The London School of Economics and Political Science

Essays on CEO Compensation

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Declaration

I certify that the thesis I have presented for examination for the PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).

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Abstract

This thesis analyzes CEO compensation contracts in a principal-agent framework with moral hazard. It focuses on two issues: the form and the timing of performance-based pay. On the one hand, if CEOs are assumed to be mean-variance maximizers, I show that it is suboptimal to provide incentives with contracts which are convex in performance. This is because these contracts make the variance of pay an increasing function of the CEO's effort, which is inefficient. Sticks are more efficient than carrots, although the latter may be used in case the agent is protected by limited liability. On the other hand, if CEOs are assumed to be not only risk averse but also prudent, convex contracts and rewards may be optimal, since they protect against downside risk. A calibration of a HARA-lognormal model shows that CEO preferences which minimize the suboptimality of the typically observed contracts (relative to the optimal contract) feature decreasing absolute risk aversion, as well as low and decreasing relative risk aversion. However, when CEO pay is contingent on a lognormally distributed stock price, it is hard to rationalize the use of convex contracts for incentive provision. The thesis then examines the optimal evaluation and payment date, when the CEO's actions materialize with a lag. Information asymmetries are progressively resolved: the precision of signals that shareholders receive regarding the final outcome is increasing with time. However, the accumulation of exogenous shocks make deferred compensation noisy. The optimal timing of CEO pay, which minimizes the extent of the mispricing at the payment date, is derived. Opportunities for two types of managerial short-termism are then introduced. To ensure that the manager does not engage in short-termist and inefficient behavior, it is often optimal to reduce the power of incentives, and to postpone the evaluation and payment date.

Contents

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Chapter 1

Introducing concavity in managerial compensation

In a moral hazard setting with additive effort and noise, symmetrically distributed noise, and an agent with mean-variance preferences, it is shown that any convex contract is dominated by a concave contract, and any undominated contract is capped. This result calls into question the widespread use of stock-options as an incentive instrument for risk averse executives. It is more efficient for the compensation contract to involve a short position in puts rather than a long position in calls. Additionally, with limited liability and step contracts, the limited liability constraint is binding at the optimum. The intuition is that it is preferable to minimize the variance of the agent's remuneration conditional on a high effort, and to maximize it conditional on a low effort. This maximizes the punishments associated to low effort, and the rewards associated to high effort. Overall, the tradeoff between risk sharing and incentives is less severe when the variance of compensation is decreasing in effort. Since low effort will not occur in equilibrium, an incentive mechanism relying on punishments is compatible with relatively low observed pay-performance sensitivities.

"The story behind the growth of pay in the 1990s is really the story of the option. (...) The chief mistake of the past 15 years was the granting of too many share options.", *The Economist*, Special report on executive pay, January 20th 2007.

This paper analyzes the optimal form of a risk averse agent's compensation profile under moral hazard, and most notably its curvature.[1] In order to exclusively focus on the effect of risk aversion on contract design, and identify the sort of contracts which are efficient at motivating an agent who is averse to the variance in his income, we assume that the agent has mean-variance preferences. In this setting, the main result is that sticks are more efficient than carrots. Dismissals without severance payment, reputation losses, even prison sentences, and other forms of harsh punishments for poor performances seem to be appropriate for eliciting effort from risk averse agents. Although many papers have recognized the superiority of punishments over rewards in specific settings, the contribution of this paper is to isolate and highlight the impact of the agent's risk aversion on the optimal structure of his incentives in a general setting.

There are two ways to induce a risk averse agent to exert effort when increasing effort increases the probability of a high performance more than it increases the probability of a low performance. First, making compensation increasing in the performance measure ensures that more effort increases the expected pay. Second, making compensation concave in the performance measure ensures that increasing effort decreases the variance of pay: this is the "variance effect". In view of these two effects, pay-performance sensitivities should be positive but decreasing in the performance measure: sticks are more effective than carrots. Relative to a linear contract, and assuming that the most informative states are at the tails of the distribution rather than in the middle, contingent pay should relatively increase in the uninformative states, and relatively decrease in the more informative states. The idea is to provide insurance in the performance region more likely when high effort is exerted, and to highly expose to risk in the performance region more likely when low effort is exerted.

A major implication of this paper's results is the suboptimality of stock-options.[2] It may

[1]It is assumed that all incentives are provided by means of contingent pay. Admittedly, incentive mechanisms can take other forms. Therefore, contingent pay at any given performance should be interpreted as the certainty equivalent of the present value of present and future gains or losses following this performance.

[2]Two important papers estimate that the optimal strike (or exercise price) of stock-options is approximately zero - which implies that granting restricted stocks is preferable to granting stock-options. Hall and Murphy (2002) define incentives as the derivative of an undiversified and risk-averse executive's valuation of the option with respect to the stock price - thus confining incentives to local pay-performance sensitivity. Consistent with this paper's results, they find that "in all cases, the exercise price that maximizes incentives is zero" when stock-options enter the agent's participation constraint - as they should in an efficient contracting framework. Dittmann and Maug (2007) calibrate a CRRA-lognormal model to a sample of U.S. CEOs. They find that

not be coincidental that private equity funds, renowned for their competence in aligning the manager's interest with the owner's, seldom use stock-options to provide incentives. Instead, they prefer to grant restricted stocks, thereby making managerial remuneration linear in the performance measure.

