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The Inflation Bias under Calvo and Rotemberg Pricing*

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Abstract

New Keynesian analysis relies heavily on two workhorse models of nominal inertia - due to (Calvo, 1983) and (Rotemberg, 1982), respectively - to generate a meaningful role for monetary policy. These are often used interchangeably since they imply an isomorphic linearized Phillips curve and, if the steady-state is efficient, the same policy conclusions. In this paper we compute time-consistent optimal monetary policy in the benchmark New Keynesian model containing each form of price stickiness using global solution techniques. We find that, due to an offsetting endogenous impact on average markups, the inflation bias problem under Calvo contracts is often significantly greater than under Rotemberg pricing, despite the fact that the former typically exhibits far greater welfare costs of inflation. The nonlinearities inherent in the New Keynesian model are significant and the form of nominal inertia adopted is not innocuous.

Keywords: New Keynesian Model; Monetary Policy; Rotemberg Pricing; Calvo Pricing; Inflation Bias; Time-Consistent Policy.

JEL codes: E52, E63

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1 Introduction

Mainstream macroeconomic analysis of both monetary and fiscal policy relies heavily on the New Keynesian model. The distinguishing feature of this model, relative to a more classical approach, is that it contains some form of nominal inertia. This allows monetary policy to have real effects, and widens the degree of interaction between monetary and fiscal policies, since monetary policy affects both the size of the tax base and real debt service costs in such models. Typically, one of two workhorse forms of nominal inertia are adopted in the literature - Calvo (1983) price contracts, and Rotemberg (1982) price adjustment costs. In the former, firms are only able to adjust their prices after random intervals of time, such that, outside of a zero inflation steady-state there will be a costly dispersion of prices across firms. In the latter, firms behave symmetrically in setting the same price but they face quadratic adjustment costs in doing so. Despite this fundamental difference, researchers have typically treated the two approaches as being equivalent since the New Keynesian Phillips Curves (NKPC) they imply are, to a first order of approximation, isomorphic when linearized around a zero inflation steady state. Moreover, when that zero inflation steady-state is also efficient (that is, it matches the output level that would be chosen by a benevolent social planner) it can be shown that the second-order approximation to welfare rewritten in terms of inflation and the output gap is also the same across the two approaches (see Nistico, 2007). Under these conditions, to a first order of approximation, the two approaches would yield the same policy implications. For these reasons the two approaches have largely been treated as synonymous within the New Keynesian literature.

However, despite this broad consensus, there are examples within the literature where the two approaches do differ. The first is where the steady-state around which we approximate the New Keynesian economy is not efficient. For example, Lombardo and Vestin (2008) relax the assumption of Nistico (2007) and consider the second order approximation to welfare when the steady state is not efficient. They find that the costs of such inefficiencies are typically larger in the Calvo economy. This mirrors the results in Damjanovic and Nolan (2011).

Moreover, there appears to be significant nonlinearities in the New Keynesian model which are affected by the size of the steady-state distortion, the degree of unindexed inflation and the type of nominal inertia adopted. For example, the trending inflation literature (see Ascari and Sbordone (2014) for a survey) finds that the presence of even a modest degree of (unindexed) steady-state inflation can radically overturn determinacy results, undermine the learnability of rational expectations equilibria, affect the monetary policy transmission mechanism and change the nature of optimal policy. In addition, these effects can differ across the two forms of nominal inertia (Ascari and Rossi, 2012), with the larger impact of trend inflation being felt under Calvo. The large costs of trend inflation under Calvo is also reflected in the analysis of Damjanovic and Nolan (2010b) where the seigniorage maximizing rate of inflation is at double digit levels under Rotemberg pricing, but only single digits under Calvo. However, this evidence largely comes from studies which linearize such economies, either to a first- or second-order approximation, after allowing for such factors.

In this paper we solve the benchmark New Keynesian model nonlinearly using the two standard approaches to model price stickiness. Since we are not imposing any kind of approximation around a steady-state, we can fully explore the nonlinearities inherent in the New Keynesian model and see clearly the extent to which the two approaches differ. Moreover, rather than consider the Ramsey problem or commitment to a simple monetary policy rule, we shall consider time-consistent optimal policy (commonly known as discretion). This in turn, given that we are not using any artificial devices to ensure the model's steady-state being efficient, implies that we can measure the extent of the inflationary bias problem under the two forms of nominal inertia.[1] In general, this paper helps us understand the nature of equilibrium without commitment - a property of monetary models that is less studied in the literature. To our knowledge, the current paper is the first to formally compare and contrast time-consistent optimal policy under the two forms of price-setting using global solution algorithms and therefore to assess how innocuous the choice of one form of price-setting over the other actually is.

The inflationary bias problem is driven by a combination of the policy maker's desire to increase an otherwise sub-optimally low level of output by inducing inflation surprises, and the fact that they cannot credibly commit not to do so. Economic agents anticipate such behavior and raise their inflation expectations until the policy maker is no longer tempted to introduce any inflation surprises. Essentially, inflation rises to a level which is sufficiently costly to prevent the policy maker from unexpectedly relaxing monetary policy, and society suffers the costs of higher inflation without reaping any benefits in terms of higher output. Since the costs of inflation are known to be higher under Calvo relative to Rotemberg pricing, ceteris paribus, it might be expected that this implies the inflationary bias problem is correspondingly lower under Calvo pricing. Our analysis shows that this is often not the case, and that the inflationary bias problem, while significant under both pricing mechanisms, can be much higher under Calvo pricing.

The possibility of a worsening inflationary bias problem under Calvo arises because of the different average markup behavior under the two models. Under Calvo higher inflation causes those firms who are able to adjust prices in a particular period to raise that price in anticipation of not being able to readjust the price for a prolonged period despite the general rise in the price level. This leads to an increase in the average markup as inflation rises. In contrast, under Rotemberg all firms set the same price, period by period, but

[1] It is well known that the optimal rate of inflation under commitment is zero, hence the inflation bias is equal to the equilibrium rate of inflation under discretion.

face adjustment costs in doing so. In discounting future profits they also discount future price adjustment costs. As a result, in the face of higher inflation the firms postpone some of the required price adjustment due to this discounting effect, which serves to reduce the average markup. Taking stock, the average markup and the associated distortion under Calvo is increasing in inflation but decreasing under Rotemberg. Other things being equal, the more the economy is distorted away from the efficient allocation, the larger the incentive of the policy maker to introduce surprise inflation. Accordingly, for a given degree of monopolistic competition which reduces output below its efficient level and thereby induces an inflation bias, this distinct average markup behavior further raises (lowers) the markup under Calvo (Rotemberg) and thereby worsens (improves) the inflationary bias problem, despite the fact that a given level of inflation is typically found to be more costly in welfare terms under Calvo. In addition, this difference in the endogenous markup effect under the two descriptions of nominal inertia is deepened as the degree of monopoly power and/or price stickiness is increased. Therefore, particularly as the flexible price markup is increased, the inflationary bias under Calvo will eventually rise above that observed under Rotemberg.

There are some recent papers using global solution techniques which also consider optimal discretionary policy in the New Keynesian model under Calvo contracts - see Van Zandweghe and Wolman (2011) and Anderson et al. (2010), which is then extended in Ngo (2014) to allow for the zero lower bound (ZLB) constraint.[2] Solving nonlinear representations of an enriched New Keynesian model is typically far more computationally intensive than conventional perturbation methods, hence some authors instead adopt the Rotemberg description of price stickiness since this reduces the number of state variables one must consider. For instance, Shibayama and Sunakawa (2012), Nakata (2013), Niemann et al. (2013) and Leeper et al. (2016) explore optimal policy in various New Keynesian models using Rotemberg pricing. In particular, Shibayama and Sunakawa (2012) use a nonlinear method to solve for the discretionary policy under Rotemberg pricing, but they do not compare and contrast the inflation bias implications between the two pricing mechanisms. A notable exception is Miao and Ngo (2016) which compares the dynamics of a fully nonlinear New Keynesian model under either Rotemberg or Calvo pricing in the presence of the ZLB constraint. They specify a simple Taylor rule for monetary policy and highlight how the government spending multiplier differs under both pricing, while we study the optimal discretionary monetary policy and focus on how

[2] Assuming a rule-based description of policy, there are papers using global solution techniques to deal with nonlinearities such as the ZLB constraint. Fernández-Villaverde et al. (2015), Wieland (2013) and Richter et al. (2013) explore equilibrium dynamics around the ZLB in variants of the New Keynesian model which adopt Calvo price contracts. Other authors also consider issues relating to the ZLB in models which use Rotemberg pricing, but also introduce extensions such as capital (see Gavin et al. (2013), Braun and Korber (2011), Johannsen (2014)), consumption habits (Gust et al. (2012) and Aruoba and Schorfheide (2013)), labor market frictions (Roulleau-Pasdeloup (2013)) or fiscal policy (Johannsen (2014)).

inflation bias differs between these two forms of nominal rigidities. Our common message is that Calvo and Rotemberg pricing can be far from equivalent in the nonlinear context.

The rest of the paper is organized as follows. In section 2, we describe the basic model under both Calvo and Rotemberg pricing. In section 3, we formulate the optimal discretionary policy problem with Rotemberg and Calvo pricing, respectively. In section 4, we explain the solution method and calibration. In section 5, we present and discuss numerical results. We conclude in section 6.

2 The Model

This section describes the basic economic structure in our model.

2.1 Households

There are a continuum of households of size one. We shall assume complete asset markets, such that, through risk sharing, they will face the same budget constraint and make the same consumption plans. As a result, at period 0 the typical household will seek to maximize the following objective function,

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \tag{1}$$

where $0 < \beta < 1$ denotes the discount factor, C_t and N_t are a consumption aggregate and labor supply at period t, respectively.

The household purchases differentiated goods in a retail market and combines them into composite goods using a CES aggregator:

$$C_t = \left(\int_0^1 C_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}, \epsilon > 1 \tag{2}$$

where $C_t(j)$ is the demand for differentiated goods of type j.

The budget constraint at time t is given by

$$\int_0^1 P_t(j) C_t(j) dj + E_t \left\{ Q_{t,t+1} D_{t+1} \right\} = \Xi_t + D_t + W_t N_t - T_t \tag{3}$$

where $P_t(j)$ is the nominal price of type j goods, D_{t+1} is the nominal payoff of the nominal bonds portfolio held at the end of period t, Ξ is the representative household's share of profits in the imperfectly competitive firms, W are wages, and T are lump-sum taxes/transfers.[3] $Q_{t,t+1}$ is the stochastic discount factor for one period ahead payoffs.

[3]In Section 5 we shall analyze cost push shocks driven by fluctuations in a revenue tax which shall be rebated to households in a lump-sum form.

The labor market is perfectly competitive and wages are fully flexible.

Households must first decide how to allocate a given level of expenditure across the various goods that are available. They do so by adjusting the share of a particular good in their consumption bundle to exploit any relative price differences—this minimizes the costs of consumption. The demand curve for each good j is,

$$C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} C_t \tag{4}$$

where the aggregate price level P_t is defined to be

$$P_t = \left(\int_0^1 P_t(j)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}. \tag{5}$$

The dynamic budget constraint at period t can therefore be rewritten as

$$P_t C_t + E_t \left\{Q_{t,t+1} D_{t+1}\right\} = \Xi_t + D_t + W_t N_t - T_t. \tag{6}$$

The representative household's decision problem can be dealt with in two stages. First, regardless of the level of C_t the household purchases the combination of individual goods that minimizes the cost of achieving this level of the composite good. Second, given the cost of achieving any given level of C_t, the household chooses C_t, D_{t+1} and N_t optimally. We have solved the first stage problem above. For tractability, we assume that (1) takes the specific form

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}\right). \tag{7}$$

where $\sigma > 0$ is a risk aversion parameter and $\varphi > 0$ is the inverse of the Frisch elasticity of labor supply.

We can then maximize utility subject to the budget constraint (6) to obtain, after taking expectations, the optimal allocation of consumption across time,

$$\beta R_t E_t \left\{\left(\frac{C_t}{C_{t+1}}\right)^{\sigma} \left(\frac{P_t}{P_{t+1}}\right)\right\} = 1, \tag{8}$$

where $R_t \equiv \frac{1}{E_t(Q_{t,t+1})}$ is the gross nominal return on a riskless one period bond paying off a unit of currency in $t+1$. This is the familiar consumption Euler equation which implies that consumers are attempting to smooth consumption over time such that the marginal utility of consumption is equal across periods (after allowing for tilting due to interest rates differing from the household's rate of time preference).

The second first order condition concerning labor supply decision is given by

$$\frac{W_t}{P_t} = N_t^{\varphi} C_t^{\sigma}. \tag{9}$$

2.2 Firms

Each firm produces a differentiated good j using a constant returns to scale production function:

$$Y_t(j) = A_t N_t(j) \tag{10}$$

where $Y_t(j)$ is the output of firm j, and $N_t(j)$ denotes the hours hired by the firm, A_t is an exogenous aggregate productivity shock at period t, and $a_t = \log(A_t)$ is time varying and stochastic.[4]

Similar to the household's problem, we first consider the cost minimization problem of firm j, which implies that the real marginal cost of production is given by

$$mc_t = \frac{W_t}{P_t A_t}. \tag{11}$$

Note that the real marginal cost described in (11) does not depend on the output level of an individual firm, since its production function exhibits constant returns to scale and prices of inputs (here labor) are fully flexible.

The demand curve the firm j faces is given by

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t,$$

where $Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}$.

The intermediate-good sector is monopolistically competitive and the intermediate good producers therefore have certain degrees of market power. In the following, we consider two alternative forms of price stickiness - firstly that due to Rotemberg (1982) and then that of Calvo (1983).

2.2.1 Rotemberg Pricing

The Rotemberg model assumes that a monopolistic firm faces a quadratic cost of adjusting nominal prices, which can be measured in terms of the final good and given by

$$\frac{\phi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 Y_t \tag{12}$$

[4]Typically, the logarithm of A_t is assumed to follow an $AR(1)$ process: $a_t = \rho_a a_{t-1} + e_{at}$, $0 \leq \rho_a < 1$ where technology shock e_{at} is an $i.i.d.$ random variable, which has a zero mean and a finite standard deviation σ_a.

where $\phi \geq 0$ measures the degree of nominal price rigidity. The adjustment cost, which accounts for the negative effects of price changes on the customer–firm relationship, increases in magnitude with the size of the price change and with the overall scale of economic activity Y_t.

The problem for firm j is then to maximize the discounted value of nominal profits,

$$\max_{\{P_t(j)\}_{t=0}^{\infty}} E_t \sum_{s=0}^{\infty} Q_{t,t+s} \Xi_{t+s}$$

where nominal profits are defined as

$$\Xi_t = P_t(j)Y_t(j) - mc_t Y_t(j) P_t - \frac{\phi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 Y_t P_t \tag{13}$$

Firms can change their price in each period, subject to their demand curve and payment of the adjustment cost. Hence, all the firms face the same problem, and thus will choose the same price, and produce the same quantity such that, $P_t(j) = P_t$ and $Y_t(j) = Y_t$ for any j. Hence, the first-order condition for a symmetric equilibrium is

$$(1-\epsilon) + \epsilon mc_t - \phi \Pi_t (\Pi_t - 1) + \phi \beta E_t \left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma} \frac{Y_{t+1}}{Y_t} \Pi_{t+1} (\Pi_{t+1} - 1) \right] = 0. \tag{14}$$

This is the Rotemberg version of the nonlinear Phillips curve that relates current inflation to future expected inflation and to the level of output.

