Dynamic Analysis of Regime Shifts under Uncertainty: Applications to Hyperinflation and Privatization

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Abstract

Many policy decisions involve discontinuous and irreversible shifts, often in the face of substantial uncertainty. This dissertation studies two types of regime shifts under uncertainty: stabilization from hyperinflation, and privatization of a government corporation. In both cases, policy-makers face decisions that involve uncertainty and irreversibility. These topics are examined using a theoretical model (Chapter 1), econometric evidence (Chapter 2), and a case study (Chapter 3).

The first chapter presents a real options model of stabilization given uncertainty over the behavior of inflation. The model is used to argue that despite the costs of hyperinflation, stabilization delays do not necessarily indicate either irrationality or the presence of political economy effects.

The second chapter constructs a comprehensive data set of hyperinflations over the past 35 years. The data set is used to examine the empirical regularities of high inflation episodes, and to test the predictions of the model presented in Chapter 1. We conclude that (1) high inflation is rare; (2) most high inflation episodes are short; (3) most economies experience only one high inflation; (4) high inflation is noisy but not necessarily explosive; (5) many high inflations end without a large fall in inflation; and (6) there is weak evidence supporting the real options approach to stabilizations.

The third chapter studies another type of regime shift, that of moving a corporation from public to private ownership. The proposed privatization of the U.S. Enrichment Corporation (USEC) illustrates the costs and benefits of privatization. While a private firm would be more efficient, privatization may endanger a crucial nuclear non-proliferation program of the U.S. government. The chapter examines the uranium enrichment market; analyzes the proposed privatization of USEC within a framework developed by Jones, Tandon and Vogelsang (1990); and extends the analysis to incorporate limited information and irreversibility.

Table of Contents

List of Tables

Page

Chapter 1

Chapter 2

Chapter 3

List of Figures

Chapter 1

Chapter 2

Chapter 3

Chapter 1: Why Are Stabilizations Delayed? A Real Options Approach

I. Introduction

Hyperinflations are costly. During a hyperinflation, relative price changes are obfuscated, uncertainty is exacerbated, and the opportunity cost of holding domestic money rises sharply.[1] These distortions can be significant: Barro (1972) suggests that the welfare costs of 50 percent monthly inflation may be as high as 15 to 22 percent of national income.[2] In Keynes's often-quoted words, "There is no subtler, no surer means of overturning the existing basis of society than to debauch the currency."[3] And popular images of people pushing shopping carts full of cash to the market and of firms using furniture vans to deliver wages to workers reinforce the notion that hyperinflations impose substantial welfare costs.

Despite the costs of hyperinflation, however, stabilizations are delayed — often for extended periods of time. As Bruno (1993) observes, "One of the most marked, and at the same times puzzling, features of the episodes...relates to the time dimension of the process of recovery once a country goes into crisis. We have seen examples of high inflation spanning periods of up to ten to twelve years before a comprehensive stabilization program was first adopted..."[4] Sachs and Larrain (1993) add that in "several high inflation experiences, stabilizations were delayed for some time before a coherent program was launched to stop inflation. Yet there is widespread evidence that the longer the wait to attack the problem, the more the damage

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[1] For a taxonomy of the costs of inflation in various institutional settings, see S. Fischer and F. Modigliani, "The Real Effects and Costs of Inflation," *Weltwirtshaftliches Archiv.*, 1978, 828-829.

[2] R. Barro, "Inflationary Finance and the Welfare Cost of Inflation," *Journal of Political Economy*, Sept.-Oct. 1972, Table 2, page 986. There is some ambiguity over the definition of a hyperinflation. Cagan defines it as "beginning in the month the rise in prices exceeds 50 per cent and as ending in the month before the monthly rise in prices drops below that amount and stays below for at least a year." He admits this definition is "purely arbitrary." See P. Cagan, "The Monetary Dynamics of Hyperinflation," in M. Friedman, *Studies in the Quantity Theory of Money* (University of Chicago Press: Chicago, 1956), page 25. Some authors make a distinction between high inflation and hyperinflation; here the terms are used interchangeably.

[3] J.M. Keynes, *The Economic Consequences of the Peace* (Harcourt, Brace & Howe: New York, 1920), page 236.

[4] M. Bruno, *Crisis, Stabilization, and Economic Reform: Therapy by Consensus* (Oxford University Press: Oxford, 1993), page 262.

to the economy, and the more costly the policy reforms eventually needed to stabilize the economy."[5]

Therein lies the puzzle: why are stabilizations delayed? The most popular explanation is given by Alesina and Drazen (1991).[6] Their model posits a war of attrition between two groups in society, which fight to determine which one will bear a disproportionate share of the costs of stabilization. Information is asymmetric, in that the preferences of a group are known only to the members of that group. Over time, the groups learn about each other's preferences, and ultimately one capitulates. Stabilization thus occurs when one group decides that the marginal benefit from stabilizing (in terms of eliminating the costs of inflation) exceeds the marginal benefit from waiting (in terms of the expected hazard function for the capitulation of the other group multiplied by the net benefit from having the other group capitulate first).

Orphanides (1996) has proposed a different explanation for the observed delays in stabilizations.[7] In his model, inflation is high and is either constant or growing in a deterministic fashion. This inflation imposes some constant welfare cost on society. The government faces a two-stage decision process: it can initiate a stabilization in any given period, and decide either to abandon it or to complete it in the next period. The success of the stabilization effort is dependent only on the level of the government's foreign reserves relative to the demand for reserves. Because the demand for reserves fluctuates stochastically, there is some uncertainty over whether a given stabilization effort will be successful. This uncertainty implies that "the government *should* optimally wait until sufficiently favorable conditions materialize before attempting a stabilization" [emphasis in original].[8]

In this chapter, we adopt a different approach to delayed

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[5] J. Sachs and F. Larrain, *Macroeconomics in the Global Economy* (Harvester Wheatsheaf: New York, 1993), page 752.

[6] A. Alesina and A. Drazen, "Why Are Stabilizations Delayed?" *American Economic Review,* December 1991, 1170-1188.

[7] A. Orphanides, "The Timing of Stabilizations," *Journal of Economic Dynamics and Control,* Jan.-March 1996, 257-79; and A. Orphanides, "Optimal Reform Postponement," Federal Reserve Board of Governors, Finance and Economics Discussion Series Number 94-25, August 1994.

[8] A. Orphanides, "The Timing of Stabilizations," op. cit.

stabilizations: we construct a simple model of stabilizations as irreversible investments in the presence of uncertainty over the future inflation rate. Assuming that future inflation is uncertain, that there is some divergence between inflationary expectations and realized inflation during the stabilization period, and that any costs (e.g., unemployment) associated with such expectational errors cannot be recovered if the stabilization is abandoned, observed delays before stabilization can simply reflect the rational decision of a benevolent government solving a stochastic dynamic optimization problem. Delayed stabilizations are therefore not necessarily the result of either political economy effects or irrationality. In this sense, the model formalizes the notion that, as Allan Drazen himself has put it, "the problem may solve itself, so that 'benign neglect' is the optimal response."[9]

The purpose of this introductory chapter is to delineate an irreversible investment approach to stabilization. Section II employs a variety of simplifying assumptions to illustrate our fundamental result. Sections III and IV explore more general versions of the model. Two appendices offer other extensions. In the next chapter, we empirically test the irreversible investment model against the Alesina-Drazen model of delay.

II. A simplified model of stabilization as an irreversible investment under uncertainty

The real options literature suggests that sunk costs combined with uncertainty generate an incentive to wait before investing in a project.[10] This is because the option to invest in such circumstances is valuable. A positive net present value (excluding the option value) for the investment project therefore does not necessarily imply that it is optimal to invest immediately: if the net present value of investing is worth less than the

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[9] A. Drazen, "The Political Economy of Delayed Reform," presented at a conference sponsored by Georgetown University and the University of Maryland entitled "Economic Reform: Latin America and the Transition Economies, " May 12-13, 1994, page 4.

[10] More precisely, irreversibility and "expandability" — the ability to invest at a finite cost in the future — generate the incentive to wait. We define "waiting" as failing to invest when the conventional net present value of the project exceeds zero. For a trenchant and comprehensive presentation of recent work on investment under uncertainty, see A. Dixit and R. Pindyck, *Investment under Uncertainty* (Princeton University Press: Princeton, 1994). For an analysis of the importance of expandability, see A. Abel, A. Dixit, J. Eberly, and R. Pindyck, "Options, the Value of Capital, and Investment," NBER Working Paper 5227, August 1995.

value of the option to invest, the option should not be exercised. In such cases, it is rational to delay the investment despite the positive net present value of investing immediately.

Following Orphanides (1996), we view stabilizations as irreversible investments under uncertainty. But here the uncertainty is over inflation itself — perhaps the most prominent manifestation of uncertainty in a hyperinflation. Governments must "invest" in a stabilization to capture the benefits of lower inflation. The costs associated with stabilizing (mostly in terms of temporarily higher unemployment), combined with uncertainty over inflation, imply that it is optimal to wait beyond the point at which stabilization is first justified by conventional cost-benefit analysis. In effect, the government waits because it hopes that it will "get lucky" — that inflation will fall without incurring the costs of stabilizing. Alternatively, but equivalently, the government is hesitant to incur the sunk costs of stabilizing because *ex post* the stabilization effort may have been unnecessary.

This section assembles the model from its various constituent components. We first posit a reduced form for the inflationary process, then evaluate the benefits and costs of stabilization, and finally derive the optimal rate of inflation at which to stabilize. The purpose is to develop the model in its most basic form; simplifying assumptions are made wherever possible.

The stochastic process for inflation

We assume that inflation (π) follows a geometric Brownian motion with drift:

$$d\pi = \alpha\pi dt + \sigma\pi dz \qquad (1)$$

where dz is the increment in a Wiener process,

$$dz = \varepsilon\sqrt{dt} \qquad (2)$$

and ε is independently and identically distributed as a standard normal (N(0,1)) variable. The dz term can be thought of as representing an amalgamation of stochastic money demand shocks, terms of trade shocks, or other shocks to the economy.[11]

While it is difficult to know whether (1) is a reasonable approximation of the inflationary process, the empirical evidence on hyperinflations in Table 1 suggests at least some sort of non-stationarity in inflation.[12] (It should be noted, however, that the power of non-stationarity tests is very low given the extremely short samples tested.[13]) In any case, the fundamental results of the paper do *not* rely on non-stationarity, as our discussion of the mean-reverting process in Appendix II emphasizes. So (1) can be justified as a modeling device that simplifies the analysis and produces results similar to other, more complicated, processes.[14]

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[11] In an approach that has some technical similarities to the one adopted here, Marcus Miller and Lei Zhang assume that fiscal deficits, which must be financed through seignorage, follow a geometric Brownian motion. See M. Miller and L. Zhang, "Hyperinflation and Stabilisation: Cagan Revisited," Federal Reserve Board of Governors, International Finance Discussion Paper No. 529, November 1995.

[12] Our model assumes that the log of inflation follows an I(1) process, which is one form of non-stationarity. Interestingly, the non-stationarity in the theoretical model below obtains only in the sense that inflation follows (1) if the government does not intervene. Given the government's stabilizations, however, inflation is a stationary process (because it is effectively subject to an upper boundary). The important point is that *during the hyperinflationary period*, inflation seems to behave as if it were non-stationary.

[13] In addition, the mapping between (1) and the empirical results for the behavior of the inflation *rate* are complicated because (1) is equivalent to the limit (as time becomes continuous) of a random walk with drift for the *log* of the inflation rate, not for the inflation rate itself. Theoretically, an ARIMA (0,1,1) process for the inflation rate can be generated by using a Cagan model with adaptive expectations and imposing a particular monetary growth rule to ensure that adaptive expectations are rational. See T. Sargent and N. Wallace, "Rational Expectations and the Dynamics of Hyperinflation," *International Economic Review*, June 1973, 328-350, and B. Friedman, "Stability and Rationality in Models of Hyperinflation," *International Economic Review*, February 1978, 45-64. Note, however, that (1) is not ARIMA(0,1,1), since dz is assumed to be independently and identically distributed.

[14] The stochastic process defined by (1) has three major advantages: First, it is analytically tractable within the stochastic control problem we consider below. Second, it ensures that there are never any "hyperdeflations": if inflation in the initial period is positive, it will remain positive in perpetuity (the process is continuous and zero is an absorbing state). Finally, it implies that the variance of inflation increases with the mean rate of inflation, as is observed empirically. See, for example, D. Logue and T. Willet, "A note on the relation between the rate and variability of inflation," *Economica*, 43: 1976, 151-158, and A. Cukierman, *Central Bank Strategy, Credibility, and Independence: Theory and Evidence* (MIT Press: Cambridge, 1992), page 439.

Table 1: Unit roots in inflation

Country, sample period (source of data)	Stochastic process followed by inflation
Austria, Jan. 1921-Aug. 1922 (Cagan)	I(1)
Austria, Jan. 1921-Dec. 1922 (Barro)	I(1)
Hungary, July 1922-Feb. 1924 (Cagan)	I(1)
Hungary, Oct. 1921-Feb. 1924 (Barro)	I(1)
Germany, Sept. 1920-July 1923 (Cagan)	I(1)
Germany, Jan. 1921-Aug. 1923 (Barro)	I(2)
Germany, Jan. 1921-Aug. 1923 (Abel et. al.)	I(2)
Greece, Jan. 1943-Aug. 1944 (Cagan)	I(0) plus linear trend
Poland, Apr. 1922-Nov. 1923 (Cagan)	I(1)
Poland, Jan. 1922-Jan. 1924 (Barro)	I(1)
Russia, Dec. 1921-Jan. 1924 (Cagan)	I(1)
Argentina, Jan. 1971-Dec. 1989 (IFS)	I(1)
Bolivia, Jan. 1971-Dec. 1987 (IFS)	I(1)
Brazil, Jan. 1971-Feb. 1986 (IFS)	I(1)
Chile, Jan. 1971-June 1985 (IFS)	I(1)
Peru, Jan. 1971-Dec. 1989 (IFS)	I(1)
Yugoslavia, Jan. 1980-Dec. 1989 (IFS)	I(1)

Sources: M. Taylor, "The Hyperinflation Model of Money Demand Revisited," *Journal of Money, Credit, and Banking*, August 1991, part 1, Table 2, page 331 and page 330, footnote 8; K. Phylaktis and M. Taylor, "Money Demand, the Cagan Model, and the Inflation Tax: Some Latin American Experience," *Review of Economics and Statistics*, February 1993, 32-37, Table 2, page 34; and J. Frenkel and M. Taylor, "Money Demand and Inflation in Yugoslavia, 1980-1989," *Journal of Macroeconomics*, Summer 1993, pages 455-481.

The benefit of stabilizing

As noted above, inflation imposes costs on society. The welfare cost of inflation at time t is assumed to be:

$$L_t = \gamma \pi_t^{\theta} \qquad (3)$$

where $\gamma > 0$ and $\theta > 0$.[15]

The benefit of stabilizing is the expected present discounted value of the welfare costs that would have been imposed on society had the

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[15] This specification for the cost function may seem somewhat ad hoc. One motivation for it is the public finance perspective: the distortion imposed by a tax is proportional to the square of the tax rate. Since inflation imposes a tax on the holders of outside money, the welfare cost of inflation could be assumed to increase with the square of the inflation rate. In that case, $\theta = 2$.

inflationary process been allowed to continue without intervention. More precisely, a stabilization at time t will be assumed to reduce the inflation rate to zero (in the next section, we assume that stabilization reduces inflation by some proportion λ, where $0 < \lambda \leq 1$). Since zero is an absorbing state in the stochastic process, inflation remains zero from the stabilization point onward. The benefit to stabilizing at time s, B_s, is thus:[16]

$$B_s = E_s\left(\int_{t=s}^{t=\infty} \gamma\pi_t^{\theta} e^{-\rho(t-s)} dt\right) \tag{4}$$

where ρ is the discount rate. We assume that $\rho > \alpha\theta + \frac{1}{2}\sigma^2\theta(\theta-1)$.

To evaluate (4), we must derive an expression for $E_s(\pi_t^{\theta})$ given that π_t follows (1). To do so, define $F=\pi^{\theta}$ and apply Ito's lemma:

$$dF = \left(\alpha\theta F + \frac{1}{2}\sigma^2\theta(\theta-1)F\right)dt + \sigma\theta F dz + o(dt) \tag{5}$$

where o(dt) contains terms that go to zero faster than dt. Thus F follows another geometric Brownian motion, and therefore:

$$E_s(F_t) = F_s e^{[\alpha\theta + \frac{1}{2}\sigma^2\theta(\theta-1)][t-s]} \tag{6}$$

Since $F=\pi^{\theta}$, we can now evaluate (4):

$$B_s = \int_{t=s}^{t=\infty} \gamma\pi_s^{\theta} e^{(\alpha\theta + \frac{1}{2}\sigma^2\theta(\theta-1) - \rho)(t-s)} dt \tag{7}$$

which is just:

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[16] This functional form ignores the effects of inflationary shocks on unemployment. With ongoing inflation, the stochastic part of (1) implies that inflation will diverge from expected inflation almost surely. Even imposing rational expectations (so that the deviation of inflationary expectations from realized inflation has mean zero), the log-linear Phillips curve posited below implies (by Jensen's inequality) that the expected cost of unemployment is positive as long as inflationary shocks continue. Since such shocks cease after stabilizing to an inflation rate of zero, there is an additional benefit to stabilizing: the expected (and actual) cost of unemployment falls to zero following the stabilization.

$$B_s = \frac{\gamma \pi_s^{\theta}}{\rho - \alpha\theta - \frac{1}{2}\sigma^2 \theta(\theta - 1)} \qquad (8)$$

The motivation for the restriction $\rho > \alpha\theta + \frac{1}{2}\sigma^2\theta(\theta-1)$ is clear from (8): it ensures that B_S is positive for positive inflation rates.

The cost of stabilizing

Despite some recent evidence to the contrary,[17] the available empirical evidence seems to suggest that stabilizations entail costs in the short run. Stabilization programs are never fully credible; disinflation is therefore costly because inflationary expectations do not fall as quickly as actual inflation. The behavior of the exchange rate on black or free markets, for example, demonstrates this lack of perfect credibility.[18] For the Israeli stabilization of 1985, moreover, there is even better evidence: inflationary expectations can be imputed from the difference between the yields on non-indexed and indexed government bonds. Cukierman (1988) reports that inflationary expectations exceeded actual inflation for six months after the stabilization, "reflecting the fear that the program would break down." [19]

The divergence between inflationary expectations and actual inflation implies that stabilizations entail costs such as increased unemployment and other dislocations. Almost all scholars of stabilization have recognized these costs. As one argues, "The notion that the stabilization of a hyperinflation could be an almost costless process is not

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[17] W. Easterly, "When is stabilization expansionary? Evidence from high inflation," *Economic Policy*, April 1996, 67-107. Easterly concludes that stabilizations are expansionary, even in the short-run, but he uses annual data and does not investigate the impact of stabilization on unemployment.

[18] In the German stabilization of November 1923, for example, the program did not represent "an immediate, obvious set of measures reflected instantly in the exchange rate in the free market...Only toward the middle of December, a full month after stabilization, did the market accept the policy." R. Dornbusch, "Lessons from the German Inflation Experience of the 1920s," reprinted in R. Dornbusch, *Exchange Rates and Inflation* (MIT Press: Cambridge, MA, 1988), pages 428-9.

[19] A. Cukierman, "The End of the High Israeli Inflation," in M. Bruno, G. DiTella, R. Dornbusch, and S. Fischer, *Inflation Stabilization: The Experience of Israel, Argentina, Brazil, Bolivia, and Mexico* (MIT Press: Cambridge, MA, 1988), page 55 and Table 2.3, page 58.

fully endorsed either by the European experiences of the 1920s or by the Bolivian experience in the mid-1980s...the short-run costs in both cases and the very slow recovery of growth in the aftermath of stabilization...suggest that stopping hyperinflation is not a simple and costless task."[20] Another noted economist adds forcefully that "with inflation stubborn or with inflation inertia — be it because of contracts, relative wages, real resistance, or credibility — reducing inflation involves *inevitably* a protracted recession" [emphasis in original].[21]

As one motivation to the cost of stabilizing, consider an expectations-augmented Phillips curve:

$$U = e^{a(\pi - \pi^{e}) + b} \tag{9}$$

where a<0 and b>0. Reverting heuristically to discrete time, assume that for a stabilization at time s, $\pi_{s+1}=(1-\lambda)\pi_s$ and $\pi^{e}_{s+1}=(1/\psi)(1-\lambda)\pi_s$, where λ is the proportional disinflation from the stabilization, and ψ is an index of the government's credibility $(1-\lambda \leq \psi \leq 1)$.[22] If $\psi=1-\lambda$, so that the government has zero credibility, expected inflation equals the pre-stabilization rate of

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[20] A. Solimano, "Inflation and the Costs of Stabilization: Historical and Recent Experiences and Policy Lessons," *The World Bank Research Observer*, July 1990, page 182.

[21] R. Dornbusch, "Inflation Stabilization and Capital Mobility," in R. Dornbusch, *Exchange Rates and Inflation*, op. cit., page 405.

[22] Two important points should be noted: First, at all times other than the stabilization point, rational expectations are assumed to hold. The deviation of inflationary expectations from inflation is therefore a mean zero variable. But, from Jensen's inequality, (9) implies a positive expected cost of unemployment even if inflation minus expected inflation has mean zero. As discussed in footnote 16, this effect is not incorporated into the present analysis. Second, expectations are only partially rational in this model: in particular, agents do not solve the government's optimization problem, and therefore do not anticipate the government's actions. In this sense, they are behaving naively (for example, by not recognizing that inflation is a regulated Brownian motion — and therefore mean-reverting — rather than a regular Brownian motion). In models such as Flood and Garber's model of stochastic process switching, agents form expectations of the government's intervention. See R. Flood and P. Garber, "A Model of Stochastic Process Switching," reprinted in R. Flood and P. Garber, *Speculative Bubbles, Speculative Attacks, and Policy Switching* (MIT Press: Cambridge, MA, 1994); R. Flood and P. Garber, "An Economic Theory of Monetary Reform," reprinted in R. Flood and P. Garber, *Speculative Bubbles, Speculative Attacks, and Policy Switching,* op. cit.; and R. Flood and P. Garber, "Process Consistency and Monetary Reform: Some Further Evidence," reprinted in R. Flood and P. Garber, *Speculative Bubbles, Speculative Attacks, and Policy Switching,* op. cit. Also see Krugman's model of exchange rate bands, in which the government's planned interventions affect agents' expectations (and which uses formal techniques similar to those applied here). See P. Krugman, "Target Zones and Exchange Rate Dynamics," *Quarterly Journal of Economics*, August 1991, 669-82.

inflation. For a fully credible government, $\psi=1$ and expected inflation equals the actual post-stabilization rate of inflation. Then stabilization does not cause a rise in unemployment.

The expectations-augmented Phillips curve (9) suggests a cost function of the form:[23]

$$C_s = \kappa(1-\psi)e^{a(1-\lambda)\pi_s\omega(1-\frac{1}{\psi})} \qquad (10)$$

where κ and ω are multipliers on the overall cost and on the responsiveness of the cost to changes in inflation, respectively. For a fully credible government ($\psi=1$), the cost function (10) implies that stabilization is costless.[24]

In order to derive the basic insights of the model, we will initially assume that $\omega=0$, so that there is a fixed cost of stabilizing that is inversely related to the credibility of the government:

$$C_s = \kappa(1-\psi) \qquad (11)$$

The fixed costs assumption is not essential to the model; it will be relaxed in Section III.

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[23] One commentator has objected to this form of the cost function, advocating instead a cost function that is linear in inflation. But a linear cost function together with a linear benefit function would imply one of three trivial possibilities for the optimal stabilization threshold: zero, infinity, or indeterminate (i.e., stabilization is optimal at any inflation rate). Which of these three cases obtains depends on the values of the parameters in the cost and benefit functions. A linear cost function together with a quadratic benefit function ($\theta=2$) would imply similar results to those presented below if r (defined in the text below) exceeds 2. If $r<2$, then the optimal threshold and the conventional threshold would coincide (see text below for definitions of these terms). If $r=2$, then the optimal threshold would not be well-defined.

[24] Although this implication may initially seem theoretically attractive, it is not clear how realistic it is. Even Sargent seems to back away from a completely costless view of disinflation: "How costly such a move would be in terms of foregone output and how long it would be in taking effect would depend *partly* on how resolute and evident the government's commitment was" (italics added). T. Sargent, "The Ends of Four Big Inflations," reprinted in T. Sargent, *Rational Expectations and Inflation* (Harper Collins: New York, 1993), page 45.

The stochastic dynamic optimization problem

The government faces a stochastic dynamic optimization problem: it must choose a strategy that maximizes the net present value of stabilizing given the stochastic process for inflation (1), the benefit function (8), and the cost function (11).[25] To solve this optimization problem, we define a value function for the value of the option to stabilize:[26]

$$V(\pi(t)) = max\left\{B(\pi(t)) - \kappa(1-\psi), \frac{1}{1+\rho dt} E\{V(\pi(t+dt))\}\right\} \qquad (12)$$

For well-behaved problems, there exists a π^* such that for $\pi < \pi^*$, the second argument on the right hand side of (12) is larger than the first, and for $\pi > \pi^*$, the first argument is larger.[27] For such problems, the state space can be split into two regions: for $\pi<\pi^*$, it is optimal to allow the hyperinflation to continue (the "continuation region"), and for $\pi>\pi^*$, it is optimal to have stopped the hyperinflation (the "stopping region"). At $\pi=\pi^*$, it is optimal to stabilize. In the region for which $\pi < \pi^*$, which has to be determined endogenously, the value of the option to stabilize must obey:

$$V(\pi(t)) = \frac{1}{1+\rho dt} E\{V(\pi(t+dt))\} \qquad (13)$$

or:[28]

$$\rho V dt = E(dV) \qquad (14)$$

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[25] More precisely, the government's objective function at time t is to choose a stabilization point, s, to maximize $E_t\{e^{-\rho(s-t)}(B(\pi_s)-C(\pi_s))\}$.

[26] In an infinite horizon problem, (12) is independent of the time unit in the sense that what matters is the level of inflation and not the time t per se. For an introduction to stochastic dynamic programming (in discrete time), see N. Stokey and R. Lucas with E. Prescott, *Recursive Models in Economic Dynamics* (Harvard University Press: Cambridge, MA, 1989).

[27] Sufficient conditions for this separation property to obtain are that $B(\pi)-C(\pi)$ is increasing in π and that the distribution function of future π conditional on a higher current π exhibits first order stochastic dominance over the distribution function conditional on a lower current π. See A. Dixit and R. Pindyck, *Investment under Uncertainty*, op. cit., pages 128-130.

[28] This is a simple "arbitrage-type" equation. The option to stabilize, V, is an asset without any dividend flow. Equation (14) therefore confirms that the return on V, $\rho V dt$, is equal to the expected capital gain.

Applying Ito's lemma to solve for dV:

$$dV = \left\{ \frac{\partial V}{\partial t} + \alpha\pi \frac{\partial V}{\partial \pi} + \frac{1}{2}\sigma^2\pi^2 \frac{\partial^2 V}{\partial \pi^2} \right\} dt + \sigma\pi \frac{\partial V}{\partial \pi} dz + o(dt) \quad (15)$$

where o(dt), as in (5), contains terms that go to zero faster than dt. In the infinite horizon problem considered here, $\frac{\partial V}{\partial t}$ =0; all of the models in this paper will be autonomous in this sense.[29] Dividing both sides of (15) by dt, letting dt go to zero, and taking expectations, we have:

$$\frac{1}{dt}E(dV) = \left\{ \alpha\pi V'(\pi) + \frac{1}{2}\sigma^2\pi^2 V''(\pi) \right\} \qquad (16)$$

Using (14) and (16), we obtain the following ordinary homogeneous differential equation defined on the interval $(0, \pi^*]$:

$$\frac{1}{2}\sigma^2\pi^2 V''(\pi) + \alpha\pi V'(\pi) - \rho V(\pi) = 0 \qquad (17)$$

This differential equation must be solved subject to the following boundary conditions:[30]

$$V(0) = 0 \qquad (18)$$

$$V(\pi^*) = B(\pi^*) - C(\pi^*) \qquad (19)$$

$$V'(\pi^*) = B'(\pi^*) - C'(\pi^*) \qquad (20)$$

Condition (18) holds because if inflation is zero, it will remain zero forever — so the option to stabilize is worthless at π=0. Equation (19) is a "value matching" condition and holds because the payoff to exercising the option must equal the value of the option at the optimal exercise level. Equation (20) is a "smooth pasting" condition that must also hold at π^*: if

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[29] In general, allowing V to be a function of time necessitates numerical rather than analytical solutions to the control problem. The non-autonomous problems also generally result in an optimal threshold that varies over time, whereas the optimal threshold is constant over time in all the examples considered here. The tradeoff between autonomous and non-autonomous problems is one of analytical tractability versus increased realism .

[30] We need *three* boundary conditions to solve (17), even though it is a second-order ordinary differential equation, because we must solve for the two constants in the general solution to the differential equation, as well as the free boundary (π^*) itself.

the value of the option to stabilize and the net benefit from stabilizing did not meet smoothly at π*, then it can be shown that small movements away from π* would produce net benefits to the government, and thus π* could not be the optimal level at which to exercise the option.[31]

The threshold inflation rate for stabilization

To solve (17), we posit a solution of the form:[32]

$$V = A\pi^{r} + K\pi^{q} \tag{21}$$

where r and q are the characteristic roots of (17):

$$r = \frac{(\frac{\sigma^2}{2} - \alpha) + \sqrt{(\frac{\sigma^2}{2} - \alpha)^2 + 2\rho\sigma^2}}{\sigma^2} \tag{22}$$

$$q = \frac{(\frac{\sigma^2}{2} - \alpha) - \sqrt{(\frac{\sigma^2}{2} - \alpha)^2 + 2\rho\sigma^2}}{\sigma^2} \tag{23}$$

Assuming ρ>α (which is implied by the restriction on ρ above if $6 \geq 1$), r>1 from (22). From (23), q<0. Since q<0, K=0 from (17) and (20). Thus the solution for π* is:

$$\pi^* = \frac{\rho - \alpha\theta - \frac{1}{2}\sigma^2\theta(\theta-1)}{\gamma}\,\frac{r}{r-\theta}\kappa(1-\psi) \tag{24}$$

Assume for simplicity that θ=1 (so that the costs of inflation are linear in inflation) and that α=0 (so that there is no trend in the inflation rate). Then (24) becomes:

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[31] For a heuristic motivation of both the value-matching and smooth-pasting conditions, see A. Dixit, "The Art of Smooth Pasting," Volume 55 in J. Lesourne and H. Sonnenschein, *Fundamentals of Pure and Applied Economics* (Harwood Academic Publishers: Chur, Switzerland, 1993).

[32] Both the Lipshitz and "restriction on growth" conditions obtain for (17). Therefore the solution to (17) exists and is unique. See Theorem 6.1 in A. Malliaris and W. Brock, *Stochastic Methods in Economics and Finance* (North-Holland: Amsterdam, 1982), pages 93-94.

$$\pi^* = \frac{\rho}{\gamma}\frac{r}{r-1}\kappa(1-\psi) \qquad (25)$$

Therein lies the crucial insight of the model: $\rho\kappa(1-\psi)/\gamma$ is the threshold inflation rate derived from equating the expected discounted benefit, $\gamma\pi/\rho$, of a stabilization to the cost, $\kappa(1-\psi)$. Under uncertainty, a benevolent social planner would not stabilize at that inflation rate, but rather at a higher one because of the option value of waiting. For example, suppose that $\sigma=1/3$ (so $\sigma^2=1/9$), $\alpha=0$, and $\rho=.15$. Then $r=2.2$ from (22), and $\frac{r}{r-1}=1.8$. In other words, if conventional cost-benefit analysis suggests stabilization is socially beneficial at 300 percent inflation, this paradigm suggests it is sub-optimal to stabilize before inflation reaches 550 percent. We refer to the former type of threshold, derived from a simple net present value calculation, as the "conventional threshold" and the latter, derived from the stochastic dynamic optimization problem, as the "optimal threshold."

Several results follow directly from (25). The optimal threshold is higher:

- The higher σ^2. Increased volatility raises the value of the option to stabilize, and thus raises the inflation rate at which the option is optimally exercised. Note also that higher volatility raises the markup between the optimal threshold and the conventional threshold, since by definition it does not affect the conventional threshold.

- The higher κ, or the lower ψ. The more expensive a stabilization, the higher the inflation rate must be before the government stabilizes.

- The lower the cost of inflation (the lower is γ). Less costly inflation makes it optimal to stop the inflationary episode later.

- The higher ρ. The benefit from stabilizing accrues over time, whereas the costs are paid immediately. A higher discount rate thus reduces the expected net benefit from stabilizing at any given inflation rate, and raises the optimal threshold.[33]

We can now use the model to examine delayed stabilizations. Given any initial value π_0, define the delay as the waiting time until π^* is reached (the so-called "first crossing" of π^* by the Brownian motion process for inflation). To solve for the distribution of the first crossing time, consider a Brownian motion process defined by:

$$dX = \mu dt + \sigma dz \tag{26}$$

Let T(y) be the first time X has risen by some amount y. Then the probability that T(y) is larger than some time t is given by:

$$P(T(y) > t) = N\left(\frac{y-\mu t}{\sigma\sqrt{t}}\right) - e^{2\mu y \sigma^{-2}} N\left(\frac{-y-\mu t}{\sigma\sqrt{t}}\right) \tag{27}$$

where N is the normal (0,1) distribution function.[34] In order to apply (27), we map (1) into a form that obeys (26) by defining X=ln(π). Then using Ito's lemma on (1):

$$dX = (\alpha - \frac{1}{2}\sigma^2)dt + \sigma dz \tag{28}$$

which is of the same form as (26). Perhaps the most natural π_0 to choose is the conventional threshold (in the example above, $\pi_0 = \frac{\rho\kappa(1-\psi)}{\gamma}$). Then let $X_0 = \ln\left\{\frac{\rho\kappa(1-\psi)}{\gamma}\right\}$ and $X^* = \ln\left\{\frac{\rho\kappa(1-\psi)}{\gamma}\right\} + \ln\left\{\frac{r}{r-1}\right\}$. Imposing the same assumptions as above, $\ln(\frac{r}{r-1})$=0.6. Then using (27) with y=0.6, $\mu = -\frac{1}{2}\left(\frac{1}{9}\right) = -0.056$, $\sigma = \frac{1}{3}$, and t=1 year:

$$\text{Probability(delay} > 1 \text{ year)} = P(T(0.6) > 1) = .95 \tag{29}$$

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[33] Note that the optimal threshold increases proportionately less than the conventional threshold when ρ rises, because an increase in ρ implies an increase in r.

