Welfare and Macroeconomic Policy in Small Open Economies

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Abstract

This thesis focuses on the analysis of welfare and macroeconomic policy in small open economies. The international dimension of monetary and fiscal policy is examined in a micro-founded New-Keynesian framework. The small open economy is characterized as a limiting case of a two-country dynamic general equilibrium model featuring imperfect competition and nominal rigidities. Under this specification, Chapter 1 formulates a utility-based loss function for a small open economy completely integrated with the rest of the world. The study investigates the role of the exchange rate in monetary policy and derives the optimal monetary policy rule. In this Chapter, the dynamics of the trade balance are shown to be crucial in determining the appropriate exchange rate regime.

Chapter 2 analyses optimal monetary policy under alternative asset market structures; more specifically, it compares and contrasts the cases of incomplete asset markets, financial autarky and complete asset markets. Furthermore, the performance of standard monetary policy rules is evaluated under these different scenarios. The results show that the degree of substitutability between domestic and foreign goods and the level of risk sharing are important factors in determining the performance of policy rules.

Finally, Chapter 3 incorporates fiscal policy in the general framework. This Chapter introduces distortionary taxation into the model and characterizes the optimal fiscal policy. In addition, a general monetary and fiscal policy problem is formulated in the presence of nominal rigidities. The Chapter demonstrates that the stabilization problem in an open economy is more complex than in a closed economy, even under flexible prices. Apart from the incentive to avoid the distortions implied by taxation, in a small open economy there is also an incentive to strategically affect the real exchange rate. That is, proportional taxation creates a distortion in the economy, but also introduces a policy instrument that can influence the terms of trade and the overall level of production and consumption in a welfare-improving manner.

To My Parents, to Bruno and to Xavi

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Introduction

A Bank of England survey documented in Fry et al. (2000) shows that, in 1997-1998, more than 70% of central banks had the exchange rate as part of their policy objectives. Many countries, although officially under a flexible exchange rate regime, do not allow a free-floating exchange rate (see Calvo and Reinhart (2002)). Moreover, as detailed in Reinhart and Rogoff (2002), the most common monetary regime throughout modern history has been an exchange rate peg. This evidence motivates the following question: should monetary authorities target the exchange rate? To deliver an answer to the question, this thesis formulates a micro-founded model of a small open economy and its corresponding welfare characterization.

The small open economy setting is derived as a limiting case of a two-country dynamic general equilibrium model. The benchmark model features imperfect competition and nominal rigidities following the New Open Economy Macroeconomic literature. Under this specification, Chapter 1 characterizes welfare in a small open economy and derives the corresponding optimal monetary policy rule. It shows that the utility-based loss function for a small open economy is a quadratic expression in domestic inflation, the output gap and the real exchange rate.

Previous work has suggested that welfare in a small open economy should not be affected by exchange rate variability and that policymakers should stabilize domestic inflation (see, for example Clarida, Galí and Gertler (2001) and Galí and Monacelli (2005)). Chapter 1, however, demonstrates that a small open economy that is completely integrated with the rest of the world can indeed be affected by exchange rate variability. Consequently, the optimal policy in a small open economy is not isomorphic to that in a closed economy and it does not prescribe a pure floating exchange rate regime. Domestic inflation targeting is optimal only for a particular parameterization, in which the unique relevant distortion in the economy is price stickiness. In the presence of an inefficient level of steady-state output and trade imbalances, exchange rate targeting arises as part of the optimal monetary plan.

The above result was obtained under the assumption that international financial markets can provide perfect risk sharing between the small open economy and the rest of the world. However, as illustrated in Obstfeld and Rogoff (1996), *"The presence of international markets for risky assets weakens and may sever the link between shocks to a country's output or factor productivity and shocks to its resident's income. Sophisticated international financial markets thus force us to rethink the channels through which macroeconomic shocks impinge on the world economy"*. That is, the assumption that domestic agents can insure against idiosyncratic risk has strong consequences for the dynamics of open economies. This issue leads us to revisit the findings of Chapter 1, and assess their robustness to different formulations of asset market structure. Chapter 2 addresses this particular question by deriving the optimal monetary policy for a small open economy under complete and incomplete asset markets, and also under financial autarky.

Our results demonstrate that the configuration of financial markets may significantly influence policy prescription. In the presence of perfect risk sharing, an exchange rate peg outperforms inflation targeting if domestic and foreign goods are substitutes in the utility function. On the other hand, in the case of incomplete markets, price stability leads to higher welfare than a fixed exchange rate regime and the optimal policy does not differ quantitatively from a pure domestic inflation targeting regime. If imported goods are complements to domestic goods in agents' utility, this conclusion is reversed.

Chapters 1 and 2 of this thesis suggest that international aspects of the economy, such as the trade balance and international financial markets, may affect the policy prescription considerably. In particular, these factors dictate whether or not there are policy incentives to affect the exchange rate. When these incentives exist, monetary policy deviates from price stability. But can fiscal policy, rather than monetary policy, be used strategically in an open economy? This issue is addressed in Chapter 3.

The analysis in Chapter 3 focuses initially on the case of flexible prices, in order to highlight the open economy dimension of the fiscal policy problem. Indeed, under this structure there are two policy incentives: reducing the inefficiency caused by movement in distortionary taxation; and managing strategically the real exchange rate. In contrast to the closed economy framework, in a small open economy it is not optimal to perfectly smooth taxes to avoid distortions in households' choices regarding consumption and leisure. Distortionary income taxes can be used to improve welfare by affecting the overall level of production and consumption and the relative price of domestic goods. For example, higher taxes could induce a smaller depreciation of the real exchange rate, allowing domestic agents to switch consumption towards foreign produced goods. Note that, in a closed economy, this mechanism is absent because a fall in the disutility of domestic production would be accompanied by a corresponding reduction in the utility of consumption.

Chapter 1

Monetary Policy and Welfare in a Small Open Economy

1.1 Introduction

Numerous papers have analysed the choice of monetary policy objectives in closed and open economies. In the former, the debate has mainly focused on whether inflation should be the unique policy target. In open economies, the characterization of optimal policy extends beyond policymaker's decision to concentrate on domestic price distortions. More specifically, the role of the exchange rate in the monetary policy framework needs to be considered. This Chapter addresses this particular issue in a small open economy setting. Our results suggest that including the exchange rate as part of the stabilization goals of monetary policy can be welfare improving for a small open economy.

We lay out a small open economy model as a limiting case of a two-country dynamic general equilibrium framework, featuring monopolistic competition and price stickiness. Moreover, the framework assumes no trade frictions (i.e. the law of one price holds) and perfect capital markets (i.e. asset markets are complete). This benchmark specification allows us to focus on the policy implications of the following factors: (a) Calvo-type staggered price setting; (b) monopolistic competition in goods' production and the resulting inefficient level of output; (c) trade imbalances;

and (d) deviations from purchasing power parity that arise from the home bias specification. The framework presented here encompasses, as special cases, the closed economy setting (as in Benigno and Woodford (2003)) and the small open economy case with a specific degree of monopolistic competition and no trade imbalances (as in Galí and Monacelli (2005)).

The small open economy representation prevents domestic policy from affecting the rest of the world and, therefore, permits us to abstract from strategic interactions between countries. We focus on understanding how monetary authorities should react to fluctuations in internal and external conditions when these reactions have no feedback effects.

Following the method developed by Benigno and Woodford (2003) and Sutherland (2002), we derive a loss function for a small open economy from the utility of the representative household. We show that the small open economy's loss function is a quadratic expression in domestic producer inflation, the output gap and the real exchange rate. The weights given to each of these variables depend on structural parameters of the model, and are hence determined by the underlying economic inefficiencies. In addition, the policy targets depend on the source of the disturbance affecting the economy, which includes an external shock.

The analytical representation of welfare allows for a precise qualitative analysis of monetary policy in a small open economy. The results obtained show that domestic inflation targeting is optimal only under specific assumptions. In cases where the economy experiences productivity and foreign shocks, a domestic inflation target is optimal only under a particular parameterization for the coefficient of risk aversion and the elasticity of substitution between domestic and foreign goods. Moreover, if fiscal disturbances are also present, the optimality of domestic price stabilization further requires a production subsidy. Conversely, in the general specification of the model, the exchange rate becomes part of monetary policy targets. Therefore, policy prescription in a small open economy is not isomorphic to a closed economy and it does not prescribe a pure floating exchange rate regime. Moreover, the quantitative results show that, for a large set of parameter specifications, an exchange rate peg outperforms a strict domestic inflation target. This result is consistent with the findings of Sutherland (2005). The author demonstrates that, for high values of the elasticity of substitution between goods, a fixed exchange rate regime leads to higher welfare than targeting domestic prices.

1.1.1 Related Literature

This work follows the New-Keynesian literature on dynamic general equilibrium models featuring imperfect competition and price rigidities. The study of these models has been extensive in the past decade.[1] Clarida, Galí and Gertler (1999) contains a survey of the early works on the closed economy literature. Important contributions include Goodfriend and King (1997) and Woodford (1999 and 2001). In addition, Woodford (2003) has a comprehensive exposition of the baseline closed economy framework and many of its extensions. In the open economy literature, the Obstfeld and Rogoff (1995) *Redux* model is generally accepted as the precursor to introducing price stickiness and imperfect competition in an open economy setting. Surveys of subsequent contributions can be found in Lane (2001), Sarno (2000) and Bowman and Doyle (2002). These authors present the benchmark *Redux* model, followed by a description of alternative specifications and extensions.

This Chapter presents a micro-founded analysis of monetary policy. It derives the loss function from the utility of the representative household. The linear quadratic approach used in the analysis follows Woodford (2001), Benigno and Woodford (2003) and Sutherland (2002b). Other works that employ similar methods include Benigno and Benigno (2003, 2003b) and Ferrero (2005), amongst others.

The optimality of the inflation target and the role of the exchange rate in monetary policy have been addressed in many previous studies. The closed economy literature contains extensive analysis of the optimality of inflation targeting. Woodford (2001) and Goodfriend and King (2001) are important contributions. Woodford and Benigno (2003) incorporate steady-state distortions created by monopolistic compe-

[1] This Section contains a non-exhaustive account of the related literature. We present a very brief exposition of works with the same line of models; studies that have followed the technical approach employed in this chapter; and papers that have addressed similar questions to the ones asked here.

tition into the analysis. Aoki (2001) analyses monetary policy in an economy with a flexible-price sector and a sticky-price sector.

In the open economy literature, several authors have investigated the role of the exchange rate in monetary policy formulation. Corsetti and Pesenti (2001) were the first to emphasize that a country might benefit from influencing its terms of trade. Benigno and Benigno (2003) illustrate the potential gains from cooperation of monetary policy between countries by analyzing the incentives of individual countries to affect the exchange rate. Corsetti and Pesenti (2005) and Sutherland (2002c) show that, with incomplete pass through, optimal monetary policy is not purely inward looking. Tille (2002) draws the same conclusion in the presence of sector specific shocks.

However, studies including Sutherland (2002c), Corsetti and Pesenti (2005), Clarida, Galí and Gertler (2001) and Galí and Monacelli (2005) have found that, under producer currency pricing and complete pass-through , there is no role for exchange rate targeting in monetary policy. Moreover, the optimal policy is shown to be completely inward looking and prescribes a pure domestic inflation target. These studies, however, analyse a characterization of a small open economy in which there are no trade imbalances. In this Chapter, we attempt to contribute to this vast literature by relaxing the last restriction, with the intention of improving our understanding of the international dimension of monetary policy.

The remainder of the Chapter is structured as follows. Section 1.2 introduces the model and derives the small open economy dynamics. Section 1.3 is dedicated to the derivation of welfare and the quadratic loss function. Section 1.4 analyses the optimal plan and the performance of a standard policy rule. Section 1.5 concludes.

1.2 The Model

The framework consists of a two-country dynamic general equilibrium model with complete asset markets. Deviations from purchasing power parity arise from the existence of home bias in consumption. This bias depends on the degree of openness and the relative size of the economy. The specification allows us to characterize the small open economy by taking the limit of the home economy size to zero. Prior to applying the limit, we derive the optimal equilibrium conditions for the general two-country model. After the limit is taken, the two countries, Home and Foreign, represent the small open economy and the rest of the world, respectively.

Monopolistic competition and sticky prices are introduced in the small open economy in order to address issues of monetary policy. We further assume that home price setting follows a Calvo-type contract, which introduces richer dynamic effects of monetary policy than in a setup where prices are set one period in advance. Moreover, we abstract from monetary frictions by considering a cashless economy as in Woodford (2003, Chapter 2).

1.2.1 Preferences

We consider two countries, H (Home) and F (Foreign). The world economy is populated with a continuum of agents of unit mass, where the population in the segment $[0, n)$ belongs to country H and the population in the segment $(n, 1]$ belongs to country F. The utility function of a consumer j in country H is given by:[2]

$$U_t^j = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[U(C_s^j) - V(y_s(j), \varepsilon_{Y,s}) \right]. \tag{1.1}$$

Households obtain utility from consumption $U(C^j)$ and contribute to the production of a differentiated good $y(j)$ attaining disutility $V(y(j), \varepsilon_{Y,})$.[3] Productivity shocks are denoted by $\varepsilon_{Y,s}$. C is a Dixit-Stiglitz aggregator of home and foreign goods, defined by

$$C = \left[v^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1-v)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{1.2}$$

[2] In the subsequent sections, we assume the following isoelastic functional forms: $U(C_t) = \frac{C_t^{1-\rho}}{1-\rho}$ and $V(y_t, \varepsilon_{Y,t}) = \frac{\varepsilon_{Y,t}^{-\eta} y_t^{1+\eta}}{1+\eta}$, where ρ is the coefficient of relative risk aversion and η is equivalent to the inverse of the elasticity of labor production.

[3] This specification would be equivalent to one in which the labour market is decentralized. These firms employ workers who have disutility of supplying labour and this disutility is separable from the consumption utility.

where $\theta > 0$ is the intratemporal elasticity of substitution and C_H and C_F are consumption sub-indices that refer to the consumption of home-produced and foreign-produced goods, respectively. The parameter determining home consumers' preferences for foreign goods, $(1-v)$, is a function of the relative size of the foreign economy, $1-n$, and of the degree of openness, λ; more specifically, $(1-v) = (1-n)\lambda$.

Similar preferences are specified for the rest of the world,

$$C = \left[v^{*\frac{1}{\theta}} C_H^{*\frac{\theta-1}{\theta}} + (1-v^*)^{\frac{1}{\theta}} C_F^{*\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{1.3}$$

with $v^* = n\lambda$. That is, foreign consumers' preferences for home goods depend on the relative size of the home economy and the degree of openness. Note that the specification of v and v^* generates a home bias in consumption, as in Sutherland (2002).

The sub-indices C_H (C_H^*) and C_F (C_F^*) are Home (Foreign) consumption of the differentiated products produced in countries H and F. These are defined as follows:

$$C_H = \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n c(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}}, \quad C_F = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\sigma}} \int_n^1 c(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}}, \tag{1.4}$$

$$C_H^* = \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n c^*(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}}, \quad C_F^* = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\sigma}} \int_n^1 c^*(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}}, \tag{1.5}$$

where $\sigma > 1$ is the elasticity of substitution across the differentiated products. The consumption-based price indices that correspond to the above specifications of preferences are given by

$$P = \left[v P_H^{1-\theta} + (1-v)(P_F)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \tag{1.6}$$

and

$$P^* = \left[v^* P_H^{*1-\theta} + (1-v^*)(P_F^*)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \tag{1.7}$$

where P_H (P_H^*) is the price sub-index for home-produced goods expressed in the domestic (foreign) currency and P_F (P_F^*) is the price sub-index for foreign produced goods expressed in the domestic (foreign) currency:

$$P_H = \left[\left(\frac{1}{n}\right)\int_0^n p(z)^{1-\sigma}dz\right]^{\frac{1}{1-\sigma}}, P_F = \left[\left(\frac{1}{1-n}\right)\int_n^1 p(z)^{1-\sigma}dz\right]^{\frac{1}{1-\sigma}}, \quad (1.8)$$

$$P_H^* = \left[\left(\frac{1}{n}\right)\int_0^n p^*(z)^{1-\sigma}dz\right]^{\frac{1}{1-\sigma}}, P_F^* = \left[\left(\frac{1}{1-n}\right)\int_n^1 p^*(z)^{1-\sigma}dz\right]^{\frac{1}{1-\sigma}}. \quad (1.9)$$

We assume that the law of one price holds, so

$$p(h) = Sp^*(h) \text{ and } p(f) = Sp^*(f), \quad (1.10)$$

where the nominal exchange rate, S_t, denotes the price of foreign currency in terms of domestic currency. Equations (1.6) and (1.7), together with condition (1.10), imply that $P_H = SP_H^*$ and $P_F = SP_F^*$. However, as Equations (1.8) and (1.9) illustrate, the home bias specification leads to deviations from purchasing power parity; that is, $P \neq SP^*$.[4] For this reason, we define the real exchange rate as $RS = \frac{SP^*}{P}$.

From consumers' preferences, we can derive the total demand for a generic good h, produced in country H, and the demand for a good f, produced in country F:

$$y_t^d(h) = \left[\frac{p_t(h)}{P_{H,t}}\right]^{-\sigma}\left\{\left[\frac{P_{H,t}}{P_t}\right]^{-\theta}\left[vC_t + \frac{v^*(1-n)}{n}\left(\frac{1}{RS_t}\right)^{-\theta}C_t^*\right] + G_t\right\}, \quad (1.11)$$

$$y_t^d(f) = \left[\frac{p_t(f)}{P_{F,t}}\right]^{-\sigma}\left\{\begin{array}{c}\left[\frac{P_{F,t}}{P_t}\right]^{-\theta}\left[\frac{(1-v)n}{1-n}C_t + (1-v^*)\left(\frac{1}{RS_t}\right)^{-\theta}C_t^*\right] \\ +G_t^*\end{array}\right\}, \quad (1.12)$$

where G and G^* are country-specific government shocks. We assume that the public sector in the Home (Foreign) economy only consumes Home (Foreign) goods and has preferences for differentiated goods analogous to the ones of the private sector (given by Equations 1.4 and 1.5). The government budget constraints in the Home and

[4]The literature investigating the empirical evidence of Purchasing Power Parity is vast and has shown that short-run deviations from PPP are large and volatile (as documented in Rogoff (1996)). Even though our model specification is in accordance with these findings, it dismisses the evidence of failures of the law of one price. Those are extensively documented in the literature (see e.g. Engel and Rogers (1999) and (2000)) and can be caused by the existence of trade barriers, transportation costs or the presence of non-traded inputs.

Foreign economy are respectively given by

$$\tau_t \int_0^n p_t(h) y_t(h) dh = n P_{H,t}(G_t + Tr_t) \tag{1.13}$$

and

$$\tau_t \int_n^1 p_t^*(f) y_t^*(f) dh = (1-n) P_{F,t}(G_t^* + Tr_t^*). \tag{1.14}$$

We consider the case in which fluctuations in proportional taxes, τ_t (τ_t^*), or government spending, G_t (G_t^*), are exogenous and completely financed by lump-sum transfers, Tr_t (Tr_t^*), made in the form of domestic (foreign) goods.

Finally, to portray our small open economy, we use the definition of v and v^* and take the limit for $n \to 0$. Consequently, conditions (1.11) and (1.12) can be rewritten as

$$y^d(h) = \left[\frac{p_t(h)}{P_{H,t}}\right]^{-\sigma} \left\{ \left[\frac{P_{H,t}}{P_t}\right]^{-\theta} \left[(1-\lambda)C_t + \lambda \left(\frac{1}{RS_t}\right)^{-\theta} C_t^*\right] + G_t \right\}, \tag{1.15}$$

$$y^d(f) = \left[\frac{p_t^*(f)}{P_{F,t}^*}\right]^{-\sigma} \left\{ \left[\frac{P_{F,t}^*}{P_t^*}\right]^{-\theta} C_t^* + G_t^* \right\}. \tag{1.16}$$

Equations (1.15) and (1.16) show that external changes in consumption affect the small open economy, but the opposite is not true. Moreover, movements in the real exchange rate do not affect the rest of the world's demand.

1.2.2 The Asset Market Structure

We assume that, as in Chari et al. (2002), markets are complete domestically and internationally. In each period t, the economy faces one of finitely many events, $s^t \in \Upsilon$ (where Υ is the set of finitely many states). We denote the history of events up to and including period t by x^t. Looking ahead from period t, the conditional probability of occurrence of state s^{t+1} is $\mu(s^{t+1} \mid x^t)$. The initial realization s^0 is given. We represent the asset structure by having complete contingent one-period nominal bonds, denominated in the home-currency. We let $B^j\left(s^{t+1}\right)$ denote home consumers' holdings of this bond, which pays one unit of the home currency if state s^{t+1} occurs and 0 otherwise. We let $Q(s^{t+1} \mid x^t)$ denote the price of one unit of such a bond at date t and state s^t in units of domestic currency. Hence, consumer j faces a sequence of budget constraints given by

$$P(s^t)C^j(s^t)+\sum_{s^t\in\Upsilon} Q(s^{t+1}\mid x^t)B^j(s^{t+1}) \leq B^j\left(s^t\right)+(1-\tau_t)p^j(s^t)y^j(s^t)+P_H(s^t)Tr(s^t). \tag{1.17}$$

A similar expression can be derived for the foreign economy. Households at home maximize (1.1) subject to (1.17), and their optimal allocation of wealth across the different state contingent bonds implies that

$$Q(s^{t+1}\mid x^t)=\beta\mu(s^{t+1}\mid x^t)\frac{U_C\left(C(s^{t+1})\right)}{U_C\left(C(s^t)\right)}\frac{P(s^t)}{P(s^{t+1})}. \tag{1.18}$$

Similarly for the foreign economy,

$$Q(s^{t+1}\mid x^t)=\beta\mu(s^{t+1}\mid x^t)\frac{U_C\left(C^*(s^{t+1})\right)}{U_C\left(C^*(s^t)\right)}\frac{S(s^t)P^*(s^t)}{S(s^{t+1})P^*(s^{t+1})}. \tag{1.19}$$

Thus, the optimal risk sharing setting implies that the intertemporal marginal rate of substitution (in nominal terms) is equalized across countries,

$$\boxed{\frac{U_C\left(C^*_{t+1}\right)}{U_C\left(C^*_t\right)}\frac{P^*_t}{P^*_{t+1}}=\frac{U_C\left(C_{t+1}\right)}{U_C\left(C_t\right)}\frac{S_{t+1}P_t}{S_tP_{t+1}}.} \tag{1.20}$$

Equation (1.20) holds in all states of nature. This specification for the asset market implies that the risk arising from movements in agent's nominal wealth is shared with the rest of the world. However, because of deviations from purchasing power parity, real exchange rate movements may lead to differences between home and foreign real income and, consequently, differences in the evolution of consumption across borders.

1.2.3 Price-setting Mechanism

Prices follow a partial adjustment rule à la Calvo (1983). Producers of differentiated goods know the form of their individual demand functions (given by Equations (1.15) and (1.16)), and maximize profits taking overall market prices and products as given. In each period a fraction, $\alpha \in [0,1)$, of randomly chosen producers is not allowed to change the nominal price of the goods they produce. The remaining fraction of firms, given by $(1-\alpha)$, chooses prices optimally by maximizing the expected discounted value of profits.[5] The optimal choice of producers that can set their price $\tilde{p}_t(j)$ at time T is, therefore:

$$E_t\left\{\sum(\alpha\beta)^{T-t}U_c(C_T)\left(\frac{\tilde{p}_t(j)}{P_{H,T}}\right)^{-\sigma}Y_{H,T}\left[\frac{\tilde{p}_t(j)}{P_{H,T}}\frac{P_{H,T}}{P_T}-\frac{\sigma V_y\left(\tilde{y}_{t,T}(j),\varepsilon_{Y,T}\right)}{(1-\tau_T)(\sigma-1)U_c(C_T)}\right]\right\}=0. \tag{1.21}$$

Monopolistic competition in production leads to a wedge between marginal utility of consumption and marginal disutility of production, represented by $\frac{\sigma}{(1-\tau_t)(\sigma-1)}$.[6] We allow for fluctuations in this wedge by assuming a time-varying proportional tax τ_t. Hereafter, we refer to these fluctuations as mark-up shocks μ_t, where $\mu_t = \frac{\sigma}{(1-\tau_t)(\sigma-1)}$.

Given the Calvo-type setup, the price index evolves according to the following law of motion,

$$(P_{H,t})^{1-\sigma} = \alpha P_{H,t-1}^{1-\sigma} + (1-\alpha)\left(\tilde{p}_t(h)\right)^{1-\sigma}. \tag{1.22}$$

The rest of the world has an analogous price setting mechanism.

In this set-up, the number of firms that can change prices in any given period is specified exogenously. The Calvo price-setting mechanism can be interpreted as a short-cut for an environment in which firms face costs of changing prices. These costs would induce firms to optimize and reset prices only periodically. As described in Eichenbaum and Fisher (2004), *"The type of costs one could have in mind is those associated with optimization (e.g., costs associated with information gathering, decision making, negotiation and communication)."* There are alternative set-ups to the Calvo approach which antagonize the price setting behavior and introduce a

[5] All households within a country (that can modify their prices at a certain time) face the same discounted value of the streams of current and future marginal costs. Thus, they choose to set the same price.

[6] Note that, if there are no proportional taxes and an infinitely elastic demand $\mu = 1$,this specification characterizes the perfect competition case.

state-dependent price setting behavior. Alternative frameworks include: models in which information (rather than prices) are sticky (as in Mankiw and Reiss (2001)); menu cost models in which the frequency of price setting is state dependent (e.g. Dotsey et al. 1999); and quadratic adjustment cost models and long-term customer relationships models (see e.g. Rotemberg (1982) and (2002)). Even though some of these models might benefit from endogenous price setting behavior, they are significantly less tractable to work with than the Calvo approach.[7]

1.2.4 A Log-linear Representation of the Model

In this Section, we present a log linear version of the model. This is done to obtain a simple representation of the optimality conditions derived above and to illustrate the dynamic properties of the model. We later solve the log-linearized model numerically using the algorithm of King and Watson (1998) and present a quantitative analysis of the model. We approximate the model around a steady state in which the exogenous variables ($\varepsilon_{y,t}$,G_t, μ_t) assume the values $\bar{\varepsilon}_y \geq 0$, $\bar{G} = 0$ and $\mu \geq 1$, while producer price inflation is set as $\Pi_{H,t} \equiv P_{H,t}/P_{H,t-1} = 1$. In this steady state, $\overline{RS} = 1$, $\bar{C} = \bar{C}^*$, $\bar{Y} = \bar{Y}^*$ and $\bar{U}_C(\bar{C}) = \mu \bar{V}_y(\bar{Y}, 0)$.[8] The log deviation of a variable from its steady-state value is denoted with a hat.

The small open economy system of equilibrium conditions derived from log linearizing Equations (1.6), (1.15), (1.20) and (1.21) is

$$(1-\lambda)\hat{p}_H + \lambda \widehat{RS} = 0, \tag{1.23}$$

$$\hat{Y}_t = -\theta \hat{p}_H + (1-\lambda)\hat{C} + \lambda \hat{C}^* + \theta\lambda \widehat{RS}_t + \hat{g}_t, \tag{1.24}$$

[7] Recent literature testing the Calvo mechanism shows contradictory results. Fuhrer and Moore (1995) study the relationship between inflation and output in the US and show that the Calvo assumption is inconsistent with the evolution of US inflation. Gali and Gertler (1999) argue that inflation should be explained by marginal costs rather than output and demonstrate that when this is considered, Calvo pricing does explain US inflation in the period after 1960. Moreover, Gali and Gertler (1999) found that the estimates of the degree of price stickiness are stable over different samples (consistent with the Calvo assumption). However, other empirical evidence suggests that the price setting decision normally depends on the state of the economy (see Fabiani at al. (2004)).

[8] This specification implies a specific level of the initial distribution of wealth across countries. Appendix A contains a full characterization of the steady state.

$$\hat{C}_t = \hat{C}_t^* + \frac{1}{\rho}\widehat{RS}_t, \qquad (1.25)$$

and

$$\widehat{\pi}_t^H = k\left(\rho\widehat{C}_t + \eta\widehat{Y}_t - \widehat{p}_H + \widehat{\mu}_t - \eta\widehat{\varepsilon}_{Y,t}\right) + \beta E_t\widehat{\pi}_{t+1}^H. \qquad (1.26)$$

Equation (1.23) describes the relationship between domestic relative prices ($p_{H,t} = P_{H,t}/P_t$) and the real exchange rate. Equation (1.24) characterizes the demand for domestic goods, with $\hat{g}_t$ defined as $\frac{Gt-\overline{G}}{\overline{Y}}$. The risk sharing condition is described in Equation (1.25). Finally, the last equation represents the small open economy Phillips Curve. We define $k = (1-\alpha\beta)(1-\alpha)/\alpha(1+\sigma\eta)$, and π_t^H denotes domestic producer price inflation; (i.e. $\pi_{H,t} \equiv \ln(P_{H,t}/P_{H,t-1})$). Moreover, as shown in the Appendix, ρ represents the coefficient of relative risk aversion, and η denotes the inverse of the elasticity of goods production. It is clear from Equation (1.26), that a policy of pure domestic price stabilization that sets $\widehat{\pi}_t^H = 0$ in every state leads to the same equilibrium allocation as the case in which prices are perfectly flexible (i.e. $\alpha = 0$, and therefore $k \to \infty$).

The system of structural equilibrium conditions is closed by specifying a monetary policy rule. In this paper, we consider the case in which monetary policy follows an optimal monetary policy. We represent the optimal plan in the form of a targeting rule. Targeting rules, as expressed in Svensson (2005), are a description of 'goal directed monetary policy'. Contrary to Taylor rules, an explicit expression for the evolution of the monetary policy instrument (i.e. the nominal interest rate) is not specified.[9] Gianonni and Woodford (2003) describe these rules as flexible inflation targets. Following this class of rules, the central bank stabilizes movements in the target variables in order to implement the most efficient allocation of resources (i.e. targeting rules are derived from a microfounded welfare maximization problem). Moreover, apart from the case in which monetary policy is represented by an optimal targeting rule, we consider the case in which the central bank follows standard policy rules. In particular, we analyse the performance of a producer price index (PPI) inflation target, an exchange rate peg, and a consumer price index

[9] For further discussion on targeting rules and instrumental rules see McCallum and Nelson (2005).

(CPI) inflation target.

The dynamics of $\widehat{Y}_t, \widehat{RS}_t, \widehat{C}_t, \widehat{\pi}_t^H$ and $\widehat{p}_{H,t}$ are determined by Equations (1.23) to (1.26) together with the specified monetary policy rule, given the domestic exogenous variables $\widehat{\varepsilon}_{y,t}$,$\widehat{g}_t$, $\widehat{\mu}_t$ and the external shock $\widehat{C}_t^*$.[10]

Foreign dynamics are governed by the foreign Phillips curve and foreign demand:

$$\widehat{\pi}_t^* = k\left(\rho\widehat{C}_t^* + \eta\widehat{Y}_t^* + \widehat{\mu}_t^* - \eta\widehat{\varepsilon}_{Y,t}^*\right) + \beta E_t\widehat{\pi}_{t+1}^* \tag{1.27}$$

and

$$\hat{Y}_t^* = \hat{C}^* + \hat{g}_t^*. \tag{1.28}$$

The specification of the foreign policy rule completes the system of equilibrium conditions, which determine the evolution of $\hat{Y}_t^*, \hat{C}^*$ and $\widehat{\pi}_t^*$. We should note that the dynamics of the rest of the world are not affected by *Home* variables. Hence, the small open economy can treat C_t^* as exogenous. The policy choice of the rest of the world modifies the way in which foreign structural shocks affect C_t^* but does not influence how the latter affects the small open economy.[11]

1.3 Welfare

The advantage of a microfounded model is that agents' discounted sum of expected utility provides a precise measure for welfare. That is, the small open economy objective function can be obtained from Equation (1.1). We follow the method developed by Woodford (2003) and Benigno and Woodford (2003) and obtain a quadratic expression for Equation (1.1). This allows us to represent the policy problem in a comprehensive manner; i.e. policymakers minimize a quadratic loss function subject to linear constraints. Moreover, the resulting optimal monetary policy can be expressed analytically. Alternative approaches to welfare evaluation include the computational methods described in Schmitt-Grohé and Uribe (2004),

[10] In order to retrieve the value of the nominal exchange rate and interest rate we can use household's intertemporal choice (i.e. the Euler equation) and the definition of the real exchange rate.

[11] For example, if the foreign authority is following a strict inflation target the evolution of foreign consumption is given by: $(\rho+\eta)\widehat{C}_t^* = \eta\widehat{\varepsilon}_{Y,t}^* - \eta\widehat{g}_t^* - \widehat{\mu}_t^*$

Collard and Juillard (2001) and Kim et al. (2003). These techniques are based on perturbation methods and deliver a numerical evaluation of the optimal policy problem.

We should note that the linear quadratic approach presented here takes into account the effect of second moments in the mean of the endogenous variables. As discussed in Benigno and Woodford (2003), this ensures that the method delivers an accurate (local) welfare evaluation tool. Another important contribution that emphasizes the relevance of second order effects on the mean of variables is Obstfeld and Rogoff (1998).

In the Appendix we derive analytically a second order approximation to Equation (1.1). In order to eliminate the discounted linear terms in the Taylor expansion, we use a second order approximation to some of the structural equilibrium conditions and obtain a complete second order solution for the evolution of the endogenous variables of interest. It follows that the final expression for the small open economy loss function can be written as a quadratic function of $\widehat{Y}_t$, $\widehat{RS}_t$, and $\widehat{\pi}_t^H$:

$$\begin{aligned} L_{to} &= U_c\bar{C}E_{t_0}\sum \beta^t \left[\frac{1}{2}\Phi_Y(\widehat{Y}_t - \widehat{Y}_t^T)^2 + \frac{1}{2}\Phi_{RS}(\widehat{RS}_t - \widehat{RS}_t^T)^2 + \frac{1}{2}\Phi_\pi(\widehat{\pi}_t^H)^2\right] \\ &\quad + t.i.p + \mathcal{O}(||\xi||^3), \end{aligned} \tag{1.29}$$

where the term $t.i.p$ stands for terms independent of policy (i.e. they are exogenous shock terms that are not affected by the policy choice). The term $\mathcal{O}(||\xi||^3)$ represents the terms of order higher than two. The policy targets $\widehat{Y}_t^T$ and $\widehat{RS}_t^T$ are functions of the various shocks and, in general, do not coincide with the flexible price allocation for output and the real exchange rate. The weights of inflation, output and the real exchange rate gap in welfare losses, Φ_π, Φ_Y and Φ_{RS}, all depend on the structural parameters of the model. The expressions for these variables are specified in Appendix B.

What are the economic forces behind these welfare losses? The small open economy specification presented in this work is characterized by two economic inefficiencies: price rigidity and monopolistic competition in production. In addition, in an open economy, domestic consumption is not necessarily equal to domestic production. In particular, movements in international relative prices can create differences between the marginal utility of consumption and the marginal disutility of production that directly affect welfare.[12] These factors create different policy incentives: the presence of staggered prices brings in gains from minimizing relative price fluctuations (justifying the presence $\Phi_\pi(\widehat{\pi}_t^H)^2$ in Equation (1.29)); monopolistic competition in production implies a suboptimal level of steady-state output and introduces an incentive to reduce steady-state production inefficiencies; and, finally, there may be incentives to manage fluctuations in the exchange rate in order to affect the wedge between the marginal utility of consumption and the marginal disutility of production (hereafter this is referred as the "U_c/V_y gap"). The last two factors imply that optimal monetary policy might deviate from price stability (and are also responsible for the presence of the terms $\Phi_Y(\widehat{Y}_t - \widehat{Y}_t^T)^2$ and $\Phi_{RS}(\widehat{RS}_t - \widehat{RS}_t^T)^2$ in Equation (1.29)).

To better understand the argument presented above, we first characterize a closed economy by setting $\lambda = 0$. In this case, Equation (1.29) can be written as:

$$L_{to}^c = U_c\bar{C}E_{t_0}\sum\beta^t\left[\frac{1}{2}\Phi_Y^c(\widehat{Y}_t - \widehat{Y}_t^{T,c})^2 + \frac{1}{2}\Phi_\pi^c(\widehat{\pi}_t^H)^2\right] + t.i.p + \mathcal{O}(||\xi||^3), \quad (1.30)$$

where the subscript c denotes the closed economy. The policymaker's problem in a closed economy can be illustrated by the relative weight of inflation with respect to output, Φ_π/Φ_Y, and by the difference between $\widehat{Y}_t^T$ and $\widehat{Y}_t^{Flex}$ (where $\widehat{Y}_t^{Flex}$ represents the flexible price allocation for output). The solution to these terms are:

$$\frac{\Phi_\pi^c}{\Phi_Y^c} = \frac{\sigma}{k(\eta+\rho)}, \quad (1.31)$$

$$\widehat{Y}_t^{T,c} = \frac{\eta\varepsilon_{Y,t}}{(\eta+\rho)} - \frac{(\mu-1)(\eta+1)\mu_t}{(\eta+\rho)(\mu\eta+\rho+(\mu-1))} + \frac{\rho(\eta\mu+\rho)g_t}{(\eta+\rho)(\mu\eta+\rho+(\mu-1))}, \quad (1.32)$$

[12] This is represented by the term $U_c\bar{C}(\widehat{C}_t - \widehat{Y}_t/\mu)$ in the Taylor expansion of the utility function (shown in the Appendix). Note that in our linear-quadratic aproach this term is expressed in terms of second moments. In particular, it can be written as a function of the variance of the real exchange rate and output gap.

and

$$\widehat{Y}_t^{Flex,c} = \frac{\eta\varepsilon_{Y,t} - \mu_t + \rho g_t}{(\eta + \rho)}. \tag{1.33}$$

As the above expressions show, $\widehat{Y}_t^{T,c} \neq \widehat{Y}_t^{Flex,c}$, so a policy of strict inflation targeting (which mimics the flexible price allocation) does not close the welfare relevant output gap. In particular, the steady-state level of the mark-up, μ, and mark-up fluctuations, μ_t, imply differences between $\widehat{Y}_t^T$ and $\widehat{Y}_t^{Flex}$. Whenever the steady-state level of production is efficient (i.e. $\mu = 1$) and there are no mark-up fluctuations, we have $\widehat{Y}_t^T = \widehat{Y}_t^{Flex}$. Therefore, there is a trade-off between stabilizing inflation and output. Moreover, the weight of inflation relative to output in the loss function depends essentially on the degree of market power, σ, and the degree of price rigidity, α (which determines the parameter k). When the elasticity of substitution between goods is infinite (i.e. the market is competitive) then the relative weight on the output gap vanishes. On the other hand, when $\alpha \longrightarrow 0$ (and consequently $k \longrightarrow \infty$), the relative weight on inflation fades away, as there are no distortions associated with price rigidity. PH directly affects this gap?

In a small open economy, real exchange rate movements, as well as domestic prices and output fluctuations, can also affect welfare. This is because the exchange rate can generate fluctuations in the so called "U_c/V_y gap". As shown in equation (1.24) and (1.23), the real exchange rate influences the relative price of Home produced goods and modifies the small open economy's demand. Secondly, in a world where purchasing power parity does not hold, real exchange rate movements generate real wealth variations, which, in turn, create fluctuations in households' ? spending and consumption (this can be seen by inspection of Equation (1.25)). It follows that the impact of the real exchange rate on output and consumption affects the wedge between the marginal utility of consumption and the marginal disutility of production. And fluctuations in this gap have an effect on the small open economy's welfare.

The value of intertemporal and intratemporal elasticities of substitution, $1/\rho$ and

θ, determine the real exchange rate effect on consumption and output through the risk sharing and demand channels explained above. Therefore, the weight of the real exchange rate in the loss function, Φ_{RS}, depends crucially on these parameters. More specifically, when $\rho\theta = 1$, the real exchange rate does not affect the "U_c/V_y gap" and $\Phi_{RS} = 0$. Section 1.4.1 explores this special case in detail. In addition, when the economy is relatively closed, the welfare implications of real exchange rate movements are small (as expected, when $\lambda \longrightarrow 0$, $\Phi_{RS} \longrightarrow 0$).

1.4 Optimal Monetary Policy

After characterizing the policy objective, we now turn to the constraints of the policy problem. The first constraint the policymaker faces is given by the Phillips Curve

$$\widehat{\pi}_t^H = k\left(\eta(\hat{Y}_t - \widehat{Y}_t^T) + (1-\lambda)^{-1}(\widehat{RS}_t - \widehat{RS}_t^T) + u_t\right) + \beta E_t \widehat{\pi}_{t+1}^H, \tag{1.34}$$

where u_t is a linear combination of the shocks defined in the Appendix. The policy problem is further constrained by the small open economy aggregate demand Equation (1.24) and the risk sharing condition (1.20). Combining these two conditions, the following relationship between output and the real exchange rate arises

$$(\widehat{Y}_t - \widehat{Y}_t^T) = (\widehat{RS}_t - \widehat{RS}_t^T)\frac{(1+l)}{\rho(1-\lambda)} + \chi u_t, \tag{1.35}$$

where $l = (\rho\theta - 1)\lambda\,(2-\lambda)$ and χ is a vector whose elements depend on the structural parameters (as shown in Appendix B). From Equation (1.34), we can see that the policy targets $\widehat{Y}_t^T$ and $\widehat{RS}_t^T$ are not necessarily the flexible price allocations of output and the real exchange rate. That is, the targets do not coincide with the allocations that would prevail if $\alpha = 0$ (and consequently $k \rightarrow \infty$). Moreover, Equation (1.35) shows that closing the output gap does not eliminate the real exchange rate gap.

We proceed by characterizing the optimal plan under the assumption that policymakers can commit to maximizing the economy's welfare. We lay out the Ramsey

(1.29) already assumes timeless perspective?

problem and derive the optimal policy response to the different shocks. The policy problem consists of choosing a path for $\{\widehat{\pi}_t^H, \widehat{Y}_t, \widehat{RS}_t\}$ to minimize (1.29), subject to the constraints (1.34) and (1.35), and given the initial conditions $\widehat{\pi}_{t_0}$ and $\widehat{Y}_{t_0}$. In effect, the constraints on the initial conditions impose that the first order conditions to the problem are time invariant. This method follows Woodford's (1999) timeless perspective approach, and thereby ensures that the policy prescription does not constitute a time-inconsistent problem.[13] The multipliers associated with (1.34) and (1.35) are, respectively, φ_1 and φ_2. Thus, the first order conditions with respect to $\widehat{\pi}_t^H, \hat{Y}_t$ and $\widehat{RS}_t$ are given by:

$$(\varphi_{1,t} - \varphi_{1,t-1}) = k\Phi_\pi \widehat{\pi}_t^H, \tag{1.36}$$

$$\varphi_{2,t} - \eta\varphi_{1,t} = \Phi_Y(\widehat{Y}_t - \widehat{Y}_t^T), \tag{1.37}$$

and

$$-\varphi_{2,t} - \frac{\rho}{(1+l)}\varphi_{1,t} = \frac{\rho(1-\lambda)}{(1+l)}\Phi_{RS}(\widehat{RS}_t - \widehat{RS}_t^T). \tag{1.38}$$

Combining equations (1.36), (1.37), and (1.38), we obtain the following [illegible]

$$(1+l)\Phi_Y\Delta(\widehat{Y}_t-\widehat{Y}_t^T)+\rho(1-\lambda)\Phi_{RS}\Delta(\widehat{RS}_t-\widehat{RS}_t^T)+(\rho+\eta(1+l))k\Phi_\pi(\widehat{\pi}_t^H) = 0, \tag{1.39}$$

where Δ denotes the first difference operator. The above expression characterizes the small open economy optimal targeting rule. It prescribes responding to movements in inflation, output and the real exchange rate.[14] Equation (1.39) stipulates how monetary policy should respond to the different shocks, according to the composition of $\widehat{Y}_t^T$ and $\widehat{RS}_t^T$. When following this policy rule, the central bank may allow some variability in inflation in order to respond to costly movements in other variables. Equation (1.39) indicates the policymaker's behavior that minimizes welfare losses generated by such fluctuations. It implements the most efficient allocation of resources, conditional on the structural characteristics of the economy.

[13] For a discussion on the timeless perspective of optimal rule, see Woodford (2003).

[14] Even if we express Equation (1.39) as a function of Consumer Price Index inflation instead of producer price inflation $\widehat{\pi}_t^H$, the targeting rule still includes the term $\Delta(\widehat{RS}_t - \widehat{RS}_t^T)$.

In the Appendix, we show which parametric restrictions are needed for the above first order conditions to lead to a determinate equilibrium. The Appendix also contains an analysis of whether the above first order conditions indeed characterize an optimal policy. That is, Section 1.D investigates if there is any alternative random policy that could improve welfare. As shown in Benigno and Woodford (2003), this approach coincides with the investigation of whether the second-order conditions of the minimization problem are satisfied. It follows that some parameter specifications violate these conditions. Those are shown in Table A.1 and A.2.

We now turn to the analysis of some special cases of the optimal plan. Further, we explore how certain economic characteristics influence the optimal monetary policy.

1.4.1 Producer Price Inflation Target

Under certain circumstances, the loss function approximation leads to clear-cut results in terms of optimal policy. In this Sec where $q_t^e = \left[1 + \lambda(\rho\theta - 1)\left(RS_t^e\right)^{\theta-1} + \frac{\lambda}{1-\lambda}\rho\theta\left(RS_t^e\right)^{\frac{\rho\theta-1}{\rho}}\right]$ and the superscript e denotes the efficient allocation. The full characterization of the efficient allocation can be obtained by combining the above equation with the constraints of the policy problem (i.e. equations (1.6), (1.11), and (1.20)). Furthermore, in steady state we have

$$U_c(\overline{Y}^e) = \frac{1}{(1-\lambda)} V_y(\overline{Y}^e). \tag{1.41}$$

On the other hand, in the decentralized problem, the equilibrium condition implied by monopolistic competition and price stickiness is given by the price setting Equation (1.21). If we assume, however, that prices are flexible, the equilibrium condition (1.21) becomes

$$p_{H,t}^{Flex} U_c(C_t^{Flex}) = \mu_t V_y\left(Y_t^{Flex}, \varepsilon_{Y,T}\right), \tag{1.42}$$

and in steady state

$$U_c(\overline{Y}^{Flex}) = \mu V_y(\overline{Y}^{Flex}). \tag{1.43}$$

Comparing conditions (1.40) and (1.42), it is clear that even with perfectly flexible prices, mark-up shocks and movements in the real exchange rate generate inefficient fluctuations in the ratio of marginal disutility of production and marginal utility of consumption. In addition, unless $\mu = 1/(1-\lambda)$, the small open economy steady-state output is inefficient (this can be seen by inspection of equations (1.41) and (1.43)). That is, in general, a policy of domestic price stabilization that mimics the flexible price allocation does not implement an efficient allocation.

