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17 INTRODUCTION

18 Water-related erosion and sedimentation processes interact with the biosphere at a global
19 scale (Walling 2009) and affect many human activities (Dotterweich 2008). One of the most
20 important aspects of sediment transport, that is also one of the most complex and difficult
21 problems in this field, is determination of the dynamics that drive the initiation of sediment
22 movement which reflect the combined effects of fluid turbulence, grain arrangement and
23 local topographic variability (Grass 1970; Buffington and Montgomery 1997; Dey 2014).
24 In spite of the large number of studies since the classic work of Shields (Buffington and
25 Montgomery 1997; Shields 1936), aspects of the problem of sediment entrainment remain
26 unsolved which partly-explains the absence of a widely accepted model for the prediction of
27 sediment transport in streams (Merritt et al. 2003).

28 Although it is generally accepted that sediment entrainment and hence material transport
29 rates increase in a non-linear manner as the flow rate increases, a unique, explicit parameter
30 capable of characterising the threshold conditions for incipient motion does not yet exist
31 (Garcia 2008; Buffington and Montgomery 1997). Furthermore, the variability of sediment
32 transport and the plethora of related parameters (e.g. the wide range of particle sizes, bed
33 surface structure, hiding and exposure, the complex history of the channel bed) lead to
34 bedload transport models and corresponding entrainment criteria that are valid only within
35 specific conditions (Habersack and Kreisler 2013).

36 Accurate modelling of bedload transport processes is complicated by this transport taking
37 place across a range of temporal and spatial scales (from the grain to catchment scale, and
38 from rapid single-grain movements to annual bed displacements). These scale ranges lead to
39 two fundamentally different descriptions of sediment transport: the Eulerian deterministic
40 approach formalized by Shields (Shields 1936), mainly applicable at reach to catchment
41 scales, and the Lagrangian stochastic step-length model introduced by Einstein (Einstein
42 1937), which is by definition relevant to the grain scale. The specification of an appropriate
43 dynamic field for these approaches (Eulerian or Lagrangian) concerns the frame of reference

44 for the water-flow generated dynamics: in Shields' model individual particles move under a
45 time-averaged mean bed shear stress, while in Einstein's model grain movement is consequent
46 on local turbulent stresses (Papanicolaou et al. 2002). Formal ways to link the two frames
47 of reference in the context of sediment transport are logical next steps in improving our
48 understanding of transport processes.

49 The most widely used criterion for incipient motion is Shields' critical shear stress (τ_c).
50 τ_c is the bed shear stress produced by the water flow (if uniform flow is assumed this is
51 approximated as a channel slope-depth product) that is capable of mobilising each specific
52 sediment size class (which, for grain sizes yielding particle Reynolds' Numbers > 70 , is
53 correlated with the median diameter of the sediment).

54 Since Shields' (1936) work, a series of empirical values have been suggested to account for
55 a range of factors including the relative depth of the flow, grain shape and protrusion (Ashida
56 and Michiue 1971; Fenton and Abbott 1977; Shvidchenko and Pender 2000). In parallel,
57 Shields' criterion has been extensively criticized for its ambiguity and limited applicability
58 (Church et al. 1998; Buffington and Montgomery 1997; Parker et al. 2003; Bunte et al. 2013)
59 and the validity of a single criterion or even the existence of measurable critical threshold
60 conditions have been questioned (Einstein 1950; Lavelle and Mofjeld 1987). Parallel work has
61 associated the effects of bed micro-topography (Kirchner et al. 1990; Buffington et al. 1992;
62 Prancevic and Lamb 2015), the near bed flow turbulence (Nelson et al. 1995; Papanicolaou
63 et al. 2002) and impulsive (Diplas et al. 2008; Valyrakis et al. 2010) or energetic (Valyrakis
64 et al. 2013) flow events on incipient motion. The combined result of these phenomena cannot
65 be accounted for within a deterministic time-averaged mean stress calculation, although such
66 approaches can still yield useful results.

67 The core problem with the inclusion of all the above phenomena in any analysis of en-
68 trainment is that each of them is difficult to measure or quantify. Advances in monitoring
69 techniques have improved the accuracy of measurements of grain scale near bed forces (Pa-
70 panielicolaou et al. 1999; Schmeeckle et al. 2007) as well as enabling monitoring of impulse

71 events and their energy potential (Valyrakis et al. 2013). These laboratory measurements
72 reveal great variability of flow dynamics at micro-scale which, combined with the random
73 character of the micro-topography, justifies the treatment of incipient motion as an inherently
74 stochastic processes (comparable to Einstein’s description). A summative review of recent
75 studies that define and explore the concept of ”pickup probability” or ”entrainment proba-
76 bility” as attributed to conditions related to both flow turbulence and sediment arrangement
77 is presented in Marion and Tregnaghi (2013).

78 Marion and Tregnaghi (2013) show how all the stochastic studies of entrainment reuti-
79 lize and extend the conceptual framework introduced by Grass (1970). Grass suggested
80 calculating the probability of entrainment as a joint probability derived from Probability
81 Density Functions (*PDF* hereafter) of critical shear stresses (connected to the resistance to
82 entrainment of the sediment grains) and of the distribution of hydrodynamic forces (derived
83 from near bed flow velocities). The probability of entrainment (P_E) is calculated as the
84 exceedance of a random near bed velocity (U_f) represented by a cumulative distribution F_{U_f}
85 having a PDF of f_{U_f} , as:

$$87 \quad P_E = P(U_f > u_f = u_g) = \int_{u_g}^{\infty} f_{U_f}(u) du = 1 - F_{U_f}(u_g) \quad (1)$$

88
89 where u_g represents the threshold velocity for grain entrainment. Note that the form of equa-
90 tion (1) is general and also applies to definitions of entrainment in terms of other relevant
91 variables such as shear stress or turbulent kinetic energy.

92
93 A development of the stochastic description of sediment transport is that the process has
94 been described using a range of mathematical approaches including state-space descriptions
95 (e.g. Markov chains, Tsai and Lai (2014)) and inference techniques in both adaptive neuro-
96 fuzzy (Valyrakis et al. 2011) and classical Bayesian (Schmelter and Stevens 2012) contexts.

97 These approaches rely on robust calculation of probabilities such as the probability of en-
98 trainment, since they can be utilized either as real transition probabilities for the state-space
99 derivations or as training functions and priors for the inference systems.

100 An aspect of the entrainment problem that has not been extensively investigated is that
101 the existing criteria for sediment entrainment are essentially implicit in the sense that they
102 are based on near particle flow features (e.g. flow turbulence) rather than characteristics of
103 each individual particle and its local arrangement. This problem has often been identified
104 (Cao et al. 2006), but has only recently been formally treated by measuring the dynamics
105 that occur in the inertial frame of the particles close to the threshold of entrainment. It is
106 now technically possible to measure inertial dynamics at scales appropriate for gravel sized
107 sediment, since the miniaturisation of sensing equipment has made the concept of a "smart
108 pebble" (a small, free-moving multi-sensor capable of measuring inertial dynamics such as
109 acceleration and angular velocity) feasible (Akeila et al. 2010; Šolc et al. 2012; Frank 2014).

110 Maniatis et al. (2013) have shown how this technology can be optimized for natural fluvial
111 environments, demonstrating the capability of the sensor to capture accurate, representative
112 and robust dynamical information over a broad range of imposed forces. However, interpre-
113 tation of the inertial data in a theoretical framework for incipient motion (Frank 2014) has
114 so far been restricted to the utilisation of Shields' conceptual model.

115 Following from these theoretical and technical developments, the contributions of this
116 paper are to provide:

- 117 • an evaluation of the mobile sensor presented by Maniatis et al. (2013) in entrainment
118 threshold experiments. These results provide supporting evidence towards the forma-
119 tion of an explicit entrainment criterion that has the potential to be utilised across
120 the range of natural river flow regimes.
- 121 • description of the derived time-series with dynamic linear models in order to make
122 space state approximations for a representative underlying entrainment process. This
123 approximation is performed by the application of a simplified Kalman filter.

- 124 • illustration of attribution of categorical variables to the approximated states, and
125 calculation of the probability of entrainment as a function of inertial acceleration
126 using logistic regression analysis. This result connects directly the inertial dynamics
127 of individual particles to the more relevant probabilistic mathematical context for the
128 description of incipient motion.
- 129 • finally we introduce a metric to evaluate the performance of the probabilistic criteria
130 that are relevant to grain incipient motion: the overlapping coefficient (OVL) (Weitz-
131 man 1970). The derivation of the OVL requires the numerical approximation of the
132 PDF of the recorded measurements (for pre- and after entrainment conditions) which
133 is achieved non-parametrically using Kernel Density Estimates (*KDE*).

