European Journal of Operational Research 259 (2017) 143–154

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Production, Manufacturing and Logistics

Speed optimization and bunkering in liner shipping in the presence of uncertain service times and time windows at ports

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ARTICLE INFO

Article history:
Received 31 August 2015
Accepted 2 October 2016
Available online 8 October 2016

Keywords:
Transportation
OR in maritime industry
Fuel cost
Dynamic programing

ABSTRACT

Recent studies in maritime shipping have concentrated on environmental and economic impacts of ships. In this regard, fuel is considered as one of the important factors for such impacts. In particular, the sailing speed of the vessels affects the fuel consumption directly. In this study, we consider a speed optimization problem in liner shipping, which is characterized by stochastic port times and time windows. The objective is to minimize the total fuel consumption while maintaining the schedule reliability. We develop a dynamic programing model by discretizing the port arrival times to provide approximate solutions. A deterministic model is presented to provide a lower bound on the optimal expected cost of the dynamic model. We also work on the effect of bunker prices on the liner service schedule. We propose a dynamic programing model for bunkering problem. Our numerical study using real data from a European liner shipping company indicates that the speed policy obtained by proposed dynamic model performs significantly better than the ones obtained by benchmark methods. Moreover, our results show that making speed decisions considering the uncertainty of port times will noticeably decrease fuel consumption cost.

1. Introduction

Sailing speed is an important decision variable affecting fuel consumption and consequently, greenhouse gas emission. Many operational strategies in container shipping have focused on reducing fuel costs (Christiansen, Fagerholt, Nygreen, & Ronen, 2013). For instance, Maersk, one of the major liner shipping company, introduced slow steaming strategy in 2008 to cope with the increasing bunker price. Although sailing with the slowest speed is favorable with respect to the fuel cost and greenhouse gas emissions, it may not be always feasible due to the uncertainties in sea transport legs and the ports that may affect service level agreement with customers. Sailing speed decision at sea transport legs mainly depends on the port time windows and the transit time between ports. Therefore, it is natural to expect that the speed decision should be made by considering both the time windows and the uncertain port service times.

Stochastic port service time and travel time play important roles on the total cruise time of vessels. Port and travel times can be highly variable due to congestion, handling and weather conditions (Notteboom, 2006). Congestion or disruption in a port may result in deviation from the planned schedule and hence, it may cause delays at the following ports along the route. Vernimmen, Dullaert, and Engelen (2007) state that only about 52% of the vessels dispatched for liner services arrived at the planned schedule. Drewry (2016) presents a detailed reliability report by trade and reports that the average percentage of on-time delivery ranges from 55% to 89%. Notteboom (2006) reports that 93.6% of the delays are caused by disruptions in port and terminal operations. citeLee15 state that container handling capacity at ports becomes insufficient for the growth in the container transport demand and the variability in port times is a vital problem for liner operators. To prevent the delays and maintain schedule reliability, vessels may increase their speeds, which in turn increases the total fuel consumption and greenhouse gas emission. In some cases like unforeseen delay in a port, vessels may not reach the next port on time even if they sail at maximum speed. To avoid poor service level and meet the planned schedule on time, uncertain port times should be considered in speed decision. Despite this expectation, numerous studies in the literature do not consider the uncertainties in liner shipping. Psaraftis and Kontovas (2013) provide a detailed review of speed models in maritime transportation and reveal that only few studies consider the uncertainties in liner

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http://dx.doi.org/10.1016/j.ejor.2016.10.002

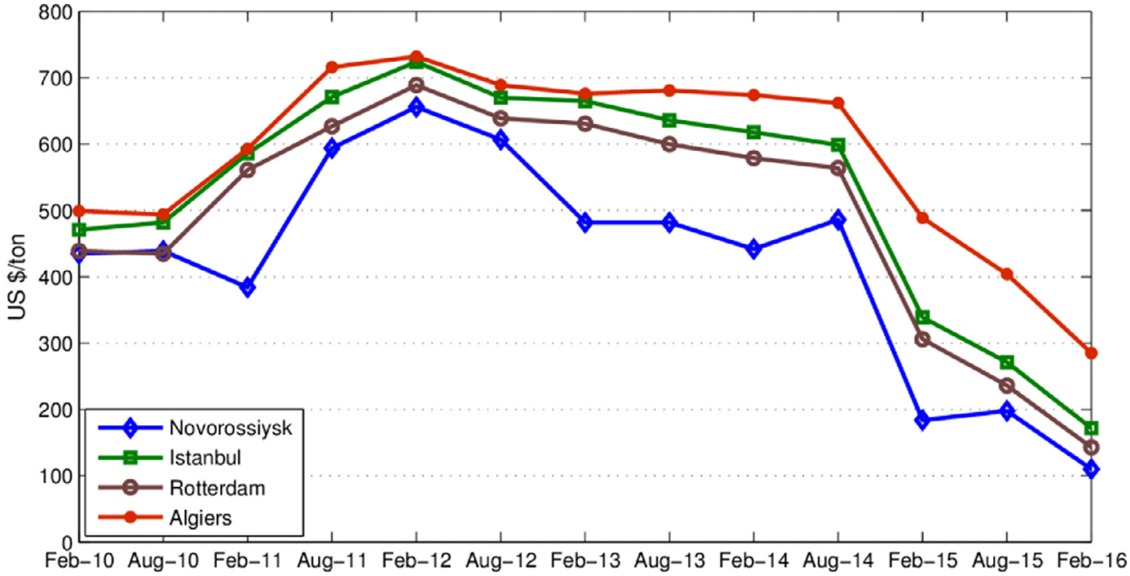


Fig. 1. Bunker fuel prices (IFO380) at various ports (source: bunkerindex.com).

shipping. In this paper, we focus on speed optimization problem with stochastic port times. We develop new models to optimize sailing speed for liner shipping.

Another important factor that affects ship liners' operating cost is bunker price. Since fuel cost constitutes a major portion of operational cost of a container ship, shipping companies focus on efficient ways to reduce bunker cost (Ronen, 2011). Through private conversation with a major liner company in the Mediterranean, it was realized that liner operators can make long term or short term contracts with the fuel suppliers. Long term contract is generally set for a year and during this period, fuel price has been fixed for the contracted amount. Liner operators who want to avoid the risks of fluctuating bunker price, prefer long term contracts. On the other hand, short term contracts are made approximately ten days before the liner service starts. The contract specifies the bunkering ports, price and quantity. Short term contracts are preferred in short-haul routes like those in the Mediterranean. Fig. 1 shows the monthly average fuel price at several bunkering ports in the Mediterranean and the Black Sea area from 2010 to 2016. As it is seen in this figure, bunker prices at different ports have significant differences. For instance, in February 2015, the average monthly prices in Novorossiysk and Algiers were 184 dollars and 489 dollars per metric ton. Bunkering ports significantly affect the total fuel cost of a liner service. Therefore, bunkering port selection is an important decision. In this paper, we also consider bunkering port selection strategy. In particular, we focus on short term bunkering contracts and study the decisions regarding where to bunker and how much to bunker? Bunkering decision is directly related to fuel consumption and affects the sailing speed. For instance, to avoid high fuel costs liner companies prefer slow steaming strategy (Maloni, Paul, & Gligor, 2013). Therefore, sailing speed and bunkering decisions are interrelated. In this paper, we study the joint speed and bunkering problem.

We make the following research contributions in this paper. First, we present a new dynamic programing formulation for the fuel consumption problem in liner shipping that can handle random port times. Our model takes into account future possible port service times in assessing the current speed decision. The objective is to optimize sailing speed along the route to minimize fuel consumption cost and the cost of delays by considering port time windows. We use the term "time window" to describe the reserved time period that a port allocates to serve the vessel. In our study, we assume that ports report the available time windows and then, the vessel determines the arrival times according to these given windows (Meng, Wang, Andersson, & Thun, 2014). The literature addressing the uncertainties at ports is limited. The proposed studies generally utilize heuristical approaches due to the difficulty in solving the stochastic problem and these approaches do not guarantee the global optimality unless the objective function has a special structure. Second, we examine the properties of the proposed dynamic model and provide some theoretical results regarding the optimal speed decision and port arrival time. Third, we work on the deterministic model formulation and we show that this model provides a lower bound on the optimal expected cost. Consequently, by using the discretized dynamic model and deterministic model, we can provide an upper bound on the optimality gap. Fourth, we develop a new model for bunkering problem. In particular, we work on where to bunker and how much to bunker decisions in short-term contracts. Different than the proposed bunkering studies in the literature, we allow liner vessel to bunker at the ports which are not on the planned route schedule. Finally, we perform a computational study by using real shipping data from a liner company. Our experiments are motivated by the fact that the liner shipping industry is interested in what-if scenarios, and this observation leads us to evaluate the impacts of problem parameters on speed and bunkering decision, fuel consumption and service level.

