

Pattern formation in the geosciences

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Pattern formation is a natural property of nonlinear and non-equilibrium dynamical systems. Geophysical examples of such systems span practically all observable length-scales, from rhythmic banding of chemical species within a single mineral crystal, to the morphology of cusps and spits along hundreds of kilometres of coastlines. This article briefly introduces the general principles of pattern formation, and argues how they can be applied to open problems in the earth sciences. Particular examples are then discussed, which summarise the contents of the rest of this theme issue.

Key words: Pattern Formation, Nonlinear Geophysics.

1. Pattern formation

Patterns exist throughout nature, and are attractive to the trained observing eye of the scientist. Many of the questions that are presented in this issue on pattern formation were already under the attention of the earliest members of the Royal Society. The origin of springs “running down by the Valleys or Guttes between the ridges of the Hills, and coming to unite, form little Rivulets or Brooks” was discussed by Edmund Halley [8], while the prismatic forms of columnar joints were first brought into the records of the Society by travellers’ reports forwarded by Sir Richard Bulkeley [2]. The quantitative investigation of such patterns, however, is now a field of active research. For example, Petroff *et al.* [15] here demonstrate that springs eroding into a slope will generically split, or bifurcate, at an angle of $2\pi/5$, while I present a crack-ordering mechanism that links columnar joints with polygonal terrain, and mud-cracks [6]. There are many other situations where regular patterns are generated, from the largest scales of geomorphology, such as the curving subduction arcs of the earth’s crust [12] or the meanders of river networks, to ripples on the beach [1], and periodic chemical precipitation patterns [9]. The details of such patterns can provide quantitative information about the systems in which they form, and the mechanisms underlying them.

The science of pattern formation, a physically motivated study of spatio-temporal patterns, and an attempt to explain how, why and when they arise, was formalised during the second half of the 20th century [3]. Feedback can do much more than amplify or control a signal, and as understanding of nonlinear dynamical systems grew, unexpected universal features of these systems were found. The complex response of even simple systems, such as a double-pendulum, or Lorenz’s first 3-component atmospheric model [11], where

arbitrarily close initial conditions rapidly diverge, is now known as chaos. In other situations complex interactions can give rise to simple regular patterns. This is the case we focus on here, although often both chaos and patterns may be found in the same system (see, for example, L’Heureux’s review [9] herein on geochemical precipitation patterns). In both cases the systems are usually dynamical, nonlinear, and out of equilibrium: the patterns are dissipative, entropy-producing states, driven by an energy flux between some external energy source and heat sink.

Patterns typically arise as the result of competition between two opposing forces. Rayleigh-Bénard convection is the result of a competition between buoyancy, which acts to lift warmer, lighter fluids, and viscous dissipation, which tends to damp out any motion. The ratios of these forces define the dimensionless groups whereby a dynamical system can be parameterised. In most cases the relevant number of such groups is small, because when groups are too dissimilar in magnitude, the weaker terms are usually (but not always) negligible in effect. Thus, for example, Wells and Cossu describe here how a balance between centrifugal and Coriolis forces is captured by a dimensionless Rossby number, and explain how the dominant driving force, and the consequent morphology, of submarine channelling differs at high and low Rossby numbers [20].

Finally, the conditions under which patterns change form have been shown to be particularly revealing. The theory of such bifurcations, or transitions between states, was originally developed by Poincaré [21], and modern physical use grew out of specific applications of these ideas to phase changes in condensed matter physics [3]. Bifurcations can frequently be classified into one of a small number of instability types, based on considerations of dimensionality and symmetry. These dictate what the leading nonlinear contributions are, and what generic type of response is expected around a critical point. For example, in many cases an instability in a system with reflection (*e.g.* left-right or up-down) symmetry leads to its classification as a pitchfork bifurcation. Near critical points of bifurcations the behaviours of diverse systems can reduce to those of a few universal responses, which are independent of the detailed physics of these systems (*i.e.* all members of the same universality class share the same behaviour close to a supercritical bifurcation). For example the transition from a straight to wavy crack [22], and the onset of oscillations during certain chemical reactions [3, 9] are Hopf bifurcations, and near their transitions these very different problems can be mapped onto each other. Just as critical points are key to understanding the behaviour of classical thermodynamic phase diagrams, they are generally a very powerful tool for establishing and testing the physics of any pattern-forming system.