METHODS

Flume experiments

Initial laboratory experiments used a prototype sensor designed specifically for flume deployment. This prototype consists of a wireless mote platform deployed with a 3-axis accelerometer with a measurement range of $\pm 4g$ precisely located at the centre of mass of the particle. The electronics were enclosed in a spherical enclosure of $111mm$ diameter and the overall assembly weighed $1.43kg$ (Maniatis et al. 2013), giving a density of $2383kg.m^{-3}$, which is within the range of natural materials. Higher density can be achieved by adding removable weights to voids designed within the case (Maniatis et al. 2013). The prototype was tested in a series of experiments in the $6m \times 0.6m$ ($L \times W$) recirculation flume in the Mountain Channel Hydraulics Experimental Laboratory (MCHHEL), University of British Columbia. The scope of the experiments was to make a first evaluation of the 3D inertial acceleration measurements from the prototype sensor under varying flow and slope conditions.

We constructed an idealized bed of hemispheres (Figure 1) of the same diameter as the the sensor ($111mm$) using the rapid prototyping technique described in Maniatis et al. (2013). The result was a bed topography with saddle and grain-top positions that approximated the model described in Kirchner et al. (1990). In each experiment, the sensor was placed in a saddle position (position A in Figure 1) and the flow initiated from zero with a steadily increasing rate of $0.014l.s^{-2}$ up to a maximum rate of $6.25l.s^{-1}$ which was reached after 446 seconds. Upstream of the bed of hemispheres, large sediment particles generated fully turbulent flows which approximated uniform conditions (equal water depth along the length of the flume) over the full range of discharges used. Flow velocities were monitored using a *Vectrino II* Acoustic Doppler Profiler (Craig et al. 2011) for the discharges where flow depths were sufficient to permit the placement of the probe to be far enough from above the bed to derive measurements. The experiments were designed to observe the following series of processes:

- 160 • Entrainment from the saddle position and subsequent deposition in the grain top
161 position (position B in Figure 1). As well as providing information on movement
162 from saddle positions, this step removes bias from the placement of the sensor, since
163 the deposition in the second grain top position can be treated as natural with minor
164 random variations in position and pebble orientation.
- 165 • A second entrainment of the instrumented particle which transports the sensor out
166 of the monitoring area.

167 At each of four different flume slopes, 0.026, 0.037, 0.044 and 0.057, nine repeat experi-
168 ments were undertaken giving 36 experiments in total. The inertial acceleration of the sensor
169 was monitored at a frequency of $4Hz$.

170 **Experimental limitations**

171 Our sensor has idealised physical characteristics compared to a natural coarse particle,
172 and the sensor was only subjected to shallow flows (up to $0.16m$) and velocities up to
173 $0.37m.s^{-1}$ (Figure 2), such that, except at the highest flows at the lowest slope (0.026),
174 flow depth was less than sensor diameter. When using the data from these experiments the
175 assumptions and the errors inherent to the sensing process need consideration.

176 The sensor has been developed for natural environments and the testing of relevant signal
177 transmission and wireless communication technologies was crucial for the evaluation for the
178 prototype. Hence for this prototype a sampling rate of $4Hz$ was used, the maximum sampling
179 rate at which acceleration data could be reliably transmitted over the underwater radio.

180 The optimal sampling frequency for entrainment of large grains in turbulent flows is less
181 than the frequency for characterising flow turbulence due to the lower sensitivity of particle
182 movement to micro-turbulence. However, the system must record particle dynamics at sam-
183 pling rates that reflect the physical meaning of the derived inertial measurements. Although
184 previous experiments with artificial pebbles have used very high sampling frequencies for
185 both inertial (e.g. Šolc et al. (2012)) and turbulence (e.g. Valyrakis et al. (2013)) mea-

186 surements, over-sampling involves redundant data storage and may lead to artefacts in the
187 data. The optimal sampling frequency for particle motion can be defined with reference
188 to the velocity of the particle. Assuming maximum displacement velocities of the order of
189 100 diameters per second, as demonstrated in experiments for particles of smaller diameter
190 (Ancey et al. 2002), we suggest a target sampling frequency of about $> 50Hz$.

191 To enhance the analysis by isolating the impact of the flow, we report the absolute
192 total acceleration after gravity compensation (CA) which is the total acceleration minus the
193 acceleration due to gravity (Equation 2):

$$194 \quad A = \sqrt{a_x^2 + a_y^2 + a_z^2} \quad , \quad CA = A - |g| \quad (2)$$

195 Where A is the magnitude of total acceleration, a_x , a_y , a_z are the accelerations recorded
196 along the x , y and z axes respectively, CA is the acceleration norm after gravity compensation
197 and g is the acceleration norm due to gravity ($9.81m.s^{-2}$ or $1g$). From this point, the term
198 absolute acceleration (acceleration norm) refers to the absolute total inertial acceleration
199 after gravity compensation (CA) as described in Equation 2.

200 More accurate compensation for gravity is possible, with the monitoring of rotation and
201 the removal of the gravitational effect from the axis parallel to the gravity vector (Nagrath
202 et al. 2008). This form of compensation was not possible using our current sensor which did
203 not contain a gyroscope and magnetometer, but its absence does not affect the calculation of
204 the absolute compensated acceleration value. The addition of a gyroscope would also enable
205 comparison with evidence for directional entrainment from saddle positions (Chin and Chiew
206 1993).

207 Another important implication of not measuring angular velocities is the inability to inte-
208 grate the accelerations in order to derive velocities, which has two aspects. Firstly, although
209 the noise threshold was identified during (manufacturer proposed) calibration and subtracted

210 from the measurement, inaccuracies remain and are highly relevant to the sensor's response
211 to gravity (Woodman 2007). Secondly, in the field of electrical engineering the error accu-
212 mulation in MEMS based IMUs is one of the most intensively researched problems (Zekavat
213 and Buehrer 2011). It is known that, without a restriction of the degrees of freedom of the
214 motion, the error propagation during integration makes the velocity (and the displacement
215 results after two integrations) unusable. To access the "velocity response" of the sensor the
216 measurement of angular velocity is necessary and we address this in a subsequent paper.

217 Finally a combination of restricted sampling frequency and absolute gravity compensation
218 leads to a masking of the pre-entrainment conditions during the statistical treatment of the
219 signal. More specifically entrainments and pre-entrainment motions occur in the same time-
220 window when the signals from individual experiments are synchronized (Figure 3). This is
221 an artefact of the data-processing in order to increase the confidence on the magnitude of
222 the recording dynamics. An other type of analysis that includes advanced filtering of the
223 individual signals (see Section 2) of higher frequency is needed to estimate accurately the
224 fluctuation of pre-entrainment dynamics.

CA and the fundamental forces in a fluid flow

Gravity compensation is important since, for the inertial frame of the sensor, gravity is fictitious force. After removing the fictitious forces from the accelerometer measurements we are left with the linear coordinate acceleration, which is the acceleration that mobilises the sensor relative to the bed (or the Eulerian frame of the flow if an explicit description is required). As a result the CA represents the magnitude of the resultant force, the (3D-tensor) force generated on the centre of mass of the particle when the force balance is disturbed.

The above can simplify significantly the mathematisation of particle entrainment. Using recent definitions of the force balance on a spherical particle that is exposed to a fluid flow (Bialik et al. 2012), the resultant force is given by:

$$\Sigma F = F_D + F_L + F_M + W_{sub} \quad (3)$$

Where F_D and F_L are drag and lift generated forces, respectively, F_M is the added mass force and W_{sub} is the submerged weight of the particle. For a parametrisation of these forces see Bialik et al. 2012. Interestingly, the CA acceleration parameterises directly the left part of Equation 3. $\Sigma F = ma$, where m is the mass of the particle and a is the acceleration tensor applied on the centre of mass of the particle. If an ideal accelerometer (without noise) was placed precisely on the centre of the mass of the exposed particle then we could write $CA = |a|$ since all the forces are non-fictitious. Unfortunately real accelerometers are not ideal and this is why, in this work, we choose to treat the acceleration measurements statistically.

Categorisation and summary of Total Acceleration (*CA*)

Pre-entrainment position grouping

The experiments produced two different modes of movement. For the two lowest slopes (0.026 and 0.037) initial entrainment from the saddle position was followed by settling in a grain top position, where the sensor remained until entrained for a second time. On the contrary, for the two higher slopes (0.044 and 0.057), although entrainment from the saddle position was clearly recorded, the sensor did not remain stationary in the grain-top position for sufficient time prior to its second, grain-top, entrainment to allow isolation of pre-entrainment conditions.