The rest of the paper is organized as follows. In Section 2, we provide an overview of the related literature. In Section 3, we develop a dynamic programing formulation of the speed optimization problem for liner shipping. Section 4 presents a deterministic approximation to the dynamic model that provides a lower bound on the optimal total expected cost. In Section 5, we extend the dynamic programing formulation by considering bunkering problem. In Section 6, we present our computational study. We conclude and discuss some future research directions in Section 7.

2. Review of related literature

There is an extensive literature on ship routing and scheduling problems in maritime transportation. For a comprehensive review of this area, we refer to Psaraftis and Kontovas (2013), Christiansen et al. (2013) and Meng et al. (2014). These review studies demonstrate that recent trend in maritime transportation have focused on sailing speeds and environmental impact of ships.

Ronen (2011) points out the importance of reducing vessels' speed on operating cost. He addresses the speed optimization problem by considering the service frequency and the required number of vessels. Fagerholt, Laporte, and Norstad (2010) and Hvattum, Norstad, Fagerholt, and Laporte (2013) work on the speed optimization problem in liner shipping with port time windows. They restrict vessel to arrive within the time window of each port to achieve 100% service level. Fagerholt et al. (2010) discretize the arrival times and solve the problem by using shortest path algorithm. Hvattum et al. (2013) develop an exact solution algorithm for the deterministic problem. Wang and Meng (2012c) work on the speed optimization problem with transshipment and container routing. They formulate the problem as a mixed-integer nonlinear model and propose outer-approximation algorithm to obtain approximate solutions. Norstad, Fagerholt and Laporte (2011) incorporate speed decision in the tramp ship routing and scheduling problem and propose a local search method. They first develop a solution algorithm for speed optimization problem with fixed route. Then, they utilize this algorithm to generate an initial solution for the proposed local search method. Zhang, Teo, and Wang (2014) extend the work of Fagerholt et al. (2010) and Norstad, Fagerholt and Laporte (2011), and study the optimality properties.

Later studies in this field focus on the schedule reliability in liner shipping. Mansouri, Lee, and Aluko (2015) provide a review on multi-objective models in maritime shipping and examine the relation between service level and fuel costs. Li, Qi, and Lee (2015) and Brouer, Dirksen, Pisinger, Plum, and Vaaben (2013) analyze the delays in planned arrival time due to the port disruption and examine speeding up, port omitting and port swapping options to catch up with the planned schedule. Both of these studies assume deterministic port service time. While Li et al. (2015) discuss single vessel problem, Brouer et al. (2013) propose a general solution method for multiple vessels on a liner network. Notteboom (2006) highlights the effects of waiting times and delays on schedule reliability and discusses the trade-off between schedule reliability and operating costs. He also points out that due to the fast growth in the volume of sea transportation, port congestion has become the main reason of port delays. Due to the port congestion, vessels may have to wait long hours for service, which may result in delays in the following ports. In case of such delays, ship managers of liner shipping companies can keep up with the planned schedule by increasing vessel speeds. However, this may result in high fuel cost. This is the main dilemma experienced by ship managers. According to SeaIntel global liner performance report, although Maersk Line is one of the top reliable carriers in terms of on time performance, their operational costs are considerably higher compared with the low-cost carriers (SeaIntel, 2015). Port delays and resultant high costs have directed shipping companies to finding ways of anticipating unexpected delays (Notteboom, 2006).

The maritime literature on fuel emission with uncertainty is scarce. Wang and Meng (2012a) study the ship route schedule design problem by considering sea contingency and uncertain port times. They do not allow late arrivals and formulate the problem as nonlinear mixed integer stochastic programing model. Wang and Meng (2012b) work on the robust design of liner shipping schedule by allowing late arrivals. They penalize the vessel if it arrives later than the published time. To recover the delays, they force vessels to sail at high speed whenever it is necessary. In other words, they decide vessel speed without considering the trade-off between fuel and delay costs. Lee, Lee, and Zhang (2015) work on the effects of slow steaming on service level and fuel consumption by considering uncertain port times. They consider fast steaming and flexible slow steaming strategies to examine the relation between delay and fuel consumption costs. In fast steaming strategy, a vessel sails at its highest speed during the entire journey. On the other hand, in flexible slow steaming strategy, a vessel usually sails at its lowest speed and it can switch to the highest speed when there is a delay.

Qi and Song (2012) propose a vessel scheduling model by considering uncertain port times. They formulate a liner shipping problem to find the optimal transit time between ports and consequently, obtain the planned arrival schedule for multiple voyages. In their problem formulation, ports do not have specific time windows for service and hence, vessels are allowed to arrive at any time. In other words, they assume that ports are always ready for service. However, this assumption may not be consistent with the practice (Wang & Meng, 2014). To achieve high service levels during the journey, Qi and Song (2012) assume that ports penalize the vessel if it arrives later than the planned arrival time of that port. In other words, they aim to minimize the deviation from the planned schedule. They use linear cost function to penalize the delays and propose a stochastic approximation algorithm to tackle the problem.

Li, Qi, and Song (2015) study the real-time schedule recovery problem in a liner shipping service. They propose a multi-stage stochastic model by dividing the sea legs into multiple segments. Their objective is to find optimal travel time at each segment by considering the uncertainties in sailing times and at ports (due to port disruption). As in the work of Qi and Song (2012), they assume that ports do not have any time windows for their contracted service. Instead, the vessel aims to arrive at the planned arrival time. Moreover, different than our work, they do not consider waiting cost for early arrivals, which is contradictory to the current practice in liner shipping. To facilitate the implementation of the model, they assume that planned arrival time to each segment in a leg is predefined. Nonetheless, due to the difficulty of solving this problem, they focus on uncertainties in sailing time and consider the case with only one disruption event. In our proposed work, we study the uncertainties at all ports by considering both early and late arrivals.

The literature on bunkering management is limited. Recently, Ronen (2011) analyzes the effects of bunker price change on liners' operational costs. He works on the tradeoff between slow steaming and increasing the fleet size. Yao, Ng, and Lee (2012) take into account the bunker price difference across different ports and propose a bunkering model to find optimal bunkering ports and bunkering amounts. However, in the proposed model they limit the number of bunkering times. They also optimize the sailing speed between ports by restricting the vessel to arrive within predefined time windows. The objective is to minimize bunker cost and revenue loss due to carrying bunker fuel weight. Sheng, Lee, and Chew (2014) extend the work of Yao et al. (2012) by considering the uncertainty in fuel consumption and bunker prices. The proposed model aims to minimize the bunker cost and bunker inventory holding cost. Kim (2014) present a bunkering model to minimize the total cost of bunker purchasing, ship time (total chartering cost of the vessel and time value of containers) and carbon tax. In the model formulation, a liner vessel is assumed to arrive ports at any time, without any restriction. Lagrangian heuristic is proposed to obtain a bunkering strategy. Ghosh, Lee, and Ng (2015) focus on long term bunker contracts. By considering fluctuating bunker prices, they propose a dynamic programing model. They assume that vessel sails at constant speed during the voyage, however the fuel consumption between ports is uncertain. The objective is to minimize total bunkering cost, penalty cost of running out of fuel and damage cost for not fulfilling the committed amount.

