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Multi-mode Propagation in 2D filters and Meta-materials

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Abstract— Meta-materials are characterised using a modal technique for the analysis of 2N- port two dimensional filter networks. It will be shown that in general, all such networks support N modes of propagation each with different propagation constants unless as with the TEM multi-wire transmission line, the inductance matrix is the inverse of the capacitance matrix. Furthermore a simple equivalent circuit for these 2N port networks is derived, enabling complete network analysis for any combination of modal excitations. An explicit formula is derived for the propagation constants of a quasi lowpass filter. This demonstrates that there are always N modes even when the transverse network is infinite in extent.

Index Terms—Meta-materials, Filters, Modes, Propagation

I. Introduction

Modal analysis of two-dimensional filter networks was introduced in [1] to explain the phenomenon of negative reflection in microwave and optical metamaterials. However, this technique only considered four possible modes of propagation, the even and odd modes for TE and TM excitation. Here we take a more general approach and demonstrate that a two dimensional 2N-port network will have N unique modes of propagation each with its own propagation constant.

In section II, starting from the basic electromagnetics of a TEM N-wire transmission line [2] it is shown that the necessary condition for a single mode of propagation is that the static inductance matrix of the structure is the inverse of the capacitance matrix. With the exception of the N wire line this is generally not the case, as it would require extremely complex networks. The propagation constants are found from the eigen values of the system matrix and the nodal voltages and currents are derived from their associated eigen vectors. This is demonstrated using the example of a simple 3 node filter.

At first glance obtaining the transfer function of a 2N-port network with N mods of propagation is a daunting prospect. However a relatively simple method for achieving this is described in section III. By diagonalising the network matrix a simple equivalent circuit is obtained comprising input and output transformers and separate uncoupled internal networks for each mode. Thus applying any individual mode at the input only excites the internal network corresponding to that mode. Consequently the response to any input may be obtained by superposition.

Finally, analysis of a lumped-element filter yields an explicit formula for the values of the modal propagation constants. This is conclusive proof that even if the structure was to be extended infinitely in the transverse plane, the behavior could never converge to a single mode, as is often reported [3].

II. Modal analysis

A. Multimode propagation in metamaterials

A transverse field component $E(z-vt)$ satisfies the wave equation everywhere in the transverse plane even after the introduction of the lossless N+1-wire line structure shown in Fig.1. The mode of propagation is a non-dispersive TEM mode. All voltages and currents propagate with velocity v.

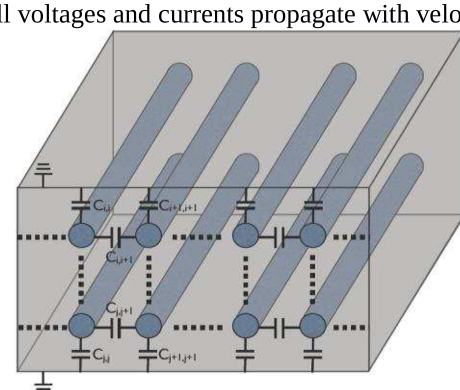


Fig. 1. Illustration of the N-wire line with coupling capacitance.

Assuming there is a ground conductor then the voltages on each of the remaining N wires can be calculated from a line integral along any path to each of the conductors producing a unique set of voltages V_r for $r = 1$ to n. Since the E field has the solution $E(z-vt)$ everywhere in the cross sectional plane then each voltage has the same argument and hence the voltage column vector is $[V(z-vt)]$. Let the current flow on each conductor be described by the vector $[I]$. The loss of charge on the wires over an incremental length dz is

$$d[Q] = dz[C][V] \quad (1)$$

where $[C]$ is the capacitance matrix with the necessary and sufficient conditions on realisability being that $[C]$ is hyperdominant i.e. all off diagonal terms are negative and the sum of all rows and columns are non-negative. In the limit

$$\frac{d[Q]}{dz} = [C][V] \tag{2}$$

and hence

$$\frac{\mathrm{d[I]}}{\mathrm{dz}} = [\mathrm{C}]\frac{\mathrm{d[V]}}{\mathrm{dt}} \tag{3}$$