For the following analysis entrainments from the saddle position for the low slopes are omitted and data are grouped in two limiting cases:

- high resistance to entrainment (low slope, grain-top position), entrainment from the grain-top position for the lowest two slopes;
- low resistance to entrainment (high slope, saddle position), entrainment from the saddle position for the two higher slopes.

This grouping avoids inconsistent comparisons and allows investigation of a wider range of pre-entrainment dynamics. Note that for the lower slopes, entrainment from the saddle position was identified from the data as an orientation change on the acceleration vector (a_x , a_y , a_z). In the total acceleration signals reported here orientation changes are masked. A representative signal at the entrainment point from an individual experimental run for one slope (0.026) and grain top position is shown in Figure 3a. Note that in this individual signal a pre-entrainment wobbling is also recorded c.10sec before the entrainment event.

Derivation of aggregated time series for each slope

The acceleration time series from each individual run were synchronised with the corresponding flow rate curve (Figure 3b). Instead of approximating the underlying process for individual runs, the individual acceleration signals for each slope have been aggregated.

271 For a time domain t_0, \dots, t_n the acceleration signal is defined by a series of A_0, \dots, A_n absolute
272 acceleration values. If k is the number of experiments for one slope the summed acceleration
273 signal is given by the set:

$$274 \quad A = \{\{A_1, \dots, A_k\}_{t_0}, \quad \dots, \quad \{A_1, \dots, A_k\}_{t_n}\} \quad (4)$$

275 The above formulation states the full range of absolute accelerations recorded in each of
276 k repeat experiments ($k = 9$ in this case) for each time point, hence discharge value (Figure
277 3c). The analysis of the aggregated signal has two benefits compared to individual signal
278 analysis:

- 279 • given that our analysis is purely statistical all the approximations are performed with
280 a larger input sample of accelerations which increases statistical confidence and thus
281 the significance of the results; and,
- 282 • the resulting individual time series for each slope is more representative of a raw signal
283 derived in a natural environment, extending the application range of the presented
284 method.

Analysis of Absolute Acceleration (CA) : Statistical Techniques

Dynamic Linear Model Filtering

The aim here is to approximate the underlying dynamical process for each slope by analysing the combined acceleration signal. Space-state estimation techniques for time series analysis are commonly used (Box et al. 2013). Here we follow (Zhang and Li 1996) and use a recursive algorithm for space state estimation to enhance numerical stability and the square root version of the Kalman filter (Kalman 1960). We used these algorithms as coded for the R-statistical software by (Petris 2010).

The summed time series is approximated with a first order polynomial model of the form:

$$\begin{cases} y_t = \theta_t + v_t, v_t \sim N(0, V) \\ \theta_t = \theta_{t-1} + \omega_t, \omega_t \sim N(0, \Omega) \end{cases} \quad (5)$$

where y_t is the vector of observed absolute accelerations after gravity compensation (CA). θ_t is a vector which represents the underlying process (the state) of the system. The observation vector is related to the process vector with the addition of Gaussian noise with variance V ($\sigma^2 = 0.2$). Similarly, the process vector, Ω , is defined as its preceding value with additional covariance ($\sigma^2 = 0.3$). The estimation of these variances is based on the calibration of the sensor under zero (non-gravitational) acceleration conditions. The recursive algorithm of Zhang and Li (1996) is then applied to compute the filtering distribution and its variance for each one of the observations. Finally, the variances are used to calculate probability intervals as shown in Figure 3c.

Note that the combination of Equation 4 with the linear Kalman filtering of the model of Equation 5 should not be confused with the Ensemble averaging presented in other works (eg. Fathel et al. 2015). Here we only group the derived synchronised accelerations (CA) and this aggregation does not represent an Expected Value or any other function.

The Kalman filter only approximates the process given the overall time series and the

310 relative variances we introduced from calibration. As a result the difference between the
 311 individual entrainment of Figure 3a and the approximated entrainment of Figure 3c is ex-
 312 plained by the fact that more entrainments happened at a later time in the aggregated data.
 313 Characteristically for the slope discussed in Figure 3 (0.026) the individual entrainments
 314 occurred in a range between between 20 and 48mg of CA as shown below (Figure 7a).

315 *Logistic Regression Calculation of the Probability of Entrainment*

316 The probability of entrainment is assessed using the derived signals for each individual
 317 run, to which a binary categorical variable was attributed with pre-entrainment and post-
 318 entrainment states (0 and 1, respectively). This allows the recorded accelerations to be
 319 grouped according to grain condition and for calculation of separate probability densities for
 320 each condition (Figure 4a). Note that as explained previously at the lower slopes, the mea-
 321 sured entrainments are from grain-top positions following initial displacement from saddle
 322 positions, whereas at higher slopes entrainment was from the saddle positions (Figure 6).
 323 The point of entrainment is shown in the derived signals as a sudden drop in the inertial
 324 acceleration, followed by high acceleration values due to impacts of the sensor with the flume
 325 bed down-stream of the monitoring area (Figure 3c). In two cases of ambiguity (for slope
 326 0.57) the time of entrainment was verified from video recordings.

327 To calculate the probability of entrainment we fitted a binomial model using logistic
 328 regression (Hosmer et al. 2000) between the binary variable that corresponds to entrainment
 329 and the accelerations derived after the space-state filtering of the time-series. As a generalized
 330 linear model, logistic regression for binomial data is expressed with the linear predictor
 331 function:

$$332 \quad \text{logit} = \left(E \left[\frac{Y_i}{n_i} \mid X_i \right] \right) = \text{logit}(p_i) = \ln \left(\frac{p_i}{1 - p_i} \right) = \beta X_i \quad (6)$$

333 where Y_i is the dependent binary variable with:

334
$$Y_i \sim B_{in}(n_i, p_i) \quad \text{for } i = 1, \dots, n \quad (7)$$

335 X_i being the vector of predictors and β being the vector of regression coefficients (Kay
336 and Little 1987).

337 In this case the model was reduced to one predictor variable, equal to the filtered acceler-
338 ation values. As an optimisation process to estimate the probabilities p_i and the regression
339 coefficients β , we implemented the Maximum Likelihood Estimation in the R-statistical pack-
340 age for the default function *glm* (Faraway 2005). The values fitted by the above process are
341 an explicit calculation of the probability of entrainment as a function of the recorded inertial
342 acceleration. The threshold of entrainment is determined by the acceleration corresponding
343 to 0.5 probability as shown in Figure 4b. The determination of the entrainment threshold as
344 a probability of 0.5 is consistent with other applications of probabilistic entrainment criteria
345 that utilize near bed turbulence measurements (Papanicolaou et al. 2002). The acceleration
346 values corresponding to probabilities < 0.5 represent dynamic conditions that act in favour
347 of the resistance of the particle to entrainment, whereas values corresponding to probabilities
348 ≥ 0.5 represent dynamic conditions where the potential for entrainment is enhanced.

349 **A statistical framework for the evaluation of incipient motion criteria**

350 *Rationale*

351 The probabilistic derivations for incipient motion differ in terms of the physical definition
352 of the conditional probability that defines the threshold of motion. In the initial framework
353 of Grass (1970) the conditional probability is expressed as a function of Shields' shear stress,
354 while in recent derivations the same probability is physically determined by point (eg. Pa-
355 panielolaou et al. (2002)) or streamwise (eg. Bottacin-Busolin et al. (2008)) flow velocities
356 (Equation 1). Furthermore, here we propose a new physical definition based on the inertial
357 dynamics of the target particle.

358 However, one observation is relevant to all the studies, including this one: the definition

359 of a non-abrupt threshold implies that the the measured physical instance (shear stress, flow
360 velocity or acceleration) is defined by two *PDFs*. One of these defines pre-entrainment
361 conditions (mainly representing the resistance to movement of the particle) and one defining
362 the post-entrainment conditions (mainly recording the mobilisation of the particle). This
363 is a representation in an inertial frame of reference of the idea introduced by Grass (1970)
364 in the form of overlapping *PDFs*, and it implies that the critical point for entrainment lies
365 within the area of overlap of these two distributions. This has been demonstrated in much
366 later work, regardless of the physical definition of this probability that was used in each case.

367 In a probabilistic context, the degree of overlap between the pre- and post-entrainment
368 *PDFs* defines the domain of the critical point, hence the exactness of the entrainment
369 criterion. A large overlap of the two distributions suggests a large domain for the threshold,
370 and is thus a less well-defined criterion. A smaller overlap shrinks the domain for the
371 threshold and the derived criterion is better defined. As a result any improvement in the
372 definition of incipient motion thresholds can be quantified by the degree of overlap of the
373 pre- and post-entrainment distributions.