[34] See J. M. Harrison, *Brownian Motion and Stochastic Flow Systems* (Krieger Publishing Company: Malabar, Florida, 1990), page 14.

In other words, the probability of reaching the optimal threshold within one year, given that the process starts at the conventional threshold of $\frac{\rho\kappa(1-\psi)}{\gamma}$, is 5 percent. The upshot is that this simple model provides a possible explanation for observed delays in stabilizations. If the delay is measured as the time that elapses between when the conventional threshold is reached and when the stabilization occurs, and if the stabilization occurs at the optimal threshold, (29) suggests that delays will often exceed a year.[35]

III. The generalized model

After identifying the basic insights of the model in Section II, we are now ready to study a more general version. This section therefore removes our previous assumption that $\omega=0$, allows partial stabilizations ($0 < \lambda \leq 1$ rather than $\lambda=1$) and relaxes the constraints on the other parameters as well.[36] If a stabilization of size λ occurs at time s, then $\pi_{s+1}=(1-\lambda)\pi_s$. So the benefit of such a stabilization will be:[37]

$$B_s = E_s\left(\int_{t=s}^{t=\infty} \gamma\pi_t^{\theta} e^{-\rho(t-s)}dt\right) - E_s\left(\int_{t=s}^{t=\infty} \gamma(1-\lambda)^{\theta}\pi_t^{\theta} e^{-\rho(t-s)}dt\right) \qquad (30)$$

Then, following the same steps as above, (30) can be evaluated to yield:

$$B_s = \frac{\gamma\pi_s^{\theta}(1-(1-\lambda)^{\theta})}{\rho-\alpha\theta-\frac{1}{2}\sigma^2\theta(\theta-1)} \qquad (31)$$

The cost function is as above, but without the restriction $\omega=0$. For reference, we repeat (10):

$$C_s = \kappa(1-\psi)e^{a(1-\lambda)\pi_s\omega(1-\frac{1}{\psi})} \qquad (10)$$

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[35] In this sense, the model actually explains too much. Given the same parameter values, the model predicts delays that are too long relative to observed lengths of hyperinflations. For example, the probability of reaching π^* within several years is also extremely small.

[36] The degree of stabilization, λ, is assumed to be exogenous here. But the model could be extended to allow the government to maximize over λ.

[37] The logic behind (30) is that for every π_t sequence from t=s to infinity that originates with π_s, there is a proportionate one, $(1-\lambda)\pi_t$, that originates with $(1-\lambda)\pi_s$.

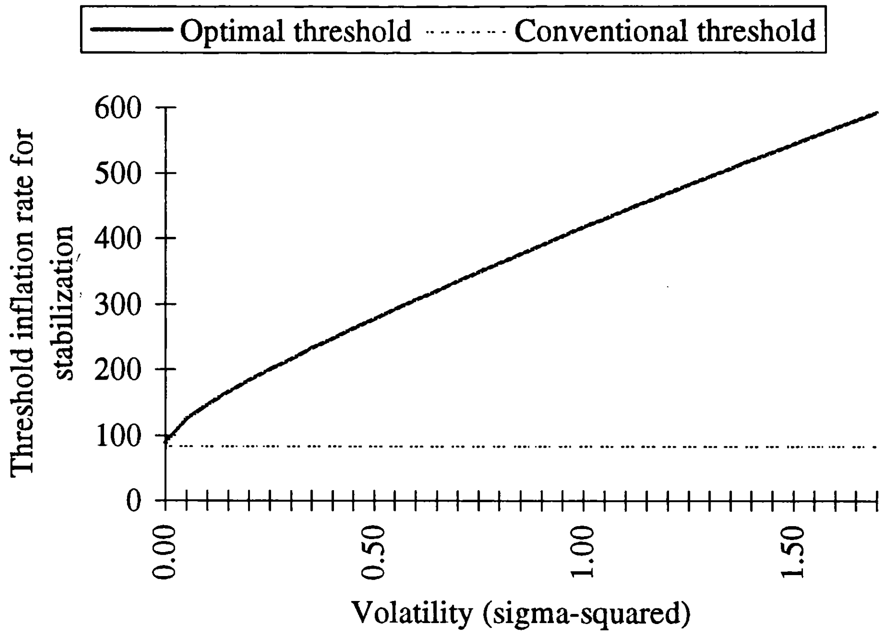
With these cost and benefit functions, an explicit solution for the optimal threshold inflation rate is not possible. But the implicit solution for π^* is:

$$\frac{\gamma\theta\pi^{*\theta}(1-(1-\lambda)^{\theta})}{\rho-\alpha\theta-\frac{1}{2}\sigma^{2}\theta(\theta-1)}-\frac{r\gamma\pi^{*\theta}(1-(1-\lambda)^{\theta})}{\rho-\alpha\theta-\frac{1}{2}\sigma^{2}\theta(\theta-1)}+r\kappa(1-\psi)e^{a(1-\lambda)\pi^{*}\omega(1-\frac{1}{\psi})}$$

$$-a(1-\lambda)\pi^{*}\omega(1-\frac{1}{\psi})\kappa(1-\psi)e^{a(1-\lambda)\pi^{*}\omega(1-\frac{1}{\psi})}=0 \qquad (32)$$

We can use (32) to examine how various parameter values affect π^*. For example, Figure 1 illustrates that the higher the volatility in the stochastic process driving inflation, the higher the optimal threshold, and the higher the ratio of the optimal threshold to the conventional threshold. As in the simplified model above, this result obtains because higher volatility raises the value of the real stabilization option, and thus raises the optimal threshold for exercising it.

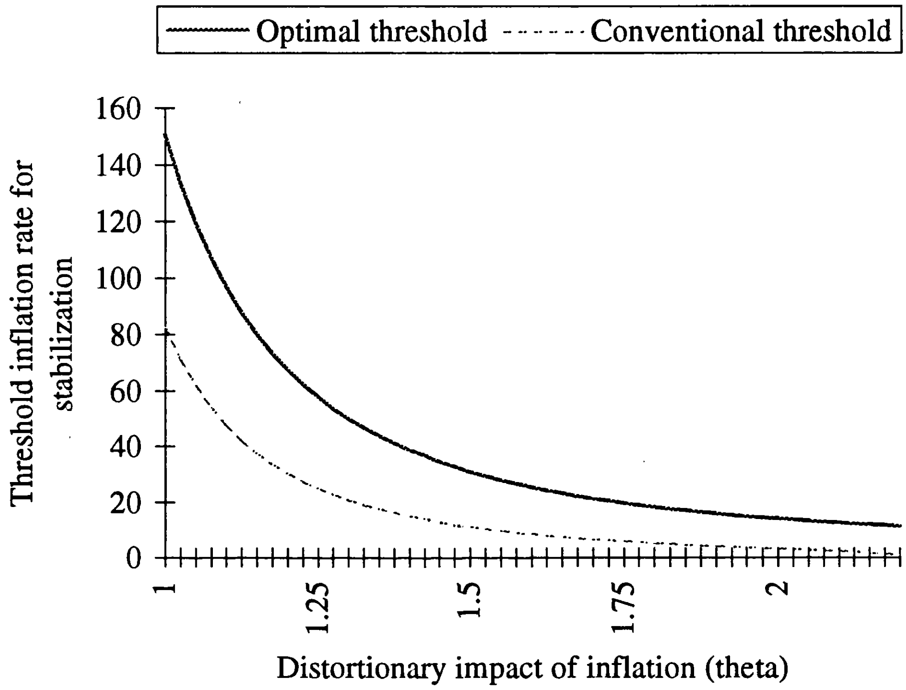
Figure 1: Threshold inflation vs. volatility



Parameter values: $\alpha=0$, $\rho=.15$, $\psi=0.9$, $\gamma=1$, $\theta=1$, $\lambda=.95$, $a=-.1$, $\kappa=5{,}000$, $\omega=1$

Figure 2 illustrates that the higher the distortions imposed by inflation, the lower the optimal and conventional thresholds. The more distortionary is inflation, the more costly it is to permit inflation to continue, and therefore the lower the intervention threshold.

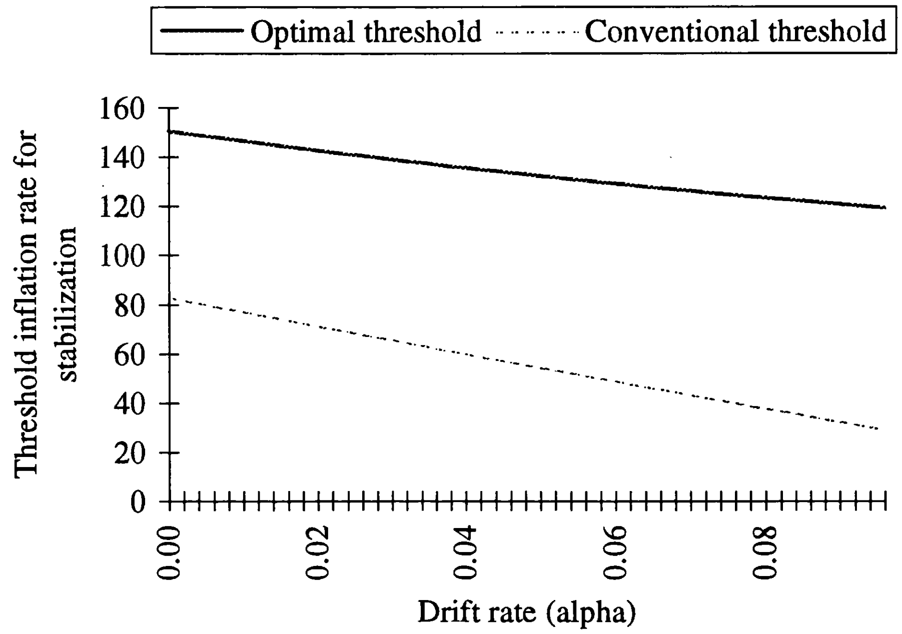
Figure 2: Threshold inflation vs. distortionary impact of inflation



Parameter values: $\sigma^2=1/9$, $\alpha=0$, $\rho=.15$, $\psi=0.9$, $\gamma=1$, $\lambda=.95$, $a=-.1$, $\kappa=5{,}000$, $\omega=1$

Similarly, Figure 3 shows that an increase in α, the drift rate, reduces both the optimal threshold and the conventional threshold. A higher drift rate raises the benefit of stabilizing, and thus induces stabilization at a lower rate of inflation. (It should be noted, however, that the optimal threshold does not monotonically decline as the drift rate rises in the version of the model presented in Section IV.)

Figure 3: Threshold inflation vs. drift rate of inflation



Parameter values: $\sigma^2=1/9$, $\rho=.15$, $\psi=0.9$, $\gamma=1$, $\theta=1$, $\lambda=.95$, $a=-.1$, $\kappa=5{,}000$, $\omega=1$

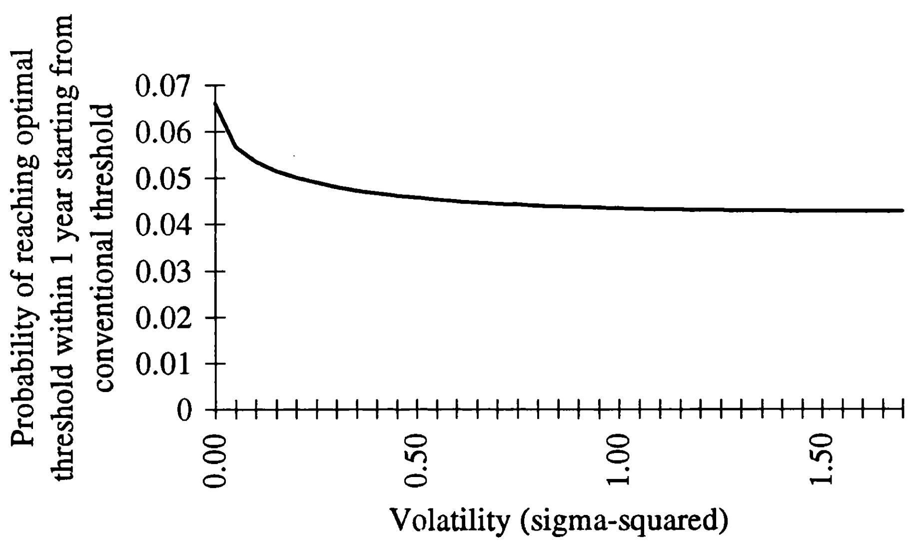
Along other dimensions, the optimal threshold is generally higher:

- The higher the discount rate;
- The lower the government's credibility;
- The lower the cost of inflation;
- The larger the extent of stabilization; and
- The larger (in absolute value) the Phillips coefficient

The precise results, however, are sensitive to the parameter assumptions. The model's numerous non-linearities produce complicated (and often non-monotonic) relationships, especially for extreme parameter values.

Given the implicit solution of (32), we can also examine how various parameters affect the delay until reaching the optimal stabilization point. For example, Figure 4 shows the probability of reaching the optimal threshold within 1 year, starting from the conventional threshold, as a function of σ^2. The increase in the optimal threshold relative to the conventional threshold as σ^2 rises (see Figure 1) reduces the probability of reaching the optimal threshold within a year.[38]

Figure 4: Probability of reaching optimal threshold within 1 year vs. volatility



Parameter values: $\alpha=0$, $\rho=.15$, $\psi=0.9$, $\gamma=1$, $\theta=1$, $\lambda=.95$, $a=-.1$, $\kappa=5{,}000$, $\omega=1$

IV. Multiple stabilization options

The model presented in Section III assumes that the government has only one option to stabilize. This section extends the model to incorporate multiple stabilization opportunities. In particular, we assume that once the government exercises one option to stabilize, it receives another.[39]

Allowing the government to stabilize as many times as desired

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[38] For a broader range of parameter values, the relationship is more complicated than may be suggested by Figure 4. An increase in volatility has two effects. First, it raises the optimal threshold relative to the conventional threshold, and thus reduces the first crossing probability. But, second, it also puts more probability mass on large movements in inflation, and thus raises the probability of reaching any given threshold within a specific period of time. The interplay between these two effects produces a complicated, non-monotonic relationship between volatility and the first-crossing probability.

[39] The government's objective at time t is thus to pick a stabilization date, s, in order to maximize $E_t\{e^{-\rho(s-t)}(B(\pi_s) - C(\pi_s) + V((1-\lambda)\pi_s))\}$.

changes the value-matching and smooth-pasting conditions (19) and (20). Now, if the government stabilizes, it receives the benefits of the disinflation engendered by that stabilization *and* another option to stabilize, valued at the inflation rate obtaining after the first stabilization. The value-matching and smooth-pasting conditions therefore become:

$$V(\pi^*) = B(\pi^*) - C(\pi^*) + V((1-\lambda)\pi^*) \tag{33}$$

$$V'(\pi^*) = B'(\pi^*) - C'(\pi^*) + V'((1-\lambda)\pi^*) \tag{34}$$

None of the other aspects of the stochastic dynamic optimization problem is affected.[40] The optimal threshold is therefore defined implicitly by:

$$\begin{aligned}
&\frac{\gamma\theta\pi^{*\theta}(1-(1-\lambda)^{\theta})}{\rho-\alpha\theta-\frac{1}{2}\sigma^2\theta(\theta-1)} - \frac{1-(1-\lambda)^{r-1}}{1-(1-\lambda)^{r}}\,\frac{r\gamma\pi^{*\theta}(1-(1-\lambda)^{\theta})}{\rho-\alpha\theta-\frac{1}{2}\sigma^2\theta(\theta-1)} \\
&+\frac{1-(1-\lambda)^{r-1}}{1-(1-\lambda)^{r}}\, r\kappa(1-\psi)e^{a(1-\lambda)\pi^*\omega(1-\frac{1}{\psi})} \\
&-a(1-\lambda)\pi^*\omega(1-\frac{1}{\psi})\kappa(1-\psi)e^{a(1-\lambda)\pi^*\omega(1-\frac{1}{\psi})} = 0
\end{aligned} \tag{35}$$

Allowing the government to stabilize multiple times does not change the fundamental precept of the model: that there is a divergence between the optimal threshold and the conventional threshold. Indeed, the optimal threshold with multiple stabilizations is often *higher* than with a single stabilization, because multiple stabilizations increase the value of the option to stabilize, since the option has embedded in it the rights to other options. The irony is thus that allowing the government more opportunities to

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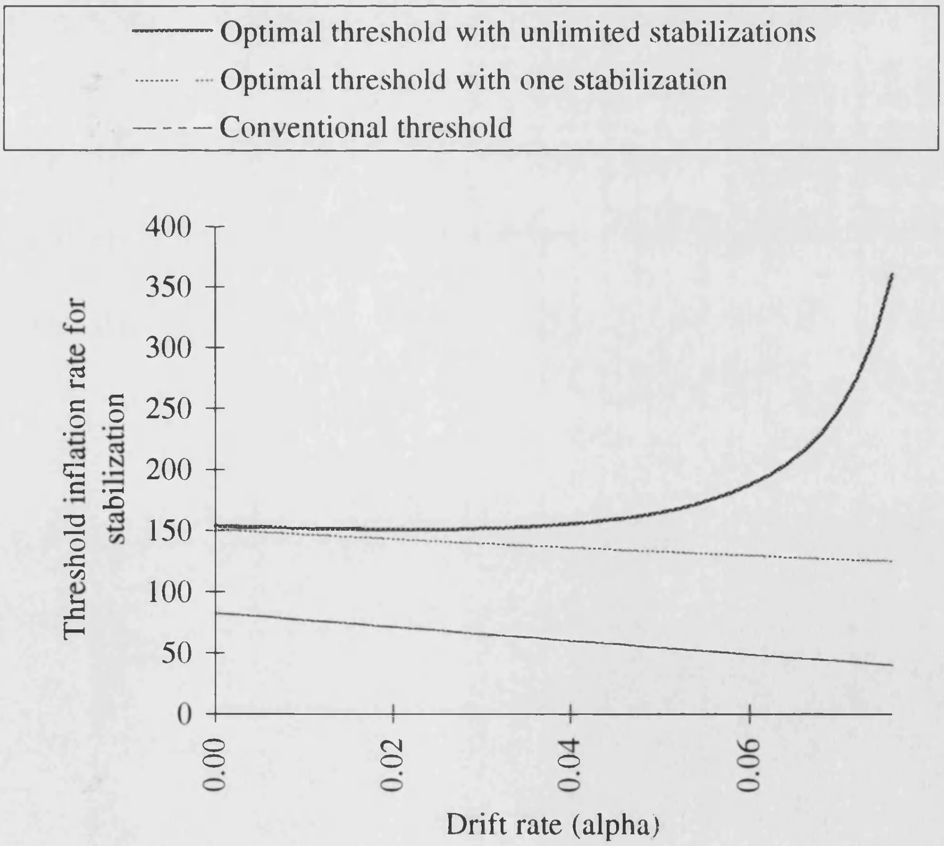
[40] In particular, the benefit function defined by (31) continues to hold, even though it had been derived under the assumption that there was only one option to stabilize. This is because for each sequence of π, there is a proportionate one at $(1-\lambda)\pi$. Therefore, if there is a stabilization subsequent to the first one, the inflation rate in the absence of the first stabilization would have risen to $\pi^*/(1-\lambda)$. Although the actual inflation rate falls back to $\pi^*(1-\lambda)$ after the second stabilization, the benefit from the first stabilization is still the welfare gain between $\pi^*/(1-\lambda)$ and π^*. The total benefit of the two stabilizations is the welfare gain between $\pi^*/(1-\lambda)$ and $\pi^*(1-\lambda)$, and continuing to use (31) correctly apportions that total benefit between the first and second stabilizations.

stabilize can *reduce* the probability of observing a stabilization within some given time period.

Multiple stabilizations can also change the nature of the relationship between the optimal threshold and various parameters. With multiple stabilizations, for example, a higher drift rate can raise the optimal threshold. Figure 5 employs the same parameter assumptions as Figure 3 above. It illustrates two effects of multiple stabilizations. First, for the parameter values shown, the optimal threshold with multiple stabilizations is strictly higher than the optimal threshold with a single stabilization. Second, for sufficiently large drift rates, an increase in the drift rate *raises* the optimal threshold when multiple stabilizations are allowed.

Figure 5: Threshold inflation vs. drift rate, multiple stabilizations



Parameter values: $\sigma^2=1/9$, $\rho=.15$, $\psi=0.9$, $\gamma=1$, $\theta=1$, $\lambda=.95$, $a=-.1$, $\kappa=5{,}000$, $\omega=1$

The intuition for the lack of a monotonic effect on the threshold is that there are two effects of a higher drift rate. The first is that a higher drift rate raises the benefit of stabilizing sooner, tending to lower the optimal threshold. But a higher drift rate also raises the value of the option to stabilize, since it reduces r. In the multiple stabilization scenario studied here, this second effect is magnified: a higher drift rate raises the value of the option to stabilize at π^* and also at $(1-\lambda)\pi^*$, which then feeds back into the value of the option to stabilize at π^*. For sufficiently large drift rates, the second effect dominates and the optimal threshold *increases* when the drift rate increases. Thus an increase in the drift rate, which raises the cost of delay, can make governments *more* averse to stabilizing at any given inflation rate. A similar counter-intuitive result is obtained, albeit through a much different motivation, by Perraudin and Sibert (1995).[41]

V. Conclusion

This paper has presented a simple model of stabilizations as an irreversible investment under uncertainty. The model offers a possible explanation for delayed stabilizations — an explanation that depends on uncertainty and costly stabilizations, not irrationality or political economy constraints. The next chapter provides a partial empirical test of the model by examining the relationship between the volatility of the inflationary process and the stabilization delay.

In addition to the extensions offered in the appendices, the model could be linked to the political economy literature on delayed stabilizations by viewing the decision-maker as a politician, rather than a benevolent social planner. The benefits and costs would then be limited to those enjoyed or incurred by the faction supported by the decision-maker, and the real option decision-making rule would be embedded within a political economy game among different factions. This extension would be in the spirit of Lambrecht and Perraudin (1996), who embed a real option

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[41] In the Perraudin-Sibert model, a higher cost of delay raises the incentives for a foreign lender to withhold loans in order to learn more about the type of borrowing government. Since the loans are necessary for reform, delays are longer in equilibrium. W. Perraudin and A. Sibert, "The Timing of Reform," Institute for Financial Research Working Paper 2, Birkbeck College, December 1995.

investment decision within a game between competing firms.[42] The approach delineated here could also be incorporated into models of designing broader reform packages under uncertainty, as in Dewatripont and Roland (1995).[43]

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[42] B. Lambrecht and W. Perraudin, "Real Options and Preemption," Institute for Financial Research Working Paper 26, Birkbeck College, July 1996.

[43] Dewatripont and Roland argue that gradualism may be preferred to big-bang reform because gradualist packages engender less initial opposition and may subsequently create stronger constituencies for further reform. See M. Dewatripont and G. Roland, "The Design of Reform Packages under Uncertainty," *American Economic Review*, December 1995, 1207-1223. Note that the irreversible investment model presented here does not address the question of gradualism versus big bang: a stabilization of a given size (λ) is assumed, and the only question is when that stabilization is undertaken.

Appendix I. A stochastic process for inflation with discrete Poisson jumps

In this section, we allow inflation to follow a mixed diffusion-jump process.[44] The benefit and cost functions are assumed to be the same as above, with θ set to 1.[45] The only difference is that (1) is modified to incorporate discrete Poisson jumps:

$$d\pi = \alpha\pi dt + \sigma\pi dz + \pi dq \tag{A1}$$

where

$$dq = \begin{cases} k-1 \text{ with probability } \xi\text{dt} \\ 0 \text{ with probability } 1 - \xi\text{dt} \end{cases} \tag{A2}$$

The Poisson process defined by (A1) is of a particularly simple form: a shock moves the process discretely from π to $k\pi$ with probability ξdt. With probability 1- ξdt, there is no discrete jump, and inflation behaves as the diffusion process in (1).

With the stochastic process for inflation defined by (A1), the differential equation for V (the analog to (17) above) is:

$$\frac{1}{2}\sigma^2\pi^2 V''(\pi) + \alpha\pi V'(\pi) - \rho V(\pi) + \xi(V(k\pi) - V(\pi)) = 0 \tag{A3}$$

Again, (A3) only holds for $\pi<\pi^*$, and we need three boundary conditions to identify the two arbitrary constants in (A3), as well as the free boundary π^*. These three boundary conditions are precisely the same as those defined by (18)-(20) above. Condition (18) holds here because the Poisson shocks are multiplicative — so if inflation is zero, it will continue to be zero in perpetuity. Condition (19) holds for the same reason as in the model above.

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[44] This type of stochastic process for inflation is similar to the stochastic process for stock returns analyzed by R. Merton in "Option Pricing When Underlying Stock Returns are Discontinuous," reprinted in R. Merton, *Continuous Time Finance* (Blackwell: Cambridge, MA, 1990), pages 309-329.

[45] We assume $\rho>\alpha+\xi(k-1)$ to ensure that the denominator in the benefit function is positive.

Condition (20) continues to hold despite the Poisson jumps.[46]

There are two ways of solving the government's stochastic dynamic optimization problem. The first posits a solution of the form $A\pi^r$, so that the characteristic equation of (A3) is:

$$\frac{1}{2}\sigma^2 r(r-1)+\alpha r-\rho+\xi+\xi k^r=0 \qquad \text{(A4)}$$

For any given value of k, (A4) can be solved implicitly for r, and then the analysis can proceed. The results are generically the same as in the second solution method, which uses a Taylor series approximation. In this second approach, we assume that k is approximately 1 and then use a first-order Taylor series approximation to V(kπ) around π:

$$V(k\pi)\approx V(\pi)+(k-1)\pi V'(\pi) \qquad \text{(A5)}$$

Thus (A3) becomes:

$$\frac{1}{2}\sigma^2\pi^2 V''(\pi)+(\alpha+\xi(k-1))\pi V'(\pi)-\rho V(\pi)=0 \qquad \text{(A6)}$$

Following the same steps as above, the optimal threshold is therefore:

$$\pi^*=\frac{(\rho-\alpha-\xi(k-1))}{\gamma}\frac{z}{z-1}\kappa(1-\psi) \qquad \text{(A7)}$$

where z>1 is the positive characteristic root of (A6):

$$z=\frac{(\frac{\sigma^2}{2}-\alpha-\xi(k-1))+\sqrt{(\frac{\sigma^2}{2}-\alpha-\xi(k-1))^2+2\rho\sigma^2}}{\sigma^2} \qquad \text{(A8)}$$

As in the section above, assume that α=0. It is then illuminating to examine whether the optimal threshold defined by (A8) is higher or lower

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[46] See discussion in A. Dixit and R. Pindyck, *Investment under Uncertainty,* op. cit., page 171. In addition, Merton applies the same boundary conditions when stock returns have a Poisson element as when they do not. See R. Merton, "Option Pricing When Underlying Stock Returns are Discontinuous," op. cit., page 318.

than the optimal threshold defined by (25). In other words, does the Poisson process induce optimally behaving governments to stabilize at a higher or lower rate of inflation? The Poisson jumps have two effects: they change both the volatility of inflation and its drift rate. The variance of the stochastic process (A3) is:

$$V(d\pi) = \sigma^2\pi^2 dt + \xi(k-1)^2\pi^2 dt > \sigma^2\pi^2 dt \qquad \text{(A9)}$$

so that the farther k is from 1 (i.e., the larger is the jump in either direction), the more volatile is the change in inflation. This higher variance tends to raise the optimal threshold.

The Poisson jumps also affect the drift rate of inflation, however, and thus also affect the optimal threshold indirectly through this conduit. We now must consider two different cases: k<1 and k>1 (when k=1, the Poisson "jump" is always zero). First consider k<1. Then the Poisson jumps diminish the drift rate of inflation, since the impact of the Poisson process on the drift rate is $\xi(k-1)$. The lower drift rate tends to raise the optimal threshold (with a lower drift rate, there is less of an incentive to stabilize at any given inflation rate, and the optimal threshold rises). For k<1, both the lower drift rate and the higher variance tend to raise the optimal threshold, and the optimal threshold is therefore unambiguously higher than without the Poisson jumps. Intuitively, the discrete shocks accentuate the incentive to wait because they introduce more uncertainty (they raise the variance of the change in inflation) and because they are inherently disinflationary (they reduce the drift rate).

For k>1, however, the impact on the optimal threshold is ambiguous. In this case, there are two opposing effects: the drift rate of inflation is higher than in the absence of discrete jumps, tending to diminish the optimal threshold, but the volatility of the change in inflation is higher, tending to raise the optimal threshold. For values of k close to 1, the first effect dominates and the optimal threshold is lower than in the absence of the Poisson jumps. For larger values of k, the second effect dominates and the optimal threshold is higher than in the absence of the Poisson jumps.

Appendix II. A mean-reverting stochastic process for inflation

As noted in the text above, the evidence for the non-stationarity of inflation during hyperinflations is at least somewhat tenuous. It is therefore necessary to consider how robust our results are to the assumption of non-stationarity. In this appendix, we posit a stationary, mean-reverting process for inflation:

$$d\pi = \alpha(\bar{\pi} - \pi)\pi dt + \sigma\pi dz \qquad \text{(A10)}$$

where $\alpha>0$ is the "speed of adjustment parameter" (determining the rate at which π reverts to $\bar{\pi}$) and $\bar{\pi}$ is the "long-run" or steady-state value of π. As one motivation for (A10), assume that monetary growth is some constant μ. Then in a wide variety of theoretical models, $\bar{\pi}=\mu$.

The benefit function to a stabilization at time s for the stochastic process defined by (A10) is analytically intractable. As an approximation to the true benefit function, we therefore consider a linearization of (A10) around $\bar{\pi}$:

$$d\pi \approx \alpha(\bar{\pi} - \pi)\bar{\pi} dt + \sigma\pi dz \qquad \text{(A11)}$$

The expected value of π_t conditional on the information set at time s is therefore given approximately by:

$$E_s(\pi_t) = \pi_s + E_s\left\{\int_{j=s}^{j=t} \alpha(\bar{\pi} - \pi)\bar{\pi} dj + \sigma\pi dz\right\} \qquad \text{(A12)}$$

Moving the expectations operator through the integration operator, differentiating with respect to time, and solving the resultant first-order differential equation (subject to the boundary condition that $E_S(\pi_S)=\pi_S$), we have:

$$E_s(\pi_t) = \bar{\pi} + (\pi_s - \bar{\pi})e^{-\alpha\bar{\pi}(t-s)} \qquad \text{(A13)}$$

The benefit to a stabilization at time s (assuming $\theta=1$) is therefore approximately:

$$B_s = E_s\left(\int_{t=s}^{t=\infty} \gamma\pi_t e^{-\rho(t-s)} dt\right) = \int_{t=s}^{t=\infty} \gamma\bar{\pi} e^{-\rho(t-s)} + \gamma(\pi_s - \bar{\pi}) e^{-(\alpha\bar{\pi}+\rho)(t-s)} dt \qquad \text{(A14)}$$

which is just:

$$B_s = \frac{\gamma\bar{\pi}}{\rho} + \frac{\gamma(\pi_s - \bar{\pi})}{\alpha\bar{\pi} + \rho} \qquad \text{(A15)}$$

We assume ω=0, so that the cost function is fixed, as in the simplified model of the main text.

Once again, we must solve for the value of the option to stabilize, V(π), as a function of the inflation rate. Applying Ito's lemma and following familiar steps, we obtain:

$$\frac{1}{2}\sigma^2\pi^2 V''(\pi) + \alpha(\bar{\pi} - \pi)\pi V'(\pi) - \rho V(\pi) = 0 \qquad \text{(A16)}$$

which must be solved subject to the boundary conditions (18)-(20). Note that (18) continues to hold because zero is an absorbing state in the stochastic process (A10).

In order to solve for the value function, we posit a solution of the form:[47]

$$V(\pi) = A\pi^r Z(\pi) \qquad \text{(A17)}$$

Taking the appropriate derivatives of (A17) and plugging them into (A16), we have:

$$\begin{aligned} &\pi^r Z(\pi)\left\{\frac{\sigma^2}{2} r(r-1) + \alpha\bar{\pi} r - \rho\right\} \\ &+ \pi^{r+1}\left\{\sigma^2 r Z'(\pi) + \frac{\sigma^2}{2}\pi Z''(\pi) + \alpha(\bar{\pi} - \pi) Z'(\pi) - \alpha r Z(\pi)\right\} = 0 \end{aligned} \qquad \text{(A18)}$$

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[47] The solution strategy follows A. Dixit and R. Pindyck, *Investment under Uncertainty*, op. cit., pages 161-167.