A new matrix which can be called an inductance matrix (3) is defined with respect to change of voltage down the lines as

$$\frac{d[V]}{dz} = [L]\frac{d[I]}{dt} \tag{4}$$

Eliminating $[I]$ we obtain

$$\frac{d^2[V]}{dz^2} = [L][C]\frac{d^2[V]}{d^2t} \tag{5}$$

The propagation along the z-axis is always of the form

$$e^{-\gamma z} \tag{6}$$

hence

$$\gamma^2[V]'' = v^2[L][C][V]'' \tag{7}$$

and in this case of the homogeneous N-wire line

$$[L] = \frac{1}{v^2}[C]^{-1} \tag{8}$$

In general this is not the case. Now define characteristic impedance and admittance matrices

$$[Z] = v[L] \tag{9}$$

$$[Y] = v[C] \tag{10}$$

And from (7), (9) and (10)

$$\gamma^2[V]'' = v^2[Z][Y][V]'' \tag{11}$$

And the propagation constants are the eigen values of

$$det|[Z][Y] - \gamma^2[1]| = 0 \tag{12}$$

i.e.

$$F(\gamma^2) \ = 0 \tag{13}$$

Thus for a single mode of propagation, $[Z] = [Y]^{-1}$ apart from a scalar multiplier. In general this is not the case and F is a complex function with $2N$ solutions for γ, with N positive solutions representing forward waves and N negative solutions representing waves travelling in the opposite direction.

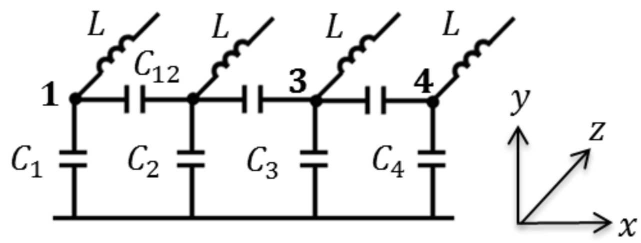


Fig. 2. 4-node lowpass filter

It is instructive to imagine constructing a circuit which supports a single mode of propagation. Consider the simplest metamaterial where the transverse network is an array of four capacitors to ground with coupling constrained to adjacent capacitors, and the series elements an array of inductors as in Fig. 2. This is analogous to a two dimensional telegraphic equivalent far a transmission line. The capacitance matrix is given by

$$[\mathrm{C}] = \begin{bmatrix} \mathrm{C}_{11} & -\mathrm{C}_{12} & 0 & 0 \\ -\mathrm{C}_{12} & \mathrm{C}_{22} & -\mathrm{C}_{23} & 0 \\ 0 & -\mathrm{C}_{23} & \mathrm{C}_{33} & -\mathrm{C}_{34} \\ 0 & 0 & -\mathrm{C}_{34} & \mathrm{C}_{44} \end{bmatrix} \tag{14}$$

And the inverse matrix is

$$[\mathrm{C}]^{-1} = \frac{1}{\Delta}\begin{bmatrix} \mathrm{C}_{22}\mathrm{C}_{33}\mathrm{C}_{44} - \mathrm{C}_{44}\mathrm{C}_{23}^2 - \mathrm{C}_{22}\mathrm{C}_{34}^2 & \mathrm{C}_{12}\left(\mathrm{C}_{33}\mathrm{C}_{44} - \mathrm{C}_{34}^2\right) & \mathrm{C}_{12}\mathrm{C}_{23}\mathrm{C}_{44} & \mathrm{C}_{12}\mathrm{C}_{23}\mathrm{C}_{34} \\ \mathrm{C}_{12}\left(\mathrm{C}_{33}\mathrm{C}_{44} - \mathrm{C}_{34}^2\right) & \mathrm{C}_{11}\left(\mathrm{C}_{33}\mathrm{C}_{44} - \mathrm{C}_{34}^2\right) & \mathrm{C}_{11}\mathrm{C}_{23}\mathrm{C}_{44} & \mathrm{C}_{12}\mathrm{C}_{23}\mathrm{C}_{34} \\ \mathrm{C}_{12}\mathrm{C}_{23}\mathrm{C}_{44} & \mathrm{C}_{11}\mathrm{C}_{23}\mathrm{C}_{44} & \mathrm{C}_{44}\left(\mathrm{C}_{11}\mathrm{C}_{22} - \mathrm{C}_{12}^2\right) & \mathrm{C}_{34}\left(\mathrm{C}_{11}\mathrm{C}_{22} - \mathrm{C}_{12}^2\right) \\ \mathrm{C}_{12}\mathrm{C}_{23}\mathrm{C}_{34} & \mathrm{C}_{12}\mathrm{C}_{23}\mathrm{C}_{34} & \mathrm{C}_{34}\left(\mathrm{C}_{11}\mathrm{C}_{22} - \mathrm{C}_{12}^2\right) & \mathrm{C}_{11}\mathrm{C}_{22}\mathrm{C}_{33} - \mathrm{C}_{33}\mathrm{C}_{12}^2 - \mathrm{C}_{11}\mathrm{C}_{23}^2 \end{bmatrix} \tag{15}$$

