over again courts have said that there is nothing sinister in so arranging one's affairs as to keep taxes as low as possible. Everybody does so, rich or poor; and all do right, for nobody owes any public duty to pay more than the law demands: taxes are enforced exactions, not voluntary contributions. To demand more in the name of morals is mere cant."

There is evidence that these traditional legal arguments continue to affect public sentiment towards tax avoidance. In the qualitative study of Kirchler, Maciejovsky, and Schneider (2003) participants relate tax avoidance to lawful acts enabling tax reduction, to cleverness, and to costs. Tax evasion, by contrast, is associated with illegal acts such as fraud, criminal prosecution, risk, tax-audits, punishment, penalty, and the risk of detection. In this sense, we argue that, for many taxpayers, tax avoidance is qualitatively preferred to evasion. Accordingly, we argue that taxpayers would exhaust the scope for legal avoidance before subsequently deciding whether or not they additionally wish to evade illegally (rather than the other way round).

3 Model

Our model is a direct extension of the canonical portfolio model of Yitzhaki (1974). A taxpayer has an income (wealth) w and faces a tax on income given by tw, where $t \in (0, 1)$. Taxpayers behave as if they maximize expected utility, where utility is denoted by $U(z) = \log z$.[4] The taxpayer's true income is not observed by the tax authority, but the taxpayer must declare an amount $x \in [0, w]$. The taxpayer can choose to avoid paying tax on an amount of income $A \in [0, w]$, and subsequently to evade illegally an amount of income $E \in [0, w - A]$, so $x = w - E - A$.

Evasion is financially costless but avoidance technology must be bought in a market in which

---

[4]Thus taxpayers are risk averse and have a constant (unit) co-efficient of relative risk aversion. We adopt the logarithmic form for reasons of analytic tractability. Other simple specifications such as constant absolute risk aversion or mean-variance utility can instead be used and yield similar results. However, the assumption of constant relative risk aversion has stronger empirical support (see, e.g., Wakker 2008; Chiappori and Paiella 2011).

"promoters" sell avoidance schemes to "users".[5] A common feature of this market is the "no saving, no fee" arrangement under which the price received by a promoter is linked to the amount by which their scheme stands to reduce the user's tax liability. Although systematic information regarding the precise contractual terms upon which avoidance schemes are typically sold is scarce, we understand from a detailed investigation in the UK that, for the majority of mass-marketed schemes, the fee is related to the reduction in the annual theoretical tax liability of the user, not the ex-post realization of the tax saved (Committee of Public Accounts 2013, 11). This implies, in particular, that the monetary risks associated with the possible subsequent detection and termination of a tax avoidance scheme are borne by the user.[6] Accordingly, we assume that the promoter's fee is a proportion $\phi \in (0,1)$ of the amount by which the taxpayer's tax liability stands to be reduced, tA. In this way, ϕ may be interpreted as measuring the degree of competition in the market for tax avoidance schemes, with lower values of ϕ indicating the presence of stronger competitive forces.

Although traditional arguments around the morality of tax avoidance continue to affect importantly public sentiment, there has nonetheless been a discernible shift in the attitudes of the judiciary, beginning in the 1980s (Stevens 2013). Increasingly, as our opening definition of tax avoidance suggests, courts apply a purposive interpretation of the law. This interpretation is summarized by Permanent Judge Ribeiro (in Collector of Stamp Revenue vs. Arrowtown Assets Ltd.) who states that "the ultimate question is whether the relevant statutory provisions, construed purposively, were intended to apply to the transaction, viewed realistically." Armed with this purposive interpretation of the law, tax authorities now routinely seek to have particular avoidance schemes ruled illegal. Yet taxpayers may legitimately continue to use an avoidance scheme while the (often lengthy) process of shutting it down is ongoing. Moreover, if the scheme is eventually declared illegal the tax authority can only seek the amount of tax that was properly due. That is, it cannot levy fines for tax evasion retrospectively once a scheme has been outlawed. Inherent in our definition of tax avoidance (as distinct from tax planning) is that – should the tax authority learn of the scheme – it will consider it illegal.

The taxpayer's income declaration is audited with probability $p \in (0,1)$. If audited, E and A

---

[5]For analyses of the market for tax advice see, e.g., Reinganum and Wilde (1991) and Damjanovic and Ulph (2010).

[6]It is apparent that such arrangements give promoters incentives to mis-represent the level of risk involved in particular schemes. Consistent with this point, Committee of Public Accounts (2013, 11) indeed finds evidence of such mis-selling.

are observed and the taxpayer has to pay $[1 + f] tE$ on account of the amount of evaded tax, where $f > 0$ is the fine rate. The tax authority mounts a legal challenge to the avoidance scheme, which is successful with probability p_L. In the event that the legal challenge is successful, the tax authority obtains the right to reclaim the tax owed (but cannot levy a fine). In this case, instead of paying tx in tax, the taxpayer must instead pay $t[x + A]$.

