Risk-sharing in heterogenous agent models with incomplete markets

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Declaration

I certify that the thesis I have presented for examination for the PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).

I warrant that this authorization does not, to the best of my belief, infringe the rights of any third party.

Declaration

I certify that chapters one and two of this thesis, "Entrepreneurship and Personal Bankruptcy Law: A Quantitative Assessment" and "Personal Bankruptcy Law, Debt Portfolios and Entrepreneurship", were coauthored with Giacomo Rodano. Jochen Mankart contributed 50 percent to the genesis of the project, 50 percent to the computational implementation, and 50 percent to the writing of the text. In addition, chapter 5 of this thesis, "Joint Search and Aggregate Fluctuations", was coauthored with Rigas Oikonomou. Jochen Mankart contributed 50 percent to the genesis of the project, 50 percent to the computational implementation, and 50 percent to the writing of the text.

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Abstract

This thesis examines the impact of different risk sharing arrangements under incomplete financial markets on macroeconomic outcomes.

The first two chapters are joint work with Giacomo Rodano. In the first chapter, we examine the effects of Chapter 7 of the US bankruptcy law on entrepreneurs. The latter are subject to production risk. They can borrow and in case they fail they can default on their debt. We examine the optimal wealth exemption level and the optimal credit market exclusion duration in this environment.

In addition to unsecured credit, entrepreneurs can also obtain secured credit in the second chapter. Secured credit lowers the cost of a generous bankruptcy regime because agents who are rationed out of the unsecured credit market can still obtain secured credit. Therefore, the optimal exemption level is relatively high.

In the third chapter, I investigate the effects of wealth exemptions on interest rates if entrepreneurs can choose the riskiness of their project. The default possibility leads to a kink in the value function which makes agents locally risk-loving.

In the fourth chapter, I focus on consumers only. In particular, I show that wealth exemptions are of particular importance in a model with expense shocks. Wealth exemptions encourage people to save more so that aggregate savings rise. The model is also consistent with the fact that consumer bankruptcy cases are not correlated with wealth exemption levels.

The fifth chapter is joint work with Rigas Oikonomou. We compare two environments: on the one hand the standard one in which a household consists of one member and, on the other hand, one in which a household consists of two members who share their risks perfectly. We investigate the differences between the two models in labor market flows and volatilities of labor market statistics in response to productivity shocks.

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Preface

This thesis examines the impact of different risk sharing arrangements under incomplete financial markets on macroeconomic outcomes. In particular, I focus on environments in which heterogeneous agents can only trade one period non-contingent contracts. I examine the degree of risk sharing that is possible in such restricted environments. In the first four chapters, I analyze how the possibility to default, which makes debt partially contingent, affects macroeconomic outcomes. In the last chapter; I analyze how perfect risk-sharing within a household consisting of two members affects business cycle fluctuations.

The first two chapters are joint work with Giacomo Rodano. In the first chapter, we examine the effects of Chapter 7 of the US bankruptcy law on entrepreneurs. We develop a quantitative general equilibrium model of occupational choice that examines the effects of the US personal bankruptcy law on entrepreneurship. The model explicitly incorporates US personal bankruptcy law and matches empirical features of the US economy regarding entrepreneurship, wealth distribution, and bankruptcy filings by entrepreneurs. The option to declare bankruptcy encourages entrepreneurship through insurance since entrepreneurs may default in bad times. However, perfectly competitive financial intermediaries take tis possibility of default into account. Consequently, they charge higher interest rates which reflect these default probabilities. Thus, personal bankruptcy provides insurance at the cost of worsening credit conditions. Our quantitative evaluation shows that in the current US bankruptcy law the latter effect dominates. Halving the wealth exemption level from the current level would increase entrepreneurship, the median firm size, welfare, and social mobility, without increasing inequality. On the other hand, we show that without the possibility to default would entrepreneurship and welfare would be reduced.

In addition to unsecured credit, entrepreneurs can also obtain secured credit in the second chapter. We show that secured credit alters the results dramatically. The reason is that if secured credit is not available, a high exemption level leads to tight endogenous borrowing limits. This implies that some, in particular poor, agents will be excluded from borrowing because their ex post incentive to default is too high. However, if they can waive their right to default by using secured credit, i.e. by providing collateral, the negative effect of a generous bankruptcy law is lessened. Consequently, the optimal exemption level is a significantly higher.

In the third chapter, I investigate the effects of wealth exemptions on interest rates if entrepreneurs can choose the riskiness of their projects. Agents can become entrepreneurs or workers. Both, the default option and the occupational choice problem lead to kinks in the value function. This makes entrepreneurs locally risk-loving. Thus, some entrepreneurs choose riskier projects than necessary. Therefore, the model predicts that interest rates are not monotonically increasing in the wealth exemption level. This is consistent with empirical evidence.

In the fourth chapter, I focus on consumer bankruptcy. I develop a heterogenous agent life-cycle model to examine the effects of the US personal bankruptcy law on bankruptcy filings and welfare. In addition to facing uncertainty over their labor income, agents also face wealth shocks that stem from unexpected changes in family composition or from unexpected medical expenses. I allow agents to borrow and save simultaneously. Under chapter 7 of the US bankruptcy law, consumers can keep all wealth up to an exemption level. I show that introducing exemption levels is of particular relevance in the presence of wealth shocks. My quantitative evaluations show that changes in the exemption level have an impact only for very low exemption levels. Thus, ignoring them might bias welfare results. But this impact fades out rather quickly. The reason is that almost no household is affected by medium to high exemption levels because those households who might default do not have much wealth. I do not find, and this is consistent with the data, a strong positive relationship between the exemption level and default rates. With less than 0.1% of annual consumption, the welfare changes due to changes in the exemption level are rather small.

The fifth chapter is joint work with Rigas Oikonomou. We compare two incomplete markets environments: We compare two environments: on the one hand the standard one in which a household consists of one member and, on the other hand, one in which a household consists of two members who share their risks perfectly. We investigate whether joint search within the household unit can help reconcile the business cycle properties of aggregate employment, unemployment and labor force participation. Our main conclusion is that joint insurance through adjustments in family labor supply can have a big impact on the business cycle properties of key labor market statistics. However, the model still falls short of completely explaining the low cyclicality in the labor force participation rate.

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Chapter 1

Personal Bankruptcy Law and Entrepreneurship: A Quantitative Assessment

1.1 Introduction

Entrepreneurs employ half of all workers in the US and they create three quarters of all new jobs.[1] Over time, successful small entrepreneurial firms grow into big firms and drive the technological progress. For example, four of the 20 largest companies in 2007, Microsoft, Cisco Systems, Google and Dell, were born in the last generation. Personal bankruptcy law is important for entrepreneurs because if an entrepreneur's firm is not incorporated he is personally liable for all the debts of his firm. And even if the firm is incorporated, the entrepreneur very often has to provide personal guarantees to secure a loan [Berkowitz / White 2004]. Ten percent of entrepreneurs go out of business each year, and out of these around twenty percent through bankruptcy.

This paper investigates quantitatively the effects of personal bankruptcy law on entrepreneurship. We focus on two key features of the personal bankruptcy procedures: the wealth exemption level and the duration of the credit market exclusion period. The wealth exemption level determines how much wealth a person can keep in case of a default. The length of the credit market exclusion period determines when someone who has defaulted in the past regains access to credit.

Bankruptcy introduces some contingency in a world of incomplete credit markets in which

[1] We thank Alex Michaelides for his continuous support and valuable comments, and Francesco Caselli and Maitreesh Ghatak for helpful comments at various stages of this research. We are also grateful to Daniel Becker, Wouter Den Haan, Emmanuel Frot, Alberto Galasso, Bernardo Guimaraes, Christian Julliard, Rachel Ngai, and participants at the LSE macro and development seminars, the 2008 EEA meeting and the 2008 SCE meeting.

only simple debt contracts are available. However, it provides only partial contingency and does not complete the markets fully. This contingency provides insurance against entrepreneurial failure at the cost of worsening credit conditions. If the bankruptcy law does not allow default under any circumstances, i.e. if there is full commitment, credit will be available at low interest rates because borrowers can not default. This comes at the expense of borrowers having no insurance against business failure. If, however, the bankruptcy law makes default very easy, borrowers might be insured against bad outcomes. But in order to compensate for the default risk banks have to charge higher interest rates or ration credit altogether. In both extreme cases the equilibrium outcome can be one of almost no credit. In the former case there is no demand for credit whereas in the latter there is no supply of credit. In this world many firms are inefficiently small, especially those owned by poorer entrepreneurs. This trade-off is at the center of recent public discussions and policy changes in Europe and the US. In Europe the bankruptcy law is much harsher than in the US. Many countries like for example Germany, Netherlands and the UK, made it more lenient with the explicit aim of fostering entrepreneurship.[2] The policy changes in the US went into the opposite direction. Following the huge increase in personal bankruptcy filings, US Congress in 2005 passed a law making personal bankruptcy less beneficial for filers. Even though the focus of the discussion has been on consumer bankruptcy, the effects on entrepreneurship are important because around 80,000 failed entrepreneurs file for bankruptcy each year. Our paper quantitatively assesses the relative strength of these two opposing forces, insurance versus credit conditions, on the number of entrepreneurs, on the access of poor agents to entrepreneurship, on firm size, and on welfare, inequality and social mobility.

We build an infinite horizon heterogeneous agent model which has an occupational choice problem at its core. Agents differ with respect to their entrepreneurial productivity and their working productivity. Each period they decide whether to become an entrepreneur or a worker, based on a noisy signal of their productivities. Cagetti / De Nardi [2006] also have this occupational choice at the center of their model. Their model is able to explain the US wealth distribution, in particular the extreme skewness at the top. However, in their model entrepreneurship is a risk-free activity because there is no uncertainty about current productivities. Thus there is no default in equilibrium and there is no insurance role for bankruptcy. In our model default exists because a significant fraction of entrepreneurs files for bankruptcy.

Starting with Athreya [2002], there is a growing literature on consumer bankruptcy. For example, Livshits et al. [2007a] compare the US system under which future earnings are exempt after consumers have declared bankruptcy with a European type of system under which future earnings are garnished to repay creditors. They find that the welfare differences between the systems depend on the persistence and variance of the shocks. Chatterjee et al. [2007] show that a recent tightening of the law in the US implies large welfare gains.[3] We

[2] In a companion paper, we are currently investigating the effects of introducing a US type of law in Europe.
[3] Other papers in this growing literature are Athreya [2006], Athreya / Simpson [2006], Li / Sarte [2006], Mateos-Planas / Seccia [2006].

differ from all these papers by focusing on entrepreneurs.[4] Moreover, as Chatterjee et al. [2007], we focuses on the wealth distribution because the benefits of bankruptcy depend crucially on the wealth of an agent.

There are two closely related papers that analyze the effects of bankruptcy on entrepreneurship in a quantitative setting similar to our paper.[5] Akyol / Athreya [2007] use an overlapping generations, partial equilibrium framework. They have heterogeneity in human capital. Their main results is that the current system is too generous. Meh / Terajima [2008] have a similar framework (partial equilibrium OLG model) in which they analyze bankruptcy decisions of both consumers and entrepreneurs. Our paper differs from these in the following way: We have two types of shocks, one persistent and one transitory. This allows us to capture the feature that many agents enter and exit entrepreneurship frequently. This fact has been emphasized by Quadrini [2000]. Our model is a general equilibrium model. The importance of general equilibrium effects has been shown by Li / Sarte [2006].

Our model is able to replicate key macroeconomic variables of the US economy: capital output ratio, fraction of entrepreneurs in the population, exit rate, bankruptcy filings of entrepreneurs, wealth of entrepreneurs compared to workers. Based on this model we conduct two experiments to assess whether the current exemption level and the current exclusion period are optimal. Our main result is that the current system is too lenient with respect to the exemption level.

There are significant welfare gains from halving the current exemption level. These are in the order of 1.4% of annual consumption per household which corresponds to $700 in 2007. The welfare gains from lowering the exemption level do not only occur from an ex ante, expected utility, perspective but also across the entire wealth distribution. Both the rich and the poor would gain. The cause of this result is that the current system provides too much insurance. This worsens credit conditions for entrepreneurs so much that there are fewer of them. Entrepreneurship increases from 7.6% of the population to 8.6% if the exemption level is halved because credit gets cheaper. However, completely abolishing bankruptcy would lead to a welfare loss in the order of $60 per household since some insurance is valuable.

The effects of changing the exclusion period are small. Reducing it from six to two years yields a welfare gain in the order of $90 annually per household. Reducing the exclusion period allows the talented entrepreneurs who have defaulted in the past to regain access to credit sooner and therefore run bigger firms. In contrast to increasing the exemption level, this form of insurance, is less harmful for credit conditions since it does not reduce the amount the banks recover in the event of default. However, since the number of talented defaulters is small compared to all defaulters, these effects are quantitatively small.

[4] Zha [2001] is a theoretical investigation of similar issues. However his model abstracts from occupational choice, that we show to be the crucial channel through which bankruptcy law affects entrepreneurship. Moreover he does not calibrate his model to the US economy. Therefore his simulations give only qualitative suggestions.

[5] These two papers and ours' were developed independently. We published our first version in June 2007 on our website.

Our results are consistent with the empirical finding of Berkowitz / White [2004] who show that in states with higher exemption levels credit conditions are worse. Our results are partially consistent with the findings of Fan / White [2003]. They show that entrepreneurship increases when the exemption level is increased from a very low level. However we differ for high exemption levels: we find that high exemption levels lead to a decline in entrepreneurship while they find the opposite.

The paper is organized as follows, Section 1.2 provides an overview of US bankruptcy law and presents data on entrepreneurial failure. In Section 1.3 we present our model and discuss the equilibrium condition. In Section 1.4 we discuss our calibration strategy and present the baseline results. Section 1.5 explains the main mechanism of the model. In Section 1.6 we conduct the policy experiments and Section 1.7 concludes.

1.2 Entrepreneurial failure and personal bankruptcy in the US

Personal bankruptcy procedures in the US consist of two different procedures: Chapter 7 and Chapter 13. Under Chapter 7, all unsecured debt is discharged immediately. Future earnings cannot be garnished. This is why chapter 7 is known as providing a "fresh start". In exchange for this a person filing for bankruptcy has to surrender all wealth in excess of an exemption level. The exemption level varies across US states, ranging from $11,000 in Maryland to unlimited for housing wealth in some states, for example Florida. Following the literature, we calculate the population-weighted average across states. The resulting average exemption level is $77,591 in 1993.[6]

Under Chapter 13 agents can keep their wealth, debt is not discharged immediately and future earnings are garnished. Entrepreneurs are better off under Chapter 7 for three reasons: they have no non-exempt wealth, their debt is discharged immediately and they can start a new business straight away, since their income will not be subject to garnishment (see White, 2007). 70% of total bankruptcy cases involving entrepreneurs are under chapter 7. Therefore we will focus on Chapter 7 only.

Persons can file for bankruptcy only once every six years. The bankruptcy filing remains public information for ten years. But there is no formal rule about bankruptcy filers being excluded from credit. However,in practice, we observe that bankruptcy filers have difficulties obtaining credit for a periods ranging from 3 to 8 years after the filing [Athreya 2002].

The US. Small Business Administration reports an exit rate of on average 9.7% per annum for small firms in the period from 1990-2005.[7] Out of these failing firms 9.3% exit through

[6] The wealth exemption level does not change much over time. We choose 1993 because it is in the middle of the sample years for our data on entrepreneurship wealth distribution and bankruptcies.

[7] The U.S. Small Business Administration splits small firms into employer and non-employer firms. Employer firms have at least one employee working in the firm. There are roughly five million employer and 15 million non-employer firms in the U.S. Since the focus of our paper is on entrepreneurs who own and manage the firm we use only the data for employer firms since non-employer firms have in many cases the owner not working in

bankruptcy, according to the official data from the Administrative Office of the Courts.[8] Unfortunately, the official data on personal bankruptcy caused by a business failure seem to be severely downward biased. Lawless / Warren [2005] estimate that the true number could be three to four times as big. Their own study is based on an in-depth analysis of bankruptcy filers in five different judicial districts. Their explanation of this discrepancy is the emergence of automated classification of personal bankruptcy cases. Almost all software used in this area has "consumer case" as the default option. Thus reporting a personal bankruptcy case as a "business related" case requires some - even though small - effort while being completely inconsequential for the court proceedings. In addition to their own study they report data from Dun & Bradstreet according to which exit through bankruptcy is at least twice the official number[9].

In the calibration of our model we set the baseline exemption level equal to $77,591. The baseline exclusion period is set to six years. We calibrate the model such that the ratio of bankruptcies over exits is equal to 20%.

1.3 The model

Our economy is populated by a unit mass of infinitely lived heterogeneous agents. Agents face idiosyncratic uncertainty, but there is no aggregate uncertainty. At the beginning of every period, agents decide whether to become workers or entrepreneurs. An entrepreneur must decide how much to invest and, if he is allowed to, how much to borrow. An entrepreneur who has defaulted in the past is not allowed to borrow for some time. Since we focus on the implications of personal bankruptcy for entrepreneurs, workers are not allowed to borrow. Agents have only a noisy signal of their productivities and are subject to uninsurable risk. After the shocks are realized, production takes place. At the end of the period borrowers decide whether to repay or whether to default and how much to consume and how much to save. If they default, they will be borrowing constrained in the next period. Thus, they cannot borrow but they can still save. Anticipating this behavior banks vary the interest rate charged for each loan taking into account the individual borrower's default probability. The remainder of this section presents the details of the model.

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the firm. To ensure consistency across our three databases, when we use data from the Survey of Consumer Finance (SCF) and the Panel Study of Income Dynamics (PSID) we define entrepreneurs as business owners who manage a firm with at least one employee.

[8] While one can obtain exit rates from the PSID data (Quadrini, 2000), it is impossible to obtain reliable bankruptcy data from the PSID. There is only one wave in which respondents were asked about past bankruptcies.

[9] Dun & Bradstreet (D&B) is a credit-reporting and business information firm. D&B compiles its own independent business failure database. Until the emergence of automated software for law firms and courts in the mid 1980s, the official business bankruptcy data and the index compiled by D&B have a positive and significant correlation of 0.73. From 1986-1998 this correlation coefficient becomes negative and insignificant. Extrapolating from the historic relationship between the D&B index and personal bankruptcy cases caused by business failures leads to the conclusion that the official data underreport business bankruptcy cases at least by a factor of two.

1.3.1 Bankruptcy law and credit status

Agents who have borrowed can declare bankruptcy. In the event of a default the agent's debt is discharged, and at the same time any assets in excess of an exemption level X are liquidated. There are transaction costs in the liquidation process so that banks can only obtain a fraction f of each unit of capital they liquidate.

An agent who has declared bankruptcy in the past can save but he cannot borrow for a certain period of time. We call this agent *borrowing constrained* and we denote his credit status as BC. We assume that every *borrowing constrained* agent, whether worker or entrepreneur, faces a credit status shock at the end of the period. This probability captures the duration of the credit market exclusion period. With probability $(1 - \varrho)$ the agent remains borrowing constrained. With probability ϱ the agent can borrow again. He becomes an *unconstrained* agent with credit status UN[10]. ϱ is calibrated such that the average exclusion period is six years, the value observed in the data.

1.3.2 Households

Our economy is populated by a unit mass of infinitely lived heterogeneous agents. Each agent differs according to the level of assets a, the entrepreneurial productivity θ, the working productivity φ, and the credit status $S \in \{UN, BC\}$.

Preferences

For simplicity we abstract from labor-leisure choice. All agents supply their unit of labor inelastically either as workers or as entrepreneurs. There is no disutility of labor. Agents discount the future at the rate β. Therefore they maximize the following utility function

$$U = E\left\{\sum_{t=0}^{\infty} \beta^t u(c_t)\right\} \tag{1.1}$$

Productivities

Each agent is endowed with a couple of stochastic productivity levels: one as an entrepreneur θ and one as a worker φ. We make the simplifying assumption that the working and entrepreneurial ability processes are uncorrelated. At the beginning of each period the agent knows only his past productivities φ_{-1} and θ_{-1}, but his productivity as a worker and as entrepreneur during the current period, denoted by φ and θ, are revealed only after he has taken the occupational choice and investment decisions.

[10] The length of the exclusion period is transformed into a probability in order to avoid an additional state variable that keeps track of the numbers of years left before the solvency status is returned to UN. This procedure is standard in the literature, see Athreya [2002] and Chatterjee et al. [2007].

The workers' ability process. Following the literature[11] we assume that labor productivity follows the following AR(1) process[12]:

$$\log \varphi_t = (1-\rho)\,\mu + \rho \log \varphi_{t-1} + \varepsilon_t \tag{1.2}$$

where ε_t is *iid* and $\varepsilon \sim N(0, \sigma_\varepsilon)$. If the agent becomes a worker his labor income during current period is given by $w\varphi$.

The entrepreneurs' ability process. In contrast to the case of working ability, there are no reliable estimates of the functional form for the case of entrepreneurial ability. Therefore, following Cagetti / De Nardi [2006], we will assume a parsimonious specification where entrepreneurial productivity follows a 2-state Markov process with $\theta^L = 0$ and $\theta^H > 0$ and transition matrix

$$P_\theta = \begin{bmatrix} p^{LL} & 1-p^{LL} \\ 1-p^{HH} & p^{HH} \end{bmatrix} \tag{1.3}$$

We calibrate the 3 parameters (θ^H, p^{HH} and p^{LL}) to match some observed features of entrepreneurial activity in the US economy.

1.3.3 Technology

Entrepreneurial sector Every agent in the economy has access to a productive technology that, depending on her entrepreneurial productivity θ, produces output according to the production function

$$Y_i = \theta_i \chi_i k_i^\nu \tag{1.4}$$

where θ_i is agent $i's$ persistent entrepreneurial productivity described above. We assume that production is subject to an *iid* idiosyncratic shock with $\chi_i \in \{0, 1\}$, where $\chi_i = 0$ happens with probability p^χ. This *iid* shock represents the possibility that an inherently talented entrepreneur (i.e. an agent with high and persistent θ_i) might choose the wrong project or could be hit by an adverse demand shock. Quadrini [2000] shows that the entry rate of workers with some entrepreneurial experience in the past, is much higher than the entry rate of those workers without any experience. Therefore it seems that entrepreneurs come mostly from a small subset of total population. If their firms fail, they are very likely to start a new firm within a few years. The *iid* shock χ_i helps us to capture this difference in the entry rates.

Corporate sector Many firms are both incorporated and big enough not to be subject to personal bankruptcy law. Therefore we follow Quadrini [2000] and Cagetti / De Nardi [2006] and assume a perfectly competitive corporate sector which is modelled as a Cobb-Douglas

[11] See for example Storesletten et al. [2004].
[12] In the simulation we discretize this process by methods based on Tauchen [1986].

production function

$$F(K_c, L_c) = AK_c^{\xi} L_c^{1-\xi} \tag{1.5}$$

where K_c and L_c are capital and labor employed in this sector. Given perfect competition and constant returns to scale the corporate sector does not distribute any dividend. Capital depreciates at rate δ in both sectors.

1.3.4 Credit market

We assume that there is perfect competition in the credit market. Therefore banks must make zero profit on any contract[13]. The opportunity cost of the lending to entrepreneurs is the rate of return on capital in the corporate sector. This is also equal to the deposit rate.[14] Banks offer one period non-contingent debt contracts. The only agent who interacts with banks is the *unconstrained* entrepreneur. Banks know everything about the agent: his assets and his productivities. For any given value of $(a, \theta_{-1}, \varphi_{-1})$ and for any amount lent b, by anticipating the behavior of the entrepreneur, the banks are able to calculate the probability of default and how much they will get in the case of default. Perfect competition implies that they set the interest rate, $r(a, \theta_{-1}, \varphi_{-1}, b)$, such that they break even. Therefore, banks offer a menu of one period debt contracts which consists of an amount lent b and a corresponding interest rate $r(a, \theta_{-1}, \varphi_{-1}, b)$ to each agent $(a, \theta_{-1}, \varphi_{-1})$.

1.3.5 Timing

At the beginning of the period, agents who have defaulted in the past and who have not received the positive credit status shock are *borrowing constrained.* The other agents are *unconstrained.* All agents face an occupational choice: they choose whether they become entrepreneurs or workers. However they make this decision without knowing their productivities (θ, φ). Since these productivities follow a Markov process they use past productivities $(\theta_{-1}, \varphi_{-1})$ to forecast their current productivities (φ, θ).

Workers deposit all their wealth at the banks, receiving a rate of return r^d. After productivities are realized and production has taken place, they choose consumption and savings. At the end of the period the *borrowing constrained* worker receives the credit status shock. With probability ϱ he remains *borrowing constrained* next period (i.e. $S' = BC$). With probability $(1 - \varrho)$ he becomes *unconstrained* next period (i.e. $S' = UN$).

The *borrowing constrained* entrepreneur can choose how much to invest in his firm before the current θ is realized. He deposits the remaining wealth at the bank. Thus the entrepreneur

[13] In many papers on consumer bankruptcy banks cross-subsidize loans. This implies however that a bank could make positive profits by denying credit to the most risky borrowers. (see Athreya [2002] and Li / Sarte [2006]). For an approach similar to ours, see Chatterjee et al. [2007].
By the law of large numbers average ex post profits will be zero too

[14] In our model the banks are isomorphic to a bond market in which each agent has the possibility to issue debt.

faces a portfolio choice between investing in his own firm (risky asset) or in a safe bank deposit. But he can not borrow. After (θ, φ) and χ are realized and production has taken place, he chooses consumption and savings. At the end of the period he receives the credit status shock.

The *unconstrained entrepreneur* can borrow from perfectly competitive banks. Before knowing (θ, φ) and χ, he chooses his capital stock by deciding how much to borrow (or invest at rate r^d). In case the entrepreneur borrows, by picking from the menu $\{b, r(a, \theta_{-1}, \varphi_{-1}.b)\}$ offered by banks, he invests everything in his own firm. After (θ, φ) and χ are realized and production has taken place, the entrepreneur can decide whether to repay his debt and be *unconstrained* next period (i.e. $S' = UN$) or whether to declare bankruptcy and be *borrowing constrained* next period(i.e. $S' = BC$). After that he chooses consumption and savings.

Summarizing, the timing is as follows:

1. The agent enters the period with a state $(a, \theta_{-1}, \varphi_{-1}, S)$;
2. The agent chooses whether to become a worker or an entrepreneur;
3. *Unconstrained* entrepreneurs choose from the menu $\{b, r(a, \theta_{-1}, \varphi_{-1}, b)\}$ offered by perfectly competitive banks;
4. Real and financial investment decisions are taken;
5. Productivities (θ, φ) and the *iid* shock $\chi \in \{0, 1\}$ are realized and production takes place;
6. Bankruptcy decisions are taken by the *unconstrained* entrepreneurs;
7. Consumption and saving decisions are taken;
8. The credit status shocks for all *borrowing constrained* agents are realized;
9. End of period: the new state is $(a', \theta, \varphi, S')$.

Since the credit state S consists only of the two states BC and UN, we define the individual state variable as $(a, \theta_{-1}, \varphi_{-1})$, and we solve for two value functions $V^{UN}(a, \theta_{-1}, \varphi_{-1})$ and $V^{BC}(a, \theta_{-1}, \varphi_{-1})$ one for each credit status.

1.3.6 The problem of the *borrowing constrained* agent

This agent cannot borrow, but he can save at an interest rate r^d. At the beginning of the period he can choose whether to become an entrepreneur, which gives utility $N^{BC}(a, \theta_{-1}, \varphi_{-1})$ or a worker which yields utility $W^{BC}(a, \theta_{-1}, \varphi_{-1})$. Therefore the value of being a *borrowing constrained* agent with state $(a, \theta_{-1}, \varphi_{-1})$ is

$$V^{BC}(a, \theta_{-1}, \varphi_{-1}) = \max\left\{N^{BC}(a, \theta_{-1}, \varphi_{-1}), W^{BC}(a, \theta_{-1}, \varphi_{-1})\right\} \tag{1.6}$$

where the "max" operator reflects the occupational choice.

Worker At the beginning of the period the *borrowing constrained* worker deposits all his wealth at the bank. Then (θ, φ) are realized, production takes place and he receives labor income $w\varphi$. At the end of the period, he chooses consumption and saving, taking into account that he will receive a credit status shock. With probability ϱ he will be still *borrowing constrained* next period with an utility $V^{BC}(a', \theta, \varphi)$, while with a probability $(1-\varrho)$ he will become *unconstrained* with an utility $V^{UN}(a', \theta, \varphi)$. Therefore the expected utility of a *borrowing constrained* worker with wealth a and productivities $(\theta_{-1}, \varphi_{-1})$ is

$$W^{BC}(a, \theta_{-1}, \varphi_{-1}) = \\ E\left\{ \begin{array}{l} \max_{c,a'} \left\{u(c) + \beta \left[\varrho V^{BC}(a', \theta, \varphi) + (1-\varrho) V^{UN}(a', \theta, \varphi)\right]\right\} \\ s.t.\ c + a' = w\varphi + \left(1 + r^d\right) a \end{array} \right\} \tag{1.7}$$

Entrepreneur At the beginning of the period the *borrowing constrained* entrepreneur chooses the amount of capital, $k \in [0, a]$, to invest in his firm and the amount $a - k$ to deposit at the bank. After (θ, φ) and the shock χ are realized he will decide how to allocate the resources $\chi\theta k^{\nu} + (1-\delta) k + \left(1 + r^d\right)(a - k)$ among consumption and savings. Therefore the optimal value of the *borrowing constrained* entrepreneur is

$$N^{BC}(a, \theta_{-1}, \varphi_{-1}) = \\ \max_{0 \le k \le a} E\left\{ \begin{array}{l} \max_{a',c} \left\{u(c) + \beta \left[\varrho V^{BC}(a', \theta, \varphi) + (1-\varrho) V^{UN}(a', \theta, \varphi)\right]\right\} \\ s.t.\ c + a' = \chi\theta k^{\nu} + (1-\delta) k + \left(1 + r^d\right)(a - k) \end{array} \right\} \tag{1.8}$$

where the expectation operator $E\{\cdot\}$ now considers also the temporary shock χ.

1.3.7 The problem of the *unconstrained* agent

At the beginning of the period the *unconstrained* agent faces the following occupational choice

$$V^{UN}(a, \theta_{-1}, \varphi_{-1}) = \max\left\{W^{UN}(a, \theta_{-1}, \varphi_{-1}), N^{UN}(a, \theta_{-1}, \varphi_{-1})\right\} \tag{1.9}$$

where $W^{UN}(a, \theta_{-1}, \varphi_{-1})$ is the utility of becoming a worker and $N^{UN}(a, \theta_{-1}, \varphi_{-1})$ of becoming an entrepreneur.

Worker The problem of the *unconstrained* worker is identical to the *borrowing constrained* one except that the agent will be *unconstrained* in the future for sure. His utility is

$$W^{UN}(a,\theta_{-1},\varphi_{-1}) = E\left\{\begin{array}{c}\max\limits_{c,a'}\left\{u(c)+\beta V^{UN}(a',\theta,\varphi)\right\} \\ s.t.\ c+a' = w\varphi + \left(1+r^d\right)a\end{array}\right\} \tag{1.10}$$

Entrepreneur The *unconstrained entrepreneur* decides how much to invest in his firm $k = a + b$ by choosing how much to borrow ($b > 0$) or save at rate r^d ($b < 0$). If he borrows he can choose from the menu $\{b, r(a, \theta_{-1}, \varphi_{-1}, b)\}$ offered by the banks. After (θ, φ) and the shock χ are realized he can choose whether to declare bankruptcy (default) or whether to repay and how much to consume and save. He solves the problem backwards.

If he repays his debt, he has to choose how to allocate his resources, $\chi\theta k^\nu + (1-\delta)k - b[1 + r(a, \theta_{-1}, \varphi_{-1}, b)]$, between consumption and savings. Given that the decision of repaying is done when current productivities (θ, φ) and the shock χ are known, his utility from repaying is given by

$$\begin{aligned} N^{pay}(a,b,\theta,\varphi,\chi) &= \max_{c,a'}\left\{u(c)+\beta V^{UN}(a',\theta,\varphi)\right\} && (1.11)\\ s.t.\ a'+c &= \chi\theta k^\nu + (1-\delta)k - b[1+r(a,\theta_{-1},\varphi_{-1},b)] && (1.12)\\ k &= a+b && (1.13)\end{aligned}$$

If he defaults, his debt is discharged. But he loses all his assets in excess of the exemption level X. Thus, the resources to allocate between consumption and savings are $\min\{\chi\theta k^\nu + (1-\delta)k, X\}$. Moreover if he defaults he will be *borrowing constrained* next period. Therefore by declaring bankruptcy he gets

$$\begin{aligned} N^{bankr}(a,b,\theta,\varphi,\chi) &= \max_{c,a'}\left\{u(c)+\beta V^{BC}(a',\theta,\varphi)\right\} && (1.14)\\ s.t.\ a'+c &= \min\{\chi\theta k^\nu + (1-\delta)k, X\} && (1.15)\\ k &= a+b && (1.16)\end{aligned}$$

He will declare bankruptcy if $N^{bankr}(a,b,\theta,\varphi\chi) > N^{pay}(a,b,\theta,\varphi,\chi)$ and vice versa. Thus, at the beginning of the period the agent choose the optimal amount of b from the menu $\{b, r(a, \theta_{-1}, \varphi_{-1}, b)\}$ anticipating his future behavior. Therefore his utility is given by

$$\begin{aligned} &N^{UN}(a,\theta_{-1},\varphi_{-1}) = \\ &\max_{\{b,r(a,\theta_{-1},\varphi_{-1},b)\}} E\left[\max\left\{N^{pay}(a,b,\theta,\varphi,\chi), N^{bankr}(a,b,\theta,\varphi,\chi)\right\}\right] \end{aligned} \tag{1.17}$$

where the max operator inside the square brackets reflects the bankruptcy decision, and the max operator outside the square brackets reflects the borrowing decision.

1.3.8 The zero profit condition for the banks

We assume that the banks observe the state variables $(a, \theta_{-1}, \varphi_{-1})$ at the moment of offering the contract. For any given state $(a, \theta_{-1}, \varphi_{-1})$ and for any given loan b, the bank knows in which states of the world the agent will declare bankruptcy by solving the problem of the agent. Therefore it is able to calculate exactly the probability that a certain agent with characteristics $(a, \theta_{-1}, \varphi_{-1})$ will default for any given loan b. Denote this probability $\pi^{bankr}(a, \theta_{-1}, \varphi_{-1}, b)$.

If the agent repays the bank receives $[1 + r(a, \theta_{-1}, \varphi_{-1}, b)]\, b$. If the agent defaults the bank sells the firm's undepreciated capital and it does not obtain the full value, but only a fraction f. This captures two features. First, since business wealth is not exempt under Chapter 7, the agent will try to move as much wealth as possible out of his firm into exempt wealth, e.g. housing. Second, as for example shown by Ramey / Shapiro [2001], the sales value of business assets is below their value with the firm. Therefore the bank receives: nothing if $\chi\theta k^{\nu} + f(1-\delta)(a-b) < X$ while it receives $\chi\theta k^{\nu} + f(1-\delta)(a+b) - X$ otherwise.

The zero profit condition for the bank is given by

$$\begin{pmatrix} \left[1 - \pi^{bankr}(a, \theta_{-1}, \varphi_{-1}, b)\right][1 + r(a, \theta_{-1}, \varphi_{-1}, b)]\, b + \\ \pi^{bankr}(a, \theta_{-1}, \varphi_{-1}, b) \max\{\chi\theta k^{\nu} + f(1-\delta)(a+b) - X, 0\} \end{pmatrix} = (1 + r^d) b \tag{1.18}$$

1.3.9 Equilibrium

Let $\eta = (a, \theta_{-1}, \varphi_{-1}, S)$ be a state vector for an individual, where a denotes assets, θ_{-1} entrepreneurial productivity, φ_{-1} working productivity and S the credit status. From the optimal policy functions (savings, capital demand, default decisions), from the exogenous Markov process for productivity and from the credit status shocks, we can derive a transition function, that, for any distribution $\mu(\eta)$ over the state provides the next period distribution $\mu'(\eta)$. A stationary equilibrium is given by

- a deposit rate of return r^d and a wage rate w
- an interest rate function $r(\eta)$
- a set of policy functions $g(\eta)$ (consumption and saving, capital demand, bankruptcy decisions and the occupational choice)
- a constant distribution over the state η, $\mu^*(\eta)$

such as, given r^d and w:

- $g(\eta)$ solves the maximization problem of the agents;
- the corporate sector representative firm is optimizing;

- capital, labor and goods market clear:
  - capital demands come both from entrepreneurs and from the corporate sector, while supply comes from saving decisions of the agents;
  - labor demand comes from corporate sector, while labor supply come from the occupational choice of the agents;
- the function $r(\eta)$ reflects the zero profit condition of the banks
- The distribution $\mu^*(\eta)$ is the invariant distribution associated with the transition function generated by the optimal policy function $g(\eta)$ and the exogenous shocks.

The model has no analytical solution and must be solved numerically. The algorithm used to solve the model and other details are presented in the appendix.

1.4 Calibration and baseline results

1.4.1 Parametrization

Fixed parameters

Following standard practice in the literature we try to minimize the number of parameters of the model used to match the data. We therefore select some parameters which have already been estimated in the literature. We choose $\rho = 0.95$ for the auto-regressive coefficient of the earnings process[15]. The variance of the earnings process is chosen to match the Gini index of labor income as in PSID data which is 0.38[16]. The process is approximated using a 4-state Markov chain, using the Tauchen [1986] method as suggested by Adda / Cooper [2003][17]. Total factor productivity is normalized to 1, while the share of capital in the Cobb-Douglas technology for the Corporate sector is set to $\xi = 0.36$. The depreciation rate is set $\delta = 0.08$. Felicity is assumed to be CRRA with coefficient of relative risk aversion $\sigma = 2$.

These parameters are summarized in table 1.1:

[15] In a life cycle setting, Storesletten et al. [2004, 2001] find ρ in the range between 0.95 and 0.98. We choose $\rho = 0.95$ to take into account that the agents in our model are infinitely lived. Since the intergenerational auto-regressive coefficient is lower. Solon [1992] estimates it around 0.4.

[16] The exact value of the variance is $\sigma_\varepsilon^2 = .08125$. This is higher than the estimate of Storesletten et al. [2004] of about 0.02. We abstract from many important factors that are empirically relevant for the earnings distribution, e.g. human capital, life-cycle savings. Therefore, in order to generate the observed inequality, we choose a higher variance of the earnings process.

[17] Floden [2008] shows that for highly correlated processes the method of Adda / Cooper [2003] achieves a higher accuracy than the original methods of Tauchen [1986] and Tauchen / Hussey [1991].

Table 1.1: The fixed parameters

Parameter	Symbol	Baseline
TFP	A	1 (normalization)
Share of capital	ξ	0.36
Depreciation rate	δ	0.08
CRRA	σ	2
Working productivities	$\varphi_1 < \varphi_2 < \varphi_3 < \varphi_4$	$\begin{bmatrix} \varphi_1 = 0.316, \varphi_2 = 0.745 \\ \varphi_3 = 1.342, \varphi_4 = 3.163 \end{bmatrix}$
Transition matrix	P_φ	$\begin{bmatrix} 0.8393 & 0.1579 & 0.0028 & 0.0000 \\ 0.1579 & 0.6428 & 0.1965 & 0.0028 \\ 0.0028 & 0.1965 & 0.6428 & 0.1579 \\ 0.0000 & 0.0028 & 0.1579 & 0.8393 \end{bmatrix}$

Bankruptcy policy parameters

The two policy parameters are the exemption level X and the probability ϱ of remaining borrowing constrained. The law does not state any formal period of exclusion from credit after bankruptcy filing. For our baseline specification, we set $\varrho = 0.2$ which corresponds to an average exclusion period from credit of 5 years[18]. The exemption level differs across US states. Using state-level data for 1993, we calculate the population-weighted average exemption level across states.[19] ("homestead" plus "personal property" exemption). The resulting average exemption level is $77,591, taking an average household labor income of $45,000 corresponds to a value of **1.72** for the exemption/wage ratio. Table 1.2 summarizes the bankruptcy parameters:

Table 1.2: the bankruptcy parameters

Parameter	Symbol	Value
Exemption/wage	X/w	1.72
Exclusion period (expressed as probability)	ϱ	0.2

Calibrated parameters

We are left with the following 7 parameters to be calibrated: high entrepreneurial productivity (θ^H), entrepreneurial productivity transition matrix (p^{HH}, p^{LL}), concavity of entrepreneurial production function (ν), capital specificity (f), discount factor (β) and the probability of the transitory shock (p^X).

We choose these 7 parameters such that the model matches the following 7 moments of

[18] This choice is in line with the consumer bankruptcy literature which sets the average length of exclusion in this range. Athreya [2002] sets this at 4 years, Li / Sarte [2006] to 5 years, Chatterjee et al. [2007] to 10 years.
[19] We took the data from Berkowitz / White [2004] and top-coded the unlimited homestead exemption to the maximum state exemption.

the US economy. First we want the model to match the *capital-output ratio* (K/Y) in US economy. In the literature we find values ranging from 2.5 to 3.1 We target it to be 2.8 and we check the sensitivity of the results to different values. We target the *fraction of exits through bankruptcy* (bankruptcy/exit). Given the discussion in Section 2 we set this equal to 20%.[20] The *fraction of entrepreneurs in the total population* is 7.6% in the Survey of Consumers Finances.[21] Based on data from the US Small Business Administration the *exit rate* of entrepreneurs is equal to 9.3%. Therefore we set the baseline target at 9.3%. However the exit rate based on the PSID is higher (around 13.6%).[22] Therefore we check the sensitivity of results to higher values.

Quadrini [2000] points out that the entry rate for workers who had some entrepreneurial experience in the past is much higher than the entry rate for those without any experience. It seems that entrepreneurs come mostly from a small subset of total population. If their firms fail, they are very likely to start a new firm within a few years. In the PSID the ratio of *entry rate of experienced entrepreneurs over the average entry rate* is 13. This is an important target because the bankruptcy law affects the possibility and the speed of re-entry for failed entrepreneurs.

Since the benefits of bankruptcy depend crucially on the wealth of an agent we match some features of the wealth distribution. The US wealth distribution is extremely skewed with the top 40% of richest households holding around 93% of total assets.[23] The Gini coefficient is very high, at around 0.8. There is a large literature that tries to match the wealth distribution in the US. The most difficult part is to match the extremely rich agents at the top end of the distribution. But, as we show below, for our model it is particularly important to match the lower end of the distribution. Therefore we target the *share of wealth held by the richest 40%*. As a last target we choose to match the *ratio of the median wealth of entrepreneurs to the median wealth in the whole population.* This target captures features of both the wealth distribution and entrepreneurial productivity and technology. We set the target to 5.6 as found in the SCF.[24]

The targets are summarized in the second column of table 4.

1.4.2 The baseline calibration

We first present the baseline version of the model. Table 1.3 reports the value of the calibrated parameters in the baseline specification

while table 1.4 reports the value of the targets and the actual results achieved in the

[20] Given the uncertainty about the estimates we check the sensitivity of results to changing this target to 10% and to 30%.
[21] See Appendix B for data sources, definitions and further details.
[22] One possible explanation for this difference could be that the PSID undersamples wealthy households. Therefore successful entrepreneurs are likely to be undersampled.
[23] See Appendix B for details.
[24] This ratio ranges from 4.8 to 5.6 in the SCF according to definitions of entrepreneurs and samples adopted.

Table 1.3: the calibrated parameters

Parameter	Symbol	Benchmark Value
High entrepreneurial productivity	θ^H	0.52
Entrepreneurial productivity transition	p^{HH}, p^{LL}	0.95 , 0.9937
Concavity of entrepreneurial technology	ν	0.875
Capital specificity	f	0.4
Discount factor	β	0.865
Probability of transitory shock	p^X	0.185

baseline specification.

Table 1.4: the baseline calibration targets

Moment	Target	Model
Fraction of Entrepreneurs (in %)	7.6	7.6
Ratio of medians (in %)	5.6	4.34
Share of net-worth of top 40%	93.0	89.4
K/Y	2.8	2.687
Exit Rate (in %)	9.3	9.4
Bankruptcy/Exit (in %)	20.0	22.0
Entry rate of experienced/Average entry rate	13.0	8.3

The equilibrium rate of return on capital in the corporate sector (r^d) is 7.81%. Since the equilibrium wage is 1.0207, each unit in our model correspond approximately to $44,000 in 1993. Less than one percent (0.79%) of the total population is borrowing constrained. Even though our model does not replicate exactly the ratio of medians and the share of the wealth held by the richest 40%, it captures the main features that entrepreneurs are several times richer than workers and that most of the wealth is held by the richest. Table 1.5 shows that our model does not replicate the wealth concentration at the top end of the wealth distribution. In particular the richest one percent hold 16% of total wealth in our model while they hold 35% in the data[25]. However for the purpose of our policy experiments it is important that the model replicates the middle and lower part of the wealth distribution since bankruptcy law affects almost exclusively these agents.

Table 1.5: wealth distribution: data and model

	percentage wealth in top				
	1%	5%	20%	40%	60%
US data (SCF 1995)	35	56	81	93	99
Benchmark model	16	38	65	84	95

Even though our model does not replicate the difference in the entry rate between experi-

[25] This is the reason that the Gini coefficient of wealth is 0.64 in the model, while it is 0.8 in the data. Cagetti / De Nardi [2006] and Castaneda et al. [2003] show that life-cycle savings and the bequest motive are essential to match the wealth distribution. Introducing these features in the model would be computationally too costly.

enced and inexperienced workers exactly it captures the fact that the former are many times more likely to enter entrepreneurship than the latter.

Quadrini [2000] reports that around 40% of total capital is invested in the entrepreneurial sector. In our baseline specification this fraction is slightly higher, around 45%. However the US. Small Business Administration estimates that the share of the entrepreneurial sector in terms of employment is 50%.

1.5 Investigating the model's mechanisms

1.5.1 Occupational choice

The key ingredient of the model is occupational choice. Figure 1.1 represents the occupational choice of an *unconstrained* agent with high entrepreneurial productivity and low working productivity. The dotted line shows the value function of becoming a worker, whereas the solid line shows the value function of becoming an entrepreneur[26].

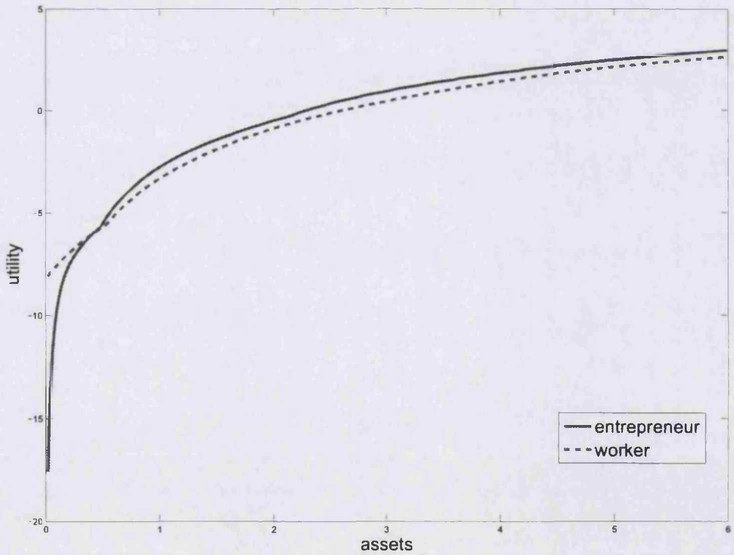

Figure 1.1: Occupational choice ($S = UN, \theta_{-1} = \theta^H, \varphi_{-1} = 0.316$)

The first result is that, otherwise identical agents choose differently according to their wealth: poor agents become workers while rich agents become entrepreneurs. This result is standard in the occupational choice under credit market imperfections literature [see for

[26] The value functions have kinks since the actual value function for an unconstrained agent is given by the upper envelop of the two functions in Figure 1.1. Therefore discounted utility tomorrow is kinked as well. The kinks do no coincide exactly with the intersection of the two functions. However the kinks must be close to the intersection of the two curves exactly because the value function tomorrow is identical for entrepreneur and worker.

example Banerjee / Newman 1993]. The main reasons are that poor agents have smaller firms and face higher interest rates. They have smaller firms because, being poor, they need to borrow more but they face higher rates on the loans. The cost of financing is higher for the poor for two reasons. First, they have a higher incentive to default. Defaulting rich agents have to give up all their wealth above the exemption level. Second, in the event of default the bank gets less when the agent is poor. Thus, to break even, the bank has to charge a higher interest rate. That is, in this model, wealth acts as collateral.

