INDUSTRY STRUCTURE AND ECONOMIC GROWTH

ESSAYS ON THE IMPACT OF THE PRODUCTION STRUCTURE AND OPENNESS ON DEVELOPMENT

Carlos Humberto Ortiz Quevedo

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ABSTRACT

This thesis contains five essays on the relationship between production structure and economic growth. The first chapter examines the conditions under which intermediate inputs can be net out from the production functions in order to relate primary factors of production with net output in each sector. We conclude that this transformation is not always possible, and this finding sets the agenda for the next three chapters.

The second chapter augments Lucas' (1988) two-sector growth model of learning-by-doing by allowing for interindustry linkages between the sectors. Under certain conditions, the model reproduces a number of the key stylized facts of the industrialization process. We also examine the advantages and disadvantages of openness to world markets and the possibilities of an import-substitution strategy as a way of development.

The third chapter analyses the effects of education on the process of economic diversification in underdeveloped countries. Under the assumption that most technological changes in underdeveloped countries are driven by imitation, we describe how a process of input-output deepening evolves and how this process increases real wages.

In the fourth chapter we use cross-country regressions to show that the correlation between some measures of interindustry dependence and the growth rate of per capita GDP seems to be positive, significant and robust.

The fifth chapter explores the consequences for economic growth of decreasing returns to scale due to fixed factors at the firm's level. By assuming that the firm's fixed factors are produced with firm creation, two possible equilibria are examined: one with entry restrictions, and another with proliferation of firms. The former is characterized by a steady state with no growth; the latter is characterized by constant returns at the aggregate level and unbounded growth. We discuss the relevance of this model for industrialization and development.

CONTENTS

Abstract 2

Contents 4

List of Tables 8

List of Figures 9

Acknowledgements 10

1 The Transformation of Gross Output Functions into Net Output Functions 12

1.1 Introduction 13

1.2 The Transformation When No Externalities Exist 14

1.2.1 A Single Good 14

1.2.2 Two Goods 16

1.3 The Transformation in the Presence of Externalities 23

1.3.1 A Single Good 23

1.3.2 Two Goods 27

1.4 Concluding Comments 30

2 Interindustry Linkages, Learning-By-Doing and Endogenous Growth 32

2.1 Introduction 33

2.2 The Model 41

2.3 The Competitive Solution (Closed Economy) 43

2.3.1 Consumer's Behaviour 43

2.3.2 Firm's Behaviour 44

2.3.3 Price and Output Determination 45

2.3.4 Workforce Allocation 47

2.3.5 Dynamics: Relative Prices and Welfare 50

2.3.6 Theory and Facts 56

2.3.7 Simulations 58

2.4 The Competitive Solution (Small Open Economy) 59

2.4.1 Terms of Trade and Specialization 59

2.4.2 Country Specialized in Good 1 61

2.4.3 Country Specialized in Good 2 64

2.4.4 World Market Equilibrium 65

2.4.4.1 The Static Equilibrium 65

2.4.4.2 The Dynamic Equilibrium 67

2.4.5 Theory and Facts Again 71

2.4.5.1 Some Previous Results 72

2.4.5.2 Dynamic Gains from International Trade 73

2.4.5.3 The Strategy of Development through "Import Substitution" 79

2.5.5.4 Trade Biases 84

2.5 Summary and Concluding Comments 85

Appendix Quality Adjusted Terms of Trade 88

Notation 89

Simulation Diagrams 90-92

3 Economic Structure, Education and Growth 93

3.1 Introduction 94

3.2 A Model of Economic Diversification through Education 97

3.2.1 The Model 97

3.2.2 The Instantaneous Equilibrium 102

3.2.3 The Dynamic Equilibrium 109

3.3 The Open Economy Case 113

3.4 An Aggregative Model of Economic Diversification 119

3.4.1 The Model 120

3.4.2 The Instantaneous Equilibrium and the Aggregate Production Function 123

3.4.3 The Dynamic Equilibrium 127

3.4.4 Summary and Concluding Comments 132

Appendix 1: Generalizing the First Model for a High Degree of Intertemporal Substitutability 134

Appendix 2: Local Stability in the Dynamic Model with Capital Accumulation 135

4 Production 'Roundaboutness' and Economic Growth: Some Empirical Evidence 139

4.1 Introduction 140

4.1.1 Objective and Theory 140

4.1.2 Related Empirical Research and Estimation Strategy 143

4.2 Cross-Country Growth Regressions 145

4.2.1 Indirect Measures of Production Roundaboutness 145

4.2.2 Direct Measures of Production Roundaboutness 150

4.3 Concluding Comments 162

Appendix Table 4.7 165-167

5 Decreasing Returns, Competition and Long-Run Growth 168

5.1 Introduction 169

5.2 The Model with No Entry 172

5.3 The Model with Proliferation of Firms 178

5.3.1 The Competitive Equilibrium with Entry Costs 178

5.3.2 The Competitive Equilibrium with Operating Costs 183

5.4 Some Final Comments on Proliferation of Firms and Industrialization 186

Appendix 1 The Saddle-Point Property of the Model with No Entry 188

Appendix 2 The Optimal Paths of Consumption and Capital Accumulation (Model with Entry Costs) 188

References 190

LIST OF TABLES

Table 2.1 Annual Growth Rates of Real GDP 1950-1983 35

Table 4.1 Regressions for Per Capita GDP Growth 1960-1988 147

Table 4.2 Unbalanced Data Set 153

Table 4.3 Growth Regressions from Unbalanced Panel 154

Table 4.4 Balanced Data Set 157

Table 4.5 Growth Regressions from Balanced Panel 158

Table 4.6 Cross-Section Data: 1950-1983 (9 Economies) 161

Table 4.7 Cross-Section Data: 1960-1988 (48 Economies) 165

Table 5.1 Average Annual Growth Rates of Number of Manufacturing Establishments and Manufacturing Output in Hong-Kong and South Korea 169

LIST OF FIGURES

Figure 2.1 Input-Output Matrix and Labour Allocation 41

Figure 2.2 Labour Market Equilibrium 48

Figure 2.3a Dynamics of the Closed Economy (High Substitutability) 52

Figure 2.3b Dynamics of the Closed Economy (Low Substitutability) 54

Figure 2.4 The International Price Line 61

Figure 2.5 Domestic Input-Output Matrix 61

Figure 2.6 Imports Matrix 62

Figure 2.7 Dynamic World Market Equilibrium 70

Figure 3.1 Input-Output Matrix 98

Figure 3.2 Demand Structure ($\sigma > 1$) 105

Figure 3.3 Employment Structure ($\sigma > 1$) 108

Figure 3.4 Labour Force Dynamics 111

Figure 3.5 Labour Supply 112

Figure 3.6 Input-Output Matrices (South and North) 114

Figure 3.7 International Allocation of Labour 117

Figure 3.8 Structure of Technologies 121

Figure 3.A1 The Determinant of the Transition Matrix 136

Figure 3.A2 Simulation of the Determinant of the Transition Matrix 138

Figure 5.1 Dynamics of Per Firm Consumption and Per Firm Capital 177

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CHAPTER 1

THE TRANSFORMATION OF GROSS OUTPUT FUNCTIONS INTO NET OUTPUT FUNCTIONS

1.1 INTRODUCTION

In the real world, production requires not only primary factors (labour, machines, land, etc.), but also intermediate inputs. The technological relations that map the use of primary factors and intermediate inputs into the maximum (gross) output are denominated gross output functions. Few microeconomists would object to this description as a general representation of the technology, yet almost the whole body of neoclassical economic theory has been built on the assumption of net production functions, i.e. production functions whose only arguments are primary factors.

In order to justify this procedure, Samuelson (1966) proved that under the assumptions of no joint production and constant returns to scale, each single good can be thought of as being produced by its total primary factors (direct factors plus indirect factors required to produce the sector's intermediate inputs). Thus, each sector is considered as an integrated industry that produces its own intermediate inputs. In a figurative sense, each sector in the economy can be considered as a kind of "black box" whose inputs are total primary factors and whose outputs are goods net of intermediate input requirements. This might be called the transformation of gross output functions into net output functions.

With this transformation Samuelson developed a trick, long ago used by the Classical economists, to express the value of goods according to the direct and indirect amount of labour involved in their production. Samuelson, of course, generalized the procedure for any number of primary factors appearing as arguments in the gross output functions.

Now, recent developments in the theory of economic growth have suggested that technological externalities may be fundamental elements in the economic mechanism inducing sustained economic growth (the seminal paper is due to Romer, 1986.) As is well-known, the presence of externalities may induce increasing returns to scale. Hence the problem we want to tackle is whether Samuelson's transformation remains valid when technological externalities are present in the economy.

The analysis of this problem is developed in section 1.3. First we will examine Samuelson's procedure in an economy without technological externalities. This is done in section 1.2. Some brief concluding comments will close this chapter in section 1.4.

1.2 THE TRANSFORMATION WHEN NO EXTERNALITIES EXIST

1.2.1 A Single Good

The transformation is trivial when only an aggregative gross output function is considered. However, it is instructive to consider this case first in order to simplify the exposition of a more complex case.

Let us assume, then, a linearly homogeneous gross output function:

$$X = F(N, K, Q) \quad . \tag{1.1}$$

where X is gross output, N is labour force size, K is capital stock and Q is the amount of intermediate input consumed in the production process. X and Q are flows in the production period, whilst N and K are given stocks.

The net output is clearly defined as the difference between gross output and intermediate input,

(1.2) $$Y \equiv X - Q \ .$$

The problem is to find a function that maps primary factors, K and N, into maximum net output:

(1.3) $$Y = \text{Max } (X - Q) = \text{Max}\,[F(N,K,Q) - Q].$$

The first order condition is simply

(1.4) $$\frac{\partial Y}{\partial Q} = F_Q(\cdots) - 1 = 0 \ ,$$

where the subscript denotes a partial derivative. Equation (1.4) defines the optimal intermediate input allocation, $Q = Q^*$, as a function of the given parameters, N and K. The concavity of $F(\cdots)$ ensures that Q^* is a well-defined maximum: the marginal product of the (own) intermediate input can be neither above 1 (in such a case it would pay to allocate more gross output as intermediate input), nor below 1 (as it would be worth to allocate a lower portion of gross output as intermediate input).

Now, substitution of Q^* into equation (1.3) yields the net output function that we are looking for:

(1.5) $$Y = \Phi(N,K) \ .$$

Hence, the transformation is complete.

Let us look at the properties of this new function. By using Euler's theorem we can rewrite the gross output function [equation (1.1)]:

(1.6) $$X = F_N(\cdots)\,N + F_K(\cdots)\,K + F_Q(\cdots)\,Q\ ,$$

and thus we also can rewrite the net output function [equation (1.3)]:

(1.7) $$Y = X - Q^* = F_N^* N + F_K^* K\ ,$$

where the starred derivatives are evaluated at Q^*. Hence, from equation (1.7) we can see that the net output function, $\Phi(\cdot\cdot)$, is also linearly homogeneous: by doubling N, K and Q, X also doubles and thus net output, Y, is doubled as well.[1] The starred derivatives in equation (1.7) are unchanged because they are homogeneous of degree zero in N, K and Q. Hence we deduce that the partial derivatives of the net output function are defined as $\Phi_K = F_K^*$ and $\Phi_N = F_N^*$, so that Euler's theorem also holds for the net output function $\Phi(\cdot\cdot)$.

1.2.2 Two Goods

We are now ready for the transformation when there are two goods in the economy. A two-sector economy allows us to

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[1] To be rigorous we have to prove that doubling primary factors doubles the optimal allocation of intermediate input. Since $F_Q(\ldots)$ is homogeneous of degree zero in N, K and Q, we have:

$$F_{QN}N + F_{QK}K + F_{QQ}Q = 0\ .$$

Partially differentiating the optimality condition [equation (1.4)] with respect to the parameters N and K gives us:

$$F_{QN} + F_{QQ}\frac{\partial Q}{\partial N} = 0\ , \quad \text{and} \quad F_{QK} + F_{QQ}\frac{\partial Q}{\partial K} = 0\ .$$

Substitution of these equations into the previous one yields

$$Q^* = \frac{\partial Q}{\partial N}N + \frac{\partial Q}{\partial K}K\ .$$

Thus, the optimal intermediate input, Q^*, is a linear homogeneous function in N and K. End of proof.

capture the existence of interindustry relations. We will look at Samuelson's procedure according to the exposition by Gandolfo (1987).

Joint production is excluded and the gross output of each sector is related to the sector's direct primary factors (labour and capital) and intermediate inputs as follows:

$$X_1 = F(N_1, K_1, X_{21}) \ , \tag{1.8}$$

$$X_2 = G(N_2, K_2, X_{12}) \ , \tag{1.9}$$

where X_j is the gross output of sector j, N_j is the amount of labour force in sector j (j = 1,2), K_j is the amount of capital in sector j, and X_{ij} is the amount of intermediate input i used in the production of sector j ($i \neq j = 1,2$).[2] $F(\cdots)$ and $G(\cdots)$ are constant returns to scale production functions.

Let us define the net output of good i, Y_i, as the excess of gross output over the fraction used as intermediate input. Thus,

$$Y_1 \equiv X_1 - X_{12} \ , \tag{1.10}$$

$$Y_2 \equiv X_2 - X_{21} \ . \tag{1.11}$$

The metamorphosis of gross output functions into net output functions should be constrained, of course, by the stock of available primary factors at a given moment in time:

[2] Notice that equations (1.8) and (1.9) do not include the own intermediate inputs as arguments. This simplifies the analysis without loss of generality: since own input derivatives should be equated to 1 in order to maximize net outputs, as proved in the previous case, we can normalized own inputs to zero.

(1.12) $$N_1 + N_2 = N_1^T + N_2^T = N ,$$

(1.13) $$K_1 + K_2 = K_1^T + K_2^T = K ,$$

where N and K are respectively the whole stocks of labour and capital in the economy, N_j and K_j are respectively the direct amounts of labour and capital used in the sector j (j = 1,2), and N_j^T and K_j^T are respectively the implied "total" amounts of labour and capital used in the sector j.

Now comes the problem. According to the neoclassical definition of a production function, the net output function is the maximum amount that can be produced given the total primary factors of the integrated sector. Thus, good 1's net output function is defined as follows:

(1.14) $$Y_1 = \text{Max}\left[F(N_1, K_1, X_{21}) - X_{12}\right] .$$

From the point of view of the integrated industry producing good 1, sector 2 is only a supplier of intermediate goods. Hence, we fix $Y_2 = 0$ in equation (1.11) and define

(1.15) $$X_{21} = G\left(N_1^T - N_1, K_1^T - K_1, X_{12}\right) .$$

Now we have to look for the optimal allocation of N_1, K_1, and X_{12} that maximizes the net output of good 1 subject to equation (1.15). The corresponding first order conditions are the following:

(1.16) $$\frac{\partial Y_1}{\partial N_1} = F_N - F_{X2}G_N = 0 \ ,$$

(1.17) $$\frac{\partial Y_1}{\partial K_1} = F_K - F_{X2}G_K = 0 \ ,$$

(1.18) $$\frac{\partial Y_1}{\partial X_{12}} = F_{X2}G_{X1} - 1 = 0 \ ,$$

where: the arguments of the functions $F(\cdots)$ and $G(\cdots)$ have been suppressed for notational simplicity; the derivatives with respect to labour and capital are denoted with the subscripts N and K respectively; and the marginal product of an intermediate good is denoted using as subscript the kind of good so that $F_{X2} \equiv \partial F(\cdots)/\partial X_{21}$, and $G_{X1} \equiv \partial G(\cdots)/\partial X_{12}$.

The interpretation of these conditions is intuitive. The derivatives with respect to primary factors [equations (1.16) and (1.17)] say that the direct marginal productivity should be equal to the indirect marginal productivity, i.e. the marginal productivity of a primary factor which is directly applied to the production of the sector j should be equal to the marginal productivity of this factor through the production of the intermediate input i used by the sector in question. Finally, the product of the intermediate crossed input derivatives [equation (1.18)] should be equal to 1, i.e. the productivity of a good which is used to produce the same good through the production of its intermediate input should be 1, as in the case of own input derivatives.

If the second order conditions for maximization hold, the equations (1.16) to (1.18) define N_1, K_1 and X_{12} in terms of the given parameters N_1^T and K_1^T. Hence, by substituting these variables into equations (1.14) and (1.15) we can solve for the net output function of good 1. This is, so far, Gandolfo's version of Samuelson's theorem.

Now, given the assumption of constant returns to scale, we can go a little farther by applying Euler's theorem and rewriting equations (1.8) and (1.15) as follows:

$$X_1 = F_N N_1 + F_K K_1 + F_{X2} X_{21} , \tag{1.19}$$

$$X_{21} = G_N (N_1^T - N_1) + G_K (K_1^T - K_1) + G_{X1} X_{12} . \tag{1.20}$$

By substituting equations (1.19) and (1.20) into equation (1.14) and using the first order conditions [equations (1.16), (1.17) and (1.18)], we obtain

$$\begin{aligned} Y_1 &= F_N^* N_1^T + F_K^* K_1^T \\ &= \Phi (N_1^T, K_1^T) , \end{aligned} \tag{1.21}$$

where the starred derivatives denote that they are evaluated at the optimal point, i.e. they are functions of the total primary factors (N_1^T and K_1^T). The function $\Phi(\cdot\cdot)$ is also a function of these parameters and it shows constant returns to scale since it satisfies Euler's theorem.[3] Clearly, by following a symmetric procedure we can also define a constant returns to scale net output function for good 2 as follows:

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[3] To prove rigorously that net output functions are linearly homogeneous we should prove that the derivatives with respect to capital and labour are unchanged with proportional changes of total factors. It turns out to be true because first derivatives of linear homogeneous functions are homogeneous of degree zero. For a formal proof see M. Chacholiades (1978).

(1.22) $$Y_2 = \Gamma(N_2^T, K_2^T) \ .$$

Hence, Samuelson's theorem seems to justify the neoclassical method. Under the assumptions of non-joint production and constant returns to scale, net output functions can be defined from a set of gross output functions, and the former preserve the qualities of the later, i.e. linear homogeneity and concavity. This theorem is obviously very important for neoclassical economists because all the basic theorems of international trade (Hecksher-Ohlin, factor-price-equalization, Stolper-Samuelson, Rybczynsky, etc.) need no modification to account for the role of intermediate inputs.

However, some caveats are necessary. First of all, although the theorem is seemingly based on relatively weak assumptions, they need not always be satisfied. In the next section we will examine some relevant situations where these assumptions are violated. Secondly, if the original technological foundation is a set of gross output functions, net output functions can only be defined for integrated industries and thus direct (observable) factor intensities may be misleading indicators of total factor intensities at the sectoral level. However, Chacholiades (1978) proved that total factor intensities are ranked according to direct (observable) intensities when constant returns to scale and full employment conditions hold, i.e. total intensities cannot show reversals with respect to direct intensities under the stated conditions. From the point of view of the applied economist, Samuelson's transformation implies that net output functions can only be estimated after total factor requirements have been calculated, but this requires one to know beforehand the set of gross output functions. Thirdly, let us recall that Samuelson's transformation yields mathematical relations between total primary factors and maximum net output for goods. Hence, the usual assumption that value added is related to primary inputs may only be valid in an aggregative model where net output is identical to aggregate value added. At the sectoral level, net output and value added are different variables: value added is the difference in value between gross output and intermediate inputs within the sector, whilst net output is the difference between gross output and the economy's absorption of this good as intermediate input. Clearly, in order to define value added at the sectoral level it is necessary to know a pre-determined vector of relative prices. Therefore, multisector models that define a sector's value added as the independent variable of a "production function" whose arguments are the primary factors of the sector are erroneous. The problem is simply worsened if instead of total primary factors, direct primary factors are used as arguments of these production functions. Last but not least, it is appropriate to point out that the reduced-form set of net output functions that stem from a given set of gross output functions do not have traces of the original interindustry linkages, thus the original equations cannot be recovered by working backwards. As a consequence, reduced-form net output functions cannot in any way give account of the exchange of intermediate inputs among sectors (or among nations, if the model is one of an open economy). Moreover, the constraints that the economy faces due to the existence of intermediate input requirements, cannot be identified with a model whose transactions of intermediate inputs have been hidden. The technology as a set of "black boxes" (or net output functions) may simplify the economic analysis, but obscures the analysis of intersectoral relations.

I believe that the previous reasons justify an approach to multisector analysis that circumvents all the problems that Samuelson's transformation might face. The obvious way of doing this is modelling the technology from the outset with a set of gross output functions. This approach is unavoidable if Samuelson's conditions do not hold.

1.3 THE TRANSFORMATION IN THE PRESENCE OF EXTERNALITIES

1.3.1 A Single Good

The previous section assumed explicitly that no economic externalities were involved in the production process. However, recent developments in growth theory have suggested a definitive role for externalities as a source of endogenous economic growth. The problem we want to tackle now is whether Samuelson's transformation remains valid when externalities are present in the economy.

Arrow (1962), Sheshinski (1967), Romer (1986) and others have shown that learning-by-doing externalities can be incorporated in a competitive model of economic growth if the individual firms are unable to exploit the process of creation and transfer of knowledge. These authors consider externalities arising from the aggregate knowledge of the economy which may be either disembodied or embodied in the aggregate stock of capital. Such externalities do not affect the transformation of gross output functions into net output functions as the transformation assumes the stock of primary factors to be instantaneously given. Thus the allocation of factors and intermediate inputs across sectors cannot affect the technological shifting coefficients. To see this, let us return to our initial aggregative production function modified with output-augmenting technological progress:

(1.23) $$X = A\,F(N,K,Q) \ ,$$

where the previous notation still hold and the technological coefficient A is defined as follows

(1.24) $$A = A(K) \ , \quad A'(K) > 0 \ .$$

We assume A to be an increasing function of the aggregate level of capital. The intuition for this approach is the learning-by-doing externality which is associated with the level of capital accumulation: the more capital is accumulated by the private sector, the more knowledge is produced and transferred through spillover effects.

Now, in order to calculate the net output function we have to follow a similar procedure to that of section 1.2.1. But then it is immediately clear that this process is not going to be different from the previous one except for the coefficient A which is taken as a given constant. Therefore, the calculation of the net output function yields the following equation:

(1.25) $$Y = A(F_N^* N + F_K^* K) ,$$

where the starred derivatives are evaluated at the optimal point and the rest of the notation is already known. This production function experiences constant returns to scale at the firm level -where the technology level is assumed as a datum- but shows increasing returns at the aggregate level. In this way, the production function is not inconsistent with the existence of competitive equilibrium. Hence, Samuelson's transformation is successful here.

Let us consider now the following seemingly innocent variation of the technological progress function A:

(1.26) $$A = A(K,Q) \quad , \quad A_K > 0, \quad A_Q > 0 .$$

This kind of variation may also be supported on the grounds of knowledge spillovers and learning by doing. We might argued that aggregate capital, K, embodies social knowledge, and aggregate intermediate input, Q, reflects the process of learning-by-doing through spillovers of technological innovations across firms and sectors. Hence, the technological coefficient A is an increasing function in these two variables. The problem again is to find the function that maximizes net output, Y, defined as the difference between gross output and intermediate input. Given the stock of physical capital, K, and the labour force size, N, the first order condition for net output maximization is simply

(1.27) $$\frac{\partial Y}{\partial Q} = AF_Q + A_Q F - 1 = 0 ,$$

where the arguments of the function $F(\cdots)$ have been suppressed for notational simplicity. This condition shows that the allocation of intermediate input has now an additional effect on its own marginal productivity which was not present in the case of no technology shift [see equation (1.4)]. This effect is precisely the contribution of the learning process to marginal productivity.

Now, expanding the gross output function, equation (1.23), by applying Euler's theorem, using the definition of net output and equation (1.27), we obtain

$$(1.28) \qquad Y = X - Q^* = A(F_N^* N + F_K^* K) + A_Q Q^* ,$$

where the starred derivatives are evaluated at the optimal allocation of intermediate input, Q^*. Thus, equation (1.28) shows that in this case we cannot reduce the gross output function into a net output function which preserves the qualities of the former. Samuelson's transformation still holds if the technological level $A(\cdot\cdot)$ is a logarithmic equation in Q, such that the term $A_Q Q^*$ in equation (1.28) is a constant and so the net output function is defined only in terms of primary factors (N and K). However, even in this particular case the transformation loses the property of linear homogeneity.

Because aggregate intermediate input is producing aggregate gross output and also intangible knowledge (or technical skills) which, in turn, increases total factor productivity, we have here a particular form of joint production and increasing returns to scale. Therefore, Samuelson's conditions are violated and the transformation fails to apply. This result requires that aggregate intermediate input be a choice variable for net output maximization (which is implied by the transformation). In the previous case, where externalities were generated by aggregate capital or aggregate knowledge, we also had joint production and constant returns to scale. However, Samuelson's transformation was unaffected because neither aggregate capital nor aggregate knowledge are choice variables for net output maximization.

1.3.2 Two Goods

It may be argued that the failure of Samuelson's transformation in the previous case hinges on the fact that aggregate intermediate input is a source of externalities. However, the key issue is whether Samuelson's transformation affects the allocation of factors and intermediate inputs which in turn affects the social technological level. Hence, an externality that has a sectoral source invalidates Samuelson's transformation. In order to show this, let us consider a two-sector economic system with labour augmenting technological progress.[4] The fundamental change in this model with respect to the simpler case presented in section 1.3.1 is that the source of efficiency gains is the level of capital accumulation in sector 2. Consider then the following linearly homogeneous gross output functions:

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[4] We could keep considering output augmenting technologies, but this variation does not modify the core of our analysis and besides it simplifies the algebra.

(1.29) $$X_1 = F(AN_1, K_1, X_{21}) \ ,$$

and

(1.30) $$X_2 = G(BN_2, K_2, X_{12}) \ ,$$

where AN_1 and BN_2 are the labour inputs in efficiency units in sectors 1 and 2. The technological coefficients A and B are defined as follows:

(1.31) $$A = A(K_2) \ , \quad A' > 0 \ ,$$

and

(1.32) $$B = B(K_2) , \quad B' > 0 \ .$$

The constraints of the total amounts of available factors hold [equations (1.12) and (1.13)], and the net output definitions are of course identical [equations (1.10) and (1.11)]. The rationale for this kind of system might be that new knowledge is generated primarily in one sector. (The paradigm we have in mind is an economy where the manufacturing sector invests in new knowledge whilst the agricultural sector obtains the new technology through learning by doing and spillover effects.) In any case, the whole system benefits from investment carried out by the firms in sector 2.

Proceeding as we did before, let us define the net output function of good 1 as follows:

(1.33) $$Y_1 = \text{Max}\left[F(AN_1, K_1, X_{21}) - X_{12}\right] ,$$

subject to

(1.34) $$X_{21} = G\left[B(N_1^T - N_1), (K_1^T - K_1), X_{12}\right] .$$

The choice variables of this problem are N_1, K_2, and X_{12}.

The corresponding first order conditions are the following:

(1.35) $$\frac{\partial Y_1}{\partial N_1} = F_E A - F_{X2} G_E B = 0 \ ,$$

(1.36) $$\frac{\partial Y_1}{\partial K_1} = F_K - F_E N_1 A' - F_{X2}\left[G_E (N_1^T - N_1) B' + G_K\right] = 0 \ ,$$

(1.37) $$\frac{\partial Y_1}{\partial X_{12}} = F_{X2} G_{X1} - 1 = 0 \ ,$$

where the subscript E denotes the partial derivative with respect to labour in efficiency units.

Now, let us expand equations (1.33) and (1.34) by using Euler's theorem:

(1.38) $$Y_1 = F_E A N_1 + F_K K_1 + F_{X2} X_{21} - X_{12} \ .$$

(1.39) $$X_{21} = G_E B (N_1^T - N_1) + G_K (K_1^T - K_1) + G_{X1} X_{12} \ .$$

Then, by substituting equation (1.39) into equation (1.38) and using the conditions for maximization, we obtain

(1.40) $$Y_1 = (F_E^*) A N_1^T + (F_{X2}^* G_K^*) K_1^T + (F_K^* - F_{X2}^* G_K^*) K_1 \ .$$

Following a parallel procedure we also obtain

(1.41) $$Y_2 = (G_E^*) B N_2^T + (G_{X1}^* F_K^*) K_2^T + (G_K^* - G_{X1}^* F_K^*) K_2 \ .$$

In these two equations the starred variables are evaluated at the optimal point and the superscript "T" indicates total requirements of the factor. Clearly, the net output functions do depend on the level of direct capital in the respective sector. By using the first order condition for capital it can be shown that the coefficients multiplying the amounts of direct capital in both sectors are different from zero if

$A'(K_2)>0$ or $B'(K_2)>0$.[5] Furthermore, it is clear that the transformed functions do not experience constant returns to scale. Hence, Samuelson's transformation fails again.

If any intuition for these results is to be given it is possibly the following: externality shifts caused by primary factor reallocations and/or intermediate input reallocations prevent the reduction of intermediate inputs in terms of primary factors. Hence, if such shifts occur Samuelson's transformation of gross output functions into net output functions fails. This is clear when a sector's technology is characterized by joint production of goods and knowledge. In this case any factor variation has a twofold effect on the technology level: a direct effect, that corresponds to the own factor productivity, plus an indirect effect through the externality "channel". Therefore, it is the varying technology which prevents the reduction of intermediate inputs in terms of primary factors.

1.4 CONCLUDING COMMENTS

Interindustry linkages matter. Sometimes we can circumvent them in macroeconomic analysis through Samuelson's transformation, but at others we cannot. If interindustry linkages determine the level of technology, they should be modelled in order to explain the workings of the economy. Furthermore, even if the technology level is independent of

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[5] Using equation (1.36) we obtain

$$(F_K^* - F_{X2}^* G_K^*) = F_E^* N_1 A' + (N_1^T - N_1) f_{X2}^* G_E^* B' \neq 0 \ .$$

A similar result holds for the net output function of good 2.

the degree of technological integration among sectors, the analysis of interindustry linkages may help us to understand the feedback effects of technological change among sectors and nations. These objectives are more easily achieved by setting up economic models with gross output functions. The next three chapters apply this approach to the study of economic growth.

CHAPTER 2

INTERINDUSTRY LINKAGES, LEARNING-BY-DOING AND ENDOGENOUS GROWTH

2.1 INTRODUCTION

The single most important feature of economic development is the continuous growth of output per worker. Furthermore, the rates of productivity growth show no obvious tendency to decline (Kaldor, 1961). Rather it has been shown that long-run growth rates of per capita GDP have if anything a tendency to increase (Romer, 1986).[1]

However, this tendency is far from being uniform across countries. Indeed it is nowadays broadly recognized that a characteristic feature of development is the persistence of differentials in long-run economic growth among nations. It is also recognized that industrialized economies tend to have higher income levels and also higher growth rates than non-industrialized economies.[2]

At the same time, it has been argued that open economies tend to be more successful than closed economies.[3] This issue is still under debate. A clear relationship between openness and growth has only been established for industrialized countries and newly industrialized countries.[4] However,

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[1] Romer showed that growth rates of per capita GDP measured over decades in a sample of eleven industrialized countries, tend to increase from decade to decade with a probability that varies between 58% and 81%. His sample includes observations as early as 1700.

[2] See Stern (1989), section II.2.

[3] See the World Bank's World Development Report (1987); see also Balassa (1989).

[4] Using data from Maddison (1982) for 16 leading industrialized countries, Romer (1989) showed a positive correlation between export growth and GDP growth that spans from 1870 to 1979. The experience of newly industrialized economies, whose successful growth has been based on manufacturing exports, is well-known.

Chenery, Robinson and Syrquin (1986) warn that there are no simple formulas in trade strategy to ensure rapid growth. Their conclusion was based on a careful comparative study of the role of industrialization in economic development for nine semi-industrial and industrial economies.

Subsequently Syrquin and Chenery (1989) claimed that on average outward-looking strategies were superior to inward-looking strategies. In order to reach this conclusion, the authors used information from 1950 to 1983 and classified 106 countries according to population size, specialization in primary or manufactured goods, and degree of openness.[5] Table 2.1 reproduces Syrquin and Chenery's calculations of annual growth rates of total GDP for categories of countries. As this table shows, outward-oriented countries grew on average at a higher rate than inward-oriented countries. However, we also can see that countries specializing in manufacturing activities grew on average at higher growth rates than countries specializing in primary activities. Therefore, given that Syrquin and Chenery provide only unweighted averages and their data are subject to great variance, which is recognized by the authors themselves, the cautionary mood of the 1986 report should not be forgotten.

Moreover, a careful assessment of individual countries shows that the performance of some moderately inward-oriented countries has been comparable to that of moderately outward-

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[5] The degree of openness was determined by Syrquin and Chenery from the sign of the difference between the actual ratio of exports to GDP and the predicted ratio after controlling for population size and income level.

Table 2.1
ANNUAL GROWTH RATES OF REAL GDP 1950-1983
Simple Averages
Sample: 106 Countries

Specialization	Openness	Population Size: Large # of obs.	Large (%)	Small # of obs.	Small (%)	Average (%)
Manufacturing	Outward	8	5.26	10	5.73	
Manufacturing	Inward	6	4.73	17	4.74	
All Manufacturing						5.09
Primary	Outward	5	5.12	23	5.01	
Primary	Inward	10	4.94	27	3.58	
All Primary						4.42
	All Outward					5.22
	All Inward					4.28

Source: Syrquin and Chenery (1989), Table 8.

oriented countries.[6] Additionally, based on a sample of 55 developing countries over the period 1964-1982, McCarthy, Taylor and Talati (1987) found that the worst performers were those countries which tend to be more open but specialized in agricultural raw materials.[7]

Therefore, the empirical evidence seems to support the hypothesis that openness without industrialization does not necessarily lead to successful and sustained economic growth. Besides, the historical evidence seems to show that some industrial base, institutional development and especially the development of technical skills, are needed in order to

[6] See World Bank, World Development Report, 1987, chapter 6. This issue is reviewed in Stern (1989), section III.3. For the Latin-American experience is also worth looking at Ocampo (1988).

[7] Similar results are also found by Taylor (1988).

achieve an open economy strategy that relies on manufacturing exports.[8]

We have stated all the previous findings because our purpose in this chapter is to build and analyze a model that generates this range of possibilities in growth performance both across countries and over time. We will follow the path set by Lucas' model of human capital accumulation through learning-by-doing (Lucas, 1988). In his model, Lucas obtains differences in economic growth arising from the difference in learning technologies between two sectors. With international trade and specialization, the countries may grow at different rates because they follow different paths of human capital accumulation depending on their initial endowments of human capital. Although physical capital is excluded from the model and the population is assumed fixed, economic growth is sustained by the average improvement in expertise that comes about as an externality from the "common" effort of production in each sector. This basic feature of the model allows both a competitive framework (because the individual firms take the efficiency level as given), and also endogenous growth (because the "common" effort of production improves the general level of efficiency and yields increasing returns to scale at the sectoral level). Some models by Arrow (1962), Matsuyama (1991), Quah and Rauch (1990), Sheshinski (1967), and Young (1991) also share this characteristic.