374 A formal measure for the overlap between two *PDFs* is the Overlapping Coefficient
375 (*OVL*) initially proposed by Weitzman (1970). *OVL* has been used since to quantify the
376 degree of overlap for a range of distributions, from theoretical normal distributions (Inman
377 et al. 1989) to empirical density functions (Schmid and Schmidt 2006; Clemons and Bradley
378 2000) which are directly relevant to the analysis presented in this work. Since the Kernel
379 Density Estimation (*KDE*) of the *PDFs* is an important step of the analysis, we discuss
380 this before the definition of *OVL*.

381 *A note on the non-parametric estimation of PDFs*

382 The *KDE* is an established technique for the approximation of *PDFs* of random vari-
383 ables, when no hypothesis can be established for the underlying distribution (non-parametric).
384 Full description of this technique is outside of the scope of this work, however it is necessary
385 to introduce it from the point of application for the coherence of the presented analysis.

386 We performed the approximations using the default routines implemented in the R-
387 statistical software which are based on the Fast Fourier transform of the kernel estimator
388 introduced by (Rosenblatt 1956). The basic algorithm was derived by Silverman (Silverman
389 1982; Silverman 1986) for Gaussian kernels, which is also the type of kernel we chose for
390 *KDE* in this paper (the default in R-statistical software).

391 The *KDE*, like all smoothing techniques, requires the selection of bandwidth. Numerous
392 automatic bandwidth selectors have been devised (see Heidenreich et al. (2013, Sheather
393 et al. (2004)) however they do not all perform equally well (Park and Turlach 1992).

394 To highlight this effect, we use two bandwidth estimators:

- 395 • for display purposes and to derive simple inferences about the data (Figure 4) we use
396 Silverman’s rule of thumb (Silverman 1986) which tends to over-smooth the data 5.
- 397 • for more accurate calculations, such as to calculate the OVL coefficient, we use the
398 data-based method proposed in Sheather and Jones (1991), which for the variability
399 in our data gives more representative approximations (Figure 5).

400 Both of these methods are options of the default library of the R-statistical software with
401 Silverman’s rule of thumb being the default method.

402 *The Overlapping coefficient (OVL)*

403 After the approximation of the *PDFs* the Overlapping Coefficient is calculated as:

$$404 \quad OVL = \int_{R_n} \min[f_1(x), f_2(x)] dx \quad (8)$$

405 where $f_1(x)$ and $f_2(x)$ are two overlapping *PDFs* and R_n is the n-dimensional space of
406 real numbers.

407 The *OVL* coefficient is always in the range $[0, 1]$ and complete overlap between $f_1(x)$ and
408 $f_2(x)$ has $OVL = 1$, while complete distinction gives $OVL = 0$ (Clemons and Bradley 2000).
409 In the context of evaluating entrainment criteria we are looking for *OVL* closer to 0 which

410 would suggest smaller overlap between the pre- and post-entrainment *PDFs*.

411 More specifically if we accept that the threshold of entrainment exists in the region where
412 pre-and post entrainment distributions overlap, then the exactness (and the significance) of
413 the threshold is related to how different the two distributions are. If the distributions were
414 normal and had the same variance the difference of the distributions would be approximated
415 by the separation of the means. The OVL coefficient quantifies this difference for empirically
416 approximated distributions.

417 A smaller OVL means that the entrainment threshold is better defined. More precisely
418 an $X\%$ reduction of the OVL coefficient represents the maximum $\%$ reduction of the variance
419 of the approximated threshold.

420 **RESULTS**

421 **Absolute Inertial Acceleration (*CA*) thresholds**

422 The methods described above for the acceleration analysis were applied to data for all
423 the slopes. Figure 6 shows the filtered acceleration signals and the fitted probabilities of
424 entrainment: these results are summarised in Table 1.

425 As slope increases the discharge at which entrainment occurs is reduced (Figure 6). How-
426 ever, inertial accelerations recorded by the sensor show a more complex pattern (Figure 6b).
427 For the lower slopes with final entrainment from grain top positions, there is considerable
428 overlap between accelerations at entrainment with the higher (0.037) slope having the high-
429 est accelerations. The steeper slopes, with entrainment from saddle positions, also show
430 considerable overlap but accelerations are significantly lower than for the lower slopes.

431 As a statistical evaluation for the derived binomial models (Figure 6b), the p-values
432 for the significance of the coefficients of the independent variable (acceleration, Table1) are
433 given. Another relevant metric is the Walden test which is used for the evaluation of single
434 predictor models, but also to evaluate competing models with different numbers of predictors.
435 The p-value of the Walden test for the four fitted probability models was $< 1 \times 10^{-20}$ which

436 increases our confidence regarding the significance of the derived models (Montgomery and
437 Runger 2010).

438 Measured inertial accelerations at the point of entrainment were of the order $50mg$ (Fig-
439 ures 3 and 6). The acceleration of mean velocity as a result of the steady increase in discharge
440 through each experiment is four orders of magnitude lower than these inertial accelerations
441 at $c.2 \times 10^{-2}mg$, justifying the assumption of gradually varied flow that has no direct influence
442 on entrainment forces.

443 **Comparison of Total Inertial Acceleration *PDFs* with Velocity *PDFs***

444 Figure 7 shows how all the derived signals are synchronised over the same time domain.
445 For the two lower slopes (0.026 and 0.037) the approximated acceleration thresholds (44
446 and $51mg$, respectively) were projected back to Threshold Discharges (6.15 and $4.3l.s^{-1}$,
447 respectively). The latter were used to separate the recorded flow velocities (Figure 2) to pre-
448 and post-entrainment distributions the *PDFs* of which were approximated by *KDE* and the
449 bandwidth selection technique of Sheather and Jones (1991). Finally the *OVL* coefficient
450 was calculated for both the velocity and total acceleration *PDFs*

451 The results in Figure 7 show that for both of the slopes the *OVL* coefficient for the
452 acceleration *PDFs* (0.36 for $slope = 0.026$ and 0.21 for $slope = 0.037$) is smaller than the
453 *OVLs* for the velocity *PDFs* (0.44 for $slope = 0.026$, and 0.33 for $slope = 0.037$).

DISCUSSION

Evaluation of applied techniques

Filters based on a Dynamic Linear Model have many advantages over traditional time series regression analysis as they can be applied without the associated assumptions of stationarity. Another important advantage is that the filtered signal corresponds to exactly the same time domain as the unfiltered series (which is not the case when some other techniques, e.g. moving average, are applied). The latter point becomes crucial since the entire time domain along with the space state characterisation of the process make the attribution of categorical variables to each one of these states both feasible and conceptually consistent. Similarly, logistic regression is a versatile technique that can be applied without the strict assumptions of linear regression and becomes very useful when categorical characterisation of states is necessary (e.g. Entrainment-No Entrainment).

Our results suggest that current technology (inertial-sensors) permits the modification of equation 1 to a form of:

$$P_E = P(A_f > a_f = a_g) = \int_{a_g}^{\infty} f_{A_f}(a) da = 1 - F_{A_f}(a_g) \quad (9)$$

where A_f is a random inertial acceleration variable for an individual pebble, represented by a cumulative distribution F_{A_f} with a probability density function of f_{A_f} , and a_g is the threshold acceleration for grain entrainment as approximated statistically in the current work. This derivation has the potential to enhance the accurate determination of P_E as it utilises the explicit dynamics of the particles being entrained instead of using implicit flow-related metrics.

Here a clarification is necessary; the fact that the above criterion is explicit does not mean that we treat the entrainment process in a non-stochastic framework. This observation is highly relevant to the use of the proposed criterion and methods under different hydrody-

479 namic conditions (eg. uniform vs non-uniform flow). There is a range of hydrodynamics that
480 can produce the same threshold of inertial acceleration (or more specifically the range of in-
481 ertial accelerations where entrainment can occur). However, this range of hydrodynamics
482 corresponds to a smaller range of inertial dynamics. This is reflected in the definition of iner-
483 tial acceleration (Section 2), and results in the approximated acceleration thresholds varying
484 in a range of only $19mg$ regardless of the distinctively different force-balance conditions
485 (slopes and initial placement). Consequently data collected under different hydrodynamic
486 regimes will improve the determination of the inertial threshold and connect it with previous
487 results.