In this paper, we first focus on speed optimization problem in the presence of uncertain port service times. To the best of our knowledge, the speed optimization problem considering port time uncertainties and port time windows for fixed liner shipping route has not been addressed in the literature. Our study is closely related to the work of Qi and Song (2012). We consider a liner schedule, which is repeated on regular basis (weekly, monthly etc.). Different from Qi and Song (2012), our study focus on speed optimization problem with time windows on a single voyage. This enables us to analyze the structure of the dynamic model and investigate the properties of optimal speed decisions. Furthermore, the deterministic approximation that we use to obtain a lower bound on the optimal expected cost, can be useful to assign the bound on the optimality gap for each problem instance. Since sailing speed decision is affected by the fuel prices, we also study the relation between sailing speed and bunkering. We focus on short term contracts at the planning level of liner shipping companies and propose a dynamic programing model to determine optimal vessel speed, bunkering ports and the bunkering quantities. Different than the previous studies, we allow a liner vessel to bunker at the ports which are not on her planned route. In other words, a vessel may detour to select a cheaper bunkering port. Based on a real shipping data, we analyze the effects of bunker prices on the liner's service schedule.

3. Dynamic speed optimization model

Motivated by the shipping operations of a major liner shipping company, we propose a dynamic speed optimization model. We consider a vessel which provides shipping services over a given sequence of ports-of-call denoted by set $N = \{0, 1, \ldots, n\}$. Port 0 shows the starting node of the network. We use leg i to denote the trip from port $(i-1)$ to port i. The vessel can visit port i within its time window. For a liner shipping company, on time delivery of the cargo is paramount as delayed cargo results in high cost by customers. Therefore, it is crucial to be served on time. If the vessel arrives earlier than the allocated time, it has to wait. If the vessel misses the reserved time, this may cause deviation from the planned schedule. Service time of a vessel in each port is a stochastic variable and denoted by S_i for port i. A vessel has for all practical purposes a lower and an upper speed limit. We have to decide the speed between ports in order to minimize total fuel consumption and maximize service level. We will use the following notation throughout the paper.

N: set of ports
S_i: random service time in hour at port i such that $l_i \leq S_i \leq u_i$
t_i^a: arrival time of vessel at port i
t_i^d: departure time of vessel at port i
$[\alpha_i, \beta_i]$: time window at port i
d_i: length of leg i in nautical mile
φ: fuel cost per hour during waiting and service time at a port
θ_i: delay penalty per hour at port i
r_s: price of fuel per ton consumed during sailing
r_p: price of fuel per ton consumed at ports
v_i: average speed at leg i (nautical mile per hour), which is limited by $[\underline{v}, \bar{v}]$

In the literature, quadratic function of sailing speed is generally used to compute fuel consumption of a vessel (Fagerholt et al., 2010). We use the equation proposed by Yao et al. (2012) to calculate fuel consumption rate (tons per day). Yao et al. (2012) present an empirical model to present the relation between bunker consumption rate and the vessel speed by considering the size of the vessels. The proposed fuel consumption rate is given as $k_1 v_i^3 + k_2$, where k_1 and k_2 are constants and their values depend on the size of the vessel. We assume that the speed of a vessel is constant between two consecutive ports. We also assume that vessel's speed is not affected by the weather conditions during sailing. Multiplication of fuel consumption rate by the time required to travel the distance between ports yields the total amount of fuel consumption (in tons). Then, the fuel consumption function for leg i is given as:

$$g(v_i) = (d_i/24v_i)k_1 v_i^3 + k_2. \tag{1}$$

The fuel consumption function $g(v_i)$ is convex and increasing with v_i for $v_i \in [\underline{v}, \bar{v}]$. As an extension to the model proposed by Qi and Song (2012), we also consider the fuel consumption during waiting and service time at ports. Due to international regulations, vessels are not allowed to use high sulfur fuel while berthing at ports as opposed to open sea cruise (EUR-Lex, 2012). Low sulfur fuel consumed at ports is more expensive than the high sulfur fuel. We assume that the vessel consumes a fixed amount of fuel per hour during waiting and service time at each port. Let κ be the average amount of fuel (tons) consumed per hour. Then, $\varphi = r_p\kappa$ gives the fuel cost per hour. Consequently, the total fuel consumption cost is given by,

$$\sum_{i=1}^{n}(r_s g(v_i) + \varphi(t_i^d - t_i^a)).$$

As in Qi and Song (2012), we also consider the service level in each port. Arriving later than the given time window will result in delay in the planned schedule. Vessel delays can be very costly due to cargo misconnecting, rerouting the delayed cargo, inventory handling cost and loss of customer goodwill (Qi & Song, 2012). To maximize the service level and avoid such delays, we penalize the vessel for each hour of the lateness. In real world applications, shipping companies also quantify costs resulting from schedule delays. For instance, Maersk pays different amounts of money to shippers depending on the length of the delay in the planned schedule (Li et al., 2015). In this study, we use a linear delay cost function. However, our model can handle any nondecreasing delay cost function. The total cost is given by:

$$\sum_{i=1}^{n} r_s g(v_i) + \sum_{i=1}^{n}\varphi(t_i^d - t_i^a) + \sum_{i=1}^{n}\theta_i[t_i^a - \beta_i]^+$$

where $[t_i^a - \beta_i]^+ = \max\{t_i^a - \beta_i, 0\}$. Given the speed decision v_i and service time S_i at port i, the states of the system at the following ports are defined by the following system dynamics equations:

$$t_i^a = t_{i-1}^d + d_i/v_i, \tag{2}$$

$$t_i^d = \max\{t_i^a, \alpha_i\} + S_i, \qquad i = 1, \ldots, n. \tag{3}$$

Since we start with port 0, we assume that $t_0^a = t_0^d = 0$. We are interested in minimizing the total expected cost over a finite horizon. In formulating this problem by using dynamic programing, we divide the problem into $(n+1)$ stages and each stage corresponds to one port. At each stage, we have to decide the average sailing speed until the next port. Our decision on speed will designate the arrival time to the next port. Since sailing speed and service time are bounded, the possible arrival time is also bounded. We use $\bar{t}_i$ and $\underline{t}_i$ to denote these upper and lower limits, respectively. To capture the state of the system at stage i, we use t_i as the arrival time at port i. Using t_i as the state variable at port i, the optimal speed policy can be found by computing the value functions $\{J_i(.)\colon i \in N\}$ through the following equation:

$$J_i(t_i) = \mathbb{E}\Big\{\min_{v_i\in[\underline{v},\bar{v}]}\{\varphi(S_i + [\alpha_i - t_i]^+) + \theta_i[t_i - \beta_i]^+ + r_s g(v_i) + J_{i+1}(t_{i+1})\}\Big\} \tag{4}$$

for every $t_i \in [\underline{t}_i, \bar{t}_i]$. Arrival time to the following port (t_{i+1}) is computed by using recursive equations given in (2) and (3). The boundary condition for every $t_n \in [\underline{t}_n, \bar{t}_n]$ is given by:

$$J_n(t_n) = \varphi(\mathbb{E}[S_n] + [\alpha_n - t_n]^+) + \theta_i[t_n - \beta_n]^+. \tag{5}$$

Since the vessel is deployed from port 0 initially, $J_0(0)$ gives the optimal expected total cost at the beginning of the journey, where 0 represents the fact that the arrival time to port 0 is $t_0 = 0$. The first two cost functions in Eq. (4) correspond to the port time cost and delay penalty, respectively. These cost functions depend on the arrival time. While port time cost function is a non-increasing function in t_i, delay penalty function is a non-decreasing function. Given the realization of the service time S_i, the arrival time to the next port is determined by the speed decision. Consequently, the state transition and decision function for the optimal policy is given by:

$$(v_i, t_{i+1}) = \underset{v_i \in [\underline{v}, \bar{v}]}{\operatorname{argmin}} \{\varphi(S_i + [\alpha_i - t_i]^+) + \theta_i[t_i - \beta_i]^+ + r_s g(v_i) + J_{i+1}(t_{i+1}) | S_i\}. \tag{6}$$

Since possible arrival times, sailing speeds and service times are continuous variables, we need to discretize time into a finite number of time points to implement the dynamic programing model. This provides an approximation to the continuous dynamic model. Since the resulting model only considers the discretized time points, its optimal solution is feasible for the continuous dynamic model. Consequently, the optimal objective value of the discrete dynamic model is greater than the continuous model and it gives an upper bound on the continuous model. Now, we will provide some information about the optimal solution.

Remark 3.1. Given a realization of service times at all ports $\{S_1, S_2, \ldots, S_n\}$, let v_i^* and t_i^* for $i \in N$ be a feasible solution for speed between ports $(i-1)$ and i, and corresponding arrival times. Consider a port p with time window $[\alpha_p, \beta_p]$ and the feasible solution $\underline{v} < v_p^* \le \bar{v}$ and $t_p^* < \alpha_p$. Then, there exists a better solution with sailing speed $\bar{v}_p$ such that $\underline{v} \le \bar{v}_p < v_p^*$.