In this chapter we will modify Lucas' model to

[8] See Chenery et al. (1986), chapters 6 and 7; this aspect is also reviewed by Stern, op. cit.

incorporate intersectoral linkages through intermediate goods. This will enable us to examine the economic implications of technological interdependence among sectors: surely the technological advances in manufacturing activities improve productivity in primary activities through the supply of better products, and vice versa. In order to capture these feedback effects we will replace the "Ricardian" net output functions in the Lucas model by Cobb-Douglas gross output functions. Thus, in our model labour power and intermediate inputs must be combined in order to produce the gross output of each sector.

Being a more general setting, our model encompasses Lucas' model and hence, not surprisingly, it is consistent with his results. Our model, however, establishes a more suitable context for comparing autarky with international trade as it takes into account both sectoral and international interdependence. Within this framework we will seek to identify the circumstances that generate the following characteristic patterns of development and industrialization:

(1) The composition of demand shifts from primary goods to manufactured goods as real income increases (Engel's law);

(2) As a general rule, labour and capital are reallocated over time from primary activities towards manufacturing activities;

(3) Over time and across countries, industrialization (and also real income) go together with technological intensity on intermediate goods, i.e. a higher share of output is sold as intermediate inputs along the path of economic development (this phenomenon is known as input-output deepening). This process occurs simultaneously with the substitution of primary inputs for manufactured inputs.[9]

These patterns of development are reported by Chenery, Robinson and Syrquin (1986), Chenery and Syrquin (1975), and corroborated by Syrquin and Chenery (1989) among others. By "calibrating" our model so that it replicates these patterns of development, the model also reproduces the long term tendency of growth rates to increase. The basic intuition for this result is that long term substitution of primary goods for manufactured goods enhances economic dynamism because the learning technology of manufacturing activities is superior to that of primary activities.

In our "calibrated" model open economies show on average a better performance than closed economies. The intuition for this result is that open economies can take full advantage of the opportunities for specialization that international trade offers. Because our model requires complete specialization under international trade, each country allocates all its available resources to the sector where it has comparative advantage; hence open economies accumulate human capital in their own activities at the fastest possible rate. Since these economies are technologically interdependent, they benefit indirectly by giving up the production of products for which

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[9] By using a comparative study of interindustry relations by Deutsch and Syrquin (1986) that derived systematic patterns of change from data on 83 input-output tables, Syrquin and Chenery (1989) found a "very significant increase in the demand for manufacturing products to be used as intermediates and a decline in the relative use of intermediate inputs from the primary sector" (page 28).

they do not have comparative advantage. As a result, the growth of open economies as a whole is enhanced because of the mutual technological feedback among countries as suppliers and consumers of intermediate goods. As in the case of static gains from international trade, these dynamic gains are brought about by the possibility of specialization. On the other hand, because of the technological constraints imposed by sectoral linkages (technological complementarities), closed economies cannot avoid some diversion of resources to activities where they do not have comparative advantage. Hence, their average rate of human capital accumulation tends to be below that of open economies.

However, in our "calibrated" model openness is not always the best solution for individual economies. First, this model implies that open economies fully specialized in fast-learning activities perform better than open economies fully specialized in slow-learning activities. Moreover, under certain circumstances, closed economies with a mix of industries biased towards fast-learning economic activities may perform better in terms of growth than open economies fully specialized in slow-learning activities. We will seek to identify the precise circumstances that yield this outcome.

At this point we should note that there is an important stylized fact that our simple model apparently contradicts, namely the long-term deterioration of relative prices of primary goods. The behaviour of primary goods prices is examined by Lewis (1989) and Balassa (1989), although without clear-cut conclusions. However, Grilli and Yang (1988) find a clear tendency for them to decline.[10] By contrast all our results hinge on the continuous fall in the relative price of high-learning industry goods. Nevertheless, in the Appendix we argue that a falling relative price of primary goods can be accommodated within the framework of our model, whilst keeping the remaining results substantially unchanged, by introducing quality upgrading of manufactured goods with respect to primary goods.

Our model will support the importance of industrialization in achieving an open development strategy which relies on manufactured exports. Hence we will examine the likelihood of success of a strategy of development through import substitution in an early stage of growth. With the assumptions corresponding to the "calibrated" model, we will show why some countries are more likely to become industrialized after a period of relative autarky and protection, and why some other countries are not. Because of this last possibility, our analysis is not a new foundation for the "infant-industry" argument; it is simply a warning that a simple rule like "open markets" is not necessarily a welfare-improving policy.

The chapter is organized as follows. Section 2.2 presents the basic model specification. Sections 2.3 and 2.4 solve for the competitive equilibrium in a closed economy and an open

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[10] By constructing a new index of primary commodity prices and two modified indices of manufactured good prices, Grilli and Yang (1988) found that the relative price of all primary commodities fell on trend by 0.5 percent per year from 1900 to 1986. When fuel products are excluded this rate is adjusted to -0.6 percent a year.

economy respectively. Sub-sections 2.3.6 and 2.4.5 compare the model with the stylized facts; readers who want to avoid the intricacies of the model solution might go directly to these sub-sections. Finally we will make some concluding comments in section 2.5. Appendix, notation and simulations are provided at the end of this chapter.

2.2 THE MODEL

Consider a two-sector economy where labour is the single primary factor. The gross output of each sector is allocated between intermediate consumption and final consumption as shown by the following input-output matrix augmented with the vector of labour force allocation.

$$
\begin{array}{c|ccc|c}
i\backslash j & 1 & 2 & C & X \\
\hline
1 & X_{11} & X_{12} & C_1 & X_1 \\
2 & X_{21} & X_{22} & C_2 & X_2 \\
\hline
n & n_1 & n_2 & & \\
 & \downarrow & \downarrow & & \\
 & X_1 & X_2 & &
\end{array}
$$

Figure 2.1 *Input-Output Matrix and Labour Allocation*

In this matrix X_i denotes the gross output of good i, X_{ij} represents the intermediate consumption of good i in sector j, C_i is the final consumption of good i, and n_j is the number of workers in sector j. All variables are flows per unit of time.

From columns 1 and 2 of the input-output matrix we can read the use of physical inputs and labour in the respective sectors. Let us assume that the production technology is Cobb-Douglas:

(2.1) $X_j = (h_j n_j)^{\alpha_j} (X_{1j})^{a_{1j}} (X_{2j})^{a_{2j}}, \ \alpha_j + a_{1j} + a_{2j} = 1, \ j = 1,2,$

where h_j is an index of efficiency per worker in activity j. The arguments of the gross output functions are the intermediate inputs 1 and 2, and effective workforce (the number of workers, n, multiplied by their index of efficiency, h). The Cobb-Douglas specification implies constant returns to scale in both sectors.

The efficiency index, h_j, reflects the level of human capital accumulated through learning-by-doing in sector j. We will assume that the rate of human capital accumulation in activity j increases proportionally with the fraction of the labour force involved in this activity. A specification of this learning technology is given by the following equation:

(2.2) $\dot{h}_j = h_j \delta_j n_j \ , \quad j = 1,2,$

where a dot denotes a time derivative, and δ_j is the learning coefficient. This is Lucas' specification of the learning-by-doing process (Lucas, 1988). It embodies constant returns to human capital: even if the workforce allocation is fixed, the rate of efficiency growth is constant. The rationale for this specification in our model may be the following. Technological know-how in activity j is a by-product of the activity of many firms in sector j; each of them may experience diminishing returns in learning-by-doing, but assuming that knowledge is a public good which is non-rival and non-excludable, every time a firm discovers a new technique this knowledge is immediately transferred with no cost to the firms of the respective sector. Then the sector as a whole increases its human capital in proportion with the number of firms. As a consequence, human capital is specific to the sector and thus appears as an externality for the component firms, i.e. individual firms take human capital levels as given. Spillover effects across sectors are only defined through intersectoral linkages as the production technologies specify.

We complete the supply side of the model by assuming that the labour force, whose size is normalized to 1, is fully employed:

$$(2.3) \qquad n_1 + n_2 = 1.$$

Now let us look at the demand side. By reading horizontally the input-output matrix, one can obtain the composition of the gross demands:

$$(2.4) \qquad X_i = X_{i1} + X_{i2} + C_i , \quad i = 1,2.$$

Since all goods are assumed to be perishable, the representative consumer does not face any inter-temporal trade-off. Hence, all we need to know in order to close the model is the instantaneous utility function. We will assume a constant elasticity of substitution (CES) utility function:

$$(2.5) \qquad u(C_1, C_2)^{\gamma} = b\, C_1^{\gamma} + (1-b)\, C_2^{\gamma} , \quad 0 < b < 1, \quad \gamma < 1 ,$$

where $\sigma = (1-\gamma)^{-1} > 0$ is the elasticity of substitution in (final) consumption between the goods 1 and 2.

2.3 THE COMPETITIVE SOLUTION (CLOSED ECONOMY)

2.3.1 Consumer Behaviour

In a competitive environment the representative consumer takes prices as given and maximizes his/her utility subject to his/her income level. Hence the relative demand function that comes from the solution of this problem is the following:

$$(2.6) \qquad C_2/C_1 = B\,q^{-\sigma}, \quad B \equiv \left(\frac{1-b}{b}\right)^{\sigma} > 0 \,,$$

where q is the relative price of good 2, Clearly the relative demand function is decreasing in q.

2.3.2 Firm's Behaviour

Perfect competition and constant returns to scale together imply zero profits (revenue equals costs):

$$(2.7) \qquad p_j X_j = p_1 X_{1j} + p_2 X_{2j} + w\,n_j \,, \quad j = 1,2,$$

where p_j denotes the sector j's output price, and w is the wage rate.

From now on we will treat good 1 as numeraire, so that everything is measured in homogeneous units. Hence we will normalize the equilibrium prices to $p_1 = 1$ and $p_2 = q$.

In order to maximize profits the firms have to choose the number of workers to be hired, and the amounts of good 1 and good 2 to be used as intermediate inputs. Since the assumption of constant returns to scale allows us to aggregate the firms in their sectors, we will calculate the sectors' first order conditions for profit maximization. For the sector 1 we have

$$(2.8) \qquad n_1 = \alpha_1 X_1 / w \,,$$

$$(2.9) \qquad X_{11} = a_{11} X_1 \,,$$

$$(2.10) \qquad X_{21} = a_{21} X_1 / q \,.$$

Similarly, the first order conditions for profit maximization of the sector 2 are

(2.11) $$n_2 = \alpha_2 q X_2 / w ,$$

(2.12) $$X_{12} = a_{12} q X_2 ,$$

(2.13) $$X_{22} = a_{22} X_2 .$$

The equations (2.8) to (2.13) are rearranged expressions for the equalization of factor prices to the respective factor marginal productivities. For these calculations the firms take prices and levels of human capital as given. Cobb-Douglas production functions are concave, thus the second order conditions for profit maximization are satisfied.

Now, by substituting the first order conditions for profit maximization into equation (2.7), the reader can check that the assumption of constant returns to scale, i.e. $\alpha_j + a_{1j} + a_{2j} = 1$, for $j = 1,2$, is consistent with zero profits. Hence, a competitive equilibrium is possible. This would not be the case if the firm could control the human capital level. In such a case the technology would yield increasing returns to scale and perfect competition would be ruled out. That is why the treatment of human capital as an externality for the firm is crucial to the model.

2.3.3 Price and Output Determination

Due to the assumption of constant returns to scale, we can solve the relative prices of this economy. By substituting equations (2.8), (2.9) and (2.10) into equation (2.1) for $j = 1$, and substituting equations (2.11), (2.12) and (2.13) into equation (2.1) for $j = 2$, we obtain the following system of equations:

$$\begin{bmatrix} \alpha_1 & -a_{21} \\ \alpha_2 & (1-a_{22}) \end{bmatrix} \begin{bmatrix} \ln w \\ -\ln q \end{bmatrix} = \begin{bmatrix} \alpha_1 \ln h_1 + \varepsilon_1 \\ \alpha_2 \ln h_2 + \varepsilon_2 \end{bmatrix}$$

where

$$\varepsilon_j = \ln\left(\alpha_j^{\alpha_j} a_{1j}^{a_{1j}} a_{2j}^{a_{2j}}\right) < 0 \,, \quad j = 1,2.$$

Before solving the price system, let us turn to the solution of gross demands in terms of final demands. By substituting equations (2.9) and (2.12) into equation (2.4) for i = 1, and substituting equations (2.10) and (2.13) into equation (2.4) for i = 2, we obtain the following system of equations:

$$\begin{bmatrix} (1-a_{11}) & -a_{12} \\ -a_{21} & (1-a_{22}) \end{bmatrix} \begin{bmatrix} X_1 \\ q X_2 \end{bmatrix} = \begin{bmatrix} C_1 \\ q C_2 \end{bmatrix}$$

which is the Leontief system of interindustry linkages. The matrix on the left hand side of this expression is obviously the Leontief matrix, denoted L. It turns out that the determinant of this matrix is also the determinant of the matrix in the price system. This is due to the fact that the interindustry linkages are common to both systems. Since the determinant of the Leontief matrix will appear frequently below, it will be convenient to note here some alternative expressions for it:

$$\begin{aligned} |L| &= (1 - a_{11})(1 - a_{22}) - a_{12}a_{21} \\ &= \alpha_1(1 - a_{22}) + \alpha_2 a_{21} \\ &= \alpha_1 a_{12} + \alpha_2(1 - a_{11}) \\ &= \alpha_1\alpha_2 + \alpha_1 a_{12} + \alpha_2 a_{21} \,. \end{aligned}$$

From these expressions we deduce that $0 < |L| < 1$. Notice, for future reference, that the maximum value of the ratio $\alpha_1\alpha_2/|L|$ is 1: it corresponds to the case where no intersectoral linkages exist, i.e. $a_{12} = a_{21} = 0$.

Now we proceed to solve the demand system:

$$\text{(2.14)} \qquad \begin{bmatrix} X_1 \\ qX_2 \end{bmatrix} = \frac{1}{|L|} \begin{bmatrix} (1-a_{22}) & a_{12} \\ a_{21} & (1-a_{11}) \end{bmatrix} \begin{bmatrix} C_1 \\ qC_2 \end{bmatrix}$$

From this system it is easily shown that the relative value of gross demands, qX_2/X_1, changes with the relative value of final demands, qC_2/C_1.

Next we solve the price system:

$$\text{(2.15)} \qquad \begin{bmatrix} \ln w \\ \ln q \end{bmatrix} = \frac{1}{|L|} \begin{bmatrix} (1-a_{22}) & a_{21} \\ \alpha_2 & -\alpha_1 \end{bmatrix} \begin{bmatrix} \alpha_1 \ln h_1 + \varepsilon_1 \\ \alpha_2 \ln h_2 + \varepsilon_2 \end{bmatrix}$$

The price solutions are given in reduced form. From equation (2.15) we can see that the wage (measured in terms of good 1), w, depends on the economy's <u>absolute</u> endowments of human capital, h_1 and h_2. On the other hand, the relative price of good 2, q, depends on the economy's <u>relative</u> endowment of human capital. We will denote the relative endowment of human capital in sector 2 by $h \equiv h_2/h_1$. For future reference note that q is a decreasing and convex function of h.[11]

2.3.4 Workforce Allocation

The equilibrium in the labour market is found by equating the fixed supply of labour to aggregate labour demand.

---

[11] The first and second derivatives of q with respect to h are

$$\frac{dq}{dh} = -\frac{\alpha_1\alpha_2}{|L|}\frac{q}{h} < 0 \,, \quad \text{and} \quad \frac{d^2q}{dh^2} = \frac{\alpha_1\alpha_2}{|L|}\frac{q}{h^2}\left(1 + \frac{\alpha_1\alpha_2}{|L|}\right) > 0 \,.$$

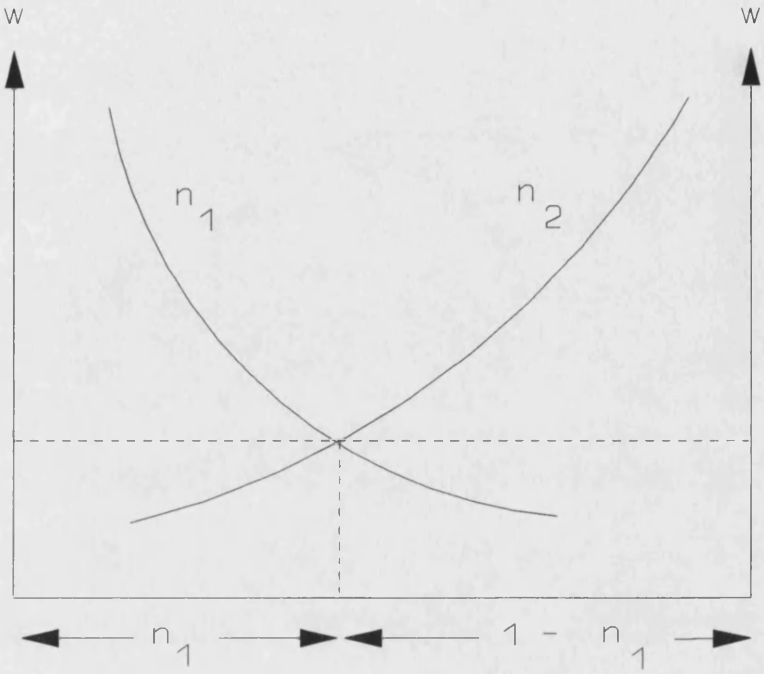

Figure 2.2 *Labour Market Equilibrium*

Substitution of equations (2.8) and (2.11) into equation (2.3) yields

(2.16) $$w = \alpha_1 X_1 + \alpha_2 q X_2 \ ,$$

and also,

(2.17) $$n_1 = \frac{\alpha_1 X_1}{\alpha_1 X_1 + \alpha_2 q X_2} \ .$$

Figure 2.2 depicts the equilibrium in the labour market.

Now, by combining equations (2.6), (2.14) and (2.17) we can solve for the workforce allocation, n_1, in terms of the relative price of good 2, q:

(2.18) $$n_1 = \frac{\alpha_1 \alpha_2}{|L|}\left[\frac{a_{12}}{\alpha_2} + \theta(q)\right],$$

where

$$\theta(q) = (1 + Bq^{1-\sigma})^{-1},$$

is the fraction of real income allocated to good 1 in final demand. If the goods are good substitutes, $\sigma > 1$, substitution effects dominate income effects and hence n_1 is increasing in q. If the goods are poor substitutes, i.e. $0 < \sigma < 1$, n_1 is decreasing in q. For both cases, the lower and upper limits of the workforce allocation to sector 1 are defined as follows:

(2.19) $$n_{1,l} = \frac{\alpha_1 a_{12}}{|L|} \geq 0,$$

and

(2.20) $$n_{1,u} = \frac{\alpha_1}{|L|}(a_{12} + \alpha_2) \leq 1.$$

These values are reached asymptotically as q tends to either zero or infinity. Notice also that the range between these extreme values is constant and equal to $\alpha_1\alpha_2/|L|$; hence if no integration exists between the sectors, this range is equal to 1, otherwise it is positive but lower than 1. Finally, in the borderline case of logarithmic preferences, that is $\sigma = 1$, substitution effects and income effects cancel each other and hence n_1 is constant as shown by the following formula:

(2.21) $$n_{1,l} \leq n_1 = \frac{\alpha_1}{|L|}(a_{12} + b\alpha_2) \leq n_{1,u}.$$

From the limit values of n_1 we can deduce that in general there will not be complete specialization if the technologies of the two sectors are mutually integrated (i.e. if each sector uses an intermediate input different to the own output). In this case there will be always an internal solution with positive production in both sectors. Therefore, complete specialization will be possible if and only if at least one sector is not integrated (i.e. if a_{12} or a_{21} or both technical coefficients are equal to zero), but even in this case complete specialization will only be achieved asymptotically (as the relative price goes to either zero or infinity).

2.3.5 Dynamics: Relative Prices and Welfare

Since we have assumed no population growth, the source of growth in this economy is through human capital accumulation. Thus the dynamics of prices should be driven by the dynamics of human capital accumulation. Differentiating the price system, equation (2.15), with respect to time and taking into account equation (2.2), we obtain

$$\textbf{(2.22)} \qquad \begin{bmatrix} \dot{w}/w \\ \dot{q}/q \end{bmatrix} = \frac{1}{|L|} \begin{bmatrix} (1-a_{22}) & a_{21} \\ \alpha_2 & -\alpha_1 \end{bmatrix} \begin{bmatrix} \alpha_1 \delta_1 n_1 \\ \alpha_2 \delta_2 (1-n_1) \end{bmatrix}$$

which is the vector of growth rates of the wage and the relative price of good 2 (both of them measured in terms of good 1).

Now, by substituting equation (2.18) into equation system (2.22) we can obtain the system of differential equations that governs the dynamics of this economy. However, we will keep these equations apart for analytical convenience.

Let us analyze first the dynamics of the relative price of good 2. This equation is very simple as it shows a positive linear relationship between the rate of change of the relative price, $(dq/dt)/q$, and the workforce allocation, n_1. From this equation we obtain the steady state value of n_1:

$$n_1^* = \frac{\delta_2}{\delta_1 + \delta_2} \; . \tag{2.23}$$

This is the value at which the relative price is stable and, thus, the system becomes stationary. Note that if the two sectors are mutually integrated (i.e. both a_{12} and a_{21} are strictly positive) the competitive economy may not reach the steady state: since the steady state allocation of the workforce to sector 1 increases with the comparative advantage of sector 2 in the learning process (i.e. n_1^* is increasing in δ_2/δ_1), it may well be that for extreme differentials in the learning coefficients, n_1^* is either above the upper limit, $n_{1,u}$, or below the lower limit, $n_{1,l}$. We will see briefly the economic consequences of this characteristic.

Let us examine the previous analysis diagramatically. As before, we will start with the case of good substitutes. In Figure 2.3a we have drawn together the diagrams of the relative price function [taken from equation system (2.15)], the function of the workforce allocation [equation (2.18)], and the function that determines the rate of change of the relative price of good 2 [taken from equation system (2.22)] for the good substitutes case. Given the initial relative endowment of human capital h (= h_2/h_1), q is determined and also is n_1. If the steady state value n_1^* is below $n_{1,l}$ (e.g. at point R), the economy will become relatively specialized in

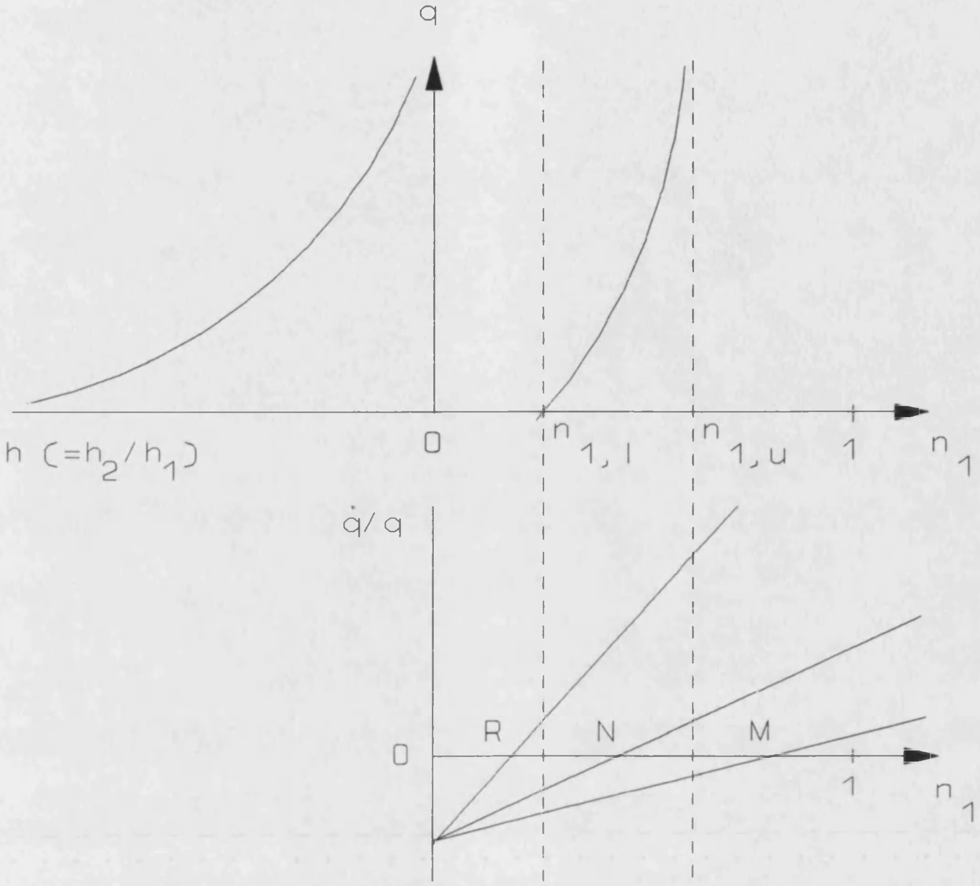

Figure 2.3a
Dynamics of the Closed Economy (High Substitutability)

the production of good 1.[12] If the steady state value n_1^* is above $n_{1,u}$ (e.g. at point M), the economy will end up relatively specialized in the production of good 2. If the steady state value n_1^* is between $n_{1,l}$ and $n_{1,u}$ (for example at point N), there exist a particular relative endowment of human capital and a particular relative price which are consistent with the steady state equilibrium. However, this equilibrium

[12] It is convenient to avoid misunderstanding by stating at once that for <u>relatively</u> specialized economy in the good 1(2), we understand the economy to be located in the vicinity of the upper(lower) limit of the workforce fraction allocated to sector 1.

is not stable: for a higher(lower) relative endowment of human capital the economy will end up relatively specialized in the production of good 1(good2). Therefore, we can conclude that in general, the economy will end up relatively specialized in one sector. This analysis confirms the results obtained by Lucas (1988). However, in our model specialization cannot be complete if intermediate goods are essential for production .

Figure 2.3b shows the poor substitutes case. In this case the sector 1's share in employment is a decreasing function of the relative price of good 2. Now, given the initial relative endowment of human capital h, q and n_1 are determined. If the steady state value n_1^* is below $n_{1,l}$ (e.g at point Q), the economy will become relatively specialized in the production of good 2. If the steady state value n_1^* is above $n_{1,u}$ (e.g. at point T), the country will be relatively specialized at producing good 1. Finally, if the steady state value n_1^* is in between $n_{1,l}$ and $n_{1,u}$ (for example at point S), the economy will converge towards the steady state equilibrium. Therefore, for sufficiently differentiated learning technologies, the economy will end up relatively specialized. This result is qualitatively different to that of Lucas' paper, because he concluded that in the poor substitutes case the economy would always converge to the steady state equilibrium. His model leads to that result because it does not allow for intersectoral linkages and, hence, the lower and upper limits of n_1 are 0 and 1 respectively.

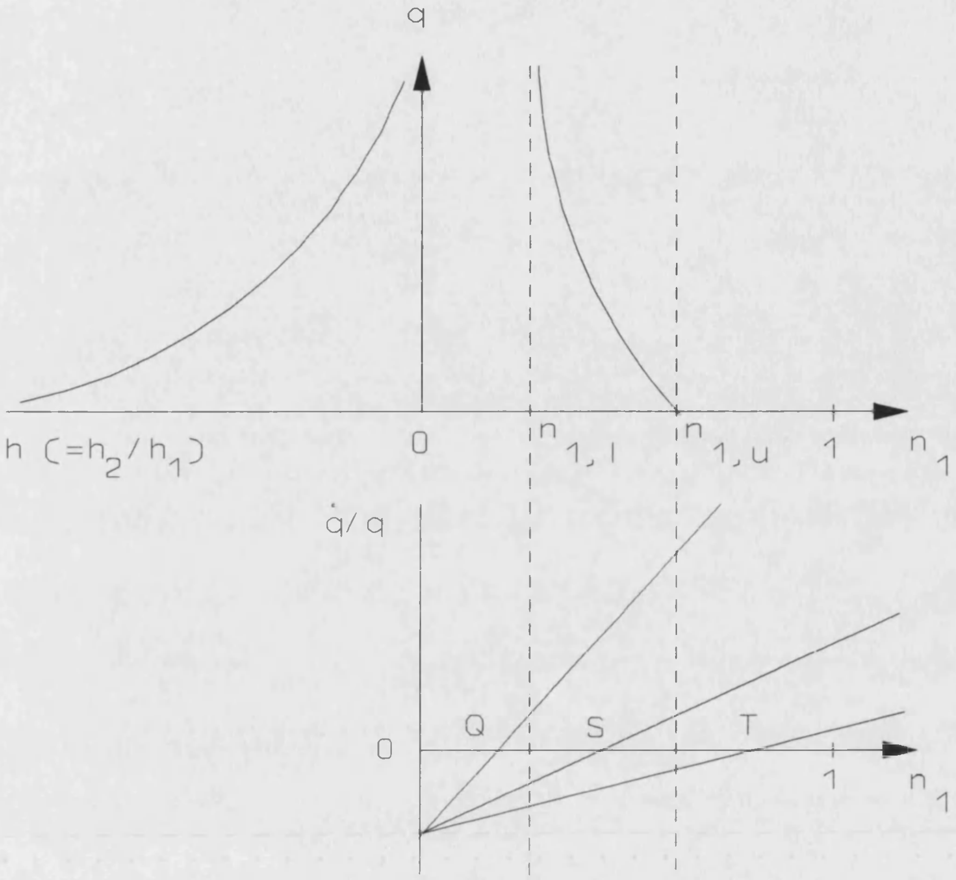

Figure 2.3b
Dynamics of the Closed Economy (Low Substitutability)

Finally, we have the borderline case of logarithmic preferences. As mentioned before, in this case the price line becomes vertical and hence the economy workforce allocation is constant. Therefore, the relative price will increase or decrease forever unless the economy happens to be exactly located at the steady state equilibrium. This also coincides with Lucas' analysis. We will omit the corresponding diagram.

Let us turn now to the analysis of the dynamic welfare gains in this economy. From equation system (2.22) we obtain the growth rate of income in terms of purchasing power of good

1, (dw/dt)/w; we can also calculate the growth rate of income in terms of purchasing power of good 2, [(dw/dt)/w-(dq/dt)/q]. As it turns out, both rates are weighted averages of the growth rates of human capital in the sectors 1 and 2: $\delta_1 n_1$ and $\delta_2(1-n_1)$, respectively [see equation (2.2)]. This is not surprising, as we know that the engine of growth in this economy is the learning-by-doing process.

Now, since the representative consumer cares about both types of goods, we should construct a proper welfare index. In order to do that it is convenient to deduce the indirect utility function. First, from the relative demand function [equation (2.6)] and the consumer's budget constraint, $w = C_1 + qC_2$, we may deduce the Marshallian demands. Substitution of these demands into the utility function [equation (2.5)] then yields the indirect utility function:

$$\text{(2.24)} \qquad u(w,q) = \frac{b^{\sigma/(\sigma-1)}\, w}{(1 + Bq^{1-\sigma})^{1/(1-\sigma)}} \; .$$

Differentiating this expression with respect to time we obtain

$$\text{(2.25)} \qquad \frac{\dot{u}}{u} = \theta(q)\left(\frac{\dot{w}}{w}\right) + [1 - \theta(q)]\left(\frac{\dot{w}}{w} - \frac{\dot{q}}{q}\right) ,$$

where $\theta(\cdot)$ is the weight function we described in equation (2.18). Equation (2.25) says that the rate of change of utility is a weighted average of the growth rate of income measured in terms of purchasing power of good 1, and the growth rate of income measured in terms of purchasing power of good 2. Since these rates are bounded by the rates of growth human capital in the sectors 1 and 2, the rate of change of utility is clearly also bounded by the same rates. We confirm this analysis by combining the equations in equation system (2.22) and equation (2.25):

$$\frac{\dot{u}}{u} = \frac{[\alpha_1 a_{12} + \alpha_1\alpha_2\theta(q)]\,\delta_1 n_1 + [\alpha_2(1 - a_{11}) - \alpha_1\alpha_2\theta(q)]\,\delta_2(1 - n_1)}{a_{12}\alpha_1 + \alpha_2(1 - a_{11})}. \quad (2.26)$$

Thus, the maximum range of variation of the growth rate in utility units is $[\delta_1 n_1, \delta_2(1-n_1)]$.[13]

2.3.6 Theory and Facts

If we choose the first sector to represent the primary sector, and the second sector to represent the manufacturing (or secondary) sector, is this model able to reproduce the perceived behaviour of closed economies? Since the previous analysis identified several dynamic possibilities, our answer can only be a qualified yes. We have to discriminate among these possibilities in order to choose the case that replicates the patterns of development mentioned in the Introduction. Hence it is clear that only the case of good substitutes with a stationary value of workforce allocation, n_1^*, above or equal to the upper limit of the workforce share of sector 1, $n_{1,u}$, satisfies this criterion (see Figure 2.3a, the case corresponding to the stationary point M).

Under these conditions the model generates the following outcomes:

(1) Continuous growth of relative human capital in the

[13] Only in the case of logarithmic preferences, $\sigma = 1$, the model gives a constant rate of utility growth. For any other case, i.e. $\sigma \neq 1$, the relative price, q, changes over time, then the weights in equation (2.26) change and thus the mentioned rate of growth varies as well.

manufacturing sector, i.e. h increases;

(2) Continuous fall of the relative price of manufactured goods (however, if quality upgrading of goods is proportional to human capital accumulation through learning by doing, the model could be modified so that the reverse result would appear without affecting any of the results mentioned here - see the Appendix);

(3) Continuous reallocation of the workforce over time in favour of the manufacturing sector, i.e. n_1 falls;

(4) Because substitution effects dominate income effects, equations (2.6) and (2.14) show that primary goods (good 1) are substituted over time by manufactured goods (good 2) both in final consumption and intermediate consumption;[14]

(5) Results (3) and (4) imply that the rate of growth of utility increases over time because of the substitution of primary goods (slow-learning technology) for manufactured goods (fast-learning technology): as the weight function $\theta(\cdot)$ and n_1 decrease with the relative price, q, the rate of welfare growth increases over time [see equations (2.18) and (2.26)]. Now, let us look for the values of the parameters which are consistent with the previously identified case (the case corresponding to the stationary point M in Figure 2.3a).