The taxpayer's expected utility is therefore given by

$$\mathbf{E}U(A, E) = [1 - p] U(w^n) + pp_L U(w^{a_s}) + p[1 - p_L] U(w^{a_u}), \tag{1}$$

where w^n is the taxpayer's wealth in the state in which they are not audited, w^{a_s} is the taxpayer's wealth in the state in which they are audited and the tax authority's legal challenge is successful, and w^{a_u} is the taxpayer's wealth in the state in which they are audited and the tax authority's legal challenge is unsuccessful:

$$
\begin{align}
w^{a_s}(A, E) &= w - t[w - E] - [1 + f] tE - \phi tA; \tag{2} \\
w^{a_u}(A, E) &= w - t[w - A - E] - [1 + f] tE - \phi tA; \tag{3} \\
w^n(A, E) &= w - t[w - A - E] - \phi tA. \tag{4}
\end{align}
$$

A key distinguishing factor between evasion and avoidance in this context is that avoidance entails a cost ϕtA in all states of the world. Thus, if avoidance is detected and the scheme closed down a taxpayer is worse-off for having chosen to avoid, even though they are not fined on avoided income. To ensure that the amount of taxes, fines and fees never exceeds a taxpayer's wealth for any $A + E \in [0, w]$ we must assume $[1 - t]/t > \max \{\phi, f\}$.

We suppose that taxpayers choose their preferred level of avoidance and evasion sequentially: gainful opportunities for tax avoidance are exhausted before the taxpayer decides whether to engage additionally in evasion. Thus, taxpayers first choose avoided income as

$$A^* = \arg \max_A \mathbf{E}U(A, 0); \tag{5}$$

and then evaded income as

$$E^* = \arg \max_E \mathbf{E}U(A^*, E). \tag{6}$$

4 Analysis

We now present an analysis of the model of the previous section. For analytic tractability, we shall consider the special case of the model with $p_L = 1$, such that legal challenges by the tax authority are always successful. In a later section we shall demonstrate numerically how the results with $p_L = 1$ relate to the results for the more general case with $p_L < 1$.

To begin, it is helpful to define the function $R(z) = [1 - z]/z$, such that, e.g., $R(p)$ is the classical odds ratio found in decision theory. We may then state our first Proposition:

Proposition 1 *An interior optimum for avoidance and evasion satisfies*

$$A^* = \frac{pR(t)}{1 - \phi} \left[R(p) R(\phi) - 1 \right] w$$

and

$$E^* = \frac{pR(t)}{1 - \phi} \frac{[1 - p][1 - fR(\phi)]}{f} w$$

where

$$R(p) R(\phi) > 1 > fR(\phi); \qquad \frac{pR(t)}{1 - \phi} \frac{[1 - p][1 - fR(\phi)] + f[R(p)R(\phi) - 1]}{f} < 1.$$

Proposition 1 gives closed-form expressions for optimal avoidance and evasion when both are at an interior maximum, and the conditions needed for a such an interior maximum to arise. The first sequence of inequalities at the bottom of the Proposition guarantee that $A^*, E^* > 0$. The left-side inequality, $R(p) R(\phi) > 1$, is the condition that the avoidance gamble be better than fair. As a necessary condition for both inequalities to hold it must be that $R(p) R(\phi) > fR(\phi)$, which implies $R(p) > f$. This is the standard restriction in the portfolio model of tax evasion that the evasion gamble be better than fair. The right side inequality at the bottom of the Proposition ensures that $A^* + E^* < w$. To gain insight into how A^* and E^* are related, note that we may write one as a (linear) function of the other:

$$E^* (A^*) = \frac{p[wR(t) - \phi A^*][R(p) - f]}{f} - pA^*. \tag{7}$$

From (7) we note that

$$\frac{\partial E^*}{\partial A^*} = -\frac{fR(\phi) + R(p)}{f[1 + R(p)][1 + R(\phi)]} < 0, \tag{8}$$

9

so the amount of evaded income E^* is negatively related with the amount of avoided income A^*. This finding matches that of Alm, Bahl, and Murray (1990) using data from Jamaica, who find that evasion and avoidance are substitutes. We now consider the comparative statics of optimal avoidance and evasion:

Proposition 2 *At an interior optimum for avoidance and evasion it holds for A^* that*

$$\frac{\partial A^*}{\partial w} = \frac{A^*}{w} > 0;$$

$$\frac{\partial A^*}{\partial t} = -\frac{A^*}{t[1-t]} < 0;$$

$$\frac{\partial A^*}{\partial f} = 0;$$

$$\frac{\partial A^*}{\partial \phi} = -\frac{wR(t)\left[p + (1-p)R(\phi)^2\right]\left[1 + R(\phi)\right]^2}{R(\phi)^2} < 0;$$

$$\frac{\partial A^*}{\partial p} = -\frac{R(t)\,w}{\phi\left[1-\phi\right]} < 0;$$

and for E^* that

$$\frac{\partial E^*}{\partial w} = \frac{E^*}{w} > 0;$$

$$\frac{\partial E^*}{\partial t} = -\frac{E^*}{t\left[1-t\right]} < 0;$$

$$\frac{\partial E^*}{\partial f} = -\frac{E^*}{\left[1 - fR(\phi)\right]f} < 0;$$

$$\frac{\partial E^*}{\partial \phi} = \frac{f\left[R(\phi)\right]^2 + 1}{\left[1-\phi\right]\left[1 - fR(\phi)\right]}E^* > 0;$$

$$\frac{\partial E^*}{\partial p} = \frac{R(p) - 1}{1 - p}E^* \gtrless 0 \Longleftrightarrow R(p) \gtrless 1.$$

