# Loss-induced transition of the Goos-Hänchen effect for metals and dielectrics

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**Abstract:** We report a unifying approach to the Goos-Hänchen (GH) shifts on external optical reflection for metals and dielectrics in particular for the case of high losses, that is for a large imaginary part of the dielectric constant. In this regime metals and dielectrics have a similar GH shift which is in contrast to the low-loss regime where the metallic and dielectric forms of the GH shift are very different. When going from the low-loss to the high-loss regime we find that metals show a much more prominent transition; we present a condition on the dielectric constant which characterizes this transition. We illustrate our theoretical analysis with a realistic example of seven lossy materials.



**OCIS codes:** (260.3910) Metal optics; (240.0240) Optics at surfaces; (160.4760) Optical properties; (160.3900) Metals; (160.6000) Semiconductor materials.

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## 1. Introduction

The Goos-Hänchen (GH) shift is a displacement of a reflected beam of light if compared to an ideal, geometric reflection [1] and has been studied for a wide range of materials, including semiconductors [2, 3, 4], photonic crystals [5] and negative refractive media [6]. The GH shift has also been investigated for various different material compositions such as multi-layered and periodic structures [7] and finite width slabs [8]. In particular the GH shift at bare metal surfaces is a topical field of research [9, 10, 11] as it promises a simple system to study large, negative GH shifts.

In this paper we focus mainly on the GH shift at an air-material interface, where the material is isotropic and can be described by complex scalar dielectric constant $\varepsilon$. The sign of the real part of the dielectric constant $\mathrm{Re}(\varepsilon) = \varepsilon_r$ distinguishes a metal ($\varepsilon_r < 0$) from a dielectric ($\varepsilon_r > 0$), while the imaginary part $\mathrm{Im}(\varepsilon) = \varepsilon_i$ indicates the amount of loss in the material [12]. With the exception of [2] this distinction is also commonly seen in the literature where authors focus on either dielectrics or metals to study the GH shift. Within the context of the GH shift it is helpful to consider the ratio $\varepsilon_i/|\varepsilon_r|$ [4], and a medium is said to be ‘low-loss’ if $\varepsilon_i/|\varepsilon_r| \ll 1$ and ‘high-loss’ if $\varepsilon_i/|\varepsilon_r| \gg 1$. We also introduce the notion of ‘intermediate-loss’ to describe materials with $\varepsilon_i/|\varepsilon_r| \approx 1$. The previous work of the GH shift for metals [2, 9, 10, 13] focussed on low-loss metals, notably noble metals such as gold and silver at frequencies where $\varepsilon_i$ is relatively small. Dielectrics have been studied in the low-loss regime [3] as well as for intermediate losses [2]. It has been shown that the analytical expression for the dielectric GH shift obtained in the low-loss case also holds for intermediate-loss dielectrics with an acceptable error [4].

It is natural to make the distinction between metals and dielectrics in the low- to intermediate-loss regime, as the form of the GH shift for light polarized parallel to the plane of incidence (p-polarization) is rather different for the two cases (see GH curves in Fig. 1(a) calculated from Eq. (6) below); the dielectric shows a characteristic, isolated (negative) resonance [2] near the Brewster angle while the metal has a maximal (negative) GH shift at grazing incidence [9]. For light polarized orthogonal to the plane of incidence (s-polarization) the GH shift is usually much smaller and the distinction between a metal and dielectric is much less pronounced (see Figure 1(b) calculated from Eq. (7) below). This is why we focus mainly on the GH shift for p-polarized light. We will show in this paper that in the high-loss regime the difference in the GH shift for p-polarized light between a metal and a dielectric becomes much less sharp and that the two curves approach each other.