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[14] Because of the assumption of homothetic preferences, the model yields an income elasticity of demand for final goods equal to 1. Hence, this model does not reflect Engel's law; the change in demand composition is brought about only through substitution effects. The model could be modified to allow for an inelastic income demand for primary goods; in order to do that we could assume a necessary minimum level of consumption of the primary good within the utility function as in Matsuyama (1990). This, however, would complicate the algebra too much and only would enforce the substitution effect.

Firstly, we need the elasticity of substitution to be greater than unity, $\sigma > 1$. Secondly, in order to have a falling relative price of good 2 for any given mix of human capital, we need the following inequality to be satisfied:

$$n_1^* = \frac{\delta_2}{\delta_1 + \delta_2} \geq n_{1,u} = \frac{\alpha_1(a_{12} + \alpha_2)}{|L|} = 1 - \frac{\alpha_2 a_{21}}{|L|} \ .$$

Two reasonably weak assumptions guarantee this inequality. First, the learning process in sector 2 is more efficient than the learning process in sector 1, $\delta_2 > \delta_1 \geq 0$. Second, the workforce allocation is biased towards the sector 2. For such a bias we need a low a_{12} coefficient and/or a high a_{21} coefficient. The latter implies a high dependence of sector 1 with respect to intermediate inputs of good 2, and the former implies a low dependence of sector 2 with respect to intermediate inputs of good 1. This is a weak assumption if we think of the first sector as mainly providing final goods (e.g. food), whilst the second sector provides both intermediate and final goods. In any event, for our results only the first assumption is fundamental: if δ_1/δ_2 tends to 0, n_1^* tends to 1, and thus the above inequality ($n_1^* \geq n_{1,u}$) holds unambiguously.

2.3.7 Simulations

By simulating the behaviour of our economy for specific parameters we will illustrate some of our results. Diagrams of these simulations are shown at the end of this chapter.

The production technology used in the simulations corresponds to the following parameters: $\alpha_1 = 0.4$, $a_{11} = 0.2$,

$a_{21} = 0.4$, $[\alpha_1 + a_{11} + a_{21} = 1]$; and $\alpha_2 = 0.35$, $a_{12} = 0.05$, $a_{22} = 0.6$, $[\alpha_2 + a_{12} + a_{22} = 1]$. The learning coefficients are assumed to be as follows: $\delta_1 = 1\%$ and $\delta_2 = 6\%$. Preferences are defined by b = 1, and the elasticity of substitution $\sigma = (1-\gamma)^{-1} = 5$.

These parameters satisfy the conditions we identified in the last section. Some modifications of these basic parameters will be used later for illustrating some specific propositions.

For the time being we will refer the reader to simulations 1 and 2 (page 90). The first simulation shows that the growth rate of utility in a closed economy increases as the relative price of manufactured goods falls [result (5) in the preceding section]. Simulation 2 shows that simultaneously the labour force is transferred from the primary sector to the manufacturing sector [result (3) in the preceding section.]

2.4 THE COMPETITIVE SOLUTION (SMALL OPEN ECONOMY)

2.4.1 Terms of Trade and Specialization

In this section we will consider a small economy trading on the international market. Hence the terms of trade are now exogenously given to the economy. These terms of trade are denoted by p, the international price of good 2 relative to good 1.[15] Additionally we make the usual assumptions of international trade models: no transport costs, free trade, and international immobility of the labour force. We also

[15] Notice that p has now a new meaning; it was used before to denote nominal prices. From now on p denotes terms of trade and q denotes the relative price of good 2 in autarky.

assume that spillover effects of learning by doing are restricted to the boundaries of the country.[16]

Let us suppose for a moment that our small open economy is willing to produce both goods. Hence the economy's relative endowment of human capital is such that the relative price in autarky, q, and the terms of trade, p, are equal. Otherwise wages would not be equalized across sectors. Substitution of q for p in equation system (2.15) yields

$$\text{(2.27)} \qquad h \equiv h_2/h_1 = e^{(\varepsilon_1/\alpha_1 - \varepsilon_2/\alpha_2)}\, p^{-|L|/(\alpha_1\alpha_2)} ,$$

where

$$\varepsilon_j = \ln\,(\alpha_j^{\alpha_j}\, a_{1j}^{a_{1j}}\, a_{2j}^{a_{2j}}) < 0\,, \quad j = 1,2.$$

Equation (2.27) defines the locus of combinations of per-worker human capital in the two sectors (h_1,h_2), which is consistent with no specialization. These combinations are represented by a line starting from the origin as shown in Figure 2.4. For higher levels of p the line moves towards the h_1-axis, and for lower levels of p the line moves towards the h_2-axis. In both cases the line rotates around the origin.

Now, if a country happens to have an initial combination of human capital along this line, the country may have a non-specialized economy; otherwise, it will become fully specialized. Those countries whose coordinates are above the line will only produce good 2, and those countries whose

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[16] The assumption of no cross-border spillovers is a strong one. However, it may be justified if the learning technology requires close contact among firms or if the knowledge of new technologies is embodied in the labour force as skills.

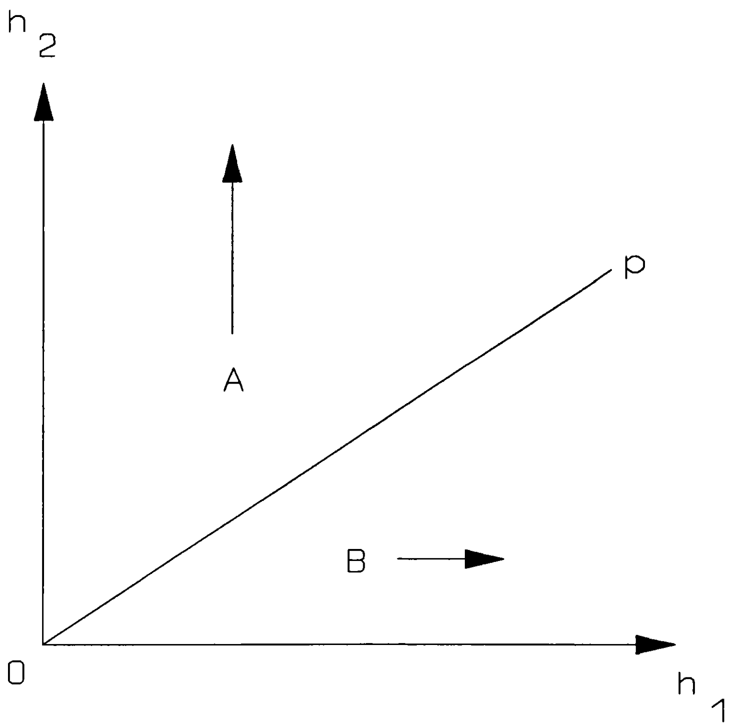

Figure 2.4 *The International Price Line*

coordinates are below the line will be specialized in theproduction of good 1. Specialization maximizes their national income. This is nothing more than a consequence of the countries' comparative advantage due to their specific combinations of human capital endowments.

2.4.2 Country Specialized in Good 1

i\j	1	2	C	E	X
1	X_{11}	0	C_1	E_1	X_1
2	0	0	0	0	0
n	1	0			

Figure 2.5 *Domestic Input-Output Matrix*

This country allocates its whole workforce to the sector 1, $n_1 = 1$. As the domestic input-output matrix shows, the production is allocated as intermediate input, final consumption and exports. This last variable is denoted with the letter E. Thus,

$$X_1 = X_{11} + C_1 + E_1 \ . \tag{2.28}$$

The imports matrix has the following form:

i\j	1	2	C	M
1	0	0	0	0
2	X_{21}	0	C_2	M_2

Figure 2.6 *Imports Matrix*

Hence the imports balance equation is

$$M_2 = X_{21} + C_2 \ . \tag{2.29}$$

This equation simply says that total imports are allocated to intermediate and final consumption.

Now, since the entire workforce is allocated to sector 1, $n_1 = 1$, and the terms of trade, p, are given, equation (2.8) yields a direct relationship between output and wages (income):

$$w = \alpha_1 X_1 . \tag{2.30}$$

Taking again into account that p is given, equations (2.9) and (2.10) determine the demands for intermediate inputs:

(2.31) $$X_{11} = a_{11}X_1 \, ,$$

(2.32) $$X_{21} = a_{21}X_1/p \, .$$

By substituting these demands and $n_1 = 1$ into the production function for good 1, we obtain the supply function of a country specialized in good 1:

(2.33) $$X_1 = h_1\left(a_{11}^{a_{11}}\, a_{21}^{a_{21}}\right)^{1/\alpha_1}/p^{a_{21}/\alpha_1} \, .$$

The behaviour of consumers is the same as in the closed economy case. Hence, the final demand function is

(2.34) $$C_2/C_1 = B\,p^{-\sigma} \, ,$$

where σ is the elasticity of substitution in consumption.

So far we have a system in 7 equations, equations (2.28) to (2.34). On the other hand we have 8 unknowns: X_1, X_{11}, X_{21}, C_1, C_2, E_1, M_2 and w. Recall that the efficiency indices, h_1 and h_2, follow independent processes and are taken as given parameters by the countries at any moment in time. Hence we only need one more equation in order to close the model; however we in fact have two. The first is the budget constraint:

(2.35) $$C_1 + p\,C_2 = w \, ,$$

and the second is the balanced trade account condition:

(2.36) $$E_1 = p\,M_2 \, .$$

This last condition is redundant by Walras Law. Hence we are left with 8 independent equations and 8 unknowns and thus the system is soluble.

For future reference we will only solve for the ratio of exports to gross output. Since in equilibrium exports and imports are balanced, this ratio can also be considered a "measure" of the country's openness to international trade:[17]

$$E_1/X_1 = a_{21} + \alpha_1 [1 - \theta(p)] \,, \tag{2.37}$$

where $\theta(\cdot)$ is the weight function defined in equation (2.18).

2.4.3 Country Specialized in Good 2

This case is symmetric to the preceding case. Hence we will omit the details. However, we will need to know the supply function of a country specialized in good 2 and its export ratio. We proceed as before. The whole workforce is now allocated to the second sector, $n_2 = 1$. Taking into account that the terms of trade are given, the intermediate input demands are given by equations (2.12) and (2.13). Substitution of these demands into the production function of good 2 then yields the supply function of a country specialized in good 2:

$$X_2 = h_2 \left(a_{12}^{a_{12}} a_{22}^{a_{22}} \right)^{1/\alpha_2} p^{\,a_{12}/\alpha_2} \,. \tag{2.38}$$

The export ratio of this country is then

$$E_2/X_2 = a_{12} + \alpha_2 \theta(p) \,, \tag{2.39}$$

where $\theta(\cdot)$ is as previously defined.

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[17] In this model countries are either closed or open. Hence, a "measure" of openness is a misleading variable. Therefore, it might be better interpreted as a measure of economic dependence on the world economy. Let us point out, however, that the ratio of exports plus imports to national product is frequently used in the literature on economic development as a measure of openness. Our aim is to examine the long run behaviour of such a measure in our model.

2.4.4 World Market Equilibrium

2.4.4.1 The Static Equilibrium

In order to consider the world market equilibrium, we can ignore those marginal countries whose human capital endowments are located along the (separating) international price line. These countries will behave as closed economies and thus their net contribution to the flow of world trade will be zero.

Let us refer to Figure 2.4. If the terms of trade increase (the price of good 2 with respect to good 1 increases), the price line will rotate around the origin towards the h_1-axis. Given a distribution of human capital endowments among countries, it follows that more countries will become producers of good 2 and less countries will produce good 1. Thus the relative world supply of good 2 increases with the relative price of good 2.

How can we determine the world relative demands? We can think about the world demands in the same way we considered demands in a closed economy. Denote X_1^w and X_2^w as the world production of goods 1 and 2, and let C_1^w and C_2^w denote the world final consumption of goods 1 and 2. In equilibrium it must be true that the relationship between world final demands and world gross demands is determined by the following Leontief system

$$(2.40)\qquad \begin{bmatrix} (1-a_{11}) & -a_{12} \\ -a_{21} & (1-a_{22}) \end{bmatrix} \begin{bmatrix} X_1^w \\ p\,X_2^w \end{bmatrix} = \begin{bmatrix} C_1^w \\ p\,C_2^w \end{bmatrix}$$

Because in equilibrium world exports of good i are cancelled out with world imports of good i, i = 1,2, the world economy inter-sectoral linkages are completely analogous to those of a closed economy (see section 2.3.3).

Now, given the assumption of homothetic preferences, the world relative final demand will also be the same function we had in the closed economy case:

$$C_2^w / C_1^w = B p^{-\sigma} . \tag{2.41}$$

This shows that the relative final demand for good 2 is a decreasing function of the international relative price of good 2, p. Is it the case that the relative gross demand function is also well-behaved? Not necessarily, as we will see immediately. By combining equations (2.40) and (2.41) we solve for the world relative gross demand:

$$\frac{p X_2^w}{X_1^w} = \frac{a_{21} + (1 - a_{11}) B p^{1-\sigma}}{(1 - a_{22}) + a_{12} B p^{1-\sigma}} . \tag{2.42}$$

The derivative of the world relative demand with respect to p is

$$\frac{d(X_2^w / X_1^w)}{dp} = - \frac{(X_2^w / X_1^w)}{p} - \frac{(\sigma - 1)|L| B p^{-\sigma}}{[(1 + a_{22}) + a_{12} B p^{1-\sigma}]^2} . \tag{2.43}$$

Hence, if $\sigma > 1$, which corresponds to the good substitutes case, the function of the world relative gross demand is downward sloping. Since the relative world supply function is upward sloping, we will have a well defined equilibrium. However, if the goods are poor substitutes, so that $0 < \sigma < 1$, there exists the possibility that for some price ranges the slope of the world gross demand turns out to be positive. In that case we could have multiple equilibria.

Now, in order to determine the world relative supply we would need to assume some kind of distribution of the countries' human capital endowments. Thus, at the static level we cannot make more progress without incorporating additional information into the model. Instead, let us turn to the dynamics of adjustment.

2.4.4.2 The Dynamic Equilibrium

The dynamics of the supply side is not difficult. Taking logs of equation (2.33) and differentiating with respect to time we obtain

$$\frac{\dot{X}_1}{X_1} = \delta_1 - \frac{a_{21}}{\alpha_1}\frac{\dot{p}}{p}, \tag{2.44}$$

which is the growth rate of supply of a country specialized in good 1.

Repeating the same procedure with equation (2.38) we obtain

$$\frac{\dot{X}_2}{X_2} = \delta_2 + \frac{a_{12}}{\alpha_2}\frac{\dot{p}}{p}, \tag{2.45}$$

which, in turn, is the growth rate of supply of a country specialized in good 2. To derive equations (2.44) and (2.45) we have used the specification of the learning process we saw at the beginning of the chapter.

By subtracting equation (2.44) from equation (2.45) we obtain the growth rate of the world relative supply of good 2:

$$(2.46) \qquad \left.\frac{d(X_2^w/X_1^w)/dt}{(X_2^w/X_1^w)}\right|_S = (\delta_2 - \delta_1) + \left(\frac{a_{12}}{\alpha_2} + \frac{a_{21}}{\alpha_1}\right)\frac{\dot{p}}{p} .$$

This equation is obtained, of course, under the assumption of no production switches; we derive below the condition for avoiding such switches. Under this condition, the growth rate of the world relative supply is a well-behaved increasing function of the rate of price change. Note that the responsiveness of the dynamic relative supply with respect to the change in the relative price depends on the degree of intersectoral integration.

Now, taking logs of equation (2.42) and differentiating with respect to time we obtain the growth rate of the world relative demand of good 2:

$$(2.47) \qquad \left.\frac{d(X_2^w/X_1^w)/dt}{(X_2^w/X_1^w)}\right|_D = -[\,1 + (\sigma - 1)|L|\,\Phi(p)\,]\,\frac{\dot{p}}{p} ,$$

where

$$\Phi(p) = \frac{Bp^{1-\sigma}}{[a_{21} + (1 - a_{11})Bp^{1-\sigma}]\,[(1 - a_{22}) + a_{12}Bp^{1-\sigma}]} > 0 .$$

It is not difficult to prove that the expression between squared brackets in equation (2.47) is non-negative; one can check that $0 \leq |L|\Phi(p) \leq 1$, since $\sigma > 0$ the proof is complete. Hence, the equilibrium growth rates of relative price and relative quantity exist and their solutions, conditional on given terms of trade, are the following:

(2.48) $$\frac{\dot{p}}{p} = -\frac{\alpha_1\alpha_2}{|L|}\left[\frac{\delta_2 - \delta_1}{1 + (\sigma - 1)\alpha_1\alpha_2\Phi(p)}\right],$$

and

(2.49) $$\frac{d(X_2^w/X_1^w)/dt}{(X_2^w/X_1^w)} = \frac{\alpha_1\alpha_2}{|L|}\left[\frac{1 + (\sigma - 1)|L|\Phi(p)}{1 + (\sigma - 1)\alpha_1\alpha_2\Phi(p)}\right](\delta_2 - \delta_1).$$

From now on we will assume, without loss of generality, that sector 2's learning technology is superior to that of sector 1, i.e. $\delta_2 > \delta_1$. In this case, the composition of the world relative supply changes over time: the countries specialized in good 2 learn more quickly and increase their supply quicker than countries specialized in good 1 [see equation (2.49)]. However, the terms of trade move against countries producing good 2 [see equation (2.48)]. Thus, if the terms of trade are worsening faster than the improvement in productivity of the countries specialized in good 2, they may be obliged to switch to the production of good 1. The condition to avoid switches is given by the following inequality:[18]

$$(\sigma - 1) \geq -\frac{\delta_1/\delta_2}{\alpha_1\alpha_2\Phi(p)}.$$

This condition is clearly satisfied when the goods are good substitutes, $\sigma > 1$. Hence, under the assumption of high substitutability the consistency of the model is guaranteed.

[18] A good-2 producing country increases its human capital at the rate δ_2, this means that in our figure 2.4 the country's human capital coordinates are moving vertically upwards. Since the terms of trade are falling, the separating line is rotating upwards. Thus, the above condition states that δ_2 is higher than the vertical speed of the separating line.

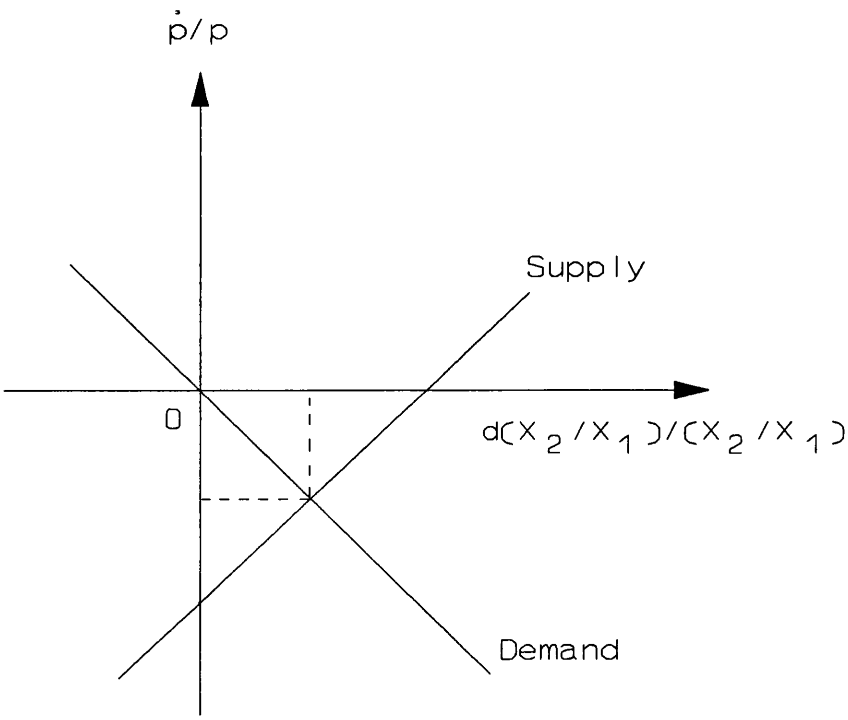

Figure 2.7 *Dynamic World Market Equilibrium*

The dynamic equilibrium under this assumption is depicted in Figure 2.7.

Given the growth of the terms of trade we can determine the dynamic welfare gains from international trade. Recall that we found the growth rate of utility to be a weighted average of the growth rate of purchasing power of good 1, and the growth rate of purchasing power of good 2 [see equation (2.25)]. Using the supply functions of countries specialized in goods 1 and 2, equations (2.33) and (2.38) respectively, we obtain the following measures of welfare growth for open economies:

$$\left.\frac{\dot{u}}{u}\right|_1 = \frac{[\alpha_1 a_{12} + \alpha_1\alpha_2\theta(p) + (\sigma-1)\alpha_1\alpha_2|L|\Phi(p)]\delta_1 + [\alpha_2(1-a_{11}) - \alpha_1\alpha_2\theta(p)]\delta_2}{\alpha_1 a_{12} + \alpha_2(1-a_{11}) + (\sigma-1)\alpha_1\alpha_2|L|\Phi(p)}$$

(2.50)

and

$$\left.\frac{\dot{u}}{u}\right|_2 = \frac{[\alpha_1 a_{12} + \alpha_1\alpha_2\theta(p)]\delta_1 + [\alpha_2(1-a_{11}) - \alpha_1\alpha_2\theta(p) + (\sigma-1)\alpha_1\alpha_2|L|\Phi(p)]\delta_2}{\alpha_1 a_{12} + \alpha_2(1-a_{11}) + (\sigma-1)\alpha_1\alpha_2|L|\Phi(p)}$$

(2.51)

where the subscript of the vertical line denotes the activity of specialization and the rest of the notation is already known. Equations (2.50) and (2.51) show that the rate of welfare growth in an open economy is given at any moment as a weighted average of the learning coefficients δ_1 and δ_2.

2.4.5 Theory and Facts Again

In section 2.3.6, in the context of autarky, we identified sector 1 with primary activities, and sector 2 with manufacturing activities. Now, for the world economy, we keep symmetry by identifying countries specialized in sector 1 with countries specialized in primary activities. Consequently those countries specialized in sector 2 are identified with countries specialized in manufacturing activities.

Here we will make the same assumptions with respect to the parameters of the model that we made for closed economies (see section 2.3.6). Let us recall briefly that we assumed high substitutability in final consumption between primary goods and manufactured goods ($\sigma > 1$). We also assumed superior learning technology in manufacturing activities with respect to primary activities ($\delta_2 > \delta_1 \geq 0$), and, complementarily, high dependency on manufacturing goods in primary activities (high a_{21}), and low dependency on primary goods in manufacturing activities (low a_{12}).

In sub-section 2.4.4.2 we showed that the assumption of high substitutability guarantees the non-switching condition which, in turn, guarantees the <u>internal</u> consistency of the model; additionally, the non-switching condition is consistent with the historic experience of development and industrialization: we do not observe industrialized countries becoming agricultural countries, rather we observe the contrary.

2.4.5.1 Some Previous Results

Some of the results that the model generates under autarky are also obtained for the world economy under the same set of assumptions:

(1) Steady increase in the human capital stock of industrial countries as compared with the human capital stock of non-industrialized countries. The corresponding evidence for this feature is the growing gap in per-capita real income between industrialized and non-industrialized economies;

(2) Substitution of primary goods for manufactured goods both in intermediate and final demand [see equations (2.41) and (2.49)];

(3) The price of manufactured goods relative to primary goods falls over time [see equation (2.48)]. However, in the Appendix we prove that if quality upgrading is proportional to human capital accumulation through learning-by-doing, the model replicates the fall in the relative price of primary goods and results (1) and (2) still hold.

We believe that our model also helps to shed light on some other features of economic development. We will consider briefly the dynamic gains from international trade, the strategy of development through import substitution and the evolution of the countries' dependence on international trade.

2.4.5.2 Dynamic Gains from International Trade

Equations (2.50) and (2.51) show that the rate of welfare growth of an open economy is a weighted average of the learning coefficients δ_1 and δ_2. Due to complete specialization, δ_1 is the rate of human capital growth in countries specialized in good 1, and δ_2 is the rate of human capital growth in countries specialized in good 2, [see equation (2.2)]. From equation (2.26) we know that the rate of welfare growth of a closed economy is a weighted average of the rates of human capital growth in sector 1 and sector 2: $\delta_1 n_1$ and $\delta_2(1-n_1)$, respectively. Hence, the range of values in which one can find the growth rates of open economies is necessarily to the right of the range of values in which one can find the growth rates of closed economies. This result is reinforced by the fact that the allocation of labour force in closed economies is confined within some given bounds -see section 2.3.4.

The last result may be useful for explaining, at least partially, why on average open economies grow faster than closed economies, which is the fact that Chenery and Syrquin (1989), and many others, have captured through cross-country analysis. This fact is unequivocally reproduced by our model under the assumption of logarithmic preferences,[19] the following proposition states this result.

PROPOSITION 1. Assuming logarithmic preferences, i.e. $\sigma = 1$, regardless of the good of specialization and over time, open economies experience superior dynamic welfare gains than closed economies. Moreover, open and closed economies grow at constant rates because logarithmic preferences imply that substitution effects and income effects cancel. **Proof:** Set the elasticity of substitution equal to unity in equations (2.26), (2.50) and (2.51). Since $\theta(\cdot) = b$ when $\sigma = 1$, it follows that $\dot{u}/u|_2 = \dot{u}/u|_1 > \dot{u}/u$. We also can see that these growth rates are time invariant. End of proof.

Our Simulation 3 (page 91) illustrates Proposition 1. The parameters of this simulation are as set in section 2.3.7, except for the elasticity of substitution which is set to unity.

Now, the assumption of logarithmic preferences is not consistent with the observed differentials in growth rates across countries, nor is it consistent with the observed long run shift of demand from primary goods to manufactured goods. Nevertheless, Proposition 1 is important because the case of

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[19] By assuming logarithmic preferences we depart temporarily from the basic assumption of high substitutability. The usefulness of this procedure will become evident below.

logarithmic preferences is the borderline of the case which is of primary interest to us (the good substitutes case). Hence, by continuity, if the degree of substitutability is not too high our model predicts that open economies must enjoy higher dynamic welfare gains than closed economies.

In the case of high substitutability, the growth rates of open and closed economies do not collapse to single values if the cross-country distribution of relative human capital endowments is not degenerate. This makes welfare comparisons a more difficult task. However, we will argue that on average the advantage of open economies is likely to be preserved. The analysis of this issue will be postponed until the end of this sub-section. First we need to analyze the welfare gains of open economies.

PROPOSITION 2. Under the maintained assumptions, which include high substitutability ($\sigma > 1$) and more efficient learning technology in manufacturing activities than in primary activities ($\delta_2 > \delta_1$), open economies specialized in manufactured goods do not experience lower rates of welfare growth than open economies specialized in primary goods.
Proof: This result is determined by the fact that the welfare growth of open economies relies more on their own learning technology, i.e. the rate of welfare growth of open economies specialized in manufactured goods is biased towards δ_2, whilst the rate of welfare growth in open economies specialized in primary goods is biased towards δ_1. These biases are deduced by comparing equations (2.50) and (2.51), taking into account that open economies face the same relative price (p), and the

elasticity of substitution is high. End of proof.

Proposition 2 is illustrated in Simulations 4 and 5 (pages 91 and 92, respectively). These simulations show that open economies specialized in manufactured goods (good 2) follow a dynamic path which is Pareto-superior to the dynamic path of open economies specialized in primary goods (good 1).[20] For Simulation 4 we use the parameter values as set in section 2.4.7; for Simulation 5 we set the parameters $a_{12} = 0$ and $\alpha = 0.4$, but everything else is unchanged. We will discuss below the consequences of this variation.

Simulations 4 and 5 also show that the welfare gains of open economies specialized in primary goods are diminished during the period of sharp substitution of primary goods for manufactured goods in the world economy. This outcome is determined by the combination of a high elasticity of substitution and a falling relative price of manufactured goods. Thus there exists a period in the world economy when the substitution process accelerates leading to lower rates of welfare growth in countries specialized in primary goods.

However, open economies converge eventually towards a common rate of welfare growth if their technologies are mutually integrated. This feature is shown in Simulation 4, where crossed technological coefficients are set as follows: $a_{12} = 0.05$ and $a_{21} = 0.4$ (see section 2.4.7).

If, however, the country specialized in primary goods is

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[20] It may be argued that dynamic welfare comparisons require a ranking of discounted streams of utility flows. However a dynamic path characterized by non-inferior growth rates of utility should Pareto-dominate other paths for any given common path of discount rates.

not integrated to the production activity of manufacturing countries, i.e. $a_{12} = 0$, which is the case illustrated by Simulation 5, the rate of welfare growth in non-industrialized economies will diverge from the rate of welfare growth in industrialized economies. This possibility implies growing welfare gaps between industrialized countries and non-industrialized countries.

The last results show the importance of intersectoral analysis for understanding the growth paths of open economies. To recap, it appears from Simulations 4 and 5 that the dynamic path of countries specialized in primary activities depends on their technological integration as suppliers of intermediate goods to industrialized countries. Those countries specialized in primary goods which are required as intermediate inputs in manufacturing activities may enjoy a better economic performance than countries specialized in primary goods which are not required in manufacturing activities.

Simulations 4 and 5 also show that open economies specialized in manufacturing (high-learning activities) tend to enjoy increasing rates of welfare growth. The economic intuition for this result is as follows: although the rates of human capital accumulation in open economies are fixed due to complete specialization, the process of substitution of primary goods for manufactured goods -both in final an intermediate consumption- implies that the world economy relies progressively more on high-learning activities. This intuition is analogous to the economic intuition for growing rates of welfare growth in closed economies [see result (5) in section 2.3.6]. However, the analogy has some limits as the benefits of the substitution process are reaped primarily by open industrialized countries.

We proceed now to compare the welfare gains of open economies and closed economies under the assumption of high substitutability. In order to do that we will assume that the relative price of manufactured goods in closed economies is not below the relative price of manufactured goods in the world market, i.e. $q \geq p$. We will justify this assumption in the next section. Hence we can formulate the following proposition.

Proposition 3. Open economies specialized in manufactured goods enjoy higher dynamic welfare gains than closed economies. **Proof:** The weight coefficient $\theta(\cdot)$ is increasing in the relative price of good 2 when high substitutability is assumed [see equation (2.18)]. If the relative price of manufactured goods is higher in closed economies than in open economies, $q > p$, then $\theta(q) > \theta(p)$. Taking into account the latter inequality, Proposition 3 follows from comparison of equations (2.26) and (2.51). End of proof.

However, open economies specialized in primary activities do not necessarily perform better than closed economies. This follows from the possibility of falling rates of growth for open economies specialized in primary activities and the fact that closed economies enjoy increasing rates of welfare growth (see feature 5 in section 2.3.6). Hence, if the degree of substitutability is sufficiently high, closed economies relatively specialized in the high-learning manufacturing activities may perform better than open economies specialized in primary goods.

We have reached the conclusion that it is not possible to claim that open economies always Pareto-dominate closed economies. However, on average open economies are bound to perform better than closed economies as the comparative advantage of the latter is most likely to be biased towards the production of primary goods. This is intrinsically connected to the assumption that $q \geq p$, as we will show in the next section.

2.4.5.3 The Strategy of Development through "Import Substitution"[21]

Is it possible to support a policy of import substitution as an strategy of development in our model? The answer again is a qualified yes.

Under the assumption of high substitutability and higher comparative advantage in learning technology of the manufacturing sector, it is likely that a closed economy will be better off by industrializing. There are two possibilities -refer to Figure 2.4: either the relative human capital in manufacturing activities is high, so that the country's human capital coordinates are above the (international) price line (e.g. point A), and thus the domestic relative price of manufactured goods is below the (international) terms of trade for manufactured goods ($q \leq p$); or the relative human capital

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[21] Import substitution here is understood as a policy such that the economy is cut off the international market through prohibitive tariffs and/or other trade barriers.

in primary activities is high, so that the country's human capital coordinates are below the international price line (e.g. point B), and the domestic relative price of manufactured goods is above the terms of trade for manufactured goods ($q \geq p$).

In the first case (point A), the country should be opened to the world markets because it has the comparative advantage to specialize in manufactured goods and enjoy a higher level of welfare gains. That is why we assumed above that closed economies tend to be intensive in human capital in primary activities and thus tend to face a higher relative price of manufactured goods than the world economy (see Proposition 3 in Section 2.4.5.2).

The second case (point B) is more interesting. If this country opens its doors to the world market, it would become absolutely specialized in agricultural activities and its welfare gains could be diminished with respect to its original situation. Instead, the country can remain in autarky whilst building the necessary relative level of human skills in manufacturing activities. This would imply simultaneously, continuous reallocation of labour force to the manufacturing sector and continuous fall in the relative price of domestic manufactured goods (this country is travelling along the line crossing point M in Figure 2.3a). If the country opens before the domestic relative price is equal to the terms of trade, the process will be reversed and the country will be specialized in primary activities. Therefore, a necessary condition for such catching-up is that the relative price of manufactured goods in autarky falls faster than the international relative price of manufactured goods [$(dq/dt)/q < (dp/dt)/p$]. By comparing equation (2.22) with equation (2.48) we deduce that the last condition is guaranteed by the following inequality:

$$\textbf{(2.52)}\quad n_1 < \bar{n}_1(p) = \frac{\delta_1 + (\sigma - 1)\,\alpha_1\alpha_2\,\Phi(p)\,\delta_2}{(\delta_1 + \delta_2)\,[1 + (\sigma - 1)\,\alpha_1\alpha_2\,\Phi(p)]}\ ,$$

where $\bar{n}_1$ is the share of primary activities in the workforce which equates the growth rate of the relative price of manufactured goods in autarky with the growth rate of the relative price of manufactured goods in the world market. Note that $\bar{n}_1$, which we will call the <u>critical</u> share of primary activities in the workforce, is a positive fraction under the assumption of good substitutability. Hence we have three possibilities -refer again to Figure 2.3a:

(1) The critical share of primary activities in the workforce is below the lower limit, i.e. $\bar{n}_1 < n_{1,l}$. Thus, a closed economy cannot pursue successfully the strategy of import substitution because it cannot allocate less labour force to primary activities than the minimum level.

(2) The critical share of primary activities in the workforce is between the lower and upper limits, i.e. $n_{1,l} < \bar{n}_1 < n_{1,u}$. Hence a closed economy relatively specialized in manufacturing (whose share of primary activities in the workforce is close to the lower limit), may be able to pursue successfully the strategy of import substitution.[22]

(3) The critical share of primary activities in the workforce is above the upper limit, i.e. $\bar{n}_1 > n_{1,u}$. Thus the closed economy seems able to pursue successfully the strategy.

At this point we should note that all the analysis in this section requires the assumption of a rational government choosing trade regime as a welfare maximizer. In making this choice the government has to consider the country's relative endowment of human capital and the conditions in the world markets.