Due to the port time window, a vessel cannot berth as soon as it arrives to the port. It has to wait until port service time starts. Therefore, by considering the fuel prices we can say that it is not beneficial to arrive at a port earlier than the time window of that port.

Lemma 3.1. *Let t_i^1 and t_i^2 denote the two possible arrival times for port i such that $\underline{t}_i \le t_i^1 \le t_i^2 \le \alpha_i$. For any given service time S_i and arrival times t_i^1 and t_i^2, let v_i^{1*} and v_i^{2*} be the optimal sailing speeds at leg i, respectively. Then, we have $v_i^{1*} = v_i^{2*}$.*

We defer the proof of the lemma to the supplementary document. Lemma 3.1 implies that the optimal speed decision does not change when the vessel arrival time is $t_i \in [\underline{t}_i, \alpha_i]$. This observation allows us to reduce the state space of dynamic programing model at each stage. In other words, sailing speed v_i at leg i will be computed for the arrival times $t_i \in [\alpha_i, \bar{t}_i]$. The optimal sailing speed for the arrival time period $t_i \in [\underline{t}_i, \alpha_i)$ will be equal to the optimal sailing speed when $t_i = \alpha_i$. Next, we present some results when $t_i \in [\alpha_i, \bar{t}_i]$. For every $t_i \in [\alpha_i, \bar{t}_i]$, the optimal value functions of the restricted problem are given as:

$$J_i^R(t_i) = \mathbb{E}\left\{ \min_{v_i \in [\underline{v}, \bar{v}]} \{\varphi S_i + \theta_i[t_i - \beta_i]^+ + r_s g(v_i) + J_{i+1}^R(t_{i+1})\} \right\} \tag{7}$$

with the boundary condition,

$$J_n^R(t_n) = \varphi \mathbb{E}[S_n] + \theta_i[t_n - \beta_n]^+. \tag{8}$$

Proposition 3.1. *$J_i^R(t_i)$ is convex and nondecreasing with t_i for every $i = 0, \ldots, n$.*

The proof of the proposition is given in the supplementary document. Next, we present the relation between the sailing speed decision and the port arrival time.

Corollary 3.1. *Let t_i^1 and t_i^2 denote the two possible arrival times for port i such that $\alpha_i \le t_i^1 \le t_i^2 \le \bar{t}_i$. For any realization of service time S_i, let v_i^{1*} and v_i^{2*} be the optimal sailing speeds at leg i given that the vessel arrival times are t_i^1 and t_i^2, respectively. Then, we have $v_i^{1*} \le v_i^{2*}$.*

The proof of the corollary is presented in the supplementary document.

4. Deterministic approximation

An alternative solution method for the speed optimization problem described in Section 3 is to solve a deterministic model. A deterministic approximation of the above dynamic programing model assumes that all random quantities are known in advance and they take their expected values. Hence, deterministic models do not capture the temporal dynamics of the problem as dynamic models do (Talluri & van Ryzin, 2005). However, they are popular in practice due to their computational efficiency. The deterministic approximation is formulated as follows:

$$\text{minimize } \sum_{i=1}^{n} r_s g(v_i) + \sum_{i=1}^{n} \varphi(t_i^d - t_i^a) + \sum_{i=1}^{n} \theta_i[t_i^a - \beta_i]^+ \tag{9}$$

$$\text{subject to } t_i^a = t_{i-1}^d + d_i/v_i, \qquad i = 1, \ldots, n, \tag{10}$$

$$t_i^d = l_i + \mathbb{E}[S_i], \qquad i = 1, \ldots, n, \tag{11}$$

$$l_i \ge \alpha_i, \qquad i = 1, \ldots, n, \tag{12}$$

$$l_i \ge t_i^a, \qquad i = 1, \ldots, n, \tag{13}$$

$$\underline{v} \le v_i \le \bar{v} \qquad i = 1, \ldots, n, \tag{14}$$

where $t_0^a = t_0^d = 0$. Constraints (10) and (11) correspond to the system dynamics equations. Constraints (12) and (13) ensure that the vessel's service starts within the time windows at all ports. Constraint (14) guarantees that the speed of the vessel is within the lower and upper limits in all legs.

Note that constraint (10) is nonlinear. We define τ_i to denote the transit time between ports $(i-1)$ and i and it is formulated as $\tau_i = d_i/v_i$. Replacing d_i/v_i by τ_i and rewriting the constraints (10) and (14), we obtain a feasible region with linear constraints. Then, the objective function in (9) is given as:

$$\sum_{i=1}^{n} r_s g(\tau_i) + \sum_{i=1}^{n} \varphi([\alpha_i - t_i^a]^+ + \mathbb{E}[S_i]) + \sum_{i=1}^{n} \theta_i[t_i^a - \beta_i]^+. \tag{15}$$

Note that the fuel consumption function $g(\tau_i)$ is convex in τ_i. Let define $h(t_i^a)$ as the waiting and penalty cost function for port i. Then, we have:

$$h(t_i^a) = \varphi([\alpha_i - t_i^a]^+ + \mathbb{E}[S_i]) + \theta_i[t_i^a - \beta_i]^+. \tag{16}$$

It is clear that the function $h(t_i^a)$ is convex in t_i^a. Consequently, the objective function in (15) is convex in (τ_i, t_i^a) for all $i = 1, \ldots, n$.

Lemma 4.1. *Deterministic model given in (9)–(14) is a convex program. Therefore, a local minimum solution is also the global minimum solution for model (9)–(14).*

Due to convexity, deterministic model (9)–(14) can be efficiently solved by a nonlinear programing solver. The solution of the model (9)–(14) provides a scheduling and speed policy to minimize total fuel consumption. Moreover, its optimal objective value provides a lower bound for the minimum expected cost. In other words, letting Z_{DM}^* be the optimal objective value of problem (9)–(14), we have $Z_{DM}^* \le J_0(0)$ as shown in the next proposition. This relation can be used to assess the optimality gap of a suboptimal policy derived from any solution algorithm such as heuristic methods. The proof of the proposition is given in supplementary document.

Proposition 4.1. *The optimal objective value of the deterministic model gives a lower bound for the dynamic programing model, that is,* $Z^*_{DM} \leq J_0(0)$.

As mentioned above, we approximate the dynamic model (4) and (5) by discretizing the time and this approximation provides an upper bound for the continuous dynamic model. Therefore, the percentage gap between the objective function values of the deterministic model and the approximate dynamic model provides an upper bound for the optimality gap of the continuous dynamic model.

5. Bunker port selection

In this section, we study the bunkering management strategy. We consider the planning level problem for short term contracts. Short term bunkering contracts are made approximately a week before the liner's planned service. In real practice, prior to the service, liner operators negotiate with the bunker suppliers and make bunkering decisions according to the quotes offered by suppliers. The key decisions in bunkering problem are where to bunker and how much to bunker?

In our model formulation, we assume that there are refueling stations at several ports-of-call along the liner service. In addition, to decrease the bunkering cost a vessel may detour to the ports which are not on her service schedule. Bunkering operation takes approximately 2–3 hours and it can be performed during the service (loading and unloading) at ports[1]. Therefore, it can be said that bunkering at one of the service ports is more time efficient than detouring to another port. We use the same notation as in Section 3 to formulate the bunkering problem. Additionally, We define d_{ij} to show the distance between ports i and j. To denote the bunkering ports, we introduce set M. The set of all ports is given by $N' = N \cup M$ and N'_i presents the set of ports that are reachable from port i. Since bunkering is a planning level problem, we assume that port time takes its expected value. As in Section 3, we assume that fuel consumption is directly related to the sailing speed for which, we use the empirical model of Yao et al. (2012) to compute the fuel consumption rate. As mentioned in Section 3, vessels generally consume two types of fuel in a voyage: low and high sulfur fuel. Since high sulfur fuel is mostly used while cruising, we focus on decisions related to this type of fuel, which has further simplified the problem. We assume that vessel has enough low sulfur fuel inventory for a voyage. The aim of the bunkering model is to find the optimal bunkering ports, bunkering quantities and sailing speed in order to minimize the total bunkering and delay cost. The optimal solution of this model can be used to decide on the bunkering ports, average bunkering quantity required to complete the service and the average sailing speed.