Proposition 2 is derived via straightforward differentiation of the expressions for A^* and E^* in Proposition 1 so we omit the proof. Beginning with the comparative statics of A^*, we see that wealthier people are predicted to avoid more income than less wealthy people. The second result is an extension of the well-known Yitzhaki paradox for evasion to the case of avoidance – avoided income falls as the tax rate is increased. The intuition for this result is analogous to that for evasion: a higher marginal tax rate makes the taxpayer feel poorer, and thereby more risk averse. As would be expected, an increase in the competitiveness of the market for avoidance schemes (a decrease in ϕ) increases avoided income, and an increase in the probability of audit decreases avoided income. Of course, knowing $\partial A^*/\partial t < 0$ does

not warrant that the total tax avoided, tA, also falls. It is straightforward to show, however, that

$$\frac{\partial tA^*}{\partial t} = -\left\{\frac{1}{t\left[1-t\right]} - 1\right\} A^* < 0. \tag{9}$$

Turning to evasion, the logic of the chain rule implies that, for an arbitrary exogenous variable z, it must hold that

$$\frac{\partial E^*}{\partial z} = \left.\frac{\partial E^*}{\partial z}\right|_{A^*=cons.} + \frac{\partial E^*}{\partial A^*}\frac{\partial A^*}{\partial z}, \tag{10}$$

where the first term on the right side is the direct effect of z on evasion, and the second term captures the indirect effect on evasion arising from the effects of z upon avoidance. Intuitively, the indirect effect is the income effect imparted upon the evasion choice by movements in avoidance. Noting from equation (8) that $\partial E^*/\partial A^* < 0$ it follows that if $\partial A^*/\partial z$ and $\partial E^*/\partial z|_{A^*=cons.}$ are of the same sign – as turns out to be the case for each of the variables $\{w, t, f, p\}$ – then the direct and indirect effects in equation (10) oppose each other. This observation notwithstanding, the first three results in Proposition 2 – those for $\{w, t, f\}$ – are each unambiguous and consistent with Yitzhaki (1974). Unambiguity in this context arises as the direct effect can be shown to always dominate the indirect effect. To take the effect of the tax rate on evasion as an example, we find the direct effect – using (7) – as

$$\left.\frac{\partial E^*}{\partial t}\right|_{A^*=cons.} = -\frac{w\left[1+R(t)\right]^2\left[R(p)-f\right]}{f\left[1+R(p)\right]} < 0, \tag{11}$$

Combining equation (11) with $\partial A^*/\partial t$ in Proposition 1 and $\partial E^*/\partial A^*$ we can rewrite the direct effect in terms of the indirect effect,

$$\left.\frac{\partial E^*}{\partial t}\right|_{A^*=cons.} = -\frac{\left[R(p)-f\right]\left[1+R(p)\right]R(\phi)}{\left[R(p)+fR(\phi)\right]\left[R(p)R(\phi)-1\right]}\frac{\partial E^*}{\partial A^*}\frac{\partial A^*}{\partial t}, \tag{12}$$

such that, by equation (10), we obtain an alternative form for $\partial E^*/\partial t$ to that given in Proposition 2:

$$\frac{\partial E^*}{\partial t} = -\frac{\left[R(p)-f\right]\left[1+R(p)\right]R(\phi)}{\left[R(p)+fR(\phi)\right]\left[R(p)R(\phi)-1\right]}\frac{\partial E^*}{\partial A^*}\frac{\partial A^*}{\partial t} + \frac{\partial E^*}{\partial A^*}\frac{\partial A^*}{\partial t} \tag{13}$$

$$= -\frac{R(p)\left[1+R(\phi)\right]\left[1-fR(\phi)\right]}{\left[R(p)+fR(\phi)\right]\left[R(p)R(\phi)-1\right]}\frac{\partial E^*}{\partial A^*}\frac{\partial A^*}{\partial t} < 0. \tag{14}$$

As well as evaded income being decreasing in the tax rate, it is straightforward to show that evaded tax, tE^*, is decreasing in the tax rate too. As it has no direct effect on evasion, the effect of competition in the market for avoidance (as captured by ϕ) is given by (10) as simply $\partial E^*/\partial \phi = [\partial E^*/\partial A^*] [\partial A^*/\partial \phi] > 0$. That is, we find that a decrease in competition in the market for avoidance increases evasion by making it more attractive relative to the alternative of avoidance.