This solution must hold for all values of $\pi<\pi^*$. Thus both terms in brackets must be equal to zero. Setting the first term in brackets equal to zero implicitly defines r, whereas equating the second term to zero, after some manipulation, produces a differential equation with a known solution. In particular, define a function $h(x)=Z(\pi)$, with $x=\frac{2\alpha\pi}{\sigma^2}$, so that $h'(x)\frac{2\alpha}{\sigma^2}=Z'(\pi)$ and $h''(x)\left(\frac{2\alpha}{\sigma^2}\right)^2=Z''(\pi)$. Substituting these into the second term in brackets in (A18), and setting the resultant expression equal to zero, we have:

$$xh''(x)+(b-x)h'(x)-rh(x)=0 \qquad \text{(A19)}$$

where $b=\frac{2(\alpha\bar{\pi}+r\sigma^2)}{\sigma^2}$. This is known as Kummer's equation, and its solution is the confluent hypergeometric function H(x,r,b):[48]

$$H(x,r,b)=1+\frac{r}{b}x+\frac{r(r+1)}{b(b+1)}\frac{x^2}{2!}+\frac{r(r+1)(r+2)}{b(b+1)(b+2)}\frac{x^3}{3!}+\ldots \qquad \text{(A20)}$$

Thus the solution is:

$$V(\pi)=A\pi^r H(\frac{2\alpha\pi}{\sigma^2},r,b) \qquad \text{(A21)}$$

We can now use (A21) together with the value-matching and smooth-pasting conditions to solve for A and π^*. The solution for π^* is defined implicitly by:

$$\left\{\frac{\gamma\bar{\pi}}{\rho}+\frac{\gamma(\pi^*-\bar{\pi})}{\alpha\bar{\pi}+\rho}-\kappa(1-\psi)\right\}\left\{r+\pi^*\frac{H'(\frac{2\alpha\pi^*}{\sigma^2},r,b)}{H(\frac{2\alpha\pi^*}{\sigma^2},r,b)}\frac{2\alpha}{\sigma^2}\right\}-\frac{\gamma\pi^*}{\alpha\bar{\pi}+\rho}=0 \qquad \text{(A22)}$$

The optimal threshold defined by (A22) is higher than the conventional threshold, and therefore the fundamental precept of the model

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[48] M. Abramowitz and I.A. Stegun, *Handbook of Mathematical Functions* (National Bureau of Standards: Washington, 1964), Section 13.1.1.

carries across even when inflation is mean-reverting.[49] For example, for $\sigma^2=1/9$, $\alpha=.1$, $\rho=.1$, $\bar{\pi}=15$ percent, $\kappa=5{,}000$, $\psi=.9$, and $\gamma=1$, $\pi^*=580$ percent.[50] The conventional threshold is 575 percent. Furthermore, the closer the inflationary process is to being non-stationary, the larger the ratio of the optimal threshold to the conventional threshold (this ratio is what determines the waiting time or delay). For $\alpha=.01$, for example, the optimal threshold is 108 percent and the conventional threshold is 102.5 percent (ratio=1.05). For $\alpha=.001$, the optimal threshold is 71 percent and the conventional threshold is 55 percent (ratio=1.29). For a non-stationary process ($\alpha=0$) with the same variance and discount parameters, the ratio of the optimal threshold to the conventional threshold is 1.28. So a stationary process that is nearly non-stationary produces results that are approximately equivalent to those produced by a non-stationary process.

In sum, the empirical evidence in Table 1 suggests that inflation is either a non-stationary process or is very nearly so. This appendix suggests that little is lost in terms of accuracy, and much is gained in terms of tractability, by simply assuming that inflation is non-stationary. None of the principal results depends on that assumption.

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[49] As noted in a footnote above, inflation will be mean-reverting, even given the Brownian motion process defined by (1), because the government's stabilizations establish an upper boundary on the process. The point here is that the stationarity or non-stationarity of inflation *in the absence of government intervention* is not crucial for the fundamental results of the paper.

[50] In calculating this value from (51), the infinite series H and H' were approximated by series containing 100 terms.

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Chapter 2: Hyperinflations and Delayed Stabilizations: Some Simple Empirical Tests

I. Introduction

Previous empirical work on high inflation has tended to focus on a select number of well-known case studies.[1] The approach undertaken here is more general: we apply several objective definitions of hyperinflation to the entire universe of monthly inflation data from the International Monetary Fund's *International Financial Statistics.* The result is a comprehensive data set of all recorded experiences of high inflation over the past 35 years. We use the data set both to expand our knowledge of the empirical regularities of high inflation, and to test two models of delayed stabilizations. The database constructed for this chapter complements the one developed by Easterly (1996), who uses annual data and applies only a single definition of hyperinflation.[2]

The chapter is organized as follows: Section II analyzes the high inflation episodes and concludes that some commonly assumed facts about high inflation seem inconsistent with the data. Section III briefly summarizes two models of delayed stabilizations; the models can be tested empirically because they offer different predictions for the relationship between the volatility of the inflationary process and the duration of an episode. The next two sections compare the two models by performing various tests of this relationship: Section IV uses ordinary least squares, and

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[1] See, for example, P. Cagan, "The Monetary Dynamics of Hyperinflation," in M. Friedman, *Studies in the Quantity Theory of Money* (University of Chicago Press: Chicago, 1956); R. Dornbusch, F. Sturzenegger, and H. Wolf, "Extreme Inflation: Dynamics and Stabilization," *Brookings Papers on Economic Activity 2: 1990* (The Brookings Institution: Washington, 1990); M. Bruno, G. Di Tella, R. Dornbusch, and S. Fischer, *Inflation Stabilization: The Experience of Israel, Argentina, Brazil, Bolivia, and Mexico* (MIT Press: Cambridge, MA, 1988); M. Bruno, S. Fischer, E. Helpman, and N. Liviatan, *Lessons of Economic Stabilization and Its Aftermath* (MIT Press: Cambridge, MA, 1991); and T. Sargent, "The Ends of Four Big Inflations," reprinted in T. Sargent, *Rational Expectations and Inflation* (Harper Collins: New York, 1993). The principal exceptions are M. Bruno and W. Easterly, "Inflation's Children: Tales of Crises that Beget Reforms," *AEA Papers and Proceedings,* May 1996, 213-217, and W. Easterly, "When is stabilization expansionary? Evidence from high inflation," *Economic Policy,* April 1996, 67-107. Unlike some other authors, we do not distinguish between high inflation and hyperinflation.

[2] W. Easterly, "When is stabilization expansionary? Evidence from high inflation," op. cit. Easterly's definition is 40 percent *annual* inflation.

Section V uses hazard rate analysis. Appendices provide additional information on, and analysis of, the inflationary episodes.

II. Episodes of high inflation, 1957-1994

Any comprehensive investigation of hyperinflation must adopt some definition of the term. The definition we use is in the spirit of Cagan (1956): a hyperinflation begins in the month in which the monthly inflation rate first exceeds some threshold level, and ends in the month before the monthly inflation rate remains below that threshold level for 12 months.[3] For example, let the threshold be 10 percent per month and consider a monthly inflation series of {5,12,7,12,12,8,8,8,8,8,8,8,8,8,8,8,8}. Then the episode begins in the second month (the first month when inflation exceeds 10 percent) and ends in the fifth month (the first month that is followed by 12 months of under 10 percent inflation). Note that the episode includes the third month, even though inflation during that month is below the threshold. The threshold levels used in this study are 10 percent, 20 percent, 30 percent, and 40 percent.[4]

The data sets of high inflation episodes were constructed from the entire universe of monthly price data between January 1957 and July 1994 from the January 1995 *International Financial Statistics.* The data on prices were converted into inflation rates by taking log differences; the monthly inflation rate for month t is thus defined as the natural logarithm of the price level in month t minus the natural logarithm of the price level in month t-1.[5] Appendix I presents summary statistics on the inflation rates from the countries included in the sample. The inflation rates are not seasonally

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[3] Cagan defines a hyperinflation as "beginning in the month the rise in prices exceeds 50 percent and as ending in the month before the monthly rise in prices drops below that amount and stays below for at least a year." He admits this definition is "purely arbitrary." See P. Cagan, "The Monetary Dynamics of Hyperinflation," op. cit., page 25. To check for robustness, we also constructed an alternative set of episodes by requiring that inflation remain below the threshold for 6 months, rather than 12. The results obtained from this alternative data set are similar to those presented in the text.

[4] These threshold inflation rates are defined in terms of the log difference of the price levels. They correspond to actual monthly inflation rates of 10.5 percent, 22.1 percent, 35.0 percent, and 49.2 percent respectively. Cagan used an actual inflation threshold of 50 percent. Dornbusch and Fischer propose a threshold of 1,000 percent per year, which corresponds to 22.1 percent per month. See R. Dornbusch and S. Fischer, *Macroeconomics* (McGraw Hill: New York, 1990), page 663.

[5] The series for Argentina, Brazil, and Zaire were spliced together from overlapping but rebased price indices. The splice dates were January 1990, August 1990, and January 1992, respectively.

adjusted.[6]

For each definition of high inflation, we extracted the individual episodes from the aggregate data set. We focus on episodes lasting 4 or more months to ensure that all the relevant volatility measures are defined in a meaningful way. Table 1 presents summary statistics on the episodes.

Table 1: Summary statistics on episodes

Monthly inflation threshold	Total number of episodes	Episodes with duration>3 months	Countries with episodes of duration >3 months (more than 1 episode of duration>3 months)	Percent of episodes in which final fall in inflation is smaller than one (two) std. dev. of change in inflation[7]
10 percent	119	42	32 (9)	42 (71)
20 percent	39	18	14 (3)	25 (67)
30 percent	15	10	8 (2)	14 (43)
40 percent	12	7	6 (1)	20 (80)

There are several striking features of Table 1:

1. A substantial proportion of episodes end within 3 months, especially at the lower thresholds. At the 10 percent and 20 percent thresholds, over half of the episodes have durations of 3 months or less. Even at the 30 and 40 percent thresholds, a third or more of the episodes do not persist for more than 3 months.

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[6] The lack of seasonal adjustment introduces a potentially serious bias. For example, consider an economy in which there is a sharp price increase every January. Assume this price increase exceeds 10 percent. Applying our definition of an episode, and setting the threshold at 10 percent, the country would perpetually be in a hyperinflation. One solution is seasonal adjustment. Seasonally adjusting the data, however, could introduce other biases, especially since it would be necessary to detrend the data as part of the process. For countries with short time series, this could lead to overfitting problems. In addition, inflation in most countries is either non-stationary or very close to non-stationary, raising the specter of introducing severe biases from spurious detrending procedures. We therefore adopted a different approach to the seasonality problem: we applied another filter to the episodes, in the form of a mean rate of inflation threshold. A mean rate of inflation substantially below the monthly inflation threshold could be indicative of severe seasonal fluctuations. In practice, this additional threshold does not affect the results. Furthermore, as the monthly inflation threshold increases, it becomes more and more likely that seasonal factors are strongly dominated by the hyperinflation process. We therefore tentatively conclude that the lack of seasonal adjustment is not a serious problem.

[7] The figures in this column and the next apply only to completed episodes of duration greater than 3 months. The mean change in inflation was subtracted from the post-episode change in inflation.

2. The number of episodes drops sharply as the definition of high inflation increases. The total number of episodes falls from 119 at the 10 percent threshold to just 15 at the 30 percent threshold. Note that this decline does not necessarily have to obtain as the threshold increases: one episode at the lower threshold can be transformed into multiple episodes at higher thresholds, so that the total number of episodes could *increase* as the threshold increases.[8]

These two conclusions seem inconsistent with a commonly held view that high inflation is necessarily explosive.[9]

3. Most countries experience only a single episode of hyperinflation, and most episodes are accounted for by countries experiencing only a single episode. At the 10 percent threshold, only 9 countries have experienced more than 1 episode of high inflation that lasted more than 3 months. At the 20 percent, 30 percent, and 40 percent thresholds, the number falls to 3, 2, and 1 respectively.

4. Many episodes end without a "large" fall in inflation. In over two-fifths of the cases at the 10 percent threshold, the mean-corrected post-episode fall in inflation is smaller than the standard deviation of the change in inflation during the episode itself.[10] And the majority of cases at all thresholds except 30 percent end with a mean-corrected fall in inflation that is smaller than twice the standard deviation of the change in inflation. This result suggests that many high inflation episodes end without dramatic stabilizations, or perhaps without any meaningful stabilization at all. In this sense, the set of post-World War II high inflations differs dramatically from the classical post-World War I hyperinflations, whose ends, as Thomas

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[8] For example, consider a monthly inflation series of {21,21,21,15,15,15,15,15,15,15,1 5, 15,15,15,15,21,21,21,....}. At the 10 percent threshold, this would constitute one episode. But at the 20 percent threshold, it would split into two episodes.

[9] As one example of the conventional wisdom, Dornbusch, Sturzenegger, and Wolf write: "We view high inflation as intrinsically unstable. Shocks to high inflation economies easily translate into explosive inflation until a fundamental stabilization program can reset the dials." See their "Extreme Inflation: Dynamics and Stabilization," op. cit., page 9. As noted below in the text, many of the episodes end without a "large" fall in inflation. This seems to undermine the potential argument that the observed decline in the number of episodes is entirely caused by fundamental stabilizations.

[10] See footnote to the relevant column in Table 1. The post-episode fall in inflation is defined as (the negative of) inflation in the first month following the episode minus inflation in the final month of the episode. There are clearly many other metrics for defining a "large" fall.

Sargent has stressed, were abrupt and dramatic.[11] Our metric confirms Sargent's conclusion: for all four of the classical hyperinflations (Austria, Germany, Hungary, and Poland), the post-episode fall in inflation was larger than the standard deviation of the change in inflation.[12]

Tables 2-5 present more detailed statistics for episodes of 4 or more months duration at each of the threshold levels. In each table, an "n" or "nc" appended to a country name indicates an episode that was not completed at the end of the available time series, and π stands for monthly inflation. An empty value in any of the cells indicates at least one missing value in the underlying data.

Table 2 provides more detailed statistics on the 10 percent threshold episodes. Note the large number of episodes for which the maximum inflation rate is under either 20 percent or 30 percent — implying that they do not qualify as episodes at the higher threshold levels. Also note that many of the episodes in Table 2 may be apocryphal, in the sense that they may reflect measurement error or severe seasonal patterns rather than a true high inflation period — a problem that seems particularly acute for some of the African cases (e.g., Niger, Equatorial Guinea, and Burkina Faso). Finally, the average (unweighted) standard deviation of the inflation rate is substantially higher for the countries listed in Table 2 (0.07) than for the complete IFS sample of countries (0.03).[13] The average standard deviation over the episodes (0.09) is even higher.[14] In other words, relative to a norm of either the average standard deviation for all countries or even the average standard deviation for those countries that have experienced a high inflation episode, recorded inflation tends to be more volatile during high inflation

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[11] See T. Sargent, "The Ends of Four Big Inflations," op. cit.

[12] As in Table 1, the mean change in inflation is subtracted from the post-episode change in inflation before comparison with the standard deviation of the change in inflation. The months included in the episodes are as defined in J. Sachs and F. Larrain, *Macroeconomics in the Global Economy* (New York: Harvester Wheatsheaf, 1993), Table 23-1, page 730. The ratios of the post-episode change in inflation (minus the mean change in inflation) to the standard deviation of the change in inflation are: Austria, -1.12; Germany, -2.89; Hungary, -2.16; and Poland, -1.34.

[13] The standard deviations given are for all recorded monthly inflation rates, not limited to those within episodes. For the countries included in Table 2, the overall standard deviation conflates the effects of the episode and non-episode months. The standard deviation for the non-episode months is lower than the figure cited in the text.

[14] The average (unweighted) mean inflation rate is 0.099. The average (unweighted) coefficient of variation is 2.71 including Equatorial Guinea and Niger, and 1.08 excluding those two economies.

episodes.[15] Although these are inexact calculations, they are consistent with empirical evidence suggesting that the variance of inflation is increasing in the mean inflation rate.[16]

Table 2: Episodes at 10 percent threshold

COUNTRY	DURATION (MONTHS)	BEG	END	STD(π)	STD (dπ)	MAX π	MEAN π
DOMINICA	4	1990 AUG	1990 NOV	0.02	0.06	0.14	0.11
SENEGALn	4	1994 JAN	1994 APR	0.04	0.02	0.12	0.06
MEXICO	5	1982 AUG	1982 DEC	0.03	0.05	0.11	0.07
GHANA	5	1977 MAR	1977 JUL	0.04	0.07	0.21	0.14
GHANA	5	1983 FEB	1983 JUN	0.04	0.06	0.21	0.14
NIGER	5	1981 JUL	1981 NOV	0.12	0.19	0.13	0.01
SIERRA_L	5	1987 JAN	1987 MAY	0.06	0.10	0.22	0.15
TANZANnc	5	1994 FEB	1994 JUN	0.08	0.15	0.22	0.08
PERU	8	1985 JAN	1985 AUG	0.02	0.03	0.13	0.10
AFGHANIS	8	1981 MAY	1981 DEC	0.06	0.10	0.11	0.02
ZAIRE	9	1988 JUN	1989 FEB	0.05	0.06	0.19	0.08
EQUATORI	10	1991 SEP	1992 JUN	0.18	0.30	0.31	0.00
TURKEY	11	1979 APR	1980 FEB	0.05	0.05	0.19	0.07
SOMALIA	11	1983 JUL	1984 MAY	0.07	0.09	0.18	0.06
NIGERIA	13	1988 JAN	1989 JAN	0.06	0.08	0.20	0.05
BURKINA_	14	1978 DEC	1980 JAN	0.05	0.06	0.11	0.02
SIERRA_L	15	1989 NOV	1991 JAN	0.05	0.08	0.18	0.07
SOMALIA	15	1988 AUG	1989 OCT	0.04	0.06	0.16	0.07
MONGOLIA	16	1992 MAR	1993 JUN	0.09	0.13	0.28	0.13
URUGUAY	17	1967 FEB	1968 JUN	0.05	0.06	0.16	0.08
POLAND	18	1989 AUG	1991 JAN	0.16	0.17	0.57	0.15

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[15] This may be at least partially due to measurement errors in the recorded inflation series if, for example, the variance of the measurement errors are an increasing function of the inflation rate.

[16] See D. Logue and T. Willet, "A note on the relation between the rate and variability of inflation," *Economica,* 43: 1976, 151-158, and A. Cukierman, *Central Bank Strategy, Credibility, and Independence: Theory and Evidence* (MIT Press: Cambridge, MA, 1992), page 439.

SURINAMnc	19	1992 OCT	1994 APR	0.06	0.07	0.22	0.10
URUGUAY	22	1972 APR	1974 JAN	0.05	0.08	0.19	0.06
BURMA	23	1973 JUL	1975 MAY	0.06	0.08	0.17	0.03
SEYCHELL	23	1971 JUL	1973 MAY	0.06	0.09	0.13	0.02
BURKINA_	27	1975 APR	1977 JUN	0.08	0.12	0.15	0.02
PERU	37	1988 JAN	1991 JAN	0.26	0.34	1.60	0.29
AFGHANIS	38	1988 JUL	1991 AUG	0.07	0.13	0.23	0.05
ARGENTIN	42	1982 JAN	1985 JUN	0.06	0.04	0.27	0.15
ARGENTIN	44	1987 AUG	1991 MAR	0.22	0.19	1.09	0.21
ROMANIAn	44	1990 NOV	1994 JUN	0.05	0.07	0.27	0.10
NICARAGnc	45	1985 DEC	1989 NOV	0.30	0.33	0.82	0.26
ZAIREnc	45	1990 OCT	1994 JUN	0.28	0.27	1.25	0.32
UGANDA	46	1984 OCT	1988 JUL	0.08	0.11	0.32	0.09
CHILE	50	1972 MAY	1976 JUN			0.63	
ZAMBIA	56	1988 DEC	1993 JUL	0.05	0.06	0.26	0.08
ARGENTIN	57	1974 DEC	1979 AUG	0.06	0.06	0.32	0.10
ISRAEL	58	1980 OCT	1985 JUL	0.05	0.05	0.24	0.09
BOLIVIA	74	1979 DEC	1986 JAN	0.19	0.16	1.04	0.15
SUDANnc	78	1988 JAN	1994 JUN	0.07	0.09	0.25	0.06
GUINEA_B	89	1986 APR	1993 AUG	0.07		0.26	0.04
BRAZILnc	133	1983 JUN	1994 JUN	0.11	0.06	0.59	0.17

As noted in the discussion of Table 1, the number of episodes included in Table 3 falls dramatically relative to Table 2 — despite splits in some of the episodes (e.g., the long Brazilian episode in Table 2 has been transformed into two shorter episodes in Table 3). The average standard deviation of inflation for the countries represented in Table 3 is 0.10 — substantially higher than either the entire IFS sample or those countries represented in Table 2. The average standard deviation for the 20 percent threshold episodes is even higher, at 0.14.[17]

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[17] The average (unweighted) mean inflation rate is .20; the average (unweighted) coefficient of variation is 0.80.

Table 3: Episodes at 20 percent threshold

COUNTRY	DURATION (MONTHS)	BEG	END	STD(π)	STD (dπ)	MAX π	MEAN π
POLAND	6	1989 AUG	1990 JAN	0.15	0.23	0.57	0.33
ROMANIA	6	1990 NOV	1991 APR	0.07		0.24	0.14
ARGENTIN	10	1975 JUL	1976 APR	0.09	0.10	0.32	0.19
ISRAEL	10	1984 OCT	1985 JUL	0.07	0.08	0.24	0.14
ARGENTIN	11	1984 AUG	1985 JUN	0.04	0.04	0.27	0.21
SUDAN	13	1991 JAN	1992 JAN	0.09	0.10	0.25	0.07
GUINEA_B	14	1991 JUL	1992 AUG	0.09	0.11	0.26	0.05
MONGOLIA	16	1992 MAR	1993 JUN	0.09	0.14	0.28	0.13
BRAZIL	19	1988 SEP	1990 MAR	0.15	0.07	0.59	0.29
CHILE	19	1972 AUG	1974 FEB			0.63	
GUINEA_B	19	1986 APR	1987 OCT	0.09		0.22	0.07
UGANDA	19	1987 JAN	1988 JUL	0.11	0.14	0.32	0.10
BOLIVIA	30	1983 AUG	1986 JAN	0.22	0.24	1.04	0.29
ARGENTIN	32	1988 JUL	1991 FEB	0.25	0.22	1.09	0.24
BRAZILnc	33	1991 OCT	1994 JUN	0.06	0.02	0.40	0.27
PERU	34	1988 MAR	1990 DEC	0.26	0.35	1.60	0.31
NICARAGnc	45	1986 MAR	1989 NOV	0.30	0.34	0.82	0.26
ZAIREnc	45	1990 OCT	1994 JUN	0.28	0.27	1.25	0.32

At a threshold of 30 percent, the number of episodes lasting 4 or more months falls to just 10. Two of the countries (Argentina and Brazil) have a pair of episodes, so that only 8 countries are represented in Table 4. In other words, only 6 percent of countries in the IFS sample (8 out of 132) have experienced a 30 percent inflation episode that lasted 4 or more months. Hyperinflations are indeed rare events. And, as is clear from the table, the average standard deviation of inflation during these episodes is substantially higher (0.20) than for the episodes defined using lower threshold levels.[18]

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[18] The average (unweighted) mean inflation rate is .31; the average (unweighted) coefficient of variation is 0.68 .

Table 4: Episodes at 30 percent threshold

COUNTRY	DURATION (MONTHS)	BEG	END	STD(π)	STD (dπ)	MAX π	MEAN π
POLAND	6	1989 AUG	1990 JAN	0.15	0.23	0.57	0.33
ARGENTIN	9	1975 JUL	1976 MAR	0.09	0.10	0.32	0.18
ARGENTIN	11	1989 MAY	1990 MAR	0.32	0.32	1.09	0.46
UGANDA	13	1987 JAN	1988 JAN	0.11	0.13	0.32	0.11
BRAZIL	15	1989 JAN	1990 MAR	0.17	0.08	0.59	0.31
BRAZILnc	15	1993 AUG	1994 JUN	0.03	0.01	0.40	0.34
BOLIVIA	18	1984 APR	1985 SEP	0.24	0.29	1.04	0.38
PERU	24	1988 SEP	1990 AUG	0.29	0.28	1.60	0.37
NICARAGnc	45	1986 MAR	1989 NOV	0.30	0.34	0.82	0.26
ZAIREnc	45	1990 NOV	1994 JUN	0.29	0.28	1.25	0.32

Finally, only six countries have experienced 40 percent episodes lasting 4 or more months, as indicated in Table 5. The standard deviation of inflation during these episodes is remarkably high: the average standard deviation is 0.26. The average mean inflation rate is 0.42. The average coefficient of variation, calculated as the simple average of each country's standard deviation divided by its mean inflation rate, is 0.65.[19] During a high inflation episode, recorded monthly inflation is quite volatile.[20]

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[19] Calculating the average coefficient of variation as the average standard deviation divided by the average mean inflation rate, which is not equivalent to the calculation in the text above because of Jensen's inequality, produces a very similar result (0.51).

[20] It is unclear whether *actual* inflation is as variable as *measured* inflation. The behavior of optimizing agents would presumably reflect the potential presence of measurement errors, even if such errors were unobservable to all relevant agents. We abstract away from such considerations in this study.

Table 5: Episodes at 40 percent threshold

COUNTRY	DURATION (MONTHS)	BEG	END	STD(π)	STD (dπ)	MAX π	MEAN π
BRAZIL	4	1989 DEC	1990 MAR	0.08	0.07	0.59	0.52
POLAND	4	1989 OCT	1990 JAN	0.19	0.27	0.57	0.35
ZAIREnc	8	1993 NOV	1994 JUN	0.47	0.40	1.25	0.61
ARGENTIN	11	1989 MAY	1990 MAR	0.32	0.32	1.09	0.46
ZAIRE	12	1991 OCT	1992 SEP	0.22	0.27	0.76	0.36
BOLIVIA	18	1984 APR	1985 SEP	0.24	0.29	1.04	0.38
NICARAGnc	45	1986 MAR	1989 NOV	0.30	0.34	0.82	0.26

Tables 1-5 suggest several conclusions. First, high inflation is rare. Second, most high inflation experiences are relatively short. Third, most economies only experience one high inflation, and most high inflations occur in countries experiencing only one such episode. Fourth, an economy experiencing high inflation is not necessarily fated to reach higher and higher rates of inflation: most high inflation episodes do not explode. Fifth, many episodes end without a large fall in inflation. Finally, the inflationary process during a high inflation episode is substantially more noisy than the inflationary process during low inflation.

III. Two models of delayed stabilizations

One of the principal stylized facts of high inflations is that stabilizations are delayed. The most popular model of such delays is given by Alesina and Drazen (1991).[21] Their model, which is introduced in Chapter 1 and is henceforth referred to as the AD model, argues that delays occur because of conflicts over how to apportion the costs of stabilization. More specifically, the model posits an economy comprising two groups and suffering from high inflation. Information is asymmetric, in that the preferences of each group are known only to members of that group. As time passes, the factions learn about each other's preferences, and ultimately one acquiesces to bearing a disproportionate share of the costs of stabilizing. Stabilization thus occurs when one group decides that the marginal benefit

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[21] A. Alesina and A. Drazen, "Why Are Stabilizations Delayed?" *American Economic Review*, December 1991, 1170-1188.

from stabilizing exceeds the marginal benefit from waiting. Anything that raises the benefit from stabilizing should therefore *ceteris paribus* cause a stabilization to occur more quickly.

An alternative explanation for observed delays, presented in Chapter 1, is based on a real options approach. If there is uncertainty over the future path of inflation, sunk costs create an incentive to delay stabilization. The fundamental prediction of such a model of stabilization as an irreversible investment under uncertainty, henceforth the IRRINV model, is that the higher the volatility of inflation, the longer governments wait before stabilizing.[22] This effect obtains because the higher the volatility of inflation, the more valuable the option to stabilize, and therefore the higher inflation must be before the option is exercised.[23] Alternatively, the higher the volatility of inflation, the more likely it is that a stabilization will *ex post* have been unnecessary, thus providing an incentive to wait.[24] The effect of volatility within the IRRINV model differs sharply from the effect within the AD model, which would predict a *negative* relationship between volatility and duration.[25]

The different predictions given by the IRRINV model and the AD model for the effect of higher volatility on the timing of stabilizations provide one method of empirically discriminating between them. But in both models, the volatility of inflation is just one of many variables that could affect delay. In particular, the drift rate of inflation is also relevant. The AD model unambiguously predicts a shorter war of attrition the higher the drift rate, since the costs of the war of attrition increase with the rate of inflation. The IRRINV model makes ambiguous predictions about the effect of a higher drift rate. In most configurations of the model, a higher drift rate

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[22] While this prediction does not hold uniformly across all parameter values in the model (see discussion in footnote describing Figure 4 in Chapter 1), it is a relatively robust result.

[23] The precise definition of the volatility of inflation will be discussed below.

[24] Assume the government wants to ensure that inflation falls below some threshold a. If inflation (π) at time t is normally distributed with mean μ and variance $\sigma^2 t$, then $P(\pi(t)<a)=\Phi\{(a-\mu)/\ \sigma^2 t\}$, where Φ is the cumulative standard normal distribution. A higher variance drives the standard normal variable toward zero. Thus if $a<\mu$, a higher variance raises the probability of observing an inflation rate below a. In the IRRINV model, inflation was typically modeled as a geometric Brownian motion, so that this analysis would be appropriate for the log of inflation, not inflation itself.

[25] Higher volatility presumably imposes higher welfare costs (through, for example, aggravated aggregate-relative confusion). The AD model would predict that the higher welfare costs, *ceteris paribus*, would shorten the war of attrition.

reduces the optimal stabilization threshold. But for some configurations and parameter values, the effect is reversed (see Figure 5 of Chapter 1). If the data suggest an inverse relationship between the drift rate and the delay, nothing can be concluded about the relative validity of the two models. But a positive relationship between the drift rate and the delay would necessarily contradict the AD model.

To test the predictions of the AD and IRRINV model, we use the high inflation data sets described in Section I and define the delay in stabilizing as the duration of the episode, measured in months.[26] This definition raises a potentially serious problem, because it is not clear that the end of an episode corresponds to a policy-induced stabilization; Table 1 suggests that many episodes end with relatively small drops in inflation. Indeed, one of the fundamental driving mechanisms of the IRRINV model is the possibility that inflation will wander below the threshold without a government-imposed stabilization. Appendix II discusses this issue more thoroughly; the conclusion is that it is unlikely to bias our results significantly.

The volatility of inflation is proxied in several different ways. The most appropriate measure given the assumptions of the basic IRRINV model in Chapter 1 is the standard deviation of the change in the log of monthly inflation. But that measure is appropriate only for a geometric stochastic process for inflation, and the fundamental prediction of the IRRINV model obtains for other stochastic processes as well.[27] For each episode, we therefore use several different measures of the volatility of inflation to test the IRRINV model in the broadest possible sense.[28] If the predictions of the model fail to obtain for *all* the measures of volatility, the model must be rejected. But a rejection for any one particular measure may reflect an

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[26] Note a subtle difference between the definition of delay in Chapter 1 and the one applied here. In Chapter 1, the delay was defined as the time until inflation reaches the optimal threshold, starting from the conventional threshold. Here, the delay is defined as the first-crossing time from any given level of inflation (in particular, the one defining a hyperinflationary episode) to the optimal threshold. Because the conventional threshold is unlikely to coincide precisely with the definition of a hyperinflation, these two definitions are not equivalent.

[27] For discrete stabilizations, the only requirement for the IRRINV model's fundamental precept is that there be some serial correlation in the inflation rate.

[28] All of our tests will be tests of the *joint* hypothesis of the validity of the IRRINV model and a class of stochastic processes for inflation. Because the data series for most episodes are so short, direct tests of the validity of any particular stochastic assumption for inflation suffer from a severe lack of power.

erroneous assumption about the stochastic process followed by inflation rather than a failure of the model itself. The specific volatility measures used in this study are:

- The standard deviation of the monthly inflation rate;
- The standard deviation of the change in the monthly inflation rate;
- The standard deviation of the percentage change in the monthly inflation rate; and
- The standard deviation of the change in the natural logarithm of monthly inflation.[29]

In order to test the AD and IRRINV models, we focus our attention on the relationships among duration, the volatility of inflation, and the drift rate of inflation. Other relevant variables (e.g., the degree of indexation within the economy, or the concentration of political power) will be assumed to be distributed independently of the volatility and drift rate factors within our sample.[30]

IV. Ordinary least squares analysis

As a first approach to analyzing the data and comparing the two models, this section uses ordinary least squares (OLS) regressions to examine the relationship between the duration of an episode and our measures of volatility and drift rates. Tables 6-9 present coefficient estimates, t-statistics, and summary statistics from OLS regressions; each table presents results for a different threshold. The regression takes the form:

$$D_i = c + \beta_1 \sigma_i + \beta_2 \alpha_i + \varepsilon_i$$

where D_i is the duration of episode i, c is a constant, σ_i is a measure of volatility for episode i, α_i is either the mean monthly change in inflation or

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[29] The last two measures should be approximately the same, with the approximation becoming more precise as the inflation rate goes to zero, since the percentage change in a variable is approximately equal to the first difference of the log of the variable (for small percentage changes). The inflation rates during high inflation episodes, however, are typically so high that there can be a substantial divergence between these two measures.

[30] This may not be a valid assumption. For example, it could be that economies subject to more volatile inflation are also the ones more likely to adopt indexation schemes — and therefore the ones in which any given rate of inflation is less distortionary.

the mean monthly change in log inflation for episode i, β_1 and β_2 are coefficients to be estimated, and ε_i is an error term.

In these regressions, no account is taken of censoring, so that D_i for uncompleted episodes is their uncompleted duration at the end of the available time series, and durations of 3 months or less are excluded. Such cross-sectional regressions seem particularly susceptible to heteroscedasticity, since it is not unreasonable to expect the variance of the error term to vary with the duration of the episode or with the explanatory variables. In the summary statistics reported for each regression, we therefore include the Cook-Weisberg test of heteroscedasticity.[31]

Table 6 presents regressions of duration for the 10 percent threshold. The standard deviation of inflation, the standard deviation of the change in inflation, and the standard deviation of the change in log inflation all attract positive coefficients, while the coefficient on the standard deviation of the percentage change in inflation is negative. These results offer only tenuous support for the IRRINV model: all of the coefficients on the volatility factors are statistically insignificant at the 5 percent significance level (t-statistics are in parentheses under each coefficient estimate).[32] The coefficient on the drift rate is negative when drift is defined as the mean change in inflation, and positive when drift is defined as the mean change in log inflation.

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[31] R.D. Cook and S. Weisberg, "Diagnostics for heteroscedasticity in regression," *Biometrika*, Vol. 70: 1970, 1-10. The test assesses multiplicative heteroscedasticity by using the OLS residuals to estimate an auxiliary regression of e_i^2 on a constant and a vector of possible explanatory variables (where e_i^2 is the square of the OLS residual for episode i). A variety of vectors were tested, but the results did not vary much. Since the fitted value of duration from the original OLS regression seemed the most intuitive explanatory variable, the tests reported in the tables reflect simple auxiliary regressions of the squared OLS residuals on a constant and the fitted values of duration. More precisely, the p-value given in the regression tables is the probability of observing the estimated coefficient on the fitted value of duration in the auxiliary regression, given that the true coefficient is zero. When the null hypothesis of a zero coefficient is rejected, the conclusion is that the variance of the error term varies (in a statistically significant manner) with the fitted value of duration from the original OLS regression. Despite our initial concerns, the Cook-Weisberg p-values indicate little evidence of multiplicative heteroscedasticity. Furthermore, hypothesis tests using White standard errors rather than OLS standard errors yield the same general conclusions as those in the text (except where specifically noted below).