Nevertheless, if we impose that $\rho\theta = 1$, the efficiency condition (1.40) and the decentralized flexible price allocation (1.42) can be written as follows:

$$(1-\lambda)\left(Y_t^e - G_t\right)^{-\rho} = \varepsilon_{Y,t}^{-\eta}\left(Y_t^e\right)^{\eta} \tag{1.44}$$

and

$$\frac{1}{\mu_t}(Y_t^{Flex} - G_t)^{-\rho} = \varepsilon_{Y,t}^{-\eta} \left(Y_t^{Flex}\right)^{\eta}. \tag{1.45}$$

The above expressions are illustrated in Figure 1.1, where $f(Y_t, \varepsilon_t) = \varepsilon_{Y,t}^{-\eta} Y_t^{\eta}$, $g^{flex}(Y_t, \mu_t, G_t) = \frac{1}{\mu_t}(Y_t - G_t)^{-\rho}$, and $g^e(Y_t, G_t) = (1 - \lambda)(Y_t - G_t)^{-\rho}$. The inefficiency of the steady-state flexible price allocation is represented by the location of $\overline{Y}^{flex}$ below $\overline{Y}^e$. Moreover, apart from the steady-state distortion, fluctuations in the wedge between g^e and g^{Flex} characterize a departure from the efficient allocation given by (1.44), and also represent distortions present in the flexible price equilibrium.

Figure 1.2 illustrates how mark-up shocks affect the wedge between g^e and g^{Flex}. It shows that even with $\rho\theta = 1$ and flexible prices, mark-up shocks generate distortions that affect welfare. Hence, there is an incentive to stabilize these shocks and depart from the flexible-price equilibrium (i.e. a strict domestic inflation target is not optimal).

Figure 1.3 shows how productivity shocks affect efficiency. In the case of $\rho\theta = 1$, the equilibrium flexible price allocation and the efficient allocation move proportionally to each other. This leaves the welfare relevant wedge unchanged. Hence, under price flexibility there is no role for policy stabilization, and, thus, producer price inflation targeting characterizes the optimal plan. The same result holds for the case of foreign shocks. External disturbances do not appear in the expressions for $f(.)$, $g^{Flex}(.)$ or $g^e(.)$. Hence, these shocks also leave the wedge unchanged when $\rho\theta = 1$. The intuition behind this result is that, under this parametrization, the marginal effect of the real exchange rate on consumption utility and labour disutility offset each other and no stabilization process is needed.

Figure 1.4 shows the case of exogenous fluctuations in government expenditure. Because fiscal shocks do not affect g^{flex} and g^e proportionally, their effect on efficiency depends on the steady-state level of output. In general, fiscal disturbances create inefficient movements in the wedge between g^{flex} and g^e, as represented in the Figure. The only circumstance in which there are no such movements is when the steady-state level of output is efficient ($\overline{Y}^{flex} = \overline{Y}^*$). This result is consistent with the findings of Benigno and Woodford (2004) in a closed economy setting.

Therefore, the assumptions needed in order to have an inflation target as the optimal plan are: (1) $\rho\theta = 1$; (2) there should be no mark-up shocks ($\mu_t = 0$, $\forall\ t$); and, in the case of fiscal shocks, (3) that the steady-state level of output ought to be efficient from the small open economy's point of view (i.e. $\mu = 1/(1-\lambda)$). These conditions guarantee that the flexible price equilibrium characterizes the efficient allocation.

Under this specification, the weights on the loss function are:

$$\frac{\Phi_Y}{(1-\lambda)} = (\eta + \rho), \tag{1.46}$$

$$\Phi_{RS} = 0, \tag{1.47}$$

and

$$\frac{\Phi_\pi}{(1-\lambda)} = \frac{\sigma}{k}. \tag{1.48}$$

The target for output is:

$$\widehat{Y}_t^T = \widehat{Y}_t^{Flex} = (\eta + \rho)^{-1} \{\eta \epsilon_{Y,t} + \rho g_t\}. \tag{1.49}$$

The relative weights specified in equations (1.46) and (1.48) are analogous to those in the closed economy, and the policy target coincides with the flexible price allocation. The assumption of $\mu = 1/(1-\lambda)$ guarantees that steady-state output is efficient from the point of view of the small open economy. In addition, the restriction $\rho\theta = 1$ ensures that exchange rate movements do not affect welfare since its marginal effect on consumption utility and labour disutility offset each other. Moreover, the optimal plan does not respond to external shocks. In what follows, under this specification, the optimal monetary policy in a small open economy is isomorphic to a closed economy. This result is consistent with the findings of Galí and Monacelli (2005).[15]

[15] The authors have characterized the loss function for a small open economy in the case in which trade imbalances and steady state monopolistic distortions are absent (i.e. $\rho = \theta = 1$ and $\mu = 1/(1-\lambda)$).

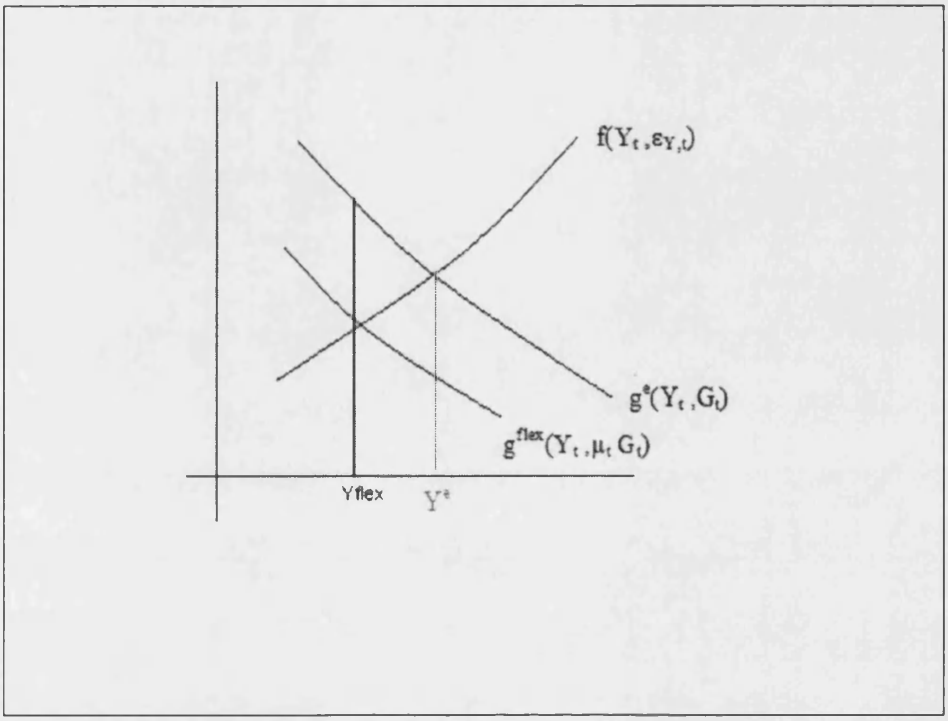

Figure 1.1: Efficiency Analysis

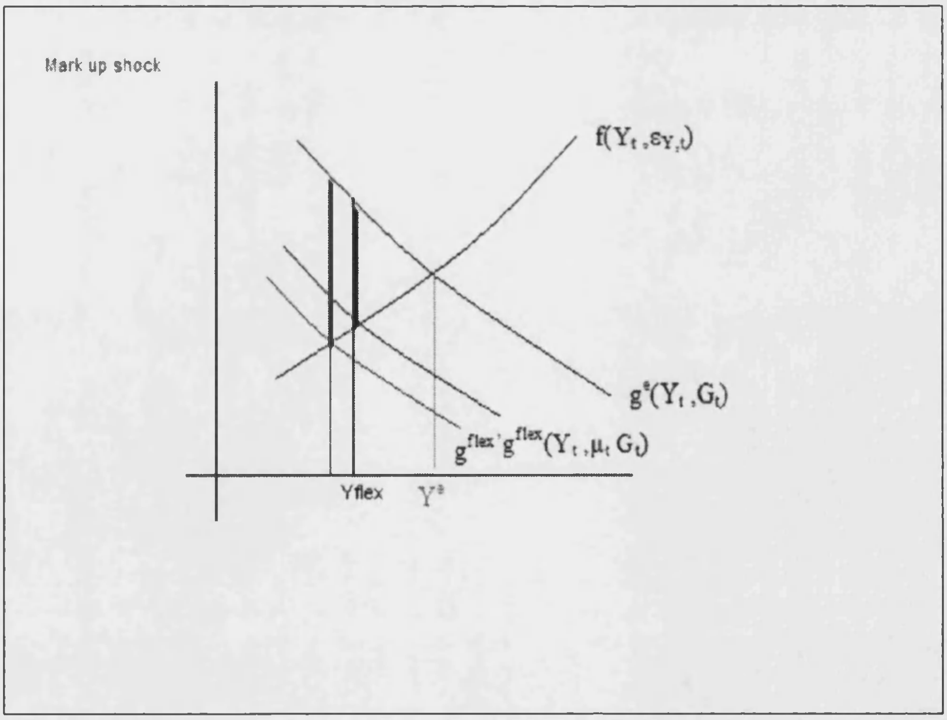

Figure 1.2: Efficiency Analysis - the Case of Mark-up Shocks

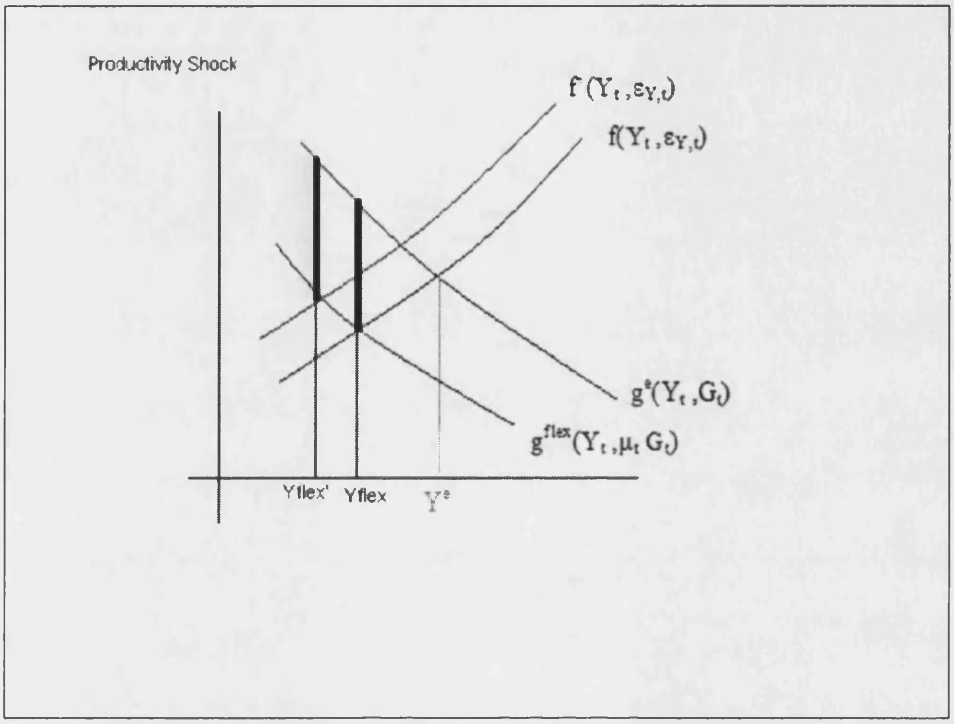

Figure 1.3: Efficiency Analysis - the Case of Productivity Shocks

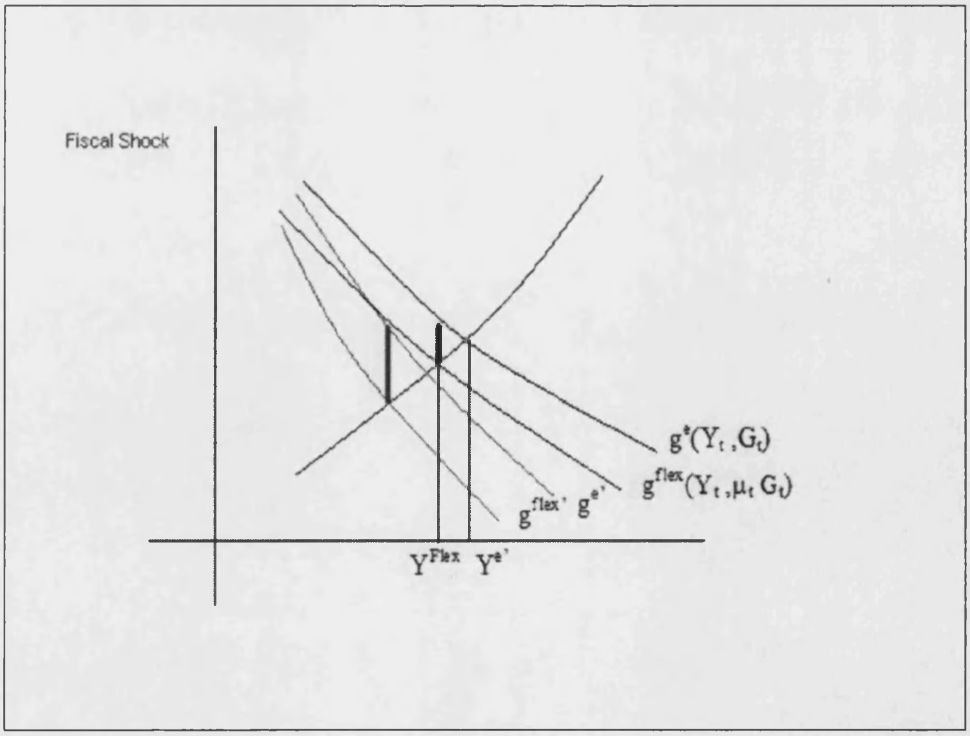

Figure 1.4: Efficiency Analysis - the Case of Fiscal Shocks

1.4.2 Quantitative results

The General Optimal Plan:

In this Section, we present some numerical analysis of the optimal monetary policy. In our benchmark specification, we assume a unitary elasticity of intertemporal substitution (i.e. $\rho = 1$). Following Rotemberg and Woodford (1997), we assume $\eta = 0.47$. Furthermore, the elasticity of substitution between home and foreign goods, θ, is assumed to be 3. Obstfeld and Rogoff (1998) argue that it should be between 3 and 6.[16] The degree of openness, λ, is assumed to be 0.2, implying a 20% import share in GDP. In addition, the baseline calibration considers the case of an "optimal subsidy" policy, where τ is set such that $\mu = 1/(1-\lambda)$. Moreover, the elasticity of substitution between differentiated goods σ is assumed to be 10, as in Benigno and Benigno (2003). To characterize an average length of price contract of 3 quarters, we assume $\alpha = 0.66$. Finally, we assume $\beta = 0.99$. Starting from this specification, we analyse how optimal monetary policy responds to the different shocks.

Figure 1.5 shows the impulse responses of consumption, output, the real exchange rate and producer price inflation following a productivity shock. Comparing the optimal policy with an inflation target highlights that there are no quantitatively significant differences between the two. Under both regimes, higher productivity at home increases domestic output and consumption. In addition, a larger supply of domestic goods leads to a depreciation in the real exchange rate.

The zero measure specification of the Home economy enables us to study how the monetary authority should respond to fluctuations in external conditions when there are no feedback effects. Figure 1.6 presents the impulse response of the various domestic variables to a foreign shock, represented by an innovation in C_t^*. Again, the optimal plan is quantitatively similar to an inflation targeting regime. Domestic consumption increases with the increase in foreign consumption and there is a real exchange rate appreciation. The impact on domestic competitiveness now leads to

[16]This leads to a specification where Home and Foreign goods are substitutes in the utility, given that $\rho\theta > 1$.

a fall in home production.

As illustrated in Figure 1.7, when the economy is subject to mark-up shocks, optimal monetary policy departs from price stabilization. The optimal plan reacts to fluctuations in the wedge between marginal utility of consumption and marginal disutility of production. The policy response to a mark-up shock implies an exchange rate depreciation and an increase in the domestic consumption of home goods. As a result, domestic output increases. As shown in Figure 1.8, this is not the case when the economy is closed. In this case, inflation stabilization is larger, requiring a contraction in the level of economic activity.

The optimal response to a fiscal shock is presented in Figures 1.9, 1.10 and 1.11.[17] Figure 1.9 compares the optimal monetary policy plan with an inflation targeting regime. It shows that the exchange rate depreciation is smaller in the former. Consequently, crowding out in consumption is smaller under the optimal regime. As a result, whereas output falls under a policy of price stability, domestic production increases under the optimal plan. Conversely, as portrayed in Figure 1.10, the optimal plan in a closed economy is closer to an inflation target and involves a larger fall in consumption.

These results change significantly when the goods are complements. As displayed in Figure 1.11, when $\theta = 0.7$, a fiscal shock leads to an exchange rate appreciation and a fall in domestic consumption.

[17] Given that $\widehat{g}_t$is defined as $\frac{(G_t - \overline{G})}{Y}$, innovations in $\widehat{g}_t$ are measured as percentages of GDP.

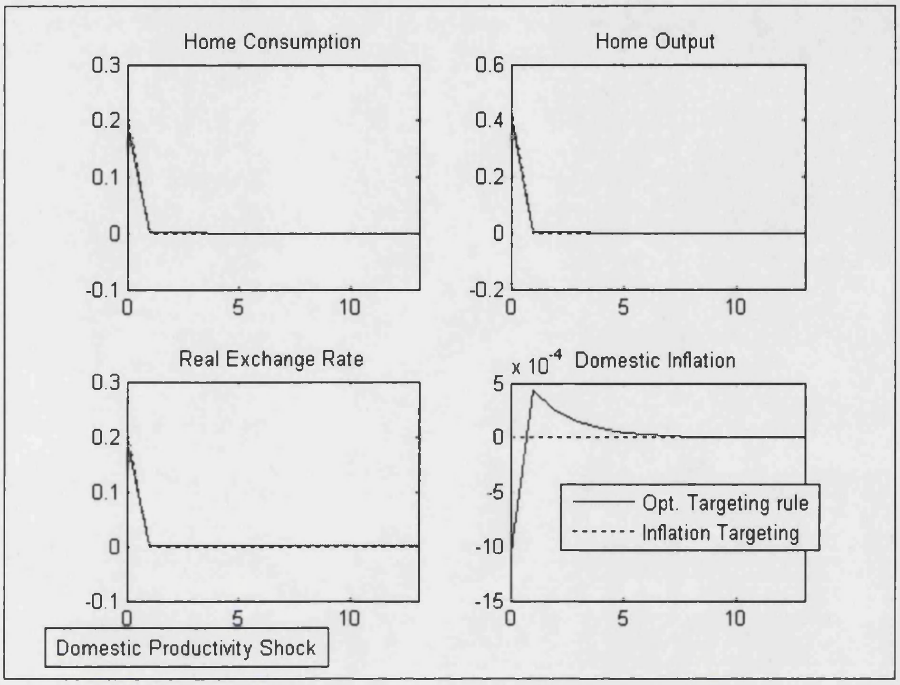

Figure 1.5: Impulse Responses following a Productivity Shock

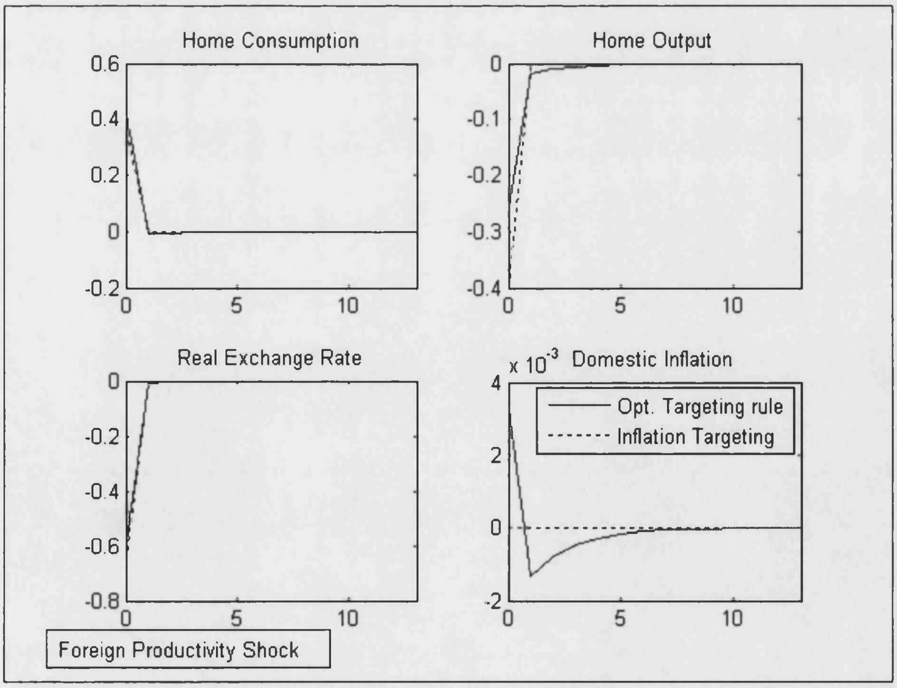

Figure 1.6: Impulse Responses following a Foreign Shock

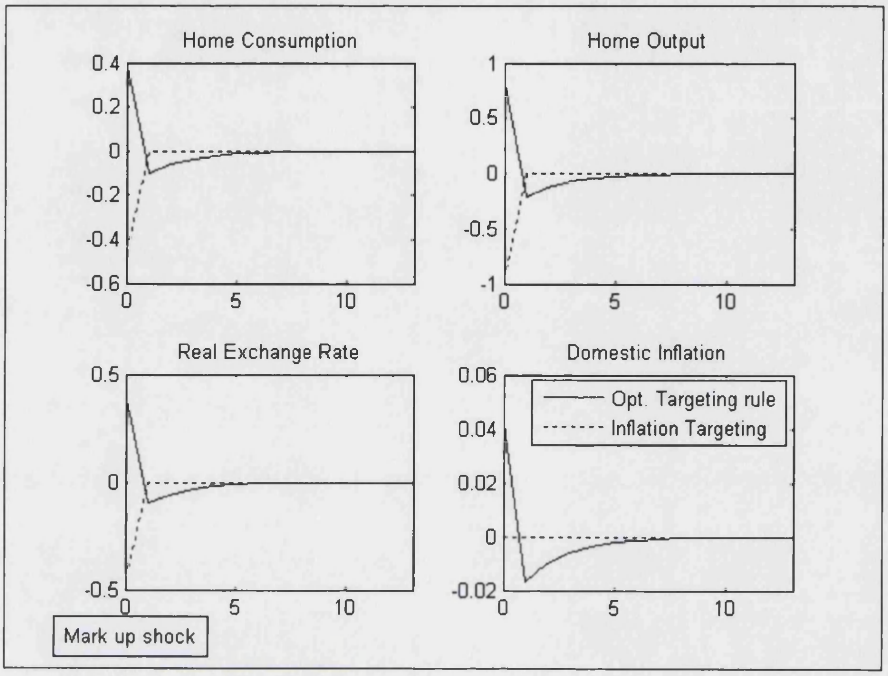

Figure 1.7: Impulse Responses following a Mark-up Shock

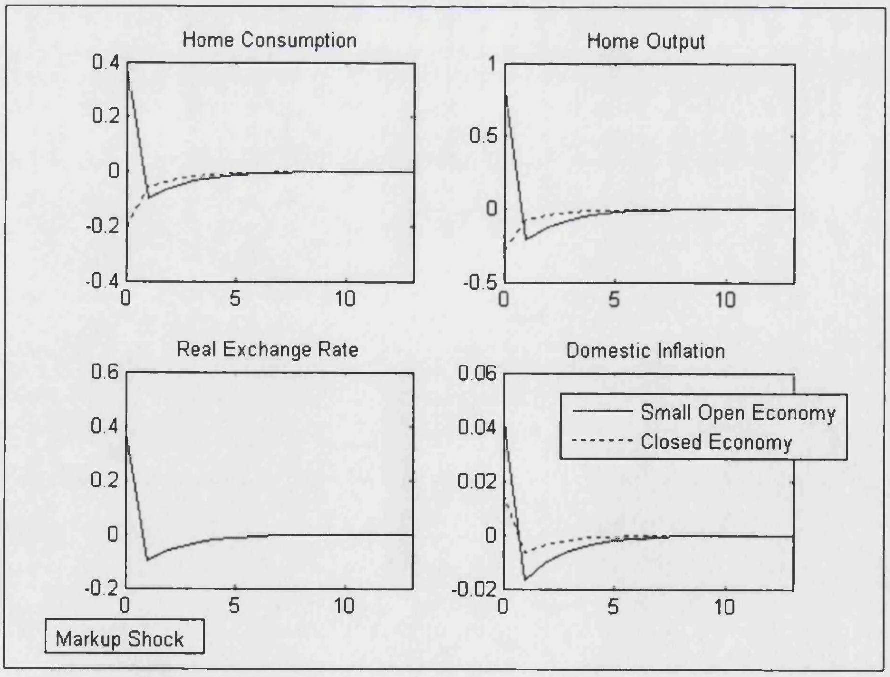

Figure 1.8: Impulse Responses following a Mark-up Shock - Open vs Closed Economy

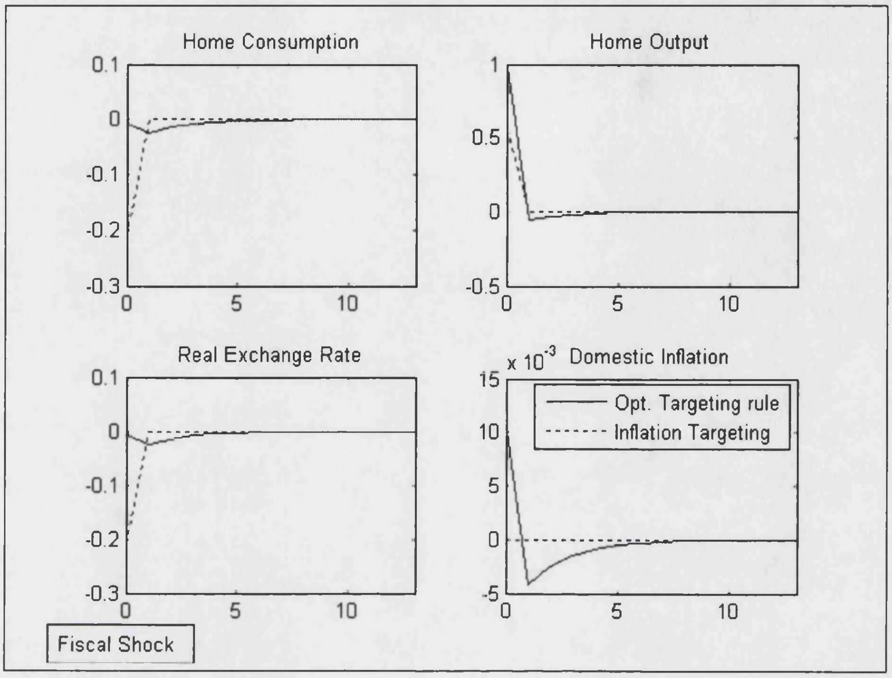

Figure 1.9: Impulse Responses following a Fiscal Shock

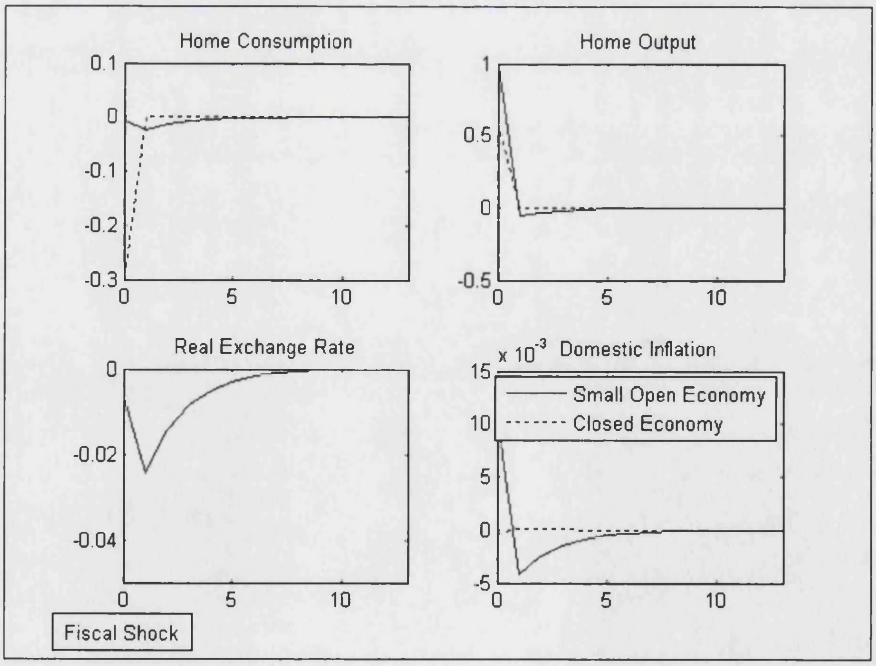

Figure 1.10: Impulse Responses following a Fiscal Shock - Open vs Closed Economy

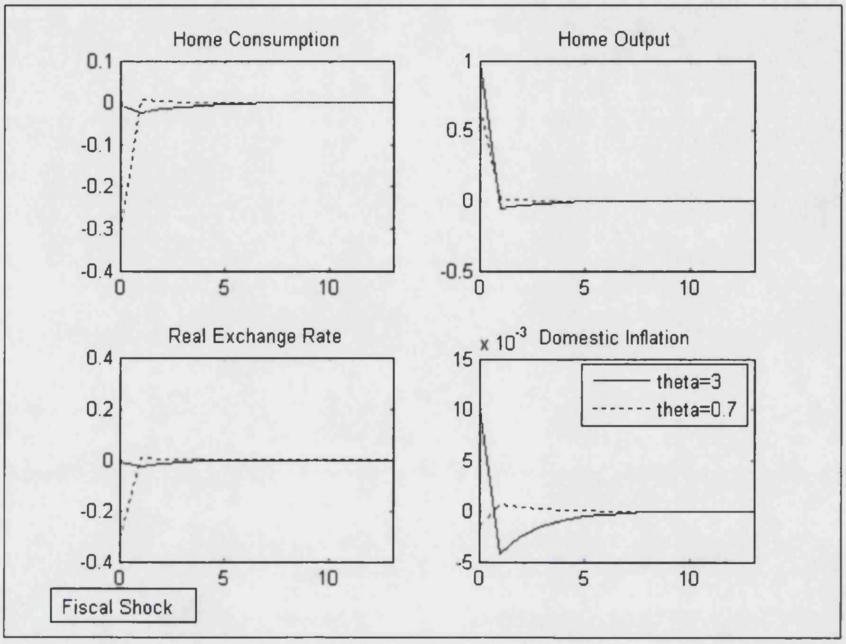

Figure 1.11: Impulse Responses following a Fiscal Shock - varying the Elasticity of Substitution between Domestic and Foreign Goods

Ranking Standard Policy Rules

Exercises such as the ones shown above demonstrate that the source of the shock affecting the economy is an important determinant of the performance of policy rules. In the optimal targeting rule, this is captured by the composition of the target variables $\widehat{Y}_t^T$ and $\widehat{RS}_t^T$, which stipulate how optimal policy should respond to different shocks. The quantitative analysis also illustrates the role of the economy's characteristics (that is, variations in the structural parameters such as λ and θ) in the policy prescription. In analytical terms, this is captured by the formulation of the weights of the variables in the loss function and

Is it OK to compute welfare ~~...~~ other than timelessly optimal policy ?

ness check has to be done by evaluating the performance of an inflation targeting regime compared with other standard policy rules for the different parameter values and types of disturbances. This exercise is also interesting *per se*, as it allows the evaluation of policies currently used by international monetary authorities.

We compute a ranking of policy rules (more specifically, domestic inflation targeting, CPI inflation targeting and exchange rate peg) for different values of ρ, θ and λ. We start by varying θ and ρ, while maintaining $\lambda = 0.4$. Alternatively, we can keep the log utility specification and analyse different scenarios for θ and λ. Further, we consider the case of 1% standard deviation productivity, fiscal and mark-up shocks.[18]

Tables 1.1 and 1.2 show the policy rule that leads to the highest level of welfare, following a productivity shock. Domestic inflation targeting is the preferred policy rule for low levels of θ, ρ and λ. A large elasticity of substitution between domestic and foreign goods increases the sensitivity of home demand to exchange rate movements. As a result, exchange rate fluctuations have a higher impact on the rate of marginal utility of consumption and marginal disutility of production. For this reason, when θ is high, the small open economy benefits from adopting an exchange rate peg. The same happens when the coefficient of risk aversion is large. Moreover, an exchange rate peg becomes superior to PPI or CPI inflation targeting when the economy is relatively open.

The gains or losses of adopting different policy regimes are represented by the following measure:

$$W_d^{a,b} = \frac{W^a - W^b}{U_c(\overline{C})} = \frac{2(1-\beta)(U_0^a - U_0^b)}{U_c(\overline{C})},$$

where U_0 is the expected life-time utility of the representative agent. $W_d^{a,b}$ measures the percentage difference in the steady-state level of consumption under regime a and b. Table 1.3 illustrates the welfare gains or losses of adopting an inflation targeting

[18]Kehoe and Perri (2000) find an estimate of 0.7% for the productivity shock standard deviation. Gali et al (2002) find a standard deviation for price mark-ups of 4.3% (implying variance of approximately 0.0016). Perotti (2005) estimates the standard deviation of a government spending shock for various countries. The estimates range from 0.8% to 3.5%. However, in the present paper we consider equally variable shocks with $\sigma^2 = 0.0001$.

rather than an exchange rate peg when the economy is subject to productivity shocks. Although an exchange rate peg is superior to an inflation targeting regime when θ, ρ and λ are large, the quantitative welfare loss is not very significant: it ranges from 0.001% to 0.004% of steady-state consumption. As shown in Section 5.1, when $\rho = \theta = 1$ and the economy is subject to productivity shocks, a domestic inflation target coincides with the optimal policy rule. In this case, the welfare losses of a fixed exchange rate regime is 0.010% of steady-state consumption. Table 1.9 shows that these costs increase to 0.013% when the economy is relatively closed ($\lambda = 1/5$).

Table 1.1: Preferred Policy Rule following a Productivity Shock - varying the Degree of Openness and the Intratemporal Elasticity of Substitution

$\lambda \setminus \theta$	1	2	3	4	5	6
1/2	IT	IT	PEG	PEG	PEG	PEG
1/3	IT	IT	IT	PEG	PEG	PEG
1/4	IT	IT	IT	CPI	PEG	PEG
1/5	IT	IT	IT	IT	PEG	PEG

Table 1.2: Preferred Policy Rule following a Productivity Shock- varying the Intertemporal and Intratemporal Elasticity of Substitution

$\rho \setminus \theta$	1	2	3	4	5	6
1	IT	IT	IT	PEG	PEG	PEG
2	IT	IT	PEG	PEG	PEG	PEG
3	IT	IT	PEG	PEG	PEG	PEG
4	IT	PEG	PEG	PEG	PEG	PEG
5	IT	PEG	PEG	PEG	PEG	PEG
6	IT	PEG	PEG	PEG	PEG	PEG

Table 1.3: Welfare Costs following a Productivity Shock- varying the Intertemporal and Intratemporal Elasticity of Substitution

ρ \ θ	1	2	3	4	5	6
1	0.010%	0.004%	0.001%	-0.001%	-0.002%	-0.002%
2	0.007%	0.001%	-0.001%	-0.003%	-0.003%	-0.003%
3	0.006%	0.000%	-0.002%	-0.003%	-0.004%	-0.004%
4	0.005%	0.000%	-0.002%	-0.003%	-0.004%	-0.004%
5	0.005%	0.000%	-0.003%	-0.003%	-0.004%	-0.004%
6	0.004%	-0.001%	-0.003%	-0.004%	-0.004%	-0.004%

$W_d^{IT,PEG}$

Table 1.4: Welfare Costs following a Productivity Shock- varying the Degree of Openness and Intratemporal Elasticity of Substitution

λ \ θ	1	2	3	4	5	6
1/2	0.008%	0.004%	0.002%	0.001%	0.001%	0.001%
1/3	0.011%	0.004%	0.002%	0.001%	0.001%	0.001%
1/4	0.012%	0.003%	0.002%	0.001%	0.001%	0.001%
1/5	0.013%	0.003%	0.002%	0.001%	0.001%	0.001%

$W_d^{IT,PEG}$

In the case of foreign shocks, figures for the preferred policy are identical to the case of domestic productivity shocks. Pegging the exchange rate outperforms an inflation targeting regime when the economy is relatively open, and demand is sensitive to exchange rate movements (i.e., θ is large) and the intertemporal elasticity of substitution is small (high levels of ρ). This is illustrated in Tables 1.5 and 1.6.

Table 1.5: Preferred Policy Rule following an External Shock - varying the Degree of Openness and the Intratemporal Elasticity of Substitution

$\lambda \setminus \theta$	1	2	3	4	5	6
1/2	IT	IT	PEG	PEG	PEG	PEG
1/3	IT	IT	IT	PEG	PEG	PEG
1/4	IT	IT	IT	CPI	PEG	PEG
1/5	IT	IT	IT	IT	PEG	PEG

Table 1.6: Preferred Policy Rule following a Foreign Shock- varying the Intertemporal and Intratemporal Elasticity of Substitution

$\rho \setminus \theta$	1	2	3	4	5	6
1	IT	IT	IT	PEG	PEG	PEG
2	IT	IT	PEG	PEG	PEG	PEG
3	IT	IT	PEG	PEG	PEG	PEG
4	IT	PEG	PEG	PEG	PEG	PEG
5	IT	PEG	PEG	PEG	PEG	PEG
6	IT	PEG	PEG	PEG	PEG	PEG

Turning to fiscal shocks, for intermediate levels of ρ, θ and λ, CPI targeting is the best of the three standard policy forms evaluated. Under these specifications, the central bank can improve welfare by targeting a weighted average of domestic inflation and exchange rate depreciation. This is illustrated in Tables 1.7 and 1.8. However, the cost of imposing an inflation targeting regime under this parametrization is insignificant; at most 0.001% loss in steady-state consumption (see highlighted statistics in Table 1.9 and 1.10). Moreover, as in the case of foreign and productivity shocks, when λ, θ and ρ are large, fixing the exchange rate is the best alternative.

Table 1.7: Preferred Policy Rule following a Fiscal Shock.- varying the Degree of Openness and the Intratemporal Elasticity of Substitution

$\lambda \setminus \theta$	1	2	3	4	5	6
1/2	IT	**CPI**	PEG	PEG	PEG	PEG
1/3	IT	**CPI**	PEG	PEG	PEG	PEG
1/4	IT	IT	**CPI**	PEG	PEG	PEG
1/5	IT	IT	**CPI**	PEG	PEG	PEG

Table 1.8: Preferred Policy Rule following a Fiscal Shock- varying the Intertemporal and Intratemporal Elasticity of Substitution

ρ \ θ	1	2	3	4	5	6
1	IT	**CPI**	PEG	PEG	PEG	PEG
2	IT	PEG	PEG	PEG	PEG	PEG
3	**CPI**	PEG	PEG	PEG	PEG	PEG
4	**CPI**	PEG	PEG	PEG	PEG	PEG
5	**CPI**	PEG	PEG	PEG	PEG	PEG
6	**CPI**	PEG	PEG	PEG	PEG	PEG

Table 1.9: Welfare Costs following a Fiscal Shock - varying the Degree of Openness and Intratemporal Elasticity of Substitution

λ \ θ	1	2	3	4	5	6
1/2	0.003%	**0.000%**	-0.002%	-0.002%	-0.003%	-0.003%
1/3	0.003%	**0.000%**	-0.001%	-0.002%	-0.002%	-0.002%
1/4	0.002%	0.000%	**-0.001%**	-0.002%	-0.002%	-0.002%
1/5	0.002%	0.000%	**-0.001%**	-0.001%	-0.002%	-0.002%

$W_d^{IT,CPI}$

Table 1.10: Welfare Costs following a Fiscal Shock - varying the Intertemporal and Intratemporal Elasticity of Substitution

ρ \ θ	1	2	3	4	5	6
1	0.003%	**0.000%**	-0.001%	-0.002%	-0.002%	-0.002%
2	0.001%	-0.002%	-0.003%	-0.003%	-0.003%	-0.003%
3	**0.000%**	-0.002%	-0.003%	-0.003%	-0.003%	-0.003%
4	**0.000%**	-0.003%	-0.003%	-0.004%	-0.004%	-0.003%
5	**-0.001%**	-0.003%	-0.003%	-0.004%	-0.004%	-0.003%
6	**-0.001%**	-0.003%	-0.004%	-0.004%	-0.004%	-0.003%

$W_d^{IT,CPI}$

In the case of mark-up shocks, an inflation target is the preferred standard policy only under the knife-edge specification where $\rho = \theta = 1$ (see Table 1.11). With unitary elasticity of substitution and $\rho > 1$, CPI targeting is the preferred policy rule. In addition, whenever $\theta > 2$, pegging the exchange rate leads to higher welfare than PPI inflation targeting. The steady-state consumption losses associated with strict domestic price stabilization compared with a fixed exchange rate regime are shown in Table 1.12. When mark-up fluctuations are the source of disturbance affecting the small open economy, $W_d^{IT,PEG}$ reaches 0.043% when $\rho = \theta = 6$.

Table 1.11: Preferred Policy Rule following a Mark-up Shock- varying the Intertemporal and Intratemporal Elasticity of Substitution

ρ \ θ	1	2	3	4	5	6
1	IT	CPI	PEG	PEG	PEG	PEG
2	CPI	PEG	PEG	PEG	PEG	PEG
3	CPI	PEG	PEG	PEG	PEG	PEG
4	CPI	PEG	PEG	PEG	PEG	PEG
5	CPI	PEG	PEG	PEG	PEG	PEG
6	CPI	PEG	PEG	PEG	PEG	PEG

Table 1.12: Welfare Costs following a Mark-up Shock - varying the Degree of Openness and Intratemporal Elasticity of Substitution

ρ \ θ	1	2	3	4	5	6
1	0.029%	-0.004%	-0.020%	-0.029%	-0.033%	-0.035%
2	0.017%	-0.014%	-0.028%	-0.035%	-0.038%	-0.040%
3	0.012%	-0.017%	-0.031%	-0.037%	-0.040%	-0.041%
4	0.009%	-0.019%	-0.032%	-0.038%	-0.041%	-0.042%
5	0.008%	-0.021%	-0.033%	-0.039%	-0.041%	-0.042%
6	0.007%	-0.021%	-0.034%	-0.039%	-0.042%	-0.043%

$W_d^{IT,PEG}$

The costs of adopting a welfare-inferior policy rule presented in the above Tables are small in magnitude. The shift in steady-state consumption is never larger than 0.05%. We should note, however, that these costs are of the same order of magnitude as the costs of business cycles reported by Lucas (1987) (who estimates a 0.1% shift in steady-state consumption).

1.5 Conclusion

In this paper, we have formalized a small open economy model as a limiting case of the two-country general equilibrium framework. We have characterized a utility-based loss function and also derived the optimal monetary plan, represented by a targeting rule, for a small open economy. The setup developed in this work encompasses, as special cases, the closed economy framework and the small open economy case with efficient levels of steady-state output. As a result, the examination of monetary policy in such environments is nested in our analysis.

The utility-based loss function for a small open economy is a quadratic expression in domestic inflation, the output gap and the real exchange rate. This paper has demonstrated that a small open economy, completely integrated with the rest of the world, should be concerned about exchange rate variability. Hence, the optimal policy in a small open economy is neither isomorphic to that in a closed economy, nor does it prescribe a pure floating exchange rate regime. Price stability (or domestic inflation targeting) has been shown to be optimal only under a specific parameterization of the model: in the cases where the economy experiences productivity and foreign shocks exclusively, domestic inflation targeting is only optimal under a particular specification for preferences; if fiscal disturbances are also present, price stability as the optimal plan further requires the presence of a production subsidy; when these restrictions on the steady-state level output and preferences are relaxed, deviations from inward looking policies arise in the optimal plan.

Nevertheless, under our benchmark calibration, when the economy experiences domestic productivity shocks and external disturbances the optimal monetary policy has been shown to closely mimic an inflation targeting regime. In the case of fiscal and mark-up shocks, the optimal plan departs from price stability. Moreover, the openness of the economy modifies the optimal responses to the referred shocks significantly.

In the sensitivity analysis exercise, we have demonstrated that inflation targeting (when compared with CPI and exchange rate targeting), is the preferred policy if the economy is relatively closed and its demand is not sensitive to exchange rate movements. Conversely, if λ, θ and ρ are large, the small open economy may improve welfare by adopting a fixed exchange rate regime.

The tools developed in this paper can be applied to different economic environments. It is important to notice that the model presented here assumes that there are complete asset markets. Relaxing such assumption can lead to a more realistic representation of the model. Moreover, the introduction of asset market imperfections and their welfare consequences would enrich the optimal monetary policy analysis. Chapter 2 of this thesis will address these issues.

Another interesting extension would involve analyzing fiscal policy by allowing proportional taxation to be an endogenous variable. This would enable the investigation of the interaction between fiscal and monetary authorities and the optimal policy mix. The small open economy representation allows for the assessment of interesting issues such as the implication of different government bond denominations

teresting

1.A Appendix: The Steady State

In this Appendix, we derive the steady-state conditions. All variables in steady state are denoted with a bar. We assume that in steady state $1 + i_t = 1 + i_t^* = 1/\beta$ and $P_t^H / P_{t-1}^H = P_t^F / P_{t-1}^F = 1$. We normalize the price indices such that $\overline{P}_H = \overline{P}_F$. This implies that $\frac{\overline{P}_H}{\overline{P}} = \overline{RS} = 1$. From the demand equation at Home, we have:

$$\overline{Y} = v\overline{C} + \frac{v^*(1-n)}{n}\overline{C^*} + \overline{G}, \tag{1.50}$$

and

$$\overline{Y}^* = \frac{(1-v)\,n}{1-n}\overline{C} + (1-v^*)\overline{C^*} + \overline{G}^*. \tag{1.51}$$

If we specify the proportion of foreign-produced goods in home consumption as $1 - v = (1-n)\lambda$, the proportion of home-produced goods in foreign consumption as $v^* = n\lambda$, and take the limiting case where $n = 0$, we have:

$$\overline{Y} = (1-\lambda)\overline{C} + \lambda\overline{C^*} + \overline{G}, \tag{1.52}$$

and

$$\overline{Y}^* = \overline{C}^* + \overline{G}^*. \tag{1.53}$$

Applying our normalization to the price setting equations we have:

$$U_C(\overline{C}) = \mu V_y\left(\lambda\overline{C^*} + (1-\lambda)\overline{C} + \overline{G}\right), \tag{1.54}$$

$$U_C(\overline{C}^*) = \mu^* V_y\left(\overline{C}^* + \overline{G}^*\right), \tag{1.55}$$

where

$$\mu = \frac{\sigma}{(1-\overline{\tau})(\sigma-1)}.$$

We also use the following definitions throughout the Appendix

$$(1-\phi) = \frac{1}{\mu}$$

$$\phi = 1 - \frac{(1-\bar{\tau})(\sigma-1)}{\sigma}$$

$$0 \leq \phi < 1; \mu > 1$$

The Symmetric steady state:

Iterating the complete asset market assumption we have:

$$RS_t = \kappa_0 \left(\frac{C_t}{C_t^*} \right)^{\rho}, \tag{1.56}$$

where

$$\kappa_0 = RS_0 \left(\frac{C_0}{C_0^*} \right)^{\rho}. \tag{1.57}$$

So if we assume an initial level of wealth such that $\kappa_0 = 1$, the steady-state version of (1.56) imply $\overline{C} = \overline{C}^*$. Further, throughout the Appendix we assume $\overline{G}^* = \overline{G} = 0$. Under this condition, equations (1.54) and (1.55) imply: $\mu = \mu^*$.