The final finding is that tax evasion is increasing in the probability of audit if $R(p) > 1$ (equivalently, $p < 0.5$) and decreasing otherwise. In this case there are again competing direct and indirect effects upon evasion, but now the direct effect does not always dominate the indirect effect. From (7) we obtain the direct effect as

$$\left.\frac{\partial E^*}{\partial p}\right|_{A^*=cons.} = -\frac{wR(t)\left[1 + R(\phi)\right]\left[1 + fR(p)R(\phi)\right]}{f\left[1 + R(p)\right]R(\phi)} < 0, \tag{15}$$

which is the pure income effect first observed by Yitzhaki (1974). This may be rewritten in the form

$$\left.\frac{\partial E^*}{\partial p}\right|_{A^*=cons.} = -\frac{\left[1 + fR(p)R(\phi)\right]}{R(p) + fR(\phi)}\frac{\partial E^*}{\partial A^*}\frac{\partial A^*}{\partial p}, \tag{16}$$

such that

$$\frac{\partial E^*}{\partial p} = -\frac{\left[1 + fR(p)R(\phi)\right]}{R(p) + fR(\phi)}\frac{\partial E^*}{\partial A^*}\frac{\partial A^*}{\partial p} + \frac{\partial E^*}{\partial A^*}\frac{\partial A^*}{\partial p} \tag{17}$$

$$= \frac{\left[R(p) - 1\right]\left[1 - fR(\phi)\right]}{R(p) + fR(\phi)}\frac{\partial E^*}{\partial A^*}\frac{\partial A^*}{\partial p}. \tag{18}$$

From (18) it is immediate that the direct effect dominates when $R(p) < 1$ and the indirect dominates when $R(p) > 1$.

How plausible is the condition $p < 0.5$ required for evasion to be increasing in the probability of audit? A-priori it appears highly plausible given that the IRS audits only around 0.96 percent of individual tax returns filed in calendar year 2012 were examined (IRS 2014). Even if we suppose that these audits were concentrated on the 20 percent or so of people in the US who are self-employed (and thus not subject to third-party reporting) this probability rises to 4.8 percent, still well below the 50 percent level.

We now wish to characterize the total level of undeclared income, $A^* + E^*$:

Proposition 3 *An interior optimum for avoidance and evasion it holds for* $A^* + E^*$ *that*

$$\frac{\partial\left[A^*+E^*\right]}{\partial w} = \frac{A^*+E^*}{w} > 0;$$

$$\frac{\partial\left[A^*+E^*\right]}{\partial t} = -\frac{A^*+E^*}{t[1-t]} < 0;$$

$$\frac{\partial\left[A^*+E^*\right]}{\partial f} = \frac{\partial E^*}{\partial f} < 0;$$

$$\frac{\partial\left[A^*+E^*\right]}{\partial \phi} = \frac{p^2 w R(t)\left\{R(p)\left\{1-f-fR(p)\left[R(\phi)\right]^2\right\}-f\right\}}{\left[1-\phi\right]^2 f} \gtrless 0$$

$$\Longleftrightarrow R(p)\left\{1-f-fR(p)\left[R(\phi)\right]^2\right\}-f \gtrless 0;$$

$$\frac{\partial\left[A^*+E^*\right]}{\partial p} = \frac{w R(t)\left[1+R(\phi)\right]\left\{R(p)\left[1-f-2fR(\phi)\right]-1-f\right\}}{f\left[1+R(p)\right]R(\phi)} \gtrless 0$$

$$\Longleftrightarrow R(p)\left[1-f-2fR(\phi)\right]-\left[1+f\right] \gtrless 0.$$

The first three results of Proposition 3 follow immediately from Proposition 2, for the comparative static effects for both A^* and E^* go in the same direction. Both of the remaining two effects – those for ϕ and p – may go in both directions, however.[7] Thus, for instance, an increase in audit probability can cause the total amount of hidden income to increase (albeit avoided income must fall). Reassuringly, however, unlike the condition for E^* to increase in p, the condition needed for this finding seems far from being satisfied empirically. In particular, it requires setting an especially low f, which, in turn, forces the tax rate to be implausibly high. Similar remarks apply to the condition needed for $A^* + E^*$ to be increasing in ϕ. The results of Proposition 3 allow us to characterize readily the comparative statics of declared income x^* ($\equiv w - A^* - E^*$). For all exogenous variables except w we obtain that the effect for declared income will take the opposite sign to the effect for total undeclared income, i.e., $\partial x^*/\partial\left(\cdot\right) = -\partial\left[A^*+E^*\right]/\partial\left(\cdot\right)$. For w, however, we obtain $\partial x^*/\partial w = x^*/w > 0$.

As a final perspective on the properties of optimal avoidance and evasion, we may characterize the properties of the proportion (or share) of unreported income that is avoided: $s_A \equiv A/\left[A+E\right]$. This shall be instructive when we come to compare our results with the existing literature.