The transition from the low-loss case, where metals and dielectrics have a very different GH shift for p-polarized light, to the high-loss case, where both kinds of materials have a similar GH shift, is depicted in Fig 2. In this series of graphs we consider an artificial pair of a metal

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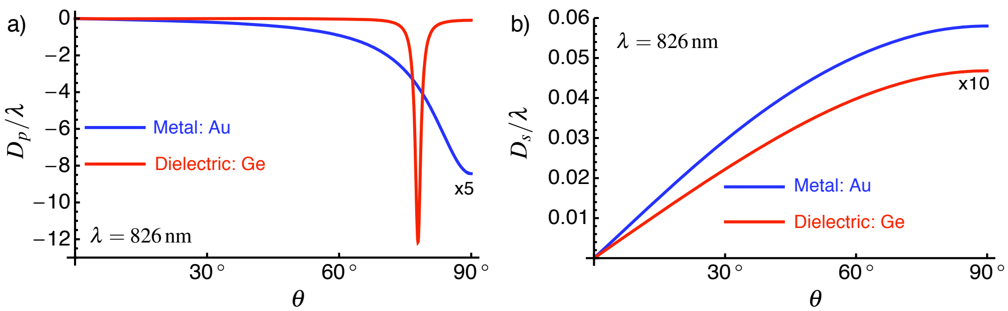


Fig. 1. Comparison of typical GH shifts $D_{p,s}$ for a low-loss metal (Au) and a low-loss dielectric (Ge) as a function of the angle of incidence $\theta$. The dielectric constants for the chosen materials are: $\varepsilon = -29.0 + 2.03\mathrm{i}$ (Au) and $\varepsilon = 21.6 + 2.77\mathrm{i}$ (Ge) at a wavelength of $\lambda = 826$ nm [14]. *a*): p-polarization: The curve for Au is 5-fold magnified. *b*) s-polarization: The curve for Ge is 10-fold magnified. The curves are calculated from Eqs. (6) and (7) below (for Au see also the experimental result in [10]).

and a dielectric which have a dielectric constant $\varepsilon$ with equal imaginary part $\varepsilon_i$ and a real part $\varepsilon_r$ of equal magnitude but opposite sign. One can clearly see that adding loss by increasing $\varepsilon_i$ causes the GH shifts for the metal to become similar to that of a dielectric. This is the main point of our paper and we will present the theoretical analysis on which these graphs are based in the following section. In the third section we will illustrate our findings with a realistic example of a series of 7 materials, 3 dielectrics and 4 metals, for which this transition can be observed. We will choose a wavelength for which all these materials are relatively lossy ($\varepsilon_i/|\varepsilon_r| > 1$) to study the GH shift far away from the low-loss regime.

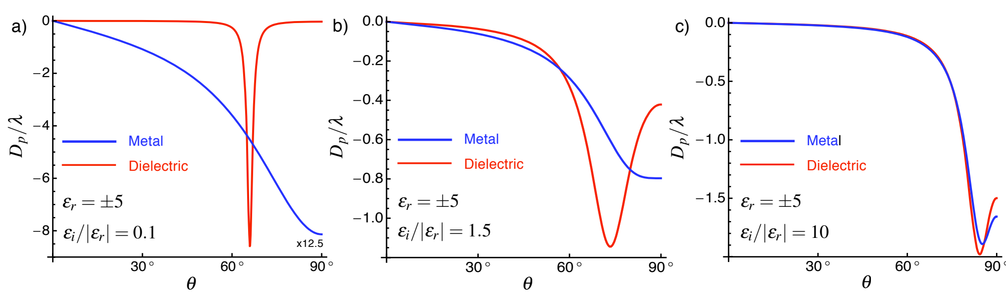


Fig. 2. Series of graphs comparing the GH shift $D_p$ as a function of the angle of incidence $\theta$ for the metallic and dielectric case on transition from the low-loss to the high-loss regime. We consider the artifical case of a metal and dielectric with a paired dielectric constant such that $\varepsilon_r$ is of equal magnitude but opposite sign and $\varepsilon_i$ is equal for the pair. The curves are calculated from Eq. (6) below.