488 The technique that we propose for the comparison of this inertial acceleration-based
489 threshold with a flow velocity based prediction (*OVL* coefficient), suggests that the overlap
490 of pre- and post-entrainment distribution is reduced by c.10%. Moreover, the reduction
491 is greater for the higher slope where the variability of the hydrodynamics is greater as
492 demonstrated from the *PDFs* of Figure 7. Based on this result, it is possible to form
493 the hypothesis that, for medium - large grain scales, the inertia of the particles exerts a
494 more significant control on their motion than flow generated forces (Bathurst 1985). It
495 also important that other geomorphological characteristics can be described by overlapping
496 distributions of dynamics (eg. Ze'ev and Schumm (1984)), which extends the applicability
497 of the *OVL* coefficient beyond the detection of incipient motion.

498 As a result, the study of inertial dynamics of the sediments has the potential to improve
499 prediction across the modes of sediment transport. Formalisation of statistical definitions
500 of entrainment can lead to further improvements to the conceptual model introduced by
501 Grass (1970) since new technologies enable dynamical measurements at high frequency and
502 accuracy.

503 Further study of the proposed criterion under varied conditions is required and is likely to
504 reveal a range of types of behaviour dependent on the same issues which lead to variability in
505 the definition of Shields' criterion (Buffington and Montgomery 1997). However, the range

506 of behaviour may be better constrained as actual forces are directly measured rather than
507 being inferentially related to measurable parameters such as grain shape and protrusion.
508 The approach therefore has the potential to lead to a general inertial force -based equivalent
509 of Shields' diagram which will not be restricted by the assumption of uniform flow (or any
510 other flow characterisation) and will have broad applicability.

511 **Future work**

512 A new prototype sensor (of diameter $< 80mm$) is under development, instrumented with
513 all the sensors required for full determination of inertial dynamics (accelerometer, gyroscope
514 and a magnetometer which contributes an extra constant reference axis). Reducing the size
515 of the overall unit is crucial for increasing the range of pebble sizes and shapes which we
516 can be tested, either by reducing the diameter of spherical pebbles, or with pebbles with
517 non-unity aspect ratios with one dimension smaller than $111mm$ (the new sensor will be
518 able to be housed in non-spherical casings, which extends its generality). This sensor will be
519 capable of higher frequency (up to c. $100Hz$) sampling allowing pre-entrainment and motion
520 dynamics to be recorded coherently in $3D$ space.

521 Finally field deployment of the sensor will contribute to a better description of all the
522 stages of sediment transport (Entrainment -Translation- Deposition). Currently the technol-
523 ogy permits the construction of robust enclosures that, in terms of physical characteristics,
524 are mainly relevant to the sediment sizes found in upland streams, debris flows and some
525 gravel beaches.

CONCLUSION

We provide a new method to approximate the probability of entrainment for individual coarse particles based on inertial acceleration measurements (Maniatis et al. 2013). This became possible after prototyping a purpose specific I.M.U. The key steps of the method are:

1. Recording of inertial dynamics at an appropriate high frequency (inertial acceleration): in this initial study, data were recorded at 4Hz for practical reasons and this has been shown to be adequate for laboratory conditions. For field deployment, recording frequencies of over 50Hz are required, although over-sampling should be avoided to ensure both efficiency and reliable interpretation of results.
2. Bayesian filtering of the derived signals (Kalman Filter): we conducted 9 replicates of each entrainment experiment. Rather than analyse each separately, combining these into one synchronised data set allows robust interpretation and specification of statistical uncertainty in the results using an appropriate process model (Equation 5). This model illustrates both the inter-experiment variability and the trends in the data (Figures 3 and 6).
3. Categorical characterisation of the filtered signals for pre- and post-entrainment conditions: the individual data sets (e.g. Figure 3a) show pre-entrainment vibration increasing through time, entrainment, and post-entrainment oscillations. Categorising these data into pre- and post-entrainment accelerations allows the probability of entrainment to be considered as a function of increasing flow intensity (approximated here by discharge).
4. Logistic approximation of the relationship: After analysing inertial accelerations pre- and post-entrainment, logistic regression provides a way of statistically expressing a gradual increase in the probability of entrainment with increasing accelerations. This also provides confidence intervals (Figures 4 and 6) which clarify the differences in entrainment processes between different grain positions (saddle vs. grain top) and

553 which also show considerable overlaps between data obtained at different slopes but
554 for the same grain positions. These differences suggest that secondary effects, such as
555 the orientation of initial grain movement, may be significant even for spherical grains.

- 556 5. Calculation of the probability of entrainment as a function of inertial acceleration
557 from the conditional threshold probability ($p_i = 0.5$): the notion of entrainment as
558 a gradational increase in probability of movement as flow intensity increases is well-
559 established (Grass 1970), but has been difficult to quantify for natural conditions.
560 Extension of our approach to natural particle shapes and positions will help to address
561 this data requirement.

562 The results support the implementation of explicit dynamical metrics with reference to
563 the inertial frame of the particle under entrainment. Further research is needed to expand
564 this type of entrainment criterion to a range of particle sizes and dynamical schemes.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

PDF = Probability Density Function

KDE = Kernel Density Estimates

OVL = Overlapping coefficient

P_E = probability of Entrainment

U_f = random near bed velocities (L/T)

F_{U_f} = cumulative distribution of U_f

f_{U_f} = Probability density function of F_{U_f}

A = Magnitude of total acceleration (mg);

g = Acceleration due to gravity ($1g$)

$|CA|$ = Absolute acceleration with gravity compensation (mg)

a_x, a_y, a_z = Accelerations on x, y and z axes respectively (mg);

y_t = Observation vector (Kalman Filter Definition)

θ_t = Unobserved vector (Kalman Filter Definition)

V, Ω = Observation and Evolution (process) covariances (Kalman Filter Definition)

p_i = Probabilities

Y_i = Binary variable (Logistic Regression)

X_i = Vector of predictors (Logistic Regression)

β = Vector of regression coefficients (Logistic Regression)

A_f = random total inertial acceleration (L/T^2)

F_{A_f} = cumulative distribution of A_f

f_{A_f} = probability density function of F_{A_f}

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TABLE 1. Summary Results

Slope	Threshold acceleration (mg)	Threshold Discharge ($l.s^{-1}$)	Type of position pre-entrainment	Significance of logistic regression model for the calculated probability
0.026	44	6.15	Grain-top	$< 2 \times 10^{-16}$
0.037	51	4.3	Grain-top	$< 2 \times 10^{-16}$
0.044	32	3.21	Saddle	$< 2 \times 10^{-16}$
0.057	25	2.2	Saddle	$< 2 \times 10^{-16}$

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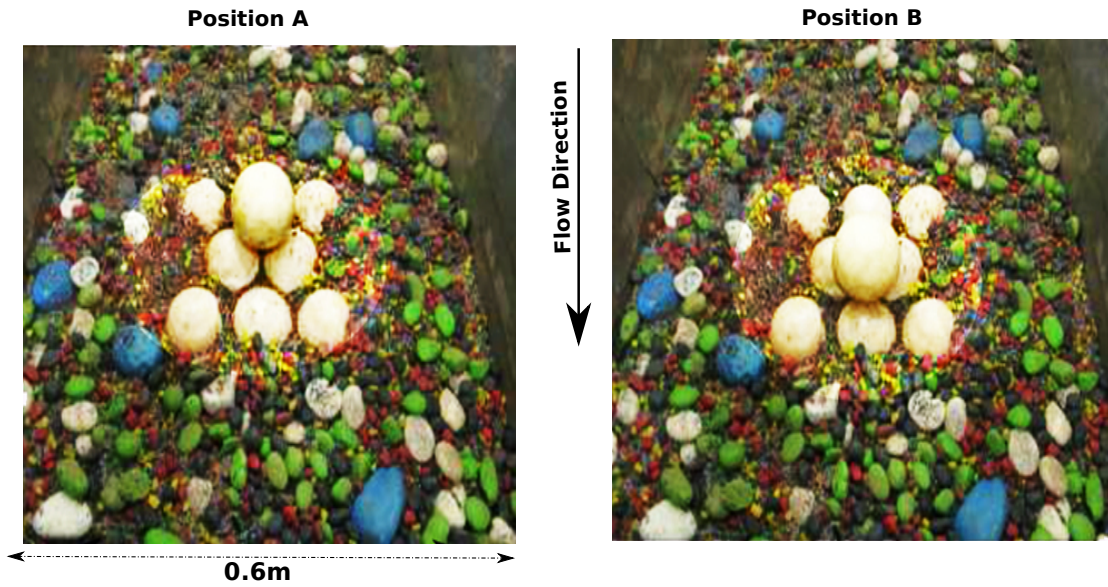


FIG. 1. Description of the experimental setting

The setting represents the theoretical model described in Kirchner et al. (1990) with two characteristic entrainment positions (saddle (A) and grain-top (B)). The physical differences of the sensor compared to natural sediment are its shape and its density. A sphere was used to enable robust determination of sensor dynamics in the inertial frame (requiring accurate definition of the center of mass and locations of points of contact) during calibration.