1.B Appendix: A Second Order Approximation to the Utility Function

In this Appendix, we derive the first and second order approximation to the equilibrium conditions of the model under the assumptions that $\overline{C} = \overline{C}^*$ and $\overline{G}^* = \overline{G} = 0$. We obtain the second order approximation to the utility function to address welfare analysis. To simplify and clarify the algebra, we use the following isoelastic functional forms:

$$U(C_t) = \frac{C_t^{1-\rho}}{1-\rho} \tag{1.58}$$

$$V\left(y_t(h), \varepsilon_{Y,T}\right) = \frac{\varepsilon_{y,t}^{-\eta} y_t(h)^{\eta+1}}{\eta+1} \tag{1.59}$$

1.B.1 Demand

As shown in the text, the home demand equation is:

$$Y_t = \left[\frac{P_{H,t}}{P_t}\right]^{-\theta}\left[(1-\lambda)C_t + \lambda\left(\frac{1}{RS_t}\right)^{-\theta} C_t^*\right] + g_t. \tag{1.60}$$

The first order approximation to demand in the small open economy is therefore:

$$\hat{Y}_t = -\theta\widehat{p}_{H,t} + (1-\lambda)\hat{C}_t + \lambda\hat{C}_t^* + \theta\lambda\widehat{RS}_t + \widehat{g}_t. \tag{1.61}$$

Note that fiscal shock $\hat{g}_t$ is defined as $\frac{G_t - \overline{G}}{\overline{Y}}$, allowing for the analysis of this shock even when the zero steady-state government consumption is zero. And the second order approximation to the demand function is:

$$\sum \beta^t \left[d_y' y_t + \frac{1}{2} y_t' D_y y_t + y_t' D_e e_t\right] + t.i.p + \mathcal{O}(||\xi||^3) = 0, \tag{1.62}$$

where

$$y_t = \left[\begin{array}{cccc} \widehat{Y}_t & \widehat{C}_t & \widehat{p}_{Ht} & \widehat{RS}_t \end{array}\right],$$

$$e_t = \left[\begin{array}{cccc} \widehat{\varepsilon}_{yt} & \widehat{\mu}_t & \widehat{g}_t & \widehat{C}_t^* \end{array}\right],$$

$$d_y' = \left[\begin{array}{cccc} -1 & 1-\lambda & -\theta & \theta\lambda \end{array}\right],$$

$$d_e' = \left[\begin{array}{cccc} 0 & 0 & 1 & \lambda \end{array}\right],$$

$$D_y' = \left[\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & \lambda(1-\lambda) & 0 & -\theta\lambda(1-\lambda) \\ 0 & 0 & 0 & 0 \\ 0 & -\theta\lambda(1-\lambda) & 0 & \theta^2\lambda(1-\lambda) \end{array}\right],$$

and

$$D_e' = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -(1-\lambda) & -\lambda(1-\lambda) \\ 0 & 0 & \theta & 0 \\ 0 & 0 & -\theta\lambda & \theta\lambda(1-\lambda) \end{bmatrix}.$$

1.B.2 Risk Sharing Equation

In a perfectly integrated capital market, the value of the intertemporal marginal rate of substitution is equated across borders:

$$\frac{U_C\left(C_{t+1}^*\right)}{U_C\left(C_t^*\right)}\frac{P_t^*}{P_{t+1}^*} = \frac{U_C\left(C_{t+1}\right)}{U_C\left(C_t\right)}\frac{S_{t+1}P_t}{S_tP_{t+1}}. \tag{1.63}$$

Assuming the symmetric steady-state equilibrium, the log linear approximation to the above condition is

$$\hat{C}_t = \hat{C}_t^* + \frac{1}{\rho}\widehat{RS}_t. \tag{1.64}$$

Given our utility function specification, Equation (1.63) gives rise to a exact log linear expression, and the first and second order approximations are therefore identical.

In matrix notation, we have:

$$\sum E_t\beta^t\left[c_y' y_t + \frac{1}{2}y_t' C_y y_t + y_t' C_e e_t\right] = 0, \tag{1.65}$$

$$c_y' = \begin{bmatrix} 0 & -1 & 0 & \frac{1}{\rho} \end{bmatrix},$$

$$c_e' = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix},$$

$$C_y' = 0,$$

and

$$C_e' = 0.$$

1.B.3 The Real Exchange Rate

Given that, in the rest of the world, $P_F = SP^*$, Equation (1.23) can be expressed as:

$$\left(\frac{P_t}{P_{H,t}}\right)^{1-\theta} = (1-\lambda) + \lambda\left(RS_t\frac{P_t}{P_{H,t}}\right)^{1-\theta}. \tag{1.66}$$

The first order approximation to the above expression is:

$$\widehat{p}_{H,t} = -\frac{\lambda\widehat{RS}_t}{1-\lambda}. \tag{1.67}$$

The second order approximation to Equation (1.66) is:

$$\sum E_t\beta^t\left[f_y'y_t + \frac{1}{2}y_t'F_yy_t + y_t'F_ee_t\right] + t.i.p + \mathcal{O}(||\xi||^3) = 0, \tag{1.68}$$

where

$$f_y' = \begin{bmatrix} 0 & 0 & -(1-\lambda) & -\lambda \end{bmatrix},$$

$$f_e' = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix},$$

$$F_y' = \lambda(\theta-1)\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & (1-\lambda/(1-\lambda)) \end{bmatrix},$$

and

$$F_e' = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

1.B.4 Price Setting

The first and second-order approximations to the price setting equation follow Benigno and Benigno (2001) and Benigno and Benigno (2003).[19] These conditions are derived from the following first order condition of sellers that can reset their prices:

$$E_t\left\{\sum(\alpha\beta)^{T-t}U_c(C_T)\left(\frac{\tilde{p}_t(h)}{P_{H,t}}\right)^{-\sigma}Y_T\left[\frac{\tilde{p}_t(h)}{P_{H,T}}\frac{P_{H,T}}{P_T}-\mu_T\frac{V_y\left(\tilde{y}_{t,T}(h),\varepsilon_{Y,t}\right)}{U_c(C_T)}\right]\right\}=0, \tag{1.69}$$

where

$$\tilde{y}_t(h)=\left(\frac{\tilde{p}_t(h)}{P_{H,t}}\right)^{-\sigma}Y_t, \tag{1.70}$$

and

$$(P_{H,t})^{1-\sigma}=\alpha P_{H,t-1}^{1-\sigma}+(1-\alpha)\left(\tilde{p}_t(h)\right)^{1-\sigma}. \tag{1.71}$$

With mark-up shocks, μ_t, defined as $\frac{\sigma}{(\sigma-1)(1-\tau_t^H)}$, the first order approximation to the price setting equation can be written in the following way:

$$\widehat{\pi}_t^H=k\left(\rho\widehat{C}_t+\eta\widehat{Y}_t-\widehat{p}_{H,t}+\widehat{\mu}_t-\eta\widehat{\varepsilon}_{Y,t}\right)+\beta E_t\widehat{\pi}_{t+1}^H, \tag{1.72}$$

where $k=(1-\alpha\beta)(1-\alpha)/\alpha(1+\sigma\eta)$.

The second order approximation to Equation (1.69) can be written as follows:

$$\begin{aligned} Q_{to} &= \phi\sum E_t\beta^t\left[a_y'y_t+\frac{1}{2}y_t'A_yy_t+y_t'A_ee_t+\frac{1}{2}a_\pi\pi_t^2\right] \\ &\quad +t.i.p+\mathcal{O}(||\xi||^3), \end{aligned} \tag{1.73}$$

[19] For a detailed derivation of the first-order approximation to the price setting see the technical appendix in Benigno and Benigno (2001). Benigno and Benigno (2003) have the details on the second-order approximation.

$$a'_y = \begin{bmatrix} \eta & \rho & -1 & 0 \end{bmatrix},$$

$$a'_e = \begin{bmatrix} -\eta & 1 & 0 & 0 \end{bmatrix},$$

$$A'_y = \begin{bmatrix} \eta(2+\eta) & \rho & -1 & 0 \\ \rho & -\rho^2 & \rho & 0 \\ -1 & \rho & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A'_e = \begin{bmatrix} -\eta(1+\eta) & 1+\eta & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

and

$$a_\pi = (\eta+1)\frac{\sigma}{k}.$$

1.B.5 Welfare

Following Benigno and Benigno (2003), the second order approximation to the utility function, U_t, can be written as:

$$U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[U(C_s) - \frac{1}{n} \int_0^n V(y_s^j, \varepsilon_{Y,s}) dj \right], \tag{1.74}$$

$$W_{to} = U_c \bar{C} E_{t_0} \sum \beta^t \left[w'_y y_t - \frac{1}{2} y'_t W_y y_t - y'_t W_e e_t - \frac{1}{2} w_\pi \pi_t^2 \right] + t.i.p + \mathcal{O}(||\xi||^3), \tag{1.75}$$

where

$$w'_\pi = \frac{\sigma}{\mu k},$$

$$w'_y = \begin{bmatrix} -1/\mu & 1 & 0 & 0 \end{bmatrix},$$

$$W'_y = \begin{bmatrix} \frac{(1+\eta)}{\mu} & 0 & 0 & 0 \\ 0 & -(1-\rho) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

and

$$W'_e = \begin{bmatrix} -\frac{\eta}{\mu} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

In addition, using the second order approximation to the equilibrium condition derived in Sections 1.B.1 to 1.B.4, we can eliminate the term $w'_y y_t$ from Equation (1.75). In order to do so, we derive the vector Lx, such that

Full rank.

$$\begin{bmatrix} \eta & -1 & 0 & 0 \\ \rho & 1-\lambda & 0 & -1 \\ -1 & -\theta & -1+\lambda & 0 \\ 0 & \theta\lambda & -\lambda & \rho^{-1} \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} \overset{4\times 1}{a_y} & \overset{4\times 1}{d_y} & \overset{4\times 1}{f_y} & \overset{4\times 1}{c_y} \end{bmatrix}}_{4\times 4} Lx = w_y,$$

Is it uniquely determined? Yes.

where a_y, d_y, f_y, c_y were previously defined in this Appendix. We have:

$$Lx_1 = \frac{1}{(\rho+\eta)+l\eta}\left[l\mu^{-1} + (1-\lambda) - \mu^{-1}\right], \tag{1.76}$$

$$Lx_2 = \frac{1}{(\rho+\eta)+l\eta}\left[\rho(\mu^{-1} - (1-\lambda)) + (1-\lambda)(\eta+\rho)\right], \tag{1.77}$$

and

$$Lx_3 = \frac{1}{(\rho+\eta)+l\eta}\left[(\rho\theta-1)(1-\lambda)\mu^{-1} - (\eta\theta+1)\right], \tag{1.78}$$

where $l = (\rho\theta - 1)\lambda(2-\lambda)$

The loss function L_{to} will have the following form:

$$L_{to} = U_c \bar{C} E_{t_0} \sum \beta^t \left[\frac{1}{2} y'_t L_y y_t + y'_t L_e e_t + \frac{1}{2} l_\pi \pi_t^2\right] + t.i.p + \mathcal{O}(||\xi||^3), \tag{1.79}$$

where:

$$L_y = W_y + Lx_1 A_y + Lx_2 D_y + Lx_3 F_y,$$

$$L_e = W_e + Lx_1 A_e + Lx_2 D_e,$$

and

$$L_\pi = w_\pi + Lx_1 a_\pi.$$

To write the model just in terms of the output, the real exchange rate and inflation, we define the matrixes N and N_e, mapping all endogenous variables into $[Y_t, T_t]$ and the errors in the following way:

Choice of N

$\widehat{RS}$?

?

$$y_t' = N\,[Y_t, T_t] + N_e e_t, \tag{1.80}$$

$$y = \begin{bmatrix} \hat{Y} \\ E \\ \hat{\pi} \\ \widehat{RS} \end{bmatrix}$$

$$N = \begin{bmatrix} 1 & 0 \\ 1 & -\frac{l+\lambda}{\rho(1-\lambda)} \\ 0 & -\frac{\lambda}{(1-\lambda)} \\ 0 & 1 \end{bmatrix},$$

which equation is this?

1.67

and

$$e = \begin{bmatrix} \varepsilon_Y \\ \mu \\ g \\ c^* \end{bmatrix}$$

$$N_e = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Equation (1.79) can therefore be expressed as:

$$\begin{aligned} L_{t_0} \;=\;& U_c \bar{C} E_{t_0} \sum \beta^t \left[\frac{1}{2} \left[\hat{Y}_t, \widehat{RS}_t\right]' L_y' \left[\hat{Y}_t, \widehat{RS}_t\right] + \left[\hat{Y}_t, \widehat{RS}_t\right]' L_e' e_t + \frac{1}{2} l_\pi \pi_t^2 \right] \\ & + t.i.p + \mathcal{O}(\|\xi\|^3), \end{aligned} \tag{1.81}$$

where:

$$L_y' = N' L_y N,$$

and

$$L_e' = N' L_y N_e + N' L_e.$$

Finally, we rewrite the previous equation with variables expressed as deviations from their targets:

$$\begin{aligned} L_{to}^i &= U_c \bar{C} E_{t_0} \sum \beta^t \left[\frac{1}{2} \Phi_Y (\widehat{Y}_t - \widehat{Y}_t^T)^2 + \frac{1}{2} \Phi_{RS} (\widehat{RS}_t - \widehat{RS}_t^T)^2 + \frac{1}{2} \Phi_\pi (\widehat{\pi}_t^H)^2 \right] \\ & +t.i.p + \mathcal{O}(||\xi||^3) \end{aligned} \tag{1.82}$$

where:

$$\begin{aligned} \Phi_Y = & (\eta + \rho)(1 - \phi) + \frac{(\rho - 1) \left[-l(1 - \phi) - (\lambda - \phi) \right]}{(1 + l)} \\ & + Lx_1 \left[(\eta + \rho) + \eta(\eta + 1) - \frac{\rho(\rho - 1)}{(1 + l)} \right] \\ & - \frac{Lx_2 (1 - \lambda)^2 \lambda (\rho\theta - 1)}{(1 + l)}, \end{aligned}$$

$$\begin{aligned} \Phi_{RS} = & -\frac{(\lambda + l)(\rho - 1)}{(1 - \lambda)\rho^2} \\ & + \frac{Lx_1 l(\rho - 1 - l)}{(1 - \lambda)^2 \rho} \\ & + \frac{Lx_2 \lambda (\rho\theta - 1) \left[\rho\theta(1 - \lambda) + \lambda + l \right]}{\rho^2} \\ & + \frac{Lx_3 \lambda (\theta - 1)}{1 - \lambda}, \end{aligned}$$

$$\Phi_\pi = \frac{\sigma}{\mu k} + (1 + \eta) \frac{\sigma}{k} Lx_1,$$

and

$$\widehat{Y}_t^T = q_y^e e_t, \text{ and } \widehat{RS}_t^T = q_{rs}^e e_t,$$

with

$$q_y^e = \frac{1}{\Phi_Y}\left[\begin{array}{cccc} \frac{\eta}{\mu} + Lx_1(1+\eta)\eta & -Lx_1(1+\eta) & \frac{(\rho-1)(1-\lambda)+Lx_2}{1+l} & 0 \end{array}\right],$$

and

$$q_{RS}^e = \frac{1}{\Phi_{RS}}\left[\begin{array}{cccc} 0 & 0 & \frac{(\rho-1-l)Lx_1}{(1-\lambda)} + \frac{Lx_2\lambda(\lambda(1-\lambda)+1)(\rho\theta-1)}{\rho(1-\lambda)} & \frac{-Lx_2\lambda(1-\lambda)(\rho\theta-1)}{\rho} \end{array}\right].$$

Moreover, we can write the constraints of the maximization problem as:

$$\widehat{\pi}_t^H = k\left(\eta(\hat{Y}_t - \widehat{Y}_t^T) + (1-\lambda)^{-1}(\widehat{RS}_t - \widehat{RS}_t^T) + u_t\right) + \beta E_t \widehat{\pi}_{t+1}^H, \tag{1.83}$$

and

$$(\widehat{Y}_t - \widehat{Y}_t^T) = (\widehat{RS}_t - \widehat{RS}_t^T)\frac{(1+l)}{\rho(1-\lambda)} + \chi u_t, \tag{1.84}$$

where

$$u_t = \left[\eta, \frac{1}{1-\lambda}\right]\left[(\widehat{Y}_t^T - \widehat{Y}_t^{Flex}), (\widehat{RS}_t^T - \widehat{RS}_t^{Flex})\right]',$$

$$\chi = \left[\frac{1}{\eta}, \frac{(1+l)}{\rho}\right],$$

and $\widehat{Y}_t^{Flex}$ and $\widehat{Y}_t^{Flex}$ are the flexible price allocation for output and the real exchange rate:

$$\widehat{Y}_t^{Flex} = [(\eta+\rho)+\eta l]^{-1}\left\{\eta(1+l)\widehat{\varepsilon}_{Y,t} - (1+l)\widehat{\mu}_t + \rho\widehat{g}_t - \rho l\widehat{C}_t^*\right\}, \tag{1.85}$$

and

$$\frac{\widehat{RS}_t^{Flex}}{(1-\lambda)} = [(\eta+\rho)+\eta l]^{-1}\rho\left\{\eta\widehat{\varepsilon}_{y,t} - \widehat{\mu}_t - \eta\widehat{g}_t - (\eta+\rho)\widehat{C}_t^*\right\}. \tag{1.86}$$

1.B.6 Special Cases

In this Section, we present the special cases described in the main text.

Special Case 1:

The assumptions are:

1. $\rho\theta = 1$
2. No mark-up or fiscal shocks.

In this case, the weights in the loss function are:

$$\Phi_Y = (\eta + \rho)(1 - \lambda) + ((1 - \lambda) - \mu^{-1})(1 - \rho),$$

$$\Phi_{RS} = 0,$$

and

$$\Phi_\pi = \frac{\sigma}{k}(1 - \lambda) + ((1 - \lambda) - \mu^{-1})(1 - \rho)\frac{\sigma}{k(\rho + \eta)}.$$

And the target variables are:

$$\widehat{Y}_t^T = q_y^e e_t = \widehat{Y}_t^{Flex} = [(\eta + \rho)]^{-1}\{\eta\widehat{\varepsilon}_{y,t}\}.$$

Special Case 2:

The assumptions are:

1. $\rho\theta = 1$
2. $\mu = 1/(1 - \lambda)$
3. No mark-up shocks.

In this case, the weights in the loss function are:

$$\Phi_Y = (\eta + \rho)(1 - \lambda),$$

$$\Phi_{RS} = 0,$$

and

$$\Phi_\pi = \frac{\sigma}{k}(1-\lambda).$$

And the target variables are:

$$\widehat{Y}_t^T = q_y^e e_t = \widehat{Y}_t^{Flex} = [(\eta+\rho)]^{-1}\left\{\eta\widehat{\varepsilon}_{y,t} + \rho g_t\right\}.$$

Special Case 3: The Closed Economy

In this case, we have:

$$\frac{\Phi_\pi^c}{\Phi_Y^c} = \frac{\sigma}{k(\eta+\rho)},$$

$$\widehat{Y}_t^{T,c} = \frac{\eta\varepsilon_{Y,t}}{(\eta+\rho)} - \frac{(\mu-1)(\eta+1)\mu_t}{(\eta+\rho)(\mu\eta+\rho+(\mu-1))} + \frac{\rho(\eta\mu+\rho)g_t}{(\eta+\rho)(\mu\eta+\rho+(\mu-1))},$$

and

$$\widehat{Y}_t^{Flex,c} = \frac{\eta\varepsilon_{Y,t} - \mu_t + \rho g_t}{(\eta+\rho)}.$$

1.C Appendix: Proof of Determinacy

In this Section, we show that the optimal targeting rule together with the policy constraints and the initial condition for inflation deliver a determinate equilibrium. The equilibrium conditions given by equations (1.34), (1.35) and (1.39) can be rewritten as:

$$\widehat{\pi}_t^H = \gamma_1(\hat{Y}_t - \widehat{Y}_t^T) + k\delta_t + \beta E_t\widehat{\pi}_{t+1}^H, \tag{1.87}$$

and

$$\gamma_2\Delta(\widehat{Y}_t - \widehat{Y}_t^T) - \gamma_3\Delta\delta_t + \gamma_4\widehat{\pi}_t^H = 0, \tag{1.88}$$

where δ_t is a linear combination of shocks following an AR(1) process

$$\delta_t = \omega\delta_{t-1} + e_t, \tag{1.89}$$

and

$$\gamma_1 = k(\eta + \rho(1+l)^{-1}),$$

$$\gamma_2 = (1+l)\Phi_Y + \frac{\rho^2(1-\lambda)^2}{(1+l)}\Phi_{RS},$$

$$\gamma_3 = \chi\frac{(1+l)}{\rho^2(1-\lambda)^2},$$

and

$$\gamma_4 = (\rho + \eta(1+l))k\Phi_\pi.$$

We can reduce the system given by conditions (1.87) and (1.88) to the following equation:

$$\beta E_t\widehat{\pi}_{t+1}^H - (1+\beta+\gamma_1\gamma_4/\gamma_2)\widehat{\pi}_t^H + \widehat{\pi}_{t-1}^H = (\gamma_1\gamma_3/\gamma_2 + k)\xi_t \tag{1.90}$$

where ξ_t is a stationary shock[20]. The characteristic polynomial associated with this equation is:

$$P(a) = \beta a^2 - (1+\beta+\gamma_1\gamma_4/\gamma_2)a + 1 \tag{1.91}$$

Equations (1.87) and (1.88) form a system with one predetermined variable and one endogenous variable. Determinacy is, therefore, guaranteed if the above polynomial has one root inside the unit circle and one outside. This is true if $\gamma_2/\gamma_4\gamma_1 > -1/2(1+\beta)$. More specifically,

$$\frac{(1+l)^2\Phi_Y + \rho^2(1-\lambda)^2\Phi_{RS}}{(1+l)^2(\rho+\eta(1+l))^2\Phi_\pi} > -\frac{k^2}{2(1+\beta)}. \tag{1.92}$$

1.D Appendix: Randomization Problem

To ensure that the policy obtained from the minimization of the loss function is indeed the best available policy, we should certify that no other random policy

[20]More specificaly $\xi_t = e_t - (1-\omega)\delta_{1-1}$

plan can be welfare improving. Equation (1.34) combined with (1.35) leads to the following expression:

$$\widehat{\pi}_t^H = k\left((\eta + \rho^{-1}(1+l))(\hat{Y}_t - \widehat{Y}_t^T) + u_t\right) + \beta E_t \widehat{\pi}_{t+1}^H, \tag{1.93}$$

or alternatively

$$\widehat{\pi}_t^H = k\left(\frac{\eta(1+l)+\rho}{\rho(1-\lambda)}(\widehat{RS}_t - \widehat{RS}_t^T) + u_t\right) + \beta E_t \widehat{\pi}_{t+1}^H. \tag{1.94}$$

Thus, a random realization that adds $\varphi_j v_j$ to π_{t+j}, also increases $\hat{Y}_t$ by $\alpha_y k^{-1}(\varphi_j - \beta\varphi_{j+1})v_j$ and $\widehat{RS}_t$ by $\alpha_{rs} k^{-1}(\varphi_j - \beta\varphi_{j+1})v_j$, where

$$\alpha_{rs} = \left(\frac{\eta(1+l)+\rho}{\rho(1-\lambda)}\right), \tag{1.95}$$

and

$$\alpha_y = (\eta + \rho^{-1}(1+l)). \tag{1.96}$$

Consequently, the total contribution to the loss function is

$$U_c \bar{C} \beta^t \sigma_v^2 E_{t_0} \sum \beta^t \left[\Phi k^{-2}(\varphi_j - \beta\varphi_{j+1})^2 + \Phi_\pi (\varphi_j)^2\right], \tag{1.97}$$

where

$$\Phi = \Phi_Y \alpha_y^2 + \Phi_{RS} \alpha_{rs}^2.$$

It follows that policy randomization cannot improve welfare if the expression given by Equation (1.97) is positive definite. Hence, the first order conditions to the minimization problem are indeed a policy optimal if Φ and Φ_π are not both equal to zero and either: (a) $\Phi \geq 0$ and $\Phi + (1-\beta^{1/2})^2 k^{-2}\Phi_\pi \geq 0$ or (b) $\Phi \leq 0$ and $\Phi + (1-\beta^{1/2})^2 k^{-2}\Phi_\pi \geq 0$ holds. This analysis follows closely Benigno and Woodford (2003). The authors also demonstrate that these conditions coincide with the second order condition for the linear quadratic optimization problem.

In the case of our small open economy conditions (a) and (b) involve complicated linear combinations of the structural parameters. Even though they are satisfied under our benchmark calibration, for many parameter combinations this is not the case. The following Tables illustrate when a randomization is never welfare improving.

$\rho \setminus \theta$	1	2	3	4	5	6
1	Yes	Yes	Yes	No	No	No
2	No	No	No	No	No	No
3	No	Yes	No	No	No	No
4	No	No	No	No	No	No
5	No	No	No	No	No	No
6	No	No	No	No	No	No

Table 1.13: Parameterization under which the 2nd Order Condition to the Minimization Problem is satisfied (1)

$\theta \setminus \lambda$	1/2	1/3	1/4	1/5
1	No	No	Yes	Yes
2	No	No	No	Yes
3	No	No	No	Yes
4	No	No	No	No

Table 1.14: Parameterization under which the 2nd Order Condition to the Minimization Problem is satisfied (2)

Main conclusion:
Asset mkt structure affects the terms-of-trade distortion?
Financial autarky → cannot move production burden to foreigners.

Chapter 2

Monetary Policy under Alternative Asset Market Structures

2.1 Introduction

How does the structure of financial markets affect monetary policy? The debate surrounding optimal monetary policy in open economies has been extensive over the past decade. Many works, including Chapter 1 of this thesis, have studied the role of the exchange rate in monetary policy and examined how the dynamics of the trade balance can affect the analysis.[1] However, technical difficulties have restricted the attention given to the capital account and

The way in which asset markets function is nevertheless a crucial determinant of an open economy's dynamics. Access to international borrowing and lending is important in determining the ability of agents to smooth consumption over time.

[1]See, for example, Corsetti and Pesenti (2000) and (2005), Benigno, G. and Benigno, P. (2003), Sutherland (2002), Gali and Monacelli (2005) and De Paoli (2004).

[2]The majority of open economy models dismiss the role of the capital account by assuming market completeness or by imposing restrictions on the structure of the economy. These assumptions ensure tractability of the models and solve the stationarity problem à la Obstfeld and Rogoff (1995), but make the structure of capital markets irrelevant. Section 1.1 contains a discussion on this matter and presents some related literature refereces.

In addition, the degree of sophistication of financial products dictates the economy's level of risk sharing with the rest of the world. Given the importance of these factors, we now incorporate them into the analysis of optimal monetary policy in a small open economy framework.

We show that the structure of asset markets can significantly affect the optimal policy prescription. A small open economy may gain from managing the exchange rate under complete markets. On the other hand, in an incomplete markets setup, we show that this incentive is absent when domestic and foreign goods are substitutes in agents' utility. In the latter case, the monetary authority should focus instead on targeting domestic inflation and reducing price dispersion distortion. Nevertheless, if the degree of substitution between the goods is significantly low, the results may be reversed.

Our model addresses the issue of optimal monetary policy under alternative asset market structures. We do this by characterizing a utility-based loss function for a small open economy in three cases: (a) incomplete capital markets, where there is a cost of borrowing from abroad that generates a country risk premium; (b) financial autarky (i.e. an extreme case of market incompleteness in which the country does not have access to international borrowing and lending); and (c) complete asset markets, which implies perfect risk sharing between the small open economy and the rest of the world.

The linear-quadratic representation of welfare presented in this work follows the method developed by Benigno and Woodford (2003) and Sutherland (2002). This approach delivers a tractable representation of the policy problem, which consists of a quadratic objective function and linear constraints. The resulting optimal plan dictates the optimal responses to productivity shocks, mark-up fluctuations, fiscal disturbances and external shocks. The policy prescription is contingent on the economic characteristics determined by the structural parameters and the configuration of asset markets. Moreover, the derived welfare criterion enables us to assess the performance of standard policy rules under different asset market structures.

It follows that the optimal monetary policy is independent of financial market structure when trade imbalances are ruled out (by the assumption of unitary elasticity of substitution between goods and log utility). In this specific case, the evolution of the current account is irrelevant to the dynamics of the small open economy, and therefore, it is also of no importance to welfare and optimal policy. However, in all other cases, the characterization of financial markets is shown to be crucial to the evaluation of monetary policy.

The optimal policy can be represented in the form of a targeting rule in the case of complete markets and financial autarky. In these cases, the optimal policy prescribes stabilizing movements in the real exchange rate and output gap as well as inflation. When asset markets are incomplete, the representation of the optimal plan is more complex and cannot be expressed in the form of a single rule. However, under the assumption ~~that there are no intermediation~~ costs, it can be shown analytically that the optimal plan consists of stabilizing *expected* movements in the exchange rate and output gap as well as *expected* inflation.

The weight of inflation variability in the small open economy's loss function is shown to depend crucially on the structure of asset markets and on the elasticity of substitution between domestic and foreign goods. When goods are substitutes in utility, inflation variability is more costly under incomplete markets than under perfect risk sharing. This result is reversed if the degree of substitutability between goods is reduced. These findings are also supported in our quantitative analysis: a domestic inflation target outperforms an exchange rate peg under incomplete (complete) markets for high (low) levels of the elasticity of substitution between goods. These different results are a consequence of the way in which the real exchange rate affects the marginal utility of consumption and the disutility of production under alternative asset market structures.

The policy prescription is also sensitive to the source of the shock hitting the economy. Under mark-up shocks, the optimal monetary policy departs from price stability regardless of the financial market arrangement. However, the optimal response to productivity, fiscal or external shocks depends on the structure of the economy, as hinted above.

2.1.1 Related Literature

In recent years there has been extensive documentation of micro-founded models of open economies featuring imperfect competition and price rigidities. Obstfeld and Rogoff (1995) (*Redux* hereafter) is commonly recognized as the pioneering contribution in the area. Since its publication, many extensions to the baseline model have been made. A comprehensive survey of these is provided in Lane (2001) and Sarno (2000).

The *Redux* model considers a dynamic general equilibrium framework in which only riskless real bonds are traded, and therefore it characterizes an environment of imperfect risk sharing. However, the model is nonstationary and as such presents an undetermined steady state. This restricts our ability to conduct quantitative analysis based on log-linear approximations. To solve this problem and make the analysis more tractable, many subsequent studies have assumed that either: (a) the intratemporal and intertemporal elasticity of substitution are unitary;[3] and/or (b) that asset markets are complete. These assumptions restrict the dynamics of open economies by either making the structure of asset markets irrelevant (assumption (a)) or by imposing an extreme case in which there is perfect risk sharing across borders (assumption (b)). Under both (a) and (b), an important dimension of open economies is ignored: the current account (and a country's net foreign asset position) plays no role in the transmission mechanism of the shocks (see for example Corsetti and Pesenti (2001) and Obstfeld and Rogoff (2000)).

Ghironi (2003) presents an extensive discussion of the consequences of these assumptions for the dynamics of open economies. Moreover, he characterizes an overlapping generations model were the stationarity issue is solved without the use of assumptions (a) or (b). Sutherland (1996) also provides an alternative formulation by incorporating costs of adjusting foreign asset stocks. Schmitt-Grohé and Uribe (2002) examine these technical difficulties in a small open economy setting.[4] A

[3]Models that consider the case in which purchasing power parity holds only require unitary elasticity of intratemporal substitution to achieve the desired tractability.

[4]Other references on alternative specifications used to solve the stationarity problem and their implication for open-economy business cycle properties can also be found in Ghironi (2003).

survey on the academic discussion surrounding the role of the current account and net foreign asset position in dynamic general equilibrium models can be found in Lane and Ganelli (2002).

At the empirical level, Lane and Milesi-Ferretti (2001) have shown that movements in the net foreign asset position are persistent and can significantly affect long run exchange rates, interest rates and international interest rate differentials. These findings suggest that current account movements and the accumulation of foreign assets can be an important factor determining open economy dynamics. For this reason, the present work incorporates the dynamics of the current account into the analysis of optimal monetary policy; we also allow for a non-zero steady-state level of the net foreign asset position.

The original *Redux* model already emphasized that dismissing current account movements and the role of net foreign asset positions can be limiting. The paper shows that an exogenous monetary disturbance can have non-neutral effects in the long run because of its initial impact on the current account and consequent permanent effect on the wealth distribution. Recent contributions have also demonstrated that the implications for monetary policy of assumptions (a) and (b) can be significant. Chapter 1 of this thesis showed that relaxing assumption (a) directly affects the optimality of domestic price stabilization in a small open economy. Using a two-country model, Benigno and Benigno (2003) study the implications of relaxing assumption (a) for the potential gains from international monetary policy coordination. The debate surrounding the relevance of policy coordination has also inspired some authors to investigate the consequences of assumption (b). In a two-country setup, Sutherland (2002) and Tille and Pesenti (2004) analyse the consequences of departing from the complete markets assumption. Both studies find that the gains from cooperation are lower when there is imperfect risk sharing. Moreover, (also using a two-country framework) Benigno (2001) investigates the welfare consequences of adopting a domestic inflation target instead of a coordinated policy when asset markets are incomplete.

In this Chapter, we aim to contribute to this vast literature by relaxing both assumptions (a) and (b), and formalizing a general micro-founded loss function for an individual country under alternative asset market structures. The remainder of the Chapter is structured as follows. Section 2.2 introduces the model. In Section 2.3, we derive the dynamics of the small open economy. Section 2.4 presents the linear-quadratic loss function. The analysis of the optimal plan and the performance of standard policy rules under alternative asset market structures are presented in Section 2.5, while Section 2.6 concludes.

2.2 The Model

The basic setup closely follows the one presented in the Chapter 1. The framework consists of a two-country dynamic general equilibrium model in which the small open economy representation is obtained by taking the limit of the size of one of the countries to zero. Preferences are characterized by home bias in consumption and, therefore, purchasing power parity does not hold.

However, the utility function considered in this Chapter is different from the one presented previously. In particular, the utility function of the representative consumer in country H is given by:

$$U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[U(C_s) - \frac{1}{n} \int_0^n V(y_s^j, \varepsilon_{Y,s}) dj \right], \tag{2.1}$$

Each household contributes to the production of all domestic goods y^j attaining disutility $\frac{1}{n} \int_0^n V(y_s^j, \varepsilon_{Y,s}) dj$. The remaining characteristics of agents' preferences are described as in Section 2 of the first Chapter. More specifically, preferences for domestic and foreign good and its varieties are represented by equations (1.2), (1.3), (1.4) and (1.5). In addition, the price indices implied by these preferences are given by equations (1.6), (1.7), (1.8) and (1.9). Equations (1.13) and (1.14) describe the government budget constraint in Home and Foreign economies, respectively. Finally, the demand functions for goods produced in the small economy and the rest of the world are described by equations (1.11) and (1.12), respectively.

As in the previous Chapter, we consider a cashless economy featuring monopolistic competition and price stickiness *à la* Calvo (1983). The price setting equation is therefore given by Equation (1.21) and the price index evolves according to Equation (1.22). However, in the present chapter, we consider alternative specifications for the structure of financial markets. These are presented in the next Section.

2.2.1 Asset Markets

The structure of financial markets can significantly alter the way idiosyncratic shocks affect consumption, output and other macroeconomic variables. As described in Obstfeld and Rogoff (1996, Chapter 5),

"[...] think about the current account effect of a temporary rise in the country's gross domestic product. In the bonds-only framework [. . .] a temporary productivity shock causes a current account surplus motivated by agents' desire to smooth consumption. But if foreigners have taken on all the country's output risk, a shock in its GDP does not affect its GNP. The increase in domestic output is matched exactly by a lower net inflow of asset income from abroad. Neither income, consumption, nor the current account changes [...]. The presence of international markets for risky assets weakens and may sever the link between shocks to a country's output or factor productivity and shocks to its resident's income. Sophisticated international financial markets thus force us to rethink the channels through which macroeconomic shocks impinge on the world economy."

In this Section, we introduce three different specifications for asset market structure and obtain the economic dynamics implied by each. First, we present the scenario in which international financial markets are incomplete, by assuming that agents can internationally trade nominal riskless bonds subject to intermediation costs. Then we lay out two benchmark cases of asset market structure: at one extreme, we analyse the case of financial autarky, in which the small open economy has no access to international financial markets; at the other, we examine the most developed form of capital markets, in which households have access to a set of contingent claims resulting in an environment of perfect risk sharing with the rest of the world.

What is the two-country setting?
How do you take limit?

Incomplete Markets

Bond only

We characterize the environment of incomplete markets by assuming that agents can trade nominal riskless bonds denominated in *Home* and *Foreign* currency. We consider that home currency-denominated bonds are only traded domestically. Moreover, following Benigno (2001), the international trade of foreign currency-denominated bonds is subject to intermediation costs. This cost is proportional to the country's aggregate net foreign asset position. If the small open economy is a net debtor, its agents pay a premium on the foreign interest rates when borrowing from abroad. On the other hand, if the country is a net creditor, households lending in foreign currency receive a rate of return lower than foreign interest rates. The spread is the remuneration of international intermediaries, and is assumed to be rebated equally among foreign households.

?

The intermediation cost assumption is introduced for technical reasons: it solves the stationarity problem à la Obstfeld and Rogoff (1995) described in Section (2.1.1). By ensuring that the model is stationary, this assumption guarantees the precision of any quantitative exercises involving a log linear version of the model. In addition, it allows for the examination of the second moments of macroeconomic variables. Nevertheless, for some of our qualitative analysis, we consider the case of zero intermediation costs. This is done in order to simplify the analytical derivation of the optimal plan and improve our intuition on the policy prescriptions under incomplete markets.

We can write the household's budget constraint at Home as follows:

Need to state the assumptions.
When SOE borrows do they issue foreign denominated bonds?

$$P_tC_t+\frac{B_{H,t}}{(1+i_t)}+\frac{S_tB_{F,t}}{(1+i_t^*)\psi\left(\frac{S_tB_{F,t}}{P_t}\right)} \leq B_{H,t-1}+S_tB_{F,t-1}+\frac{(1-\tau_t)\int_0^n p_t(h)y_t(h)dh}{n}+P_{H,t}Tr_t, \tag{2.2}$$

where $B^i_{H,t}$ and $B^i_{F,t}$ denote domestic-currency and foreign-currency denominated nominal bonds and Tr_t are government transfers, made in the form of domestic

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a zero steady-state risk premium by setting $\psi(\bar{b}) = 1$. Moreover, in specifying the budget constraint (2.2), we also assume that households in a given country produce all goods and share the revenues from production in equal proportions. We also consider the case in which the initial wealth of all households within a country are equal. These two assumptions ensure that households in the same country face the same budget constraints in every period and state of the world. Therefore, we can consider a representative consumer for each economy. We should note that, even though idiosyncratic risk is pooled among households from the same country, there is imperfect risk sharing across borders.

Foreign households are assumed to trade only in foreign currency bonds: therefore their budget constraint can be written as

why do you need this?

$$P_t^* C_t^{*i} + \frac{B_{F,t}^{*i}}{(1+i_t^*)} \leq +B_{F,t-1}^{*i} + \frac{(1-\tau_t^*)\int_{1-n}^{1} p(f) y_t(f) dh}{1-n} + P_t^* Tr_t^* + \frac{K}{1-n}. \quad (2.3)$$

The intermediation profits K, which are shared equally among foreign households, can be written as

$$K = \frac{B_{F,t}}{P_t^*(1+i_t^*)} \left[1 - \frac{RS_t}{\psi\left(\frac{S_t B_{F,t}}{P_t}\right)} \right]. \quad (2.4)$$

Given the above specification, we can write the consumer's intertemporal optimal choices as

$$U_C(C_t) = (1+i_t)\beta E_t \left[U_C(C_{t+1}) \frac{P_t}{P_{t+1}} \right], \quad (2.5)$$

$$U_C(C_t^*) = (1+i_t^*)\beta E_t \left[U_C(C_{t+1}^*) \frac{P_t^*}{P_{t+1}^*} \right], \quad (2.6)$$

and

$$U_C(C_t) = (1+i_t^*)\psi\left(\frac{S_t B_{F,t}^i}{P_t}\right) \beta E_t \left[U_C(C_{t+1}) \frac{S_{t+1} P_t}{S_t P_{t+1}} \right], \quad (2.7)$$

where (2.5) and (2.7) are Home and Foreign Euler equations derived from the optimal choice of foreign currency denominated bonds. Equation (2.6) results from the small open economy optimal choice of home currency denominated bonds. Moreover, Equations (2.5) and (2.7) imply that there is an interest rate differential across countries. Empirically, this assumption is supported by the findings of Engel (2002) and Kollman (2002), who show that allowing for interest rate differentials can improve the fit of the data.

Financial Autarky

In this setup, the economy does not have access to international borrowing or lending. Consequently, there is no risk sharing across borders. Risk is pooled internally to the extent that agents participate in the production of all goods and receive an equal share of production revenue. Moreover, as in the previous Section, we assume that there is a symmetric initial distribution of wealth across domestic agents.

The household budget constraints, at Home and abroad, can be written as

$$P_t C_t \leq \frac{(1-\tau_t)\int_0^n p_t(h) y_t(h) dh}{n} + Tr_t \tag{2.8}$$

and

$$P_t^* C_t^{*i} \leq \frac{(1-\tau_t^*)\int_{1-n}^1 p(f) y_t(f) dh}{1-n} + Tr_t^*. \tag{2.9}$$

Under financial autarky, the value of domestic production has to be equal to the level of public and private consumption in nominal terms. Aggregating private and public budget constraints, we have:

$$P_H(Y_t - G_t) = P_t C_t \tag{2.10}$$

The inability to trade bonds with the rest of the world imposes that the value of imports should equal the value of exports:

$$(1-n) S_t P_{H,t}^* C_{H,t}^* = n P_{F,t} C_{F,t}. \tag{2.11}$$

Complete Markets

We characterize the most developed form of capital markets following Chari et al. (2002). As in Chapter 1, we introduce the complete market environment by assuming that agents have access to state contingent nominal claims that deliver a unit of Home currency in each state of the world. In this setup, the rate of marginal utilities is equalized across countries at all times and states of nature.

$$\frac{U_C\left(C_{t+1}^*\right)}{U_C\left(C_t^*\right)}\frac{P_t^*}{P_{t+1}^*}=\frac{U_C\left(C_{t+1}\right)}{U_C\left(C_t\right)}\frac{S_{t+1}P_t}{S_tP_{t+1}} \tag{2.12}$$

2.3 A Log-Linear Representation of the Model

In this Section, we derive the log-linear approximation to the structural equilibrium conditions for the small open economy and the rest of the world. In what follows, a hat denotes log deviations from the steady state, i.e. $\widehat{x}=\frac{X-\overline{X}}{\overline{X}}$. A full characterization of the steady state is presented in Appendix A.

2.3.1 The Dynamics of the Small Open Economy:

The small open economy dynamics can be described by the aggregate supply condition, the aggregate demand equation, the equilibrium condition(s) implied by the financial market structure, and a monetary policy rule. Tables 2.1 to 2.3 present the system of log-linear equations under the different asset market assumptions.

Table 2.1: Equilibrium Conditions under Incomplete Markets

$\widehat{\pi}_t^H = k(\rho\widehat{C}_t + \eta\widehat{Y}_t + \frac{\lambda}{1-\lambda}\widehat{RS}_t + \widehat{\mu}_t - \eta\widehat{\varepsilon}_{Y,t}) + \beta E_t\widehat{\pi}_{t+1}^H$	AS
$\widehat{Y}_t = (1-\lambda)\widehat{C}_t + \lambda\widehat{C}_t^* + b_{rs}\widehat{RS}_t + \widehat{g}_t$	AD
$\rho E_t(\widehat{C}_{t+1} - \widehat{C}_t) = \rho E_t(\widehat{C}_{t+1}^* - \widehat{C}_t^*) + E_t\Delta\widehat{RS}_{t+1} - \delta\widehat{b}_{F,t}$	IM
$\beta\widehat{b}_{F,t} = \widehat{b}_{F,t-1} + \widehat{Y}_t - \widehat{C}_t - \frac{\lambda}{1-\lambda}\widehat{RS}_t - \widehat{g}_t$	IM2

Table 2.2: Equilibrium Conditions under Financial Autarky

$\widehat{\pi}_t^H = k(\rho\widehat{C}_t + \eta\widehat{Y}_t + \frac{\lambda}{1-\lambda}\widehat{RS}_t + \widehat{\mu}_t - \eta\widehat{\varepsilon}_{Y,t}) + \beta E_t\widehat{\pi}_{t+1}^H$	AS
$\widehat{Y}_t = (1-\lambda)\widehat{C}_t + \lambda\widehat{C}_t^* + b_{rs}\widehat{RS}_t + \widehat{g}_t$	AD
$\widehat{Y}_t - \frac{\lambda}{1-\lambda}\widehat{RS}_t = \widehat{C}_t$	FA

Table 2.3: Equilibrium Conditions under Complete Markets

$\widehat{\pi}_t^H = k(\rho\widehat{C}_t + \eta\widehat{Y}_t + \frac{\lambda}{1-\lambda}\widehat{RS}_t + \widehat{\mu}_t - \eta\widehat{\varepsilon}_{Y,t}) + \beta E_t\widehat{\pi}_{t+1}^H$	AS
$\widehat{Y}_t = (1-\lambda)\widehat{C}_t + \lambda\widehat{C}_t^* + b_{rs}\widehat{RS}_t + \widehat{g}_t$	AD
$\widehat{C}_t = \widehat{C}_t^* + \frac{1}{\rho}\widehat{RS}_t$	CM

The aggregate supply condition (AS) is derived from the pricing Equation (1.21).

Producer price inflation is denoted by π_t^H and $k = (1-\alpha\beta)(1-\alpha)/\alpha(1+\sigma\eta)$. Moreover, as shown in the Appendix, ρ represents the coefficient of relative risk aversion and η the inverse of the elasticity of goods production. This is the usual Open Economy New-Keynesian Phillips Curve and represents the supply side relationship between relative prices, output and consumption. Fluctuations in this condition are driven by productivity and mark-up shocks.

The small open economy demand equation (AD) is a log-linear version of Equation (1.12). The fiscal shock $\hat{g}_t$ is defined as $\frac{Gt-\overline{G}}{\overline{Y}}$ and $b_{rs} = \frac{\theta\lambda(2-\lambda)}{1-\lambda}$. Equation (AD) summarizes the demand conditions in the small open economy and it is affected by real external shocks and fiscal disturbances.

In the case of market incompleteness, Equations (2.6) and (2.7) determine the evolution of the consumption differential between the two economies. The combination of these conditions can be expressed in log-linear terms by Equation (IM). Moreover, in this setup, agents can trade domestic-currency and foreign-currency denominated bonds. We assume that bonds denominated in domestic currency are in zero net supply. Consequently, the aggregate budget constraint of the economy, including private agents and government, can be written as (IM^2). This expression represents the small open economy current account equation, where $b_{F,t} = \frac{S_t B_{F,t}}{P_t}$, $\widehat{b}_{F,t} = \frac{(b_{F,t}-\bar{b})}{\overline{Y}}$ and $\delta = -\psi'(\bar{b})\overline{Y}$.

In the case of financial autarky, the aggregate resource constraint (2.10) can be written in log-linear terms as (FA). And, if asset markets are complete, (CM) represents the risk sharing condition (2.12).[5]

Furthermore, when asset markets are incomplete, we allow for a non-zero steady-state net foreign asset position and an asymmetric steady-state level of consumption ($\bar{C} \neq \bar{C}^*$).[6] As documented in Lane and Milesi-Ferretti (2002), countries have different levels of net foreign assets, and this is an important determinant of economic dynamics. This specification can therefore lead to a better approximation of the open economies business cycles. In this case, the system of equilibrium condition is

[5] In the above equations, the price index Equation (1.6) was used to solve for the relative prices $\frac{P_{H,t}}{P_t}$ in terms of the real exchange rate RS_t.