2.2.2 Calvo Pricing

Each period, the firms that adjust their price are randomly selected, and a fraction $1-\theta$ of all firms adjust while the remaining θ fraction do not adjust. Those firms that do adjust their price at time t do so to maximize the expected discounted value of current and future profits. Profits at some future date $t+s$ are affected by the choice of price at time t only if the firm has not received another opportunity to adjust between t and $t+s$. The probability of this is θ^s.

The adjusting firm's pricing decision problem then involves picking $P_t(j)$ to maximize discounted nominal profits. Using the demand curve for the firm's product, this objective function can be written as

$$E_t \sum_{s=0}^{\infty} \theta^s Q_{t,t+s} \left[P_t(j) \left(\frac{P_t(j)}{P_{t+s}}\right)^{-\epsilon} Y_{t+s} - mc_{t+s} \left(\frac{P_t(j)}{P_{t+s}}\right)^{-\epsilon} Y_{t+s} P_{t+s} \right],$$

where the discount factor $Q_{t,t+s}$ is given by $\beta^s \left(\frac{C_t}{C_{t+s}}\right)^{\sigma} \frac{P_t}{P_{t+s}}$, and mc_{t+s} is the marginal cost of production.

Let P_t^* be the optimal price chosen by all firms able to reset their price at time t. The first order condition for the optimal choice of P_t^* is,

$$\frac{P_t^*}{P_t} = \left(\frac{\epsilon}{\epsilon - 1}\right) \frac{K_t^p}{F_t^p} \tag{15}$$

where

$$K_t^p = C_t^{-\sigma} mc_t Y_t + \theta\beta E_t \left[\left(\frac{P_{t+1}}{P_t}\right)^{\epsilon} K_{t+1}^p\right]$$
$$F_t^p = C_t^{-\sigma} Y_t + \theta\beta E_t \left[\left(\frac{P_{t+1}}{P_t}\right)^{\epsilon-1} F_{t+1}^p\right].$$

The price index evolves according to

$$1 = (1-\theta)\left(\frac{P_t^*}{P_t}\right)^{1-\epsilon} + \theta(\Pi_t)^{\epsilon-1} \text{ with } \Pi_t \equiv \frac{P_t}{P_{t-1}}. \tag{16}$$

and price dispersion is described by

$$\Delta_t \equiv \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} dj = (1-\theta)\left(\frac{P_t^*}{P_t}\right)^{-\epsilon} + \theta \left(\frac{P_t}{P_{t-1}}\right)^{\epsilon} \Delta_{t-1}. \tag{17}$$

2.3 Aggregate Conditions

Under Rotemberg pricing, as all the firms will employ the same amount of labor, the aggregate production function is simply given by

$$Y_t = A_t N_t,$$

and the aggregate resource constraint is given by

$$Y_t = C_t + \frac{\phi}{2}\left(\Pi_t - 1\right)^2 Y_t.$$

Note that the Rotemberg adjustment cost creates an inefficiency wedge ψ_t^R between output and consumption

$$C_t = \left(1 - \psi_t^R\right) Y_t = \left(1 - \psi_t^R\right) A_t N_t \tag{18}$$

where $\psi_t^R = \frac{\phi}{2}\left(\Pi_t - 1\right)^2$.

In the case of Calvo pricing, firms changing prices in different periods will generally have different prices. Thus, the model features price dispersion. When firms have different relative prices, there are distortions that create a wedge between the aggregate output measured in terms of production factor inputs and aggregate demand measured in terms of the composite goods. Specifically,

$$N_t(j) = \frac{Y_t(j)}{A_t} = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} \frac{Y_t}{A_t}$$

which yields,

$$N_t = \int_0^1 N_t(j)dj = \frac{Y_t}{A_t}\int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} dj = \frac{Y_t \Delta_t}{A_t}$$

after integrating across firms. $\Delta_t \geq 1$ implies that price dispersion is always costly in terms of aggregate output: the higher Δ_t, the more labor is needed to produce a given level of output. Moreover, under Calvo different firms with different prices will employ different amounts of labor. This explains why higher price dispersion acts as a negative productivity shift in the aggregate production function: $Y_t = (A_t/\Delta_t)N_t$. In addition, price dispersion is a backward-looking variable which introduces an inertial component into the model.

Under Calvo, the aggregate resource constraint is simply given by

$$Y_t = C_t.$$

Hence, after defining $\psi_t^c = \Delta_t - 1$ as an inefficiency wedge under Calvo, we have

$$C_t = Y_t = \frac{A_t N_t}{(1+\psi_t^c)}. \tag{19}$$

Comparing (18) and (19), it is illuminating to note that the Rotemberg adjustment cost creates a wedge ψ_t^R between aggregate consumption and aggregate output, while the Calvo price dispersion creates a wedge ψ_t^c between aggregate hours and aggregate output. In addition, both wedges are nonlinear functions of inflation. They are minimized at one when steady-state net inflation equals zero ($\Pi = 1$), and increase as trend inflation moves away from zero. See Ascari and Rossi (2012) for a discussion.

Appendix C.1 summarizes the models under Rotemberg and Calvo pricing.

3 Optimal Policy Problem Under Discretion

In this section, following Woodford (2003) and Anderson et al. (2010), we interpret the monetary authority's problem without commitment as an optimal planning problem, as opposed to choosing a particular policy instrument. Under discretion, the monetary authority solves a sequential or period-by-period optimization problem, which maximizes the representative household's expected discounted utility subject to the optimality conditions from market participants, the aggregate conditions, and the law of motion for the state variables. Therefore, under optimal discretion, the policymaker cannot commit to a plan in the hope of influencing economic agents' expectations.

3.1 Rotemberg Pricing

Let $V(A_t)$ represents the value function at period t in the Bellman equation for the optimal policy problem. The optimal monetary policy then solves the following optimization problem:

$$V(A_t) = \max_{\{C_t, Y_t, \Pi_t\}} \left\{ \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{(Y_t/A_t)^{1+\varphi}}{1+\varphi} + \beta E_t \left[V(A_{t+1}) \right] \right\}, \tag{20}$$

subject to,

$$C_t = \left[1 - \frac{\phi}{2} \left(\Pi_t - 1 \right)^2 \right] Y_t, \tag{21}$$

and,

$$(1-\epsilon) + \epsilon Y_t^{\varphi} C_t^{\sigma} A_t^{-\varphi-1} - \phi \Pi_t \left(\Pi_t - 1 \right) + \phi \beta E_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \frac{Y_{t+1}}{Y_t} \Pi_{t+1} \left(\Pi_{t+1} - 1 \right) \right] = 0. \tag{22}$$

Defining an auxiliary function,

$$M(A_{t+1}) \equiv C_{t+1}^{-\sigma} Y_{t+1} \Pi_{t+1} \left(\Pi_{t+1} - 1 \right),$$

we can rewrite the Phillips curve (22) as,

$$(1-\epsilon) + \epsilon Y_t^{\varphi} C_t^{\sigma} A_t^{-\varphi-1} - \phi \Pi_t \left(\Pi_t - 1 \right) + \phi \beta C_t^{\sigma} Y_t^{-1} E_t \left[M(A_{t+1}) \right] = 0,$$

which captures the fact that the policy maker recognizes that any change in the state variable will affect expectations, but cannot promise to behave in a particular way tomorrow in order to influence expectations today. The optimal policy problem can then be formulated as the following Lagrangian,

$$\begin{aligned} \mathcal{L} = & \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{(Y_t/A_t)^{1+\varphi}}{1+\varphi} + \beta E_t \left[V(A_{t+1}) \right] + \lambda_{1t} \left\{ \left[1 - \frac{\phi}{2} \left(\Pi_t - 1 \right)^2 \right] Y_t - C_t \right\} \\ & + \lambda_{2t} \left\{ (1-\epsilon) + \epsilon Y_t^{\varphi} C_t^{\sigma} A_t^{-\varphi-1} - \phi \Pi_t \left(\Pi_t - 1 \right) + \phi \beta C_t^{\sigma} Y_t^{-1} E_t \left[M(A_{t+1}) \right] \right\}, \end{aligned}$$

where λ_{1t} and λ_{2t} are the Lagrange multipliers. The first order conditions are given as follows:

consumption,

$$C_t^{-\sigma} = \lambda_{1t} - \lambda_{2t} \left\{ \sigma \epsilon Y_t^{\varphi} C_t^{\sigma-1} A_t^{-\varphi-1} + \sigma \phi \beta C_t^{\sigma-1} Y_t^{-1} E_t \left[M(A_{t+1}) \right] \right\}; \tag{23}$$

output,

$$Y_t^{\varphi} A_t^{-1-\varphi} = \lambda_{1t}\left[1-\frac{\phi}{2}\left(\Pi_t-1\right)^2\right] + \lambda_{2t}\left\{\epsilon\varphi Y_t^{\varphi-1} C_t^{\sigma} A_t^{-\varphi-1} - \phi\beta C_t^{\sigma} Y_t^{-2} E_t\left[M(A_{t+1})\right]\right\}; \tag{24}$$

and inflation,

$$\lambda_{1t}\phi\left(\Pi_t-1\right) Y_t = -\lambda_{2t}\phi\left(2\Pi_t-1\right). \tag{25}$$

Note that the consumption Euler equation serves only to define the nominal interest rate.

The fully nonlinear problem is then to find five policy functions which relate the three choice variables $\{Y_t, C_t, \Pi_t\}$ and two Lagrange multipliers $\{\lambda_{1t}, \lambda_{2t}\}$ to the state variable A_t, that is, $Y_t = Y(A_t)$, $C_t = C(A_t)$, $\Pi_t = \Pi(A_t)$, $\lambda_{1t} = \lambda_1(A_t)$, and $\lambda_{2t} = \lambda_2(A_t)$. We will use the Chebyshev collocation method to approximate these five time invariant policy rules.

3.2 Calvo Pricing

Let $V(\Delta_{t-1}, A_t)$ denote the value function at period t in the Bellman equation for the optimal policy problem. The optimal monetary policy under discretion then can be described as a set of decision rules for $\{C_t, N_t, \Pi_t, K_t^p, F_t^p, \Delta_t\}$ which maximize,

$$V(\Delta_{t-1}, A_t) = \max\left\{\frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} + \beta E_t\left[V(\Delta_t, A_{t+1})\right]\right\},$$

subject to the following constraints:

$$\frac{N_t A_t}{\Delta_t} = C_t,$$

$$\frac{K_t^p}{(1-\epsilon^{-1})} = \left(\frac{1-\theta\Pi_t^{\epsilon-1}}{1-\theta}\right)^{\frac{1}{1-\epsilon}} F_t^p,$$

$$F_t^p = C_t^{1-\sigma} + \theta\beta E_t\left[L(\Delta_t, A_{t+1})\right],$$

$$K_t^p = \frac{N_t^{\varphi+1}}{\Delta_t} + \theta\beta E_t\left[M(\Delta_t, A_{t+1})\right],$$

$$\Delta_t = (1-\theta)\left(\frac{1-\theta\Pi_t^{\epsilon-1}}{1-\theta}\right)^{\frac{-\epsilon}{1-\epsilon}} + \theta\Pi_t^{\epsilon}\Delta_{t-1},$$

where we have utilized two auxiliary functions,

$$M(\Delta_t, A_{t+1}) = \Pi_{t+1}^{\epsilon} K_{t+1}^p,$$

and

$$L(\Delta_t, A_{t+1}) = \Pi_{t+1}^{\epsilon-1} F_{t+1}^p,$$

which again captures the fact that the policy maker recognizes that any change in the state variable will affect expectations, but cannot make credible promises about their future behavior.

As before, the policy problem can be written in Lagrangian form as follows:

$$\mathcal{L} = \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t{}^{1+\varphi}}{1+\varphi} + \beta E_t \left[V(\Delta_t, A_{t+1}) \right] + \lambda_{1t} \left[\frac{N_t A_t}{\Delta_t} - C_t \right]$$

$$+ \lambda_{2t} \left[C_t^{1-\sigma} + \theta \beta E_t \left[L(\Delta_t, A_{t+1}) \right] - F_t^p \right]$$

$$+ \lambda_{3t} \left[\frac{N_t^{\varphi+1}}{\Delta_t} + \theta \beta E_t \left[M(\Delta_t, A_{t+1}) \right] - K_t^p \right]$$

$$+ \lambda_{4t} \left[(1-\theta) \left(\frac{1-\theta \Pi_t{}^{\epsilon-1}}{1-\theta} \right)^{\frac{-\epsilon}{1-\epsilon}} + \theta \Pi_t^{\epsilon} \Delta_{t-1} - \Delta_t \right]$$

$$+ \lambda_{5t} \left[\left(\frac{1-\theta \Pi_t{}^{\epsilon-1}}{1-\theta} \right)^{\frac{1}{1-\epsilon}} F_t^p - \frac{K_t^p}{(1-\epsilon^{-1})} \right],$$

where λ_{jt} $(j = 1, .., 5)$ are the Lagrange multipliers. The first order conditions are given as follows:

consumption,

$$C_t^{-\sigma} - \lambda_{1t} + (1-\sigma) C_t^{-\sigma} \lambda_{2t} = 0; \tag{26}$$

labor,

$$-\Delta_t N_t^{\varphi} + A_t \lambda_{1t} + (1+\varphi) N_t^{\varphi} \lambda_{3t} = 0; \tag{27}$$

inflation,

$$-\epsilon \left(\left(\frac{1-\theta \Pi_t{}^{\epsilon-1}}{1-\theta} \right)^{\frac{1}{\epsilon-1}} - \Pi_t \Delta_{t-1} \right) \lambda_{4t} = \left(\frac{1-\theta \Pi_t{}^{\epsilon-1}}{1-\theta} \right)^{\frac{\epsilon}{1-\epsilon}} \frac{F_t^p \lambda_{5t}}{1-\theta}; \tag{28}$$

numerator of optimal price K_t^p,

$$\lambda_{3t} + \frac{\lambda_{5t}}{(1-\epsilon^{-1})} = 0; \tag{29}$$

denominator of optimal price F_t^p,

$$-\lambda_{2t} + \left(\frac{1-\theta \Pi_t{}^{\epsilon-1}}{1-\theta} \right)^{\frac{1}{1-\epsilon}} \lambda_{5t} = 0; \tag{30}$$

and price dispersion,

$$0 = \beta \frac{\partial E_t \left[V(\Delta_t, A_{t+1}) \right]}{\partial \Delta_t} - \frac{N_t A_t \lambda_{1t}}{\Delta_t^2} + \theta\beta \frac{\partial E_t \left[L(\Delta_t, A_{t+1}) \right]}{\partial \Delta_t} \lambda_{2t} + \left(\theta\beta \frac{\partial E_t \left[M(\Delta_t, A_{t+1}) \right]}{\partial \Delta_t} - \frac{N_t^{\varphi+1}}{\Delta_t^2} \right) \lambda_{3t} - \lambda_{4t}.$$

Note that the envelope theorem yields

$$\frac{\partial V(\Delta_{t-1}, A_t)}{\partial \Delta_{t-1}} = \theta \Pi_t^{\epsilon} \lambda_{4t},$$

which allows us to rewrite the first order condition for price dispersion as,

$$0 = \frac{C_t \lambda_{1t}}{\Delta_t} + \frac{N_t^{\varphi} C_t}{A_t \Delta_t} \lambda_{3t} + \lambda_{4t} - \theta\beta\lambda_{2t} \frac{\partial E_t \left[L(\Delta_t, A_{t+1}) \right]}{\partial \Delta_t} - \theta\beta\lambda_{3t} \frac{\partial E_t \left[M(\Delta_t, A_{t+1}) \right]}{\partial \Delta_t} - \theta\beta E_t \left[\Pi_{t+1}^{\epsilon} \lambda_{4t+1} \right]. \tag{31}$$

We can solve the nonlinear system consisting of these six first order conditions and the five constraints to yield the time-consistent optimal policy under Calvo pricing. Specifically, without commitment, we need to find these eleven time-invariant policy rules which are functions of the two state variables $\{\Delta_{t-1}, A_t\}$. That is, we need to find policy functions such as $F_t^p = F^p(\Delta_{t-1}, A_t)$, $K_t^p = K^p(\Delta_{t-1}, A_t)$, and $\Pi_t = \Pi(\Delta_{t-1}, A_t)$. Similar to the Rotemberg case, the Chebyshev collocation method will be used to approximate these policy functions.