2. Why are geophysical patterns interesting?

The Earth is not in thermodynamic equilibrium. Heat flows from the molten core to the crust, through a convecting mantle. Winds blow, rain falls, and mountains erode, as the sun’s energy is processed and re-radiated to the cooler background of space. Life happens, and in doing so, changes the world. The means that shaped the geography that we see today are dynamic, complex, non-equilibrium, and nonlinear. These are the natural conditions in which to expect self-organisation, and the patterns that can be found tell us about the physics of these systems.

Further, pattern-forming mechanisms can be very robust; they are not only seen in controlled laboratory situations, but also survive (and can even thrive on) the noise of real-world geomorphic environments.

One strength of the modern approach to patterns is its universality. The same instability can appear in a great variety of situations. Turing's seminal paper *On the chemical basis of morphogenesis*, for example, outlined the conditions necessary for generating a linear instability in a system with two reacting, diffusing, chemical morphogens [19]. In general, it applies to any two (or by a straightforward generalisation, more) interacting fields u and v , where

$$\partial_t u = f(u, v) + D_u \nabla^2 u$$

$$\partial_t v = g(u, v) + D_v \nabla^2 v.$$

This simple system is unstable to periodic stripes, or spots, when the ratio of diffusivities is sufficiently high, and the interaction terms f and g are alternatively excitatory (*e.g.* $\partial f / \partial u > 0$), and inhibitory ($\partial g / \partial v < 0$). Furthermore, the values of these terms, which can derive from microscopic interactions, predict a most unstable length-scale, at which patterns arise. This mechanism directly underlies about a third of the articles in our volume. L'Heureux applies a form of it, close to its original intent, to show how periodic precipitation patterns are found in a variety of rocks [9]. This banding is thus a permanent chemical record of the dynamical processes of its formation, and can be read only with an appropriate understanding of the nonlinear feedbacks and instabilities of the system. The interacting variables can, however, be more subtle. Zelnik *et al.* study the process of desertification in a series of models of dry-land vegetation, where moisture and biomass interact to give rise either to patterns vegetative spots, or bare soil [23]. Penny, Daniels, and Thompson share an observational study of a similar pattern, where symmetry is broken by hills, and the down-slope drainage of the water leads to oriented stripes [14]. Finally, Da Lio, D'Alpaos and Marani describe wetlands, where interactions between sediment flux and biomass stabilise a series of stepped vegetated platforms [10].

The universality of pattern-forming systems can also be a challenge. The patterned ground of permafrost soils has evident structure. Detailed numerical models can reproduce the shapes of this terrain well. However, as discussed here by Hallet [7], such models can generate a similar pattern as the result of either frost heave, or ground-water convection. The frost-heave model is now preferred, but only as the result of cycles of prediction and validation. In another example, certain labyrinthine patterns in grasses look superficially like the reaction-diffusion based vegetation patterns of larger plants, but may result from the entirely different mechanism of porous-media convection [18]. Similarity of form alone, no matter how beautiful, is insufficient for proof - a useful model of pattern formation is necessarily quantitative, and describes additional features such as wavelength selection, scaling, rates, or the location of bifurcation points between patterns in parameter space. In turn, these quantitative details guide further testing of models against field measurements, and aid in the design of meaningful analogue experiments. Once understanding is gained, and tested, one can turn to interpretation, and here patterns can be powerful diagnostics of conditions that no longer exist, or which (*e.g.* other planets, long timescales) are difficult to access directly.

3. Contents of this issue

This issue is dedicated to the application of the tools of pattern formation to geophysical problems. Richard Feynman recalled of his student days that he acquired a “reputation for doing integrals, only because my box of tools was different from everybody else’s, and they had tried all their tools on it before giving the problem to me.” [4] Although pattern formation in physics and applied mathematics is a mature field, and the phenomena included here are mostly very well known, it is only relatively recently that this box of tools, including linear stability analysis, bifurcation theory, symmetries and symmetry breaking, and dynamical systems theory, has been brought to bear on geophysical problems. Progress has been impressive, as will hopefully be demonstrated. However, despite drawing on similar methods, the applications here have also tended to be developed piecemeal, and represent a growing community with considerable opportunity to learn from each other. One additional aim of this theme issue is to draw these topics together.