---

[7]To demonstrate this by example, setting $t = 0.85$, $p = 0.40$, $\phi = 0.51$, $f = 0.085$ and $w = 10$ we obtain $\{A^*, E^*\} = \{0.64, 9.36\}$ and $\partial\left[A^*+E^*\right]/\partial p = 0.74$. Alternatively, setting $t = 0.70$, $p = 0.40$, $\phi = 0.51$, $f = 0.02$ and $w = 10$ we obtain $\{A^*, E^*\} = \{1.54, 8.46\}$ and $\partial\left[A^*+E^*\right]/\partial p = -10.10$. Turning to ϕ, setting $t = 0.80$, $p = 0.40$, $\phi = 0.51$, $f = 0.12$ and $w = 10$ we obtain $\{A^*, E^*\} = \{0.90, 9.10\}$ and $\partial\left[A^*+E^*\right]/\partial\phi = 13.35$. Alternatively, setting $t = 0.80$, $p = 0.40$, $\phi = 0.25$, $f = 0.10$ and $w = 10$ we obtain $\{A^*, E^*\} = \{4.67, 5.33\}$ and $\partial\left[A^*+E^*\right]/\partial\phi = -5.87$.

Proposition 4 *At an interior optimum for avoidance and evasion it holds that*

$$\frac{\partial s_A}{\partial t} = \frac{\partial s_A}{\partial w} = 0;$$

$$\frac{\partial s_A}{\partial f} = \phi p^2 \left[1 - p - \phi\right] R(p)\vartheta > 0;$$

$$\frac{\partial s_A}{\partial \phi} = -p^2 \left[1 - p - fp\right] R(p)\vartheta < 0;$$

$$\frac{\partial s_A}{\partial p} = -f \left[\phi p\right]^2 \left[1 - fR(\phi)\right] \left[1 + R(p)^2 R(\phi)\right] \vartheta < 0;$$

where $\vartheta \equiv \left\{ \frac{A^*}{f[A^*+E^*][1-p-\phi]} \right\}^2 > 0.$

Proposition 4 clarifies that the share of undeclared income that is avoided is independent of the tax rate (t) and the taxpayer's wealth (w). This follows from the observation in Proposition 1 that w and $R(t)$ enter both avoidance and evasion as multiplicative factors. We find that the probability of audit (p) unambiguously reduces the share of undeclared income that is avoided, even though the effect of p on evasion can be of either sign. The results for the effects of the fine rate (f) and the cost of avoidance (ϕ) follow immediately from Proposition 2 (as these variables affect avoidance and evasion in opposing ways).

5   Comparison with the literature

We now compare the findings of the previous section to the existing literature. First, where our results overlap with the empirical findings on the interrelationship between evasion and avoidance of Alm, Bahl, and Murray (1990), we observe agreement. These authors find that the quantity $[1 - ft]^{-1}$ is negatively related to evaded income, implying that an increase in either f or t reduces evasion (consistent with Proposition 2). They also find, again consistent with Proposition 2, that avoided income is decreasing in the cost of avoidance (as measured in our model by the parameter ϕ). A caveat to these agreement in findings, however, is that Alm, Bahl, and Murray identify avoidance as a riskless asset, somewhat different from the definition of avoidance as a risky asset we employ here. Second, we may compare our findings for optimal evasion to those of Yitzhaki's (1974) canonical model of tax evasion. Our findings for the effect of wealth, the tax rate and the fine rate on evasion are consistent with Yitzhaki, but the finding that evasion may increase in the probability of audit is different from that in Yitzhaki (where evasion is always decreasing in the probability of audit).

Third, we may compare our findings to those of Alm and McCallin (1990), who report comparative statics results for reported income (x) and for the share of undeclared income that is avoided (s_A). Like these authors, we find that higher fines for evasion increase reported income and increase the share of undeclared income that is avoided. Different from these authors, however, we retain the well-known result of Yitzhaki (1974) that a tax rate rise will increase reported income (whereas Alm and McCallin report the opposite relationship) and, whereas Alm and McCallin find that a tax rate rise increases the share of undeclared income that is avoided, we find that this share is independent of the tax rate. It can be shown that this independence is robust to allowing for $p_L < 1$, and allowing for a coefficient of constant relative risk aversion that is different from unity. We are unable, however, to follow Alm and McCallin (1990) in examining the comparative statics effects of quantities such as the mean and variance of the return to evasion and avoidance, however, as in our model these quantities are determined endogenously. Last, we may compare our findings to those of the theoretical model of Alm (1988), albeit an important difference between his model and ours is that he models avoidance as a riskless asset. Alm presents comparative statics results for the quantities $w - E$ and the share $s_x \equiv [w - A - E] / [w - E] = x / [w - E]$, i.e., the fraction of income net of evasion that is avoided. In his very general framework, Alm finds all comparative statics for the share s_x to be ambiguous in sign. We similarly find that the effects of ϕ and p on s_x are ambiguous, but we find that the effects of f and t on s_x are unambiguous – in both cases proportional to a positive constant:

$$
\begin{aligned}
\frac{\partial s_x}{\partial f} &\propto \phi^2 p^3 [1 - t]^2 R(p) [R(p)R(\phi) - 1] > 0; \\
\frac{\partial s_x}{\partial t} &\propto p f^2 \phi^3 R(\phi) [R(p)R(\phi) - 1] > 0.
\end{aligned}
$$

We find that s_x is independent of a taxpayer's wealth: $\partial s_x / \partial w = 0$. The only other two clear-cut results in Alm (1988) are that evasion is decreasing in the fine rate and in the audit probability. In our model the first of these effects is preserved, but we find that evasion can be increasing in the probability of audit.