4 Numerical Analysis

This section starts with a description of the global solution method to numerically solve for the discretionary equilibrium. Then, the calibration of parameters is discussed.

4.1 Solution Method

We use the Chebyshev collocation method with time iteration to globally approximate the policy functions.[5] In contrast to the linear-quadratic approximation method, this projection method can capture the extent to which the two approaches modeling price stickiness differ, due to the nonlinearities inherent in the New Keynesian model. First, we discretize the state space into a set of collocation nodes. In the Rotemberg model, there is one state variable (A_t), while in the Calvo model there are two state variables (Δ_{t-1}, A_t). Accordingly, the space of the approximating functions for the Rotemberg pricing consists of one-dimensional Chebyshev polynomials. In comparison, the space of approximating

[5] Judd (1992), Judd (1998) and Miranda and Fackler (2004) are good references.

functions for the Calvo pricing is two-dimensional, and is, given by the tensor products of two sets of Chebyshev polynomials. Then we define the residual functions based on the equilibrium conditions. Gaussian-Hermite quadrature is used to approximate expectation terms. Under Calvo pricing, the partial derivatives with respect to price dispersion are approximated by differentiating the Chebyshev polynomials. Finally, we solve the resultant system of nonlinear equations consisting of the residual functions evaluated at each collocation node.[6] See appendix C.2 for details.

4.2 Calibration

The benchmark parameters for Calvo pricing are taken from Anderson et al. (2010) and are standard. Table 1 summarizes the relevant parameter values. We set $\beta = (1/1.04)^{1/4} = 0.99$, which is a standard value for models with quarterly data and implies a 4% annual real interest rate. The intertemporal elasticity of substitution is set to one ($\sigma = 1$) which is in the middle of the parameter range typically considered in the literature. Labor supply elasticity is set to $\varphi^{-1} = 1$. The elasticity of substitution between intermediate goods is chosen as $\epsilon = 11$, which implies a monopolistic markup of approximately 10%. The technology parameters are set to $\rho_a = 0.95$ and $\sigma_a = 0.01$. To make the results from Rotemberg pricing comparable, the value of price adjustment cost ($\phi = 116$) is calibrated so that the linear quadratic approximation for both cases are equivalent.[7] This implies an equivalence between the two forms of pricing provided the steady-state is undistorted with a rate of inflation of zero. Admittedly, this commonly used strategy is more debatable in the nonlinear context, since it is not guaranteed that the equivalence of the two forms of nominal inertia is retained in nonlinear solutions of the New Keynesian model where the steady-state is distorted and the rate of inflation will typically not be zero. We consider an alternative calibration strategy of matching relative inflation/output volatilities in the sensitivity section 5.3 below.

With this benchmark parameterization, we solve the fully nonlinear models via the Chebyshev collocation method. Following Anderson et al. (2010), relative price dispersion Δ_t is bounded by $[1, 1.02]$, and logged productivity a_t takes values from $[-2\sigma_a/(1-\rho_a), 2\sigma_a/(1-\rho_a)] = [-0.4, 0.4]$. For the Rotemberg case, the order of approximation n_a is chosen to be 6, and the number of nodes for Gauss-Hermite quadrature $q = 12$. This combination is quite accurate, since the maximum Euler equation error is of the order of 10^{-8}. For the Calvo case, the order of approximation n_a and n_Δ are both assigned to be 6, and $q = 12$ for Gauss-Hermite quadrature. The maximum Euler equation error over

[6] In contrast, Anderson et al. (2010) solve a large system of nonlinear equations consisting of the residual functions evaluated over all collocation nodes. We use the time iteration method which naturally divides the large root-finding problem into small problems that can be solved independently. A big advantage of this method is that parallel computing can be used as the size of system of nonlinear equations increases.

[7] That is, $\phi = \frac{(\epsilon-1)\theta}{(1-\theta)(1-\beta\theta)}$.

the full range under all the cases is of the order of 10^{-7}. As suggested by Judd (1998), this order of accuracy is reasonable.

5 Results

This section presents the main results of this paper. First, the inflation bias in the steady state between Calvo pricing and Rotemberg pricing is compared. Second, we explain why the inflation bias under Calvo can be significantly greater, even though the welfare costs of the resulting inflation are substantially greater under Calvo. Finally, sensitivity analysis is conducted.

5.1 Steady State Inflation Bias

Figure 1 illustrates how the steady-state inflation bias under the two forms of nominal rigidities differs as we vary the degree of nominal inertia.[8] The black solid line plots the annualized inflation bias (in percent) against the Rotemberg adjustment parameter (ϕ). For comparison, the red dashed line plots the inflation bias under Calvo pricing with θ ranging from 0.5 to 0.75 - implying average price duration of 2 and 4 quarters, respectively, which are reasonable empirical bounds for price adjustment. The mapping between the Calvo and Rotemberg parameters, ϕ for θ, is such that the LQ approximations of the two models are equivalent. We can see that the inflation bias problem under Calvo pricing (about 2.2%) is more severe than that under Rotemberg pricing (about 1.89%) for the benchmark parameters. However, the Calvo pricing does not always imply higher inflation bias, as the case with $\theta = 0.5$ shows. More generally, the inflation bias problem gradually worsens under Calvo as the degree of nominal inertia is increased, but is largely insensitive to the price adjustment cost parameter under Rotemberg, improving very slightly as inertia is increased. Conditional on the benchmark values of other calibrated parameters, the inflation bias under Calvo rises above that under Rotemberg when the probability of price change rises to $\theta = 0.62$ (equivalently $\phi = 42$), an expected duration of price contracts of 8 months. This is well within the range of conventional estimates of the degree of nominal inertia.

We now turn to assessing the effect of the monopolistic competition distortion $\epsilon/(\epsilon-1)$ on the equilibrium inflation bias by changing the value of ϵ. We interchangeably describe this measure of the monopolistic competition distortion as the flexible-price markup since it measures the markup that would be observed under flexible prices. This approach is

[8] In general, the steady state here should be called the risky steady state in the sense of Coeurdacier et al. (2011) since economic agents still anticipate shocks hitting the economy, even if none are actually realized. When the variance of the productivity shock is set to be effectively zero, the risky steady state collides with the nonstochastic steady state. In fact, the benchmark productivity shock barely affects the steady state values. As a double-check, we also shut down the shock and essentially find the same steady state values.

based on the fact that the size of the inflation bias depends on the degree of monopolistic distortion, which makes steady state (even flexible-price) output inefficient and generates the temptation on the part of the policy maker to inflate the economy. Figure 2 shows how the size of inflation bias changes as the markup is varied for the Calvo and Rotemberg pricing, respectively. The benchmark $\epsilon = 11$ yields a gross flexible-price markup of 1.1. When ϵ decreases, the corresponding monopolistic competition distortion and inflation bias increases. This has a dramatic impact on the relative size of the inflation bias under the two forms of nominal inertia, with the bias under Calvo pricing rising significantly above that under Rotemberg.[9]

We find that the inflationary bias problem can become significantly greater under Calvo as nominal inertia is increased, but especially as the monopolistic competition distortion is increased. At the same time Figure 2 shows that consumption falls by more, and hours worked by less under Calvo as we increase this distortion, and the average markup rises above the flexible price markup under Calvo, while decreasing under Rotemberg as a result of the nonlinear effects of the inflation bias. It is also striking that the welfare costs of the resulting inflation are substantially greater under Calvo, which might have been thought to mitigate the desire to increase inflation in the first place. We must, therefore, explain why the high costs of inflation under Calvo do not appear to inhibit the inflationary bias problem.

5.2 Discussion

In understanding the apparently counterintuitive result that the inflation bias can often be significantly higher under Calvo pricing relative to Rotemberg, despite the significantly higher welfare costs of inflation under the former, it is helpful to consider the effects of inflation on the two models. Ascari and Rossi (2012) discuss how inflation affects both models through a 'wedge' effect as well as an average markup effect. We shall consider the wedge effect first, before turning to the average markup effect, which will turn out to be key. Under both forms of nominal inertia the 'wedge' implies that the representative household's aggregate consumption will be lower for a given level of labor input as inflation rises. Under Calvo, due to price dispersion, the representative household consumes relatively more of the cheaper goods to compensate for the expensive goods, given diminishing marginal utility in the consumption of each good. As Goodfriend and King (1997) and Damjanovic and Nolan (2010a) note, this is akin to a negative productivity shock. We can combine the resource and aggregate production function to

[9] To illustrate the importance of nonlinearities in this context, the inflation bias for both cases under the linear-quadratic approximation (LQ) are also presented in Figures 3 and 4 in the online appendix. The traditional linear-quadratic method becomes increasingly inaccurate for larger distortions.

yield,

$$C_t = \frac{A_t}{(1+\psi_t^c)} N_t,$$

where the inefficient wedge under Calvo, $\psi_t^c = \Delta_t - 1$, captures the extent to which price dispersion has been raised above one.

Under Rotemberg the micro-foundation of the wedge is different - adjusting prices uses up consumption goods directly. However, we can similarly combine the aggregate production function and resource constraint to obtain a similar expression under Rotemberg,

$$C_t = A_t(1-\psi_t^R)N_t,$$

where the Rotemberg wedge, $\psi_t^R = \frac{\phi}{2}(\Pi_t - 1)^2$, reflects the costs per unit of output of changing prices. Therefore in both cases the labor costs of attaining a particular level of aggregate consumption are higher, ceteris paribus, as inflation rises.

In order to assess how this affects the inflation bias problem facing the policy maker, it is helpful to imagine how a social planner would respond to a technology shock in the presence of such wedges. Given the form of household utility, the social planner would choose an optimal level of labor input of

$${N_t}^{\sigma+\varphi} = \left(\frac{A_t}{(1+\psi_t^c)}\right)^{1-\sigma}$$

under Calvo, and

$${N_t}^{\sigma+\varphi} = \left(A_t(1-\psi_t^R)\right)^{1-\sigma}$$

under Rotemberg. Therefore, for our benchmark calibration of $\sigma = 1$ the social planner would not seek to adjust the labor input into the production process as a result of increases in either of the wedges, but would simply allow consumption to fall. In other words, for our benchmark calibration the efficiencies implied by these wedges do not give the policy maker a further desire to generate a surprise inflation, ceteris paribus. While if $\sigma > 1$ the social planner would seek to reduce the labor input as either of these inefficiency wedges increased. That is, in this case the wedges would *reduce* the desire to encourage firms to employ more workers, ceteris paribus. We can see this from Table 2 where raising the inverse of the intertemporal elasticity of substitution, σ, reduces the inflation bias under both pricing models. Therefore the different inefficiency wedges under Calvo and Rotemberg are not responsible for the observed inflation biases.

Instead the differences in inflation bias across the two models are generated by their average mark-up behavior, which is fundamentally different. Consider the steady-state of the average markup (equal to the inverse of real marginal cost) under Rotemberg which is obtained by rearranging the deterministic steady state of the NKPC as,

$$mc^{-1} = \left[\frac{\epsilon - 1}{\epsilon} + \frac{(1-\beta)}{\epsilon}\phi(\Pi - 1)\Pi\right]^{-1}.$$

The second term within the square brackets exists as a combination of steady-state inflation and discounting on the part of firms (on behalf of their owners, the representative household). Essentially as the firms discount future profits they also discount future price adjustment costs. As a result, in the face of ongoing inflation, they will opt to partially delay the required price adjustment such that the average markup is decreasing in inflation. This effect, whereby the average markup under Rotemberg is falling in inflation, is enhanced as the degree of nominal inertia and/or flexible-price markup increase.

The effect of inflation on the average markup under Calvo is,

$$mc^{-1} = \frac{\epsilon}{\epsilon - 1}\left(\frac{1 - \theta\beta\Pi^{\epsilon-1}}{1 - \theta\beta\Pi^{\epsilon}}\right)\left(\frac{1 - \theta\Pi^{\epsilon-1}}{1-\theta}\right)^{\frac{1}{\epsilon-1}}.$$

In this case the effects of inflation on the average markup are ambiguous. However, following King and Wolman (1996) the average markup can be decomposed into two elements - the marginal markup,

$$\frac{P^*}{MC} = \frac{\epsilon}{\epsilon - 1}\left(\frac{1 - \theta\beta\Pi^{\epsilon-1}}{1 - \theta\beta\Pi^{\epsilon}}\right),$$

and the price adjustment gap,

$$\frac{P}{P^*} = \left(\frac{1 - \theta\Pi^{\epsilon-1}}{1-\theta}\right)^{\frac{1}{\epsilon-1}}.$$

Here we can see that higher inflation raises the marginal markup. Firms facing the possibility of being stuck with the current price for a prolonged period will tend to raise their reset price when that price is likely to be eroded by inflation throughout the life of that contract. The effect of inflation on the price-adjustment gap will tend to reduce this element of the average markup. However, except at very low rates of inflation, the effect of inflation on the average markup through the marginal markup channel is positive. Additionally, in contrast to what we find under Rotemberg, greater price stickiness and/or a flexible price markup further increase the average markup under Calvo.

Therefore, we would expect to see average markups rise with inflation under Calvo, but fall under Rotemberg, especially as price stickiness and/or the flexible-price markup are increased. This, in turn, implies that the inflationary bias problem is worsened under Calvo as the rising markups increase the policy makers incentives to introduce a surprise inflation, ceteris paribus, at the same time as it is mitigated under Rotemberg. As a result, the inflation bias problem can become significantly higher under Calvo where consumption falls by more and hours by less than it does under Rotemberg, despite the fact that a given level of inflation is typically found to be more costly in welfare terms under Calvo.