[32] Statistical significance throughout this study will be evaluated at the 5 percent significance level unless explicitly noted otherwise.

Table 6: OLS regressions for 10 percent threshold

Regressand: duration of episode				
SD (inflation)	109.65			
	(1.79)			
SD (change in inflation)		28.75		
		(0.53)		
SD (change in log inflation)				1.23
				(0.21)
SD (percentage change in inflation)			-0.35	
			(-0.92)	
Mean change in inflation		-405.73	-2186.2	
		(-0.86)	(-3.78)	
Mean change in log inflation				84.54
				(2.13)
N	41	40	31	31
F-test (p-value)	0.08	0.59	0.00	0.11
R^2	0.08	0.03	0.35	0.14
Cook-Weisberg test (p-value)	0.48	0.58	0.06[33]	0.15

At the 20 percent threshold, the coefficients on the standard deviation of inflation and the standard deviation of the change in inflation, which remain positive, become statistically significant (Table 7). In terms of explanatory power, cross-sectional variation in the standard deviation of the change in inflation and the mean change in inflation explain 65 percent of the cross-sectional variation in duration, as shown by the second regression. But as is evident from the third and fourth regressions, the coefficients on the other two volatility proxies remain statistically insignificant. The coefficient on the mean change in inflation remains negative and the coefficient on the mean change in log inflation remains positive, as in Table 6. But the drift coefficient is not statistically significant except in the second regression.

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[33] Using White standard errors rather than OLS standard errors changes the t-statistic on the estimated coefficient for the standard deviation of the percentage change in inflation to -2.05, and the t-statistic on the estimated coefficient for the mean change in inflation to -4.94.

Table 7: OLS regressions for 20 percent threshold

Regressand: duration of episode				
SD (inflation)	117.45			
	(5.22)			
SD (change in inflation)		72.27		
		(3.57)		
SD (change in log inflation)				-7.86
				(-0.71)
SD (percentage change in inflation)			0.57	
			(0.40)	
Mean change in inflation		-331.90	-192.46	
		(-2.96)	(-1.41)	
Mean change in log inflation				104.10
				(0.85)
N	17	15	12	8
F-test (p-value)	0.00	0.00	0.30	0.70
R^2	0.65	0.65	0.23	0.13
Cook-Weisberg test (p-value)	0.26	0.39	0.33	0.61

At the 30 percent threshold, the coefficient on the standard deviation of inflation remains positive and statistically significant at the 10 percent significance level. The coefficients on all the other volatility proxies are statistically insignificant (see Table 8). The coefficients on the drift rate terms are negative, but once again statistically significant only in the second regression.

Table 8: OLS regressions for 30 percent threshold

Regressand: duration of episode				
SD (inflation)	78.34			
	(1.98)			
SD (change in inflation)		62.07		
		(1.76)		
SD (change in log inflation)				-14.47
				(-1.73)
SD (percentage change in inflation)			-0.24	
			(-0.07)	
Mean change in inflation		-265.74	-29.20	
		(-2.08)	(-0.21)	
Mean change in log inflation				-119.95
				(-1.33)
N	10	8	7	6
F-test (p-value)	0.08	0.22	0.98	0.29
R^2	0.33	0.45	0.01	0.56
Cook-Weisberg test (p-value)	0.09	0.11	0.25	0.28

Finally, at the 40 percent threshold level, all the coefficients on the volatility regressors are positive. But only the coefficients on the standard deviation of the change in log inflation and the standard deviation of the percentage change in inflation are statistically significant. These results contrast with those in Table 7, where the first two volatility proxies had higher t-statistics than the second two. The coefficients on the drift rate terms are negative and statistically significant in the third and fourth regressions.

Table 9: OLS regressions for 40 percent threshold

Regressand: duration of episode				
SD (inflation)	27.76			
	(0.54)			
SD (change in inflation)		68.96		
		(0.84)		
SD (change in log inflation)				8.54
				(2.81)
SD (percentage change in inflation)			4.02	
			(6.87)	
Mean change in inflation		41.43	-138.33	
		(0.36)	(-7.18)	
Mean change in log inflation				-37.09
				(-3.51)
N	7	7	5	5
F-test (p-value)	0.61	0.71	0.01	0.11
R^2	0.06	0.16	0.99	0.89
Cook-Weisberg test (p-value)	0.39	0.27	0.97	0.16

In summary, the OLS regressions provide weak evidence of some positive association between duration and volatility. The relationship is certainly not robust to changes in the threshold level or the definition of volatility. Nevertheless, all the statistically significant results do support a positive volatility-duration effect.[34] The results also generally point to a negative drift rate-duration effect.

V. Hazard rate analysis

Two major shortcomings of the simple OLS regressions reported above are that they could produce negative predicted durations, a result that

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[34] The results for episodes defined to end the month before inflation remains below the threshold for 6 months similarly provide weak support for a positive correlation.

is difficult to interpret, and that they ignore censoring, which obtains because we treated the episodes that had not been completed at the end of the available time series as if they had been completed. A slightly more sophisticated approach, which ensures non-negative predicted durations and easily accommodates censored data, is to use a "hazard rate" or "duration model" approach. We briefly review the terminology and theory of hazard analysis before presenting our results.[35]

Assume that the duration of an episode is represented by the random variable T, and suppose that T has probability density f(t). Then the cumulative distribution function is given by:

$$F(t) = \int_{s=0}^{s=t} f(s)ds = \text{Prob}(T \leq t)$$

F(t) thus represents the probability that the duration T is less than or equal to t. Hazard rate analysis also relies on two other related concepts, the survival function and the hazard rate. The survival function, S(t), is given by 1-F(t) and represents the probability that the duration is greater than or equal to t.[36] Finally, the hazard rate, $\lambda(t)$, represents the conditional probability of the episode's ending immediately after t, given that it has survived up until to t. More explicitly, the hazard function is given by:

$$\lambda(t) = \lim_{\varepsilon \to 0} \frac{\text{Prob}(t \leq T \leq t+\varepsilon)}{\varepsilon} = \lim_{\varepsilon \to 0} \frac{F(t+\varepsilon) - F(t)}{\varepsilon S(t)} = \frac{f(t)}{S(t)} = -\frac{d \ln S(t)}{dt}$$

There are a variety of different models of the hazard rate (and therefore of the survival function and cumulative distribution function). The most popular are the Weibull and the exponential models. The Weibull hazard function is given by:

$$\lambda(t) = \lambda p(\lambda t)^{p-1}$$

where p determines whether the hazard rate is increasing (p>1) or decreasing

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[35] For a comprehensive presentation of duration analysis, see J. D. Kalbfleisch and R. L. Prentice, *The Statistical Analysis of Failure Time Data* (John Wiley: New York, 1980).

[36] For continuous density functions f(t), the probability of a duration exactly equal to t is zero. So both F(t) and S(t) can be defined with weak inequalities.

(p<1) over time. Since $\lambda(t)$ is also equal to the negative of the time derivative of ln(S(t)), we can solve for S(t):

$$\frac{d\ln S(t)}{dt} = -\lambda p(\lambda t)^{p-1} \Rightarrow \ln S(t) = K - (\lambda t)^p \Rightarrow S(t) = e^{K-(\lambda t)^p}$$

Since S(0)=1, K=0. Therefore:

$$S(t) = e^{-(\lambda t)^p}$$

The exponential hazard function is the Weibull function with p=1. In that case, the hazard rate is a constant over time, and the survival function is simply $S(t)=e^{-\lambda t}$.[37] Thus far the analysis has emphasized the *form* of the hazard rate. But our primary concern is the impact of our volatility and drift rate variables on the *level* of the hazard rate. Such exogenous covariates do not change the shape of the hazard rate as a function of time. Instead, we can think of the covariates as shifting the entire hazard function up or down; we are interested in the direction and size of this shift. In particular, we assume a proportional hazards model, so that the hazard function for episode i can be written as:

$$\lambda_i(t) = p(\lambda t)^{p-1} e^{\beta x_i}$$

where x is a vector of time-invariant variables and β is a set of coefficients to be estimated.[38] It is assumed that the first component of the x vector is a set of 1's, so that $\lambda=\exp(\beta_1)$. The effect of the covariates is thus to scale up

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[37] There are a plethora of possible hazard functions in addition to the Weibull (and the associated exponential), and for each application of hazard analysis, it is necessary to justify the use of one distribution over another. See J.D. Kalbfleisch and R.L. Prentice, *The Statistical Analysis of Failure Time Data,* op. cit., pages 21-30, or W. Greene, *Econometric Analysis* (Macmillan Publishing Company: New York, 1993), page 718. One test for the Weibull is derived from its survival function, which is given by $\exp\{(-\lambda t)^p\}$. Therefore $\log(S(t))=-(\lambda t)^p$, and $\log(-\log(S(t))=p\log(\lambda)+p\log(t)$. Since p is a constant, a graph of log(-logS(t)) against log(t) would yield a straight line with intercept $p\log(\lambda)$ and slope p. A simple test for the appropriateness of the Weibull is thus to plot log(-logS(t)) against log(t). If the underlying data are consistent with the Weibull, then this plot should yield a straight line. If the line has slope 1, moreover, the exponential distribution is appropriate. Such "loglog" plots for the various thresholds used in this paper indicate that the Weibull assumption seems reasonable.

[38] Formally, adapting the model in this way is equivalent to changing the unit of time. The model with covariates is therefore sometimes referred to as the "accelerated failure time" model. See J.D. Kalbfleisch and R.L. Prentice, *The Statistical Analysis of Failure Time Data,* op. cit., pages 33-34, or W. Greene, *Econometric Analysis*, op. cit., page 721.

or down the Weibull hazard rate.

Estimation proceeds via maximum likelihood. Defining $\delta_i=1$ if episode i has been completed and $\delta_i=0$ if the episode has been censored, defining t_i as the duration of episode i, assuming an independent censoring mechanism,[39] and letting s=1/p be the so-called shape parameter, the log-likelihood of the observed sample is:[40]

$$\ln L = \sum_i \delta_i \ln\left\{\frac{1}{s}\exp\left[\frac{\ln t_i + \beta' x_i}{s} - e^{\frac{\ln t_i+\beta' x_i}{s}}\right]\right\} + (1-\delta_i)\ln\left\{\exp\left[-e^{\frac{\ln t_i+\beta' x_i}{s}}\right]\right\}$$

which can be written as:

$$\ln L = \sum_i \delta_i\left(\frac{\ln t_i + \beta' x_i}{s} - \ln s\right) - e^{\frac{\ln t_i+\beta' x_i}{s}}$$

This log-likelihood is then maximized over the parameters s and the components of β by using standard non-linear optimization techniques.[41]

Tables 10-14 present the results for the various thresholds and a variety of covariates (the coefficients on the constant term are not shown; similar results obtain with heteroscedasticity-corrected standard errors).[42] A positive coefficient on a covariate indicates that the partial effect of that variable is to raise the hazard rate and thus reduce the expected survival time.[43] Conversely, a negative coefficient indicates that the variable reduces the hazard rate and raises the expected survival time. The t-statistics reflect standard errors conditional on the value of the estimated shape parameter.

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[39] Independent censoring basically requires that episodes are not censored, at any given duration t, because "they appear to be at unusually high (or low) risk of failure." See J.D. Kalbfleisch and R.L. Prentice, *The Statistical Analysis of Failure Time Data,* op. cit., pages 40-41. Because censoring in our data arises solely because a time series ends, rather than because episodes are purposefully excluded, independent censoring seems a reasonable assumption to adopt. Nevertheless, the assumption could be fallacious if, for example, countries with very low probabilities of ending a hyperinflationary episode tended to drop out of (or be dropped from) the *International Financial Statistics* database.

[40] See W. Greene, *Econometric Analysis*, op. cit., page 721.

[41] Estimation was undertaken using the Newton-Raphson procedure built into the STATA econometric package.

[42] The χ^2 p-values presented in the tables are the probability of observing a χ^2 value of $2(L_1-L_0)$ under a null hypothesis that all the additional (non-intercept) variables have zero coefficients.

[43] The expected survival time for the Weibull distribution is $(1/\lambda)^p$.

The "pseudo-R^2" statistics are defined as $1-L_1/L_0$, where L_1 is the log-likelihood of the estimated regression and L_0 is the log-likelihood of a constant-only regression.

As Table 10 indicates, the higher the standard deviation of inflation, the lower the hazard rate, and therefore the longer the duration.[44] The standard deviation of the change in inflation also enters with a negative sign, but is not statistically significant. The other two volatility proxies enter with positive coefficients, implying a negative impact on duration, but again are not statistically significant. The regressions also suggest that the higher the mean change in monthly inflation, the higher the hazard rate, a result that is statistically significant. The shape parameters are consistently less than one ($p>1$), so the hazard rate is increasing in time.

Table 10: Weibull regressions for 10 percent threshold, hazard rate form

SD (inflation)	-7.42			
	(-2.02)			
SD (change in inflation)		-2.95		
		(-1.05)		
SD (change in log inflation)				0.89
				(1.06)
SD (percentage change in inflation)			0.02	
			(.50)	
Mean change in inflation		234.51	189.22	
		(7.12)	(5.37)	
Mean change in log inflation				-12.39
				(-6.08)
N	41	40	31	17
χ^2-test (p-value)	0.02	0.00	0.00	0.00
Pseudo R^2	0.04	0.30	0.24	0.34
s	0.90	0.65	0.07	0.58

At the 20 percent threshold, the standard deviation of inflation and the standard deviation of the change in inflation continue to enter with negative coefficients, and are statistically significant. The other two volatility parameters continue to lack statistical significance.

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[44] The results presented in Tables 10-14 are similar to those obtained from both exponential and Cox regressions, which are available upon request to the author.

Table 11: Weibull regressions for 20 percent threshold, hazard rate form

SD (inflation)	-10.10			
	(-3.25)			
SD (change in inflation)		-5.13		
		(-1.98)		
SD (change in log inflation)				1.31
				(1.07)
SD (percentage change in inflation)			-0.04	
			(-0.24)	
Mean change in inflation		59.92	52.97	
		(4.81)	(3.87)	
Mean change in log inflation				-16.85
				(-1.48)
N	17	15	12	8
χ^2-test (p-value)	0.00	0.00	0.05	0.45
Pseudo R^2	0.26	0.38	0.25	0.09
s	0.49	0.40	0.35	0.48

The results are similar at the 30 percent threshold, except that the *positive* coefficient on the standard deviation of the change in log inflation becomes statistically significant (see the final column of Table 12). Such a positive coefficient contradicts the basic prediction of the IRRINV model.

Table 12: Weibull regressions for 30 percent threshold, hazard rate form

SD (inflation)	-7.12			
	(-2.05)			
SD (change in inflation)		-11.20		
		(-2.42)		
SD (change in log inflation)				5.83
				(3.73)
SD (percentage change in inflation)			0.03	
			(0.06)	
Mean change in inflation		39.63	-8.84	
		(2.39)	(-0.31)	
Mean change in log inflation				50.85
				(3.51)
N	10	8	7	6
χ^2-test (p-value)	0.10	0.03	0.88	0.04
Pseudo R^2	0.23	0.38	0.06	0.44
s	0.60	0.42	0.35	0.22

At the 40 percent threshold, all of the regressions are highly significant (see the row of χ^2 statistics for the probability that all of the non-intercept coefficients are zero), all of the volatility parameters enter with negative coefficients, and all of the drift rates enter with positive coefficients. These regressions offer perhaps the strongest evidence in favor of the IRRINV model, but they are based on an extremely limited number of observations.[45]

Table 13: Weibull regressions for 40 percent threshold, hazard rate form

SD (inflation)	-19.21 (-3.98)			
SD (change in inflation)		-16.79 (-3.39)		
SD (change in log inflation)				-16.81 (-12.14)
SD (percentage change in inflation)			-2.82 (-1.77)	
Mean change in inflation		237.92 (14.11)	177.11 (4.40)	
Mean change in log inflation				66.77 (20.07)
N	7	7	5	5
χ^2-test (p-value)	0.01	0.00	0.00	0.00
Pseudo R^2	0.43	0.86	0.77	0.86
s	0.54	0.13	0.12	0.07

In sum, the Weibull regressions presented in this section offer mixed support for the IRRINV model. Higher volatility tends to be associated with a lower hazard function, and thus a longer duration, but the results are not robust across all threshold levels and definitions of volatility. Furthermore, while the Weibull regressions ensure non-negative predicted durations and account for censored observations, and are thus superior to the OLS regressions presented above, they are certainly not immune from criticism. Perhaps most important, they assume that the volatility parameters are time-invariant variables. But if volatility is time-varying, the Weibull regressions presented above are invalid. Appendix III explores this issue in more detail by splitting the episodes into 6-month intervals and examining the behavior of the volatility parameter across these intervals. It concludes that the

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[45] Another potential error is that a Weibull distribution may not be an appropriate assumption for these episodes. But the results seem robust to an error of this type, since Cox regressions produce similar estimates.

assumptions upon which the OLS and hazard rate analysis are predicated appear to obtain, although the evidence is admittedly mixed.

VI. Conclusion

This chapter has constructed a monthly database of high inflation experiences. These experiences are rare, do not necessarily explode into ever higher inflation, often end without a large fall in inflation, and usually visit a country only once. They are also very noisy: inflation tends to be much more variable during a high inflation episode than it is, on average, across the set of IFS countries.

The chapter has also attempted to discriminate between the Alesina-Drazen model of delayed stabilizations and an irreversible investment under uncertainty model. The irreversible investment model generally predicts a positive relationship between the volatility of inflation and the delay before stabilization occurs, whereas the Alesina-Drazen model predicts the opposite. Simple OLS regressions provide tenuous support for a positive volatility-duration nexus, as do hazard rate regressions using a Weibull distribution. The conclusion is that there seems to be weak evidence in support of the irreversible investment model.

Appendix I. Summary statistics of monthly inflation rates by country

Country	Months of data	Mean π	Std. dev. π	Min π	Max π
UNITED_S	454	0.0037	0.0032	-0.0046	0.0179
UNITED_K	452	0.0056	0.0068	-0.0163	0.0422
AUSTRIA	452	0.0033	0.0074	-0.0267	0.0500
BELGIUM	453	0.0035	0.0038	-0.0125	0.0161
DENMARK	333	0.0055	0.0067	-0.0258	0.0572
FRANCE	454	0.0050	0.0044	-0.0086	0.0328
GERMANY	454	0.0027	0.0033	-0.0130	0.0123
ITALY	450	0.0063	0.0060	-0.0086	0.0310
LUXEMBOU	453	0.0033	0.0043	-0.0096	0.0196
NETHERLA	453	0.0035	0.0071	-0.0285	0.0464
NORWAY	453	0.0048	0.0058	-0.0141	0.0557
SWEDEN	453	0.0051	0.0060	-0.0054	0.0328
SWITZERL	454	0.0029	0.0039	-0.0074	0.0207
CANADA	453	0.0039	0.0040	-0.0076	0.0259
JAPAN	453	0.0039	0.0078	-0.0156	0.0399
FINLAND	453	0.0054	0.0058	-0.0158	0.0366
GREECE	453	0.0087	0.0136	-0.0214	0.0578
ICELAND	142	0.0139	0.0163	-0.0156	0.0999
MALTA	452	0.0028	0.0109	-0.0677	0.0953
PORTUGAL	453	0.0089	0.0126	-0.0277	0.0848
SPAIN	453	0.0075	0.0139	-0.1521	0.1791
TURKEY	309	0.0292	0.0292	-0.0644	0.2208
SOUTH_AF	453	0.0071	0.0059	-0.0062	0.0412
ARGENTIN	453	0.0649	0.0986	-0.0618	1.0873
BOLIVIA	444	0.0328	0.0953	-0.1043	1.0394
BRAZIL	452	0.0731	0.0916	-0.0328	0.5919
CHILE	453	0.0307	0.0479	-0.0840	0.6289
COLOMBIA	444	0.0141	0.0138	-0.0605	0.0779
COSTA_RI	430	0.0091	0.0143	-0.0522	0.1019
DOMINICA	443	0.0088	0.0222	-0.0668	0.1374
ECUADOR	453	0.0138	0.0174	-0.0410	0.1003
EL_SALVA	452	0.0075	0.0118	-0.0346	0.0466
GUATEMAL	451	0.0066	0.0201	-0.0620	0.1347
HAITI	416	0.0048	0.0275	-0.0944	0.1826
HONDURAS	453	0.0056	0.0111	-0.0391	0.0523
MEXICO	453	0.0170	0.0222	-0.0175	0.1438
NICARAGU	56	0.2211	0.3003	-1.2132	0.8180
PANAMA	238	0.0026	0.0053	-0.0094	0.0519
PARAGUAY	448	0.0101	0.0185	-0.0600	0.1305
PERU	454	0.0469	0.1059	-0.0282	1.6033
URUGUAY	453	0.0359	0.0291	-0.0566	0.1852
VENEZUEL	453	0.0103	0.0181	-0.0586	0.1928
ANTIGUA_	95	0.0080	0.0151	-0.0291	0.0476
BAHAMAS	265	0.0050	0.0047	-0.0173	0.0261
ARUBA	103	0.0037	0.0024	0.0000	0.0144
BARBADOS	347	0.0066	0.0127	-0.0761	0.0892

DOMINIC1	341	0.0062	0.1751	-2.2467	2.2879
GRENADA	219	0.0065	0.0104	-0.0343	0.0553
GUYANA	407	0.0071	0.0191	-0.0523	0.2887
JAMAICA	449	0.0114	0.0157	-0.0239	0.1256
NETHERL1	315	0.0045	0.0050	-0.0106	0.0230
ST__KITT	167	0.0038	0.0074	-0.0138	0.0405
ST__LUCI	363	0.0059	0.0104	-0.0255	0.0586
S_VINCEN	191	0.0057	0.0100	-0.0297	0.0448
SURINAME	303	0.0155	0.0328	-0.0589	0.2193
TRINIDAD	449	0.0066	0.0094	-0.0299	0.0632
BAHRAIN	212	0.0036	0.0161	-0.0428	0.0855
CYPRUS	450	0.0038	0.0119	-0.0306	0.1387
IRAN__I_	417	0.0080	0.0163	-0.0443	0.0562
ISRAEL	453	0.0247	0.0376	-0.0493	0.2430
JORDAN	224	0.0061	0.0186	-0.0662	0.0785
KUWAIT	220	0.0041	0.0111	-0.0307	0.0482
SAUDI_AR	174	0.0004	0.0059	-0.0491	0.0179
SYRIAN_A	414	0.0076	0.0305	-0.1391	0.1269
EGYPT	453	0.0073	0.0190	-0.0725	0.0946
AFGHANIS	151	0.0090	0.0646	-0.3209	0.2276
BANGLADE	241	0.0072	0.0198	-0.0604	0.1073
BURMA	364	0.0084	0.0277	-0.1199	0.1742
SRI_LANK	453	0.0059	0.0113	-0.0357	0.0853
HONG_KON	306	0.0067	0.0128	-0.0463	0.0541
INDIA	451	0.0061	0.0109	-0.0290	0.0420
INDONESI	319	0.0100	0.0207	-0.1252	0.2003
KOREA	297	0.0082	0.0101	-0.0193	0.0515
MALAYSIA	452	0.0025	0.0060	-0.0151	0.0318
MALDIVES	29	0.0041	0.0509	-0.1507	0.0810
NEPAL	371	0.0071	0.0196	-0.0680	0.0826
PAKISTAN	453	0.0060	0.0126	-0.0645	0.0821
PHILIPPI	453	0.0082	0.0128	-0.0359	0.0856
SINGAPOR	318	0.0031	0.0090	-0.0197	0.0629
THAILAND	354	0.0048	0.0079	-0.0163	0.0419
ALGERIA	236	0.0104	0.0254	-0.0510	0.1095
BOTSWANA	232	0.0091	0.0067	-0.0087	0.0470
BURUNDI	248	0.0083	0.0229	-0.0472	0.1576
CAMEROON	266	0.0072	0.0197	-0.1249	0.0794
CAPE_VER	31	0.0042	0.0138	-0.0465	0.0298
CENTRAL_	158	0.0025	0.0197	-0.0399	0.0922
CHAD	113	-0.0003	0.0349	-0.0933	0.0934
CONGO	348	0.0049	0.0173	-0.0808	0.0896
ZAIRE	91	0.1959	0.2495	-0.0189	1.2528
EQUATORI	83	-0.0012	0.0775	-0.3106	0.3106
ETHIOPIA	339	0.0058	0.0244	-0.0592	0.0879
GABON	341	0.0046	0.0186	-0.1262	0.1083
GAMBIA__	398	0.0074	0.0286	-0.0856	0.1204
GHANA	378	0.0221	0.0417	-0.2138	0.2163
GUINEA_B	102	0.0380	0.0668	-0.1140	0.2622
COTE_D_I	407	0.0052	0.0238	-0.0988	0.1555

KENYA	321	0.0104	0.0161	-0.0197	0.1227
LIBERIA	269	0.0059	0.0161	-0.0581	0.1285
MADAGASC	368	0.0093	0.0187	-0.0947	0.1059
MALAWI	173	0.0131	0.0250	-0.0723	0.1106
MALI	77	0.0001	0.0136	-0.0369	0.0374
MAURITAN	108	0.0060	0.0233	-0.0840	0.0993
MAURITIU	387	0.0068	0.0139	-0.0277	0.1398
MOROCCO	452	0.0046	0.0103	-0.0381	0.0406
MOZAMBIQ	13	0.0439	0.0317	-0.0053	0.0888
NIGER	314	0.0049	0.0309	-0.1842	0.1443
NIGERIA	413	0.0124	0.0250	-0.1066	0.2042
ZIMBABWE	200	0.0131	0.0172	-0.0166	0.1451
RWANDA	344	0.0068	0.0176	-0.0592	0.0812
SEYCHELL	304	0.0063	0.0269	-0.1057	0.1278
SENEGAL	315	0.0059	0.0254	-0.0735	0.1188
SIERRA_L	96	0.0415	0.0520	-0.0697	0.2162
SOMALIA	313	0.0184	0.0355	-0.0563	0.1788
NAMIBIA	26	0.0076	0.0061	-0.0064	0.0180
SUDAN	449	0.0185	0.0463	-0.1555	0.2493
SWAZILAN	333	0.0089	0.0162	-0.1032	0.1280
TANZANIA	9	0.0583	0.0728	0.0017	0.2223
TOGO	287	0.0048	0.0237	-0.0752	0.1261
TUNISIA	87	0.0051	0.0045	-0.0051	0.0220
UGANDA	162	0.0445	0.0637	-0.1094	0.3635
BURKINA_	423	0.0044	0.0380	-0.1427	0.1467
ZAMBIA	320	0.0249	0.0359	-0.0218	0.2583
SOLOMON_	198	0.0092	0.2312	-2.2748	2.3057
FIJI	310	0.0064	0.0115	-0.0110	0.1462
WESTERN_	333	0.0077	0.0164	-0.0496	0.0775
TONGA	56	0.0047	0.0152	-0.0321	0.0355
CHINA__P	14	0.0170	0.0087	0.0030	0.0373
CZECHOSL	23	0.0134	0.0159	-0.0009	0.0688
HUNGARY	222	0.0097	0.0156	-0.0148	0.0890
MONGOLIA	37	0.0829	0.0754	-0.0134	0.2842
POLAND	81	0.0627	0.0907	0.0006	0.5728
ROMANIA	47	0.0926	0.0552	0.0157	0.2657

Appendix II. The definition of "delay" and the theoretical predictions from an irreversible investment model

By identifying the duration of an episode as the delay before stabilizing, we are implicitly assuming that all episodes end with a stabilization, an assumption that may be invalid. It is not even clear that the IRRINV model predicts a positive relationship between the delay as measured here and the volatility of inflation. The model's predictions focus on the delay until hitting some upper boundary (the optimal stabilization threshold), not the delay until hitting either that upper boundary *or* some lower boundary.

There are two possible solutions to this problem. First, we can examine whether the model predicts a positive relationship between the volatility of inflation and the delay as defined here. Unfortunately, Monte Carlo simulations suggest that it is not possible to derive an unambiguous prediction.[46]

To conduct the Monte Carlo simulations, 1,000 geometric random walks were simulated for a given variance of the innovation to the random walk. For each of these 1,000 random walks, the waiting time until either (1) hitting the optimal threshold, or (2) hitting the lower threshold and then remaining below it for 12 months, was recorded. These waiting times were then averaged. The entire process was then repeated for a higher variance.

Figures A1 and A2 present the Monte Carlo results. They suggest that if the drift rate is small enough, and if the process starts above but sufficiently close to the lower threshold, the average waiting time until the end of an episode is *lower* the higher the variance of the change in log inflation (see the bottom series of Figure A1). But if the drift rate is sufficiently positive, or if the process starts sufficiently close to the upper boundary, the average waiting time until the end of the episode is higher the higher the variance (see the second series in Figure A1). Finally, the Monte Carlo simulations confirm that duration is increasing in volatility in the absence of a lower boundary (i.e., when the lower boundary is zero). Figure A2 shows the percentage of the simulations hitting the optimal threshold —

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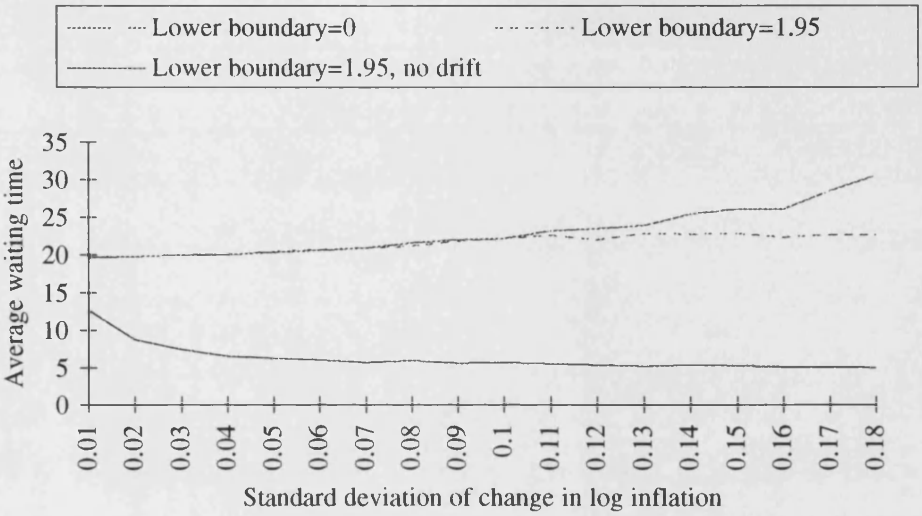
[46] An analytical solution involves a complicated boundary condition involving the arccos law. We therefore resort to Monte Carlo simulations to study the relationship.

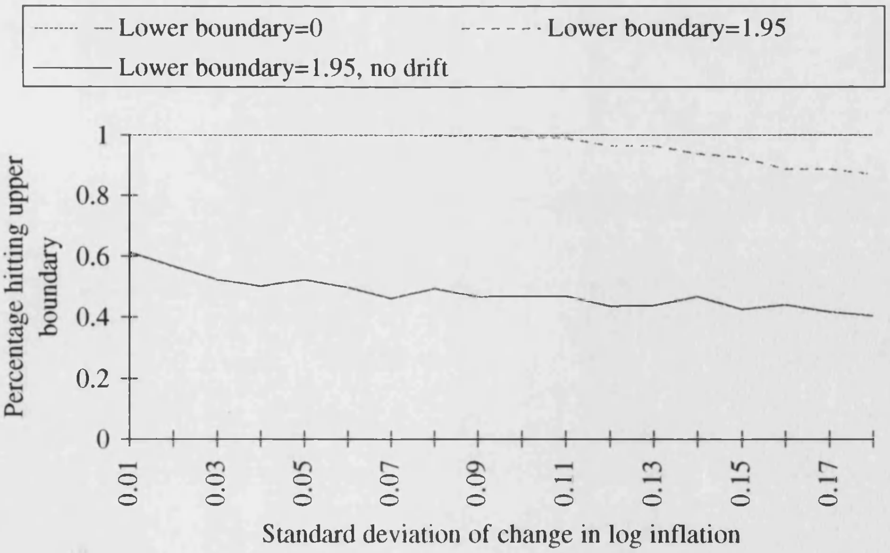
that is, the percentage of completed episodes corresponding to "stabilizations." Not surprisingly, this result is a function of the parameters of the model and starting position of inflation relative to the two boundaries.

Figure A1: Average waiting time vs. standard deviation



Note: The figure illustrates simulations assuming ρ=.1, a starting value of π of 0.2, a conventional threshold of 2, and a drift rate of 0.075 (unless otherwise noted). For each volatility parameter, 1000 geometric random walks were generated, and the average waiting time until either (a) hitting the optimal threshold or (b) hitting the lower boundary and then remaining under that boundary for 12 periods, was observed. The optimal threshold is predicated on the fixed cost model of Chapter 1.

Figure A2: Probability of hitting upper boundary



Note: The figure shows the percentage of simulated inflationary episodes that hit the optimal threshold before the lower boundary (and then remain under the lower boundary for 12 periods).

A second possible approach is to filter out the episodes that do not correspond to policy-induced stabilizations by applying various tests. For example, if the drop in inflation between the final month of the episode and the first month following the episode is sufficiently large, then it may seem more likely that a stabilization occurred (since very large disinflationary shocks are unlikely to obtain serendipitously). After applying such a filter, we can study the relationship between the delay and the volatility of inflation. In practice, such adjustments do not seem to alter significantly the results presented in the text above. As one example, Table A1 presents the results of the Weibull regressions at the 10 percent threshold for those cases in which the post-sample change in inflation exceeded (in absolute value) the standard deviation of the change in inflation during the episode.[47] Table A2 reproduces Table 10, and gives the corresponding estimates from the full sample of episodes. Filtering the data to exclude those with small drops in inflation changes some of the point estimates, but none of the general conclusions.