However, even accepting that governments are rational, closed economies may be unable to pursue successfully a strategy of import substitution -which they may consider as optimal- because their strategies interact. Suppose that given the technological conditions, the preferences and the terms of trade, all economies have the possibility of pursuing a successful policy of import substitution as a strategy of development. Nevertheless, we will argue that this strategy <u>cannot</u> be generalized. We can think of at least two reasons within the framework of this model:

(1) The world equilibrium should be preserved. A successful policy of import substitution can only be possible for individual economies whose size is small enough so that the international market equilibrium is not affected. A massive

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22 Actually, under this circumstance a closed economy may become industrialized if it waits sufficiently to build up its human capital in manufacturing activities before switching to an open trade regime. But clearly this strategy is more likely to be successfully pursued by those economies with a higher comparative advantage in manufacturing activities.

process of industrialization would increase the relative supply of manufactured commodities, hence the relative price of primary goods would increase and some countries would find it optimal to return to primary activities. Furthermore, even if some countries are successful in pursuing a policy of import substitution, followers will find it increasingly difficult to become exporters of manufactured goods and simultaneously competitive in the world market. Hence, here first movers benefit and, to that extent, history matters.

(2) The second reason we can think of is also related to history: a policy of import substitution is more likely to be successful during the period of quick substitution of primary goods for manufactured goods. It is during this period that the world demand for manufactured goods expands quicker. Afterwards the substitution process is relatively exhausted and the speed of expansion is diminished.

This last argument could be supported analytically by examining the behaviour of the critical share of primary activities in the workforce, $\bar{n}_1$. From equation (2.52) we deduce that $\bar{n}_1$ depends on the terms of trade, p, the elasticity of substitution, σ, and the interindustry linkages. If the elasticity of substitution is high, $\sigma > 1$, and manufacturing and primary activities are technologically linked, the behaviour over time of $\bar{n}_1$ is as follows: for high values of p, $\bar{n}_1$ is closed to zero; for a brief interval of prices (which is shorter the larger the degree of substitutability) $\bar{n}_1$ climbs rapidly; afterwards $\bar{n}_1$ falls to levels close to zero as p goes to zero. The exhaustion of the substitution process is reflected in the fall of the critical share at low relative prices of manufactured goods; at this stage, a closed economy requires a higher level of human capital accumulation in manufacturing activities for pursuing successfully the policy of import substitution.

It is important to emphasize that the period of quick substitution in the world economy only occurs if interindustry linkages exist and the elasticity of substitution is high. From equation (2.52) we see that, if the elasticity of substitution is equal to unity, the critical share $\bar{n}_1$ is constant. On the other hand, if no interindustry linkages exist, i.e. $a_{12} = a_{21} = 0$, it can be proved that the critical share is also constant.[23]

Simulation 6 (page 92) shows the typical behaviour of the critical share under the assumption of high substitutability and mutually integrated sectors. The parameters used in this simulation are as set in section 2.3.7.

2.4.5.4 Trade Biases

Under the maintained assumptions, our model implies that the technological interdependence of open economies changes as the international relative price of manufactured goods falls. Open economies specialized in primary commodities experience a growing ratio of exports to output, thus these countries become increasingly dependent on international

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[23] If no interindustry linkages exist, one can prove that $\alpha_1\alpha_2\Phi(p) = 1$. Hence from equation (2.52) we deduce that $\bar{n}_1$ is constant.

trade. This result follows from equation (2.37) and the fact that the weight coefficient $\theta(\cdot)$ is increasing in the relative price of manufactured goods when high substitutability is assumed [see equation (2.18)]. On the other hand, from equation (2.39) we deduce that open economies specialized in manufactured goods experience a decreasing ratio of exports to output.

2.5 Summary and Concluding Comments

After all this analysis we have found that a two-sector model with human capital accumulation through learning by doing, where interindustry linkages are explicitly considered, can replicate some features of economic development within a competitive environment characterized by endogenous growth. The features are the following: **(1)** the long run reallocation of factors (labour in our model) out of the primary sector; **(2)** the long term shift of final demand in favour of manufactured goods; **(3)** the growing share of intermediate goods in gross demand (input-output deepening) through the substitution of primary goods for manufactured goods; and **(4)** the long term deterioration of relative prices of primary goods when quality upgrading is proportional to capital accumulation through learning by doing (refer to the Appendix). Features (2) and (3) do not conflict with feature (4) because the relative price of manufactured goods adjusted by quality improvements falls continuously (the quality corrected relative price of primary goods increases).

Our model reproduces these patterns of industrialization under the following assumptions: **(1)** primary goods and manufactured goods are good substitutes (this assumption is also necessary for the internal consistency of the model, it guarantees that no industrialized country switches to primary activity); **(2)** manufacturing activities induce a higher rate of human capital accumulation than primary activities; and, **(3)** the technological interdependence between the sectors is such that the workforce allocation is biased towards the manufacturing sector (this assumption strengthens the replication of the stylized facts but it is not necessary). These maintained assumptions are consistent with the experience of economic development.

Under these assumptions we found that long term rates of welfare growth tend to increase along the path of industrialization.

We also found, as in Lucas (1988), that open industrialized economies grow faster than open economies specialized in primary activities. By comparing the performance of open economies against closed economies, we found that on average open economies experience higher dynamic welfare gains than closed economies.

Since our model generates a tendency for the quality-corrected relative price of manufactured goods to fall, there exists a historic period in which a sharp substitution of primary goods for manufactured goods takes place. During this period, the welfare growth of open economies specialized in primary goods may fall. However, open economies will eventually converge towards a common rate of welfare growth if they are integrated through intermediate linkages. If primary inputs are not required in manufacturing activities, the rate of welfare growth of open economies specialized in primary goods will diverge from the rate of welfare growth of open industrialized countries.

Under the maintained assumptions the model predicts that open economies specialized in primary activities will become increasingly dependent on international trade for their process of growth. This dependence is basically due to their need for manufactured goods.

Finally, we explored the possibility of an import substitution strategy as a way of development and industrialization. Our model gives a theoretical background to the claims of development economists who argue that an initial period of import substitution of manufactured goods and even protection to infant industries is necessary for an economy to pursue a successful development strategy based on manufacturing exports. However, we also found that not all countries specialized in primary goods can follow this strategy and hence the infant-industry argument is not general. Favourable initial conditions, which include a high relative human capital endowment in manufacturing activities, and the timing of entry in the market of manufactured goods -which implies an expanding world market for these goods- are decisive in the possibility of successful development relying on manufacturing exports.

APPENDIX: QUALITY ADJUSTED TERMS OF TRADE

Our model is solved under the assumption that the quality of goods is constant. However, in the real world, quality upgrading of goods and improvements in labour productivity often go together. If we assume that quality upgrading is proportional to capital accumulation through learning by doing, the production function [equation (2.1)] could be rewritten as follows

$$Y_j = (h_j n_j)^{\alpha_j} (Y_{1j})^{a_{1j}} (Y_{2j})^{a_{2j}}, \qquad \alpha_j + a_{1j} + a_{2j} = 1, \quad j = 1, 2.$$

where $Y_j = h_j X_j$ is the gross output of good j corrected by quality upgrading, and $Y_{ij} = h_i X_{ij}$ is the quality corrected intermediate good i used in sector j. If we assume that consumers' utility is also augmented by quality upgrading, we would obtain a symmetric model to the model we have developed here. In such a case q and p would denote the quality-corrected relative price of good 2 for the closed economy and the open economy, respectively. The relative prices in physical terms would then be given by qh and ph.

Using equation (2.2) and equation (2.22) we calculate the growth rate of the relative price of good 2 in physical terms under autarky:

$$\frac{\dot{q}}{q} + \frac{\dot{h}}{h} = \left(\frac{\alpha_1 a_{12} + \alpha_2 a_{21}}{|L|} \right) [\delta_2 - (\delta_1 + \delta_2) n_1] .$$

For this equation to imply a positive trend in the relative price of good 2, we require the same condition we imposed for our model to fit the facts, namely that the share of the primary sector in the labour force is below the steady state level: $n_1 < n_1^* = \delta_2/(\delta_1+\delta_2)$ (see section 2.3.6). Interindustry linkages are also necessary for this result, either a_{12} or a_{21}, or both technical coefficients should be positive.

In the international market and under the non-switching condition (see section 2.4.4.2), the terms of trade of manufactured goods also improve, provided quality upgrading is taken into account. Take one country specialized in good 2 and another specialized in good 1. Since we are not taking into account population growth, the human capital stock of the country specialized in good 2 is increasing relative to the human capital stock of the country specialized in good 1, at the rate $(\delta_2-\delta_1)$. Therefore, the terms of trade for the country producing good 2, ph, are changing at the following rate:

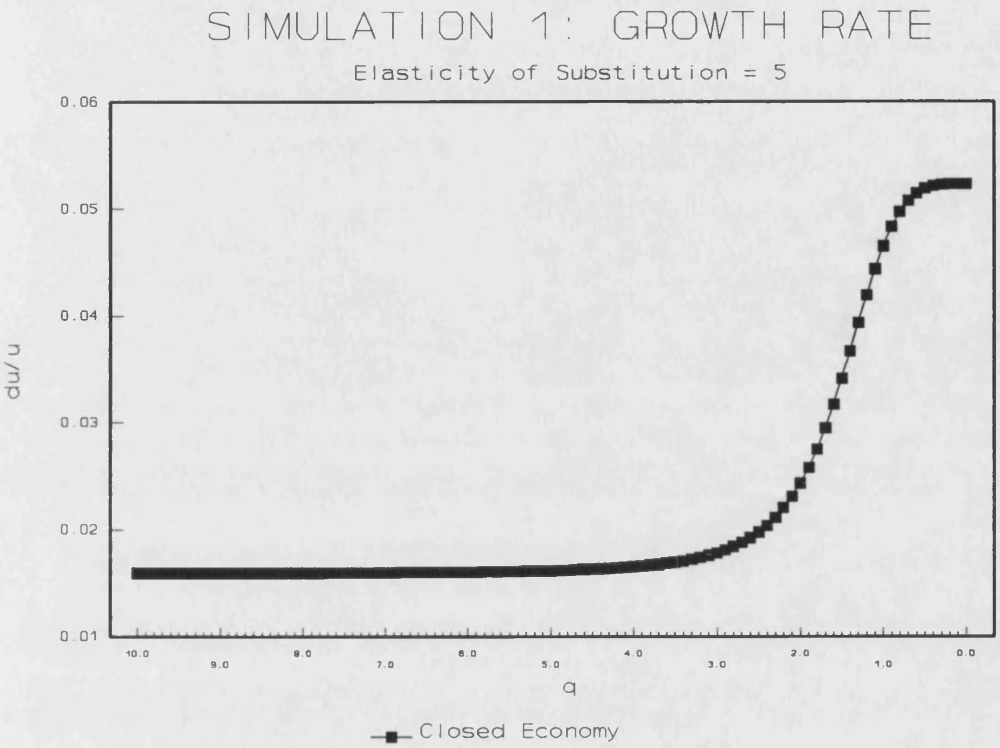
$$\frac{\dot{p}}{p} + \frac{\dot{h}}{h} = \frac{\alpha_1 \alpha_2}{|L|} \left[\frac{a_{12}}{\alpha_2} + \frac{a_{21}}{\alpha_1} + \frac{(\sigma - 1) |L| \Phi(p)}{1 + (\sigma - 1) \alpha_1 \alpha_2 \Phi(p)} \right] (\delta_2 - \delta_1) .$$

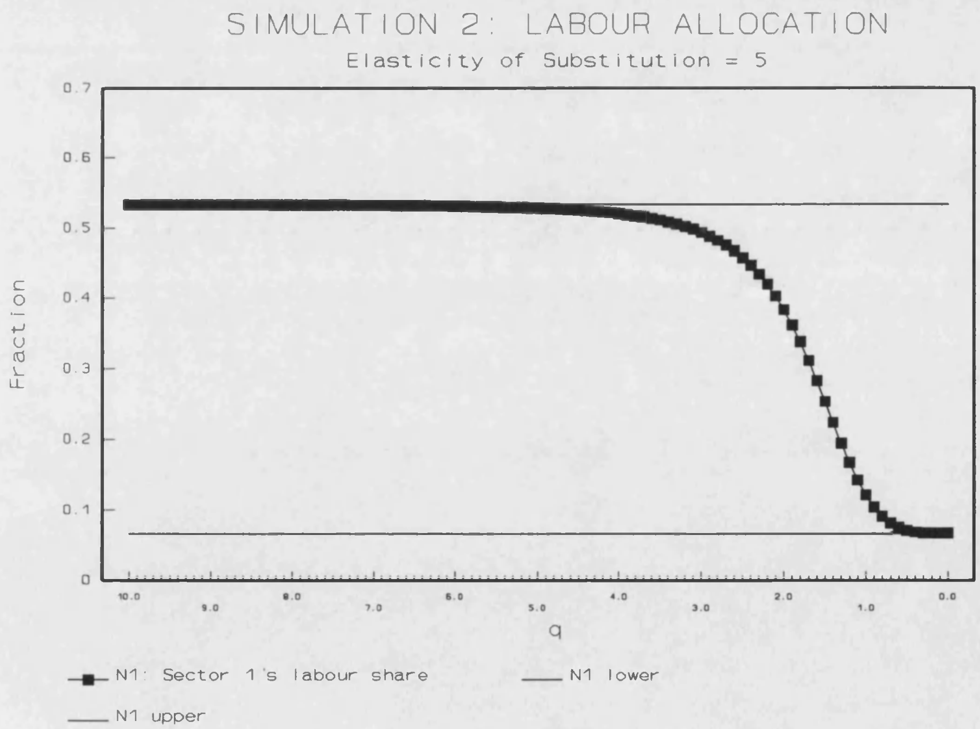
Clearly, the terms of trade for the country with higher learning technology improve over time. This improvement is strengthened with the degree of international complementarity in production (intersectoral linkages) and the degree of substitutability between goods.

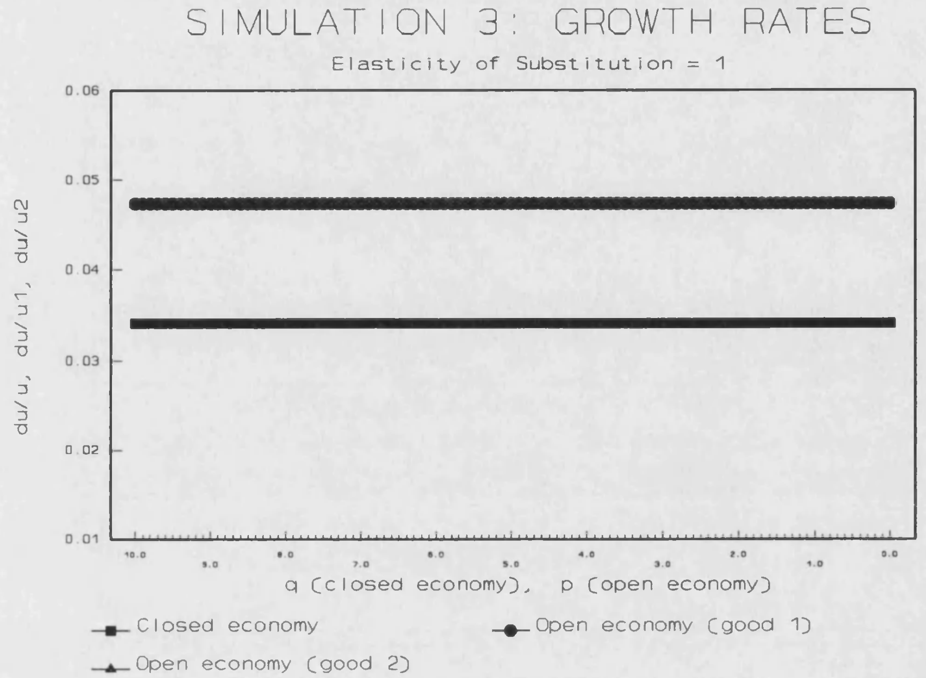
Therefore, both in autarky and international markets, we could have an increasing relative price of manufactured goods whilst the quality-adjusted relative price of manufactured goods falls over time, inducing substitution in favour of manufactured goods.

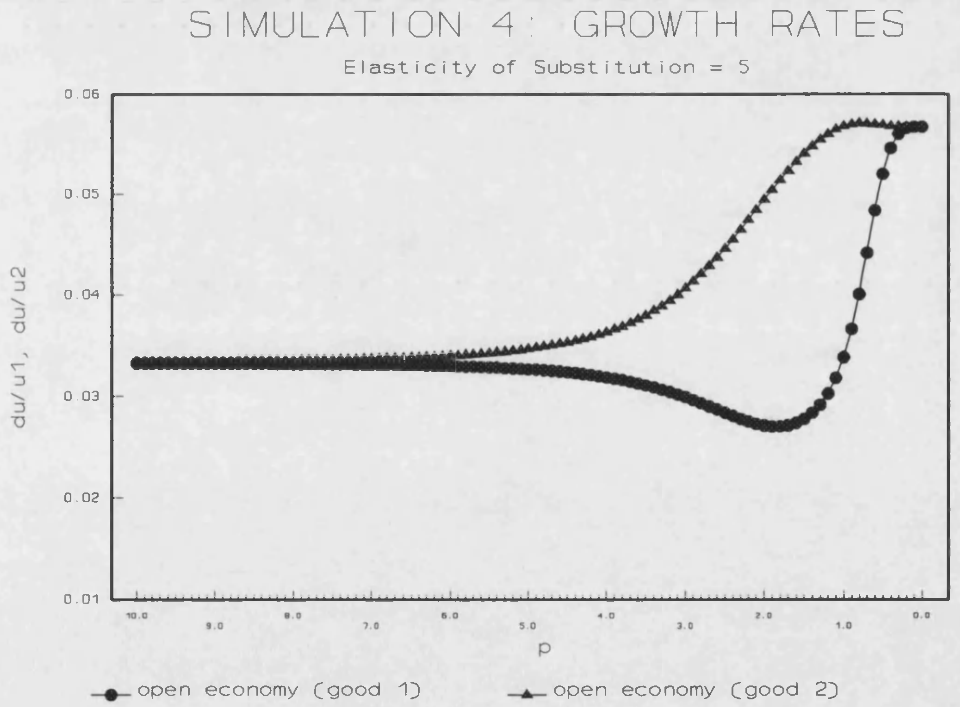
NOTATION

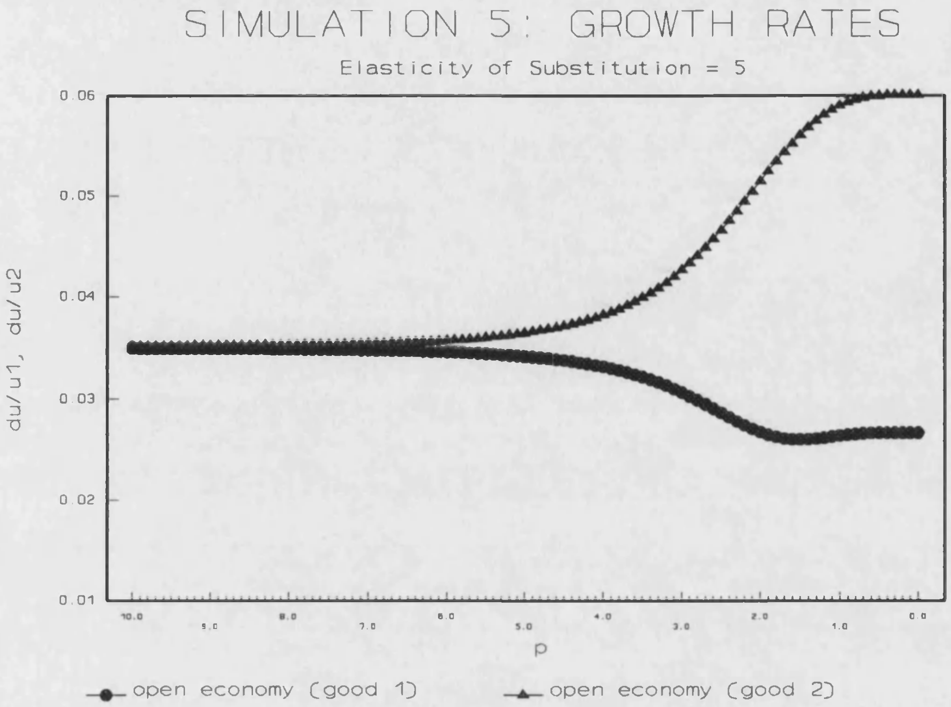
a_{ij} : Output elasticity of good i with respect to intermediate input j.
α_j : Output elasticity of good j with respect to labour.
b : Weight of good 1 in the utility function.
B : Constant [equation (2.6)].
C_i : (Final) consumption of good i .
γ : Exponent of the CES utility function.
δ_j : Learning coefficient in sector j.
E_i : Exports of good i.
ε_j : Constant [equation (2.15)].
$\Phi(\cdot)$: Function [equation (2.47)].
h_j : Human capital stock in sector j.
h : Relative endowment of human capital in sector 2 ($\equiv h_2/h_1$).
L : Leontief Matrix.
$|L|$: Determinant of the Leontief Matrix.
M_i : Imports of good i.
n_j : Workforce fraction allocated to sector j.
$n_{1,l}$: Lower limit of n_1 (closed economy)
$n_{1,u}$: Upper limit of n_1 (closed economy)
n_1^* : Steady state value of n_1 (closed economy)
$\bar{n}_1$: Critical workforce allocation to sector 1 (closed economy).
p_i : Nominal price of good i.
p : (Relative) terms of trade of good 2.
q : Relative price of good 2 (closed economy).
σ : Elasticity of substitution in (final) consumption.
t : time.
$\theta(\cdot)$: Fraction of real income allocated to final consumption of good 1.
u : Utility.
w : Wage rate.
X_i : Gross output of good i.
X_{ij} : Intermediate consumption of good i in sector j.

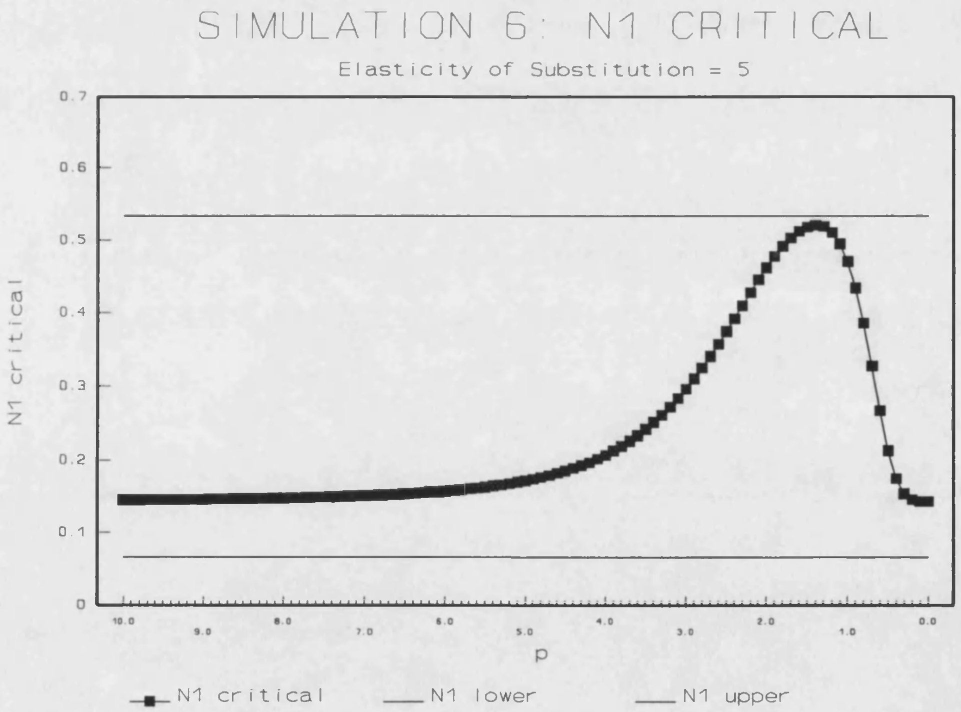

SIMULATION 1: GROWTH RATE
Elasticity of Substitution = 5
0.06
0.05
0.04
0.03
0.02
0.01
du/u
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
q
Closed Economy


SIMULATION 2: LABOUR ALLOCATION
Elasticity of Substitution = 5
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Fraction
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
q
N1: Sector 1's labour share
N1 lower
N1 upper


SIMULATION 3: GROWTH RATES
Elasticity of Substitution = 1
0.06
0.05
0.04
0.03
0.02
0.01
du/u, du/u1, du/u2
q (closed economy), p (open economy)
Closed economy
Open economy (good 1)
Open economy (good 2)


SIMULATION 4: GROWTH RATES
Elasticity of Substitution = 5
0.06
0.05
0.04
0.03
0.02
0.01
du/u1, du/u2
p
open economy (good 1)
open economy (good 2)


SIMULATION 5: GROWTH RATES
Elasticity of Substitution = 5
0.06
0.05
0.04
0.03
0.02
0.01
du/u1, du/u2
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
p
open economy (good 1)
open economy (good 2)


SIMULATION 6: N1 CRITICAL
Elasticity of Substitution = 5
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
N1 critical
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
p
N1 critical
N1 lower
N1 upper

CHAPTER 3

ECONOMIC STRUCTURE, EDUCATION AND GROWTH

3.1 INTRODUCTION

A casual examination of any country's input-output matrix shows different degrees of backward technological integration across sectors: some activities require more inputs and some activities require less. Similarly there exist different degrees of forward integration: some sectors provide more intermediate inputs than final goods, and some sectors providing intermediate goods serve more activities than others. This structure of interindustry linkages is usually very stable even when the economy is shocked by strong changes in relative prices. Hence, in the framework of a closed economy, it is natural to think that sectors with higher backward technological integration should develop later than those sectors with lower backward integration. This intuition is right: one of the more robust features of economic development is the evolution of the economic structure towards more technologically integrated forms of production (Leontief, 1963). Hence, along the path of development the economy enjoys a wider availability of goods that reflects an increasing process of social division of work. This process is reflected in an increasingly complicated structure of interindustry linkages. Following Chenery, Syrquin and Robinson (1986), we shall call this process input-output deepening.

If division of work increases labour productivity, as in the pin factory of Adam Smith, we may have social gains in productivity coming from the multiplication of economic activities across the society. Romer (1987, 1990) and others have explored this intuition and have shown the possibility of sustained growth in models where technological progress increases permanently the social division of labour. However, they do not capture the phenomenon of input-output deepening as they assume *ab initio* identical (and simple) technologies for all inputs of a (single) final good technology. Besides, these models have focused on the process of technological change through innovation. Hence they are more suitable to the analysis of economic development in industrialized economies where the possibilities of increasing the range of available goods (and the degree of social division of work) come basically from technological innovation.

However, for developing countries the main source of economic diversification is the copy, transfer and adaptation of existing technologies from developed countries. This is not to deny the possibility of important technological breakthroughs in developing countries, but clearly the non-rival character of technological information and also the limited possibility of excluding developing countries from using the technologies previously discovered in developed countries, make it cheaper and more advantageous for developing countries to become specialized in copying existing technologies. Since we are primarily interested in modelling the economics of developing countries, our model will be based on technological change through copy and adaptation. We will ignore the existence of patents and assume that adaptation of technologies can be done by investing in know-how. We will also consider an economic structure that experiences a growing degree of backward technological integration among sectors.

In the first stage of our research we will analyze a closed economy where economic diversification is brought about by technological transfers. It may seem odd to assume both autarky in trade and the possibility of technological transfers. However, such a scenario arises naturally where there is initially a high degree of protection due to transport costs or prohibitive tariffs. This means that the nature of foreign goods is known, but their consumption is restricted until the country starts its own production. In other words, we will assume that technology transfer is much cheaper than transfer of goods.

Technological transfers are, however, limited by the process of accumulation of human capital. In our model human capital is interpreted as the knowledge of a given number of technologies: we may understand the technology as a "recipe", one for each good, that allows the transformation of some "ingredients" into new goods.[1] Before starting cooking the chefs must learn the recipes. Hence increasing human capital (i.e. learning new recipes) depends on the quality and efficiency of education and the allocation of some effort. Thus, the transfer and adaptation of technology is a process that requires continuous education of the country's workforce.

In our model, as in Lucas (1988), education is a condition for improvement of human capital. But education diverts resources from productive activities. If no effort is allocated to education, the range of goods (and sectors) is unchanged but the current level of output is maximum. On the

[1] The idea is taken from Leontief (1963).

contrary, if the whole workforce is allocated to education, the growth of human capital is maximum (the learning rate of recipes is maximum), but output is zero. Hence there exists a trade-off between education and production.

Now, once a recipe is learned it stays with us forever. Furthermore, as we learn recipes we get to know more ingredients and then it is easier to learn even more recipes. Hence, we will assume that the technology of education is linear in the current level of human capital (the number of recipes known) and the amount of effort allocated to education. This linearity is the source of sustained diversification in our economy.

The chapter is organized as follows. In section 3.2 we consider a closed economy where economic diversification is brought about by education. Section 3.3 extends this model to consider the effects of international trade. In section 3.4 we return to the closed economy case but introduce physical capital accumulation.

3.2 A MODEL OF ECONOMIC DIVERSIFICATION THROUGH EDUCATION

3.2.1 The Model

The economic structure is represented instantaneously by an input-output matrix augmented with the vector of workforce allocation (see Figure 3.1). There is no joint production and all sectors (and goods) are indexed according to the degree of backward technological integration between 0 and N. This integration is assumed to increase linearly with the sector's index: the sector j only uses as intermediate

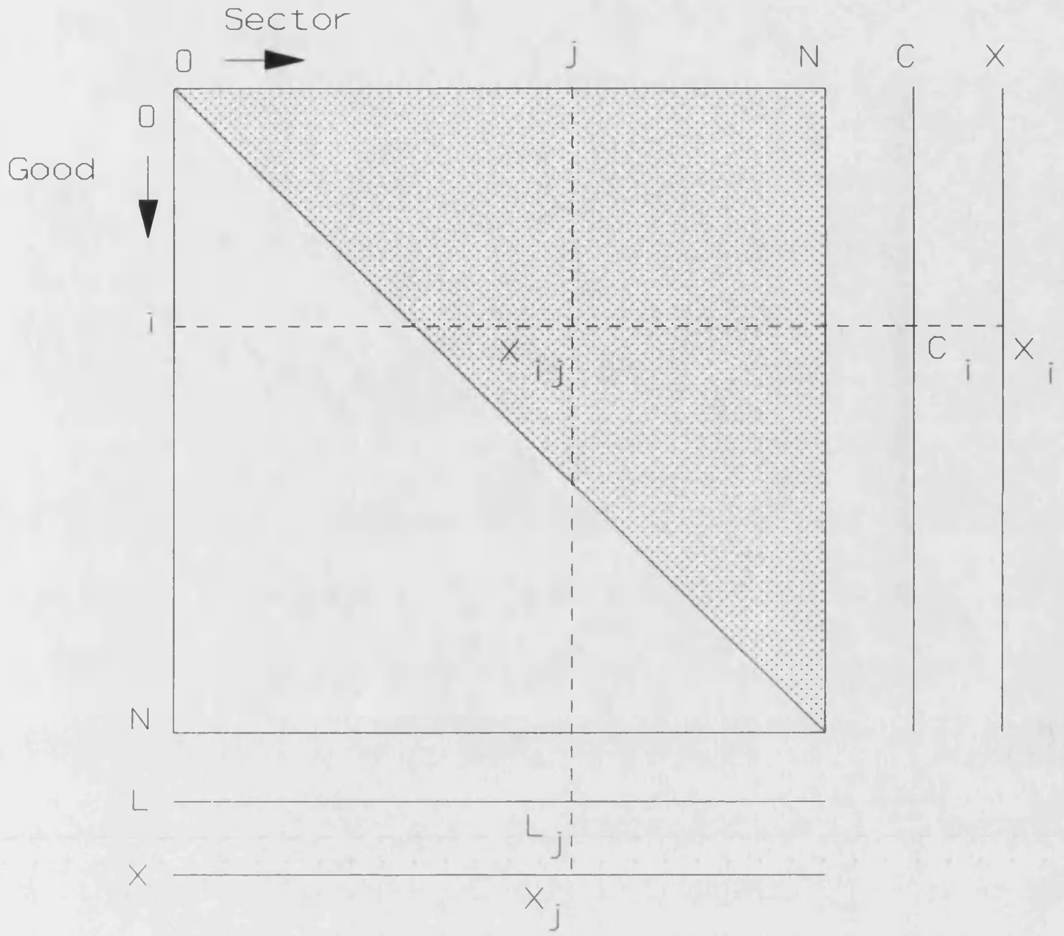


Figure 3.1 *Input-Output Matrix*

inputs the goods with lower index. This feature guarantees that the input-output matrix is perfectly triangular. The intermediate inputs of any sector can be read vertically off the input-output matrix. The labour force is indexed according to its allocation among sectors.

The technology of each activity is defined by a modified Cobb-Douglas production function:

(3.1) $$X_j = A L_j^{\alpha} \int_0^j X_{ij}^{1-\alpha} di ,$$

where X_j is the gross output of good j, A is a technological parameter, X_{ij} is the intermediate consumption of good i in sector j (i < j), and L_j is the workforce allocated to sector j. The technology is characterized by constant returns to scale and perfect substitutability among intermediate inputs. Equation (3.1) implies that all goods are produced with the same technology, the only difference comes from the size of the range of intermediate inputs used by each sector.

At any given moment in time a fraction m of the labour force is offered inelastically:

$$\int_0^N L_j \, dj = m , \tag{3.2}$$

where N measures the current range of existing goods. The labour force is assumed to be constant and normalized to 1.

All goods are perishable and all of them are suitable for final consumption. Hence, the gross demand of good i is made up of intermediate demands and final consumption:

$$X_i = \int_i^N X_{ij} \, dj + C_i , \tag{3.3}$$

where C_i is the final demand for good i. Notice that the i-th sector is integrated forward only with sectors of higher backward integration ($X_{ij} > 0$ for $i < j$; $X_{ij} = 0$ for $i \geq j$).

We will assume that the representative consumer derives utility from the consumption of any good and maximizes the discounted stream of utility over an infinite horizon. The objective function is defined as follows:

$$\int_0^\infty e^{-\rho t} u\left(\left\{ C_i(t) \right\}\right) dt , \tag{3.4}$$

where ρ is the discount rate, $u(\cdot)$ is the instantaneous utility function and $\{C_i(t)\}$ is the vector of current final consumption over the range $[0,N(t)]$.