1.5.2 The behavior of the unconstrained agents

The second important ingredient is the decision of the *unconstrained* entrepreneurs. The solution of the entrepreneurs' problem is represented in Figure 1.2:

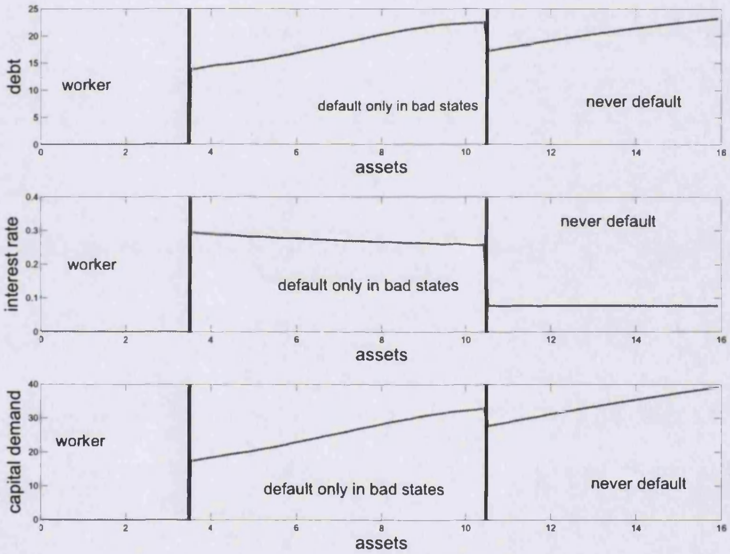

Figure 1.2: interest rate and firm size ($\theta_{-1} = \theta^H$, $\varphi_{-1} = 1.341$)

The upper panel shows credit demand (debt) of the entrepreneur, the middle panel represents the corresponding interest rate charged and the lower panel capital demand (firm size). As shown above the poorer agents (e.g. agents with assets $a = 2$) become workers while all the others become entrepreneurs ($a > 3.5$). The very rich entrepreneurs (e.g. $a = 14$) will never find it profitable to default. Their wealth is so high that defaulting is too costly for them. Therefore they can borrow at rate r^d. The "middle class" entrepreneurs (e.g. $a = 6$) will instead default if their productivity θ drops to θ^L or a bad shock ($\chi = 0$) happens, since the cost of bankruptcy is lower for them. Then the bank, in order to break even, must charge a higher interest rate. The interest rate depends (negatively) on the assets of the entrepreneur, because in the event of default the bank will be able to seize the difference between the assets of the entrepreneur and the exemption level. Capital demand for the "middle-class" entrepreneurs is increasing because the cost of borrowing is declining. The discontinuity in all three functions between "middle-class" and rich entrepreneurs (around $a = 10.5$) is due to the change in the default decision. Those who default are insured against the bad outcome whereas those who do not default are not. This explains why relatively poorer agents (e.g. $a = 10$) have slightly bigger firms than relatively richer agents (e.g. $a = 11$).

1.5.3 A first look at the effects of bankruptcy

Bankruptcy affects the problem of the unconstrained agents, because it changes credit conditions and the extent of insurance available. We examine these effects with the following experiment. We compare the behavior of the unconstrained agents and the banks in two different situations: one in which bankruptcy is allowed and one in which bankruptcy is absent. Figure 1.3 shows the capital demand function and the interest rate function in these situations.

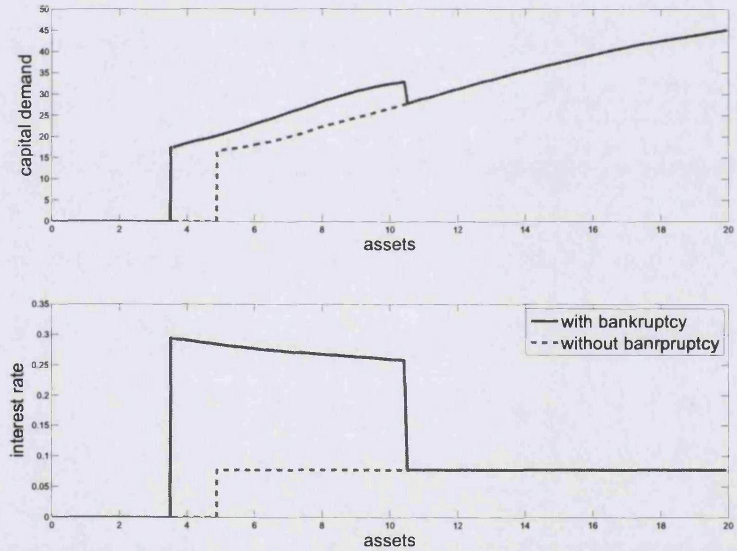

Figure 1.3: Firm size and interest rate ($S = UN, \theta_{-1} = \theta^H, \varphi_{-1} = 1.314$)

The effects of allowing bankruptcy depend on the wealth of the agent. First, the behavior of the very rich (e.g. $a = 12$) is not affected. They are entrepreneurs and they repay their debt even in the bad states. As explained above, even if bankruptcy is available, it is too costly for them. Second, allowing bankruptcy affects the behavior of the less rich agents (e.g. $a = 8$). They are entrepreneurs in both situations. But when bankruptcy is allowed they borrow more because they can and will default in the bad states. Therefore their firms are bigger (upper panel). This insurance comes at expense of higher interest rates (lower panel). Anticipating default in the bad states the banks have to charge higher interest rates in order to break even. We call this increase in the firm size the *intensive margin.* Third, the occupational choice of even less rich agents (e.g. $a = 4$) is affected. When bankruptcy is not allowed they are not insured against bad outcomes. Therefore they do not want to borrow, even though they could borrow at rate r^d. They become workers. When bankruptcy is allowed they are insured against bad outcomes. Therefore they borrow, even though they have to pay a high interest rate. This increases the rewards of entrepreneurship enough to change their occupational choice. We call this increase of the number of entrepreneurs the *extensive margin.* Fourth, the occupational choice of the very poor agents (e.g. $a = 2$) is not affected, they are workers in both situations.

In this particular experiment abolishing bankruptcy reduces entrepreneurship and firm size, the intensive and the extensive margins are negative. The negative effect of lowering the amount of insurance available dominates the positive effect of better credit conditions.

1.6 The effects of bankruptcy reforms

We now turn to analyze the effects of changes in the bankruptcy law. We conduct 2 different experiments:

1. we change the exemption level from zero, which corresponds to eliminating bankruptcy completely, to a very high level, twice the current level;
2. we change the length of the credit market exclusion period from two to 20 years.[27]

We will focus our attention mainly on changes in the following variables: entrepreneurship, the poors' access to entrepreneurship, welfare, distributional issues and social mobility.

1.6.1 Changing the exemption level

Our first policy experiment is to analyze the effects of changing the exemption level. First we inspect the changes in the policy functions and later we analyze the quantitative results. Figure 1.4 reports capital demand (upper panel) and the interest rate (lower panel) for 3 different values of X/w. It shows the effects of increasing the exemption level from $X/w = 0$, which corresponds to completely eliminating bankruptcy to an intermediate one ($X/w = 0.875$) and to the actual one ($X/w = 1.72$).

Increasing the exemption level, from zero to 0.875 has two effects. Both, the firms get bigger (intensive margin) and more agents enter entrepreneurship (extensive margin). The insurance effect is dominating. Further increasing the exemption level, to the current level of 1.72, has three effects. First, agents with assets around 3, who were entrepreneurs before, become workers because credit conditions worsen so much that they outweigh the increase

[27] In the model this corresponds to changing the probability of receiving a positive solvency shock ϱ from 0.5 to 0.05.

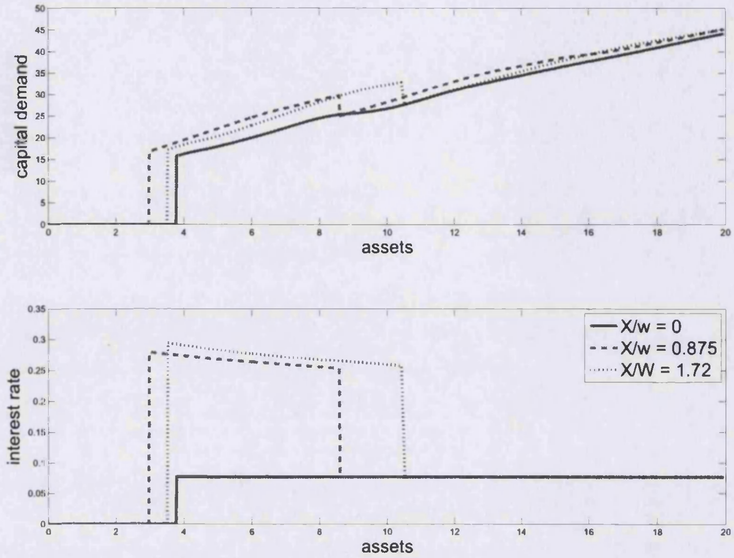

Figure 1.4: Firm size and interest rates, different exemption levels ($\theta_{-1} = \theta^H$,$\varphi_{-1} = 1.342$)

in insurance. The extensive margin is negative. Second, agents with assets around 6 are charged higher interest rates for the same reasons. Thus they run smaller firms. For these agents the intensive margin is negative. Third, agents with assets around 10 switch from never defaulting to defaulting in the bad states. Now they runs bigger firms, even if credit conditions are worse, because of the insurance effect. For these agents the intensive margin is positive.

The magnitude of these effects depends on the number of agents affected. The extensive margin is unambiguously positive. The sign of the intensive margin, however, is ambiguous. It depends on the wealth distribution. The increase in capital demand of agents with asset around 10 is bigger than the decrease in capital demand of agents with asset around 6. But the overall effects depend on the number of agents in these areas of the wealth distribution.

Table 1.6 reports the variables of interest for 5 values of X/w. Column 2 reports results when bankruptcy is absent ($X/w = 0$). Column 4 reports results for the baseline calibration ($X/w = 1.72$) and column 6 for doubling the current exemption level ($X/w = 3.5$).

The first pattern to notice is that no bankruptcy and extremely generous bankruptcy law produce very similar results (see column 2 and column 6). When bankruptcy is absent the demand for risky loans (loans with high interest rate due to high positive default probability) is zero. Entrepreneurial activity is so risky that only relatively rich agents, who always repay and get credit at rate r^d, become entrepreneurs. When bankruptcy law is very generous, the banks have to charge such high interest rates on risky loans that nobody demands them. Again, only rich agents become entrepreneurs. This also explains that the ratio of medians is highest in the case of no bankruptcy and very generous bankruptcy law. Even though for

Table 1.6: the effects of changes in the exemption level

X/w	**0**	**0.875**	**1.72**	**2.625**	**3.5**
Exit rate (in %)	9.5	9.9	**9.4**	9.6	9.6
Fraction of Entrepreneurs (in %)	7.4	8.1	**7.6**	7.4	7.4
Bankruptcy/Exit (in %)	0	45.9	**22.2**	0.2	0.3
Capital/Output	2.677	2.693	**2.677**	2.677	2.677
Median assets of Entr/ Median assets	4.467	4.157	**4.347**	4.429	4.429
Share of Capital in entr. sector (in %)	47.8	49.4	**47.9**	47.8	47.8
Gini of Assets	0.635	0.636	**0.635**	0.635	0.635
Share of assets in top 40% of pop (in %)	89.0	89.3	**89.0**	89.0	89.0
Median output in entrepreneurial sector	15.05	14.55	**14.58**	15.05	15.05
Welfare (%-change in cons.-equivalent)	-0.07	1.26	**0**	-0.05	-0.05
Welfare of the POOR	-0.09	1.27	**0**	-0.07	-0.06
Welfare of the RICH	-0.02	1.23	**0**	0.03	0

each level of assets entrepreneurs borrow less and therefore have smaller firms, the median firm size is bigger under extreme bankruptcy laws, see Figure 1.5[28]. The reason for this result is again that only rich agents, who have bigger firms, become entrepreneurs.

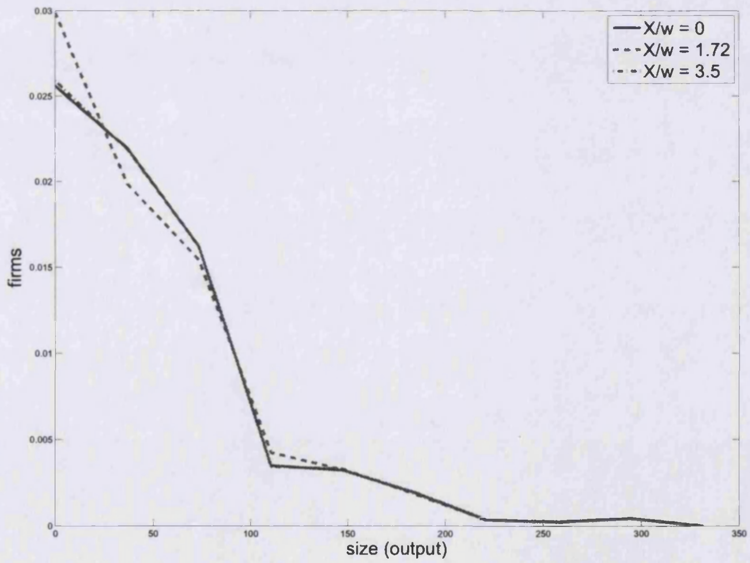

Figure 1.5: Firm size distribution for different exemption levels

Next we investigate the effects of increasing the exemption level gradually from $X/w = 0$ to $X/w = 3.5$ on entrepreneurship, the poors' access to entrepreneurship, welfare, wealth distribution and social mobility. As can be seen in table 1.6 almost all variables follow a hump-shaped pattern.

[28] We smoothed the firm size distribution by creating ten equally sized bins to make the figure easier to read.

Entrepreneurship Increasing the exemption level first increases and then decreases the fraction of entrepreneurs. The insurance effect dominates the credit market conditions effect for low exemption levels. The opposite is true for high exemption levels. The exit rate and the fraction of exits through bankruptcy follow the behavior of the fraction of entrepreneurs. The fraction of exits through bankruptcy first increases from zero percent to 46% when the exemption level increases from $X/w = 0$ to $X/w = 0.875$. As insurance is higher, a bigger fraction of exits happens through bankruptcy. When the exemption level increases further, from $X/w = 0.875$ to $X/w = 3.5$ the fraction falls gradually back to zero percent because only the rich, who never default, become entrepreneurs.

The impact of different exemption levels on the investment behavior of entrepreneurs can be understood from the firm size distribution, see Figure 1.6[29].

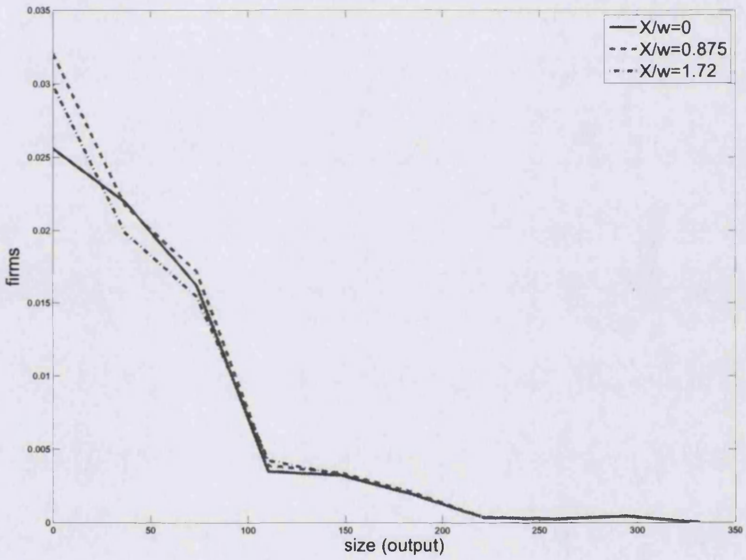

Figure 1.6: Firm size distribution (different exemption levels)

Increasing the exemption level from $X/w = 0$ to $X/w = 0.875$ leads to the creation of more small firms due to positive extensive and intensive margins, see also Figure 1.4. When we further increase the exemption level to $X/w = 1.72$ some of these new small firms disappear because the negative effect on credit market conditions dominates.

Access to entrepreneurship of the poor Next we turn to how bankruptcy law affects the determinants of entry into entrepreneurship. There is allocative inefficiency in our model because insurance markets are missing. Part of this inefficiency is reflected in some poor highly productive agents not becoming entrepreneurs, either because they receive too little

[29] As shown in Figure 1.5, the firm size distribution for higher exemption levels is identical to the case $X/w = 0$. Therefore in Figure 1.6 we report only the cases: $X/w = 0$, $X/w = 0.875$, and $X/w = 1.72$.

insurance or because the conditions at which credit is available are too bad. Table 1.7 reports the effects of different exemption levels on the minimum assets needed for the highly productive ($\theta_{-1} = \theta^H$) agent to become an entrepreneur.

Table 1.7: minimum wealth for entrepreneurship

X/w	0	0.875	1.72	2.625	3.5
φ_{-1}= 0.316	0.481	0.160	0.421	0.381	0.361
φ_{-1}= 0.745	1.323	0.842	1.263	1.323	1.323
φ_{-1}= 1.342	3.768	2.946	3.507	3.768	3.768
φ_{-1}= 3.163	16.032	15.030	15.230	16.032	16.032

The rows show these values for the levels of working productivity (φ_{-1}). The attractiveness of becoming a worker is increasing in working productivity. Thus, in order to enter entrepreneurship, the expected profits must be higher for an agent with high working productivity. Since richer agents need to borrow relatively less and since they receive better credit conditions, their expected profits are higher. This implies that, to become an entrepreneur, an agent with high working productivity must be richer than an agent with low working productivity.

At each level of working productivity the wealth level at which an agent enters entrepreneurship is lowest when $X/w = 0.875$. Thus, even from an efficiency point of view a less generous bankruptcy law would improve upon the status quo. However, abolishing bankruptcy completely would make it more difficult for the poor to become entrepreneurs, thereby worsening allocative efficiency.

Welfare Following Aiyagari / Mcgrattan [1998], to assess welfare we first calculate expected utility in each bankruptcy policy regime separately

$$V = \int_{\eta} V(\eta) d\mu^* (\eta) \tag{1.19}$$

where $\eta = (a, \theta_{-1}, \varphi_{-1}, S)$ and $\mu^* (\eta)$ is the equilibrium steady state distribution. Thus, expected utility is measured over all asset levels, productivities and the credit status. This utilitarian social welfare function weights all households equally. Then we calculate the constant, at all states and dates, amount of consumption, *consumption equivalent*, that yields expected utility V.[30] We compare two bankruptcy policy regimes by calculating the percentage change in consumption equivalent that makes agents indifferent between the two regimes. For example, for a given regime Q, that yields utility V^Q, this percentage change in

[30] Thus, we first calculate a constant $\bar{c}$ that yields that same utility as V. Given CRRA preferences this is the solution to:

$$\left(\frac{\bar{c}^{(1-\sigma)} - 1}{1 - \sigma} \right) \frac{1}{1 - \beta} = V \ .$$

consumption equivalent is given by

$$\lambda^Q = \left(\frac{V^Q + 1/\left[(1-\sigma)(1-\beta)\right]}{V^{bench} + 1/\left[(1-\sigma)(1-\beta)\right]} \right)^{1/(1-\sigma)} - 1 \tag{1.20}$$

where a positive λ^Q implies that regime Q increases welfare with respect to the baseline regime.

Table 1.6 shows that welfare follows the same hump-shaped pattern as the other variables. In particular welfare is highest for exemption level $X/w = 0.875$. Thus, halving the current exemption level would increase welfare by 1.26%, which corresponds to an increase in annual consumption of approximately $700 for the average household.

Table 1.6 also shows that there are no adverse distributional effects. Both, rich and poor agents[31] gain from reducing the exemption level from the current one.

Wealth distribution and social mobility Entrepreneurs are relatively less rich compared to the entire population when $X/w = 0.875$. This is shown by the ratio of median assets in table 1.6. This is again due to the fact that there are more poor entrepreneurs when $X/w = 0.875$ than for any other exemption level. However changing the exemption level has little effect on the wealth distribution: it does not change significantly the Gini coefficient and the share of wealth held by the richest agents. The changes in entrepreneurship and firm sizes are too small to significantly affect the wealth distribution.

We investigate the effects on social mobility by dividing all agents in 3 wealth classes: poor, middle-class and rich, where each class accounts for 1/3 of total population. Then we compute the transition between these classes over a 10 year horizon for the different values of the exemption level. The results are reported in tables 1.8 to 1.10[32].

Table 1.8: 10-years transition matrix: $X/w = 0$

	poor	middle-class	rich
poor	0.721	0.246	0.033
middle-class	0.277	0.482	0.241
rich	0.004	0.270	0.726

These tables show that there is slightly more mobility in the intermediate case ($X/w = 0.875$) since the probabilities along the main diagonal are smaller. As shown in table 1.7, for intermediate exemption levels poorer agents have more insurance and therefore enter entrepreneurship. Thus, in our model, entrepreneurship is a vehicle of social mobility. This is consistent with the findings of Quadrini [2000].

[31] We define a poor agent as one with assets less than the median. Comparing the top and bottom quintiles yields similar results.

[32] Again, results for $X/w = 2.625$ and $X/w = 3.5$ are not reported. They are very similar to the case with $X/w = 0$.

Table 1.9: 10-years transition matrix: $X/w = 0.875$

	poor	middle-class	rich
poor	0.717	0.249	0.034
middle-class	0.279	0.478	0.243
rich	0.004	0.274	0.722

Table 1.10: 10-years transition matrix: $X/w = 1.72$

	poor	middle-class	rich
poor	0.720	0.248	0.032
middle-class	0.276	0.480	0.244
rich	0.005	0.271	0.724

1.6.2 Changing the exclusion period

The second policy experiment we conduct is to change the length of time an agent who has defaulted is excluded from borrowing[33]. As discussed above we model this as changes in the probability of a favorable credit status shock: ϱ. Therefore a low ϱ represents a long exclusion period while a high ϱ represents a short exclusion period.

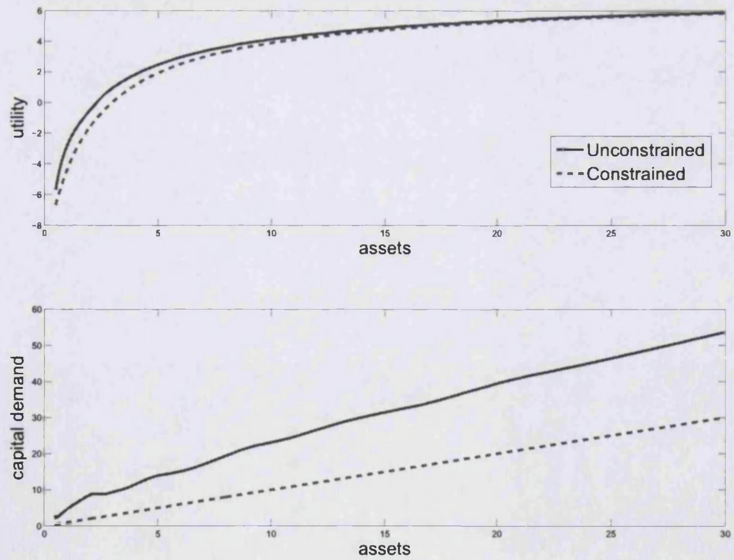

Figure 1.7: Utility and capital demand of *borrowing constrained* and *unconstrained* entrepreneur

Table 1.11 reports the effects of gradually increasing the exclusion period from two years ($\varrho = 0.5$) to 20 years ($\varrho = 0.05$) on the main variables. The baseline value of five years

[33] The length of the exclusion period is determined mainly by banks in the US, but in principle this could be regulated by a law.

($\varrho = 0.2$) is reported in column four.

Table 1.11: the effects of changes in the exclusion period

ϱ	**0.5**	**0.25**	**0.2**	**0.1**	**0.05**
Exit rate (in %)	9.3	9.4	**9.4**	9.6	9.8
Fraction of Entrepreneurs (in %)	7.7	7.6	**7.6**	7.5	7.4
Bankruptcy/Exit (in %)	23.6	22.7	**22.2**	21.1	20.1
Capital/Output	2.686	2.680	**2.678**	2.668	2.654
Median assets of Entr/ Median assets	4.431	4.388	**4.329**	4.225	4.156
Share of capital in entr. sector (in %)	48.8	48.0	**47.8**	46.7	45.4
Gini of Assets	0.065	0.065	**0.063**	0.064	0.063
Share of assets in top 40% of pop.(in %)	89.2	89.1	**89.0**	88.8	88.6
Median output in entrepreneurial sector	14.991	14.535	**14.576**	13.701	12.289
Welfare (%-change in cons.-equivalent)	0.12	0.02	**0**	-0.18	-0.43
Welfare of the POOR	0.05	-0.04	**0**	-0.09	-0.28
Welfare of the RICH	0.34	0.21	**0**	-0.46	-0.84

Table 1.11 shows that reducing the length of the exclusion period increases welfare, and the fraction of entrepreneurs monotonically. However these changes are quantitatively much smaller than in the case of changing the exemption level. The main implication of increasing ϱ is to allow highly productive, failed agents to regain access to credit earlier. Figure 1.7 shows the difference in utility and the difference in firm size for a highly productive agent between being borrowing constrained and being unconstrained.

One important difference between changing the exemption level and changing the exclusion period is that the credit market conditions effects are smaller. Both, increasing the exemption level and lowering the exclusion period, increase the attractiveness of defaulting. However, the latter does not affect the amount recovered by banks in the event of a default. Therefore the interest rates charged by banks do not change for most agents, see for example the agents with assets between four and ten in Figure 1.8. These agents default in the bad states for all values of ϱ. However agents with assets around 10.5 change their behavior. Instead of repaying their debt in all states, as they do when $\varrho = 0.05$, they default in the bad states when $\varrho = 0.5$ because defaulting is more attractive. Therefore they borrow more and have bigger firms. For similar reasons, agents with assets around 3.5 enter entrepreneurship only when ϱ increases.

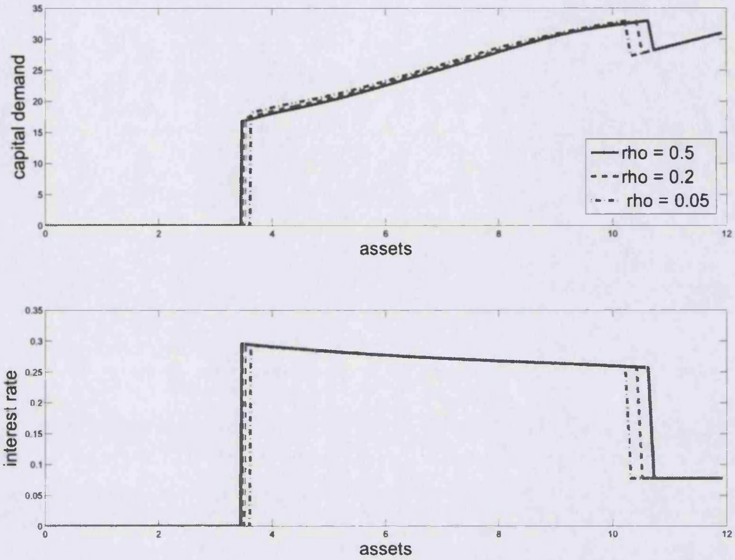

Figure 1.8: Capital demand and interest rate, different ϱ ($\theta_{-1} = \theta^H$,$\varphi_{-1} = 1.341$)

Some of the defaulters are hit by the very persistent change in entrepreneurial productivity. Therefore only a fraction of defaulters are still highly productive as entrepreneurs. This implies that the overall effects are small.

Next we investigate the effects of increasing the exclusion period from 2 years ($\varrho = 0.5$) to 20 years ($\varrho = 0.05$) on entrepreneurship, the poors' access to entrepreneurship, welfare, wealth distribution and social mobility in detail.

Increasing the exclusion period from 2 years ($\varrho = 0.5$) to 20 years ($\varrho = 0.05$) lowers the fraction of entrepreneurs. As shown in Figure 1.8, poorer agents do not enter entrepreneurship as often as before because the cost of defaulting is higher. The median firm size decreases because relatively rich entrepreneurs change their behavior. When they are hit by a bad shock they do not default anymore. This implies that they are fully exposed to the production risk. Therefore they operate smaller firms.

The wealth levels needed to become an entrepreneur, one for each level of working productivity, are reported in table 1.12.

Increasing the exclusion period implies that more wealth is needed to enter entrepreneurship. Therefore it makes access to entrepreneurship more difficult for poor but highly productive agents. But these changes are small, in particular when compared to the changes when the exemption level is lowered.

Increasing the exclusion period also reduces welfare. Note that even though the Gini coefficient is highest for the shortest exclusion period ($\varrho = \mathbf{0.5}$), welfare for both, rich and

Table 1.12: minimum wealth for entrepreneurship

ϱ	0.5	0.25	0.2	0.1	0.05
φ_{-1}= 0.316	0.38	0.42	0.42	0.46	0.48
φ_{-1}= 0.745	1.26	1.28	1.28	1.28	1.28
φ_{-1}= 1.342	3.47	3.53	3.53	3.59	3.63
φ_{-1}= 3.163	15.63	15.73	15.73	15.73	15.63

poor, is highest in this case as well. Lowering the exclusion period from the current five years to two years would increase welfare by 0.12%, which corresponds to an increase in annual consumption of approximately $70 for the average household. Increasing the exclusion period to 20 years would yield a welfare loss of approximately 0.43%, which corresponds to a decrease in annual consumption of approximately $230 for the average household.

As tables 1.13 to 1.15 show there are hardly any changes in social mobility.

Table 1.13: 10-years transition matrix: $\varrho = 0.5$

	poor	middle-class	rich
poor	0.721	0.248	0.032
middle-class	0.276	0.480	0.244
rich	0.005	0.270	0.725

Table 1.14: 10-years transition matrix: $\varrho = 0.2$

	poor	middle-class	rich
poor	0.721	0.247	0.032
middle-class	0.276	0.479	0.244
rich	0.005	0.271	0.724

Table 1.15: 10-years transition matrix: $\varrho = 0.05$

	poor	middle-class	rich
poor	0.720	0.248	0.032
middle-class	0.276	0.480	0.244
rich	0.005	0.271	0.724

1.7 Conclusion

We explore quantitatively the effects of personal bankruptcy law on entrepreneurship in a general equilibrium setting with heterogeneous agents. We developed a dynamic general equilibrium model with occupational choice which explicitly incorporates the US bankruptcy law. Our model endogenously generates interest rates that reflect the different default probabilities of the agents. It accounts for the main facts on entrepreneurial bankruptcy, entrepreneurship, wealth distribution and macroeconomic aggregates in the US.

We used the model to quantitatively evaluate the effects of changing the US bankruptcy law. The simulation results show that reducing the exemption level would increase the fraction of entrepreneurs and welfare. These effects are significant: halving the exemption level would have positive welfare effects in the order of 1.4% of average consumption. All households, rich and poor, would be better off. However eliminating bankruptcy completely would reduce the number of entrepreneurs and welfare. The key mechanism driving most of our results is the occupational choice of agents. The fraction of entrepreneurs would increase by one percentage point if the exemption level were reduced by 50%.

We are currently extending our research program along two dimensions. First, we are incorporating the transition to the new steady state. So far, our results are based on a comparison of steady-states. Transitional effects might be important to evaluate welfare. In addition it might explain why the current law is too lenient. It could be that some groups lose during the transition and therefore oppose changes.

Second, we are expanding our model to incorporate explicitly a European type of bankruptcy law. The laws in European countries are much harsher than the law in the US. For example in Italy, debt is never discharged. A defaulter is liable forever. We are analyzing the effects of introducing a US type of law on the Italian economy.

1.8 Appendix

1.8.1 Computational strategy

The state vector for an individual is given by $\eta = (a, \theta_{-1}, \varphi_{-1}, S)$. The aggregate state variable is a density $\mu_t(a, \theta_{-1}, \varphi_{-1}, S)$ over the states. We assume that a take value on a grid G_a of dimension n_a. Therefore the dimension of the individual state space is $n = n_a \times n_\theta \times n_\varphi \times 2$ where $n_\theta = 2$ is the number of states for the entrepreneurial productivity and $n_\varphi = 4$ is the number of states for the working productivity.

In order to solve the model we use the following approach:

Algorithm 1 *Our solution algorithm is:*

1. *Assign all parameters values*
2. *Guess a value for the endogenous variable r.*
3. *Given r the FOC of the corporate sector uniquely pin down the wage rate w. The representative competitive firm in the corporate sector will choose K_c and L_c such that*

$$r^d = \xi A K_c^{\xi-1} L_c^{1-\xi} = \xi A \left(\frac{K_c^d}{L_c^d}\right)^{\xi-1} \quad (1.8\text{-}1)$$

$$w = (1-\xi) A K_c^{\xi} L_c^{-\xi} = (1-\xi) A \left(\frac{K_c^d}{L_c^d}\right)^{\xi} \quad (1.8\text{-}2)$$

Therefore r uniquely pins down $\left(\frac{K_c}{L_c}\right)$ and in turn uniquely pins down w.

4. *Given (r, w) we solve for the optimal value functions and corresponding policy functions by value function iteration. The details of the zero profit conditions for the banks are presented in the next subsection.*

   a) First we solve for the following policy functions[34]*:*

   - *Saving policy function:* $\mathsf{a}'(a, \theta_{-1}, \varphi_{-1}, \theta, \varphi, S, OCC)$ *which for any state today $(\theta_{-1}, \varphi_{-1})$ and for any state tomorrow (θ, φ), for any given level of assets a, for any given credit status $S \in \{UN, BC\}$ and for any occupational choice $OC \in \{W = 0, E = 1\}$ gives us the optimal saving decision of the agent;*
   - *Capital demand function* $\mathsf{k}(a, \theta_{-1}, \varphi_{-1}, S, OCC)$ *for entrepreneurs;*
   - *default decision* $\mathsf{d}(a, \theta_{-1}, \varphi_{-1}, \theta, \varphi, S, OCC)$ *for unconstrained entrepreneur;*

[34] Note that given our timing the saving and bankruptcy decisions are taken when the uncertainty about $\theta'$and φ' has been resolved, therefore they appear as argument of the policy function.

b) *The above policy functions allow us to calculate the implied value functions* $V(a, \theta_{-1}, \varphi_{-1}, S, OCC)$

c) *This in turn allows us to solve for the occupational choice function*

$$\mathsf{OC}^*(a, \theta_{-1}, \varphi_{-1}, S) = \begin{cases} = 1 & V(a, \theta_{-1}, \varphi_{-1}, S, E) \geq V(a, \theta_{-1}, \varphi_{-1}, S, W) \\ = 0 & \textit{otherwise} \end{cases} \tag{1.8-3}$$

5. *The policy functions, the exogenous transition matrix for the shocks (both for* θ_{-1} *and for* φ_{-1}*) and the credit status shock* ϱ *allow us to derive the probability that an agent in a certain state* η *will be in the state* η' *next period, for any give state* η*. Given the dimension of the state, all these probabilities form a transition matrix* P_η *of dimension* $n \times n$.

6. *The transition matrix* P_η *maps the current distribution*[35] μ_η *into a next period distribution* μ'_η

$$\mu_{\eta,t+1} = P'_\eta \times \mu_{\eta,t} \tag{1.8-4}$$

We calculate the steady state distribution over the state μ^*_η *by solving for a*

$$\mu^*_\eta = P'_\eta \times \mu^*_\eta \tag{1.8-5}$$

7. *From this we can derive the market clearing conditions*

- *the saving for the whole economy*

$$SA(r) = \sum_{i=1}^{na} \sum_{j=1}^{n_\theta} \sum_{v=1}^{n_\varphi} \sum_{u=1}^{2} a_i \times \mu^*(a_i, \theta_{-1j}, \varphi_{-1v}, S_u) \tag{1.8-6}$$

- *the supply of labor*

$$L^s(r) = \sum_{i,j,v,u} \mu^*(a_i, \theta_{-1j}, \varphi_{-1v}, S_u) \times [1 - \mathsf{OC}^*(a_i, \theta_{-1j}, \varphi_{-1v}, S_u)] \varphi_{-1v} \tag{1.8-7}$$

- *the demand of capital from the entrepreneurial sector*

$$\begin{aligned} K^d_{ENTR}(r) = \sum_{i,j,v,u} & \mathsf{p}^*(a_i, \theta_{-1j}, \varphi_{-1v}, S_u) \\ & \times \mathsf{OC}^*(a_i, \theta_{-1j}, \varphi_{-1v}, S_u) \times \mathsf{k}^*(a_i, \theta_{-1j}, \varphi_{-1v}, S_u) \end{aligned} \tag{1.8-8}$$

where $\mathsf{k}^*(a_i, \theta_{-1j}, \varphi_{-1v}, S_u) = \mathsf{k}[a_i, \theta_{-1j}, \varphi_{-1v}, S_u, OC^*(a_i, \theta_{-1j}, \varphi_{-1v}, S_u)]$

[35] Note that in our framework the distribution of household over the state μ_η, is vector of dimension n whose elements sum up to 1.

8. *Labor market clearing implies that labor supply $L^s(r)$ is equal to labor demand L_c^d. Plugging this into the FOC (1.8-1) of the corporate sector we get the capital demand in the corporate sector:*

$$K_c^d(r) = \left(\frac{r}{\xi A}\right)^{\frac{1}{\xi-1}} L^S(r) \tag{1.8-9}$$

9. *Now we look at capital market clearing:*

$$K_{ENTR}^d(r) + K_c^d(r) = SA(r) \tag{1.8-10}$$

10. *If there is not equilibrium at point 9 we adjust the interest rate, go back to point 3, and iterate until the market clears*[36].

Value function iteration

Given the presence of kinks in the problem we use a value function iteration algorithm to solve for the value functions. We approximate the value functions using cubic splines.

The iteration goes as follows.

1. We guess a value function both for the UN and the BC agent: $V^{BC0}(a, \theta_{-1}, \varphi_{-1})$ and $V^{UN0}(a, \theta_{-1}, \varphi_{-1})$
2. Given the guesses, we solve for 4 value functions, two for the workers ($W^{BC}(a, \theta_{-1}, \varphi_{-1})$ and $W^{UN}(a, \theta_{-1}, \varphi_{-1})$) and two for the entrepreneurs ($N^{BC}(a, \theta_{-1}, \varphi_{-1})$ and $N^{UN}(a, \theta_{-1}, \varphi_{-1})$). The only non standard problem is to find $N^{UN}(a, \theta_{-1}, \varphi_{-1})$ where we take the zero profit condition of the bank into account. The solution is described in the next subsection.
3. Form the function we can derive a new guess for the value function

$$V^{BC1}(a, \theta_{-1}, \varphi_{-1}) = \max\left\{N^{BC}(a, \theta_{-1}, \varphi_{-1}), W^{BC}(a, \theta_{-1}, \varphi_{-1})\right\} \tag{1.8-11}$$

$$V^{UN1}(a, \theta_{-1}, \varphi_{-1}) = \max\left\{N^{UN}(a, \theta_{-1}, \varphi_{-1}), W^{UN}(a, \theta_{-1}, \varphi_{-1})\right\} \tag{1.8-12}$$

4. Therefore we can construct an iteration of the form

$$\begin{bmatrix} V^{BCj}(a, \eta) \\ V^{UNj}(a, \eta) \end{bmatrix} \rightarrow \begin{bmatrix} V^{BCj+1}(a, \eta) \\ V^{UNj+1}(a, \eta) \end{bmatrix} \tag{1.8-13}$$

[36] In practice we first run a grid search over different values for r and then bisect until we get market clearing.

The zero profit condition

In the derivation of the optimal choice of the unconstrained entrepreneur we assume that he can borrow from a perfectly competitive banking sector: that is there is free entry in the sector. This implies that the bank makes zero profit on each contract. What we need is a menu of contracts that the bank offers, where each contract is an amount lent $b\left(a, \theta_{-1}, \varphi_{-1}\right)$ and an interest rate $r\left(a, \theta_{-1}, \varphi_{-1}, b\right)$ that, give the assumption of perfect symmetric information, can depend on the individual state of the agent

Banks will get repaid if the type-$(a, \theta_{-1}, \varphi_{-1})$ agent finds it optimal not to declare bankruptcy at the end of the period, given the amount lent. We denote the probability of bankruptcy as $\pi^{bankr}\left(a, \theta_{-1}, \varphi_{-1}, b\right)$. Therefore the zero profit condition is given by

$$\left(\begin{array}{c} \left[1 - \pi^{bankr}\left(a, \theta_{-1}, \varphi_{-1}, b\right)\right]\left[1 + r(a, \theta_{-1}, \varphi_{-1}, b)\right] b + \\ + \pi^{bankr}(a, \eta) \max\left\{\chi \theta k^{\nu} + f\left(1 - \delta\right)\left(a - b\right) - X, 0\right\} \end{array}\right) = (1 + r) b \qquad (1.8\text{-}14)$$

In order to find the equilibrium interest rate $r\left(a, \theta_{-1}, \varphi_{-1}, b\right)$ charged to each type of agent we must find the probability that the agent defaults. However, it is important to note that the contracts the bank offers must all make zero profits in expectations, also the out-of-equilibrium contracts (i.e. those the agent does not choose).

We solve the problem of unconstrained entrepreneurs over a grid. For any given type $(a, \theta_{-1}, \varphi_{-1})$ we find the optimal choice given a grid of possible levels of loans: $b_i \in [b_{\min}, b_{\max}]$. Given each value of $b_i > 0$ (if $b_i < 0$ the agent saves so he does not need the bank and gets an interest rate r) there are only three possibilities[37]:

- The agent always repays, both in the event of bad and in the event of a good shock to entrepreneurial productivity. In this case $\pi^{bankr}\left(a, \theta_{-1}, \varphi_{-1}, b\right) = 1$, and therefore the only interest rate compatible with zero profits is r.
- The agent repays only in the case of a bad shock. In this case we know that $\pi^{bankr}\left(a, \theta^{H}, \varphi_{-1}, b\right) = 1 - p^{HH}$ and $\pi^{bankr}\left(a, \theta^{L}, \varphi_{-1}, b\right) = 1 - p^{LL}$ and we can calculate, for any b, the unique interest rate $r\left(a, \theta_{-1}, \varphi_{-1}, b\right)$ such that the bank breaks even.
- The agent never repays so he never gets credit.

Therefore our strategy is, for any $b_i \in [b_{\min}, b_{\max}]$

1. First we check what happens if the agent is offered the rate r^d.
2. If the agent always repays we are done.
3. If the agent does not repay we check what would he do if he was offered the unique

[37] This is under the assumption of only two state for entrepreneurial talent and that this is the only case that matters.

interest compatible with his defaulting only in the bad state. If he actually defaults only in the bad state, we are done.

4. If at point 3 we find out that given the interest rate the agent will always default (in the good and in the bad state) we know that the agent will never get credit so we set his utility to $-\infty$.

5. We do this for all the $b_i \in [b_{\min}, b_{\max}]$ and then the agent picks the b_i that maximizes his utility.

1.8.2 Data on Entrepreneurship

To calibrate the model and to select a value for the targets we need a definition of an entrepreneur. Given the need to target bankruptcy, we are bounded in the choice by the availability of data on business bankruptcy filings. The main source for data on business bankruptcy is *The Small Business Economy* (2006) by the US Small Business Administration, Office of Advocacy[38]. Their definition of entrepreneurs (see Table 1.8-1) is a business owner who actually runs his business and has at least one employee. Given this definition the main data on entrepreneurs, entrepreneurs' termination and bankruptcy are reported in table 1.8-1.

To get the fraction of entrepreneurs in the population we apply the same definition of entrepreneurs to several waves of the Survey of Consumer Finances (1989-2004). We define an household as entrepreneurial if the head owns and runs a business with at least one employee. The fraction of the population engaged in entrepreneurial activity, for several waves of the SCF is reported in the last column of table 1.8-2. According to our definition, the fraction of entrepreneurial household in total population is given by **7.62%**. This number does not differ from the numbers obtained by using other definitions of entrepreneurship used in the literature[39]

Using the same definition we calculate the median net worth for entrepreneurial household and for the total population, using data from the Survey of Consumer Finances. The results are reported in table 1.8-3 which reports the median wealth based on other definition of entrepreneurship as well.

The corresponding ratio of the median entrepreneurial wealth to the median wealth in total population is **5.66**[40].

[38] The original sources of data are:

- for the employers, from the Bureau of Census and U.S. Department of Commerce
- for employer' births and terminations, from the Census Bureau
- for bankruptcies. from the Administrative Office of the U.S. Courts (business bankruptcy filings).

[39] Cagetti / De Nardi [2006] define as entrepreneurial an household whose head owns and runs a business and declares herself as self-employed. Gentry / Hubbard [2004] define as entrepreneurial an household who owns and runs a business with a total market value of at least 5000$.

[40] Using other definitions of entrepreneurship the ratio of median wealth of entrepreneurs is lower: 4.8 and 5.3

Table 1.8-1: entrepreneurship exit and bankruptcy

Year	Entrepreneurs	Exit	Exit Rate	Bankruptcy	Bankruptcy/Exit
1990	5073795	531400	0.105	64853	0.122
1991	5051025	546518	0.108	71549	0.131
1992	5095356	521606	0.102	70643	0.135
1993	5193642	492651	0.095	62304	0.126
1994	5276964	503563	0.095	52374	0.104
1995	5369068	497246	0.093	51959	0.104
1996	5478047	512402	0.094	53549	0.105
1997	5541918	530003	0.096	54027	0.102
1998	5579177	540601	0.097	44367	0.082
1999	5607743	544487	0.097	37884	0.070
2000	5652544	542831	0.096	35472	0.065
2001	5657774	553291	0.098	40099	0.072
2002	5697759	586890	0.103	38540	0.066
2003	5767127	540658	0.094	35037	0.065
2004	5865400	544300	0.093	34317	0.063
2005	5992400	544800	0.091	39201	0.072
Average	**5493734**	**533328**	**0.097**	**49136**	**0.093**

SOURCE: US Small Business Administration, Office of Advocacy (2006)

Table 1.8-2: fraction of entrepreneurs in total population

year	Cagetti and De Nardi	Gentry-Hubbard	Our definition
1989	0.076	0.067	0.085
1992	0.081	0.096	0.081
1995	0.067	0.071	0.068
1998	0.074	0.074	0.073
2001	0.078	0.081	0.076
2004	0.075	0.084	0.075
Average	**0.075**	**0.079**	**0.076**

SOURCE: Survey of Consumer Finances (1989-2004)

Table 1.8-3: median net worth of total population and of entrepreneurial household

year	Tot Population	Cagetti and De Nardi	Gentry-Hubbard	Our definition
1989	47060	265000	318680	275500
1992	49600	208680	234250	300100
1995	57650	213300	226820	245801
1998	71700	331650	342600	371800
2001	86610	458000	495400	528900
2004	93001	536000	562500	606160
average	**67603.5**	**335438.3**	**363375**	**388043.5**

SOURCE: Survey of Consumer Finances (1989-2004)

In the literature another source of data on entrepreneurship is the Panel Study on Income Dynamics [Quadrini 2000]. Given the panel structure it is particularly useful to calculate exit and entry rates. However, one major drawback is that it undersamples rich households, and therefore entrepreneurs. Unfortunately PSID does not report the number of employees per firm. We cannot use our definition. In the literature on entrepreneurship that uses PSID, Quadrini [2000], two definitions are adopted. According to the first an entrepreneur is someone who declares himself self employed (SELF). According to the second an entrepreneur is someone who owns a business (OWN). Both these definitions are less stringent than the one adopted above. Column 2 and 3 of table 1.8-4 report the fraction of entrepreneurs in PSID according to these definitions. The first definition yields an average fraction of entrepreneurs of 11%. The second definition yields a fraction of 13%. This is much higher than the figure derived from SCF data. Therefore we also use a third definition which is more restrictive: an agent is an entrepreneur if both he owns a business and is self employed. This yields a lower fraction of entrepreneurs, equal to 8%.

Given this discrepancy we avoid using PSID data unless it is strictly necessary. As a check of the SBA data we calculate the exit and entry rates according to the 3 definitions above. The entry rate in period t is defined as the ratio of the number of total households in the sample who were workers in period $t-1$ and were entrepreneurs in period t over the total number of workers in period $t-1$. The exit rate in period t is the ratio of those who were entrepreneurs in period $t-1$ and are worker in period t over the total number of entrepreneurs in period $t-1$. The results are reported in table A5.

These numbers are much higher than the number from the number of SBA. The reason is that the PSID undersamples rich household. Since successful entrepreneurs are richer and do not exit, this results could be biased. Therefore, we choose as the target for the exit rate 9.3%.