[6] As shown in Appendix A the steady-state relationship between the net foreign asset position and the consumption differential is given by $(1-\beta)\overline{B} = \lambda(\bar{C} - \bar{C}^*)$.

summarised in Table 2.4:

Table 2.4: Equilibrium Conditions under Incomplete Markets and Non-zero Steady-state Net Foreign Asset Position

$\widehat{\pi}_t^H = k(\rho(1+a)\widehat{C}_t + \eta\widehat{Y}_t + \frac{\lambda}{1-\lambda}\widehat{RS}_t + \widehat{\mu}_t - \eta\widehat{\varepsilon}_{Y,t}) + \beta E_t\widehat{\pi}_{t+1}^H$	AS'
$\widehat{Y}_t = (1-\lambda)\widehat{C}_t + \lambda\widehat{C}_t^* + b_{rs}\widehat{RS}_t + \widehat{g}_t$	AD'
$\rho E_t(\widehat{C}_{t+1} - \widehat{C}_t) = \rho E_t(\widehat{C}_{t+1}^* - \widehat{C}_t^*) + E_t\Delta\widehat{RS}_{t+1} - \delta\widehat{b}_{F,t}$	IM
$\beta E_t\widehat{q}_{F,t+1} = \widehat{q}_{F,t} + \widehat{Y}_t - (1+a)\widehat{C}_t + \rho a\widehat{C}_t - \frac{\lambda}{1-\lambda}\widehat{RS}_t - \widehat{g}_t$	$\mathrm{IM}^{2\prime}$

where $\widehat{q}_{F,t} = \widehat{b}_{F,t-1} + a/(1-\beta)(\Delta RS_t - \pi_t^* - \rho\widehat{C}_t)$. The existence of a non-zero steady-state net foreign asset position modifies the aggregate supply, demand and current account equations. The aggregate supply and demand equations change because the log-linearization is made around a different steady state for $\bar{C}$ and $\bar{C}^*$. Moreover, Equation ($\mathrm{IM}^{2\prime}$) is different from Equation (IM^2) because when $\overline{B} \neq 0$ the debt burden affects the small open economy current account. In Table (2.4) we denote $a = (1-\beta)\frac{\overline{B}}{\overline{Y}}$, $d_b = (1+a)(1-\lambda)$, and $d_rs = \frac{\theta(\lambda+(1-d_b)(1-\lambda))}{(1-\lambda)}$.

Given the domestic exogenous variables $\widehat{\varepsilon}_{y,t}$,$\widehat{g}_t$, $\widehat{\mu}_t$ and the real and nominal external conditions $\widehat{C}_t^*$ and $\widehat{\pi}_t^*$, the small open economy system of equilibrium conditions is closed by specifying the monetary policy rule. The current Chapter examines different specifications for the monetary policy rule. Apart from analyzing the optimal monetary policy regime in the form of a targeting rule, we evaluate the performance of alternative standard policy rules such as an exchange rate peg, and both CPI and PPI inflation targets.

2.3.2 Foreign Dynamics

Foreign dynamics are governed by the foreign Phillips curve and foreign demand:

Table 2.5: Foreign Equilibrium Conditions

$\widehat{\pi}_t^* = k(\rho\widehat{C}_t^* + \eta\widehat{Y}_t^* + \widehat{\mu}_t^* - \eta\widehat{\varepsilon}_{Y,t}^*) + \beta E_t\widehat{\pi}_{t+1}^*$	AS*
$\hat{Y}_t^* = \hat{C}_t^* + \hat{g}_t^*$	AD*

The specification of the foreign policy rule completes the system of equilibrium conditions which determine the evolution of $\hat{Y}_t^*, \hat{C}^*$ and $\widehat{\pi}_t^*$. We should note that the dynamics of the rest of the world are not affected by *Home* variables. Therefore, the small open economy can treat C_t^* and π_t^* as exogenous.

The policy choice in the rest of the world determines how the endogenous variables respond to the structural shocks. Hence, it affects the correlation between C_t^* and π_t^*. In the case of a symmetric steady state, the system of equilibrium conditions in the small open economy is only affected by C_t^*. Therefore, the policy choice of the *Foreign* economy is irrelevant for the policy analysis in the small open economy. However, if the steady-state consumption profile is asymmetric, π_t^* also affects *Home* dynamics (this can be seen by inspection of Equation ($\mathrm{IM}^{2'}$)). Consequently, in this case, the choice of foreign policy rule is not irrelevant for the small open economy.

2.4 Welfare

In a micro-founded model, a precise measure of welfare can be obtained from agents' level of utility. Therefore, the policy objective for the small open economy consists of agents' life time expected utility, given by Equation (2.1). Following the linear-quadratic approach developed by Benigno and Woodford (2003) and Sutherland (2002), we derive a second order approximation to the policy objective in the Appendix. It follows that the welfare criterion can be approximated by

$$W_{to} = U_c\bar{C}E_{t_0}\sum\beta^t\left[\begin{array}{c}\widehat{C}_t - \frac{1}{\overline{\mu}(1+a)}\widehat{Y}_t + \frac{1}{2}(1-\rho)\widehat{C}_t^2 \\ -\frac{1}{2}\frac{(\eta+1)}{\overline{\mu}(1+a)}(\widehat{Y}_t - \frac{\eta}{(\eta+1)}\widehat{\varepsilon}_{Y,t})^2 - \frac{1}{2}q_\pi\left(\pi_t^H\right)^2\end{array}\right] + t.i.p + \mathcal{O}(||\xi||^3), \tag{2.13}$$

where $\overline{\mu} = \frac{\sigma}{(1-\tau)(\sigma-1)}$ represents the steady-state wedge between the marginal utility of consumption and the marginal disutility from production. The weight given to inflation in the above expression is $q_\pi = \frac{\sigma}{k\overline{\mu}(1+a)}$. The term $t.i.p$ again stands for terms independent of policy (i.e. they are exogenous shock terms that are not affected by the policy choice). Finally, the element $\mathcal{O}(||\xi||^3)$ refers to terms of order strictly higher than two.

In order to derive a purely quadratic representation of welfare, the discounted linear terms in Equation (2.13) have to be eliminated. In Appendix B, we derive second order approximations to some of the structural equilibrium conditions and obtain a complete second order solution for the evolution of the endogenous variables of interest. Because alternative asset market characterizations imply different equilibrium conditions, the final expression for welfare varies according to the structure of the asset market.

The loss function for our small open economy under the different asset market structures (denoted with a superscript m) can be expressed as

$$L_{to} = U_c\bar{C}E_{t_0}\sum\beta^t\left[\begin{array}{c}\frac{1}{2}l_{yy}^m\widehat{Y}_t^2 + \frac{1}{2}l_{rr}^m\widehat{RS}_t^2 + l_{yr}^m\widehat{Y}_t\widehat{RS}_t \\ +L_{ey}^{m'}\widehat{e}_t\widehat{Y}_t + L_{er}^{m'}\widehat{e}_t\widehat{RS}_t + \frac{1}{2}l_\pi^m\left(\pi_t^H\right)^2\end{array}\right] + t.i.p + \mathcal{O}(||\xi||^3). \tag{2.14}$$

In what follows, we let the superscript $m = c$ signify the case of complete markets, while $m = fa$ is the financial autarky setup and the incomplete market case is denoted by $m = i$. e_t denotes the vector containing the following exogenous variables:

$$\widehat{e}_t = \left[\begin{array}{cccc}\widehat{\varepsilon}_{yt} & \widehat{\mu}_t & \widehat{g}_t & \widehat{C}_t^*\end{array}\right].$$

The weights $l_{yy}^m, l_{rr}^m, l_{yr}^m, l_\pi^m$, and the vectors L_{ey}^m and L_{er}^m depend on the structural parameters of the model and on the asset market configuration (see Appendix B). We should note that the nominal external shock $\widehat{\pi}_t^*$ does not appear in our welfare characterization. Even though $\widehat{\pi}_t^*$ (and the policy choice of the rest of the world) can affect the dynamics of the small open economy when $\overline{B} \neq 0$ and asset markets are incomplete, it is of no importance for the loss function formulation. That is, $\widehat{\pi}_t^*$ can affect the constraints of the policy problem, but it does not change the policy objective.

This comparison makes sense only if l_{ys} are kept constant across 3 cases.

2.4.1 The Weight of Domestic Inflation in the Loss Function

The expression for l_π is a complex function of the structural parameters. However, if we assume that the level of output is efficient in the steady state (for the small open economy)[7] and that the net foreign asset position is zero, the expressions for l_π^i, l_π^{fa} and l_π^c can be easily compared. In particular, under this specification:

$$l_\pi^i = l_\pi^{fa} = \frac{\sigma(1-\lambda)}{k}\left(1 + \frac{l_i\lambda(1-\lambda)^{-1}(\eta+1)}{l_i(\rho+\eta)+\rho(1-\lambda)+\eta+\lambda}\right)$$

and

$$l_\pi^c = \frac{\sigma(1-\lambda)}{k}\left(1 - \frac{l_c(\eta+1)}{(\rho+\eta)+\eta l}\right)$$

with $l_i = (\theta-1)(2-\lambda)$ and $l_c = (\rho\theta-1)\lambda(2-\lambda)$.

Therefore, when domestic and foreign goods are substitutes in the utility function (more specifically, when $\rho\theta > 1$ and $\theta > 1$), we have that $l_\pi^i = l_\pi^{fa} > q_\pi$ and $l_\pi^c < q_\pi$. So when we rewrite welfare as a purely quadratic expression, the weight on inflation under incomplete markets and financial autarky increases, while in the case of complete markets it decreases. That is, with complete markets, the linear term $\widehat{C}_t - \frac{1}{\overline{\mu}(1+a)}\widehat{Y}_t$ in Equation (2.13) can be written as an increasing function of inflation variability. On the other hand, with imperfect risk sharing, either in the case of financial autarky or market incompleteness, this term is a decreasing function of $\left(\pi_t^H\right)^2$. Now, if $\rho\theta < 1$ and $\theta < 1$, the conclusion is reversed: $l_\pi^i = l_\pi^{fa} < q_\pi$ and $l_\pi^c > q_\pi$. The differences in the weight on inflation have direct implications for

[7] As shown Chapter 1, this can be achived by setting $\overline{\mu} = 1/(1-\lambda)$.

optimal monetary policy. These are explored in the next Section.

2.5 Monetary Policy

In this Section, we analyse optimal monetary policy under alternative asset market structures. Firstly, the optimal monetary policy plan for the different financial markets settings is formalized. Secondly, we carry out quantitative exercises which illustrate the optimal responses to different shocks and evaluate how these change with the characterization of the small open economy. Finally, we conduct a welfare evaluation of different standard policy rules. The performances of a domestic inflation target, a CPI inflation target and a fixed exchange rate regime are ranked based on our welfare measure.

2.5.1 Optimal Monetary Policy under Alternative Asset Market Structures:

We proceed by characterizing the optimal plan under the assumption that policymakers can commit to maximizing the economy's welfare. The policy problem consists of minimizing the loss function given the equilibrium conditions and the initial conditions $\widehat{\pi}_{t_0}$ and $\widehat{Y}_{t_0}$. In effect, the constraints on the initial conditions impose that the first order conditions to the problem are time invariant. This method follows Woodford's (1999) timeless perspective approach and ensures that the policy prescription does not constitute a time inconsistent problem.

In the case of complete markets and financial autarky, the policy problem consists of choosing the path of $\{\widehat{\pi}_t^H, \widehat{Y}_t, \widehat{C}_t, \widehat{RS}_t\}$ in order to minimize (2.14), subject to the equilibrium conditions given by Tables 2 and 3, respectively. The first order conditions to the minimization problem (shown in the Appendix) can be written in the form of the following targeting rules:

$$Q_y^c \Delta(\widehat{Y}_t - \widehat{Y}_t^{T,c}) + Q_{rs}^c \Delta(\widehat{RS}_t - \widehat{RS}_t^{T,c}) + Q_\pi^c \widehat{\pi}_t^H = 0 \tag{2.15}$$

and

$$Q_y^{fa}\Delta(\widehat{Y}_t - \widehat{Y}_t^{T,fa}) + Q_{rs}^{fa}\Delta(\widehat{RS}_t - \widehat{RS}_t^{T,fa}) + Q_\pi^{fa}\widehat{\pi}_t^H = 0, \tag{2.16}$$

where the superscript c denotes the complete market case and fa refers to the financial autarky setting. Δ denotes first difference operator. The above targeting rules set the objectives for monetary policy. This is done by specifying the targets $\widehat{Y}_t^T$ and $\widehat{RS}_t^T$ as functions of the various shocks (excluding the nominal external disturbance $\widehat{\pi}_t^*$). Moreover, according to these equations, policymakers should respond to real exchange rate and output movements, as well as inflation. The coefficients Q_y, Q_{rs} and Q_π depend on the weights of each of the variables on welfare and therefore are determined by the underlying distortions in the economy. In general, the weights as well as the target variables are different depending of the asset market specification. These expressions are shown in Appendix B and are complex functions of the structural parameters.

What is the tax policy?

In the case of incomplete markets, the policy problem consists of choosing the path of $\{\widehat{\pi}_t^H, \widehat{Y}_t, \widehat{C}_t, \widehat{RS}_t, \widehat{b}_{F,t}\}$ in order to minimize (2.14) subject to the equations specified in Table 4.[8] The resulting first order conditions are:

$$l_\pi \widehat{\pi}_t^H + \Delta\varphi_{1,t} = 0, \tag{2.17}$$

$$0 = l_{yy}\widehat{Y}_t + l_{yr}\widehat{RS}_t + l_{ye}\widehat{e}_t - k\eta\varphi_{1,t} + \varphi_{2,t} - \varphi_{4,t}, \tag{2.18}$$

$$\begin{aligned} 0 = {} & l_{yr}\widehat{Y}_t + l_{rr}\widehat{RS}_t + l_{re}\widehat{e}_t - k\frac{\lambda}{(1-\lambda)}\varphi_{1,t} - d_{rs}\varphi_{2,t} + \varphi_{3,t} - \beta^{-1}\varphi_{3,t-1} \\ & + \frac{\lambda}{(1-\lambda)}\varphi_{4,t} - a_\beta\Delta\varphi_{4,t} + a_\beta\beta\Delta E_t\varphi_{4,t+1}, \end{aligned} \tag{2.19}$$

[8] We compute the optimal plan for the general case where the steady state level of foreign assets is non-zero. The case of a symmetric steady state can be obtained by setting $a = 0$.

$$0 = -\rho k(1+a)\varphi_{1,t} - (1+a)(1-\lambda)\varphi_{2,t} - \rho\varphi_{3,t} + \rho\beta^{-1}\varphi_{3,t-1} + (1+a)\varphi_{4,t} + \rho a_{\beta}\Delta\varphi_{4,t} \tag{2.20}$$

and

$$E_t\Delta\varphi_{4,t+1} = \beta^{-1}\delta\varphi_{3,t}. \tag{2.21}$$

The characterization of the optimal policy under incomplete markets is more complicated because of the intertemporal representation of the constraints (IM) and (IM^2). The presence of intermediation costs also adds to the complexity of the problem. In general, the optimal plan for a small open economy with incomplete markets is the solution to a system of linear stochastic difference equations given by the above first order conditions and the equations specified in Table 4.

Nevertheless, in the special case in which there are no intermediation costs involved in the international trade of bonds (i.e. $\delta = 0$), the above first order conditions imply

$$Q_y^i E_t\Delta(\widehat{Y}_{t+1} - \widehat{Y}_{t+1}^{T,i}) + Q_{rs}^i E_t\Delta(\widehat{RS}_{t+1} - \widehat{RS}_{t+1}^{T,i}) + Q_\pi^i E_t\widehat{\pi}_{t+1}^H = 0. \tag{2.22}$$

We should note that the above equation is not a targeting rule. The dynamics of the small open economy are not *determined* by this equation together with ? the other expressions in Table (2.4).[9] Equation (2.22) is simply an equilibrium condition implied by the optimal plan, but does not represent a monetary policy rule. Comparing Equation (2.22) with (2.15) and (2.16), we can see that, while under incomplete markets the optimal policy stabilizes *expected* movements in the variables, under complete markets and financial autarky the targeting rule reacts to actual fluctuations in these variables. Behind this result lies the fact that under complete markets consumption is determined by the risk sharing condition which

[9]The determinancy conditions are checked numerically using the algorithm of King and Watson (1998).

holds in each period. Similarly, under financial autarky, domestic consumption at period t is financed by the domestic output in that period. On the other hand, in the case of incomplete markets, agents smooth consumption given their expectation of future income.

Is it the reason?

2.5.2 Irrelevance of Asset Market Structure

Under certain parameter specifications, the dynamics of the small open economy are independent of the asset market structure. This is the case when the elasticity of intertemporal and intratemporal substitution are unitary and the initial level of debt is zero. These specifications imply that the economy never experiences trade imbalances, regardless of asset market structure. Therefore, the value of domestic production is always equal to the value of domestic consumption.

As shown in Appendix B, if we impose the restrictions $\rho = \theta = 1$ and $\bar{b}_F = \widehat{b}_{F,-1} = 0$, the second order approximation to the equilibrium conditions for the three different asset market structures can also be expressed as

wealth effect disappears

$$\widehat{C}_t = \widehat{RS}_t + \widehat{C}_t^* \tag{2.23}$$

The equilibrium conditions for the alternative forms of asset market configuration are therefore identical.[10] Consequently, the welfare characterization is also independent of the degree of risk sharing. Furthermore, the utility-based loss function becomes isomorphic to a closed economy loss function and can be represented as a quadratic expression of domestic inflation and the output gap only (see Appendix B). If we assume that the steady-state level of output is efficient (i.e. $\mu = 1/(1-\lambda)$), the optimal policy consists of a strict domestic inflation target. In this case, the loss function can be written as:

$$L_{to}^i = U_c \bar{C} E_{t_0} \sum \beta^t \left[\frac{1}{2}(\eta + 1)(\widehat{Y}_t - \widehat{Y}_t^{Flex})^2 + \frac{1}{2}\frac{\sigma}{k}(\widehat{\pi}_t^H)^2 \right] + t.i.p + \mathcal{O}(||\xi||^3), \tag{2.24}$$

[10]The irrelevance of the asset market structure under this specification has been extensively discussed in the literature (e.g. Obstfeld and Rogoff (1995) and Benigno (2001), among others).

where $\widehat{Y}_t^{Flex} = \frac{\eta}{(\eta+1)}\widehat{\varepsilon}_{Yt} + \frac{1}{(\eta+1)}\widehat{g}_t$ denotes the flexible price allocation, or equivalently, the equilibrium output when a strict domestic inflation target is implemented. Under this specification, the first order conditions of the minimization problem under incomplete markets, given by equations (2.17) through to (2.21), can be expressed as:

$$0 = \Delta(\widehat{Y}_t - \widehat{Y}_t^{Flex}) + \sigma\widehat{\pi}_t^H \tag{2.25}$$

The same expression arises if we impose these parameter specifications on the targeting rule under complete markets and financial autarky, given by equations (2.15) and (2.16). Under this parameterization, a policy of complete domestic price stabilization closes the welfare relevant output gap. Hence, it is optimal to target producer price inflation, regardless of the asset market structure.

We should note that, in the case of financial autarky, a domestic inflation target is the optimal policy under a less restrictive assumption. In particular, Equation (2.25) holds when $\theta = 1$, regardless of the value of ρ. In other words, when asset markets are characterized by financial autarky and the elasticity of substitution between goods is unitary, the flexible price allocation coincides with the efficient allocation.[11]

2.5.3 Quantitative Analysis

We solve the log-linearized model using the algorithm of King and Watson (1998), which also checks numerically if the determinacy conditions are satisfied. To apply this numerical method, we consider the following benchmark specification for the small open economy structural parameters. We start with a unitary coefficient of risk aversion, i.e. $\rho = 1$. This specification implies a log utility function of aggregate consumption and is extensively used in the literature. However, many studies have estimated different values for this parameters. Eichenbaum et al. (1988), finds that this parameter should range between 0.5 and 3. On the other hand, Hall (1988)

[11] In order to ensure that a strict inflation target is optimal when the economy is subject to fiscal shocks we should also assume that $\mu = 1/(1-\lambda)$. Moreover, we note that domestic price stabilization is never optimal when mark-up shocks are present.

suggested a value higher than 5. We analyse the cases of ρ ranging from 1 to 6.

The elasticity of substitution between domestic and foreign goods, θ, is another crucial parameter in our analysis. Our benchmark specification assumes that $\theta = 3$. This is consistent with Obstfeld and Rogoff (1998), who argue that this parameter should be between 3 and 6. Many other papers have estimated this parameter: Chari, Kehoe and McGrattan (1998) suggest it should range between 1 and 2; Trefler and Lai (1999) estimates are at around 5. Therefore we consider a robustness analysis, with θ ranging 0.5 to 6. Our benchmark case sets $\rho = 1$ and $\theta = 3$, and therefore implies that Home and Foreign goods are substitutes in utility. The specification of $\rho\theta < 1$ is also analysed in order to evaluate the implications of assuming that domestic and foreign goods are complements.

In addition, the elasticity of substitution between differentiated goods is assumed to be 10, as in Benigno and Benigno (2003). Following Rotemberg and Woodford (1997), we assume that the elasticity of labour supply to real wages is $\eta = 0.47$. Moreover, we set $\beta = 0.99$, which implies steady-state annual returns of 4%. Furthermore, in order to characterize an average length of price contract of 3 quarters, we assume that $\alpha = 0.66$. This is consistent with the findings of Galí and Gertler (1998) for the U.S. economy.

λ is assumed to be 0.25, implying a 25% import share of GDP. We also vary λ from 0.2 to 0.5. This range includes the degree of openness in countries like Canada, where the imports as a percentage of GDP are around 40%, but also encompasses lower levels of openness, such as those found in New Zealand, Chile or Peru, which have import to GDP ratios of around 20%.

Following Benigno (2001), we assume $\delta = 0.01$, which implies a spread between home and foreign interest rates of 1%. We also consider the case of zero interest rate spreads. Under our baseline specification, we consider a zero steady-state foreign asset position. However, in order to illustrate more realistic values of the debt to GDP ratio, we analyse the cases of $\frac{-\overline{B}}{\overline{Y}}$ of up to 50%, as in Benigno (2001).

Finally, we assume an efficient steady-state level of output, i.e. $\overline{\mu} = 1/(1-\lambda)$. This specification is imposed in order to abstract from policy incentives that may arise from inefficiencies in the steady-state level of output. (The analysis of this factor can be found in the Chapter 1 and is deeply explored in Benigno and Woodford (2003)). The mark-up implied by our benchmark specification for λ is therefore 33% and can reach 100% when we consider $\lambda = 0.5$. Erceg et al. (2000) find the value of 77% for the steady-state mark-up and Benigno and Woodford (2003) consider the case of a 50% mark-up.[12]

Illustrating the Optimal Monetary Policy under Alternative Asset Market Structures

In this Section, we portray the optimal responses to the different shocks for each of the asset market configurations, under our benchmark calibration. Figure 2.1 shows the impulse responses of consumption, output, the real exchange rate and producer price inflation following a domestic productivity shock. These pictures illustrate that, regardless of the asset market characterization, the optimal policy does not differ quantitatively from a producer price inflation target. Following the domestic productivity improvement, output and consumption increase. In addition, the larger supply of domestic goods induces a real exchange rate depreciation. However, the more 'sophisticated' is the asset market structure, the smaller is the consumption reaction to this shock. On the other hand, output and the real exchange rate react less strongly when the degree of risk sharing is smaller.

Figure 2.2 presents the case of a real external shock $\widehat{C}_t^*$. In particular, we consider the case of a foreign productivity shock $\widehat{\varepsilon}_{Y,t}^*$ when the rest of the world is following strict inflation targeting. By inspection of foreign equilibrium conditions (Table 5) we can see that, when $\widehat{\pi}_t^* = 0$, a 1% productivity shock results in a $\frac{\eta}{\eta+\rho}$% shift in $\widehat{C}_t^*$. As in the case of a domestic productivity shock, following a foreign shock the optimal plan also prescribes domestic inflation stabilization. This holds for all asset market structures. In this case, higher foreign productivity leads to a real exchange rate appreciation, a fall in output and an increase in consumption.

[12] The numerical exercise is pursued in order to obtain a qualitative analysis of the optimal policy and the performance of policy rules. Even though we follow the literature on the specification of the parameters, the choice of values in some of the sensitivity analysis has the objective of attaining an intuitive evaluation of optimal policy.

The latter effect is smaller under financial autarky, because in this case the domestic resource constraint is always binding.

The optimal responses to a fiscal shock are shown in Figure 2.3. If asset markets are complete, the positive demand shock implies an increase in consumption and, therefore, the positive response of output to the shock is more than proportional. Under financial autarky and incomplete markets, there is a 'crowding out' effect on domestic consumption. In these two cases, as is clear from the impulse response of domestic inflation, the optimal plan is very close to a producer price inflation target.

Figure 2.4 shows that when the economy is subject to mark-up shocks, the optimal monetary policy departs from price stabilization under all asset market settings. The optimal plan reacts to the fluctuations in the wedge between the marginal utility of consumption and the marginal disutility of production. For the cases of financial autarky and incomplete markets, the shock reduces output and consumption. On the other hand, when the asset market is complete, the optimal policy implies an increase in domestic output. In this case, the exchange rate depreciation improves the small open economy's competitiveness. Furthermore, inflation is more volatile when asset markets are complete.

Figures 2.5, 2.6, 2.7 and 2.8 show the impulse responses of the variables for the case of incomplete asset markets with a non-zero steady-state net foreign asset position. In this case, we calibrate the net foreign asset position in steady state as $\frac{\overline{B}}{\overline{Y}} = -0.5$, implying an external debt to GDP ratio of 50%. By inspection of the pictures, it is clear that the results exposed previously do not change and deviations from domestic inflation target only happen in the presence mark-up shocks.

Figures 2.9 to 2.12 illustrate the impulse responses of the variables for the case of incomplete asset markets with no intermediation costs ($\delta = 0$), contrasting them to the case where $\delta = 1\%$. The specification of δ does not seem to affect the optimal plan, regardless of the source of disturbance. However, as discussed earlier, when $\delta = 0$, the incomplete market model is characterized by a nonstationary net foreign asset position.

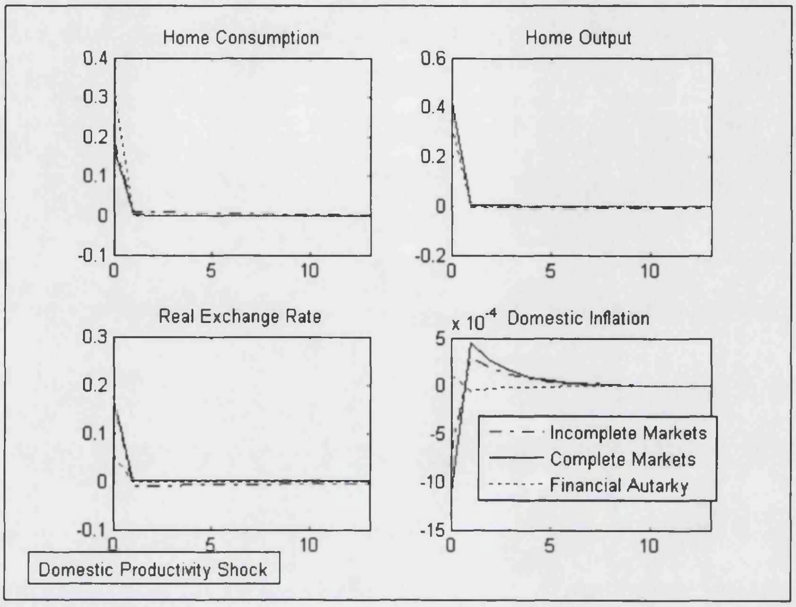


Figure 2.1: Optimal Monetary Policy following a Domestic Productivity Shock

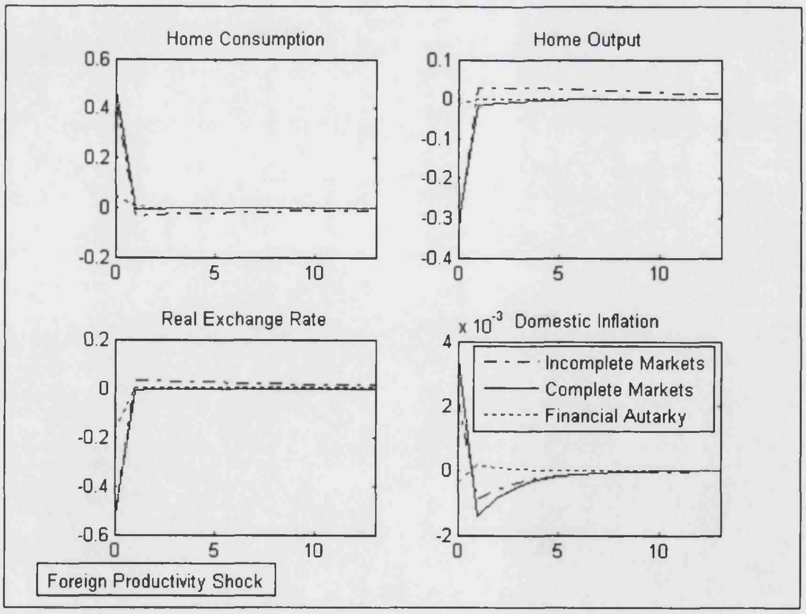


Figure 2.2: Optimal Monetary Policy following a Foreign Shock

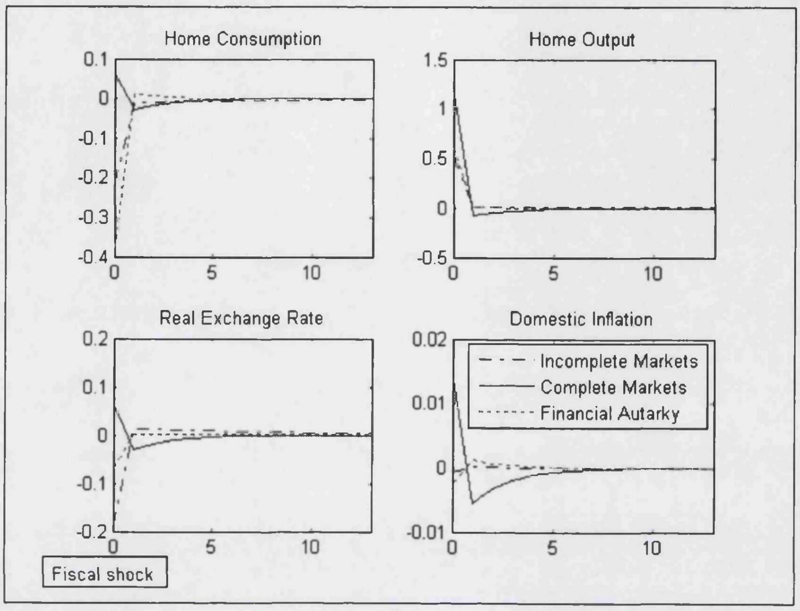
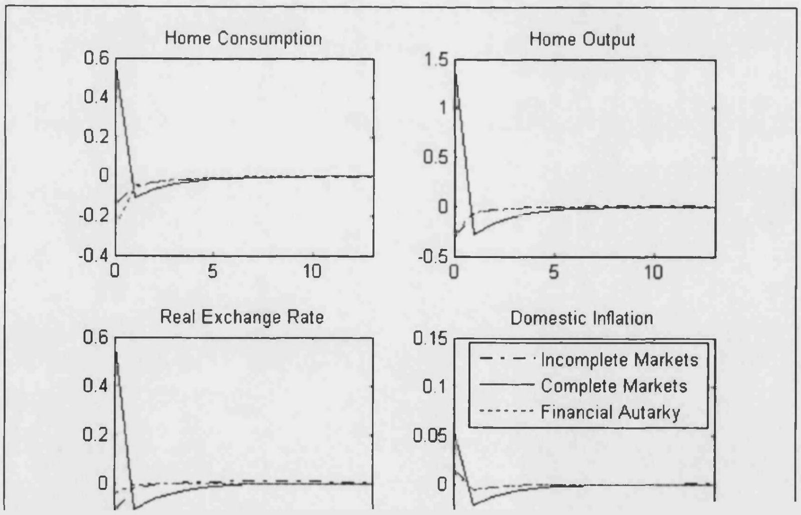


Figure 2.3: Optimal Monetary Policy following a Fiscal Shock



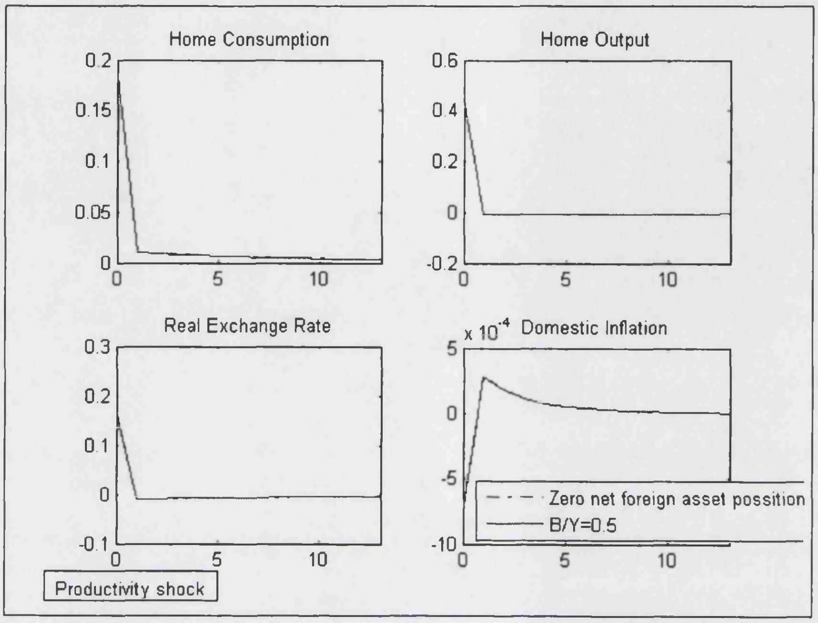


Figure 2.5: Productivity Shock (Non-Zero Steady-state Net Foreign Asset Position)

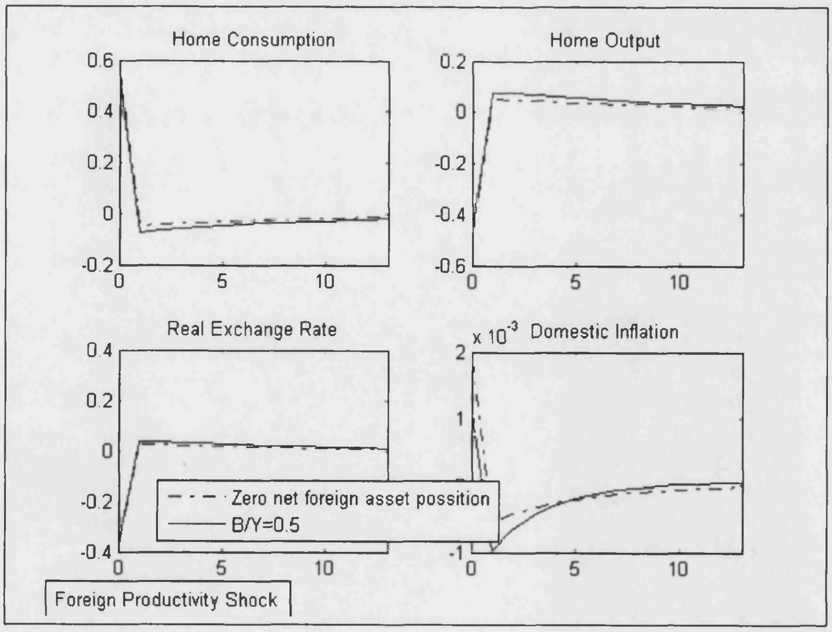


Figure 2.6: Foreign Shock (Non-zero Steady-state Net Foreign Asset Position)

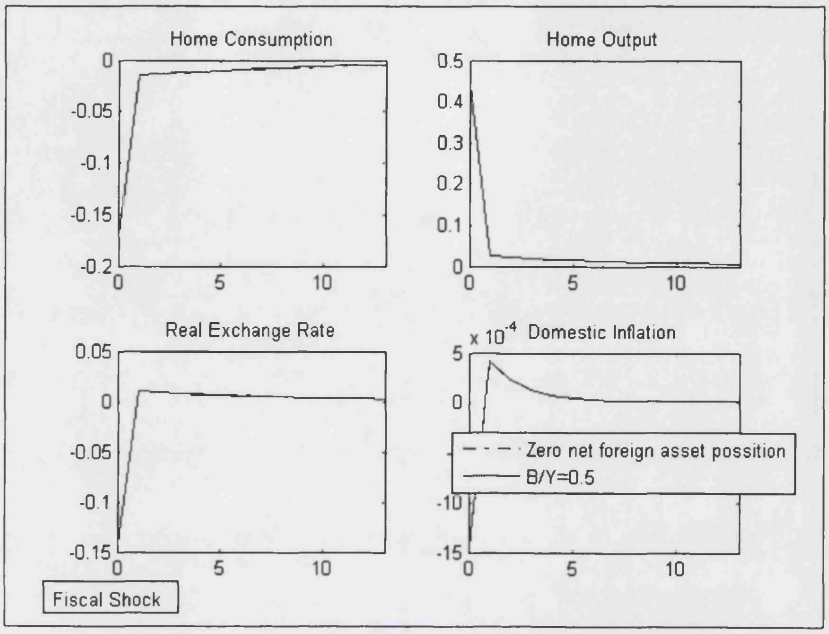


Figure 2.7: Fiscal Shock (Non-zeroSsteady-state Net Foreign Asset Position)

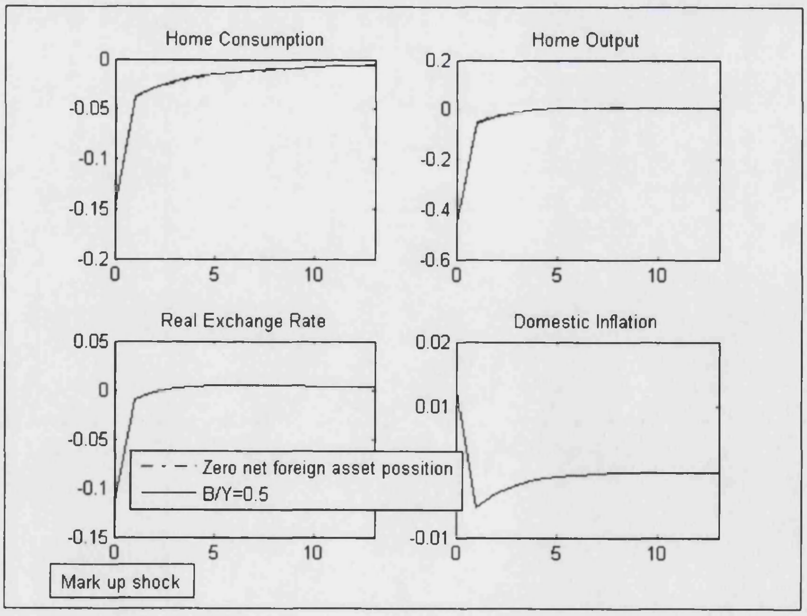


Figure 2.8: Mark-up Shock (Non-zeroSsteady-state Net Foreign Asset Position)

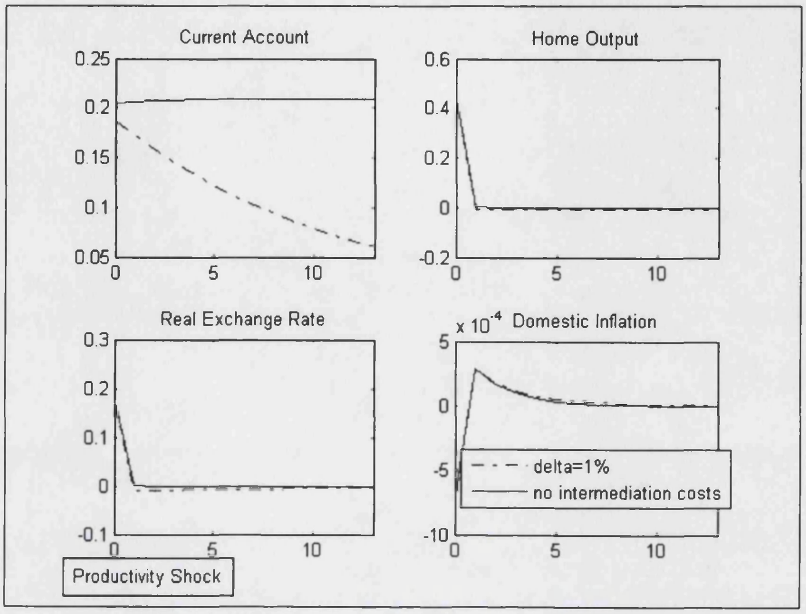


Figure 2.9: Productivity Shock (No Intermediation Costs)

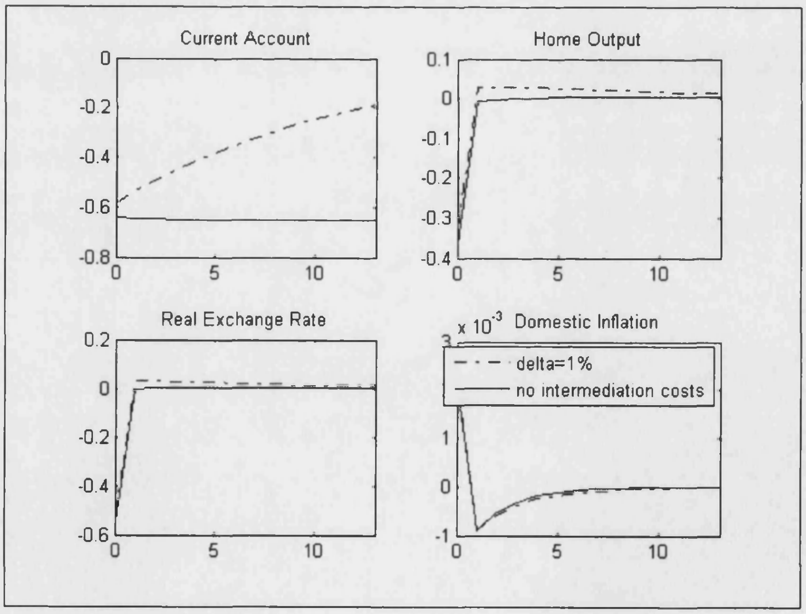


Figure 2.10: Foreign Shock (No Intermediation Costs)

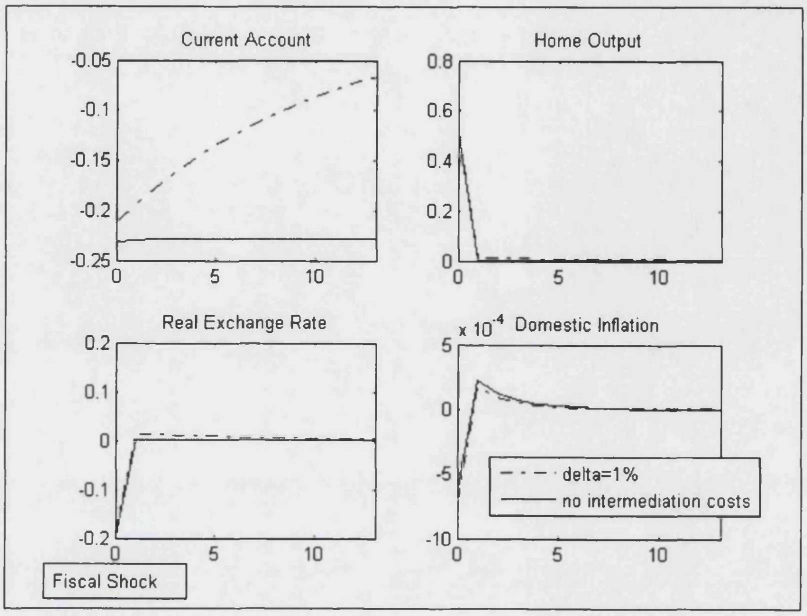


Figure 2.11: Fiscal Shock (No Intermediation Costs)

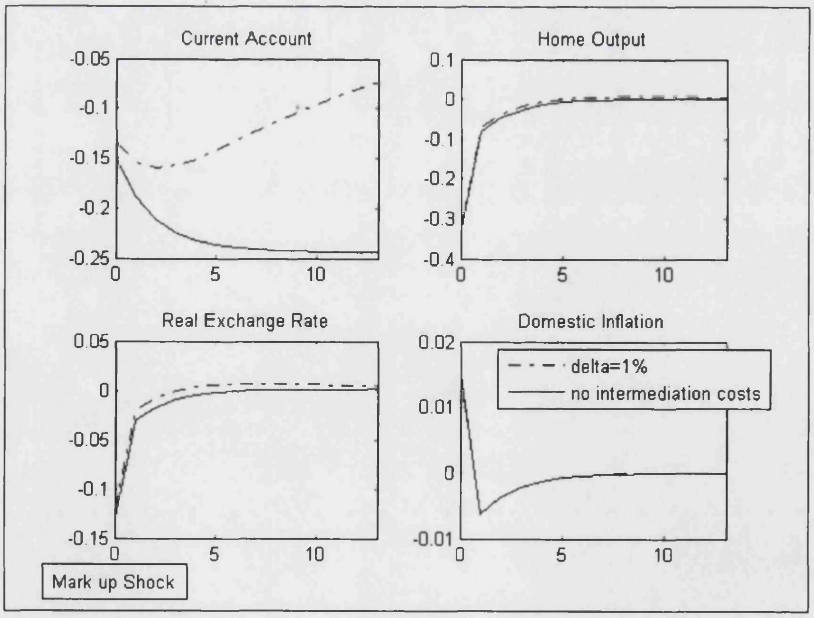


Figure 2.12: Mark up Shock (No Intermediation Costs)

these results sensitive to changes in the specification of parameters? In the next session we proceed to compute the ranking of standard policy rules for the different parameter values and types of disturbances. We also test the robustness of the policy prescription of domestic inflation targeting.

Evaluating Policy Rules

Our welfare characterization is a precise tool for measuring the performance of different policy rules. In this Section, we compare welfare under a producer price inflation target, a CPI (consumer price index) inflation target and an exchange rate peg. We also report the preferred policy for different parameter specifications.

Tables 2.6 to 2.13 indicate the preferred policy rule under alternative values of ρ, θ and λ. We start by varying θ and ρ, and maintain $\lambda = 0.4$. Alternatively, we keep the log utility specification and analyse different scenarios for θ and λ.

As shown in Tables 2.6 to 2.11, in the case of imperfect risk sharing (financial autarky or incomplete markets), producer price inflation targeting (denoted by IT in the Tables below) is the best policy available regardless of the parameter values. This is true for all types of disturbances excluding mark-up shocks. On the other hand, when asset markets are complete, large values of λ, θ and ρ modify the performance of the different policy rules. Economies that are more open or more sensitive to exchange rate movements (i.e. with large values of λ and θ) may benefit from fixing the exchange rate or targeting a weighted average of exchange rate and domestic inflation (i.e. CPI targeting, denoted by CPI).

If the economy experiences mark-up fluctuations, CPI targeting may outperform a policy of domestic price stability, even in the case of imperfect risk sharing (see Tables 2.12 and 2.13). Nevertheless, a fixed exchange rate regime (denoted by PEG) is only the best policy for extreme values of θ under financial autarky. Similarly, in the case of incomplete markets, an exchange rate peg is only the preferred policy when $\theta \geq 5$ and $\lambda \leq 1/5$.