In order to complete the characterization of instantaneous equilibrium we require a specification for instantaneous preferences. We will assume the following modified constant elasticity of substitution utility function:

$$(3.5)\quad u = \begin{cases} \dfrac{\left(\int_0^N C_i^\gamma \, di\right)^{(1-\varepsilon^{-1})}\gamma^{-1} - 1}{1-\varepsilon^{-1}}, & \text{for } \varepsilon > 0, \neq 1,\ \gamma > 0, \\ \dfrac{1}{\gamma}\ln\left(\int_0^N C_i^\gamma \, di\right), & \text{for } \varepsilon = 1,\ \gamma > 0, \end{cases}$$

where ε is the intertemporal elasticity of substitution of the given bundle of goods, and σ $[=1/(1-\gamma)]$ is the (instantaneous) intratemporal elasticity of substitution among goods. Although the orthodox CES function is usually assumed to be homogeneous of degree 1 ($\varepsilon^{-1} = 0$), we assume the utility function to be strictly concave ($\varepsilon^{-1} > 0$) with a high intertemporal elasticity of substitution ($0 < \varepsilon^{-1} \leq 1$, or $\varepsilon \geq 1$). These functional forms imply that the representative consumer experiences diminishing marginal utility with respect to any given bundle of goods. This assumption ensures an interior solution to the dynamic path. We also assume a high intratemporal elasticity of substitution among goods ($0 < \gamma < 1$, or $\sigma > 1$). This last assumption is necessary for a positive marginal utility from diversification ($\gamma > 0$),[2] and also for obtaining well-behaved demand functions for individual goods.

The previous equations complete the static model. Before

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[2] It can be checked that $\partial u(\cdot)/\partial N > 0$ if $\gamma > 0$.

characterizing the corresponding equilibrium, we proceed to define the technology of human capital accumulation. This will provide the dynamics of our model.

Human capital is simply the accumulated knowledge of technologies defined by the number of existing sectors (goods): N(t). We assume that our economy's human capital is small compared to more advanced economies. We also assume that technological knowledge is non-excludable. Hence, our economy specializes in appropriating foreign technologies. However, this process requires educated agents. Furthermore, the appropriation of new technologies requires new skills. Hence, the process of economic diversification continues as long as the agents allocate some effort to education. Since knowledge is not subject to depreciation, the technology of education is defined by the following function:

(3.6) $$\dot{N}(t) = N(t)[1 - m(t)]\delta ,$$

where a dot denotes a time derivative. Thus the rate of creation of new sectors (goods) is proportional to the current level of knowledge, N(t), and the amount of effort allocated to education as measured by the fraction of workforce which is not working, 1 - m(t). The parameter δ is an index of productivity in education.

Given the possibility of education the agents in this economy face an intertemporal trade-off: it pays to invest in education today -working less and producing a lower output- in order to enjoy a broader range of goods tomorrow. This assumes, of course, that the productivity in education is sufficiently high: the rate of diversification of goods must be sufficiently high in order to compensate for the lower level of current consumption. Additionally, for an interior solution of the dynamic problem, we need the instantaneous utility function to be concave in its arguments, namely the set of goods currently available. That is why we assume a high intertemporal elasticity of substitution ($\varepsilon \geq 1$).

3.2.2 The Instantaneous Equilibrium

The representative consumer maximizes his instantaneous utility, equation (3.5), subject to the instantaneous budget constraint which is defined by the following expression:

$$\int_0^N p_i C_i \, di = mw \, , \tag{3.7}$$

where w is the wage rate, mw is current income, and p_i is the (unit) price of good i.

The consumer takes as given income and prices, generating the following relative demand function:

$$\frac{C_i}{C_j} = \left(\frac{p_i}{p_j}\right)^{-\sigma} , \qquad \sigma = \frac{1}{1-\gamma} > 1 \, . \tag{3.8}$$

Note that the inequality $\gamma < 1$, or $\sigma > 1$, guarantees that relative demands fall with relative prices.

Firms' profits in sector j are defined as follows:

$$\pi_j = p_j X_j - w L_j - \int_0^j p_i X_{ij} \, di \, .$$

Due to the assumption of constant returns to scale one can aggregate the firms in each sector. In order to maximize profits, firms in sector j choose the amount of labour force to be hired and the intermediate inputs from the range [0,j].

The factor demands are calculated assuming the wage and the input prices as given. The first order conditions for this problem are the following:

(3.9) $$L_j = \alpha \, p_j X_j / w \ ,$$

and

(3.10) $$X_{ij} = \left[(1-\alpha) A \, p_j / p_i\right]^{1/\alpha} L_j \ , \qquad i \in [0, j] \ .$$

Now we can straightforwardly calculate the price of each good. Substitution of equations (3.9) and (3.10) into equation (3.1) yields

(3.11) $$p_j^{-1/\alpha} = \frac{a}{w} \int_0^j p_i^{1-1/\alpha} di, \ a \equiv [\alpha^\alpha (1-\alpha)^{1-\alpha} A]^{1/\alpha} > 0 \ .$$

Differentiating with respect to j gives

$$\frac{dp_j}{dj} = -\frac{\alpha a}{w} p_j^2 \ .$$

Integrating between 0 and i we find

(3.12) $$p_i = \frac{w}{\alpha a i} \ .$$

We are able to obtain such a simple equation for the relative price of good i because under the technological assumptions, see equation (3.1), the output of good 0 is zero: non integrated sectors do not produce output, hence the only meaningful price of good zero is infinity.[3] Equation (3.12) shows that the relative prices decrease asymptotically towards zero with the degree of backward technological integration.

Given the structure of relative prices we can solve for the technical coefficients. Substitution of equation (3.12) into equation (3.9) yields the technical coefficient for labour in sector j:

$$\frac{L_j}{X_j} = \frac{1}{a\,j} \, . \tag{3.13}$$

Now, by combining equations (3.10), (3.12) and (3.13) we obtain the intermediate input coefficients of sector j:

$$\frac{X_{ij}}{X_j} = \frac{1-\alpha}{\alpha}\,\frac{i^{\,1/\alpha}}{j^{\,1+1/\alpha}} \qquad \text{for all } i \in [0, j] \, . \tag{3.14}$$

The last two equations show that given the degree of technological integration, j, the technical coefficients are "fixed" as in a Leontief technology. Note, however, that we do not assume fixed technological coefficients. Actually, intermediate inputs in each activity are assumed to be perfect substitutes [see equation (3.1)]. Fixed technological coefficients in this model are due to fixed relative prices. Thus, in our economy the workers learn only one way of making

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[3] The technology may be modified allowing sector zero to be autonomous, e.g.

$$X_j = A\,L_j^{\alpha}\left(\int_0^j X_{ij}^{1-\alpha}\,di + X_{0j}^{1-\alpha}\right), \qquad \text{so that} \qquad X_0 = A\,L_0^{\alpha}\,X_{00}^{1-\alpha} \, .$$

This technology would yield a positive output of good zero that would be supported by a positive price. However, it does not seem that we gain much by complicating the model in this way.

each good and the "recipes" are never modified (not even in composition).

Let us solve now for the final demand for good i. By combining equations (3.7), (3.8) and (3.12) we deduce

(3.15) $$C_i = \alpha a \sigma m (i/N)^{\sigma} .$$

This equation shows that the final demand structure is biased in favour of sectors with high backward technological integration (i close to N). This result is not surprising as relative prices fall with the degree of backward integration [see equation (3.12)]. The bias in the final demand structure is stronger the higher the intratemporal elasticity of substitution. The structure of final demand is illustrated in Figure 3.2.

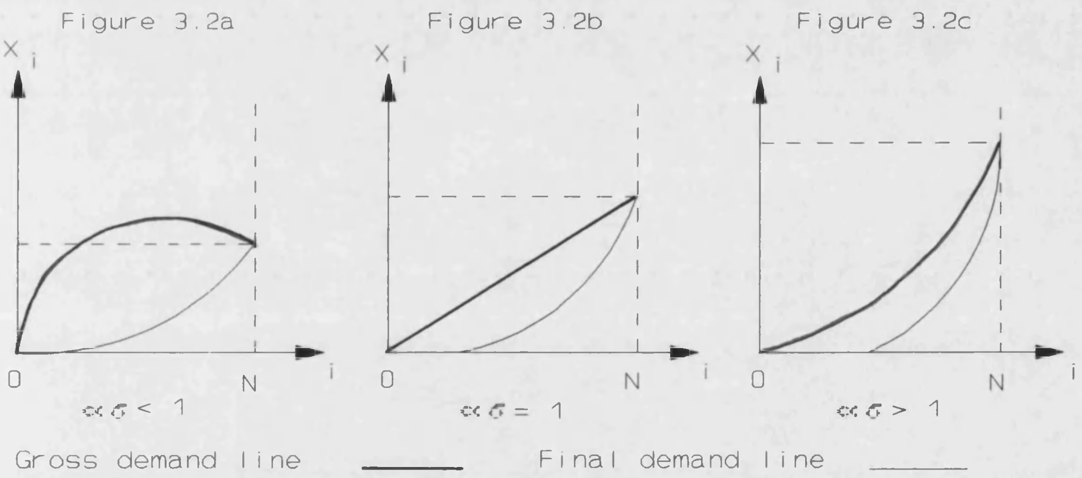

Figure 3.2 *Demand Structure (σ>1)*

Equation (3.15) also implies that the final demand structure shifts in favour of newer goods as the number of sectors increases. Thus the final demand for sectors with a low degree of backward technological integration ($i \approx 0$) becomes negligible. Again, the higher is the elasticity of substitution the stronger is this effect.

Let us solve now for the structure of gross demands. Substitution of equations (3.14) and (3.15) into equation (3.3) yields

$$\text{(3.16)} \qquad X_i = \frac{1-\alpha}{\alpha} i^{1/\alpha} \int_i^N \frac{X_j}{j^{1+1/\alpha}} dj + \frac{\alpha a \sigma m}{N^\sigma} i^\sigma .$$

Differentiating twice with respect to i yields

$$\frac{d^2 X_i}{d i^2} = \frac{a \sigma^2 (\alpha \sigma - 1)}{N^\sigma} i^{\sigma - 2} .$$

This is a second order differential equation whose general solution has the form $X_i = \phi_0 + \phi_1 i + \phi_2 i^\sigma$, where ϕ_0, ϕ_1 and ϕ_2 are constant coefficients to be determined. By substituting this solution into equation (3.16) we can identify these coefficients and obtain the solution for the gross demand of good i:

$$\text{(3.17)} \qquad X_i = \frac{a\sigma}{\sigma - 1} \left[(1-\alpha)\left(\frac{i}{N}\right) + (\alpha\sigma - 1)\left(\frac{i}{N}\right)^\sigma \right] m .$$

From this equation we deduce that the economic structure profile depends on the relationship between the elasticity of intratemporal substitution in final consumption, σ, and the output elasticity of labour, α. Figure 3.2 shows the possible shapes of the gross demand structure. The economic intuition for these shapes is as follows. The final demand always increases with the degree of backward economic integration, i, because highly integrated sectors produce cheaper goods. Given "fixed" technological coefficients [see equation (3.14)], the gross demand tend to increase with final demand. However, the bias of the final demand structure towards highly integrated goods needs not determine the bias of the gross demand structure: even if the final demand for lower integrated goods is negligible, they are still required as intermediate inputs in the production of highly integrated sectors. These derived demands will be higher the larger is the intensity of intermediate input in the production technology, i.e. the lower α. Thus, if the bias toward final goods is not too high (the elasticity of substitution is not too high), and production is intensive in intermediate goods (α low), so that $\alpha\sigma < 1$, the gross demand may be biased towards sectors with an intermediate degree of technological integration. This case is illustrated in Figure 3.2a. On the other hand, high elasticity of substitution and/or low production intensity in intermediates, so that $\alpha\sigma > 1$, determine a bias in gross demand towards highly integrated sectors. This case is illustrated in Figure 3.2c. Figure 3.2b illustrates the borderline case.

Now, by combining equations (3.13) and (3.17) we deduce the labour demand in sector j:

$$\textbf{(3.18)} \qquad L_j = \frac{\sigma}{\sigma - 1}\left[(1-\alpha) + (\alpha\sigma - 1)\left(\frac{j}{N}\right)^{\sigma - 1}\right]\frac{m}{N} .$$

Figure 3.3 shows the different possibilities of labour allocation across sectors.

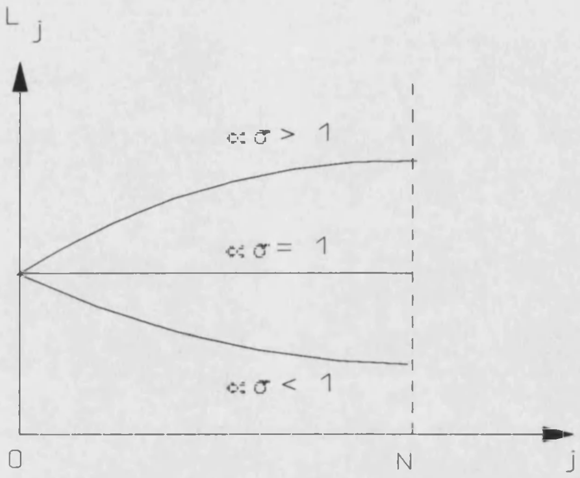

Figure 3.3 *Employment Structure (σ>1)*

The structure of employment is clearly related to the structure of gross demand. Even sectors with the lowest backward technological integration are demanded at least as intermediate inputs. Thus they require some allocation of labour. If production intensity in intermediates is high, the labour demand is biased towards sectors with low technological integration (the labour profile is downward sloping); if production intensity in intermediates is low, the labour demand is biased towards sectors with high technological integration; in the borderline case all sectors hire identical number of workers.

Given the structure of final demand we can solve for the instantaneous level of utility. For simplicity we will choose the case of logarithmic preferences (ε = 1). However, in Appendix 1 we show that our main results are not significantly changed by allowing for a higher degree of intertemporal substitution. Now, plugging equation (3.15) into equation

(3.5), for $\varepsilon = 1$, yields

$$\textbf{(3.19)} \qquad u = \ln[\beta m N^{\sigma/(\sigma-1)}], \quad \beta \equiv \alpha a \sigma^{-1/(\sigma-1)} > 0 .$$

Hence, the instantaneous level of utility depends on the fraction of labour force allocated to productive activity (m), and the range of existing goods in the economy (N). Equation (3.19) shows why it is natural to assume a high degree of intratemporal substitutability among goods ($\sigma > 1$): only in this case the society's welfare increases with the range of available goods, N.

3.2.3 The Dynamic Equilibrium

The consumer maximizes equation (3.4) subject to the instantaneous utility function [equation (3.19)] and the transition equation of education [equation (3.6)]. The Hamiltonian equation associated with this problem is

$$H(\cdots) = \text{Max}\left\{\ln\left[\beta\, m(t)\, N(t)^{\frac{\sigma}{\sigma-1}}\right]e^{-\rho t} + \lambda(t)\, N(t)\, [1 - m(t)]\, \delta\right\},$$

where the arguments of the Hamiltonian are m(t), N(t) and the multiplier $\lambda(t)$.

The first order conditions for maximization are

$$\textbf{(3.20)} \qquad H_m(\cdots) = 0 : \quad e^{-\rho t} m(t)^{-1} = \delta\, \lambda(t)\, N(t) ,$$

and

$$\textbf{(3.21)} \qquad \dot{\lambda}(t) = -H_N(\cdots)$$

$$= -\left(\frac{\sigma}{\sigma-1} N(t)^{-1} e^{-\rho t} + \lambda(t)\, [1 - m(t)]\, \delta\right).$$

The equilibrium path of this economy should satisfy the following transversality condition:

(3.22) $$\lim_{t \to \infty} \lambda(t)\, N(t) = 0 \; .$$

Now we proceed to find the equilibrium. By combining equations (3.20) and (3.21) we obtain

$$\frac{\dot{\lambda}}{\lambda} = -\delta\left(1 + \frac{m(t)}{\sigma - 1}\right) .$$

By differentiating equation (3.20) with respect to time, and using the last equation and equation (3.6), we deduce the differential equation that drives workforce allocation:

$$\frac{\dot{m}(t)}{m(t)} = -\rho + \frac{\delta\,\sigma}{\sigma - 1}\, m(t) \; .$$

The phase picture corresponding to this equation is in Figure 3.4. Rest points are $m(t) = 0$, and the following steady state equilibrium:

(3.23) $$m^* = \frac{\rho}{\delta}\,\frac{\sigma - 1}{\sigma} \; .$$

Under the assumption of interior solution, m^* is the only solution consistent with the transversality condition. Hence there is no transitional dynamics in this model, i.e. forward-looking agents choose at once the level of labour supply m^* given by equation (3.23).

With logarithmic preferences ($\varepsilon = 1$), and a high elasticity of intratemporal substitution among goods ($\sigma > 1$), the workforce allocation to productive activities is always positive ($m^* > 0$). On the other hand, the allocation of time

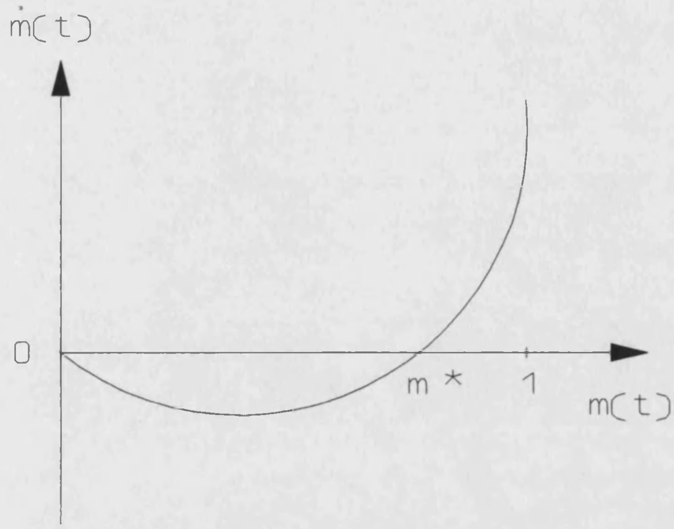

Figure 3.4 *Labour Force Dynamics*

to education might be positive ($m^* < 1$), if the following inequality holds: $\delta > \rho(\sigma-1)/\sigma$. This means that given some degree of impatience, $\rho > 0$, the workforce will get educated if the degree of intratemporal substitutability among goods is high and the education system is sufficiently efficient. If the last condition does not hold, i.e. $\delta < \rho(\sigma-1)/\sigma$, no time is allocated to education.[4]

This analysis implies a relationship between labour supply, education efficiency and welfare gains. Refer to Figure 3.5. Below the threshold level of efficiency in education no education takes place and hence economic diversification does not progress. For high levels of education efficiency, some effort is allocated to education

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[4] A rigorous deduction of this result implies restricting our Hamiltonian equation to solutions for $m \leq 1$. This procedure however is reduced to yield a Kuhn-Tucker multiplier equal to zero (and so $m = 1$) for education efficiency lower than or equal to the threshold level identified in the text.

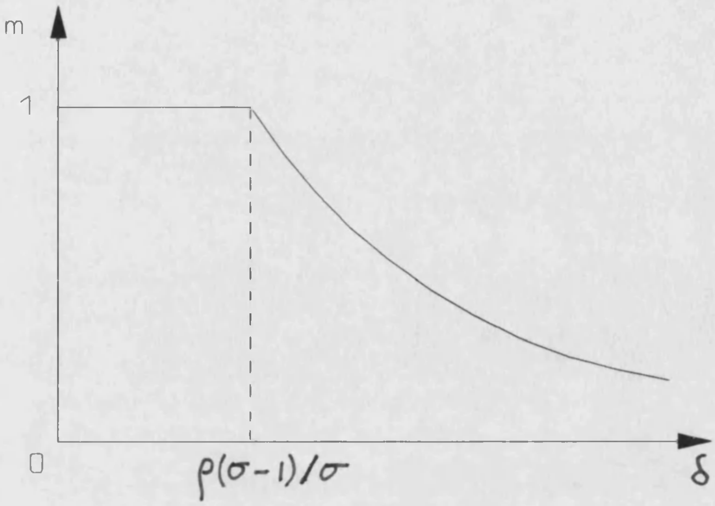

Figure 3.5 *Labour Supply*

(the labour supply is lower), but the number of sectors increases at the following rate:

$$\frac{\dot{N}}{N} = \delta - \frac{\sigma - 1}{\sigma}\rho , \tag{3.24}$$

and the utility level increases permanently:

$$\dot{u} = \frac{\sigma\delta}{\sigma - 1} - \rho . \tag{3.25}$$

These results define the trade-off between education and labour supply.

At this point we should note that equation (3.25) implies that the growth rate of welfare gains ($\dot{u}/u$) falls steadily towards zero. This is a consequence of assuming an intertemporal elasticity of substitution equal to 1 ($\varepsilon = 1$); however, if this elasticity is larger than one ($\varepsilon > 1$), the growth rate of welfare gains falls asymptotically towards the following positive minimum (see Appendix 1):

$$\text{(3.25')} \qquad \lim_{t\to\infty} \frac{\dot{u}(t)}{u(t)} = (\varepsilon - 1)\left(\frac{\sigma\,\delta}{\sigma - 1} - \rho\right).$$

The intertemporal elasticity of substitution measures the willingness to postpone consumption today for consumption tomorrow. Thus a higher elasticity reflects a propensity to allocate a higher level of effort in education, which yields a higher rate of welfare growth.

Finally, if an interior solution exists the transversality condition boils down to the requirement that the discount factor be positive, $\rho > 0$.

3.3 THE OPEN ECONOMY CASE

Consider our model of economic diversification in the context of international trade. Refer to Figure 3.6. Two economies, South and North, are initially in autarky and afterwards they are joined through international trade. The population is mobile within the countries but international migration is prohibited. The single factor that can be accumulated is human capital, which is here the same as the cumulative knowledge of technologies. We will assume that the North owns a higher level of human capital and thus has a more diversified economy; i.e. the South produces N goods and the North produces N^* goods, such that $N^* > N > 0$. From now on all variables related to the North will be starred.

For relative prices we obtain the same solutions as in the closed economy case because they are determined solely by the degree of backward technological integration of each sector [equation (3.12)]. With international trade the prices

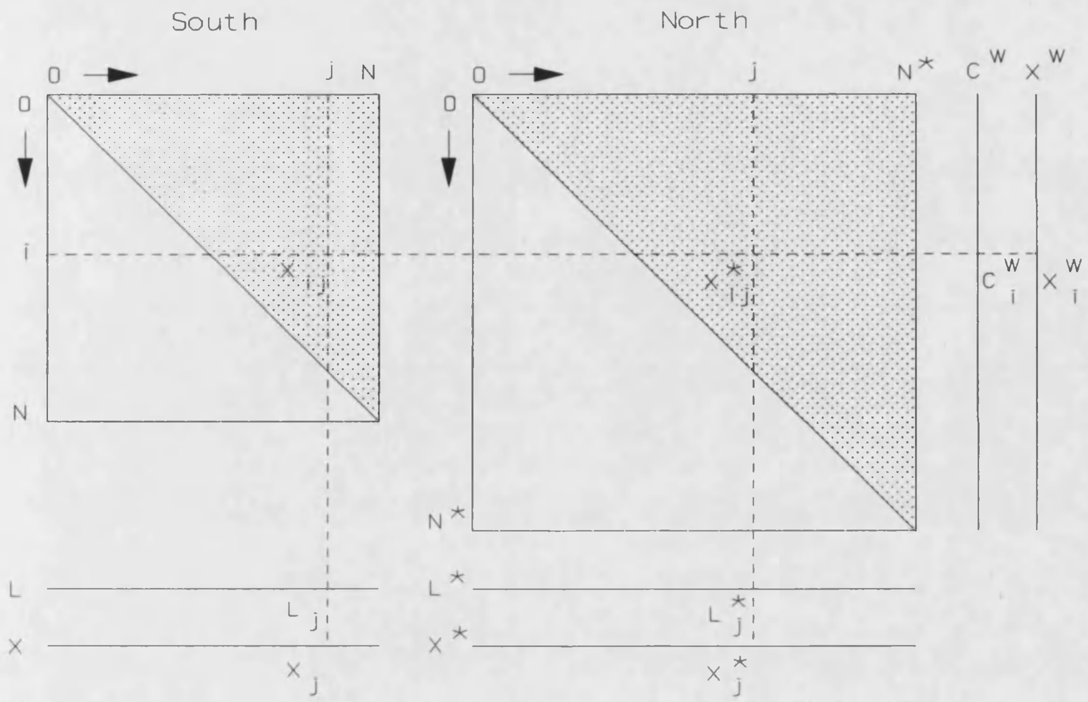

Figure 3.6 *Input-Output Matrices (South and North)*

of identical goods are equalized. With common production activities, the factor price equalization theorem implies that the wage rate is identical in the South and the North.

Let us now turn to the determination of the world gross demands. As Figure 3.6 shows the world demands are given by

(3.26)
$$X^W_i = X_i + X^*_i = \int_i^N X_{ij}\,dj + \int_i^{N^*} X^*_{ij}\,dj + C_i + C^*_i, \ i \le N,$$
$$X^W_i = X^*_i = \int_i^{N^*} X^*_{ij}\,dj + C_i + C^*_i, \ N \le i \le N^*,$$

where the superscript W denotes world demand. Note that the equilibrium condition for goods within the range $[N, N^*]$ includes only the intermediate demands of the North as we know that the South does not produce this range of goods. But we should include the final demand for these goods from South as well as those from the North.

Let us note briefly the fundamental asymmetric relationship between the South and the North. Whilst the North may be specialized in those sectors with higher backward integration, it nevertheless can produce the goods with lower backward integration which the South produces. However, the South cannot produce the higher backward integrated goods because of its lack of human capital.

Next we need to obtain expressions for the constituents parts of equations (3.26). We will start with the final demands. All consumers share the same utility function [see equation (3.5)], and all of them have access to the consumption of N^* goods. This means that South can consume goods it does not produce through international exchange.

The final demands are given by the following formulas which are equivalent to equation (3.15):

(3.27)

$$C_i = \alpha a \sigma (i/N^*)^{\sigma} m L ,$$
$$C_i^* = \alpha a \sigma (i/N^*)^{\sigma} m^* L^* ,$$

where m is the fraction of the workforce in productive activities in the South, and L is the workforce in the South. Starred variables again correspond to the North.

Now, labour demand and intermediate inputs are proportional to the gross output in each sector, as we saw in the previous section [see equations (3.13) and (3.14)].

Substituting these demands into equations (3.26) we solve for the gross demands. The result is the following:

$$(3.28) \quad X_i^W = \frac{a\sigma}{\sigma - 1}\left[(1-\alpha)\left(\frac{i}{N^*}\right) + (\alpha\sigma - 1)\left(\frac{i}{N^*}\right)^{\sigma}\right](mL + m^*L^*),$$

which is analogous to the solution in the closed economy case [see equation (3.18)]. Equation (3.28) applies to all goods within the range $[0,N^*]$. Hence, there are no discontinuities in the world demand structure at the level of the Nth good, as one might believe by looking at Figure 3.6. The intuition for this feature is that the world final demand structure is smooth. Hence, given "fixed" intermediate input coefficients, the gross demand structure should be smooth as well. For a graphical intuition of this result it may be helpful to add the input-output matrices as well as the vectors of labour and gross product in Figure 3.6. Thus, we are back to the "closed economy" case, and the smoothness of the world demand structure follows.

Given the solution for the world gross demands we can solve for the world demand for labour in industry j by using equation (3.13):

$$(3.29) \quad L_j^W = \frac{\sigma}{\sigma - 1}\left[(1-\alpha) + (\alpha\sigma - 1)\left(\frac{j}{N^*}\right)^{\sigma - 1}\right]\frac{mL + m^*L^*}{N^*}.$$

Then, integrating between O and N and dividing by the world labour demand, $mL + m^*L^*$, we deduce

$$(3.30) \quad d\left(\frac{N}{N^*}\right) = \frac{\sigma}{\sigma - 1}\left[(1-\alpha)\left(\frac{N}{N^*}\right) + \frac{\alpha\sigma - 1}{\sigma}\left(\frac{N}{N^*}\right)^{\sigma}\right],$$

which is the fraction of the world labour demand in the range of activities $[0,N]$. Note that this fraction increases with

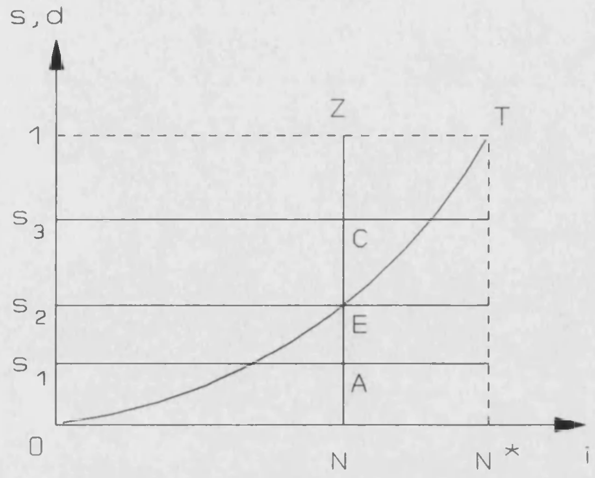

Figure 3.7 *International Allocation of Labour*

the relative level of human capital in the South, N/N^*. The line OET in Figure 3.7 depicts the fraction of world labour demand corresponding to activities with a degree of backward integration lower than i, $i \in (0,N^*)$. In drawing this line we assume that $\alpha\sigma > 1$, so that employment demand increases more than proportionally with the degree of backward integration [see Figure 3.3]. However, the important issue is that the labour demand line OET is increasing in i for the relevant case of high degree of intratemporal substitutability ($\sigma > 1$).

Now, if the fraction of labour supply corresponding to the South is denoted by s $(=mL/(mL+m^*L^*))$, we have three possibilities (refer again to Figure 3.7):

(1) If the South supplies the fraction of labour s_1, the North employs a fraction of its workforce equal to the ratio AE/AZ in activities with backward integration lower than N, the remainder, given by the ratio EZ/AZ, is employed in activities with higher backward integration. The actual distribution of output supply in common activities (with backward integration lower than N) is not determined.

(2) If the South happens to supply the fraction of labour s_2, it will be specialized in activities with backward integration lower than N. The North, of course, will be specialized in activities with higher backward integration. In this case there will be only one activity in common, the marginal activity with backward integration equal to N.

(3) If the South provides the fraction of labour supply s_3, the southern wage will fall in order to correct the excess supply of labour given by the distance EC.

In the last case the factor price equalization theorem does not apply because the South will be completely specialized in products with backward integration lower than N. Because relative prices are proportional to wages [see equation (3.12)], the prices of Southern goods fall relative to Northern goods.

Figure 3.7 shows that, given a high relative supply of labour force in the South as s_3, the excess supply, EC, is larger the lower the level of human capital in the South, N, relative to the level of human capital in the North, N^*. This follows from the fact that the relative demand line, OET, is increasing in the degree of backward integration. Thus the wage adjustment is stronger the lower is N/N^*.

These results may help to explain why countries with low school enrolment ratios -which are usually taken as good proxies for human capital accumulation- have less diversified economies and low real incomes. Evidence on the correlation between growth and school enrolment ratios is found in Barro (1989a, 1989b). Evidence of the relationship between economic structure and real income is found in Leontief (1963), Chenery, Robinson and Syrquin (1986), and Syrquin and Chenery (1989).

Figure 3.7 also exhibits an important property: if the relative supply of labour from South is high, so that the Southern wage is below the Northern wage, the South may increase its real income by increasing its human capital level relative to the human capital level of the North.

In section 3.2.3 we found that individual decisions to undertake education efforts depend on the efficiency of the education system. There we assumed that efficiency in education is a structural parameter. However, if the government has some influence on establishing standards of education, this models predicts that a country is more likely to start a process of diversification and welfare growth by setting high standards of education.

3.4 AN AGGREGATIVE MODEL OF ECONOMIC DIVERSIFICATION

The model developed in the last sections yields endogenous welfare growth. This is due exclusively to diversification since output in each sector decreases steadily as the fixed amount of labour is allocated among an increasing number of activities [see equation (3.18)]. This happens because labour productivity is assumed to be unaffected by the accumulation of human capital. In that sense our model is different from Lucas' (1988) model of human capital accumulation: in his model, Lucas assumed an abstract force called human capital that augmented labour efficiency through education and also through external effects captured by the technological parameter A [see equation (3.1)]. We also could resort to this procedure and have endogenous growth. However, we only need to introduce physical capital accumulation and a final good sector in order to achieve this feature.

3.4.1 The Model

The structure of this economy is shown in Figure 3.8. The final good technology is given by

$$Y = K_y^{\alpha} L_y^{\beta} \int_0^N Q_i^{1-\alpha-\beta} \, di \, , \tag{3.31}$$

where K_y is the amount of physical capital allocated to the final good sector, L_y is the labour force allocated to the same sector and Q_i is the intermediate good i used in the same sector. The final good technology is assumed to use the whole range of intermediate goods [0,N].

Forgone consumption of the final good is transformed one to one into capital. Hence, the equilibrium in the final good market is given by

$$Y = C + \dot{K} \, , \tag{3.32}$$

where C is final good consumption, and $\dot{K}$ is investment in physical capital (the time derivative of physical capital).