Quadrini [2000] points out that the entry rate of workers who have some entrepreneurial

when using Cagetti / De Nardi [2006] and Gentry / Hubbard [2004] definitions respectively.

Table 1.8-4: fraction of entrepreneurs

year	SELF	OWN	BOTH
1969	0.11	0.08	0.06
1970	0.10	0.09	0.06
1971	0.10	0.09	0.06
1972	0.10	0.09	0.05
1973	0.10	0.09	0.06
1974	0.10	0.08	0.05
1975	0.10	0.08	0.06
1976	0.10	0.09	0.07
1977	0.10	0.09	0.06
1978	0.10	0.10	0.06
1979	0.10	0.10	0.06
1980	0.10	0.09	0.07
1981	0.10	0.10	0.06
1982	0.11	0.10	0.07
1983	0.11	0.11	0.07
1984	0.12	0.12	0.08
1985	0.13	0.14	0.09
1986	0.12	0.15	0.09
1987	0.13	0.15	0.09
1988	0.13	0.16	0.10
1989	0.13	0.15	0.09
1990	0.13	0.14	0.09
1991	0.13	0.14	0.09
1992	0.13	0.15	0.09
1993	0.13	0.13	0.08
1994	0.13	0.14	0.08
1995	0.12	0.13	0.08
1996	0.12	0.16	0.09
1997	0.13	0.17	0.09
average	**0.11**	**0.12**	**0.08**

SOURCE: PSID (1969-1997)

Table 1.8-5: Exit and entry rates (different definitions of entrepreneurship)

year	$EXIT_{OWN}$	$EXIT_{SELF}$	$EXIT_{BOTH}$	$ENTRY_{OWN}$	$ENTRY_{SELF}$	$ENTRY_{BOTH}$
1970	0.17	0.13	0.13	0.02	0.02	0.01
1971	0.16	0.11	0.13	0.02	0.02	0.01
1972	0.19	0.15	0.18	0.02	0.02	0.01
1973	0.22	0.14	0.15	0.03	0.02	0.02
1974	0.28	0.13	0.21	0.02	0.02	0.01
1975	0.22	0.10	0.14	0.02	0.02	0.02
1976	0.15	0.08	0.11	0.03	0.01	0.02
1977	0.20	0.12	0.21	0.03	0.02	0.01
1978	0.22	0.10	0.13	0.03	0.02	0.02
1979	0.18	0.11	0.15	0.03	0.02	0.01
1980	0.27	0.10	0.12	0.02	0.01	0.01
1981	0.22	0.10	0.16	0.03	0.01	0.01
1982	0.23	0.07	0.14	0.03	0.02	0.02
1983	0.16	0.09	0.11	0.03	0.02	0.01
1984	0.20	0.11	0.13	0.03	0.01	0.01
1985	0.18	0.12	0.13	0.04	0.03	0.02
1986	0.20	0.14	0.13	0.04	0.02	0.02
1987	0.18	0.12	0.11	0.04	0.02	0.01
1988	0.20	0.13	0.13	0.05	0.03	0.02
1989	0.24	0.15	0.16	0.04	0.02	0.02
1990	0.20	0.13	0.15	0.04	0.02	0.02
1991	0.22	0.11	0.15	0.04	0.03	0.02
1992	0.23	0.12	0.17	0.05	0.02	0.02
1993	0.25	0.13	0.20	0.03	0.02	0.02
1994	0.22	0.15	0.21	0.04	0.02	0.02
1995	0.25	0.13	0.18	0.04	0.02	0.02
1996	0.19	0.10	0.12	0.04	0.02	0.02
1997	0.16	0.09	0.15	0.03	0.02	0.01
average	0.21	0.12	0.15	0.03	0.02	0.02

SOURCE: PSID (1969-1997)

experience in the past is much higher than the entry rate of those who has not got any experience. Using the PSID data we replicate his results. An agent is defined as "experienced" worker in $t-1$ if he is a worker in period t-1 and has been an entrepreneur in any of the three periods before ($t-2,\ t-3,\ t-4$). All the remaining workers in period $t-1$ are defined as non-experienced. The entry rate for experienced and non experienced workers, as well as the overall entry rate are reported in table 1.8-6.

Table 1.8-6: Entry rates (experienced and non experienced workers)

year	total pop	non experienced	experienced
1974	0.015	0.009	0.313
1975	0.017	0.012	0.298
1976	0.014	0.010	0.280
1977	0.018	0.012	0.311
1978	0.014	0.009	0.216
1979	0.013	0.010	0.171
1980	0.011	0.008	0.190
1981	0.015	0.010	0.268
1982	0.014	0.010	0.197
1983	0.014	0.009	0.265
1984	0.023	0.017	0.324
1985	0.019	0.014	0.264
1986	0.014	0.010	0.182
1987	0.020	0.017	0.136
1988	0.017	0.012	0.192
1989	0.018	0.013	0.140
1990	0.017	0.013	0.167
1991	0.019	0.013	0.196
1992	0.017	0.012	0.185
1993	0.018	0.010	0.230
1994	0.019	0.011	0.247
1995	0.017	0.011	0.200
1996	0.012	0.008	0.167
average	**0.016**	**0.011**	**0.223**

SOURCE: PSID (1969-1997)

The entry rate of experienced workers is 14 times higher than the entry rate of the total population.[41]

1.8.3 Formal definition of equilibrium

In our model the state space is given by 4 elements: the asset level a, the entrepreneurial productivity θ, the worker productivity φ and the credit status S. We discretize the asset

[41] If we restrict the sample period to 1989 to 1996,in order to be compatible with other data sources the ratio falls to 11. We set this as the target.

state space, assuming that assets can values on a grid of n_a elements $G_a \subseteq \Re_+^{n_a}$. Given the Markov approximation for the productivities processes we have that θ can take $n_\theta = 2$ values, $\theta \in \Theta \equiv \left\{0, \theta^H\right\}$, and φ can take $n_\varphi = 4$ values $\varphi \in \{\varphi_1, \varphi_2, \varphi_3, \varphi_4\} \equiv \Gamma$. Moreover $S \in \{BC, UN\} \equiv \Xi$. Following Huggett [1993], we can define the state space for the households as $\Omega = G_a \times \Theta \times \Phi \times \Xi$. Letting σ_Ω be the Borel σ-algebra on Ω and letting the optimal policy functions $PF(\omega), \omega \in \Omega$,(assets decisions, occupational choice, capital demand, bankruptcy decision) we have that the policy functions and the exogenous stochastic process imply a **transition function** $T(\omega, \varsigma), \forall \varsigma \in \sigma_\Omega$ on the measurable space (Ω, ω). This transition function implies a stationary probability measure $\mu(\varsigma), \forall \varsigma \in \sigma_\Omega$ that describes the distribution of households' assets holdings, productivity levels, and credit status. Stationarity implies

$$\mu(\varsigma) = \int_\Omega T(\omega, \varsigma)\, d\mu \tag{1.8-15}$$

After this bit of notation we can formally state the following definition of stationary equilibrium:

Definition 2 *A* ***stationary equilibrium*** *of the model is a four-tuple* $\{PF(\omega), \mu(\varsigma), (r, w), r(\omega)\}$ *such that:*

1. $PF(\omega)$ *is optimal for given* (r, w)
2. $\mu(\varsigma)$ *is the stationary distribution associated with the transition function generated by* $PF(\omega)$*, given* (r, w)
3. *The corporate sector representative firm is optimizing, given* (r, w)

$$r = \xi A K_c^{\xi-1} L_c^{1-\xi} = \xi A \left(\frac{K_c}{L_c}\right)^{\xi-1} \tag{1.8-16}$$

$$w = (1-\xi) A K_c^{\xi} L_c^{-\xi} = (1-\xi) A \left(\frac{K_c}{L_c}\right)^{\xi} \tag{1.8-17}$$

4. $r(\omega)$ *reflects the zero profit condition for the banking sector*
5. *Labor market and capital market clears.*

Chapter 2

Personal Bankruptcy Law, Debt Portfolios and Entrepreneurship

2.1 Introduction

Entrepreneurs employ half of all workers in the US and they create three quarters of all new jobs.[1] Over time, successful entrepreneurs, for example Bill Gates in 1978 or Larry Page and Sergey Brin in 1997, grow their small firms into big enterprises, for example Microsoft and Google today. Personal bankruptcy law is important for entrepreneurs because if an entrepreneur's firm is not incorporated he or she is personally liable for all the unsecured debts of this firm.[2] Many entrepreneurs fail each year, and around 60,000 file for bankruptcy.

This paper investigates quantitatively the effects of personal bankruptcy law on entrepreneurship. Bankruptcy introduces some contingency in a world of incomplete credit markets where only simple debt contracts are available. This contingency provides insurance against entrepreneurial failure at the cost of worsening credit conditions. If the bankruptcy law does not allow default under any circumstances, credit will be available at lower interest rates because borrowers will not default. This comes at the expense of borrowers having no insurance against business failure. If, however, the bankruptcy law makes default very easy, borrowers might be insured against bad outcomes. But in order to compensate for the default risk, banks have to charge higher interest rates or ration credit all together. In our model, as in the real world, entrepreneurs can also obtain secured credit. This modifies the trade-off between insurance and credit conditions by allowing agents, if they want to, to obtain cheap

[1] We thank Alex Michaelides for his continuous support and valuable comments, and Francesco Caselli and Maitreesh Ghatak for helpful comments at various stages of this research. We are also grateful to Orazio Attanasio, Daniel Becker, Chris Caroll, Wouter Den Haan, Eric Hurst, Bernardo Guimaraes, Christian Julliard, Winfried Koeniger, Tom Krebs, Dirk Krueger, Rachel Ngai, Vincenzo Quadrini, Victor Rios-Rull, Alwyn Young and participants at Fifth European Workshop in Macroeconomics, the Heterogenous Agent Models in Macroeconomics workshop in Mannheim 2009 and the NBER 2009 Summer Institute EFACR workshop.

[2] Meh / Terajima [2008] report that unsecured debt accounts for around on ethird of all debt.

(secured) credit even in a world with a very generous bankruptcy law. We find that allowing entrepreneurs to obtain both, secured and unsecured credit, has quantitatively important effects on the model economy.

The trade-off between insurance and credit conditions is at the center of recent public discussions and policy changes in Europe and the US. In Europe, the bankruptcy law is much harsher than in the US. Many countries, for example Germany, the Netherlands and the UK, have made legislation more lenient with the explicit aim of fostering entrepreneurship.[3] The policy changes in the US went in the opposite direction. Following the huge increase in personal bankruptcy filings, US Congress in 2005 passed a law making personal bankruptcy less beneficial for filers. Even though the focus of this discussion has been on consumer bankruptcy, the effects on entrepreneurship are important because around 60,000 failed entrepreneurs file for bankruptcy each year. Our paper quantitatively assesses the relative strength of these two opposing forces: insurance versus credit conditions, on the number of entrepreneurs, on the access of poor agents to entrepreneurship, on firm size, and on welfare, inequality and social mobility.

We build an infinite horizon heterogeneous agent model, which has an occupational choice problem at its core. Agents differ with respect to their entrepreneurial and working productivity. During each period, they decide whether to become an entrepreneur or a worker. Cagetti / De Nardi [2006] also have this occupational choice at the center of their model, which is able to explain US wealth distribution, in particular its extremely skewed nature at the top. However, in their model, entrepreneurship is a risk-free activity because there is no uncertainty about current productivities. Thus there is no default in equilibrium and there is no insurance role for bankruptcy. We have default in our model because in the US 2.25% of all entrepreneurs file for bankruptcy.

Despite the importance of personal bankruptcy law for entrepreneurship, there is little quantitative literature on this topic. Starting with Athreya [2002], the literature so far has focused almost exclusively on consumer bankruptcy. For example, Livshits et al. [2007a] compare the US system under which future earnings are exempt after consumers have defaulted with a European type of system under which future earnings are garnished to repay creditors. They find that the welfare differences between the systems depend on the persistence and variance of the shocks. Chatterjee et al. [2007] show that the recent tightening of the law in the US implies large welfare gains.[4] In this literature there are few papers that focus on secured and unsecured borrowing. Athreya [2006] finds that welfare is increasing in the wealth exemption level. Hintermaier / Koeniger [2008] examine the reasons for the increase in consumer bankruptcies in a model with durable and nondurable goods.

There are three closely related papers that analyze the effects of bankruptcy on entrepreneurship in a quantitative setting similar to our paper. Akyol / Athreya [2007] use an overlapping

[3] In a companion paper, we are currently investigating the effects of introducing a US type of law in Europe.
[4] Other papers in this growing literature are Athreya [2006], Athreya / Simpson [2006], Li / Sarte [2006], Mateos-Planas / Seccia [2006].

generations, partial equilibrium framework with heterogeneity in human capital. Their main results is that the current system is too generous. Meh / Terajima [2008] have a similar framework (partial equilibrium OLG model) in which they analyze bankruptcy decisions of both consumers and entrepreneurs. Mankart / Rodano [2007] have a model with temporary and permanent productivity shocks. The main result of all three papers is that the current system is too generous.[5]

Our model is able to replicate key macroeconomic variables of the US economy: the capital output ratio, the fraction of entrepreneurs in the population, the exit rate, the bankruptcy filings of entrepreneurs, the wealth of entrepreneurs compared to workers. Based on this model, we can conduct a policy experiment to assess whether the current exemption level (how much wealth a person can keep in case of a default) is optimal.

Our main result is that the current system is too harsh with respect to the exemption level. There are welfare gains from increasing the current exemption level to the optimal one. Entrepreneurship would increase from 7.2% of the population to 7.4% if the exemption level were increased because of the increased insurance effect. Moreover, eliminating bankruptcy exemptions would lead to a reduction of welfare and a reduction in entrepreneurship to 6.6% of the population.

Our results are strikingly different from other papers in the literature. Meh / Terajima [2008], Akyol / Athreya [2007] and Mankart / Rodano [2007] find that the current system is too generous[6]. The main difference is that all these paper do not allow entrepreneurs to obtain secured, in addition to unsecured, credit.

In a counterfactual experiment we find that if we exclude secured credit we get similar results as the previous literature: the current law appears to be too lenient. The reason is the following. When we exclude secured credit some agents are credit rationed because their incentive to default is too high. Therefore they become workers. Increasing the exemption level worsens this problem. If instead these agents can obtain secured credit (i.e. pledge collateral), they can run bigger firms and therefore find it profitable to become entrepreneurs. Excluding secured credit from the analysis overstates the role of credit rationing. Thus, the policy conclusion reached in the previous literature might be premature.

Our results, as those from Meh / Terajima [2008], Akyol / Athreya [2007] and Mankart / Rodano [2007] are consistent with the empirical finding of Berkowitz / White [2004] who show that in states with higher exemption levels, credit conditions are worse. But our paper is also consistent with the findings of Fan / White [2003] that show that entrepreneurship is higher in states with a more lenient bankruptcy law. This is not true in the work of Meh / Terajima [2008], Akyol / Athreya [2007] and Mankart / Rodano [2007].

Moreover, we use Epstein-Zin preferences. This allows us to distinguish between risk aversion

[5] Zha [2001] is a theoretical investigation of similar issues. However his model abstracts from occupational choice, which we show to be the crucial channel through which bankruptcy law affects entrepreneurship.
[6] This result is also common to most papers in the consumer bankruptcy literature.

and intertemporal elasticity of substitution. This is particularly interesting, given that the costs of a generous bankruptcy system, in terms of higher interest rates, depend mainly on the elasticity of intertemporal substitution, while the benefits, in terms of insurance, depend on risk aversion. Our choice of preferences allows us to examine these effects separately. We find that the optimal exemption level increases with the elasticity of intertemporal substitution. This result is quite intuitive since agents who are more willing to substitute consumption across time are less affected by the higher borrowing rates resulting from higher exemption levels. We also find that the optimal exemption level increases with risk aversion. The more risk averse agents are the more they value insurance.

The paper is organized as follows: Section 2.2 show the importance of secured credit. Section 2.3 provides an overview of US bankruptcy law and presents data on entrepreneurial failure. In Section 2.4 we present our model and discuss the equilibrium condition. In Section 2.5 we discuss our calibration strategy and present our results. Section 2.6 concludes. Details of the computational algorithm are presented in section 2.7.

2.2 A static example

In this section, we develop a static example that demonstrates the importance of secured borrowing in any analysis of the optimal wealth exemption level. In particular, we show that secured credit mitigates the consequences of credit rationing.

2.2.1 Environment

Entrepreneurs have access to the following linear technology with fixed project size: $\bar{K} = 1$, $Y = A\bar{K} = A$ and

$$A = \begin{cases} 2 & w.p. \quad 0.5 \\ 1 & w.p. \quad 0.5 \end{cases}$$

Suppose there are two risk-averse entrepreneurs in this economy: a poor one with $a = 0.1$ and a rich one with $a = 0.5$. The former has to borrow $b = 0.9$ and the latter $b = 0.5$ to finance the project. We assume that the risk free rate is zero. The first-best outcome would be one in which entrepreneurs are fully insured against the production risk and therefore would enjoy the same amount of consumption in both states.

2.2.2 Policy regime 1: X=0

If the wealth exemption level is 0, both agents will always repay. Therefore, they borrow at the risk-free rate. The outcome for the poor agent is $c_p^B = 1 - 0.9 = 0.1$ in the bad state and

$c_p^G = 2 - 0.9 = 1.1$ in the good state. The outcome for the rich agent is $c_r^B = 1 - 0.5 = 0.5$ in the bad state and $c_r^G = 2 - 0.5 = 1.5$ in the good state. Note that while both agents get credit none of them is insured against the bad outcome.

2.2.3 Policy regime 2: X=1 and no secured credit

If the wealth exemption level is 1, both agents have an incentive to default in the bad state. Therefore, they can not borrow at the risk-free rate. Financial intermediaries will set the interest rate according to their zero profit condition. Under the assumption that agents default only in the bad state the interest rate is given by:

$$\pi^D 0 + \left(1 - \pi^D\right)(1 + r)\, b = b \Rightarrow r = 1$$

The question is now whether the agents repay in the good state or whether they prefer to default also in the good state. If the poor agent repays he obtains $c_p^G = 2 - (1 + 1) \times 0.9 = 0.2$. Therefore, he will also default in the good state. This implies that financial intermediaries cannot break even on a poor agent; consequently he will get no credit at all. The rich agent obtains, upon repaying, $c_p^G = 2 - (1 + 1) \times 0.5 = 1.0$. Thus, he is just indifferent between repaying and defaulting, therefore, he repays. The rich agent is better off with $X = 1$ than with $X = 0$ because now he is fully insured against production risk. His consumption is the same in both states.

This demonstrates the trade-off of a generous bankruptcy law. Some, relatively rich, agents gain because they obtain more insurance. Others, relatively poor, agents lose because they are unable to obtain credit because their ex post default incentive is too high.

2.2.4 Policy regime 3: X=1 and secured credit

Now, we also introduce secured credit. Secured credit in this example means that the agent offers his project returns as collateral, i.e. he waives his right to default. This is in line with the law. Secured debt can not be discharged in a bankruptcy case. The rich agent does not want to borrow secured because he achieves his first-best outcome with unsecured credit. The poor agent, however, can now borrow $b = 0.9$ secured. This means he will have the same consumption allocation as in the case of $X = 0$.

Thus, secured credit lowers the cost of a high (generous) exemption level. Therefore, the optimal exemption level in a model with secured credit will be higher than in a model with unsecured credit only.

2.3 Entrepreneurial failure and personal bankruptcy in the US

Personal bankruptcy procedures in the US consist of two different procedures: Chapter 7 and Chapter 13. Under Chapter 7, all unsecured debt is discharged immediately, while a secured creditor can fully seize the assets pledged as collateral. Future earnings cannot be garnished. This is why Chapter 7 is known as providing a "fresh start". At the same time, a person filing for bankruptcy has to surrender all wealth in excess of an exemption level. The exemption level varies across US states, ranging from $11,000 in Maryland to unlimited for housing wealth in some states, for example Florida. Therefore, we calculate the population-weighted median across states. The resulting average exemption level is $47,800 in 1993.[7]

Under Chapter 13 agents can keep their wealth, debt is not discharged immediately and future earnings are garnished. Entrepreneurs are better off under Chapter 7 for three reasons: they have no non-exempt wealth, their debt is discharged immediately and they can start a new business straight away, since their income will not be subject to garnishment [see White 2007]. 70% of total bankruptcy cases involving entrepreneurs are under Chapter 7. Therefore we will focus on Chapter 7 only.

Persons can file for bankruptcy only once every six years. The bankruptcy filing remains public information for ten years. Therefore, agents have difficulties obtaining unsecured credit for some time after having defaulted. Secured credit, credit that is collateralized, is always available.

The US. Small Business Administration reports an exit rate of on average 9.7% per annum for small firms in the period from 1990-2005.[8] Out of these failing firms 9.3% file for bankruptcy, according to the official data from the Administrative Office of the Courts.[9] Unfortunately, the official data on personal bankruptcy caused by a business failure seem to be severely downward biased. Lawless / Warren [2005] estimate that the true number could be three to four times as big. Their own study is based on an in-depth analysis of bankruptcy filers in five different judicial districts. Their explanation of this discrepancy is the emergence of automated classification of personal bankruptcy cases. Almost all software used in this area has "consumer case" as the default option. Thus reporting a personal bankruptcy case as a "business related" case requires some - even though small - effort while being completely inconsequential for the court proceedings. In addition to their own study they report data

[7] The wealth exemption level does not change much over time. We choose 1993 because it is in the middle of the sample years for our data on entrepreneurship wealth distribution and bankruptcies.

[8] The U.S. Small Business Adminstration splits small firms into employer and non-employer firms. Employer firms have at least one employee working in the firm. There are roughly five million employer and 15 million non-employer firms in the U.S. Since the focus of our paper is on entrepreneurs who own and manage the firm we use only the data for employer firms since non-employer firms have in many cases the owner not working in the firm. To ensure consistency across our three databases, when we use data from the Survey of Consumer Finance (SCF) and the Panel Study of Income Dynamics (PSID) we define entrepreneurs as business owners who manage a firm with at least one employee.

[9] While one can obtain exit rates from the PSID data [Quadrini 2000], it is impossible to obtain reliable bankruptcy data from the PSID. There is only one wave in which respondents were asked about past bankruptcies.

from Dun & Bradstreet according to which business bankruptcies are at least twice the official number.[10]

In the calibration of our model we set the baseline exemption level equal to $47,800. The baseline exclusion period is set to two year.[11] We calibrate the model such that the default rate of entrepreneurs is 2.25%.

2.4 The model

Our economy is populated by a unit mass of infinitely lived heterogeneous agents. Agents face idiosyncratic uncertainty, but there is no aggregate uncertainty. At the beginning of every period, agents decide whether to become workers or entrepreneurs. An entrepreneur must decide how much to invest, how much to borrow secured and, if he is allowed to, how much to borrow unsecured. An entrepreneur who has defaulted on unsecured credit is excluded from unsecured credit for two year but is allowed to obtain secured credit. Since we focus on the implications of personal bankruptcy for entrepreneurs, workers are not allowed to borrow. Agents productivities evolve over time and agents are subject to uninsurable production risk. After the shocks are realized, production takes place. At the end of the period unsecured borrowers decide whether to repay or whether to default and how much to consume and how much to save. If they default, they will be borrowing constrained in the next period. Anticipating this behavior, banks who give unsecured credit vary the interest rate charged for each loan taking into account the individual borrower's default probability. The remainder of this section presents the details of the model.

2.4.1 Credit and bankruptcy law

Agents can get two types of credit: secured and unsecured. Both types of credit are subject to a limited commitment problem.[12] After getting credit, all borrowers have two options: take all liquid assets, their own wealth plus the amount borrowed, and run or start the entrepreneurial activity. If they run the agents can keep a fraction λ of the liquid assets. If the agents start the entrepreneurial activity then the only difference is that secured credit

[10] Dun & Bradstreet (D&B) is a credit-reporting and business information firm. D&B compiles its own independent business failure database. Until the emergence of automated software for law firms and courts in the mid 1980s, the official business bankruptcy data and the index compiled by D&B have a positive and significant correlation of 0.73. From 1986-1998 this correlation coefficient becomes negative and insignificant. Extrapolating from the historic relationship between the D&B index and personal bankruptcy cases caused by business failures leads to the conclusion that the official data under report business bankruptcy cases at least by a factor of two.

[11] We choose a short exclusion period because there is evidence that entrepreneurs obtain unsecured credit even after defaulting. However as a robustness check, we set the exclusion period to six years and the results do not change much.

[12] We introduce this limited commitment problem to obtain reasonable leverage ratios. As pointed out by Heaton / Lucas [2002] models without information asymmetries yield counterfactually large leverage ratios.

must be repaid (and it has priority in the bankruptcy proceedings), while unsecured credit is subject to Chapter 7 bankruptcy procedure, if the agent exercises his default option.

In the event of a default the agent still must repay her secured debt. Unsecured debt, however, is discharged. Any assets remaining after repaying the secured debt which is in excess of an exemption level X are liquidated.

An agent who has defaulted in the past is excluded from the market for unsecured credit for a certain period of time. During this period he still can obtained secured credit and can become an entrepreneur. We call this agent *borrowing constrained* and we denote his credit status as BC. It is important to note that this agent is not fully excluded from the credit market. He can still obtain *secured* credit. However he cannot obtain *unsecured* credit. We assume that every *borrowing constrained* agent, whether worker or entrepreneur, faces a credit status shock at the end of the period. With probability $(1-\varrho)$ the agent remains *borrowing constrained.* With probability ϱ the agent regain access to *unsecured* credit. He becomes an *unconstrained* agent with credit status UN.[13] This probability ϱ captures the duration of exclusion period from the market of unsecured borrowing. It is calibrated such that the average exclusion period is two year.

2.4.2 Households

Our economy is populated by a unit mass of infinitely lived heterogeneous agents. Agents differ with respect to their level of assets a, their entrepreneurial productivity θ, their working productivity φ, and their credit market status $S \in \{UN, BC\}$.

Preferences

For simplicity we abstract from labor-leisure choice. All agents supply their unit of labor inelastically either as workers or as entrepreneurs. In order to disentangle the effects of risk aversion from that of the elasticity of inetertemporal substitution we assume that agents have Epstein-Zin preferences. A stochastic consumption stream $\{c_t\}_{t=0}^{\infty}$ generates an utility $\{u_t\}_{t=0}^{\infty}$ according to

$$u_t = U(c_t) + \beta U\left(\mathbb{CE}_t\left[U^{-1}(u_{t+1})\right]\right)$$

where β is the discount rate and $\mathbb{CE}_t\left[U^{-1}(u_{t+1})\right] \equiv \Gamma^{-1}\left[\mathbb{E}_t\Gamma(u_{t+1})\right]$ is the consumption equivalent of u_{t+1} given information at period t. The utility function $U(c) = c^{1-\frac{1}{\psi}}/\left(1-\frac{1}{\psi}\right)$ aggregates consumption across dates and ψ is the intertemporal elasticity of substitution. The utility function $\Gamma(c) = c^{1-\gamma}/(1-\gamma)$ aggregates consumption across states and γ is the coefficient of relative risk aversion.

[13] The length of the exclusion period is transformed into a probability in order to avoid an additional state variable that keeps track of the numbers of years left before the solvency status is returned to UN. This procedure is standard in the literature, see Athreya [2002] and Chatterjee et al. [2007].

Productivities

Each agent is endowed with a couple of stochastic productivity levels which are known at the beginning of the period: one as an entrepreneur θ, and one as a worker φ. We make the simplifying assumption that the working and entrepreneurial ability processes are uncorrelated.

The workers' ability process Following the literature, we assume that labor productivity follows the following AR(1) process

$$\log \varphi_t = (1-\rho)\,\mu + \rho \log \varphi_{t-1} + \varepsilon_t$$

where ε_t is *iid* and $\varepsilon \sim N(0, \sigma_\varepsilon)$. If the agent becomes a worker his labor income during current period is given by $w\varphi$.

The entrepreneurs' ability process In contrast to the case of working ability, there are no reliable estimates of the functional form for the case of entrepreneurial ability. Therefore, following Cagetti / De Nardi [2006], we will assume a parsimonious specification where entrepreneurial productivity follows a 2-state Markov process with $\theta^L = 0$ and $\theta^H > 0$ and transition matrix

$$P_\theta = \begin{bmatrix} p^{LL} & 1-p^{LL} \\ 1-p^{HH} & p^{HH} \end{bmatrix}$$

We calibrate the 3 parameters (θ^H, p^{HH} and p^{LL}) to match some observed features of entrepreneurial activity in the US economy.

2.4.3 Technology

Entrepreneurial sector Every agent in the economy has access to a productive technology that, depending on her entrepreneurial productivity θ, produces output according to the production function

$$\begin{aligned} Y &= \theta k^{\nu} \\ k &= \chi I \end{aligned}$$

where θ is the agent's persistent entrepreneurial productivity described above.

We assume that investment is subject to an *iid* idiosyncratic shock. Each unit of the *numeraire* good which is invested in the entrepreneurial activity is transformed in χ units of capital with $log\chi \sim N(0, \sigma_\chi)$ This *iid* shock represents the possibility that an inherently talented entrepreneur (i.e. an agent with high and persistent θ) might choose the wrong project or could be hit by an adverse demand shock. Quadrini [2000] shows that the entry rate of workers with some entrepreneurial experience in the past, is much higher than the entry rate of those workers without any experience. Therefore it seems that entrepreneurs come mostly from a small subset of total population. If their firms fail, they are very likely to start a new firm within a few years. The *iid* shock χ helps us to capture this difference in the entry rates.

Corporate sector Many firms are both incorporated and big enough not to be subject to personal bankruptcy law. Therefore we follow Quadrini [2000] and Cagetti / De Nardi [2006] and assume a perfectly competitive corporate sector which is modeled as a Cobb-Douglas production function

$$F(K_c, L_c) = AK_c^{\xi} L_c^{1-\xi}$$

where K_c and L_c are capital and labor employed in this sector. Given perfect competition and constant returns to scale the corporate sector does not distribute any dividend. Capital depreciates at rate δ in both sectors.

2.4.4 Credit market

We assume that there is perfect competition (free entry) in the credit market. Therefore banks must make zero expected profit on any contract. The opportunity cost of lending to entrepreneurs is the rate of return on capital in the corporate sector. This is also equal to the deposit rate.[14] Agents can get two types of credit: *secured* credit and *unsecured* credit. *Secured* credit represents collateralized borrowing. Thus, it is available at the risk free rate plus a small transaction cost ($r^s = r^s + \tau^s$). *Unsecured* credit requires higher transaction costs ($\tau^u > \tau^s$) that reflect the higher information costs which are present in the real world and from which we abstract in the model.

Both types of contracts are subject to a limited commitment constraint. Instead of investing the money in the entrepreneurial firm the agent can take the money and run away with a fraction λ of the credit plus assets. Anticipating this behavior, banks will never lend any amount such that the agent prefers to run.[15]

There are no information asymmetries in the credit market. Banks know the agent's assets, the amount he borrowed secured s and his productivities. For any given value of (a, s, θ, φ) and for any amount lent unsecured b, by anticipating the behavior of the entrepreneur, banks are able to calculate the probability of default and the recovery rate in case of default. Perfect competition implies that they set the interest rate, $r(a, s, \theta, \varphi, b, X)$, such that they expect to break even. This interest rate depends on the exemption level X because it affects the incentives to default and the amount the bank recovers in this event. Therefore banks offer a

[14] In our model, banks are isomorphic to a bond market in which each agent has the possibility to issue debt.
[15] This means that running with the money is an out of equilibrium behavior. We introduce it to limit the leverage ratio to empirically plausible levels.

menu of one period debt contracts which consist of an amount lent b and a corresponding interest rate $r(a, s, \theta, \varphi, b, X)$ to each agent (a, s, θ, φ).

2.4.5 Timing

Figure 2.1 shows the timing of the model. Given the focus of the paper we choose the timing such that workers can never default. Entrepreneurs' borrowing and default decisions are taken within the period. At the beginning of the period all agents face an occupational choice: they choose whether they become entrepreneurs or workers. Agents know their current productivities (φ, θ).

Workers deposit all their wealth at the banks, receiving a rate of return r^d. After production has taken place, they choose consumption and savings. At the end of the period the *borrowing constrained* worker receives the credit status shock. With probability ϱ he remains *borrowing constrained* next period (i.e. $S' = BC$). With probability $(1 - \varrho)$ he becomes *unconstrained* next period (i.e. $S' = UN$).

The *borrowing constrained* entrepreneur chooses how much secured credit s to obtain or whether to save. After having obtained secured credit s, the *borrowing constrained* entrepreneur decides whether to take s and his own wealth a and run (with a fraction λ of it). In this case the bank receives nothing. Anticipating this, the bank will never lend an amount s with which the agent would run. The entrepreneur decides how much to invest before the *iid* shock χ is realized. After χ is realized and production has taken place, he chooses consumption and savings. At the end of the period he receives the credit status shock.

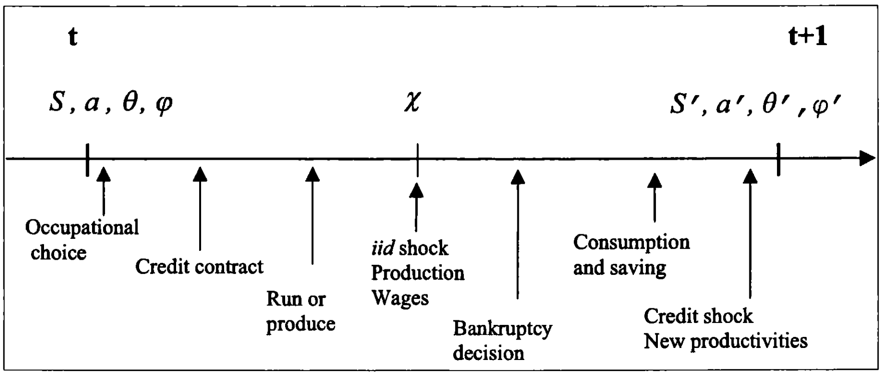

Figure 2.1: Timing of the model

The *unconstrained entrepreneur* can obtain both: secured credit s and unsecured credit b. Before knowing χ, he chooses his capital stock by deciding how much to borrow (or invest at rate r^d). He obtains secured credit s at the interest rate r^s. Unsecured borrowing is done by picking from the menu $\{b, r(a, \theta, \varphi, s, b, X)\}$ offered by the the banks. As for the *borrowing constrained*, the *unconstrained constrained* can take $a + b + s$ and run. And as before, the bank will never lend in a way that induces the agent to run. After χ is realized and production has taken place, the entrepreneur must repay his secured debt. Then he can decide whether to repay his unsecured debt as well and be *unconstrained* next period (i.e. $S' = UN$) or whether to declare bankruptcy and be *borrowing constrained* next period(i.e. $S' = BC$). After that he chooses consumption and savings.

Since the credit status S consists only of the two states BC and UN, we define the individual state variable as (a, θ, φ), and we solve for two value functions $V^{UN}(a, \theta, \varphi)$ and $V^{BC}(a, \theta, \varphi)$ one for each credit status.

2.4.6 The problem of the *borrowing constrained* agent

The *borrowing constrained* agent can only obtain secured credit. Therefore he can either save or borrow at a rate r^d subject to the limited commitment constraint. At the beginning of the period he can choose whether to become an entrepreneur, which gives utility $N^{BC}(a, \theta, \varphi)$ or a worker which yields utility $W^{BC}(a, \theta, \varphi)$. Therefore the value of being a *borrowing constrained* agent with state (a, θ, φ) is

$$V^{BC}(a, \theta, \varphi) = \max\left\{N^{BC}(a, \theta, \varphi), W^{BC}(a, \theta, \varphi)\right\}$$

where the 'max' operator reflects the occupational choice.

Worker At the beginning of the period the *borrowing constrained* worker deposits all his wealth at the bank and he receives labor income $w\varphi$. At the end of the period, he chooses consumption and saving, taking into account that he will receive a credit status shock in addition to productivity shocks. With probability ϱ he will be still *borrowing constrained* next period which yields utility $V^{BC}(a', \theta, \varphi)$, while with probability $(1-\varrho)$ he will become *unconstrained* which yields utility $V^{UN}(a', \theta, \varphi)$. His saving problem is the following

$$\begin{aligned} W^{BC}(a, \theta, \varphi) &= \max_{c, a'}\left\{U(c) + \beta U\left(\mathbb{CE}_t\left[\varrho V^{BC}(a', \theta', \varphi') + (1-\varrho) V^{UN}(a', \theta', \varphi')\right]\right)\right\} \\ s.t.\ c + a' &= w\varphi + \left(1 + r^d\right) a \\ a' &\geq 0 \end{aligned}$$

Entrepreneur At the beginning of the period the *borrowing constrained* entrepreneur decides how much to invest in his firm $I = a + s$ by choosing how much secured credit $(s > 0)$ or save, at rate r^d $(s < 0)$. Each unit of investment is transformed in χ units of capital, $(k = \chi I)$. After he has got credit he could take the money and run away with a fraction λ. If he does so his utility is given by

$$\begin{aligned} \Upsilon[a+s,\theta,\varphi] &= \max_{c,a'}\left\{U(c)+\beta U\left[\mathbb{CE}_t V^{BC}(a',\theta',\varphi')\right]\right\} \\ s.t.\ c+a' &= \lambda(a+s) \\ a' &\geq 0 \end{aligned}$$

After the shock χ is realized he will decide how to allocate the resources $(\chi I)^{\nu}\theta+(1-\delta)\chi I-\left(1+r^{d}\right)s$ among consumption and savings. His saving problem, after uncertainty is resolved,[16] is

$$\begin{aligned} \tilde{N}^{BC}(a,\theta,\varphi,\chi,s) &= \max_{a',c}\left\{U(c)+\beta U\left(\mathbb{CE}_t\left[\varrho V^{BC}(a',\theta',\varphi')+(1-\varrho)V^{UN}(a',\theta',\varphi')\right]\right)\right\} \\ s.t.\ c+a' &= [\chi(a+s)]^{\nu}\theta+(1-\delta)\chi(a+s)-\left(1+r^{d}\right)s \\ a' &\geq 0 \end{aligned}$$

Therefore the optimal investment decisions of the agent at the beginning of the period is

$$\begin{aligned} N^{BC}(a,\theta,\varphi) &= \max_{s} U\left(\mathbb{CE}_t\left\{\tilde{N}^{BC}(a,\theta',\varphi',\chi,s)\right\}\right) \\ s.t.\ N^{BC}(a,\theta,\varphi) &> \Upsilon[a+s,\theta,\varphi] \end{aligned}$$

2.4.7 The problem of the *unconstrained* agent

At the beginning of the period the *unconstrained* agent faces the following occupational choice

$$V^{UN}(a,\theta,\varphi)=\max\left\{W^{UN}(a,\theta,\varphi),N^{UN}(a,\theta,\varphi)\right\}$$

where $W^{UN}(a,\theta,\varphi)$ is the utility of becoming a worker and $N^{UN}(a,\theta,\varphi)$ of becoming an entrepreneur.

Worker The problem of the *unconstrained* worker is identical to the *borrowing constrained* one except that the agent will be *unconstrained* in the future for sure. His saving problem is the following

$$\begin{aligned} W^{UN}(a,\theta,\varphi) &= \max_{c,a'} U(c)+\beta U\left(\mathbb{CE}_t\left[V^{UN}(a',\theta',\varphi')\right]\right) \\ s.t.\ c+a' &= w\varphi+\left(1+r^{d}\right)a \\ a' &\geq 0 \end{aligned}$$

[16] We denote with a "~" all the value functions, *after* uncertainty (about χ) is resolved. The value functions without "~" are *before* uncertainty is resolved.

Entrepreneur The *unconstrained entrepreneur* decides how much to invest in his firm $I = a + b + s$ by choosing how much to borrow from secured credit ($s > 0$) from unsecured credit ($b > 0$) or save at rate r^d ($s < 0$). If he borrows unsecured credit he can choose from the menu $\{b, r(a, \theta, \varphi, b, s, X)\}$ offered by competitive banks. After the shock χ is realized he can choose whether to declare bankruptcy (default) or whether to repay and how much to consume and save. He solves the problem backwards.

If he repays his unsecured debt, he has to choose how to allocate his resources, $\theta[(a+b+s)\chi]^{\nu} + (1-\delta)(a+b+s)\chi - b[1 + r(a, \theta, \varphi, b, s, X)] - (1 + r^d)s$, between consumption and savings. Given that the decision of repaying is done when current productivities (θ, φ) and the shock χ are known, his utility from repaying is given by

$$\begin{aligned}
\tilde{N}^{pay}(a, b, s, \theta, \varphi, \chi) &= \max_{c, a'} \left\{ U(c) + \beta U\left(\mathbb{CE}_t \left[V^{UN}(a', \theta', \varphi') \right] \right) \right\} \\
s.t.\ a' + c &= \theta[(a+b+s)\chi]^{\nu} + (1-\delta)(a+b+s)\chi - \cdots \\
& \quad -b[1 + r(a, \theta, \varphi, b, s, X)] - (1 + r^d)s \\
a' &\geq 0
\end{aligned}$$

If he defaults, his unsecured debt is discharged. But he must repay any secured debt he had and he loses all assets in excess of the exemption level X. Thus, the resources to allocate between consumption and savings are $\min\left\{\theta[(a+b+s)\chi]^{\nu} + (1-\delta)(a+b+s)\chi - (1+r^d)s, X\right\}$. Moreover if he defaults he will be *borrowing constrained* next period. Therefore by declaring bankruptcy he gets

$$\begin{aligned}
\tilde{N}^{bankr}(a, b, s, \theta, \varphi, \chi) &= \max_{c, a'} \left\{ U(c) + \beta U\left(\mathbb{CE}_t \left[V^{BC}(a', \theta', \varphi') \right] \right) \right\} \\
s.t.\ a' + c &= \min\left\{ \theta[(a+b+s)\chi]^{\nu} + (1-\delta)(a+b+s)\chi - (1+r^d)s, X \right\} \\
a' &\geq 0
\end{aligned}$$

He will declare bankruptcy if $N^{bankr}(a, b, s, \theta, \varphi\chi) > N^{pay}(a, b, s, \theta, \varphi, \chi)$ and vice versa. Thus, at the beginning of the period the agent choose the optimal amount of b from the menu $\{b, r(a, \theta, \varphi, b, X)\}$ and the optimal s anticipating his future behavior. Therefore his utility is given by

$$\begin{aligned}
N^{UN}(a, \theta, \varphi) &= \max_{\{b, r(\cdot)\}, s} \mathbb{CE}_t \left[\max\left\{ \tilde{N}^{pay}(a, b, s, \theta, \varphi, \chi), \tilde{N}^{bankr}(a, b, s, \theta, \varphi, \chi) \right\} \right] \\
s.t.\ N^{UN}(a, \theta, \varphi) &\geq \Upsilon^{UN}[a + s + b, \theta, \varphi]
\end{aligned}$$

where the "max" operator inside the square brackets reflects the bankruptcy decision, and the "max" operator outside the square brackets reflects the borrowing decision. The last equation represents the limited commitment constraint where

$$\begin{aligned}\Upsilon\left[a+s+b,\theta,\varphi\right] &= \max_{c,a'}\left\{U\left(c\right)+\beta U\left[\mathbb{C}\mathbb{E}_t V^{BC}\left(a',\theta',\varphi'\right)\right]\right\} \\ s.t.\ c+a' &= \lambda\left(a+s+b\right) \\ a' &\geq 0\end{aligned}$$

2.4.8 The zero profit condition of the banks

Banks observe the state variables (a,θ,φ) at the moment of offering the contract.There is perfect competition (free entry) in the credit market therefore banks make zero profit on each secured and unsecured loan contract. Therefore the bank is indifferent between issuing secured and unsecured loans. For each unit of secured credit the bank know that the agent will repay for sure: free entry will push the interest rate on secured credit to the risk free rate plus the transaction cost τ^s. For any given state (a,θ,φ) and for any given amount of secured borrowing the agent is doing (s) and for any unsecured loan (b), banks know in which states of the world the agent will file for bankruptcy. Therefore, they are able to calculate the probability that a certain agent with characteristics (a,θ,φ), and secured loan s, will default for any given amount b. This default probability, $\pi^{bankr}\left(a,\theta,\varphi,b,s,X\right)$, depends on the exemption level X because X affects the incentive to default directly.

If the agent repays banks receive $\left[1+r(a,\theta,\varphi,b,s,X)\right]b$. If the agent defaults banks sells the firm's un-depreciated capital. Therefore they receive: nothing if $\theta\left[(a+b+s)\chi\right]^{\nu}+(1-\delta)(a+b+s)\chi-\left(1+r^d\right)s<X$, while banks receive $\theta\left[(a+b+s)\chi\right]^{\nu}+(1-\delta)(a+b+s)\chi-\left(1+r^d\right)s-X$ otherwise.

The zero profit condition of the banks is given by

$$\begin{pmatrix}\left[1-\pi^{bankr}(a,\theta,\varphi,b,s,X)\right]\left[1+r(a,\theta,\varphi,b,s,X)\right]b+ \\ +\pi^{bankr}(a,\theta,\varphi,b,s,X) \\ \max\left\{\theta\left[\chi I\right]^{\nu}+(1-\delta)\chi I-\left(1+r^d\right)s-X,0\right\}\end{pmatrix}=(1+r^d)(1+\tau^u)b,$$

where $I=a+b+s$

2.4.9 Equilibrium

Let $\eta=(a,\theta,\varphi,S)$ be a state vector for an individual, where a denotes assets, θ entrepreneurial productivity, φ working productivity and S the credit status. From the optimal policy functions (savings, capital demand, default decisions), from the exogenous Markov process for productivity and from the credit status shocks, we can derive a transition function, that, for any distribution $\mu(\eta)$ over the state provides the next period distribution $\mu'(\eta)$. A stationary equilibrium is given by

- a deposit rate of return r^d and a wage rate w
- an interest rate function
- a set of policy functions $g(\eta)$ (consumption and saving, secured and unsecured borrowing, capital demand, bankruptcy decisions and occupational choice)
- a constant distribution over the state η, $\mu^*(\eta)$

such that, given r^d and w and a bankruptcy regime X and ϱ:

- $g(\eta)$ solves the maximization problem of the agents;
- the corporate sector representative firm is optimizing;
- capital, labor and goods market clear:
  - capital demand comes from both, entrepreneurs and the corporate sector, while supply comes from the saving decisions of the agents;
  - labor demand comes from the corporate sector, while labor supply comes from the occupational choice of the agents;
- the interest rate function reflects the zero profit condition of the banks
- The distribution $\mu^*(\eta)$ is the invariant distribution associated with the transition function generated by the optimal policy function $g(\eta)$ and the exogenous shocks.

The model has no analytical solution and must be solved numerically. The algorithm used to solve the model and other details are presented in the appendix.

2.5 Results

2.5.1 Parametrization

Fixed parameters

Following standard practice in the literature we try to minimize the number of parameters of the model used to match the data. We therefore select some parameters which have already been estimated in the literature. We choose $\rho = 0.95$ for the auto-regressive coefficient of the earnings process.[17] The variance of the earnings process is chosen to match the Gini index of labor income as observed in the PSID, where it is 0.38.[18] The process is approximated

[17] In a life cycle setting, Storesletten et al. [2004] and Storesletten et al. [2001] find ρ in the range between 0.95 and 0.98. We choose $\rho = 0.95$ to take into account that the agents in our model are infinitely lived and that the intergenerational auto-regressive coefficient is lower. Solon [1992] estimates it around 0.4.

[18] The exact value of the variance is $\sigma_\varepsilon^2 = .08125$. This is higher than the estimate of Storesletten et al. [2004] of about 0.02. We abstract from many important factors that are empirically relevant for the earnings

using a 4-state Markov chain, using the Tauchen [1986] method as suggested by Adda / Cooper [2003].[19] Total factor productivity is normalized to 1, while the share of capital in the Cobb-Douglas technology for the corporate sector is set to $\xi = 0.36$. The depreciation rate is set $\delta = 0.08$. These parameters are summarized in table 2.5-1.