Productivity shock						
Incomplete Markets						
λ \ θ	1	2	3	4	5	6
1/2	IT	IT	IT	IT	IT	IT
1/3	IT	IT	IT	IT	IT	IT
1/4	IT	IT	IT	IT	IT	IT
1/5	IT	IT	IT	IT	IT	IT
Financial Autarky						
λ \ θ	1	2	3	4	5	6
1/2	IT	IT	IT	IT	IT	IT
1/3	IT	IT	IT	IT	IT	IT
1/4	IT	IT	IT	IT	IT	IT
1/5	IT	IT	IT	IT	IT	IT
Complete Markets						
λ \ θ	1	2	3	4	5	6
1/2	IT	IT	IT	PEG	PEG	PEG
1/3	IT	IT	IT	PEG	PEG	PEG
1/4	IT	IT	IT	CPI	PEG	PEG
1/5	IT	IT	IT	IT	PEG	PEG

Table 2.6: Preferred Policy Rule following a Productivity Shock: varying the Elasticity of Intratemporal Substitution and the Degree of Openness

Productivity shock						
Incomplete Markets						
ρ \ θ	1	2	3	4	5	6
1	IT	IT	IT	IT	IT	IT
2	IT	IT	IT	IT	IT	IT
3	IT	IT	IT	IT	IT	IT
4	IT	IT	IT	IT	IT	IT
5	IT	IT	IT	IT	IT	IT
6	IT	IT	IT	IT	IT	IT
Financial Autarky						
ρ \ θ	1	2	3	4	5	6
1	IT	IT	IT	IT	IT	IT
2	IT	IT	IT	IT	IT	IT
3	IT	IT	IT	IT	IT	IT
4	IT	IT	IT	IT	IT	IT
5	IT	IT	IT	IT	IT	IT
6	IT	IT	IT	IT	IT	IT
Complete Markets						
ρ \ θ	1	2	3	4	5	6
1	IT	IT	IT	PEG	PEG	PEG
2	IT	IT	PEG	PEG	PEG	PEG
3	IT	IT	PEG	PEG	PEG	PEG
4	IT	PEG	PEG	PEG	PEG	PEG
5	IT	PEG	PEG	PEG	PEG	PEG
6	IT	PEG	PEG	PEG	PEG	PEG

Table 2.7: Preferred Policy Rule following a Productivity Shock: varying the Elasticity of Intertemporal and Intratemporal Substitution

Foreign shock
Incomplete Markets

λ \ θ	1	2	3	4	5	6
1/2	IT	IT	IT	IT	IT	IT
1/3	IT	IT	IT	IT	IT	IT
1/4	IT	IT	IT	IT	IT	IT
1/5	IT	IT	IT	IT	IT	IT

Financial Autarky

λ \ θ	1	2	3	4	5	6
1/2	IT	IT	IT	IT	IT	IT
1/3	IT	IT	IT	IT	IT	IT
1/4	IT	IT	IT	IT	IT	IT
1/5	IT	IT	IT	IT	IT	IT

Complete Markets

λ \ θ	1	2	3	4	5	6
1/2	IT	IT	IT	PEG	PEG	PEG
1/3	IT	IT	IT	PEG	PEG	PEG
1/4	IT	IT	IT	CPI	PEG	PEG
1/5	IT	IT	IT	IT	PEG	PEG

Table 2.8: Preferred Policy Rule following a Foreign Shock: varying the Elasticity of Intratemporal Substitution and the Degree of Openness

Foreign Shock
Incomplete Markets

ρ \ θ	1	2	3	4	5	6
1	IT	IT	IT	IT	IT	IT
2	IT	IT	IT	IT	IT	IT
3	IT	IT	IT	IT	IT	IT
4	IT	IT	IT	IT	IT	IT
5	IT	IT	IT	IT	IT	IT
6	IT	IT	IT	IT	IT	IT

Financial Autarky

ρ \ θ	1	2	3	4	5	6
1	IT	IT	IT	IT	IT	IT
2	IT	IT	IT	IT	IT	IT
3	IT	IT	IT	IT	IT	IT
4	IT	IT	IT	IT	IT	IT
5	IT	IT	IT	IT	IT	IT
6	IT	IT	IT	IT	IT	IT

Complete Markets

ρ \ θ	1	2	3	4	5	6
1	IT	IT	IT	PEG	PEG	PEG
2	IT	IT	PEG	PEG	PEG	PEG
3	IT	IT	PEG	PEG	PEG	PEG
4	IT	PEG	PEG	PEG	PEG	PEG
5	IT	PEG	PEG	PEG	PEG	PEG
6	IT	PEG	PEG	PEG	PEG	PEG

Table 2.9: Preferred Policy Rule following a Foreign Shock: varying the Elasticity of Intertemporal and Intratemporal Substitution

Fiscal Shock

Incomplete Markets

$\lambda \backslash \theta$	1	2	3	4	5	6
1/2	IT	IT	IT	IT	IT	IT
1/3	IT	IT	IT	IT	IT	IT
1/4	IT	IT	IT	IT	IT	IT
1/5	IT	IT	IT	IT	IT	IT

Financial Autarky

$\lambda \backslash \theta$	1	2	3	4	5	6
1/2	IT	IT	IT	IT	IT	IT
1/3	IT	IT	IT	IT	IT	IT
1/4	IT	IT	IT	IT	IT	IT
1/5	IT	IT	IT	IT	IT	IT

Complete Markets

$\lambda \backslash \theta$	1	2	3	4	5	6
1/2	IT	CPI	PEG	PEG	PEG	PEG
1/3	IT	CPI	PEG	PEG	PEG	PEG
1/4	IT	IT	CPI	PEG	PEG	PEG
1/5	IT	IT	CPI	PEG	PEG	PEG

Table 2.10: Preferred Policy Rule following a Fiscal Shock: varying the Elasticity of Intratemporal Substitution and the Degree of Openness

Fiscal Shock

Incomplete Markets

ρ \ θ	1	2	3	4	5	6
1	IT	IT	IT	IT	IT	IT
2	IT	IT	IT	IT	IT	IT
3	IT	IT	IT	IT	IT	IT
4	IT	IT	IT	IT	IT	IT
5	IT	IT	IT	IT	IT	IT
6	IT	IT	IT	IT	IT	IT

Financial Autarky

ρ \ θ	1	2	3	4	5	6
1	IT	IT	IT	IT	IT	IT
2	IT	IT	IT	IT	IT	IT
3	IT	IT	IT	IT	IT	IT
4	IT	IT	IT	IT	IT	IT
5	IT	IT	IT	IT	IT	IT
6	IT	IT	IT	IT	IT	IT

Complete Markets

ρ \ θ	1	2	3	4	5	6
1	IT	CPI	PEG	PEG	PEG	PEG
2	IT	PEG	PEG	PEG	PEG	PEG
3	CPI	PEG	PEG	PEG	PEG	PEG
4	CPI	PEG	PEG	PEG	PEG	PEG
5	CPI	PEG	PEG	PEG	PEG	PEG
6	CPI	PEG	PEG	PEG	PEG	PEG

Table 2.11: Preferred Policy Rule following a Fiscal Shock: varying the Elasticity of Intertemporal and Intratemporal Substitution

Mark up shock						
Incomplete Markets						
$\lambda \backslash \theta$	1	2	3	4	5	6
1/2	IT	IT	IT	CPI	CPI	CPI
1/3	IT	CPI	CPI	CPI	CPI	CPI
1/4	CPI	CPI	CPI	CPI	CPI	PEG
1/5	CPI	CPI	CPI	CPI	PEG	PEG
Financial Autarky						
$\lambda \backslash \theta$	1	2	3	4	5	6
1/2	IT	IT	IT	CPI	CPI	CPI
1/3	IT	CPI	CPI	CPI	CPI	CPI
1/4	CPI	CPI	CPI	CPI	CPI	PEG
1/5	CPI	CPI	CPI	CPI	PEG	PEG
Complete Markets						
$\lambda \backslash \theta$	1	2	3	4	5	6
1/2	IT	CPI	PEG	PEG	PEG	PEG
1/3	IT	CPI	PEG	PEG	PEG	PEG
1/4	CPI	CPI	PEG	PEG	PEG	PEG
1/5	CPI	CPI	CPI	PEG	PEG	PEG

Table 2.12: Preferred Policy Rule following a Mark-up Shock: varying the Elasticity of Intratemporal Substitution and the Degree of Openness

Mark up shock

Incomplete Markets

$\rho \setminus \theta$	1	2	3	4	5	6
1	IT	CPI	CPI	CPI	CPI	CPI
2	IT	CPI	CPI	CPI	CPI	CPI
3	IT	CPI	CPI	CPI	CPI	CPI
4	IT	CPI	CPI	CPI	CPI	CPI
5	IT	CPI	CPI	CPI	CPI	CPI
6	IT	CPI	CPI	CPI	CPI	CPI

Financial Autarky

$\rho \setminus \theta$	1	2	3	4	5	6
1	IT	CPI	CPI	CPI	CPI	CPI
2	IT	CPI	CPI	CPI	CPI	CPI
3	IT	CPI	CPI	CPI	CPI	CPI
4	IT	CPI	CPI	CPI	CPI	PEG
5	IT	CPI	CPI	CPI	CPI	PEG
6	IT	CPI	CPI	CPI	CPI	PEG

Complete Markets

$\rho \setminus \theta$	1	2	3	4	5	6
1	IT	CPI	PEG	PEG	PEG	PEG
2	CPI	PEG	PEG	PEG	PEG	PEG
3	CPI	PEG	PEG	PEG	PEG	PEG
4	CPI	PEG	PEG	PEG	PEG	PEG
5	CPI	PEG	PEG	PEG	PEG	PEG
6	CPI	PEG	PEG	PEG	PEG	PEG

Table 2.13: Preferred Policy Rule following a Mark-up Shock: varying the Elasticity of Intertemporal and Intratemporal Substitution

Tables 2.14 to 2.17 display the preferred policy rule for the cases in which $\theta < 1$ (maintaining $\rho = 1$). Under this specification, domestic and foreign goods are complements. The Tables show that a domestic inflation target is the preferred policy if asset markets are complete. However, when there is imperfect risk sharing, inflation targeting is no longer optimal when $\theta < 0.8$. These results hold for all types of disturbances.

Productivity Shock						
θ	0.5	0.6	0.7	0.8	0.9	1
Incomplete Markets	PEG	PEG	IT	IT	IT	IT
Complete Markets	IT	IT	IT	IT	IT	IT
Financial Autarky	PEG	PEG	IT	IT	IT	IT

Table 2.14: Preferred Policy Rule following a Productivity Shock: when Domestic and Foreign Goods are Complements

Foreign Shock						
θ	0.5	0.6	0.7	0.8	0.9	1
Incomplete Markets	PEG	PEG	IT	IT	IT	IT
Complete Markets	IT	IT	IT	IT	IT	IT
Financial Autarky	PEG	PEG	IT	IT	IT	IT

Table 2.15: Preferred Policy Rule following a Foreign Shock: when Domestic and Foreign Goods are Complements

Fiscal Shock						
θ	0.5	0.6	0.7	0.8	0.9	1
Incomplete Markets	PEG	PEG	CPI	IT	IT	IT
Complete Markets	IT	IT	IT	IT	IT	IT
Financial Autarky	PEG	PEG	CPI	IT	IT	IT

Table 2.16: Preferred Policy Rule following a Fiscal Shock: when Domestic and Foreign Goods are Complements

Mark up Shock						
θ	0.5	0.6	0.7	0.8	0.9	1
Incomplete Markets	PEG	PEG	CPI	IT	IT	IT
Complete Markets	IT	IT	IT	IT	IT	IT
Financial Autarky	PEG	PEG	CPI	IT	IT	IT

Table 2.17: Preferred Policy Rule following a Mark-up Shock: when Domestic and Foreign Goods are Complements

Finally, Table 2.18 ranks the policy rules for values of $\delta = 0.00001$ to $\delta = 0.01$ and $\frac{\overline{B}}{Y}$ ranging from 0 to -0.5, for the case in which asset markets are incomplete. The results are unchanged: an inflation target is the preferred policy rule following productivity, fiscal and foreign shocks, regardless of the values of δ and $\overline{B}$. Moreover, as in our baseline specification (see Tables 2.12 and 2.13), CPI targeting is the preferred policy in the case of mark-up shocks.

Productivity Shock			
(B/Y) \ δ	0.001%	0.5%	1%
0	IT	IT	IT
0.3	IT	IT	IT
0.5	IT	IT	IT
Fiscal Shock			
(B/Y) \ δ	0.001%	0.5%	1%
0	IT	IT	IT
0.3	IT	IT	IT
0.5	IT	IT	IT
Foreign Shock			
(B/Y) \ δ	0.001%	0.5%	1%
0	IT	IT	IT
0.3	IT	IT	IT
0.5	IT	IT	IT
Marck Up Schock			
(B/Y) \ δ	0.001%	0.5%	1%
0	CPI	CPI	CPI
0.3	CPI	CPI	CPI
0.5	CPI	CPI	CPI

Table 2.18: Sensitivity Analysis: varying the Steady-state Debt to GDP Ratio and Risk Premium

The quantitative results can be summarized as follows. In a small open economy characterized by incomplete asset markets, the optimal monetary policy is not quantitatively different from domestic inflation targeting under our benchmark specification. This result holds for the case of fiscal, productivity and foreign shocks. Moreover, following these shocks, a domestic inflation targeting regime outperforms an exchange rate peg or CPI targeting when home and foreign goods are substitutes in utility. On the other hand, under perfect risk sharing, for a large set of parameter values, an exchange rate peg is preferred to a domestic inflation targeting regime, regardless of the source of the shock.

Conversely, if the degree of substitutability between Home and Foreign goods is significantly low (more specifically, $\theta \leq 0.7$), this result is reversed. Under incomplete markets, it can be optimal to target the exchange rate, but domestic inflation targeting is the preferred policy under perfect risk sharing. Moreover, the presence of intermediation costs or a non-zero steady-state net foreign asset position does not seem to affect the conclusions.

These findings are consistent with the results shown in Section 2.4.1. As demonstrated in this Section, when home and foreign goods are substitutes in utility, the coefficient of inflation variability in the loss function is smaller under perfect risk sharing than it is under incomplete markets. As a result, under complete markets, a more flexible policy towards inflation that permits policymakers to manage the exchange rate optimally, can be welfare improving. In particular, allowing greater inflation variability increases welfare by increasing the term $\widehat{C}_t - \frac{1}{\overline{\mu}(1+a)}\widehat{Y}_t$ in the welfare Equation (2.13). On the other hand, with imperfect risk sharing this term decreases $\left(\pi_t^H\right)^2$ and, therefore, a more flexible form of inflation targeting reduces welfare. The exact opposite conclusion holds for the case in which domestic and foreign goods are complements.

If we consider, for example, the case of a positive productivity shock, the above results can be explained as follows. When asset **markets are complete** and goods are substitutes, restricting the exchange rate depreciation caused by the productivity improvement might be beneficial for the small open economy. A smaller depreciation diverts some output production to the foreign economy and therefore reduces the disutility of producing at home. At the same time, the complete market specification ensures that consumption at home does not suffer significantly with the policy of diverting production. A policy that constrains the exchange rate depreciation (e.g. a fixed exchange rate) can therefore outperform an inflation targeting regime. When goods are complements, however, it is no longer possible to shift consumption towards foreign goods by inducing a greater appreciation in the exchange rate. In this case, domestic inflation targeting is the preferred policy.

In the case of **incomplete markets**, consumption is more responsive to output movements. In the extreme case of financial autarky, for example, consumption has to be fully financed by domestic production. Consequently, a policy that tries to reduce the disutility of production will inevitably reduce consumption utility. When the elasticity of substitution between the goods is high, restricting the exchange rate movements has a strong impact on output and consequently on consumption. Therefore, it does not lead to welfare gains. In this case, the authorities should focus on stabilizing inflation and on minimizing the distortions that price dispersion brings. On the other hand, lowering the degree of substitutability between the goods reduces output sensitivity to exchange rate movements. Hence, the income effect on consumption of restricting the depreciation is smaller. In addition, a relatively appreciated exchange rate can improve the small open economy's purchasing power under market incompleteness (see equations (FA) and (IM)). When θ is sufficiently low, the income effect in consumption is small and therefore its negative impact on welfare is smaller than the positive welfare effect from an improvement in purchasing power. Hence, in this case, an exchange rate peg outperforms a domestic inflation target.

We should note that when there are mark-up shocks, the optimal plan differs from an inflation targeting regime, regardless of the asset market structure and the degree of substitutability between domestic and foreign goods. These shocks always disturb the pricing decision of firms and create an incentive to depart from price stabilization.

2.6 Concluding Remarks

In this Chapter, we have formalized the dynamics of the small open economy under different degrees of international risk sharing. We have compared the previously documented complete market characterization with two forms of market incompleteness: the extreme case of financial autarky; and the case of an intermediate level of risk sharing, in which the country is allowed to trade riskless bonds internationally subject to intermediation fees. We have shown that the different dynamics implied by alternative asset market structures have significant implications for monetary policy.

When a country can perfectly share risk with the rest of the world, and home and foreign goods are substitutes, optimally managing the exchange rate may improve its welfare. Following a productivity shock, for example, we have shown that a country may gain from restricting the exchange rate movement in order to divert some production towards the rest of the world. At the same time, perfect risk sharing ensures that consumption levels are maintained. This policy therefore reduces the disutility of producing domestically without decreasing agents' utility derived from consumption. As a consequence, the monetary authority has an incentive to manage the exchange rate besides the objective of price stability. Conversely, if goods are complements, the ability to shift consumption towards foreign goods is restricted and inflation targeting is the preferred policy rule.

Under imperfect risk sharing, however, the results are entirely reversed: when goods are substitutes, we have shown that inflation targeting is a robust policy prescription; but if the degree of substitutability is considerably low, an exchange rate peg can be the preferred policy. Under market incompleteness (including the extreme case of financial autarky), when $\theta > 0.7$, the monetary authority is not able to improve welfare by switching production towards the rest of the world. This is because the latter has a direct effect on consumption and consumption utility. However, this effect is reduced when the elasticity of substitution between goods is significantly small. In this case, it might be beneficial to manage the exchange rate in order to improve the country's purchasing power ~~parity~~.

The exception to the above analysis is the case of mark-up shocks. These fluctuations affect monopolistic competition distortions and create a direct trade-off between stabilizing inflation and smoothing such movements. Therefore, following this type of shock, optimal policy might deviate from price stability regardless of the degree of substitutability between goods or the level risk sharing in the economy.

Moreover, optimal monetary policy is independent of the financial market structure only when the latter is entirely irrelevant for the economy's dynamics. This is the case when trade imbalances are ruled out and the steady-state level of net foreign assets is zero. Under this specification, and provided there are no mark-up shocks or steady-state inefficiencies in output, price stability coincides with the optimal plan, regardless of the degree of risk sharing.

This work has shown that the level of sophistication of the financial market has clear implications for monetary policy. The conclusions also vary according to the degree of substitutability between domestic and imported goods. An interesting exercise would be to test these findings empirically by studying how monetary policy has behaved in countries with different import profiles and different asset market structures. For example, a prediction of the model is that, in countries where imported goods are complements to domestic production, the lower is the degree of risk sharing, the larger is the gain from targeting the exchange rate. The opposite should hold in countries in which imports and domestic products are close substitutes.

2.A Appendix: The Steady State

In this Appendix, we derive the steady-state conditions. In contrast with Chapter 1. we allow for an *asymmetric* steady state in the analysis of the incomplete market case. All variables in steady state are denoted with a bar. We assume that in steady state $1 + i_t = 1 + i_t^* = 1/\beta$ and $P_t^H / P_{t-1}^H = P_t^F / P_{t-1}^F = 1$. We normalize the price indexed such that $\overline{P}_H = \overline{P}_F$.

The steady-state versions of the demand equation at Home and in the rest of the world are

$$\overline{Y} = (1-\lambda)\overline{C} + \lambda\overline{C^*} + \overline{G} \tag{2.26}$$

and

$$\overline{Y}^* = \overline{C}^* + \overline{G}^*. \tag{2.27}$$

From the household and government budget constraints we have

$$(1-\beta)\overline{B} = \overline{C} - \overline{Y}(1-\overline{\tau}) + \overline{Tr} \tag{2.28}$$

and

$$\overline{G} = \overline{\tau}\overline{Y} - \overline{Tr} \tag{2.29}$$

We can therefore write the following relationship between the steady-state foreign asset position and the consumption differential:

$$(1-\beta)\overline{B} = \lambda(\overline{C} - \overline{C}^*). \tag{2.30}$$

Finally, applying our normalization to the price setting equations we have

$$U_C(\overline{C}) = \mu V_y\left(\lambda\overline{C^*} + (1-\lambda)\overline{C} + \overline{G}\right) \tag{2.31}$$

and

$$U_C(\overline{C}^*) = \mu^* V_y\left(\overline{C}^* + \overline{G}^*\right), \tag{2.32}$$

where

$$\mu = \frac{\sigma}{(1-\overline{\tau})(\sigma-1)};\ \mu > 1$$

Equations (2.30), (2.31) and (2.32) determine the relationship between $\overline{B}$, $\overline{G}$, $\overline{G}^*$ and $\overline{\mu}^*$. In particular, when $\overline{G} = \overline{G}^* = \overline{B} = 0$, $\mu^* = \mu$.

2.B Appendix: Approximating the Model

In this Appendix, we derive first and second order approximations to the equilibrium conditions of the model. Moreover, we show the second order approximation to the utility function in order to conduct our welfare analysis. We assume $\overline{G} = 0$ and use isoelastic functional forms for the utility functions, as specified in Chapter 1 by (1.58) and (1.59). The first and second order approximations to the optimal price setting condition and the price index are identical to the ones presented in Section (1.B.4) and (1.B.3) in Chapter 1. The approximation to the other equilibrium conditions described in the text are shown below.

2.B.1 Demand

The first order approximation to the small open economy demand is

$$\hat{Y}_H = -\theta\widehat{p}_H + d_b\hat{C} + (1-d_b)\hat{C}^* + \theta(1-d_b)\widehat{RS} + \widehat{g}, \tag{2.33}$$

where $d_b = (1-\lambda)(1+a)$ and $a = \frac{\lambda(\overline{C}-\overline{C}^*)}{\overline{Y}}$. Moreover, Home relative prices are denoted by $p_H = P_H/P$ and the fiscal shock $\hat{g}_t$ is defined as $\frac{Gt-\overline{G}}{\overline{Y}}$, allowing for the analysis of this shock even when steady-state government consumption is non-zero. In the symmetric steady state, in which $d_b = 1-\lambda$, Equation (2.33) becomes

$$\hat{Y}_H = -\theta\widehat{p}_H + (1-\lambda)\hat{C} + \lambda\hat{C}^* + \theta\lambda\widehat{RS} + \widehat{g}. \tag{2.34}$$

The second order approximation to the demand function is

$$\sum \beta^t \left[d'_y y_t + \frac{1}{2} y'_t D_y y_t + y'_t D_e e_t \right] + t.i.p + \mathcal{O}(||\xi||^3) = 0, \tag{2.35}$$

where

$$y_t = \left[\begin{array}{cccc} \widehat{Y}_t & \widehat{C}_t & \widehat{p}_{Ht} & \widehat{RS}_t \end{array} \right],$$

$$e_t = \left[\begin{array}{cccc} \widehat{\varepsilon}_{yt} & \widehat{\mu}_t & \widehat{g}_t & \widehat{C}^*_t \end{array} \right],$$

$$d'_y = \left[\begin{array}{cccc} -1 & d_b & -\theta & \theta(1-d_b) \end{array} \right],$$

$$D'_y = \left[\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & (1-d_b)d_b & 0 & -\theta(1-d_b-d_g)d_b \\ 0 & 0 & 0 & 0 \\ 0 & -\theta(1-d_b)d_b & 0 & \theta^2(1-d_b)d_b \end{array} \right],$$

and

$$D'_e = \left[\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -d_b & -(1-d_b)d_b \\ 0 & 0 & 0 & \theta & 0 \\ 0 & 0 & 0 & -\theta(1-d_b) & \theta(1-d_b)d_b \end{array} \right].$$

2.B.2 Incomplete Markets: Approximating the Current Account equation

We assume that home currency denominated bonds are in zero net supply. The net foreign asset position is therefore fully denominated in foreign currency. Aggregating private and public budget constraints, the law of motion for $B_{F,t}$ can be written as

$$P_tC_t + \frac{S_tB_{F,t}}{(1+i_t^*)\,\psi\left(\frac{S_tB_{F,t}}{P_t}\right)} = S_tB_{F,t-1} + P_{H,t}(Y_t - G_t) \tag{2.36}$$

Defining $\frac{B_{F,t}S_t}{P_t} \equiv b_{F,t}$ we can rewrite the government budget constraint as

$$b_{F,t} = b_{F,t-1}\,(1+i_t^*)\,\psi\left(\frac{S_tB_{F,t}}{P_t}\right) + \frac{P_{H,t}}{P_t}(Y_t - G_t)\,(1+i_t^*)\,\psi\left(\frac{S_tB_{F,t}}{P_t}\right) - C_t\,(1+i_t^*)\,\psi\left(\frac{S_tB_{F,t}}{P_t}\right). \tag{2.37}$$

From agents' intertemporal choice,

$$U_C\,(C_t) = (1+i_t^*)\psi\left(\frac{S_tB_{F,t}^i}{P_t}\right)\beta E_t\left[U_C\,(C_{t+1})\,\frac{S_{t+1}P_t}{S_tP_{t+1}}\right]. \tag{2.38}$$

We can therefore write (2.37) as

$$\beta E_t\left[U_C\,(C_{t+1})\,b_{F,t}\frac{S_{t+1}P_t}{S_tP_{t+1}}\right] = b_{F,t-1}\frac{S_tP_{t-1}}{S_{t-1}P_t}U_C\,(C_t) + \frac{P_{H,t}}{P_t}Y_tU_C\,(C_t) - C_tU_C\,(C_t). \tag{2.39}$$

And the log linear representation of the above equation, defining $a_\beta = \frac{a}{1-\beta}$, is

$$\begin{aligned} &-\rho a_\beta\widehat{C}_t + \widehat{b}_{F,t-1} + a_\beta(\Delta RS_t - \pi_t^*) \qquad (2.40)\\ &= -\widehat{Y}_t + (1+a)\widehat{C}_t - \rho a\widehat{C}_t - \widehat{p}_{H,t} + \widehat{g}_t \\ &+ \beta E_t\left[-\rho a_\beta\widehat{C}_{t+1} + \widehat{b}_{F,t} + a_\beta(\Delta RS_{t+1} - \pi_{t+1}^*)\right]. \end{aligned}$$

Furthermore, if we allow $B_{W,t} = b_{t-1}\frac{P_{t-1}}{P_t}\frac{S_t}{S_{t-1}}U_C\,(C_t)$ and $s_t = -\frac{P_{H,t}}{P_t}(Y_t - G_t) + C_t$, the intertemporal government solvency condition (2.39) can be written as

$$\widehat{B}_{W,t} = U_C\,(C_t)\,s_t + E_t\beta\widehat{B}_{W,t+1} = E_t\sum_{T=t}^{\infty}\beta^{T-t}U_C\,(C_T)\,s_t, \tag{2.41}$$

and the term $U_C\left(C_T, \xi_{C,T}\right)s_t$ can be approximated, up to the second order, by

$$U_C\overline{Y}\left\{\begin{array}{c} a-\widehat{Y}_t+(1+a(1-\rho))\widehat{C}_t-\widehat{p}_{H,t}-\frac{1}{2}\widehat{Y}_t^2+\rho\widehat{Y}_t\widehat{C}_t-\widehat{Y}_t\widehat{p}_{H,t} \\ +\frac{1}{2}(a\rho^2+(1+a)(1-2\rho))\widehat{C}_t^2+\rho\widehat{C}_t\widehat{p}_{H,t}-\frac{1}{2}\widehat{p}_{H,t}^2 \end{array}\right\}.$$

Thus, defining $\widehat{B}_{W,t}=\frac{B_{W,t}-\overline{B}_W}{\overline{B}'_W}$ and $\overline{B}'_W=\frac{U_c\overline{Y}}{(1-\beta)}$, we have,

$$\begin{aligned}\widehat{B}_{W,t} &= (1-\beta)\left[b_y^{i\prime}y_t+\frac{1}{2}y_t'B_y^i y_t+y_t'B_e^i e_t\right]+\beta E_t\widehat{B}_{W,t+1} \\ &\quad +t.i.p+\mathcal{O}(||\xi||^3) \end{aligned} \tag{2.42}$$

where

$$b_y^{i\prime}=\left[\begin{array}{cccc} -1 & 1+a(1-\rho) & -1 & 0 \end{array}\right],$$

$$B_y^{i\prime}=\left[\begin{array}{cccc} -1 & \rho & -1 & 0 \\ \rho & a(1-\rho)^2+(1-2\rho) & \rho & 0 \\ -1 & \rho & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right],$$

and

$$B_e^{i\prime}=\left[\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & -\rho & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

Special Case: Note that if $\rho=\theta=1$, $a=0$ and $\widehat{b}_{-1}=0$, the second order current account approximation becomes

$$\widehat{C}_t=\widehat{Y}_t+\widehat{p}_{H,t}-\widehat{g}_t-\widehat{g}_t\widehat{p}_{H,t}+\widehat{g_t C_t}, \tag{2.43}$$

which combining with the demand equation implies

$$\widehat{C}_t = \widehat{RS}_t + \widehat{C}_t^*. \tag{2.44}$$

This is identical to the perfect risk sharing condition.

2.B.3 Financial Autarky: the Extreme Case of Market Incompleteness

In this case we assume that there is no risk sharing between countries. The inability to trade bonds across borders impose that the value of imports equates the value of exports:

$$(1-n)SP_{H,t}^*C_{H,t}^* = nP_{F,t}C_{F,t}, \tag{2.45}$$

given the preference specification, we can write:

$$C_{H,t} = v\left[\frac{P_{H,t}}{P_t}\right]^{-\theta} C_t,\ C_{F,t} = (1-v)\left[\frac{P_{F,t}}{P_t}\right]^{-\theta} C_t, \tag{2.46}$$

$$C_{H,t}^* = v^*\left[\frac{P_{H,t}^*}{P_t^*}\right]^{-\theta} C_t^*, C_{F,t}^* = (1-v^*)\left[\frac{P_{F,t}^*}{P_t^*}\right]^{-\theta} C_t^*. \tag{2.47}$$

Substituting in Equation (2.45):

$$C_t = \left[\frac{p_{H,t}}{RS_t}\right]^{1-\theta} [RS_t]^{\theta}\, C_t^*. \tag{2.48}$$

And using the definition of the consumption indexes and market clearing, condition (2.48) implies

$$P_{H,t}(Y_t - G_t) = P_tC_t. \tag{2.49}$$

Assuming $\overline{C} = \overline{C}^*$, the second order approximation is

$$\widehat{p}_{H,t} + \widehat{Y}_t - \widehat{g}_t + \widehat{Y}_t\widehat{g}_t = \widehat{C}_t, \tag{2.50}$$

and can be represented as follows:

$$\sum E_t \beta^t \left[b_y^{fa\prime} y_t + \frac{1}{2} y_t' B_y^{fa} y_t + y_t' B_e^{fa} e_t \right] + t.i.p + \mathcal{O}(||\xi||^3) = 0,$$

$$b_y^{fa\prime} = \begin{bmatrix} -1 & 1 & -1 & 0 \end{bmatrix},$$

$$B_y^{fa\prime} = 0,$$

and

$$B_e^{fa\prime} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Special Case: when $\theta = 1$, Equation (2.50) combined with the demand equation becomes

$$\hat{C}_t^* = \hat{C}_t + \widehat{RS}_t. \tag{2.51}$$

2.B.4 Complete markets: the Risk Sharing Equation

Assuming a symmetric steady-state equilibrium, the log linear approximation to the risk sharing Equation (2.12) is

$$\hat{C}_t^* = \hat{C}_t + \frac{1}{\rho} \widehat{RS}_t. \tag{2.52}$$

Given our utility function specification, Equation (2.12) gives rise to an exact log linear expression and therefore the first and second order approximations are identical. In matrix notation, we have

$$\sum E_t \beta^t \left[b_y^{c\prime} y_t + \frac{1}{2} y_t' B_y^c y_t + y_t' B_e^c e_t \right] = 0,$$

where

$$b_y^{c\prime} = \begin{bmatrix} 0 & -1 & 0 & \frac{1}{\rho} \end{bmatrix},$$

$$b_y^{c\prime} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix},$$

$$B_y^{c\prime} = 0,$$

and

$$B_e^{c\prime} = 0.$$

2.B.5 Welfare with Incomplete Asset Markets:

Following Benigno and Benigno (2003), the second order approximation to the utility function, U_t, can be written as:

$$U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[U(C_s) - \frac{1}{n} \int_0^n V(y_s^j, \varepsilon_{Y,s}) dj \right], \tag{2.53}$$

$$W_{to} = U_c \bar{C} E_{t_0} \sum \beta^t \left[w_y' y_t - \frac{1}{2} y_t' W_y y_t - y_t' W_e e_t - \frac{1}{2} w_\pi \pi_t^2 \right] + t.i.p + \mathcal{O}(||\xi||^3), \tag{2.54}$$

where

$$w_\pi' = \frac{\sigma}{\mu k},$$

$$w_y' = \begin{bmatrix} -1/\mu(1+a) & 1 & 0 & 0 \end{bmatrix},$$

$$W_y' = \begin{bmatrix} \frac{(1+\eta)}{(1+a)\mu} & 0 & 0 & 0 \\ 0 & -(1-\rho) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

and

$$W_e' = \begin{bmatrix} -\frac{\eta}{\mu(1+a)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Using the second order approximation to the equilibrium condition, we can eliminate the term $w_y' y_t$. We derive the vector Lx, such that

$$\begin{bmatrix} a_y & d_y & f_y & b_y^i \end{bmatrix} Lx = w_y.$$

Thus, the loss function can be written as

$$L_{to} = U_c \bar{C} E_{t_0} \sum \beta^t \left[\frac{1}{2} y_t' L_y^i y_t + y_t' L_e^i e_t + \frac{1}{2} l_\pi^i \pi_t^2 \right] + t.i.p + \mathcal{O}(||\xi||^3), \qquad (2.55)$$

where

$$L_y^i = W_y + Lx_1^i A_y + Lx_2^i D_y + Lx_3^i F_y + Lx_4^i B_y^i,$$

$$L_e^i = W_e + Lx_1^i A_e + Lx_2^i D_e + Lx_4^i B_e,$$

and

$$l_\pi^i = w_\pi + Lx_1^i a_\pi.$$

Given the values of a_y, d_y, b_y^i, f_y, defined in this Appendix, we have:

$$Lx_1^i = \frac{(1+a)\lambda(l_1(2-\lambda) + (\phi - \lambda)) - a(l_1 + \phi)}{(1+a)\lambda(l_2(2-\lambda) - (\rho+\eta)(1-\lambda) - \lambda(\rho-1)) - al_2},$$

$$Lx_2^i = \frac{-\lambda((\rho+\eta) - \phi(\rho-1))}{(1+a)\lambda(l_2(2-\lambda) - (\rho+\eta)(1-\lambda) - \lambda(\rho-1)) - al_2},$$

$$Lx_3^i = \frac{-(\lambda + (1-\lambda)a)\theta((\rho+\eta) - (\rho-1)(\phi - (\phi-1)a))}{(1+a)\lambda(l_2(2-\lambda) - (\rho+\eta)(1-\lambda) - \lambda(\rho-1)) - al_2},$$

and

$$Lx_4^i = \frac{(1+a)\lambda(l_3(2-\lambda) - (\phi-\lambda)) - a(l_3-\phi)}{(1+a)\lambda(l_2(2-\lambda) - (\rho+\eta)(1-\lambda) - \lambda(\rho-1)) - al_2},$$

where $l_1 = \theta a(\rho - \phi(\rho-1)) + \phi(\theta-1)$,$l_2 = \theta((\rho+\eta) - \eta(\rho-1)a)$, $l_3 = \theta((\rho+\eta(1+a) - (\rho-1)\phi)$ and $(1-\phi) = \frac{1}{\mu}$.

We write the model just in terms of the output, real exchange rate and inflation, using the matrixes N and N_e, as follows:

$$y_t' = N\left[Y_t, T_t\right] + N_e e_t,$$

$$N = \begin{bmatrix} 1 & 0 \\ \frac{1}{d_b} & -\frac{d_{rs}}{d_b} \\ 0 & -\frac{\lambda}{(1-\lambda)} \\ 0 & 1 \end{bmatrix},$$

and

$$N_e = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{d_b} & -\frac{(1-d_b)}{d_b} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

where $d_{rs} = \frac{\theta(\lambda+(1-d_b)(1-\lambda))}{(1-\lambda)}$.

Equation (2.55) can therefore be expressed as

$$L_{to} = U_c\bar{C}E_{t_0}\sum\beta^t \begin{bmatrix} \frac{1}{2}\left[\widehat{Y}_t, \widehat{RS}_t\right]' L_y^{i'}\left[\widehat{Y}_t, \widehat{RS}_t\right] \\ +\left[\widehat{Y}_t, \widehat{RS}_t\right]' L_e^{i'} e_t + \frac{1}{2} l_\pi^i \pi_t^2 \end{bmatrix} + t.i.p + \mathcal{O}(||\xi||^3), \quad (2.56)$$

where:

$$L_y^{i\prime} = N' L_y^i N,$$

$$L_e^{i\prime} = N' L_y^i N_e + N' L_e,$$

$$\left[\widehat{Y}_t, \widehat{RS}_t\right]' L_y^{i\prime} \left[\widehat{Y}_t, \widehat{RS}_t\right] = \left[\widehat{Y}_t, \widehat{RS}_t\right]' \begin{bmatrix} l_{yy}^i & l_{yr}^i \\ l_{yr}^i & l_{rr}^i \end{bmatrix} \left[\widehat{Y}_t, \widehat{RS}_t\right]$$

$$\left[\widehat{Y}_t, \widehat{RS}_t\right]' L_e^{i\prime} e_t = \left[\widehat{Y}_t, \widehat{RS}_t\right]' \begin{bmatrix} l_{ye}^i \\ l_{re}^i \end{bmatrix} L_e' e_t, \tag{2.57}$$

$$l_{ye}^i = \begin{bmatrix} l_{y\varepsilon}^i & l_{y\mu}^i & l_{yg}^i & l_{yc^*}^i \end{bmatrix},$$

$$l_{re}^i = \begin{bmatrix} l_{r\varepsilon}^i & l_{r\mu}^i & l_{rg}^i & l_{rc^*}^i \end{bmatrix},$$

$$l_{yy}^i = \frac{(\eta+1)(1-\phi)}{(1+a)} + \frac{\rho-1}{d_b^2} + \left(\frac{\rho\left(\rho-2d_b\right)}{d_b^2} + \eta(2+\eta)\right) Lx_1 + \frac{(1-d_b)}{d_b^2} Lx_2 + \frac{\rho\left(a\rho + 2(a(1-\lambda)-\lambda) + (1+a)^{-1}\rho^{-1}\right)}{d_b^2} Lx_4,$$

$$l_{yr}^i = l_{ry}^i = \frac{\rho(1-\lambda)^{-1} + (1+a)}{d_b^2} \left((r_1-\lambda)(1+a) + a\rho\theta\right) Lx_1 + \frac{\theta(a-\lambda\,(1-\lambda)^{-1})}{d_b^2} Lx_2 + \left((1-\lambda)^{-1}\lambda + \frac{\rho(1+a)\lambda}{d_b^2}\right) Lx_4 + \left(\left(\rho d_b^{-1} + r_3\right) Lx_4 - 1\right) \frac{r_2}{d_b},$$

$$l^i_{rr} = r_2^2(\rho-1) + (\lambda(1-\lambda)^{-1} - r_2\rho)^2 Lx_1 + \theta d_b(1-d_b)(1+r_2)Lx_2$$
$$+ (1-\lambda)^{-1}\lambda(\theta-1)Lx_3$$
$$+ \left((\lambda(1-\lambda)^{-1} + r_2r_3)r_2 + \lambda(1-\lambda)^{-1}(-\lambda(1-\lambda)^{-1} + r_2\rho)\right) Lx_4,$$

$$l^i_{\pi} = \frac{\sigma}{k}\left(\frac{(1-\phi)}{(1+a)} + (\eta+1)Lx_1\right),$$

$$l^i_{y\epsilon} = \frac{-\eta(1-\phi)}{(1+a)} - \eta(\eta+1)Lx_1,$$

$$l^i_{y\mu} = (\eta+1)Lx_1,$$

$$l^i_{yg} = \frac{-(\rho-1)}{d_b^2} + \frac{\rho\,(d_b-\rho)}{d_b^2}Lx_1 - \frac{Lx_2}{d_b} - \frac{(r_3+\rho)Lx_4}{d_b},$$

$$l^i_{yc*} = \frac{-(\rho-1)\,(1-d_b)}{d_b^2} + -\frac{\rho\,(d_b-\rho)}{d_b^2}\frac{(1-d_b)}{d_b}Lx_1 - \frac{(\lambda(1+a)-a)Lx_2}{d_b} - \frac{(1-d_b)}{d_b}\left(\rho + \frac{r_3}{d_b}\right)Lx_4,$$

$$l^i_{r\epsilon} = 0,$$

$$l^i_{r\mu} = 0,$$

$$l^i_{rg} = \frac{\theta\,(\rho-1)\,(a-\lambda(2-\lambda)(1+a))}{(1-\lambda)d_b^2} + \frac{((r_1+\lambda)(1+a) - a\rho\theta)\,\rho}{(1-\lambda)d_b^2}(Lx_1 + Lx_4)$$
$$+ \frac{(\lambda(1+a)-a)Lx_2}{(1-\lambda)d_b} - \frac{((r_3+\rho)(2-\lambda)+\lambda)(1+a) + a\theta r_3)\,\rho}{(1-\lambda)d_b^2}\frac{Lx_4}{d_b},$$

and

$$l^i_{rc^*} = \frac{\theta\,(\rho-1)\,(\lambda(1+a)-a)((1-\lambda)^{-1}+d_b)}{d_b^2} - \frac{((r_1+\lambda)(1+a)-a\rho\theta)\,(\lambda(1+a)-a)Lx_1}{(1-\lambda)d_b^2}$$
$$+\frac{\theta(\lambda(1+a)-a)Lx_2}{(1-\lambda)d_b} + \frac{(\lambda(1+a)-a)}{d_b}\left(\rho\lambda(1-\lambda)^{-1}+r_2r_3\right)Lx_4,$$

where $r_1 = \lambda(2-\lambda)(\rho\theta-1)$, $r_2 = \frac{\theta((1-d_b)+(1-\lambda)^{-1}\lambda)}{d_b}$ and $r_3 = (1+\rho^2)a+1-2\rho$

2.B.6 Optimal Plan with Incomplete Asset Markets:

The optimal plan consists of minimizing (2.14) subject to equations in Table 4. Therefore, the first order conditions with respect to $\widehat{\pi}_t^H, \widehat{Y}_t, \widehat{RS}_t, \widehat{C}_t$, and $\widehat{b}_{F,t}$ are:

$$l^i_\pi \widehat{\pi}_t^H + \Delta\varphi_{1,t} = 0, \tag{2.58}$$

$$0 = l^i_{yy}\widehat{Y}_t + l^i_{yr}\widehat{RS}_t + l^i_{ye}\widehat{e}_t - k\eta\varphi_{1,t} + \varphi_{2,t} - \varphi_{4,t},$$

$$0 = l^i_{yr}\widehat{Y}_t + l^i_{rr}\widehat{RS}_t + l^i_{re}\widehat{e}_t - k\frac{\lambda}{(1-\lambda)}\varphi_{1,t} - d_{rs}\varphi_{2,t} + \varphi_{3,t} - \beta^{-1}\varphi_{3,t-1}$$
$$+\frac{\lambda}{(1-\lambda)}\varphi_{4,t} - a_\beta\Delta\varphi_{4,t} + a_\beta\beta\Delta E_t\varphi_{4,t+1}, \tag{2.59}$$

$$0 = -\rho k(1+a)\varphi_{1,t} - (1+a)(1-\lambda)\varphi_{2,t} - \rho\varphi_{3,t} + \rho\beta^{-1}\varphi_{3,t-1} + (1+a)\varphi_{4,t} + \rho a_\beta\Delta\varphi_{4,t}, \tag{2.60}$$

and

$$E_t\Delta\varphi_{4,t+1} = \beta^{-1}\delta\varphi_{3,t}. \tag{2.61}$$

The case of no intermediation costs:

When $\delta = 0$, the first order conditions can be written as

$$Q^i_y E_t \Delta(\widehat{Y}_{t+1} - \widehat{Y}^{T,i}_{t+1}) + Q^i_{rs} E_t \Delta(\widehat{RS}_{t+1} - \widehat{RS}^{T,i}_{t+1}) + Q^i_\pi E_t \widehat{\pi}^H_{t+1} = 0, \qquad (2.62)$$

with

$$Q^i_y = l^i_{yr} + (d_{rs} + (1+a)(1-\lambda)\rho^{-1}) l^i_{yy},$$

$$Q^i_{rs} = l^i_{rr} + (d_{rs} + (1+a)(1-\lambda)\rho^{-1}) l^i_{yr},$$

$$Q^i_\pi = k\left[(1+a)(\rho + \eta(1-\lambda)\rho^{-1}) + \eta d_{rs} + \lambda(1-\lambda)^{-1}\right] l^i_\pi,$$

$$\widehat{RS}^{T,i}_t = \frac{-l_{re}}{Q^i_r} \widehat{e}_t,$$

and

$$\widehat{Y}^{T,i}_t = \frac{-(d_{rs} + (1+a)(1-\lambda)\rho^{-1}) l^i_{ye}}{Q^i_y} \widehat{e}_t.$$

Special Case: Incomplete markets, symmetric steady state, no trade imbalances and specific level of steady-state output

In the case we have

1. $\mu = 1/(1-\lambda)$
2. $\rho = \theta = 1$
3. $a = 0$

In this case, the first order conditions can be written as:

$$0 = (l^i_{yy} + l^i_{yr}(1-\lambda))\Delta\widehat{Y}_t + ((1-\lambda)l^i_{rr} + l^i_{yr})\Delta\widehat{RS}_t + (l^i_{ye} + l^i_{re}(1-\lambda))\Delta\widehat{e}_t + k(\eta+1) l^i_\pi \widehat{\pi}^H_t \qquad (2.63)$$

Moreover:

$$Lx_1 = 0;\ Lx_2 = -1;\ Lx_3 = -1;\ \text{and } Lx_4 = 2 - \lambda.$$

And therefore:

$$l_{yy} + l_{yr}(1-\lambda) = (\eta+1)(1-\lambda)$$
$$(1-\lambda)l_{rr} + l_{yr} = 0$$
$$l_{\pi} = (1-\lambda)\sigma/k$$
$$(l_{ye} + l_{re}(1-\lambda)) = \begin{bmatrix} -\eta(1-\lambda) & 0 & -(1-\lambda) & 0 \end{bmatrix}$$

Hence, the targeting rule can be written as

$$0 = \Delta\left(\widehat{Y}_t - \frac{\eta}{(\eta+1)}\widehat{\varepsilon}_{Yt} - \frac{1}{(\eta+1)}\widehat{g}_t\right) + \sigma\widehat{\pi}_t^H. \tag{2.64}$$

In addition, using Equation (2.43), we can write the Phillips Curve as follows:

$$\widehat{\pi}_t^H = k\left((\eta+1)\hat{Y}_t - \eta\widehat{\varepsilon}_{Yt} - \widehat{g}_t + \widehat{\mu}_t\right) + \beta E_t\widehat{\pi}_{t+1}^H. \tag{2.65}$$

By inspection of Equation (2.64) and (2.65), we can see that domestic inflation target is the optimal plan if there are no mark-up shocks, $\widehat{\mu}_t$.