## 2. Theoretical analysis

In this article we make use of an expression for the GH shift which was first derived by Artmann [15]. In Artmann's work the GH shift results from different phase shifts that the constituent plane waves in a decomposition of the incident beam experience on reflection [16, 17]. The GH shift can thus be expressed as

$$D_\mu = -\frac{\lambda}{2\pi}\frac{\partial \phi_\mu}{\partial \theta}, \tag{1}$$

where $\phi_\mu$ is the phase of the complex reflection coefficient $r_\mu$ with $\mu = s, p$ indicating the polarization of the light and $\theta$ is the angle of incidence. Artmann's formula has been applied in the past to explain the GH shift for metals and dielectrics alike [2, 3, 9, 13] but not for the high-loss case. The complex reflection coefficients for p- and s- polarized light are given by [18]:

$$r_p = \frac{\varepsilon\cos(\theta) - \sqrt{\varepsilon - \sin^2(\theta)}}{\varepsilon\cos(\theta) + \sqrt{\varepsilon - \sin^2(\theta)}}, \quad r_s = \frac{\cos(\theta) - \sqrt{\varepsilon - \sin^2(\theta)}}{\cos(\theta) + \sqrt{\varepsilon - \sin^2(\theta)}}. \tag{2}$$

Artmann's expression Eq. (1) can be elegantly rewritten in the following form which has originally been used by Wolter [13]:

$$D_\mu = -\frac{\lambda}{2\pi}\mathrm{Im}\frac{r'_\mu}{r_\mu}. \tag{3}$$

Here, the prime indicates the derivative with respect to $\theta$. Subsituting the expression for the reflection coefficient into Eq. (3) we find for the GH shifts:

$$D_p = -\frac{\lambda}{2\pi}\mathrm{Im}\left(-\frac{2\varepsilon\sin(\theta)}{\sqrt{\varepsilon - \sin^2(\theta)}(\varepsilon\cos^2(\theta) - \sin^2(\theta))}\right), \tag{4}$$

$$D_s = -\frac{\lambda}{2\pi}\mathrm{Im}\left(\frac{2\sin(\theta)}{\sqrt{\varepsilon - \sin^2(\theta)}}\right). \tag{5}$$

The imaginary parts can be evaluated which allows us to give the following expressions for the GH shift in case of $\varepsilon_i \neq 0$:

$$D_p = -\frac{\lambda}{\sqrt{2}\pi}\sin(\theta)\varepsilon_i\frac{|\varepsilon|^2\cos^2(\theta) + \sin^2(\theta)(|\varepsilon - \sin^2(\theta)| - \sin^2(\theta))}{|\sin^2(\theta) - \varepsilon\cos^2(\theta)|^2|\varepsilon - \sin^2(\theta)|\sqrt{|\varepsilon - \sin^2(\theta)| + \varepsilon_r - \sin^2(\theta)}}, \tag{6}$$

$$D_s = \frac{\lambda}{\sqrt{2}\pi}\frac{\sin(\theta)\varepsilon_i}{|\varepsilon - \sin^2(\theta)|\sqrt{|\varepsilon - \sin^2(\theta)| + \varepsilon_r - \sin^2(\theta)}}. \tag{7}$$

These equations are exact and make no assumption about the lossiness of the material. We can use Eqs. (6) and (7) to determine the ratio of the GH shifts for p- and s-polarized light:

$$\frac{D_p}{D_s} = -\frac{|\varepsilon|^2\cos^2(\theta) + \sin^2(\theta)(|\varepsilon - \sin^2(\theta)| - \sin^2(\theta))}{|\sin^2(\theta) - \varepsilon\cos^2(\theta)|^2}. \tag{8}$$

This ratio approaches $-|\varepsilon - 1| + 1$ for grazing incidence ($\theta = 90^\circ$), so that we obtain the appealing result $D_p/D_s \approx -|\varepsilon|$ for $|\varepsilon| \gg 1$.