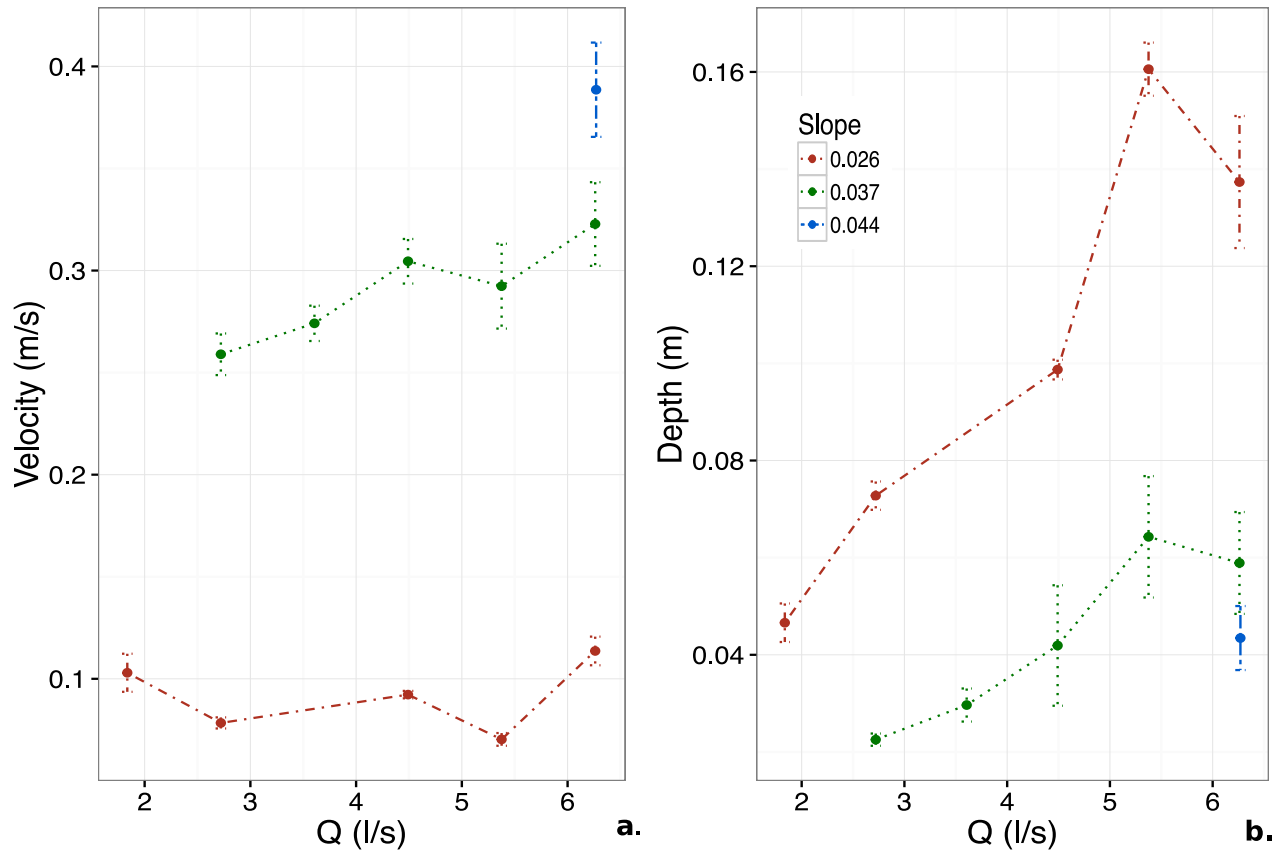


FIG. 2. Description of hydraulic conditions

Near bed flow velocities (a), measured c.0.5m upstream of the bed of hemispheres. Figure 2b. shows water depths at the measurement points on the bed of hemispheres in the absence of the sensor particle. This was possible for the lower two slopes (0.026 and 0.037) at a range of discharges. At slope = 0.044 velocities could only be measured for the highest discharge ($6.25 l.s^{-1}$) and depths were too low at slope =0.057 for any velocity measurements. In all cases the lower end of the probe was placed at a distance of 15mm from the bed. The calculated Froude number ($F = u/\sqrt{gd}$) was subcritical, in the range 0.57 to 0.71. The low depth:sediment diameter ratios mean that it was not possible to use a uniform flow approximation of the Shields stress (τ) for cross comparison with the inertial accelerations (Shields 1936). These low ratios also account for some variability in the responses of depth and velocity to increasing discharge as larger roughness elements were progressively drowned out.

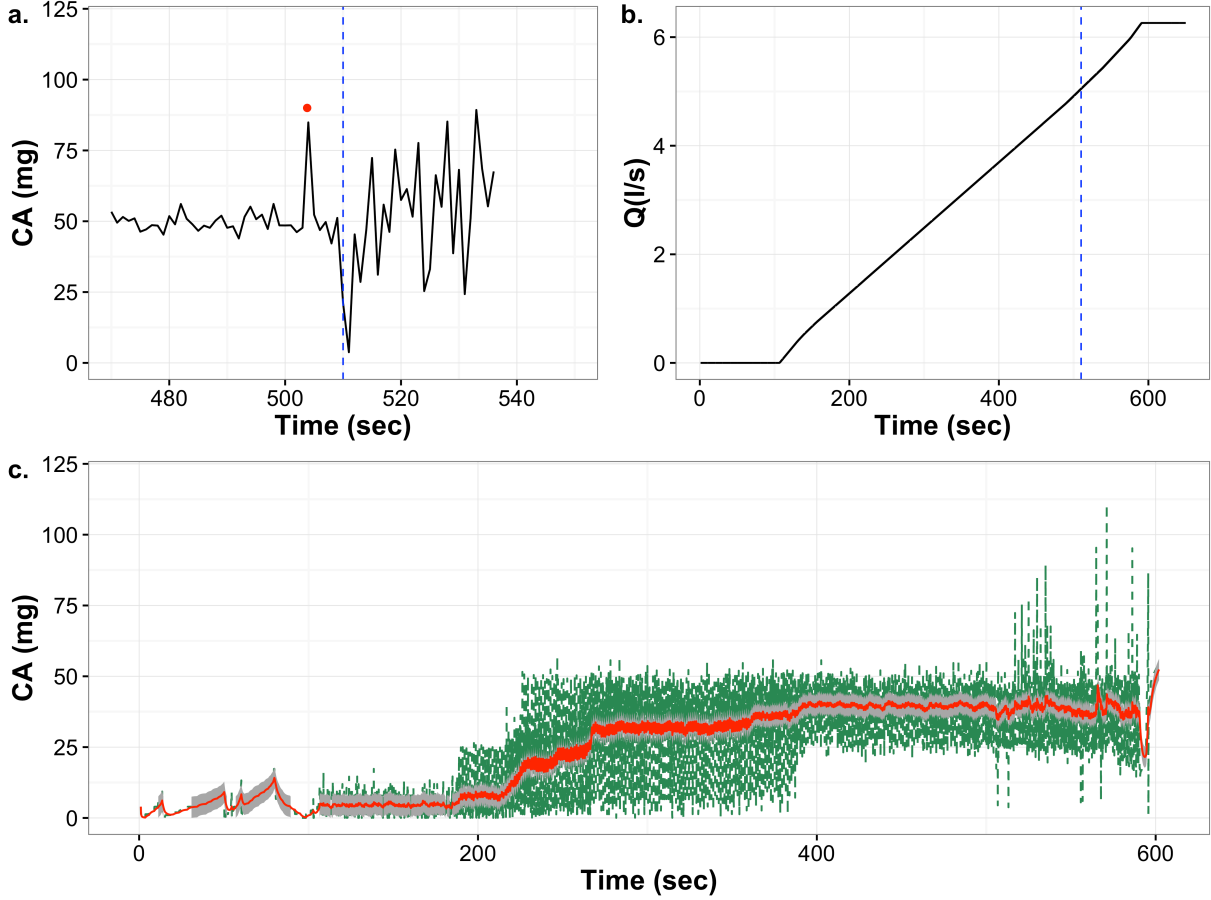


FIG. 3. Definition of the underlying entrainment process

Figure 3a shows the total absolute acceleration close to the point of entrainment from one run at slope = 0.026. The sampling frequency is $4Hz$ and the point of entrainment ($50mg$ at $t = 507sec$, blue-dotted line) is shown as a sudden reduction of the acceleration (dislodging) followed by variable smaller accelerations due to subsequent vibrations. The red dot indicates a distinct pre-entrainment vibration c.10sec before the entrainment event for this experiment. **Figure 3b** shows the corresponding flow increase (steady increase rate of $0.014l.s^{-2}$ up to a maximum rate of $6.25l.s^{-1}$ for all experiments) and the entrainment point from Figure 3a expressed as discharge ($5.17l.s^{-1}$ at $t = 507sec$, blue line). **Figure 3c** shows the summary signal derived by the process described in Equations 4 and 5 for all nine replicates with slope = 0.026. The red line is the underlying process as approximated after the application of the Kalman filter (Eq. 5). The grey band shows the process noise which is modelled as a Gaussian distribution with $\sigma^2 = 0.3$.