Finally, we do some comparative statics with the model under both pricing approaches, in order to explore how other parameters affect the severity of the inflation bias problem and the sensitivity of the results obtained from the linear-quadratic approach. Table 2 summarizes the robustness outcomes for the Calvo and Rotemberg pricing. Notably, there are cases where the inflation bias problem is much worse under Calvo pricing, especially as the endogenous markup effect is enhanced by raising the flexible-price markup and/or the degree of price stickiness.

5.3 Sensitivity Analysis

Table 2 considers the robustness of our results across various parameters for Calvo and Rotemberg pricing. The first three rows of the table increase the degree of nominal inertia (where the Rotemberg price adjustment parameter is adjusted in line with the changes in the Calvo parameter such that the linearized NKPC is equivalent across both forms of nominal inertia). As we increase the degree of nominal inertia, we find that the inflation bias rises under Calvo, but falls under Rotemberg. Figure 1 illustrates this point as well. This is for the reasons discussed above. Under Calvo greater price stickiness means that firms are likely to be stuck with their current prices for longer, such that they aggressively raise prices when given the opportunity to do so. This will tend to raise average markups and worsen the inflationary bias problem.[10] In contrast, under Rotemberg higher price adjustment costs result in firms wishing to delay price adjustment which reduces average markups and reduces the inflation bias problem.

The next piece of sensitivity analysis looks at various parameterizations of the inverse of the intertemporal elasticity of substitution, σ. As noted above, at the benchmark value of $\sigma = 1$, the social planner would not wish to expand employment as either of the efficiency wedges due to the two forms of nominal inertia increase. While if $\sigma < (>)$ 1 then they would wish to increase (decrease) the labor input as either efficient wedge increased. Therefore we see the inflationary bias falling as σ increases across both forms of nominal inertia. Finally, we consider an increase in the inverse of the Frisch elasticity of labor supply, φ, which serves to reduce the inflationary bias problem across both types of price stickiness. As labor supply becomes less elastic, there is less desire to use costly inflation surprises to achieve only marginal increases in the level of output and therefore the inflation bias falls.

[10] It is only at extremely high levels of price stickiness ($\theta = 0.9$, or an average price contract duration of two and a half years) that the steady state rate of inflation begins to fall as the costs of price dispersion begin to overturn the average markup effect.

Alternative Calibration Strategy

In conducting the analysis above, we followed the standard approach in the literature of calibrating the Rotemberg price adjustment parameter to ensure the linearized NKPC isomorphic across both descriptions of nominal inertia. However, it is not obvious that this approach is valid given our model features several key nonlinearities. We therefore follow an alternative strategy of choosing the Rotemberg price adjustment cost parameter to ensure the relative volatilities of output and inflation are the same across the two models.[11] To do so, we move away from the technology shock considered in Anderson et al. (2010) and introduce a cost-push shock, since the technology shock does not generate a plausible degree of inflation volatility.[12] We adopt the estimated shock process from Chen et al. (2014) which is modeled as a revenue tax rate fluctuating around a steady state value of zero,

$$\ln(1-\tau_{pt}) = (1-\rho^{\tau_p})\ln(1-\tau_p) + \rho^{\tau_p}\ln(1-\tau_{pt-1}) - e_{\tau t}$$

where $e_{\tau t} \sim N(0, 0.00486^2)$ and $\rho^{\tau_p} = 0.939$. In a log-linearized model this is equivalent to allow for fluctuations in the desired markup through variations in ϵ. However, in our nonlinear model allowing ϵ to be time varying has a direct impact on the measure of price dispersion in a way which would not normally be considered to be an inherent part of a cost-push shock. Therefore we focus on variations in a revenue tax as a means of generating an autocorrelated cost-push shock which is consistent with the data. The complete model with this time-varying revenue tax rate is presented in appendix C.4.

Under the calibration strategy for Calvo parameters of 0.613, 0.615 and 0.617, the corresponding Rotemberg adjustment costs parameters are 50, 116 and 250. These are increasingly above the conventionally calibrated Rotemberg parameters of around 40, 41 and 41. Since increasing price-stickiness reduces average markups under Rotemberg, this reduces the associated inflation bias to 1.92%, 1.88% and 1.86%, respectively, while implying a relative worsening of the inflation bias under Calvo to 1.93% across all variants. In this sense, the possible inapplicability of the standard calibration strategy does not seem to be responsible for our results - if anything it leads them to be under-reported. However, it is also important to stress that the impact of changing the Rotemberg adjustment cost function has only a limited impact on the resultant inflation bias, such that regardless of the calibration strategy it is relatively easy to find cases where the inflation bias rises significantly under Calvo relative to Rotemberg through the endogenous markup effects highlighted in the paper. It is not possible to apply this alternative calibration

[11] We thank the associate editor Andre Kurmann for suggesting this alternative approach.

[12] The technology shock already present in our model does not create meaningful policy trade-offs under our benchmark calibration since the policy maker will apply offsetting interest rate movements regardless of the form of nominal inertia. Hence, we shut down the technology shock in this section and focus on the effects of the cost-push shock.

strategy outside of this range as for higher or lower values of the Calvo parameter, there is no Rotemberg cost adjustment parameter that can deliver the same relative volatilities of output and inflation. We therefore follow the conventional approach throughout the rest of the paper.

6 Conclusion

In this paper we have contrasted the properties of the Calvo and Rotemberg forms of nominal inertia which are commonly used in New Keynesian analyses of macroeconomic policy. They are often treated as being interchangeable, largely because they generate equivalent NKPCs and policy implications when linearized around an efficient zero-inflation steady state. However, our nonlinear solution of the discretionary policy problem reveals some striking differences across the two models of price stickiness, which have significant implications for the importance of nonlinearities in New Keynesian policy analyses more generally.

The inflation bias problem is often far greater under Calvo pricing than Rotemberg pricing, despite the fact that the costs of inflation are significantly higher under the former. The reason for this is that inflation raises the average markup under Calvo pricing as firms seek to raise their prices more aggressively whenever they can to avoid the erosion of their relative price due to inflation. This increase in average markups worsens the inflationary bias problem. In contrast, under Rotemberg pricing firms can adjust prices every period, and will moderate their average markups as inflation rises as they attempt to delay some of the costs of price adjustment due to the discounting inherent in their objective function. These endogenous markup effects, which move in opposite directions across Rotemberg and Calvo pricing, are enhanced as the flexible-price markup and/or the degree of nominal inertia are increased.

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A Tables

Table 1: Parameterization

Parameter	Value	Definition
β	0.99	Quarterly discount factor
σ	1	Relative risk aversion coefficient
φ	1	Inverse Frisch elasticity of labor supply
ϵ	11	Elasticity of substitution between varieties
θ	0.75	Probability of fixing prices in each quarter
ρ_a	0.95	AR-coefficient of technology shock
σ_a	0.01	Standard deviation of technology shock
ϕ	116	Rotemberg adjustment cost

Table 2: Sensitivity analysis

Parameter	Values				Nonlinear solution		LQ solution	
	θ	σ	φ	ϵ	Calvo	Rotemberg	Calvo	Rotemberg
θ	0.5	1	1	11	1.84	1.93	1.66	1.83
	0.75	1	1	11	2.18	1.90	1.65	1.82
	0.85	1	1	11	3.01	1.83	1.64	1.80
σ	0.75	0.3	1	11	5.64	2.95	2.54	2.54
	0.75	1	1	11	2.18	1.90	1.65	1.82
	0.75	5	1	11	0.60	0.64	0.56	0.56
φ	0.75	1	0.36	11	4.31	2.83	2.42	2.42
	0.75	1	1	11	2.18	1.90	1.65	1.82
	0.75	1	4.75	11	0.60	0.64	0.56	0.56

Note: the nonlinear solution and LQ solution contain the annualized inflation rate in percentage solved by the projection method and the LQ method, respectively. The numbers are rounded up.

B Figures

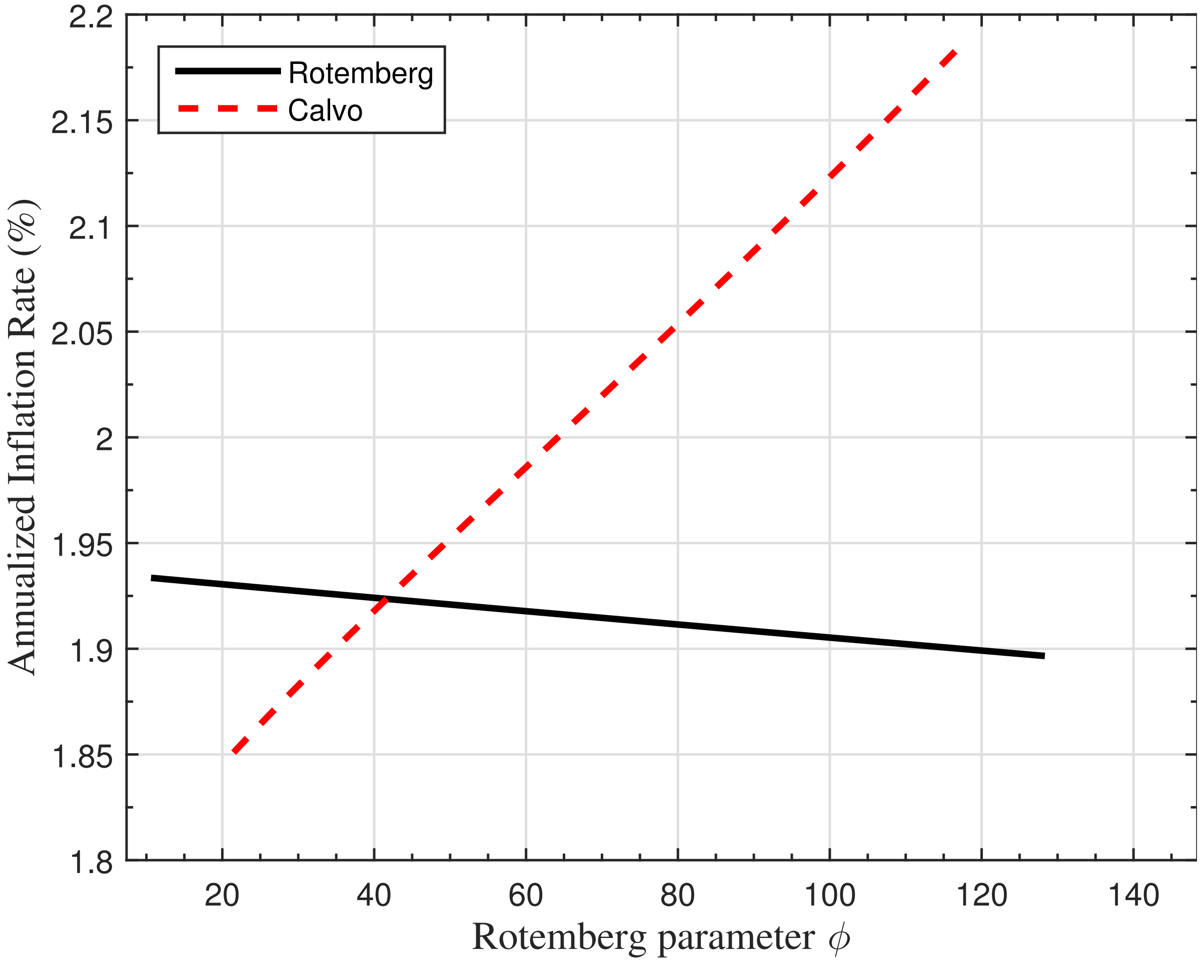

Figure 1: This figure plots annualized inflation bias (in percent) against the Rotemberg adjustment parameter (ϕ). For Calvo, the inflation bias is calculated with respect to θ ranging from 0.5 to 0.75. The corresponding values of ϕ are determined such that the LQ approximations of the two models are equivalent.

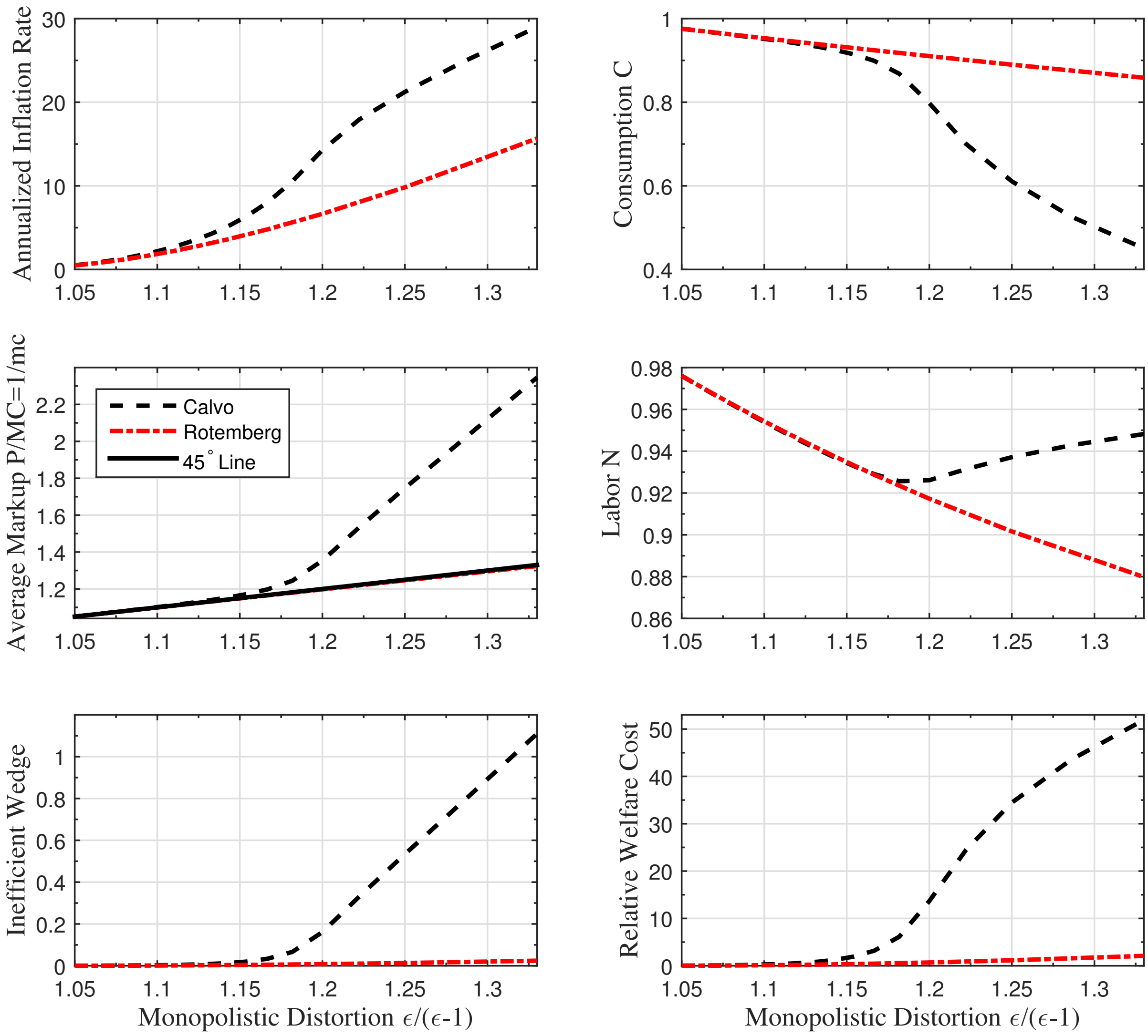

Figure 2: This figure contrasts the effect of monopolistic distortion under Calvo pricing and Rotemberg pricing. The monopolistic distortion is measured by markup at the deterministic steady state with zero inflation rate.