On considering the first derivative of $D_p$ we can see that the GH shift for p-polarized light will always have an extremum at $\theta = 90^\circ$. For a low-loss dielectric this extremum is usually not the global maximum of (the absolute value of) the GH shift (see Figs. 1 and 2). For a low-loss metal, however, the extremum at $\theta = 90^\circ$ does coincide with the maximum of the GH shift. If we then go to a high-loss metal, we see the development of a second extremum at a smaller angle (see Fig. 2(c)) in addition to the extremum at $\theta = 90^\circ$. The maximum GH shift for a high-loss metal occurs at the second extremum, similarly to the dielectric case. In the dielectric case the transition from the high-loss to the low-loss regime is less striking; however, one can generally see a broadening of the resonance on transition from the low-loss to the intermediate

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loss regime [3, 4]. On increasing the loss further we reach the high-loss regime in which the resonance seems to become narrower again (see Figure 2(c)) and the maximum of the GH shift moves closer to grazing incidence; here, for high-loss dielectrics, the analytic formulae from [4] are no longer valid. The resonance in the dielectric case is dominated by the term

$$\frac{1}{|\sin^2(\theta)-\varepsilon\cos^2(\theta)|^2}=\frac{1}{(\sin^2(\theta)-\varepsilon_r\cos^2(\theta))^2+(\varepsilon_i\cos(\theta))^2}. \tag{9}$$

For a dielectric ($\varepsilon_r > 0$) the resonant behaviour is similar to a forced harmonic oscillator with damping. The damping term $\varepsilon_i\cos(\theta)$ in Eq. (9) contains the loss of the material and also depends on the angle. This explains why an increased loss leads to a broadening and a shift of the resonance away from the Brewster angle. For high losses this simple picture does not longer hold and the full expression in Eq. (6) has to be taken into account. Generally, for external dielectric reflection we do not see a transition induced by loss as in the metallic case; the maximum GH shift is always at an angle smaller than $\theta = 90^\circ$.

It is interesting to determine the amount of loss at which for a metal the transition from the low-loss to the high-loss regime occurs. Finding this point from $D'_p$ (the prime indicating the derivative with respect to $\theta$) directly is not trivial, but we can look instead at the second derivative of $D_p$ which describes the curvature of the GH curve. As can be seen from Figs. 1(a) and 2(a) the curvature $D''_p$ at $\theta = 90^\circ$ is of opposite sign for a low-loss metal and a low-loss dielectric. In the high-loss regime the curves for a metal and a dielectric are similar and the curvatures have the same sign. Therefore there has to be a point for a metal where the curvature at $\theta = 90^\circ$ changes sign, which leads to the second extremum for a high-loss metal. The curve in the $(\varepsilon_r, \varepsilon_i)$ parameter space for which the second derivative is zero can be approximated by the hyperbola

$$\frac{(\varepsilon_r+0.67)^2}{(0.12)^2}-\frac{\varepsilon_i^2}{(0.21)^2}=1, \tag{10}$$

which is shown in Fig. 3. For a metal ($\varepsilon_r < 0$) this hyperbola separates approximatively the low-loss behaviour, with a single extremum and a maximum GH shift at $90^\circ$ (left hand side (LHS) of Eq. (10) $> 1$), from the high-loss behaviour, with an additional extremum (LHS of Eq. (10) $< 1$). The parameters of the hyperbola have been determined from a numerical fit and are given in Eq. 10 to 2 significant numbers. This is an excellent approximation, in particular for $\varepsilon_r < -1.5$, where the relative error drops below 1.5% and decreases quickly if $\varepsilon_r \to -\infty$. However, special care has to be taken when the LHS of Eq. (10) is close to 1. One can clearly see that the 'low-loss' behaviour only sets in for $\varepsilon_r \lesssim -0.8$ and the 'high-loss' behaviour extends even to very small values of $\varepsilon_i$ if the real part of the dielectric constant is sufficiently small ($|\varepsilon_r| \lesssim 0.8$). For $\varepsilon_r < -1.5$ the hyperbola can be very well approximated by its asymptotic behaviour which is given by the straight line $\varepsilon_i = 1.73(\varepsilon_r + 0.67)$.