The intermediate good technology is defined by

$$X_j = K_j^{\alpha} L_j^{\beta} \int_0^j X_{ij}^{1-\alpha-\beta} \, di \, , \tag{3.33}$$

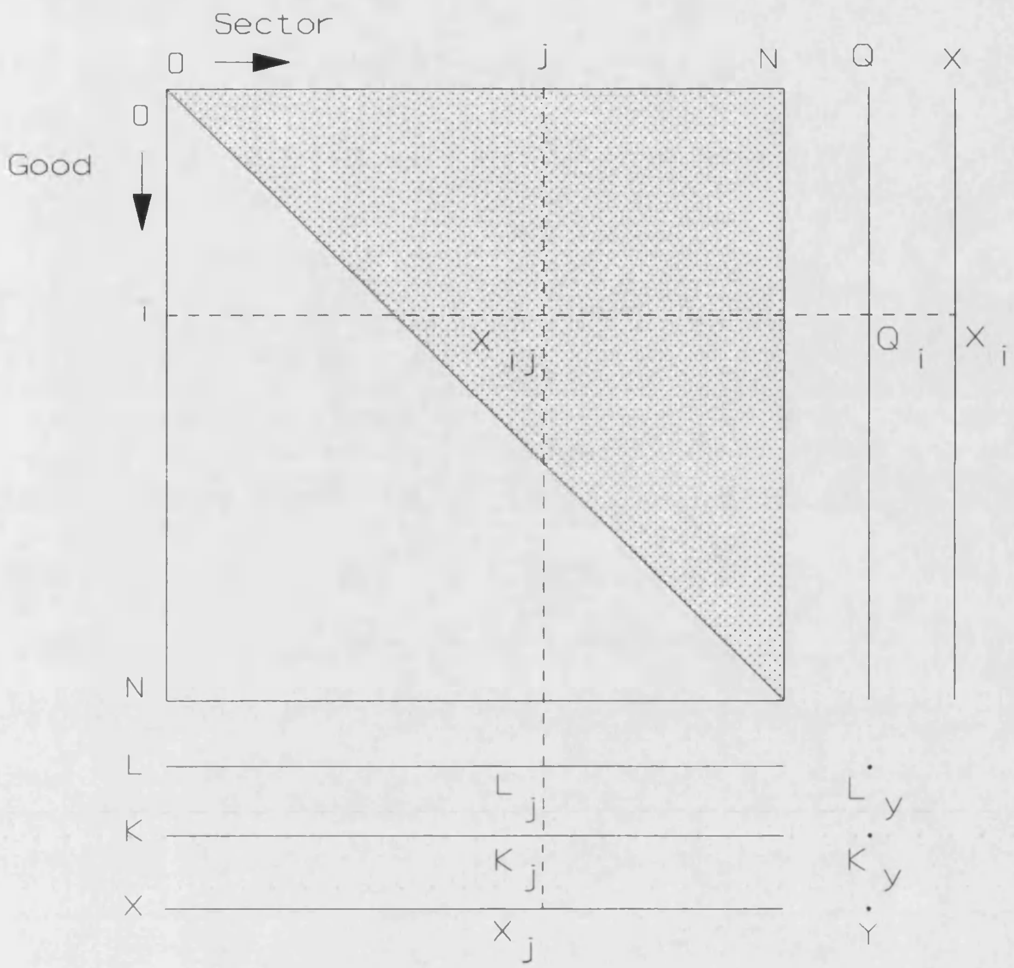

Figure 3.8 *Structure of Technologies*

where K_j is the amount of physical capital allocated to the production of intermediate good j, L_j is the labour force allocated to the same activity, and X_{ij} is the quantity of intermediate good i used in the production of good j. Notice that all technologies, including the final good technology, are identical except for the fact that some sectors are allowed to use a longer list of intermediate inputs. In this way the model captures the observed differences in backward technological integration across sectors. The assumption of identical technologies is not realistic, of course, but simplifies the analytical solution of our model.

This time we will assume that intermediate goods are not suitable for final consumption. Hence the total demand for intermediate good i is

$$X_i = \int_i^N X_{ij}\, dj + Q_i \ . \tag{3.34}$$

This equation says that intermediate good i is used by the final good sector and those sectors producing intermediate goods with a higher degree of backward technological integration ($j > i$).

Besides the knowledge of a given range of technologies, the economy is constrained by the availability of labour and capital. Labour must be allocated between the final good sector and the intermediate goods sector:

$$L_y + \int_0^N L_j\, dj = m \ , \tag{3.35}$$

where m is the fraction of the workforce allocated to productive activities. For physical capital we also have a similar constraint:

$$K_y + \int_0^N K_j\, dj = K \ , \tag{3.36}$$

where K is the total amount of physical capital to be distributed across sectors.

The number of available technologies is constrained by the learning process described by equation (3.6) in section 3.2. Let us recall that learning there was interpreted as the result of formal education through diversion of agents from productive activities. We will keep this interpretation.

There is only one source of "felicity" for the representative consumer: the consumption of the final good. We will assume that the consumer is infinitely lived and his/her preferences are additive and separable. A suitable utility function is the following:

$$\int_0^\infty e^{-\rho t} \ln[C(t)]\,dt \;. \tag{3.37}$$

where ρ is the discount rate.

3.4.2 The Instantaneous Equilibrium and the Aggregate Production Function

We will solve first the instantaneous equilibrium in a competitive environment. Let us start with the intermediate sector j. Firms are price takers and maximize profits. The solution of this problem yields the following factor demands:

$$K_j = \alpha\, p_j X_j / r \;, \tag{3.38}$$

$$L_j = \beta\, p_j X_j / w \;, \tag{3.39}$$

$$X_{ij} = \left[(1 - \alpha - \beta)\, p_j / p_i \right]^{\frac{1}{\alpha+\beta}} K_j^{\frac{\alpha}{\alpha+\beta}} L_j^{\frac{\beta}{\alpha+\beta}}, \; i \in [0, j] \,. \tag{3.40}$$

where r is the rental price of capital, w is the wage rate, and p_j is the price of good j.

Now we proceed as follows. Substitution of these factor demands into the production function of good j [equation (3.33)] yields a function for the price of this good in terms of the rental price of capital, the wage rate and all prices of intermediate goods used in the production of good j:

$$(3.41)\qquad p_j^{-1/(\alpha+\beta)} = a\, r^{-\alpha/(\alpha+\beta)}\, w^{-\beta/(\alpha+\beta)} \int_0^j p_i^{1-1/(\alpha+\beta)}\, di ,$$

where

$$a = \left[\alpha^\alpha \beta^\beta (1-\alpha-\beta)^{1-\alpha-\beta}\right]^{1/(\alpha+\beta)} > 0 .$$

Note that the constant a has been redefined. Differentiating this equation with respect to j and integrating afterwards between zero and i yields the following general expression:

$$(3.42)\qquad p_i = \frac{r^{\frac{\alpha}{\alpha+\beta}}\, w^{\frac{\beta}{\alpha+\beta}}}{(\alpha+\beta)\, a\, i} .$$

Again this equation is obtained by exploiting the fact that the price of good zero tends to infinity.

Let us turn now to the final good sector. Here as well firms are price takers and maximize profits. We will choose the final good as numeraire ($p_y = 1$). The factor demands in this sector are given by the following equations:

$$(3.43)\qquad K_y = \alpha Y / r ,$$

$$(3.44)\qquad L_y = \beta Y / w ,$$

$$(3.45)\qquad Q_i = \left[(1-\alpha-\beta)/p_i\right]^{\frac{1}{\alpha+\beta}} K_Y^{\frac{\alpha}{\alpha+\beta}} L_Y^{\frac{\beta}{\alpha+\beta}}, \quad i \in [0,N] .$$

Now we substitute these factor demands into the production function of the final good [equation (3.31)], and obtain a function relating all the prices in this economy. Using equation (3.42) we solve

(3.46) $$r^{\frac{\alpha}{\alpha+\beta}} w^{\frac{\beta}{\alpha+\beta}} = (\alpha + \beta) a N \ .$$

Now we combine the last equation and equation (3.42) in order to deduce the relative price of good j:

(3.47) $$p_j = \frac{N}{j} \ .$$

Let us now solve for the associated quantities. First let us see the determination of intermediate good demands. Combining equations (3.34), (3.38), (3.39), (3.40), (3.45), (3.46) and (3.47) we deduce that

$$X_i = \frac{1-\alpha-\beta}{\alpha+\beta} \, i^{\frac{1}{\alpha+\beta}} \int_i^N \frac{X_j}{j^{1+1/(\alpha+\beta)}} dj + \frac{1-\alpha-\beta}{\alpha+\beta} \frac{Y}{N} \left(\frac{i}{N}\right)^{\frac{1}{\alpha+\beta}} .$$

By differentiating this expression with respect to i we find that the gross output of good i, X_i, is unit elastic with respect to its degree of backward technological integration, i. Hence we can solve

(3.48) $$X_i = \frac{1-\alpha-\beta}{\alpha+\beta} \frac{Y}{N} \frac{i}{N} , \quad \text{for all } i \in [0, N] \ .$$

Given prices and quantities of intermediate goods, we can find the prices of final factors from the equilibrium conditions of the primary factor markets [equations (3.35) and (3.36)]. Thus we obtain

(3.49) $$r = \frac{\alpha}{\alpha+\beta} \frac{Y}{K} ,$$

and

(3.50) $$w = \frac{\beta}{\alpha+\beta} \frac{Y}{m} .$$

Given these results and equation (3.46) we deduce the aggregate final-good production function of this economy:

$$Y = B K^{\frac{\alpha}{\alpha+\beta}} m^{\frac{\beta}{\alpha+\beta}} N, \tag{3.51}$$

where

$$B = (\alpha + \beta)^2 (1 - \alpha - \beta)^{\frac{1-\alpha-\beta}{\alpha+\beta}} > 0 .$$

The aggregate technology is homogeneous of degree one in physical capital and labour. However, taking into account human capital (the number of sectors and also the degree of backward technological integration), this economy experiences increasing returns at the aggregate level. Since instantaneously agents take as given the number of sectors, a competitive equilibrium can be supported.

Equation (3.51) shows that the whole technological structure represented by Figure 3.8 is reduced to the aggregate production function in a competitive environment. This reduction corresponds precisely to Samuelson's transformation of gross output functions into net output functions (see Chapter 1). As proved by Samuelson (1966), the necessary conditions for this transformation are constant returns to scale in the production of intermediate inputs and final goods, no joint production and perfect competition. All these conditions are satisfied by our model.

The aggregate production function shows that in this economy the degree of diversification -which here coincides with the degree of technological integration- affects positively the productivity level. Thus if international trade is not a good substitute for economic diversification, equation (3.51) implies that total factor productivity must be higher in countries with more developed technological structures.

3.4.3 The Dynamic Equilibrium

The last section describes the instantaneous supply side of the model. Let us now turn to the demand side. The consumer maximizes the discounted stream of utility [equation (3.37)] subject to the (aggregate) technology [equation (3.51)], the transition equation of physical capital [equation (3.32)], and the transition equation of education [equation (3.6)]. The Hamiltonian related to this problem is as follows:

$$H(\cdots) = \text{Max}\Big\{ e^{-\rho t} \ln C(t) + \lambda_1(t) \Big[B\, K(t)^{\frac{\alpha}{\alpha+\beta}}\, m(t)^{\frac{\beta}{\alpha+\beta}}\, N(t) - C(t)\Big] + \lambda_2(t)\, N(t)\, [1 - m(t)]\, \delta \Big\} .$$

The necessary conditions of maximization are the following:

$$\textbf{(3.52)} \qquad H_C(\cdots) = 0 , \quad \text{or} \quad C(t)^{-1} = \theta_1(t) ,$$

and

$$\textbf{(3.53)} \qquad H_m(\cdots) = 0 , \quad \text{or} \quad \theta_1(t) \frac{\beta B}{\alpha+\beta} \left[\frac{K(t)}{m(t)}\right]^{\frac{\alpha}{\alpha+\beta}} = \theta_2(t)\, \delta .$$

where $\theta_i(t) = e^{\rho t}\lambda_i(t)$, i = 1,2, is the present value multiplier of the state variables, i.e. K and N. Along the optimal path these multipliers obey the following processes:

$$\textbf{(3.54)} \qquad \frac{\dot{\theta}_1}{\theta_1} = \rho - \frac{\alpha B}{\alpha+\beta} \left[\frac{m(t)}{K(t)}\right]^{\frac{\beta}{\alpha+\beta}} N(t) ,$$

and

(3.55) $$\frac{\dot{\theta}_2(t)}{\theta_2(t)} = \rho - \delta - \frac{\alpha\delta}{\beta} m(t) \ .$$

We also need a couple of transversality conditions:

(3.56) $$\lim_{t \to \infty} e^{-\rho t} \theta_1(t) K(t) = 0 \ ,$$

and

(3.57) $$\lim_{t \to \infty} e^{-\rho t} \theta_2(t) N(t) = 0 \ .$$

In general, the dynamics of two-sector models of endogenous growth is complex and not well understood. Although some advances have been made on this subject (Lucas, 1988; Mulligan and Sala-i-Martin, 1992; Barro and Sala-i-Martin, 1993), in many cases analytical solutions are not feasible at all. As one might expect, the dynamics of our model is also complex. Hence, in this section we will examine only the balanced growth path, i.e. the dynamic path characterized by constant growth rates of consumption, physical capital and human capital. However, local convergence to this path is examined in Appendix 2. There we deduce that our economy might converge locally to the balanced path (with unbounded growth) if the education system is sufficiently efficient (high δ). However, we also show that if education efficiency is too high, the economy does not converge to the balanced growth path -this gives the possibility of increasing growth rates. If education efficiency is low, the economy might experience either negative growth or convergence to a steady state with no growth.

The balanced growth path is characterized by a constant allocation of time between production and education, and constant growth rates of final consumption, physical capital and human capital [see again Appendix 2]. We will drop the time subscript from variables moving along the balanced path and denote them with a star.

First we notice from equation (3.52) that, along the balanced path, the shadow price of capital, θ_1, grows at a constant rate equal to minus the growth rate of consumption:

$$\frac{\dot{C}^*}{C^*} = - \frac{\dot{\theta}_1^*}{\theta_1^*} .$$

Hence, from equation (3.54) we deduce that, along the balanced path, human capital, N, grows at a fraction $\beta/(\alpha+\beta)$ of the growth rate of physical capital:

$$\frac{\dot{N}^*}{N^*} = \frac{\beta}{\alpha + \beta} \frac{\dot{K}^*}{K^*} .$$

Using equation (3.6) we obtain

$$\frac{\dot{K}^*}{K^*} = \frac{\alpha + \beta}{\beta} (1 - m^*) \delta ,$$

where m^* is the fraction of workforce allocated to productive activities along the balanced path. Now, differentiating equations (3.52) and (3.53) with respect to time and using the last equation plus equation (3.55) we obtain

$$\text{(3.58)} \qquad \frac{\dot{C}^*}{C^*} = \frac{\alpha + \beta}{\beta}\delta - \rho ,$$

which is the balanced growth rate of consumption.

Now, dividing the transition equation of physical capital [equation (3.32)] by K(t), and taking into account the aggregate technology [equation (3.51)] we obtain

$$\frac{C(t)}{K(t)} + \frac{\dot{K}(t)}{K(t)} = B\left[\frac{m(t)}{K(t)}\right]^{\frac{\beta}{\alpha+\beta}} N(t) .$$

Notice that along the balanced path the right hand side of this expression is constant, it corresponds to the marginal product of capital. Since the growth rate of physical capital is also constant along this path, we deduce that physical capital and consumption grow equiproportionally. Thus we can deduce the balanced-path allocation of workforce to production:

$$\text{(3.59)} \qquad m^* = \frac{\beta}{\alpha + \beta}\frac{\rho}{\delta} > 0.$$

Note that this fraction is always positive. An interior solution ($m^* < 1$) requires the productivity parameter in education to be sufficiently high: $\delta > \rho\beta/(\alpha+\beta)$. Note that this inequality is also the condition for a positive growth rate. As in our previous model, we reach the conclusion that education effort, education efficiency and economic growth are closely associated.

Since the marginal product of capital is constant along the balanced path, the balanced-path interest rate is also constant. By combining equations (3.49), (3.51), (3.52) and (3.54) we deduce

(3.60) $$r^* = \frac{\alpha + \beta}{\beta}\delta \ .$$

Thus, this model predicts that the balanced-path rental price of capital is high in countries with high education efficiency (high δ). This result may help to explain why capital does not flow from industrialized countries to non-industrialized countries in order to take advantage of the usually much cheaper available labour -the returns to capital are not necessarily higher in countries with cheap labour, but tend to be high in countries with high education standards.

Now, combination of equations (3.49), (3.50), (3.59) and (3.60) yields

(3.61) $$w^* = \frac{(\alpha + \beta)^2}{\alpha\beta}\frac{\delta^2}{\rho}K^* \ .$$

Thus, along the balanced growth path, the wage rate grows at the same rate of physical capital and final output. Since along the balanced path physical capital and human capital (the number of sectors) grow together, the last equation implies that countries with a high degree of interindustry relations and economic diversification enjoy high wages. Therefore, our model also explains why there exists such a migration pressure from underdeveloped countries to industrialized countries. The latter have a more developed economic structure and thus enjoy higher real wages.

Finally, let us add that the two transversality conditions hold if the discount rate, ρ, is positive. This completes the characterization of the dynamic behaviour of this economy in the steady state.

3.4.4 Summary and Concluding Comments

In section 3.4.1 we set up an economy composed of a continuum set of industries characterized by a growing degree of interdependence. They only produce intermediate goods. We also add a single final-good sector that uses the whole range of intermediate inputs. The final good can be used for final consumption or for capital accumulation. Physical capital and labour are the primary factors used by all sectors.

In section 3.4.2 we deduced this economy's aggregate production function, which is understood as the function that maps primary factors (physical capital, labour and human capital) into maximum net output. In other words, as discussed in Chapter 1, we performed Samuelson's transformation. As proved by Samuelson, the necessary conditions for this transformation are constant returns to scale in the production of basic inputs, no joint production and perfect competition (Samuelson, 1966). All these conditions are satisfied by our model.

Given the structure of our economy, the aggregate production function shows that the higher the degree of intersectoral integration (which is here equal to the number of intermediate sectors), the higher the average level of productivity. Because private agents take the number of sectors (and goods) as given, they perceive constant returns to scale in their own activities; that makes a competitive equilibrium possible. But the allocation of effort to education increases the number of sectors and then the society's aggregate technology is characterized by increasing returns to scale.

In section 3.4.3 we show that increasing returns with respect to physical capital, labour and human capital make unbounded growth possible. Since we assumed that the expansion of economic activities is related to the expansion of knowledge through education, the model yields a direct relationship between real wages and the degree of intersectoral integration in the economy. We also find that the steady-state rental rate of physical capital is constant and increases with the coefficient of education efficiency.

These results are interesting because they may explain, at least partially, why capital does not flow from industrialized countries to non-industrialized countries and why there exist migration pressures from underdeveloped countries to industrialized countries.

The analysis in this section has focused on the characteristics of the balanced growth equilibrium. However, in Appendix 2 we support the hypothesis that for very high coefficients of education efficiency, the economy might enjoy increasing rates of growth. We also postulate that for very low coefficients of education efficiency the economy might converge to zero growth or experience negative growth. An analysis of global stability in these extremes cases could provide valuable insights into the process of economic development.

Appendix 1: Generalizing the First Model for a High Degree of Intertemporal Substitutability

In section 3.2 we assumed an intertemporal elasticity of substitution equal to 1. Here we show that the main results of that section are not affected provided that this elasticity is high, i.e. $\varepsilon > 1$ [see equation (3.5)].

Substitution of equation (3.15) into equation (3.5), for $\varepsilon > 1$, yields the instantaneous level of utility:

$$u(m, N) = \frac{1}{1 - \varepsilon^{-1}}\{[\alpha a \sigma^{1/(1-\sigma)} m N^{\sigma/(\sigma-1)}]^{1-\varepsilon^{-1}} - 1\} .$$

For $\varepsilon = 1$, the last equation collapses to our equation (3.19). Now, as before we want to maximize the discounted sum of utility [equation (3.4)], subject to the transition equation of education [equation (3.6)]. The first order conditions for this problem are the following:

$$e^{-\rho t}\frac{\partial u(m(t), N(t))}{\partial m(t)} = \lambda(t) N(t) \delta ,$$

$$\frac{\dot{\lambda}}{\lambda} = -\delta\left[1 + \frac{m(t)}{\sigma - 1}\right] ,$$

$$\lim_{t\to\infty} \lambda(t) N(t) = 0 .$$

where λ is the shadow value of the stock of knowledge.

Following the same procedure as in section 3.2 we deduce the workforce allocation to productive activities:

$$m = \varepsilon\left(\frac{\sigma - 1}{\sigma}\frac{\rho}{\delta} - 1 + \varepsilon^{-1}\right) .$$

Afterwards we obtain the rate of utility growth:

$$\frac{\dot{u}}{u} = \frac{(1 - \varepsilon^{-1})\sigma}{\sigma - 1}\frac{\alpha a \sigma^{1/(1-\sigma)} m N^{(1-\varepsilon^{-1})\sigma/(\sigma-1)}}{\alpha a \sigma^{1/(1-\sigma)} m N^{(1-\varepsilon^{-1})\sigma/(\sigma-1)} - 1}\frac{\dot{N}}{N} ,$$

where

$$\frac{\dot{N}}{N} = \varepsilon\left(\delta - \frac{\sigma - 1}{\sigma}\rho\right) .$$

From these equations we deduce that the interiority condition is as before $\delta > \rho(\sigma-1)/\sigma$; hence the conclusions of section 3.2 are valid here. Notice also that as the number of sectors grows, the growth rate of utility falls towards the minimum value shown in equation (3.25′).

Appendix 2: Local Stability in the Dynamic Model with Capital Accumulation

The system of differential equations driving the dynamics of this economy is as follows:

$$\dot{C}/C = \hat{\alpha} K^{\hat{\alpha}-1} m^{1-\hat{\alpha}} N - \rho,$$

$$\dot{m}/m = \delta/\hat{\alpha} + \delta m/(1-\hat{\alpha}) - C/K,$$

$$\dot{K} = K^{\hat{\alpha}} m^{1-\hat{\alpha}} N - C,$$

$$\dot{N} = N(1-m)\delta.$$

In order to obtain this system we have made the following transformations. First the coefficient **B** in the production function [equation (3.51)] has been normalized to 1. We also denote $\hat{\alpha} \equiv \alpha/(\alpha+\beta)$. The first two differential equations are obtained by differentiating equations (3.52) and (3.53) with respect to time, afterwards we substitute for the growth rates of the shadow prices of physical capital and human capital [equations (3.54) and (3.55), respectively]. The last two differential equations in the system above are the transition equation of physical capital [equation (3.32)] and the transition equation of education [equation (3.6)].

If the allocation of workforce to production (m) is constant, the second differential equation implies that physical capital and consumption grow at the same rate. From the first and the third equation we deduce that this growth rate is constant. In section 3.4.3 we showed that the balanced growth rate of physical capital and consumption is given by $g \equiv (1-\hat{\alpha})^{-1}\delta - \rho$, the balanced growth rate of human capital is $g_N = (1-\hat{\alpha})g$, and the balanced path allocation of workforce to production is $m^* = (1-\hat{\alpha})\rho/\delta$. We will adopt the usual convention that starred variables are on the equilibrium growth path. We will also find it convenient to define the following balanced path constants: $\Delta = (K^*/m^*)^{\hat{\alpha}-1}N^* = \delta/(\hat{\alpha}(1-\hat{\alpha}))$ and $(C^*/K^*) = \rho + \delta/\hat{\alpha}$.

Linearizing the above system around the balanced path equilibrium, we obtain the following system of transition equations:

$$\begin{bmatrix} \dot{C} \\ \dot{m} \\ \dot{K} \\ \dot{N} \end{bmatrix} = \begin{bmatrix} g & \frac{\hat{\alpha}(1-\hat{\alpha})C^*\Delta}{m^*} & -\frac{\hat{\alpha}(1-\hat{\alpha})C^*\Delta}{K^*} & \frac{\hat{\alpha}C^*\Delta}{N^*} \\ -\frac{m^*}{K^*} & \frac{\delta m^*}{1-\hat{\alpha}} & \frac{m^*C^*}{(K^*)^2} & 0 \\ -1 & \frac{(1-\hat{\alpha})\Delta K^*}{m^*} & \hat{\alpha}\Delta & \frac{\Delta K^*}{N^*} \\ 0 & -\delta N^* & 0 & (1-\hat{\alpha})g \end{bmatrix} \begin{bmatrix} C-C^* \\ m-m^* \\ K-K^* \\ N-N^* \end{bmatrix} + \begin{bmatrix} gC^* \\ 0 \\ gK^* \\ g_N N^* \end{bmatrix}$$

With two predetermined variables, the stocks of physical and human capital -K and N respectively, and two forward-looking variables -the choices of workforce allocation and consumption -m and C, saddle-path stability requires that the transition matrix should be characterized by two negative eigenvalues and two positive eigenvalues when evaluated along the balanced path. Thus the determinant of the transition matrix along the balanced path should be positive for saddle-path stability. Analytical solutions for the eigenvalues are extremely difficult to obtain -if not impossible. Hence, we will only examine the sign of the matrix determinant. After some algebra we obtain:

$$\det = \frac{\delta}{\hat{\alpha}} g \left\{ -\frac{(1-\hat{\alpha})}{\hat{\alpha}} \delta^2 + \rho\left[(1-\hat{\alpha})\left(\frac{2}{\hat{\alpha}} + 1\right) + \frac{\hat{\alpha}}{1-\hat{\alpha}}\right]\delta - \rho^2[2(1-\hat{\alpha}^2) + \hat{\alpha}] \right\},$$

Because the growth rate, g, depends on δ, the determinant is a polynomial expression of the fourth order in the coefficient of education efficiency, δ. The discriminant of the quadratic expression between curly brackets depends exclusively on $\hat{\alpha}$ and is always positive, i.e. the roots of this expression are real. All roots are positive except δ=0. These features determine the shape of the determinant function which is shown by the following figure:

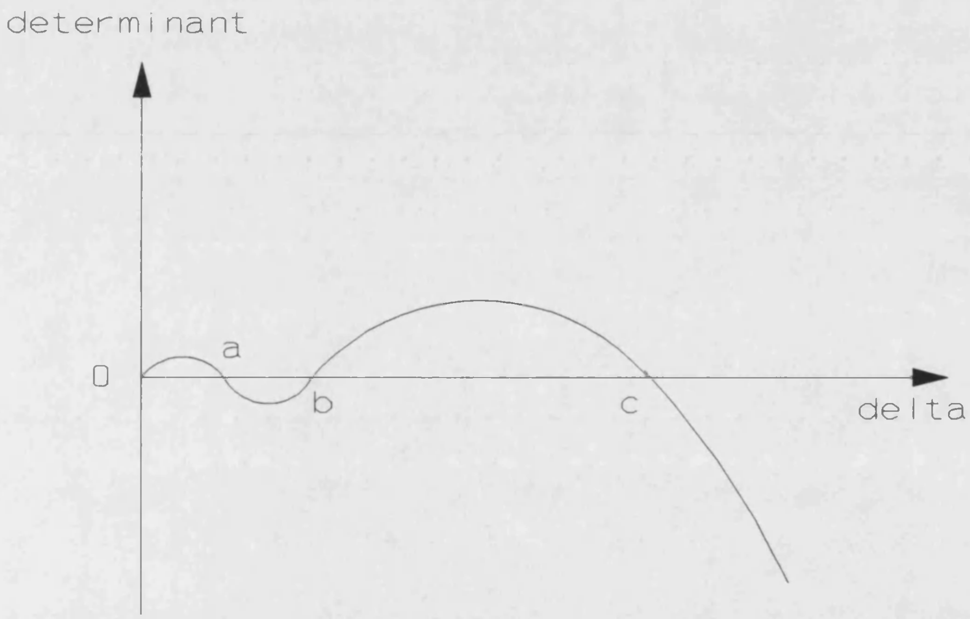

Figure 3.A1 *The Determinant of the Transition Matrix*

Assuming that the coefficient of efficiency in education is positive (δ>0), one of the positive roots, **a**, **b** or **c**, corresponds to zero growth (g=0). The highest root of the quadratic expression in curly brackets is higher than $(1-\hat{\alpha})\rho$, which is the root value associated with zero growth. Thus, the delta value associated with zero growth is either the value at point **a** or the value at point **b**. Three situations may arise: **(1)** Efficiency coefficients below the root **a** would imply negative growth. Agents might allocate instead all their time to production and converge to zero growth. Notice that if all the workforce is allocated to production, m=1, human capital, N, would be fixed. Thus the dynamic system is reduced to two variables, physical capital, K, and consumption, C, and the transition matrix would be characterized by two eigenvalues of opposite signs, which gives a negative determinant. The dynamics then might resemble the dynamics of the standard Ramsey-Cass-Koopmans model of growth with decreasing returns.

(2) If the zero-growth root is at point **a**, only coefficients of education efficiency in the range **bc** might be consistent with balanced growth; in this case the economy as a whole might experience constant returns to scale as the coefficient Δ is constant, implying constant marginal productivity of physical capital. For coefficients in the range **ab** the efficiency in education is so low that the system might be trapped again in a corner solution with no effort in education; in this case the economy might converge to a steady state with no growth. Now, for efficiency coefficients above the point **c**, the economy cannot converge locally to the balanced growth path. In this case the economy might experience increasing growth rates because the growth rate of human capital is so high that the economy might experience increasing returns to scale.

(2) If the zero-growth root is at point **b** there are only two possibilities. For delta values in the range **bc** the economy might be consistent with balanced growth, as analyzed in the preceding situation. For delta values above the point **c**, the economy is not consistent with balanced growth; again, this situation might induce increasing rates of growth.

The Figure 3.2A is a simulation of the determinant of the transition matrix for the parameters $\hat{\alpha}$=0,5 and ρ=0.04. In this case the zero-growth root is δ=0.03. Saddle-path stability around the balanced growth path is only feasible for $\delta \in$ (0.03,0.08). For higher values of delta the economy is not consistent with balanced growth, and perhaps the economy experiences increasing growth rates.

The above analysis is focused on local stability. The issue of global stability is not addressed at all. Thus, we do not know the behaviour of this economy for dynamic paths which are outside of the vicinity of the balanced growth path.

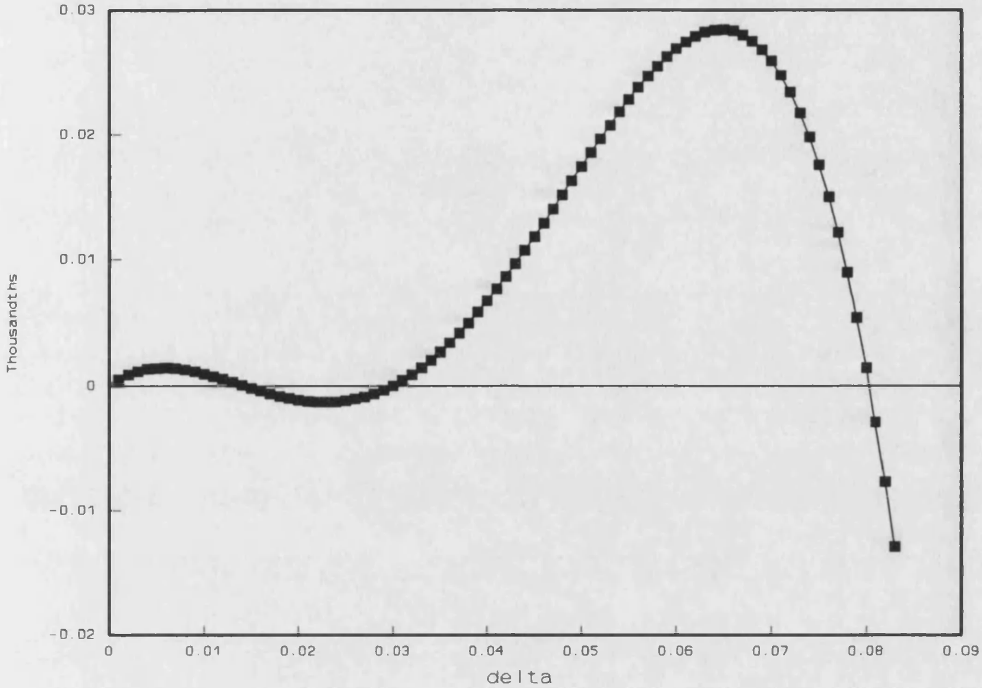

Figure 3.A2 *Simulation of the Determinant of the Transition Matrix*

CHAPTER 4

PRODUCTION 'ROUNDABOUTNESS' AND ECONOMIC GROWTH: SOME EMPIRICAL EVIDENCE

4.1 INTRODUCTION

4.1.1 Objective and Theory

The explanation of the worldwide disparities in income and economic growth has become one of the central issues in the recent wave of endogenous growth literature. This chapter is also focused on that topic. Our hypothesis is that a partial explanation of these disparities lies in the degree of maturity of the countries' economic structure. Specifically we claim that the tightness of the net of interindustry linkages is a significant element in the growth process.

It has been known for a long time that advanced industrialized countries enjoy a higher degree of technological interdependence across sectors and industries:

> "Displayed in the input-output table, the pattern of transactions between industries and other major sectors of the system shows that the more developed the economy, the more its internal structure resembles that of other developed economies. (...).
>
> Recent advances in input-output analysis and in the bookkeeping of underdeveloped countries have made it possible to apply the technique to a number of these economies. Their input-output tables show that in addition to being smaller and poorer they have internal structures that are different, because they are incomplete, compared with the developed economies. From such comparative studies a fundamental analytical approach to the structure of economic development is now emerging" (W. Leontief, "The Structure of Development", 1963, reprinted in Leontief, 1986, p. 163).

But the links between economic structure and growth had not been properly identified until recent developments in economic analysis made it possible for growth economists to use the concepts of externalities and specialization through expansion of varieties in mathematically tractable economic models.

Endogenous growth models centred on specialization have pointed out the role of expansion of varieties of goods and factors as a source of dynamic increasing returns. The leading papers on this subject have emphasized endogenous technological change (Romer, 1987, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1991). In this type of model, forward-looking entrepreneurs bear the responsibility of innovating by investing in R&D; they have incentives to do so because it is assumed that free patent mechanisms exist to enforce property rights on new designs. In contrast to this line of research, we develop a model where expansion of varieties is brought about by the copying and adaptation of other's ideas (see Chapter 3). In this model, the force driven growth is education; if workers invest in education they enable themselves to copy others' ideas and also to adapt others' technologies. The fundamental assumption of our approach is that knowledge and technologies are not completely excludable, but the worker needs to be sufficiently educated for appropriation to occur.[1]

Both kinds of models capture different mechanisms in the process of economic growth. The assumption of knowledge excludability by patent is more appropriate for modelling the process of R&D in industrialized countries. Our model, on the other hand, is perhaps more suitable to explain economic development through technological imitation, which is probably the main source of technological development in non-

[1] There is no need to interpret our models as emphasizing labour force education. In our models everyone supplies labour inelastically, hence workers and economic agents are just the same.

industrialized countries.

Both kinds of models stress the costs associated with technological diversification. Models of endogenous technological change emphasize the investment cost in R&D; our model emphasizes the investment costs in education. Thus, it is clear that these models omit the role of knowledge externalities associated with the process of economic diversification.[2] However, knowledge may be non-rival and non-excludable, thus technological breakthroughs may be diffused with no cost (or negligible cost) to the whole economy. These externalities then become a potential source of aggregate increasing returns which may induce sustained economic growth at increasing rates. As is well known, the seminal paper in the new growth literature models this intuition (Romer, 1986). Moreover, using time series from 1700 to 1979 for the world leading capitalist countries, Romer finds that the hypothesis of increasing long-run growth rates of per capita GDP is not obviously contradicted by the data.

In line with Romer's work, the purpose of this chapter is to explore the cross-country relationship between economic structure diversification and economic growth. Our hypothesis is that the social division of work increases total factor productivity and the rate of economic growth.