C Technical Appendix (Not for Publication)

C.1 Summary of Models

C.1.1 Rotemberg Pricing

The equilibrium conditions are given as follows:

Consumption Euler equation:

$$\beta R_t E_t \left\{ \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right\} = 1$$

Labor supply:

$$\left(\frac{W_t}{P_t} \right) = N_t^{\varphi} C_t^{\sigma}$$

Resource constraint:

$$\left[1 - \frac{\phi}{2} \left(\Pi_t - 1 \right)^2 \right] Y_t = C_t$$

Phillips curve:

$$(1-\epsilon) + \epsilon mc_t - \phi \Pi_t \left(\Pi_t - 1 \right) + \phi \beta E_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \frac{Y_{t+1}}{Y_t} \Pi_{t+1} \left(\Pi_{t+1} - 1 \right) \right] = 0$$

Technology:

$$Y_t = A_t N_t$$

Marginal costs:

$$mc_t = \frac{W_t}{P_t A_t} = \frac{N_t^{\varphi} C_t^{\sigma}}{A_t} = \frac{(Y_t/A_t)^{\varphi} C_t^{\sigma}}{A_t} = Y_t^{\varphi} C_t^{\sigma} A_t^{-\varphi-1}$$

We can simplify these equilibrium conditions by eliminating the interest rate and labour supply from the constraints, so that consumption can be considered as the monetary policy instrument. Specifically,

Resource constraint:

$$\left[1 - \frac{\phi}{2} \left(\Pi_t - 1 \right)^2 \right] Y_t = C_t$$

Phillips curve:

$$(1-\epsilon) + \epsilon Y_t^{\varphi} C_t^{\sigma} A_t^{-\varphi-1} - \phi \Pi_t \left(\Pi_t - 1 \right) + \phi \beta E_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \frac{Y_{t+1}}{Y_t} \Pi_{t+1} \left(\Pi_{t+1} - 1 \right) \right] = 0$$

while the objective function is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{(Y_t/A_t)^{1+\varphi}}{1+\varphi} \right)$$

Note that the state variables are productivity (and any other exogenous shock processes we choose to add).

C.1.2 Calvo Pricing

The equilibrium conditions are given below:

Consumption Euler equation:

$$\beta R_t E_t \left\{ \left(\frac{C_t}{C_{t+1}} \right)^\sigma \left(\frac{P_t}{P_{t+1}} \right) \right\} = 1$$

Labor supply:

$$\left(\frac{W_t}{P_t} \right) = N_t^\varphi C_t^\sigma$$

Resource constraint:

$$Y_t = C_t = \frac{A_t N_t}{\Delta_t}$$

Phillips curve:

$$\frac{P_t^*}{P_t} = \left(\frac{\epsilon}{\epsilon - 1} \right) \frac{K_t^p}{F_t^p}$$

Inflation:

$$1 = (1-\theta) \left(\frac{P_t^*}{P_t} \right)^{1-\epsilon} + \theta (\Pi_t)^{\epsilon - 1}$$

Price dispersion:

$$\begin{aligned} \Delta_t &= (1-\theta) \left(\frac{P_t^*}{P_t} \right)^{-\epsilon} + \theta \left(\frac{P_t}{P_{t-1}} \right)^{\epsilon} \Delta_{t-1} \\ &= (1-\theta) \left(\frac{P_t^*}{P_t} \right)^{-\epsilon} + \theta \left(\Pi_t \right)^{\epsilon} \Delta_{t-1} \end{aligned}$$

Marginal costs:

$$mc_t = \frac{W_t}{P_t A_t} = \frac{N_t^\varphi C_t^\sigma}{A_t} = (Y_t \Delta_t)^\varphi C_t^\sigma A_t{}^{-\varphi - 1}$$

Note that the state variables are not just productivity, but also price dispersion.

C.2 Numerical Algorithm

C.2.1 Algorithm for Rotemberg Pricing

In the following, let s_t denote the state of the economy at time t. There are five functional equations associated with five endogenous variables $\{C_t, Y_t, \Pi_t, \lambda_{1t}, \lambda_{2t}\}$.

The state is $s_t = a_t \equiv \ln A_t$, which evolves according to the following motion equation:

$$a_{t+1} = \rho_a a_t + e_{at}$$

where $0 \leq \rho_a < 1$ and technology innovation e_{at} is an *i.i.d.* normal random variable, which has a zero mean and a finite standard deviation σ_a.

Let's define a new function $X : \mathbb{R} \to \mathbb{R}^5$, in order to collect the policy functions of endogenous variables as follows:

$$X(s_t) = (C_t(s_t), Y_t(s_t), \Pi_t(s_t), \lambda_{1t}(s_t), \lambda_{2t}(s_t))$$

Given the specification of the function X, the equilibrium conditions can be written more compactly as,

$$\Gamma(s_t, X(s_t), E_t [Z (X(s_{t+1}))]) = 0$$

where $\Gamma : \mathbb{R}^{1+5+1} \to \mathbb{R}^5$ summarizes the full set of dynamic equilibrium relationship, and $Z (X(s_{t+1})) = M(A_{t+1})$. Then the problem is to find a vector-valued function X that Γ maps to the zero function. Projection methods, hence, can be used.

Following the notation convention in the literature, we simply use $s = (a)$ to denote the current state of the economy $s_t = (a_t)$, and s' to represent next period state that evolves according to the law of motion specified above. The Chebyshev collocation method with time iteration which we use to solve this nonlinear system can be described as follows:

1. Define the collocation nodes and the space of the approximating functions:

   - Choose an order of approximation (i.e., the polynomial degrees) n_a for the state space $s = (a)$, then there are $N_s = (n_a + 1)$ nodes in the state space. Let $S = (S_1, S_2, ..., S_{N_s})$ denote the set of collocation nodes.
   - Compute the $n_a + 1$ roots of the Chebychev polynomial of order $n_a + 1$ as

     $$z_a^i = \cos\left(\frac{(2i-1)\pi}{2(n_a+1)}\right)$$

     for $i = 1, 2, ..., n_a + 1$.
   - Compute collocation points

     $$a_i = \frac{\overline{a} + \underline{a}}{2} + \frac{\overline{a} - \underline{a}}{2} z_a^i = \frac{\overline{a} - \underline{a}}{2}\left(z_a^i + 1\right) + \underline{a}$$

     for $i = 1, 2, ..., n_a + 1$, which map $[-1, 1]$ into $[\underline{a}, \overline{a}]$. Note that the collocation nodes is given by

     $$S = \{a_i \mid i = 1, 2, ..., n_a + 1\}$$

   - Formulate the approximating policy functions. Let $T_i(z) = \cos(i \cos^{-1}(z))$ denote the Chebyshev polynomial of order i, $z \in [-1, 1]$, and let ξ denote a linear function mapping the domain of $x \in [\underline{x}, \overline{x}]$ into $[-1, 1]$. In this way, $T_i(\xi(x))$ are Chebyshev polynomials adapted to $x \in [\underline{x}, \overline{x}]$ for $i = 0, 1, ...$ Apparently, $\xi(x) = 2 (x - \underline{x}) / (\overline{x} - \underline{x}) - 1$. Then, the space of the approximating functions, denoted as Ω, is a matrix of one-dimensional Chebyshev polynomials given by

     $$\Omega(S) = \begin{bmatrix} \Omega(S_1) \\ \Omega(S_2) \\ \vdots \\ \Omega(S_{N_s}) \end{bmatrix} = \begin{bmatrix} 1 & T_1(\xi(a_1)) & T_2(\xi(a_1)) & \cdots & T_{n_a}(\xi(a_1)) \\ 1 & T_1(\xi(a_2)) & T_2(\xi(a_2)) & \cdots & T_{n_a}(\xi(a_2)) \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & T_1(\xi(a_{n_a+1})) & T_2(\xi(a_{n_a+1})) & \cdots & T_{n_a}(\xi(a_{n_a+1})) \end{bmatrix}_{N_s \times N_s}$$

     .
   - Then, at each node $s \in S$, policy functions $X(s)$ are approximated by $X(s) = \Omega(s)\Theta_X$,
     where

     $$\Theta_X = [\theta_y, \theta_c, \theta_\pi, \theta_{\lambda_1}, \theta_{\lambda_2}]$$

     is a $N_s \times 5$ matrix of the approximating coefficients.

2. Formulate an initial guess for the approximating coefficients, Θ_X^0, and specify the stopping rule ϵ_{tol}, say, 10^{-6}.

3. At each iteration j, we can get an updated Θ_X^j by implement the following time iteration step:

   - At each collocation node $s \in S$, compute the possible values of future policy functions $X(s')$ for $k = 1, ..., q$. That is,

   $$X(s') = \Omega(s')\Theta_X^{j-1}$$

   where q is the number of Gauss-Hermite quadrature nodes. Note that

   $$\Omega(s') = T_{j_a}\left(\xi(a')\right)$$

   is a $q \times N_s$ matrix, with $a' = \rho_a a + z_k\sqrt{2\sigma_a^2}$, $j_a = 0, ..., n_a$.

   - Now calculate the expectation terms $E\left[Z\left(X(s')\right)\right]$ at each node s. Let ω_k denote the weights for the quadrature, then

   $$E\left[M(s')\right] \approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q}\omega_k\left(\widehat{C}(s';\theta_c)\right)^{-\sigma}\widehat{Y}(s';\theta_y)\widehat{\Pi}(s';\theta_\pi)\left(\widehat{\Pi}(s';\theta_\pi)-1\right) \equiv \Psi\left(s',q\right)$$

   The hat symbol indicates the corresponding approximate policy functions, so $\widehat{C}$ is the approximate policy for consumption, for example.

   - At each collocation node s, solve for $X(s)$ such that

   $$\Gamma\left(s, X(s), E\left[\widehat{Z}\left(X(s')\right)\right]\right) = 0$$

   The Matlab equation solver *csolve.m* written by Christopher A. Sims is employed to solve the resulted system of nonlinear equations. With $X(s)$ at hand, we can get the corresponding coefficient

   $$\widehat{\Theta}_X^j = \left(\Omega\left(S\right)^T\Omega\left(S\right)\right)^{-1}\Omega\left(S\right)^T X(s)$$

   - Update the approximating coefficients, $\Theta_X^j = \eta\widehat{\Theta}_X^j + (1-\eta)\,\Theta_X^{j-1}$, where $0 \le \eta \le 1$ is some dampening parameter used for improving convergence.

4. Check the stopping rules. If $\left\|\Theta_X^j - \Theta_X^{j-1}\right\| < \epsilon_{tol}$, then stop, else update the approximation coefficients and go back to step 3.

When implementing the above algorithm, we start from lower order Chebyshev polynomials and some reasonable initial guess. Then, we increase the order of approximation and take as starting value the solution from the previous lower order approximation. This informal homotopy continuation idea ensures us to find a solution.

C.2.2 Algorithm for Calvo Pricing

Now the state space is $s_t = (\Delta_{t-1}, A_t)$, where price dispersion Δ_{t-1} is endogenous and technology A_t is exogenous and respectively, with the following law of motion:

$$\Delta_t = (1-\theta)\left(\frac{1-\theta\Pi_t{}^{\epsilon-1}}{1-\theta}\right)^{\frac{-\epsilon}{1-\epsilon}} + \theta\Pi_t^{\epsilon}\Delta_{t-1}$$
$$a_t = \rho_a a_{t-1} + e_{at}$$

There are 6 endogenous variables and 5 Lagrangian multipliers, hence 11 functional e-quations. Similar to Rotemberg pricing, we can rewrite this nonlinear system a more compact form,

$$\Gamma\left(s_t, X(s_t), E_t\left[Z\left(X(s_{t+1})\right)\right], E_t\left[Z_\Delta\left(X(s_{t+1})\right)\right]\right) = 0$$

where $\Gamma : \mathcal{R}^{2+11+3+3} \to \mathcal{R}^{11}$ summarizing the equilibrium relationship,

$$X(s_t) = (C_t(s_t), N_t(s_t), \Pi_t(s_t), K_t^p(s_t), F_t^p(s_t), \Delta_t(s_t), \lambda_{1t}(s_t), \lambda_{2t}(s_t), \lambda_{3t}(s_t), \lambda_{4t}(s_t), \lambda_{5t}(s_t))$$

collecting the policy functions we need to solve, with $X : \mathcal{R}^2 \to \mathcal{R}^{11}$, and

$$Z\left(X(s_{t+1})\right) = \begin{bmatrix} Z_1\left(X(s_{t+1})\right) \\ Z_2\left(X(s_{t+1})\right) \\ Z_3\left(X(s_{t+1})\right) \end{bmatrix} = \begin{bmatrix} M(\Delta_t, a_{t+1}) \\ L(\Delta_t, a_{t+1}) \\ \left(\Pi_{t+1}\right)^{\epsilon}\lambda_{4t+1} \end{bmatrix}$$

and

$$Z_\Delta\left(X(s_{t+1})\right) = \begin{bmatrix} \frac{\partial Z_1(X(s_{t+1}))}{\partial \Delta_t} \\ \frac{\partial Z_2(X(s_{t+1}))}{\partial \Delta_t} \\ \frac{\partial Z_3(X(s_{t+1}))}{\partial \Delta_t} \end{bmatrix} = \begin{bmatrix} \frac{\partial M(\Delta_t, a_{t+1})}{\partial \Delta_t} \\ \frac{\partial L(\Delta_t, a_{t+1})}{\partial \Delta_t} \\ \frac{\partial[(\Pi_{t+1})^{\epsilon}\lambda_{4t+1}]}{\partial \Delta_t} \end{bmatrix}$$
$$= \begin{bmatrix} \epsilon\left(\Pi_{t+1}\right)^{\epsilon-1} K_{t+1}^p \frac{\partial \Pi_{t+1}}{\partial \Delta_t} + \left(\Pi_{t+1}\right)^{\epsilon} \frac{\partial K_{t+1}^p}{\partial \Delta_t} \\ (\epsilon-1)\left(\Pi_{t+1}\right)^{\epsilon-2} F_{t+1}^p \frac{\partial \Pi_{t+1}}{\partial \Delta_t} + \left(\Pi_{t+1}\right)^{\epsilon-1} \frac{\partial F_{t+1}^p}{\partial \Delta_t} \\ \epsilon\left(\Pi_{t+1}\right)^{\epsilon-1} \lambda_{4t+1} \frac{\partial \Pi_{t+1}}{\partial \Delta_t} + \left(\Pi_{t+1}\right)^{\epsilon} \frac{\partial \lambda_{4t+1}}{\partial \Delta_t} \end{bmatrix}$$

Note we are assuming $E_t\left[Z_\Delta\left(X(s_{t+1})\right)\right] = \partial E_t\left[Z\left(X(s_{t+1})\right)\right]/\Delta_t$, which is normally valid using the Interchange of Integration and Differentiation Theorem.