The results are similar for other thresholds, estimation procedures, and reasonable filtering levels. Since the results do not seem to be strongly affected by whether or not the filtering is undertaken, it seems unlikely that our conclusions are strongly affected by conflating non-stabilizations with policy-induced stabilizations.

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[47] In the construction of Table A1, the mean change in inflation over the episode was *not* subtracted from the post-episode change before comparison to the standard deviation. If the mean change in inflation is subtracted, so that the metric of a large fall is the same as in Section I of the paper, the results are nevertheless broadly similar. Relative to the full sample, the restricted sample point estimates change (with some that were previously statistically significant becoming statistically insignificant), but the general conclusions are unaffected. The most important coefficient changes involve the standard deviation of the change in inflation, but even these changes are not substantial enough to affect the conclusions. For example, in the Weibull regressions at the 10 percent threshold, the coefficient on the standard deviation of the change in inflation moves from a statistically insignificant value of -2.77 for the full sample to a statistically insignificant value of 38.66 for the sample restricted using the mean-corrected metric. The other changes tend to be less dramatic. Essentially, regardless of the filter applied, the limited number of "non-stabilizations" do not seem to be driving the results obtained in the main section of the text.

Table A1: Weibull regressions for 10 percent threshold episodes with post-sample fall in inflation greater (in absolute value) than the std. dev. of change in inflation

Regressand: duration of episode				
SD (inflation)	-16.2			
	(-1.59)			
SD (change in inflation)		-10.02		
		(-1.50)		
SD (change in log inflation)				0.61
				(1.39)
SD (percentage change in inflation)			-0.21	
			(-1.44)	
Mean change in inflation		298.60	277.90	
		(6.26)	(5.90)	
Mean change in log inflation				-16.60
				(-5.28)
N	17	16	16	18
χ^2-test (p-value)	0.04	0.00	0.00	0.00
Pseudo R^2	0.09	0.41	0.40	0.34
s	0.78	0.48	0.48	0.55

Table A2 (Table 10): Weibull regressions for 10 percent threshold

Regressand: duration of episode				
SD (inflation)	-7.42			
	(-2.02)			
SD (change in inflation)		-2.95		
		(-1.05)		
SD (change in log inflation)				0.89
				(1.06)
SD (percentage change in inflation)			0.02	
			(.50)	
Mean change in inflation		234.51	189.22	
		(7.12)	(5.37)	
Mean change in log inflation				-12.39
				(-6.08)
N	41	40	31	17
χ^2-test (p-value)	0.02	0.00	0.00	0.00
Pseudo R^2	0.04	0.30	0.24	0.34
s	0.90	0.65	0.07	0.58

Appendix III. Analysis of 6-month intervals within episodes

As noted in Section V, the analysis in the main text is predicated on a time-invariant volatility parameter over an episode. One method of testing whether this assumption obtains is to split each episode into intervals, calculate the volatility parameter for each interval, and then test whether there is a statistically significant difference among them. This appendix therefore takes each episode, splits it into 6-month intervals, and conducts a variety of tests on these intervals.

We focus our attention on the first two volatility parameters: the standard deviation of inflation and the standard deviation of the change in inflation. One intuitively appealing test of whether these variables are constant is to conduct a regression, for each episode, of volatility during interval t on t itself. In other words, for each episode i we regress:

$$\sigma_t = c + \beta t + \varepsilon_t$$

where σ_t is the volatility parameter estimated over interval t, c is a constant, t is the interval number, β is a coefficient to be estimated, and ε_t is the error term for interval t. If our estimate of β is statistically insignificant, we conclude that there is no obvious linear relationship between the volatility parameter and time (measured in 6-month intervals.) The results presented in Table A3 suggest that the volatility parameters exhibit no statistically significant duration dependence.

Table A3: OLS regressions by episode of σ_t on a constant and t

Threshold	Number of episodes tested	Std(inflation) as dependent variable: episodes for which time coefficient is statistically significant	Std(change in inflation) as dependent variable: episodes for which time coefficient is statistically significant
10 percent	22	1	1
20 percent	9	0	0
30 percent	3	0	0
40 percent	2	0	0

Although the results of Table A3 imply the absence of a time trend, they do not necessarily indicate that volatility is constant because the volatility parameters could be varying from interval to interval without trending over time. Other tests are therefore necessary. One more rigorous method for testing the equality of the volatility parameters is to assume that inflation is normally distributed.[48] It is then possible to use Hartley's test for the equality of k variances. For each episode, let L be the largest value of the volatility parameter across the intervals, and S be the smallest value of the volatility parameter. Kanji (1993) gives critical values of L^2/S^2 for the null of equal variances,[49] so the test is particularly easy to implement.

In about half the episodes, Hartley's test rejects the joint null that the volatility parameter is a constant *and* that the underlying distribution is normal (see Table A4). But it is not clear whether rejection obtains so frequently because of non-constancy of the volatility parameter, because of non-normality of the distribution, or both.

Table A4: Hartley's test for constant volatility within episodes

Threshold	Number of episodes tested	Rejections of joint null of normality and constant std (inflation)	Rejections of joint null of normality and constant std (change in inflation)
10 percent	22	13	14
20 percent	9	5	6
30 percent	3	2	3
40 percent	1	0	1

In order to test for normality of the distribution (of either inflation or the change in inflation) within an episode, we apply a skewness-kurtosis test.[50] Normality is rejected for about half the episodes at each threshold, and normality is rejected for the vast majority of the episodes for which Hartley's test also rejects its null (see Table A5).

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[48] If inflation is normally distributed at duration t and duration t-1, then the change in inflation is also normally distributed (since a linear combination of normals is normal).

[49] See G. K. Kanji, *100 Statistical Tests* (Sage: London, 1993), pages 64 and 181-2.

[50] The skewness-kurtosis test is described in R.B. D'Agostino, A. Balanger, and R.B. D'Agostino, Jr., "A suggestion for using powerful and informative tests of normality," *The American Statistician,* Vol. 44, No. 4, 1990, 316-321.

Table A5: Normality tests for rejected episodes

Threshold	Number of rejections of joint null under Hartley's test for std (inflation)	Of these, number for which normality is rejected	Number of rejections of joint null under Hartley's test for std (change in inflation)	Of these, number for which normality is rejected
10 percent	13	9	14	10
20 percent	5	3	6	4
30 percent	2	2	3	3
40 percent	0	0	1	1

Tables A4 and A5 together suggest the need for a statistical test of the equality of k different variances that does not assume normality of the underlying distribution. Unfortunately, such tests suffer from particularly low power.[51] We are thus unsure whether the constant volatility assumption is valid.

As an alternative way of proceeding, we adopt a different estimation technique, a fixed effects logit model on the six-month intervals, that does not rely on constancy of the volatility parameters.

Assuming that the volatility parameter is not constant over an episode, we can examine how the probability of stabilizing within a given 6-month interval varies with the volatility for that interval. If volatility were time-varying, the IRRINV model would predict that higher volatility *ceteris paribus* would reduce the probability of observing a stabilization during that interval. But if true volatility were constant, we would not expect any such relationship, since any observed changes in the volatility parameter would represent noise, to which an optimally behaving government would not respond.

In order to examine the relationship between the probability of stabilization and volatility within a 6-month interval, we would like to apply a discrete-response regression model to our panel of episodes. Unfortunately, if there are episode-specific effects, ordinary probit or logit regressions would produce inconsistent estimators (with consistency

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[51] One non-parametric test for the equality of k variances is described in W.J. Conover, *Practical Nonparametric Statistics* (John Wiley: New York, 1980), pages 239-248.

evaluated as the number of episodes goes to infinity).[52] Even worse, the standard fixed-effects approach of removing the episode-specific effect by taking differences from the episode average is inapplicable because of the non-linearity of the discrete-response model.

Despite these problems, it is possible to estimate a fixed-effects logit model.[53] The approach, as proposed by Chamberlain (1980),[54] is based on maximizing a conditional likelihood function rather than the unconditional likelihood function. To fix ideas, let $y_{it}=1$ if episode i ends in interval t, and $y_{it}=0$ otherwise. Then assume that the probability of episode i ending in interval t is given by:

$$P(y_{it}=1)=\frac{e^{c_i+\beta' x_i}}{1+e^{c_i+\beta' x_i}}$$

The intuition behind the approach is clearest for t=2.[55] In that case, the unconditional likelihood function is:

$$L=\prod_{i=1}^{2} P(Y_{i1}=y_{i1}, Y_{i2}=y_{i2})$$

This likelihood function includes the individual-specific effects (c_i), and therefore using it to estimate the β parameters of interest would yield inconsistent estimators. Instead, Chamberlain suggests maximizing the conditional likelihood function:

$$L^c=\prod_{i=1}^{2} P(Y_{i1}=y_{i1}, Y_{i2}=y_{i2} | y_{i1}+y_{i2})$$

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[52] This difficulty arises because of the "incidental-parameter" problem: there is an additional individual-specific parameter to be estimated for each additional episode, so that expanding the number of episodes does not increase the degrees of freedom with which the parameters are estimated. For a proof of the inconsistency of a logit model in the presence of individual-specific effects, see C. Hsiao, *Analysis of Panel Data* (Cambridge University Press: Cambridge, 1986), pages 159-161.

[53] The probit model is not amenable to a fixed effects approach. See the discussion in W. Greene, *Econometric Analysis*, op. cit., page 655, or B. Baltagi, *Econometric Analysis of Panel Data* (John Wiley: New York, 1995), pages 178-182.

[54] G. Chamberlain, "Panel Data," in Z. Griliches and M. Intrilligator, *Handbook of Econometrics, Volume II* (Elsevier Science Publishers: Amsterdam, 1984), pages 1247-1318.

[55] The discussion follows W. Greene, *Econometric Analysis*, op. cit., page 656.

Note that the probability of $y_{i1}=y_{i2}=0$, given that their sum is zero, is 1. The probability that $y_{i1}=y_{i2}=1$, given that their sum is 2, is also 1. Taking the log of the conditional likelihood function, these two terms drop out (since ln(1)=0). We are therefore left with a term for the probability that $y_{i1}=0$ and $y_{i2}=1$, given that their sum is 1, and another term for the probability that $y_{i1}=1$ and $y_{i2}=0$, again given that their sum is 1. The former probability is:

$$P(y_{i1}=0,y_{i2}=1|y_{i1}+y_{i2}=1)=\frac{P(y_{i1}=0,y_{i2}=1)}{P(y_{i1}+y_{i2}=1)}$$

$$=\frac{\dfrac{1}{1+e^{c_i+\beta' x_{i1}}}\dfrac{e^{c_i+\beta' x_{i2}}}{1+e^{c_i+\beta' x_{i2}}}}{\dfrac{1}{1+e^{c_i+\beta' x_{i1}}}\dfrac{e^{c_i+\beta' x_{i2}}}{1+e^{c_i+\beta' x_{i2}}}+\dfrac{e^{c_i+\beta' x_{i1}}}{1+e^{c_i+\beta' x_{i1}}}\dfrac{1}{1+e^{c_i+\beta' x_{i2}}}}=\frac{e^{\beta' x_{i2}}}{e^{\beta' x_{i1}}+e^{\beta' x_{i2}}}$$

which is independent of the individual-specific effect. The probability of $y_{i1}=1$ and $y_{i2}=0$, given a sum of 1, is also independent of the individual-specific effect: it is just 1 minus the probability given above. Thus the conditional log-likelihood function is independent of the individual-specific effects, and consistent estimators of β can be obtained by maximizing the conditional log-likelihood.

For general t, the conditional log-likelihood is given by:[56]

$$\ln L^c=\sum_i \ln\left\{\frac{\exp(\beta'\sum_t x_{it}y_{it})}{\sum_t \exp(\beta' x_{it}d_t)}\right\}$$

where

$$B_i=\left\{d=(d_1,\ldots,d_t)=0 \text{ or } 1 \text{ and } \sum_t d_t=\sum_t y_{it}\right\}$$

This function is in the form of McFadden's (1974) conditional logit model, and can be estimated by standard econometric packages.[57] The

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[56] See G. Chamberlain, "Panel Data," op. cit., page 1276.

[57] D. McFadden, "Conditional Logit Analysis of Qualititative Choice Behavior," in P. Zarembka, *Frontiers in Econometrics* (Academic Press: New York, 1974). A procedure for estimating such models is included in the STATA econometric package.

results of running such conditional logit regressions on the volatility parameters and the drift rates are nugatory (see Table A6 for the 10 percent threshold; other thresholds are similar). Ordinary logit regressions also indicate no relationship between the probability of stabilizing and any of the variables of interest, which obviates the need to test for the appropriateness of the fixed effects approach, since the conclusion is invariant to the results of such a test.[58]

Table A6: Conditional and ordinary logit regressions, 10 percent threshold

Probability of stabilization in interval t			
	Conditional logit		
Std dev of inflation	2.88	2.71	
	(.97)	(0.63)	
Std dev of change in inflation			2.26
			(1.05)
Mean inflation		0.26	
		(0.06)	
Mean change in inflation			-2.69
			(-0.32)
N	187	186	186
χ^2 (p-value)	0.35	0.64	0.50
Pseudo R^2	0.01	0.01	0.02
	Ordinary logit		
Std dev of inflation	-0.48	1.77	
	(-0.21)	(0.70)	
Std dev of change in inflation			0.49
			(0.75)
Mean inflation		-3.91	-4.09
		(-1.61)	(0.53)
Mean change in inflation			
N	187	186	186
χ^2 (p-value)	0.83	0.24	0.75
Pseudo R^2	0.00	0.02	0.00

This appendix suggests several conclusions. First, there is no statistically significant time trend in volatility during high inflation episodes. Second, a joint test of normality and constant volatility rejects that null hypothesis in about half the cases, but normality itself is rejected for the vast majority of these cases. Logit analysis fails to indicate any relationship between the variables of interest and the probability of stabilizing in any given interval. It is difficult, however, to know what to conclude from this

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[58] In principle, a Hausman test could be used for such a purpose.

finding. If volatility is indeed time-varying, the logit analysis would contradict the IRRINV model. But if volatility is constant over an episode, the results are consistent with the IRRINV model. The absence of a significant time trend in volatility is at least suggestive that the latter view may be the correct one.

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Chapter 3: Privatization of the U.S. Enrichment Corporation: An Economic Analysis[1]

I. Introduction

The Energy Policy Act of 1992 created the United States Enrichment Corporation (USEC), a government-owned corporation that inherited the Department of Energy's role in enriching uranium for use in nuclear power reactors. With annual revenues of approximately $1.5 billion per year, USEC would — if privately owned — easily qualify for the Fortune 500.[2] And it may soon make it onto the Fortune 500 list: the USEC Privatization Act of 1996 directed that, following approval from the Secretary of the Treasury, the firm should be privatized.[3] The proposed privatization of USEC is significant not only because it would be the largest privatization in the United States since the sale of the Conrail railroad in 1987, but also because it highlights fundamental questions about the conditions under which privatization is socially beneficial.[4]

Arguments over the costs and benefits of privatization have a long and venerable history. In *The Wealth of Nations,* Adam Smith argued that selling the crown lands would improve their management:

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[1] The author thanks Thomas Neff, Mark Shankerman, and staff members at the Uranium Institute for helpful conversations; and Diane Whitmore, for her first-rate fact-finding skills. The author — who previously served on the staff of the Council of Economic Advisers — participated in U.S. government deliberations on the privatization of USEC, and has therefore taken great care to avoid divulging classified or proprietary information. The views expressed here do not necessarily represent those of the Council of Economic Advisers.

[2] Based on fiscal year 1994 revenue, USEC would rank 286th on the Fortune 500. See opening statement of Senator Frank Murkowski, Committee on Energy and Natural Resources, U.S. Senate, 104th Congress, 1st Session, "Hearing on S. 755, A Bill to Amend the Atomic Energy Act of 1954 to Provide for the Privatization of the United States Enrichment Corporation," June 13, 1995, page 1.

[3] Section 3101 of the USEC Privatization Act states that USEC's Board, "with the approval of the Secretary of the Treasury, shall transfer the interests of the United States in the United States Enrichment Corporation to the private sector in a manner that provides for the long-term viability of the Corporation, provides for the continuation by the Corporation of the operation of the Department of Energy's gaseous diffusion plants, provides for the protection of the public interest in maintaining a reliable and economical domestic source of uranium mining, enrichment, and conversion services, and, to the extent not inconsistent with such purposes, secures the maximum proceeds to the United States."

[4] For our purposes, privatization refers to the sale of the majority of equity in a public-sector asset or enterprise. A discussion of the various definitions of "privatization," and a history of the word, can be found in R. Hemming and A. Mansoor, "Privatization and Public Enterprises," International Monetary Fund Occasional Paper No. 56, January 1988.

> The crown lands of Great Britain do not at present afford the fourth part of the rent which could probably be drawn from them if they were the property of private persons...When the crown lands had become private property, they would, in the course of a few years, become well improved and well cultivated.[5]

This traditional perspective — that privatization improves internal efficiency by bolstering incentives — remains popular.[6] *The Economist*, for example, recently argued that the "real benefits of privatization...flow from letting managers do their job properly...better managers, encouraged by competition and held accountable for their successes and failures, will create lasting increases in productivity, year after year."[7] Policy-makers across the globe seem to be convinced by these arguments, as demonstrated by the increasing frequency of privatization over the past two decades.[8]

Some economists, however, view privatization somewhat more critically. They argue that privatization can reduce allocative efficiency, that the separation of ownership and control exists in private firms as well as public ones, and that corporatization is more important than privatization per se in boosting internal efficiency. As one example of the potential problems with privatization, Vickers and Yarrow (1995) note that:

> ...profit-maximizing monopolists may engage in a variety of business practices that run counter to the public interest, and, while it may be feasible to limit such behavior via the provisions of competition or regulatory policies, the complexities of this type of exercise in conditions of

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[5] A. Smith, *An Inquiry into the Nature and Causes of the Wealth of Nations*, Volume II (J.M. Dent & Sons: London, 1924 edition), pages 304-306.

[6] For arguments emphasizing the efficiency benefits of privatization, see, for example, M. Boycko, A. Shleifer, and R.W. Vishny, "A Theory of Privatization," *Economic Journal*, Vol. 106, March 1996, 309-319; World Bank, *Bureaucrats in Business: The Economics and Politics of Government Ownership* (IBRD: Washington, 1995); and World Bank, *Privatization: The Lessons of Experience* (IBRD: Washington, 1992).

[7] "A Case of the DTs: Privatization in Europe will fail to live up to its promise unless it is done right," *The Economist*, November 23, 1996, pages 19-20.

[8] According to the World Bank, there were 2,655 privatizations globally between 1988 and 1993, relative to 696 between 1980 and 1987 (despite the greater number of years in the earlier period). And evidence suggests that the average size of the firms being privatized was larger in the later period. See World Bank, *Bureaucrats in Business,* op. cit., Summary, pages 5 and 6. The scale of privatization efforts, even outside the former Soviet Union and Eastern Europe, is momentous. *The Economist* suggests that $250 billion worth of government assets may be sold within the next few years in Europe. See "Privatisation in Europe: Is the price right?" *The Economist,* November 23, 1996, page 125.

> asymmetric information may render public ownership the preferred framework in which to tackle the problem.[9]

Similarly, Fudenberg and Tirole (1994) argue that "a benefit from public ownership is that the government can impose socially desirable arrangements to the firm in unforeseen contingencies, while it must bargain with a private firm."[10] Given imperfect regulatory contracts and private information, there may thus be a tradeoff between internal (productive) efficiency and other objectives.[11] While the default policy prescription may be to favor privatization because of its internal efficiency benefits, in particular cases other factors may dominate.

The proposed privatization of USEC dramatically illustrates the potential benefits and costs of privatization. By almost all accounts, the Department of Energy and its predecessors were inefficient producers of enrichment services. While corporatization seems to have improved efficiency, there are likely to be further gains from privatization. For example, the Federal government is legally prohibited from developing a new, more efficient enrichment technology, at least partly because of opposition from a politically effective public union. It is therefore likely that a private firm would operate with more internal efficiency.[12]

On the other hand, USEC is a domestic monopoly — with approximately 90 percent of the domestic market and 40 percent of the world market — so that privatization may reduce allocative efficiency. And more important, a privatized USEC may endanger a crucial non-

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[9] J. Vickers and G. Yarrow, *Privatization: An Economic Analysis* (MIT Press: Cambridge, MA, 1995), page 28.

[10] J. Laffont and J. Tirole, *A Theory of Incentives in Procurement and Regulation* (MIT Press: Cambridge, MA, 1994), page 644.

[11] For empirical studies of the effects of privatization, see L. Clements, "Privatization American Style: The 'Grand Illusion'" in T. Clarke, *International Privatisation: Strategies and Practices* (Walter de Gruyter: Berlin, 1994), pages 95-96; J.A. Kay and D.J. Thompson, "Privatisation: A Policy in Search of a Rationale," *Economic Journal*, Volume 96, March 1986, 18-32, pages 22-25; J. Vickers and G. Yarrow, *Privatization: An Economic Analysis*, op. cit., page 3 and Chapters 3-4; and S. Domberger and J. Piggot, "Privatization Policies and Public Enterprise: A Survey," in M. Bishop, J. Kay, and C. Mayer, *Privatization and Economic Performance* (Oxford University Press: Oxford, 1994), pages 32-61.

[12] For a discussion of the conditions under which privatization rather than corporatization induces restructuring despite political pressures for over-employment, see A. Shleifer and R. Vishny, "Politicians and firms," *Quarterly Journal of Economics*, November 1994, 995-1025.

proliferation program of the U.S. government: the highly enriched uranium (HEU) deal with Russia, under which 500 metric tons of Russian weapons-grade uranium is blended into reactor fuel and sold to U.S. utilities. USEC serves as the U.S. government's executive agent with the Russians, but has not signed contracts for the entire 500 metric tons. Privatization could endanger the deal because USEC's marginal cost of producing enrichment is significantly lower than the cost of the Russian material, so that the more it imports, the higher its costs. Privatization — which imposes a profit-maximizing objective function on the firm — therefore creates a potentially momentous divergence in incentives between the principal (the U.S. government) and agent (USEC) in the HEU deal.

This chapter is organized as follows. The second section reviews the institutional details of the uranium enrichment market. The third presents the divestiture framework developed by Jones, Tandon and Vogelsang (1990); the fourth uses that framework to analyze the proposed privatization of USEC. The fifth extends the framework to incorporate the irreversibility of the privatization and uncertainty over its net benefits. The sixth further extends the analysis to reflect private information by modeling the implications of privatization for the HEU deal within a dynamic game of asymmetric information.

II. The enrichment industry

Before evaluating the merits of privatization, it is important to understand the institutional details of the uranium enrichment industry and the role USEC plays in it. This section explores the enrichment production technology, the market structure, and the significance of the HEU agreement.

The enrichment technology

Nuclear power reactors account for approximately 20 percent of the electricity generated in the OECD as a whole, as well as in the United States by itself.[13] The vast majority of these reactors are of the so-called "light

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[13] OECD, Nuclear Energy Agency, *Nuclear Power Economics and Technology: An Overview* (OECD: Paris, 1992), Table 1, page 14; and Bureau of the Census, *Statistical*

water" variety, which must use enriched uranium rather than natural uranium as fuel.[14] Uranium enrichment is thus a critical part of the nuclear energy market.

Light water reactors must use enriched uranium because they cannot sustain an ongoing nuclear reaction with natural uranium. The reason is slightly technical: light water reactors rely on ordinary (or "light") water as the moderator in the fission process. The moderator slows emitted neutrons to raise the probability that they interact with a fissionable atom before escaping from the reactor core, thereby ensuring a sustained chain reaction. But a light water moderator does not slow the neutrons sufficiently to sustain a chain reaction with natural uranium, which contains only 0.7 percent of the fissionable U-235 isotope.[15] To generate electricity, light water reactors must therefore use "low enriched uranium" (LEU) fuel, which has been enriched to between 3 and 5 percent U-235. USEC's business — uranium enrichment — transforms natural uranium into LEU by increasing the concentration of U-235. The LEU is then fabricated into fuel for use in nuclear utilities.

The typical nuclear fuel cycle for these light water reactors is depicted in Figure 1. After mining, uranium is separated from other minerals through chemical extraction and recovered as a solid concentrate (U_3O_8) known as "yellowcake" or natural uranium. This yellowcake contains 0.711 percent of fissionable U-235, 0.006 percent of the non-fissionable isotope U-234, and 99.283 percent of the non-fissionable isotope U-238.[16] The yellowcake is converted into uranium hexaflouride (UF_6) prior to the

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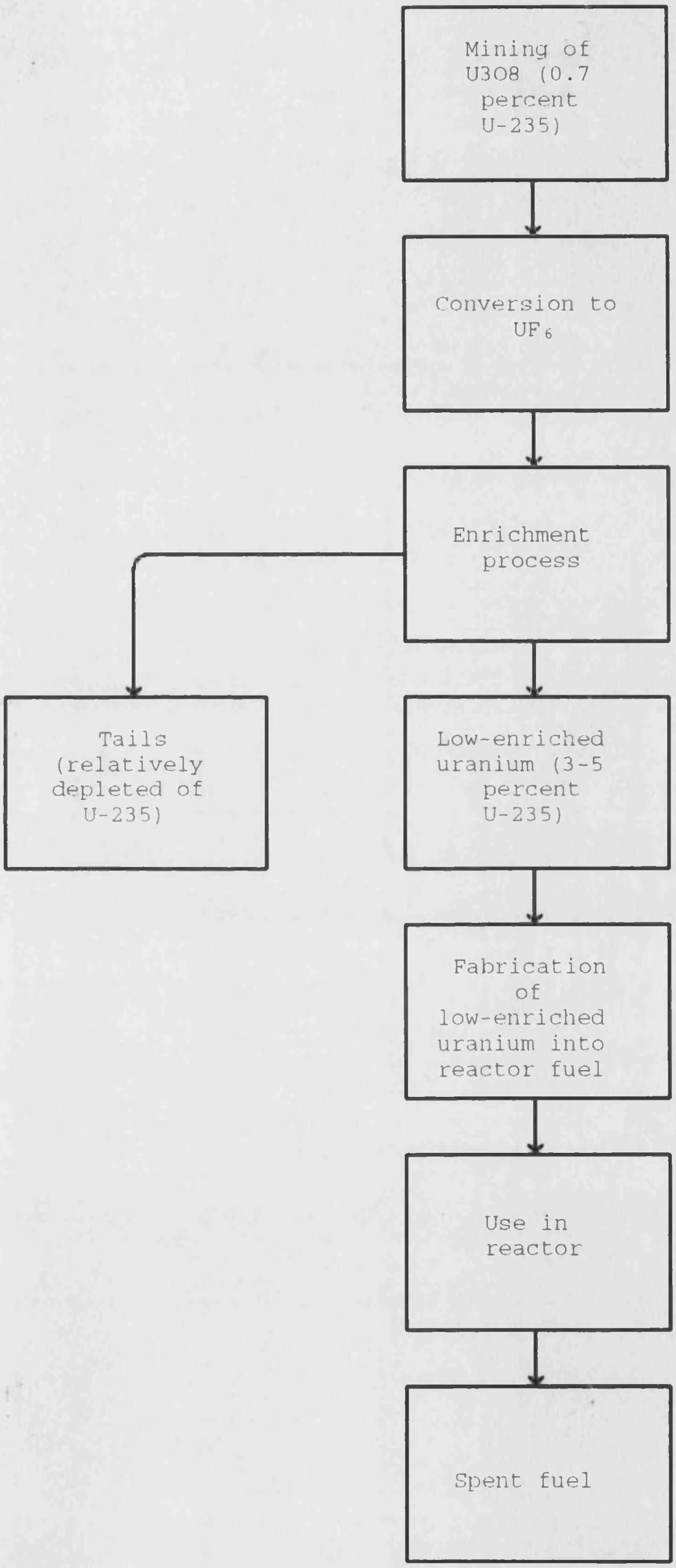
Abstract of the United States 1995 (Government Printing Office: Washington, 1995), Table 962, page 601.

[14] The database maintained by the Uranium Institute in London indicates that light water reactors account for 87 percent of current nuclear generating capacity.

[15] The Canadian CANDU reactor uses "heavy water" (D_2O) as a moderator. Because D_2O is more effective than light water in slowing the emitted neutrons, heavy water reactors can use natural uranium as fuel (since the neutrons are slowed sufficiently to sustain a chain reaction despite the relatively low percentage of fissionable atoms in natural uranium). "Fast" reactors use fuel with such a high percentage of fissionable atoms that they do not require any moderator at all. For a discussion of the relationship between moderator and fuel, see A. D. Owen, *The Economics of Uranium* (Praeger: New York, 1985), pages 16-26; A.J. Hyett, "The Structure and Economics of the Nuclear Fuel Cycle Service Industry," in L. Brookes and H. Motamen, eds., *The Economics of Nuclear Energy* (Chapman and Hall: London, 1984); and OECD, Nuclear Energy Agency, *Nuclear Power Economics and Technology*, op. cit., Chapter 3.

[16] A. D. Owen, *The Economics of Uranium*, op. cit., page 15.

Figure 1: The nuclear fuel cycle for a light water reactor



Source: Nuclear Energy Agency, *Nuclear Power Economics and Technology: An Overview* (OECD: Paris, 1992), page 30.

enrichment process, which is described below. After enrichment, the LEU is converted into uranium dioxide (UO_2), which is then formed into ceramic pellets and stacked in thin, sealed metal tubes. Finally, these tubes are bundled together into fuel assemblies and shipped to a reactor.

There are currently two principal commercial technologies for enriching uranium.[17] The first, gaseous diffusion, filters the UF_6 through a semiporous membrane. Since U-235 is lighter than U-238, a higher fraction of U-235 isotopes diffuse through the barrier — leaving a slightly enriched product on the far side and a slightly depleted product behind the barrier. The process is then repeated more than a thousand times in order to obtain LEU with 3 percent concentration of U-235.[18] The second commercial technology, gas centrifuge, injects the UF_6 into a spinning centrifuge, causing the heavier U-238 isotopes to concentrate on the outer edge of the centrifuge and the lighter U-235 isotopes to concentrate closer to the center. This process is then repeated approximately 10 times until the gas stream near the center is sufficiently enriched.[19] The final product of both enrichment technologies is LEU and a uranium residue, called the "tails," which has a relatively low concentration of U-235. The concentration of U-235 in the tails is called the "tails assay."

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[17] There are other enrichment technologies, but they are not in widespread commercial use. The other technologies include one developed by South Africa that relies on the different aerodynamic paths followed by gaseous uranium isotopes as they flow around a curved nozzle, and a calutron technology developed by the United States. See O. R. Cote, Jr., "A Primer on Fissile Materials and Nuclear Weapon Design," in G. T. Allison, O. R. Cote, R. A. Falkenrath, and S. E. Miller, *Avoiding Nuclear Anarchy: Containing the Threat of Loose Russian Nuclear Weapons and Fissile Material* (MIT Press: Cambridge, MA, 1996), pages 214-15. The advanced vapor laser isotope separation (AVLIS) technology, described in the text below, has not yet reached commercial stages.

[18] Each stage raises the U-235 share by a factor of only 1.00429. See A. D. Owen, *The Economics of Uranium*, op. cit., page 28.

[19] Each stage of the centrifuge process boosts the U-235 share by a factor of 1.1 to 1.4, so that substantially fewer stages are required than in the gaseous diffusion process. But the flow capacity of a single centrifuge is much lower than a gaseous diffusion plant: the optimal scale of a gaseous diffusion plant (9 million SWU) may be 3 to 5 times higher than the optimal scale of a centrifuge plant (2 to 3 million SWU). See A.J. Hyett, "The Structure and Economics of the Nuclear Fuel Cycle Service Industry," op. cit., page 170. The centrifuge technology uses less than 5 percent of the energy required in the gaseous diffusion process. See also statement of W. Howard Arnold, "Hearing on S. 755, A Bill to Amend the Atomic Energy Act of 1954 to Provide for the Privatization of the United States Enrichment Corporation," op. cit., page 64.

Enrichment services are measured in Separative Work Units (SWU), a metric of the effort required to separate U-235 from U-238.[20] There is a tradeoff between the tails assay and SWU: The higher the tails assay, the fewer SWUs are needed to produce a given quantity of LEU at a given U-235 concentration level.[21] To some extent, natural uranium and SWU are therefore substitutes in the production process — and the optimal mix depends on the relative price of natural uranium and SWU.[22]

It is worth emphasizing that enrichment technology represents a significant nuclear non-proliferation risk, since it can be used to produce highly enriched uranium (HEU) for nuclear weapons. Indeed, the process of enriching uranium into weapons-grade HEU, which usually has a U-235 concentration of about 90 percent, is identical to the enrichment process for LEU; the only difference is the degree of enrichment involved.[23] And obtaining HEU (or fissionable plutonium) is *the* key to building a nuclear weapon. As one scholar has written, "because only a few countries have

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[20] Technically, the unit is "kilogram separative work unit," a metric of the separative work done when the natural uranium input and LEU output are expressed in kilograms. The unit was devised by Dirac during World War II. For a discussion of the term, see R. F. Mozley, "Uranium Enrichment and Other Technical Problems Relating to Nuclear Weapons Proliferation," Center for International Security and Arms Control, Stanford University, July 1994.

[21] This tradeoff obtains because the more natural uranium used as an input, the less separation work required, and thus the higher the tails assay. For example, 6.6 kg of natural uranium combined with 3.4 SWU would produce 1 kg of LEU (enriched to 3 percent) and 5.6 kg of tails with a tails assay of 0.3 percent. But 5.5 kg of natural uranium combined with 4.3 SWU would produce the same amount of LEU, with 4.5 kg of tails and a tails assay of 0.2 percent. See A. D. Owen, *The Economics of Uranium*, op. cit., page 183.