5.1   Audit probability vs. fine rate

As a final comparison to the literature, we consider the finding of Christiansen (1980) that, for a constant expected return to evasion, the amount evaded is always reduced by increasing the fine rate and by decreasing the audit probability. Following Christiansen (1980), we first

restrict analysis solely to evasion. For a given level of avoidance the expected return to evasion is given by $\mu_E \equiv p[1+f] - 1$. Holding this constant by appropriate variation of f, and differentiating E^* with respect to p, we obtain:

Proposition 5 *An interior optimum for avoidance and evasion it holds that*

$$\left.\frac{\partial E^*}{\partial p}\right|_{\mu_E=cons.} = \frac{wR(t)\left([1-R(p)]f^2R(\phi) + [1+2f]R(p) - f\right)}{f^2[1-\phi][1+R(p)]} > 0.$$

According to Proposition 5 we are able to replicate Christiansen's finding: it always worsens evasion to raise the audit probability and lower the fine rate, holding the expected return to evasion fixed. In the context of a model containing both avoidance and evasion, however, what will be relevant to a tax authority seeking to maximize tax revenue is the effect of varying p and f on the total level of income that does not get taxed. On this question we have that:

$$\left.\frac{\partial[A^* + E^*]}{\partial p}\right|_{\mu_E=cons.} = \frac{wR(t)\{[1-p]\{1 + f[3 - 2fR(\phi)]\} - f[1+f]\}}{[1-\phi]f^2} \gtrless 0. \qquad (19)$$

As the right side of (19) can take either sign, depending upon parameter values, we now no longer find that raising fines is always superior to raising audit probability. Intuitively, this finding stems from the observation that increasing the fine rate only affects the evasion decision, whereas increasing the audit probability affects both avoidance and evasion. The range of parameters for which audit probability can dominate the fine rate is limited by the fact that evasion is increasing in p below $p = 0.5$. We know of numerical examples, however, that confirm that audit probability can dominate the fine rate even in the region where $\partial E^*/\partial p > 0$.[8]

6 Probabilistic Anti-Avoidance Outcomes

Up until this point, the analysis has been undertaken with the simplifying assumption that, if the tax authority mounts a legal challenge to the avoidance scheme, its challenge is always successful. While important in securing a tractable model, clearly tax authorities are not

---

[8]To demonstrate this by example, setting $t = 0.50$, $p = 0.40$, $\phi = 0.53$, $f = 0.05$ and $w = 10$ we obtain $\{A^*, E^*\} = \{2.93, 5.60\}$ and $\partial[A^* + E^*]/\partial p|_{\mu_E=cons.} = 40.65$. Alternatively, setting $t = 0.30$, $p = 0.055$, $\phi = 0.94$, $f = 2.5$ and $w = 10$ we obtain $\{A^*, E^*\} = \{2.07, 6.80\}$ and $\partial[A^* + E^*]/\partial p|_{\mu_E=cons.} = -91.56$.

always successful in their attempts to shut-down avoidance schemes, so it is of interest to understand how this consideration affects our findings.

Solving for A^* using the definition in equation (5) and the full expression for expected utility given in (1) we obtain

$$A^* = \frac{pp_L R(t)}{1 - \phi} [R(pp_L) R(\phi) - 1] w, \qquad (20)$$

which can be obtained from the solution for A^* given in Proposition 1 (for the case of $p_L = 1$) simply by replacing p with pp_L.[9] It follows that the comparative statics results for A^* given in Proposition 2 continue to hold, and that the effects of p_L upon avoidance are analogous to those of p. The solution for E^* coming from (6) is complex, however. We note, though, that the the taxpayer's wealth and the tax rate both still enter the solution multiplicatively – as they do also for A^* in equation (20) – so these two variables stay independent of the share of undeclared income that is avoided, s_A.

To make further progress we assess the properties of optimal evasion via a numerical optimization procedure that locates optimal avoidance and evasion for a specified set of parameter values. Figure 1 depicts optimal avoidance and evasion as p_L is allowed to vary on the unit interval.[10] For very low values of p_L in the interval denoted $[0, \hat{p}_L|_{E=0}]$ avoidance is seen to be maximal, and the taxpayer does not evade. In a second interval, denoted in the figure by $[\hat{p}_L|_{E=0}, \hat{p}_L|_{A+E=w}]$, the taxpayer both avoids and evades, and reports no income ($A^* + E^* \leq w$ is binding). In a third interval, denoted $[\hat{p}_L|_{A+E=w}, 1]$, the taxpayer again both avoids and evades, but now $A^* + E^* < w$ (this is the case to which our comparative statics analysis applies). Within this interval we observe that optimal evasion increases as the probability of a successful legal challenge increases. This is as expected, for an increase

---

[9]Whereas the expression for A^* given in Proposition 1 is the unique solution to a first order condition linear in A^*, the expression for A^* in (20) is one of a pair of solutions to a first order condition quadratic in A^*. The other solution to this first order condition is $A^* = -wR(t)/[1 - \phi] < 0$, which however, may be dismissed as, by definition, $A^* \geq 0$.