Again, let $s = (\Delta, a)$ to denote the current state of the economy $s_t = (\Delta_{t-1}, a_t)$, and s' to represent next period state that evolves according to the law of motion specified above. The Chebyshev collocation method with time iteration for solving the nonlinear system can be described as follows:

1. Define the collocation nodes and the space of the approximating functions:

   - Choose an order of approximation n_Δ and n_a for each dimension of the state space $s = (\Delta, a)$, then there are $N_s = (n_\Delta + 1) \times (n_a + 1)$ nodes in the state space. Let $S = (S_1, S_2, ..., S_{N_s})$ denote the set of collocation nodes.
   - Compute the $n_\Delta + 1$ and $n_a + 1$ roots of the Chebychev polynomial of order

$n_\Delta + 1$ and $n_a + 1$ as

$$z_\Delta^i = \cos\left(\frac{(2i-1)\pi}{2(n_\Delta+1)}\right), \text{ for } i = 1, 2, ..., n_\Delta + 1.$$

$$z_a^i = \cos\left(\frac{(2i-1)\pi}{2(n_a+1)}\right), \text{ for } i = 1, 2, ..., n_a + 1.$$

- Compute collocation points a_i as

$$a_i = \frac{\overline{a} + \underline{a}}{2} + \frac{\overline{a} - \underline{a}}{2} z_a^i = \frac{\overline{a} - \underline{a}}{2}\left(z_a^i + 1\right) + \underline{a}$$

for $i = 1, 2, ..., n_a + 1$, which map $[-1, 1]$ into $[\underline{a}, \overline{a}]$. Similarly, compute collocation points Δ_i as

$$\Delta_i = \frac{\overline{\Delta} + \underline{\Delta}}{2} + \frac{\overline{\Delta} - \underline{\Delta}}{2} z_\Delta^i = \frac{\overline{\Delta} - \underline{\Delta}}{2}\left(z_\Delta^i + 1\right) + \underline{\Delta}$$

for $i = 1, 2, ..., n_\Delta + 1$, which map $[-1, 1]$ into $[\underline{\Delta}, \overline{\Delta}]$. Note that

$$S = \{(\Delta_i, a_j) \mid i = 1, 2, ..., n_\Delta + 1,\ j = 1, 2, ..., n_a + 1\}$$

that is, the tensor grids, with $S_1 = (\Delta_1, a_1)$, $S_2 = (\Delta_1, a_2)$, ..., $S_{N_s} = (\Delta_{n_\Delta+1}, a_{n_a+1})$.

- The space of the approximating functions, denoted as Ω, is a matrix of two-dimensional Chebyshev polynomials. More specifically,

$$\Omega(S) = \begin{bmatrix} \Omega(S_1) \\ \Omega(S_2) \\ \vdots \\ \Omega(S_{n_a+1}) \\ \vdots \\ \Omega(S_{N_s}) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & T_0(\xi(\Delta_1)T_1(\xi(a_1)) & T_0(\xi(\Delta_1)T_2(\xi(a_1)) & \cdots & T_{n_b}(\xi(\Delta_1)T_{n_a}(\xi(a_1)) \\ 1 & T_0(\xi(\Delta_1)T_1(\xi(a_2)) & T_0(\xi(\Delta_1)T_2(\xi(a_2)) & \cdots & T_{n_b}(\xi(\Delta_1)T_{n_a}(\xi(a_2)) \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & T_0(\xi(\Delta_1)T_1(\xi(a_{n_a+1})) & T_0(\xi(\Delta_1)T_2(\xi(a_{n_a+1})) & \cdots & T_0(\xi(\Delta_1)T_{n_a}(\xi(a_{n_a+1})) \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & T_0(\xi(\Delta_{n_\Delta+1})T_1(\xi(a_{n_a+1})) & T_0(\xi(\Delta_{n_\Delta+1})T_2(\xi(a_{n_a+1})) & \cdots & T_0(\xi(\Delta_{n_\Delta+1})T_{n_a}(\xi(a_{n_a+1})) \end{bmatrix}_{N_s \times N_s}$$

where $\xi(x) = 2(x - \underline{x}) / (\overline{x} - \underline{x}) - 1$ maps the domain of $x \in [\underline{x}, \overline{x}]$ into $[-1, 1]$.

- Then, at each node $s \in S$, policy functions $X(s)$ are approximated by $X(s) = \Omega(s)\Theta_X$,
where

$$\Theta_X = [\theta_c, \theta_n, \theta_\pi, \theta_k, \theta_f, \theta_\Delta, \theta_{\lambda_1}, \theta_{\lambda_2}, \theta_{\lambda_3}, \theta_{\lambda_4}, \theta_{\lambda_5}]$$

is a $N_s \times 13$ matrix of the collocation coefficients.

2. Formulate an initial guess for the approximating coefficients, Θ_X^0, and specify the stopping rule ϵ_{tol}, say, 10^{-6}.

3. At each iteration j, we can get an updated Θ_X^j by implement the following time iteration step:

   - At each collocation node $s \in S$, compute the possible values of future policy functions $X(s')$ for $k = 1, ..., q$. That is,

   $$X(s') = \Omega(s')\Theta_X^{j-1}$$

   where q is the number of Gauss-Hermite quadrature nodes. Note that

   $$\Omega(s') = T_{j_\Delta}(\xi(\Delta'))T_{j_a}(\xi(a'))$$

   is a $q \times N_s$ matrix, with $\Delta' = \widehat{\Delta}(s;\theta^\Delta)$, $a' = \rho_a a + z_k\sqrt{2\sigma_a^2}$, $j_\Delta = 0, ..., n_\Delta$, and $j_a = 0, ..., n_a$. The hat symbol indicates the corresponding approximate policy functions, so $\widehat{\Delta}$ is the approximate policy for price dispersion, for example. Similarly, the two auxiliary functions can be calculated as follows:

   $$M(s') \approx \left(\widehat{\Pi}(s';\theta_\pi)\right)^\epsilon \widehat{K}(s';\theta_k)$$

   and,

   $$L(s') \approx \left(\widehat{\Pi}(s';\theta_\pi)\right)^{\epsilon-1} \widehat{F}(s';\theta_f).$$

   - Now calculate the expectation terms $E\left[Z\left(X(s')\right)\right]$ at each node s. Let ω_k denote the weights for the quadrature, then

   $$E\left[M(s')\right] \approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q}\omega_k\left(\widehat{\Pi}(s';\theta_\pi)\right)^\epsilon \widehat{K}(s';\theta_k) \equiv \overline{M}\left(s',q\right)$$

   $$E\left[L(s')\right] \approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q}\omega_k\left(\widehat{\Pi}(s';\theta_\pi)\right)^{\epsilon-1} \widehat{F}(s';\theta_f) \equiv \overline{L}\left(s',q\right)$$

   and

   $$E_t\left[\left(\Pi_{t+1}\right)^\epsilon \lambda_{6t+1}\right] \approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q}\omega_k\left(\widehat{\Pi}(s';\theta_\pi)\right)^{\epsilon-1} \widehat{\lambda}_4(s';\theta_{\lambda_6}) \equiv \Lambda\left(s',q\right).$$

   Hence,

   $$E\left[Z\left(X(s')\right)\right] \approx E\left[\widehat{Z}\left(X(s')\right)\right] = \begin{bmatrix} \Psi\left(s',q\right) \\ \overline{L}\left(s',q\right) \\ \Lambda\left(s',q\right) \end{bmatrix}$$

   - Next calculate the partial derivatives under expectation $E\left[Z_\Delta\left(X(s')\right)\right]$. Note that we only need to compute $\partial\Pi_{t+1}/\partial\Delta_t$, $\partial K_{t+1}^p/\partial\Delta_t$ and $\partial F_{t+1}^p/\partial\Delta_t$, which are given as follows:

   $$\frac{\partial\Pi_{t+1}}{\partial\Delta_t} \approx \sum_{j_\Delta=0}^{n_\Delta}\sum_{j_a=0}^{n_a}\frac{2\theta_{\pi,j_\Delta j_a}}{\overline{\Delta}-\underline{\Delta}}T'_{j_\Delta}(\xi(\Delta_i))T_{j_a}\left(\xi(a_j)\right) \equiv \widehat{\Pi}_\Delta\left(s'\right)$$

$$\frac{\partial K_{t+1}^{p}}{\partial \Delta_t} \approx \sum_{j_\Delta=0}^{n_\Delta} \sum_{j_a=0}^{n_a} \frac{2\theta_{k,j_\Delta j_a}}{\overline{\Delta} - \underline{\Delta}} T'_{j_\Delta}(\xi(\Delta_t)) T_{j_a}(\xi(a_{t+1})) \equiv \widehat{K}_\Delta(s')$$

$$\frac{\partial F_{t+1}^{p}}{\partial \Delta_t} \approx \sum_{j_\Delta=0}^{n_\Delta} \sum_{j_a=0}^{n_a} \frac{2\theta_{f,j_\Delta j_a}}{\overline{\Delta} - \underline{\Delta}} T'_{j_\Delta}(\xi(\Delta_t)) T_{j_a}(\xi(a_{t+1})) \equiv \widehat{F}_\Delta(s')$$

Hence, we can approximate the two partial derivatives under expectation

$$\frac{\partial E[M(s')]}{\partial \Delta} \approx \frac{1}{\sqrt{\pi}} \sum_{k=1}^{q} \omega_k \left[\epsilon \left(\widehat{\Pi}(s';\theta_\pi)\right)^{\epsilon-1} \widehat{K}(s';\theta_k)\widehat{\Pi}_\Delta(s') + \left(\widehat{\Pi}(s';\theta_\pi)\right)^{\epsilon} \widehat{K}_\Delta(s')\right] \equiv \widehat{M}_\Delta(s', q),$$

$$\frac{\partial E[L(s')]}{\partial \Delta} \approx \frac{1}{\sqrt{\pi}} \sum_{k=1}^{q} \omega_k \left[(\epsilon-1) \left(\widehat{\Pi}(s';\theta_\pi)\right)^{\epsilon-2} \widehat{F}(s';\theta_f)\widehat{\Pi}_\Delta(s') + \left(\widehat{\Pi}(s';\theta_\pi)\right)^{\epsilon-1} \widehat{F}_\Delta(s')\right] \equiv \widehat{L}_\Delta(s', q).$$

That is,

$$E[Z_\Delta(X(s'))] \approx E\left[\widehat{Z}_\Delta(X(s'))\right] = \begin{bmatrix} \widehat{M}_\Delta(s', q) \\ \widehat{L}_\Delta(s', q) \end{bmatrix}$$

- At each collocation node s, solve for $X(s)$ such that

$$\Gamma\left(s, X(s), E\left[\widehat{Z}(X(s'))\right], E\left[\widehat{Z}_\Delta(X(s'))\right]\right) = 0$$

  The Matlab equation solver *csolve.m* written by Christopher A. Sims is employed to solve the resulted system of nonlinear equations. With $X(s)$ at hand, we can get the corresponding coefficient

$$\widehat{\Theta}_X^j = \left(\Omega(S)^T \Omega(S)\right)^{-1} \Omega(S)^T X(s)$$

- Update the approximating coefficients, $\Theta_X^j = \eta\widehat{\Theta}_X^j + (1-\eta)\Theta_X^{j-1}$, where $0 \leq \eta \leq 1$ is some dampening parameter used for improving convergence.

4. Check the stopping rules. If $\left\|\Theta_X^j - \Theta_X^{j-1}\right\| < \epsilon_{tol}$, then stop, else update the approximation coefficients and go back to step 3.

When implementing the above algorithm, we start from lower order Chebyshev polynomials and some reasonable initial guess. Then, we increase the order of approximation and take as starting value the solution from the previous lower order approximation. This informal homotopy continuation idea ensures us to find a solution.

C.3 Welfare Comparison

In order to compare the social welfare under Calvo and Rotemberg pricing in a fully nonlinear model, we first describe the second-order approximation to welfare. Then we transform the welfare as the fraction of the consumption path under the Ramsey allocation that must be given up in order to equalize welfare under the Ramsey policy and discretionary policy.

C.3.1 The Quadratic Approximation to Welfare

Individual utility in period t is

$$U_t \equiv U(C_t, N_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

Let $\widehat{X}_t \equiv \log\left(X_t/\overline{X}\right)$ denote the log-deviation of variable X_t from its steady state $\overline{X}$. In addition, let $\widetilde{X}_t = X_t - \overline{X}$ denote the linear deviation of X_t around its steady state value. Then using a second-order Taylor approximation,

$$\frac{X_t - \overline{X}}{\overline{X}} = \frac{\widetilde{X}_t}{\overline{X}} = \widehat{X}_t + \frac{1}{2}\widehat{X}_t^2 + o(2) \tag{32}$$

where $o(2)$ represents terms that are of order higher than 2 in the bound on the amplitude of the relevant shocks. We will repeatedly use (32) to derive a second-order approximation to the social welfare.

Now consider the second-order approximation to per period utility,

$$U_t = \overline{U} + \overline{C}^{1-\sigma}\left[\widehat{C}_t + \frac{1-\sigma}{2}\widehat{C}_t^2\right] - \overline{N}^{1+\varphi}\left[\widehat{N}_t + \frac{1+\varphi}{2}\widehat{N}_t^2\right] + o(2)$$

where

$$\overline{U} = \frac{\overline{C}^{1-\sigma} - 1}{1-\sigma} - \frac{\overline{N}^{1+\varphi}}{1+\varphi}$$

Rotemberg Pricing The second-order approximation to market clearing condition, $C_t = \left[1 - \frac{\phi}{2}\left(\Pi_t - 1\right)^2\right] Y_t$, is

$$\widehat{C}_t + \frac{1}{2}\widehat{C}_t^2 = \widehat{Y}_t + \frac{1}{2}\widehat{Y}_t^2 - \frac{\phi}{2}\widehat{\Pi}_t^2 + o(2)$$

such that,

$$\begin{aligned} U_t = \overline{U} &- \frac{(\sigma+\varphi)\,\overline{C}^{1-\sigma}}{2}\left[(x_t - x^*)^2 + \frac{\phi}{\varphi+\sigma}\widehat{\Pi}_t^2\right] \\ &+ \overline{C}^{1-\sigma}\left[\begin{array}{c} \frac{\Phi^2}{2(\varphi+\sigma)} - \frac{(1-\sigma)(1-\Phi)-(1+\varphi)}{1+\varphi}\widehat{Y}_t^f \\ +\frac{(1-\sigma)(\sigma+\varphi)}{2(1+\varphi)}\left(\widehat{Y}_t^f\right)^2 \end{array}\right] + o(2) \end{aligned} \tag{33}$$

where $\widehat{Y}_t^f = \log\left(Y_t^f/\overline{Y}^f\right)$ denote the log-deviation of output from its steady state under flexible price, $x_t \equiv \widehat{Y}_t - \widehat{Y}_t^f$ is the output gap, $x^* \equiv \ln \overline{Y} - \ln \overline{Y}^f = -\ln\left(1-\Phi\right)/\left(\sigma+\varphi\right) \approx \Phi/\left(\sigma+\varphi\right)$ is a measure of the distortion created by the presence of monopolistic competition alone, $t.i.p.$ are terms independent of policy, and terms like $\Phi\left(\widehat{Y}_t^f\right)^2$ and $\Phi\widehat{Y}_t\widehat{Y}_t^f$ are omitted[13]. In addition, the fact that $\overline{N}^{1+\varphi} = \left(1-\epsilon^{-1}\right)\overline{C}^{1-\sigma} \equiv \left(1-\Phi\right)\overline{C}^{1-\sigma}$, and $\widehat{A}_t = \left(\varphi+\sigma\right)/\left(1+\varphi\right)\widehat{Y}_t^f$ is used in deriving (33).