[22] For given characteristics of LEU, the tails assay implicitly defines the amount of natural uranium and SWU used in the enrichment process. The cost-minimizing tails assay, given uranium and enrichment prices, is defined implicitly by $\frac{P_u}{P_{SWU}} = 4.87 + .986 \ln\left(\frac{t}{1-t}\right) - \frac{(.00711 - t)(2t-1)}{t(1-t)}$, where P_u is the price of natural uranium per kilogram, P_{SWU} is the cost of enrichment services (per SWU), and t is the optimal tails assay. For a derivation of the general formula for the optimal tails assay, see T. L. Neff, *The International Uranium Market* (Ballinger Publishing Company: Cambridge, MA, 1984), Appendix B.

[23] The original gaseous diffusion plant, the K-25 facility at Oak Ridge, Tennessee, was constructed as part of the Manhattan Project in World War II. It contributed some of the enrichment to the uranium bomb dropped on Hiroshima. Interestingly, however, the first known separation of uranium isotopes was achieved using the centrifuge process. For a discussion of the K-25 plant, see A. S. Krass, P. Boskma, B. Elzen, and W. A. Smit, *Uranium Enrichment and Nuclear Weapon Proliferation* (Taylor & Francis Ltd: London, 1983), pages 13-16. About 3000 stages of the gaseous diffusion process are required to produce 90 percent enrichment, relative to about 1200 stages for 4 percent enrichment. See R. F. Mozley, "Uranium Enrichment and Other Technical Problems Relating to Nuclear Weapons Proliferation," op. cit., page 30.

perfected the technology to enrich uranium to bomb-grade levels, obtaining the material has been far more difficult than building non-nuclear components for fabricating an explosive device."[24]

The structure of the enrichment market

Given the sensitive nature of the technology, and the long lead times involved in the nuclear power industry, it may not be surprising that the enrichment market is not perfectly competitive. It has substantial excess capacity: global demand amounts to approximately 35 million SWU per year, while capacity is roughly 50 million SWU per year.[25] Market demand is relatively inelastic.[26] And since fuel interruptions are extremely costly, long-term contracts are the norm: only about 10 percent of the projected demand for SWU between 1996 and 2000 is not already under contract.[27] These long-term contracts are generally of a "tolling" nature. Figure 2 illustrates a transaction under the type of contract dominant in the

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[24] G. MacLean, "Trade for Security: The Interplay of Domestic Economic and International Security Issues in the HEU Agreement," speech at the Nuclear Energy Institute conference "Fuel Cycle '95," San Diego, California, April 4, 1995, page 11.

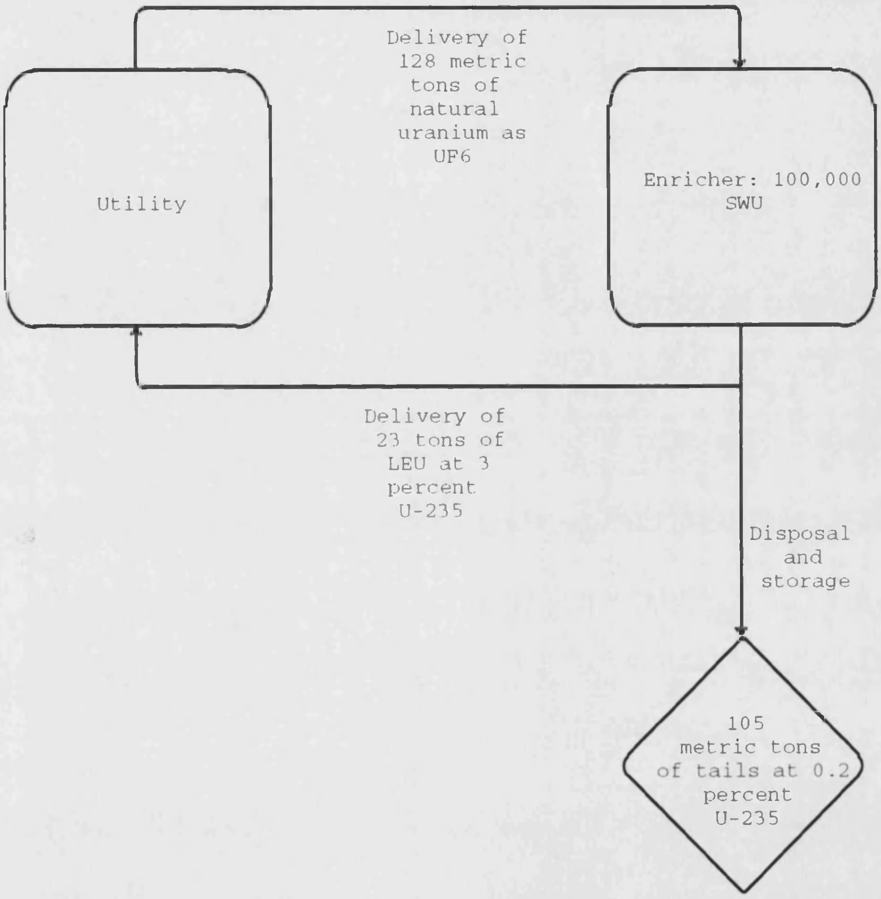
[25] "The New Birth of Urenco," *NUKEM Market Report*, June 1994, page 7-9; statement of James Phillips, "Hearing on S. 755, A Bill to Amend the Atomic Energy Act of 1954 to Provide for the Privatization of the United States Enrichment Corporation," op. cit., page 58; and United States Enrichment Corporation, *USEC Annual Report 1994* , page 7.

[26] Demand is inelastic because of the high cost of interruptions and because the share of enrichment in overall utility costs is relatively low. One of Marshall's laws of derived demand, as amended by Hicks, states that the derived demand for an input is less elastic the lower the share of the input, provided that the elasticity of substitution is less than the price elasticity of demand for output. The elasticity of substitution between LEU and other factors of nuclear power production is zero: LEU is a physical requirement for a given level of output. The elasticity of demand for *nuclear* power output, furthermore, may be relatively high. Roughly one-tenth the cost of nuclear power is accounted for by enrichment services. "Uranium Enrichment," Nuclear Issues Briefing Paper 33, January 1996, http://www.uic.com, page 2. Yarrow reports that total fuel costs account for only 30 percent of nuclear power costs. See "The price of nuclear power," *Economic Policy* 6 (April 1988), pages 111-112.

[27] For the cost of fuel interruptions, see M. Morman, *An Empirical Analysis of Uranium Spot Prices* (Ph.D. dissertation, Georgia State University, May 1988), page 37; and OECD, Nuclear Energy Agency, *Nuclear Power Economics and Technology: An Overview*, op. cit., Chapter 6. For the prevalence of long-term contracts, see J. A. Paleit, "The World Enrichment Industry Since 1987, and the Outlook to 2005," The Uranium Institute, Twenty-First Annual Symposium, 1996, page 4. In September 1996, uncontracted demand was projected to be just over 1 million SWU for 1997 and 1998. "Whither the Spot Enrichment Market?" *The U_X Weekly*, Volume 10, Issue 39, September 23, 1996, page 1. Long-term contracts have dominated the market since the U.S. policy change in 1973 described below. Neff reported that in the early 1980s, 70 percent of enrichment contracts held by non-U.S. utilities could not be adjusted within 5 years. T. L. Neff, *The International Uranium Market*, op. cit., page 23.

enrichment industry.[28] The first step is for a utility to deliver natural uranium to an enricher. The enricher then increases the concentration of U-235 to the desired level, and returns the LEU to the utility, charging for the SWU required to transform the natural uranium into the LEU. At current market prices of about $100 per SWU, approximately two-thirds of the cost of LEU is the embodied enrichment; the other third is the cost of the natural uranium and conversion.[29]

Figure 2: Operation of the enrichment market



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[28] The contract is for 100,000 SWU — roughly the requirements of a light-water reactor for one year. A 1300 MWe light water reactor requires roughly 130,000 SWU for one year of operation. United States Enrichment Corporation, *Serving the World*, 1995.

[29] At a tails assay of 0.2 percent, 5.5 kgU of UF_6 is combined with 4.3 SWU to produce 1 kg of LEU enriched to 3 percent. The market price of UF_6 is approximately $45 per kgU; the market price of enrichment is approximately $100 per SWU. Thus, the total cost of producing LEU is approximately $675 per KG, of which $430 (or 64 percent) is the cost of enrichment.

In addition to being dominated by long-term contracts, the enrichment market is also an oligopoly almost entirely served by four enterprises: USEC; Eurodif, a French-led consortium; Urenco, a UK-German-Dutch consortium; and MINATOM, the Russian Ministry of Atomic Energy. Table 1 lists their respective enrichment capacities.

Table 1: Global enrichment capacity

	Capacity (million SWU per year)
USEC (U.S.)	19.3
Eurodif (France)	10.8
Urenco (U.K., Germany, the Netherlands)	3.5
MINATOM (Russia)[30]	10 - 20
Others[31]	1.1
Total	44.7 - 54.7

Source: Uranium Institute, *The Global Nuclear Fuel Market: Supply and Demand, 1995-2015*, Table 10.3, page 120. The Uranium Institute quotes a figure of 19.0 million SWU per year for MINATOM.

USEC runs two gaseous diffusion plants: one in Kentucky, and the other in Ohio, which collectively employ approximately 4,350 workers.[32] Its shares are currently owned solely by the U.S. Treasury. The corporation has enrichment contracts with 63 utilities in 14 countries, generating net income of $373 billion.[33] It sells 14 million SWU per year, implying a

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[30] The precise Russian capacity figure is not known. Neff, writing in 1984, reported a figure of 10 million SWU per year. See T. Neff, *The International Uranium Market,* op. cit., page 21. Falkenrath quotes a figure of 20 to 25 percent of world enrichment capacity, which would imply about 10 to 12.5 million SWU per year. See R. Falkenrath, "The HEU Deal," in G. T. Allison, O. R. Cote, R. A. Falkenrath, and S. E. Miller, *Avoiding Nuclear Anarchy* op. cit., page 278. But Oleg Bukharin suggests a figure at 20 million SWU per year. See O. Bukharin, "Analysis of the Size and Quality of Uranium Inventories in Russia," *Science and Global Security,* Volume 6, 1996, page 67. And the Uranium Institute cites a figure of 19 million SWU per year. See Uranium Institute, *The Global Nuclear Fuel Market: Supply and Demand, 1995-2015*, Table 10.3, page 120. The differences may result at least partly from different treatment of the Russian enrichment centrifuges contaminated with plutonium, which precludes them from producing LEU that would meet international specifications.

[31] The two prominent others are Japan Nuclear Fuel Limited, which has capacity of 0.6 million SWU, and a Chinese enterprise with 0.5 million SWU capacity.

[32] The plants are operated by a subcontractor, Lockheed Martin Utility Services, Inc. See United States Enrichment Corporation, *Annual Report 1995*, page 10.

[33] United States Enrichment Corporation, *Annual Report 1995*, pages 6 and 24.

global market share of about 40 percent.[34] Two-thirds of its sales are in the U.S. market, and its U.S. market share is in the range of 90 percent.[35] As mentioned above and discussed in detail below, USEC is also the U.S. government's executive agent in the HEU deal with the Russian government. An appendix describes the operations of the other three major suppliers in more detail.[36]

The HEU deal

The HEU deal, originally signed in February 1993, will have a dramatic impact on the enrichment market. The agreement is intended to transform 500 metric tons of Russian HEU into LEU, import it into the United States over a period of 20 years, and sell it to domestic utilities.[37] The LEU "enriched down" from HEU and imported into the United States is a direct substitute for LEU "enriched up" from natural uranium.[38] The amounts involved in the transaction are substantial. The original deal called for the Russians to export LEU derived from at least 10 metric tons of HEU per year for five years, and then at least 30 metric tons of HEU per year for the next 15 years. At 4.4 percent U-235 concentration, the LEU derived from 30 metric tons of HEU embodies about 6 million SWU. This is about half of the projected annual enrichment requirements of U.S. utilities in

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[34] Testimony of William H. Timbers, Jr., "Hearing on S. 755, A Bill to Amend the Atomic Energy Act of 1954 to Provide for the Privatization of the United States Enrichment Corporation," op. cit., page 28.

[35] United States Enrichment Corporation, *Annual Report 1995*, pages 10 and 28, and opening statement of Senator Frank Murkowski, "Hearing on S. 755, A Bill to Amend the Atomic Energy Act of 1954 to Provide for the Privatization of the United States Enrichment Corporation," op. cit., page 1.

[36] The enrichment market outside the Soviet Union was monopolized by the U.S. government until the mid-1970s. In 1973, as a blandishment for utilities to enter into long-term contracts, the U.S. government promulgated the Long-Term Fixed-Commitment Contract, which required utilities to sign contracts eight years in advance of delivery. Utilities rushed to sign contracts, and by 1974 all of the U.S. government's capacity was committed, which opened the door for foreign suppliers to enter the market. Perceptions that the U.S. government had acted peremptorily spurred the foreigners' entrance.

[37] For a thorough description and analysis of the HEU deal until September 1995, see R. Falkenrath, "The HEU Deal," op. cit.

[38] To produce LEU from HEU, the HEU is "de-enriched" by blending it with uranium having a much lower concentration of U-235. The Russian HEU is blended with uranium containing 1.5 percent U-235, rather than the 0.7 percent U-235 found in yellowcake, to reduce the contamination of U-234 in the ultimate LEU product. For a discussion, see T. Neff, address before the World Nuclear Fuel Market Annual Meeting, Edinburgh, Scotland, May 21, 1996, footnote 3, page 5.

2000.[39] According to Thomas Neff, who originally proposed it, "the HEU deal between the U.S. and Russia has perhaps the greatest, but also the most uncertain, potential to affect the supply and pricing of uranium, enrichment, and conversion services in this new era."[40]

But the HEU deal offers much more than simply another source of fuel for nuclear reactors. The 500 metric tons of HEU covered by the deal is enough for 25,000 simple nuclear weapons.[41] If fully executed, the agreement would transform roughly 40 percent of Russian HEU inventories into LEU, significantly reducing the risk of proliferation from imperfectly secured HEU stocks in Russia.[42] It has been hailed as "second only to the START I and START II agreements in terms of its implications for post-Cold War nuclear non-proliferation and international security."[43] Neff (1996) notes that the "purpose of the HEU deal was to carry out an international security objective through commercial forces: not to disarm anyone, but to create economic incentives to keep surplus fissile material from falling into the wrong hands."[44]

The deal would also provide the Russian government with billions of dollars in export earnings, and potentially lower the cost of electricity within the United States, albeit probably only very slightly.[45] Furthermore, these benefits would accrue at no direct budgetary cost, since the Russian LEU would be sold to domestic utilities. As a well-known foreign policy

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[39] OECD, Nuclear Energy Agency, *Nuclear Energy and Its Fuel Cycle: Prospects to 2025*, op. cit., Table 12 and Table 13.3.

[40] T. Neff, address before the World Nuclear Fuel Market Annual Meeting, op. cit., page 3. Neff proposed the basic outline of the HEU deal in an October 1991 op-ed piece in the *New York Times*. For further discussion of the effects of the HEU deal on the enrichment market, see A. Max, "Long-Term Impacts of Weapons-Grade HEU on the Uranium, Conversion, and Enrichment Markets," speech at the International Uranium Fuel Seminar, October 8-11, 1995, Williamsburg, Virginia.

[41] R. Falkenrath, "The HEU Deal," op. cit., page 235.

[42] The precise size of the extant Russian HEU stock is unknown. Bukharin estimates that Russia held 1,300 metric tons of HEU in 1995. See O. Bukharin, "Analysis of the Size and Quality of Uranium Inventories in Russia," op. cit., page 71. In late 1993, the head of MINATOM suggested that Russia had up to 1,250 metric tons of HEU. See G. MacLean, "Trade for Security," op. cit., footnote 2, page 2.

[43] G. MacLean, "Trade for Security," op. cit., pages 1-2.

[44] T. Neff, address before the World Nuclear Fuel Market Annual Meeting, op. cit., page 3.

[45] Assume that the HEU deal produces a 10 percent fall in the price of enrichment services. Since nuclear power represents 20 percent of U.S. electricity generation, and enrichment represents less than 25 percent of the cost of nuclear power, the direct effect would reduce electricity prices by only 0.5 percent.

analyst, Jessica Mathews, wrote in the *Washington Post*:

> This was a classic swords into plowshares, win-win deal. Russia was to get desperately needed dollars...and an enormous security threat was to be removed, not just from Russia but from the world, and turned into civilian use. Some of the dollars would keep Russian nuclear scientists from going to work for bomb-seeking governments or terrorists. No charity was involved — this was trade, not aid.[46]

But as the Mathews's use of the past tense intimates, there are concerns about the status of the HEU deal. The concerns stem from the naming of USEC as the U.S. government's exclusive executive agent in the HEU deal, in combination with its proposed privatization. They will be discussed in detail below.

III. A perfect-information framework for analyzing privatization

Our focus now shifts to the proposed privatization of USEC. This section presents a modified version of the analytical framework for selling public enterprises developed by Jones, Tandon, and Vogelsang (1990).[47] Section IV will apply the framework to USEC's privatization.

Jones, Tandon, and Vogelsang propose a fundamental formula of divestiture, which — after adding a term for transaction costs — is given by:[48]

$$\Delta W = V_{sp} - V_{sg} - V_{ss} + (\lambda_g - \lambda_p)Z \qquad (1)$$

where ΔW is the change in social welfare resulting from the privatization, V_{sp} is the social value of the enterprise under private ownership, V_{sg} is the social value under government ownership, V_{ss} is the social value of the resources used to effect the sale, λ_g is the shadow value of government

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[46] J. Mathews, "National Security Blunder," *Washington Post*, May 5, 1995, page A25.

[47] L. P. Jones, P. Tandon, and I. Vogelsang, *Selling Public Enterprises: A Cost-Benefit Methodology* (MIT Press: Cambridge, MA, 1990).

[48] Jones, Tandon, and Vogelsang subsume transactions costs under the $(\lambda_g-\lambda_p)Z$ term. This effectively treats the transactions costs as if they were transfers rather than real resource costs. If the financial underwriting market is competitive, the transaction fees should reflect the resource costs of the transaction.

revenue in terms of consumption-equivalents, λ_p is the shadow value of private profits in terms of consumption-equivalents, and Z is the transaction price.

The shadow multipliers may require some explanation. In most models, we assume $\lambda_g=\lambda_p=1$. But with distortionary taxation and imperfect capital markets, it is likely that $\lambda_g>1$ and $\lambda_p>1$.[49] Jones, Tandon, and Vogelsang suggest that $\lambda_g=1.3$ and that $\lambda_g>\lambda_p$.[50] To avoid basing the analysis on point estimates of these potentially crucial variables, our results are presented for a variety of values between 1 and 2 for both λ_g and λ_p. Note that if $\lambda_g=\lambda_p$, the sale price is irrelevant to the social benefit from privatization, whereas if $\lambda_g>\lambda_p$, social welfare is increasing in the sale price.

V_{sp} can be written as the present value of consumer surplus under private ownership, plus private after-tax profits weighted by λ_p, plus tax payments weighted by λ_g:

$$V_{sp} = \sum_{t=0}^{t=\infty} \rho^t \left[S_p(t) + \lambda_p \left\{ \pi_p(t) - X(t) \right\} + \lambda_g X(t) \right] \qquad (2)$$

where ρ is the social discount factor, $S_p(t)$ is consumer surplus under private ownership in year t, $\pi_p(t)$ is private pre-tax profits in year t, and X(t) is corporate tax payments in year t.

Similarly, V_{sg} can be written as:

$$V_{sg} = \sum_{t=0}^{t=\infty} \rho^t \left[S_g(t) + \lambda_g \pi_g(t) \right] \qquad (3)$$

Therefore, (1) can be written as:

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[49] For analyses of the debate over the use of shadow prices, see Part I of R. Layard and S. Glaister, eds., *Cost-Benefit Analysis*, Second Edition (Cambridge University Press, Cambridge, England, 1994).

[50] L. P. Jones, P. Tandon, and I. Vogelsang, *Selling Public Enterprises*, op. cit., Chapter 3. Laffont and Tirole concur that $\lambda_g=1.3$ "is a reasonable mean estimate for the U.S. economy." J. Laffont and J. Tirole, *A Theory of Incentives in Procurement and Regulation*, op. cit., page 38.

$$\Delta W = \sum_{t=0}^{t=\infty} \rho^t \left[S_p(t) - S_g(t) + \lambda_g \{ \pi_p(t) - \pi_g(t) \} \right] - (\lambda_g - \lambda_p) \left\{ \sum_{t=0}^{t=\infty} \rho^t \left[\pi_p(t) - X(t) \right] - Z \right\} - V_{ss} \tag{4}$$

Let $\Delta S = \sum_{t=0}^{t=\infty} \rho^t \left[S_p(t) - S_g(t) \right]$, and $\Delta\pi = \sum_{t=0}^{t=\infty} \rho^t \left[\pi_p(t) - \pi_g(t) \right]$. Define the private sector's maximum willingness to pay for the enterprise as Z_p, which is given by:

$$Z_p = \sum_{t=0}^{t=\infty} r^t \left[\pi_p(t) - X(t) \right] \tag{5}$$

where r is the private sector's discount factor.

Thus (4) can be written as:

$$\Delta W = \Delta S + \lambda_g \Delta\pi - (\lambda_g - \lambda_p) \{ Z_p - Z + \Psi \} - V_{ss} \tag{6}$$

where $\Psi = \sum_{t=0}^{t=\infty} (\rho^t - r^t) \left[\pi_p(t) - X(t) \right]$.

Equation (6) delineates the social implications of privatization. For example, assume $\lambda_g = \lambda_p = 1$, and that no resources are required to sell the enterprise ($V_{ss} = 0$). Then (6) becomes:

$$\Delta W = \Delta S + \Delta\pi \tag{7}$$

which illustrates one of the possible tradeoffs inherent in privatization. It is often asserted that public managers lack proper incentives for internal efficiency, and that enterprises are therefore more efficient under private management. If so, $\Delta\pi > 0$. But if the enterprise has market power, and if a private firm maximizes profits while a public firm prices at marginal cost, then privatization (without regulation) creates allocative inefficiency — and ΔS is negative. As (7) emphasizes, the social benefit of privatization in such circumstances depends on whether the improvement in internal efficiency dominates the deterioration in allocative efficiency.

The Jones, Tandon, and Vogelsang framework delineated above identifies many of the costs and benefits of privatization. In the next section, we use it to examine the proposed privatization of USEC.

IV. Analysis of USEC privatization

In June 1995, the Board of Directors of USEC concluded that the corporation "was ready to be privatized" and that "privatization can be accomplished in a manner that will serve the interests of the United States and its taxpayers."[51] The purpose of this section is to evaluate these claims by applying the general form of (6) to the proposed privatization of USEC. As a preview of the results, the analysis suggests that privatization is likely to raise USEC's internal efficiency. But the benefit from that increase in internal efficiency may not be sufficient to offset the potential social welfare costs of privatization — particularly with respect to the HEU deal.

We will examine each term of (6) in turn. Because the ΔS term is particularly difficult to quantify, we temporarily ignore it and address it at the end of the section. We begin with the $\Delta\pi$ term.

Profits ($\Delta\pi$)

The U.S. government has a history of seemingly poor decision-making in managing its enrichment enterprise. For example, the Department of Energy continued construction of a gas centrifuge enrichment plant for several years after it became clear that additional enrichment capacity was not needed — ultimately spending about $3 billion on the project, which was never completed. And it continued to operate a gaseous diffusion plant in Oak Ridge, Tennessee until 1985, well after it had ceased to be economically viable.[52] Marketing and finance operations were not particularly imaginative: the enterprise, for example, had a single type of contract that applied to every customer.[53]

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[51] United States Enrichment Corporation, *Plan for the Privatization of the United States Enrichment Corporation*, submitted to the President and the Congress of the United States, June 1995, page ii and page 1.

[52] These examples are taken from Office of Management and Budget, "History of Bad Business Decisions Made by the U.S. Enrichment Enterprise, " December 15, 1995.

[53] S. Barr, "William Timbers's Nuclear Test: Turn A Bureaucracy into a Profitable Business," *Washington Post*, June 28, 1993, page A17.

Operations have improved since USEC was created in 1993. The Energy Policy Act of 1992 exempted USEC from many of the restrictions imposed on the Department of Energy, including the need to obtain annual appropriations from Congress and to observe various Federal procurement rules.[54] Partly as a result of this added flexibility, the management of the public-sector corporation has achieved record production levels and cut unit production costs.[55] As the president of USEC has asserted, "We've transformed a government bureaucracy into a corporate culture."[56]

Privatization may produce even more efficiency improvements than those already effected by corporatization. A privatized USEC would have more flexibility in its employment and pricing policies than the government-owned corporation currently does, as predicted by Shleifer and Vishny (1994) for profitable firms transferred into the private sector.[57] For example, the General Accounting Office has estimated that a private-sector USEC could achieve 11 percent higher revenue than a public-sector USEC because of the latter's reduced operating flexibility.[58]

A potentially important efficiency benefit from privatization involves the development of a new enrichment technology, AVLIS (Atomic Vapor Laser Isotope Separation). The U-235 and U-238 isotopes of uranium have slightly different excitation energies. AVLIS exploits this difference by using an extremely precise laser to give a positive charge to the U-235 atoms, but not the U-238 atoms. The positively charged U-235 ions are then attracted to a negatively charged plate and collected. Although AVLIS has not yet been tested on a commercial scale, it appears to be a promising technological development, with a very high separation factor and one-twentieth the energy requirements of the gaseous diffusion process.[59] A USEC officer believes that AVLIS offers potential cost savings of up to 12

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[54] General Accounting Office, "Uranium Enrichment: Process to Privatize the U.S. Enrichment Corporation Needs to be Strengthened," GAO/RCED-95-245, September 1995, page 4.

[55] See United States Enrichment Corporation, *Annual Report 1995*, op. cit., pages 4-5; and United States Enrichment Corporation, *Plan for the Privatization of the United States Enrichment Corporation*, op. cit., pages 7-8.

[56] Quoted in P. Passell, "U.S. Goals at Odds In a Plan to Sell Off Nuclear Operation," *The New York Times*, July 25, 1995, page A1.

[57] A. Shleifer and R. Vishny, "Politicians and firms," op. cit.

[58] General Accounting Office, "Uranium Enrichment," op. cit., pages 53 and 54.

[59] A. D. Owen, *The Economics of Uranium*, op. cit., page 29, and United States Enrichment Corporation, *Annual Report 1995*, op. cit., page 12.

percent.[60] Nonetheless, because of statutory restrictions that were undoubtedly the result of employment-driven political pressures, a public-sector USEC is prohibited from fully developing the AVLIS technology.[61] (It is interesting that legislators have prohibited a public-sector firm from developing a technology, but approved a privatization which will result in the adoption of that same technology.)

These differences in internal efficiency imply that $\Delta\pi$ is likely to be positive. Unfortunately, the numerous analyses that have been conducted on the sign and size of $\Delta\pi$ are not in the public domain.[62] Public documents do permit rough present value estimates of both $\pi_p(t)$ and $\pi_g(t)$, and thus of $\Delta\pi$. But before we examine estimates of these variables, we must address a central difficulty in any present value analysis: the social discount rate.

There is a large economics literature on the social discount rate, and whether it is best approximated by the return to private capital, the consumer's rate of time preference, or some other rate.[63] There is a related literature on adjusting for risk in social welfare cost-benefit analysis.[64] This literature has failed to reach consensus on the best approach, although the most favored approach seems to be (1) converting all flows into consumption-equivalents using shadow prices; (2) adjusting the converted

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[60] R. Kingdon, "Carpe Diem: Opportunity in the Future Fuel Cycle Markets," The Uranium Institute, Twenty-First Annual Symposium, 1996, page 7. The 12 percent figure is calculated for 4.4 percent enriched product, and includes cost savings in other stages of the fuel cycle (according to Kingdon, AVLIS will not require conversion of U_3O_8 to UF_6, for example).

[61] Section 1605 of the Atomic Energy Act of 1954, as amended by the Energy Policy Act of 1992, provides authority for AVLIS of only $364 million and specifically states that the "Corporation may not incur any obligation, or expend any amount, with respect to AVLIS or alternative technologies for uranium enrichment " beyond that amount.

[62] These analyses were conducted to evaluate the statutory requirement that privatization "result in a return to the United States at least equal to the net present value of the Corporation," which is embodied in Section 1502 of the Energy Policy Act of 1992.

[63] See R.C. Lind, ed., *Discounting for Time and Risk in Energy Policy* (Johns Hopkins Press: Baltimore, 1982); *Journal of Environmental Economics and Management*, Volume 18, Number 2, March 1990, Part 2; and Part I of R. Layard and S. Glaister, eds., *Cost-Benefit Analysis*, op. cit.

[64] See R. Wilson, "Risk Measurement of Public Projects," in R.C. Lind, ed., *Discounting for Time and Risk in Energy Policy*, op. cit. Current government practice on this issue lacks consistency — perhaps because OMB Circular A-94 does not provide clear guidance on how (or whether) to adjust for risk. See also the discussions in "Economic Analysis of Federal Regulations under Executive Order No. 12866," Executive Office of the President, January 11, 1996, and R. M. Lyon, "Federal Discount Rate Policy, the Shadow Price of Capital, and Challenges for Reform," *Journal of Environmental Economics and Management*, Volume 18, Number 2, March 1990, S-29 to S-50.

flows for uncertainty by expressing them as certainty-equivalents; and (3) discounting at the consumption rate of interest.[65] Nevertheless, since there is continuing debate over the proper methodology, we present a range of figures for $\Delta\pi$ using different social discount rates.

We begin with the present value of π_p, which we calculate as the sum of (1) the present value of projected tax revenue (X(t)), and (2) the present value of projected after-tax profits ($\pi_p(t)$-X(t)). Projected tax revenue estimates are readily available: The General Accounting Office, using data derived from J.P. Morgan projections and discounting at the government's borrowing rate, estimates that the present value of X(t) is $1.1 billion.[66] Estimates of after-tax profits are implicit in estimates of Z_p, which is defined as the present value of $\pi_p(t)$-X(t) using the private sector's discount rate. J.P. Morgan estimates that Z_p is between $1.5 billion and $1.8 billion.[67]

Whenever a relevant figure is not published for different discount rates, we make a rough adjustment by assuming the underlying flow is a real perpetuity, and then adjusting the published figure for the change in discount rates.[68] For example, J.P. Morgan's estimate of Z_p is predicated on a 15 percent nominal discount rate; figures for other discount rates are not published. To convert the estimate to one based on a 7 percent nominal discount rate, we assume the underlying private-sector after-tax cash flow is a perpetuity in real terms and therefore multiply Z_p by $\frac{.12}{.04} = 3$, to reflect a 15 percent nominal private-sector rate, a 7 percent nominal social discount rate, and 3 percent expected inflation.[69]

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[65] See, for example, R. Hartman, "One Thousand Points of Light Seeking a Number: A Case Study of CBO's Search for a Discount Rate Policy," *Journal of Environmental Economics and Management*, Volume 18, Number 2, March 1990, page S-3.

[66] General Accounting Office, "Uranium Enrichment," op. cit., Table 1, page 8.

[67] United States Enrichment Corporation, *Plan for the Privatization of the United States Enrichment Corporation*, op. cit., page 18.

[68] Any adjustment of this type is an extremely rough approximation, since the time profile of the cash flows may be incongruous with the perpetuity assumption. It would be much more precise to discount the projected annual after-tax cash flows at the social discount rate; unfortunately, the annual flows are not published.

[69] The 15 percent nominal private-sector rate is roughly the midpoint of the 10.4 to 20.0 percent range used by J.P. Morgan; see General Accounting Office, "Uranium Enrichment," op. cit., page 44. J.P. Morgan identifies the range of discount rates applied, but not the present value estimates resulting from them. The 7 percent nominal social discount rate is roughly the 30-year Treasury yield.

Table 2 presents the estimates of the present value of after-tax profits plus tax payments at different nominal discount rates. The figures in the table use the upper bound of J.P. Morgan's estimate for Z_p; they may thus be an overestimate. In any case, these are extremely rough estimates; many of the assumptions underlying them have been criticized by U.S. government agencies, and the real perpetuity assumption necessary to generate some of the estimates is likely to be fallacious. But the estimates should at least reflect the orders of magnitude of the pre-tax profits likely to accrue under privatization.

Table 2: Present value of $\pi_p(t)$ for different social discount rates ($ billion)

Nominal social discount rate	7 percent	9 percent	11 percent	13 percent	15 percent
Taxes	$1.10	$0.73	$0.55	$0.44	$0.37
After-tax profits	$5.40	$3.60	$2.70	$2.16	$1.80
Total (pre-tax profits)	$6.50	$4.33	$3.25	$2.60	$2.17

Note: Publicly available estimates are used when possible. Other figures have been adjusted by assuming the underlying flow is a real perpetuity (assumed inflation rate is 3 percent per year).

The General Accounting Office has also estimated the net present value of $\pi_g(t)$, and done so for different discount rates.[70] There have been serious questions raised about the methodology and assumptions used to obtain these estimates, however,[71] and a downward adjustment seems appropriate. To be conservative, we assume the GAO estimates are exaggerated by a factor of 2 in our "corrected" estimates.

Table 3: Present value of π_g for different discount rates ($ billion)

Nominal social discount rate	7 percent	9 percent	11 percent	13 percent	15 percent
GAO estimate	$3.30	$2.90	$2.60	$2.40	$2.20
"Corrected" estimate	$1.65	$1.45	$1.30	$1.20	$1.10

Using the conservative estimates of π_g, we obtain the following rough estimates of $\Delta\pi$:

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[70] General Accounting Office, "Uranium Enrichment," page 49.

[71] See letter from Mozelle Thompson to Charles Bowsher in General Accounting Office, "Uranium Enrichment," op. cit.

Table 4: $\Delta\pi$ at different social discount rates ($ billion)

Nominal social discount rate	7 percent	9 percent	11 percent	13 percent	15 percent
π_p	$6.50	$4.33	$3.25	$2.60	$2.17
π_g	$1.65	$1.45	$1.30	$1.20	$1.10
$\Delta\pi$	$4.85	$2.88	$1.95	$1.40	$1.07

Although these estimates are subject to tremendous uncertainty, they suggest that it is unlikely that $\Delta\pi$ is negative. This result reflects the principal benefit of privatization: it raises the internal efficiency of the enrichment operation. But the estimates also suggest that the difference in internal efficiency is unlikely to exceed a few billion dollars.