Where

$$\Delta = |C| = C_{11}C_{22}C_{33}C_{44} + C_{12}^2C_{34}^2 - C_{33}C_{44}C_{12}^2 - C_{11}C_{44}C_{23}^2 - C_{11}C_{22}C_{34}^2 \tag{16}$$

Thus although the capacitance matrix is sparse the inductance matrix is full, with each inductor coupling to all the others. Any attempt to construct a metamaterial with a single mode of propagation would require the construction of this complex matrix with all inductors coupling all the others despite the capacitors not coupling.

B. Analysis of simple 3-node lowpass filter

Fig. 3 shows a simple 3-node lowpass filter.

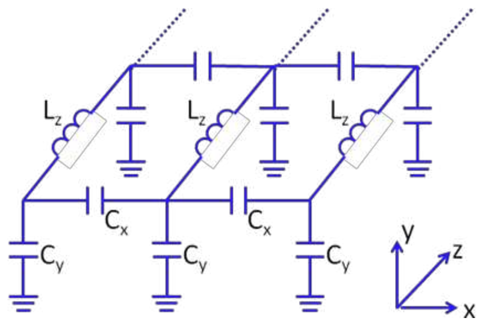


Fig. 3 3 node lowpass filter

Let $L_z = 2, C_x = 1$ and $C_y = 2$

$$[Z][Y] - \gamma^2[1]$$

$$= 2p^2\begin{bmatrix}2p & 0 & 0\\ 0 & 2p & 0\\ 0 & 0 & 2p\end{bmatrix}\begin{bmatrix}3p & -p & 0\\ -p & 4p & -p\\ 0 & -p & 3p\end{bmatrix} - \gamma^2\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \quad (17)$$

$$= 2p^2\begin{bmatrix}3 - \frac{\gamma^2}{2p^2} & -1 & 0\\ -1 & 4 - \frac{\gamma^2}{2p^2} & -1\\ 0 & -1 & 3 - \frac{\gamma^2}{2p^2}\end{bmatrix} \quad (18)$$

$$= 2p^2\begin{bmatrix}3+z & -1 & 0\\ -1 & 4+z & -1\\ 0 & -1 & 3+z\end{bmatrix} = 2p^2[A] \quad (19)$$

Where

$$z = -\frac{\gamma^2}{2p^2} \quad (20)$$

The eigen values are found from

$$\begin{vmatrix}3+z & -1 & 0\\ -1 & 4+z & -1\\ 0 & -1 & 3+z\end{vmatrix} = 0 \quad (21)$$

And

$$z_i = -2, -3, -5 \quad (22)$$

$$\gamma_i = 2p, \sqrt{6}p, \sqrt{10}p \quad (23)$$

And for sinusoidal signals

$$\gamma_i = j\beta_i = j2\omega, j\sqrt{6}\omega, j\sqrt{10}\omega \quad (24)$$

The circuit supports three modes of propagation, each of which has dc cutoff frequency.