Figure 3 not only shows the separation between low-loss and high-loss regime for a metal, but also additional contours of the curvature $D''_p$ at $\theta = 90^\circ$ as indicated by the small number next to a contour line. The contour lines are plotted numerically from an analytical expression of $D''_p$ at $\theta = 90^\circ$. In this figure we have excluded a small parameter range for $0 \leq \varepsilon_r \leq 1$ (gray area in Fig 3) which corresponds to internal reflection. The GH shift of internal reflection in presence of loss is partially discussed in [4]. Our paper, however, is mainly concerned with external reflection on an interface between air and a dielectric or a metal and this special case is therefore excluded. The point in the $(\varepsilon_r, \varepsilon_i)$ parameter space where internal reflection sets in ($\varepsilon_r = 1, \varepsilon_i = 0$) is critical for the GH shift. This is why in Fig. 3 the contours of of the curvature $D''_p$ collapse at this point.

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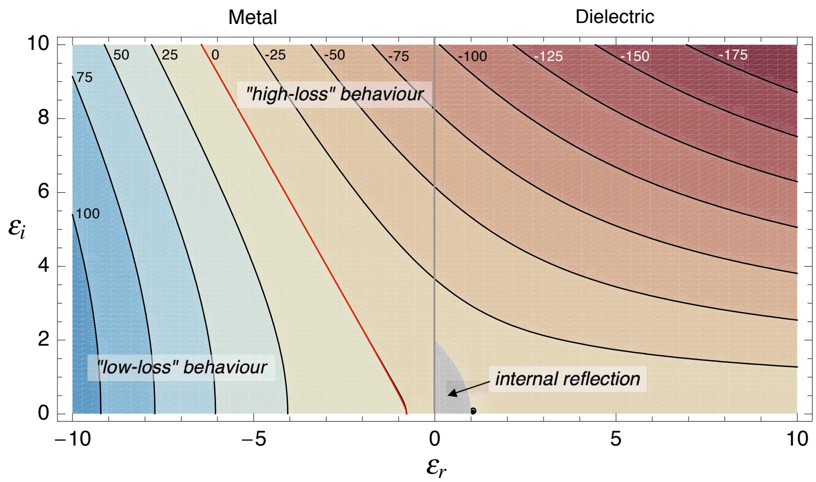


Fig. 3. Contour plot of the curvature $D_p''$ at $\theta = 90^\circ$ for metals ($\varepsilon_r < 0$) and dielectrics ($\varepsilon_r > 0$). The curvature is calculated from Eq. (6) and the contour lines are numerically evaluated. The small numbers give the value of $D_p''$ at $\theta = 90^\circ$ along the contour. The sign of this number and hence the sign of the curvature determines whether the metal has the maximum GH shift at $\theta = 90^\circ$ or if the maximum GH shift occurs at a second extremum. The red curve shows the approximated separation between the two regimes according to Eq. (10); it is almost a straight line. We have excluded a small, gray area which corresponds to internal reflection. The change from external to internal reflection causes a concentration of contour curves at the point $\varepsilon_r = 1, \varepsilon_i = 0$.