2.B.7 Welfare under Financial Autarky

Using an analogous derivation for welfare, but substituting the matrices b_y^i, B_y^i and B_e^i for b_y^{fa}, B_y^{fa} and B_e^{fa}, the loss function under financial autarky has the following weights[13]:

$$\begin{aligned} l_{yy}^{fa} &= (\eta+1)(1-\phi) + \frac{\rho-1}{d_b^2} \\ &+ \left(\frac{\rho\,(\rho-2d_b)}{d_b^2} + \eta(2+\eta)\right) Lx_1^{fa} + \frac{(1-d_b)}{d_b^2}Lx_2^{fa}, \end{aligned}$$

$$\begin{aligned} l_{yr}^{fa} = l_{ry}^{fa} &= \frac{\rho(1-\lambda)^{-1}}{d_b^2}\left((r_1-\lambda)(1+a) + a\rho\theta\right) Lx_1^{fa} \\ &+ \frac{\theta(a-\lambda\,(1-\lambda)^{-1})}{d_b^2}Lx_2^{fa} - \frac{r_2}{d_b}, \end{aligned}$$

[13] Note that for the derivation of wefare under Complete Market and Financial autarky, we assume $a = 0$.

$$l_{rr}^{fa} = r_2^2(\rho-1) + (\lambda(1-\lambda)^{-1} - r_2\rho)^2 Lx_1^{fa} + \theta d_b(1-d_b)(1+r_2)Lx_2^{fa}$$
$$+ (1-\lambda)^{-1}\lambda(\theta-1)Lx_3^{fa},$$

$$l_{\pi}^{fa} = \frac{\sigma}{k}\left((1-\phi) + (\eta+1)Lx_1^{fa}\right),$$

$$l_{y\varepsilon}^{fa} = -\eta(1-\phi) - \eta(\eta+1)Lx_1^{fa},$$

$$l_{y\mu}^{fa} = (\eta+1)Lx_1^{fa},$$

$$l_{yg}^{fa} = -\frac{(Lx_2(1-\lambda) + \rho Lx_4^{fa})}{d_b},$$

$$l_{yc^*}^{fa} = \frac{-(\rho-1)(1-d_b)}{d_b^2} - \frac{\rho(d_b-\rho)}{d_b^2}\frac{(1-d_b)}{d_b}Lx_1^{fa} - \frac{(\lambda(1+a)-a)Lx_2^{fa}}{d_b},$$

$$l_{r\varepsilon}^{fa} = 0,$$

$$l_{r\mu}^{fa} = 0,$$

$$l_{rg}^{fa} = \frac{(r_2+\lambda)Lx_4^{fa}}{d_b^2},$$

$$l_{yc^*}^{fa} = \frac{\theta(\rho-1)(\lambda(1+a)-a)((1-\lambda)^{-1}+d_b)}{d_b^2} - \frac{((r_1+\lambda)(1+a)-a\rho\theta)(\lambda(1+a)-a)Lx_1^{fa}}{(1-\lambda)d_b^2}$$
$$+ \frac{\theta(\lambda(1+a)-a)Lx_2^{fa}}{(1-\lambda)d_b},$$

and

$$l_{yc^*}^{fa} = -\frac{(Lx_2^{fa}(1-\lambda) + (-r_2 - \lambda + \rho)Lx_4^{fa})}{d_b},$$

with

$$Lx_1^{fa} = \frac{\lambda(l_1^{fa}(2-\lambda) + (\phi - \lambda))}{\lambda(l_2^{fa}(2-\lambda) - (\rho+\eta)(1-\lambda) - \lambda(\rho-1))},$$

$$Lx_2^{fa} = \frac{-\lambda((\rho+\eta) - \phi(\rho-1))}{\lambda(l_2^{fa}(2-\lambda) - (\rho+\eta)(1-\lambda) - \lambda(\rho-1))},$$

$$Lx_3^{fa} = \frac{-\lambda\theta((\rho+\eta) - (\rho-1)\phi)}{\lambda(l_2^{fa}(2-\lambda) - (\rho+\eta)(1-\lambda) - \lambda(\rho-1))},$$

and

$$Lx_4^{fa} = \frac{\lambda(l_3^{fa}(2-\lambda) - (\phi - \lambda))}{\lambda(l_2^{fa}(2-\lambda) - (\rho+\eta)(1-\lambda) - \lambda(\rho-1))},$$

where $l_1^{fa} = \phi(\theta - 1)$,$l_2^{fa} = \theta((\rho+\eta)$ and $l_3^{fa} = \theta((\rho + \eta - (\rho-1)\phi)$.

2.B.8 Optimal Plan under Financial Autarky

We can write the system of equations given in Table 2 in terms of $\widehat{Y}_t$ and $\widehat{RS}_t$ as follows:

$$\widehat{\pi}_t^H = \phi\left((\eta+\rho)\hat{Y}_t - (\rho-1)\lambda(1-\lambda)^{-1}\widehat{RS}_t + \widehat{\mu}_t - \eta\widehat{\varepsilon}_{Y,t}\right) + \beta E_t \widehat{\pi}_{t+1}^H, \tag{2.66}$$

and

$$\widehat{Y}_t = \widehat{RS}_t \frac{(1+l_i)}{(1-\lambda)} + \hat{C}_t^* + \lambda^{-1}\widehat{g}_t. \tag{2.67}$$

The policymaker minimizes the loss function subject to the problem (2.66) and (2.67). Given that the multipliers associated with (2.66) and (2.67) are, respectively, φ_1 and φ_2, the first order conditions with respect to $\widehat{\pi}_t^H, \hat{Y}_t$ and $\widehat{RS}_t$ are given by:

$$(\varphi_{1,t} - \varphi_{1,t-1}) = kl_\pi^{fa}\widehat{\pi}_t^H, \tag{2.68}$$

$$\varphi_{2,t} - (\eta + \rho)\varphi_{1,t} = l^{fa}_{yy}\widehat{Y}_t + l^{fa}_{yr}\widehat{RS}_t + l^{fa}_{ye}\widehat{e}_t, \tag{2.69}$$

and

$$-(1 + l_i)(1 - \lambda)^{-1}\varphi_{2,t} + (\rho - 1)\lambda(1 - \lambda)^{-1}\varphi_{1,t} = l^{fa}_{rsy}\widehat{Y}_t + l^{fa}_{rr}\widehat{RS}_t + l^{fa}_{re}\widehat{e}_t. \tag{2.70}$$

The last 3 equations can be combined, giving rise to the following targeting rule

$$Q^{fa}_y\Delta(\widehat{Y}_t - \widehat{Y}^{T,fa}_t) + Q^c_{rs}\Delta(\widehat{RS}_t - \widehat{RS}^{T,fa}_t) + Q^{fa}_\pi\widehat{\pi}^H_t = 0, \tag{2.71}$$

where

$$Q^{fa}_y = (l^{fa}_{yy} + l^{fa}_{ry}(1 - \lambda)(1 + l_i)^{-1}),$$

$$Q^{fa}_r = ((1 - \lambda)(1 + l_i)^{-1}l^{fa}_{rr} + l^{fa}_{ry}),$$

$$Q^{fa}_\pi = k((\eta + \rho) - (\rho - 1)\lambda(1 - \lambda)(1 + l_i)^{-1})l^{fa}_\pi,$$

$$\widehat{Y}^{T,fa}_t = (Q^{fa}_y)^{-1}l^{fa}_{ye}\widehat{e}_t,$$

and

$$\widehat{RS}^{T,fa}_t = (Q^{fa}_r)^{-1}l^{fa}_{re}(1 - \lambda)(1 + l_i)^{-1}\widehat{e}_t.$$

Special Case: when $\mu = 1/(1-\lambda)$ and $\rho = \theta = 1$, the targeting rule is identical to (2.64). Also, in the less restrictive case that only $\theta = 1$, the targeting rule would be given by

$$0 = \Delta\left(\widehat{Y}_t - \widehat{Y}^{Flex}_t\right) + \sigma\widehat{\pi}^H_t, \tag{2.72}$$

where $\widehat{Y}^{Flex}_t = \frac{\eta}{(\eta+\rho)}\widehat{\varepsilon}_{Yt} + \frac{\rho}{(\eta+\rho)}\widehat{g}_t$. In other words, producer price stability consists the optimal plan under the assumptions of $\mu = 1/(1 - \lambda)$ and $\theta = 1$, regardless of the value of ρ.

2.B.9 Welfare with Complete Markets

The welfare and optimal policy derivation under complete markets are presented in the first Chapter. However, to use the same notation as in the other asset market structures, we present the solution under this alternative notation. Therefore, following the derivation in the previous Chapter, the loss function with complete markets can be written as

$$
\begin{aligned}
L^i_{to} &= U_c \bar{C} E_{t_0} \sum \beta^t \left[\frac{1}{2} l^c_{yy} (\widehat{Y}_t - \widehat{Y}^{T,c}_t)^2 + \frac{1}{2} l^c_{rr} (\widehat{RS}_t - \widehat{RS}^{T,c}_t)^2 + \frac{1}{2} l^c_{\pi} (\widehat{\pi}^H_t)^2 \right] \\
&\quad + t.i.p + \mathcal{O}(||\xi||^3)
\end{aligned} \tag{2.73}
$$

where:

$$
\begin{aligned}
l^c_{yy} = (\eta + \rho)(1 - \phi) &+ \frac{(\rho - 1)\left[-l^c(1 - \phi) - (\lambda - \phi)\right]}{(1 + l^c)} \\
&+ Lx^c_1 \left[(\eta + \rho) + \eta(\eta + 1) - \frac{\rho(\rho - 1)}{(1 + l^c)} \right] \\
&- \frac{Lx^c_2 (1 - \lambda)^2 \lambda (\rho\theta - 1)}{(1 + l^c)},
\end{aligned}
$$

$$
\begin{aligned}
l^c_{rr} = &- \frac{(\lambda + l^c)(\rho - 1)}{(1 - \lambda)\rho^2} \\
&+ \frac{Lx^c_1 l^c (\rho - 1 - l^c)}{(1 - \lambda)^2 \rho} \\
&+ \frac{Lx^c_2 \lambda (\rho\theta - 1)\left[\rho\theta(1 - \lambda) + \lambda + l^c\right]}{\rho^2} \\
&+ \frac{Lx^c_3 \left[1 + \lambda^2 (2 - \lambda)\right] \lambda (\theta - 1)}{1 - \lambda},
\end{aligned}
$$

$$
l^c_{\pi} = \frac{\sigma}{\mu k} + (1 + \eta)\frac{\sigma}{k} L x^c_1,
$$

$$\widehat{Y}_t^{T,c} = q_y^e e_t,$$

and

$$\widehat{RS}_t^{T,c} = q_{rs}^e e_t,$$

where

$$q_y^e = \frac{1}{\Phi_Y}\begin{bmatrix} \frac{\eta}{\mu} + Lx_1^c(1+\eta)\eta & -Lx_1^c(1+\eta) & \frac{(\rho-1)(1-\lambda)+Lx_2^c}{1+l^c} & 0 \end{bmatrix},$$

$$q_{RS}^e = \frac{1}{\Phi_{RS}}\begin{bmatrix} 0 & 0 & \frac{(\rho-1-l^c)Lx_1^c}{(1-\lambda)} + \frac{Lx_2^c\lambda(\lambda(1-\lambda)+1)(\rho\theta-1)}{\rho(1-\lambda)} & \frac{-Lx_2^c\lambda(1-\lambda)(\rho\theta-1)}{\rho} \end{bmatrix},$$

$$Lx_1^c = \frac{1}{(\rho+\eta)+l^c\eta}\left[l\mu^{-1} + (1-\lambda) - \mu^{-1}\right],$$

$$Lx_2^c = \frac{1}{(\rho+\eta)+l^c\eta}\left[\rho(\mu^{-1} - (1-\lambda)) + (1-\lambda)(\eta+\rho)\right],$$

$$Lx_3^c = \frac{1}{(\rho+\eta)+l^c\eta}\left[(\rho\theta-1)(1-\lambda)\mu^{-1} - (\eta\theta+1)\right], \tag{2.74}$$

and $l^c = (\rho\theta-1)\lambda(2-\lambda)$.

2.B.10 Optimal Plan with Complete Markets

The optimal plan consists of minimizing the loss function subject to

$$\widehat{\pi}_t^H = k\left(\eta\hat{Y}_t + (1-\lambda)^{-1}\widehat{RS}_t + \mu_t - \eta\varepsilon_{Y,t} + \rho\widehat{C}_t^*\right) + \beta E_t\widehat{\pi}_{t+1}^H, \tag{2.75}$$

and

$$\widehat{Y}_t = \widehat{RS}_t\frac{(1+l^c)}{\rho(1-\lambda)} + g_t + \widehat{C}_t^*. \tag{2.76}$$

The multipliers associated with (2.75) and (2.76) are, respectively, φ_1 and φ_2. The first order conditions with respect to $\widehat{\pi}_t^H, \hat{Y}_t$ and $\widehat{RS}_t$ are, therefore, given by

$$(\varphi_{1,t} - \varphi_{1,t-1}) = kl^c_\pi \widehat{\pi}^H_t, \tag{2.77}$$

$$\varphi_{2,t} - \eta\varphi_{1,t} = l^c_{yy}(\widehat{Y}_t - \widehat{Y}^T_t), \tag{2.78}$$

and

$$-\varphi_{2,t} - \frac{\rho}{(1+l)}\varphi_{1,t} = \frac{\rho(1-\lambda)}{(1+l)} l^c_{rr}(\widehat{RS}_t - \widehat{RS}^T_t). \tag{2.79}$$

To obtain a targeting rule for the small open economy, we combine equations (2.77), (2.78), and (2.79):

$$Q^c_y \Delta(\widehat{Y}_t - \widehat{Y}^{T,c}_t) + Q^c_{rs}\Delta(\widehat{RS}_t - \widehat{RS}^{T,c}_t) + Q^c_\pi \widehat{\pi}^H_t = 0, \tag{2.80}$$

where

$$Q^c_y = (1 + l^c) l^c_{yy},$$

$$Q^c_r = \rho(1-\lambda) l^c_{rr},$$

and

$$Q^c_\pi = (\rho + \eta(1+l))kl^c_\pi.$$

Special Case: when $\mu = 1/(1-\lambda)$ and $\rho = \theta = 1$, the targeting rule is identical to (2.64). This confirms that, under these circumstances, the asset market structure is irrelevant for monetary policy.

2.C Appendix: Randomization Problem - the Financial Autarky Case

To ensure that the policy obtained from the minimization of the loss function is indeed the best available policy, we should confirm that no other random policy plan can be welfare improving. In the first Chapter, we analyse the case of complete markets. We present the conditions under which no random policy can enhance welfare. As shown in Woodford and Benigno (2003), these conditions coincide with the second order condition for the linear quadratic optimization problem. In the present Section, we study the case of financial autarky.

Following the same steps as in Section 1.D, we characterize the relationship between inflation and output and inflation and the real exchange rate. Equation (AS) combined with Equation (FA) leads to the following expression:

$$\widehat{\pi}_t^H = k\left(\frac{(\eta+\rho)d_1-(\rho-1)\lambda}{d_1}\widehat{Y}_t+\mu_t+\eta\varepsilon_{Y,t}\right)+\beta E_t\widehat{\pi}_{t+1}^H, \tag{2.81}$$

where $d_1 = (\theta-1)(1-\lambda)-\lambda\theta$. Alternatively,

$$\widehat{\pi}_t^H = k\left(\frac{(\eta+\rho)d_1-(\rho-1)\lambda}{(1-\lambda)}\widehat{RS}_t+\mu_t+\eta\varepsilon_{Y,t}\right)+\beta E_t\widehat{\pi}_{t+1}^H. \tag{2.82}$$

A random sunspot realization that adds $\varphi_j v_j$ to π_{t+j}, will, therefore, add a contribution of $\alpha_y k^{-1}(\varphi_j-\beta\varphi_{j+1})v_j$ to $\hat{Y}_t$ and $\alpha_{rs}k^{-1}(\varphi_j-\beta\varphi_{j+1})v_j$ to $\widehat{RS}_t$, where

$$\alpha_{rs}^{fa} = \frac{(\eta+\rho)d_1-(\rho-1)\lambda}{(1-\lambda)}, \tag{2.83}$$

and

$$\alpha_y^{fa} = \frac{(\eta+\rho)d_1-(\rho-1)\lambda}{d_1}. \tag{2.84}$$

To obtain what is the contribution to the loss function of the realization $\varphi_j v_j$ to π_{t+j}, we rewrite the loss function as follows. Noticing that $(d_{rs}\lambda^{-1}-1)\widehat{RS}_t = \widehat{Y}_t + t.i.p$, the loss function under financial autarky can be written as

$$L_{to} = U_c\bar{C}E_{t_0}\sum\beta^t\left[\begin{array}{c}\frac{1}{2}(l_{yy}^{fa}+(d_{rs}\lambda^{-1}-1)^{-1}l_{yr}^{fa})(\widehat{Y}_t-\widehat{Y}_t^T)^2\\ +\frac{1}{2}(l_{rr}^{fa}+(d_{rs}\lambda^{-1}-1)l_{yr}^{fa})(\widehat{RS}_t-\widehat{RS}_t^T)^2+\frac{1}{2}l_\pi^{fa}\pi_t^2\end{array}\right]+t.i.p, \tag{2.85}$$

where

$$\widehat{Y}_t^T = l_y^e e_t,$$

$$l_y^e = \frac{1}{(l_{yy}^{fa} + (d_{rs}\lambda^{-1} - 1)^{-1} l_{yr}^{fa})} \left[\begin{array}{cccc} l_{ye}^{fa} & l_{y\mu}^{fa} & l_{yg}^{fa} & l_{yc^*}^{fa} \end{array} \right],$$

$$\widehat{RS}_t^T = l_{rs}^e e_t,$$

and

$$l_y^e = \frac{1}{(l_{yy}^{fa} + (d_{rs}\lambda^{-1} - 1)l_{yr}^{fa})} \left[\begin{array}{cccc} l_{re}^{fa} & l_{r\mu}^{fa} & l_{rg}^{fa} & l_{rc^*}^{fa} \end{array} \right].$$

Consequently, the contribution to the loss function of a random realization in $\varphi_j v_j$ is

$$U_c \bar{C} \beta^t \sigma_v^2 E_{t_0} \sum \beta^t \left[\Phi^{fa} k^{-1} (\varphi_j - \beta \varphi_{j+1})^2 + l_\pi^{fa} (\varphi_j)^2 \right], \tag{2.86}$$

where

$$\Phi^{fa} = \Phi_Y^{fa} \alpha_y^2 + \Phi_{RS}^{fa} \alpha_{rs}^2,$$

$$\Phi_Y^{fa} = (l_{yy}^{fa} + (d_{rs}\lambda^{-1} - 1)^{-1} l_{yr}^{fa}),$$

and

$$\Phi_{RS}^{fa} = (l_{rr}^{fa} + (d_{rs}\lambda^{-1} - 1) l_{yr}^{fa}).$$

It follows that policy randomization cannot improve welfare if the expression given by Equation (2.86) is positive definite. Hence, the first order conditions to the minimization problem are indeed a policy optimal if Φ^{fa} and l_π^{fa} are not both equal to zero and either: (a) $\Phi^{fa} \geq 0$ and $\Phi^{fa} + (1 - \beta^{1/2})^2 k^{-2} l_\pi^{fa} \geq 0$, or (b) $\Phi^{fa} \leq 0$ and $\Phi^{fa} + (1 - \beta^{1/2})^2 k^2 l_\pi^{fa} \geq 0$ holds.

Chapter 3

Optimal Monetary and Fiscal Policy for a Small Open Economy

3.1 Introduction

Chapters 1 and 2 of this thesis suggest that international aspects of an economy, such as the trade balance and international financial markets, can affect the policy prescription considerably. In particular, these factors dictate whether or not there are policy incentives to affect the exchange rate. When these incentives exist, monetary policy deviates from price stability. But can fiscal, rather than monetary policy be used strategically in an open economy? This Chapter addresses this particular issue by incorporating distortionary taxation into the framework presented in Chapter 1. When prices are perfectly flexible, our results show that, contrary to the closed economy case, the optimal tax rate is time varying in a small open economy. Moreover, it demonstrates that the introduction of price stickiness reduces the optimal variability of both taxes and inflation.

The small open economy characterization closely follows the one presented in Chapter 1. We assume that there are no trade frictions (i.e. the law of one price holds) and that capital markets are perfect (i.e. asset markets are complete). On the other hand, following recent contributions by Benigno and Woodford (2003), Schmitt-Grohé and Uribe (2001) and Siu (2001), we allow for distortionary income taxation.

To highlight the open economy dimension of the fiscal policy problem, our analysis focuses initially on the case of flexible prices. Indeed, in this structure, there are two policy incentives: reducing the inefficiency caused by movement in distortionary taxation; and strategically managing the real exchange rate. In a closed economy framework, it is optimal to perfectly smooth taxes so as to avoid distortions in households' marginal rate of substitution between consumption and leisure. In an open economy, however, varying the level of proportional taxes may improve welfare by affecting the overall level of consumption utility and production disutility. For example, higher income taxes can lower domestic disutility of production without a corresponding decline in the utility of consumption. This is possible because a higher level of taxation reduces the depreciation in the real exchange rate, allowing domestic agents to switch consumption towards foreign produced goods.[1] Note that, in a closed economy, this mechanism is absent because a fall in the disutility of domestic production would be accompanied by a corresponding reduction in the utility of consumption.

The denomination of government debt does not alter the key mechanism described above, but it is important in determining the dynamic properties of our variables of interest. If public debt is indexed to consumer price inflation, and therefore yields real returns, taxes are nonstationary. On the other hand, if inflation can affect the level of real debt, taxes will follow a stationary process.

Once we allow for sticky prices, a further distortion is added to the economy, namely the inefficiency in the allocation of resources caused by positive domestic producer inflation (as in Woodford (2003)). Also, under sticky prices, both inflation and taxes affect the agent's consumption-leisure decision. Like income taxes, domestic producer inflation can also be used in a strategic way to affect the terms of

[1] This fact depends on the specified values for the structural parameter (in particular the elasticities of intertemporal and intratemporal substitution).

trade and the overall level of utility. Consequently, the introduction of price rigidity reduces the variability of taxes because, in this case, domestic producer inflation is strategically used to affect the exchange rate.

The quantitative results suggest that the cost of inflation overshadows the inefficiency caused by varying distortionary taxation and, therefore, changes in domestic producer inflation are quantitatively small. Note that this result holds even in a model with real bonds and is a consequence of the conflict between price stability and the incentive to affect strategically the real exchange rate. This is different, however, from the trade-off (emphasized by the Benigno and Woodford (2003), Schmitt-Grohé and Uribe (2001) and Siu (2001)) between price stability and the use of inflation as insurance that arises only in models in which the government issues nominal bonds.

From a methodological point of view, our analysis follows the technique developed in Benigno and Woodford (2003). Specifically, we propose a linear quadratic approach to the optimal policy problem. The present work encompasses, as special cases, the closed economy framework (Benigno and Woodford (2003)) and the small open economy case in which there are endogenous lump sum taxes (De Paoli (2004)). Under price flexibility, our loss function is quadratic in the output and real exchange rate gaps. With price rigidities, the variability of inflation also affects welfare.

The linear quadratic approach allows us to derive simple policy rules that prescribe the optimal state-contingent responses to shocks. We do so by specifying targeting rules as in Svensson (2003). In particular, the optimal plan is composed of two rules: one that specifies targeting a linear combination of domestic producer inflation, domestic output growth and changes in the real exchange rate; the other seeking to stabilize expected producer inflation at zero.

3.1.1 Related literature

Following the work of Ramsey (1927), the traditional optimal taxation literature has focused on closed economy frameworks. These studies have examined how distortionary taxes should respond fiscal shocks. Lucas and Stokey (1983) have shown that, in an environment in which the government can issue state-contingent debt, it is optimal to smooth taxes, and the resulting tax variance is small relative to fiscal shocks. Chari, Christiano and Kehoe (1991) extend the analysis to economies with risk-free debt and show that, in this environment, it is optimal to use state-contingent inflation to absorb the fiscal shock.

Recent contributions to the optimal taxation literature (Correia, Nicolini and Teles (2003), Schmitt-Grohé and Uribe (2004), Siu (2004) and Benigno and Woodford (2003)) incorporate monopolistic competition and nominal price rigidity into the analysis. Correia et al. (2003) assume that state-continent bonds are available and determine the conditions in which inflation is irrelevant to the optimal plan. Schmitt-Grohé and Uribe (2004), Siu (2004) and Benigno and Woodford (2003) consider an economy in which there is no state-contingent debt and show that optimal inflation volatility is close to zero, even for a small degree of price rigidity.

Our work is related to some recent contributions that have analysed the interaction of monetary and fiscal policy in open economies. Beetsma and Jensen (2005) analyse monetary and fiscal policy interaction in a two-country monetary union model. The authors assume that per capita public spending delivers utility to the consumer and that taxes are lump sum. Similar fiscal policy assumptions are adopted in Galí and Monacelli (2005b), who consider a continuum of small economies in a currency union setting. Lombardo and Sutherland (2004) investigate the costs and benefits from fiscal cooperation in a two-period version of Beetsma and Jensen (2004).

Ferrero (2005) lays out a currency union model in which lump sum taxes are not available to fiscal authorities. The paper analyses the optimal fiscal and monetary plan under commitment. In another interesting paper, Adao, Correia and Teles (2005) examine the implications of the choice of exchange rate regimes for fiscal policy. The authors find that the assumption of lack of labor mobility is crucial in establishing that the choice of exchange rate regime is irrelevant.

We aim to contribute to this vast literature by characterizing an integrated framework for fiscal and monetary policy in a small open economy under alternative assumptions regarding inefficiencies created by the policy instruments.

The remainder of the Chapter is structured as follows. Section 3.2 describes the structure of the model. Section 3.3 presents the log-linear version of the model. Section 3.4 discusses the policy problem while the analysis of the optimal policy plan is conducted in Section 3.5. Section 3.7 concludes.

3.2 The Model

We lay out a two-country dynamic general equilibrium framework, which follows closely the one presented in Chapter 1. We consider a very simple small open economy model in which markets are complete and producer currency pricing holds. As in the previous chapters, the goods markets are characterized by monopolistic competition and the price setting follows Calvo (1983). However, in this Chapter, fiscal policy is endogenous. In particular, we assume that production taxation is chosen optimally.

Household Behavior

There is a measure n of agents in our small open economy, who have a utility function of the same form:

$$U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[U(C_s) - V(y_s(h), \varepsilon_{Y,s}) \right]. \tag{3.1}$$

Households obtain utility from consumption $U(C)$ and contribute to the production of a differentiated good $y(h)$, attaining disutility $V(y(h), \varepsilon_{Y,})$. Productivity shocks are denoted by $\varepsilon_{Y,s}$.

As in Chapter 1, we assume that markets are complete domestically and internationally. This assumption implies that the marginal utilities of income are equalized across countries at all times and in all states of nature:

$$\frac{U_C\left(C_{t+1}^*\right)}{U_C\left(C_t^*\right)} \frac{P_t^*}{P_{t+1}^*} = \frac{U_C\left(C_{t+1}\right)}{U_C\left(C_t\right)} \frac{S_{t+1} P_t}{S_t P_{t+1}}. \tag{3.2}$$

The preference specification for home and foreign goods is identical to that described in Chapter 1. The demand conditions in the small open economy and the rest of the world can therefore also be represented by the following equations:

$$y_t^d(h) = \left[\frac{p_t(h)}{P_{H,t}}\right]^{-\sigma} \left\{ \left[\frac{P_{H,t}}{P_t}\right]^{-\theta} \left[(1-\lambda)\,C_t + \lambda \left(\frac{1}{RS_t}\right)^{-\theta} C_t^*\right] + G_t \right\}, \quad (3.3)$$

$$y_t^d(f) = \left[\frac{p_t(f)}{P_{F,t}}\right]^{-\sigma} \{C_t^* + G_t^*\}. \quad (3.4)$$

Price Setting

Following Calvo (1983), in each period a fraction, $\alpha \in [0,1)$, of randomly picked firms is not allowed to change the nominal price of the good it produces. The remaining fraction of firms, $(1-\alpha)$, chooses prices optimally by maximizing the expected discounted value of profits. Therefore, the optimal choice of producers that can set their price $\tilde{p}_t(j)$ at time T is therefore

$$E_t \left\{ \sum (\alpha\beta)^{T-t} U_c(C_T) \left(\frac{\tilde{p}_t(j)}{P_{H,T}}\right)^{-\sigma} Y_{H,T} \left[\frac{\tilde{p}_t(j)}{P_{H,T}} \frac{P_{H,T}}{P_T} - \frac{\sigma \widehat{mc}_T V_y\left(\tilde{y}_{t,T}(j), \varepsilon_{Y,T}\right)}{(1-\tau_T)(\sigma-1)U_c(C_T)}\right] \right\} = 0. \quad (3.5)$$

Monopolistic competition in production leads to a wedge between marginal utility of consumption and marginal disutility of production, represented by $\frac{\sigma \widehat{mc}_t}{(1-\tau_t)(\sigma-1)}$. Movements in the tax rate, τ_t, affect this wedge and generate distortions in agents' choices between consumption and labour. However, changes in the tax rate are no longer exogenous, which is different from the case studied in the previous chapters. We allow for exogenous fluctuations in this wedge by assuming a time-varying mark-up shock $\widehat{mc}_t$.[2]

Thus, the price index evolves according to the following law of motion:

$$(P_{H,t})^{1-\sigma} = \alpha P_{H,t-1}^{1-\sigma} + (1-\alpha)\left(\tilde{p}_t(h)\right)^{1-\sigma}. \quad (3.6)$$

[2]This mark-up shock is introduced in order to allow for the evaluation of pure cost-push shocks. It can be interpreted as a shock to the level of monopolistic power of firms. Alternatively, it may be thought as a shock to wage mark-up in an environment where the labour market is also characterized by imperfect competition and there is a set of differentiated labour inputs.

Government Budget Constraint

We consider two alternative specifications for government debt. In particular, we consider the cases in which the government issues bonds denominated in domestic currency and in which public debt is denoted in real terms (or, equivalently, debt is indexed to consumer price inflation). The structure of debt denomination is exogenously given. We also abstract from the existence of seigniorage revenues.

In the first case, we focus on the situation in which the government issues one-period nominal risk-free bonds expressed in local currency units, collects taxes and faces exogenous expenditure streams. The law of motion of government debt, expressed in nominal terms, D_t^n, is

$$D_t^n = D_{t-1}^n(1 + i_{t-1}) - P_{H,t}s_t, \tag{3.7}$$

where s_t is the real primary budget surplus

income tax

$$s_t \equiv \tau_t Y_t - G_t - Tr_t,$$

and G_t and Tr_t are exogenously given government purchases and (lump-sum) government transfers, respectively, and τ_t denotes the income tax rate.

In addition to the case of nominal bonds, we consider the case in which the government issues riskless real one-period bonds D_t^r. Under this specification, the government budget constraint can be written as

$$D_t^r = D_{t-1}^r(1 + r_{t-1}) - \frac{P_{H,t}}{P_t}s_t. \tag{3.8}$$

The implication for fiscal and monetary policy of the different debt characterizations are explored later in the text.

Note that, expressions analogous to (3.5), (3.7) and (3.8) can also be derived for the foreign economy.

How about the solvency condition?

3.3 A Log-linear Representation of the Model

As in the previous chapters, we approximate the model around the steady state (details are in the Appendix). The log-linear system of equilibrium conditions for the small open economy is given in Table 3.1.

Table 3.1: System of Log-linear Equilibrium Conditions

Phillips Curve

$$\widehat{\pi}_t^H = k\left(\rho\widehat{C}_t + \eta\widehat{Y}_t - \widehat{p}_H + \widehat{mc}_t + \omega\widehat{\tau}_t - \eta\widehat{\varepsilon}_{Y,t}\right) + \beta E_t\widehat{\pi}_{t+1}^H$$

Demand

$$\hat{Y}_t = -\theta\hat{p}_H + (1-\lambda)\hat{C} + \lambda\hat{C}^* + \theta\lambda\widehat{RS}_t + \hat{g}_t$$

Risk Sharing Condition

$$\hat{C}_t = \hat{C}_t^* + \frac{1}{\rho}\widehat{RS}_t$$

Government Budget Constraint

$$\widetilde{d}_t = d_{ss}(1-\beta)(-\rho\widehat{C}_t - \frac{\lambda}{1-\lambda}RS_t) + \overline{\tau}(\widehat{\tau}_t + \widehat{Y}_t) - \widehat{g}_t + \beta E_t\widetilde{d}_{t+1}$$

Price Index

$$(1-\lambda)\hat{p}_H + \lambda\widehat{RS} = 0$$

*where $\widetilde{d}_t = \widehat{d}_{t-1} - d_{ss}(a\Delta RS_t + b\widehat{\pi}_t^H) - \rho d_{ss}\widehat{C}_t$

The first equation is derived from the price setting condition (3.5) and represents the small open economy Phillips Curve.[3] The demand equation is derived from log-linearizing condition (3.3) and the price index equation is derived from the linearized condition (1.6) of Chapter 1. The government budget constraint is represented in a compact form to allow for different types of bonds. We specify $a = \lambda/(1-\lambda)$ and $b = 1$ for the case of nominal bonds, and $a = b = 0$ for the case of real bonds.[4] Note that, in the case of zero steady-state government debt, the denomination of government debt is irrelevant for the dynamics of the small open economy. In this case, the government budget constraint becomes

$$\widehat{d}_{t-1} = \overline{\tau}(\widehat{\tau}_t + \widehat{Y}_t) - \widehat{g}_t + \beta\widehat{d}_t. \tag{3.9}$$

The system of structural equilibrium conditions is closed by specifying monetary and fiscal policy rules. Given the domestic exogenous variables $\widehat{\varepsilon}_{y,t}$,$\widehat{g}_t$, $\widehat{mc}_t$ and the external shock $\widehat{C}_t^*$, we can determine the dynamics of $\widehat{Y}_t, \widehat{RS}_t, \widehat{C}_t, \widehat{\pi}_t^H, \widehat{d}_t$ and $\widehat{p}_{H,t}$.

Foreign dynamics are governed by the foreign Phillips curve, demand condition and government budget constraint:

---

[3]We denote $\widehat{p}_{H,t} = \ln(P_{H,t}/P_t)$, $\widehat{\pi}_{H,t} = \ln(P_{H,t}/P_{H,t-1})$, $\hat{g}_t = \frac{Gt-\overline{G}}{Y}$ and $\widehat{d}_t = \frac{dt-\overline{d}}{Y}$. ρ represents the coefficient of relative risk aversion and η the inverse of the elasticity of goods production. Also, we define $k = \frac{(1-\alpha\beta)(1-\alpha)}{\alpha(1+\sigma\eta)}$, $\omega = \frac{\overline{\tau}}{1-\overline{\tau}}$ and $d_{ss} = \frac{\overline{d}}{Y}$. See the Appendix for a detailed derivation of the the approximations.

[4]To obtain the value of the interest rates in equations (3.7) and (3.8) we use households' intertemporal choice (i.e. the Euler equation). See the Appendix for a detailed derivation.

Table 3.2: Foreign System of Log-linear Equilibrium Conditions

Phillips Curve

$$\widehat{\pi}_t^* = k\left(\rho\hat{C}_t^* + \eta\widehat{Y}_t^* + \omega\widehat{\tau}_t^* + \widehat{\mu}_t^* - \eta\widehat{\varepsilon}_{Y,t}^*\right) + \beta E_t\widehat{\pi}_{t+1}^*$$

Demand

$$\hat{Y}_t^* = \hat{C}^* + \hat{g}_t^*$$

Government Budget Constraint

Is it equal to zero when the counterpart is the small open economy?

$$\widetilde{d}_t^* = -\rho d_{ss}(1-\beta)\hat{C}_t^* + \overline{\tau}^*(\widehat{\tau}_t^* + \widehat{Y}_t^*) - \widehat{g}_t^* + \beta E_t\widetilde{d}_{t+1}^*$$

*where $\widetilde{d}_t^* = \widehat{d}_{t-1}^* - d_{ss}^*\widehat{\pi}_t^* - \rho d_{ss}^*\hat{C}_t^*$

The specification of the foreign policy rules completes the system of equilibrium conditions that determine the evolution of $\hat{Y}_t^*$, $\hat{C}^*$, $\widehat{d}_t^*$, $\widehat{\tau}_t^*$ and $\widehat{\pi}_t^*$. Note that the dynamics of the rest of the world are not affected by *Home* variables. Therefore, the small open economy can treat C_t^* as an exogenous shock. Neither the policy choice of the rest of the world, nor the denomination of foreign public debt, can influence how C_t^* affects the small open economy.

3.4 Welfare Measure

The policy objective for the small open economy is given by the expected utility of the agents belonging to the economy:

$$W = E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left[U(C_t) - \frac{1}{n}\int_0^n V(y_t(h), \varepsilon_{Yt})dh\right]\right\}$$

We assume that policymakers can commit to maximize this objective function and that they are committed to past promises following a timeless perspective commitment (as in Woodford (2003, Ch.7)).

We derive a second order approximation to the policy objective around the steady state in the Appendix. The second order Taylor expansion of the utility function can be written as:

$$W_{to} = U_c\bar{C}E_{t_0}\sum\beta^t\left[\begin{array}{c}\widehat{C}_t - \frac{1}{\mu}\widehat{Y}_t + \frac{1}{2}(1-\rho)\widehat{C}_t^2 \\ -\frac{1}{2}\frac{(\eta+1)}{\mu}(\widehat{Y}_t - \frac{\eta}{(\eta+1)}\widehat{\varepsilon}_{Y,t})^2 - \frac{1}{2}\frac{\sigma}{k\mu}\left(\pi_t^H\right)^2\end{array}\right] + t.i.p + O(||\xi||^3), \tag{3.10}$$

where the term $t.i.p$ stands for terms independent of policy (i.e. constants or functions of exogenous shocks that are not affected by the policy choice). The term $O(||\xi||^3)$ refers to terms of order strictly higher than two. And the parameter μ denotes the steady-state degree of monopolistic distortion, (i.e. $\mu = \frac{\sigma\overline{mc}}{(1-\bar{\tau})(\sigma-1)}$).

To eliminate the discounted linear terms that appear in the Taylor approximation, we follow the method developed by Benigno and Woodford (2003) and Sutherland (2002). We use a second order approximation to some of the structural equilibrium conditions to obtain a complete second order solution for the evolution of the endogenous variables of interest. It follows that the loss function for our small open economy can be expressed as a quadratic function of $\widehat{Y}_t, \widehat{RS}_t$, and $\widehat{\pi}_t^H$:

$$L_{to}^i = U_c\bar{C}E_{t_0}\sum\beta^t\left[\frac{1}{2}\Phi_Y\widehat{y}_t^2 + \frac{1}{2}\Phi_T\widehat{rs}_t^2 + \frac{1}{2}\Phi_\pi(\widehat{\pi}_t^H)^2\right] + t.i.p + O(||\xi||^3) \tag{3.11}$$

where $\widehat{rs}_t = (\widehat{RS}_t - \widehat{RS}_t^T)$, $\widehat{y}_t = (\widehat{Y}_t - \widehat{Y}_t^T)$. The target variables $\widehat{Y}_t^T$ and $\widehat{RS}_t^T$ are functions of the various exogenous shocks, and the weights Φ_Y, Φ_{RS} and Φ_π depend on the structural parameters of the model (these are defined in the Appendix).

Equation (3.11) indicates that policymakers should seek to minimize both the discounted value of a weighted sum of squared deviations of inflation from zero and the squared fluctuations in the output and real exchange rate gap. As in Chapter 1, the open economy dimension of the model gives rise to a real exchange rate term as a policy objective. The only case in which there is no concern for the real exchange rate in the policy objective function is the special parametric case in which $\rho\theta = 1$ and $d_{ss}(\mu - (1-\lambda)^{-1}) = 0$. This is consistent with the results of Chapter 1 and Galí and Monacelli (2005), where there is no fiscal stabilization problem. Galí and Monacelli (2005) find that, when $\rho = \theta = 1$ and $\mu = (1-\lambda)^{-1}$, the small open economy is isomorphic to a closed economy, and a producer price inflation target is optimal. Φ_π Φ_Y Φ_{RS} independent of fiscal policy regime?

By inspection of the weights Φ_π, Φ_Y, and Φ_{RS} in the Appendix, we can analyse what determines welfare losses in the small open economy. Our small open economy is characterized by three frictions that are common to the closed economy framework: (a) monopolistic competition with an inefficient output level; (b) the staggered price setting mechanism that creates dispersion of output across the differentiated goods; and (c) distortionary income taxation that generates inefficiencies in agents' labour/leisure decisions. Therefore, factors such as the degree of monopolistic competition μ, the degree of price stickiness α, and the steady-state level of government taxes $\overline{\tau}$ are important determinants of the weights Φ_π and Φ_Y in the loss function.

In an open economy, however, another policy incentive arises. As first emphasized by Corsetti and Pesenti (2001), *"In an open economy there exists an economic distortion that is directly associated with openness, namely, a country's power to affect its terms of trade by influencing the supply of labour product. [...] the improved terms of trade allow domestic agent to finance higher consumption for any given level of labour effort."*

In our framework, there is a similar incentive because policymakers may wish to increase the unconditional mean consumption for a given level of domestic production (or alternatively, decrease the unconditional mean of output without an equivalent fall in the unconditional mean of consumption). Equation (3.10) illustrates analytically the above argument. It shows that welfare in a small open economy is affected by the unconditional means of consumption and output, and those are directly affected by the real exchange rate. In particular, if we abstract from the steady-state monopolistic distortion,[5] the term $E[\widehat{C}_t - \frac{\widehat{Y}_t}{\mu}]$ can be rewritten as a function of $E[(1-\rho\theta)\widehat{RS}_t]$. That is, the unconditional mean of the real exchange rate has a direct impact on the small open economy's welfare. Hence, there exists a "real exchange rate" externality (see Corsetti and Pesenti (2001), Benigno and Benigno (2003) and Arseneau (2004), amongst others).

Following the linear quadratic approach, we express the unconditional means present in the Taylor expansion (3.10) in terms of the variance of the endogenous variables. This delivers a simple representation of the loss function and allows for a derivation of the optimal plan that is time invariant (and therefore abstracts from time-inconsistent problem). As derived in the Appendix, the term $E[\widehat{C}_t - \frac{\widehat{Y}_t}{\mu}]$ can be written as a function of the real exchange rate, output and inflation variability. That is, there is an incentive to affect the variance of the real exchange rate. In the present framework, both taxes (if distortionary) and inflation (in the case of sticky prices) can affect those variances by influencing the supply of domestic goods.

3.5 Optimal Policy

In this Section, we analyse optimal monetary policy under different specifications of the model. We start by characterizing the case of flexible prices and later turn to the case of sticky prices. Throughout the analysis, we explore the implications of having a non-zero steady-state level of government debt. This implies that inflation has direct fiscal consequences; i.e. it affects the real value of government debt. Under these alternative environments, we first explore the international dimension of fiscal policy. We do so by comparing the optimal plan for the small open economy with a closed economy.[6]

The quantitative analysis of the optimal policy is presented in this Section. In particular, we illustrate the optimal response to productivity shocks under the different scenarios. The parameter values used in the numerical exercise are shown in Table 3.3. Note that the incentive to affect the inefficient level of output given

[5] As shown in the first chapter, this can be done by setting $\mu = (1-\lambda)^{-1}$.

[6] More specifically, we contrast the general optimal policy prescription with the case in which the small open economy is isomorphic to the closed economy (as alluded to above).

Table 3.3: Parameter Values used in the Quantitative Analysis

Parameter	**Value**	**Notes:**
β	0.99	Specifying a quarterly model
η	0.47	Following Rotemberg and Woodford (1997)
θ	3	Following Obstfeld and Rogoff (1998) (unless specified otherwise)
λ	0.4	This implies a 40% import share of the GDP
ρ	0.157	Specifying a Log utility function
α	0.66	Characterizing an average length of price contract of 3 quarters
σ	10	Following Benigno and Woodford (2003)
d_{ss}	2.4	steady-state debt to GDP of 60% (unless specified otherwise)
τ	0.2	steady-state taxes of 20% of GDP

by monopolistic competition is ruled out in the quantitative analysis. This is done by assuming a specific level of steady-state mark-up ($\mu = (1-\lambda)^{-1}$) that ensures efficiency in the steady-state level of output in the small economy. We impose this restriction in order to concentrate on the roles of distortionary taxation, sticky prices and the terms of trade externality.

3.5.1 The Case of Flexible Prices

In this Section, we start by considering the case in which prices are perfectly flexible (that is, $\alpha = 0$). Our objective is to understand the open economy dimension of the optimal (fiscal) problem. The assumption that $\alpha = 0$ implies that $\Phi_\pi = 0$. The loss function for our small open economy becomes

$$\min\ U_c\bar{C}E_{t_0}\sum\beta^t\left[\frac{1}{2}\Phi_Y\widehat{y}_t^2 + \frac{1}{2}\Phi_T\widehat{rs}_t^2\right] + t.i.p + O(||\xi||^3). \tag{3.12}$$

When prices are perfectly flexible, domestic producer inflation is not costly and, hence, it does not appear in the objective function. In this environment, policy-makers have the following policy incentives: (1) minimizing distortions created by distortionary taxation; (2) reducing inefficiencies in the steady-state level of output (implied by monopolistic competition); and (3) strategically managing the terms of trade. By using the relationship between distortionary taxes and output dictated by the Phillips curve, it is possible to rewrite the objective function (3.12) as

$$\min\ U_c\bar{C}E_{t_0}\sum\beta^t\left[\frac{1}{2}\Phi_\tau(\widehat{\tau}_t-\widehat{\tau}_t^{T'})^2+\frac{1}{2}\Phi_T\widehat{rs}_t^2\right]+t.i.p+O(||\xi||^3),$$

where $\Phi_\tau = -\frac{\omega}{\eta+\rho(1+l)^{-1}}\Phi_Y$. The constraints of the policy problem are given by the equilibrium conditions presented in Table 3.1 (under the assumption that $\alpha = 0$ and, thus, $k^{-1} = 0$). We define φ_t as the Lagrange multiplier associated with the government budget constraint. In other words, φ_t represents the marginal value, measured in utility terms, of one unit of the government revenue in any given period. As shown in the Appendix, the optimal plan can be summarized by the following conditions:

$$\Phi_T\widehat{rs}_t+\frac{(1+l)}{\rho(1-\lambda)}\Phi_Y\widehat{y}_t=-m_0\varphi_t+(a+1)d_{ss}(\varphi_t-\varphi_{t-1}), \tag{3.13}$$

$$-bd_{ss}(\varphi_t-\varphi_{t-1})=0, \tag{3.14}$$

and

$$E_t\varphi_{t+1}=\varphi_t, \tag{3.15}$$

where m_0 is defined in the Appendix. The above equations represent the small open economy optimal plan under price flexibility. As shown in the Appendix, we impose further constraints, associated with initial conditions $\widehat{\pi}_{t_0}$ and $\widehat{RS}_{t_0}$, to ensure that the first order conditions to the problem are time invariant. This method follows Woodford's (1999) timeless perspective approach and ensures that the policy prescription does not constitute a time inconsistent problem.