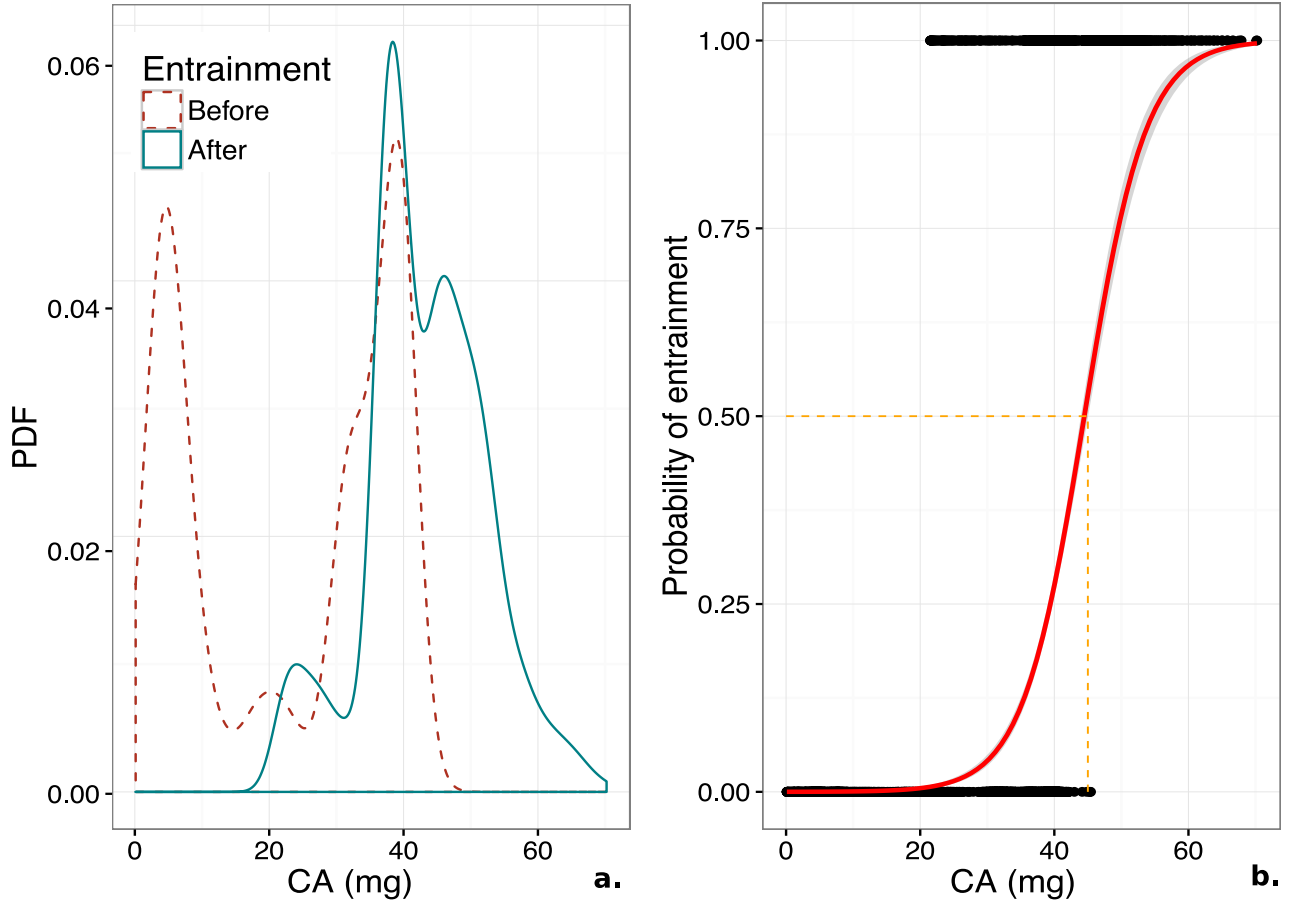


FIG. 4. Calculation of the probability of entrainment

Figure 4a shows the classification of acceleration according to the pre- and post-entrainment conditions ($slope = 0.026$). The probability density of the post-entrainment accelerations shows that the grain is subject to greater forces than pre-entrainment which is consistent with the experimental procedure. **Figure 4b** shows the calculation of the probability of entrainment after the application of the logistic regression model (Equations 5 and 6). The orange-dotted line indicates the acceleration threshold of entrainment ($44mg$ at 0.5 probability). The acceleration value corresponding to 0.5 probability is interpreted as the acceleration where the dynamics start to act in favour of entrainment. The grey band indicates the 95% confidence bands of the logistic regression model.

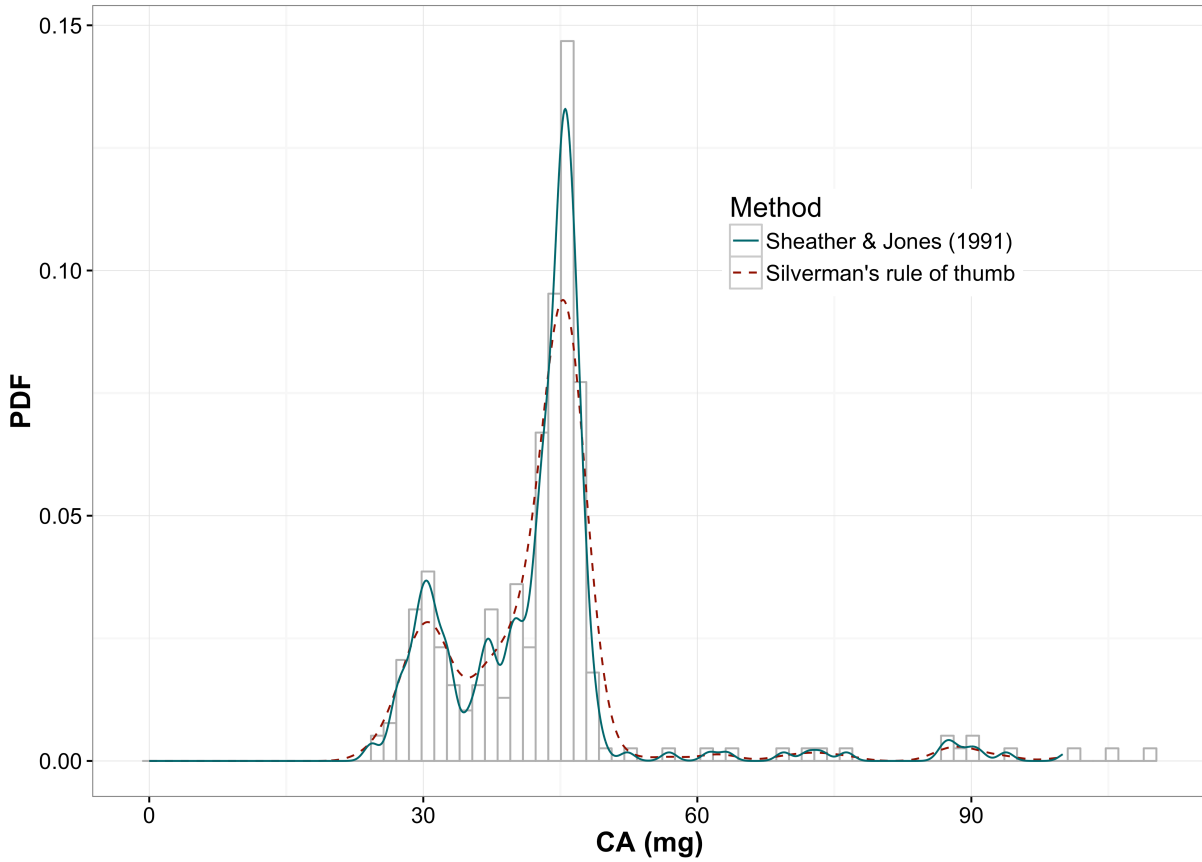


FIG. 5. KDE approximation for inertial acceleration

Figure 5 shows two *KDE* approximations for the absolute accelerations recorded after the entrainment point for $slope = 0.037$ (histogram). The technique proposed by Silverman (1986) is used for display purposes (Figure 4) while for the calculation of the Overlapping coefficient (Figure 7) the *PDFs* are approximated using the bandwidth selection method proposed in Sheather and Jones (1991).