## 3. Realistic example

We choose a series of 4 metals and 3 dielectrics with an increasing $\varepsilon_i/|\varepsilon_r|$ in each group and a wavelength of $\lambda = 496$ nm, at which our selected metals (Au, Ni, Cr, and Mo) are rather lossy ($\varepsilon_i/|\varepsilon_r > 1$). As dielectrics we choose Os, W and Ge; at $\lambda = 496$ nm Os and W have a positive $\varepsilon_r$ which makes them dielectrics and we complement the series with Ge, a more conventional dielectric. The dielectric constants for these materials is listed in Tab. 1. In addition to the real and imaginary part of $\varepsilon$ we present the ratio $\varepsilon_i/|\varepsilon_r|$ in this table. We also list the angle of incidence at which the maximum GH shift $\theta_{\max}$ occurs; we have determined this angle numerically from the curves in Fig. 4.

The 'low-loss' behaviour of the metallic GH shift is shown in Figs. 1(a) and 2(a). The GH curve in this case is monotonically decreasing and the maximum of the (absolute values of the) GH shift occurs at an angle of incidence $\theta = 90^\circ$. In contrast all the metals used in Fig. 4 show the 'high-loss' behaviour; the maximum GH shift does not occur at $\theta = 90^\circ$ (see Tab. 1) and the coordinate pairs $(\varepsilon_r, \varepsilon_i)$ lie outside of the hyperbola opening specified in Eq. (10). But one can clearly see that with an increasing ratio $\varepsilon_i/|\varepsilon_r|$ the curves for the GH shifts of metals and dielectric approach each other. In fact, they overlap because the curves are not completely determined by the ratio but depend slightly on the absolute values of $\varepsilon_r$ and $\varepsilon_i$.

It has been stressed in [9], that it is advantageous to study the GH shift on metals as the large, negative shift does not occur at the Brewster angle. The experimental observation would thus not be hampered by a low reflectivity. It is interesting to investigate if this statement, that has been made in the context of low losses, is also valid in the high-loss regime studied in the present paper. To this end we have determined the Brewster angle $\theta_B$ numerically as the position of the dip in the reflectivity $\mathscr{R}_p = |r_p|^2$ [9] from the curves in Fig. 5; these $\theta_B$ values

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Table 1. Table listing the dielectric constants $\varepsilon = \varepsilon_r + \mathrm{i}\varepsilon_i$ at $\lambda = 496$ nm and calculated values based upon these dielectric constants for the materials shown in Figs. 4 and 5. The angle $\theta_{\max}$ denotes the angle of the maximum GH shift, while $\theta_B$ is Brewster angle as defined by the minimum in the reflectivity. Both quantities have been determined numerically from the graphs in Figs. 4 and 5. An estimate of the Brewster angle according to [20] is given by $\theta_{|\varepsilon|}$. The residual reflectivity $\mathscr{R}_{p,s}$ and the differential reflectivity $\Delta\mathscr{R}$ are all evaluated at the Brewster angle $\theta_B$. The entries for 'Source' refer to the dielectric constants.

| Material | $\varepsilon_r$ | $\varepsilon_i$ | $\varepsilon_i/\lvert\varepsilon_r\rvert$ | $\theta_{\max}$ | $\theta_B$ | $\theta_{\lvert\varepsilon\rvert}$ | $\mathscr{R}_p$ | $\mathscr{R}_s$ | $\Delta\mathscr{R}$ | Source |
|---|---|---|---|---|---|---|---|---|---|---|
| Au | -2.55 | 3.37 | 1.32 | 86.6° | 61.0° | 64.1° | 0.33 | 0.72 | 0.54 | [14] |
| Ni | -5.80 | 9.79 | 1.69 | 86.8° | 72.6° | 73.5° | 0.31 | 0.85 | 0.63 | [14] |
| Cr | -3.33 | 18.2 | 5.45 | 82.6° | 76.5° | 76.9° | 0.20 | 0.87 | 0.77 | [19] |
| Mo | -2.62 | 25.1 | 9.55 | 83.1° | 78.5° | 78.7° | 0.18 | 0.90 | 0.79 | [14] |
| W | 4.24 | 18.1 | 4.27 | 79.9° | 76.7° | 76.9° | 0.11 | 0.84 | 0.86 | [14] |
| Ge | 13.2 | 20.7 | 1.57 | 80.0° | 78.5° | 78.6° | 0.06 | 0.86 | 0.92 | [14] |
| Os | 22.2 | 25.1 | 1.13 | 81.1° | 80.2° | 80.2° | 0.04 | 0.90 | 0.95 | [14] |