In order to support this hypothesis we will refer the

[2] This is recognized by Romer in his seminal paper on growth and specialization: "The model (...) ignores increasing returns from investment in knowledge and external effects due to spillovers of knowledge. It focuses exclusively on the role of specialization. A more realistic and more ambitious model would examine both effects" (Romer, 1987, page 56).

reader to the supply side of our aggregative model of economic diversification (Chapter 3, sections 3.4.1 and 3.4.2). There we deduced the aggregate output function of a competitive economy where the interdependence of intermediate good technologies increases with the number of varieties. Hence, assuming that the number of varieties of intermediate goods is taken as given by firms, the final good technology exhibits constant returns in primary factors, but increasing returns in primary factors and the number of varieties. If there are no costs in appropriating at least some of the new technologies, it follows that technological interdependence might be directly associated with economic growth. This feature must be true both in time series and cross-country data.

4.1.2 Related Empirical Research and Estimation Strategy

Existing empirical research has already reported significant evidence for the existence of external economies across industries in West Germany, France, the United Kingdom and Belgium (Caballero and Lyons, 1990). They also report very little evidence of internal increasing returns economies in the industries of these countries. Besides, they show that failure to take into account external economies leads to upward-biased estimates of internal economies at the industry level. These findings suggests that external economies are an important part of aggregate increasing returns as emphasized in Romer's (1986) paper.

Building on Caballero and Lyons' paper, Bartelsman, Caballero and Lyons (1991) have explored the relationship between input-output linkages and total factor productivity. They found that external effects operate through interindustry linkages in United States manufacturing industry. Specifically they found evidence that the relationship between an industry and its suppliers of intermediate inputs is important in the transmission of external effects leading to total productivity growth. The mechanisms leading to the generation of externalities are thought to be specialization through input diversification and knowledge embodied in intermediate products.

In this chapter we want to go a step further to test the relationship between economic diversification and economic growth. Now, how can we test this theory? A direct approach would involve constructing an index of good variety across countries. But then one must solve the difficult problem of comparability; this task is not within our possibilities. However, based on Chapter 3, we can circumvent this problem by using instead direct and indirect measures of production roundaboutness. Notice then that our approach rests on the assumption that economic diversification and technological interdependence are intimately related. Such an assumption is consistent with the stylized fact that a country's production structure becomes more roundabout as industrialization takes place.

As is well known, cross-country regressions are usually subject to problems of heteroscedasticity and measurement error that might render invalid estimates. Cross-country analysis is also subject to problems of heterogeneity across countries (Stern, 1989). In the spirit of Levine and Renelt's (1991) sensitivity analysis of cross-country growth regressions, we will try to minimize the heterogeneity problem by including a basic set of regressors that have proved to be robustly correlated with economic growth. Hence, our objective is to check whether our measures of roundaboutness appear to be significantly correlated with economic growth after controlling for the effects of Levine and Renelt's basic set of regressors.

4.2 CROSS-COUNTRY GROWTH REGRESSIONS

4.2.1 Indirect Measures of Production Roundaboutness

Our dependent variable is the annual growth rate of real per capita GDP from 1960 to 1988 (G). This is calculated from Summers and Heston's data set (Summers and Heston, 1991) for a sub-set of 48 countries [see Table 4.7 in the Appendix].

Direct measures of the degree of interindustry integration are difficult to obtain. Indeed we obtained comparable direct measures for only nine countries. Hence, we will postpone the analysis of this information until section 4.2.2. Here we will use instead two proxies which are likely to be closely correlated with production roundaboutness: the ratio of intermediate consumption to gross output for the whole economy in 1980 (IO), and the ratio of intermediate consumption to gross output for the manufacturing sector in 1980 (IOMAN). (The term IO stands for aggregate input-output coefficient, whilst IOMAN stands for input-output coefficient in the manufacturing sector.) These variables were calculated for 48 countries from the United Nations' *National Account Statistics*; we chose the year 1980 because information for all countries in our sample is not available for earlier years.

Levine and Renelt's basic set of regressors are the following: real gross domestic product per capita in 1960 (RGDP60), the ratio of investment to GDP (I), the secondary-school enrolment ratio in 1960 (SEC60), and the average annual growth rate of population (GN). These variables are thought to be robustly correlated with the growth rate and have been theoretically motivated by many models in the growth literature. We also add an index of openness to this basic set of regressors: the average ratio of the sum of exports and imports to GDP (OPEN).

All this data is reported in Table 4.7 in the Appendix. All variables with exception of IO and IOMAN are taken or calculated from Summers and Heston (1991).

We first estimate linear regressions by ordinary least squares. Because there is evidence of heteroscedasticity, we check the significance of our coefficients by using White's (1980) heteroscedasticity-consistent variance-covariance matrix. Hence we only report t-statistics based on this matrix. The results are shown in Table 4.1.

Regression (1) shows that the aggregate input-output coefficient (IO) does not seem to be correlated with economic growth, although the corresponding slope coefficient has the right sign. However, the input-output coefficient in the manufacturing sector (IOMAN) seems to be strongly correlated with economic growth. Because both IO and IOMAN are proxies

Table 4.1
Regressions for Per Capita GDP Growth 1960-1988
Ordinary least squares estimation based on White's (1980) heteroscedasticity-consistent covariance matrix
(t-statistics in parentheses)

Equation	(1)	(2)	(3)	(4)
Constant	-4.81*	-1.16	-4.75*	-5.84*
	(-2.98)	(-0.95)	(-3.18)	(-3.87)
RGDP60	-0.31E-3*	-0.35E-3**	-0.31E-3*	-0.22E-3*
	(-1.82)	(-1.79)	(-2.02)	(-2.50)
I	0.15*	0.13*	0.15*	0.15*
	(3.86)	(3.43)	(4.94)	(6.93)
IO	4.2E-3	0.04	--	--
	(0.14)	(1.22)		
IOMAN	0.08*	--	0.08*	0.09*
	(3.37)		(3.54)	(3.62)
SEC60	-3.8E-3	-4.2E-3	-2.4E-3	--
	(-0.19)	(-0.19)	(-0.12)	
OPEN	3.0E-3	0.01	2.3E-3	--
	(0.34)	(1.18)	(0.28)	
GN	-0.27	-0.17	-0.26	--
	(-1.63)	(-0.86)	(-1.62)	
R^2	0.674	0.607	0.678	0.648
S.E.	1.037	1.123	1.001	0.997
Sample	45	45	47	48

Note
* : coefficient statistically significant at the 5% level (two-tailed test).
** : coefficient statistically significant at the 10% level (two-tailed test).

for production roundaboutness, we drop the regressor IOMAN in regression (2). This regression shows again that the slope coefficient on IO does not appear to be significant. In regression (3) we only use IOMAN as regressor, the slope coefficient corresponding to this regressor has the expected positive sign and is statistically significant.

The initial level of real income per capita (RGDP60), and the average investment ratio (I) are both significant and also appear with the expected signs. The former captures any tendency for catching up, whilst the investment rate should be positively correlated with economic growth for obvious reasons. These two variables appear as strongly robust in Levine and Renelt's (1992) analysis of cross-country regressions.

The school-enrolment ratio (SEC60), the index of openness (OPEN) and the growth rate of population (GN) do not appear to be significantly correlated with economic growth in our sample.[3] They are not significant either as a set. Hence in regression (4) we exclude these regressors and find that the regressors RGDP60, I and IOMAN explain 65% of the total variation of cross-country economic growth in our sample.

Why does IOMAN seem to be correlated with economic growth whilst the aggregate measure IO, does not seem to be? This is particularly interesting if one takes into account that IO shows a higher degree of variation than IOMAN in our sample:

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[3] This is probably due to the smallness of our sample and the inclusion of small economies and oil exporting countries. Excluding these countries reduces further our sample and does not modify the initial result.

the respective coefficients of variation are 0.21 and 0.10. We will advance two tentative explanations. The first has to do with the characteristics of these proxies; it is obvious that IO is more sensitive to composition problems as it is a weighted average of all the ratios of intermediate consumption to gross output across sectors. Hence, IO may be subject to greater measurement error which renders this variable less reliable as a proxy for economic interdependence. The second reason is based on Rebelo's analysis of two-sector models of economic growth characterized by linearly homogeneous technologies (Rebelo, 1991). In this paper Rebelo proves that the requisite for sustained economic growth at a constant rate in a competitive environment is the existence of a "core" set of reproducible factors whose technologies are characterized by constant returns to scale. Hence, it follows that increasing returns to scale technologies in the "core" set of reproducible factors yields increasing growth rates, as in Romer's model of knowledge externalities (Romer, 1986). This result may explain why the relevant proxy of economic integration for explaining economic growth is the manufacturing index (IOMAN). After all, the manufacturing sector is by definition the sector that provides most intermediate and capital goods in the economy. Additionally, the manufacturing sector is an intensive user of manufacturing intermediates, so that the growth externalities of input-output linkages identified by Bartelsman, Caballero and Lyons (1991) are likely to accrue primarily to the manufacturing sector.

If Rebelo's and Romer's models provide the clue for understanding the strongly positive association between the degree of economic roundaboutness and economic performance, here we have further evidence that economic structure matters for economic development.

Some caution is required, however, in interpreting our results. Our proxies may be related to the degree of technological interdependence, but they also may reflect the degree of industrialization or related processes:

> "As countries industrialize, their productive structures become more "roundabout" in the sense that a higher proportion of output is sold to other producers rather than to final users" (H. Chenery and M. Syrquin, "Typical Patterns of Transformation", Chapter 3, page 57, in H. Chenery, S. Robinson and M. Syrquin, 1986).

Therefore, we would like to check whether our results are reproduced when a direct measure of economic integration is used instead of our proxy IOMAN. This leads us to the next section.

4.2.2 Direct Measures of Production Roundaboutness

Based on Kubo's work on cross-country comparisons of interindustry linkages (Kubo, 1985), Kubo, De Melo, Robinson and Syrquin (1986) calculated comparable indices of aggregate interindustry linkages using information from 30 input-output matrices for nine countries and different years from 1950 to 1975.

The procedure to calculate these indices was the following. First, the authors rearranged each matrix into 14 comparable economic sectors and calculated the matrix of technical coefficients $\mathbf{A} = [a_{ij}]$, where a_{ij} is the technical coefficient measuring the amount (in value terms) of input i which is consumed in the production process of one unit of good j. Subsequently, they calculated the Leontief matrix, $\mathbf{L} = \mathbf{I} - \mathbf{A}$, where $\mathbf{I}$ denotes the identity matrix of the same order as matrix $\mathbf{A}$. Finally they obtained an index of overall linkages (OL) as follows: $(OL) = \mathbf{f}'(\mathbf{L}')^{-1}\mathbf{i}$, where (OL) is a scalar, $\mathbf{f}$ is a 14x1 weight vector whose elements add up to 1, $\mathbf{i}$ is a 14x1 unit vector, the apostrophe (′) denotes matrix transposition, and the power -1 denotes matrix inversion. Let us decompose this expression: $(\mathbf{L}')^{-1}\mathbf{i}$ is a 14x1 vector whose elements measure the degree of backward technological integration of the corresponding sectors, i.e. each element measures the proportion of gross output which is produced in the economy per unit value of final demand in the corresponding sector. The final expression (OL) is then a weighted average of these measures, where the weights are taken from the representative structure of the final demand vector for a semi-industrial country (see Chenery, Robinson and Syrquin, Chapter 4, 1986). These authors also obtain an index of domestic linkages (DL) by excluding imported intermediate inputs from the input-output matrix, the calculation is completely analogous to the previous one.

These measures of interindustry linkages are shown in Table 4.2, where we also show the equivalent annual growth rates of per capita GDP during 10 years (G10), the real per capita GDP (RGDP), the secondary-school enrolment ratio (SEC), the index of openness (OPEN), the equivalent annual growth rate of population in the following decade (GN10), and the average investment ratio in the next decade (I10). The choice of variables was determined by the same reasons stated in the Introduction. Sources and explanations of these variables are provided in the table.

Table 4.2 contains a small unbalanced panel. Using this information we run the growth regressions in Table 4.3. We estimate by ordinary least squares. Since we cannot reject the assumption of homoscedasticity, the associated OLS covariance matrix is used to calculate significance levels. The first three regressions use the measure of overall linkages (OL), whilst the last three use the measure of domestic linkages (DL). Because of the oil shocks of the 70's we add an interactive dummy in order to account for the apparent downward jump of growth rates during this period. It is likely that the oil shocks reduced the positive externalities of interindustry linkages because oil is perhaps the most important intermediate input for the current technology.[4] In the first and fourth regressions we also add country dummies in order to capture possible fixed effects. However, none of the country dummies appears to be significant, either in regressions (1) or in regression (4). The country dummies are not jointly significant either. When they are excluded we find that the measures of interindustry linkages appear to be strongly correlated with economic growth [see regressions (2) and (5)].

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[4] Without a dummy for the seventies our regressions exhibit lower determination coefficients, but the significance of other regressors does not change significantly.

Table 4.2
Unbalanced Data Set
Sample: 9 Countries, 30 Observations, Different Periods.

Economy	Year	G10	OL %	DL %	RGDP 1985 US$	SEC %	OPEN %	GN10 %	I10 %
Colombia	1953	0.80	50.0	37.2	1760	7	17.0	0.80	21.3
	1966	3.20	65.4	52.3	2126	19	15.5	3.20	17.7
	1970	3.39	69.0	53.9	2387	25	16.9	3.39	16.6
Mexico	1950	2.58	54.3	40.5	2224	5	14.8	2.58	16.4
	1960	3.53	68.9	51.3	2870	11	11.7	3.53	18.7
	1970	3.55	63.9	52.0	4061	22	10.1	3.55	21.6
	1975	1.15	69.5	54.2	4755	35	11.8	1.15	21.4
Turkey	1963	3.32	52.1	46.4	1884	16	7.7	3.32	18.9
	1968	3.78	56.7	51.5	2181	21	7.0	3.78	22.4
	1973	1.62	59.6	52.8	2612	28	9.3	1.62	23.8
Yugos-lavia	1962	5.71	82.2	67.9	1815	60	12.7	5.71	37.2
	1966	4.97	79.5	61.9	2324	64	16.4	4.97	35.4
	1972	3.78	87.3	59.4	3126	70	22.3	3.78	36.5
Japan	1955	8.26	89.9	81.3	1865	70	7.6	8.26	23.3
	1960	9.49	94.5	82.7	2701	74	7.7	9.49	29.5
	1965	6.62	94.6**a**	82.4	4125	82	9.1	6.62	33.5
	1970	3.70	106.3	88.7	6688	86	10.9	3.70	34.2
Korea	1963	7.44	89.9	60.9	1041	31	15.2	7.44	22.4
	1970	5.82	89.8	58.7	1722	42	21.7	5.82	29.3
	1973	5.22	92.8	54.6	2133	50	30.2	5.22	29.6
Taiwan	1956	4.92	76.5	42.6	852	34	17.1	4.92	13.6
	1961	7.21	85.9	55.0	1001	38	18.4	7.21	18.4
	1966	7.48	92.9	55.7	1377	43	25.9	7.48	24.3
	1971	6.90	93.7	55.2	2099	48	33.1	6.90	28.2
Israel	1958	4.68	83.7	53.8	3575	44	22.3	4.68	30.3
	1965	4.73	78.6	50.5	5280	48	26.9	4.73	28.9
	1972	1.17	101.5	48.1	7643	60	39.4	1.17	26.1
Norway	1953	2.71	66.7	40.8	4709	46	34.7	2.71	32.7
	1961	3.61	77.9	47.8	5673	59	35.6	3.61	33.2
	1969	4.21	87.2	47.6	7628	76	41.0	4.21	34.6

Sources. G10: Equivalent annual growth rate of real gross domestic product per capita during 10 years (calculated from Summers and Heston, 1991). **OL:** Overall linkage measure; DL: Domestic linkage measure (Chenery et al., Table 7-3, 1986). See text for explanation on linkage measures. **RGDP:** Real gross domestic product per capita (Summers and Heston, 1991). **SEC:** Secondary-school enrolment ratio (taken or estimated from the World Bank's World Tables 1980 and 1983). **OPEN:** Openness measure, ratio of the sum of exports and imports to total supply on the domestic market (Chenery et al., Table 7-5, 1986). **I10:** Average Investment-to-GDP ratio during 10 years (calculated from Summers and Heston, 1991). **GN10:** Equivalent annual growth rate of population during 10 years (calculated from Summers and Heston, 1991).
Note. a: Using Kubo's estimation (1985) we corrected this figure from Kubo, De Melo, Robinson and Syrquin (1986).

Table 4.3
Growth Regressions from Unbalanced Panel
Sample=30 (t-statistics in parentheses)

Equation	(1)	(2)	(3)	(4)	(5)	(6)
Constant	4.39	-0.72	-2.70*	-3.16	-4.00	-4.20*
	(0.99)	(-0.36)	(-2.67)	(-0.44)	(-1.40)	(-3.42)
RGDP	-.8E-3*	-.6E-3*	-.6E-3*	-.9E-3*	-.7E-3*	-.7E-3*
	(-2.02)	(-3.61)	(-5.68)	(-2.25)	(-4.81)	(-5.39)
OL	0.07	0.13*	0.12*	--	--	--
	(1.53)	(3.92)	(9.25)			
OL*D70	-0.02*	-0.02*	-0.02*	--	--	--
	(-2.74)	(-3.93)	(-4.34)			
DL	--	--		0.15	0.16*	0.16*
				(1.84)	(3.60)	(8.72)
DL*D70	--	--		-0.02	-0.02*	-0.02*
				(-1.60)	(-2.77)	(-2.91)
OPEN	-0.06	-0.02		0.06	0.14*	0.13*
	(-0.99)	(-0.91)		(0.57)	(3.49)	(5.17)
SEC	4.6E-3	-4.9E-3		0.04	0.02	
	(0.09)	(-0.15)		(0.74)	(0.67)	
I10	0.06	-0.02		-0.02	-0.06	
	(0.97)	(-0.47)		(-0.31)	(-1.18)	
GN10	-1.78	-0.48		-0.26	0.19	
	(-2.09)	(-1.24)		(-0.22)	(0.55)	
Mexico	1.63			1.72		
	(1.89)			(1.80)		
Turkey	-0.14			0.53		
	(-0.13)			(0.40)		
Yugos-lavia	-3.13			-1.69		
	(-1.50)			(-0.66)		
Japan	-0.78			-1.52		
	(-0.33)			(-0.55)		
Korea	0.50			0.78		
	(0.28)			(0.49)		
Taiwan	2.02			1.39		
	(0.96)			(0.64)		
Israel	2.73			1.78		
	(1.45)			(0.77)		
Norway	-0.85			1.70		
	(-0.41)			(0.71)		
R^2	0.914	0.835	0.808	0.896	0.818	0.797
S.E.	0.921	1.019	1.011	1.015	1.071	1.061

* Coefficient significant at the 5% level (two-tailed test).

The initial level of per capita GDP (RGDP), and the 70's interactive dummy variables (OL*D70 and DL*D70) are also significant and exhibit the expected negative signs.

Interestingly, regressions (1) and (2) show that when the measure of overall linkages (OL) is included some of the traditional explanatory variables do not seem to be significant. However, regressions (5) and (6) show that the measure of openness (OPEN) appears to be significant when the measure of domestic linkages (DL) is included. Since the difference between the measures of overall linkages and domestic linkages is accounted for by the exclusion of imported intermediates, the previous result suggests that openness is correlated with economic growth to the extent in which it proxies the role of imported intermediates in the degree of economic integration. Now, by excluding the non significant regressors we are left with regressions (3) and (6). Again it is most interesting that these two regressions explain similar proportions of cross-country growth performance: around 80%.

The sets of excluded variables in going from regression (2) to regression (3), and from regression (5) to regression (6), do not appear to be statistically significant at the 5% level.

There is one reason to be uncomfortable with the last set of regressions. Our dependent variable is the annual growth rate calculated over a period of 10 years. We proceed in this way in order to eliminate, at least partially, the cyclical effects. However, the periods between observations for the same country are usually smaller than 10 years (see Table 4.2). Hence the regressors may partially "explain" the behaviour of consecutive dependent variables, which introduces some correlation between regressors and disturbances of each country. Because of this feature we may obtain biased estimates of the regression coefficients.

We try to solve this problem by choosing only observations at the beginning of each decade for which there is available information. The cost of this procedure is the loss of observations. The new data set contains only 23 observations and is displayed in Table 4.4. Although the number of observations is not the same for each country, we call this data set "balanced" because the period between observations is the same for all countries in our sample. This use of the term is unconventional, but it is useful to distinguish this panel data from the panel in Table 4.2. The balanced panel contains the same set of variables as the unbalanced panel. Taking advantage of the strong time trend behaviour of the linkage measures (OL and DL), we estimated some of the new observations by linear interpolation or least squares from the original unbalanced panel.

The regressions corresponding to Table 4.4 are shown in Table 4.5. They yield the same results as the regressions in Table 4.3: the set of country dummies is not significant at conventional levels, and the measures of interindustry linkages (OL and DL) appear again with positive and significant coefficients. The initial level of per capita GDP (RGDP) is also significant and its coefficient appears with

Table 4.4
Balanced Data Set
Sample: 9 Countries, 23 Observations, Decadal Periods.

Economy	Year	G10 %	OL %	DL %	RGDP 1985US$	SEC %	OPEN %	I10 %	GN10 %
Colombia	1950	1.26	46.7	34.5	1653	5	17.0	21.6	3.11
	1960	2.45	58.1	44.7	1874	12	15.5	19.2	3.05
	1970	3.39	69.0	53.9	2387	25	16.9	16.6	1.99
Mexico	1950	2.58	54.3	40.5	2224	5	14.8	16.4	3.26
	1960	3.53	68.9	51.3	2870	11	11.7	18.7	3.28
	1970	3.55	63.9	52.0	4061	22	10.1	21.6	2.93
Turkey	1960	3.23	50.1	45.1	1669	14	7.7	17.8	2.53
	1970	2.73	57.6	51.5	2293	27	7.0	23.0	2.41
Yugosla-via	1960	5.66	79.2	68.5	1690	58	12.7	37.7	1.02
	1970	4.62	84.9	60.3	2932	63	22.3	36.3	0.91
Japan	1950	7.80	84.0	78.3	1275	66	7.6	18.1	1.19
	1960	9.49	94.5	82.7	2701	74	7.7	29.5	1.04
	1970	3.70	106.3	88.7	6688	86	10.9	34.2	1.13
Korea	1960	6.43	88.8	63.0	923	27	15.2	18.0	2.58
	1970	5.82	89.8	58.7	1722	42	21.7	29.3	1.79
Taiwan	1950	4.35	71.4	41.7	630	30	17.1	11.1	3.73
	1960	6.64	83.1	49.4	964	37	18.4	17.5	3.12
	1970	7.52	94.9	57.1	1833	47	33.1	27.6	1.95
Israel	1960	5.31	81.6	52.8	3958	48	22.3	29.6	3.47
	1970	2.49	94.3	48.8	6645	57	39.4	28.0	2.69
Norway	1950	2.47	63.2	40.8	4263	42	34.7	32.7	0.93
	1960	3.61	76.0	45.0	5443	57	35.6	32.6	0.80
	1970	4.41	88.8	49.2	7761	84	41.0	34.8	0.54

Sources.
G10: Equivalent annual growth rate of real gross domestic product per capita during 10 years (calculated from Summers and Heston, 1991). **OL:** Overall linkage measure; **DL:** Domestic linkage measure (taken or estimated from Chenery et al., Table 7-3, 1986). See text for explanation on linkage measures. **RGDP:** Real GDP per capita (Summers and Heston, 1991). **SEC:** Secondary-school enrolment ratio (taken or estimated from the World Bank's World Tables 1980 and 1983). **OPEN:** Openness measure, ratio of the sum of exports and imports to total supply on the domestic market (Chenery et al., Table 7-5, 1986). I10: Average Investment-to-GDP ratio (Summers and Heston, 1991). **GN10:** Equivalent annual growth rate of population during 10 years (calculated from Summers and Heston, 1991).

Table 4.5
Growth Regressions from Balanced Panel
Sample = 23
(t-statistics in parentheses)

Equation	(1)	(2)	(3)	(4)	(5)	(6)
Constant	-5.38*	-1.91	-0.95	-5.04	-3.28*	-3.24
	(-2.47)	(-1.63)	(-1.01)	(-1.91)	(-2.46)	(-2.44)
RGDP	-1.4E-3*	-.6E-3*	-.7E-3*	-.9E-3*	-.7E-3*	-.7E-3*
	(-4.70)	(-4.60)	(-4.57)	(-3.37)	(-4.62)	(-4.61)
OL	0.18*	0.11*	0.10*	--	--	--
	(4.13)	(6.68)	(5.52)			
DL	--			0.18*	0.14*	0.14*
				(3.22)	(6.95)	(6.90)
OPEN	--			0.09	0.12*	0.12*
				(1.22)	(3.75)	(3.73)
Mexico	1.54			1.60		
	(1.88)			(1.54)		
Turkey	1.36			0.73		
	(1.50)			(0.60)		
Yugoslavia	-1.11			-0.52		
	(-0.84)			(-0.35)		
Japan	0.13			-0.05		
	(0.09)			(-0.02)		
Korea	-2.80			0.09		
	(-1.58)			(0.06)		
Taiwan	-1.89			1.59		
	(-1.24)			(1.35)		
Israel	0.70			2.02		
	(0.59)			(1.40)		
Norway	3.18*			2.77		
	(3.01)			(1.77)		
R^2	0.877	0.716	0.644	0.851	0.741	0.739
S.E.	0.974	1.148	1.052	1.121	1.125	1.122

* Coefficient significant at the 5% level (two-tailed test).

the expected negative sign. The other regressors of the basic set are not significant as a whole, except for the openness measure (OPEN) when accompanied with the index of domestic linkages (DL).

Regressions (3) and (6) are estimated under the assumption of random effects in the countries' intercept. We estimate by applying an equivalent procedure to generalized least squares (see, for instance, Johnston, 1984, ch. 10). The regression model is defined as usual by $y_{it}=W_{it}\beta+u_{it}$, where the independent variable, y_{it}, is the growth rate at time t in country i, W_{it} is the matrix of independent variables, β is the vector of coefficients, and $u_{it}=\varepsilon_i+\eta_{it}$ is the disturbance term composed of a fixed part plus a random component. In order to obtain the random effects estimator we calculate the means for countries as follows: $y_{i\bullet}=(T_i)^{-1}\sum_t y_{it}$, where t goes from 1 to T_i, T_i being the number of observations for country i. Using this information the original observations are modified as follows: $\hat{y}_i=y_{it}-\gamma_i y_{i\bullet}$, where $\gamma_i=1-(\sigma^2_\eta/(\sigma^2_\eta+T_i\sigma^2_\varepsilon))^{1/2}$, σ^2_η is the variance of the errors for the fixed effects regression, and $\sigma^2_\eta+\sigma^2_\varepsilon$ is the variance of the errors obtained from the regression with a single intercept. Finally we estimate $\hat{y}_i=\hat{W}_{it}\beta+e_{it}$ by OLS, which yields the random effects coefficients. Now, judging from the t-statistics, regressions (3) and (6) seem to yield more efficient estimates than the corresponding fixed effects estimates [regressions (1) and (4)]. However, this evidence is non conclusive. A Hausman test for the regressor OL in regressions (1) and (3), on the null hypothesis that the random effects estimator is consistent and efficient, yields a test statistic of 2.02; the same test for the coefficient DL in regressions (4) and (6) yields a test statistic of 0.768. The Hausman statistics is asymptotically distributed as a normal (0,1) when the null hypothesis is valid. Thus the first test rejects marginally the random effects assumption at the 5% significance level; the second test fails to reject the random effects assumption. These results imply that the test is inconclusive. Moreover, since the Hausman test is only valid asymptotically, and our sample is small, its application here does not allow any definitive inference. However, for our purposes it is enough to show that all regressions in Table 4.5 yield similar estimates for the coefficients associated with the linkage measures and they appear to be strongly significant.

Finally we run pure cross-country growth regressions for the nine countries on which we have direct information on interindustry integration. For such a small sample the power of this exercise is minimal, but we avoid all sort of potential problems from time series estimation. The dependent variable is the annual growth rate of per capita GDP between 1950 and 1988 (G). The data set is displayed in Table 4.6.

Table 4.6
Cross-Section Data: 1950-1988
Nine economies

Economy	G %	RGDP60 1985 US$	OL70 %	DL70 %	I %	GN %	SEC %	OPEN %
Colombia	2.05	1874	69.0	53.9	18.3	2.75	12.0	28.1
Mexico	2.15	2870	63.9	52.0	18.8	3.20	11.0	24.8
Turkey	3.18	1669	57.6	51.5	19.1	2.75	14.0	20.7
Yugoslavia	3.80**a**	1690	84.9	60.3	36.9**a**	0.90**a**	58.0	41.1
Japan	6.13	2701	106.3	88.7	27.5	1.01	74.0	22.5
Korea	5.39**b**	958	89.8	58.7	22.0**b**	2.07**b**	27.5	41.3
Taiwan	5.97	964	94.9	57.1	19.6	2.54	37.0	52.3
Israel	3.76**b**	3958	94.3	48.8	27.5**b**	3.05**b**	48.0	62.6
Norway	3.36	5443	88.8	49.2	32.8	0.66	57.0	82.9

Sources.
G: Annual growth rate of per capita GDP 1950-1988 (Summers and Heston, 1991). **RGDP60:** Real per capita GDP in 1960 (Summers and Heston, 1991). **I:** Average investment ratio to GDP 1950-1988 (Summers and Heston, 1991). **GN:** Annual population growth rate (Summers and Heston, 1991). **OL70:** Overall linkage measure in 1970, and **DL70:** Domestic linkage measure in 1970 (the linkage measures are taken or estimated from Chenery et al., 1986). **SEC:** secondary-school enrolment ratio in 1960 (World Tables, 1980 and 1983). **OPEN:** Average ratio of exports plus imports to Real GDP from 1955 to 1988 (World Bank, World Tables; information for Taiwan between 1980 and 1988 was taken from *National Income in Taiwan Area of the Republic of China, 1992*, Directorate-General of Budget, Accounting and Statistics, Executive Yuan).

Notes. a: 1953-1988; **b:** 1969-1987.

We obtain the following results:

$$(1)\quad \hat{G} = \underset{(-1.34)}{-1.76}\ CONST. - \underset{(-2.75)}{0.47E\text{-}3^{*}}RGDP60 + \underset{(5.32)}{0.083^{*}}OL70, \qquad R^2=0.842,\ S.E.=0.702$$

$$(2)\quad \hat{G} = \underset{(0.05)}{0.12}\ CONST. - \underset{(-0.62)}{0.20E\text{-}3}\ RGDP60 + \underset{(1.94)}{0.075}\ DL70, \qquad R^2=0.446,\ S.E.=1.316$$

$$(3)\quad \hat{G} = \underset{(-1.81)}{-3.21}\ CONST. - \underset{(-2.95)}{0.69E\text{-}3^{*}}RGDP60 + \underset{(4.41)}{0.108^{*}}DL70 + \underset{(3.53)}{0.063^{*}}OPEN, \qquad R^2=0.841,\ S.E.=0.771$$

*: Coefficient significant at the 5% level (two-tailed test).

Regression 1 shows that the measure of overall linkages in 1970 (OL70) appears with a positive and significant coefficient. By comparing regressions (2) and (3) we can see that the measure of domestic linkages in 1970 (DL70) appears to be significant if accompanied by the index of openness (OPEN). No other regressor reported in Table 4.6 seems to be significant when the set of regressors includes the level of per capita GDP in 1960 (RGDP60) and the measure of overall linkages in 1970 (OL70).

4.3 CONCLUDING COMMENTS

In this chapter we explored the relationship between interindustry linkages, or production roundaboutness, and economic growth. We tried different ways of tackling this: first with indirect measures of roundaboutness and afterwards with direct measures.

We found that the cross-country relationship between production roundaboutness and economic growth appears to be robust. In all our regressions the indices of technological integration or their proxies have positive coefficients that seem to be strongly significant.

Our regressions provide some support to economic theories that emphasize the need for industrialization as a necessary condition for economic take-off and sustained economic growth. Recall that we found that the relevant proxy for economic integration as an "explanatory" variable of growth performance is the proxy for technological integration in the manufacturing sector (see section 4.2.1).

Our paper also sheds some light on the newly established wisdom that trade liberalization is a condition for improving economic performance. Our results suggest that trade liberalization might be an important condition for successful economic growth in so far as it leads to a more diversified and technologically integrated economic structure (see section 4.2.2). In that sense a policy of import substitution may be as effective if it achieves the same goal. The important issue seems to be whether the trade regime enhances the possibility of dynamic increasing returns by augmenting the degree of interindustry integration.

Due to the likely existence of positive externalities from technological integration, it is highly probable that government intervention is needed. Subsidies to activities leading to technological integration, like R&D and technological education, might achieve better results than direct government investment. However, in preindustrial stages of development direct public intervention may be unavoidable. Hopefully we will see further research on this topic in the near future.

Our research is clearly limited by the availability and quality of data. For our larger data set (see Table 4.7 in the Appendix) we are forced to use proxies for the degree of technological integration (see section 4.2.1). We could improve the quality of the index of technological integration but only for a small sample (see Table 4.2). Hence, our results must be interpreted cautiously. It would be highly desirable to check the robustness of our findings for a larger set of countries using comparable direct measures of economic technological integration.