Hence,

[13] When $\Phi = 1/\epsilon$ is so small that the product of Φ with a second-order term can be ignored as negligible.

$$W_R \equiv E_0 \sum_{t=0}^{\infty} \beta^t U_t = \frac{\overline{U}}{1-\beta} - \frac{(\sigma+\varphi)\,\overline{C}^{1-\sigma}}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[(x_t - x^*)^2 + \frac{\phi}{\sigma+\varphi} \widehat{\Pi}_t^2 \right]$$

$$+ \left[\frac{\Phi^2 \overline{C}^{1-\sigma}}{2(\varphi+\sigma)(1-\beta)} - \frac{(1-\sigma)(1+\varphi)\,\overline{C}^{1-\sigma}\sigma_a^2}{2(\varphi+\sigma)(1-\beta)(1-\rho_a)} \right] + o(2)$$

$$= \frac{\overline{U}}{1-\beta} - \Omega_R E_0 \sum_{t=0}^{\infty} \beta^t \left[\lambda_R (x_t - x^*)^2 + \widehat{\Pi}_t^2 \right] \tag{34}$$

$$+ \left[\frac{\Phi^2 \overline{C}^{1-\sigma}}{2(\varphi+\sigma)(1-\beta)} - \frac{(1-\sigma)(1+\varphi)\,\overline{C}^{1-\sigma}\sigma_a^2}{2(\varphi+\sigma)(1-\beta)(1-\rho_a)} \right] + o(2)$$

where

$$\Omega_R \equiv \frac{\phi \overline{C}^{1-\sigma}}{2}$$

$$\lambda_R \equiv \frac{\sigma+\varphi}{\phi}$$

Note that we derive the LQ welfare function explicitly retaining the relevant $t.i.p$ in order to make a legitimate comparison with the social welfare obtained from the fully nonlinear model.

In order to calculate the inflation bias under LQ method, we write down the log-linearized IS equation and NKPC below. The IS curve is,

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}\left(\widehat{R}_t - E_t \widehat{\Pi}_{t+1} \right) + \frac{1+\varphi}{\varphi+\sigma}(\rho_a - 1)\widehat{A}_t$$

and the NKPC is,

$$\widehat{\Pi}_t = \beta E_t \widehat{\Pi}_{t+1} + \frac{(\epsilon-1)(\varphi+\sigma)}{\phi} x_t$$

Calvo Pricing The second-order approximation to market clearing condition is

$$\widehat{C}_t + \frac{1}{2}\widehat{C}_t^2 = \widehat{Y}_t + \frac{1}{2}\widehat{Y}_t^2 + o(2)$$

and it can be shown (see Woodford, 2003, chap 6) that,

$$\widehat{N}_t = \left(\widehat{Y}_t - \widehat{A}_t \right) + \frac{\epsilon}{2} var_j \left(\widehat{P}_t(j) \right) + o(2)$$

Hence, similar to the Rotemberg case,

$$U_t = \overline{U} - \frac{(\varphi+\sigma)\,\overline{C}^{1-\sigma}}{2} \left[(x_t - x^*)^2 + \frac{\epsilon}{\varphi+\sigma} var_j \left(\widehat{P}_t(j) \right) \right]$$

$$+\overline{C}^{1-\sigma} \left[\frac{\Phi^2}{2(\varphi+\sigma)} - \frac{(1-\sigma)(1-\Phi)-(1+\varphi)}{(1+\varphi)} \widehat{Y}_t^f + \frac{(1-\sigma)(\sigma+\varphi)}{2(1+\varphi)} \left(\widehat{Y}_t^f \right)^2 \right] + o(2)$$

The next step is to relate price dispersion $\Delta_t \equiv var_j\left(\widehat{P}_t(j)\right)$ to the average inflation rate across all firms. Walsh (2003, p.554) shows that

$$\Delta_t \approx \theta\Delta_{t-1} + \left(\frac{\theta}{1-\theta}\right)\pi_t^2$$

which implies

$$\sum_{t=0}^{\infty}\beta^t\Delta_t = \frac{\theta}{(1-\theta)(1-\theta\beta)}\sum_{t=0}^{\infty}\beta_t\pi_t^2$$

Therefore,

$$W_C = \frac{\overline{U}}{1-\beta} - \Omega_C E_0\sum_{t=0}^{\infty}\beta^t\left[\lambda_C(x_t - x^*)^2 + \pi_t^2\right]$$
$$+\left[\frac{\Phi^2\overline{C}^{1-\sigma}}{2(\varphi+\sigma)(1-\beta)} - \frac{(1-\sigma)(1+\varphi)\overline{C}^{1-\sigma}\sigma_a^2}{2(\varphi+\sigma)(1-\beta)(1-\rho_a)}\backslash\right] + o(2)$$

where

$$\Omega_C \equiv \frac{(\sigma+\varphi)\overline{C}^{1-\sigma}}{2}\frac{\epsilon}{\kappa}$$
$$\lambda_C \equiv \kappa/\epsilon$$
$$\kappa \equiv \frac{(1-\theta)(1-\theta\beta)(\sigma+\varphi)}{\theta}$$

The log-linearized IS equation and NKPC are given, respectively, as follows:

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}\left(\widehat{R}_t - E_t\widehat{\Pi}_{t+1}\right) + \frac{1+\varphi}{\varphi+\sigma}(\rho_a - 1)\widehat{A}_t$$

$$\widehat{\Pi}_t = \beta E_t\widehat{\Pi}_{t+1} + \kappa x_t$$

Note that when

$$\phi = \frac{(\epsilon-1)\theta}{(1-\theta)(1-\theta\beta)}$$

the NKPC under both Rotemberg pricing and Calvo pricing are the same. Also note that $\lambda_R = \left(\frac{\epsilon}{\epsilon-1}\right)\lambda_C$, and $\Omega_R = \left(\frac{\epsilon-1}{\epsilon}\right)\Omega_C$. The inflation weights λ_R and λ_C differ only marginally, since ϵ usually takes values between 7 and 10 in the applied literature.

C.3.2 Inflation Bias Under LQ Method

We can rewrite the above LQ model as follows, using $\pi_t = \Pi_t - 1 \approx \ln(\Pi_t) - \ln\left(\overline{\Pi}\right) = \widehat{\Pi}_t$ and $i_t = R_t - 1 \approx \ln(R_t) - \ln\left(\overline{R}\right) = \widehat{R}_t$:

$$\max_{\{x_t,\pi_t\}} -\Omega_j E_0\sum_{t=0}^{\infty}\beta^t\left[\lambda_j(x_t - x^*)^2 + \pi_t^2\right]$$

subject to

$$\pi_t = \beta E_t\pi_{t+1} + \kappa x_t \tag{35}$$

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}(i_t - E_t\pi_{t+1}) + \frac{1+\varphi}{\varphi+\sigma}(\rho_a - 1)\widehat{A}_t$$

where $j = R, C$. Woodford (2003, p.471) shows that the equilibrium inflation under optimal discretion is

$$\pi_t = \frac{\lambda_j}{\lambda_j + \kappa^2} \left(\beta E_t \pi_{t+1} + \kappa x^* \right)$$

hence the steady state $\overline{\pi}$ under rational expectation satisfies

$$\overline{\pi} = \frac{\lambda_j}{\lambda_j + \kappa^2} \left(\beta \overline{\pi} + \kappa x^* \right)$$

that is,

$$\overline{\pi} = \frac{\lambda_j \kappa}{(1-\beta)\lambda_j + \kappa^2} x^* = \frac{\lambda_j \kappa}{(1-\beta)\lambda_j + \kappa^2} \frac{\Phi}{(\sigma + \varphi)}$$

with $j = R, C$. $\overline{\pi}$ is the so-called inflation bias, relative to the targeted zero rate of inflation which is optimal under perfect commitment.

Note that $\lambda_R \geq \lambda_C$ and

$$\frac{d\overline{\pi}}{d\lambda_j} = \frac{\Phi}{\sigma + \varphi} \frac{\kappa^3}{\left[(1-\beta)\lambda_j + \kappa^2 \right]^2} > 0$$

which imply that $\overline{\pi}_R \geq \overline{\pi}_C$. That is, the inflation bias problem using the LQ approximation is always worsen under the Rotemberg pricing than that under the Calvo pricing.

C.3.3 Relative Welfare Cost

The welfare under discretion from the LQ method is calculated as follows. Unless stated otherwise, the superscript d denotes the discretion case, and subscripts R and C represent the Rotemberg and Calvo pricing, respectively. From (35), $\overline{x} = (1-\beta)\overline{\pi}/\kappa$, then using the log-linearized model we can solve for steady state values for deviations $\widehat{C}_t$ and $\widehat{N}_t$, denoted as $\widehat{C}$ and $\widehat{N}$, respectively. It is straightforward to show that $\widehat{C} = \widehat{N} = \widehat{Y} = \overline{x}$. Finally, the steady state values for levels C_t and N_t, are

$$\overline{C}_j^d = \overline{C}^r e^{\widehat{C}} \approx \overline{C}^r \left(1 + \widehat{C} \right) = \overline{C}^r (1 + \overline{x})$$

$$\overline{N}_j^d = \overline{N}^r e^{\widehat{N}} \approx \overline{N}^r \left(1 + \widehat{N} \right) = \overline{N}^r (1 + \overline{x})$$

where $j = R, C$, and

$$\overline{C}^r = \overline{N}^r = \left(\frac{\epsilon - 1}{\epsilon} \right)^{1/(\sigma+\varphi)}$$

are the Ramsey steady states around which we log-linearize the model. Therefore,

$$W_j = \frac{1}{1-\beta} \left[\frac{\left(\overline{C}_j^d\right)^{1-\sigma} - 1}{1-\sigma} - \frac{\left(\overline{N}_j^d\right)^{1+\varphi}}{1+\varphi} \right] - \frac{\Omega_j}{1-\beta} \left[\lambda_j \left(\frac{(1-\beta)\overline{\pi}}{\kappa} - \frac{\Phi}{(\varphi+\sigma)} \right)^2 + \overline{\pi}^2 \right]$$

$$+ \frac{\left(\overline{C}_j^d\right)^{1-\sigma}}{2(\varphi+\sigma)(1-\beta)} \left[\Phi^2 - \frac{(1-\sigma)(1+\varphi)\sigma_a^2}{(1-\rho_a)} \right]$$

where $j = R, C$.

For the fully nonlinear method, the welfare under discretion is calculated by adding corresponding policy functions into optimal policy problem and then approximated by the Chebyshev collocation method. That is,

$$W^d_{R,t} = W^d_R(A_t) = \frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{(Y_t/A_t)^{1+\varphi}}{1+\varphi} + \beta E_t\left[W^d_R(A_{t+1})\right]$$

$$W^d_{C,t} = W^d_C(\Delta_{t-1}, A_t) = \frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{(\Delta_t Y_t/A_t)^{1+\varphi}}{1+\varphi} + \beta E_t\left[W^d_C(\Delta_t, A_{t+1})\right]$$

and the steady state welfare, denoted as W^d_R and W^d_C for ease of notation, can be correspondingly found.

Note that W_R, W^d_R and W_C, W^d_C which represent the conditional expectation of lifetime utility, are absolute welfare measures under Rotemberg pricing and Calvo pricing, respectively. However, the utility function is ordinal, so a welfare measure based on the value function is not very revealing. Hence, we calculate the relative welfare cost in terms of the consumption equivalent units under the Ramsey allocation. Specifically, we want to find ξ such that

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C^d_t, N^d_t) = E_0 \sum_{t=0}^{\infty} \beta^t U((1-\xi)C^r_t, N^r_t)$$

where the r superscript denotes the Ramsey allocation (under full commitment), and the d superscript stands for the allocation under discretion. Given the utility function adopted, the expression for ξ in percentage terms is

$$\xi = \left\{1 - \exp\left[(1-\beta)\left(W^d - W^r\right)\right]\right\} \times 100 \tag{36}$$

where

$$W^d \equiv E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C^d_t - \frac{\left(N^d_t\right)^{1+\varphi}}{1+\varphi}\right)$$

represents the unconditional expectation of lifetime utility in the economy under discretion, and

$$W^r \equiv E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C^r_t - \frac{\left(N^r_t\right)^{1+\varphi}}{1+\varphi}\right) = \frac{1}{1-\beta}\left[\ln \overline{C}^r - \frac{\left(\overline{N}^r\right)^{1+\varphi}}{1+\varphi}\right]$$

is the unconditional expectation of lifetime utility associated with the economy under full commitment. Recall that $\sigma = 1$ is the benchmark case in our paper.

Hence, under the Rotemberg case,

$$\xi_R = \begin{cases} \{1 - \exp\left[(1-\beta)\left(W_R - W^r\right)\right]\} \times 100 & \text{, using LQ method} \\ \left\{1 - \exp\left[(1-\beta)\left(W^d_R - W^r\right)\right]\right\} \times 100 & \text{, using projection method} \end{cases}$$

and under the Calvo case,

$$\xi_C = \begin{cases} \{1 - \exp\left[(1-\beta)\left(W_C - W^r\right)\right]\} \times 100 & \text{, using LQ method} \\ \left\{1 - \exp\left[(1-\beta)\left(W^d_C - W^r\right)\right]\right\} \times 100 & \text{, using projection method} \end{cases}$$

C.4 The Model With Time-Varying Tax Rate

To indirectly introduce cost push shock, we consider the revenue tax τ_{pt} which is assumed to follow the following autoregressive process,

$$\ln(1-\tau_{pt}) = (1-\rho^{\tau_p})\ln(1-\tau_p) + \rho^{\tau_p}\ln(1-\tau_{pt-1}) - e_{\tau t}$$

$$e_{\tau t} \overset{i.i.d}{\sim} N\left(0, \sigma_\tau^2\right)$$

With revenue tax τ_{pt}, the expected discounted sum of nominal profits under Rotemberg pricing is given by

$$E_t \sum_{s=0}^{\infty} Q_{t,t+s}\left[(1-\tau_{pt})P_t(j)Y_t(j) - mc_t Y_t(j)P_t - \frac{\phi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 Y_t P_t\right]$$

and under Calvo it can be written as

$$E_t \sum_{s=0}^{\infty} \theta^s Q_{t,t+s}\left[(1-\tau_{pt})P_t(j)Y_{t+s}(j) - mc_{t+s}Y_{t+s}(j)P_{t+s}\right]$$

Based on the derivation of the benchmark model, it is quite straightforward to write down the complete system of non-linear equations describing the discretionary equilibrium. We will use Chebyshev collocation with time iteration method to solve the models with time-varying tax for optimal policy functions.