Table 2.5-1: The fixed parameters

Parameter	Symbol	Baseline
TFP	A	1 (normalization)
Share of capital	ξ	0.36
Transaction cost secured credit	τ^s	0.01
Transaction cost unsecured credit	τ^u	0.05
Depreciation rate	δ	0.08
Working productivities	φ	$\begin{bmatrix} \varphi_1 = 0.316, \varphi_2 = 0.745 \\ \varphi_3 = 1.342, \varphi_4 = 3.163 \end{bmatrix}$
Transition matrix	P_φ	$\begin{bmatrix} 0.8393 & 0.1579 & 0.0028 & 0.0000 \\ 0.1579 & 0.6428 & 0.1965 & 0.0028 \\ 0.0028 & 0.1965 & 0.6428 & 0.1579 \\ 0.0000 & 0.0028 & 0.1579 & 0.8393 \end{bmatrix}$

Preference parameters

The option to default provides agents with an insurance against bad outcomes. The value of this insurance depends crucially on the agents attitudes towards risk. As described above, the price of this insurance are worsened credit conditions. Agents who still borrow face higher interest rates. Thus, the value of the costs of the insurance depends mainly on the agents elasticity of intertemporal substitution. Therefore, we separate these two parameters and conduct our main policy experiment for different values of these parameters. In the baseline model, we set the coefficient of relative risk aversion $\sigma = 3$ and the elasticity of intertemporal substitution $\psi = 1.1$. Later on we investigate values for σ ranging from 1.5 to 4.5 and ψ ranging from 0.5 to 1.5. Table 2.5-2 summarizes preferences.

Table 2.5-2: Preference parameters

Parameter	Symbol	Value
CRRA	σ	3
IES	ψ	1.1

distribution, e.g. human capital, life-cycle savings. Therefore, in order to generate the observed inequality, we need a higher variance of the earnings process.

[19] Floden [2008] shows that for highly correlated processes the method of Adda / Cooper [2003] achieves a higher accuracy than the original methods of Tauchen [1986] and Tauchen / Hussey [1991].

Bankruptcy policy parameters

The two policy parameters are the exemption level X and the probability ϱ of being able to obtain unsecured credit again. The law does not state any formal period of exclusion from unsecured credit after a bankruptcy filing. For our baseline specification, we set $\varrho = 0.5$ which corresponds to an average exclusion period from credit of two years. This is lower than most values in the consumer bankruptcy literature.[20] We think that this is warranted since there is evidence the entrepreneurs have access to unsecured credits relatively fast after having defaulted, see for example Lawless / Warren [2005]. However, we conduct a robustness check and also investigate a considerably longer exclusion period of six years. The exemption level differs across US sates. Using US state-level data for 1993 we calculate the median across states of the total exemption[21] ("homestead" plus "personal property" exemption). The resulting median exemption level is \$47,800, taking an average household labor income of \$48,600 corresponds to a value of **0.98** for the exemption/wage ratio.[22] Table 2.5-3 summarizes the bankruptcy parameters.

Table 2.5-3: the bankruptcy parameters

Parameter	Symbol	Value
Exemption/wage	X/w	0.98
Unsecured credit exlsuion (expressed as probability)	ϱ	0.5

Calibrated parameters

We are left with the following 7 parameters to be calibrated: high entrepreneurial productivity (θ^H), entrepreneurial productivity transition matrix (p^{HH}, p^{LL}), concavity of entrepreneurial production function (ν), fraction of cash on hand with which an agent can run (λ), discount factor (β) and the variance of the transitory shock (σ_χ).

We choose these 7 parameters such that the model matches the following 7 moments of the US economy. First we want the model to match the *capital-output ratio* (K/Y) in the US economy. In the literature we find values ranging from 2.8 to 3.1. We target it to be 3.0. We target the *fraction of defaults.* Given the discussion in Section 2 we set this equal to 2.25%. The *fraction of entrepreneurs in the total population* is 7.3% in the Survey of Consumers Finances.[23] Based on PSID data the *exit rate* of entrepreneurs is equal to 15%. The median leverage ratio of entrepreneurs [24] in the SCF is around 15%.

[20] Athreya [2002] sets the exclusion period to 4 years, Li / Sarte [2006] to 5 years, Chatterjee et al. [2007] to 10 years.
[21] We took the data from Berkowitz / White [2004] and top-coded the unlimited homestead exemption to the maximum state exemption.
[22] As a further robustness check, we increase the exemption level by 50% and the results do not change
[23] See Mankart / Rodano [2007, appendix B] for data sources, definitions and further details.
[24] Leverage is defined as the ratio of debt to the sum of debt and equity.

Since the benefits of bankruptcy depend crucially on the wealth of an agent we match some features of the wealth distribution. The US wealth distribution is extremely skewed with the top 40% of richest households holding around 94% of total assets. As a last target we choose to match the *ratio of the median wealth of entrepreneurs to the median wealth in the whole population.* This target captures features of both the wealth distribution and entrepreneurial productivity and technology. We set the target to 6.3 as found in the SCF. The targets are summarized in the second column of Table 2.5-5.

2.5.2 The baseline calibration results

We first present the baseline version of the model. Table 2.5-4 reports the value of the calibrated parameters in the baseline specification.

Table 2.5-4: the calibrated parameters

Parameter	Symbol	Benchmark Value
High entrepreneurial productivity	θ^H	0.662
Entrepreneurial productivity transition	p^{HH}, p^{LL}	0.890 , 0.989
Concavity of entrepreneurial technology	ν	0.876
Fraction with which agent can run	λ	0.963
Discount factor	β	0.895
Variance of transitory shock	σ_X	0.346

Table 2.5-5 reports the value of the targets and the actual results achieved in the baseline specification.

Table 2.5-5: the baseline calibration targets

Moment	Target	Model
Fraction of Entrepreneurs (in %)	7.3	7.3
Ratio of medians (in %)	6.3	6.1
Share of net-worth of top 40%	94.0	94.1
K/Y	3.0	3.0
Exit Rate (in %)	15.0	15.0
Bankruptcy Rate (in %)	2.25	2.25
Median leverage (in %)	15.0	15.0

The marginal product of capital in the corporate sector (r^d) is 2.9%. Less than one percent (0.79%) of the total population is in the constrained state. Our model does replicate the ratio of medians and the share of the wealth held by the richest 40% fairly well. It captures the main features that entrepreneurs are several times richer than workers and that most of the wealth is held by the richest. The Gini coefficient of wealth is 0.83 in the model, slightly higher than the data (0.8). For the purpose of our policy experiments it is important that the model replicates the middle and lower part of the wealth distribution since bankruptcy law affects almost exclusively these agents.

Another feature that we do not target but that our model captures fairly well is the difference in the entry rate between workers with previous business experience and those without previous business experience. Based on PSID data[25], those who had some experience within the past three years are 13 times as likely to enter entrepreneurship than the average worker. In the model this ratio is 10.

Quadrini [2000] reports that around 35-40% of total capital is invested in the entrepreneurial sector. In our baseline specification this fraction is slightly lower, around 31.3%.

2.5.3 Investigating the model's mechanisms

Occupational choice

The key ingredient of the model is occupational choice. Figure 2.2 represents the occupational choice of an *unconstrained* agent with high entrepreneurial productivity and low working productivity.

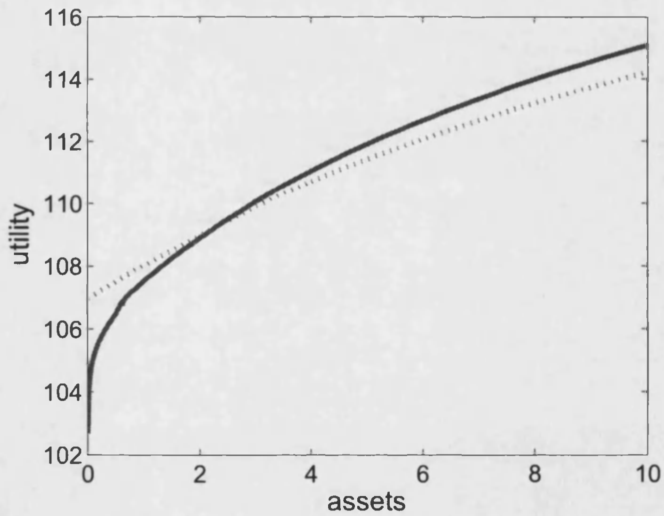

Figure 2.2: Occupational choice ($S = UN, \theta = \theta^H, \varphi = \varphi_3$)

The dotted line shows the value function of becoming a worker, whereas the solid line shows the value function of becoming an entrepreneur.

The first result is that, otherwise identical agents choose differently according to their wealth: poor agents become workers while rich agents become entrepreneurs. This result is standard in the occupational choice under credit market imperfections literature [see e.g. Banerjee / Newman 1993]. The main reasons are that poor agents have smaller firms and

[25] See Mankart / Rodano [2007, appendix B]

face higher interest rates. They have smaller firms because, being poor, they need to borrow more but they face higher rates on the loans. The cost of financing is higher for the poor for two reasons. First, they have a higher incentive to default. Defaulting rich agents have to give up all their wealth above the exemption level. Second, in the event of default the bank gets less when the agent is poor. Thus, to break even, the bank has to charge a higher interest rate. That is, in this model, wealth acts as collateral.

The behavior of the unconstrained agents

The second important ingredient is the decision of the *unconstrained* entrepreneurs. The solution of the entrepreneurs' problem is represented in Figure 2.3.

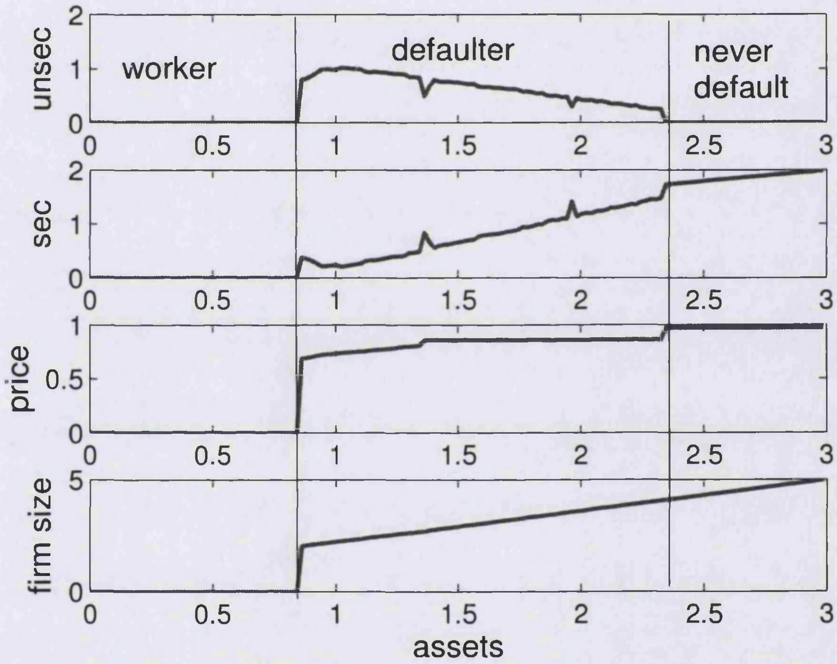

Figure 2.3: interest rate and firm size ($\theta = \theta^H$, $\varphi = \varphi_2$)

The top panel shows demand for unsecured debt (b). The second panel shows demand for secured debt (s). The third panel shows the corresponding price of unsecured credit[26] The bottom panel shows the resulting firm size ($(a + b + s)$). Poorer agents (e.g. agents with assets $a < 0.8$) become workers while all the others become entrepreneurs ($a > 0.8$). The very rich entrepreneurs ($a > 2.4$) will never find it profitable to default. Their wealth is so high that defaulting is too costly for them. Therefore they borrow only secured since secured credit is cheaper than unsecured.[27] The "middle class" entrepreneurs (e.g. $a = 2$) will instead default if the shock is sufficiently bad, since the cost of bankruptcy is lower for them. In order to break even, the bank charges a higher interest rate, i.e. the unsecured credit is more

[26] For readability, we show the price of credit instead of the interest rate.

[27] The transaction cost for secured credit is lower than for unsecured credit.

expansive. The interest rate depends (negatively) on the assets of the entrepreneur, because in the event of default the bank will be able to seize the difference between the assets of the entrepreneur and the exemption level. Capital demand for the "middle-class" entrepreneurs is increasing because of the cost of borrowing is declining. The spikes in the demand for unsecured credit reflect the discretization of the investment shock.

A first look at the effects of bankruptcy

Bankruptcy affects the problem of the unconstrained agents, because it changes credit conditions and the amount of insurance available. We examine these effects with the following experiment. We compare the behavior of the unconstrained agents in two different situations: one in which bankruptcy is allowed and one in which bankruptcy is absent. Figure 2.4 shows the policy functions in these situations.

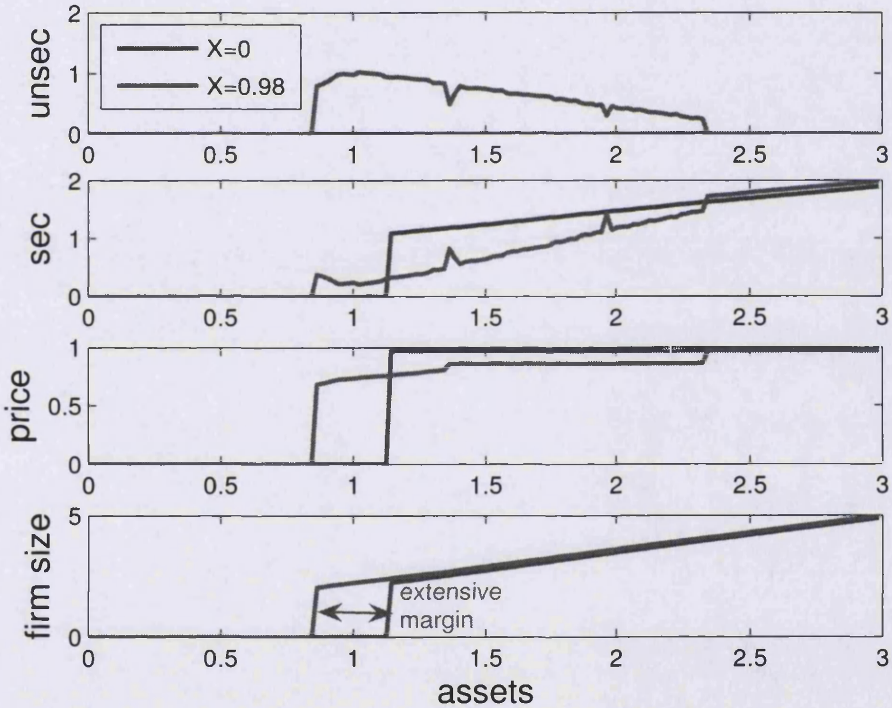

Figure 2.4: Firm size and interest rate ($S = UN, \theta = \theta^H, \varphi = \varphi_2$)

The effects of allowing bankruptcy depend on the wealth of the agent. First, the default behavior of the rich (e.g. $a > 2.4$) is not affected. They are entrepreneurs and they repay their debt even in the bad states. As explained above, even if bankruptcy is available, it is too costly for them. They demand a little bit more secured credit due to a general equilibrium effect. Second, allowing bankruptcy affects the behavior of the less rich agents (e.g. $a = 1.5$). They are entrepreneurs in both situations. But when bankruptcy is allowed they borrow more unsecured because they are better insured at cost of more expansive credit. We call this increase in the firm size the *intensive margin*. Third, the occupational choice of even less rich agents (e.g. $a = 1$) is affected. When bankruptcy is not allowed they are not insured against bad outcomes. Therefore they do not want to borrow, even though they could borrow at rate r^s. They become workers. When bankruptcy is allowed they are insured against bad outcomes. Therefore they borrow, even though they have to pay a high interest rate. This increases the rewards of entrepreneurship enough to change their occupational choice. We call this increase in the number of entrepreneurs the *extensive margin*. Fourth, the occupational choice of the very poor agents (e.g. $a < 0.7$) is not affected, they are workers in both situations.

In this particular experiment abolishing bankruptcy reduces entrepreneurship and firm size, the intensive and the extensive margins are negative. The negative effect of lowering the amount of insurance available dominates the positive effect of better credit conditions.

2.5.4 Changing the exemption level

Our main policy experiment is to analyze the effects of changing the exemption level.

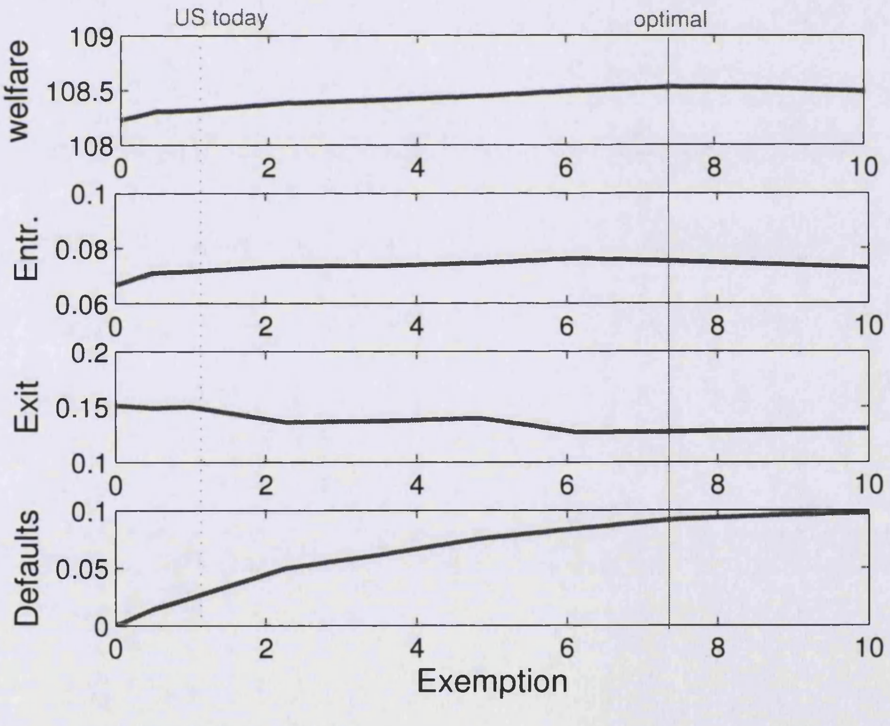

Figure 2.5: Changes in the exemption levels

Figure 2.5 shows the effects of changing the exemption level on welfare, entrepreneurship, exit rates and defaults. Table 2.5-6 reports the variables of interest for 3 values of X/w. Column 2 reports results when bankruptcy is very harsh ($X/w = 0$). Column 3 reports results for the baseline calibration ($X/w = 0.98$) and column 4 for the optimal exemption level ($X/w = 7.3$).

Welfare Increasing the exemption level from zero increases welfare. The insurance effect is dominating the worsening credit market effect. More agents become entrepreneurs (see also Table 2.5-6) and welfare increases. However, increasing the exemption level beyond the optimal level worsens credit market conditions so much that agents borrow less, and therefore fewer agents find it profitable to become entrepreneurs. The current exemption level in the US, $X/w = 0.98$, is too low. The bankruptcy law is too harsh. The welfare gains in increasing the exemption level are substantial. The change in consumption equivalent (see row 10 in 2.5-6) is 2.2% of annual consumption. The rich and the poor both gain from increasing the exemption level.

Entrepreneurs Increasing the exemption level increases the fraction of entrepreneurs by 0.2 percentage points. Thus, there is a positive extensive margin. In particular, the optimal exemption level allows entrepreneurs who have defaulted to remain entrepreneurs because they can keep more assets in the default case. However, as can be seen in figure 2.5, the entrepreneurship rate peaks earlier than welfare. This implies that the intensive margin, i.e. bigger firms, is important in explaining the welfare results. As expected the default rate is increasing in the exemption level. The exit rate however is declining in the exemption level. The reason for this is that entrepreneurs who have defaulted keep enough assets to remain entrepreneurs despite being excluded from unsecured credit.

Table 2.5-6: the effects of changes in the exemption level

X/w	**0**	**0.98**	**7.3**
Exit rate (in %)	15.1	15.0	12.9
Fraction of Entrepreneurs (in %)	6.7	7.2	7.4
Bankruptcy/Exit (in %)	0	15.0	73.8
Capital/Output	3.02	3.02	3.02
Median assets of Entr/ Median assets	7.2	6.3	7.3
Share of Capital in entr. sector (in %)	30.9	31.4	33.2
Gini of Assets	0.84	0.84	0.83
Share of assets in top 40% of pop (in %)	94.6	94.6	94.5
Median output in entrepreneurial sector	9.7	8.9	11.4
Welfare in CE	-0.5	0.0	2.2
Welfare of rich in CE	-0.9	0.0	2.46
Welfare of poor in CE	0.1	0.0	2.02

Access to entrepreneurship of the poor Next we turn to how bankruptcy law affects the determinants of entry into entrepreneurship. There is allocative inefficiency in our model because insurance markets are missing. Part of this inefficiency is reflected in some poor highly productive agents not becoming entrepreneurs, either because they receive too little insurance or because the conditions at which credit is available are too bad. Table 2.5-7 reports the effects of different exemption levels on the minimum assets needed for the highly productive ($\theta_{-1} = \theta^H$) agent to become an entrepreneur.

The rows show these values for the levels of working productivity (φ). The attractiveness of becoming a worker is increasing in working productivity, i.e. the outside option of entrepreneurs is increasing in working productivity. Thus in order to enter entrepreneurship, the expected profits must be higher for an agent with high working productivity. Since richer agents need to borrow relatively less and since they receive better credit conditions, their expected profits are higher. This implies that, to become an entrepreneur, an agent with high working productivity must be richer than an agent with low working productivity to enter entrepreneurship.

Increasing the exemption level to the optimal induces agents with high levels of labor productivity to enter entrepreneurship earlier. Poorer agents however will enter only when they are richer. The reason for this is that the credit market conditions worsen so much that they can obtain only secured credit and therefore lose the insurance coming from unsecured credit.

Table 2.5-7: minimum wealth for entrepreneurship

MINIMUM WEALTH FOR ENTREPRENEURSHIP

X/w	0	0.98	7.3
$\varphi = 0.316$	0.32	0.28	0.32
$\varphi = 0.745$	1.14	0.86	1.08
$\varphi = 1.342$	2.34	2.24	2.20
$\varphi = 3.163$	6.87	6.83	6.75

2.5.5 Modeling of credit markets matter

Almost all paper in the bankruptcy literature allow unsecured borrowing only.[28] Notable exceptions are Athreya [2006] and Hintermaier / Koeniger [2008] in the consumption literature. The reason is that the computational burden of allowing for secured credit as well is considerable. However, according to data from Sullivan et al. [1989], secured borrowing is as important as unsecured borrowing.[29].

Not only is secured credit empirically relevant, but also, as we show in this section, it is crucial for the results. To show this, we set up a model identical to the one discussed so far except that there is no secured credit available, neither for the *borrowing constrained* nor for the *unconstrained* entrepreneur. This implies that the former cannot borrow at all and must finance his projects with his own wealth. We first recalibrate the model and then conduct the same policy experiment as before. The results in figure 2.6 are striking. The optimal bankruptcy law now would be to abolish bankruptcy completely. This would increase welfare and lead to a higher number of entrepreneurs.

[28] See for example Akyol / Athreya [2007], Meh / Terajima [2008], Athreya [2002], Livshits et al. [2007a], Chatterjee et al. [2007], Athreya / Simpson [2006], Li / Sarte [2006], Mateos-Planas / Seccia [2006].

[29] Mean secured debt over mean total debt is about 55%

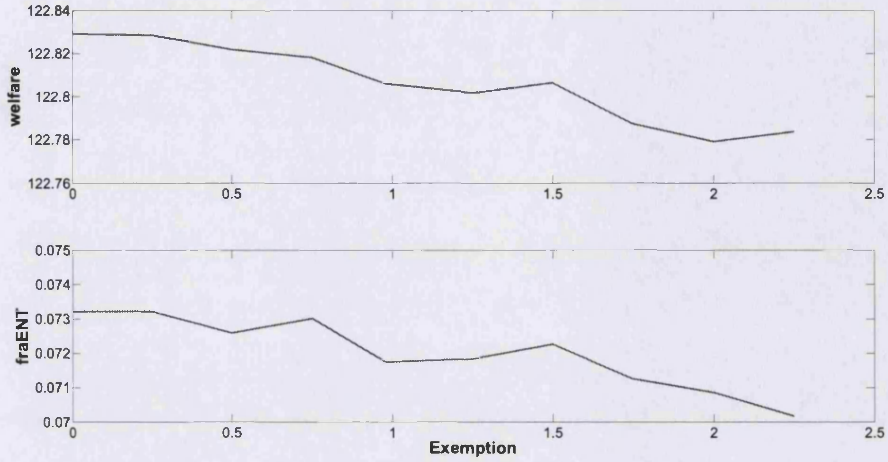

Figure 2.6: Welfare effects of changes in X if only secured credit available

Table 2.5-8: calibration unsecured credit only

Moment	Target	Unsec credit only	Sec and Unsec
Entrepreneurs (in %)	7.3	7.17	7.44
Exit Rate (in %)	15.0	13.55	12.76

Table 2.5-8 shows what happens if we use the calibrated parameters of the model without secured borrowing and now allow secured borrowing. Since the financial market is now relatively more complete, we see that there are more entrepreneurs and fewer exits.

The reason for this can be seen in figure 2.7. All agents is region (2) are not able to obtain unsecured credit because their default incentive is too high. If secured credit is not available, these agents become workers. However, if secured credit is available, these agents can borrow secured and so become entrepreneurs. Agents in region (3) use secured credit to run bigger firms.

This mechanism explains why the optimal exemption level in a model with secured and unsecured credit is much higher than the optimal exemption level in a model with only unsecured credit. Absent secured credit, an increase in the exemption level prices out many more agents. It would expand regions (1) and (2). Thus, the agents become workers because they are credit rationed. The availability of secured credit dampens this negative effect.

Another way of looking at this is the following. The optimal policy is a very harsh bankruptcy law. This implies that the agents do not value the insurance that is provided by the bankruptcy law. They would like to have less insurance but therefore have better credit market conditions. This means essentially that the agents want a commitment device that takes away the default option. One way to achieve his is to make the law harsher. Another way, however, is to use secured credit. Secured credit is the commitment device that the agents want.

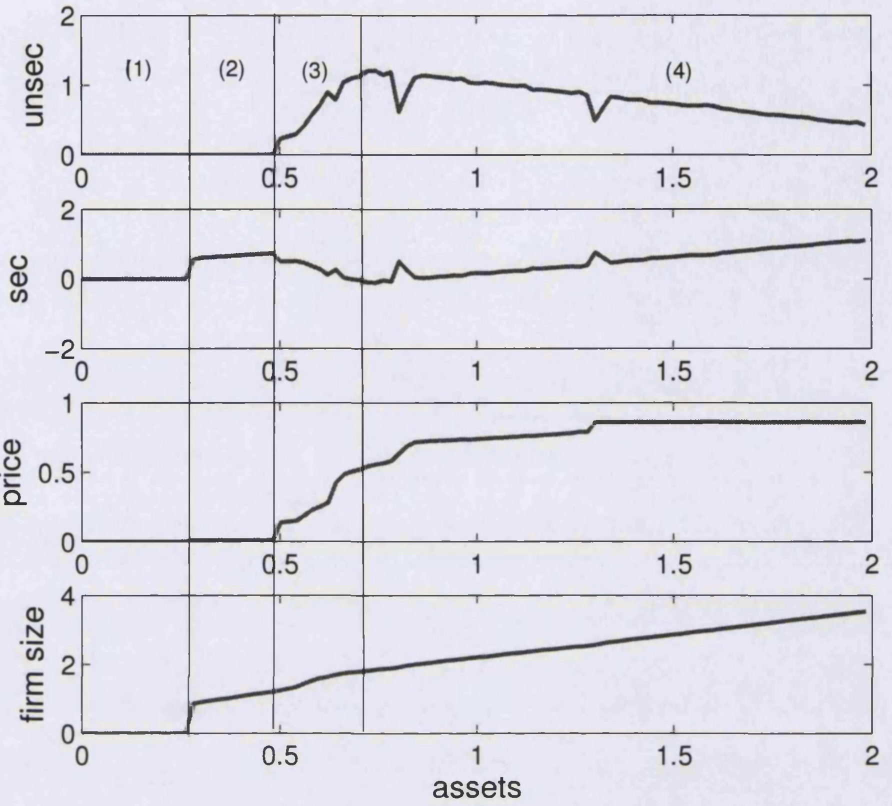

Figure 2.7: Policy functions with secured credit

As already mentioned, most previous papers do not include secured credit in their models and most of them find that the current bankruptcy law is too lenient.[30] Our results imply that these results might not be robust towards including secured borrowing.

2.5.6 Robustness

In this section, we show the effects of changing the agent's preferences. We separate the elasticity of intertemporal substitution from the coefficient of relative risk aversion because they have different effects. With a standard utility function, one is the inverse of the other. In this case, an increase in risk aversion as for example examined in Athreya [2006] conflates two effects. On the one hand, since agents are more risk averse, they value insurance more so the optimal exemption level is likely to be higher. On the other hand, with standard preferences, an increase in risk aversion simultaneously lowers the elasticity of intertemporal substitution. Thus, agents are less willing to transfer consumption across time. But a higher exemption level will increase the interest rate agents face because banks have to charge higher interest rates in order to break even. Thus, a decrease in the elasticity of intertemporal substitution is likely to lead to a lower optimal exemption level. By not separating the two, one examines

[30] The two other papers (Akyol / Athreya [2007],Meh / Terajima [2008]) in the entrepreneurial bankruptcy literature find significant welfare gains from making the law harsher. The papers in the consumer bankruptcy literature reach similar conclusions.

only their net effect. It is possible that each of these two effects is big but that they cancel each other so that the net effect is small.

Changing EIS

In this subsection we investigate the robustness of the results towards different values of elasticity of intertemporal substitution. The costs of a lenient bankruptcy law are higher interest rates which make substitution across time more costly. If agents' willingness to substitute consumption across period is low (i.e. eis is small), higher interest rates will be particularly costly. Therefore the optimal exemption level should be an increasing function of the elasticity of intertemporal substitution. We recalibrate the model once with a low elasticity of intertemporal substitution, ($\psi = 0.6$) and once with a high elasticity of intertemporal substitution ($\psi = 1.4$). We keep the coefficient of risk aversion constant. The results are shown in table 2.5-9. The optimal exemption level is increasing in the elasticity of intertemporal substitution as expected. While the magnitude of the effects is not huge, they are quantitatively significant.

Table 2.5-9: Optimal exemption level for different EIS

CRRA	Optimal X
0.6	6.7
1.1	7.3
1.4	7.9

Changing RRA

In this subsection we investigate the robustness of the results towards different degrees of risk aversion. The possibility to default provides insurance against bad outcomes. The value agents attach to this insurance depends on their risk aversion. We recalibrate the model once with a low coefficient of risk aversion, ($\sigma = 1.5$) and once with a high coefficient of relative risk aversion ($\sigma = 4.5$). We keep the elasticity of intertemporal substitution constant since we want to isolate the importance of risk attitudes.

The optimal exemption level, the amount of insurance, is increasing in σ. This result is qualitatively not surprising. However it is also quantitatively important. If agents were less risk averse, the optimal exemption level would be 13% lower. However, the effects are rather small in welfare terms. Welfare never changes by more than a fraction of a percent. This is due to the fact that all exemption levels are pretty high.

Table 2.5-10: Optimal exemption level for different CRRA values

CRRA	Optimal X
1.5	6.3
3.0	7.3
4.5	8.7

2.6 Conclusion

This is the first paper to explore quantitatively the effects of personal bankruptcy law on entrepreneurship in a general equilibrium setting with heterogeneous agents and secured and unsecured credit. First, we developed a dynamic general equilibrium model with occupational choice which explicitly incorporates the US bankruptcy law. Our model endogenously generates interest rates that reflect the different default probabilities of the agents. Our model accounts for the main facts on entrepreneurial bankruptcy, entrepreneurship, wealth distribution and macroeconomic aggregates in the US.

Then, we used the model to quantitatively evaluate the effects of changing the US bankruptcy law. The simulation results show that increasing the exemption level would increase the fraction of entrepreneurs and welfare. These effects are significant: increasing the exemption level to the optimal one has positive welfare effects in the order of 2.2% of average consumption. All households, rich and poor, would be better off.

The most important contribution of our paper is to show that the modeling of the credit market matters. Investigating the optimal exemption level in a model without secured credit gives misleading results because it overstates credit rationing.

We are currently extending our research program along two dimensions. First, we are incorporating the transition to the new steady state. So far, our results are based on a comparison of steady-states. Transitional effects might be important to evaluate welfare. In addition it might explain why the current law is too lenient. It could be that some groups lose during the transition and therefore oppose changes.

Second, we are expanding our model to incorporate explicitly a European type of bankruptcy law. The laws in European countries are much harsher than the law in the US. For example in Italy, debt is never discharged. A defaulter is liable forever. We are analyzing the effects of introducing a US type of law on the Italian economy.

2.7 Appendix

2.7.1 Computational strategy

The state vector for an individual is given by $\eta = (a, \theta, \varphi, S)$. The aggregate state is a density $\mu_t(a, \theta, \varphi, S)$ over the individual state variables. We assume that a takes on value on a grid G_a of dimension n_a. Therefore the dimension of the individual state space is $n = n_a \times n_\theta \times n_\varphi \times 2$ where $n_\theta = 2$ is the number of states for the entrepreneurial productivity and $n_\varphi = 4$ is the number of states for the working productivity.

In order to solve the model we use the following:

Algorithm 3 *Our solution algorithm is:*

1. *Assign all parameters values*
2. *Guess a value for the endogenous variable r.*
3. *Given r the FOC of the corporate sector uniquely pin down the wage rate w. The representative competitive firm in the corporate sector will choose K_c and L_c such as*

$$r^d = \xi A K_c^{\xi-1} L_c^{1-\xi} = \xi A \left(\frac{K_c^d}{L_c^d}\right)^{\xi-1} \tag{2.7-1}$$

$$w = (1-\xi) A K_c^{\xi} L_c^{-\xi} = (1-\xi) A \left(\frac{K_c^d}{L_c^d}\right)^{\xi} \tag{2.7-2}$$

Therefore r uniquely pins down $\left(\frac{K_c}{L_c}\right)$ and in turn uniquely pins down w.

4. *Given (r, w) we solve for the optimal value functions and corresponding policy functions by value function iteration. Within the period we solve backwards in time.*
   a) *We guess a value function $\mathsf{V}(\eta)$*
   b) *We solve the consumption-savings problem of the constrained and unconstrained agent for a grid of cash on hand.*
   c) *We approximate the resulting continuation value functions.*
   d) *Since the worker faces no uncertainty within the period, these value functions give us the values for the workers.*
   e) *Given the continuation value, we solve the problem of the unconstrained entrepreneur:*
      - *We set up a grid for secured credit.*
      - *For each value of secured credit, we set up a grid for unsecured credit.*

- *For each value of unsecured credit, we price the credit according to the zero profit condition.*
- *We identify the optimal grid point and then bisect around that optimal point to get a more accurate choice of unsecured credit.*
- *We calculate the value for each combination of secured.*

f) *The problem of the constrained entrepreneur is solved similarly.*

g) *Occupational choice gives us the updated value functions* $\hat{V}(\eta)$.

h) *We iterate until convergence.*

i) *As a byproduct we obtain the policy functions.*

5. *The policy functions, the exogenous transition matrix for the shocks (both for* θ *and for* φ*), the* iid *investment shock and the credit status shock* ϱ *induce a transition matrix* P_η *over the state* η.

6. *The transition matrix* P_η *maps the any current distribution*[31] μ_η *into a next period distribution* μ'_η *by simply*

$$\mu_{\eta,t+1} = P'_\eta \times \mu_{\eta,t}$$

We calculate the steady state distribution over the state μ^*_η *by solving for a*

$$\mu^*_\eta = P'_\eta \times \mu^*_\eta$$

7. *From the policy functions and the steady state distribution, we derive the market clearing conditions*

8. *Labor market clearing implies that labor supply* $L^s(r)$ *is equal to labor demand (that comes from corporate*L^d_c*). Plugging this into the FOC (2.7-1) of the corporate sector we get capital demand from corporate sector:*

$$K^d_c(r) = \left(\frac{r}{\xi A}\right)^{\frac{1}{\xi-1}} L^S(r)$$

9. *Now we look at capital market clearing:*

$$K^d_{ENTR}(r) + K^d_c(r) = SA(r)$$

10. *If there is not equilibrium at point 9 we adjust interest rate, we go back to point 3 and we iterate until market clears*[32].

[31] Note that in our framework the distribution of household over the state μ_η, is vector of dimension n whose elements sum up to 1.

[32] In practice we first run a grid search over different values for r and then bisect until we get market clearing.

Chapter 3

Bankruptcy Reform and Endogenous Risk-taking by Entrepreneurs

3.1 Introduction

In recent years many European countries have adopted policies to encourage entrepreneurship. One of the policy changes has been to make the bankruptcy law more lenient. For example in Germany, prior to a law change in 1999, a person who was unable to repay a loan was liable for his debt forever. Creditors could garnish part of the income forever. The garnishment period is now reduced to six years. In the UK this period was reduced to three years. A similar law changed was introduced in the Netherlands. One of the stated objectives was to encourage risk-taking by entrepreneurs.

I analyze the effect of changes in the bankruptcy law on risk-taking by entrepreneurs when risk-taking is endogenous. In particular, I investigate whether a more lenient bankruptcy law really encourages risk-taking. I show that this is not necessarily the case. Consequently, the policy changes mentioned above might not have the effects envisioned by policy makers.

It is well understood that making the bankruptcy law more lenient worsens credit market conditions and therefore can lower entrepreneurship rates and / or firm size. However, the possibility to default in bad states provides entrepreneurs with insurance. Akyol / Athreya [2007], Mankart / Rodano [2007, 2009], and Meh / Terajima [2008] analyze this trade-off in quantitative models of the US economy. However in their papers risk-taking is exogenous in the sense that agents can not influence the success probabilities of their projects. In this paper, I allow agents to influence the riskiness of their projects. They can increase the return of their project in good states by accepting lower returns in bad states.

Risk-taking behavior of agents facing an occupational choice problem has been analyzed by Hopenhayn / Vereshchagina [2009]. In their model, entrepreneurs have to finance their projects themselves. The occupational choice of the agents leads to a kink in the value function. Agents with assets in the neighborhood of this kink will be locally risk-loving. Therefore, they will take on more risk even when the expected value of the projects does not change.

In my model agents can borrow and default. This introduces another kink in the value function. This can also lead agents to take on more risk. In my quantitative model, I indeed find that some entrepreneurs choose risky projects. But this result is due to the kink stemming from the default option and not due to the occupational choice as in Hopenhayn / Vereshchagina [2009]. The reason that occupational choice alone does not lead to increased risk-taking is that agents face some uncertainty about their entrepreneurial and labor productivity next period. This randomness convexifies the continuation value functions to such a degree that the kink due to the occupational choice problem gets smoothed out. The kink due to the default possibility,however, is not smoothed out and therefore some entrepreneurs are locally risk-loving and choose risky projects.

The main result of the paper is that making the bankruptcy law more lenient can lead agents, keeping their wealth and productivity constant, to take on less, and not more, risk. The reason for this counter-intuitive effect is that agents are able to borrow a lot and choose risky projects when the exemption level is low. As the exemption level is increased, the default incentives increase and banks might fail to break even on the original loan. There is no finite interest rate for which a bank would break even on the original loan size. Therefore banks offer the entrepreneur only smaller loans. That makes defaulting less attractive because there will be less undepreciated capital left. In this situation an entrepreneur might decide to take on less risk. This lower risk-taking will lead to a lower interest rate on the (smaller) amount borrowed.

This effect might be an explanation of a finding by Berkowitz / White [2004]. They examine the effects of different exemption levels across US states on credit conditions for small firms. Their main finding is that the harsher the bankruptcy law, i.e. the more assets the bank can seize in the case of a default, the lesser borrowing constraints, and the lower are the interest rates that firms have to pay. However, this relationship is non-monotonic. This non-monotonicity might be due to a selection effect, the entrepreneurs with the highest default probabilities are rationed out of the credit market, or due to less risk-taking by entrepreneurs. In the model, I show that the result is not driven by the selection effect but by the risk-taking incentives of the entrepreneurs. The latter makes bankruptcy more attractive might lead agents to take on less risks.[1]

The paper is organized as follows. In section 3.2, I lay out the baseline model. In section 3.3, I present the result for the benchmark case. In section 3.4, I analyze the effects of changing the exemption level. In particular, I show that the model can reproduce the worsening of credit market conditions observed by Berkowitz / White [2004]. In section 3.5, I investigate

[1] I tried to obtain the data used by Berkowitz / White [2004]. But, unfortunately, the state level information is available only to members of the Federal Reserves Board of Governors.

the robustness of the results. Section 3.6 concludes.

3.2 A Quantitative Model

I investigate the optimal risk-taking decision of entrepreneurs in an occupational choice model calibrated to the US economy. The model in this section is built on the model in Mankart / Rodano [2007].[2]

The economy is populated by a unit mass of infinitely lived heterogeneous agents. Agents face idiosyncratic uncertainty, but there is no aggregate uncertainty. At the beginning of every period, agents decide whether to become workers or entrepreneurs. An entrepreneur must decide how much to invest, in what technology to invest and, if he is allowed to, how much to borrow. The agent can choose among different technologies. The technologies differ in their riskiness and returns. An entrepreneur who has defaulted in the past is excluded from unsecured credit for some time. Since I focus on the implications of personal bankruptcy for entrepreneurs, workers are not allowed to borrow. Agents' productivities evolve over time and agents are subject to uninsurable production risk. However, the amount of risk taken is partially endogenous. After the shocks are realized, production takes place. At the end of the period borrowers decide whether to repay or whether to default and how much to consume and how much to save. If they default, they will be borrowing constrained in the next period. Anticipating this behavior, banks who give unsecured credit vary the interest rate charged for each loan taking into account the individual borrower's default probability. The remainder of this section presents the details of the model.

3.2.1 Credit and bankruptcy law

Entrepreneurs can obtain unsecured credit to finance their firms. If the entrepreneur decides to default he will be subject to a Chapter 7 bankruptcy procedure. All unsecured debt will be discharged. But, any assets in excess of the exemption level X are liquidated and used to repay creditors. In order to limit leverage to empirically plausible levels, I introduce a borrowing limit as multiple of the entrepreneur's own assets. This multiple is calibrated to obtain a leverage ratio of 15%.[3]

An agent who has defaulted in the past is excluded from the market for unsecured credit for a certain period of time. During this period he still can become an entrepreneur. He has to self-finance his project though. I call this agent *borrowing constrained* and I denote his credit

[2] One caveat is in order however, Mankart / Rodano [2009] show that including secured credit in the analysis matters. Due to the computational burden this would imply, I consider only unsecured credit. However, the focus of the analysis here is not so much normative as positive. Thus, the emphasis is on the effects of different exemption level on risk-taking and not on the optimal exemption level.

[3] As has been pointed out by Heaton / Lucas [2002], models without uncertainty about the type of the agent feature implausibly high leverage ratios.

status as BC. It is important to note that this agent is not fully excluded from the credit market. He can still save. I assume that every *borrowing constrained* agent, whether worker or entrepreneur, faces a credit status shock at the end of the period. The agent remains *borrowing constrained* with probability $(1-\varrho)$. He regains access to credit markets with probability ϱ and therefore becomes an *unconstrained* agent with credit status UN.[4] This probability ϱ captures the duration of the exclusion period from the market of unsecured borrowing. It is calibrated such that the average exclusion period is two years.

3.2.2 Households

The economy is populated by a unit mass of infinitely lived heterogeneous agents. Agents differ with respect to their level of assets a, their entrepreneurial productivity θ, their working productivity ϑ, and their credit market status $S \in \{UN, BC\}$.

Preferences

For simplicity I abstract from the labor-leisure choice. All agents supply their unit of labor inelastically either as workers or as entrepreneurs. As has been shown by Mankart / Rodano [2009], disentangling the effects of risk aversion from that of the elasticity of intertemporal substitution can yield interesting insights. Therefore, I assume that agents have Epstein-Zin preferences. A stochastic consumption stream $\{c_t\}_{t=0}^{\infty}$ generates an utility $\{u_t\}_{t=0}^{\infty}$ according to

$$u_t = U(c_t) + \beta U\left(\mathbb{CE}_t\left[U^{-1}(u_{t+1})\right]\right)$$

where β is the discount rate and $\mathbb{CE}_t\left[U^{-1}(u_{t+1})\right] \equiv \Gamma^{-1}\left[\mathbb{E}_t\Gamma(u_{t+1})\right]$ is the consumption equivalent of u_{t+1} given information at period t. The utility function $U(c) = c^{1-\frac{1}{\psi}}/\left(1-\frac{1}{\psi}\right)$ aggregates consumption across dates and ψ is the intertemporal elasticity of substitution. The utility function $\Gamma(c) = c^{1-\gamma}/(1-\gamma)$ aggregates consumption across states and γ is the coefficient of relative risk aversion.

Productivities

Each agent is endowed with a couple of stochastic productivity levels which are known at the beginning of the period: one as an entrepreneur θ, and one as a worker ϑ. I make the simplifying assumption that the working and entrepreneurial ability processes are uncorrelated.

[4] The length of the exclusion period is transformed into a probability in order to avoid an additional state variable that keeps track of the numbers of years left before the solvency status is returned to UN. This procedure is standard in the literature, see Athreya [2002] and Chatterjee et al. [2007].

The workers' ability process Following the literature, I assume that labor productivity follows the following AR(1) process

$$\log \vartheta_t = (1-\rho)\,\mu + \rho \log \vartheta_{t-1} + \varepsilon_t$$

where ε_t is *iid* and $\varepsilon \sim N(0, \sigma_\varepsilon)$. If the agent becomes a worker his labor income during current period is given by $w\vartheta$.

The entrepreneurs' ability process In contrast to the case of working ability, there are no reliable estimates of the functional form of entrepreneurial ability. Therefore, following Cagetti / De Nardi [2006], I assume a parsimonious specification where entrepreneurial productivity follows a 2-state Markov process with $\theta^L = 0$ and $\theta^H > 0$ and transition matrix

$$P_\theta = \begin{bmatrix} p^{LL} & 1-p^{LL} \\ 1-p^{HH} & p^{HH} \end{bmatrix}$$

I calibrate the 3 parameters (θ^H, p^{HH} and p^{LL}) to match some observed features of entrepreneurial activity in the US economy.

3.2.3 Technology

Entrepreneurial sector Every agent in the economy has access to a set of productive technologies that, depending on her entrepreneurial productivity θ, produces output according to the production function

$$\begin{aligned} Y &= \theta k^{\nu} \\ k &= \chi I \end{aligned}$$

where θ is the agent's persistent entrepreneurial productivity described above, and

$$log\chi \sim N\left(0, \sigma_\chi^{2,j}\right).$$

The key innovation is that the variance of the project is a choice variable, i.e. agents can choose different levels of riskiness. They might want to do this due to the non-convexities resulting from the default option and from the occupational choice. Thus, entrepreneurs can choose σ_χ^j. However, there is a lower bound on σ_χ^j, chosen to match observed default rates.

Corporate sector Many firms are both incorporated and big enough not to be subject to personal bankruptcy law. Therefore, I follow Quadrini [2000] and Cagetti / De Nardi [2006] and assume a perfectly competitive corporate sector which is modeled as a Cobb-Douglas production function

$$F(K_c, L_c) = AK_c^{\xi} L_c^{1-\xi}$$

where K_c and L_c are capital and labor employed in this sector. Given perfect competition and constant returns to scale the corporate sector does not distribute any dividend. Capital depreciates at rate δ in both sectors.

3.2.4 Credit market

I assume that there is perfect competition (free entry) in the credit market. Therefore, banks make zero expected profits on any contract. The opportunity cost of lending to entrepreneurs is the rate of return on capital in the corporate sector. This is also equal to the deposit rate.[5] In addition, producing credit incurs a transaction cost τ. Agents can obtain unsecured credit on which they can default. As already mentioned, there is also an endogenously calibrated borrowing limit. While there are no information asymmetries in the credit market about the borrowers' types, there is a hidden action problem. Thus, while banks know the agent's assets, and his productivity levels, banks cannot control the technology the entrepreneur will invest in. In particular, they have to take into account that the entrepreneur might choose a riskier project. But since banks can predict the behavior of the agent, they know what the agent will do and therefore price every loan accordingly. Thus, for any value of (a, θ, ϑ) and for any amount lent b, by anticipating the behavior of the entrepreneur, banks are able to calculate the probability of default and the recovery rate in case of default. Perfect competition implies that they set the interest rate, $r(a, \theta, \vartheta, b, X)$, such that they expect to break even. This interest rate depends on the exemption level X because it affects the incentives to default and the amount the bank recovers in this event. Therefore banks offer a menu of one period debt contracts which consist of an amount lent b and a corresponding interest rate $r(a, \theta, \vartheta, b, X)$ to each agent (a, θ, ϑ).

3.2.5 Timing

Figure 3.1 shows the timing of the model. Given the focus of the paper I choose the timing such that workers can never default. Entrepreneurs' borrowing and default decisions are taken within the period. At the beginning of the period all agents face an occupational choice: they choose whether they become entrepreneurs or workers. Agents know their current productivity levels (ϑ, θ).