We begin with a review of one of the most well-known, and commonly seen geophysical patterns, that of sand dunes. When fluid flows over a granular bed, the grains can be picked up and transported with the flow. If the bed has an uneven surface, grains can be preferentially eroded from regions of higher wind stress, and deposited elsewhere. Andreotti and Claudin [1] review how this mechanism leads to the linear instability of dunes of well-defined wavelength. They show how such dunes scale under different fluids, such as liquid water or the atmospheres of Earth, Mars, and Venus, and how these small-amplitude dunes grow and coarsen until they are limited by the thickness of the flowing fluid layer. Finally, they discuss the current challenges in modelling subaqueous ripples, which may lie between turbulent and laminar flow conditions, and summarise extensions of these ideas to situations such as alternating river bars.

Thomas *et al.* consider the same hydrodynamic forcing that give rise to dunes, but apply it to the viscoelastic substrate of microbial mats. In their paper on *Kinneyia* [17] they consider the formation of a class of wrinkled fossil biofilms that are common in the Precambrian fossil record, but absent from more modern times. By coupling linear stability theory to analogue experiments and field observations, they show how *Kinneyia* may form through a generic Kelvin-Helmholtz type instability of a sheared interface. This raises interesting questions of whether these fossils can be used to interpret the prevailing conditions under which they formed, and as to why they are no longer observed.

Just as sands and biofilms are moved by flow, near-shore sediments can be transported by wave action. This leads to an instability of coastlines to periodic perturbations, and the growth of small bumps into cusped spits and bays, for example. Murray and Ashton [13] review recent work on self-organised coastline patterns. They show how the along-shore sediment flux can be simplified into the form of a nonlinear diffusion equation, which becomes unstable at locations, or conditions, where the effective diffusivity changes sign. They then show how the finite-amplitude forms of this instability have been explored numerically, in a range of conditions ranging from open coastlines with symmetric or highly asymmetric wave forcing, to enclosed lakes, where coastline features on either side of a body of water can interact in surprising ways.

The near-shore sediment flux in tidal regions can also be controlled by more local conditions, including vegetation. Da Lio, D'Alpaos and Marani describe how competition between specialised species in such wetlands can lead to the stabilisation of a series of salt marsh platforms of different heights, each dominated by one species of vegetation [10]. They explore a dynamical model that couples biomass of different species with sediment production and transport, and show how stable, attractive solutions that describe a series of platforms would naturally develop. As sea-level rise is a timely concern for tidal marshes, they also study the effects of the relative rate of rise on the stability of the structure of salt marshes, and demonstrate that the bio-geomorphic feedback may make such wetland patterns more resilient than previously thought.

In other situations, the feedback between life and its environment can also give rise to patterns, and vegetation patterns in landscapes have become well studied in recent years. In arid environments these typically take the form of spots, stripes, or labyrinths of alternating vegetation and bare soil (see *e.g.* [16]). Zelnik *et al.* survey a broad range of the models that have developed to describe the interaction of vegetative spots with water, given a limiting precipitation rate [23]. In particular, they study the process of desertification, which in these models results from multiple stable states, and hysteresis, in the vegetative cover. By investigating multiple models, they attempt to answer the question of whether desertification should proceed by gradual growth of desert patches, or by the rapid non-local shift between patterns. In terms of a perhaps more familiar phase change like freezing, these are equivalent to heterogeneous and homogeneous nucleation, respectively.

Although pattern formation in vegetation is common, observational tests of the theoretical predictions regarding such landscapes are few. Penny, Daniels, and Thompson present a field study of patterns in the drylands of Texas, where alternating stripes of vegetative cover and bare soil cover several hillsides [14]. Using Fourier methods, they analyse the directions of stripes, and compare them to local slopes. The stripes generally run perpendicular to the slopes, but they show local excursions from this behaviour that depend on other heterogeneities of the environment. For example, they show that soil depth or type may be important in the feedbacks of vegetative stripes, features that have not previously been considered important.

Having now considered pattern formation in deserts, dry lands, wetlands and shorelines, we turn to surface patterns in icy regions. Hallet presents a field study of sorted circles in permafrost [7]. This pattern is thought to arise from a convection-like overturning of the soil over many years, an instability driven by freeze-thaw cycles. Interestingly, in contrast to vegetation patterns, it can also manifest as stripes on hillsides, but which lie perpendicular to the topographic contours. Hallet has studied the dynamics of this pattern on the flat plains of Spitzbergen for over 20 years, and he presents here a long-timescale series of measurements of the evolution of their shape, the displacement and rotation of tracers in their surface and interior, along with local temperature records. These compare well to numerical models of instabilities of sorted circles, and help quantify our understanding of the near-surface transport of soils in polar environments.