are given in Tab. 1. For comparison we also list in Tab. 1 an estimate of the Brewster angle as given by $\theta_{|\varepsilon|} = \tan^{-1}(\sqrt{|\varepsilon|})$ [20]. We find that in particular for large $|\varepsilon|$ this estimate is in good agreement with the numerically determined Brewster angle. The most prominent aspect of Fig. 5 is that for all our (high-loss) materials the residual reflectivity at the Brewster angle, $\mathscr{R}_p(\theta_B)$, is still at least one order of magnitude higher than for low-loss dielectrics [3]. The behaviour of the Brewster dip with loss can be undertstood if we start from a loss-free metal with a finite, negative $\varepsilon_r$ which reflects all incident light fully, regardless of the angle of incidence, as there is no absorption and no transmission. The formation of a Brewster dip, even if very weakly formed, is a consequence of the small, but finite loss. Higher losses lead to an overall reduced reflectivity and a more strongly formed Brewster dip. This in contrast to the dielectric case where the Brewster dip is perfectly formed in the case of zero loss.

On comparing Figs. 4 and 5 and also from Tab. 1 it can be seen that for the high-loss metals the Brewster angle $\theta_B$ is a few degrees away from the angle of the maximum of the GH shift $\theta_{\max}$. But for increasing loss this separation becomes less. This is in contrast to the behaviour of dielectrics where an increase in loss leads to an increased separation between $\theta_B$ and $\theta_{\max}$. It is interesting to note that adding loss has the opposite effect on the reflectivity for dielectrics as can be seen from Fig. 5. Whereas adding loss to a metal reduces the reflectivity, adding loss to a dielectric enhances the reflectivity. As pointed out in [10] the differential reflectivity $\Delta\mathscr{R} = (\mathscr{R}_s - \mathscr{R}_p)/\mathscr{R}_s$ is an important quantity for the experimental observability of the GH shift. We have listed the values for the realistic materials in Tab. 1. As one can see the values are higher than for Au at $\lambda = 826$ nm, where it was 0.06 [10], but not to an extent which seriously complicates the observation in an experiment.

## 4. Conclusion

The Goos-Hänchen (GH) shifts for metal and dielectric are often treated seperately. This separation arises naturally for low loss materials, that is for small values of the imaginary part $\varepsilon_i$ of the complex dielectric constant $\varepsilon$, as the form for the GH shift is very different for the two cases. We have shown that for high losses, i.e. for a large imaginary part $\varepsilon_i$ the GH shifts for metals and dielectrics have a similar form. We have illustrated the similarity in the GH shifts for high losses using a realistic example of 7 materials, 4 metals and 3 dielectrics. This makes it interesting to study the transition in the GH shift from the low-loss regime to the high-loss regime. We have studied the nature of this transition, in particular for metals, and we have given