To include varying fuel prices, we extend the dynamic programing model given in (4) and (5). Since bunkering decision depends on the remaining bunker on the vessel, we need to store the bunker inventory. We define w_i to denote the bunker inventory when vessel arrives at port i. The state space for port i is given by (i, t_i, w_i). At each stage, we have to decide whether to detour for bunkering (if possible), the sailing speed to the next port and how much to bunker at the current port (if possible)? We use r_i to denote the bunker price per ton at port i and x_i to denote the bunker order-up-to-level at port i. If vessel does not bunker at port i, then $x_i = w_i$. Let $V(i, t_i, w_i)$ denote the minimum cost from port i up to port n given that the arrival time to port i is t_i and the remaining bunker is w_i. Then, we have the following DP recursion,

$$V(i, t_i, w_i) = \min_{\substack{v_{ij} \in [\underline{v}, \bar{v}] \\ x_i \leq C, j \in N'_i}} \Big\{ \varphi(E[S_i] + [\alpha_i - t_i]^+) + r_i(x_i - w_i) + \theta_i[t_i - \beta_i]^+ + V(j, t_j, w_j) \Big\} \tag{17}$$

where $t_j = \max\{\alpha_i, t_i\} + E[S_i] + d_{ij}/v_{ij}$ and $w_j = x_i - g(v_{ij})$. The boundary condition is given by:

$$V(n, t_n, w_n) = \varphi(\mathbb{E}[S_n] + [\alpha_n - t_n]^+) + \theta_i[t_n - \beta_n]^+. \tag{18}$$

Now, we will provide some structural results for the bunkering decision. The cost function $V(i, t_i, w_i)$ is decreasing in w_i for every t_i and i. In other words, as the initial bunker inventory at port i increases, the total cost from port i to port n decreases. Since carried bunker does not affect the total cost, increase in the initial bunker inventory does not increase the total cost.

Lemma 5.1. *Let assume a vessel visits ports i and j consecutively in the optimal schedule and the bunker prices (dollars per ton) at ports i and j are denoted by* r_i *and* r_j *and* $r_i > r_j$. *Let* v^*_{ij} *denote the optimal value of the sailing speed at leg* $i - j$, *and* x^*_i *denote the optimal bunker up-to-level at port i when the initial bunker inventory is* w_i. *Then,* $x^*_i = g(v^*_{ij})$, *if* $w_i < g(v^*_{ij})$ *and* $x^*_i = w_i$, *if* $w_i \geq g(v^*_{ij})$.

The proof of the lemma is given in the supplementary document. Lemma 5.1 implies that we can reduce the search space for the bunkering decision variable in special cases.

6. Computational results

We devote this section to a computational study for discussing different aspects of our proposed solution methods. We conduct experiments by using real data from a major European liner shipping company. In particular, we evaluate performances of dynamic programing and deterministic models and investigate managerial insights obtained from these models. We next explain our simulation setup in detail and then present our numerical results.

6.1. Experimental settings

We design experiments by using real data from the case shipping company including three routes with 16, 11 and 8 ports, respectively. These data include the distances between ports, vessel arrival and berthing times, and port service times. The detailed schedules of these routes are presented in Tables 1–3.

As proposed by Qi and Song (2012), we assume the service times S_i follow the uniform distribution. The lower bound on the service times at each port are set to the port times given in Tables 1 and 2. We apply sensitivity analysis with respect to the port times and set the range between upper (u_i) and lower (l_i) bound values as 6 and 10. The cost function of our proposed model can be divided into three parts: fuel consumption cost, port time cost (waiting and service) and delay cost. We use the empirical formula of Yao et al. (2012) to compute the fuel consumption per time unit. Their model presents the relation between sailing speed and fuel consumption rate by considering the size of the vessel. We test the empirical model by using the historical fuel consumption data of the case shipping company. The results are demonstrated in Fig. 2. In our numerical experiments, we set the constants in fuel consumption function as $k_1 = 0.004595$ and $k_2 = 16.42$. Fuel consumption and port time cost is directly related to the fuel price. As mentioned before, international regulations limit the use of high sulfur fuels near ports. According to the data obtained from the shipping company, their vessels consume two types of fuel with different sulfur contents. The fuel consumed at ports (low sulfur

[1] Private conversation with a liner operator, 2016.

Table 1
An existing schedule of the shipping company with 16 ports.

Port	Distance	Arrival	Service start	Departure	Port time (hours)	Weight of port
P0	–	–	–	18/09/15 23:00		
P1	40	19/09/15 02:00	19/09/15 02:00	19/09/15 13:00	11.0	7
P2	125	19/09/15 22:30	19/09/15 23:00	20/09/15 08:00	9.0	2
P3	190	20/09/15 21:30	20/09/15 22:30	21/09/15 15:30	17.0	9
P4	1149	25/09/15 02:00	25/09/15 03:30	25/09/15 17:30	14.0	10
P5	249	26/09/15 13:00	26/09/15 14:00	27/09/15 00:00	10.0	7
P6	182	27/09/15 13:00	27/09/15 13:00	28/09/15 00:00	11.0	8
P7	97	28/09/15 07:00	28/09/15 08:30	28/09/15 17:30	9.0	4
P8	40	28/09/15 20:45	28/09/15 22:00	29/09/15 04:30	6.5	3
P9	50	29/09/15 08:30	29/09/15 09:30	29/09/15 16:30	7.0	7
P10	112	30/09/15 01:00	30/09/15 01:00	30/09/15 13:30	12.5	8
P11	136	01/10/15 00:00	01/10/15 01:00	01/10/15 11:30	10.5	2
P12	966	04/10/15 17:15	04/10/15 18:00	05/10/15 01:00	7.0	5
P13	416	06/10/15 10:00	06/10/15 11:30	07/10/15 01:00	13.5	3
P14	190	07/10/15 14:30	07/10/15 15:00	08/10/15 00:30	9.5	1
P15	125	08/10/15 09:30	08/10/15 10:00	08/10/15 16:00	6.0	9

Table 2
An existing schedule of the shipping company with 11 ports.

Port	Distance	Arrival	Service start	Departure	Port time (hours)	Weight of port
P0	–	–	–	22/01/15 08:30		
P1	116	22/01/15 16:00	22/01/15 17:05	23/01/15 10:30	17.5	3
P2	410	24/01/15 10:00	24/01/15 11:30	24/01/15 19:00	7.5	7
P3	35	24/01/15 21:30	24/01/15 21:30	25/01/15 09:30	12.0	5
P4	1546	29/01/15 07:00	29/01/15 08:00	29/01/15 19:00	11.0	9
P5	1681	03/02/15 15:30	03/02/15 18:30	04/02/15 18:00	23.5	3
P6	537	06/02/15 07:00	06/02/15 07:00	07/02/15 04:00	21.0	8
P7	142	07/02/15 13:00	07/02/15 14:00	07/02/15 23:00	9.0	6
P8	617	09/02/15 07:30	09/02/15 07:30	09/02/15 21:30	14.0	10
P9	143	10/02/15 07:00	10/02/15 07:30	11/02/15 01:00	17.5	4
P10	180	11/02/15 13:00	11/02/15 13:30	11/02/15 21:30	8.0	1

Table 3
An existing schedule of the shipping company with 8 ports.