The voltage vectors for each of the three nodes may be found by evaluating the eigen vectors of matrix A [4] by solving

$$[A(z_i)][\mathrm{V}]_i = 0 \quad (25)$$

e.g. for mode 1 with $z_i = -2$

$$[A(-2)][\mathrm{V}]_1 = \begin{bmatrix}1 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 1\end{bmatrix}\begin{bmatrix}V_1\\ V_2\\ V_3\end{bmatrix} = 0 \quad (26)$$

Giving

$$[v]_1 = \begin{bmatrix}1\\ 1\\ 1\end{bmatrix} \quad (27)$$

Similarly,

$$[v]_2 = \begin{bmatrix}1\\ 0\\ -1\end{bmatrix} \quad (28)$$

and

$$[v]_3 = \begin{bmatrix}1\\ -2\\ 1\end{bmatrix} \quad (29)$$

The eigen vectors represent the input voltage at the three nodes which will excite the eigen mode with corresponding eigen values. The first eigen vector represents equal voltages at the three input nodes. This case is equivalent to a common(even) mode excitation, an even or common mode analysis can be applied as shown in Fig. 4a providing the equivalent circuit with propagation constant of $\gamma = \pm \mathrm{p}\sqrt{\mathrm{L_z C_y}} = \pm 2\mathrm{p}$. The second eigen vector represents a zero voltage at the second node and the equivalent circuit is shown in Fig. 4b. The propagation constant of this mode is $\gamma = \pm \mathrm{p}\sqrt{\mathrm{L_z(C_y + C_x)}} = \pm\sqrt{6}\mathrm{p}$. For the third eigen vector, as in Fig. 4c, the voltage across C_y is V_0 and the voltage across C_x is $3V_0$ which provides an equivalent capacitance of $3C_x$. As a result, the propagation constant is $\gamma = \pm \mathrm{p}\sqrt{\mathrm{L_z(C_y + 3C_x)}} = \pm\sqrt{10}\mathrm{p}$.

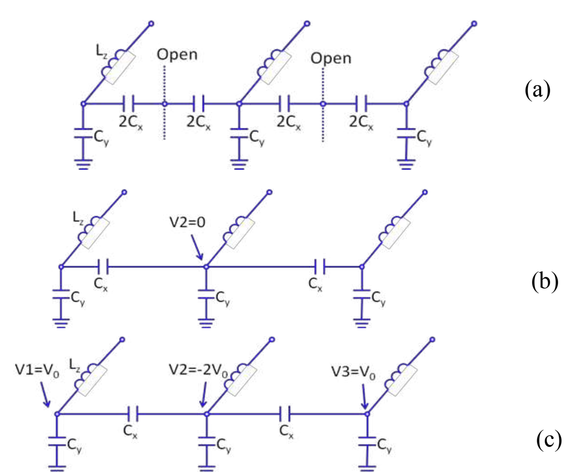


Fig. 4. Equivalent circuit model FOR A 3 node network with the excitation for each mode.

C. Derivation of network transfer matrix

The 2N-port circuit can be greatly simplified by diagonalising $[A]$ [4]. To do this form matrix $[P]$ whose columns are the eigen vectors. Then

$$[D] = [P]^{-1}[A][P] \tag{30}$$

For example using the 3-node lowpass filter from section II(B)

$$[P] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -2 \\ 1 & -1 & 1 \end{bmatrix} \tag{31}$$

and

$$[D] = \frac{1}{6}\begin{bmatrix} 2 & 2 & 2 \\ 3 & 0 & -3 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} 3+z & -1 & 0 \\ -1 & 4+z & -1 \\ 0 & -1 & 3+z \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -2 \\ 1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 3+z & 0 & 0 \\ 0 & 2+z & 0 \\ 0 & 0 & 5+z \end{bmatrix} \tag{32}$$

[D] is a nodal matrix consisting of a set of three individual single node two-port networks as in Fig. 5

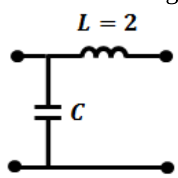


Fig. 5 Equivalent circuit for a particular diagonal element in [D]

And since

$$[Z][Y] - \gamma^2 = 2Cp^2 - \gamma^2 = 2p^2(C + z) \tag{33}$$

From (20) and (33)

$$C_i = 3{,}2 \text{ and } 5 \tag{34}$$

Diagonalization makes the input vector for a specific mode appear only at the particular two-port which is associated with that mode. Thus the entire networks can be represented by the following equivalent circuit.