Sale price and discount rates

Assuming that the government is able to capture the full Z_p in the sale price, so that $Z=Z_p$, the next term in (6) collapses to $(\lambda_g-\lambda_p)\Psi$.[72] Ψ is non-zero only to the extent that the private-sector discount rate differs from the social discount rate. As noted above, these two discount rates coincide under the assumptions of the perfectly competitive model. In the presence of capital market imperfections and taxation, however, the situation becomes more complicated. As above, we therefore assume the underlying flow is a real perpetuity (at an assumed inflation rate of 3 percent and a nominal private sector rate of 15 percent) and evaluate Ψ for different social discount rates.

Table 5: Estimates of ψ ($ billion)

Nominal social discount rate	7 percent	9 percent	11 percent	13 percent	15 percent
ψ	$3.60	$1.80	$0.90	$0.36	$0.00

Transaction costs (V_{SS})

The next term, V_{SS}, represents the social cost of the resources used in the privatization process. J.P. Morgan estimates that privatization

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[72] Capturing the full private value of the enterprise in the sale price is not a trivial matter. We assume the existence of an optimal auction mechanism, but the design of that mechanism is a substantial challenge in reality.

transaction fees will be $100 million.[73] Such a fee level corresponds to about 7 percent of projected gross proceeds, which is consistent with the transaction fees in major privatizations from other countries.[74] Assuming that these fees are true resource costs, which would obtain if the financial placement market is competitive, and that the government ultimately incurs the costs, V_{SS} is thus equal to $100 million multiplied by λ_g.

Summary of non-ΔS terms

We can now summarize the net effects of the final three terms in (6) at different discount rates and various shadow prices. Note that our figures are intentionally biased toward privatization, since our estimates of $\Delta\pi$ use the upper bound of estimates for private-sector profits and a conservative estimate for public-sector profits. Also note that for a social discount rate of 15 percent, λ_p does not affect the results; this invariance obtains because λ_p only enters via the Ψ term, and $\Psi=0$ when $\rho=r$.

Table 6: Social benefits of privatization, not including ΔS ($ billion)

Nominal social discount rate = 7 percent per year	λ_g			
λ_p	1	1.25	1.5	2
1	$4.75	$5.04	$5.33	$5.90
1.25	$5.65	$5.94	$6.23	$6.80
1.5	$6.55	$6.84	$7.13	$7.70
2	$8.35	$8.64	$8.93	$9.50

Nominal social discount rate = 15 percent per year	λ_g			
λ_p	1	1.25	1.5	2
1	$0.97	$1.21	$1.45	$1.93
1.25	$0.97	$1.21	$1.45	$1.93
1.5	$0.97	$1.21	$1.45	$1.93
2	$0.97	$1.21	$1.45	$1.93

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[73] United States Enrichment Corporation, *Plan for the Privatization of the United States Enrichment Corporation*, op. cit., page 18.

[74] Vickers and Yarrow (1995), for example, report that the expenses from the major privatizations in the U.K. amounted to between 3.1 percent (Cable and Wireless) and 11.2 percent (Associated British Ports) of proceeds. See *Privatization: An Economic Analysis*, op. cit., page 181.

Table 6 suggests that for a variety of parameter values, the effect from privatization of all terms in (6) except ΔS is somewhere between \$1 billion and \$10 billion. In other words, if ΔS is smaller (in absolute value) than \$1 billion, Table 6 suggests that privatization should proceed because ΔW is clearly positive. If ΔS is larger (in absolute value) than \$10 billion, then $\Delta W<0$. If it is between \$1 billion and \$10 billion, then the social impact is unclear. We now turn to an examination of ΔS.

Consumer surplus

Privatization may result in $\Delta S<0$ for two reasons: (1) the exercise of USEC's market power may raise prices for the nuclear power industry and ultimately for consumers, and (2) the lack of incentives for USEC to implement fully the HEU deal increases the risk of nuclear weapons proliferation.[75] We will assume that these two effects are additively separable, and that we may therefore write $\Delta S= \Delta S_m+ \Delta S_n$, where ΔS_m is the change in consumer surplus due to USEC's market power, and ΔS_n is the loss of consumer surplus due to heightened nuclear proliferation risks.

A. USEC's market power (ΔS_m)

As mentioned above, USEC is a domestic monopoly with 90 percent of the U.S. market in an industry with relatively inelastic demand. As a profit-maximizing private enterprise, it may leverage this monopoly power more than a public-sector USEC would, in which case ΔS_m may be negative.[76] Proponents of privatization propose three reasons for why USEC's market position is less powerful than it might appear. First, the enrichment market is a global business, and foreign producers can compete in the U.S. market. But the principal foreign producers have only limited

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[75] Another element of S is the momentous environmental costs — estimated at as much as \$17.8 billion — of decontaminating the gaseous diffusion plants. See General Accounting Office, "Uranium Enrichment," op. cit. While this issue may have a large effect on S, it is not clear that it has any effect on ΔS: the costs must be borne regardless of whether USEC is privatized. (The portion of the costs that accrued prior to July 1993 will remain with the government after privatization, although the cost allocation process may be a difficult one.) The costs of cleaning up the gaseous diffusion plants are therefore not considered in the privatization analysis.

[76] This concern has been brought to the attention of the Federal Trade Commission and the Antitrust Division of the Department of Justice. See, for example, the letter from Senator Orrin Hatch to Robert Pitofsky, Chairman of the Federal Trade Commission, *Inside U.S. Trade*, August 11, 1995, page 4.

access to the domestic market. Except for the HEU exports that are channeled through USEC, MINATOM is effectively barred from the U.S. enrichment market because of an anti-dumping case on both natural uranium and enrichment.[77] And the European producers would, in all likelihood, become targets of anti-dumping and countervailing duty cases if they were to threaten USEC's market share; such cases would have a high probability of succeeding.[78]

The second reason to suspect that USEC's market power may be limited in the future is because of the possible introduction of a new domestic competitor, Louisiana Energy Services (LES). LES is a partnership among three U.S. utilities, an engineering firm, and Urenco; it has applied for licenses necessary to operate a gas centrifuge plant with the capacity to supply about 15 percent of the U.S. market.[79] But because it has not yet obtained the licenses, and because its prospects are uncertain, it is not yet a credible competitor to USEC.[80] Indeed, the LES experience highlights the substantial barriers to entry in the enrichment market: the capital requirements and licensing process are onerous.

The final potential competitive threat to USEC is the HEU deal, since the flow of LEU from Russia is effectively another source of enrichment for the domestic utility market. But USEC is itself the conduit for the imports from Russia. Thus, under current plans, the HEU deal is unlikely to provide much, if any, competitive pressure for USEC. The next sub-section examines the HEU agreement in more detail.

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[77] The suspension agreement to the anti-dumping case allowed a quota of 2.0 million SWU in Russian exports outside of the HEU deal in 1994 and 1995. The quota for 1996 was zero. The quota for future years is being re-negotiated, but is unlikely to be substantial. For a discussion of the U.S. anti-dumping and countervailing laws, see P. Orszag and J. Stiglitz, "Dumping on Free Trade: The U.S. Import Trade Laws," Centre for Economic Performance Discussion Paper No. 210, London School of Economics, October 1994.

[78] Domestic firms have been extremely successful in securing trade restrictions under the anti-dumping and countervailing duty laws, which many analysts believe have become grossly biased toward protectionism. For example, I.M. Destler finds that over 60 percent of the anti-dumping cases filed between 1980 and 1993 resulted in a finding of dumping, and were withdrawn by the petitioner (usually because some other form of protection was proffered). See I.M. Destler, *American Trade Politics*, Third Edition (Institute for International Economics: Washington, 1995), Table 6.2.

[79] See statement of W. Howard Arnold, "Hearing on S. 755, A Bill to Amend the Atomic Energy Act of 1954 to Provide for the Privatization of the United States Enrichment Corporation," op. cit., pages 64-68.

[80] A. Max, "Long-Term Impacts of Weapons-Grade HEU on the Uranium, Conversion, and Enrichment Markets," op. cit., page 11.

In sum, USEC is likely to maintain a significant degree of market power following privatization. This market power would imply $\Delta S_m<0$ if USEC charged a higher markup over marginal cost following privatization. But privatization could also imply $\Delta S_m>0$ because of improved internal efficiency, which lowers marginal cost and, for any given markup, the enrichment price. If the percentage reduction in marginal cost is smaller than the percentage increase in the markup, $\Delta S_m<0$, and vice versa.

Table 7 presents rough estimates of ΔS_m from the reduction in marginal cost following privatization, and the potential increase in the markup for USEC's enrichment services.[81] For simplicity, the calculations in the table assume a linear demand curve, current domestic sales of 10 million SWU, a current average sales price of $125 per SWU, a current marginal cost of $60 per SWU. We assume that USEC has a perfect monopoly in the U.S. market, and therefore do not incorporate any strategic behavior. Our attention is limited to domestic sales because our focus is U.S., not global, welfare. The results suggest that the magnitude of ΔS from this factor is likely to be relatively small — somewhere between +$0.2 billion and -$0.5 billion.

Table 7: Estimates of ΔS_m ($ billion)

Cost reduction (percent)	Elasticity of demand for USEC services (absolute value)			
	1.4	1.6	1.8	2
5	-$0.43	-$0.25	-$0.08	$0.06
7.5	-$0.42	-$0.23	-$0.05	$0.10
10	-$0.41	-$0.20	-$0.01	$0.13
12.5	-$0.40	-$0.17	$0.02	$0.17

B. The HEU deal (ΔS_n)

On January 14, 1994, USEC (the U.S. government's executive agent) signed a contract with Tenex (MINATOM's executive agent) to implement the HEU agreement. Given fixed demand, which is the most

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[81] The assumptions in Table 7 are given in terms of USEC's elasticity of demand, which determines the profit-maximizing markup. Maximizing $P(Q)Q-c(Q)$ with respect to Q yields $P(1+1/\varepsilon)=MC$, where $\varepsilon<0$ is the price elasticity of demand and $MC=c'(Q)$. The markup of P over MC is thus determined by the inverse of $1+1/\varepsilon$. Note that the elasticity of demand for USEC's product is higher than the industry elasticity of demand.

realistic short-run assumption for the enrichment industry, every SWU imported from Russia displaces a SWU of domestic production. As George Rifakes, USEC's executive vice president for operations, has been quoted as saying: the "HEU deal will be viewed like the rest of our capacity...and we will try to meet the requirement at the least cost. So the HEU is just going to come into the cost picture."[82]

The danger, as has been pointed out by Falkenrath (1996), Neff (1996), and others, is that a simple cost comparison is inconsistent with purchasing the Russian material.[83] USEC's marginal cost of production at the gaseous diffusion plants is somewhere between $60 and $70.[84] The Russians demand about $85 per SWU, roughly $10 to $20 below the spot enrichment price, which is itself below USEC's average sales price.[85] Purchasing Russian SWU, although objectively profitable (i.e., the cost is less than the price at which it is sold), is therefore $10 to $20 less profitable per SWU than domestic production. The effect is substantial: fully executing the HEU deal would reduce USEC's net present value by about 20 percent.[86] As Neff (1996) has noted: "There have sometimes been tradeoffs between the national security interest of the HEU deal and the objective of maximizing the commercial value of the Corporation."[87]

The formal optimization problem is trivial. Assuming total revenue

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[82] Quoted in R. Falkenrath, "The HEU Deal," op. cit., page 265.

[83] Neff notes the potential difficulty in his address before the World Nuclear Fuel Market Annual Meeting, op. cit., page 12. The incentive problem has also been noted in the press. See, for example, P. Passell, "U.S. Goals at Odds in A Plan to Sell Off Nuclear Operation," op. cit., page A1.

[84] Falkenrath (1996) quotes a figure of $60 per SWU. R. Falkenrath, "The HEU Deal," op. cit., page 277. James K. Phillips, vice president of the workers' union, cites a figure of $68 per SWU. See "Hearing on S. 755, A Bill to Amend the Atomic Energy Act of 1954 to Provide for the Privatization of the United States Enrichment Corporation," op. cit., page 56.

[85] "USEC, Russians Conclude Five-Year SWU Deal, As Pleiades Lobbies for USEC Ownership," *FreshFuel*, Volume 13, No. 386, November 25, 1996, page 1. The spot price is between $95 and $105 per SWU, although the spot market is extremely thin and thus the price may not be a reliable indicator of market conditions. USEC often earns between $125 and $150 per SWU on its extant contracts. See *The Nuclear Review*, September 1996, Figure 3.

[86] USEC had originally hoped to be compensated by the U.S. government for the differential in costs, calling it a "national security premium." See, for example, K. H. Bacon, "Buy High, Sell Low is Marching Order of Firm Purchasing Ex-Soviet Uranium," *Wall Street Journal*, January 14, 1994, page C10. USEC officials have since dropped this request.

[87] T. Neff, address before the World Nuclear Fuel Market Annual Meeting, op. cit., page 12.

is given and that inventories are not held, USEC's profit-maximizing objective function is simply to minimize total costs, $\underset{H,D}{Min}\{c_h H + c_d D\}$, subject to $H + D \geq Q$, and $H \geq 0, D \geq 0$, where Q is quantity of SWU sold, H is SWU imports from Russia, D is SWU domestic production, c_h is the cost per SWU from Russia, and c_d is the cost per SWU for domestic production. Then since $c_h > c_d$, the cost-minimizing solution is clearly D=Q, the non-negativity constraint on H is binding at the maximum, and H*=0.[88]

None of these incentive problems would matter if there were a signed, enforceable contract for the entire 500 metric tons of HEU. But there is not. The February 1994 contract promulgated a system under which the price would be renegotiated each October.[89] In November 1996, USEC reached a 5-year deal with the Russians for 132 metric tons of HEU.[90] The recent switch from an annual to a 5-year basis attenuates, but does not eliminate, the underlying concern: that in 5 years, after it has been privatized, USEC will not find it profit-maximizing to implement the rest of the deal. Even if the recent agreement is fully implemented over the next 5 years, contracts for more than 60 percent of the original 500 metric ton target will still have to be signed. And as Nick Timbers, the president of USEC, has stated that "the Russian deal, after all, is cancelable."[91]

Even while USEC has remained in the public sector, there have been two incidents that illustrate a divergence in incentives between expediting the HEU deal and promoting USEC's commercial interests — a divergence that would undoubtedly be exacerbated after privatization, when USEC's directors would have a fiduciary responsibility to maximize profits. The first was a series of disagreements that erupted over the natural uranium component of the LEU imported from Russia. The LEU imports embody both natural uranium and SWU, and under the agreement these two

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[88] For a discussion of linear programming such as the cost minimization problem faced by USEC, see A. Chiang, *Fundamental Methods of Mathematical Economics* (McGraw Hill: New York, 1984), Chapter 19. Given that the capacity constraint is not binding in the relevant region, USEC's variable cost curve is approximately linear.

[89] R. Falkenrath, "The HEU Deal," op. cit., page 263. If no price was agreed upon, then deliveries would continue for an additional year under the previous price. But if agreement were not reached by the end of the second year, imports would cease. See USEC-Russia LEU Contract Can be Terminated, " *FreshFuel*, September 5, 1994, page 3.

[90] "USEC, Russians Conclude Five-Year SWU Deal, As Pleiades Lobbies for USEC Ownership," op. cit., page 1.

[91] Quoted in R. Falkenrath, "The HEU Deal," op. cit., page 263.

components are purchased separately. But an anti-dumping settlement restricts imports of natural uranium from the former Soviet Union. Since the natural uranium embodied in the Russian LEU is treated as imported, it falls under the auspices of the anti-dumping order and may only be sold in specified ways.[92]

The HEU deal had been predicated on an assumption that the natural uranium component of the LEU would be used by USEC to boost the tails assay — in effect, to allow USEC to reduce its SWU requirement for given deliveries of natural uranium and LEU orders.[93] Such a use was permitted under the anti-dumping order. But following its formation, USEC concluded that this use of the natural uranium component was "impractical under current economic conditions."[94] It therefore decided, as it was entitled to under the contract, not to purchase that part of the LEU from the Russians. This decision caused substantial problems for the implementation of the deal. Viktor Mikailov, the head of MINATOM, called it "robbery in broad daylight," adding "we do not need such a contract."[95] After several costly attempts at a solution, the issue was eventually resolved.[96] All of this trouble was caused by USEC's unwillingness to incur about $50 to $200 million in additional annual costs — roughly the same additional cost that it would incur by importing the Russian SWU rather than producing domestically.

The second incident occurred in 1996. In January 1996, the Russians offered to sell 18 metric tons of HEU during 1997, but USEC refused. In July 1996, the Russians again offered to sell 18 metric tons in

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[92] Technically, since the natural uranium embodied in the LEU is not tangible, the anti-dumping authorities require that an equivalent amount of natural uranium from other domestic sources be set aside. For an extensive discussion of the natural uranium anti-dumping case and its effect on the HEU deal, see R. Falkenrath, "The HEU Deal," op. cit.

[93] T. Neff, "Integrating Uranium from Weapons into the Civil Fuel Cycle," *Science and Global Security,* Vol. 3 (1993), pp. 215-222.

[94] Statement of William H. Timbers, Jr., "Hearing on S. 755, A Bill to Amend the Atomic Energy Act of 1954 to Provide for the Privatization of the United States Enrichment Corporation," op. cit., page 21.

[95] S. Efron, "Russian Says U.S. Not Paying for Uranium," *Los Angeles Times,* June 14, 1995, page 10.

[96] See discussion in "Hearing on S. 755, A Bill to Amend the Atomic Energy Act of 1954 to Provide for the Privatization of the United States Enrichment Corporation," op. cit. The problem was "fixed" by giving title of the material to the Russians and promulgating a rising schedule of permitted forward sales in the U.S. market. See Section 3112 of the USEC Privatization Act of 1996.

1997, but USEC rejected the offer, indicating that it was only willing to accept 12 metric tons. The reasons for the rejection, and the degree of involvement of other U.S. government officials, are the subject of some controversy.[97] But one concerned party, Senator Pete Domenici, wrote on July 31 that he was "convinced that the USEC is acting directly contrary to the national-security interests of the United States."[98] The U.S. government subsequently instructed USEC to purchase the full 18 metric tons of HEU during 1997.[99] The incident seems to illustrate clearly USEC's limited incentives to import LEU derived from Russian HEU.

There are several possible solutions to the incentive problem. First, the U.S. government's ability to change executive agents could provide an incentive for USEC to implement the HEU deal: although its profits are lower if it buys SWU from the Russians, its profits would be even lower if someone else imported the SWU. It may therefore be willing to import Russian material in order to safeguard its status as sole executive agent. This argument will be evaluated in Section VI of the chapter; to preview our results, it is not obvious that this "threat effect" aligns USEC's incentives with society's non-proliferation objective.

The second possibility would be to subsidize the imports from Russia. The problem with this solution is that the HEU deal is supposed to be budget neutral.[100] Perhaps more important, there is no need to subsidize the deal: the market price of SWU, at about $100 even in the spot market, is well above what the Russians are asking. As Dr. Klaus Messer, CEO of Urenco, has stated: "The USEC will be paying about $82 per SWU. If the U.S. utilities have access to a substantial amount of material *at such low prices*, it will hurt us."[101] [emphasis added] In other words, the price the Russians are demanding is well below the market price. With any other executive agent but an enricher, there would be no need to subsidize the deal.

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[97] P. Passell, "Profit Motive Clouding Effort to Buy Up A-Bomb Material," *New York Times*, Wednesday, August 28, 1996, page A1.

[98] Quoted in P. Passell, "Profit Motive Clouding Effort to Buy Up A-Bomb Material," op. cit., page A1.

[99] Text of letter from Charles Curtis, Deputy Secretary of Energy, to Senator Domenici, quoted in *Nuclear Fuel*, September 9, 1996, pages 8-9.

[100] R Falkenrath, "The HEU Deal," op. cit., pages 277-278.

[101] Interview with Dr. Klaus Messer, *NUKEM Market Report*, op. cit., page 18.

The final possibility would be to name another executive agent, possibly including the U.S. Department of Energy itself. The benefits of such an option are that another agent would not face USEC's incentive problem, and that competition within the United States enrichment market would be enhanced. The cost is that no other U.S. entity has the market experience, extant contracts, and backup production capability that USEC does, although it is not clear that any of these variables is crucial to being a successful broker for the Russian LEU. This final option remains viable, but has not been adopted.

In sum, while there are possible solutions to the incentive problem, none has been implemented. Without such a solution, the possibility that a profit-maximizing USEC will not fully pursue the nation's non-proliferation objectives must therefore be incorporated into the cost-benefit analysis of privatization.

It is difficult to quantify ΔS_n, the value of the risk to national security from endangering the HEU deal. But it is potentially momentous. For example, Secretary of Defense William Perry has identified preventive nuclear non-proliferation efforts as being "perhaps our most important tool in protecting American interests from the special dangers that characterize the post-Cold War era."[102] A prominent group of Harvard foreign policy experts under the leadership of Graham Allison has identified the risk "that former Soviet nuclear weapons and materials will leak of out Russia, finding their way into the hands of rogue states or terrorist group" as the "most serious direct threat to U.S. vital interests today and for the foreseeable future."[103]

A revealed preference argument may be helpful in evaluating the potential cost in terms of ΔS_n. According to one recent estimate, the *annual* cost of maintaining the U.S. nuclear deterrent is $20 billion.[104] The present

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[102] "Message of the Secretary of Defense: The Dangers of the Post-Cold War World, " *1996 Annual Report to the President and the Congress*, http://www.dtic.mil/execsec/adr96, page 2.

[103] G. T. Allison, O. R. Cote, R. A. Falkenrath, and S. E. Miller, *Avoiding Nuclear Anarchy*, op. cit., page 7.

[104] S. Sestanovich, "High Time to Scrap the Nuclear Legacy of the Cold War, " *International Herald Tribune*, December 31, 1996, page 6. Some analysts believe that the figure is lower. Since we are only interested in orders of magnitude, the precise figure is

value of these costs is therefore hundreds of billions of dollars. Assuming that the costs reflect the implicit value of nuclear protection, it is clear that ΔS_n could dominate the internal efficiency from privatizing USEC. After all, the benefits from privatization presented in Table 6 are equivalent to the average cost of merely a couple B-2 bombers.[105]

As another estimate of the possible costs involved, consider the following back-of-the-envelope calculation. Assume that privatizing USEC under current conditions raises the probability by just 1 percent that terrorists are able to acquire 100 pounds of HEU for use in an attack on the United States. A bomb with 100 pounds of HEU could devastate a three-square-mile urban area,[106] a tenth the area of the Manhattan borough in New York. About 1.5 million people live in Manhattan,[107] and personal income per capita is at least $28,000.[108] Assuming a real discount rate of 3 percent, zero real per capita income growth, and an average life expectancy of 30 years, the human capital of Manhattan's residents is therefore worth at least $825 billion.[109] In addition, there are 265 million square feet of occupied commercial real estate in Manhattan, and the average rent per square foot for midtown office space is about $30 per year.[110] Assuming a real 8 percent

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not crucially important; we do not multiply by λ_g for the same reason. To put the estimated cost of the nuclear deterrent in perspective, note that t he United States spends over $250 billion annually on all defense activities. Office of Management and Budget, *A Citizen's Guide to the Federal Budget*, FY 1997, Table 2-2 (Government Printing Office: Washington, 1996).

[105] The B-2 bomber program is expected to cost $45 billion for 21 bombers. See General Accounting Office, "B-2 Bomber: Status of Efforts to Acquire 21 Operational Aircraft," October 22, 1996, GAO/NSIAD-97-11. The average cost per bomber is therefore approximately $2 billion.

[106] G. T. Allison, O. R. Cote, R. A. Falkenrath, and S. E. Miller, *Avoiding Nuclear Anarchy*, op. cit., page 1.

[107] Bureau of the Census, *Statistical Abstract of the United States 1995*, op. cit.,, Table 46, page 45.

[108] Per capita personal income in the New York Consolidated Metropolitan Statistical Area was $28,122 in 1993. Bureau of the Census, *Statistical Abstract of the United States 1995*, op. cit., Table 715, page 463. The personal income per capita in Manhattan is undoubtedly higher than for the New York City area as a whole.

[109] Valuing human capital as the present value of future earnings over 30 years at a real discount rate of 3 percent yields a figure of about $550,000 per person. This is at the lower end of the "reasonable" range proposed in R. Layard and S. Glaister, "Introduction" in Layard and Glaister, eds., *Cost-Benefit Analysis*, op. cit., page 25. The total human capital figure may be conservative for two reasons. First, we are not accounting for the loss of future generations. To the extent that human capital earns supernormal returns, this omission leads to an underestimate of the costs involved. Second, the assumption of zero real per capita growth is unduly pessimistic: a reasonable long-run estimate of real per capita growth for the United States as a whole is about 1 percent per year.

[110] In 1995, there were 319.2 million square feet of commercial real estate in Manhattan (excluding government-owned, owner-occupied, and medical buildings), and the vacancy

rate of time preference plus depreciation, and a constant occupancy rate, Manhattan's commercial real estate is therefore worth about $100 billion. The expected cost — in terms of lost human and physical capital — from a 1 percentage point increase in the threat of a nuclear terrorist attack that would obliterate a tenth of Manhattan is thus about a billion dollars.[111] This figure is not intended as a serious estimate of the non-proliferation risk from USEC's privatization; rather, it is intended to illustrate that ΔS_n *could* well be sufficiently large that $\Delta W<0$, even for small changes in probabilities.

In sum, despite the significant internal efficiency benefits that may accrue from privatization of USEC ($\Delta\pi>0$), the social implications may be deleterious ($\Delta W<0$) even in a full-information framework because other factors ($\Delta S<0$) may dominate. Indeed, there may be a lesson in the fact that, despite massive privatization efforts undertaken by both the British and Russian governments, neither has privatized its uranium enrichment enterprises.[112]

V. Privatization under uncertainty and irreversibility

The framework introduced in Sections III and applied in Section IV, while quite useful in identifying the various tradeoffs inherent in privatization, nevertheless has several shortcomings. For example, it implicitly assumes that privatization is "reversible," in the sense that the enterprise, once privatized, could be costlessly nationalized. But in reality, privatization would be difficult and costly to reverse. Both the transactions costs involved and the government's loss of credibility imply that privatization is at least partially irreversible, and that the Jones, Tandon, and Volgesang framework is therefore biased toward approving privatizations

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rate was 17.0 percent. Bureau of the Census, *Statistical Abstract of the United States 1996* (Government Printing Office: Washington, 1996), Table 1204. The Cushman and Wakefield office market report indicates that the average annual rental rate in the third quarter of 1995 for midtown office space in Manhattan was $30 per square foot.

[111] The expected cost is 0.01($825 billion+$100 billion)/10=$0.925 billion.

[112] The May 1995 review of nuclear power privatization in the U.K. held open the possibility of privatizing British Nuclear Fuels, Limited, which owns a third of the shares in Urenco. See Department of Trade and Industry and the Scottish Office, *The Prospects for Nuclear Power in the UK*, Presented to Parliament, May 1995, pages 59-60. The Russian 1993 privatization law explicitly prohibited privatization of firms producing fissionable or radioactive materials. For a discussion of the Russian privatization program, see M. Boycko, A. Shleifer, and R. Vishny, *Privatizing Russia* (MIT Press: Cambridge, MA, 1995).

that are not socially optimal.[113] This section explores the ramifications of privatization's irreversibility.

Privatization can be thought of as a social investment in the privatized enterprise. The cost of the investment is the social value of the forgone public-sector production and the resources necessary to effect the transaction. At least part of these costs are sunk. The benefit is the social value of the corporation under private ownership. As with many investments, the return is uncertain since the social benefit from private ownership is not precisely known *ex ante.* Therefore, privatization is effectively an investment under uncertainty, and the literature on such investments suggests that a simple net present value rule, such as the one employed in Section IV, is potentially misleading.[114]

Intuitively, the government has an incentive to "wait and see" how privatization will affect social welfare to avoid situations in which privatization, *ex post,* turns out to have been a serious — and difficult to reverse — mistake. For example, if the AVLIS technology turns out not to be commercially operable, the social benefits of privatization will be less than had been expected. Or if some shock to the HEU deal raises the cost of having a private-sector executive agent, the social benefits of privatization will again be less than had been expected.

Somewhat more formally, irrreversibility and uncertainty imply that the government owns a "privatization option." When privatization takes place, the option is exercised.[115] The price of the option must therefore be included in the cost-benefit analysis. In order to incorporate uncertainty over both V_{sp} and V_{sg}, assume that both follow geometric Brownian motions with drift:

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[113] If the government's credibility is a social asset in that it facilitates a preferred equilibrium in the economy, then "spending" that credibility can be just as socially cos tly as incurring real resource costs.

[114] More precisely, irreversibility and *expandability* imply that a simple net present value rule sets too low a threshold. See A. Abel, A. Dixit, J. Eberly, and R. Pindyck, "Options, the Value of Capital, and Investment," NBER Working Paper No. 5227, August 1995. For a general introduction to the investment under uncertainty literature, see A. Dixit and R. Pindyck, *Investment under Uncertainty* (Princeton University Press: Princeton, 1994).

[115] We assume that the government sells all its shares. For a discussion of why governments often do not sell all their shares immediately, see E. Perotti, "Credible Privatization," *American Economic Review*, Vol. 85, No. 4, September 1995, pages 847-859.

$$dV_{sp} = \alpha_p dt + \sigma_p dz_p \qquad (8)$$
$$dV_{sg} = \alpha_g dt + \sigma_g dz_g \qquad (9)$$

where $dz_p = \varepsilon_p \sqrt{dt}$ and $dz_g = \varepsilon_g \sqrt{dt}$ are increments to Wiener processes, ε_p and ε_g are standard normal variables, and $E(dz_p dz_g)=\upsilon$.

For simplicity, assume $\lambda_g=\lambda_p=1$, and that V_{ss} is a constant. Define $F(V_{sp},V_{sg},V_{ss})$ as the value of the enrichment enterprise. Then, since the government can choose to privatize at any point, F is equal to the immediate exercise value, or the expected continuation value, whichever is higher:

$$F\{V_{sp}(t), V_{sg}(t), V_{ss}\} = \max\left\{V_{sp}(t) - V_{sg}(t) - V_{ss}, \frac{1}{1+\rho dt} E[F(t) + dF(t)]\right\} \quad (10)$$

Assuming that the problem is well-behaved, (10) implies $\rho F dt = E(dF)$ in the region for which the enterprise remains within the public sector.[116] Applying Ito's lemma to compute dF, taking expectations, and setting the result equal to $\rho F dt$:

$$\frac{1}{2}\{\sigma_p^2 V_{sp}{}^2 F_{pp} + \sigma_g^2 V_{sg}{}^2 F_{gg} + 2\upsilon\sigma_p V_{sp}\sigma_g V_{sg} F_{pg}\} - \alpha_p V_{sp} F_p - \alpha_g V_{sg} F_g - \rho F = 0 \quad (11)$$

where F_i is the first partial derivative of F with respect to V_{si}, F_{ii} is the second partial derivative, and F_{ij} is the cross-partial derivative. The partial differential equation defined by (11) holds in the region for which the enterprise is kept within the public sector. At the boundary of this region, the "value-matching" and "smooth-pasting" conditions must obtain:

$$F\{V_{sp}(t)^*, V_{sg}(t)^*, V_{ss}\} = V_{sp}(t)^* - V_{sg}(t)^* - V_{ss} \qquad (12)$$
$$F_p\{V_{sp}(t)^*, V_{sg}(t)^*, V_{ss}\} = 1 \qquad (13)$$
$$F_g\{V_{sp}(t)^*, V_{sg}(t)^*, V_{ss}\} = -1 \qquad (14)$$

In addition, the geometric Brownian motion assumptions in (8) and (9) ensure that:

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[116] See Chapter 1 for a discussion of sufficient conditions for the problem to be well-behaved.

$$F\{0,0,0\}=0 \tag{15}$$

In general, the free-boundary partial differential equation system defined by (11) through (15) is difficult to solve, even with numerical methods. To simplify the mathematics and permit an analytical solution, we therefore assume that the resource costs of privatizing are proportional to the social value of the enterprise within the government (the larger the value of the enterprise, the larger the resource costs of the transaction). In other words, we assume $V_{ss}=uV_{sg}$. In this case, F is homogeneous of degree 1 in V_{sp} and V_{sg} (doubling V_{sp} and V_{sg} will double both the benefit and cost of holding the option, thus doubling its worth). We can therefore reduce the partial differential equation to an ordinary differential equation in the ratio of V_{sp} to V_{sg}. Define $R=\frac{V_{sp}}{V_{sg}}$. Then the homogeneity implies:

$$F(V_{sp},V_{sg},uV_{sg})=V_{sg}f(R) \tag{16}$$

Using (16) and the definition of R to calculate the partial derivatives of F, and then substituting into (11), we obtain an ordinary differential equation in R:

$$\frac{1}{2}\{\sigma_p^2-2\upsilon\sigma_p\sigma_g+\sigma_g^2\}R^2f''(R)+\{\alpha_g-\alpha_p\}Rf'(R)-\{\alpha_g+\rho\}F=0 \tag{17}$$

Adapting the boundary conditions, we have:[117]

$$f(R^*)=R^*-(1+u) \tag{18}$$

$$f'(R^*)=1 \tag{19}$$

$$f(R^*)=R^*f'(R^*)-(1+u) \tag{20}$$

$$f(0)=0 \tag{21}$$

Notice that any two of (18) through (20) imply the third.