Port	Distance	Arrival	Service start	Departure	Port time (hours)	Weight of port
P0	–	–		04/01/15 05:30		
P1	430	05/01/15 06:30	05/01/15 09:30	06/01/15 10:00	24.5	4
P2	593	07/02/15 21:00	07/01/15 21:00	08/01/15 00:30	3.5	3
P3	90	08/01/15 05:30	08/01/15 06:00	08/01/15 19:00	13.0	6
P4	484	10/01/15 05:00	10/01/15 08:00	10/01/15 17:30	9.5	5
P5	435	11/01/15 20:30	11/01/15 21:30	12/01/15 09:00	11.5	10
P6	100	12/01/15 15:30	12/01/15 15:30	12/01/15 19:30	4.0	2
P7	625	14/01/15 14:00	14/01/15 14:00	14/01/15 23:00	9.0	7

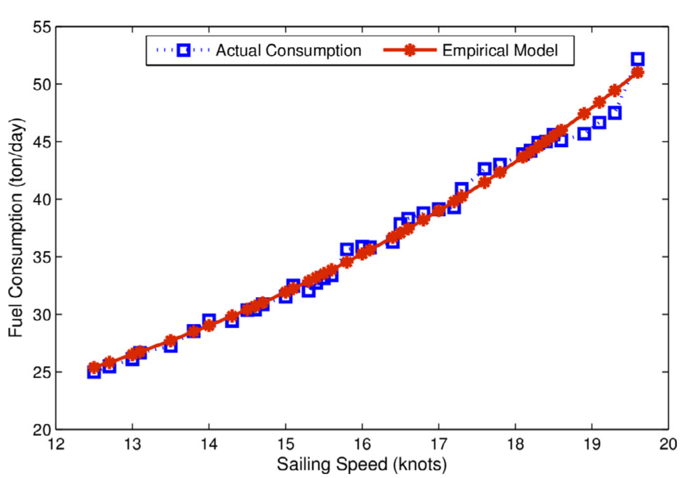


Fig. 2. Comparison of actual and estimated fuel consumption.

fuel) is more expensive than the one consumed during sailing (intermediate sulfur fuel). We refer readers to Bunkerworld (2016)'s web site for the price relationship between various fuel types. In our experiments, fuel prices are set by considering the average prices charged to the liner shipping company. The price of fuel used in the ports and during sailing are set to $r_p = 380$ dollars per ton and $r_s = 185$ dollars per ton, respectively. To measure the impact of waiting, we apply sensitivity analysis with respect to the waiting cost. Waiting cost per hour is directly related to the price of fuel consumed at the ports. According to the information obtained from the shipping company, a vessel consumes 2.0 tons per day fuel on average while waiting at a port which approximately corresponds to 31 dollars per hour. Considering this information, we apply sensitivity analysis with respect to the waiting cost selected as $\varphi \in$ [30 dollars, 50 dollars].

On the other hand, it is difficult to estimate the monetary value of delay penalty since it is also related to the loss of customer goodwill. Since customers served in each port are different, our delay penalty for each port is also different. We define weights for each port to show the importance of that port. As the real data

Table 4
Upper bound on the optimality gap.

Instances			Discrete DM	Deterministic model	Relative gap
n	δ	φ	UB	LB	(UB − LB)/LB (%)
8	50	30	51,328	50,779	1.08
		50	53,247	52,699	1.04
	100	30	51,548	50,779	1.52
		50	53,468	52,699	1.46
11	50	30	100,579	100,000	0.58
		50	103,998	103,427	0.55
	100	30	101,807	100,303	1.50
		50	105,228	103,723	1.45
16	50	30	73,834	72,402	1.98
		50	77,807	76,372	1.88
	100	30	74,687	72,405	3.15
		50	78,661	76,375	2.99

is not available, we generate these weights randomly between 1 and 10; 1 being the lowest and 10 being the highest priority. The weights of the ports are given at the last column in Tables 1 and 2. Let κ_i be the weight of port i. Then, the delay penalty of port i is given by $\theta_i = \kappa_i\delta$, where δ can be defined as the delay coefficient. Since we do not have liner data regarding delay penalty, we set δ to [50, 100] to represent low and high penalty values. This leads delay cost to change between 50 dollars and 1000 dollars per hour (Qi & Song, 2012).

Another important parameter that affects the speed policy is the time window. The schedule of the vessel is planned with respect to the available time slots of the ports. These time windows are defined according to the contractual agreements with customers and ports. To test the performances of our models, we define two values for the width of time window $t^w \in [3, 6]$ corresponding to tight and loose window. In our experiments, time window of each port is extracted from the real data of the shipping company. The starting time (α_i) of the time window is set to the service start time at port i and the latest service start time is computed as $\beta_i = \alpha_i + t^w$. Lastly, we assume that sailing speed ranges from 12.5 knots to 19.5 knots (Yao et al., 2012). To implement dynamic programing, we discretize time by an interval of 5 minutes. We present detailed results on the effect of discretization in supplementary document. For our experiments, we used a computer with 1.8 gigahertz Intel Core i5 with 4 processors and 8 gigabytes of RAM. The solution methods are coded in MATLAB 8.2.0 and run under Windows 8.1 operating system.

6.2. Results analysis

As discussed before, the optimal objective values of the deterministic model and the discretized dynamic programing model provide lower and upper bounds for the continuous model, respectively. We compare the percentage gaps between these lower and upper bounds in Table 4. We use UB and LB to denote the upper and lower bounds provided by the discretized dynamic model and the deterministic model, respectively. The first three columns indicate the characteristics of the test instances. The next two columns give the optimal objective values of dynamic and deterministic models. The last column presents the percentage gap between the objective function values of these two bounding models. Our aim is to measure the effects of delay and waiting costs on the tightness of these bounds. In this experiment, we set the range between the upper and lower bounds on the service time to 6 hours and to emphasize the effect of the delays, we set the width of the time window to 3 hours.

The first observation from the results in Table 4 is that the optimality gap is less than 4.0%. The results show that the relative differences are mostly affected by the delay penalties. As delay penalty increases, optimal objective value of the dynamic model increases significantly. In our problem setting, we assume that delays are resulted from the uncertainty of port times. In other words, more variation in port times increases the probability of delays. Dynamic model considers all possible realizations of the port times and hence, it is sensitive to cost parameters. On the other hand, deterministic model computes the optimal sailing speeds by only considering the expected port times. Therefore, it is slightly affected by the changes in delay penalty. Consequently, the percentage gap tends to increase with the delay penalty. Regarding the impact of the problem size, we have not observed any correlation between the number of ports visited and the percentage gap. We conjecture that the percentage gap is significantly affected by the problem structure. When we examine the test problems with 11 and 16 ports, we see that the distance between ports is shorter in 16 port test. Since dynamic model considers the uncertainty in the problem, it can estimate the effects of delays in shorter distances better than the deterministic model.

In the next experiment, we test the effect of delay penalty on the speed decision. We use the shipping data given in Table 3 and set the width of the time window to 3 hours and waiting time to 30 dollars per hour. To observe the cumulative effect of the delay, we set the service time realizations of ports P1 and P2 to the upper bound on their service time. The service times of the remaining ports are set to their expected values. The realization of port service time for this experiment is given as {30.5, 9.5, 16, 12.5, 14.5, 7, 12} for ports P1 to P7, respectively. In this experiment, we analyze the reaction of the dynamic model to this long port time. Table 5 presents the results of this experiment. The first column correspond to the delay penalty parameter. The next columns present the resulting delay time (in hours) and sailing speeds according to the port time realization. The first observation we made is that the speed decision is significantly affected by the delay penalty. When delay penalty is high, vessel is only late at port P3. This delay is resulted from the long port time in P1 and P2. When delay penalty is low, vessel arrives at ports P3 and P6 later than the time windows. Moreover, we observe that sailing speed decision depends on the fuel consumption cost and the penalty paid for being late. When delay penalty is low, sailing at slow speed and arriving late to ports can be more cost effective than sailing at high speed. This behavior can be attributed to the impact of port weights. As mentioned before, each port has different delay penalty and these penalties are computed with respect to the port weights. Weight of port P6 is 2 while the maximum weight is 10 and hence, the vessel prefers to arrive later than the time window instead of sailing at higher speed. We can deduce from this result that in unexpected events, being late to some ports can be less costly.

6.3. Simulation results

In this section, we test the performances of dynamic and deterministic models. We also provide a sensitivity analysis with respect to various problem parameters. For our analysis, we generate another speed strategy based on a heuristic method. This heuristic method aims at maximizing service level by avoiding delays. It computes the sailing speed in such a way that the vessel arrives at ports at the middle of respective time windows. On the other hand, to improve the performance of deterministic model, we refine its speed policy during the simulation. In our simulation experiments, we estimate the expected total costs of these models by simulating the service time realizations over 250 sample paths. We use common random numbers when simulating the performances of the different solution methods. In the sequel, we refer to the average costs obtained by the optimal policy of the discretized dynamic model, deterministic model and the heuristic method as DDM, DTM and HM, respectively.