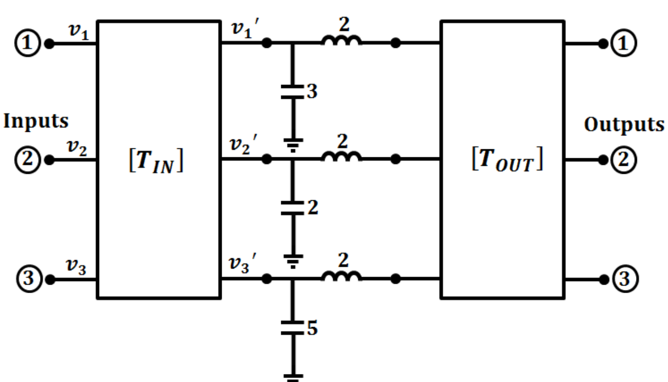


Fig. 6. Equivalent circuit of 3-node network of Fig. 3

The input and output networks are 2N-port ideal transformers with transfer matrices

$$[T_{IN}] = \begin{bmatrix} [P] & [0] \\ [0] & [P]^{-1} \end{bmatrix} \tag{35}$$

and

$$[T_{OUT}] = [T_{IN}]^{-1} = \begin{bmatrix} [P]^{-1} & [0] \\ [0] & [P] \end{bmatrix} \tag{36}$$

For this example

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ I_1 \\ I_2 \\ I_3 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -2 \\ 1 & -1 & 1 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 2 & 2 & 2 \\ 3 & 0 & -3 \\ 1 & -2 & 1 \end{bmatrix} \end{bmatrix}\begin{bmatrix} V_1' \\ V_2' \\ V_3' \\ I_1' \\ I_2' \\ I_3' \end{bmatrix} \tag{37}$$

And

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -2 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} V_1' \\ V_2' \\ V_3' \end{bmatrix} \tag{38}$$

For mode 1, with $v_1 = 1, v_2 = 1, v_3 = 1$ then

$$v_1' = 1, v_2' = 0, v_3' = 0 \tag{39}$$

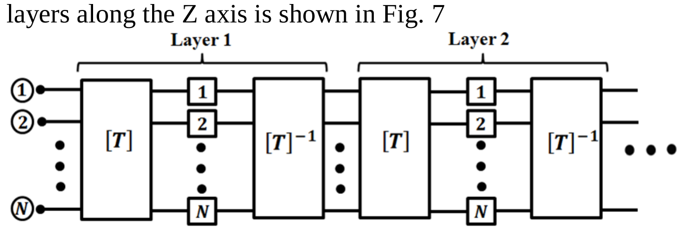
For mode 2, with $v_1 = 1, v_2 = 0, v_3 = -1$ then

$$v_1' = 0, v_2' = 1, v_3' = 0 \tag{40}$$

And for mode 3 with $v_1 = 1, v_2 = -2, v_3 = 1$ then

$$v_1' = 0, v_2' = 0, v_3' = 1 \tag{41}$$

The procedure can be applied to any two dimensional filter or metamaterial with uniform cross section. The frequency response for any combination of modes at the input may be obtained by simple superposition. Furthermore extending the analysis to include multiple layers along the z-axis is relatively trivial. The equivalent circuit for an N node circuit with M layers along the Z axis is shown in Fig. 7



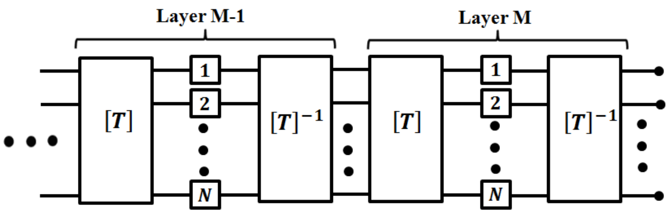


Fig. 7 N node M layer circuit equivalent

And since $[T]^{-1}[T] = [I]$ the circuit reduces to Fig. 8. The cascade of M identical modal subnetworks may then easily be dealt with using image parameters for example.