Appendix: Table 4.7
Cross-Section Data: 1960-1988
Sample: 48 Countries

Country	G %	RGDP 60 85US$	I %	IO MAN %	IO TOT %	SEC 60 %	OPEN %	GN %
Benin	-0.4	1075	5.5	68.4	30.6	2	17	-.4
Botswana[a]	6.2	474	23.9	74.5	42.2	1	44.1	3.2
Burkina Faso[b]	1.8	346	16.7	64.1	31.8	1	18.7	2.3
Burundi	0.6	473	7.9	60.9	30.4	1	14.7	2
Cameroon	2.8	736	10.3	67.8	39	2	22	2.7
Cape Verde	1.9	893	28.6	58.7	32.8	n.a.	30	2.0
The Gambia[c]	2.3	411	3.2	72.4	33.2	3	73.7	2.8
Ghana	-0.6	1049	7.6	53.1	28.1	5	26	2.6
Mauritius	2.8	2113	12.3	67.8	46.1	24	35.8	1.7
Nigeria	-0.4	1133	11.7	57.7	29.8	4	20.5	2.7
Rwanda	0.7	538	4.5	64.8	33.9	2	10.6	3.2
Sierra Leone[d]	0.2	871	2.2	74	28.4	2	14.2	1.9
Sudan	-0.3	975	1.8	60.6	33.1	3	13.3	2.7
Swaziland[c]	2.4	1182	22.9	73.1	49.5	5	49.1	3.2
Zimbabwe	1.1	937	17.6	63.6	49.9	6	32.5	3.4
Canada	2.7	7758	22.9	75.8	54.1	46	46	1.4
Costa Rica	2	2160	14.2	68.3	n.a.	21	32.0	2.7
El Salvador	1	1305	7.7	60.6	33.3	13	26.5	2.4

Country	G %	RGDP 60 85US$	I %	IO MAN %	IO %	SEC 60 %	OPEN %	GN %
Jamaica[d]	1	1829	21.6	73.2	56.5	45	54.3	1.5
Mexico	2	2870	19.6	58.6	37.1	11	9.2	2.8
Argentina	0.6	3381	11.8	58.4	n.a.	32	12.7	1.5
Bolivia	0.6	1142	16.8	64.7	36.8	12	21.8	2.5
Chile	1	3103	13.3	63.5	46.1	24	20.5	1.8
Colombia	2.3	1874	17.2	64.3	42	12	11.7	2.3
Ecuador	2.3	1461	24.8	64.8	46.1	12	19.7	2.9
Peru	1	2130	15.9	68	48.5	15	16.7	2.7
Uruguay	0.6	4401	15.7	62.6	45.9	37	15.5	0.6
Venezuela[d]	1.4	3899	16.5	61.8	40	21	37.4	3.5
Bangladesh[c]	0.5	621	5.9	69.1	31.4	8	7	2.6
Japan	5.5	2701	31	70.4	53.4	74	23.2	0.9
Jordan	2.1	1328	16.6	67.7	47.8	25	50	3.1
Korea, South Rep.	6.3	923	24.7	76.4	56.7	27	28	2.0
Sri Lanka[d]	1.3	1389	21.2	43.5	31.6	27	18.1	1.9
Syria	3	1787	17.0	80.5	38.3	16	16.5	3.4
Austria	3.3	4476	27.5	64.4	49.1	50	60.9	0.3
Cyprus	4.9	2039	31.6	67	45	47	60	0.6
Denmark	2.6	5900	27.8	67.5	45.1	65	68.3	0.4

Country	G %	RGDP 60 85US$	I %	IO MAN %	IO TOT %	SEC 60 %	OPEN %	GN %
Finland	3.5	4718	34.2	69.3	50.7	74	54.6	0.4
France	3	5344	25.9	64.4	45.9	46	36.4	0.7
Germany, Fed. Rep.	2.7	6038	26.9	70.2	65.1	53	52.5	0.3
Iceland	3.3	5352	25.8	69.1	47.6	61	92.0	1.2
Nether-lands	2.6	5587	24	72.6	48.2	58	91.8	0.9
Norway	3.7	5443	32.8	71.3	46.3	57	95.1	0.6
Portugal	4.3	1618	23.7	68.4	54.9	20	36.4	0.5
Spain	3.7	2701	26.2	63.1	48.3	23	22.5	0.9
Sweden	2.5	6483	22.7	65.9	48.9	55	65.7	0.4
New Zealand	1.1	7222	22	67.7	55.8	73	41.9	1.2
Fiji	1.3	2354	22.4	74.7	45.7	15	43.2	1.3

Notation and sources. **G**: Average annual growth rate of real gross domestic product per capita (Summers and Heston, 1991). **RGDP60**: Real gross domestic product in 1960 (Summers and Heston,1991). **I**: average investment to GDP ratio (Summers and Heston,1991). **IOMAN**: Aggregate input-output coefficient of the manufacturing sector in 1980; **IO**: Aggregate input-output coefficient in 1980 (National Accounts Statistics, United Nations). **SEC60**: Secondary-school enrolment ratio in 1960 (World Tables 1980 and 1983, World Bank). **OPEN**: Average ratio of the sum of exports and imports to GDP (Summers and Heston, 1991). **GN**: Average annual growth rate of population (Summers and Heston, 1991).
Notes: (a) 1960-86, (b) 1965-88, (c) 1960-85, (d) 1960-87, n.a. : non available information.

CHAPTER 5

DECREASING RETURNS, COMPETITION AND LONG-RUN GROWTH

5.1 INTRODUCTION

The role of firm proliferation in the process of economic development has not been analyzed, to my best knowledge, in a model of economic growth. However, Table 5.1 shows empirical evidence of the potential importance of firm proliferation in the growth process of two recently industrialized countries. Hong-Kong and South Korea experienced a huge explosion in the number of manufacturing establishments during the period of industrial take-off. How big those "explosions" are can be gauged by noting that the number of manufacturing establishments in the United States grew from 1958 to 1987 at the average annual rate of 0.5% (same sources of Table 5.1).

Table 5.1
Average Annual Growth Rates of Number of Manufacturing Establishments and Manufacturing Output in Hong-Kong and South Korea
(selected periods)

Period\Country	Hong-Kong Growth %		South Korea Growth %	
	# Establ.	Output	# Establ.	Output
1958-1963	-	-	7.2	12.3
1963-1970	10.9	14.0	3.9	21.5
1970-1980	10.6	7.3	2.6	15.7
1980-1988	1.7	5.7	8.7	13.2

Calculations based on the following sources: Establishments: *Industrial Statistical Yearbook*, United Nations, several issues; and *Growth of World Industry*, United Nations, several issues. Real Manufacturing GDP: *World Tables*, The World Bank, several issues. Hong-Kong real manufacturing GDP was estimated by using the implicit GDP deflator.

In this chapter we build a competitive general equilibrium model where proliferation of firms is a necessary condition for sustained economic growth. The economic mechanism leading to this condition hinges on the relationship between decreasing returns to scale at the firm level and industry structure under competitive conditions. The idea is simple: if new firms are continuously allowed to enter the market so that the average amount of factors per firm is constrained within some limits, then the marginal productivity of these factors will have a lower limit and thus aggregate economic growth might be unbounded. Such a situation requires a competitive economic structure so that no firm or group of firms is sufficiently powerful to preclude the entry of new competitors through pre-emptive price cutting or any other device leading to oligopolistic control of the market. We will develop this idea below.

Most models of economic growth are built under the assumption of constant returns to scale. The case for constant returns to scale is well-known and can be summarized in the "replication" argument: if no factors are fixed, constant returns to scale follow naturally because, indivisibility constraints aside, it is always possible to replicate the production unit (plant, firm, etc.). Usually these growth models go a step further and assume (explicitly or implicitly) that constant returns to scale hold at the firm level. Hence, the number of firms is indeterminate; with constant returns to scale at the firm level one firm does as well as many. Thus it is also implicitly assumed that firm proliferation is irrelevant for economic growth.

However, constant returns to scale at the firm level are not a necessary condition for having constant returns at the aggregate level. Even if some factors are fixed at the firm level, and the firm's technology is characterized by decreasing returns to scale, constant returns at the aggregate level may hold if the firm's fixed factors are produced in the process of creation of firms. In an economy with these technological characteristics, proliferation of firms is a necessary condition for sustained growth; without proliferation of firms, i.e. with entry restrictions, there is no expansion of the firm's fixed factors, hence the tendency to decreasing returns dominates and the economy converges to a steady state with no growth. We will analyze these two polar cases in a competitive model of growth characterized by decreasing returns at the firm level.

Our model depends crucially on the mechanism regulating firms' entry into the economy. We will propose three mechanisms which yield three different versions of the model. The first one is simple: there is no entry, i.e. the number of incumbent firms is fixed. The second mechanism is given by investment costs; the idea here is that some fixed investment must be made in the process of creating the firm; for new firms these investment costs represent entry costs. The third mechanism is given by operating costs: we assume no entry costs, but postulate that running the firm imposes some fixed costs.

Our assumptions are not unrelated to Kaldor's hypothesis that entrepreneurial ability is the ultimate factor leading to decreasing returns at the firm level (Kaldor, 1934). However, we do not attempt to explain the firm's technology; we just assume, as we explained before, that any factor that remains fixed at the firm level can be reproduced in the process of creation of firms.

This chapter is organized as follows. In section 5.2 we will examine the model with a fixed number of firms. In section 5.3 we will consider the two versions of the model with entry of firms. Some final comments conclude the chapter in section 5.4.

5.2 THE MODEL WITH NO ENTRY

The economy produces a single good which can be consumed or accumulated. At any moment in time a given amount of capital, K, is inherited; assume that K is owned by the consumers who rent it to the firms. The firm's production technology is characterized by decreasing returns with respect to capital, which is the single variable factor of production at the firm level. As in Rebelo (1991), we may consider K as an index of all forms of physical and human capital. A general representation of this production function is the following

$$(5.1) \qquad Y_{i,t} = f(K_{i,t}), \quad f' > 0, \; f'' < 0 ,$$

where $Y_{i,t}$ is the output of the i-th firm at time t, and $K_{i,t}$ is the capital rented by the i-th firm at the same time. The number of firms, denoted M, is fixed.

Because competition ensures that capital is rewarded at the same rate across firms, and the production function is concave, profit maximization ensures that all firms yield the same marginal product of capital and thus should be equal in size. Hence, at any moment in time, each firm rents a fraction of capital equal to K/M. Adding up across firms we obtain aggregate output as follows:

$$(5.2) \qquad Y_t = \sum_{i=1}^{M_t} f(K_{i,t}) = \sum_{i=1}^{M_t} f(K_t/M_t) = M_t f(K_t/M_t) \ .$$

Notice that aggregate output is a linearly homogeneous function in physical capital and the number of firms.[1] [2]

Firm's profits are defined as follows:

$$(5.3) \qquad \pi_{i,t} = f(K_{i,t}) - r_t K_{i,t} \ ,$$

where $\pi_{i,t}$ denotes the profits of the i-th firm at time t, $K_{i,t}$ is the firm's demand for capital at time t, and r_t is the rental price of capital at time t. Profit maximization under price-taking behaviour implies that firms rent capital until its marginal product equates its price: $r_t = f'(\hat{K}_{i,t})$. Capital market equilibrium requires that the aggregate demand for capital, $M_t K_{i,t}$, equals the aggregate supply of capital, K_t. Hence, the (common) equilibrium rental rate of capital is given by

$$(5.4) \qquad r_t = f'(K_t/M_t) = f'(k_t/m_t) \ ,$$

and the rate of profit (dividend per firm) is

---

[1] In order to ensure continuity of the aggregate output function we could resort to consider K and M as continuous variables. This implies indexing the firms from 0 up to M_t. For such a variation we should replace the summation term by an integral, but the form of the function will not change.

[2] If the firm's technology includes efficient labour, $Y_i = F(K_i, \epsilon L_i)$, where ϵ is the efficiency level and L_i is the firm's employment, we obtain the same aggregate output function if the efficiency level is proportional to the degree of mechanization: $\epsilon = K/L$ (non-indexed letters represent aggregate variables), and the firm's production function, $F(\cdot)$, is subject to decreasing returns to scale; thus output per firm is $Y/M = F[K/M, (K/L)(L/M)] = f(K/M)$, which gives again equation (5.2).

(5.5) $$\pi_t = f(k_t/m_t) - (k_t/m_t)f'(k_t/m_t) \ .$$

In the last two equations we denote per capita variables with lower-case letters. We will keep that convention throughout from now on.

Let us turn now to the demand side of the model. The representative consumer maximizes the following standard additive separable utility function over an infinite horizon:

(5.6) $$U_0 = \int_0^\infty e^{-\rho t} \frac{c_t^{1-\sigma} - 1}{1 - \sigma} dt \ ,$$

where c_t is per capita consumption at time t, ρ is the instantaneous discount rate and σ is the constant coefficient of risk aversion (the inverse of the intertemporal elasticity of substitution).

This objective is constrained by the consumer's income. People in this economy obtain income from the rental payments on physical capital and/or from the profits (dividends) they obtain as shareholders in the firms. Since setting up new firms is forbidden, income is only allocated to consumption and capital accumulation. We will assume that foregone consumption is transformed one-for-one into capital. Assume as well that capital depreciates at the constant rate δ. The aggregate budget constraint embodying the preceding assumptions is the following:

$$C_t + \dot{K}_t + \delta K_t = r_t K_t + \pi_t M_t \ ,$$

where a dot over a variable denotes the corresponding time derivative. In per capita terms the budget constraint can be rewritten as follows:

(5.7) $$c_t + \dot{k}_t = (r_t - n - \delta) k_t + \pi_t m_t ,$$

where n is the constant rate of population growth.

Thus the consumer's general problem is to maximize equation (5.6) subject to equation (5.7). The consumer solves this problem taking the rate of interest, r_t, and the dividend, π_t, as given. The related Hamiltonian equation is

$$H = e^{-\rho t} \frac{c_t^{1-\sigma} - 1}{1 - \sigma} + \lambda_t [(r_t - n - \delta) k_t + \pi_t m_t - c_t] ,$$

where λ is the multiplier of per capita net savings. The first order conditions for this problem are as follows:

(5.8) $$H_c = 0 \quad : \quad \lambda_t = e^{-rt} c_t^{-\sigma} ,$$

(5.9) $$-\dot{\lambda}_t = H_k \quad : \quad -\dot{\lambda}_t / \lambda_t = (r_t - n - \delta) .$$

Equation (5.8) gives the level of the shadow price of savings: it says that the present discounted value of marginal utility equals the current value of the unit of savings. Equation (5.9) gives the growth rate of the shadow price of savings: it implies that the net return to a unit of capital plus capital gains equals the return to a consumption loan.

Besides the previous first order conditions, the optimal choice of consumption should satisfy the following transversality condition:

(5.10) $$\lim_{t \to \infty} \lambda_t k_t = 0 \ .$$

Now we proceed to solve the model. Differentiating equation (5.8) with respect to time and using equations (5.4) and (5.9) we deduce the growth rate of per capita consumption:

(5.11) $$\frac{\dot{c}_t}{c_t} = \frac{r_t - n - \delta - \rho}{\sigma} = \frac{f'(k_t/m_t) - n - \delta - \rho}{\sigma} \ .$$

Dividing the budget constraint by k_t and using equations (5.4) and (5.5) we deduce

(5.12) $$\dot{k}_t/k_t = f(k_t/m_t)/(k_t/m_t) - (n + \delta) - c_t/k_t \ .$$

The system of differential equations (5.11) and (5.12) governs the dynamics of this economy. It will be convenient to transform this system in terms of consumption per firm (c_t/m_t) and capital per firm (k_t/m_t):

(5.11′) $$\dot{\left(\frac{c_t}{m_t}\right)} \div \left(\frac{c_t}{m_t}\right) = \frac{f'(k_t/m_t) - n - \delta - \rho}{\sigma} - n \ ,$$

(5.12′) $$\dot{\left(\frac{k_t}{m_t}\right)} \div \left(\frac{k_t}{m_t}\right) = \frac{f(k_t/m_t)}{(k_t/m_t)} - \frac{(c_t/m_t)}{(k_t/m_t)} - \delta \ .$$

In doing this transformation we use the fact that the number of firms per capita, m_t, grows at the rate $-n$, given that we assume no proliferation of firms. The phase-plane diagram corresponding to the system of equations (5.11′) and (5.12′) is shown in Figure 5.1; notice that it exhibits the same mathematical behaviour as the standard Ramsey-Cass-Koopmans

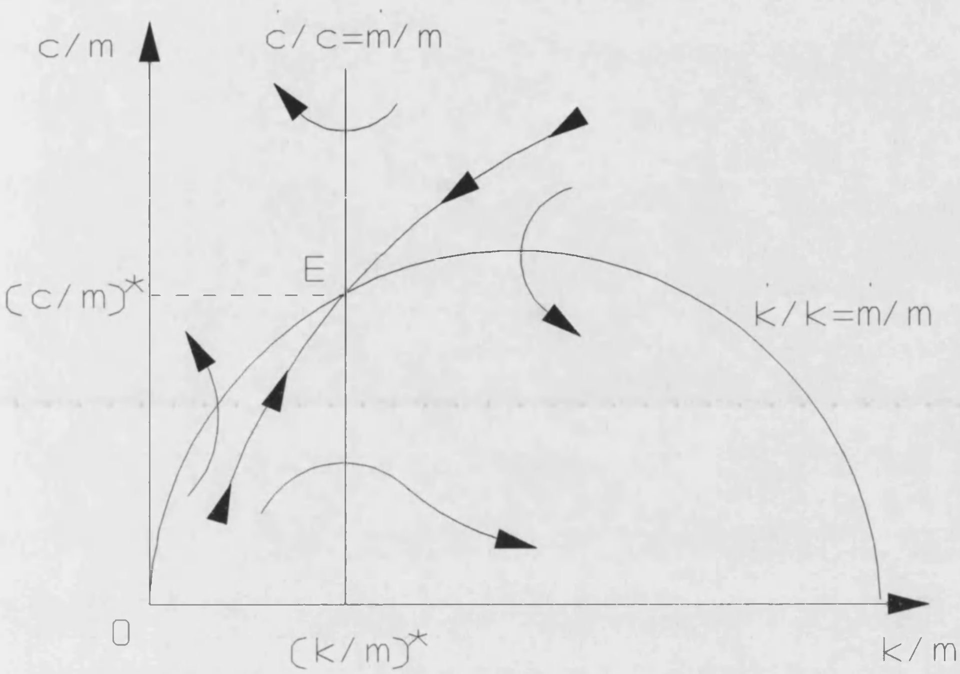

Figure 5.1
Dynamics of Per Firm Consumption and Per Firm Capital

model of growth. Given capital per firm, forward-looking agents choose the saddle-path level of consumption per firm and so the economy converges to the steady state equilibrium at point E. At this point economic growth ceases and so consumption per capita falls at the growth rate of the number of firms per capita [(dc/dt)/c = (dm/dt)/m = -n].

Convergence to the steady state is driven by decreasing returns to capital; this is shown in Appendix 1. However, equations (5.11) and (5.12) show that if capital per firm were constant, so that the marginal product of capital were also constant, the economy might experience unbounded endogenous growth. In the next section we will explore possible microfoundations for that outcome.

5.3 THE MODEL WITH PROLIFERATION OF FIRMS

5.3.1 The Competitive Equilibrium with Entry Costs

Let us analyze briefly the aggregate output function [equation (5.2)]. If capital were perfectly divisible, aggregate output could always be augmented through the mechanical process of fragmenting the same capital stock among an infinite number of firms.[3] In this section we will rule this possibility out by assuming that creating firms is costly.

The basic structure of the model is as before, but now we add two non-exclusive alternatives for savings: the agents invest in producing capital, or invest in setting up new firms. In order to close the model we need to assume that both alternatives are costly. Hence, assume as before that output can be transformed one-for-one into capital; assume also that β units of output are consumed in setting up a new firm. The investment in firms should be understood in a general sense as

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[3] Partially differentiating aggregate output [equation (2)], with respect to the number of firms, M, yields

$$\partial Y_t/\partial M_t = f(K_t/M_t) - (K_t/M_t)\, f'(K_t/M_t) > 0 ,$$

as decreasing returns implies that marginal product is below average product. If $f(\cdot)$ satisfies the following Inada condition

$$\lim_{K_i \to 0} f'(K_i) = \infty ,$$

aggregate output would go to infinity as M goes to infinity.

the necessary investment in those physical and human factors which are fixed at the firm level; this investment is assumed to be intrinsically linked to the actual process of firm creation. Given these assumptions it will be convenient to distinguish between reproducible factors which are variable at the firm level, which we will keep denoting K, and reproducible factors which are fixed at the firm level. For simplicity we will also assume that both forms of capital depreciate at the common rate δ. Taking into account the preceding assumptions the aggregate budget constraint is modified as follows:

$$C_t + (\dot{K}_t + \delta K_t) + \beta(\dot{M}_t + \delta M_t) = r_t K_t + \pi_t M_t .$$

We can rewrite this budget constraint in per capita terms:

(5.13) $c_t + \dot{k}_t + \beta \dot{m}_t = (r_t - n - \delta) k_t + [\pi_t - \beta(n + \delta)] m_t .$

The consumer's general problem is again to maximize equation (5.6), but now this objective is subject to equation (5.13). In this case the consumer takes the rate of interest, r_t, and the dividend, π_t, as given. The related Hamiltonian equation is

$$H = e^{-\rho t} \frac{c_t^{1-\sigma} - 1}{1 - \sigma} + \lambda_t [(r_t - n - \delta) k_t + (\pi_t - \beta n - \beta \delta) m_t - c_t)]$$

The first order conditions for this problem are as follows:

$$\text{(5.14)} \qquad H_c = 0 \quad : \quad \lambda_t = e^{-rt} c_t^{-\sigma} ,$$

$$\text{(5.15)} \qquad -\dot{\lambda}_t = H_k \quad : \quad -\dot{\lambda}_t/\lambda_t = (r_t - n - \delta) ,$$

$$\text{(5.16)} \qquad -\beta\dot{\lambda}_t = H_m \quad : \quad -\beta\dot{\lambda}_t/\lambda_t = (\pi_t - \beta n - \beta\delta) .$$

Equation (5.14) gives the level of the shadow price of savings. Equations (5.15) and (5.16) give the growth rate of the shadow price of savings.

Given that we have here two different forms of capital, we also need two transversality conditions in order to determine the optimal path of this economy:

$$\text{(5.17)} \qquad \lim_{t\to\infty} \lambda_t k_t = 0 ,$$

$$\text{(5.18)} \qquad \lim_{t\to\infty} \lambda_t m_t = 0 .$$

Now we have all the information needed to solve the model. From equations (5.15) and (5.16) we obtain

$$\text{(5.19)} \qquad r_t = \pi_t/\beta .$$

Combining the last equation with the expressions we found above for r and π [equations (5.4) and (5.5), respectively], we solve implicitly for the capital-to-firm ratio:

$$\text{(5.20)} \qquad f(k/m) = (\beta + k/m)\, f'(k/m) .$$

If the firm's production function satisfies the Inada conditions, there exists a positive solution for k/m.[4] This

[4] In footnote 3 we mentioned the first Inada condition, the second one is as follows:

$$\lim_{k_i\to\infty} f'(K_i) = 0 .$$

solution is increasing in the entry cost, ß,[5] hence the solution is unique.

Capital per firm is constant for three reasons. First, the net return to capital is decreasing in K/M, whilst the net return to setting up a new firm is increasing in K/M. Second, along the optimal path these returns should be equalized. Third, given no adjustment costs and reversibility of investment the optimal allocation can be achieved instantaneously. The latter assumption implies that any excess of capital can be eliminated by transforming it into firms and vice versa. This is not very realistic, of course, but very useful to avoid complex transitional dynamics. It has been proved that this kind of two-capital growth models converge in the long run to the steady state (Mulligan and Sala-i-Martin, 1992); since we are primarily interested in the long run characteristics of the model, the reversibility assumption is therefore not problematic.

We can find now the equilibrium path. In order to obtain closed form solutions let us assume that the firm's production function is homogeneous of degree α:

(5.1') $$f(K_i) = A K_i^{\alpha} ,$$

where $0 < \alpha < 1$, and A is a positive constant. Hence, the aggregate output function adopts the following "Cobb-Douglas" type form:

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[5] Implicitly differentiating equation (20) with respect to β one obtains

$$\frac{\partial (k/m)}{\partial \beta} = \frac{-f'(k/m)}{(\beta + k/m) f''(k/m)} > 0 .$$

(5.2′) $$Y_t = A K_t^{\alpha} M_t^{1-\alpha} .$$

Thus the equilibrium capital-to-firm ratio, the rental price of capital and the rate of profit are determined as follows:

(5.20′) $$K/M = \alpha\beta/(1-\alpha) ,$$

(5.4′) $$r = A^* \equiv \alpha^{\alpha}(1-\alpha)^{1-\alpha} A/\beta^{1-\alpha} ,$$

(5.5′) $$\pi = \beta A^* .$$

These equations show that the higher the entry cost, β, the higher the capital per firm, the lower the rental price of capital and the higher the profit rate.

We also can solve for the output-capital ratios. By combining equations (5.2′), (5.4′) and (5.20′) we obtain

(5.21) $$Y_t = \frac{A^*}{\alpha} K_t = \frac{\beta A^*}{1-\alpha} M_t .$$

Hence our model reproduces the stylized fact that the capital-output ratio is constant.

Now, substitution of equation (5.4′) into equation (5.15), solves for the growth rate of the shadow price of either form of capital

(5.22) $$-\dot{\lambda}_t/\lambda_t = A^* .$$

Differentiating the first order condition for consumption, equation (5.14), with respect to time, and using the last equation, solves for the growth rate of per capita consumption:

(5.23) $$\dot{c}_t/c_t = \gamma \equiv \sigma^{-1}(A^* - n - \delta - \rho) .$$

Hence economic growth is sustained if the following inequality is satisfied: $A^*-n-\delta > \rho$. Given that per capita consumption grows at the constant rate γ, utility is bounded if the following inequality holds: $\gamma(1-\sigma) < \rho$, or $(A^*-n-\delta)(1-\sigma) < \rho$. Therefore the marginal productivity of capital should be high enough in order to generate unbounded growth, but not too high as to generate unbounded utility.[6] The latter inequality also implies $A^*-n-\delta > \gamma$; we will use this inequality below.

The optimal choice of consumption is given by the following equation (see Appendix 2):

$$\textbf{(5.24)} \quad C_t = \frac{(A^* - n - \delta)(\sigma - 1) + \rho}{\alpha\,\sigma} K_t = \frac{A^* - n - \delta - \gamma}{\alpha} K_t \ .$$

Therefore the average propensity to consume is equal to

$$\textbf{(5.25)} \quad \frac{C_t}{Y_t} = \frac{(A^* - n - \delta)(\sigma - 1) + \rho}{\sigma A^*} = \frac{A^* - n - \delta - \gamma}{A^*} \ .$$

We also calculate the gross savings ratio for financing firm proliferation

$$\textbf{(5.26)} \quad s_M \equiv \frac{\beta(\dot{M}_t + \delta M_t)}{Y_t} = \frac{(1 - \alpha)}{\sigma} \frac{A^* - \rho + (n + \delta)(\sigma - 1)}{A^*} ,$$

and the gross savings ratio for financing capital accumulation

$$\textbf{(5.27)} \quad s_K \equiv \frac{\dot{K}_t + \delta K_t}{Y_t} = \frac{\alpha}{\sigma} \frac{A^* - \rho + (n + \delta)(\sigma - 1)}{A^*} \ .$$

These equations completely characterize the dynamic path of the competitive economy. We have shown here that the industry structure is determined together with the economy output and the rate of growth. The above equations show that,

[6] The latter case is only relevant if the intertemporal elasticity of substitution is low enough ($1 > 1/\sigma \geq 0$).

as expected, the higher entry costs decrease welfare: they increase firm size and decrease the growth rate.

5.3.2 The Competitive Equilibrium with Operating Costs

Suppose that entry is free and costless. If everything else remains unchanged, the preceding version of the model would be quite unstable; for if new firms keep entering so long as profits are positive, the firm's size would be driven to zero whilst, given the stock of capital, total output would increase with each new firm.[7] Even though the return to capital would go to infinity, no one would produce capital because the return to setting up firms would also be infinite. This scenario is clearly not interesting.

However, we still can assume that entry is costless and have positive firm size in equilibrium if there exists some other costs related with the firm's fixed factors. Hence let us assume that running the firm imposes some fixed costs. We will model these costs as a constant deduction of firms' output per period. So assume that the firm's production technology is the following:

$$Y_{i,t} = A K_{i,t}^{\alpha} - \mu \ , \qquad (5.28)$$

where μ is the fixed cost of running the firm; the other notation is as before.

The profits of the firm at time t are given by

[7] If the Inada condition stated in footnote 3 holds, as it holds for the aggregate output function (2'), aggregate output goes to infinity as the number of firms goes to infinity.

(5.29) $$\pi_{i,t} = A K_{i,t}^{\alpha} - \mu - r_t K_{i,t} \ .$$

Profit maximization implies that the rental price of capital is equalized to the marginal product of capital. As before, perfect competition and capital market equilibrium imply that all firms must be identical, so that all of them demand the same amount of capital, $K_{i,t}=K_t/M_t$. Hence we solve for the rental rate of capital,

(5.30) $$r_t = \alpha A (K_t/M_t)^{\alpha-1} ,$$

and also for the profit rate,

(5.31) $$\pi_t = (1-\alpha) A (K_t/M_t)^{\alpha} - \mu \ .$$

But now the free entry condition imposes zero profits. Thus the required level of capital per firm is given by

(5.32) $$\frac{K}{M} = \left[\frac{\mu}{(1-\alpha) A}\right]^{1/\alpha} .$$

Notice that the firm size is determined if and only if decreasing returns to capital are assumed ($0 < \alpha < 1$). This element is again a determinant of the equilibrium of the competitive firm. We also need the firm's fixed costs to be positive ($\mu > 0$).

Given the firm size, and following the same steps for aggregating across firms [see equation (5.2)], we deduce the aggregate output function:

(5.33) $$Y_t = \hat{A} K_t , \quad \hat{A} = [\alpha^{\alpha} (1-\alpha)^{1-\alpha} A / \mu^{1-\alpha}]^{1/\alpha} > 0 \ .$$

This function is equivalent to the AK production technology of Rebelo's growth model (Rebelo, 1991), as the marginal product of capital is constant.

This aggregate technology, together with preferences as given by equation (5.6), can yield unbounded economic growth in a competitive environment. The per capita growth rate is given by the following expression:

$$\dot{c}/c = \dot{k}/k = \sigma^{-1}(\hat{A} - n - \delta - \rho) \quad (5.34)$$

Because the firm's size increases with the firm's fixed costs, the productivity of capital diminishes with those costs, as shown by equation (5.33). Hence, the rate of growth is also decreasing in the firm's fixed costs.

This model, as the previous one, shows the importance of proliferation of firms and competition in economic growth and development.

5.4 SOME FINAL COMMENTS ON PROLIFERATION OF FIRMS AND INDUSTRIALIZATION

Some minimum rate of proliferation of firms is necessary in our model to generate constant returns to scale at the aggregate level and induce sustained economic growth. If the dynamics of firm creation is below that minimum rate, the dynamic path converges to a steady state with no growth -as in the standard Ramsey-Cass-Koopmans model of economic growth.

The factors leading to one case or the other are very important, for they might mean the difference between development and stagnation. Very likely candidates for restricting the process of proliferation of firms are imperfections of all sort in the goods market and capital markets. Extensions of this model incorporating these imperfections could provide important insights for understanding the process of development.

Now, if the economic conditions are as assumed in our model, i.e. perfect markets and forward-looking agents, our model explains how a general, "irrational", proclivity to creating firms may induce a self-sustained process of growth and development. "Generalized optimism" in the process of development has been analyzed by Murphy, Shleifer and Vishny (1989), in a multisector economic model where coordination of industrialization across sectors induces technological shifts towards increasing returns through self-enforcing demand spillovers. Their model focuses on the demand side of the industrialization process in the context of imperfect competition. In contrast, our model focuses on the supply side of economic growth in a competitive framework. Perhaps each model explains partial aspects of the process of industrial take-off.

From the viewpoint of economic policy, our model suggests that the government should encourage competition in markets where the firm's technology is characterized by decreasing returns to scale. This policy should be understood as a general commitment to eliminate entry barriers, but not to subsidize entry.

Table 5.1 shows that the beginning of industrialization processes may be related to proliferation of firms. However, it also shows that the process of proliferation of manufacturing establishments tends to diminish over time after the take-off, perhaps because after some threshold of development the limited supply of the entrepreneur ability becomes less important, perhaps because after some degree of industrialization the countries tend to experience a structural transformation towards service activities (Chenery et al, 1986) -which implies that firm proliferation may continue in sectors different to manufacturing activities.

Therefore, more information and analysis are needed to clarify the behaviour of firms and their role in the process of development. However, the evidence in Table 5.1 suggests that proliferation of firms may play an important role in defeating the intrinsic tendency to decreasing returns. This process may be particularly important in inducing a process of industrial take-off in countries where an important fraction of economic activity is carried out by small firms.

APPENDIX 1: The Saddle Point Property of the Model with No Entry

The steady-states values of per firm consumption, $(c/m)^*$, and per firm capital, $(k/m)^*$, are deduced by equating equations (5.11′) and (5.12′) to zero. Differentiating the same equations around the steady state one obtains the following system of equations:

$$\begin{pmatrix} (\dot{c/m}) \\ (\dot{k/m}) \end{pmatrix} = \begin{bmatrix} 0 & (c/m)^* f''[(k/m)^*]/\sigma \\ -1 & f'[(k/m)^*] \end{bmatrix} \begin{pmatrix} (c/m) - (c/m)^* \\ (k/m) - (k/m)^* \end{pmatrix}$$

Since the determinant of the transition matrix is negative $[(c/m)^* f''[(k/m)^*]/\sigma < 0]$, which is guaranteed in the case of decreasing returns to capital, the saddle path exists. Q.E.D.

APPENDIX 2: The Optimal Paths of Consumption and Capital Accumulation (Model with Entry Costs).

Using the aggregate output function [equation (5.2′)], the equilibrium values of the rental price of capital and the profit rate [equations (5.4′) and (5.5′) respectively], the consumer's budget constraint [equation (5.13)], and the ratio between human and physical capital [equation (5.20′)], we obtain the differential equation driving the accumulation of per capita physical capital

$$\dot{k}_s - (A^* - n - \delta)\, k_s = -\alpha\, c_s \; .$$

Integrating between period 0 and period t we obtain

$$k_t = k_0\, e^{(A^* - n - \delta)\, t} - \alpha\, c_0 \frac{e^{\gamma t} - e^{(A^* - n - \delta)\, t}}{\gamma - (A^* - n - \delta)} \; .$$

Because the shadow price of capital falls at the rate (A^*-n-δ) [see equation (5.22)], the price of physical capital at any moment in time can be expressed as follows

$$\lambda_t k_t = \lambda_o \left[k_0 - \alpha\, c_0 \frac{e^{[\gamma - (A^* - n - \delta)]t} - 1}{\gamma - (A^* - n - \delta)} \right] .$$

Optimization requires that this value goes to zero as time goes to infinity [equation (5.17)]; hence the optimal choice of per capita consumption at the beginning of times is

$$c_0 = \frac{(A^* - n - \delta)(\sigma - 1) + \rho}{\alpha\, \sigma}\, k_0 = \frac{(A^* - n - \delta) - \gamma}{\alpha}\, k_0 \; ,$$

where use has been made of the assumption of bounded utility (i.e. A^*-n-δ > γ). By substituting this value back into the formula for physical capital we find that per capita physical capital also grows at the rate of per capita consumption, γ. Hence the last equation is valid at any moment in time, which is expressed in equation (5.24).

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