Workers deposit all their wealth at the banks, receiving a rate of return r^d. After production has taken place, they choose consumption and savings. At the end of the period, the *borrowing constrained* worker receives the credit status shock. With probability ϱ he remains *borrowing constrained* next period (i.e. $S' = BC$). With probability $(1 - \varrho)$ he becomes *unconstrained* next period (i.e. $S' = UN$).

[5] In the model banks are isomorphic to a bond market in which each agent has the possibility to issue debt.

The *borrowing constrained* entrepreneur chooses two things: he must decide the riskiness of the project σ_χ^j and how much of his own wealth he invests in the project respectively how much he saves. He must take these decisions before the *iid* shock χ is realized. After χ is realized and production has taken place, he chooses consumption and savings. At the end of the period he receives the credit status shock.

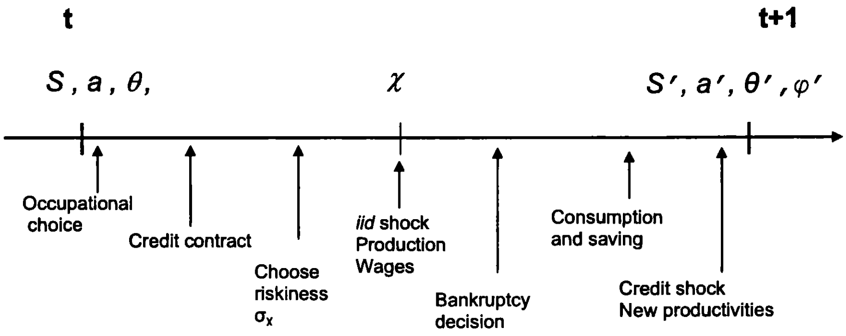

Figure 3.1: Timing of the model

The *unconstrained entrepreneur* has to make two additional decisions. At the beginning of the period, he has to decide how much to borrow. Competitive banks will offer fairly priced loans that take the entrepreneur's subsequent technology risk choice into account. Thus, they offer a menu $\{b, r(a, \theta, \vartheta, b, X)\}$ of credit contracts. The entrepreneur will pick the one that implies the highest utility. After χ is realized and production has taken place, the entrepreneur must decide whether to repay his debt and be *unconstrained* next period (i.e. $S' = UN$) or whether to file for bankruptcy and be *borrowing constrained* next period(i.e. $S' = BC$). After that he chooses consumption and savings.

Since the credit status S consists only of the two states BC and UN, I define the individual state variable as (a, θ, ϑ), and I solve for two value functions $V^{UN}(a, \theta, \vartheta)$ and $V^{BC}(a, \theta, \vartheta)$ one for each credit status.

3.2.6 The problem of the *borrowing constrained* agent

At the beginning of the period, the *borrowing constrained* agent can choose whether to become an entrepreneur, which gives utility $N^{BC}(a, \theta, \vartheta)$ or a worker which yields utility $W^{BC}(a, \theta, \vartheta)$. Therefore, the value of being a *borrowing constrained* agent with state (a, θ, ϑ) is

$$V^{BC}(a, \theta, \vartheta) = \max\left\{N^{BC}(a, \theta, \vartheta), W^{BC}(a, \theta, \vartheta)\right\}$$

where the "max" operator reflects the occupational choice.

Worker At the beginning of the period a *borrowing constrained* worker deposits all his wealth at the bank. He receives labor income $w\vartheta$. At the end of the period, he chooses consumption and saving, taking into account that he will receive a credit status shock in addition to productivity shocks. With probability ϱ he will be still *borrowing constrained* next period which yields utility $V^{BC}(a', \theta', \vartheta')$, while with probability $(1-\varrho)$ he will become *unconstrained* which yields utility $V^{UN}(a', \theta', \vartheta')$. His saving problem is the following

$$\begin{aligned} W^{BC}(a,\theta,\vartheta) &= \max_{c,a'} \left\{ U(c) + \beta U \left(\mathbb{CE}_t \left[\varrho V^{BC}(a',\theta',\vartheta') + (1-\varrho) V^{UN}(a',\theta',\vartheta') \right] \right) \right\} \\ s.t.\ c + a' &= w\vartheta + \left(1 + r^d\right) a \\ a' &\geq 0 \end{aligned}$$

Entrepreneur At the beginning of the period the *borrowing constrained* entrepreneur decides how much to invest in his firm $I\ (\leq a)$ and how much to save $a-I$. In addition, he must decide how much risk σ_χ^j to take on. After he has invested, each unit of investment is transformed in χ units of capital $(k = \chi I)$.

After the shock χ is realized he will decide how to allocate his resources $(\chi I)^\nu \theta + (1-\delta)\chi I - \left(1 + r^d\right)(a - I)$ between consumption and savings. His saving problem, after uncertainty is resolved,[6] is

$$\begin{aligned} \tilde{N}^{BC}(a,\theta,\vartheta,\chi) &= \max_{a',c} \left\{ U(c) + \beta U \left(\mathbb{CE}_t \left[\varrho V^{BC}(a',\theta',\vartheta') + (1-\varrho) V^{UN}(a',\theta',\vartheta') \right] \right) \right\} \\ s.t.\ c + a' &= (\chi I)^\nu \theta + (1-\delta)\chi I - \left(1 + r^d\right)(a - I) \\ a' &\geq 0. \end{aligned}$$

Therefore the optimal investment decisions of the agent at the beginning of the period is

$$\begin{aligned} N^{BC}(a,\theta,\vartheta) = \max_{I,\sigma_\chi^j} U \left(\mathbb{CE}_t \left\{ \tilde{N}^{BC}(a,\theta',\vartheta',\chi) \right\} \right) \\ s.t.\ I \leq a \text{ and } \sigma_\chi^j \in \left[\sigma_\chi^1, \sigma_\chi^2, ..., \sigma_\chi^n\right]. \end{aligned}$$

Thus, the agent chooses the size of his project and the variance of the distribution from which the shock will be drawn.

3.2.7 The problem of the *unconstrained* agent

At the beginning of the period the *unconstrained* agent faces the following occupational choice

$$V^{UN}(a,\theta,\vartheta) = \max \left\{ W^{UN}(a,\theta,\vartheta), N^{UN}(a,\theta,\vartheta) \right\}$$

[6] I denote with a "~" all the value functions, *after* uncertainty (about χ) is resolved. The value functions without "~" are *before* uncertainty is resolved.

where $W^{UN}(a,\theta,\vartheta)$ is the utility of becoming a worker and $N^{UN}(a,\theta,\vartheta)$ of becoming an entrepreneur.

Worker The problem of the *unconstrained* worker is identical to the *borrowing constrained* one except that the agent will be *unconstrained* in the future for sure. His saving problem is

$$\begin{aligned} W^{UN}(a,\theta,\vartheta) &= \max_{c,a'} U(c) + \beta U\left(\mathbb{CE}_t\left[V^{UN}(a',\theta',\vartheta')\right]\right) \\ s.t.\ c + a' &= w\vartheta + \left(1 + r^d\right) a \\ a' &\geq 0. \end{aligned}$$

Entrepreneur The *unconstrained entrepreneur* decides how much to invest in his firm $I = a + b$ by choosing how much to borrow ($b > 0$) or save ($b < 0$). If he borrows he can choose from the menu $\{b, r(a,\theta,\vartheta,b,X)\}$ offered by competitive banks. If he saves, i.e. $b > 0$, $r(\cdot) = r^d$.[7] In addition, just as the *constrained entrepreneur*, he has to decide the riskiness of his project. After the shock χ is realized he can choose whether to default or whether to repay and how much to consume and save. He solves the problem backwards.

If he repays his debt, he has to choose how to allocate his resources, $\theta[(a+b)\chi]^{\nu} + (1-\delta)(a+b)\chi - [1 + r(a,\theta,\vartheta,b,X)]\,b$, between consumption and savings. Given that the decision of repaying is done when current productivities (θ,ϑ) and the shock χ are known, his utility from repaying is given by

$$\begin{aligned} \tilde{N}^{pay}(a,b,\theta,\vartheta,\chi) &= \max_{c,a'} \left\{U(c) + \beta U\left(\mathbb{CE}_t\left[V^{UN}(a',\theta',\vartheta')\right]\right)\right\} \\ s.t.\ a' + c &= \theta[(a+b)\chi]^{\nu} + (1-\delta)(a+b)\chi - [1 + r(a,\theta,\vartheta,b,s,X)]\,b \\ a' &\geq 0 \end{aligned}$$

If he defaults, his unsecured debt is discharged. But he loses all assets in excess of the exemption level X. Thus, the resources to allocate between consumption and savings are $\min\{\theta[(a+b)\chi]^{\nu} + (1-\delta)(a+b)\chi, X\}$. Moreover if he defaults he will be *borrowing constrained* next period. Therefore, by declaring bankruptcy he gets

$$\begin{aligned} \tilde{N}^{bankr}(a,b,\theta,\vartheta,\chi) &= \max_{c,a'} \left\{U(c) + \beta U\left(\mathbb{CE}_t\left[V^{BC}(a',\theta',\vartheta')\right]\right)\right\} \\ s.t.\ a' + c &= \min\{\theta[(a+b)\chi]^{\nu} + (1-\delta)(a+b)\chi, X\} \\ a' &\geq 0 \end{aligned}$$

[7] The entrepreneur is not allowed to borrow and save in the risk-free asset simultaneously. Thus, he cannot engage in hidden savings. This assumption is made to simplify the computations. And, it is unlikely to be restrictive since the marginal product of capital is, particularly for poor agents so high that they will invest as much as they can in the project.

He will file for bankruptcy if $N^{bankr}(a,b,\theta,\vartheta\,\chi) > N^{pay}(a,b,\theta,\vartheta,\chi)$ and vice versa. Thus, at the beginning of the period the agent choose the optimal amount of b from the menu $\{b, r(a,\theta,\vartheta,b,X)\}$ and the optimal σ_χ^j anticipating his future behavior. Therefore his utility is given by

$$\begin{aligned} N^{UN}(a,\theta,\vartheta) &= \max_{\{b,r(a,\theta,\vartheta,b)\},\sigma_\chi^j} \mathbb{CE}_t\left[\max\left\{\tilde{N}^{pay}(a,b,\theta,\vartheta,\chi),\tilde{N}^{bankr}(a,b,\theta,\vartheta,\chi)\right\}\right] \\ s.t.\ b &\geq \lambda a \text{ and } \sigma_\chi^j \in \left[\sigma_\chi^1,\sigma_\chi^2,...,\sigma_\chi^n\right]. \end{aligned}$$

where the "max" operator inside the square brackets reflects the bankruptcy decision, and the "max" operator outside the square brackets reflects the borrowing decision. λ is an exogenous borrowing limit that will be calibrated to match the observed leverage ratio.

3.2.8 The zero profit condition of the banks

Banks observe the state variables (a,θ,ϑ) at the moment of offering the contract. There is perfect competition (free entry) in the credit market. Therefore, banks make zero profit on each loan contract. However, banks cannot condition the contract on the subsequent technology choice of the entrepreneur. For any given state (a,θ,ϑ) and for any loan (b), banks know which technology σ_χ^j the entrepreneur will choose and in which states of the world he will default. Therefore, they are able to calculate the probability that a certain agent with characteristics (a,θ,ϑ) will default for any given amount b. This default probability,$\pi^{bankr}(a,\theta,\vartheta,b,X)$, depends on the exemption level X because X affects the incentive to default directly.

If the agent repays, banks receive $[1+r(a,\theta,\vartheta,b,X)]\,b$. If the agent defaults, banks sell the firm's un-depreciated capital. In this case banks receive: nothing if $\theta\left[(a+b)\chi\right]^\nu + (1-\delta)(a+b)\chi < X$ or $\theta\left[(a+b)\chi\right]^\nu + (1-\delta)(a+b)\chi - X$ otherwise. The zero profit condition of the banks is given by

$$\begin{pmatrix} \left[1-\pi^{bankr}(a,\theta,\vartheta,b,X)\right][1+r(a,\theta,\vartheta,b,X)]\,b+ \\ +\pi^{bankr}(a,\theta,\vartheta,b,X)\max\left\{\theta\left[\chi I\right]^\nu + (1-\delta)\chi I - X, 0\right\} \end{pmatrix} = (1+r^d+\tau)b,$$

where $I = a+b$ and τ is a resource cost banks incur when producing credit.

3.2.9 Equilibrium

Let $\eta = (a,\theta,\vartheta,S)$ be a state vector for an individual, where a denotes assets, θ entrepreneurial productivity, ϑ working productivity and S the credit status. From the optimal policy functions (savings, capital demand, default decisions), from the exogenous Markov process for productivity and from the credit status shocks, we can derive a transition function, that, for any distribution $\mu(\eta)$ over the state provides the next period distribution $\mu'(\eta)$. A stationary equilibrium is given by

- a deposit rate of return r^d and a wage rate w
- an interest rate function
- a set of policy functions $g(\eta)$ (consumption and saving, borrowing, capital demand, bankruptcy decisions and occupational choice)
- a constant distribution over the state η, $\mu^*(\eta)$

such that, given r^d and w and a bankruptcy regime X and ϱ:

- $g(\eta)$ solves the maximization problem of the agents;
- the corporate sector representative firm is optimizing;
- capital, labor and goods market clear:
  - capital demand comes from both, entrepreneurs and the corporate sector, while supply comes from the saving decisions of the agents;
  - labor demand comes from the corporate sector, while labor supply comes from the occupational choice of the agents;
- the interest rate function reflects the zero profit condition of the banks
- The distribution $\mu^*(\eta)$ is the invariant distribution associated with the transition function generated by the optimal policy function $g(\eta)$ and the exogenous shocks.

3.3 Results

3.3.1 Parametrization

Fixed parameters

Some of the parameters are fixed while others are calibrated to match some observed features of the US economy. Essentially, I follow the calibration strategy of Mankart / Rodano [2009]. I choose $\rho = 0.95$ for the auto-regressive coefficient of the earnings process.[8] The variance of the earnings process is chosen to match the Gini index of labor income as observed in the PSID, where it is 0.38.The process is approximated using a 4-state Markov chain, using the Tauchen [1986] method as suggested by Adda / Cooper [2003].[9] Total factor productivity is normalized to 1, while the share of capital in the Cobb-Douglas technology for the corporate

[8] In a life cycle setting, Storesletten et al. [2004] and Storesletten et al. [2001] find ρ in the range between 0.95 and 0.98. I choose $\rho = 0.95$ to take into account that the agents in the model are infinitely lived and that the intergenerational auto-regressive coefficient is lower. Solon [1992] estimates it to be around 0.4.

[9] Floden [2008] shows that for highly correlated processes the method of Adda / Cooper [2003] achieves a higher accuracy than the original methods of Tauchen [1986] and Tauchen / Hussey [1991].

sector is set to $\xi = 0.36$. The depreciation rate is set $\delta = 0.08$. The cost of credit production is set to $\tau = 4\%$. These parameters are summarized in table 3.3-1.

Table 3.3-1: The fixed parameters

Parameter	Symbol	Baseline
TFP	A	1 (normalization)
Share of capital	ξ	0.36
Depreciation rate	δ	0.08
Resource cost	τ	0.04
Working productivities	$\varphi_1 < \varphi_2 < \varphi_3 < \varphi_4$	$\begin{bmatrix} \varphi_1 = 0.316, \varphi_2 = 0.745 \\ \varphi_3 = 1.342, \varphi_4 = 3.163 \end{bmatrix}$
Transition matrix	P_φ	$\begin{bmatrix} 0.8393 & 0.1579 & 0.0028 & 0.0000 \\ 0.1579 & 0.6428 & 0.1965 & 0.0028 \\ 0.0028 & 0.1965 & 0.6428 & 0.1579 \\ 0.0000 & 0.0028 & 0.1579 & 0.8393 \end{bmatrix}$

Preference parameters

Following Mankart / Rodano [2009], I separate the utility function parameters. The option to default provides agents with an insurance against bad outcomes. The value of this insurance depends crucially on the agents attitudes towards risk. But the price of this insurance are worsened credit conditions. Agents who still borrow face higher interest rates. Thus, the value of the costs of the insurance depends mainly on the agents elasticity of intertemporal substitution. In the baseline model, I set the coefficient of relative risk aversion $\sigma = 3$ and the elasticity of intertemporal substitution $\psi = 0.8$. In the robustness section, I investigate the cases $\sigma = 1.5$ and $\psi = 0.5$. Table 3.3-2 summarizes the preference parameters.

Table 3.3-2: Preference parameters

Parameter	Symbol	Value
CRRA	σ	3
EIS	ψ	0.8

Bankruptcy policy parameters

The two policy parameters are the exemption level X and the probability ϱ of being able to obtain unsecured credit again. Following Mankart / Rodano [2009], I set $\varrho = 0.5$ which corresponds to an average exclusion period from credit of two years. This low value is warranted since there is evidence the entrepreneurs have access to unsecured credits relatively fast after having defaulted, see for example Lawless / Warren [2005]. The exemption level differs across US sates. Using US state-level data for 1993, I calculate the median across states of the total exemption ("homestead" plus "personal property" exemption). The resulting median exemption level is \$47,800. Taking an average household labor income of \$48,600, this corresponds to a value of **0.98** for the exemption/wage ratio. Table 3.3-3 summarizes the bankruptcy parameters.

Table 3.3-3: The bankruptcy parameters

Parameter	Symbol	Value
Exemption/wage	X/w	0.98
Unsecured credit exclusion (expressed as probability)	ϱ	0.5

Calibrated parameters

This leaves 7 parameters that have to be calibrated: high entrepreneurial productivity (θ^H), entrepreneurial productivity transition matrix (p^{HH}, p^{LL}), concavity of entrepreneurial production function (ν), multiple of assets an agent can borrow (λ), discount factor (β) and the minimum variance of the transitory shock (σ_χ^1). The last one warrants a discussion. I allow agents to take on more risk, i.e. choose a higher (σ_χ^j).[10] But it is not plausible that agents can choose completely safe projects. Therefore, I set a minimum level of risk that each entrepreneur has to face.[11]

I choose these 7 parameters so that the model matches the following 7 moments of the US economy.[12] First, I want the model to match the *capital-output ratio* (K/Y) in the US economy. I target it to be 3.0. The target for the *fraction of defaults.* is 2.25% of the entrepreneurs. The *fraction of entrepreneurs in the total population* is 7.3% in the Survey of Consumers Finances.[13] Based on PSID data the *exit rate* of entrepreneurs is equal to 15%. The median leverage ratio of entrepreneurs [14] in the SCF is around 15%.

Since the benefits of bankruptcy depend crucially on the wealth of an agent I match some features of the wealth distribution. The US wealth distribution is extremely skewed with the top 40% of richest households holding around 94% of total assets. As a last target I choose to match the *ratio of the median wealth of entrepreneurs to the median wealth in the whole population.* This target captures features of both the wealth distribution and entrepreneurial productivity and technology. I set the target to 6.3 as found in the SCF. The targets are

[10] This implies a mean preserving spread in logs and therefore a positive relationship in levels. However the concavity in the production function counters this effect. The net effect is very small. The advantage of log-normality is that levels are guaranteed to be positive. The increase in the expected return is so small that rich agents will never choose a high σ_χ^j. If it were otherwise, the interpretation of the results would be more difficult.

[11] I discretize the shock realizations with seven nodes. The risk levels the agents can choose are discretized with five equidistant nodes between σ_χ^1 and $\sigma_\chi^5 = 1.5 * \sigma_\chi^1$. Later on I show that increasing the highest possible risk level from 1.5 to 2 does not change the results.

[12] In this section, I also follow Mankart / Rodano [2009].

[13] See Mankart / Rodano [2007, appendix B] for data sources, definitions and further details.

[14] Leverage is defined as the ratio of debt to the sum of debt and equity.

summarized in the second column of Table 3.3-5.

3.3.2 The baseline calibration results

Table 3.3-4 shows the value of the calibrated parameters in the baseline specification. Most of the values are similar to those of earlier studies, for example Mankart / Rodano [2007].

Table 3.3-4: The calibrated parameters

Parameter	Symbol	Benchmark Value
High entrepreneurial productivity	θ^H	0.739
Entrepreneurial productivity transition	p^{HH}, p^{LL}	0.863 , 0.988
Concavity of entrepreneurial technology	ν	0.802
Borrowing multiple	λ	18.58
Discount factor	β	0.883
Min variance of transitory shock	σ_X	0.025

Table 3.3-5 reports the value of the targets and the actual results achieved in the baseline specification.

Table 3.3-5: The baseline calibration targets

Moment	Target	Model
Fraction of Entrepreneurs (in %)	7.30	7.30
Ratio of medians (in %)	6.30	6.31
Share of net-worth of top 40%	94.0	90.7
K/Y	3.00	3.02
Exit Rate (in %)	15.0	15.1
Bankruptcy rate (in %)	2.25	2.27
Mean debt/asset ratio (in %)	0.57	0.55

The model reproduces the chosen targets rather well. Thus, it is able to reproduce the extreme skewness in the wealth distribution and the fact that entrepreneurs are more than six times richer than workers. It also gets the dynamics of entrepreneurship right.

In order to assess the model, table 3.3-6 reports some statistics that were not targeted. The marginal product of capital in the corporate sector is 4.7% in the model, somewhat above the 4% estimated by McGrattan / Prescott [2001]. The share of capital in the non-corporate sector is slightly higher than what has been reported by Quadrini [2000]. The ratio of the mean to median firm size, measured in terms of capital, is below the data. The mean interest rate in the model is 9.05%, slightly below the estimate by Berkowitz / White [2004]. Note, however, that their rate is an average across US states, i.e. across different exemption levels whereas the result here is for one particular exemption level. I will discuss the relationship between interest rates and the exemption level at length in the next sections.

Table 3.3-6: Model implications

Moment	Data	Model
MPK (in %)	4.0	4.7
Share of cap in non-corp sector (in %)	35.0-40.0	46.6
Gini of wealth distribution	0.80	0.74
Mean/median ratio of firm size	1.7	1.3
Mean interest rate (in %)	9.27	9.05

3.3.3 The behavior of the unconstrained agents

In this section, I first explain the workings of the model by showing some of the agent's policy functions. I show that some entrepreneurs choose risky projects. This is due to the default possibility which introduces a non-convexity into the value function. The non-convexity coming from occupational choice as emphasized by Hopenhayn / Vereshchagina [2009] plays no role here.

Figure 3.2 shows the policy function of an unconstrained agent in the benchmark case of $X = 0.98$. The panels on the left hand side show agents with low labor productivity $\vartheta = \vartheta_1$ and with assets between $0 < a < 5$, corresponding to \$0-\$250,000. The panels on the right hand side show the same policy function but focus on poor agents $0 < a < 0.4$. In addition, they also show agents with a higher labor productivity, $\vartheta = \vartheta_3$.

First, I will describe the panels on the left hand side: Very poor agents (a<0.1) do not become entrepreneurs. The firms these agents could operate are so small that they are better off becoming workers. All others become entrepreneurs. Rich agents (a>4.9) have a lot of wealth to lose in case of a default. Therefore, they choose the least risky projects available. Thus, the variance of the shock they choose in the first panel of figure 3.2 is the lowest possible, i.e. σ^1_χ. For the same reason, they will never default. Thus, their default probability in the second panel of figure 3.2 is zero and therefore they borrow at the risk-free rate.

Slightly less rich agents (2.7<a<4.9) also choose σ^1_χ. But, they default when the worst outcome is realized. The insurance provided by the default possibility leads these agents to borrow more than those with a>4.9 who will never default. While these slightly less rich agents behave in the same way over the entire range, the amount borrowed is increasing in these agent's assets. This is because richer agent can bear more risk.

Agents with asset (0.22<a<2.7) are similar. They also choose σ^1_χ. But since they have fewer assets, they use the insurance provided by the default possibility in the two worst states. This higher default probability is reflected in a (slightly) higher interest rate. Note that these agents borrow as much as they can while still obeying the borrowing constraint $b \leq \lambda a$.

The behavior of the poor agents, shown in the panels on the right hand side, is the most interesting. Entrepreneurs with assets around $a = 0.2$ also borrow up to $b \leq \lambda a$. Thus, the

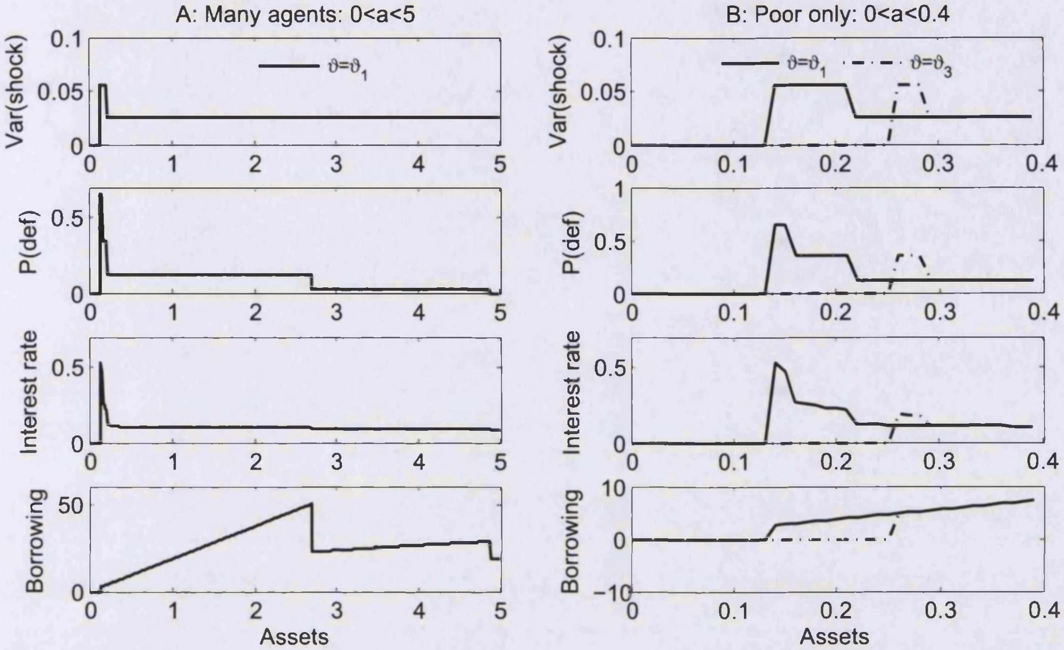

Figure 3.2: Risk taking, interest rates and borrowing

size of their firms, the sum of borrowing and own assets, is relatively small. These agents choose projects with a higher variance. The reason for this behavior is a non-concavity in their continuation value functions.[15] This non-concavity has two sources: First, the default possibility, which limits the amount the agents can loose. This leads agents to take on more risk because they are insured against the downside. Second, as in Hopenhayn / Vereshchagina [2009], occupational choice alone also leads to a non-concavity. Both reasons for the non-concavity imply a kink in the value function and make the entrepreneurs (locally) risk-loving.

The non-convexity of the problem coming from the default possibility is essential for the result. The firms of the marginal entrepreneurs employ capital around $k = 3$. Thus, even if the worst shock, which implies a loss of capital of about 50%, hits them, they are still left with undepreciated capital in excess of the exemption level. Thus, after filing for bankruptcy, they will be left with resources equal to the exemption level. This gives these entrepreneurs an incentive to choose projects with a high variance after having obtained credit. Banks will anticipate this behavior and therefore offer credit based on this. This explains the higher default probability and the higher interest rates in panels 2 and 3 on the right hand side of figure 3.2.

In Hopenhayn / Vereshchagina [2009] agents have to self-finance. The intersection of the value functions of workers and entrepreneurs creates a non-convexity. Rich agents become entrepreneurs and poor ones workers. If entrepreneurs can choose the riskiness of their project,

[15] Note that while the problem is a non-convex problem, the continuation value function becomes non-concave.

rich agents will not choose any additional risk since they remain entrepreneurs. This is also the case here. All entrepreneurs with $a > 0.22$ choose the lowest possible risk.

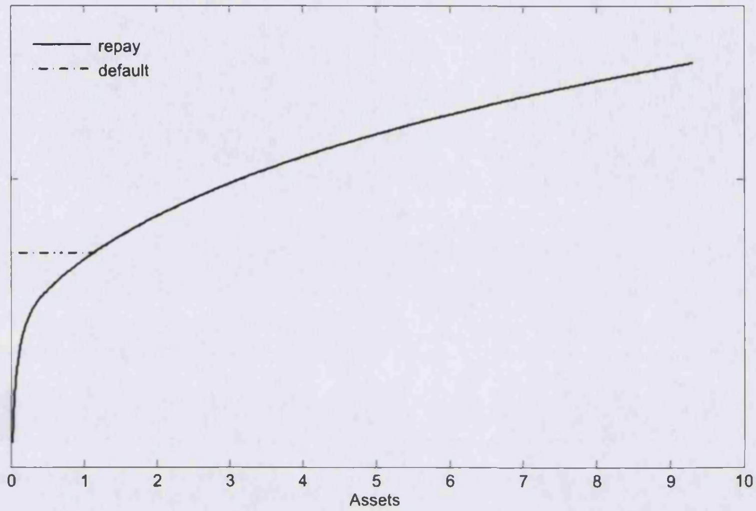

Figure 3.3: Continuation value functions for repaying and defaulting

A lower labor productivity implies, since labor productivities are persistent, a lower worker value function. The value function as an entrepreneur is not affected that much. Therefore, agents with low labor productivity $\vartheta = \vartheta_1$ enter entrepreneurship at a lower level of assets than agents with higher labor productivity, e.g. $\vartheta = \vartheta_3$. The difference in the point of the intersection of the two value function and therefore entry behavior of agents can be seen in the left hand panels of 3.2. Agents with $\vartheta = \vartheta_1$ enter entrepreneurship already with $a = 0.12$, while agents with $\vartheta = \vartheta_3$ enter only with $a = 0.25$.

Since the exemption level X is the same for everyone, the fact that agents with higher labor productivity choose riskier projects at $a = 0.28$ and not at $a = 0.2$ could be read as evidence for the importance of the non-convexity coming from the kink in the value function due to the occupational choice. However, this is not necessarily true. Agents with higher labor productivity have, ceteris paribus, a higher default incentive since their outside option is better. Therefore, their incentive to take on more risk happens at higher levels of assets, here at $a = 0.28$. The outside option of agents with low labor productivity is worse, therefore their incentive to choose riskier projects matters at a lower level of wealth, here $a = 0.2$.

Figure 3.3 shows the continuation value function in the middle of the period after the shocks have been realized but before the agents have consumed. The continuation value function for a defaulter is drawn under the assumption that the defaulting entrepreneur has resources that exceed the exemption level so that he will be left with $a = X$. [16] The project choice, i.e. the decision about how much risk to take on is taken before the production uncertainty is revealed. Therefore, entrepreneurs essentially decide over points on the continuation value function when they decide in which project to invest.

[16] This is indeed the case in the benchmark case.

The non-convexity coming from the default possibility is clearly present in figure 3.3. The non-convexity due to occupational choice is not visible. One reason for this is that the timing of the model is different from the timing in Hopenhayn / Vereshchagina [2009]. A related reason is that agents in the current model face a lot of uncertainty at the moment of deciding whether to default or whether to repay. At that moment, they do not yet know neither their labor productivity nor their entrepreneurial productivity next period. This uncertainty convexifies the continuation value function. Therefore, the additional risk-taking is not due to the occupational choice problem but due to the default possibility.

In addition, in the next section when I investigate different exemption levels, I will show the result of a model without default. In this case the only potential non-convexity is due to the occupational choice problem. But, in this case, every agent, including the marginal entrepreneur, chooses the least risky project, i.e. $\sigma_{\chi}^{j} = \sigma_{\chi}^{1}$. Thus, the fact that agents with a higher labor productivity choose riskier projects depends on the non-convexity due to the default option and not on the one due to occupational choice.

3.4 Policy experiment: changing the exemption level

In this section, I investigate the consequences of varying the exemption level from \$0 to \$250,000, the values observed across US states. In particular, I show that increasing the exemption level can increase the incentive to take on risk to such an extent that the endogenous borrowing constraint gets tightened so much that entrepreneurs choose the lowest level of riskiness and therefore have to pay relatively low interest rates. First, I focus on individual agents. Then, I show aggregate outcomes.

3.4.1 The impact on individual agents

Figure 3.4 shows entrepreneurial policy functions of agents with low labor productivity, $\vartheta = \vartheta_1$, for three different values of the exemption level: $X = 0$ which implies abolishing bankruptcy altogether,[17] $X = 0.98$ which is the benchmark case and a higher level $X = 2.7$.

If defaulting is not allowed, all entrepreneurs choose the lowest level of risk, i.e. σ_{χ}^{1}. Their default probability is zero.[18] Therefore, they borrow at the risk free rate. As mentioned in the previous section, the occupational choice problem alone does not lead to increased risk-taking. The bottom panel in figure 3.4 reveals that except for the very poor entrepreneurs, no entrepreneur is close to the exogenous borrowing limit. The absence of the insurance of bankruptcy makes these entrepreneur cautious in their borrowing and investment decision.

[17] This is the case because all agents have to finance their consumption after they default or repay. Thus, if an entrepreneur would default in the case of $X = 0$, his consumption would be $c = 0$ too. This would yield utility of negative infinity. Therefore, no entrepreneur will ever choose a project which might yield that outcome.
[18] The policy function in the second panel in figure 3.4 coincides with the x-axis.

If the exemption level is increased to $X = 0.98$, some of the poorest agents actually become workers. This is because, the default possibility leads to a tightening of the endogenous borrowing constraint. Poor agents have a strong incentive to default, therefore they can borrow only very small amounts. This makes entrepreneurship for them less attractive and so they become workers, despite having a low labor productivity. Thus, there is a negative extensive margin.[19] When the exemption level is increased even further, the extensive margin gets even more negative.

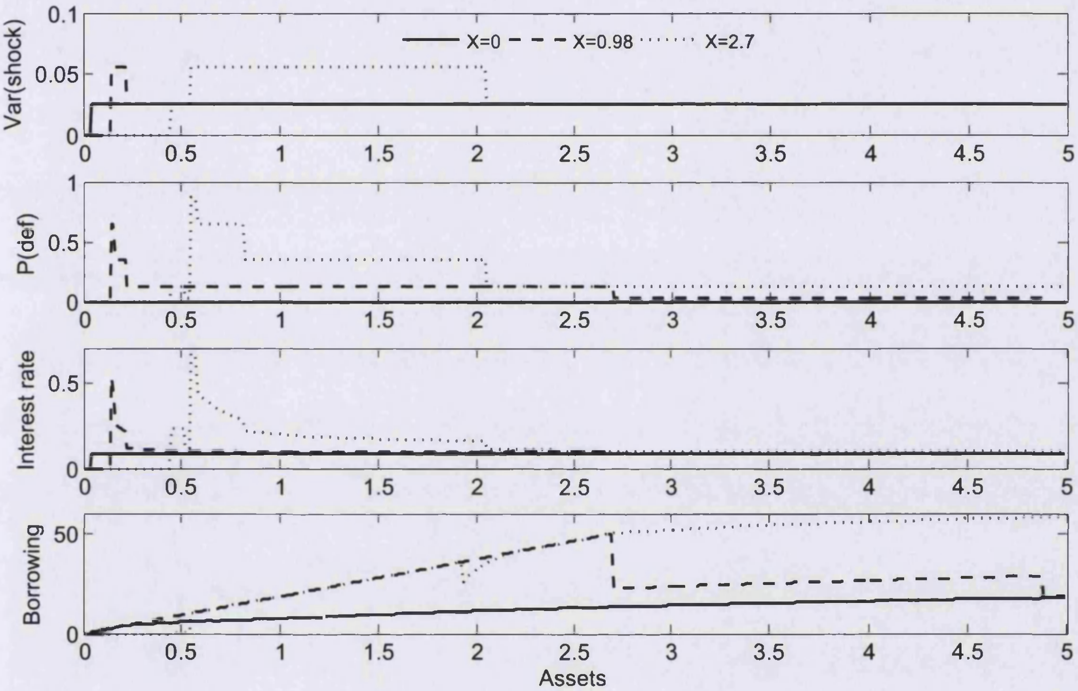

Figure 3.4: Policy functions for different exemption levels

For $X = 0.98$, poor entrepreneurs, assets around a=0.2, choose risky projects. They do this because the non-convexity in the continuation value function makes these agents locally risk-loving. If the exemption level is increased to $X = 2.7$, the non-convex region expands since now failed entrepreneur can keep more wealth. Therefore, the number of entrepreneurs who choose high risk projects increases. This can be seen in the top panel where agents with wealth $0.55 < a < 2.1$ choose the high risk project. These agents are also likely to default. This is reflected in the interest rates, they have to pay, see the second and third panel in figure 3.4.

While the extensive margin here is negative, there is a strong positive intensive margin,

[19] For agents with higher labor productivity, the extensive margin works exactly in the opposite direction (not shown). As shown in figure 3.2, agents with higher labor productivity enter entrepreneurship at a higher level of assets since their outside option is better. Since these agents are richer, they run bigger firms and with a positive exemption level they are insured against the downside. This leads them to run even bigger projects. This makes entrepreneurship more attractive to some agents who have been workers for $X = 0$. However, for high levels of the exemption level, the extensive margin is also negative for these agents.

see the bottom panel. The default possibility provides entrepreneurs with partial insurance against bad outcome. Therefore, they can run bigger firms, i.e. they borrow more.

The interest rate in the case of $X = 2.7$ is non-monotonic. In particular, agents with assets around $a = 0.5$ pay a lower interest rate than richer agents. The reason for this can be seen in the top panel. These agents choose the lowest risk level possible, therefore their default rate is lower. This occurs because the endogenous borrowing constraint for these agents become so tight because banks severely ration their access to credit. This credit rationing is consistent with the findings by Berkowitz / White [2004] who find that credit rationing is more sever in states with higher exemption levels.

These entrepreneurs can borrow only small amounts because if they were allowed to borrow as much as richer agents, they would choose high risk projects and default most of the times, making it impossible for banks to break even. By limiting the amount they lend, banks ensure that entrepreneurs will choose a low risk project and therefore their default probability is lower.

In figure 3.5, I show the behavior of three different agents for different exemption levels. The exemption level, which is depicted on the x-axis,[20] varies from 0 to 5, corresponding to \$0-\$250,000, which is the range of exemption levels observed in the US.

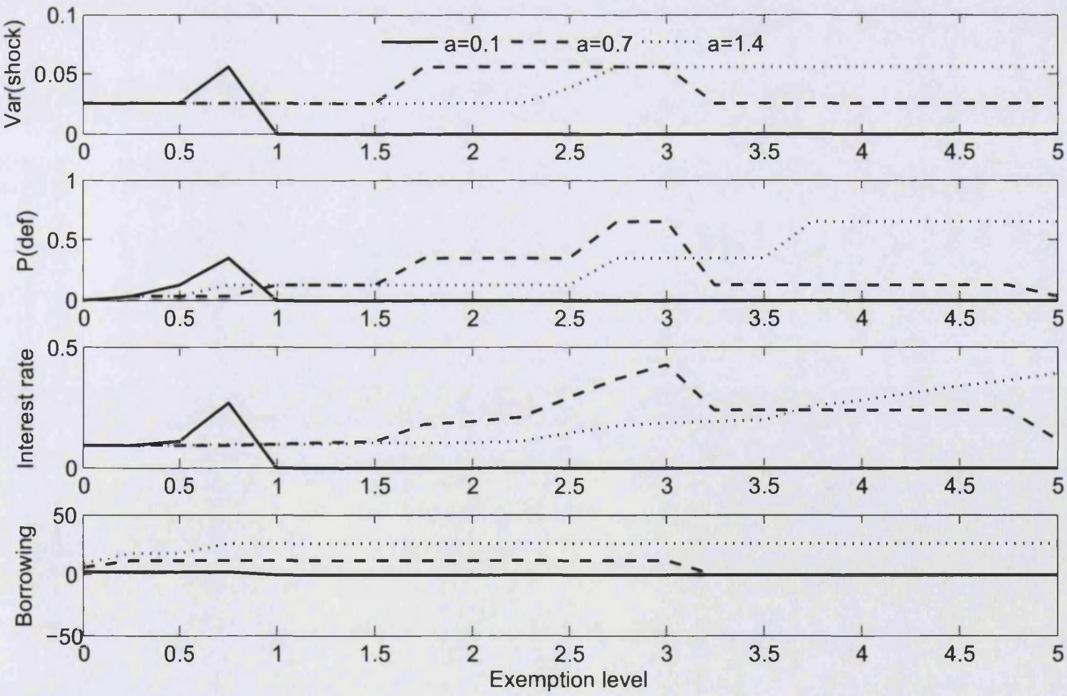

Figure 3.5: Risk taking, interest rates and borrowing for selected agents across different exemption levels

[20] Thus, these are not policy function in the asset space

All agents in figure 3.5 choose the lowest risk level for low exemptions, i.e. for $X < 0.5$. The poorest agent, $a = 0.1$, chooses a high risk project for $X = 0.75$. But if the exemption level is increased further, the endogenous borrowing constraint gets so tight that the agent can borrow only very little. This means the scale of the project he could run is so small that he prefers to be a worker. Thus, for any exemption level, $X > 1$, this agent is not an entrepreneur.

The most interesting agent is the one with $a = 0.7$. This agent chooses a low risk project for low exemption levels. The reason is that low levels of X do not offer much insurance against bad outcomes. Therefore, this agent wants to avoid default. If X is increased, this agents chooses a higher amount of risk. This is because the higher exemption level, which implies more resources in the case of a default, implies that the randomization region now becomes relevant for this agent. Therefore, a higher exemption level leads some agents to take on more risk.

However, if the exemption level is increased beyond $X = 3.25$, then the agent chooses a lower level of risk. The reason for this is that the endogenous borrowing limit, i.e. credit rationing becomes so severe that the agent cannot borrow much. This can be seen in the bottom panel where borrowing falls for this agent around $X = 3.25$. The scale of the firm is so small that the agent would be left with resources that are less than the exemption level in bad states. Therefore this agent does not want to take on additional risk. He chooses the low risk project. Therefore, the interest rate in the third panel is actually declining in the exemption level for this agent. This non-standard result might explain the finding by Berkowitz / White [2004] that the average interest rate is non-monotonic in the exemption level. While it is increasing for most values of the exemption level. It is declining around the 90th percentile.

3.4.2 The impact on aggregate outcomes

In this section I analyze the effects of changing the exemption level on several aspects of entrepreneurship and on welfare. I compare the outcomes of the model with the findings by Berkowitz / White [2004]. The model, in particular, also generates non-monotonic increases in the average interest rate across exemption levels. Another interesting result is that the optimal exemption level is the same in the current model with endogenous risk-taking as in a model with only exogenous risk-taking.

All figures in this section have the exemption level on the x-axis. Figure 3.6 shows several aspects of the effect of changing the exemption level. Panel A shows that entrepreneurship is declining almost monotonically with the exemption level. There are several reasons for this. First, as can be seen in panel B, the fraction of borrowers declines more that the fraction of entrepreneurs. This means that more and more entrepreneurs have to self-finance. There are two reasons for this. Firstly, credit rationing gets more severe. This is shown in panel D which shows the wealth level of the marginal entrepreneur who is just indifferent between

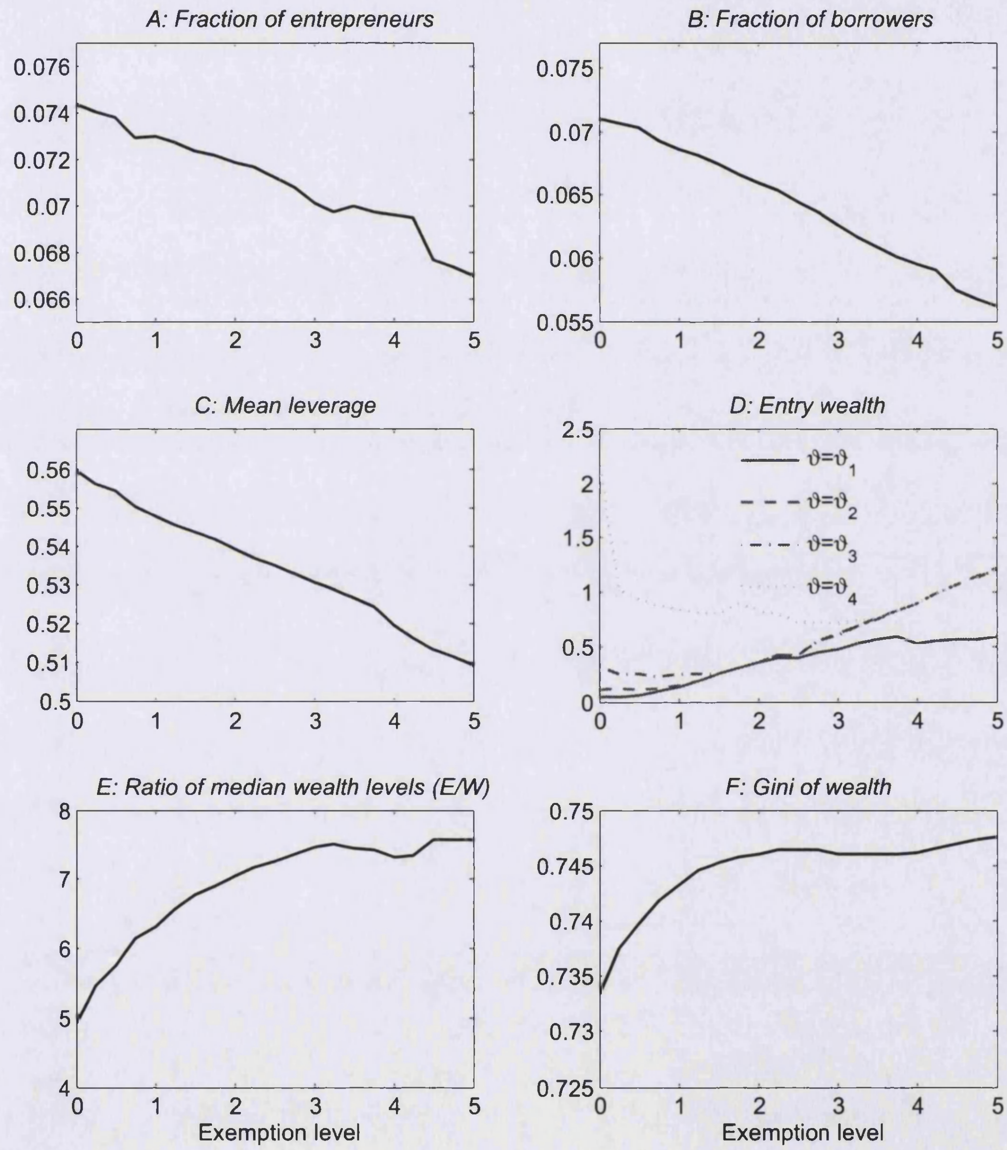

Figure 3.6: Entrepreneurship, borrowing, and wealth for different exemption levels

remaining a worker or entering entrepreneurship. The higher is an agent's labor productivity, the higher is his outside option. Therefore, the minimum wealth level at which agents become entrepreneurs depends positively on their labor productivity. This is the extensive margin discussed above. The only case where the extensive margin is positive is for agents with very high labor productivity in the case of low exemption levels. The positive effect of an increase in the exemption level leads these agents to borrow more because of the higher insurance provided by the higher exemption level. Secondly, as the exemption level is increased further more agents default and therefore do not have access to credit markets for some time.

Panel C shows that the mean leverage ratio of continuing entrepreneurs is declining monotonically. Thus, there is a negative intensive margin. These negative effects of higher exemption levels on availability and size of credit are in line with the evidence provided by Berkowitz / White [2004]. The negative effect of higher exemption levels hits relatively poor agents hardest because it is poor agents who are rationed out of the market and who subsequently become workers. The consequence is that the remaining entrepreneurs are even richer, as can be seen in panel E. Inequality, as measured by the Gini coefficient of the wealth distribution, also increases significantly, see panel F.

Figure 3.7 shows the average interest rate for different exemption levels. The solid line shows the average interest rate for the benchmark model with endogenous risk-taking. The broken line shows the average interest rate for a version of the model in which there is only exogenous risk, i.e. when entrepreneurs can not choose σ^j_χ.[21]

Berkowitz / White [2004] find that the interest rate is increasing in the exemption level, except for high exemption levels where interest rates actually fall in case of a further increase in the exemption level. The model reproduces this effect. However, the model also gets a non-monotonicity around $X = 1.8$ which is not in the data. There are two reasons for these instances of non-monotonicity. First, some poor agents do not become entrepreneurs anymore because credit rationing becomes so severe that the scale of their potential firms is so small that they are better off becoming workers. And, ceteris paribus, poor agents face the highest interest rates. But, countering this selection effect is the fact that the default incentive of remaining entrepreneurs increases with the exemption level. Therefore, interest rates increase with the exemption level, see figure 3.5.