Table 5
Analysis on delay penalty and waiting cost.

Instances	Delay time (hours)/	Port index						
δ	speed (knots)	P1	P2	P3	P4	P5	P6	P7
50	Delay	–	–	3.83	–	–	0.38	–
	v_i	15.35	19.33	19.29	17.23	17.23	17.66	16.00
100	Delay	–	–	3.58	–	–	–	–
	v_i	15.35	19.50	19.29	17.34	17.40	16.66	15.82

Table 6
Simulation results over 250 runs.

Instances				DDM		DTM		HM		% gap with DDM	
n	t^w	δ	φ	Mean	Std dev	Mean	Std dev	Mean	Std dev	DTM (%)	HM (%)
8	3	50	30	51,424	1260	52,072	1583	52,315	1528	1.26	1.73
		50	50	53,356	1350	54,031	1650	54,246	1614	1.27	1.67
		100	30	51,640	1382	53,005	2459	52,845	1995	2.64	2.33
		100	50	53,572	1469	54,963	2521	54,776	2074	2.60	2.25
	6	50	30	50,484	1079	51,088	1716	51,913	1403	1.20	2.83
		50	50	52,416	1169	53,028	1791	53,845	1490	1.17	2.73
		100	30	50,549	1094	52,198	3081	52,156	1715	3.26	3.17
		100	50	52,481	1184	54,137	3151	54,088	1795	3.16	3.06
11	3	50	30	100,800	1799	102,666	2601	103,207	2182	1.85	2.39
		50	50	104,228	1890	106,134	2663	106,636	2270	1.82	2.31
		100	30	102,124	2698	105,374	4889	105,906	3866	3.18	3.70
		100	50	105,553	2781	108,843	4947	109,335	3945	3.11	3.58
	6	50	30	98,931	1312	101,000	2662	101,629	1734	2.09	2.73
		50	50	102,361	1408	104,453	2732	105,058	1824	2.04	2.64
		100	30	99,392	1531	103,184	4724	103,124	3015	3.81	3.75
		100	50	102,821	1623	106,637	4790	106,552	3094	3.71	3.63
16	3	50	30	73,941	1856	77,085	4594	76,742	3712	4.25	3.79
		50	50	77,916	1958	81,132	4662	80,713	3798	4.13	3.59
		100	30	74,830	2915	81,386	8871	79,804	7010	8.76	6.65
		100	50	78,806	3000	85,426	8991	83,775	7084	8.40	6.31
	6	50	30	72,548	1204	74,992	3736	75,513	3113	3.37	4.09
		50	50	76,521	1317	79,011	3808	79,484	3200	3.25	3.87
		100	30	72,748	1471	77,989	7203	77,576	5826	7.20	6.64
		100	50	76,722	1573	82,068	7306	81,547	5897	6.96	6.29

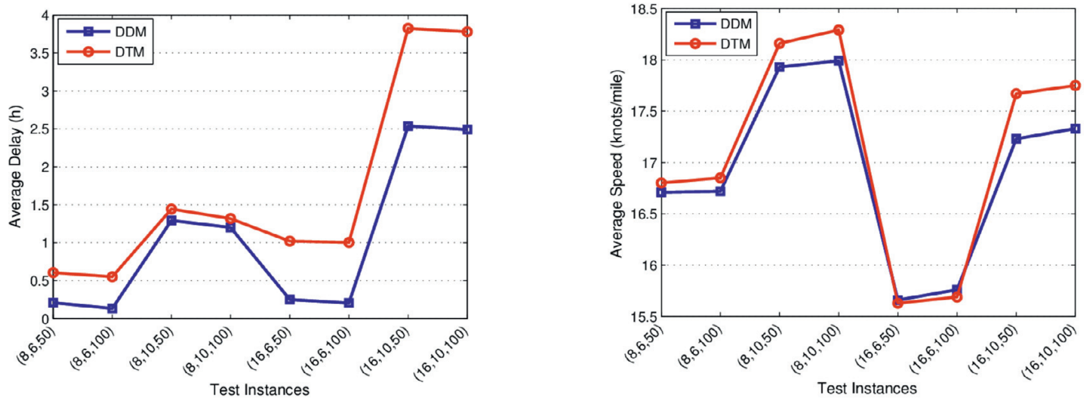
Table 6 presents average costs over all simulation runs for varying factors. In this experiment, we set the range between the upper and lower bounds on the service time to 6 hours. We aim to compare the performances of speed optimization methods with respect to width of time windows, delay, waiting cost and size of the problem. The total cost obtained by DDM is used as a base approach to report the relative performances of the solution methods. It is validated that the percent gaps between the total expected costs obtained by DDM and the remaining solutions methods are statistically significant at 99% level. The details of ANOVA and the post-hoc analyses are presented in supplementary document.

Comparing the percentage gaps in Table 6, we observe that the dynamic model constantly outperforms the other solution strategies. Moreover, the performance gaps between DDM and the remaining solution methods are statistically significant across all problems instances with 8, 11 and 16 ports, respectively (see Tables B.6–B.8 in supplementary document). When we look into DTM and HM, we observe that performance of DTM is better for low delay penalty. The performance of deterministic model deteriorates as the delay penalty increases. The speed decision of deterministic model is static and it does not change with the realization of port time. Therefore, delay in one port leads to delays in the successive ports. On the other hand, heuristic method aims to avoid delays and hence, its speed policy is advantageous for high delay penalty. As it is seen in Table 6, DTM performs worse than HM in almost all instances when the delay penalty is high. We caution the reader that the performances of DTM and HM deteriorate when the problem size is large. As we mentioned before, uncertainty in the problem increases with the number of ports. Therefore, DDM performs significantly better than the other methods.

Next, we apply sensitivity analysis with respect to the uncertainty in port times and analyze its effect on delay and sailing speed. In this experiment, we compare the speed policy of dynamic and deterministic model with respect to service time uncertainty. We set waiting cost to 30 dollars per hour and time window length to 3 hours. We test the models by setting the range between upper and lower bound values of the service time $\Delta = u_i - l_i$ as 6 and 10. Fig. 3 illustrates the trend in average delay and sailing speed. In this figure, the horizontal axes correspond to the test instances denoted by (n, Δ, δ). As delay penalty coefficient δ increases, optimal policies of the dynamic and deterministic model impose to sail at high speed so that the overall delay cost is minimized. We can observe this result by comparing Fig. 3(a) and (b). The effect of delay penalty is more significant when the variation at port service time is high and problem size is large. An interesting result is that the average sailing speed of the deterministic model is generally higher than the one in dynamic model. Average sailing speeds of these models are closer as the variation in the service time decreases. Although the deterministic model imposes a higher sailing speed than the dynamic model, the delay resulted from its speed policy is higher than the delay resulted from the dynamic speed policy.

6.4. Bunkering problem results

In this section, we test the bunkering policy of dynamic model presented in Section 5. We consider the liner service given in Table 2. The vessel is allowed to bunker at all ports along the service. Moreover, it may detour to other ports that provide cheaper bunker. The alternative bunkering port information is given in



(a) Delay Time (b) Sailing Speed

Fig. 3. Analysis on delay and sailing speed.

Table 7
Alternative bunkering ports.

Alternative	Distances (nautical miles)			
Bunkering ports	P0	P1	P4	P5
A1	225	100	–	–
A2	–	–	200	1600

Table 7. In this experiment, we assume that alternative bunker ports do not have any time windows. We set the average bunkering time to 3 hours and the bunker prices in port B1 and B2 to 150 dollars and 125 dollars, respectively. We assume that bunker prices at the scheduled ports gradually decrease and increase along the service route (see Table 8). In this study, we look into details of the optimal speed and bunkering decisions. In particular, we analyze the effect of port time windows, delay cost and bunker price on these decisions.

We first study the effect of port time windows and delay cost. Although detour for bunkering is an effective way to reduce fuel cost, it may delay the planned arrival time for the scheduled ports. In this experiment, we analyze the bunkering decision with respect to different delay cost and time window parameters. We set the waiting cost to 30 dollars per hour and initial bunker inventory to 50 tons. Tables 8 and 9 show optimal bunkering and speed decisions for tight and wide time windows, respectively. As seen from these tables, vessel generally detours to reduce fuel cost. When the delay penalty is low, detour for bunkering is more cost effective even if it increases the total traveled distance. We can observe this result by comparing the total bunkering cost. It is important to note that the total bunkering amounts in low and high delay penalty cases are different due to the additional distance covered during detour. Moreover, vessel increases its speed following the detour to prevent the delays. This result shows that although detour leads to additional bunker cost, it may decrease the total bunkering cost of the service. By comparing Tables 8 and 9, we observe that the total bunkering cost is lower in wide time window case. Wide time windows allow a vessel to sail at slower speed and hence, total required bunker amount for the service decreases.