Polar terrain is patterned by two different mechanisms, the freeze-thaw mechanism just considered, and thermal contraction cycles. The ice-cemented

soil that frequently underlies the active layer of permafrost is hard and tough, but can crack under the stresses caused by seasonal temperature variations. Here, I consider the ordering of these fracture patterns, and other hexagonally-ordered crack structures, through analogue experiments on drying clay [6]. I show how a simple model of cracking, where cracks open and heal repeatedly, guided by their previous positions but changing their order of appearance, can explain quantitative details of such networks mechanistically.

Similar networks can also be found in surprising places, such as in fluid mixing. Fu, Cueto-Felgueroso and Juanes present here a numerical study of the dissolution of CO₂ that is expected as a result of carbon capture and storage in underground aquifers [5]. The convective mixing initially selects a most unstable wavelength, but the pattern rapidly develops into a well-defined cellular network of columnar fingers of CO₂-rich fluid. The coarsening of this network occurs by discrete cellular rules that are shown to be similar to how foams and crack patterns coarsen, and gives rise to a non-equilibrium steady state with universal scaling.

Petroff *et al.* here also consider the dynamics of a network, when analysing the branching of springs and streams [15]. By applying a complex potential method to the seepage of groundwater around a spring, they map this dynamics to the more general problem of growth in a potential field. They find an instability of the stream tip, and predict that it should give rise to a stream branching angle of $2\pi/5$. This result is compared to other tip-splitting systems, such as solidification of a melt, or vascular networks, and the branch angle of several thousand streams is measured, and found to agree precisely with their model.

We close this issue with two further review pieces. The sinuous meander of rivers is another of the more well-known instabilities seen in nature. However, sinuosity is not only present in fluvial cases, but is seen in submarine channeling as well. Wells and Cossu [20] review interesting recent observations which show that high-latitude channels are significantly straighter than low-latitude channels. They connect this with the symmetry-breaking mechanism of the Coriolis force, and show how a dimensionless Rossby number, which describes the relative importance of this force, becomes significant at high-latitudes.

Finally, L'Heureux reviews a class of geochemical patterns that are related to the case of periodically precipitating Liesegang rings [9]. He shows that the same generic reaction-diffusion instability can lead to the rhythmic banding of precipitates in volcanic, sedimentary, and metamorphic rocks, and presents a particularly detailed novel account of this process in sapropels. These processes can also occur during the precipitation of single crystals, and a brief review of this oscillatory banding is also given.

4. Concluding remarks

We have attempted to gather a representative collection of topics in this theme issue on pattern formation in the geosciences. It is not exhaustive, but gives an overview of what is possible when the techniques of pattern formation and nonlinear dynamics are applied to geophysical problems. Some topics have been well-studied over several decades, such as sand dunes and sinuous rivers. The results that are presented here show how understanding of these ideas, once developed, can be fluidly applied in other contexts, such as sand bars

[1] or submarine channels [20]. The inclusion of Kinneya [17], shows how new applications of linear stability theory can give insight into processes that happened long ago in our planet's past. Similarly, the papers on permafrost [6, 7] and vegetation patterns [14] are relevant to the study of remote locations, such as Earth's polar and desert regions, or Mars, where satellite imaging is available, but direct studies can be challenging.

The systems discussed here are dynamic, and their steady states can be expected to evolve as conditions change. Prediction, and in some cases even control, of geophysical patterns is an area where future developments can be expected to be particularly influential. Pattern-forming systems can be susceptible to catastrophic regime changes [16], such as the desertification process studied by Zelnik *et al* [23], when a bifurcation point is reached. Alternatively, the approaches of Murray and Ashton [13] to coastline migration, or Da Lio, D'Alpaos and Marani to wetland development use nonlinear feedback to make explicit testable predictions, which will be important in attempts to minimise disruption of coastal regions by changing climates. Finally, large-scale experiments on carbon capture and storage will require understanding and engineering patterns of reactive porous media flow over many cubic kilometres, and inspired modelling such as that of Fu, Cueto-Felgueroso and Juanes [5].

It is our hope that this collection serves to demonstrate the strengths of the current research of this highly multidisciplinary field, and to encourage its potential for future development.

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