Second, increases in the exemption level can lead agents to take on less risk. Since they take on less risk, their incentive to default gets lower and consequently, they have to pay lower interest rates. As explained in the previous section, this happens because banks ration access to credit by so much that the entrepreneurs are not well insured against the bad outcome. Therefore, they choose low levels of risk.

Since this effect occurs at different levels of wealth depending on the agents' labor productiv-

[21] I recalibrated the model with only exogenous risk to the identical targets as the benchmark model. The parameters I obtained for this model are very similar to the parameters of the benchmark model. The reason for this is that not many entrepreneurs choose (or are tempted to choose) high levels of risk. Figure 3.7 then shows the result of the policy experiment using the parameters for this new calibration.

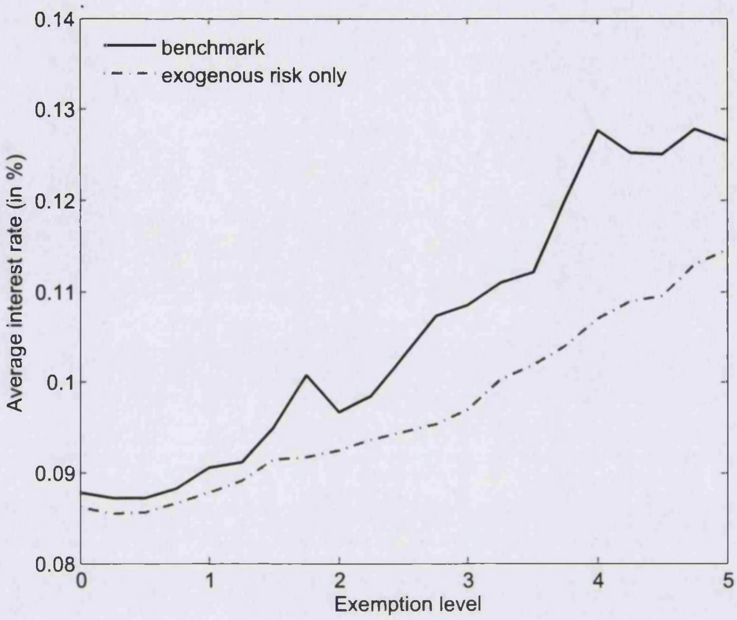

Figure 3.7: Average interest rates for different exemption levels

ity, it is difficult to predict the exact value of the exemption level where this counter-intuitive effect will occur. As I have shown in figure 3.5, only some agents will take on less risk. Other agents will take on more. The average effect will depend on the distribution of the population over the state space.

The broken line in figure 3.7 shows the average interest rate in a model without endogenous risk-taking, i.e. in a model with exogenous risk only. In this variant of the model the average interest rate increases monotonically. The selection effect, i.e. the effect that some poor agents who previously faced the highest interest rates are rationed out of the market is still existent in this version of the model. But, by definition, the endogenous risk-taking channel is not present. The fact that the average interest rate in this variant of the model is monotonically increasing supports the hypothesis that it is the endogenous risk-taking channel that explains the non-monotonicity of the interest rate.

Figure 3.8 shows the effect of different exemption levels on welfare for both versions of the model. Welfare is expressed in terms of the change in annual consumption that would make agents indifferent between living in the benchmark economy which features endogenous risk-taking and has an exemption level of $X = 0.98$ or in an economy with a different exemption level.[22] Welfare in the economy with endogenous risk-taking is uniformly higher than welfare in the model with exogenous risk-taking. This is not obvious ex ante since the possibility to take on more risk acts as a constraint on lending by banks. If endogenous risk-taking is possible, banks have to ensure that an agent who is applying for a certain

[22] Note that welfare of the model with exogenous risk-taking only is also expressed relative to the benchmark exemption level $X = 0.98$ in the model with endogenous risk-taking.

amount of credit under the assumption that he will choose a low risk project will behave accordingly. If that agent found it optimal to choose more risk, the contract would not be incentive compatible. And therefore the contract would not be viable anymore. But, since markets are still far from being complete in this model, most agents do not want to take on more risk. Thus, the additional incentive compatibility constraint is slack for most agents.

Welfare in the model with endogenous risk-taking is higher because it expands the choice set of agents while the additional constraint it imposes does not matter. As shown in the previous section, it is the marginal entrepreneurs, i.e. relatively poor agents, who engage in additional risk-taking. This is because these agents are the ones who are closest to the non-convexity in the continuation value function and therefore they want to take on more risk. The possibility to take on more risk leads poorer agents to enter entrepreneurship earlier in the model with endogenous risk-taking compared to a model with only exogenous risk-taking. Thus, the wealth level of those who enter entrepreneurship in panel D of figure 3.6 is uniformly higher in the model with exogenous risk-taking.

This is similar to Hopenhayn / Vereshchagina [2009]. In their model entrepreneurs choose higher risk due to the non-concavity of the continuation value function that comes from the occupational choice problem. There is no borrowing and therefore no default in their model. But the non-concavity of the value function in their model also matters mostly for agents who are close to the entry threshold, i.e. who are relatively poor. Richer agents have sufficient wealth to remain entrepreneurs even when they are hit by a bad shock. This implies that their continuation value function is concave and therefore rich entrepreneurs do not engage in additional risk-taking. The possibility to take on more risk, i.e. the randomization device of choosing a higher level of risk increases the welfare of those agents who are close to the non-concavity.

The second surprising result is that the two welfare lines are almost parallel to each other. In particular, the optimal exemption level is almost the same in both models. This implies that models that do not incorporate endogenous risk-taking when investigating the optimal exemption level, i.e. as has been done in the previous literature, might give unbiased results. But, of course, the current model is just one example and it remains to be seen whether this result also holds for other environments.

The optimal exemption level with endogenous risk-taking would be twice as high as the current one. Increasing the exemption level to $X = 2$ would yield welfare gains of 0.45% of annual consumption, a small but not negligible increase in welfare.

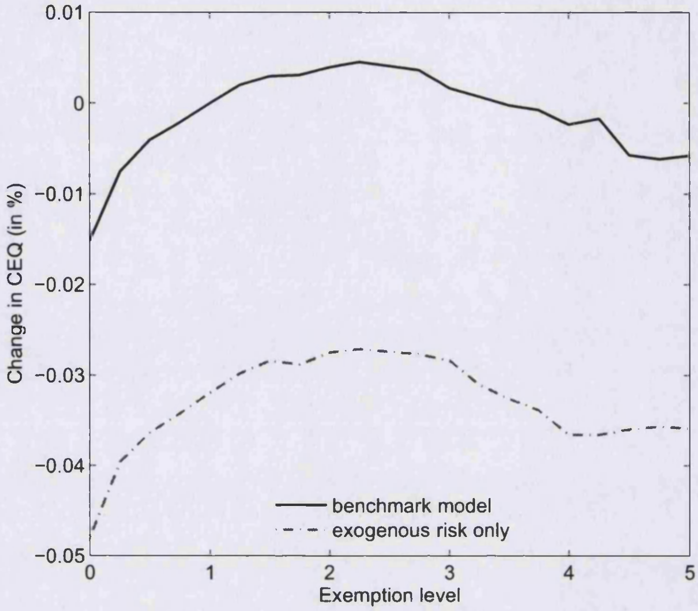

Figure 3.8: Welfare for different exemption levels

3.5 Robustness checks

In this section, I briefly discuss slight variations of the model. First, I show how the results change if the upper limit of risk-taking is increased from 50% of the exogenous risk level to 100% of the exogenous risk level. This change does not affect the results much. Then, I change the risk aversion coefficient to 1.5 and change the elasticity of intertemporal substitution to 0.5.

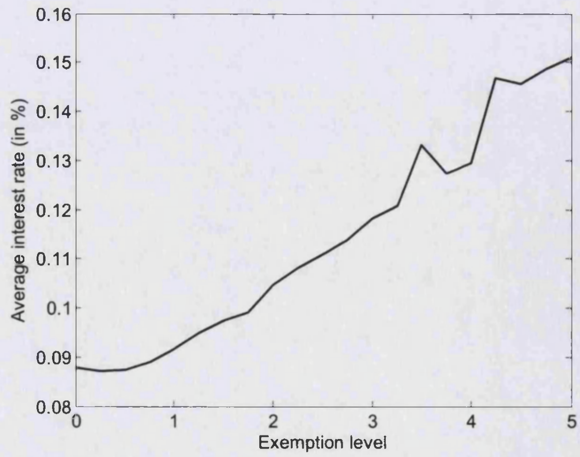

Figure 3.9: Average interest rates for higher risk levels

3.5.1 Higher upper limit of risk-taking

In the main section, entrepreneurs could choose among risk levels ranging from a minimum amount of (exogenous) risk,$\sigma_\chi^j = \sigma_\chi^1$ and a maximum amount that was set at 50% above the minimum level, $\sigma_\chi^{nj} = 1.5\sigma_\chi^1$. In this robustness check this maximum amount is doubled to 100%.

The policy functions are very similar to the benchmark case. Those agents who are credit rationed and still remain entrepreneurs choose high levels of risk. Therefore the interest rate is non-monotonic in the exemption level. Figure 3.9 shows that the average interest rate increases with the exemption level. But, as in the benchmark model, this increase is non-monotonic. Comparing figure 3.9 with figure 3.7, one can see that the increase in the interest rates is higher in the former case. This is intuitive since now the agents take on more risk and therefore are more likely to default. Other results are similar to the benchmark case. Welfare, for example, follows an inverted U and peaks around $X = 2$.

3.5.2 Lower risk aversion

Risk aversion in the benchmark model was set to 3. In this section, I lower it to 1.5 while keeping the elasticity of intertemporal substitution constant. Figure 3.10 shows that in this case the non-monotonicity at high exemption level disappears and the one at intermediate levels is lessened. The policy functions reveal that only those agents with the lowest labor productivity show a decrease in risk-taking when the exemption level is increased further. One reason could be that the value function has less curvature with lower risk aversion.

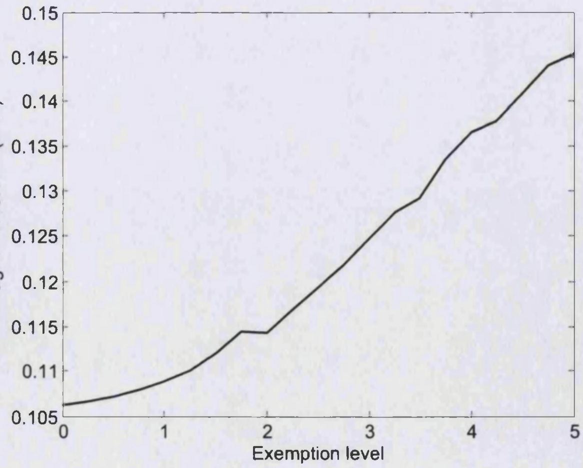

Figure 3.10: Average interest rates for lower risk aversion

Welfare (not shown), again, follows an inverted U, albeit the effect is more dramatic in this case. And, it peaks at a lower exemption level. This is consistent with Mankart / Rodano [2009] who also find this effect. The reason is that, the less risk averse the agents are, the less they value the insurance provided by the exemption level. Therefore, they demand less insurance so that the optimal exemption level is lower.

3.5.3 Lower elasticity of intertemporal substitution

The effects of lowering the elasticity of intertemporal substitution from 0.8 to 0.5, while keeping the coefficient of relative risk aversion constant, is shown in figure 3.11. Again, the non-monotonicity at high exemption levels disappears while the one at intermediate exemption levels gets lessened. If the elasticity of intertemporal substitution is increased to 1.1 (not shown), the non-monotonicity is the same as in the benchmark case.

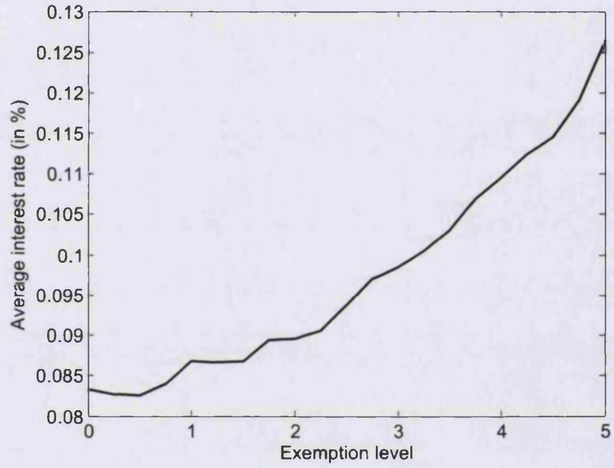

Figure 3.11: Average interest rates for lower elasticity of intertemporal substitution

Thus, the non-standard result that an increase in the exemption level can lead to less risk-taking by entrepreneurs is fragile when it comes to average interest rates. However, it is always present in some entrepreneurs' policy functions. Welfare is not affected much by changes in the elasticity of intertemporal substitution. The optimal exemption level is increasing in the elasticity of intertemporal substitution. The quantitative effects are small however. For $\psi = 0.5$, the optimal exemption level is 1.6. The reason for this result is that a higher exemption level leads to higher interest rates. The value agents attach to this negative effect depends negatively on the agents' elasticity of intertemporal substitution.

3.6 Conclusion

This is the first paper that investigates the effects of different exemption levels in a model with entrepreneurs when these choose the riskiness of their projects. The model features tow potential sources of non-convexity: occupational choice and the default option. I show that the non-convexity due to occupational choice is not present because the uncertainty at the time of the default decision convexifies the continuation value function sufficiently. The non-convexity due to the default option is present and leads some agents to choose risky projects.

An increase in the exemption level leads some agents, ceteris paribus, to take on more risk at first. As the exemption level is increased further, credit rationing becomes sever for some agents so that they run only very small firms and therefore will be left with only very few resources in bad states of the world. This leads these agents to choose the least risky projects. I have shown that while the average interest rate is increasing in the exemption level most of the time, occasionally it is decreasing.

In order to investigate further whether the mechanism outlined in my paper is behind the non-monotonicity in the average interest rate, it would be useful to analyze firm level data as in Hopenhayn / Vereshchagina [2009]. But, due to confidentiality issues, these data are not publicly available. Also, it would be interesting to see whether the effect on risk-taking is also present in models of European economies where the main difference is the length of the garnishment period.

Chapter 4

The optimal Chapter 7 exemption level in life-cycle model with asset portfolios

4.1 Introduction

The steep increase in consumer bankruptcy filings in the 1990ies and early 2000s led to an increased interest in the workings of personal bankruptcy laws.[1] On the one hand there has been a public debate leading to a reform of the US bankruptcy law. On the other hand there has been a growing interest among economists in models that are able to explain observed behavior and that can be used to evaluate different bankruptcy policies. I contribute to this debate by examining the effects of different wealth exemption levels on economic outcomes and welfare in a life-cycle model.

In order to investigate the effects of changing the exemption level, I use a heterogenous agent life-cycle model. In addition to facing uncertainty over their labor income, agents also face wealth shocks that stem from unexpected changes in family composition or from unexpected medical expenses. The latter are an important reasons for bankruptcies [Sullivan et al. 2000] in the US. The model features incomplete financial markets. But, I allow for two assets: unsecured debt and savings. The possibility to default introduces some contingency and therefore moves the financial system closer to complete markets.[2]

The default option gives consumers insurance against the economic consequences of the aforementioned shocks to their income or wealth. A more generous bankruptcy system, respectively a higher exemption level to be precise, will provide more of this insurance. This comes however at the cost of deteriorated credit market conditions, higher interest rates and possibly complete credit rationing.

In addition, the possibility to shield some assets in the case of default increases the incentive

[1] I am grateful to Alex Michaelides, Giacomo Rodano and Stephan Mankart for very helpful comments.

[2] For a theoretical evaluation of that trade-off see Dubey et al. [2005]

to save of those agents who might default. Suppose that the wealth exemption level is zero and an agent knows that he will default if he is hit by a bad shock. This agent will lose his savings when he defaults, i.e. the realized rate of return in this state of the world is -1. Therefore, he is unlikely to save in the first place. If, however, the exemption level is positive, the agent is more likely to save since the realized rate of return in the bad states now is the normal risk-free interest rate, at least for savings up to the exemption level.

The exemption level differs widely across US states, ranging from a few thousand US dollars, for example, in Maryland to an unlimited amount in, for example, Florida. Higher exemption levels increase the incentive to default. Therefore one would expect to see a positive relationship between the exemption level and default rates, unless credit rationing becomes so severe that a significant number of households are excluded from borrowing altogether. A strong positive relationship is predicted by previous papers that investigated the optimal exemption level of the Chapter 7 consumer bankruptcy code, see for example Athreya [2006].

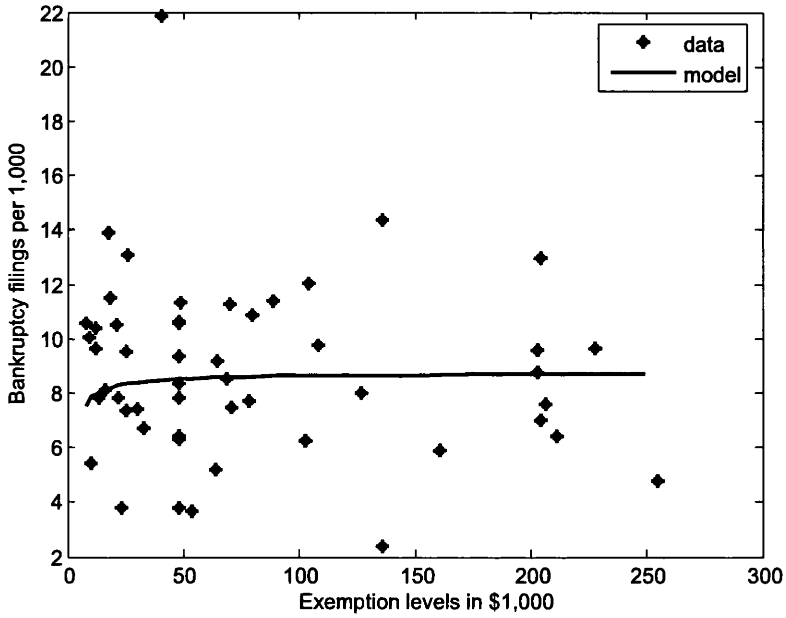

Figure 4.1: Bankruptcy rates for different exemption levels

Figure 4.1 however shows that there is no significant relationship between the exemption level and bankruptcy filings. [3] Even though my model, which is built on Livshits et al. [2007b], also has a positive relationship between the exemption level and bankruptcy rates, the effect is very small, in particular for exemption levels above a relatively modest value of $20,000. Thus, the model is consistent with the data. Moreover, my quantitative evaluations show that the welfare difference between different exemption levels are rather small, less than 0.1% of annual consumption. This might explain why the differences in the exemption level across US states have persisted for such a long time. If the welfare differences are small, voters are almost indifferent between different exemption levels. Thus, if the political process

[3] The data are are the average per capita filing rates between 1995-2003.

is rational, there is no pressure to change the law.

Almost all variables of interest follow a similar pattern as the bankruptcy rate. There are significant changes when the exemption is increased from zero to some small positive level. However, there are almost no further effects when the exemption level is increased further. The main reason for this is that almost no household is affected by an even higher exemption level. Households who default when they are hit by bad shocks have assets that are below the exemption level. And households who have assets above the exemption level do not default even for intermediate exemption levels. Therefore, if the exemption level is increased further, asset holdings and consequently none of the results change by much.

The quantitative literature on consumer bankruptcy has increased substantially since Athreya's original paper in 2002. He found that eliminating the default option would be welfare increasing. Chatterjee et al. [2007] show that the recent tightening of the law in the US implies large welfare gains. Livshits et al. [2007b] compare the US system under which future earnings are exempt after having declared bankruptcy with a European type of system under which future earnings are garnished to repay creditors. They find that the welfare differences between the systems depends on the persistence and variance of the shocks. Mateos-Planas / Seccia [2006] compare different credit market exclusion periods in a model with endogenous borrowing limits stemming from the default option with a model with exogenous borrowing limits. All of these models are one goods models. Hintermaier / Koeniger [2008], however, examine the reasons for the increase in consumer bankruptcies in a model with durable and nondurable goods.

My paper is closest to Livshits et al. [2007b] in that I follow their set up with labor income uncertainty and wealth shocks. Livshits et al. [2007b], however, have only one asset in their model and ignore the exemption level all together. This means that they set it implicitly to zero. However, as my result indicate, this omission makes their welfare results slightly spurious since a positive exemption level has a particularly positive effect for very low exemption levels. My modeling of the asset market is close to Li / Sarte [2006] who also have both, unsecured debt and savings. Their model, however, uses an infinite horizon framework with the only uncertainty coming from changes in labor productivity. This is also the case in Athreya [2006] who investigates the optimal exemption level in a model with secured and unsecured debt.

The paper is structured as follows. Section 4.2 explains key features of the US bankruptcy code. Section 4.3 shows the importance of including the exemption level in a model the features wealth shocks. In particular, I focus on the impact on savings. Section 4.4 lays out the model and the computational algorithm. Section 4.5 shows the benchmark calibration and discusses the main mechanisms of the model. Section 4.6 shows the main policy experiment. I examine the impact of changing the exemption level on default rates, default reasons, borrowing and savings decisions and welfare. Section 4.7 concludes.

4.2 Consumer bankruptcy

Personal bankruptcy law in the US consists of two different procedures: Chapter 7 and Chapter 13. Under Chapter 7, all unsecured debt is discharged immediately, and future earnings cannot be garnished. This is why Chapter 7 is known as providing a "fresh start". At the same time, a person filing for bankruptcy has to surrender all wealth in excess of an exemption level. The exemption level varies across US states, ranging from \$8,000 in Maryland to unlimited for housing wealth in some states, for example Florida. A person can file for Chapter 7 only once every six years.

Under Chapter 13 agents can keep their wealth, debt is not discharged immediately, and future earnings are garnished. A person can file for Chapter 13 every six moths. In the model, I follow Livshits et al. [2007b] and let agents file first for Chapter 7 and if they want to default again soon afterwards they have to file for Chapter 13.

4.3 An illustration of the importance of wealth exemptions

In this section, I show a simple example that demonstrates that wealth exemption levels are particularly important in the presence of wealth shocks, stemming from, for example, unexpected medical expenditures. The reason for this is that the option to default can encourage savings of, particularly, the poor.

Suppose an agent lives for two periods. His initial wealth is $a_0 = 1$ For simplicity, I assume that the agent has no income, that the risk free interest rate $r^f = 0.0$ and that his discount factor $\beta = 1.0$.[4].

In the second period the agent might face a health shock that requires him to spend e in order to survive. This health shocks acts as a wealth shock and occurs with probability p. However, the agent can default on these expenses. If he defaults, he will lose all his assets up to the exemption level X. For simplicity, I set the value of the health shock $e = 1$. This ensures that the agent will always default on his expense debt.

The agent maximizes lifetime utility

$$\begin{aligned}\max_s U &= \log(c_1) + \mathbb{E}\log(c_2) \\ &= \log(a_0 - s) + (1-p)\log(s) + p\log(\min[s, X])\end{aligned}$$

The solution to this problem will depend on the exemption level X in a non-trivial way. There are three possible cases. First, the agent might choose a level of savings that is higher

[4] None of the results of this section hinges on any of these assumptions. Their sole purpose is to make the analysis more transparent.

than the exemption level. In this case, the solution is

$$s = \frac{(1-p)\,a0}{2-p} = \bar{X}_L$$

This situation is likely for low exemption levels. Note, that this implicitly defines a value for the exemption level until which this situation can occur, call this $\bar{X}_L$ If the exemption level is higher, the second case might occur and the optimal saving will be equal to the exemption level. The last case is when the exemption is so high that it becomes non-binding and the agent implements his first best level of savings

$$s = s^{FB} = \frac{1}{2}a_0 = \bar{X}_H$$

Note again, that this implicitly defines the critical exemption level $\bar{X}_H$ until which this situation can occur. This yields the following result.

Result 4 *If the expense shock is so high that the agent cannot repay it, his savings are (weakly) increasing in the exemption level.*

$$s^* = \begin{cases} \frac{(1-p)a0}{2-p} & \textit{for} \quad X \leq \frac{(1-p)a0}{2-p} \\ X & \textit{for} \quad \frac{(1-p)a0}{2-p} < X < \frac{1}{2}a_0 \\ \frac{1}{2}a_0 & \textit{for} \quad \frac{1}{2}a_0 \leq X \end{cases}$$

Formal details are relegated to the appendix. In the first and in the last case, an increase in the exemption level has no effect. In the second case, however, savings increase one-for-one with the exemption level.

Of course, a higher exemption level can induce people to default even if they could afford to repay their expense debts. If they plan to do so, their savings will be lower than in a situation in which they had to self-insure. This is the well-understood negative effect of generous exemption levels on savings. It is important to note that the positive effect is particularly relevant for poor agents, whereas the negative effect is relevant for richer agents. Thus, it is important to use a heterogenous agent framework in order to investigate the aggregate effects on savings and welfare. In the next section, I will present a heterogenous agent model in which agents face uncertainty with regards to income and expense (wealth) shocks. This is not the case in Li / Sarte [2006] and Athreya [2006]. They both infinite horizon models without expense shocks. On the other hand, setting the exemption level to zero as is implicitly done by Livshits et al. [2007b] will probably bias welfare results since positive exemptions are particularly valuable if agents face wealth shocks.

4.4 The model

My model framework is a partial equilibrium overlapping generations model based on the model by Livshits et al. [2007b]. Each household lives for J periods. Each generation consists of households of measure 1. All households are born equally without any wealth. There is no bequest motive. Therefore each household maximizes its own expected lifetime utility. There is no disutility of labor. Households face uncertainty with respect to their future labor productivity and with respect to small and large wealth shocks, reflecting family risks and health risks.

Financial markets are incomplete, in particular there are no insurance markets in which households could insure themselves against their labor income, family (divorce, unwanted pregnancy) or health risks. There are two assets in the economy. First, households can save any non-negative amount by buying a risk-free bond. This bond pays an exogenous interest rate r^f.

Second, they can also borrow non-negative amounts from financial intermediaries. This borrowing is done through notionally non-contingent debt contracts on which the household, however, can default. This default option makes these bonds partially contingent. I abstract from any issues of informational asymmetries. At the point of signing the debt contract, financial intermediaries have the same information set as the households themselves. Therefore all debt contracts are household specific in that financial intermediaries price these debts according to the characteristics of each specific household.

4.4.1 Households

Preferences

Households live for J years. For simplicity I abstract from labor-leisure choice.[5] All agents supply their unit of labor inelastically, i.e. there is no disutility of labor. Households maximize their discounted expected utility of consumption. However, in order to take varying household sizes into account, household size is expressed in terms of equivalence scale units n_j. Felicity is standard, non-decreasing and concave

$$U = \sum_{j=1}^{J} \beta^{j-1} u\left(\frac{c_j}{n_j}\right).$$

[5] I ignore labor-leisure choice mainly because the current model is already quite complicated and not because it is not interesting (see on that matter for example Li / Sarte [2006].)

Productivity

Labor productivity of household i at age j is the product of three components: an age specific component e_j, a household specific persistent component z_j^i, and a household specific component transitory component η_j^i

$$y_j^i = e_j z_j^i \eta_j^i.$$

The age specific component e_j is chosen to reflect life cycle income patterns that are common across households. The household specific components reflect uncertainties over the life cycle. For example, Storesletten et al. [2004] estimate an AR(1) for log earnings of the following form

$$\begin{aligned} \ln\left(y_j^i\right) &= \ln\left(z_j^i\right) + \ln\left(\eta_j^i\right) + \ln g\left(x_j^i\right) \\ \ln\left(z_j^i\right) &= \rho \ln\left(z_j^i\right) + \varepsilon_j^i \end{aligned}$$

where $g(\cdot)$ reflects the deterministic component of earnings. The persistent component z_j^i follows an AR(1) process with a very high autocorrelation. Storesletten et al. [2004] estimate it to be 0.99. The variance of the transitory shock is about six times as high a the variance of the persistent shock. As is standard in the literature, I will discretize the income process by using a Markov chain.

Wealth shocks

In addition to income uncertainty, households also face idiosyncratic wealth shocks. These wealth shocks represent expenditures that have to be incurred due to, for example, a divorce or some necessary medical treatment. It is important to note that these expenditures do not yield any utility, therefore they simply reduce the wealth of the household. If the household does not hold sufficient wealth he will have to default on these expenditures. As in Livshits et al. [2007b], I assume that these shocks are i.i.d. and uncorrelated to income. It is important to note that these expenditures are due to third parties, e.g. hospitals in the case of a health shock. Thus if the agent files for bankruptcy, and he does not repay the expenditure shock, it is the hospital that loses money and not that bank. The bank only loses the amount of unsecured credit. [6]

[6] While this assumption is plausible for health shocks, it is less plausible for e.g. the expenditures for a divorce since households have to pay these costs themselves. One way to model this would be to model the debt contract as a credit card contract with a pre-specified credit line (credit limit) that the household can draw on in case he faces an expenditure shock.

4.4.2 Credit market

I assume perfect competition (free entry) in the credit market. Therefore, banks must make zero expected profit on each contract. The opportunity cost of lending is the safe rate of return on capital which is taken as exogenous. I assume that financial intermediation incurs real resource costs.

Households, who have not defaulted in the past, can hold two types of assets or any one of them: savings a and unsecured debt d. Savings earns the household rate of return r^F on his savings. Unsecured credit requires transaction costs τ^u that reflects the higher information costs that banks incur in the data when producing unsecured debt.

Furthermore, I abstract from information asymmetries in the credit market.[7] Each bank knows the borrower's age j and his the persistent component of his labor productivity z^i.[8] Therefore, by anticipating the behavior of the borrower, the banks are able to calculate the probability of default and how much they will get in the case of default. Perfect competition implies that they set the interest rate, $r(j, z, d, \gamma, X)$, such that they expect to break even. This interest rate depends on the exemption level X because it affects the incentives to default and the amount the bank recovers in this event. The banks offer a menu of one period debt contracts which consist of an amount lent d and a corresponding interest rate $r(j, z, d, \gamma, X)$ to each agent (j, z).

Households who have just defaulted are excluded from borrowing. However they can still save.

4.4.3 Timing

The sequence of events is shown in figure 4.2. A household of age j brings forward from last period: a certain credit record S, a value for the persistent component of labor productivity z_{t-1}, debt d and savings a. At the beginning of the period the expenditure shock k and his labor productivity z, η is realized. Since the household can default on the expenditure shock, this expenditure shock is simply added to the household's debt holdings. All households who carry some debt, either because they have borrowed in the previous period and (or) they have been hit by an expenditure shock then decide whether to repay or whether to default. The credit record of households who had not defaulted in the previous period and who repay also this period remains clean. The credit record of households who default this period is reflects their default. A household who had defaulted in the previous period will have no debt $d = 0$. If this household is however hit by an expense shock, it might default again. This behavior will also be reflected by the credit record.

All households with a clean credit record can borrow d in the unsecured credit market.

[7] For an analysis of bankruptcy under asymmetric information see Athreya et al. [2007]

[8] It is immaterial whether the bank also knows how much the borrower will save. The bank can always anticipate the decision of the borrower.

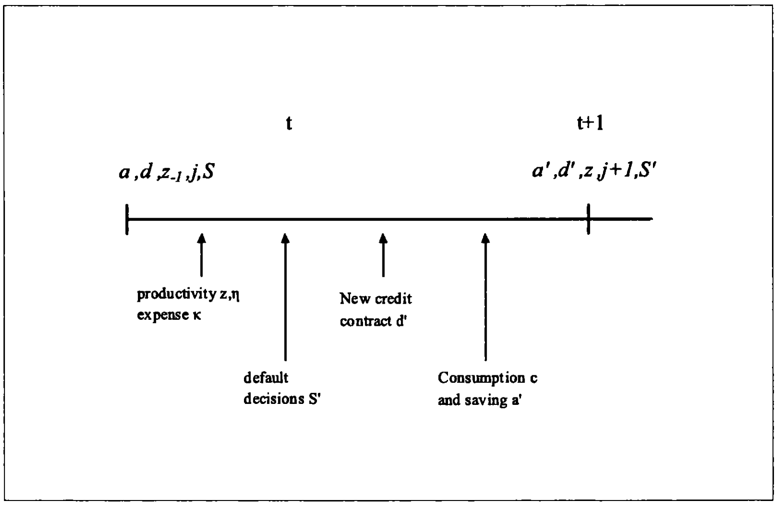

Figure 4.2: Timing

At the end of the period, each household can decide how much to save a' and how much to consume c.

4.4.4 The Household's problems

As usual, the household's problem is defined recursively. In order to describe the problem three value functions are needed. V^R is the value of repaying the debt, V^D is the value of defaulting the first time under Chapter 7, and V^{DD} is the value of defaulting again after the household has already defaulted in the previous period. This last value function is needed since a household can default under Chapter 7 only once every six years.

An unconstrained agent of age j with savings a, current productivity z, η, expense shock realization κ, and debt d has to decide whether to repay or whether to default.

The value of repaying is given by

$$
\begin{aligned}
V_j^R(a,d,z,\eta,\kappa) &= \max_{c,a',d'} \left\{ \begin{array}{c} u\left(\frac{c}{n_j}\right) + \\ \beta \mathbb{E} \max \left[V_{j+1}^R(a',d',z',\eta',\kappa'), V_{j+1}^D(a',z',\eta') \right] \end{array} \right\} \\
\text{s.t.} \quad c + d + \frac{a'}{1+r^s} + \kappa &\leq e_j z \eta + a + \frac{d'}{1+r(j,z,d',a',X)}, \\
a' &\geq 0, \qquad d' \geq 0.
\end{aligned}
$$

where savings a' and new debt d' have to be non-negative. Since the agent repays, he will be unconstrained tomorrow and therefore has the option to default tomorrow, i.e. he can choose the maximum of defaulting or repaying. If the agent's debt repayment and expenditures on the expense shock exceed his income and potential new borrowing, the constraint set is empty, i.e. consumption would have to be negative. In this case the value function is set to negative infinity and the agent will have to default.

The value of defaulting is given by

$$\begin{aligned} V_j^D(a,z,\eta) &= \max_{c,a'} \left\{ \begin{array}{c} u\left(\frac{c}{n_j}\right) - \Psi + \\ \beta \mathbb{E} \max \left[V_{j+1}^R(a',0,z',\eta',\kappa'), V_{j+1}^{DD}(a,z',\eta',\kappa') \right] \end{array} \right\} \\ \text{s.t. } c + \frac{a'}{1+r^s} &\leq (1-\gamma) e_j z \eta + \min[a, X] \\ a' &\geq 0. \end{aligned}$$

Since the household defaults on all unsecured debt d and all expenditures κ, their values play no role here. However if the agent defaults, he can keep assets only up to the exemption level X. In addition part of his labor income will be garnished. Event though a household who defaults cannot borrow in the current period, the household can save. This is in contrast to Livshits et al. [2007b] who do not allow defaulters to save. In their paper a household is in financial autarky after a default. This assumption has been used by other authors as well because it simplifies the analysis. However, this financial autarky assumption clearly overstates the punishment from a default since there is no evidence that households who have defaulted in the past are precluded from saving. Ψ is a utility cost of defaulting and reflects both pecuniary costs and non-pecuniary costs. The pecuniary costs, for example court fees and lawyer fees, have been estimated to exceed \$1,000. In addition Ψ reflects the cost of the stigma of having had to declare bankruptcy. I use this parameter in the calibration to tie down the default rate. If the continuation value of defaulting exceeds the value of repaying, i.e. $V_j^D(a,z,\eta) > V_j^R(a,d,z,\eta,\kappa)$, the household will default. I denote this decision by $I_j^D(a,d,z,\eta,\kappa)$.

In the next period, the household will have no debt but he might be hit by an expense shock. If he is unable to repay the expense shock, he will have to default again. In that case, I assume that he has to surrender all his wealth and part of his income will be garnished. Therefore the value of not repaying expense debt after having already defaulted is

$$\begin{aligned} V_j^{DD}(a,z,\eta,\kappa) &= u\left(\frac{c}{n_j}\right) - \Psi + \beta \mathbb{E} \max[\left[V_{j+1}^R(0,d',z',\eta',\kappa'), V_{j+1}^D(0,z',\eta') \right] \\ \text{where } c &= (1-\gamma) e_j z \eta, \qquad d' = (\kappa - a - \gamma e_j z \eta)(1+\bar{r}) \end{aligned}$$

where debt is rolled over to the next period at an exogenous interest rate $\bar{r}$. This agent has no choice problem. Similarly to the agent who has not defaulted in the past, if the continuation value from defaulting exceeds the value of paying off expense debt, i.e. $V_j^{DD}(a,z,\eta,\kappa) > V_{j+1}^R(a',0,z',\eta',\kappa')$, the agent will default a for second time. I denote this decision by $I_j^{DD}(a,z,\eta,\kappa)$.

4.4.5 The zero profit condition of the banks

There is perfect competition (free entry) in the credit market. Banks make zero profit on each savings contract and on each unsecured loan contract. All agents, except those who default twice, can save at the risk free interest rate r^f.

Since I abstract from asymmetric information, banks observe the household fully. This means they know the household's age j, cash on hand $e_j z\eta + a - d - \kappa$ and persistent component of productivity z. In addition they know how much the household is going to save, i.e. they know $\frac{a'}{1+r^s}$. Given a savings level $\frac{a'}{1+r^s}$ and productivity level z the bank knows in which future states of the world the household will be willing to repay and in which the household will default Therefore, for each amount of unsecured credit d', the bank can calculate the default probability $\pi(d', a', z, j, \gamma, X)$ and the amount the bank recovers in each default state. This will depend on the exemption level X , the fraction of labor income that can be garnished γ[9] and on the amount the household owes in expense debt κ'. I assume that all assets above the exemption level X and the garnished labor income are split proportionally in the repayment of the bank and expense debt. So, the bank receives a fraction $\frac{d'}{d'+\kappa'}$ of labor income $e_j z\eta$ and of the savings above the exemption level, if these savings exceed the exemption level. In addition, the credit production process incurs real costs τ^u which are assumed to be proportional to the loan size. The zero profit condition is given by

$$\begin{aligned}\left(1+r^f+\tau^u\right)d' = \\ &(1-\pi(d',a',z,j,\gamma,X))\,(1+r(d',a',z,j,X))\,d' \\ &+\pi(d',a',z,j,\gamma,X)\,\mathbb{E}\left(\frac{d'}{d'+\kappa'}\left(\gamma e_j z\eta+\max\left[a'-X,0\right]\right)\right).\end{aligned}$$

4.4.6 Equilibrium

Let $=(a, d, z, j, S)$ be a state vector for an individual, where a denotes savings, d unsecured borrowing, z the persistent component of labor productivity, j, the age of the household, and S the credit status. Let r^f be the exogenous interest rate, τ^u the resource costs of producing unsecured credit, γ the proportional garnishment and X the wealth exemption level. A competitive recursive equilibrium is then given by:

- value functions V_j^R, V_j^D, V_j^{DD} that solve the households problem and lead to optimal policy functions c, d', a', I^D, I^{DD},
- an interest rate function that satisfies the zero profit condition,

[9] If there was no garnishment, households would not repay any fraction of the loan. This is not according to the US bankruptcy law which requires bankruptcy filers to have acted in good faith and therefore denies filing for bankruptcy immediately after having taken out a credit.

- and correct default probabilities $\pi(d', a', z, j, \gamma, X) = E\left(I_j^D(a', d', z', \eta', \kappa')\right)$

4.4.7 Computational algorithm

In this section, I present an overview of the computational algorithm. As is standard in the literature, I solve this program by backward induction. I assume that the household faces no uncertainty in the last period of his life and cannot default. I discretize the asset space, the persistent state of productivity, the temporary productivity and the expense shock.

Algorithm 5 *1. Solve the value functions for the last period T.*

2. Given the values in T, I solve the households problems in $T-1$. Since agents are not allowed to default in period T, this is simple.

3. In period $T-2$, I set up a grid of savings values a' and borrowing values d'. These two grids form a matrix.

   a) Then, I calculate the default probabilities and associated recoveries for each entry of this matrix and each level or persistent productivity by using the continuation values in period $T-1$ by looping over persistent and transitory productivity and the expense shock. These default probabilities imply an interest rate for each pair of values a', d', z.

   b) Given this array, I solve for the optimal decision of the households. Since households who have defaulted cannot borrow, calculating their value functions is standard.

4. I repeat this until the first period.

5. I simulate the model for 10 million households in order to obtain default rates, default reasons, savings and borrowing rates etc.

4.5 Calibration

In this section, I first show the parametrization. Afterwards, I describe the results and compare the model's implications to the data.

4.5.1 Parametrization

Fixed parameters

Since I want to compare my results to the results obtained by Livshits et al. [2007b], I follow their parametrization in all respects with one addition. Their model does not have the utility cost Ψ which I need to calibrate the default rate. In essence, all of the parameter values are based on empirical studies. The only exceptions being the garnishment rate γ and the utility cost Ψ which are calibrated to match the average debt to income ratio and the observed default rate.

In order to simplify computations, each model period corresponds to three years. Households are born at age 20, retire with 65 and die at age 74. This implies that a life in the model has 18 periods, where the last three periods are spent in retirement. As already mentioned, households face no uncertainty in retirement.[10] In the following, I report only annual values since these are more familiar than triennial values.

The felicity function features constant relative risk aversion

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

where σ is the coefficient of relative risk aversion and set to 2.0. The discount factor is equal to 0.94. The family size life-cycle comes from Fernandez-Villaverde / Krueger [2007] which in turn is based on the US Census data for 1990.

The interest rate on saving r^f is set to 4.0 percent. This is in line with estimates of the average return to capital in the US. The transaction cost for unsecured credit τ^u is based on the costs of producing credit card debt and is also set to 4.0 percent Evans / Schmalensee [1999].

The AR(1) income process

$$\begin{aligned}\ln\left(y_j^i\right) &= \ln\left(z_j^i\right) + \ln\left(\eta_j^i\right) + \ln g\left(x_j^i\right)\\ \ln\left(z_j^i\right) &= \rho \ln\left(z_j^i\right) + \varepsilon_j^i\end{aligned}$$

is parameterized by the following values which are based on Storesletten et al. [2004]: The autocorrelation coefficient ρ is set to 0.99. Its innovation is assumed to be normal, $\varepsilon_j^i \sim N(0, \sigma_\varepsilon)$, with variance $\sigma_\varepsilon^2 = 0.007$. That transitory shock η is also assumed to be normal $\eta_j^i \sim N(0, \sigma_\eta)$ with variance $\sigma_\eta^2 = 0.043$. All these annual values are mapped into triennial values and then discretized into a Markov process with five states . The transition matrix $\Pi(z'|z)$ is assumed to be age-independent. The transitory shock is discretized using three states where ten percent of the population receive a positive and ten percent a negative shock.

Upon entering retirement, there are no further shocks. In order to make retirement income (social security) dependent on earnings, it has two components: first a lump-sum component

[10] This assumption is innocuous for income uncertainty since retirees receive social security benefits. It is less plausible for expense shocks. But I maintain it for computational simplicity. Since old people do not borrow much and hardly ever default in the data, this assumption is unlikely to bias the results.

Table 4.5-1: The fixed parameters

Parmeter	Symbol	Value
CRRA	σ	2
Risk free rate	r^f	4%
trasaction cost	τ^u	4%
expense shocks	κ_1, κ_2	$\$10,973, \$34,154$
probability of expense shocks	π_1, π_2	$2.369\%, 0.153\%$
Transitory states	η_1, η_2, η_3	$[0.6151, 0.9785, 1.5568]$
Transitory probabilties	$p_{\eta_1}, p_{\eta_2}, p_{\eta_3}$	$[0.1, 0.8, 0.1]$
Persistent states	z_1, z_2, z_3, z_4, z_5	$\begin{bmatrix} 0.3799 & 0.6311 & 0.8613 & 1.1754 & 1.9523 \end{bmatrix}$
Transition matrix	$\Gamma(z'\|z)$	$\begin{bmatrix} 0.8638 & 0.1351 & 0.0011 & 0.0 & 0.0 \\ 0.1351 & 0.6778 & 0.1838 & 0.0034 & 0.0 \\ 0.0011 & 0.1838 & 0.6302 & 0.1838 & 0.0011 \\ 0.0 & 0.0034 & 0.1838 & 0.6778 & 0.1351 \\ 0.0 & 0.0 & 0.0011 & 0.1351 & 0.8638 \end{bmatrix}$
Exemption level	X	$\$47,800$

equal to 35% of average earnings and then an individual specific component consisting of 30 percent of the last earning.

The expense shock κ can take three values $\{0, \kappa_1, \kappa_2\}$. The first value means no shock. The small shock, κ_1, is set to $\$10,973$ annually and has probability $\pi_1 = 2.368$ percent. Livshits et al. [2007b] aggregate three different shocks of similar size. First, a divorce shock which has probability 1.244 percent which leads to expenditures on the divorce and a loss in economies of scale. Second, an unwanted pregnancy which occurs with 0.5 percent. Lastly, medical shocks that are not too big and which affect 0.625 percent of households each year. The big shock κ_2 is purely a large medical expense shock. This is set to $\$34,154$ annually and has probability $\pi_2 = 0.153$ percent. This means that a small fraction of households are hit by very large medical expenditure shocks.[11]

As shown in fig 4.1, the wealth exemption level X. varies tremendously across states. It ranges from a few thousand dollars to an unlimited amount in some states. I set it to the population weighted median value of $\$47,800$. This value is a higher than what is mostly used in the literature (see for example Athreya [2006]). One reason for the difference is that I include all exemption levels and not only the homestead exemption. Thus while it is true that the homestead exemption in Maryland is zero, there is an exemption on personal belongings of $\$2,500$ and one on tools of trade of $\$5,000$. Moreover, since it is easy to hide some assets, I think that very low exemption levels are not plausible. A second reason is that I use a higher value for top-coding of the exemption levels in states that have an unlimited homestead exemption. However as a robustness check, I recalibrated the model to a low exemption level and the results do not change much. Table 4.5-1 summarizes all the parameters.

[11] For details on these data see Livshits et al. [2007b], and also the working paper version.

Calibrated parameters

The remaining parameters to be set are the utility cost of bankruptcy Ψ and the garnishment parameter γ. Livshits et al. [2007b] do not include the utility cost in their model. They calibrate γ in order to match the observed debt to earnings ratio of 8.4 percent. The default rate is then endogenously determined in their model. Their model does very well and manages to explain 85 percent of observed chapter 7 defaults. However, the result is not robust. My model nests their model as a special case. In particular, if I set the exemption level to zero, I am able to reproduce their results. But if I set the exemption level to a plausible one, I get way too many defaults. Therefore, I calibrate Ψ to match the observed default rate of 0.84 percent. The calibrated parameters are shown in table 4.5-2.

Table 4.5-2: The calibrated parameters

Parmeter	Symbol	Value
Garnishment	γ	34.6%
Utility cost	Ψ	0.1

As can be seen in table 4.5-3, the model matches the targeted moments very well. The bankruptcy target are all non-business related Chapter 7 bankruptcies, averaged between 1995-1999.

Table 4.5-3: The targets

Target	Data	Model
Debt to earnings ratio	8.4%	8.39%
Chapter 7 bankruptcies	0.84%	0.85%

4.5.2 Benchmark model

In this subsection, I first present some implications of the model. Then, I will discuss some policy functions. Lastly, I will show some life-cycle implications of the model.

Model implications

In order to assess the model, I present some further comparisons between the model and data in table 4.5-4. The average interest rate is similar to the average interest rate the Federal Reserve Board reports on two-year personal loans. The fraction of households with negative net wealth is too high in the model. Wolff [2007] reports that 18 percent of households have negative net wealth. However, he also shows that 27 of households have wealth of less than \$5,000.

The earnings of defaulters are about one half in the model and in the data. The amount of debt households hold at the time of filing for bankruptcy is a bit too high in the model

Table 4.5-4: Model implications

Variable	Data	Model
Average borrowing rate	11.2%	10.3%
Households with negative net worth	18%	27.5%
Relative earning of defaulters	49.1%	52.9%
Average debt to income ratio of defaulters	187%	232.5%
Recovery probability	< 5%	1.8%

compared to the data. However, the numbers from Sullivan et al. [2000] are based on a relatively small sample of bankruptcy cases. In the model, the lender almost never collects anything in case the household defaults. This is because households who default do not have savings in excess of the exemption level. This is consistent with the data. Empirically, there are very few cases in which the lenders recover anything. Thus, overall the model fits the data rather well.