Solving (17) subject to the two binding boundary conditions in (18)

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[117] Applying Ito's lemma indicates that $dR=(\alpha_p-\alpha_g+\frac{1}{2}\sigma_g^2-\upsilon\sigma_g\sigma_p)Rdt+(\sigma_p dz_p-\sigma_g dz_g)R$, so that R is itself a geometric Brownian motion.

through (20) and subject to (21), we obtain:[118]

$$R^* = (1+u)\frac{a}{a-1} \qquad (22)$$

where a is the characteristic root of (17) that is larger than 1:

$$a = \frac{\frac{1}{2}\left\{\sigma_p^2 - 2\upsilon\sigma_p\sigma_g + \sigma_g^2\right\} - \left\{\alpha_g - \alpha_p\right\}}{\sigma_p^2 - 2\upsilon\sigma_p\sigma_g + \sigma_g^2} + \frac{\sqrt{\left[\frac{1}{2}\left\{\sigma_p^2 - 2\upsilon\sigma_p\sigma_g + \sigma_g^2\right\} - \left\{\alpha_g - \alpha_p\right\}\right]^2 + 2(\alpha_g + \rho)\left\{\sigma_p^2 - 2\upsilon\sigma_p\sigma_g + \sigma_g^2\right\}}}{\sigma_p^2 - 2\upsilon\sigma_p\sigma_g + \sigma_g^2} \qquad (23)$$

The solution (22) illustrates the fundamental point of this section. Under the simple net present value rule as applied in Section IV, the government should privatize whenever V_{sp}-$V_{sg}(1+u)>0$. In other words, whenever $R>1+u$, privatization seems socially beneficial. But (22) indicates that such a rule is misleading if there is uncertainty and irreversibility: the optimal threshold for R is larger than 1+u, since $\frac{a}{a-1} > 1$ for $a>1$.

The gap between R* and (1+u), which determines the degree to which the conventional analysis is misleading, is larger the larger is the variance of the innovations to V_{sp} or V_{sg}, the *smaller* is the correlation between those innovations, and the larger is α_g relative to α_p. The privatization option is worth more the more uncertainty there is, and the less that uncertainty is common to both the privatized and government enterprise scenarios (since it is the *difference* between social welfare under the two cases that is relevant to the government's decision). It is relatively more important to account for this effect when there is more uncertainty over future events, and when the social implications of those future developments are likely to vary significantly across the privatized and public-sector cases — both of which seem likely to be relevant for the proposed privatization of

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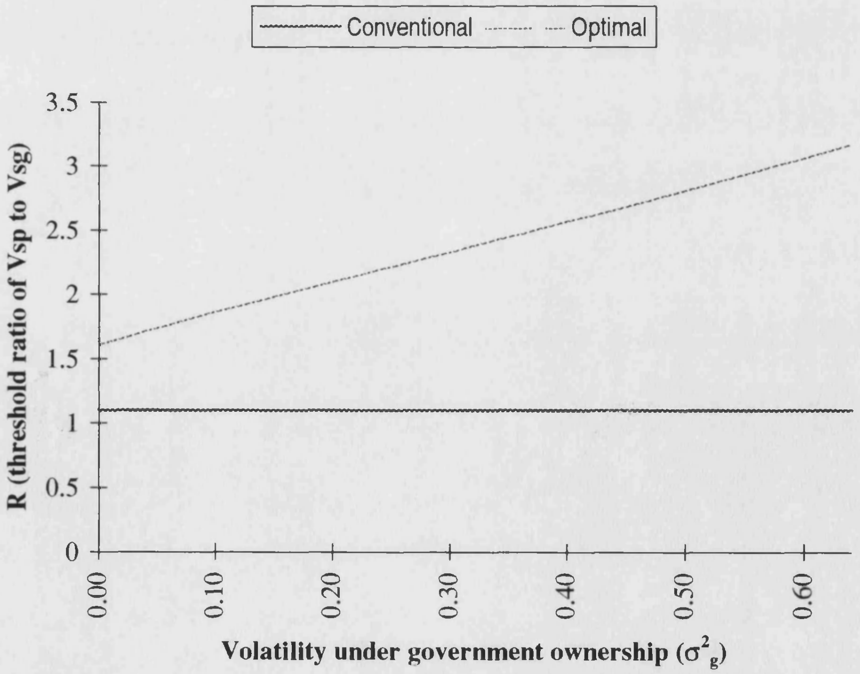
[118] The solution method, which follows the method used in Chapter 1, involves positing a solution of $f=A_1R^a+A_2R^b$, plugging the proposed solution into (17), using the f(0)=0 condition to set $A_2=0$ (since $b<0$), and using two of the conditions from (26) through (28) to solve for R*.

USEC.

To consider the potential importance of accounting for the real privatization option in USEC's case, assume $\Delta S=0$ and $\lambda_g=\lambda_p=1$. Then from (6), $\Delta W=\Delta\pi-V_{ss}$, and from Table 4, R=2 at a social discount rate of 15 percent per year. Assuming u=.1, the threshold R for privatization under a conventional cost-benefit analysis (the "conventional threshold") is 1.1. Since R>1.1, the conventional approach would thus suggest that privatization is socially beneficial. But the real options approach delineated in this section shows that such a privatization *could* be a mistake. For internal consistency, set $\rho=.15$. Then assume $\alpha_g=\alpha_p=0$, $\sigma^2_p=1/9$, and $\nu=0$. Given these assumptions and after taking account of the real privatization option, Figure 3 shows how the threshold for privatization (the"optimal threshold") varies with σ^2_g. The optimal threshold is above 2 for $\sigma^2_g>.25$.

Figure 3: Privatization threshold vs. volatility under government ownership



Parameter values: $\rho=.15$, $\sigma^2_p=.1$, $\nu=0,\alpha_g=0$, $\alpha_p=0$, u=.1

In other words, if the proportional change in the social value of USEC within the public sector varies, on average, by about 15 percent per month or more, then privatization is not socially optimal. (A standard deviation of 0.15 per month corresponds to an annual variance of about 0.25.) This example illustrates the importance of accounting for the real privatization option in some settings. The exercise is admittedly tendentious because it examines the case of ρ=.15, which has a relatively low R, and assumes ν=0. But on the other hand, it also assumes ΔS=0 and thus no nuclear non-proliferation risk from privatization (note that with regard to the HEU deal, ν is likely to be negative).

In summary, this section shows that failing to account for the real privatization option *could* bias the cost-benefit analysis. The closer the conventional ΔW is to zero, the more volatile are V_{sp} and V_{sg}, and the lower the correlation between shocks in the public and private sectors, the more likely it is that the real privatization option could actually change the optimal social decision regarding the ownership of the enterprise.

VI. Privatization under asymmetric information

Section V illustrated one shortcoming of the framework employed in Section IV. This section emphasizes another: the failure to account for asymmetric information, which is crucial to evaluating the costs and benefits of privatization. Indeed, without private information, the performance of a public-sector enterprise and an optimally regulated private-sector enterprise should coincide. As Shapiro and Willig (1990) argue, "the presence of such information is necessary for privatization to have a genuine effect."[119] Asymmetric information is particularly important in connection with the impact of privatization on the HEU deal.

As discussed in Section II, USEC's profit-maximizing behavior may not be consistent with full implementation of the HEU deal. Advocates of

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[119] C. Shapiro and R. Willig, "Economic Rationales for the Scope of Privatization," in E. N. Suleiman and J. Waterbury, eds., *The Political Economy of Public Sector Reform and Privatization* (Westview Press: Boulder Colorado, 1990), page 56.

privatization, however, argue that a simple marginal cost comparison is misleading. The HEU agreement promulgates that either party can change its executive agent with 30 days' notice. USEC argues that a breakdown in negotiations would therefore lead the U.S. government to name another agent, and that this threat provides sufficient incentive for it to purchase the Russian material.[120] Other analysts question USEC's argument. Neff (1996), for example, argues that:

> the possibility of replacement as executive agent is probably not enough [to assure implementation of the HEU deal] — the U.S. does not have a clear mechanism for choosing a new one...nuclear fuel markets are complex and the inventiveness of market participants can easily outclass the second-guessing of well-intentioned but under-informed government officials.[121]

The purpose of this section is to evaluate the supposed "threat effect," USEC's claim that the risk of being replaced as executive agent is sufficient to align its profit-maximizing interests with the non-proliferation objective of aggressively importing Russian LEU. We model the HEU negotiations with the Russians as a dynamic game under asymmetric information.[122] We first examine a scenario in which the U.S. government bargains directly with the Russians; abstracting from internal agency problems, this scenario can be interpreted as equivalent to keeping USEC within the public sector. We then extend the model to include USEC as the U.S. government's agent.

Direct U.S. government negotiations with Russia

We consider a three period game. At period zero, nature determines the level of domestic enrichment production costs per SWU, c, which are high (c_H) with probability q and low (c_L) with probability 1-q. The probability distribution of c is known to both the United States and Russia, but its realization is known only to the U.S. government. Nature also

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[120] See quotations from Nick Timbers in P. Passell, "U.S. Goals at Odds in a Plan to Sell Off Nuclear Operation," op. cit., page A1.

[121] T. Neff, address before the World Nuclear Fuel Market Annual Meeting, op. cit., page 13.

[122] For an informal discussion of dynamic asymmetric information games, see R. Gibbons, *Game Theory for Applied Economists* (Princeton University Press: Princeton, 1992), Chapter 4.

independently determines the aversion of the Russian government to selling its HEU inventories to the United States, which is affected by factors such as internal political opposition.[123] This value is proxied by α, which is either high or low (α_H or α_L) with probability k and 1-k respectively. The realization of α is independent of the realization of c and known only by Russia.

In the first period of the game, the U.S. government makes an offer to Russia for the purchase of HEU amounting to L million SWU. For simplicity, we normalize L to 1. The U.S. government offers a price of either A_H or A_L per SWU, where we assume:

$$c_L < \alpha_L < c_H < A_L < \alpha_H < A_H < P \tag{24}$$

This structure captures the realistic notion that domestic costs are lower than the cost of the Russian material, both of which are lower than the domestic price. In the second period, Russia either accepts or rejects the offer.

After the initial round of negotiations, the U.S. learns how important non-proliferation risks (N) are to its national security. We assume that N is either high or low (N_H or N_L), with probability t and 1-t respectively. Both sides know the probability distribution of N but only the U.S. learns its realization, which is independent of c and α. If the initial offer had been rejected by Russia, the U.S. can make a second offer, of either A_H or A_L, after learning the value of N. If this second offer is accepted, the non-proliferation benefit accruing to the U.S. is $N(1-\varepsilon)$, where ε is the proportionate non-proliferation risk from delaying agreement. We assume:

$$\frac{A_H - c_H}{1-\varepsilon} < N_L < \frac{A_H - qc_H - (1-q)c_L}{1-\varepsilon} < N_H < A_H - c_L \tag{25}$$

$$P - A_H > \varepsilon E(N) \tag{26}$$

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[123] Evgenij Mikerin, Head of MINATOM'S Technology Department, has noted the political pressures involved in the HEU deal: "some even say we jeopardize our national security by dismantling these warheads in order to receive money from the United States." See E. Mikerin, "Russia's Enrichment Program: New Challenges," speech delivered at U.S. Council for Energy Awareness, International Conference on Enrichment, Washington, DC, June 13-15, 1993.

$$k > \frac{A_H - A_L}{\varepsilon E(N) + A_H - A_L} \tag{27}$$

Because Russian HEU displaces domestic enrichment production, the net benefit to the U.S. from an accepted offer is the non-proliferation benefit (N) minus the premium of the offer over domestic production costs (A-c). In the absence of a deal, the U.S. payoff is zero. The payoff to Russia is A if agreement is reached on the first offer, A(1-δ) if agreement is reached on the second offer, and α if agreement is not reached. The δ loss from delaying agreement reflects the costs to Russia (e.g., the time value of money) from postponing the sale at a given price; we assume δ is small enough that $A_H(1-\delta)>\alpha_H$.

Figure 4 presents the game in extensive form. A *strategy* for the U.S. government, σ(US), defines a probability distribution over its choice(s) of A, given its private information and its beliefs. Its beliefs, μ(US), are its expectations about the Russian aversion factor and, until it is revealed, the value of N. A strategy for Russia, σ(Russia), defines a probability distribution over rejecting or accepting the U.S. offer, conditional on the U.S. action(s), Russia's private information regarding α, and its beliefs. Russia's beliefs, μ(Russia), are its expectation of N and its conditional expectation of c given A.

A *perfect Bayesian Nash equilibrium* is a set of strategies and beliefs such that:

- The U.S. strategy maximizes its expected payoff conditional on Russia's optimal strategy, its own beliefs about α and N, and its private information about c and possibly N;

- The Russian strategy maximizes its expected payoff given the optimal U.S. strategy, the offer of A, its posterior belief about c conditional on observing A, its expectation of N, and its private information about α;

- The U.S. belief about the Russian aversion type reflects the probability distribution of α and uses Bayes' rule if the initial offer is rejected; Russia's posterior beliefs use Bayes' rule for an A that has non-zero probability mass in equilibrium (any posterior beliefs are acceptable for any A with zero probability mass in equilibrium); and both beliefs about E(N) reflect the probability distribution of N.[124]

The unique pure-strategy perfect Bayesian Nash equilibrium of the game is defined by:

- σ(US)={Offer A_H if c= c_H; offer A_L if c= c_L; offer A_L again if c= c_L and initial offer is rejected};

- σ(Russia)={Accept all offers of A_H; accept first offer of A_L if $A_L>\alpha$; reject first offer of A_L if $A_L<\alpha$; reject all second offers of A_L};

- μ(US)={$E(N)=tN_H+(1-t)N_L$; P(Russia rejects first offer of A_L)=k; P(Russia rejects second offer of A_L)=1; $P(\alpha_H|\text{first offer of } A_L \text{ is rejected})=1$}; and

- μ(Russia)={$E(N)=tN_H+(1-t)N_L$; $P(c_H|A_H)=1; P(c_L|A_L)=1$}.

To prove this is a perfect Bayesian Nash equilibrium, start by considering the U.S. government's final move if its initial offer of A_L has been rejected. Russia will always accept a final offer of A_H, since it then receives a return of $A_H(1-\delta)$, which lies above the support of α. Since a second offer of A_H is always accepted, the U.S. receives $N(1-\varepsilon)-(A_H-c)$ from such an offer. We will show that Russia will always reject a second offer of A_L, so the U.S. return from a final offer of A_L is zero. The U.S. will therefore offer A_H in the final round when $N(1-\varepsilon)-(A_H-c)>0$. Since $N_H<A_H-c_L$ from (25), N_H is also less than $\frac{A_H-c_L}{1-\varepsilon}$ for $0<\varepsilon<1$. Thus the U.S. would never offer A_H in the final round if $c=c_L$. But since $N_L>\frac{A_H-c_H}{1-\varepsilon}$, the U.S. would always offer A_H in the final round if $c=c_H$.

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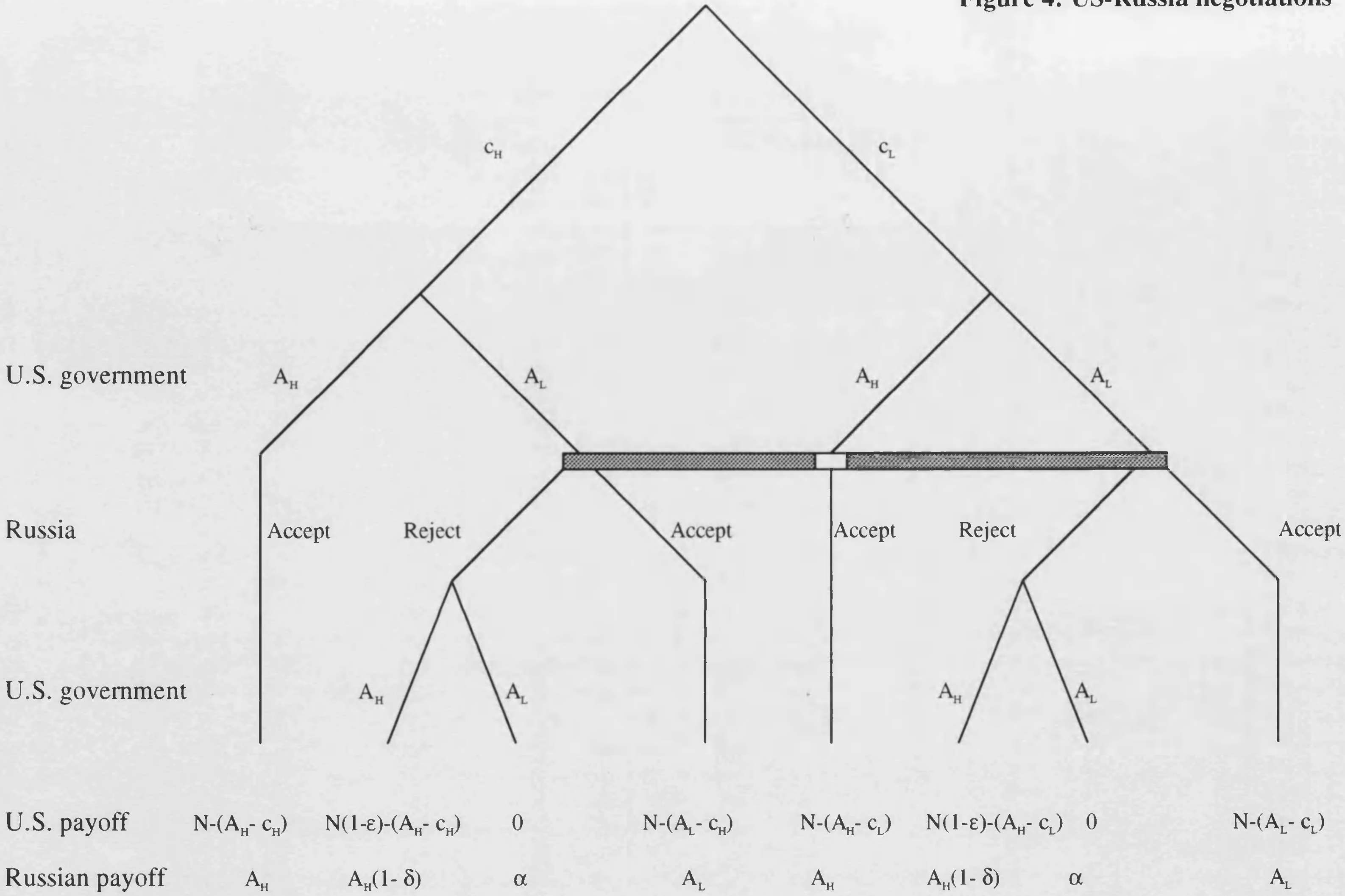
[124] For a formal exposition of the general conditions for a perfect Bayesian Nash equilibrium, see D. Fudenberg and J. Tirole, *Game Theory* (MIT Press: Cambridge, MA, 1995), pages 325-26.

Figure 4: US-Russia negotiations



Now consider the first round of the game. For c=c_H, the U.S. will offer A_H if $E(N)-(A_H-c_H)>(1-r)\{E(N)-(A_L-c_H)\}+r\{E(N(1-\varepsilon))-(A_H-c_H)\}$, where r is the probability that Russia will reject an offer of A_L. For $r > \frac{A_H - A_L}{\varepsilon E(N) + A_H - A_L}$, this condition obtains and therefore the U.S. always offers A_H if c=c_H. The intuition is that with high domestic costs, the non-proliferation benefits of purchasing Russian material are relatively inexpensive. The U.S. will therefore ultimately offer A_H to secure the Russian HEU. And if r is high enough, it is not worth the risk of waiting until the final round. The U.S. therefore offers A_H immediately, and Russia accepts.

When c=c_L, the U.S. never offers A_H, since (25) implies that A_H-c_L lies above the support for N and therefore an offer of A_H always results in a negative payoff. Russia can therefore infer that if A=A_L, U.S. costs must be low. And Russia thus also knows that if it rejects the initial offer of A_L, the U.S. will not offer A_H in the second round. Russia therefore rejects the initial offer of A_L if $\alpha > A_L$, which happens with probability k, since $\alpha_L < A_L < \alpha_H$ from (24). And if Russia rejects the first offer of A_L, then $\alpha > A_L$ and it therefore also rejects any second offer of A_L, since $\alpha > A_L$ clearly implies $\alpha > A_L(1-\delta)$. Thus, r=k, ensuring from (27) that the U.S. does indeed offer A_H in the high-cost scenario.

In this perfect Bayesian Nash equilibrium, the HEU deal is concluded 1-(1-q)k percent of the time. The U.S. government's expected payoff is:

$$q\{E(N) - (A_H - c_H)\} + (1-q)(1-k)\{E(N) - (A_L - c_L)\} \qquad (28)$$

and the Russian government's expected payoff is:

$$qA_H + (1-q)\{k\alpha_H + (1-k)A_L\} \qquad (29)$$

A private-sector USEC as the U.S. government's executive agent

The game is now extended to incorporate a private-sector USEC serving as the U.S. government's executive agent. In this extended game, we model the U.S. government's loss of information following privatization by assuming that it can no longer observe the realization of domestic enrichment costs. Instead, c is revealed to USEC alone, while the U.S. government continues to be the only party that knows N.

In the first period of the game, USEC makes an offer of either A_H or A_L to the Russians, who either accept it or reject it in the second period. If the offer is accepted, the deal is implemented. If the Russians reject the offer, the U.S. government can allow the deal to collapse, or can replace USEC as the executive agent. In the latter case, it is assumed that an alternative agent offers A_H (still below P) and the deal is consummated. Figure 5 presents the extensive form of the game.

As Figure 5 illustrates, *the key change following privatization is that the U.S. government no longer knows whether a breakdown of the negotiations occurs in the high or low cost state of the world.* This ambiguity reflects the fact that government officials are unable to monitor USEC perfectly. The government is thus forced to form its best Bayesian guess as to USEC's private information, conditional on the actions (A_L and rejection) it has observed. It therefore replaces USEC if:

$$qnr\left[E(N)(1-\varepsilon)-(A_H-c_H)\right]+(1-q)mr\left[E(N)(1-\varepsilon)-(A_H-c_L)\right]>0 \quad (30)$$

where n is the probability that USEC offers A_L conditional on the high-cost state; m is the probability that USEC offers A_L conditional on the low-cost state; and r is the probability that the Russians reject an offer of A_L.

Assume t=0, so that N is always equal to N_L. Then a perfect Bayesian Nash pooling equilibrium exists in which USEC offers A_L in the first round regardless of whether domestic costs are high (n=1) or low (m=1), the Russians reject the offer of A_L with probability k (r=k), and the U.S. government always allows the deal to collapse if the Russians reject it. To see why this is an equilibrium, note that t=0 implies that the replacement condition (30) never obtains when n=1 and m=1: from (25), N_L lies below $\frac{A_H-qc_H-(1-q)c_L}{1-\varepsilon}$. Thus the Russians face a simple choice of A_L or α, and they therefore accept A_L if $\alpha<A_L$, which occurs with probability 1-k. USEC is also behaving optimally with n=1 and m=1, since it benefits from offering a lower price while never being replaced. In this equilibrium, the

HEU deal is consummated only 1-k percent of the time. The equilibrium payoff to the U.S. government is:

$$(1-k)\{N_L-(A_L-qc_H-(1-q)c_L)\} \tag{31}$$

while the equilibrium payoff to the Russians is:

$$k\alpha_H+(1-k)A_L \tag{32}$$

Relative to the non-privatized scenario, the HEU deal is consummated qk percent less frequently. The change in U.S. welfare, relative to (28), is $q\{(A_H-A_L)-k(N_L-(A_L-c_H))\}$, which is unambiguously negative if $k>\frac{A_H-A_L}{N_L-(A_L-c_H)}$, which in turn is implied by (27) and (25). The fall in U.S. welfare occurs only because A_L is offered when $c = c_H$; thus the higher q, the larger the decline in U.S. welfare. Russian welfare is also lower under privatization. The Russian payoff in (29) is a weighted average of (32) and A_H; since A_H is greater than (32), (29) is also.

This example illustrates that a breakdown in negotiations will not necessarily induce the U.S. government to replace USEC. The "threat effect" thus does not exert enough discipline to align USEC's incentives with the U.S. government's, and privatization reduces both national and global welfare.

Now consider t=1.[125] Then another perfect Bayesian Nash equilibrium exists in which the U.S. government always replaces USEC if the Russians reject the initial offer, the Russians always reject an offer of A_L, and USEC offers A_H regardless of whether domestic costs are high or low. The HEU deal is consummated with probability 1. Thus, Timbers's argument is corroborated in this case: despite the cost differential between domestic production and Russian HEU, USEC is induced to offer A_H to

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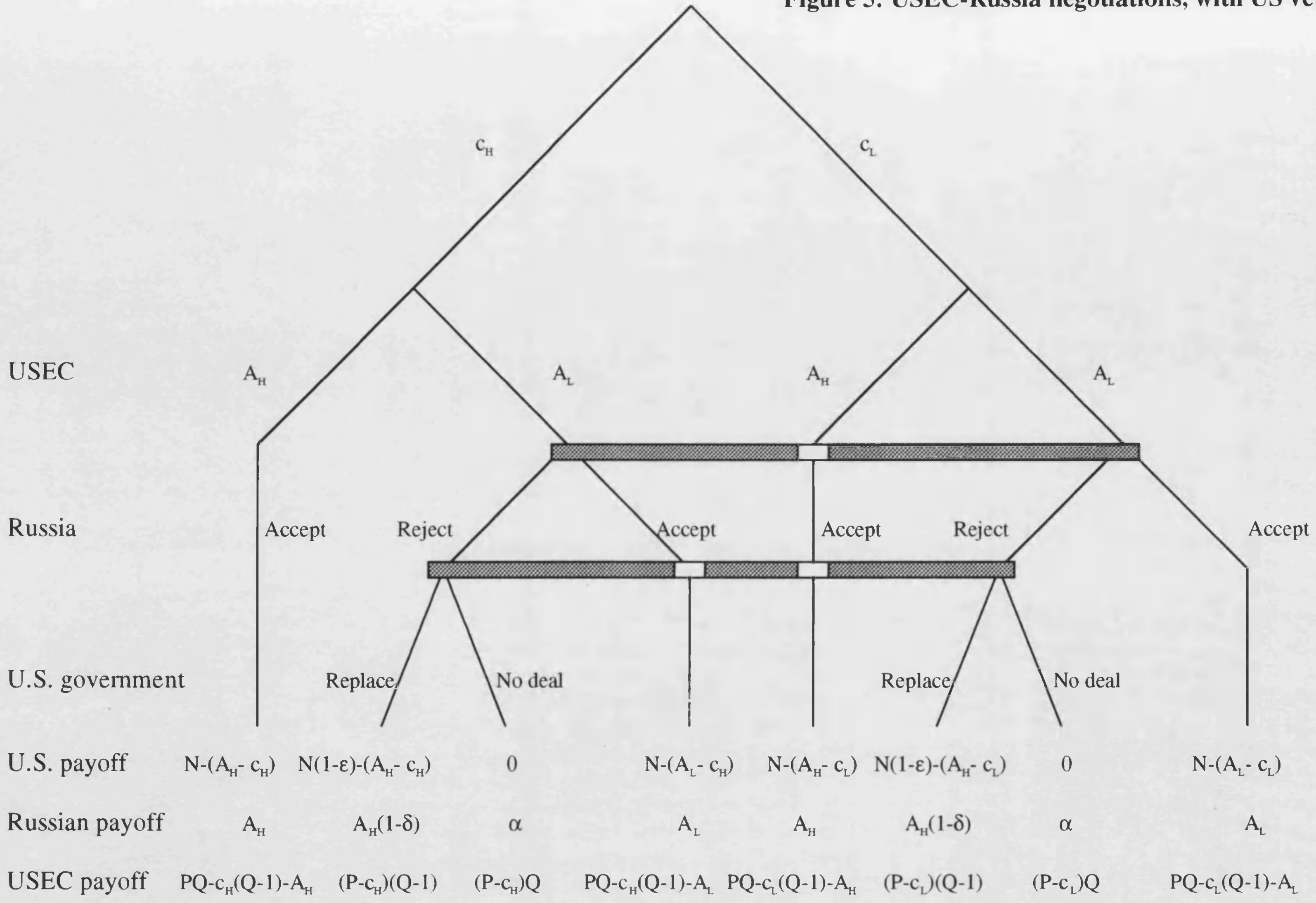
[125] The choices of t=0 and t=1 are merely for ease of exposition. The critical t is defined by the replacement condition (30).

Figure 5: USEC-Russia negotiations, with US veto



avoid being replaced as executive agent. Comparing domestic costs to the cost of the Russian material is therefore a misleading indicator of incentives, as USEC argues.

Even in this case, privatizing makes the U.S. government worse off. In equilibrium, the U.S. government's expected payoff is $N_H - (A_H - qc_H - (1-q)c_L)$. Relative to the payoff in the non-privatized case as given by (28), the change in U.S. welfare is $(1-q)\{k(N_H - (A_H - c_L)) - (A_H - A_L)(1-k)\}$, which is unambiguously negative since (1-q) is positive and both terms within the brackets are negative from (25). Ironically, the problem is that the U.S. government's loss of information leads the HEU deal to be implemented too often, which hurts U.S. welfare. The decline in welfare arises only when $c=c_L$, the state in which the U.S. government would never offer A_H if negotiations were direct.[126]

This section illustrates how the government's loss of private information following privatization results in $\Delta S<0$. The result can be viewed in light of the Sappington and Stiglitz (1987) fundamental theorem of privatization: that the principal difference between public ownership and privatization is the transaction costs of government intervention in the enterprise's activities. As they conclude, "When the task is particularly novel and complex, unforeseen contingencies are likely to arise. If rapid adaptation to these events is crucial (as in the case of national defense, for example), ease of intervention to redirect activities may be relatively important; under such circumstances, public provision is more likely to be the preferred mode of organization."[127] The HEU negotiations are both novel and complex, and have already demonstrated the need for rapid adaptation to changing events. It is unlikely that government oversight will provide as much flexibility to respond to these events as government

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[126] Note that Russia's welfare is *higher* than under the government-only scenario (12), since it always receives A_H for its uranium. This equilibrium would also be achieved by appointing an alternative executive agent, which does not know c and could either purchase LEU from Russia or be forced to buy it from USEC. Such an alternative executive agent always prefers to offer A_H to Russia if $r > \frac{A_H - A_L}{P - A_L}$. This restriction on the rejection probability is credible in equilibrium given the conditions (26) and (27).

[127] D. Sappington and J. Stiglitz, "Privatization, Information, and Incentives," *Journal of Policy Analysis and Management*, Vol. 6, No. 4, Summer 1987, page 581.

ownership.

VII. Conclusion

Privatization is usually seen as a method of bolstering internal incentives and thus improving internal efficiency. But it often has broader implications than its internal efficiency effects. For example, privatizing a monopoly creates a potential trade-off between internal efficiency ($\Delta\pi$) and allocative efficiency (ΔS).

In the specific case study examined here, privatization under current institutional constraints may involve a trade-off between internal efficiency and a national security objective. The full information analysis in Section IV suggests that USEC's internal efficiency may be significantly higher in the private sector, but that the expected costs from the national security concerns could potentially dominate the gain in internal efficiency. Section V raises further questions about privatization by demonstrating that the analysis in Section IV does not take sufficient account of uncertainty and irreversibility. If privatization is irreversible, and if there is uncertainty over its social returns, the "threshold" of $\Delta W=0$ proposed in Section IV will be biased toward approving privatizations that are not truly socially optimal. Given that ΔW could be close to zero even under full information, this section raises additional doubts about the net social benefits from privatizing USEC under current conditions. The dynamic game studied in Section VI highlights the principal-agent problem in implementing the HEU deal after USEC is privatized. The results of the game indicate that the government's loss of information following privatization reduces social welfare. The conclusion is that privatization is not always a panacea. Indeed, in USEC's case, it may be quite costly.

Appendix. The three major non-U.S. enrichment suppliers

This appendix briefly describes the three major enrichment suppliers outside of the United States: Eurodif, Urenco, and MINATOM. Eurodif runs a gaseous diffusion plant in Tricastin, France, with employment of about 1,400. The majority of its shares are owned by Cogema, a subsidiary of the French government's Atomic Energy Commission.[128] Eurodif sold 7 million SWU in 1994 (a global market share of roughly 20 percent). According to the company itself, its "economic production capacity is limited to 8 million SWU, after which the use of very high cost incremental power becomes prohibitive."[129]

Urenco operates three gas centrifuge plants: one in Capenhurst, the United Kingdom; the second in Almelo, the Netherlands, and the third in Gronau, Germany. Its shares are divided equally between the U.K. government (British Nuclear Fuels, Limited), the Dutch government, and private German utilities. Most of its business is in the U.K. and Germany, where it has a market share of about 75 percent, although it also has contracts in 13 countries and supplies 4 percent of the U.S. market.[130] The nature of the centrifuge technology, in which the optimal plant size is relatively small, implies that Urenco is usually near capacity in the short run. As Dr. Klaus Messer, the Chief Executive Officer of Urenco, has put it: "Our customer orders and capacity expansion always go hand in hand."[131]

MINATOM runs three principal enrichment facilities: Chelyabinsk-65, Krasnoyarsk-26, and Tomsk-7.[132] MINATOM uses the gas centrifuge technology, having closed its last gaseous diffusion plant (at Tomsk) in 1991.[133] With assistance from its government-owned marketing agent, TENEX, it has been supplying enrichment services — on the order of 2 to 4

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[128] Cogema holds 52 percent of Eurodif's shares. The remainder is held by the Italians (16 percent), the Belgians (11 percent), the Spanish (11 percent), and the Iranians (10 percent). The Italian and Iranian shares are inactive and do not carry voting rights. See Eurodif, "Fact Sheet on Eurodif," transmitted to the author on October 17, 1995.

[129] Eurodif, "Fact Sheet on Eurodif"

[130] "The New Birth of Urenco," *NUKEM Market Report*, op. cit., page 9.

[131] Interview with Dr. Klaus Messer, *NUKEM Market Report*, op. cit., page 17.

[132] Ex-USSR Nuclear Technology and the World, "Russian Republic," http://www.ida.net, page 1.

[133] O. Bukharin, "Analysis of the Size and Quality of Uranium Inventories in Russia," op. cit., page 67.

million SWU per year — to Western Europe since 1974.[134] But there are strict quotas on MINATOM's exports to the European market, and an anti-dumping case limits exports to the United States.[135] MINATOM is also the Russian government's representative in the HEU deal.

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[134] A.J. Hyett, "The Structure and Economics of the Nuclear Fuel Cycle Service Industry," op. cit., page 171; and A. D. Owen, *The Economics of Uranium*, op. cit., page 188. Bukharin cites a figure of 40 million SWU in Soviet exports to the West between 1973 and 1988; see "Analysis of the Size and Quality of Uranium Inventories in Russia," op. cit., page 67.

[135] The anti-dumping case in the United States exempts SWU imported under the HEU deal.

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