[10]The parameter values that produce Figure 1 are: $w = 10, p = 0.5, t = 0.8, f = 0.1, \phi = 0.22$. We note that these values are chosen purely to illustrate cleanly the full range of possible outcomes of the model. As is well-known, models such as ours, which implicitly assume taxpayers know the true probability of audit, significantly over-predict non-compliance if calibrated realistically (see, e.g., Alm, McClelland, and Schulze (1992), footnote 3). This difficulty does not appear especially consequential in this context, however, for insights such as probability weighting (Kahneman and Tversky 1979) have been shown to dramatically reduce predicted levels of non-compliance, while not importantly affecting its comparative static properties (these being our interest in this paper).

in p_L leaves the returns to evasion unaffected, but reduces the returns to avoidance, making evasion more attractive relative to avoidance.

In Figure 2 we explore the effect of varying p_L on our earlier finding that evasion can be increasing in the probability of audit.[11] On the interval of Figure 2 where both optimal evasion and avoidance are interior, we see that reducing p_L below unity reduces to a value below one-half the threshold audit probability above which evasion is decreasing in p. Thus lower values of p_L imply a smaller set of parameter values for which evasion is observed to be increasing in audit probability.

Other numerically generated results we have analyzed – which we do not report here for brevity – indicate that the qualitative nature of the results given in propositions 2-4 continue to hold. In particular, the taxpayer's wealth and the tax rate continue to act as multipliers in the expressions for optimal avoidance and evasion, and $A^* + E^*$ may be either an increasing or decreasing function in ϕ and p.

7 Conclusion

Although the economic literature has largely limited itself to the study of tax evasion, tax avoidance is empirically observed alongside tax evasion. We therefore examine the choice of a taxpayer of how much tax to avoid and how much to evade, under the assumptions that (i) a taxpayer narrow brackets the joint avoidance/evasion decision – breaking the decision down into separate avoidance and evasion sub-decisions, and taking the first of these two sub-decisions in isolation from the second; and (ii) that a taxpayer will decide first on whether and how much tax to avoid legally before deciding whether and how much tax to evade illegally. Among our results are, first, that an analogous finding to the so-called Yitzhaki puzzle for evasion also holds for tax avoidance – an increase in the tax rate decreases the level of avoided income and the level of avoided tax. Second, for a small enough audit probability, evasion is an increasing function of the audit probability. Although tax avoidance is always decreasing in the probability of audit, in some circumstances even the total amount of income lost to evasion and avoidance can be increasing in the probability of audit. Last, holding constant the expected return to evasion, it is not always the case that combined loss of

---

[11]The parameter values that produce Figure 2 are: $w = 10, t = 0.85, f = 0.11, \phi = 0.17, p_L \in \{0.7, 1\}$. The same qualitative conclusions obtain if the parameter values used to draw Figure 1 are instead used, but these alternative parameter values yield improved visual clarity.

reported income due to avoidance and evasion can be stemmed by increasing the fine rate and decreasing the audit probability.

We finish with some possible avenues for future research. First, it would be of interest to allow for imperfect audit effectiveness, as in Rablen (2014) and Snow and Warren (2005a,b), for it might be that evasion and avoidance differ in the amount of tax inspector time required to detect them. Second, it might also be of interest to model more carefully the market for avoidance. In practice there are a range of providers of tax advice, ranging from those that offer solely tax planning, to those that are willing to offer aggressive (or even criminal) methods, making it important to understand the separate supply- and demand-side effects. A last suggestion is to embed the model within a general equilibrium framework (see, e.g., Alm and Finlay 2013; Neck, Wächter, and Schneider 2012), for the partial equilibrium setting explored here may miss some important wider interactions between avoidance and evasion that should properly be accounted for.

Appendix

Proof of Proposition 1: From (1) we have that

$$\frac{\partial \mathbf{E}U\left(A,0\right)}{\partial A} = \frac{[1-p][1-\phi]}{A[1-\phi]+wR(t)} - \frac{\phi p}{wR(t)-\phi A} \tag{A.1}$$

$$= \phi p \left\{ \frac{R(p)R(\phi)}{A[1-\phi]+wR(t)} - \frac{1}{wR(t)-\phi A} \right\}. \tag{A.2}$$

Solving for the point $\partial \mathbf{E}U\left(A,0\right)/\partial A = 0$ gives

$$A^* = \arg\max_A \mathbf{E}U\left(A,0\right) = \frac{pR\left(t\right)}{1-\phi}\left[R\left(p\right)R\left(\phi\right)-1\right]w. \tag{A.3}$$