Policy functions

In this subsection, I first show how the possibility to save affects the price and availability of credit. Then I show examples of policy functions of the household.

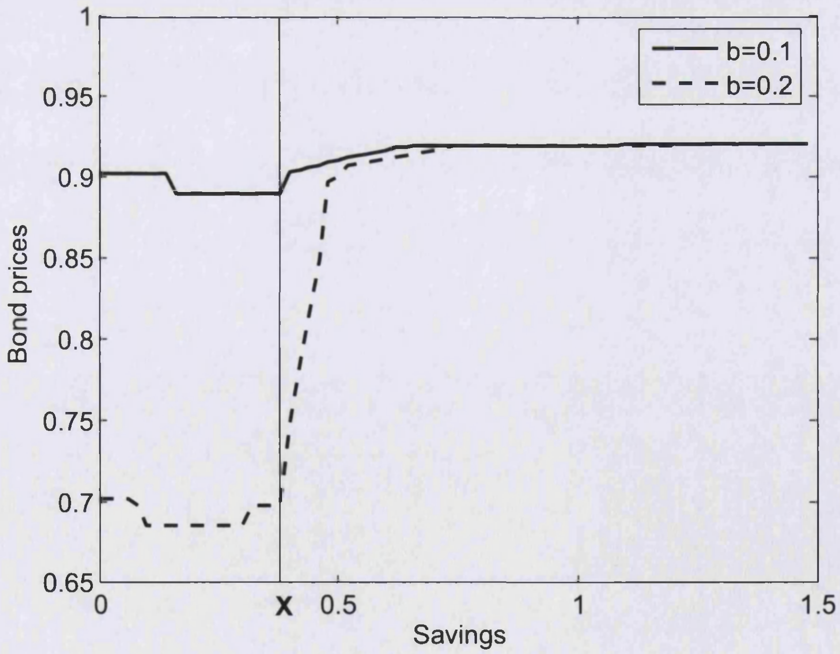

Figure 4.3: Bond prices for $b = 0.1$ and $b = 0.2$ for different levels of savings

The prices at which households can borrow depend on their default incentives. As usual, the more households borrow the higher is the incentive to default and therefore the lower will the price of a bond that is a promise to repay a fixed amount be. In addition, the incentive to default also depends on the exemption level. There are two opposing effects that play a role. On the one hand, the more a household has saved, to better it can afford to repay its debt. On the other hand, for savings up to the exemption level, the more the household has saved, keeping the repayment requirement constant, the better off the household will be in default.

Figure 4.3 shows the price of bonds with repayment requirement $b = 0.1$ and $b = 0.2$ respectively, for different levels of savings of agents with low labor productivity. If the household has no savings, it will default in some states. If savings are positive but below the exemption level, the household is better off defaulting, therefore the bond price falls. In this case banks will recover nothing in case of a default. Once savings exceed the exemption level, banks recover part of the loan because assets in excess of the exemption level go to the bank. This is reflected in an increase in the price of the bond. Since the household loses all its assets above the exemption level, default becomes costlier. This lowers the incentive to default. If the household has saved a lot, it will actually never default and therefore it will be able to borrow at the risk free rate.

Figure 4.3 also shows that these incentives depend on the amount borrowed. If repayment requirements are high, as is the case when the household borrows $b = 0.2$, then the default incentive is already high without savings. An increase in savings increases the default incentive further. And, similar to the case of borrowing only $b = 0.1$, once the household saves more than the exemption level, the recovery rate of the bank increases and therefore the loan price increases. It is important to note that these pricing functions pertain to particular bonds. Agents might or might not choose an element of this particular pricing function. Equilibrium choices are shown in figure 4.4.

Figure 4.4 shows several policy functions of households aged 26, have a low persistent labor productivity and that have not defaulted in the previous period. All panels have the cash on hand of the agent after the current shocks were realized, production has taken place and previous debt (or assets) has been repayed on the x-axis. Panel A shows the amount they borrow. Panel B shows the (annual) interest rate they have to pay on these loans. Panel C shows their savings decision. Recall that they save at the constant risk-free interest rate. Panel D shows their consumption decision.

Poor agents, those with cash on hand less than -0.075 cannot borrow more than 0.18. Since they will default when they receive a bad shock, they have to pay a higher interest rate. These agents prefer not to save since their marginal utility of consuming immediately is so high. Agents with cash on hand of more than -0.075 start borrowing less. Therefore the interest rate declines.

The most interesting agents are those with cash on hand between 0.08 and 0.27. These agents borrow at an (annual) interest rate around 10.5% and save at the lower rate of 4% simultaneously. They do this because they can default on the unsecured debt in bad states. Agents with cash on hand around 0.25 are actually net savers. Nevertheless, they are willing to pay the high interest rate on their debt because of the insurance offered by its partial contingency. If they default, they can keep all of their savings since these are less than the exemption level. And, conversely, the banks will recover nothing in case of a default.

Agents with cash on hand above 0.27 do not borrow. Therefore, the interest rate in panel *B* is not shown. These agents only save at the risk-free rate. However, those who have cash on hand less than the worst expenditure shock, which has a value around 0.82, might have negative cash on hand next period. In this case, they might default on this expense debt. After defaulting they can keep their savings up to the exemption level.

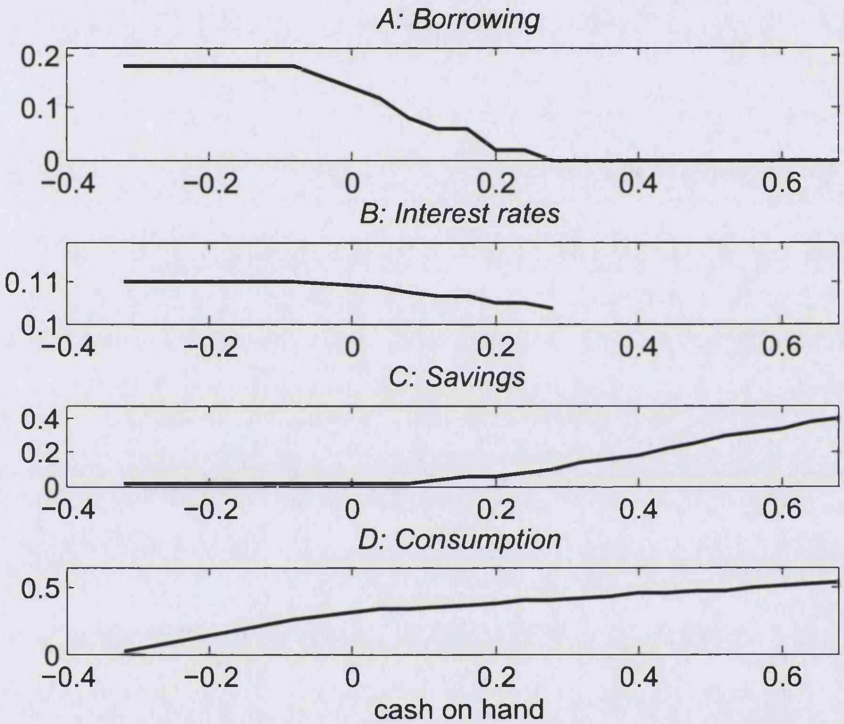

Figure 4.4: Policy functions of an agent aged 26 and with low persistent labor productivity

Life-cycle implications

Income and consumption of the model have the observed hump over the life-cycle. In this subsection, I show the fraction of borrowers, the amount borrowed and the interest rates over the life-cycle. Since the key contribution of this paper is to introduce two assets into a life-cycle model, I show how these variables differ across agents who either only borrow or who borrow and save simultaneously. The left column of figure 4.5 shows households who only borrow. The right column shows households who borrow and save.

The first row of figure 4.5 shows that the fraction of agents who only borrow is higher across all age groups. The second row shows that these household borrow substantially more than households who borrow and save. Both these quantities show a hump-shape with an increase at the end of the working lives.

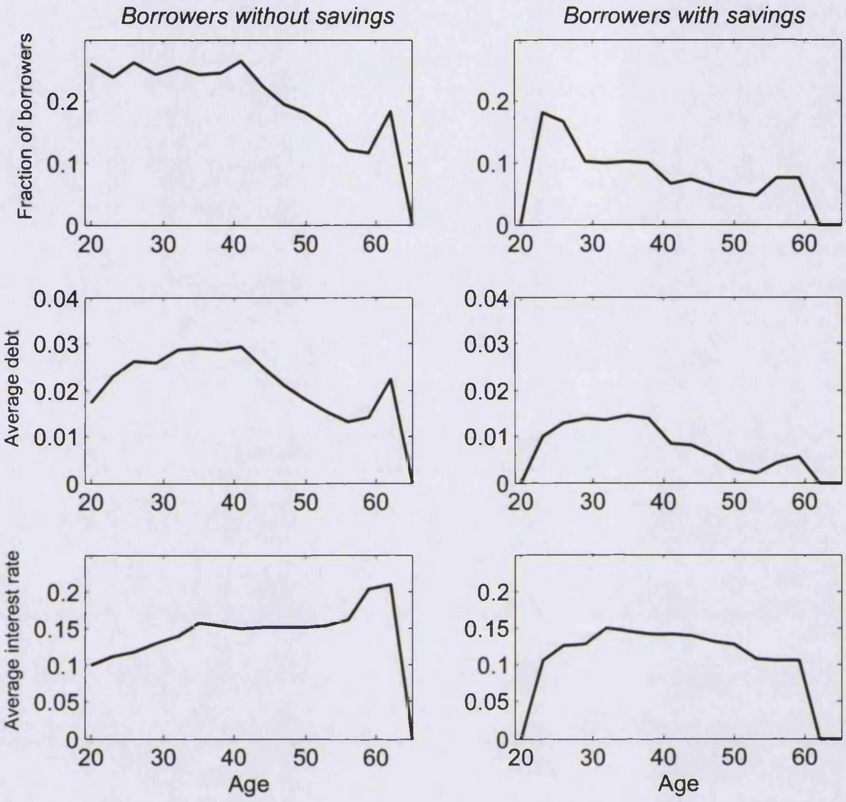

Figure 4.5: Fraction of borrowers, average debt and interest rates of borrowers who do not save and of borrowers who save

In figure 4.4, we saw that it is the very poor agents who only borrow. They do this because their current marginal utility of consumption is so high that they prefer not to save at all. Since these agents are also likely to default, they have to pay higher interest rates, as can be seen in the last row of 4.5. The incentive to default of these agents rises almost monotonically over their life cycle. The main reason for this is that there is less time left and therefore the punishment of being excluded from the unsecured credit market has a declining impact. The second row shows that an increase in borrowers at age 62, and in particular an increase in the amount they borrow. This higher level of borrowing combined with no further concern for the future, because there is no uncertainty after 65, explains the sharp increase in interest rates in the last periods of life.

The interest rates of borrowers who also save is lower, even though conditional on the loan size, they have a higher incentive to default. But as we have seen in 4.4, agents who borrow and save are relatively richer than those who only borrow. Their default incentives are relatively constant over their life-cycle, therefore the interest rate they have to pay do not change much.

4.6 Policy experiment

In this section, I present the implications of varying the exemption level from zero to high levels. In particular, I will investigate the following exemption levels: The lowest level is $X = 0$, the case implicitly analyzed by Livshits et al. [2007b] because they do not have an exemption level in their model. A low levels $X = 0.063$ which corresponds to the observed minimum of $8,000 found in Maryland. Then the benchmark value of $X = 0.38$ corresponding to $47,800 which is the population weighted median level. And a higher level $X = 1$ which corresponds to about $124,700. I will report the maximum of almost $250,000 found in Kansas only occasionally since these results are usually the same as the one obtained for $X = 1$.

4.6.1 Default rates

Since the incentive to default increases with the exemption level, default should be positively correlated with the exemption level.[12] However, as shown in figure 4.1 there is no positive relationship between the exemption level and the occurrence of default. In fact, the correlation is slightly negative. My model does not produce a negative relationship. It produces a small positive one with non-linear effects. However, a regression on the data shows that a positive coefficient cannot be excluded. In particular, the correlation my model produces is within the 95 percent confidence interval.

Previous models, for example Athreya [2008] for consumers or Mankart / Rodano [2007] for entrepreneurs, find a strong positive relationship between the exemption level and default rates. The reason my model predicts only modest increases in the default rate when the exemption is very low and almost no increase once it exceeds $30,000 is that almost no household is affected by such high exemption levels. Households that might default have assets that are below the exemption level. This is explained in more detail later on.

The non-linear effect at low exemption levels indicates that not including the exemption level might lead to spurious results. My model nests the model by Livshits et al. [2007b]. In particular, setting the exemption level to zero makes my model equivalent to their model. Therefore, I also contrast their results with mine.[13]

In addition to looking at average default rates, it is instructive to look at default rates over the life-cycle. Figure 4.6 shows default rates for three different exemption levels and the default rates observed in the data.[14] While the benchmark model gets the hump-shape over

[12] This is unless credit rationing is so severe that the most risky borrowers are excluded from the market completely. In this case the selection effect might overturn the positive relationship between the exemption level and the default rate.
[13] Since I do not recalibrate the model for each exemption level, the results that I report for $X = 0$ and their results differ. However, these differences are very small since the value of my calibrated variables are very close to the values in their calibration.
[14] For observed bankruptcies, I used the data from Sullivan et al. [2000] and adjusted the mean.

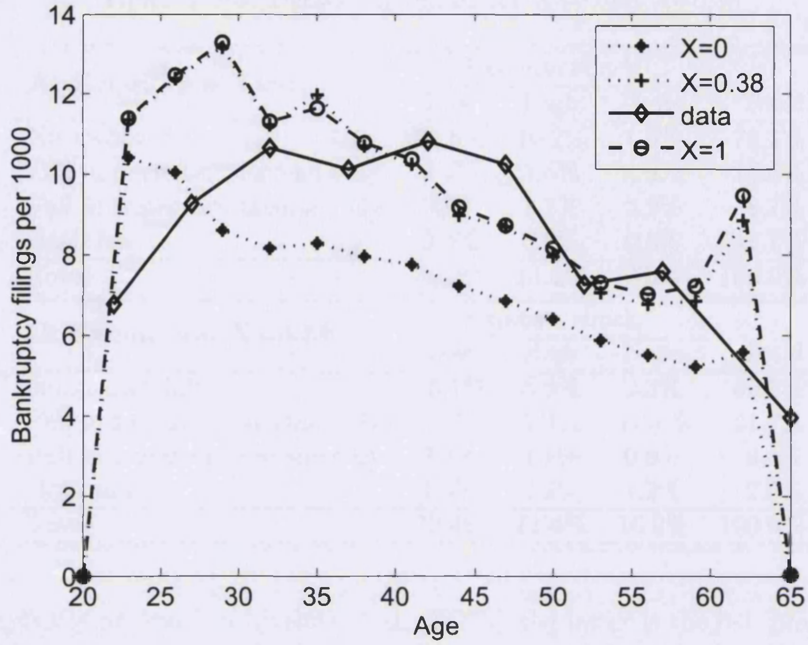

Figure 4.6: Defaults over the life cycle: Data, benchmark model ($X = 0.38$), low exemption ($X = 0$) and high exemption ($X = 1$)

the life-cycle right, the peak in the default rate occurs too early compared to the data. In addition, defaults pick up at the end of the life in the model.[15]

The case of $X = 0$ is the case analyzed by Livshits et al. [2007b]. In this case the peak occurs in the first period in which default is possible in the model. [16] Thus, this version of the model does worse in describing default rates over the life-cycle. Even a very small exemption level 0.06 (not shown) already implies a peak in the third model period, i.e. at age 29.

The main result that increases in the exemption level beyond an intermediate level do not lead to an increase in bankruptcies can also be seen in figure 4.6. The case of a high exemption level $X = 1$ is almost indistinguishable from the benchmark case $X = 0.38$. Default rates are only marginally higher during the last periods of life.

4.6.2 Default reasons

Households in the model are exposed to three types of uncertainty: expense (wealth) shocks, changes in persistent labor productivity and transitory income shocks. In this section, I compare the default reasons across two exemption levels, $X = 0$ and $X = .38$. The former is

[15] The model produces this marked increase because everyone retires for sure at 65 and I assume that there is no further uncertainty. If the model included heterogeneity with respect to retirement age and additional uncertainty this peak would flatten out.

[16] See also figure 1 in Livshits et al. [2007b].

Table 4.6-5: Default by reason for X=0 and X=0.38

A: Exemption X=0	Expense shock			
	Low	High	None	Total
No income fall	63.8%	10.2%	1.0%	75.1%
Fall in persistent income only	8.1%	1.6%	4.9%	14.6%
Fall in transitory income only	7.3%	1.2%	0.2%	8.7%
Both fall	0.9%	0.2%	0.6%	1.7%
Total	80.2%	13.2%	6.6%	100.0%
B: Exemption X=0.38	Expense shock			
	Low	High	None	Total
No income fall	55.1%	8.8%	2.3%	66.1%
Fall in persistent income only	8.6%	1.4%	11.7%	21.7%
Fall in transitory income only	7.4%	1.1%	0.9%	9.4%
Both fall	1.3%	0.2%	1.3%	2.8%
Total	72.4%	11.4%	16.2%	100.0%

the case implicitly analyzed in Livshits et al. [2007b], the latter is the benchmark case.

The biggest difference between the two panels in table 4.6-5 is that the fraction of defaulters who have experienced no wealth shock more than doubles from 6.6% to 16.2% when the exemption level is increased from $X = 0$ to $X = 0.38$. In particular, defaulters who have experienced no wealth shock but whose persistent productivity has dropped compared to the previous period increases from 4.9% to 11.7%. In the data [17], the fraction of defaulters who report job reasons is at least as high as those reporting expense shocks. Thus, the benchmark model with a positive exemption level probably still overstates the role of expense shocks. But it is a significant improvement over a model without any exemption. Further increases in the exemption level (not shown) do not lead to any significant changes in the default reasons.

4.6.3 Debt and savings

Figure 4.7 shows the average debt over the life-cycle. Panel A shows debt of agents who do not save. Panel B shows debt of agents who save. Panel C shows total debt.

First, if the exemption is 0, no borrower will ever save. This is because, in case of a default, he has to surrender all assets above the exemption level which in this case simply means all assets. Therefore, in panel B the line for $X = 0$ corresponds with the x-axis. If the exemption is positive, for example $X = 0.38$ as in the benchmark case, some agents will borrow and save simultaneously (see panel B). But since this is mainly a substitution, this lowers the borrowing amount in panel A of figure 4.7.

The net effect can be seen in panel C. A positive exemption leads to slightly more borrowing in the first half of the life-cycle and to slightly less in the second half. Figure 4.7 shows again

[17] See figure 1.2 on page 16 in Livshits et al. [2007b].

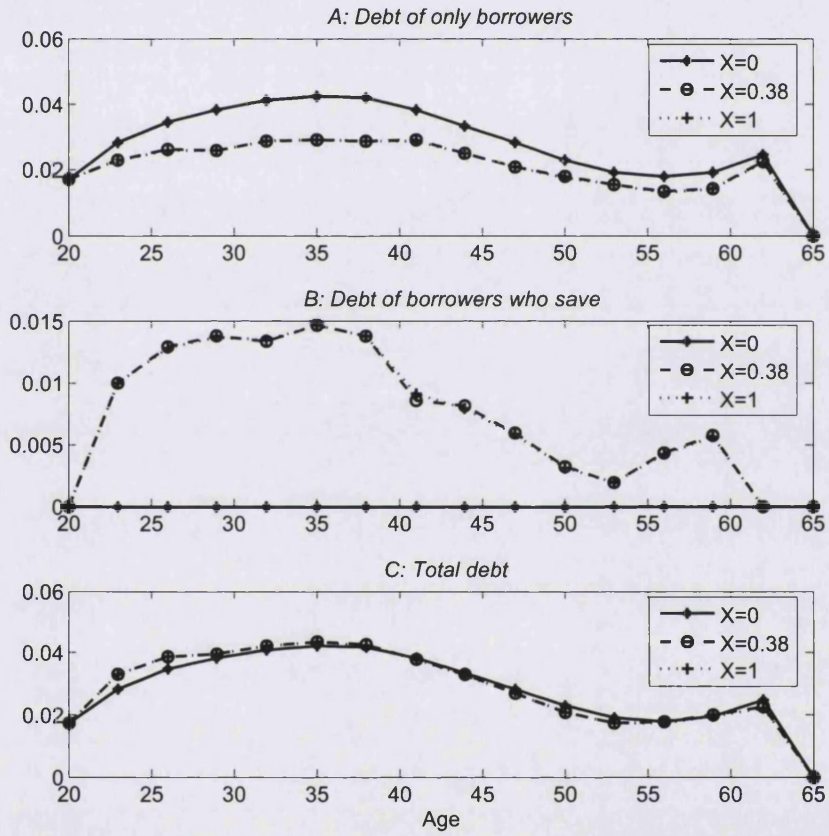

Figure 4.7: Debts over the life cycle: Benchmark model ($X = 0.38$), low exemption ($X = 0$) and high exemption ($X = 1$)

that a further increase in the exemption level from $X = 0.38$ to $X = 1$ leads to almost no change. The two lines are indistinguishable in all three panels. This can also be seen in figure 4.9.

Figure 4.8 shows the assets held by households at the moment of filing for bankruptcy. If the exemption level is zero, less than six percent of households have positive savings at the time of default. If the exemption level is increased to a still very low level of 0.06, the distribution shifts outwards. This means agents hold more wealth at the moment of default. The reason for this is that being able to keep some wealth leads agents who might default to increase their savings since they can keep it. Further increasing the exemption level to $X = 0.38$ alters the distribution somewhat. But an increase to $X = 1$ has again almost no additional effect.

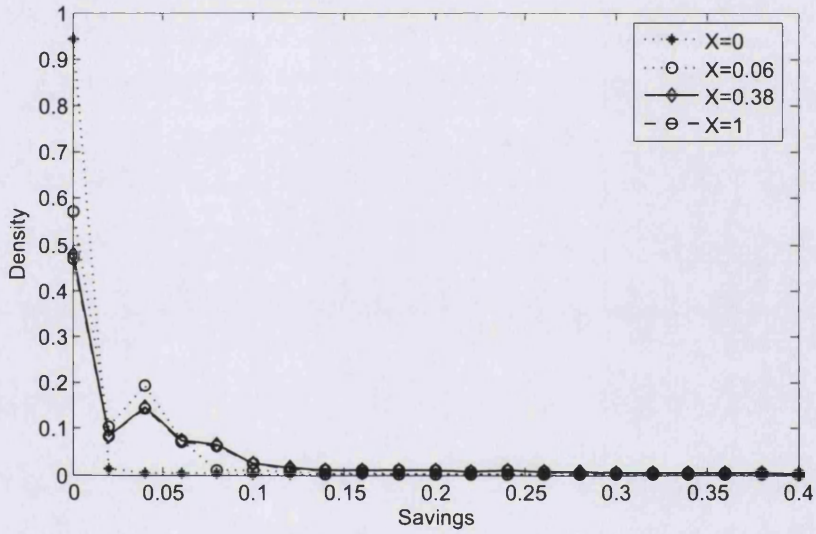

Figure 4.8: Distribution of assets of defaulters for different exemption levels

Figure 4.9 shows aggregate savings and aggregate borrowing for exemption levels, ranging from \$0 to \$250,000. Borrowing increases rapidly for low levels of savings before it falls back to a smaller level. And then, it remains unchanged for exemption levels higher than $X = 0.2$.

Savings however keep on increasing for all levels of the wealth exemption, even though the increases get smaller. Nevertheless, it is almost the only variable that keeps changing even for high exemption levels. This confirms the discussion in section 4.3 that savings can increase when the exemption level increases. The reason is that the insurance through a high exemption level now is available also for relatively richer households. This leads these households to increase their savings supply.

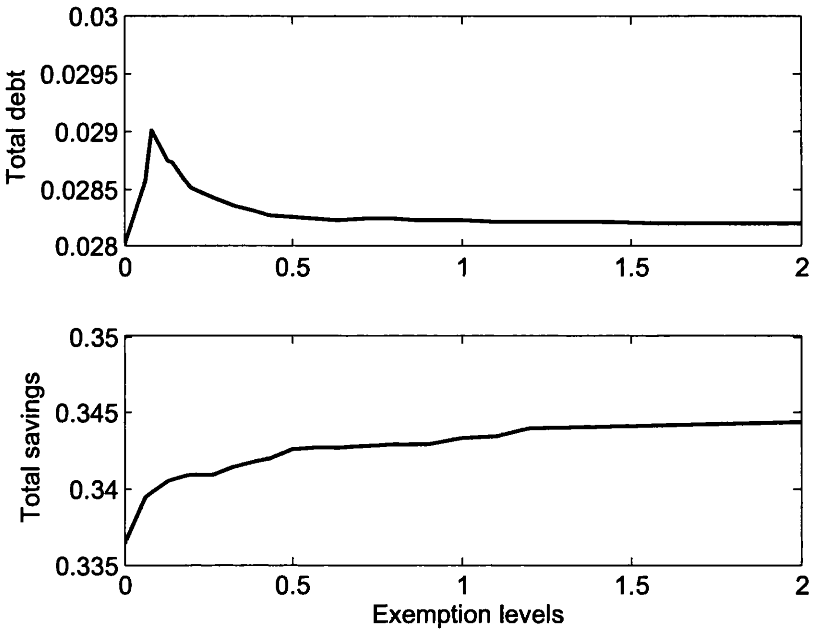

Figure 4.9: Debts and savings for different exemption levels.

4.6.4 Welfare

The top panel in figure 4.10 shows the welfare impacts of changing the exemption level. As utilitarian welfare measure, I use the percentage increase in lifetime consumption necessary to make the households equally well off under both regimes (ECV). $X = 0.38$ is the benchmark case. A negative number here means that this particular exemption level is worse than the benchmark and vice versa. The bottom panel shows the variance of log consumption. A lower variance means that consumption is more equally distributed which, from an ex ante perspective, makes (ceteris paribus) households better off.

Figure 4.10 shows that there are welfare gains from moving from a very low level of the exemption ($X = 0$) to an intermediate level ($X = 0.3$). The bottom panel shows that these welfare gains are obtained by decreasing the variance of log consumption, i.e. by distributing consumption more equally. Thus, a positive exemption level allows for more risk-sharing in this economy. The welfare gains from further increases are extremely small.

Livshits et al. [2007b] compare a US style system in which debt is wiped out upon default (fresh start) with a European style system where this is not the case. They find that the fresh start system is better by about 0.06% in terms of ECV. Incorporating the second important feature of the US system, a positive exemption level, doubles this welfare differences.

The net supply of savings, aggregate savings minus aggregate debt, in figure 4.9, is increasing in this model. While the model is a partial equilibrium model, this, at least, suggests that general equilibrium effects are unlikely to overturn the case for high exemption levels.

In figure 4.1, I have shown that the model implies only small differences in bankruptcy rates for different exemption levels. I have also shown that this is consistent with the data. In addition, figure 4.10 shows that the welfare differences between positive exemption levels are very small. The very fact that there are huge differences in exemption levels across US states suggests that the welfare implications are probably not that big. Otherwise, at least if the political process were efficient, a convergence of exemption levels should have occurred over the last decades.

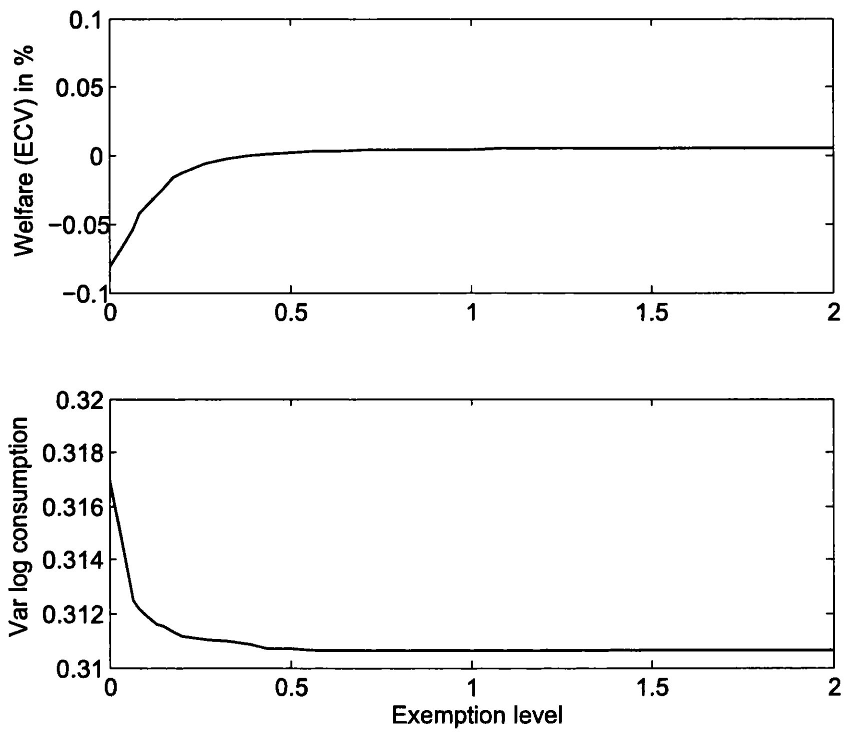

Figure 4.10: Welfare and variance of log consumption for different exemption levels.

4.7 Conclusion

In this paper, I develop a heterogenous agent life-cycle model in which agents are subject to three types of shocks: persistent and transitory labor productivity shocks and expense shocks. Financial markets in the model are incomplete but agents can insure themselves against risks by holding a portfolio of unsecured debt and savings. I show that including the possibility to keep some of the assets, i.e. a positive wealth exemption level, as is the case in all US states, is important.

A positive exemption level increases aggregate savings and welfare. However, I also show that increases in the exemption level beyond \$25,000-\$30,000 have hardly any effect. The default rate does not increase any further. This is consistent with the data which show no positive correlation between the exemption level and default rates. Additionally, the wide variation in exemption levels across US states is consistent with a rational political process if welfare differences are small. This is indeed the case in the model.

One limitation of the model is that it assumes that both types of expenditure shocks are born by someone else in case of default. This is plausible for high medical expense debts where hospitals often do not get paid. However, for divorces, unwanted pregnancies etc. it is more reasonable to assume that households have to bear these costs themselves. They could do this if they had access to a credit line on which they could draw in these situations. In that case the loan pricing would be more difficult since the loan would actually resemble a credit card with a credit card limit. Incorporating this into a life-cycle model with expenditure shocks is left for future research.

4.8 Appendix

4.8.1 Derivations of result 1 in section 3

This is a partial proof of result 1 in the section 3. The utility function is

$$\begin{aligned}\max_s U &= \log(c_1) + \mathbb{E}\log(c_2) \\ &= \log(a_0 - s) + (1-p)\log(s) + p\log(\min[s, X])\end{aligned}$$

Proof. There are three possible cases

$$s^* = \begin{cases} \frac{(1-p)a0}{2-p} & \text{for} \quad X \leq \frac{(1-p)a0}{2-p} \\ X & \text{for} \quad \frac{(1-p)a0}{2-p} < X < \frac{1}{2}a_0 \\ \frac{1}{2}a_0 & \text{for} \quad \frac{1}{2}a_0 \leq X \end{cases}$$

If $\min[s, X] = X$, then the last term in (4.8-1) is independent of s, therefore the first order condition with respect to s is

$$\frac{1}{a_0 - s}(-1) + \frac{(1-p)}{s} = 0$$

solving for s yields

$$s^* = \frac{(1-p)\,a0}{2-p}$$

Now, note that this was obtained under the assumption that $\min[s, X] = X$. Therefore, this results holds only for $\frac{(1-p)a0}{2-p} \geq X$

If $\min[s, X] = s$, then the problem is actually a standard problem

$$\max_s U = \log(a_0 - s) + \log(s)$$

with first order condition

$$\frac{-1}{a_0 - s} + \frac{1}{s} = 0$$

and therefore

$$s^* = \frac{1}{2}a_0$$

Note, that this case was obtained under the assumption that $\min[s, X] = s$. Therefore, it holds only $\frac{1}{2}a_0 \leq X$.

Lastly, note that for any $p \in (0,1)$

$$\frac{1-p}{2-p} < \frac{1}{2},$$

therefore, as long as there is a positive probability for an expense shock, there will be an exemption level such that $\frac{(1-p)a0}{2-p} < X < \frac{1}{2}a_0$. In that case $s^* = X$. ■

Chapter 5

Joint Search and Aggregate Fluctuations

5.1 Introduction

The idea that economic agents lack sufficient access to markets to insure against misfortune has been one of the founding blocks of modern macroeconomics.[1] By now the literature assigning a central role to heterogeneity and postulating that risk sharing is far from perfect is voluminous and has addressed many interesting aspects of macroeconomic theory.[2] For instance the baseline incomplete markets paradigm in the tradition of Aiyagari [1994] and Krusell / Smith [1998] builds on the assumption that households are formed by bachelor agents who by trading claims on the aggregate capital stock can self-insure against shocks in labor income.

Over time alternative sources of insurance (either private or government provided) have been introduced into this framework. Much less common is the idea that a considerable amount of insurance against employment and productivity risk can be provided in the form of adjustments of the family members' labor supplies. This form of insurance within the family is the focus of our paper. We investigate the differences between economies where risk sharing is limited because agents stand alone against uncertain contingencies, and those where households are formed by unions of two ex ante identical (ex post heterogeneous) members who can mutually insure against economic risks.

Any realistic modeling of labor supply decisions should take the importance of trading frictions into account. Therefore, we are taking stock of the large literature of search models of the labor market. The risks that arise naturally in this environment are uncertainty about the match quality and the possibility of rationing of employment opportunities. On both these margins joint labor supply decisions present households with economically meaningful

[1] We thank Chris Pisssarides, Franceso Caselli and Alex Michaelides for their continuous support and valuable comments. We are also grateful to Wouter Den Haan, Rachel Ngai and participants at the LSE macroeconomics seminar.
[2] See Heathcote et al. [2008] for a survey.

opportunities that we explore.

Naturally this analysis lends itself to the possibility of tackling many interesting questions with which modern macro labor economics is confronted. From our part we want to use this model to understand whether granting joint insurance and labor supply possibilities to couples can help disassociate the search behavior of economically active (those in the labor force) from that of economically passive (out of the labor force) agents.

Generally traditional theories of the labor market have had a hard time matching the low procyclicality of the labor force and in the presence of frictions the business cycle correlations of key labor market statistics Veracierto [2008]. The reason is that the strong intertemporal substitution motive convinces agents to flow into the labor force in good times and abandon it in bad. Our main theme here is that if recessions are periods of high incidences of unemployment or low opportunities to find work then this induces household members to search jointly and intensively to insure against potential earnings losses. By contrast in bachelor household frameworks inactive workers are either those who have experienced a sequence of bad shocks or those who have accumulated sufficient wealth to finance leisure or both. We do not believe that either is realistic but rather view inactivity as part of a specialization pattern in which one partner provides market income and the other focuses on producing home goods.

In section 5.2 of our paper we use the data from the Current Population Survey to show that joint insurance though adjustments of labor supplies of household members can explain the low procyclicality of the US labor force. Then we turn to theory and build a general equilibrium framework to contrast the properties of two economies (bachelors and families) and demand that they be consistent with a broad range of empirical targets. When we introduce aggregate fluctuations in this environment we find that joint insurance and in general the structure of the household unit can have a large impact on the business cycle properties of key labor market statistics. Nonetheless, the effect on the cyclical properties of the labor force is limited. Our couple economy can do better in terms of employment and unemployment volatilities and correlations with output but only marginally reduces the cyclicality of the labor force. We argue that these predictions are consistent with two main implications of our theory: joint insurance against unemployment risk within the household and a reduction in the reliance on precautionary savings to self-insure.

5.1.1 Related literature

This paper is related to several strands in the literature: First, there has been an enormous interest on the implications of heterogeneity and incomplete insurance markets for the aggregate labor fluctuations, for example Gomes et al. [2001] and Chang / Kim [2007] . These papers, however, build on the bachelor household paradigm which is precisely our point of departure. Interestingly, Chang / Kim [2007] develop a framework where families consist of two members (a male and female) and use it to address how individual supply rules affect the value of the aggregate elasticity of labor supply. As far as incomplete markets models go, this work is closest to our intentions but many of the ingredients are different.

First, we emphasize the role of family in circumventing frictions in the labor market such as the limited availability of job opportunities whilst in Chang / Kim [2007] the role assigned to frictions is secondary. Second, contrasting the properties of two economies, one with bachelor households and one with couple households, in various environments is one of the main themes that we pursue. Most importantly none of the models of incomplete insurance markets from this literature takes up seriously the task of matching the patterns of worker reallocation between employment unemployment and inactivity. This is exactly what we do. For this reason we introduce a different shocks to make our model consistent with the relevant empirical flows.

There is a sizeable literature that highlights the role of family labor supply as a mean of insuring against idiosyncratic labor income risks. In Attanasio et al. [2005, 2008] and Heathcote et al. [2008] an additional margin of insurance provided by female labor market participation becomes a valuable instrument to buffer shocks to labor income. These papers analyze the effects of various changes in the economic environment on the historical trends of female labor supply. This is not the interpretation we want to give to our theory however.

Our model is one of complete markets within the household and incomplete markets outside the household. Closer to our attempt is recent work by Guler et al. [2008] who characterizes the effects of joint search on optimal reservation wage policies. In contrast to them, we use a more realistic search model and we build a general equilibrium framework with realistic heterogeneity that accounts for observed labor market flows as well as for the effects of shocks to aggregate productivity.

The paper is organized as follows: Section 5.2 uses the estimated flows from the Current Population Survey (CPS) to provide evidence that joint insurance and labor supply are key factors that explain the low procyclicality of the US labor force participation. In section 5.3, we develop the bachelor household model and the couple household model. In section 5.4, we show and discuss the basic results and implications of our theory. Section 5.5 concludes.

5.2 Labor Market Flows in the US

Table 5.2-1 summarizes the US labor market business cycle statistics. The data are constructed from the CPS and they correspond to observations spanning the years 1976 to 2005. They are logged and HP filtered and all quantities refer to quarterly aggregates and are expressed relative to a de-trended measure of GDP. Unemployment is extremely counter-cyclical and more than six times as volatile as aggregate output. Aggregate employment has two thirds of the volatility of output at business cycle frequencies and is very procyclical. The labor force is not volatile and its contemporaneous correlation with GDP is low (0.22).

Table 5.2-1: US Business Cycle: Labor Market Statistics

	Employment	Unemployment	LF	LF Couples	LF Wifes
	Aged 16 and Above			Aged 22 to 55	
$\frac{\sigma_x}{\sigma_y}$	0.66	6.68	0.34	0.35	0.47
$\rho_{x,y}$	.81	-.88	.22	.05	.2

The last columns of Table 5.2-1 present a breakdown of the relevant quantities into demographic groups that are of particular interest to us. For married couples aged 22 to 55 in our sample, aggregate statistics are no different than those of the full population, aged 16 and above. The labor force for this demographic is somewhat less procyclical, and hence even more puzzling from the point of view of theory, owning to the strong acyclical attachment of males in the sample, but also to the low contemporaneous correlation with GDP of female labor force participation. The volatility of both males (not shown) and females are higher than the aggregate volatility for this demographic group as can be seen in column 4. In turn this might suggest that there is some negative correlation of labor force participation of wives and husbands in our sample.

We note that this breakdown corresponds to an imperfect measure of our notion of couples in the model. Ideally we would like to have duads of agents who are linked with near perfect insurance opportunities and make labor supply decisions jointly, but the data preclude us from doing so. In what follows we treat household units that are comprised of two spouses as an ideal ground to provide evidence for our theory.

5.2.1 Implications for models: Fixed participation?

Are these observations consistent with the tendency of macro labor market theory to restrict attention to environments where economic agents can be either employed or unemployed at any point in time? We provide an answer to this question by looking at the monthly transitions of the US workforce across adjacent labor market states.

In table 5.2-2 we summarize the relevant flows estimated from the CPS. Each month roughly 7 % of out of the labor force workers join the labor force , and 3 % of employed workers quit and become inactive. Further on to dilute the suspicion that these results are driven by demographics Table 5.2-3 presents the analogous matrix for the sub-sample of married couples aged 22 to 55.

A point that merits some attention is the fact that roughly 5 % of out of the labor force workers find a job and become employed in the following month. There are two relevant possibilities: The first is that this is an immediate consequence of time aggregation since monthly horizons are more than enough for a worker to make a transition between inactivity and employment without having a recorded unemployment spell. The second pertains to the search behavior of passive searchers, marginally attached and discouraged workers. For

Table 5.2-2: Matrix for Flow rates of Agents Aged Above 16

	E	U	I
E	.9543	.0146	.0311
U	.2743	.4983	.2274
I	.0466	.0245	.9289

Table 5.2-3: Matrix for Flow rates of Married Couples Aged 22 to 55

	E	U	I
E	.9662	.0112	.0226
U	.2891	.5159	.195
I	.0623	.0282	.9095

these groups the work by Jones / Riddell [1999] demonstrates that they have transition probabilities into employment that are half as large as those of unemployed workers which in turn implies that some of the flows between states U and I can be broadly interpreted as evidence of optimal time variation in search intensity for these groups. These implications have already been explored in the literature, see for example Hornstein et al. [2007], and adjusting the transition probabilities to embrace the idea that marginally attached workers should be treated as unemployed rather than inactive doesn't make a big difference in the matrices of tables 5.2-2 and 5.2-3.

Hence we draw two conclusions from these calculations. First that the line between economically active and inactive workers is somewhat arbitrarily drawn by the theoretical models of the labor market and second that our model, calibrated at monthly frequencies should allow all agents (independent of their labor market status) to receive job offers and to experience transitions between nonemployment and employment.

5.2.2 Empirical evidence

One possibility to investigate our mechanism empirically would be to run limited dependent variable models (such as linear probability or probit models) and to estimate the effect of the husbands employment status, on the wife's labor force transitions, and this would allow us to control for some relevant aspects of heterogeneity. Such attempts to determine the magnitude of added worker effects are numerous in the literature and we can summarize these estimates without relying on our own empirical work. Moreover, this kind of analysis would have very little to say about the contribution of the joint labor supply on the low procyclicality of the labor force which is our focal point here.

Contrasting the cyclical behavior of singles versus couples, even after controlling for demographic characteristics, would fare no better as an alternative, since our notion of singles is a very different one from what the data could potentially suggest. In our framework singles are those agents who have an own idiosyncratic productivity and more importantly don't possess ties with any other agent in the economy that could alleviate the risk from this process. In the data however, unwed agents or even those who form a household unit on their own, are very likely to have joint insurance with other agents in the economy, for example their extended family. This consideration would cloud the conclusions we could potentially draw.

We treat the two spouses, husband and wife, in the household unit as the closest data analogue to our notion of partnerships with joint labor supply and insurance. Using data on individual transitions we want to test the following prediction: If it weren't for employment fluctuations over the business cycle of primary household earners, the labor force participation of secondary earners would be considerably more procyclical. We focus on individuals aged 22 to 55 and for this demographic group married agents account for roughly 60 % of the population. [3] In our sample we treat husbands as primary and wives as secondary earners.

For each period t we estimate the transition probability of a wife from state i to state j conditional on her spouse making a transition from state k to l. We denote this probability by $p_t^f(i,j,k,l)$ and analogously we let $p_t^m(k,l)$ be the unconditional probability that the husband, who is the household head in our sample, makes the transition from state k to state l over the course of a month. Due to data limitations we cannot define conditional transition probabilities for all relevant labor market states. For this reason we restrict our attention to $i, j \in \{LF, OLF\}$, that is wives can either be in the labor force or inactive, and husbands can either be employed or not (we denote this by $k, l \in \{E, N\}$). Finally, let $n_t(i,k)$ be the share of the population of couples with a secondary earner is state i and a primary earner in state k. The evolution of these measures is central to our experiment. With these estimates we construct counterfactual Markov transition matrices for couples over the relevant state space $\{LF, OLF\} \times \{E, N\}$.

The typical element of these matrices is given $p_t^f(i,j,k,l)\overline{p}^m(k,l)$ where $\overline{p}^m(k,l)$ denotes the transition probability of the husband averaged over all periods. What we mean to accomplish by that is to have data on household transitions whereby the probability distribution of primary earners across labor market states is independent of time. Notice that other than the business cycle variation there is no secular trend or other discernible pattern of time variation for the transition probabilities of males in our sample.

We use these matrices to construct population measures at one and three month ahead horizons. That is to say we feed the actual populations n_t once and track the measures over the relevant horizon using our constructed matrices. To make our comparison meaningful we also compute populations based on the actual transition probabilities, i.e $p_t^f(i,j,k,l)p_t^m(k,l)$, since small errors that compile over time may cloud the conclusions from this experiment. We denote by $\overline{n}_t$ the constructed measure based on the time averaged probabilities for husbands, and by $\tilde{n}_t$ the analogous object based on the actual estimated transitions. Figure 5.1 shows

[3] For the entire sample of agents aged above 16 they form 36% of all individuals.

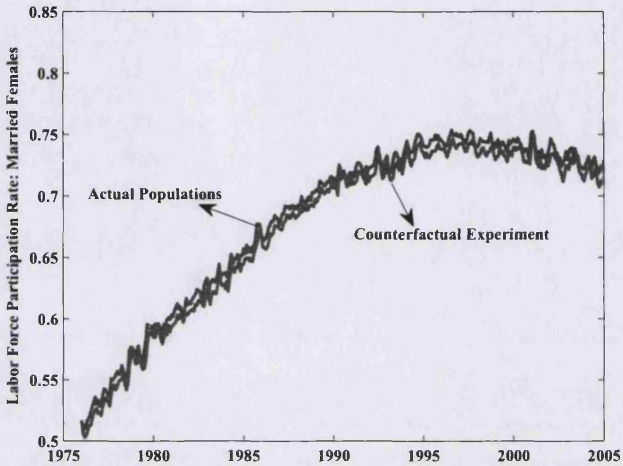

Figure 5.1: Actual and Counterfactual Labor Force Participation Rates of Married Women

the labor force participation rate, based on measure n_t that we draw from the data, for wives over the sample period with the three month ahead counterfactual time series, based on $\overline{n}_t$. Reassuringly the correlations between actual and counterfactual measures is high above .99 at our longest horizon. The correlation between n_t and $\tilde{n}_t$ is even higher.

Table 5.2-4: Experiments

	Actual Population	Actual (One Month Horizon)	Counterfactual (One Month Horizon)	Actual (Three Month Horizon)	Counterfactual (Three Month Horizon)
$\frac{\sigma_x}{\sigma_y}$	.3604	.3770	.3805	.4294	.4362
$\rho_{x,y}$	.2963	.2988	.3703	.2570	.3216

In table 5.2-4, we summarize the results from this experiment. We compare the relative standard deviations and contemporaneous correlation with a de-trended measure of GDP. The first column refers to the cyclical properties of the labor force participation rate of married wives based on the actual population measure n_t.[4] Columns 2 to 3 and 4 to 5 compare the analogous objects based on the measures $\tilde{n}_t$ and $\overline{n}_t$, for one and three months horizons respectively. As the horizon expands the errors that compile over time make the processes display considerably more volatility and smaller cyclical correlation with GDP.

[4] The differences in the quantities $\frac{\sigma_n}{\sigma_y}$ and $\rho_{n,y}$ relative to tables 5.2-2 and 5.2-3 stem from the fact that the population is normalized to unity.

The result is both qualitatively and quantitatively encouraging. The cyclical correlation of labor force participation for wives jumps from 0.2988 to 0.3703 in columns 2 and 3 and from 0.257 to 0.3216 in columns 4 and 5. Further on in light of this higher correlation with GDP we can argue that the increase in volatility from our actual to our counterfactual measures is mostly due to the business cycle.

5.2.3 Implications for the model

Our hypothesis is that in recessions secondary earners flow into the labor force to insure the family against the possibility of a job loss of it's main earner. We find indeed that the probability of a wife entering the labor force, or the probability that she experiences a $OLF \rightarrow LF$ transition is higher when the husband looses his job, and effectively this defines an added worker effect. But recessions are times when the event of a job loss becomes considerably more likely, and analogously the possibility that a job is found becomes considerably lower. With more husbands in the pool of nonemployed in recessions, wives flow in the labor force with higher rates. By shutting down time variation in the transition of primary earners we seek to isolate this effect and in fact our results confirm our hypothesis.