Next, we analyse if these conditions deliver a *determinate* equilibrium for all the endogenous variables. Also, we assess under what circumstances the variables of interest for our small open economy inherit the stochastic properties of the exogenous shocks.

The Case of Nominal Government Debt

When inflation influences the burden of government debt (i.e. when $d_{ss} \neq 0$ and

$b \neq 0$), the optimal plan can be summarized by

$$\Phi_T \widehat{rs}_t + \frac{(1+l)}{\rho(1-\lambda)} \Phi_Y \widehat{y}_t = 0, \tag{3.16}$$

or, alternatively,

$$\Phi_T \widehat{rs}_t + \frac{(1+l)}{\rho(1-\lambda)} \Phi_\tau (\widehat{\tau}_t - \widehat{\tau}_t^{T'}) = 0. \tag{3.17}$$

Also, in this case, φ_t is time-invariant. The above equation can be interpreted as the small open economy targeting rule *à la* Gianonni and Woodford (2003) and Svensson (2003). The optimal plan prescribes stabilizing a linear combination of the output gap and the real exchange rate gap.

In this case, the evolution of expected producer inflation and government debt are indeterminate; we can only determine the evolution of real debt. This can be verified by inspection of the government budget constraint.

The Case in which the Government Solvency Condition is Independent of Inflation

When the government only issues real debt (i.e. $b = 0$), or the steady-state debt is zero (i.e. $d_{ss} = 0$), inflation does not affect the government budget constraint. Furthermore, under flexible prices, the system of equilibrium conditions specified in Table 3.1 is completely independent of domestic inflation. In other words, producer price inflation is *indeterminate.*

By combining equations (3.13) and (3.15) we obtain the following expression:

$$\Phi_T E_t \Delta \widehat{rs}_{t+1} + \frac{(1+l)}{\rho(1-\lambda)} \Phi_Y E_t \Delta \widehat{y}_{t+1} = 0. \tag{3.18}$$

what do you mean?

In this case, the optimal plan entails stabilizing expected movements in the output and real exchange rate gaps. Note that the above equation is no what do you mean?
Indeed, the equilibrium dynamics of the small open economy under the optimal plan are not *determined* by this equation together with the other expressions in Table 3.1.[7] Rather, Equation (3.18) is simply an equilibrium condition implied by the

[7] The determinancy conditions are checked numerically using the algorithm of King and Watson

optimal plan - it does not represent a policy rule that policymakers should follow.

In contrast to the case in which inflation affects the government budget constraint, government debt now follows a unit root process (see the Appendix). Moreover, taxes, output and the real exchange rate face permanent changes following a temporary shock to fiscal conditions. That is, these variables are *nonstationary.*

Assessing the International Dimension of Fiscal Policy under Flexible Prices

To gain some intuition on the open economy dimension of the fiscal policy, we can assume $\theta = \rho = 1$ and $\mu = 1/(1-\lambda)$. The first restriction implies that there are no trade imbalances, whilst the second implies that monopolistic distortions are at an efficient level from the small open economy's perspective. Under this specification, using a model with lump sum taxes, Galí and Monacelli (2005) and De Paoli (2004) find that the small open economy is isomorphic to a closed economy and that the flexible price allocation is equivalent to the constrained efficient allocation.

In this case, the real exchange rate vanishes from the loss function. Furthermore, the targeting rule for the small open economy is analogous to the closed economy case (i.e. the case in which $\lambda = 0$). In particular, Equation (3.16) is reduced to

$$\widehat{y}_t = (\widehat{\tau}_t - \widehat{\tau}_t^{T'}) = 0. \tag{3.19}$$

The output gap is fully stabilized (as in Benigno and Woodford (2003)) and the first best outcome can be achieved. Under these restrictions, there are no terms of trade externalities. This is because the assumption $\theta = \rho = 1$ implies that the small open economy is isolated with respect to terms of trade changes. Also, there are no steady-state distortions in the level of output, given the assumption of $\mu = 1/(1-\lambda)$. Finally, there are no welfare costs associated with inflation, because prices are perfectly flexible. Hence, there is only one policy incentive: to smooth taxes across states and across periods in order to minimize distortions in agents' labour-leisure decisions.

(1998).

Since inflation is not costly, it is possible to use unexpected variations in domestic prices in order to equilibrate the government budget constraint after idiosyncratic shocks. The resulting allocation is the same as the one that would prevail if lump-sum taxes were available. This result is consistent with the findings of Bohn (1990), Chari, Christiano and Kehoe (1991) and Benigno and Woodford (2003).

Figure (3.1) illustrates the optimal response to a positive productivity shock. The productivity shock generates a fiscal primary surplus. But unexpected deflation increases the real value of public debt in order to equilibrate the government's budget constraint. Hence, in this case, taxes can be perfectly smoothed across states and times and the welfare relevant output gap is closed.

On the other hand, when the steady-state level of debt is zero, the first best outcome cannot be achieved. In this case, the optimal plan implies

$$E_t \Delta \widehat{y}_{t+1} = E_t \Delta (\widehat{\tau}_{t+1} - \widehat{\tau}^T_{t+1}) = 0. \tag{3.20}$$

Even though tax smoothing is the only policy objective, this cannot be implemented. Taxes have to adjust when disturbances affect the government budget constraint. When $d_{ss} = 0$, inflation cannot act as a "shock absorber" because it has no impact on the fiscal side of the economy. This result is consistent with the findings of Barro (1979) and Aiyagari et al. (2002). As shown in Figure (3.2), the tax rate follows a random walk. Taxes vary across states, but they remain constant after the shock hits the economy. In other words, the best policy available entails a "jump" in the tax rate in order to adjust the level of the primary surplus after the shock. Subsequently, taxes are kept constant so as to minimize distortions in the trade-off between consumption and labour.

We now examine the case in which $\theta \neq 1$ and $\rho \neq 1$. This allows an analysis of the open economy dimension of the stabilization problem. This specification allows for trade imbalances and introduces a terms of trade externality into the policy problem. In this case, the optimal plan is given by Equation (3.17). Hence, it is no longer optimal to keep taxes constant because of the incentive to exploit the real exchange rate externality.

Figure (3.3) illustrates the optimal response to a positive productivity shock when $\theta \neq 1$ and $\rho \neq 1$. Following the shock we observe an increase in the level of taxes, a fall in domestic output and a depreciation in the real exchange rate. But higher taxation limits the real exchange rate depreciation (that is, the real exchange rate would have depreciated by even more had taxes remained constant). The relatively higher level of the real exchange rate diverts domestic consumption toward foreign-produced goods. Hence, varying taxes reduces the disutility of domestic production without an equivalent fall in the utility of consumption.

In the small open economy, movements in the real exchange rate help to redirect demand towards foreign produced goods. This is because changes in the real exchange rate can improve the purchasing power of domestic households and can modify the relative price of domestic goods versus foreign goods. Note that, when $\rho = \theta = 1$, the income and the substitution

3.5.2 The Case of Sticky Prices

We now turn to the optimal policy problem in the case of sticky prices (i.e. $\alpha > 0$). We characterize the general optimal fiscal and monetary plans, in which there are two policy instruments (inflation and taxes), and four policy objectives: minimizing the distortions created by sticky prices and taxation; managing the real exchange rate; and reducing steady-state inefficiencies generated by monopolistic competition. As before, we allow for different types of bond denominations.

The policy problem consists of choosing the path of $\{\widehat{\pi}_t^H, \widehat{Y}_t, \widehat{RS}_t, \widehat{d}_t, \widehat{\tau}_t\}$ so as to minimize (1.29), subject to the following: the equilibrium conditions specified in Table 3.1: the initial condition for $\widehat{d}_{t_0-1}$; and the constraints on $\widehat{\pi}_{t_0}$ and $\widehat{RS}_{t_0}$ that ensure a time-invariant policy problem *à la* Woodford (2003) (see Appendix for details).

As before, we express the optimal state-contingent response to shocks in the form of targeting rules. In particular, the optimal plan can be written as follows:[8]

$$\left[\frac{(1+l)\Phi_y}{(1-\lambda)\rho}\right]\Delta\widehat{y}_t + \Phi_T\Delta\widehat{rs}_t + \left[\frac{k\Phi_\pi}{(1-\overline{\tau})+bd_{ss}k}\right](\gamma\widehat{\pi}_t^H + d_{ss}(a+1)\widehat{\pi}_{t-1}^H) = 0, \quad (3.21)$$

and

$$E_t\widehat{\pi}_{t+1}^H = 0, \quad (3.22)$$

where $\gamma = \left[d_{ss}\left(\frac{(1-\beta)}{(1-\lambda)} - (1+a)\right) + \left(\frac{(1+l)(\eta(1-\overline{\tau})-\overline{\tau})}{\rho(1-\lambda)} + \frac{1-\overline{\tau}}{(1-\lambda)}\right)\right]$.

We first note that the variables of interest in this targeting rule are: current and past domestic producer inflation; the rate of change in the real exchange rate gap; and the rate of change of the output gap. Also, Equation (3.22) states that expected producer inflation is set to zero under the optimal plan.

We can compare the targeting rule (3.21) with the one in Chapter 1, in which there is no fiscal stabilization problem. The relative weights on the target variables are different now, since Φ_y, Φ_T, and Φ_π are affected by the degree of distortionary taxation in steady state. Moreover, past producer inflation enters the targeting

[8] See Appendix for a full derivation of the optimal policy problem.

criteria, except in the case of zero steady-state government debt.

Furthermore, Equations (3.21) and (3.22), together with the constraints specified in Table 3.1, imply that output, the real exchange rate and taxes follow nonstationary processes.[9] This result contrasts with the case of flexible prices. In the latter, the system is stationary when bonds are denominated in nominal terms and the steady-state debt is non-zero. In that case, domestic producer inflation varies freely in order to adjust the level of real debt. As a result, taxes, the output gap and the real exchange rate gap are stationary.

The Case in which a Small Open Economy is Isomorphic to a Closed Economy

To understand how the open economy dimension changes the stabilization problem under sticky prices, we first focus on the special case in which $d_{ss} = 0$ and $\theta = \rho = 1$ and $\mu = (1 - \lambda)^{-1}$. Under these assumptions, there are only two policy incentives: reducing the inefficiencies created by distortionary taxations; and minimizing the relative price distortion. These restrictions imply that the loss function is only affected by inflation and output gap variability. However, even though there are two policy incentives and two policy instruments, the first best outcome cannot be achieved. That is, it is not possible to keep inflation and taxes simultaneously constant across states and over time. In this case, the optimal plan implies

$$\omega E_t \Delta(\widehat{\tau}_{t+1} - \widehat{\tau}^{T'}_{t+1}) + k^{-1}\pi^H_t = 0. \tag{3.23}$$

Here taxes cannot be smoothed over time, which is different from the stabilization problem under flexible prices (this can be seen by comparing the above equation with Equation (3.20)). By inspection of the Phillips curve we note that, when prices are sticky, a permanent change in taxes would imply a nonstationary process for inflation (and an explosive path for the domestic price level).

Figure (3.4) illustrates the optimal response to a positive productivity shock for the special case in which the small open economy is isomorphic to a closed economy.

[9] See the Appendix for a detailed derivation of the dynamic properties of the variables of interest.

Taxes vary in order to satisfy the intertemporal solvency condition. And changes in taxes are accompanied by changes in inflation, as dictated by the Phillips curve. If, on the other hand, lump sum taxes were available, as in Chapter 1, these could adjust in response to the shock, closing the output gap and stabilizing domestic inflation.

Optimal Fiscal and Monetary Interactions in a Small Open Economy

When $\theta \neq 1$ and $\rho \neq 1$, in our small open economy, there is an incentive to divert production toward the rest of the world, as we have emphasized before. If we impose this assumption along with $d_{ss} = 0$, Equations (3.21) and (3.22) imply that

$$\Phi_T E_t \Delta \widehat{rs}_{t+1} + \frac{(1+l)}{\rho(1-\lambda)} \Phi_Y E_t \Delta \widehat{y}_{t+1} = 0, \tag{3.24}$$

and the optimal plan also specifies that

$$E_t \widehat{\pi}_{t+1} = 0. \tag{3.25}$$

Equation (3.24) is identical to Equation (3.18) obtained under flexible prices. When there is no nominal burden from existing debt (i.e. $d_{ss} = 0$), the optimal policy under both flexible and sticky prices therefore prescribes the stabilization of the expected growth rate of the output gap and expected change in the real exchange rate gap. The difference between these two cases arises in the use of the two stabilization tools (i.e. inflation and taxes). As shown in Figure (3.5), taxes are more volatile under flexible prices than under sticky prices. This happens because under flexible prices domestic producer inflation does not affect the supply of home produced goods. Thus, inflation cannot be used as an instrument to redirect production toward the rest of the world. Hence, only taxes can be used as a policy instrument to exploit the terms of trade externality.

Figure (3.6) compares the closed and open economy cases. Whereas in a closed economy taxes are procyclical, the incentive to divert production makes taxes move in a countercyclical way in our small open economy.

Figure (3.7) contrasts the case of flexible and sticky prices under the assumption that $d_{ss} \neq 0$. Under sticky prices, domestic producer inflation is costly and therefore it varies less under the optimal plan. However, in an open economy framework with price stickiness, the variability of taxes is also reduced when compared to the case in which prices are flexible. As mentioned earlier, taxes move less because domestic producer inflation can be used to manipulate the consumption-leisure choice.

From a quantitative point of view, however, our framework suggests that the cost of inflation will overshadow the inefficiency caused by varying distortionary taxation and, therefore, changes in domestic producer inflation are quantitatively small. Note that this result holds even in a model with real bonds, and is a consequence of the conflict between price stability and the incentive to strategically affect the real exchange rate. This is different, however, from the trade-off (emphasized by Benigno and Woodford (2003), Schmitt-Grohé and Uribe (2004) and Siu (2001)) between price stability and the use of inflation as insurance that arises only in models in which the government issues nominal bonds.

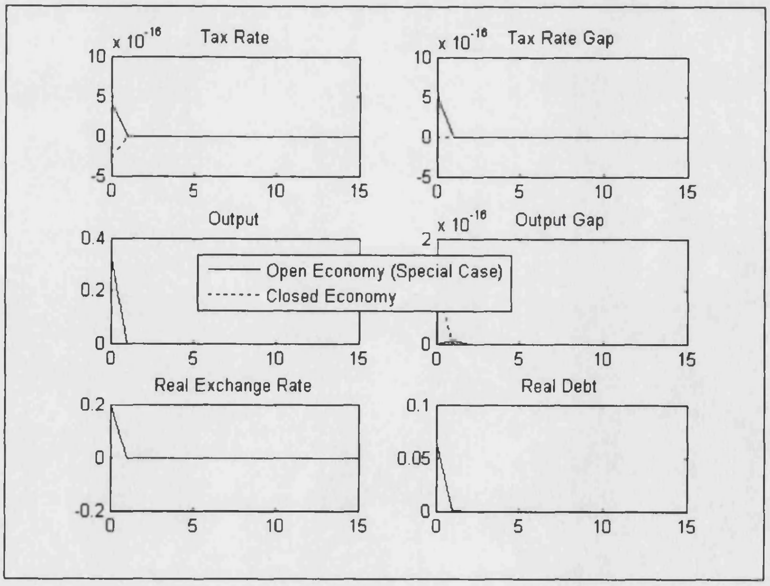


Figure 3.1: Impulse responses following a productivity shock- the case of flexible prices, nominal bonds and $\rho = \theta = 1$

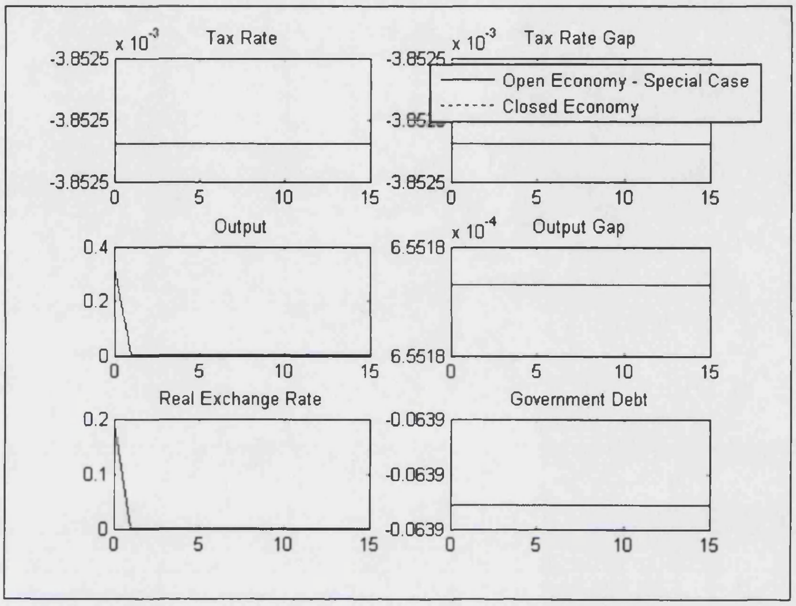


Figure 3.2: Impulse responses following a productivity shock- the case of flexible prices, $\rho = \theta = 1$ and $d_{ss} = 0$

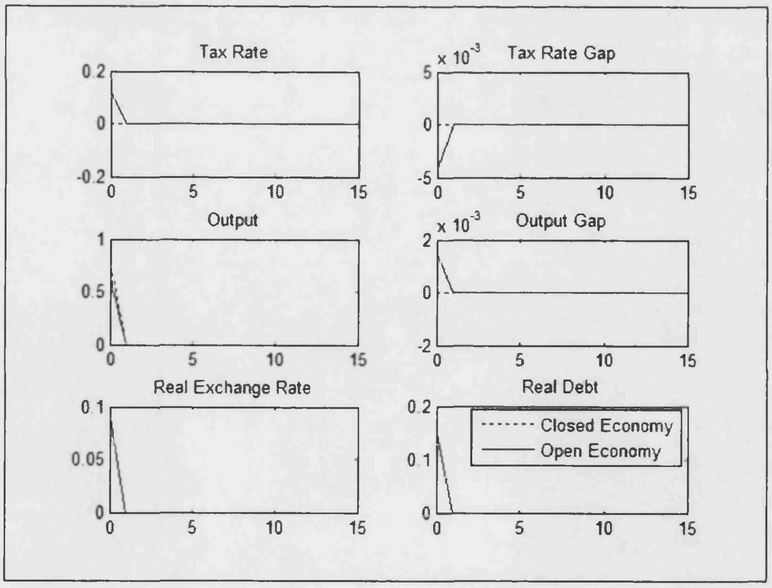


Figure 3.3: Impulse responses following a productivity shock- the case of flexible prices and nominal bonds

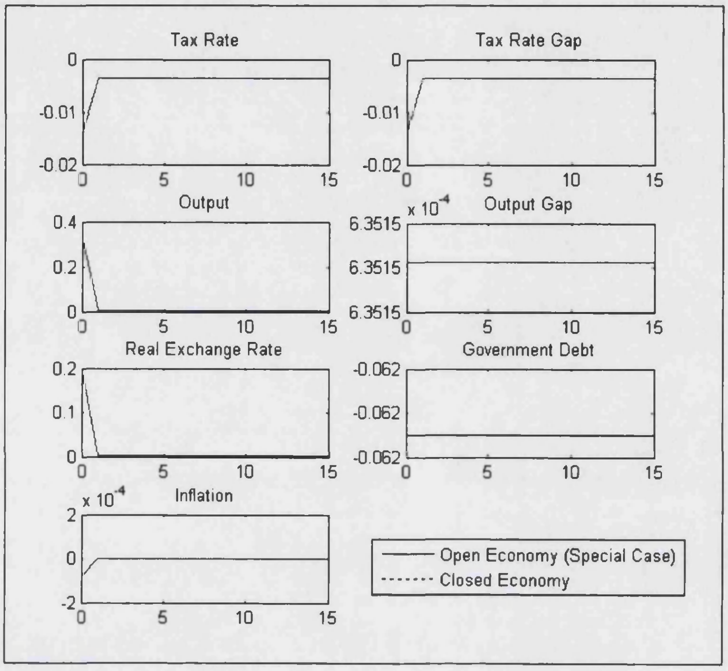


Figure 3.4: Impulse responses following a productivity shock- the case of sticky prices, $\rho = \theta = 1$ and $d_{ss} = 0$

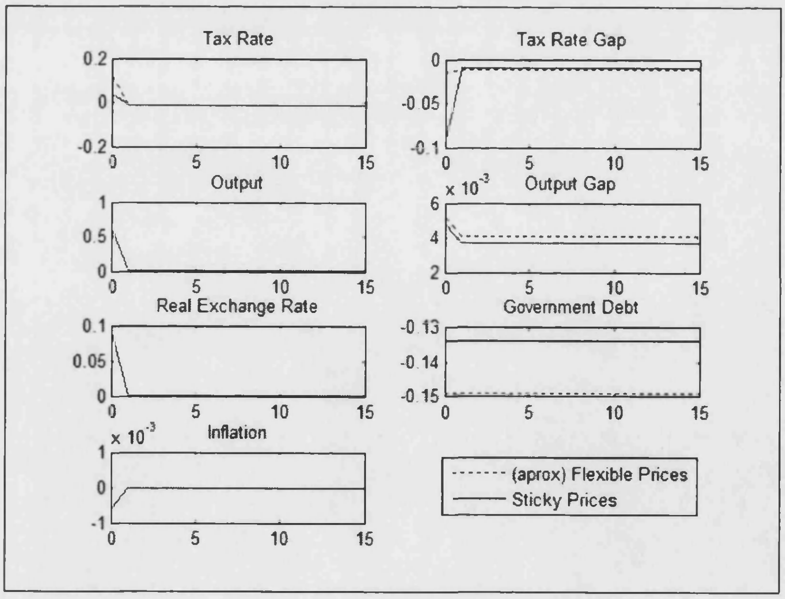


Figure 3.5: Impulse responses following a productivity shock- the case of $d_{ss} = 0$

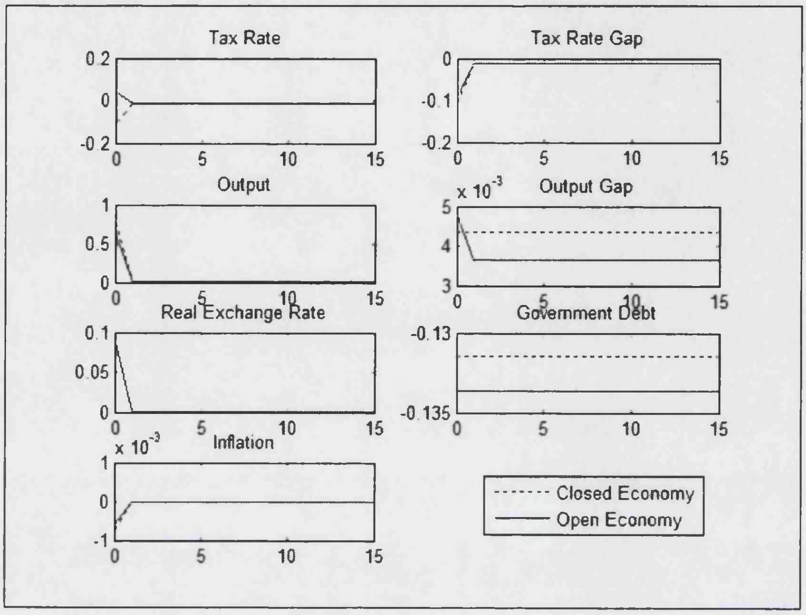


Figure 3.6: Impulse responses following a productivity shock- the case of sticky prices and $d_{ss} = 0$

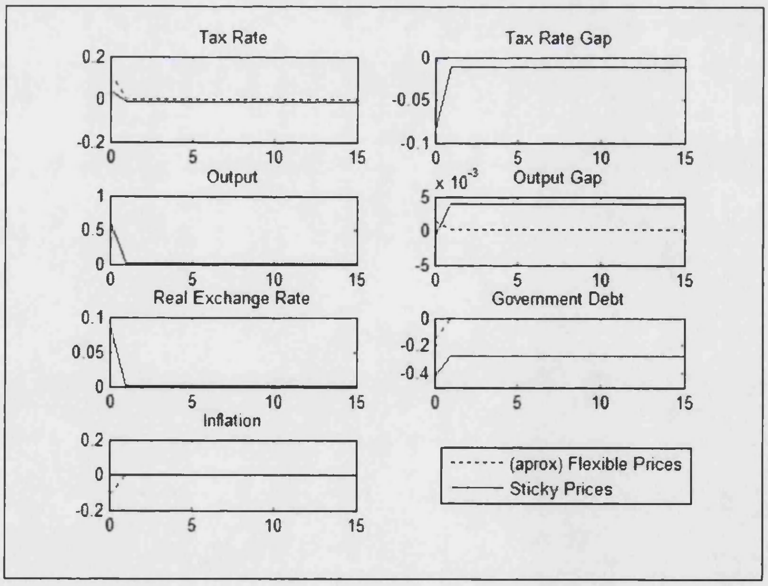


Figure 3.7: Impulse responses following a productivity shock- the case of nominal bonds

3.6 Optimal Inflation and Tax Variability

In the previous Section, we analysed the main policy incentives for a small open economy when fiscal and monetary policy tools are available. We now explore how the degree of openness and nominal rigidities affect the volatility of taxes, the real exchange rate and domestic producer inflation under the optimal plan. To do so, we use the parameter values specified in Table 3.1. For the calibration of the shocks, we follow Galí and Monacelli (2005), who fit an AR(1) processes to (log) labor productivity in Canada (their proxy for domestic productivity). Using quarterly, HP-filtered data over the sample period 1963:1 2002:4, the authors obtain the following estimates: $\widehat{\varepsilon}_{Y,t} = 0.66(0.06)\widehat{\varepsilon}_{Y,t-1} + a_t$ and $\sigma_{a_t} = 0.0071$.

We compute the moments based on these Monte Carlo simulations because, under certain specifications, our model is nonstationary. We first generate simulated time series of length T for the variables of interest and compute the standard deviation. We repeat this procedure J times and then compute the average of the moments. We set T equal to 400 quarters and J equal to 500.

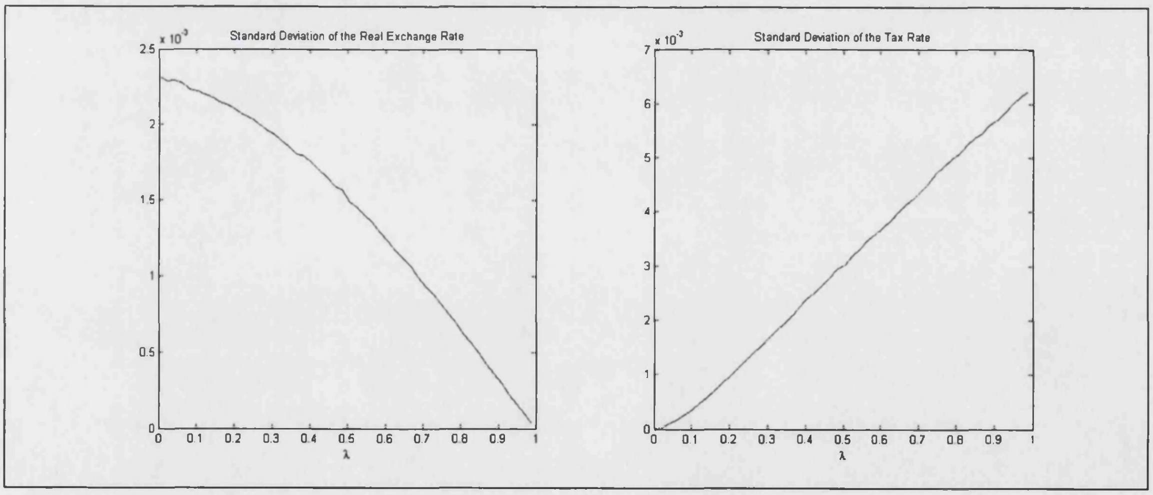


Figure 3.8: Optimal Volatilities under Flexible prices

previous Section suggests that the degree of openness has direct implications for the optimal volatilities of taxes and the real exchange rate. In particular, whereas taxes are perfectly smoothed in closed economies, this is not the case in a small open economy. Figure (3.8) displays the relationship between the sample standard deviation of our variables of interest and the openness parameter, λ. Higher levels of openness imply higher tax variability and lower real exchange rate volatility. This is because the incentive to use taxes to affect the real exchange rate increases as the economy becomes more open.

When nominal price rigidities are introduced, both taxes and inflation can affect agents' labour supply, and therefore, both instruments can be used to exploit the terms of trade externality. Hence, it is interesting to see how the degree of nominal rigidities affects the variability of taxes and inflation. Figure (3.9) presents the optimal volatility of taxes and domestic producer inflation for the parametrization specified in Table 3.1. Not surprisingly, optimal inflation volatility is decreasing in the degree of nominal rigidity. On the other hand, the volatility of taxes initially decreases with α and then rises as the degree of price rigidity becomes extreme. When price rigidities are introduced, inflation can be used to affect the real exchange rate, and this fact reduces the required movement in taxes. But for significantly high levels of price rigidity, this conclusion does not hold. In these cases, inflation is practically constant (given the welfare costs associated with price dispersion) and only taxes are used to exploit the terms of trade externality.

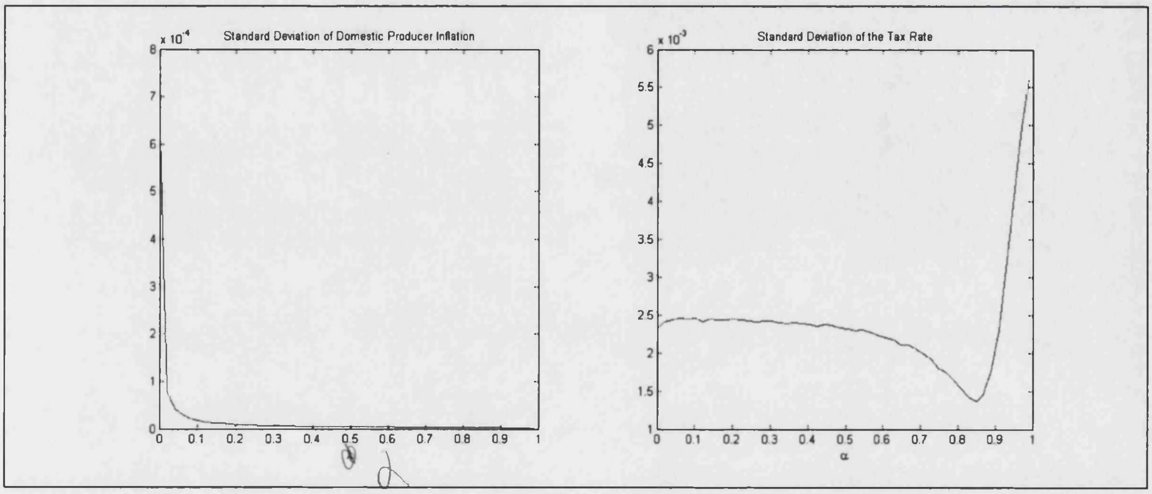


Figure 3.9: Optimal Volatilities

3.7 Conclusion

This Chapter presents an integrated analysis of fiscal and monetary policy in a small open economy. The literature on optimal policy in open economies has extensively analysed the monetary stabilization problem when inflation is costly and taxation is non-distortionary. In this Chapter, we start our analysis by characterizing the opposite scenario. That is, we study the optimal policy problem in an environment in which prices are perfectly flexible (and therefore inflation is costless) and production taxation affects households labour-leisure trade off (i.e. taxes are distortionary). We lay out this specification in order to highlight the international dimension of fiscal policy. Our results show that, whereas it is optimal to perfectly smooth taxes in a closed economy, the optimal tax rate varies over tim rigidity reduces the required variability of tax rates. But the presence of nominal rigidities also reduces the policy incentive to use inflation to affect the level of real government debt. Consequently, the variability of inflation is also smaller when nominal rigidities are present.

Our quantitative exercise shows that the optimal response to a positive productivity shock in a closed economy implies a fall in taxes (i.e. fiscal policy is procyclical). On the other hand, in our small open economy, under the benchmark specification for the parameter values, the optimal plan prescribes a countercyclical fiscal response to the productivity shock. In this case, higher taxation reduces the positive impact of the technological shock on output.

Finally, we follow the approach of Schmitt-Grohé and Uribe (2005) and calculate the second moments of the variables based on Monte Carlo simulations. We show that the optimal variability of taxes increases with the degree of openness. This is because the higher is the degree of openness, the larger is the incentive to use taxes to exploit the terms of trade externality. On the other hand, the introduction of price stickiness initially reduces, and then increases, tax variance. This is a result of the optimal trade off between using taxes and inflation to affect the terms of trade.

An interesting exercise could be to investigate empirically the prediction of the model. In particular, one could examine the variability of taxes in countries with different levels of openness.

3.A Appendix

We derive the second order approximation to the equilibrium conditions of the model. We assume that in steady state $\overline{G} = \overline{Tr} = 0$, $1 + i_t = 1 + i_t^* = 1/\beta$ and $P_t^H/P_{t-1}^H = P_t^F/P_{t-1}^F = 1$. We normalize the price index such that $\overline{P}_H = \overline{P}_F$. In addition, we assume an initial level of wealth such that $\overline{C} = \overline{C}^*$. The approximation for the demand equation, the risk sharing conditions and the price index are described in Sections (1.B.1), (1.B.2) and (1.B.3) of Chapter 1. In this Appendix, we derive the second order approximation to the price setting with endogenous production taxation, the government budget constraint, and the utility function. As in the previous Chapter, we use isoelastic function, forms for the utility function with ρ representing the coefficient of risk aversion and η the inverse of the elasticity of goods production.

3.A.1 Price Setting

The first and second-order approximations to the price setting equation follow Benigno and Benigno (2003). The introduction of the tax component is done in the same manner as in Benigno and Woodford (2003). The optimal price setting condition of sellers that can reset their prices is

$$E_t \left\{ \sum (\alpha\beta)^{T-t} U_c(C_T) \left(\frac{\tilde{p}_t(h)}{P_{H,T}} \right)^{-\sigma} Y_{H,T} \left[\frac{\tilde{p}_t(h)}{P_{H,T}} \frac{P_{H,T}}{P_T} - \frac{\sigma mc_t V_y\left(\tilde{y}_{t,T}(h), \varepsilon_{Y,t}\right)}{(1-\sigma)(1-\tau_t) U_c(C_T)} \right] \right\} = 0, \tag{3.26}$$

where

$$\tilde{y}_t(h) = \left(\frac{\tilde{p}_t(h)}{P_{H,t}} \right)^{-\sigma} Y_{H,t}. \tag{3.27}$$

Income taxes are represented by $\tau_t$and mc_t is a markup shock. The evolution of the domestic price level is therefore

$$(P_{H,t})^{1-\sigma} = \alpha P_{H,t-1}^{1-\sigma} + (1-\alpha) \left(\tilde{p}_t(h)\right)^{1-\sigma}. \tag{3.28}$$

We can write the second order approximation to Equation (3.26) as follows:

$$V_o = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t z_t + \frac{1}{2} z_t X_t + \frac{1}{2} \frac{\sigma(1+\eta)}{k} \left(\pi_t^H\right)^2 \right\} + t.i.p + O(||\xi||^3), \tag{3.29}$$

where

$$z_t = \eta \widehat{Y}_t + \rho \widehat{C}_t - \widehat{p}_H + \widehat{mc}_t - \widehat{q}_t - \eta \widehat{\varepsilon}_{Y,t},$$

and

$$X_t = (2+\eta)\widehat{Y}_t - \rho \widehat{C}_t + \widehat{p}_H + \widehat{mc}_t + \widehat{q}_t - \eta \widehat{\varepsilon}_{Y,t}.$$

We define $q_t = 1 - \tau_t$ and, therefore,

$$\widehat{q}_t = -\omega \widehat{\tau}_t - \frac{1}{2} \frac{\omega}{1-\overline{\tau}} \widehat{\tau}_t^2 + O(||\xi||^3),$$

where $\omega = \frac{\overline{\tau}}{1-\overline{\tau}}$.

The first order approximation to the price setting equation can be written in the following way:

$$\widehat{\pi}_t^H = k\left(\rho \widehat{C}_t + \eta \widehat{Y}_t - \widehat{p}_H + \widehat{mc}_t + \omega \widehat{\tau}_t - \eta \widehat{\varepsilon}_{Y,t}\right) + \beta E_t \widehat{\pi}_{t+1}^H, \tag{3.30}$$

where $k = (1-\alpha\beta)(1-\alpha)/\alpha(1+\sigma\eta)$.

And the second order approximation to the price setting can be written as follows:

$$Q_{to} = \phi \sum E_t \beta^t \left[a_y' y_t + \frac{1}{2} y_t' A_y y_t + y_t' A_e e_t \right] + t.i.p + O(||\xi||^3), \tag{3.31}$$

with

$$a_y' = \begin{bmatrix} \eta & \rho & -1 & \omega & 0 \end{bmatrix},$$

$$A'_y = \begin{bmatrix} \eta(2+\eta) & \rho & -1 & \omega & 0 \\ \rho & -\rho^2 & \rho & -\rho\omega & 0 \\ -1 & \rho & -1 & \omega & 0 \\ \omega & -\rho\omega & \omega & \omega & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

and

$$A'_e = \begin{bmatrix} -\eta(1+\eta) & 1+\eta & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

3.A.2 Government Budget Constraint

We assume at first that all public debt consists of riskless nominal one-period bonds. The law of motion of government debt is

$$D_t = D_{t-1}(1 + i_{t-1}) - P_{H,t}s_t,$$

where

$$s_t \equiv \tau_t Y_t - G_t - Tr_t.$$

Defining

$$d'_t \equiv \frac{D_t\,(1 + i_t)}{P_t},$$

we can rewrite the government budget constraint as

$$d'_t = d'_{t-1}\frac{(1 + i_t)}{\Pi_t} + \frac{P_{H,t}}{P_t}s_t(1 + i_t). \tag{3.32}$$

In log-linear terms the government budget constraint is given by

$$\beta \hat{d}'_t = \hat{d}'_{t-1} + \frac{\beta s^{-1}}{1-\beta} i_t - s^{-1}\hat{p}_{H,t} - s_\tau s^{-1}\left(\hat{Y}_t + \hat{\tau}_t\right) + \hat{g}_t + \widehat{Tr} - \frac{s^{-1}}{1-\beta}\left(\widehat{\pi}^H_t + \lambda/(1-\lambda)\Delta RS_t\right).$$

To derive a second order approximation to the intertemporal government solvency condition, we define,

$$NW_t \equiv \frac{d'_{t-1}}{\Pi_t} U_C\left(C_T, \xi_{C,T}\right), \tag{3.33}$$

and use the individual intertemporal choice (i.e. the Euler condition):

$$U_C\left(C_t\right) = (1+i_t)\beta E_t\left[U_C\left(C_{t+1}\right)\frac{P_t}{P_{t+1}}\right]. \tag{3.34}$$

Substituting the Euler equation for interest rates, Equation (3.32) implies:

$$NW_t = E_t \sum_{T=t}^{\infty} U_C\left(C_T, \xi_{C,T}\right) s_t p_{H,t}. \tag{3.35}$$

The second order approximation to condition (3.35) is

$$\begin{aligned} U_C\left(C_T, \xi_{C,T}\right) s_t p_{H,t} \quad &= \quad U_c \bar{s} \left\{ \begin{array}{c} 1 + s_\tau \widehat{Y} - \rho \widehat{C} + \widehat{p}_H + s_\tau \widehat{\tau} + \frac{1}{2} s_\tau \widehat{Y}^2 - \rho s_\tau \widehat{Y}\widehat{C} + s_\tau \widehat{Y}\widehat{p}_H \\ + s_\tau \widehat{\tau}\widehat{Y} + \frac{1}{2}\rho^2 \widehat{C}^2 + \frac{1}{2}\widehat{p}_H^2 + s_\tau \widehat{\tau}^2 \\ -\rho s_\tau \widehat{\tau}\widehat{C} + s_\tau \widehat{\tau}\widehat{p}_H + \rho s \widehat{g}\widehat{C} - s\widehat{p}_H\widehat{g} \end{array} \right\} \\ &\quad + t.i.p + O(||\xi||^3) \end{aligned}$$

Defining $\widetilde{NW}_t = \frac{NW_t - \overline{NW}}{\overline{NW}}$, we have:

$$\widetilde{NW}_t = (1-\beta)\left[b'_y y_t + \frac{1}{2} y'_t B_y y_t + y'_t B_e e_t\right] + \beta E_t \widetilde{NW}_{t+1} + t.i.p + O(||\xi||^3),$$

$$b'_y = \left[\begin{array}{ccccc} s_\tau & -\rho & 1 & s_\tau & 0 \end{array}\right],$$

$$B_y' = \begin{bmatrix} s_T & -\rho s_T & s_T & s_T & 0 \\ -\rho s_T & \rho^2 & 0 & -\rho s_T & 0 \\ s_T & 0 & 1 & s_T & 0 \\ s_T & -\rho s_T & s_T & s_T & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

and

$$B_e' = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \rho s & 0 \\ 0 & 0 & -s & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Note that

$$\pi_t = \widehat{\pi}_t^H + \frac{\lambda}{1-\lambda}\Delta RS_t.$$

The first order approximation to condition (3.33) is therefore

$$\widetilde{NW}_t = -\rho\widehat{C}_t + \widehat{d}_{t-1} - \widehat{\pi}_t = -\rho\widehat{C}_t + \widehat{d}_{t-1} - (\frac{\lambda}{1-\lambda}\Delta RS_t + \widehat{\pi}_t^H).$$

Hence, the first order approximation to the intertemporal budget constraint can be written as:

$$\begin{aligned} -\rho\widehat{C}_t + \widehat{d}_{t-1} - (\frac{\lambda}{1-\lambda}\Delta RS_t + \widehat{\pi}_t^H) &= (1-\beta)(-\rho\widehat{C}_t + \widehat{p}_{H,t} + s_T(\widehat{\tau}_t + \widehat{Y}_t) - s\widehat{g}_t) \\ &\quad + \beta E_t\left[-\rho\widehat{C}_{t+1} + \widehat{d}_t - (\frac{\lambda}{1-\lambda}\Delta RS_{t+1}^* + \widehat{\pi}_{t+1}^H)\right]. \end{aligned}$$

Throughout the text, we use an alternative representation of the budget constraint in order to allow for a zero steady-state government debt. The above equation is rescaled, using $\widehat{d}_t = d_{ss}\widehat{d}_t'$ (note that $s_T = \frac{\overline{\tau}}{d_{ss}(1-\beta)}$). The final expression is

$$-\rho d_{ss}\widehat{C}_t + \widehat{d}_{t-1} - (\frac{\lambda}{1-\lambda}d_{ss}\Delta RS_t + d_{ss}\widehat{\pi}_t^H) = (1-\beta)d_{ss}(-\rho\widehat{C}_t + \widehat{p}_{H,t}) + \overline{\tau}(\widehat{\tau}_t + \widehat{Y}_t) - \widehat{g}_t$$
$$+\beta E_t d_{ss}\left[-\rho\widehat{C}_{t+1} + \widehat{d}_t - (\frac{\lambda}{1-\lambda}\Delta RS^*_{t+1} + \widehat{\pi}^H_{t+1})\right]$$

An analogous derivation can be conducted in the case of **Real Bonds**. In this case, the flow government budget constraint is,

$$D_t^r = D_{t-1}^r(1 + r_{t-1}) - \frac{P_{H,t}}{P_t}s_t, \tag{3.36}$$

or, alternatively,

$$d_t^{r'} = d_{t-1}^{r'}(1 + r_t) + \frac{P_{H,t}}{P_t}s_t(1 + r_t), \tag{3.37}$$

where

$$d_t^{r'} \equiv D_t^r(1 + r_t).$$

In order to derive the second order approximation to the government budget constraint we use a the recursive formulation, in which

$$RW_t = d_{t-1}^{r'}U_C\left(C_t\right), \tag{3.38}$$

and

$$RW_t = E_t\sum_{T=t}^{\infty}\beta^{T-t}U_C\left(C_T\right)s_t p_{H,t}. \tag{3.39}$$

Defining $\widetilde{RW}_t = \frac{RW_t - \overline{RW}}{\overline{RW}}$, we have:

$$\widetilde{RW}_t = (1-\beta)\left[rb_y' y_t + \frac{1}{2}y_t' RB_y y_t + y_t' RB_e e_t\right] + \beta E_t\widetilde{RW}_{t+1} + t.i.p + O(||\xi||^3), \tag{3.40}$$

$$rb_y' = \begin{bmatrix} s_T & -\rho & 1 & s_T & 0 \end{bmatrix},$$

$$RB_y' = \begin{bmatrix} s_T & -\rho s_T & s_T & s_T & 0 \\ -\rho s_T & \rho^2 & 0 & -\rho s_T & 0 \\ s_T & 0 & 1 & s_T & 0 \\ s_T & -\rho s_T & s_T & s_T & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

and

$$RB_e' = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \rho s & 0 \\ 0 & 0 & -s & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The first order approximation to the intertemporal budget constraint is:

$$\begin{aligned} -\rho\widehat{C}_t + \widehat{d^r}'_{t-1} &= (1-\beta)(-\rho\widehat{C}_t + \widehat{p}_{H,t} + s_T(\widehat{\tau}_t + \widehat{Y}_t) - s\widehat{g}_t) \\ &\quad +\beta E_t\left[-\rho\widehat{C}_{t+1} + \widehat{d^r_t}'\right]. \end{aligned} \tag{3.41}$$

As it will be shown in the next Section, the welfare function depends only on rb_y', RB_y' and RB_e', which are equal to b_y', B_y' and B_e'. This implies that the loss function formulation is independent of the denomination of government debt. However, the first order approximation to the government budget constraint changes with the bond denomination. Hence, the constraint of the policy problem varies according to the type of bond being issued by the government.

Moreover, we can write the budget constraint as follows:

$$
\begin{aligned}
-\rho\widehat{C}_t + \widehat{d}^l_{t-1} - (a\Delta RS_t + b\widehat{\pi}^H_t) &= (1-\beta)(-\rho\widehat{C}_t - \frac{\lambda}{1-\lambda}RS_t + s_\tau(\widehat{\tau}_t + \widehat{Y}_t) - s\widehat{g}_t) \\
&+\beta E_t\left[-\rho\widehat{C}_{t+1} + \widehat{d}^l_t - (a\Delta RS_{t+1} + b\widehat{\pi}^H_{t+1})\right],
\end{aligned}
$$

where $a = \lambda/(1-\lambda)$ and $b = 1$ in the case of nominal bonds, and $a = b = 0$ in the case of real bonds (in this case $\widehat{d}^l_t = \widehat{d}^{r\prime}_t$). Alternatively, rescaling the above equation using $\widehat{d}^l_t = d_{ss}\widehat{d}_t$, we obtain the government budget constraint as specified in the text (see Table 3.1).