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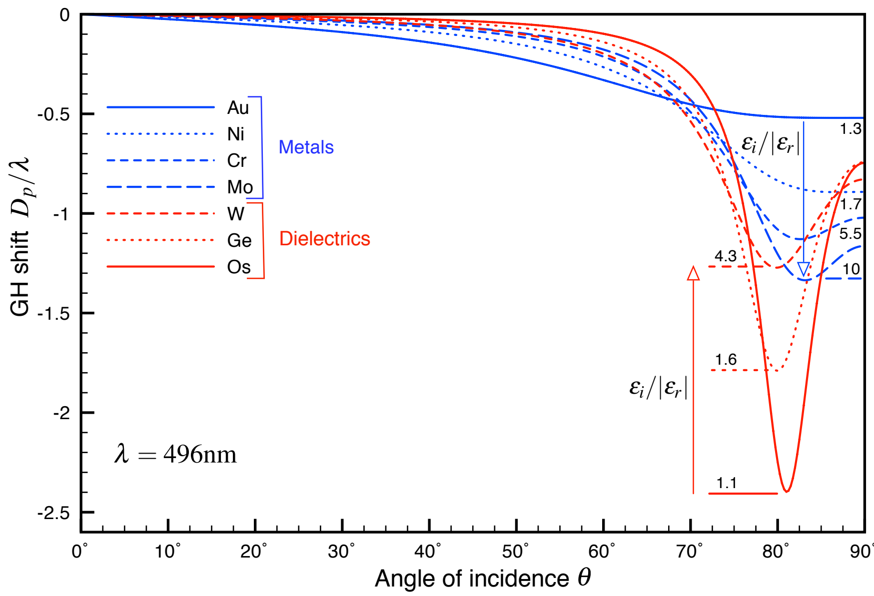


Fig. 4. Plot of the GH shift for p-polarized light for the 7 materials specified in Tab. 1. The figure shows that for an increased loss the GH curves for metals and dielectrics are becoming more similar. The arrows indicate the direction of an increasing ratio $\varepsilon_i/|\varepsilon_r|$.

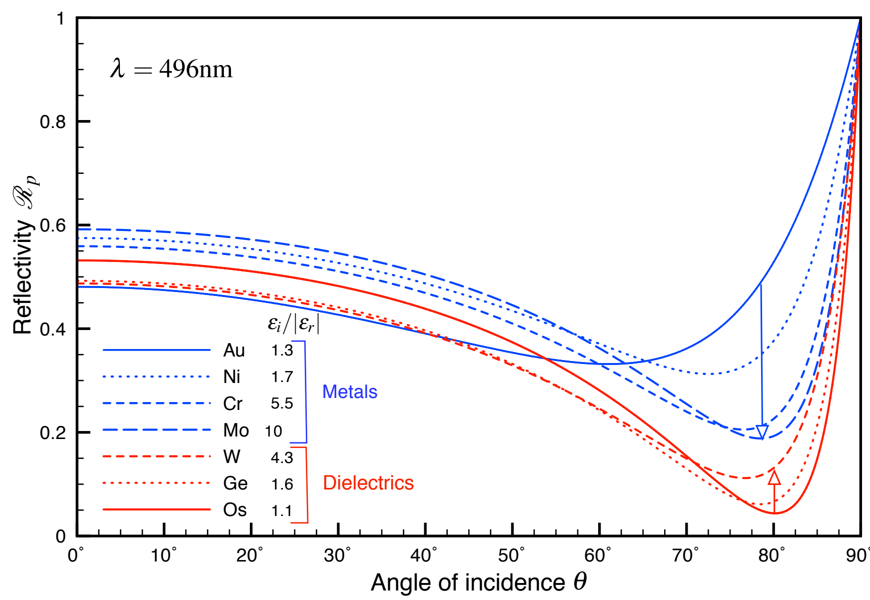


Fig. 5. Plot of the reflectivity for p-polarized light for the materials listed in Tab. 1. As in Fig. 4 the arrows indicate the increase in the ratio $\varepsilon_i/|\varepsilon_r|$. One can see that an increased loss effects metals and dielectrics differently; whereas the reflectivity of a metal is reduced for increased loss, a dielectric shows an enhanced reflectivity.

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an approximate criterion in terms of the real and imaginary part of $\varepsilon$ for this transition (see Fig. 3). We can thus give an argument under which conditions it will be possible to observe a reasonably large ($|D_p| \approx \lambda$), negative GH shift in external reflection: Whereas in the 'low-loss' regime it is advantageous to study the effect on metals, in the 'high-loss' regime metals and dielectrics are equally suited.

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