We next study the effect of bunker prices on the sailing speed and detour decisions. We set the width of the time window to 3 hours and delay parameter to 100 dollars per hour. We consider two scenarios corresponding to low and high bunker prices. While in the first scenario bunker prices are closer, the differences are more significant in the high bunker price scenario. Optimal bunkering and speed decisions are presented in Table 10. When bunker prices are high, detour is more cost efficient even for high delay penalty. On the other hand, when bunker prices are closer, detour for bunkering may not cover the additional bunker cost due to detouring and cost of delay. Therefore, vessel may prefer to bunker at her planned schedule. These results indicate that

Table 8
Optimal bunkering and speed decision with respect to low and high delay cost ($t^w = 3$).

Port	Bunker price	Low delay cost		High delay cost	
	(dollars per ton)	Bunkering amount (tons)	Sailing speed (knots)	Bunkering amount (tons)	Sailing speed (knots)
P0	300		18.75		13.64
A1	150	204	19.33		
P1	275		19.07		15.47
P2	250		17.50		17.50
P3	225		17.37	142	16.02
P4	200		15.38	18	15.38
A2	125	306	16.24	306	16.24
P5	175		15.13		15.13
P6	200		15.77		15.77
P7	225		17.38		17.38
P8	250		14.30		14.30
P9	375		14.40		14.40
P10	300		–		–
Total bunkering Cost		68,850		73,800	

Table 9
Optimal bunkering and speed decision with respect to low and high delay cost ($t^w = 6$).

Port	Bunker price	Low delay cost		High delay cost	
	(dollars per ton)	Bunkering amount (tons)	Sailing speed (knots)	Bunkering amount (tons)	Sailing speed (knots)
P0	300		18.75		13.64
A1	150	200	19.33		
P1	275		19.07		15.47
P2	250		17.50		17.50
P3	225		16.80	140	15.61
P4	200		14.81	18	15.38
A2	125	306	16.24	305	16.09
P5	175		15.79		15.79
P6	200		15.77		15.77
P7	225		16.90		16.90
P8	250		14.30		14.30
P9	375		14.40		14.40
P10	300		–		–
Total bunkering Cost		68,250		73,225	

Table 10
Optimal bunkering and speed decision at different bunker price.

Port	Low bunker price			High bunker price		
	Bunker price (dollars per ton)	Bunkering amount (tons)	Sailing speed (knots)	Bunker price (dollars per ton)	Bunkering amount (tons)	Sailing speed (knots)
P0	240		13.64	600		18.75
A1	120			300	204	19.32
P1	220		15.47	550		19.07
P2	200		17.50	500		17.50
P3	180	142	16.02	450		17.37
P4	160	18	15.38	400		15.38
A2	100	306	16.24	250	306	16.24
P5	140		15.13	350		15.12
P6	160		15.77	400		15.77
P7	180		17.38	450		17.38
P8	200		14.30	500		14.30
P9	220		14.40	550		14.40
P10	240		–	600		–

delay penalty should be determined by considering the variation in fuel prices. Assigning high delay penalty to a port forces vessel to arrive within the time window, which may increase the fuel consumption cost significantly. On the other hand, assigning low delay penalty may result in arriving much later than the time window of important ports, which leads to high operational costs and damages the reliability of liner company. Moreover, bunker decision is directly related to bunker prices which also affect the service route and sailing speed.

7. Conclusion

In this paper we addressed the speed optimization and bunkering problem in liner shipping. We first focus on the stochastic speed optimization problem. Despite the fact that vessel scheduling and routing has been well studied in recent years, stochastic aspects of the problem has not explicitly been considered in the literature. This paper aimed to fill this gap. By considering the uncertain port times, we formulated the problem as a dynamic program. The proposed model incorporates possible costs resulted from waiting and delay in ports, and take the port service times into consideration when making speed decisions. To implement the dynamic model, we discretize the state space, which provides an upper bound on the optimal expected cost. We also formulate a deterministic model by assuming that the stochastic port times take on their expected values and we show that this model provides a lower bound on the optimal expected cost. In practical implementations, these lower and upper bounds designate the estimated minimum and maximum total cost of a vessel service. We also study the properties of the optimal value functions and investigate the variation of sailing speed with respect to the vessel arrival time.

Fuel consumption cost is one of the major operational costs of liner companies. Sailing speed and bunker prices have direct impact on this cost. Therefore, in the second part of the paper, we study the relationship between sailing speed and bunkering. We mainly focus on where to bunker, how much to bunker and determining the vessel speed at each leg along the service route. We develop a dynamic programing model for joint bunkering and speed optimization problem. Different than the proposed bunkering models in the literature, our formulation allows a vessel to bunker at the ports that are not on the planned service route. The aim of the proposed model is to plan the bunkering operation. In other words, optimal solution of this model can be used to decide on the bunkering ports, average bunkering quantity required to complete the service and the average sailing speed. Since bunker contracts are set before the vessel departs for a voyage, our model is formulated by considering the expected service times. However, stochastic dynamic model proposed in Section 3 can be used to assign the sailing speed to the next port if vessel deviates from the planned schedule.

We conduct a computational study by using real-life cases from a liner shipping company. We test the performances of the proposed speed optimization and bunkering models. The numerical experiments for the stochastic speed optimization problem demonstrate that determining sailing speed by taking into account the uncertain port times can bring significant cost improvements. This result is consistent with the real life examples. As Notteboom (2006) stated, fast growth in cargo volumes has increased the possibility of port congestion. Planning the service schedule only based on the expected port times can disrupt the whole liner service schedule. Therefore, sailing speeds and port arrival times should be computed by considering the uncertainties at ports to mitigate associated risks.

Our computational study also provides a number of useful practical insights. Many studies in maritime literature restrict vessels to arrive within the time window and compute the optimal sailing speed by only considering the fuel consumption cost. However, delays are very common in real-life cases as reported by Vernimmen et al. (2007). Our numerical experiments indicate that sailing speed is significantly affected by the delay penalty. Even waiting cost can influence on the speed decision. Due to uncertainties in ports, a vessel can prefer to sail at high speeds to avoid delays or sail at slow speed to arrive late at some ports in order to avoid arriving early to following ports. Thus, ship operators are likely to see clear benefits when making decisions considering both cost types. On the other hand, our numerical results for bunkering problem show that bunkering amount and ports should be determined by considering the planned schedule of vessel, delay cost and bunker prices. For tight schedules, it can be more cost effective to bunker at ports on the planned schedule of a vessel, even if the fuel prices at the alternative bunkering ports are cheaper. Moreover, delay penalty significantly affects the bunkering port selection. Therefore, delay penalty parameter should be assigned by considering the bunker prices. We also analyze the implications of bunker prices on the speed decision. Our results show that bunkering port and quantity decisions are directly related to bunker prices which also affect the service route and sailing speed. When bunker prices are closer at all ports, vessel may not prefer detour for bunkering due to the additional bunker cost of detouring.

There are a number of research directions to pursue. Although we focus on the uncertainties in liner shipping, we assume that weather does not affect the speed decision. Further research is needed to develop new models and solution methods that avoid this assumption. This line of research can benefit from the studies proposed for weather routing. Another future research direction is to incorporate port swapping and port skipping options. In practical applications, ship operators can deal with delays by reshuffling the order of ports or skip some ports. Even there are cases that a vessel can leave the port before completing its service to avoid delay to next ports. A promising direction of future research would be to model these decisions as a dynamic program by considering the uncertainties along the journey.

Acknowledgment

This research is supported in part by EU FP7 project MINI-CHIP (Minimising Carbon Footprint in Maritime Shipping) under Grant number PIAP-GA-2013-611693.

Supplementary material

Supplementary material associated with this paper can be found, in the online version, at 10.1016/j.ejor.2016.10.002.

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