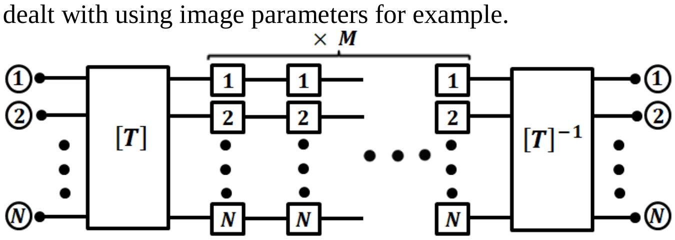


Fig. 8 Final equivalent circuit of $N \times M$ filter

The techniques developed in this section will be demonstrated by analysis of the two dimensional lowpass filter in the next section.

III. Modal Analysis of 2-D Lowpass Filters

A. Analysis

We will apply the theory to the analysis of a two dimensional quasi-lowpass filter as shown in Fig.9. This is a simple approximate equivalent circuit for the classic Waffle iron filter [5].

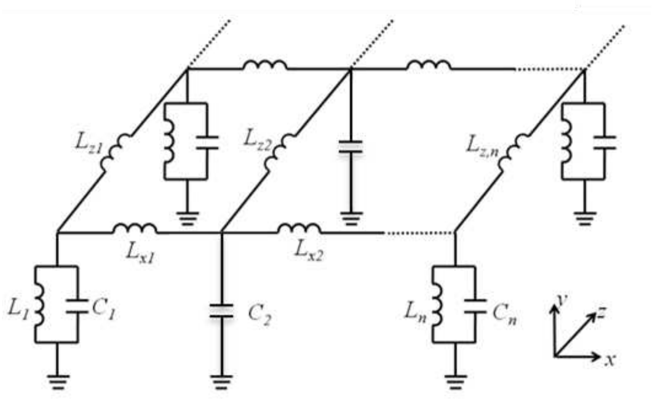


Fig. 9 Lumped 2-D quasi-lowpass filter, with N transversal sections and M longitudinal section.

The theory developed in section II has been used to simulate a filter with N=5 and M=10 sections. For a practical design a half inductance was added to the basic section at both ends of the transverse section as in Fig. 10

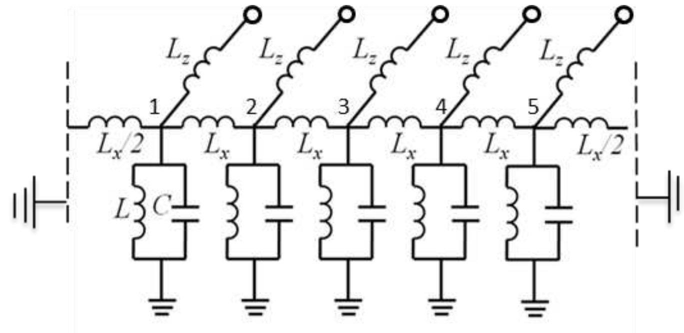


Fig. 10 Circuit model of a single layer of N=5 quasi lowpass filter

The eigen vectors of this N=5 circuit are listed below and are independent of specific element values, although obviously the eigen values are dependent.

$v_1 = -0.1954, -0.5117, -0.6325, -0.5117, -0.1954$
$v_2 = -0.3717, -0.6015, 0, 0.6015, 0.3717$
$v_3 = -0.5117, -0.1954, 0.6325, -0.1954, -0.5117$
$v_4 = -0.6015, 0.3717, 0, -0.3717, 0.6015$
$v_5 = 0.4472, -0.4472, 0.4472, -0.4472, 0.4472$

As discussed in the previous section when a particular eigen vector is input to the circuit only the corresponding mode will be excited, and analysis of the equivalent circuit shown in Fig. 8 may easily be performed.