An important consideration pertains to heterogeneity in our sample which arguably we are unable to control for. If, for example, wives that are more likely to make transitions between labor market states are matched with husbands that are more probable to experience a loss of employment, then the added worker effect that we identify could be spurious. We address this possibility by making the following points:

First, not all of the relevant conditional probabilities in the data tilt in our direction: For instance we find that in our sample the probability that a wife exits the labor force is higher when her spouse experiences an employment to nonemployment transition, which is a counterfactual implication according to our reasoning. Heterogeneity therefore could go either way. More substantively we can argue the case based on the empirical estimates from the relevant literature. Working with the CPS flows, the closest to our attempt, Spletzer [1997] identifies sizable and significant added worker effects. In his sample the probability that a wife flows in the labor force when her husband looses his job is estimated to be .08 log points higher than the average of the population in a limited dependent variable model and when he adds household characteristics this difference falls to .06, still sizable nonetheless.[5]

Further on, although our calculations may be fraught with some heterogeneity, they are also unable to identify other important issues such lags in labor adjustments of household members or even responses to anticipated shocks which might reinforce our results. Both of these possibilities are clearly relevant. Stephens [2002] uses data from the PSID to argue that

[5] In this study the husband's recent history of unemployment spells (the number of weeks spend in unemployment in the previous year) appears to be a key variable that reduces the relevant probability. In our view this should not be interpreted as evidence against the existence of an added worker effect since even delayed responses to past shocks or front-loaded labor adjustments to anticipated shocks may be interpreted as joint insurance.

post displacement, substantial earnings losses due to unemployment spells of male earners lead to near permanent increases in female labor supply.

5.3 The model

We develop two related models in which households face uninsurable idiosyncratic labor income risk. In the first model a household consists of one agent, a bachelor. In the second model, and this is the key contribution of our paper, a household consists of two agents, a couple, who share their income risk.

5.3.1 Bachelor economy

We consider an economy populated by a unit mass of strictly risk averse bachelor households that are identical in preferences and value the consumption of a general multipurpose market good c. We denote the discount factor for these agents by β_S and the period utility deriving from consumption by $u(c)$.

At any point in time a household member can be either employed, unemployed or out of the labor force. We assume that labor supply decisions are formed at the extensive margin and are subject to the frictions that impede instantaneous transitions across these adjacent labor market states. In particular employed agents spend a fraction $\overline{h}$ of their unitary time endowment each period in market activities associated with a utility cost which we denote by $\Phi(\overline{h})$. Job availability for non-employed agents in the economy is limited: They are endowed with a technology that transforms units of search effort s into arrival rates of job opportunities $p(s)$ at a cost $k(s)$ per unit of time. As will become clearer below, based on these optimal choices, we classify household members as either unemployed (active searchers) or out of labor force workers.

Further on we assume that households face idiosyncratic labor productivity risks and we summarize this risk in two independent stochastic processes ϵ and x. The former,ϵ, is an agent specific process, an own labor productivity component, that is a persistent state variable in the agents value function independent of her labor market status. The latter x is a job specific component that pertains to the quality of active jobs and available job opportunities in the economy. These characteristics evolve stochastically over time according to the transition cumulative distribution functions $\pi_{\epsilon',\epsilon} = Pr(\epsilon_{t+1} < \epsilon', \epsilon_t = \epsilon)$ and $\pi_{x',x} = Pr(x_{t+1} < x', x_t = x)$ respectively. Further on, we assume that the initial assignment of job quality x derives from a general density $H(x)$.

Financial markets are incomplete and agents can self-insure by trading non contingent claims on the aggregate capital stock. These earn a return R_t each period. Agents are subject to an ad hoc borrowing limit $a_t \geq \overline{a}\ \ \forall t$. Wages per efficiency units of labor w_t as well as rental rates R_t are determined in competitive factor markets where it is assumed that a representative firm aggregates all inputs into a multipurpose final good. The technology is of the standard form $Y_t = K_t^{\alpha}(L_t\lambda_t)^{1-\alpha}$, where capital K_t depreciates at rate δ each period and $L_t = \int\int\int \epsilon x h_{a,\epsilon,x} I_{h_{a,\epsilon,x}=\overline{h}} d\Gamma_t$ denotes the aggregate efficiency units of the labor input. Finally, Γ_t is the density over the relevant state space of employment status, productivity and wealth and λ_t is the TFP process which evolves according to the transition equation $\pi_{\lambda'|\lambda'} = Prob(\lambda_{t+1} < \lambda'|\lambda_t = \lambda')$. The law of motion for the distribution of workers is defined as: $\Gamma_{t+1} = \mathcal{T}(\Gamma_t, \lambda_t)$ where $\mathcal{T}$ is the relevant transition operator.

Timing of events

Each period t, and after the resolution of all relevant uncertainty, a non-employed agent chooses optimally the number of search units s_t to exert and finances her consumption out of the current stock of savings. Her choice of s_t maps into a probability $p(s_t)$ of receiving a job offer in the next period. When this opportunity arrives the new values ϵ_{t+1} and x_{t+1} are sampled and the aggregate state vector $\{\Gamma_{t+1}, \lambda_{t+1}\}$ is revealed and the agent will decide whether she wants to give up search and become employed. Notice that given that all jobs entail a fixed cost $\Phi(\overline{h})$ the realization of the relevant state vector might be such that the prospective match, i.e. the job, does not generate a positive surplus for the worker.

Similarly for an employed agent, the sampling of the new values for x_{t+1} and ϵ_{t+1} generates the risk of separation. The worker may decide that it is not worthwhile to spend $\overline{h}$ of her time working and would rather search for new opportunities next period. For this worker, optimal consumption and savings decisions are borne out of the stock of wealth and labor earnings, conditional on her keeping her current employment status.

Value functions

Consider the problem of an agent with a stock of wealth a_t and a productivity endowment ϵ_t who is currently not employed. She must optimally allocate resources between current consumption and savings and choose the number of units of search effort to exert to maximize her well-being. We denote the lifetime utility for this worker by V^n. We also define an auxiliary object $Q^e = \max\{V^n, V^e\}$ which is the outer envelope over the relevant menu of choices for this worker conditional on her receiving a job offer next period.

Applying standard arguments we can represent her program recursively as:

$$\begin{aligned} V^n(a,\epsilon,\Gamma,\lambda) &= \max_{a'\geq\overline{a},s} u(c) - k(s) + \beta_S\Big(\int_{\epsilon',\lambda'} p(s) \int x' Q^e(a',\epsilon',x',\Gamma',\lambda'))\, d\, H(x') \\ &+ (1-p(s))\, V^n(a',\epsilon',\Gamma',\lambda')\ d\pi_{\epsilon'|\epsilon} d\pi_{\lambda'|\lambda} \end{aligned} \quad (5.3\text{-}1)$$

subject to the constraint set:

$$a' = R_{\lambda,\Gamma} a - c. \tag{5.3-2}$$

Notice that the distribution Γ becomes a state variable in the workers value function. In order to forecast factor prices next period and to make optimal savings and labor market search decisions knowledge of Γ' is necessary since this object determines the economy's aggregate capital stock and effective labor in the next period. [6]

In a similar fashion we can represent the employed workers lifetime utility as a solution to the following functional equation:

$$\begin{aligned} V^e(a,\epsilon,x,\Gamma,\lambda) &= \max_{a'\geq\overline{a}} u(c) - \Phi(\overline{h}) \\ &+ \beta_S\left(\int_{\epsilon',\lambda',x'} Q^e(a',\epsilon',x',\Gamma'|\lambda')\right)\ d\pi_{\epsilon'|\epsilon} d\pi_{\lambda'|\lambda} d\pi_{x'|x}) \end{aligned} \tag{5.3-3}$$

subject to

$$a' = R_{\lambda,\Gamma} a + w_{\lambda,\Gamma}\overline{h}\epsilon x - c \tag{5.3-4}$$

.

Our classification criterion for nonemployed workers is the following:

$$\text{if} \quad s^* \begin{cases} < s_{min} & \text{Worker is OLF} \\ \geq s_{min} & \text{Worker is Unemployed} \end{cases}$$

We classify an agent as unemployed if she chooses effort above a given threshold s_{min}, and as out of the labor force otherwise. This mapping is consistent with the notion that inactive agents search less intensively in the labor market and as coarse as this classification rule may be it is very close to the analogous criterion used in the CPS.

We normalize the flow value of income for both unemployed and out of the labor force agents to zero so that their consumption is financed exclusively out of the stock of savings. This assumption is made mainly to avoid the complications of having to deal with eligibility for a government insurance scheme as it is not clear how benefits would be distributed across the population. For instance inactive workers in principle should not receive any sort of replacement income but in our model there is a considerable amount of mobility between the two non employment states. In turn keeping track of benefit histories would add to the computational burden of our exercise without being clear how it would affect the main results.

[6] We use primes to denote next period variables. Furthermore, we chose to use integrals instead of the conventional expectation operators to highlight that the relevant uncertainties faced by employed and non employed workers differ in the current context. The initial draws of x derive from the general distribution $H(x)$ and the continuation match qualities are determined by $\pi_{x'|x}$ so that in general:

$$\left(\int_{x'} Q^e(a',\epsilon',x',\Gamma',\lambda')\right) d\pi_{x'|x} d\,x') \neq \int_{x'} Q^e(a',\epsilon',x',\Gamma',\lambda'))\ d\,H(x')$$

.

Competitive Equilibrium

The equilibrium consists of a set of value functions $\{V^n, V^e\}$, and a set of decision rules for consumption, asset holdings $a'_e(a, x, \epsilon, \lambda, \Gamma$ and $a'_n(a, \epsilon, \lambda, \Gamma)$, search $s(a, \epsilon, \lambda, \Gamma)$ and labor supply $h(a, x, \epsilon, \lambda, \Gamma)$. It also consists of a collection of quantities $\{K_t, L_t\}$ and prices $\{w_t, R_t\}$ and a law of motion of the distribution $\Gamma_{t+1} = T(\Gamma_t, \lambda_t)$ [7]

such that:

- given prices, households solve the maximization program in 5.3-1 and 5.3-3 and optimal policies derive;
- final goods firms maximize their profits;

$$w_t = K_t^{\alpha} {\lambda_t}^{1-\alpha} L_t^{-\alpha} \quad \text{and} \quad r_t = K_t^{-\alpha} {\lambda_t}^{1-\alpha} L_t^{1-\alpha}$$

- goods and factor markets clear;

$$\begin{aligned} Y_t + (1-\delta)K_t &= \int \mathcal{I}_{h=\bar{h}}(a'_w(a_t, \epsilon_t, x_t, \Gamma_t, \lambda_t) + c_w(a_t, \epsilon_t, x_t, \Gamma_t, \lambda_t)) \; d\Gamma_t \\ &+ \int \mathcal{I}_{h=0}(a'_n(a_t, \epsilon_t, \Gamma_t, \lambda_t) + c_n(a_t, \epsilon_t, \Gamma_t, \lambda_t)) \; d\Gamma_t \qquad (5.3\text{-}5) \\ L_t &= \int \epsilon x h_{a,\epsilon,x,\lambda,\Gamma} \mathcal{I}_{(h_{a,\epsilon,x,\lambda,\Gamma}=\bar{h})} \; d\Gamma_t \qquad (5.3\text{-}6) \\ K_t &= \int a_t \; d\Gamma_t \qquad (5.3\text{-}7) \end{aligned}$$

- and individual behavior is consistent with the aggregate behavior.

The equilibrium conditions are standard. Equation 5.3-5 is the resource constraint of the economy. Equation 5.3-6 represents the labor market equilibrium. Equations 5.3-7 represent

[7] The law of motion of the measure Γ can be represented as follows:

$$\begin{aligned} \Gamma'_e(\mathcal{A}, \mathcal{E}, \mathcal{X}) &= \int_{a'_e \in \mathcal{A}, \epsilon' \in \mathcal{E}, x' \in \mathcal{X}} \mathcal{I}_{h(a',\epsilon',x',\Gamma',\lambda')=\bar{h}} \; d\pi_{\epsilon'|\epsilon} d\pi_{x'|x} d\Gamma_e \\ &+ \int_{a'_n \in \mathcal{A}, \epsilon' \in \mathcal{E}, x' \in \mathcal{X}} \mathcal{I}_{h(a',\epsilon',x',\Gamma',\lambda')=\bar{h}} \; p(s(a, \epsilon, x, \Gamma, \lambda)) \; d\pi_{\epsilon'|\epsilon} \; dH(x') \; d\Gamma_n \\ \Gamma'_n(\mathcal{A}, \mathcal{E}) &= \int_{a'_e \in \mathcal{A}, \epsilon' \in \mathcal{E}} \mathcal{I}_{h(a',\epsilon',x',\Gamma',\lambda')=0} \; d\pi_{\epsilon'|\epsilon} d\pi_{x'|x} d\Gamma_e \\ &+ \int_{a'_n \in \mathcal{A}, \epsilon' \in \mathcal{E}} \mathcal{I}_{h(a',\epsilon',x',\Gamma',\lambda')=0} \; p(s(a, \epsilon, x, \Gamma, \lambda)) d\pi_{\epsilon'|\epsilon} dH(x') d\Gamma_n \\ &+ \int_{a'_n \in \mathcal{A}, \epsilon' \in \mathcal{E}} \mathcal{I}_{h(a',\epsilon',x',\Gamma',\lambda')=0} \; (1 - p(s(a, \epsilon, x, \Gamma, \lambda))) d\pi_{\epsilon'|\epsilon} dH(x') d\Gamma_n, \end{aligned}$$

where Γ_n and Γ_e denote the marginal cdfs for non-employed and employed workers respectively and $\mathcal{A}$ $\mathcal{E}$ $\mathcal{X}$ are subsets of the relevant state space.

the capital market equilibrium.

5.3.2 Couple households economy

We introduce households that consist of two members in the economy retaining as many elements from the singles environment as possible. In particular we have a measure one of agents (so a total mass of one half of households) and each one of them is endowed with a unit of time. Household members derive utility from the consumption and the felicity function is given again by the general form $u(c_t)$. We denote the time preference parameters for households in this case by β_C.

As far as intra-household allocations are concerned we adopt the unitary model whereby the household as a whole is treated as a decision unit and the members share the same common utility function. Income and wealth are pooled and consumption and labor supply or search decisions are formed jointly to maximize the households' well being. Each agent in the economy has her own idiosyncratic productivity and consequently household members differ in their productive endowments and we denote by ϵ_t and x_t the vector of productivities of the members of a generic household. To economize on notation, we let $\Pi_{\epsilon'|\epsilon}$ be the joint cdf for the household members' own productivities.

Having labor supply decisions formed jointly within households that are comprised of two members gives rise to opportunities of specialization in market and non-market work that were absent in a world of bachelor agents . Ideally a household would like to have at any point in time, the most productive agent in the market but it cannot do so without confronting the frictions that impede instantaneous transitions across labor market states. In what follows we adopt the convention that the array (k, l) $k, l \in \{E, N\}$ denotes a household whose first and second member are in states k and l respectively. Also, it will prove useful to define the following objects beforehand:

$$Q^{en} = \max\{V^{nn}, V^{en}\} \tag{5.3-8}$$
$$Q^{ne} = \max\{V^{nn}, V^{ne}\} \tag{5.3-9}$$
$$Q^{ee} = \max\{Q^{en}, Q^{ne}, V^{ee}\} \tag{5.3-10}$$

These objects define the relevant menu of choices for our households. For instance a household with one employed member can in any given period decide to withdraw her from the labor market and allocate both agents to search. This option is described in equation 5.3-8. Analogously in equation 5.3-10 a household with both members employed, can withdraw them to non-employment, or keep one working or both. With these definitions we can represent the dynamic programming problem of a household with two non-employed members as:

$$
\begin{aligned}
V^{nn}_{a,\epsilon,\lambda,\Gamma} &= \max_{a' \geq \bar{a}, s_1, s_2} \; u(c_t) - \sum_i k(s_i) \\
&+ \beta_C \Big(\int_{\epsilon',\lambda'} p(s_1)p(s_2) \int_{x'_1,x'_2} Q^{ee}(a',\epsilon',x'_1,x'_2,\lambda',\Gamma')dH(x'_1),dH(x'_2) \\
&+ p(s_1)(1-p(s_2)) \int_{x'_1} Q^{en}(a',\epsilon',x'_1,\lambda',\Gamma')dH(x'_1) \\
&+ p(s_2)(1-p(s_1)) \int_{x'_2} Q^{ne}(a',\epsilon',x'_2,\lambda',\Gamma')dH(x'_2) \\
&+ (1-p(s_2))(1-p(s_1))Q^{nn}(a',\epsilon',\lambda',\Gamma')d\pi_{\epsilon'|\epsilon}d\pi_{\lambda'|\lambda}\Big) \qquad (5.3\text{-}11)
\end{aligned}
$$

subject to:

$$
a' = R_{\lambda,\Gamma}a - c. \qquad (5.3\text{-}12)
$$

Optimal choices for these agents consist of current consumption and a pair of search intensity levels. Note that nothing precludes household members from setting $s_i \neq s_j$, although with standard convexity assumptions this can only be the case if the productivity endowments ϵ_i and ϵ_j are unequal. Further on, with probability $p(s_1)p(s_2)$ both members receive an offer and the sampling from the distribution of qualities $H(x)$ is independent. Both, joint search coupled with the limited availability of job opportunities, and the independent sampling introduce risk sharing possibilities to households through adjustments of labor supply that were nonexistent in the singles economy.

The lifetime utility for a household with the first member employed solves the following functional equation:

$$
\begin{aligned}
V^{en}_{a,\epsilon,x_1,\lambda,\Gamma} &= \max_{a' \geq \bar{a}, s_2} \; u(c_t) - k(s_2) - \Phi(\bar{h}) \\
&+ \beta_C \Big(\int_{\epsilon',\lambda'} (p(s_2) \int_{x'_1,x'_2} Q^{ee}(a',\epsilon',x'_1,x'_2,\lambda',\Gamma')d\pi_{x'_1|x_1}dH(x'_2) \\
&+ (1-p(s_2)) \int_{x'_1} Q^{en}(a',\epsilon',x'_1,\lambda',\Gamma')d\pi_{x'_1|x_1})d\pi_{\epsilon'|\epsilon}d\pi_{\lambda'|\lambda}\Big) \qquad (5.3\text{-}13)
\end{aligned}
$$

subject to

$$
a' = R_{\lambda,\Gamma}a + w_{\lambda,\Gamma}\bar{h}x_1\epsilon_1 - c. \qquad (5.3\text{-}14)
$$

For the sake of brevity we omit the value function V^{ne} since the recursive representation is similar to that of equation 5.3-13. Finally, for a household with both members employed we can write:

$$V^{ee}_{a,\epsilon,x_1,x_2,\lambda,\Gamma} = \max_{a' \geq \overline{a}} u(c_t) - \sum_i \Phi(\overline{h}) + \beta_C \Big(\int_{\epsilon',\lambda'} \Big(\int_{x'_1,x'_2} Q^{ee}(a',\epsilon',x'_1,x'_2,\lambda',\Gamma') d\pi_{x'_1|x_1} d\pi_{x'_2|x_2}\Big) d\pi_{\epsilon'|\epsilon} d\pi_{\lambda'}\Big) \tag{5.3-15}$$

subject to

$$a' = R_{\lambda,\Gamma} a + w_{\lambda,\Gamma} \overline{h} \sum_i x_i \epsilon_i - c. \tag{5.3-16}$$

The definition of a competitive equilibrium is similar to the one in section 5.3.1 and for the sake of brevity is omitted here.

5.3.3 Shocks and search technology

Our model builds on Chang / Kim [2007] and Gomes et al. [2001] who assess the labor market implications of models with heterogeneous agents and aggregate uncertainty. There, as well as in our case, the distribution of match (job) rents is governed by the idiosyncratic productivity endowments. And, according to their realizations agents adjust their labor market status in each period. Our model goes beyond that by adding the following features: we introduce both own productivity shocks ϵ and match quality shocks x. And we assume that search in the labor market is subject to a technology that maps search effort s into arrival rates of job offers $p(s)$.

Shocks

We introduce this rich structure of shocks for two reasons: First, decomposing the overall labor market risk into these two processes seems to be empirically relevant since in the data firm effects as well as individual effects both account for substantial fractions of the individual earnings uncertainty.[8] Second, we want to match the worker flows reported in tables 5.2-2 and 5.2-3. Since our model has to disassociate the behavior of agents who make frequent transitions between employment and unemployment from those who move in and out of the labor force it is imperative that we introduce both own productivity and match quality shock. For instance in our calibration we choose the moments of the two processes in such a way that the transitions between unemployment and employment are governed by the x type shocks and those between unemployment an inactivity by the ϵ type shocks.

[8] See Abowd et al. [1999].

Search technology

We adopt a very parsimonious representation of the search technology. In particular we assume that there are two levels of search intensity that a worker can exert $s \in \{s_I, s_U\}$ where the subscripts I stands for inactive agents, i.e. those out of the labor force and the subscript U stands for unemployment agents, i.e. active job searchers. Associated with these choices are the following probabilities of receiving a job offer next period:

$$p(s) = \begin{cases} p_I & \text{if } s = s_I \\ p_U & \text{if } s = s_U \end{cases}$$

The search costs are assumed to be of the form: $k(s) = 0$ if $s = s_I$ and $k(s) = k$ if $s = s_U$. These discrete choices are enough to capture our division between workers who search actively, and hence are counted as unemployed, and those whose optimal choice of search does not translate into a large enough contact rate with potential employers and hence are considered to be out of the labor force. Adding more thresholds would in general complicate things for us by requiring that the model be consistent with a larger set of targets. For instance, if we included two thresholds of search for inactive workers we would have to make the model match the populations of agents who don't search at all (and this is a large fraction of respondents in the CPS) and those who do search albeit in a passive way. We do not believe that these considerations are important and that they would alter our results. Notice that there is in general nothing that precludes us from setting $p_I = 0$. But in order to match the observed flows from inactivity to employment in our model's horizon it must be that $p_I > 0$.

We give the following interpretation to our technology: p_U and p_I are treated as technological upper bounds to the number of matches that are possible each period from states U and I respectively. When we increase the values of these parameters we also need to increase the variance of the x shocks to keep the transition rates close to the data, since by the standard intuition a mean preserving spread in a match quality distribution would make searchers more selective.

Generally, we set $p_U < 1$ in all our calibrations. The reason being that with limited job availability we want to give couples meaningful insurance opportunities against unemployment spells. Further on, these bounds must not be too tight since in our model these probabilities are constant over the business cycle. That is, we do not assume exogenous changes in this probability. If we were to set $p_U = .28$, the steady state unemployment to employment (UE) transition rate in the data, there would be no room for an increase job finding rates when an expansion arrives, and unemployment in the economy would be counter-factually procyclical.

This last point merits some attention. If in our model the flows between labor market states were governed by the firms' willingness to create jobs over the business cycle, as for example in Mortensen / Pissarides [1994], then the probabilities p_I and p_U would change over time. However the implications of such a model would be no different from ours since search and matching models generate procyclical search intensity (so agents would flow from inactivity to unemployment) which is precisely what we want to avoid by introducing couple households. In addition, our model generates endogenous separations and job findings by virtue of the processes x and ϵ and the fixed cost of participation in the labor market. Whether firms bear the costs of investment in search, as is the case in Mortensen / Pissarides [1994], or workers as we assume here is completely irrelevant. The only thing that matters is how these investments change over the business cycle.

5.4 Calibration and Results

5.4.1 Parametrization

We briefly discuss our choice of parameters and functional forms. We adopt a period utility function of the form:

$$u(c_t) = \log(c_t).$$

Following Chang / Kim [2007] we set the disutility from working equal to $B\frac{\overline{h}^{1+\gamma}}{1+\gamma}$ and we normalize γ to unity.[9] Parameter B is chosen to target the average employment population ratio of 60 % in the data. Since we draw no distinction between male and female population in the economy we don't have to worry about matching the division of employment between these two demographics and we set the disutility of labor for a household that comprises of two employed members equal to $2B\frac{\overline{h}^{1+\gamma}}{1+\gamma}$. We do, however, show how the model fares in terms of the specialization of home vs market activity in primary and secondary earners against the data.

For the search technology we set $p_U = 0.5$ and $p_I = 0.1$ in our benchmark which, given the empirical labor market flows, are reasonable values. We also experiment with a model where p_U is set equal to 0.4. The cost of search for unemployed workers k is chosen to target the fraction of the population of nonemployed workers that are unemployed, i.e. those that set $s = s_U$. In turn in the US data the unemployment rate is 5.5 % over our sample period.

Given that the models horizon is one month, we fix the time preference parameter for couple households β_C to 0.995 and the depreciation rate δ to 0.0083. These values turn out to be consistent with an average (steady state) interest rate $R = 1 + r - \delta$ of 1.0041. The annual analogue is 5 %. The discount factor for singles β_S is chosen so that the resulting capital labor ratios (and hence the interest rates) in the two economies are equal.

The capital share α is calibrated to 0.33 and we assume that the employed agents spend roughly one third of their time endowment in market work $\overline{h} = 0.33$. Following Chang / Kim [2007] the aggregate TFP process is calibrated such that the quarterly first order

[9] This is unimportant in the current context.

autocorrelation is $\rho_\lambda = 0.95$ and the conditional standard deviation is $\sigma_\lambda = 0.007$. Table 5.4-5 summarizes these choices.

Finally our idiosyncratic labor productivity processes are of the following form [10]:

$$\log(x_t) = \rho_x \log(x_{t-1}) + v_{x,t}$$
$$\log(\epsilon_t) = \rho_\epsilon \log(\epsilon_{t-1}) + v_{\epsilon,t}$$

Table 5.4-5: The model parameters (quarterly values)

Parameter	Symbol	Baseline
std of TFP shock	σ_λ	0.007
AR1 of TFP shock	ρ_λ	0.95
Share of capital	α	0.33
Depreciation rate	δ	0.025
Discount Factor Couples	β_C	0.995
Fraction of time working	$\overline{h}$	0.33
Offer Rate: OLF	p_I	.1
Offer Rate: Unemployed	p_U	policy paratmeters
Labor Disutility	B	
Discount Factor Singles	β_S	
Search cost	k	jointly calibrated
Moments of x	σ_x, ρ_x	(see text)
Moments of ϵ	σ_ϵ, ρ_ϵ	

Our calibration procedure is as follows: For each of the models (singles, couples $p_U \in \{0.4, 0.5\}$), we choose the moments of the idiosyncratic productivity processes ρ and σ along with B and k to match the observed labor market flows. We have six parameters for six targets. We treat $p_U = 1/2$ as the economy where frictions are important gives rise to risk sharing opportunities against prolonged non-employment spells in the couple economy.

Solution method

We solve the model with aggregate uncertainty using the bounded rationality approach whereby agents forecast future prices using a finite set of moments of the distribution Γ_t. As in Krusell / Smith [1998], we find that 1st moments (means) are sufficient for very accurate forecasts in our context. This means approximate aggregation holds. A detailed description of the algorithm is delegated to the appendix.

[10] This choice is standard in the literature, see for example Heathcote et al. [2008]

5.4.2 Steady State Findings

We use this section to provide information on the models' performances in a number of relevant dimensions.

Table 5.4-6: Estimated Labor Market Flows: Singles vs. Couples

	Bachelor Households			Couples Households		
	E	U	I	E	U	I
E	.9507	.00432	.0450	.9515	.00567	.0428
U	.2801	0.5831	.1368	.2830	.5051	.2119
I	.0503	.0322	.9175	.0507	.0381	.9112

In Table 5.4-6 we summarize the estimated worker flows from the bachelor and the couple economy.[11] In both cases the decomposition between movements in and movements out of labor market states is such that the model output is consistent with an employment population ratio of 60 %, an unemployment rate of 5.5 %, and a outflow rate from unemployment to employment of 28 %. These values correspond to the respective values in the data.

Both models can match the total outflow from employment to non-employment. But the composition between the number of workers who leave their jobs to search intensively (unemployed) and those who leave their jobs but do not search is off target. In particular, in the data the EU rate is around 1.4% on average and the EI rate is 3.11% but even the couple economy produces values of 0.567% and 4.28% respectively. This in turn might suggest that our model already featuring two independent stochastic processes for labor income risk is yet too parsimonious to match some aspects of the data.

A striking difference in terms of the performance of the two models is the resulting UI flows. We find that the couple model economy can easily attain a target of 21%, which corresponds to the value found in the data, whilst with bachelor households the best we can do is a value of 14%. This discrepancy is at the center of our notion of joint insurance. In the steady state there is a large fraction of families where one member is employed and another not. For the non-employed members the choice of search intensity, and consequently the choice of their labor market status, is affected by their own productivity state and the composite productivity of their partners. Changes in household income in this case entail a wealth effect on the labor supply of non-employed agents which could induce them to drop out of the labor force.[12] In contrast, in a bachelor household economy this channel is absent and the two factors that determine the choice of labor market status are wealth and productivity. Since

[11] We use the CPS flows of table 5.2-2 as our targets, i.e the ones that refer to all agents aged above 16 independent of marital status. The reason is that we don't possess aggregate statistics (especially GDP) decomposed for different demographic groups. It would be meaningless to target say flows for married agents aged 22 to 55 as in table 5.2-3 and calibrate technology parameters (factor shares and Solow residuals) based on the estimates of an aggregate production functions

[12] The same applies when there are two searchers in the family and one of them receives a job offer. In most cases the second earner will drop out of the labor force.

wealth is run down in non-employment, productivity must be less persistent to match the data.

To see how specialization in market work respectively leisure is determined within the household consider the decomposition of inactivity and unemployment in the steady state summarized in Table 5.4-7. Roughly 35% of all agents who are out of the labor force in the economy live in households where both members are inactive and the remaining 65 % percent are in families where one member is either unemployed or employed. In the data the analogous fractions are 24 % and 76 % respectively for a population aged between 16 and 65 and 50 % for ages 16 and above. Clearly demographics play a significant role here but we think that our model strikes a good balance between the two samples in the data.

An important insight from table 5.4-7 is that the vast majority of the working age population in the US live together with someone who provides a market income. Models of labor force participation that incorporate only bachelor households cannot address this important feature.

Finally, insofar as the cross section of unemployed agents is concerned we observe that our model overestimates the fraction of agents that are part of households where both members are unemployed. Again this probably is symptomatic of the fact that independent shocks and identical agents exacerbate the role of insurance in the couple economy.

Table 5.4-7: Decompositions of Unemployment and Inactivity

Unemployed	**UU**	**UI**	**UE**
Bechmark Model	.2	.27	.53
US Data: Ages 16-65	.07	.19	.74
US Data: Ages >16	.1	.22	.68
OLF	**II**	**UI**	**IE**
Bechmark Model	.354	.026	.62
US Data: Ages 16-65	.24	.026	.734
US Data: Ages >16	.5	.02	.48

How readily can household members substitute in terms of their labor income?

To answer this question we look at the persistence of employment status over time in a cohort of agents simulated from the steady state distribution. We use a sample of 5000 families for the simulation. For each period we assume that a family's primary earner is the agent that had the highest recorded annual labor income. Annual horizons on the other hand serve to mitigate the effect of frictions on recorded employment histories. To uncover the persistence we simply estimate the Markov transition matrix of primary and secondary earners. This is the probability that the identity of the household head changes from one year to the next. Using this metric we find that roughly 30% of our families alternate roles as primary and secondary earners in the labor market each year. Furthermore, when we use the number of hours as our index, and drop productivity from the calculation we find that this rate decreases to 20%.

Arguably the employment status of agents is a much more persistent state, and the reason that our theory cannot match this aspect of the data is precisely that we put two ex ante identical agents within each household. In reality agents differ in fixed productivity and command different rewards in the labor market based on age, sex, and experience among other things. We can only do so much as to summarize some of these features in our two stochastic processes. However, our model requires low persistence in the ϵ risk to match the flows between inactivity and unemployment.

The implied stochastic processes and other calibrations

Since in our model idiosyncratic labor incomes confound risks from various sources (search frictions and the joint stochastic processes of productivity) the only way we can assess whether our choices are consistent with the data is by estimating the realized profiles of wages for individuals in our economy. We use a simple representation of the logarithm of annual (time aggregated) wages: $\ln w_t = \phi \ln w_{t-1} + v_t$ and use a sample of 10,000 individuals over 20 years to estimate the implied values for ϕ and the variance of the shock σ_v. Since in our model the distinction between household heads and secondary earners seems to be virtually irrelevant, given that we have two ex ante identical agents, we pool the estimates from all household members in the simulated population.

Both of these values are far away from the data. Our estimates are $\phi_S = 0.1$ for the singles economy and $\phi_C = 0.4$ for the couple economy. Notice that the value for couple economy is much closer to the high persistence process that is empirically relevant. Further on, there is a wealth of estimates for the data analogues for these statistics, see for example Heathcote et al. [2008], and all of them yield a value for ϕ in the neighborhood of 0.9.[13] Given that both of our models imply that labor income is less persistent than what is found in the data we conclude that only temporary components of shocks are important in matching the labor market flows. In the data for instance the UI flows are high at monthly horizons but their quarterly counterparts are much smaller indicating that there is a lot of temporary variations in search intensity for a large group of non-employed workers.

What happens when we increase the persistence of the process to match the data?

In this case the distinction between heads and spouses becomes more important in the sense that the most productive agents are allocated to market activities and less productive agents to leisure. When the value of ρ_ϵ is 0.7 (it is 0.3 in the baseline calibration of the couple economy) the UI flow becomes 0.14, which is what we get in the bachelor households economy,

[13] In Chang / Kim [2007] a model that accounts for selection effects yields a value for the persistence component of 0.73.

and the EU flow rate increases to 0.008. Thus the model clearly features a tradeoff between matching these two targets. The reason is that with a low persistence of the ϵ shock only match quality shocks x matter for employment decisions. The market allocation is such that on average equally productive, in the ϵ dimension, agents are employed and non-employed and substitution possibilities in the identity of the main earner of the household are ample. In contrast when shocks are persistent each agent's own productivity becomes an important determinant of his labor market status, and agents who loose their jobs remain in the labor force to get a new draw next period. We use this alternative calibration below to contrast the cyclical properties of the two economies, one where insurance is more (when ρ_ϵ is low) or less meaningful.

What happens when we change the p_U parameter?

With tighter frictions $p_U = .4$ we need to recalibrate the stochastic processes to match the worker flows. Joint search becomes more important for couples that want to maximize the probability of receiving a job offer but also unemployment is a less attractive state since the duration of spells is higher. It turns out that the properties of the calibrated shocks are similar (lower volatilities of the two shocks are required since frictions are tighter) to the baseline in this case. The implications of this model version are no closer to (or further from) the data than our benchmark. In section 5.4.3, however, we treat this case as an economy that helps us understand whether the extend of frictions matters for our results.

5.4.3 Cyclical properties

Table 5.4-8 presents the results from our baseline calibration with $p_U = 0.5$ and $p_I = 0.1$ for both the couple and the bachelor household economies. We restrict attention to key labor market statistics and all quantities are expressed relative to a detrended measure of GDP.[14] The data are quarterly aggregates of the simulated aggregate paths.

In the single economy unemployment is extremely procyclical (contemporaneous correlation with GDP is 0.65) and so is the labor force. It is clear that this is due to the joint impact of search frictions and intertemporal substitution. When an expansion occurs agents flow in the labor force since job opportunities are relatively more attractive. Due to the existence of frictions the reallocation of these workers to employment takes time and the pool of unemployed searchers increases.

The couple model (columns 3-4) produces a different set of statistics. Unemployment now becomes acyclical. The contemporaneous correlation with GDP is near zero. In addition, unemployment also becomes more volatile than with bachelor households. It is closer to the data, even though it is still not high enough. Aggregate employment is more volatile and

[14] They are logged and HP filtered with a parameter $\lambda = 1600$.

equally procyclical and the labor force is more volatile and only marginally less procyclical than in the previous case.

Table 5.4-8: Results with search friction p=0.5

	Bachelors		Couples Benchmark		Couples Calibration 2	
	$\frac{\sigma_x}{\sigma_y}$	$\rho_{x,y}$	$\frac{\sigma_x}{\sigma_y}$	$\rho_{x,y}$	$\frac{\sigma_x}{\sigma_y}$	$\rho_{x,y}$
Unemployment	1.78	.65	2.7	.02	3.5	-.15
Employment	0.54	0.96	.85	0.97	.77	.96
Labour Force	0.32	0.97	.62	0.93	.41	.9

These results can be explained by the following observations: Firstly, our calibration is such that the idiosyncratic process of labor productivity is considerably more volatile in the bachelor household economy. In this case aggregate shocks don't have an important effect in guiding individuals optimal decisions. Although the family in the couple model, may face a comparable amount of uncertainty overall, there are more instruments, the additional labor supply margin, to hedge and not all of this risk is important for individual labor supply decisions which are formed primarily on the basis of an agent's own productivity process. Hence cyclical volatilities are indeed lower in the bachelor economy.

Secondly, the distribution of agents across the relevant state space differs considerably in the two models. For instance in the bachelor household economy inactive agents have accumulated sufficient wealth to finance leisure, whilst in the couple economy there is a large fraction of households where wealth is low and one member is employed and the other one inactive. Therefore, in the former case a smaller fraction of agents is induced to participate in the labor force when the expansion arrives and hence the cyclical volatility is lower.

Although our couple model engineers a lower contemporaneous correlation of the labor force participation with GDP, the difference is rather small. We anticipated that the joint insurance would induce inactive agents to flow into the labor force in bad times to minimize the duration of non-employment spells but this does not seem to happen in our baseline calibration. The result suggests that there is indeed a wealth effect from employed household members to non-employed ones which reduces their desired supply of labor (when wages rise consumption of non employed members rises since risk sharing is perfect within the family) but this is overwhelmed by the motive to increase search intensity in view of the higher returns in expansions.

Alternative calibrations

Columns 5-6 of table 5.4-8 present the outcome of an alternative calibration in which the ϵ shocks are more persistent (we set $\rho_\epsilon = 0.7$). There unemployment is more counter-cyclical since the household allocates the most productive member to the market and this member's labor market status is more persistent. Adjustments in family labor supply take place by withdrawing productive agents from employment in recessions and placing them in the pool of unemployed until aggregate conditions improve. In equilibrium the allocation entails fewer movements in and out of the labor force and hence the labor force participation volatility drops. But there is only a minor improvement in the correlation of the latter quantity with GDP. Notice that when these flows become zero, which is equivalent to having a fixed labor force, the model should have no difficulty in matching the data. But, off course, this will have nothing to do with joint insurance in families.

Further on when we lower p_U, thus making job opportunities more scarce we find that the model's implications are not too different compared to our baseline. Unemployment is slightly more procyclical in both the singles and the couple economies since now workers that enter the labor force in expansions encounter tighter frictions and spend more time in unemployment and the cyclical properties of the labor force are similar to the benchmark. In this economy the idiosyncratic uncertainty faced by agents is lower, we decrease σ_ϵ and σ_x to hit our targets since job opportunities are limited exogenously and reservation wage policies are not important to match the average job finding rate in the economy. This seems not to make a difference for the results and we conclude that the way frictions are modeled is irrelevant here.

Other Models

Can insurance markets within the household be the answer to the low procyclicality of the labor force? Certainly the evidence presented in section 5.2 seem to suggest that joint insurance is important. But our model cannot yield reasonable elasticities. We use the following paragraphs to investigate what key ingredients need to be added to the framework to bring the statistics closer to their empirical counterparts.

In the discussion of section 5.4.2, we saw that the model features too much insurance possibilities due to the fact that agents are ex ante identical and shocks are not persistent enough and joint labor supply adjustments can effectively remove a large fraction of the underlying risk. In reality, however, household members differ in permanent components of earnings and allocations to market activities, which member works and which does not, is much more persistent. By calibrating the earnings risk to match the worker flows we have taken our economy away from a realistic account of family structure but there is no reason to anticipate that this will be crucial for our results. Search intensity would continue to be procyclical for the marginal worker and so would be the labor force.

A more promising avenue is to reconsider the nature of labor market risks in our model. Since separations and job finding rates represent partly risks and partly choices, movements in and out of employment are the outcomes of endogenous decisions which makes the joint insurance channel we highlight less meaningful. In our model workers sample their own productivity and the match quality each period and determine their labor supply. But in reality there are involuntary aspects associated with job losses, which our modeling of frictions does not allow us capture. This is a possibility that we address in future work. In a companion paper we analyze a model where frictions play a dominant role. They are summarized by a search technology as here. However, there are no match quality shocks. We experiment with a model where both job finding probabilities and job separations are exogenous, and in some cases vary exogenously over the business cycle. We document the differences in the search behavior of the couple and the single economies.

Finally another important extension is to incorporate a departure for complete insurance within the household unit. There is a lot of empirical evidence against that model, see for example Chiappori [1988] and Bourguignon et al. [2009]. In our model allocations in the household unit feature too much risk sharing since we use the unitary framework but if this assumption is abandoned (in favor of a limited commitment or collective alternative) it is not so clear how the equilibrium quantities will be affected.

5.5 Conclusion

In this paper we contrast the properties of economies where lack of insurance possibilities means that agents stand alone against uncertain contingencies with those where risk sharing opportunities exist within households that are comprised of two members. We ask how the implications for the labor market in an otherwise standard incomplete market model with search frictions and endogenous labor force participation are affected depending on the structure of the household, and especially how the bachelor and the couple economy respond to fluctuations in aggregate productivity.

We find that with these ingredients the economy with bachelor agents produces counterfactual cyclical correlations and volatilities for key labor market statistics, upon which the couple model is able to improve. However, the cyclical properties of the labor force still lie away from the data.

Admittedly, there is a number of dimensions in which our theory is incomplete. Our framework abstracts from many important features in the sense that we have two ex ante identical agents in the household that differ in their labor market productivity. In reality some components of the differences in productivity of household members are permanent and this generally reduces risk sharing opportunities relative to what our findings suggest. Furthermore, agents command different rewards in the labor market based on their age, gender, experience etc or even idiosyncratic risks may be correlated. We think of our economy as one that simply allows for complete insurance markets within the household and incomplete outside.

Patterns of specialization in work versus leisure and in general intra-household allocations have a large impact on equilibrium allocations and thus far these features have been cast aside from macroeconomic theory. Our model can be viewed as a necessary step to a more ambitious research agenda. We do not have a clear understanding of how allocations are affected in economies where insurance is abundant in the family. What are the implications of having one, two or even an infinity, as in the social planning economies, family members? There are many policy or welfare related questions where these alternative environments produce different answers. For instance, in incomplete market models with bachelor households wealth encodes the history of productivity and those agents who build up a stock of wealth can finance leisure and drop out of work. In contrast in social planning economies most productive agents are always send to work. We suspect that allocations in couple economies will be somewhere in between these two extremes.

5.6 Appendix

5.6.1 Computational strategy for steady-state equilibrium

In steady state, factor prices are constant and the distribution of agents over the relevant state space Γ is time invariant. The calibration consists of three nested loops. The outer loop is the estimation loop where we set the endogenous parameters $\{B, k, \rho_\epsilon, \sigma_\epsilon, \rho_x, \sigma_x\}$. We solve the model and check whether the generated moments (labor market flows) are close enough to their empirical counterparts. If not, we try a new set of parameters.

The middle loop is the market clearing loop. We guess an interest rate r which implies a wage rate w and then solve for the value functions and the steady state distribution Γ). The steady sate distribution yields an aggregate savings supply. If the implied marginal product of capital is equal to the guessed interest rate, we found the equilibrium. If not, we update our interest rate guess. For the singles version of our model instead of changing interest rates to clear the market of savings we adjust the discount factor β_S and keep constant the aggregate rate of return R. The inner loop is the value function iteration. Details are as follows:

1. We choose an unevenly spaced grid for asset holdings (a) (with more nodes near the borrowing constraint) and a grid for individual productivities ϵ and x. We experiment with different numbers of nodes for the asset grid, usually between $N_a = 101$ and $N_a = 161$.The number of nodes for the idiosyncratic labor market risks are $N_\epsilon = 5$ and $N_x = 2$. These are equally spaced and the transition matrix of idiosyncratic shocks is obtained by the discretization procedure described by Adda / Cooper [2003].

2. Given our guess for the interest rate r, we solve for the individual value functions, V^n, V^e in the bachelor model and V^{nn}, V^{en}, V^{ee} in the couple model. This is done by finding the optimal savings and search intensity choice at each node. Values that fall outside the grid are interpolated with cubic splines. Once the value functions have converged we recover the optimal policy functions of the form $a'(a, \epsilon)$, $s(a, \epsilon)$ and $h(a, \epsilon)$.

3. The final step is to obtain the invariant measure Γ over the relevant state space (asset productivities and employment status).

    a) We first approximate the optimal policy rules on a finer grid which $N_{a_{BIG}} = 2000$ nodes and we initialize our measure Γ_0.

    b) We update it and obtain a new measure Γ_1

    c) The invariant measure is found when the maximum difference between Γ_0 and Γ_1 is smaller than a pre-specified tolerance level.

    d) By using the invariant measure, we compute aggregate labor supply and asset supply. This implies a new marginal product of capital which we then compare to our initial guess.

5.6.2 Computational strategy for equilibrium with aggregate fluctuations

Aggregate shocks imply that factor prices are time varying. When solving their optimization program agents have to predict future factor prices. Therefore they have to predict all the individual policy decisions in all possible future states. This requires agents to keep track of every other agent. Thus in order to approximate the equilibrium in the presence of aggregate shocks, one has to keep track of the measure of all groups of agents over time. Since Γ is an infinite dimensional object it is impossible to do this directly. We therefore follow Krusell / Smith [1998] and assume that agents are boundedly rational and use only the mean of wealth and aggregate productivity to forecast future capital K and factor prices w and R.

Compared to the steady-state algorithm we now have two additional state variables that we must add in the list of the existing state variables in the inner loop: aggregate productivity λ and aggregate capital K. As the outer loop, we iterate on the forecasting equations for aggregate capital and factor prices.[15] The details are as follows:

1. We approximate the aggregate productivity process with 2 nodes and use again the methodology of Adda / Cooper [2003] to obtain the values and transition probabilities. We choose a capital grid around the steady-state level of capital $K^s s$, particularly we $N_k = 6$ equally spaced nodes to form a grid with range $[0.95 * K^{ss}; 1.05K^{ss}]$.

2. As already mentioned, we choose the means of aggregate capital and aggregate productivity as explanatory variables in the forecasting equations. We use a log-linear form

$$ln\, K_{t+1} = \kappa_0^0 + \kappa_1^0 ln\, K_t + \kappa_2^0 ln\, \lambda_t \qquad (5.6\text{-}1)$$
$$ln\, w_t = \omega_0^0 + \omega_1^0 ln\, K_t + \omega_2^0 ln\, \lambda_t \qquad (5.6\text{-}2)$$
$$ln\, R_t = \varrho_0^0 + \varrho_1^0 ln\, K_t + \varrho_2^0 ln\, \lambda_t \qquad (5.6\text{-}3)$$

3. We initialize the coefficients so that K_{t+1}, w, R are equal to their steady state values.

4. Given the forecasting equations, we solve the value function problems as before, just that now the state vector is four-dimensional. Values that are not on the asset grid are interpolated using cubic splines. Values that are not on the aggregate capital grid are interpolated linearly.

5. Instead of simulating the economy with a large finite number of agents we use the procedure of Young [2010] and simulate a continuum of agents. This procedure has the advantage of avoiding cross-sectional sampling variation. We simulate the economy for 10,000 periods and discard the first 2,000. In each period we get an observation for K, w and R. We use the simulated data to run OLS regressions on the forecasting equations which yield new coefficient estimates κ^1's, ω^1's, ϱ^1's. If these coefficients are

[15] In the steady state algorithm, there were three loops. Since we use the steady state values for the endogenous parameters, we do not have an estimation loop here.

close to the previous ones we stop, otherwise we update the forecasting equations with the new coefficients and solve the problem again.

The convergent solutions for the forecasting equations our models are as follows:

Table 5.6-1: Couples $p = 1/2$

Equation	**Constant**	$\ln(K_t)$	$\ln(\lambda_t)$	R^2
$\ln(K_{t+1})$	0.0841	0.9802	0.0379	0.9999
$\ln(w_t)$	-0.59839	0.4304	0.4598	0.9917
$\ln(R_t)$	0.0504	-0.0108	0.0136	0.9874

Table 5.6-2: Singles $p = 1/2$

Equation	**Constant**	$\ln(K_t)$	$\ln(\lambda_t)$	R^2
$\ln(K_{t+1})$	0.0734	0.9820	0.0303	0.9999
$\ln(w_t)$	-04691	0.4427	0.4766	0.9805
$\ln(R_t)$	0.0387	-0.0090	0.0107	0.9714

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If available for a reference, the DOI-number (DOI = Digital Object Identifier) is listed. Digital Object Identifiers are similar to ISBN-numbers for books. They are stable over time and provide in the case of academic journals a way to link to the publisher-website associated with a published paper. The link between a doi and a website is provided by http://dx.doi.org/. If you are reading the pdf-Version of this document, the doi-entries are clickable. If a doi does not exist, but a stable URL exists, the URL is given.

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