C.4.1 Rotemberg Pricing

Since we want to focus on the effect of tax rate, then the technology shock can be shut down by setting $A_t \equiv 1$. This, in fact, can simplify numerical computation.

The Lagrangian is

$$\mathcal{L} = \frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} + \beta E_t\left[V(\tau_{pt+1})\right] + \lambda_{1t}\left\{\left[1-\frac{\phi}{2}(\Pi_t-1)^2\right]N_t - C_t\right\}$$
$$+ \lambda_{2t}\left\{(1-\epsilon)(1-\tau_{pt}) + \epsilon C_t^{\sigma}N_t^{\varphi} - \phi\Pi_t(\Pi_t-1) + \phi\beta C_t^{\sigma}Y_t^{-1}E_t\left[M(\tau_{pt+1})\right]\right\}$$

where λ_{jt} $(j=1,2)$ are the Lagrange multipliers, and

$$M(\tau_{pt+1}) \equiv C_{t+1}^{-\sigma}N_{t+1}\Pi_{t+1}(\Pi_{t+1}-1)$$

The equilibrium conditions for time-consistent policy are,

$$C_t^{-\sigma} = \lambda_{1t} - \lambda_{2t}\left\{\epsilon\sigma C_t^{\sigma-1}N_t^{\varphi} + \sigma\phi\beta C_t^{\sigma-1}N_t^{-1}E_t\left[M(\tau_{pt+1})\right]\right\}$$

$$N_t^{\varphi} = \lambda_{1t}\left[1-\frac{\phi}{2}(\Pi_t-1)^2\right] + \lambda_{2t}\left\{\begin{array}{c}\epsilon\varphi N_t^{\varphi-1}C_t^{\sigma} \\ -\phi\beta C_t^{\sigma}N_t^{-2}E_t\left[M(\tau_{pt+1})\right]\end{array}\right\}$$

$$\lambda_{1t}\phi(1-\Pi_t)N_t = \lambda_{2t}\phi(2\Pi_t-1)$$

$$C_t = \left[1-\frac{\phi}{2}(\Pi_t-1)^2\right]N_t$$

$$0 = (1-\epsilon)(1-\tau_{pt}) + \epsilon C_t^{\sigma}N_t^{\varphi} - \phi\Pi_t(\Pi_t-1) + \phi\beta\frac{C_t^{\sigma}}{N_t}E_t\left[M(\tau_{pt+1})\right].$$

C.4.2 Calvo Pricing

Similar to the Rotemberg case, we solve a simpler question by shutting down the technology shock. Then, there are two state variables, τ_{pt} and Δ_{t-1}. The Lagrangian is given as follow:

$$
\begin{aligned}
\mathcal{L} = & \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} + \beta E_t \left[V(\Delta_t, \tau_{pt+1}) \right] \\
& + \lambda_{1t}[N_t/\Delta_t - C_t] \\
& + \lambda_{2t} \left[(1-\tau_{pt}) \frac{N_t}{\Delta_t C_t^{\sigma}} + \theta\beta E_t \left[L(\Delta_t, \tau_{pt+1}) \right] - F_t \right] \\
& + \lambda_{3t} \left[\frac{N_t^{\varphi+1}}{(1-\epsilon^{-1})\Delta_t} + \theta\beta E_t \left[M(\Delta_t, \tau_{pt+1}) \right] - S_t \right] \\
& + \lambda_{4t} \left[(1-\theta) \left(\frac{1-\theta\Pi_t{}^{\epsilon-1}}{1-\theta} \right)^{\frac{\epsilon}{\epsilon-1}} + \theta \Pi_t^{\epsilon} \Delta_{t-1} - \Delta_t \right] \\
& + \lambda_{5t} \left[F_t \left(\frac{1-\theta\Pi_t{}^{\epsilon-1}}{1-\theta} \right)^{\frac{1}{1-\epsilon}} - S_t \right]
\end{aligned}
$$

where λ_{jt} $(j = 1, 2, 3, 4, 5)$ are the Lagrange multipliers, and

$$
L(\Delta_t, \tau_{pt+1}) \equiv \Pi_{t+1}^{\epsilon-1} F_{t+1}
$$

$$
M(\Delta_t, \tau_{pt+1}) \equiv \Pi_{t+1}^{\epsilon} S_{t+1}
$$

The equilibrium conditions for time-consistent policy are,

$$
\begin{aligned}
C_t &= N_t/\Delta_t \\
F_t &= (1-\tau_{pt}) C_t^{1-\sigma} + \theta\beta E_t \left[\Pi_{t+1}^{\epsilon-1} F_{t+1} \right] \\
S_t &= \frac{N_t^{\varphi+1}}{(1-\epsilon^{-1})\Delta_t} + \theta\beta E_t \left[\Pi_{t+1}^{\epsilon} S_{t+1} \right] \\
\Delta_t &= (1-\theta) \left(\frac{1-\theta\Pi_t{}^{\epsilon-1}}{1-\theta} \right)^{\frac{\epsilon}{\epsilon-1}} + \theta \Pi_t^{\epsilon} \Delta_{t-1} \\
S_t &= F_t \left(\frac{1-\theta\Pi_t{}^{\epsilon-1}}{1-\theta} \right)^{\frac{1}{1-\epsilon}} \\
0 &= 1 - \lambda_{1t} C_t^{\sigma} - \sigma(1-\tau_{pt})\lambda_{2t} \\
0 &= \Delta_t C_t^{\sigma} N_t{}^{\varphi} - C_t^{\sigma} \lambda_{1t} - (1-\tau_{pt})\lambda_{2t} - \frac{(\varphi+1)\, C_t^{\sigma} N_t^{\varphi} \lambda_{3t}}{(1-\epsilon^{-1})} \\
0 &= \lambda_{2t} - \lambda_{5t} \left(\frac{1-\theta\Pi_t{}^{\epsilon-1}}{1-\theta} \right)^{\frac{1}{1-\epsilon}} \\
0 &= \lambda_{3t} + \lambda_{5t}
\end{aligned}
$$

$$0 = \epsilon \left(\left(\frac{1 - \theta {\Pi_t}^{\epsilon - 1}}{1 - \theta} \right)^{\frac{1}{\epsilon - 1}} - \Delta_{t-1} \Pi_t \right) \lambda_{4t}$$

$$- \frac{1}{1 - \theta} \left(\frac{1 - \theta {\Pi_t}^{\epsilon - 1}}{1 - \theta} \right)^{\frac{\epsilon}{1 - \epsilon}} \lambda_{5t} F_t$$

$$0 = \frac{C_t}{\Delta_t} \lambda_{1t} + (1 - \tau_{pt}) \frac{C_t^{1-\sigma}}{\Delta_t} \lambda_{2t} + \frac{N_t^{\varphi} C_t}{(1 - \epsilon^{-1}) \Delta_t} \lambda_{3t}$$

$$+ \lambda_{4t} - \theta \beta \lambda_{2t} E_t \left[L_1(\Delta_t, \tau_{pt+1}) \right] - \theta \beta \lambda_{3t} E_t \left[M_1(\Delta_t, \tau_{pt+1}) \right] - \theta \beta E_t \left[\Pi_{t+1}^{\epsilon} \lambda_{4t+1} \right]$$

D Additional Figures (Not for Publication)

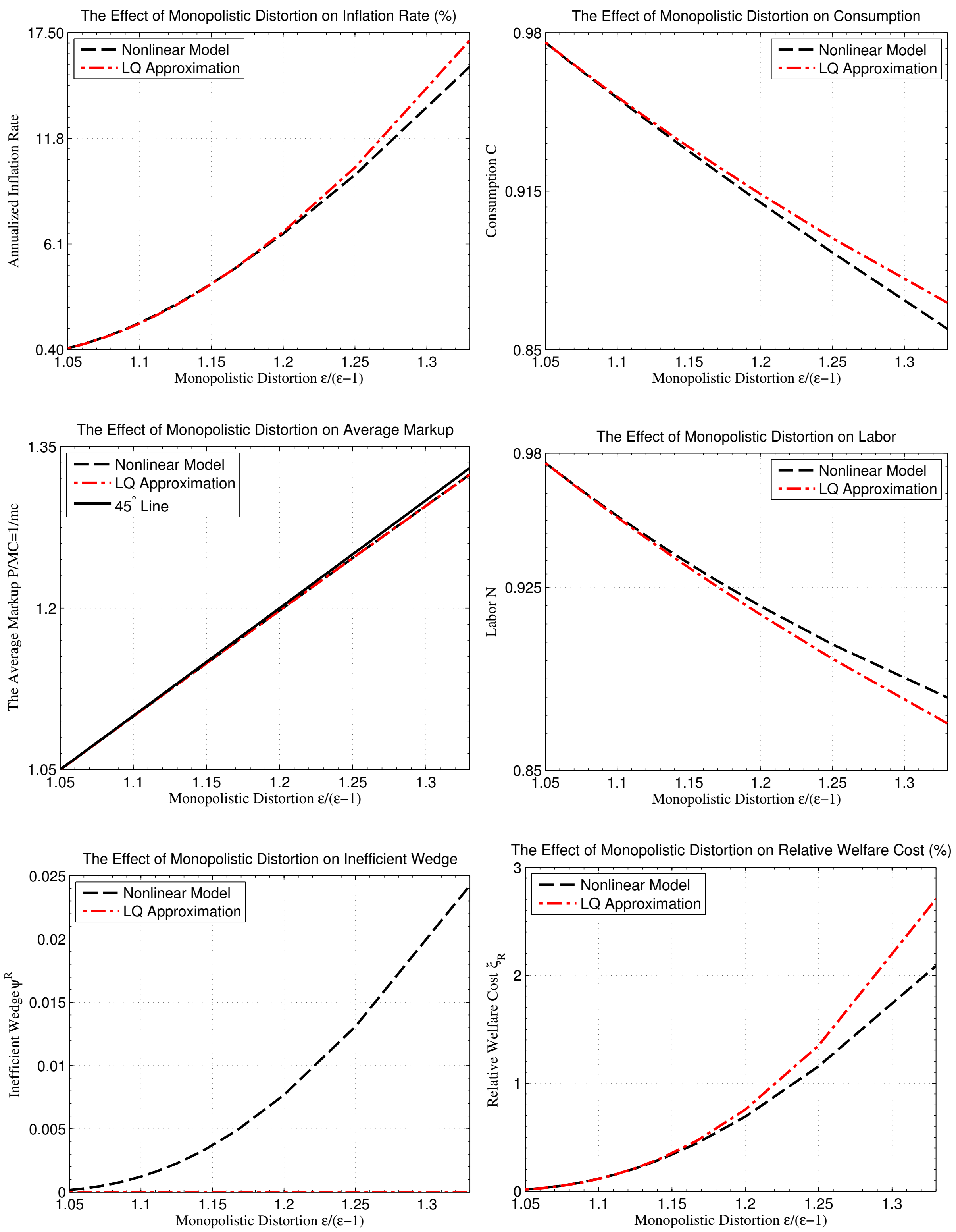

Figure 3: This figure shows the effect of monopolistic distortion under Rotemberg pricing. The monopolistic distortion is measured by markup at the deterministic steady state with zero inflation rate. The results from LQ and projection method are compared.

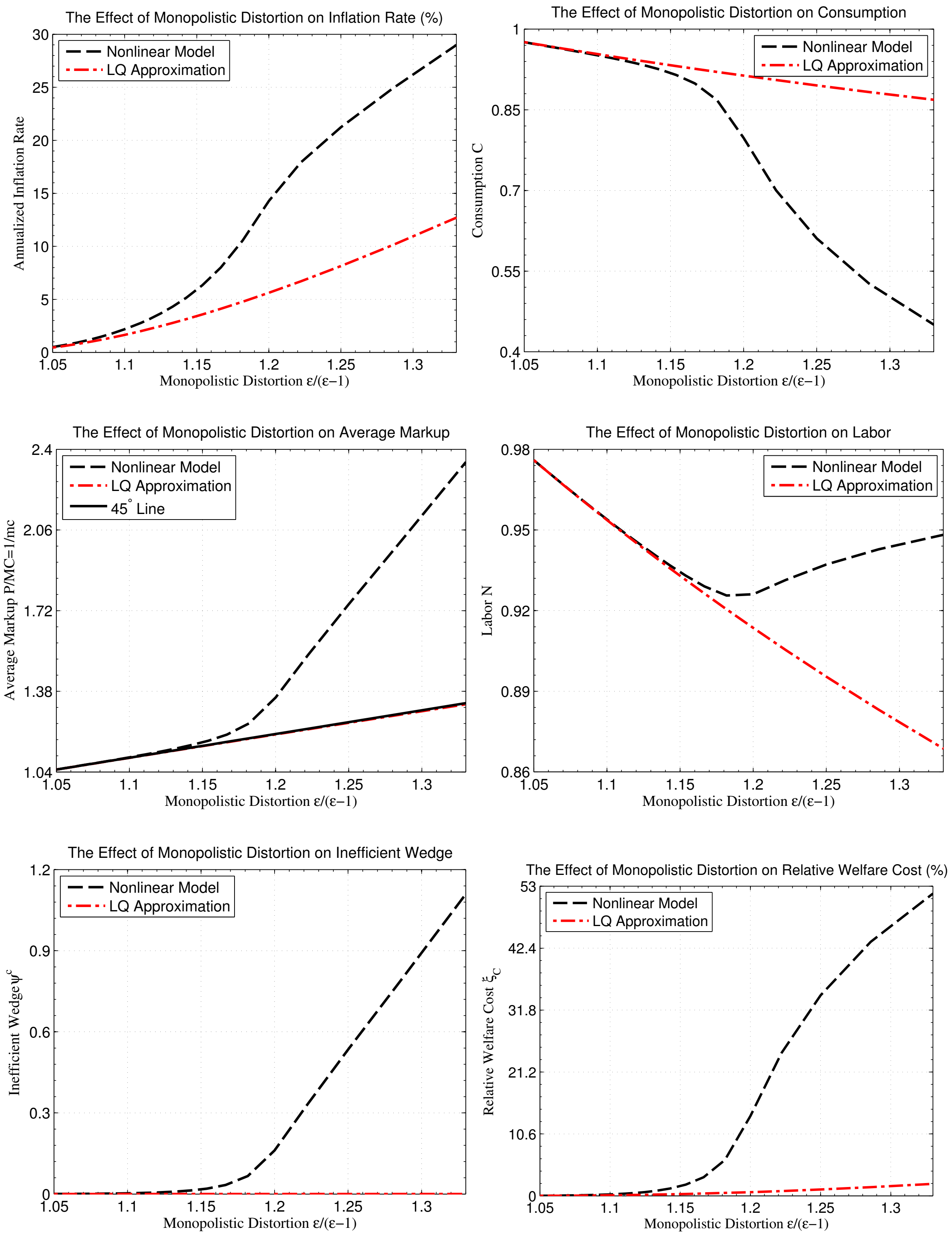

Figure 4: This figure shows the effect of monopolistic distortion under Calvo pricing. The monopolistic distortion is measured by markup at the deterministic steady state with zero inflation rate. The results from LQ and projection method are compared.