In this example we chose $C = 2$, $L = 2$, $L_x = 1$ and $L_z = 1$

The transfer function may be evaluated by summing the voltages from each of the output ports and is shown in Fig. 11

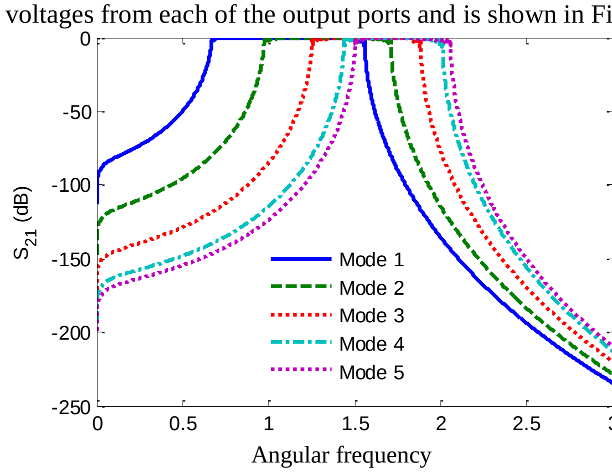


Fig. 11 $|S_{21}|^2$ for the 5 modes of $N = 5$, $M = 10$ waffle-iron filter circuit model

For any other input voltage the resulting transfer function is a combination of all 5 modes. For example with $V_{in} = 1,2,3,0,-2$ the transfer function is shown in Fig. 12

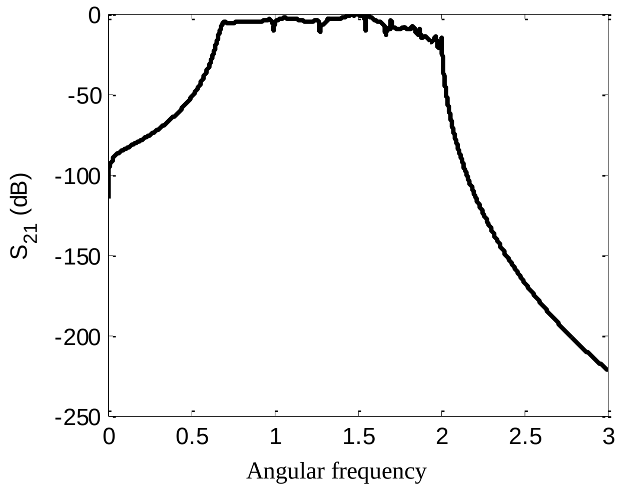


Fig. 12 Transfer function with input voltage, $V_{in} = 1,2,3,0,-2$

B. Explicit formulae for eigen values of quasi lowpass filter

The equivalent circuit for a transverse section of the waffle iron filter is shown in Fig. 13

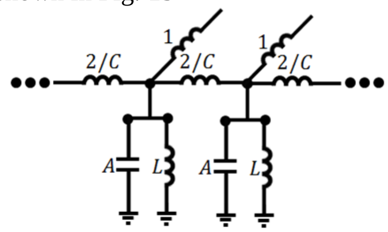


Fig.13 Transverse section of quasi lowpass filter

Now

$$[A] = [Z][Y] - \gamma^2[1] \tag{42}$$

$$[A] = p\begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}\begin{bmatrix} Ap+B/p & -C/p & 0 & \dots & 0 \\ -C/p & Ap+B/p & -C/p & \dots & 0 \\ 0 & -C/p & Ap+B/p & \dots & 0 \\ \vdots & \vdots & \vdots & & -C/p \\ 0 & 0 & 0 & -C/p & Ap+B/p \end{bmatrix} - \gamma^2\begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix} \tag{43}$$

$$= \begin{bmatrix} z+B & -C & 1 & \dots & 1 \\ -C & z+B & -C & \dots & 1 \\ 1 & -C & z+B & \dots & 1 \\ \vdots & \vdots & \vdots & & -C \\ 1 & 1 & 1 & -C & z+B \end{bmatrix}$$

Where

$$z = Ap^2 - \gamma^2 \tag{44}$$

This is the admittance matrix of the network shown in Fig. 14

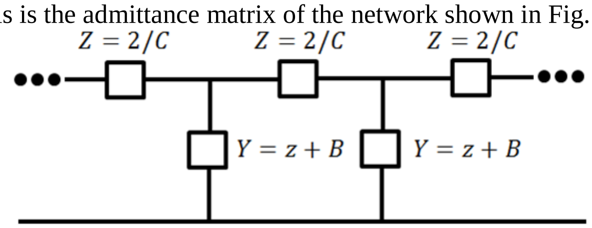


Fig. 14 Equivalent circuit of $[A]$

Which is a cascade of basic sections shown in Fig. 15.