3.A.3 Welfare

Following Benigno and Benigno (2003), the second order approximation to the utility function can be written as:

$$
U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t}\left[U(C_s) - \frac{1}{n}\int_0^n V(y^j_s, \varepsilon_{Y,s})dj\right], \tag{3.42}
$$

$$
W_{to} = U_c\bar{C}E_{t_0}\sum \beta^t\left[w'_y y_t - \frac{1}{2}y'_t W_y y_t - y'_t W_e e_t - \frac{1}{2}w_\pi \pi^2_t\right] + t.i.p + O(||\xi||^3), \tag{3.43}
$$

where,

$$
w'_\pi = \frac{\sigma}{\mu k},
$$

$$
w'_y = \begin{bmatrix} -1/\mu & 1 & 0 & 0 & 0 \end{bmatrix},
$$

$$W_y' = \begin{bmatrix} \frac{(1+\eta)}{\mu} & 0 & 0 & 0 & 0 \\ 0 & -(1-\rho) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

and

$$W_e' = \begin{bmatrix} -\frac{\eta}{\mu} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Using the second order approximation to the equilibrium condition, we can eliminate the term $w_y' y_t$. Do do so, we will derive the vector Lx, such that:

$$\begin{bmatrix} a_y & d_y & f_y & c_y & b_y \end{bmatrix} Lx = w_y.$$

Given the values of a_y, b_y, f_y, c_y, and d_y defined in this Appendix, we have:

$$Lx_1 = \frac{\left((-1+\phi)(1-\theta\rho)\lambda^2 + (-(-1+\phi)2\theta\rho - 1 + 2\phi)\lambda - \phi\right)(-1+\tau)}{\Omega},$$

$$Lx_2 = \frac{\Sigma\rho(-1+\phi) + (\Psi(-1+\lambda))}{\Omega},$$

$$Lx_3 = \frac{(-1+\phi)\Sigma(1-\theta\rho)\lambda - \theta(-1+\phi)\Sigma\rho + \Psi\theta + \phi\Sigma}{\Omega},$$

and

$$Lx_5 = \frac{(-1+\phi)(1-\theta\rho)\lambda^2 + (-(-1+\phi)2\theta\rho - 1 + 2\phi)\lambda - \phi}{\Omega},$$

where, $\Psi = ((\eta+1)\tau - \eta)$, $\Sigma = (-1 + \tau - d_{ss} + d_{ss}\beta)$, $\Omega = -\Psi l - \Sigma\rho - \Psi$, $l = (\rho\theta - 1)\lambda(2-\lambda)$ and $1 - \phi = 1/\mu$.

The loss function L_{to} can be written as follows:

$$L_{to} = U_c \bar{C} E_{t_0} \sum \beta^t \left[\frac{1}{2} y_t' L_y y_t + y_t' L_e e_t + \frac{1}{2} l_\pi \pi_t^2 \right] + t.i.p + O(||\xi||^3), \tag{3.44}$$

where,

$$L_y = W_y + Lx_1 A_y + Lx_2 D_y + Lx_3 F_y + Lx_5 B_y,$$

$$L_e = W_e + Lx_1 A_e + Lx_2 D_e,$$

and

$$L_\pi = w_\pi + Lx_1 a_\pi.$$

Note that Lx_4 is irrelevant since $C_y = 0$

To write the model just in terms of the output, real exchange rate, taxes and inflation, we define the matrices N and N_e that map all endogenous variables into $[Y_t, T_t]$ and the errors in the following way:

$$y_t' = N \left[Y_t, RS_t, \tau_t \right] + N_e e_t, \tag{3.45}$$

where

$$N = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -\frac{l+\lambda}{\rho(1-\lambda)} & 0 \\ 0 & -\frac{\lambda}{(1-\lambda)} & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix},$$

and

$$N_e = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Equation (3.44) can therefore be expressed as

$$\begin{aligned} L_{to} &= U_c \bar{C} E_{t_0} \sum \beta^t \left[\frac{1}{2} \left[\widehat{Y}_t, \widehat{RS}_t, \widehat{\tau}_t \right]' L_y' \left[\widehat{Y}_t, \widehat{RS}_t, \widehat{\tau}_t \right] + \left[\widehat{Y}_t, \widehat{RS}_t, \widehat{\tau}_t \right]' L_e' e_t + \frac{1}{2} l_\pi \pi_t^2 \right] \\ &\quad + t.i.p + O(||\xi||^3), \end{aligned} \tag{3.46}$$

where,

$$L_y' = N' L_y N,$$

and

$$L_e' = N' L_y N_e + N' L_e.$$

The last step is to eliminate the cross variables terms $\widehat{Y}_t \widehat{RS}_t$. For that we use the following identity (derived from combining the demand function with the risk sharing condition):

$$2\widehat{Y}_t \widehat{RS}_t = \frac{\rho(1-\lambda)}{(1+l)} \widehat{Y}_t^2 + \frac{(1+l)}{\rho(1-\lambda)} \widehat{RS}_t^2 + t.i.p + O(||\xi||^3), \tag{3.47}$$

and, therefore,

$$\left[\widehat{Y}_t, \widehat{RS}_t, \widehat{\tau}_t\right]' L_y' \left[\widehat{Y}_t, \widehat{RS}_t, \widehat{\tau}_t\right] = \left[\widehat{Y}_t, \widehat{RS}_t, \widehat{\tau}_t\right]' \begin{bmatrix} l_{yy} & l_{yt} & 0 \\ l_{yr} & l_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix} \left[\widehat{Y}_t, \widehat{RS}_t, \widehat{\tau}_t\right]$$
$$= (l_{yy} + \frac{\rho(1-\lambda)}{(1+l)} l_{yt}) \widehat{Y}_t^2 + (l_{tt} + \frac{(1+l)}{\rho(1-\lambda)} l_{yr}) \widehat{RS}_t^2 + t.i.p + O(||\xi||^3). \quad (3.48)$$

Substituting (3.48) into (3.46), we have:

$$L_{to} = U_c \bar{C} E_{t_0} \sum \beta^t \begin{bmatrix} \frac{1}{2}(l_{yy} + \frac{\rho(1-\lambda)}{(1+l)} l_{yt}) \widehat{Y}_t^2 + (l_{tt} + \frac{(1+l)(1-\lambda)}{\rho} l_{yr}) \widehat{RS}_t^2 \\ + \left[\widehat{Y}_t, \widehat{RS}_t\right]' L_e' e_t + \frac{1}{2} l_\pi \pi_t^2 \end{bmatrix} + t.i.p + O(||\xi||^3). \quad (3.49)$$

Finally, we rewrite the previous equation as deviations from the target variables:

$$L_{to} = U_c \bar{C} E_{t_0} \sum \beta^t \left[\frac{1}{2}\Phi_Y (\widehat{Y}_t - \widehat{Y}_t^T)^2 + \frac{1}{2}\Phi_T (\widehat{RS}_t - \widehat{RS}_t^T)^2 + \frac{1}{2}\Phi_\pi (\widehat{\pi}_t^H)^2\right] + t.i.p + O(||\xi||^3), \quad (3.50)$$

where,

$$\begin{aligned} \Phi_Y = {} & Lx5 \left\{ -2\rho\tau + \rho^2 d_{ss}(1-\beta) + \tau + \frac{\left[\rho\tau - \rho^2 d_{ss}(1-\beta)\right](l+\lambda) - \tau\lambda\rho}{1+l} \right\} \\ & + Lx2 \left\{ (1-\lambda)\lambda + \frac{(-\lambda)(l+\lambda)(1-\lambda) - \theta\lambda(1-\lambda)^2 \rho}{1+l} \right\} \\ & + Lx1 \left\{ 2\rho - \rho^2 + (2+\eta)\eta + \frac{-(\rho-\rho^2)(l+\lambda) - (-1+\rho)\lambda\rho}{1+l} \right\} \\ & + (\eta+1)(1-\phi) - 1 - \frac{(-1+\rho)(l+\lambda)}{1+l} + \rho 2, \end{aligned}$$

$$\begin{aligned}\Phi_{RS} &= Lx5\left\{\frac{d_{ss}\left(1-\beta\right)\left[\left(l+\lambda\right)^2+\lambda^2-\left(1+l\right)\left(l+\lambda\right)\right]+\frac{1}{\rho}\left(1+l\right)l\tau}{\left(1-\lambda\right)^2}\right\}\\ &+Lx3\left\{\frac{\lambda\left(\theta-1\right)}{1-\lambda}\right\}\\ &+Lx2\left\{\frac{\left(l+\lambda\right)\lambda\left(\theta-\frac{1}{\rho}\right)-\lambda\theta\left(1-\lambda\right)}{\rho}+\theta^2\lambda\left(1-\lambda\right)\right\}\\ &+Lx1\left\{\frac{l\left(l+2\lambda\right)+\frac{1+l}{\rho}l\left(\rho-1\right)}{\left(1-\lambda\right)^2}\right\}\\ &-\frac{\left(l+\lambda\right)\left(-1+\rho\right)}{\left(1-\lambda\right)\rho^2},\end{aligned}$$

$$\Phi_\pi = \frac{\sigma(1-\phi)}{k} + (1+\eta)\frac{\sigma}{k}Lx_1,$$

$$\widehat{Y}_t^T = q_y^e e_t,$$

and

$$\widehat{RS}_t^T = q_{rs}^e e_t,$$

with

$$q_y^e = \frac{1}{\Phi_T}\left[\ \frac{\eta}{\mu} + Lx_1(1+\eta)\eta \quad -Lx_1(1+\eta) \quad -q_{yg}^e \quad Lx2\left(1-\lambda\right)\lambda\ \right],$$

$$\begin{aligned}q_{yg}^e &= Lx5\left\{\rho\left(1+\tau\right)-\rho^2 dss\left(1-\beta\right)\right\}\\ &+Lx1\left\{\rho\left(\rho-1\right)\right\}\\ &+Lx2\left\{\lambda^2-1\right\}\\ &+1-\rho,\end{aligned}$$

$$q_t^e = \frac{1}{\Phi_T}\left[\begin{array}{cccc} 0 & 0 & -q_{rg}^e & -Lx2\left\{\frac{(l+\lambda)\lambda}{\rho}+\theta\lambda\left(1-\lambda\right)\right\}\end{array}\right],$$

and

$$\begin{aligned} q_{rg}^e &= Lx5\left\{\frac{(l+\lambda)\,\rho dss\,(1-\beta)-l}{1-\lambda}\right\} \\ &\quad +Lx2\left\{\frac{(l+\lambda)\,(1+\lambda)}{\rho}-\frac{\lambda\theta}{1-\lambda}-\theta\lambda^2\right\} \\ &\quad +Lx1\left\{\frac{-l\rho}{1-\lambda}\right\} \\ &\quad +\frac{(l+\lambda)\,(-1+\rho)}{\rho\,(1-\lambda)}. \end{aligned}$$

Special Case

***Assumptions: $\rho=\theta=1$ and $\phi=\lambda$**

In this case, the weights in the loss function are

$$\frac{\Phi_Y}{1-\lambda}=(\eta+1),$$

$$\frac{\Phi_{RS}}{1-\lambda}=0,$$

$$\frac{\Phi_\pi}{1-\lambda}=\frac{\sigma}{k},$$

and the target output is

$$\widehat{Y}_t^T=q_y^e e_t,$$

where

$$q_y^e=\frac{1}{1+\eta}\left[\begin{array}{cccc}\eta & 0 & -1 & 0\end{array}\right].$$

***Note:** In the text we use the above specification (as in Galí and Monacelli, 2005). However, by inspection of the weights presented in this Appendix, we can verify that the necessary conditions for a zero weight of the exchange rate in the loss function are $\rho\theta=1$ and $d_{ss}(\phi-\lambda)=0$.

3.A.4 Optimal Fiscal Policy under Flexible Prices

The optimal policy can be represented by the following Lagrangian:

$$\mathcal{L} = E_{t_0} \sum \beta^{t-t_0} \left[\begin{array}{c} \frac{1}{2}\Phi_Y(\widehat{Y}_t - \widehat{Y}_t^T)^2 + \frac{1}{2}\Phi_T(\widehat{RS}_t - \widehat{RS}_t^T)^2 + \varphi_{1,t}\left(\eta \hat{Y}_t + (1-\lambda)^{-1}\widehat{RS}_t - \omega\widehat{\tau}_t\right) + \\ +\varphi_{2,t} \left(\begin{array}{c} -d_{ss}\widehat{RS}_t + \widehat{d}_{t-1} - d_{ss}(a\Delta RS_t + b\widehat{\pi}_t^H) + d_{ss}(1-\beta)(\frac{1}{1-\lambda}\widehat{RS}_t) \\ -\overline{\tau}(\widehat{\tau}_t + \widehat{Y}_t) + \beta E_t\left[d_{ss}\widehat{RS}_{t+1} - \widehat{d}_t + d_{ss}(a\Delta RS_{t+1} + b\widehat{\pi}_{t+1}^H)\right] \end{array} \right) \\ +\varphi_{3,t}\left(\widehat{Y}_t - \frac{(1+l)}{\rho(1-\lambda)}\widehat{RS}_t\right) \\ +bdss\varphi_{2,t_0-1}\widehat{\pi}_{t_0}^H + d_{ss}(a+1)\varphi_{2,t_0-1}\widehat{RS}_{t_0}^H \end{array} \right.$$

$$+t.i.p + O(||\xi||^3).$$

The last line of the Lagrangian contains the constraints for the optimal policy that ensure that the problem is time invariant. Note that, in the text, we denote the Lagrange multiplier on the government budget constraint as φ_t. In this Appendix, we use the notation $\varphi_{2,t}$.

These are the first order conditions:

$$-bd_{ss}(\varphi_{2,t} - \varphi_{2,t-1}) = 0, \tag{3.51}$$

$$\Phi_y(\widehat{Y}_t - \widehat{Y}_t^T) + \eta\varphi_{1,t} - \overline{\tau}\varphi_{2,t} + \varphi_{3,t} = 0, \tag{3.52}$$

$$\begin{aligned} &\Phi_y(\widehat{RS}_t - \widehat{RS}_t^T) + \frac{1}{(1-\lambda)}\varphi_{1,t} - d_{ss}(a+1)(\varphi_{2,t} - \varphi_{2,t-1}) \\ &+d_{ss}\frac{(1-\beta)}{(1-\lambda)}\varphi_{2,t} + \beta a d_{ss}(E_t\varphi_{2,t+1} - \varphi_{2,t}) - \frac{(1+l)}{\rho(1-\lambda)}\varphi_{3,t} = 0, \end{aligned} \tag{3.53}$$

$$\omega\varphi_{1,t} - \overline{\tau}\varphi_{2,t} = 0, \tag{3.54}$$

and

$$-\varphi_{2,t} + E_t\varphi_{2,t+1} = 0. \tag{3.55}$$

In addition, the first order condition at time t_0 implies $bd_{ss}(\varphi_{2,t_0} - \varphi_{2,t_0-1}) = 0$. Substituting $\hat{C}_t = \hat{C}_t^* + \frac{1}{\rho}\widehat{RS}_t$ into the government budget constraint, we have

$$\begin{aligned} -\rho d_{ss}\hat{C}_t^* - d_{ss}\widehat{RS}_t + \hat{d}'_{t-1} - d_{ss}(a\Delta\widehat{RS}_t + b\hat{\pi}_t^H) = \\ d_{ss}(1-\beta)(-\rho\hat{C}_t^* - \frac{1}{1-\lambda}\widehat{RS}_t) + \tau(\hat{\tau}_t + \hat{Y}_t) - \hat{g}_t \\ +\beta E_t\left[-\rho d_{ss}\hat{C}_{t+1}^* - d_{ss}\widehat{RS}_{t+1} + \hat{d}'_t - d_{ss}(a\Delta\widehat{RS}_{t+1} + b\hat{\pi}_{t+1}^H)\right]. \end{aligned} \tag{3.56}$$

Furthermore, under the assumption that $\alpha = 0$, the Phillips curve implies

$$-\omega^{-1}\left(\frac{\eta(1+l)+\rho}{(1+l)}\right)(\hat{Y}_t - \widehat{Y}_t^T) = (\widehat{\tau}_t - \widehat{\tau}_t^{T'}). \tag{3.57}$$

By integrating Equation (3.56) forward we can rewrite the intertemporal budget constraint of the government as

$$\begin{aligned} \hat{d}'_{t-1} - d_{ss}b\hat{\pi}_t^H = \widehat{f}_t + d_{ss}a\Delta\frac{\rho(1-\lambda)}{(1+l)}(\widehat{Y}_t - \widehat{Y}_t^T) + \\ \frac{d_{ss}\rho(1-\lambda)}{1+l}(\hat{Y}_t - \widehat{Y}_t^T) + \frac{(1-\beta)}{(1+l)}d_{ss}E_t\sum_{s=0}^{\infty}\beta^{t+s}\left[m\left(\widehat{Y}_t - \widehat{Y}_t^T\right)\right], \end{aligned} \tag{3.58}$$

where,

$$m \equiv \left(\frac{\overline{\tau}(1-\omega^{-1}\eta)(1+l)}{(1-\beta)} - d_{ss}\rho(1+\omega^{-1})\right),$$

and

$$\begin{aligned} \widehat{f}_t \equiv d_{ss}a\Delta\frac{\rho(1-\lambda)}{(1+l)}\left[-\widehat{C}_t^* - \widehat{g}_t + \widehat{Y}_t^T\right] + \frac{\rho(l+\lambda)}{1+l}d_{ss}\hat{C}_t^* - \frac{\rho(1-\lambda)}{1+l}d_{ss}g_t + \frac{\rho(1-\lambda)}{1+l}d_{ss}Y_t^T + \\ +(1-\beta)d_{ss}E_t\sum_{s=0}^{\infty}\beta^{t+s}\left[\left(\frac{\overline{\tau}}{(1-\beta)\,d_{ss}} - \frac{\rho}{1+l}\right)\widehat{Y}_t^T + \frac{\overline{\tau}}{(1-\beta)\,d_{ss}}\widehat{\tau}_t^{T'} + \frac{\rho - \overline{s}(1+l)}{1+l}g_t - \frac{\rho l}{1+l}C_t^*\right]. \end{aligned}$$

The combination of the first order conditions implies:

$$\frac{\Phi_T \rho(1-\lambda)}{(1+l)}(\widehat{Y}_t - \widehat{Y}_t^T - \delta_t) + \frac{(1+l)}{\rho(1-\lambda)}\Phi_Y(\widehat{Y}_t - \widehat{Y}_t^T) \tag{3.59}$$
$$+m_0\varphi_{2,t} - d_{ss}(1+a)(\varphi_{2,t} - \varphi_{2,t-1}) = 0,$$

where $\delta_t \equiv \widehat{C}_t^* + \widehat{g}_t - \widehat{Y}_t^T + \frac{(1+l)}{\rho(1-\lambda)}\widehat{RS}_t^T$ and $m_0 = \frac{1}{1-\lambda}\left(\bar{s} + \frac{\eta(1+l)\bar{\tau}}{\omega\rho} + \frac{\bar{\tau}}{\omega} - \frac{\bar{\tau}(1-l)}{\rho}\right)$. Alternatively, we can write:

$$(\widehat{Y}_t - \widehat{Y}_t^T) = m_1\delta_t - m_2\varphi_{2,t} + (1+a)d_{ss}m_3(\varphi_{2,t} - \varphi_{2,t-1}), \tag{3.60}$$

where

$$m_1 = \left(\frac{(1+l)^2\Phi_Y + \Phi_T\rho^2(1-\lambda)^2}{\Phi_T\rho^2(1-\lambda)^2}\right)^{-1},$$

$$m_2 = \left(\frac{(1+l)^2\Phi_Y + \Phi_T\rho^2(1-\lambda)^2}{\rho(1+l)}\right)^{-1} m_0,$$

and

$$m_3 = \left(\frac{(1+l)^2\Phi_Y + \Phi_T\rho^2(1-\lambda)^2}{\rho(1-\lambda)(1+l)}\right)^{-1}.$$

Substitution of Equation (3.60) into Equation (3.58), we have:

$$\varphi_{2,t} = \frac{-(1+l)}{(n_1+n_2)}\widehat{d}_{t-1} + \frac{1}{(n_1+n_2)}\widehat{f}_t + \frac{n_1}{(n_1+n_2)}\varphi_{2,t-1}, \tag{3.61}$$

and using (3.55) we have

$$\varphi_{2,t} = \frac{-(1+l)}{n_2}\widehat{d}_t + \frac{1}{n_2}E_t\widehat{f}_{t+1}, \tag{3.62}$$

where

$$n_1 = -((1-\beta)m - d_{ss}\rho(1-\lambda))m_3 d_{ss},$$

and

$$n_2 = (d_{ss}\rho(1-\lambda) + m)m_2.$$

Therefore the evolution of the Lagrange multiplier and government debt can be written as

$$\varphi_{2,t} = \frac{1}{(n_1+n_2)}(\widehat{f}_t - E_{t-1}\widehat{f}_t) + \varphi_{2,t-1} \tag{3.63}$$

and

$$\widehat{d}_t = \widehat{d}_{t-1} - \frac{n_2}{(1+l)}\frac{1}{(n_1+n_2)}(\widehat{f}_t - E_{t-1}\widehat{f}_t) + \frac{1}{(1+l)}E_t\widehat{f}_{t+1}. \tag{3.64}$$

Case (i) : $\mathbf{d}_{ss} = 0$

In this case, Equation (3.61) implies

$$\varphi_{2,t} = \frac{-(1+l)}{n_2}\widehat{d}_{t-1} + \frac{1}{n_2}\widehat{f}_t \tag{3.65}$$

and

$$\widehat{d}_t = \widehat{d}_{t-1} + \frac{1}{(1+l)}E_t\Delta\widehat{f}_{t+1}. \tag{3.66}$$

Thus, under this specification, government debt has a unit root.

Case (ii) : $\mathbf{d}_{ss} \neq 0$ and of Nominal Bonds

In this case, we define $a = \frac{\lambda}{(1-\lambda)}$ and $b = -1$. The first order condition at time t_0 combined with (3.51) implies that $\varphi_{2,t}$ is constant over time. In this case, the first order conditions (3.51) to (3.55) can be expressed as the following targeting rule:

$$\Phi_T(\widehat{RS}_t - \widehat{RS}_t^T) + \frac{(1+l)}{\rho(1-\lambda)}\Phi_Y(\widehat{Y}_t - \widehat{Y}_t^T) = 0.$$

Moreover, in the special case where $\rho = \theta = 1$ and $\mu = 1/(1-\lambda)$,

$$\widehat{Y}_t = \widehat{Y}_t^T. \tag{3.67}$$

Equation (3.58) implies

$$\hat{d}_{t-1} - d_{ss}b\hat{\pi}_t^H = \widehat{f}_t; \tag{3.68}$$

that is, if the underlying structural disturbances composing $\widehat{Y}_t^T$ are stationary, output and the real exchange rate will also be stationary. Finally, Equation (3.68) determines the evolution of real debt.

3.A.5 Optimal Fiscal and Monetary Policy when Prices are Sticky:

In this case the Lagrangian is:

$$\mathcal{L} = E_{t_0}\sum \beta^{t-t_0}\left[\begin{array}{c} \frac{1}{2}\Phi_Y(\widehat{Y}_t-\widehat{Y}_t^T)^2+\frac{1}{2}\Phi_T(\widehat{RS}_t-\widehat{RS}_t^T)^2+\frac{1}{2}\Phi_\pi\left(\widehat{\pi}_t^H\right)^2 \\ +\varphi_{1,t}\left(-k^{-1}\pi_t^H+\eta\hat{Y}_t+(1-\lambda)^{-1}\widehat{RS}_t-\omega\widehat{\tau}_t+\beta E_t\pi_t^H\right)+ \\ +\varphi_{2,t}\left(\begin{array}{c} -d_{ss}\widehat{RS}_t+\widehat{d}_{t-1}-d_{ss}(a\Delta RS_t+b\widehat{\pi}_t^H)+d_{ss}(1-\beta)(\frac{1}{1-\lambda}\widehat{RS}_t) \\ -\overline{\tau}(\widehat{\tau}_t+\widehat{Y}_t)+\beta E_t\left[d_{ss}\widehat{RS}_{t+1}-\widehat{d}_t+d_{ss}(a\Delta RS_{t+1}+b\widehat{\pi}_{t+1}^H)\right]\end{array}\right) \\ +\varphi_{3,t}\left(\widehat{Y}_t-\frac{(1+l)}{\rho(1-\lambda)}\widehat{RS}_t\right) \\ +\varphi_{1,t_0-1}k^{-1}\widehat{\pi}_{t_0}^H+bdss\varphi_{2,t_0-1}\widehat{\pi}_{t_0}^H+d_{ss}(a+1)\varphi_{2,t_0-1}\widehat{RS}_{t_0}^H \end{array}\right]$$

$$+t.i.p+O(||\xi||^3).$$

As before, the last line in the Lagrangian contain constraints in the initial conditions that ensure a time-invariant policy problem. The first order conditions are:

$$\Phi_\pi\widehat{\pi}_t^H-(\varphi_{1,t}-\varphi_{1,t-1})k^{-1}-bdss(\varphi_{2,t}-\varphi_{2,t-1})=0, \tag{3.69}$$

$$\Phi_y(\widehat{Y}_t-\widehat{Y}_t^T)+\eta\varphi_{1,t}-\overline{\tau}\varphi_{2,t}+\varphi_{3,t}=0, \tag{3.70}$$

$$\begin{aligned}&\Phi_y(\widehat{RS}_t-\widehat{RS}_t^T)+\frac{1}{(1-\lambda)}\varphi_{1,t}-d_{ss}(a+1)(\varphi_{2,t}-\varphi_{2,t-1}) \\ &+d_{ss}\frac{(1-\beta)}{(1-\lambda)}\varphi_{2,t}+\beta ad_{ss}(E_t\varphi_{2,t+1}-\varphi_{2,t})-\frac{(1+l)}{\rho(1-\lambda)}\varphi_{3,t}=0,\end{aligned} \tag{3.71}$$

$$-\omega\varphi_{1,t}=\overline{\tau}\varphi_{2,t}, \tag{3.72}$$

and

$$-\varphi_{2,t} + E_t\varphi_{2,t+1} = 0. \tag{3.73}$$

These equations imply that,

$$E_t\widehat{\pi}^H_{t+1} = 0. \tag{3.74}$$

And the first order conditions can be combined and written as

$$\left[\frac{(1+l)\Phi_y}{(1-\lambda)\rho}\right]\Delta\widehat{y}_t + \Phi_T\Delta\widehat{rs}_t + \left[\frac{k\Phi_\pi}{(1-\overline{\tau})+bd_{ss}k}\right](\gamma\widehat{\pi}^H_t + d_{ss}(a+1)\widehat{\pi}^H_{t-1}) = 0, \tag{3.75}$$

where $\gamma = \left[d_{ss}\left(\frac{(1-\beta)}{(1-\lambda)} - (1+a)\right) + \left(\frac{(1+l)(\eta(1-\overline{\tau})-\overline{\tau})}{\rho(1-\lambda)} + \frac{1-\overline{\tau}}{(1-\lambda)}\right)\right]$. We define $\widehat{rs}_t = (\widehat{RS}_t - \widehat{RS}^T_t)$ and $\widehat{y}_t = (\widehat{Y}_t - \widehat{Y}^T_t)$.

Combining the first order condition with the government budget constraint and the Phillips Curve leads to the following expressions:

$$\varphi_{2,t} = \frac{f'_t - E_{t-1}f'_t}{(n'_1 + n'_2)} + \varphi_{2,t-1}, \tag{3.76}$$

$$\widehat{d}_t = \frac{Ef'_{t+1}}{1+l} - \frac{n'_2\varphi_{2,t}}{1+l} + \frac{n_3 d_{ss}}{(1+l)}(\varphi_{2,t} - \varphi_{2,t-1}) \tag{3.77}$$

and

$$\Phi_T\widehat{rs}_t + \frac{(1+l)}{\rho(1-\lambda)}\Phi_Y\widehat{y}_t = -m_0\varphi_{2,t} + (a+1)d_{ss}(\varphi_{2,t} - \varphi_{2,t-1}), \tag{3.78}$$

where,

$$n'_1 = -\left((1-\beta)m + d_{ss}\rho(1-\lambda)\right)(a+1)m_3 d_{ss} + d_{ss}m_4 + m_5),$$

$$n'_2 = (d_{ss}\rho(1-\lambda) + m)m_2,$$

$$n_3 = -a\rho(1-\lambda)(a+1)d_{ss}m_3,$$

$$m_4 = \left[a\rho(1-\lambda)(-m_2 + (a+1)d_{ss}m_3) + b\frac{(1+l)}{\Phi_\pi}\left(\frac{\overline{\tau}}{k\omega} + bd_{ss}\right)\right],$$

and

$$m_5 \equiv \frac{\overline{\tau}\,(1+l)}{(1-\beta)}\frac{\omega^{-1}}{k}\frac{1}{\Phi_\pi}\left(\frac{\overline{\tau}}{k\omega} + bd_{ss}\right).$$

Conclusion

This thesis analyses how macroeconomic policy should be conducted in a small open economy. Firstly, it characterizes the optimal monetary policy in the benchmark case in which the small economy can perfectly share its risk with the rest of the world. Under this specification, Chapter 1 demonstrates that a small open economy, completely integrated with the rest of the world, should be concerned about exchange rate variability. The optimal policy in a small open economy is neither isomorphic to that in a closed economy, nor does it prescribe a pure floating exchange rate regime. Price stability (or domestic inflation targeting) has been shown to be optimal only under a particular specification for preferences and in the presence of a production subsidy. When these restrictions on the steady-state level output and preferences are relaxed, deviations from inward looking policies arise in the optimal plan.

The model presented in Chapter 1 assumes that the asset markets are complete. Chapter 2 relaxes this assumption to deliver a more realistic representation of the model. The Chapter studies how the introduction of asset market imperfections affects welfare and the optimal monetary policy analysis. Our analysis compares the complete market characterization presented in Chapter 1 with the following forms of market incompleteness: the extreme case of financial autarky; and the case of an intermediate level of risk sharing, in which the country is allowed to trade riskless bonds internationally subject to intermediation fees. It shows that the different dynamics implied by alternative asset market structures have significant implications for optimal monetary policy. The degree of substitutability between domestic and imported goods is also an important determinant of the optimal policy

Chapters 1 and 2 of this thesis concentrate on the analysis of monetary policy when inflation is costly (due to sticky prices) and taxation is non-distortionary (because taxes are lump-sum) Chapter 3 starts by analyzing the opposite scenario, in which prices are flexible and production taxation affects households' labour-leisure trade off. The results show that, whereas in a closed economy it is optimal to perfectly smooth taxes, the optimal tax rate in an open economy varies over time. Chapter 3 also investigates the optimal fiscal and monetary policy mix when prices are sticky. It demonstrates that the introduction of price rigidity reduces the required variability of tax rates as well as inflation.

As first emphasized by Corsetti and Pesenti (2001), *"In an open economy there exists an economic distortion that is directly associated with openness, namely, a country's power to affect its terms of trade by influencing the supply of labour product"*. This thesis analyses the policy implications of this distortion in a small open economy setting. It does so under alternative asset market specifications and in the presence of different policy instruments.

Bibliography

[1] Adao, B., Correia, I. and Teles, P. (2001), "Gaps and Triangles", CEPR Discussion Papers 2668.

[2] Adao, B., Correia, I. and Teles, P. (2005), "On the Relevance of Exchange Rate Regimes for Stabilization Policy", mimeo, Banco de Portugal/CEPR Conference.

[3] Aguiar, M., Amador, M. and Gopinath, G. (2005), "Efficient Fiscal Policy and Amplification", NBER Working Paper 11490.

[4] Aoki, K. (2001), "Optimal Monetary Policy Response to Relative Price Changes", *Journal of Monetary Economics* 48, 55-80.

[5] Arseneau, D. (2004), "Optimal inflation in an open economy with imperfect competition" Finance and Economics Discussion Series 2004-25, Board of Governors of the Federal Reserve System (U.S.),

[6] Barro, R. (1979), "On the Determination of the Public Debt", *Journal of Political Economy* 87(5), 940-71.

[7] Baxter, M. and King, R. (1993), "Fiscal Policy in General Equilibrium", *American. Economic Review* 83, 315-335.

[8] Baxter, M. and King, R. (2005), "Fiscal Externalities and Optimal Taxation in an Economic Community", NBER International Seminar on Macroeconomics.

[9] Benigno, G. and Benigno,P. (2003), "Price Stability in Open Economies", *Review of Economic Studies* 70, 245.

[10] Benigno, G. and Benigno,P. (2003), "Designing Targeting rule for International Monetary Policy Coordination" ECB working paper 279.

[11] Benigno, G. and Thoenissen, C, (2005), "On the Consumption-real Exchange Rate Anomaly", Bank of England Working Paper 254.

[12] Benigno, P. (2001), "Price Stability with Imperfect Financial Integration" CEPR Discussion Paper 2854.

[13] Benigno, P. and Woodford, M. (2003), "Inflation Stabilization and Welfare: the Case of Large Distortions" NBER Working Papers 10838.

[14] Benigno, P. and Woodford, M. (2004), "Optimal Stabilization Policy when Wages and Prices are Sticky: The Case of a Distorted Steady State", CEPR Discussion Paper 4740.

[15] Benigno, P. and Woodford, M. (2004), "Optimal Monetary and Fiscal Policy: A Linear-Quadratic Approach", FRB International Finance Discussion Paper 806.

[16] Benigno, P. and Woodford, M. (2004), "Optimal Taxation in an RBC Model: A Linear-Quadratic Approach", CEPR Discussion Papers 4764.

[17] Bowman, D. and Doyle, B. (2003), "New-Keynesian, Open-Economy Models and Their Implications for Monetary Policy", FRB International Finance Discussion Paper 762.

[18] Calvo, G. (1983), "Staggered Prices in a Utility-Maximizing Framework", *Journal of Monetary Economics* 12, 383-398.

[19] Calvo, G. and Reinhart, C. (2000), "Fear of Floating" NBER Working Paper 7993.

[20] Cavallo, M. and Ghironi, F. (2002), "Net foreign assets and the exchange rate: Redux revived", *Journal of Monetary Economics* 49(5), 1057-1097.

[21] Chari, V., Christiano, L. and Kehoe, P. (1991), "Optimal Fiscal and Monetary Policy: Some Recent Results", *Journal of Money, Credit and Banking* 23(3), 519-539.

[22] Chari, V. and Kehoe, P. (1999), "Optimal Fiscal and Monetary Policy", NBER Working Paper 6891.

[23] Chari, V., Kehoe, P. and McGrattan, E., (2002), "Can Sticky Price Models Generate volatile and Persistent Real Exchange Rates?", *Review of Economic Studies* 69(3), 533-63.

[24] Clarida, R., Galí, J, Getler, M. (1999), "The Science of Monetary Policy: A New Keynesian Perspective", *Journal of Economic Literature* 37(4), 1661-1707.

[25] Clarida, R., Galí, J, Getler, M. (2001), "Optimal Monetary Policy in Closed versus Open Economies: An Integrated Approach" NBER Working Paper 8604

[26] Cole, H., Obstfeld, M. (1991), "Commodity trade and international risk sharing: how much do financial markets matter?" *Journal of Monetary Economics* 28, 3-24.

[27] Cole, H. (1993), "The Macroeconomic Effects of World Trade in Financial Assets", *Federal Reserve Bank of Minneapolis Quarterly Review*17(3),

[28] Collard, F. and Juillard, M, (2001) "Accuracy of stochastic perturbation methods: The case of asset pricing models", *Journal of Economic Dynamics and Control* 25(6-7), 979-999.

[29] Correia, I., Nicolini, J. and Teles, P. (2003), "Optimal Fiscal and Monetary Policy: Equivalence Results", CEPR Discussion Paper 3730.

[30] Corsetti, G., Dedola, L. and Leduc, S. (2003), "International risk-sharing and the transmission of productivity shocks" Working Papers 03-19, Federal Reserve Bank of Philadelphia.

[31] Corsetti, G. and Pesenti, P. (2001), "Welfare and Macroeconomic Interdependence", *Quarterly Journal of Economics* 116(2),

[32] Corsetti, G. and Pesenti, P. (2005), "International dimensions of optimal monetary policy", *Journal of Monetary Economics* 52(2), 281-305.

[33] Devereux, M., and Engel, C. (2000), "Monetary Policy in the Open Economy Revisited: Price Setting and Exchange Rate Flexibility", NBER Working Paper 7665.

[34] Dotsey, M., King, R. and Wolman, A. (1999), "State-Dependent Pricing And The General Equilibrium Dynamics Of Money And Output", *The Quarterly Journal of Economics* 114(2), 655-690.

[35] Eichenbaum, M. and Fisher, J. (2004), "Evaluating the Calvo Model of Sticky Prices" NBER Working Papers 10617.

[36] Engel, C. (2002), "The Responsiveness of Consumer Prices to Exchange Rates And the Implications for Exchange-Rate Policy: A Survey Of a Few Recent New Open-Economy" NBER Working Papers 8725.

[37] Engel, C. and Rogers, J. (1999), "Deviations from Purchasing Power Parity:Causes and Welfare Costs" Discussion Papers in Economics at the University of Washington 0038.

[38] Engel, C. and Rogers, J. (2000), "Deviations from purchasing power parity: causes and welfare costs" International Finance Discussion Papers 666, Board of Governors of the Federal Reserve System (U.S.).

[39] Fabiani, S., Gattulli, A. and Sabbatini, R. (2004), "The Pricing Behavior of Italian Firms: New Survey Evidence on Price Stickiness", ECB Working Paper 333.

[40] Ferrero, A. (2005), "Fiscal and Monetary Rules for a Currency Union" ECB Working Paper 502.

[41] Fry, M., D. Julius, L. Mahadeva, S. Roger and G. Sterne (2000), "Key issues in the choice of monetary policy framework", *L. Mahadeva and G. Sterne (eds.), Monetary policy frameworks in a global context*, 1-216.

[42] Fuhrer, J. and Moore, G., (1995), "Inflation Persistence" *The Quarterly Journal of Economics* 110(1), 127-59.

[43] Galí, J. and Gertler, M. (1999), "Inflation Dynamics: A Structural Econometric Analysis", *Journal of Monetary Economics* 44, 195-222.

[44] Galí, J., Gertler, M. and Lopez-Salido, J. (2002), "Markups, Gaps, and the Welfare Costs of Business Fluctuations", NBER Working Paper 8850.

[45] Galí, J. and Monacelli, T. (2005), "Monetary Policy and Exchange Rate volatility in a Small Open Economy", *Review of Economic Studies* 72(3), 707-734.

[46] Galí, J. and Monacelli, T. (2005), "Optimal Monetary and Fiscal Policy in a Currency Union", CEPR Discussion Paper 5374.

[47] Ganelli, G. and Lane, P, (2002), "Dynamic General Equilibrium Analysis: The Open Economy Dimension" CEPR Discussion Papers 3540.

[48] Ghironi, F. (2003), "Macroeconomic Interdependence under Incomplete Markets", Boston College Working Papers in Economics.

[49] Giannoni, M. and Woodford, M. (2003), "Optimal Inflation Targeting Rules", NBER Working Paper 9939.

[50] Goodfriend, M. and. King, R. (1997), "The new neoclassical synthesis and the role of monetary policy", Federal Reserve Bank of Richmond Working Paper 98-05

[51] Goodfriend, M. and. King, R. (2001), "The Case for Price Stability", NBER Working Papers 8423.

[52] Gourinchas, P. and Rey, H. (2005), "International Financial Adjustment", UC Berkeley Center for International and Development Economic Research Working Paper C05-139.

[53] Khan, A., King, R. and Wolman, A. (2003), "Optimal Monetary Policy", *Review of Economic Studies* 70(4), 825-860.

[54] Kim, J., Kim, S. and Levin, A. (2003), "Patience, persistence, and welfare costs of incomplete markets in open economies", *Journal of International Economics* 61(2), 385-396.

[55] King, R. and Watson, M., (1998), "The Solution of Singular Linear Difference Systems under Rational Expectations" *International Economic Review* 39(4), 1015-1026

[56] Kollmann, R. (2002), "Monetary Policy Rules in the Open Economy: Effects on Welfare and Business Cycles", CEPR Discussion Papers 3279.

[57] Kollmann, R. (2003), "Welfare Maximizing Fiscal and Monetary Policy Rules", mimeo, University of Bonn.

[58] Lane, P. (2001), "The New Open Economy Macroeconomics: A Survey", *Journal of International Economics* 54, 235-266.

[59] Lane, P. and Milesi-Ferretti, G. (2001), "Long-Term Capital Movements", NBER Working Paper W8366.

[60] Loungani, P. and Razin, A. (2005), "Globalization and Disinflation: The Efficiency Channel" CEPR Discussion Paper 4895.

[61] Lucas, R. (1987), Models of business cycles, Basil Blackwell, New York.

[62] Lucas, R. and Stokey, N. (1983), "Optimal Fiscal and Monetary Policy in an Economy Without Capital", *Journal of Monetary Economics* 12, 55-94.

[63] Mankiw, N. and Reis, R. (2001), "Sticky Information Versus Sticky Prices: A Proposal to Replace the New-Keynesian Phillips Curve", NBER Working Papers 8290.

[64] McCallum, B. and Nelson, E. (2004), "Timeless perspective vs. discretionary monetary policy in forward-looking models", *Federal Reserve Bank of St. Louis Review*, 43-56.

[65] McCallum, B. and Nelson, E. (2005), "Targeting vs. Instrument Rules for Monetary Policy", *Federal Reserve Bank of St. Louis Review* 87(5), 597-611.

[66] Monacelli, T. and Faia, E. (2004), "Ramsey Monetary Policy and International Relative Prices", ECB Working Paper 344.

[67] Obstfeld, M, and Rogoff, K. (1995), "Exchange Rate Dynamics Redux", *Journal of Political Economy* 103, 624-660.

[68] Obstfeld, M. and Rogoff, K. (1996), *Foundations of International Macroeconomics*, MIT Press.

[69] Obstfeld, M. and Rogoff, K. (1998), "Risk and Exchange Rates", NBER Working Papers 6694.

[70] Obstfeld, M. and Rogoff, K. (2000), "New Directions for Stochastic Open Economy Models", *Journal of international Economics* 50, 117-53.

[71] Perri, F. and Heathcote, J. (2002), "Financial Autarky and International Real Business Cycles", *Journal of Monetary Economics* 49, 601-27.

[72] Perotti, R. (2005), "Estimating the Effects of Fiscal Policy in OECD Countries", ECB Working Paper 168.

[73] Pesenti, P. and Tille, C. (2004), "Stabilization, Competitiveness, and Risk-Sharing: A Model of Monetary Interdependence", mimeo, Federal Reserve Bank of New York.

[74] Ramsey, F. (1927), "A Contribution to the Theory of Taxation", *Economic Journal* 37, 47-61.

[75] Ravn, M. (2001), "Consumption Dynamics and Real Exchange Rates", CEPR Discussion Paper 2940.

[76] Reinhart, C. and Rogoff, K. (2002), "The Modern History of Exchange Rate Arrangements: A Reinterpretation", NBER Working Paper 8963.

[77] Rogoff, K. (1996), "The Purchasing Power Parity Puzzle", *Journal of Economic Literature* 34(2), 647-668.

[78] Rotemberg, J. (1982), "A Monetary Equilibrium Model with Transactions Costs", NBER Working Papers 0978.

[79] Rotemberg, J. and Woodford, M. (1997), "An Optimization-Based Framework for the Evaluation of Monetary Policy", in Ben S. Bernanke and Julio J. Rotemberg (eds.), NBER Macroeconomics Annual, MIT Press.

[80] Rotemberg, J. (2002), "Customer Anger at Price Increases, Time Variation in the Frequency of Price Changes and Monetary Policy", NBER Working Papers 9320.

[81] Sarno, L. (2000), "Towards a New Paradigm in Open Economy Modeling: Where do we Stand?", *Federal Reserve Bank of St. Louis Review* 83, 21-36.

[82] Schmitt-Grohé, S and Uribe, M. (2002), "Closing Small Open Economies", NBER working paper 9270.

[83] Schmitt-Grohé, S and Uribe, M. (2001), "Solving Dynamic General Equilibrium Models using a Second Order Approximation to the Policy Function", CEPR discussion paper 2963.

[84] Schmitt-Grohé, S and Uribe, M. (2004), "Optimal fiscal and monetary policy under imperfect competition", *Journal of Macroeconomics* 26(2), 183-209.

[85] Siu, H. (2004), "Optimal fiscal and monetary policy with sticky price", *Journal of Monetary. Economics* 51, 575-607

[86] Sondergaard, J. and Cova, P. (2004), "When Should Monetary Policy Target The Exchange Rate?", Royal Economic Society Annual Conference 51, Royal Economic Society.

[87] Sutherland, A. (1996), "Exchange Rate Dynamics and Financial Market Integration", CEPR Discussion Papers 1337.

[88] Sutherland, A. (2002), "International Monetary Policy Coordination and Financial Market Integration", ECB Working Paper 174.

[89] Sutherland, A. (2002), "A Simple Second-Order Solution Method For Dynamic General Equilibrium Models", CEPR Discussion Papers 3554.

[90] Sutherland, A. (2002), "Incomplete Pass-Through and the Welfare Effects of Exchange Rate Variability", CEPR Discussion Papers 3431.

[91] Sutherland, A. (2002), "Cost-Push Shocks and the Optimal Choice of Monetary Target in an Open Economy", mimeo, St. Andrews University.

[92] Sutherland, A. (2005), "The Expenditure Switching Effect, Welfare and Monetary Policy in a Small Open Economy" The Institute for International Integration Studies Discussion Paper Series.

[93] Svensson, L. (2003), "The Magic of the Exchange Rate: Optimal Escape from a Liquidity Trap in Small and Large Open Economies", Princeton working paper.

[94] Svensson, L. (2002), "Inflation Targeting: Should It Be Modeled as an Instrument Rule or a Targeting Rule?", *European Economic Review* 46 .. 771-780.

[95] Svensson, L. (2003), "What Is Wrong with Taylor Rules? Using Judgment in Monetary Policy through Targeting Rules", *Journal of Economic Literature* 41(2), 426-477.

[96] Taylor, J. "The Role of the Exchange Rate in Monetary Policy ", mimeo, Stanford University.

[97] Thoenissen, C, (2004), "Real exchange rates, current accounts and the net foreign asset position", Money Macro and Finance (MMF) Research Group Conference 71, Money Macro and Finance Research Group.

[98] Tille, C. (2002), "How valuable is exchange rate flexibility? Optimal Monetary policy under sectorial shocks", Staff Reports 147, Federal Reserve Bank of NY.

[99] Tille, C. (2000), "Is the integration of world asset markets necessarily beneficial in the presence of monetary shocks?" Staff Reports 114, Federal Reserve Bank of New York.

[100] Tuladhar, A. (2003), "Monetary Policy under Imperfect Capital Markets in a Small Open Economy", *American Economic Review* 93(2), 266-270

[101] Wincoop, E. (1999), "How big are potencial gains from international risk sharing", *Journal of International Economics* 47, 109 135.

[102] Wolman, A. (1999), "Sticky Prices, Marginal Cost, and the Behavior of Inflation", *Federal Reserve Bank of Richmond Economic Quarterly* 85(4).

[103] Woodford, M. (1999), "Optimal Monetary Policy Inertia", NBER working paper 7261.

[104] Woodford, M. (2003), *Interest Rates and Prices: Foundation of a Theory of Monetary Policy*, Princeton University Press.

[105] Woodford, M. (2001), "Inflation Stabilization and Welfare", NBER working paper 8071

[106] Woodford, M. (2001), "Fiscal Requirements for Price Stability", *Journal of Money, Credit and Banking* 33(3), 669-728.

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I declare that the work presented in this thesis is my own except where the collaboration with coauthors is explicitly acknowledge.

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I agree with the above statement.

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I have worked with Bianca De Paoli on the paper "Optimal Monetary and Fiscal Policy for a Small Open Economy" which forms Chapter 3 of her thesis. I declare that Bianca De Paoli was responsible for approximately 80% of the work involved in the paper.

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