Rewriting A^* in (A.3) as $A^* = wR(t)\left[1-p-\phi\right]\phi^{-1}\left[1-\phi\right]^{-1}$, substituting this expression for A^* into (1), and differentiating with respect to E gives

$$\frac{\partial \mathbf{E}U\left(A^*,E\right)}{\partial E} = \frac{[1-p]\phi}{[1-p]wR(t)+E\phi} - \frac{pf\left[1-\phi\right]}{pwR(t)-Ef\left[1-\phi\right]} \tag{A.4}$$

$$= \phi p \left\{ \frac{R\left(p\right)}{[1-p]wR(t)+E\phi} - \frac{fR(\phi)}{pwR(t)-Ef\left[1-\phi\right]} \right\}. \tag{A.5}$$

Evaluating at $\partial \mathbf{E}U\left(A^*,E\right)/\partial E = 0$ and solving for E gives

$$E^* = \arg\max_E \mathbf{E}U\left(A^*,E\right) = \frac{pR(t)}{1-\phi}\left[\frac{\left[1-p\right]\left[1-fR(\phi)\right]}{f}\right]w. \tag{A.6}$$

From (A.3) and (A.6) we see that

$$A^* > 0 \iff R\left(p\right)R\left(\phi\right) > 1; \qquad E^* > 0 \iff fR(\phi) < 1. \tag{A.7}$$

From (A.3) and (A.6) we compute

$$A^* + E^* = \frac{pR(t)}{1-\phi}\left[\frac{\left[1-p\right]\left[1-fR(\phi)\right]+f\left[R\left(p\right)R\left(\phi\right)-1\right]}{f}\right]w, \tag{A.8}$$

so

$$A^* + E^* < w \iff \frac{pR(t)}{1-\phi}\left[\frac{R(p)-f+fR(p)\left[R(p)R(\phi)-1\right]}{f}\right] < 1. \tag{A.9}$$

Proof of Proposition 5: Noting that

$$\left.\frac{\partial^2 E^*}{\partial p \partial f}\right|_{\mu_E=cons.} = -\frac{wR(t)\left\{2\left[1+f\right]R(p)-f\right\}}{f^3\left[1-\phi\right]\left[1+R(p)\right]} < -\frac{wR(t)\left[1+2f\right]}{f^2\left[1-\phi\right]\left[1+f\right]} < 0, \tag{A.10}$$

it follows that if $\partial E^*/\partial p|_{\mu_E=cons.} > 0$ when f approaches its maximum possible value, i.e., $f \to R\left(\phi\right)^{-1}$, then $\partial E^*/\partial p|_{\mu_E=cons.} > 0$ for all $f < R\left(\phi\right)^{-1}$. At $f \to R\left(\phi\right)^{-1}$ we indeed have

$$\left.\frac{\partial E^*}{\partial p}\right|_{\mu_E=cons.} = \frac{wR(p)R(t)R(\phi)\left[1+R(\phi)\right]}{\left[1-\phi\right]\left[1+R(p)\right]} > 0. \tag{A.11}$$
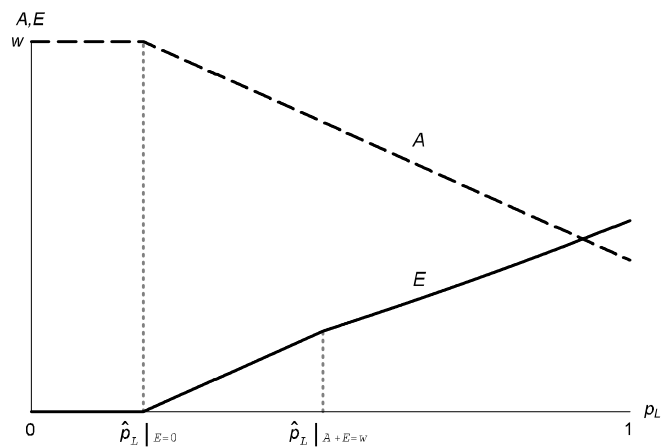
Figures

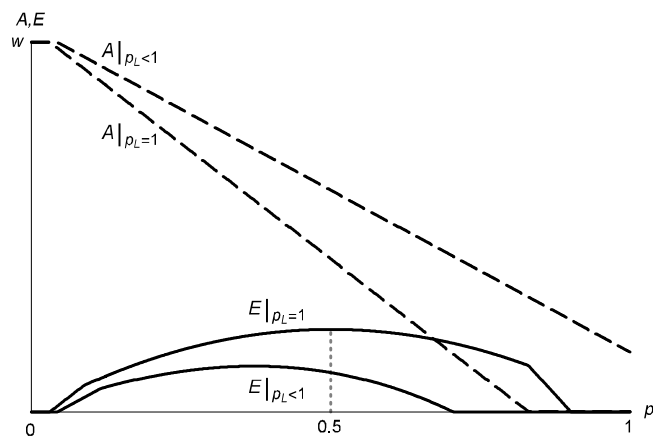Figure 1: Optimal avoidance and evasion for $p_L \in [0, 1]$.

Figure 2: Optimal avoidance and evasion for $p_L < 1$ and $p_L = 1$.