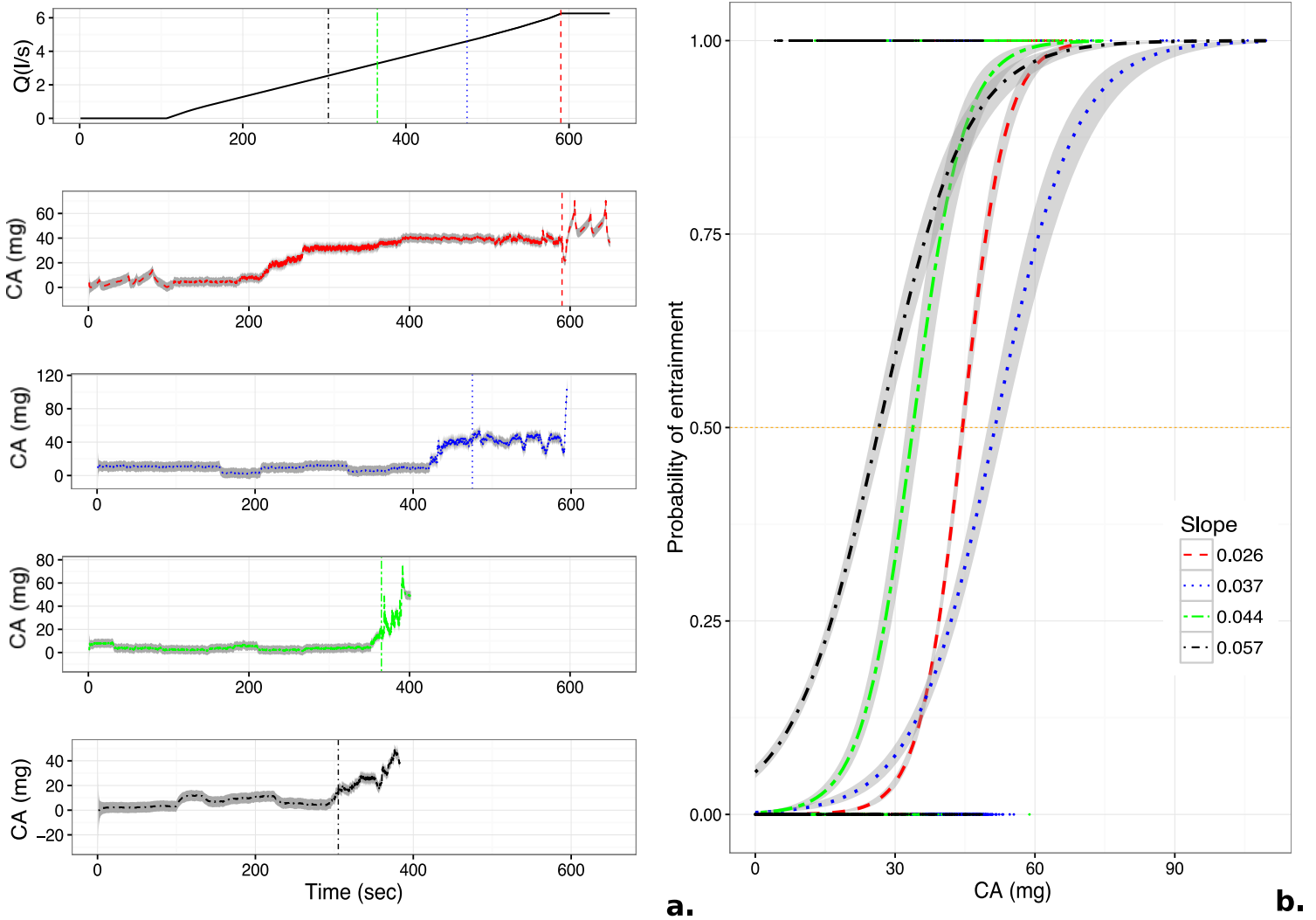


FIG. 6. Summary results

Figure 6a total inertial acceleration and the thresholds of entrainment (vertical lines). The same thresholds are synchronised with the corresponding flow increase which is the same for all experiments (top diagram). Slope changes are colour coded as in Figure 6b. **Figure 6b** shows the calculation of the probability of entrainment for all slopes by logistic regression. Grey bands indicate the 95% confidence bands.

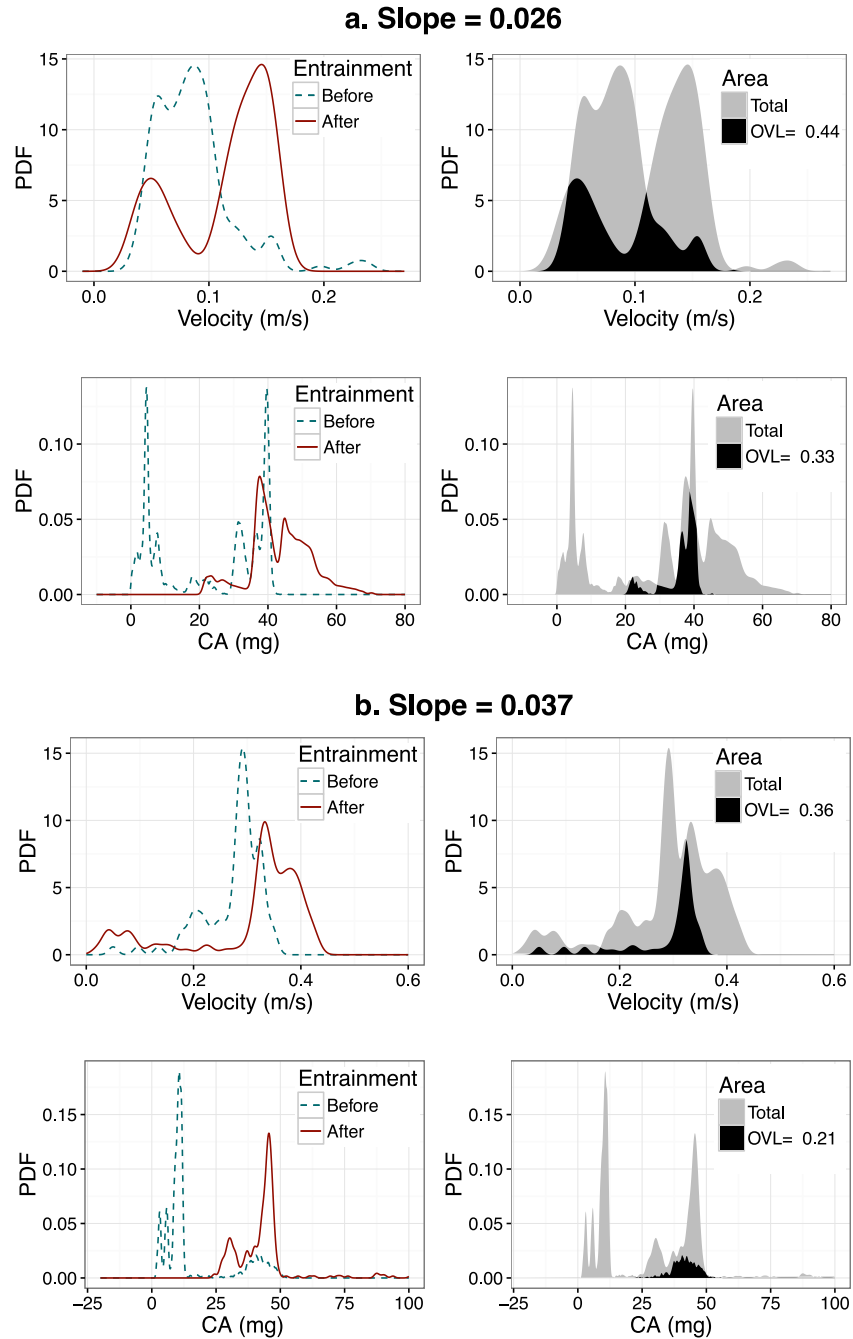


FIG. 7. Comparison of velocity and acceleration

Figures 7a and **b** show pre- and post-entrainment *PDFs* for two slopes (0.026 and 0.037). Plots on the left side are smoothed *PDFs*, and the right hand plots show the areas of overlap used to calculate *OVL* values. The calculation of the *OVL* coefficient suggests that an incipient motion criterion based on inertial acceleration has the potential to improve prediction as the overlap is reduced at an order of 10% in both cases.