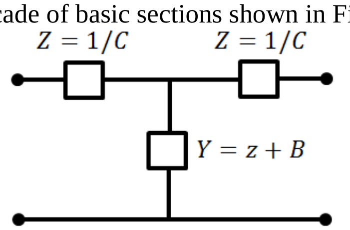


Fig. 15 Basic section in equivalent circuit of $[A]$

The basic section has the transfer matrix

$$[T] = \begin{bmatrix} 1 & 1/C \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ z+B & 1 \end{bmatrix}\begin{bmatrix} 1 & 1/C \\ 0 & 1 \end{bmatrix} \tag{45}$$

$$[T] = \begin{bmatrix} 1 + \frac{z+B}{C} & \frac{1}{C}\left(2 + \frac{z+B}{C}\right) \\ z+B & 1 + \frac{z+B}{C} \end{bmatrix} \tag{46}$$

And using image parameters [6]

$$[T] = \begin{bmatrix} cosh(\varphi) & Z_I sinh(\varphi) \\ Y_I sinh(\varphi) & cosh(\varphi) \end{bmatrix} \tag{47}$$

Where

$$\varphi = cosh^{-1}\left(1 + \frac{z+B}{C}\right) \tag{48}$$

and

$$Z_I = \sqrt{\frac{1}{C(z+B)}\left(2 + \frac{z+B}{C}\right)} \tag{49}$$

And cascading N sections we obtain

$$[T]^N = \begin{bmatrix} cosh(N\varphi) & Z_I sinh(N\varphi) \\ Y_I sinh(N\varphi) & cosh(N\varphi) \end{bmatrix} \tag{50}$$

The A parameter of $[T]^N$ will be zero at the eigen values hence

$$cosh\left[Ncosh^{-1}\left(1+\frac{z+B}{C}\right)\right]=0 \tag{51}$$

and

$$cos\left[Ncos^{-1}\left(1+\frac{z+B}{C}\right)\right]=0 \tag{52}$$

The zeros of this Chebyshev polynomial occur when

$$1+\frac{z+B}{C}=\cos\left(\frac{(2r-1)\pi}{2N}\right), \text{for } r=1,2,\dots,N \tag{53}$$

and

$$\frac{z+B}{C}=\cos\left(\frac{(2r-1)\pi}{2N}\right)-1\leq 0 \tag{54}$$

The values of z lie on the negative real axis as in an RC network. The roots z are distinct even when N tends to infinity, Now

$$z=\mathrm{C}\left(\cos\left(\frac{(2r-1)\pi}{2N}\right)-1\right)-\mathrm{B} \tag{55}$$

and from (44)

$$\gamma^2=Ap^2-z \tag{56}$$

z is negative and real so for sinusoidal signals

$$\gamma=\pm\sqrt{k^2-A\omega^2} \tag{57}$$

Each of the modes has a distinct cutoff frequency below which γ is real, i.e. evanescent and above which γ is imaginary i.e. in the modes passband. The same analysis to obtain the explicit formula could be applied to any other metamaterials.

IV. Conclusion

Two dimensional filters and meta-materials with N nodes in the transverse direction support N distinct modes of propagation each with their own unique propagation constant. The propagation constants are the eigen values of the transverse nodal matrix of the network and the modal voltages are the eigen vectors.

An equivalent circuit for an $N\times M$ filter has been derived. This enables the frequency response to be determined for any arbitrary modal excitation. The analysis technique has been applied to 2-D lowpass filters. An explicit formula for its eigen values is derived and demonstrates conclusively that there are also N modes even when N tends to infinity.

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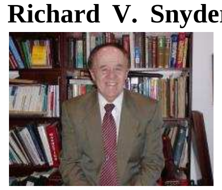

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