Another major implication of this paper is that observed pay-performance sensitivities are a poor measure of incentive provision. Following the seminal paper of Jensen and Murphy (1990), and as documented in Murphy's review (1999), the literature has tended to equate incentives with the sensitivity of the manager's pay to his firm's performance - thereby neglecting the incentives potentially provided through changes in the variance of compensation. However, for a risk averse agent, concavifying the compensation profile when the MLRP holds reduces the average pay-performance sensitivity required in equilibrium to induce a given level of effort. In this sense, the average pay-performance sensitivity and the concavity of the compensation profile are at the margin substitutable ways of inducing effort: for a given level of effort, making the compensation profile more concave translates into a diminution of the average pay-performance sensitivity. This may contribute to explaining both the apparent paucity of explicit incentive schemes and low observed pay-performance sensitivities, as observed by Jensen and Murphy (1990) and many others. If incentives are primarily provided through punishments for poor performances, and since a high effort is likely to result in a good performance, it should be acknowledged that incentives can be strong even if pay-performance sensitivities for good performances are low. In other words, low pay-performance sensitivities as measured in equilibrium do not indicate that few incentives are provided.[3] In this regard, the widespread use of stock-options and the associated rise in pay-performance sensitivities throughout the 1990s may indicate a deterioration in the efficiency of compensation rather than an increase in the amount of explicit incentives provided.

This paper derives properties of efficient incentive structures.[4] Under fairly general condi-

optimal unconstrained contracts feature high stock holdings and *negative* call option holdings for 98.7 percent of all CEOs in their sample. That is, compensation profiles are concave, and equivalently combine short positions in puts with long positions in stocks. Consistent with the results of this paper, they also find zero pay-performance sensitivities for high stock prices to be optimal for "a large number" of CEOs.

[3]All the more that a survival bias may be at play: we do not observe the pay-performance sensitivity of bad CEOs who are terminated and do not find another CEO job, whereas good CEOs keep on "enjoying" a low pay-performance sensitivity.

[4]This article overlooks the issue of selecting the optimal effort level. It focuses on efficient contracting, which consists in minimizing the agency cost of implementing a given level of effort. Since we do not know what form efficient contracts take in this setting, it merely shows that certain classes of contracts are dominated. Doing so disqualifies commonly used contracts (in theory and in practice), and restricts the set of contenders for efficient contracts.

tions, the model shows that it is suboptimal for the variance of compensation to be increasing in effort at the equilibrium effort. This implies that convex contracts are dominated.[5] Besides, in line with rewards being a poor instrument to elicit effort, compensation should be capped. This notably implies that linear contracts are suboptimal.

With options, contracts based on a short position in a put option are more efficient than contracts based on a long position in a call option (i.e., stock-options). We show that a contract taking the form of a short position in a put option with an arbitrarily small exercise price is approximately optimal when the performance measure is normally distributed. With step contracts characterized by two levels of payments and with a lower bound on payments, we find that the low payment is equal to the lower bound at the optimum. In the limit, with an arbitrarily low lower bound on payments, we show that the optimal contract is the Mirrlees (1975) approximation of the optimal contract. The fact that punishments make the variance of pay a decreasing function of effort makes them more efficient than rewards. However, for a sufficiently high lower bound on payments, we also find that it is impossible to use punishments for incentive purposes without giving a rent to the agent. In this case, it may be optimal to use rewards instead. Contracts with rewards features may thus be constrained-efficient.

Some papers have already recognized that punishments are more efficient than rewards to incentivize risk averse agents. Jewitt (1988) shows that the superiority of punishments over rewards in a monitoring problem is driven by properties of the utility function. Mirrlees (1975) proposes a contract based on extreme punishments to approximate the first-best in a moral hazard problem when the agent is risk averse. The contribution of this paper is to isolate and highlight the impact of the agent's aversion to the variability of his pay on the optimal structure of his incentives in a simple but general setting.

Other papers argue that rewards are more efficient than punishments: as shown notably in Innes (1990) and in the baseline model in Tirole (2006), it is optimal to offer a convex, call-option like contract to a risk-neutral agent protected by limited liability. The risk aversion models of moral hazard and the risk-neutrality-limited liability models of moral hazard thus tend to generate different predictions regarding the form of the optimal contract. Punishments can be optimal with the former, while rewards are generally optimal with the latter. Whereas it

[5]However, keep in mind that the results of this paper apply to the whole compensation package, not to one compensation instrument in isolation. In particular, a monetary contract only captures a limited part of rewards and punishments. An agent can miss a promotion, lose his job or tarnish his image, which steeply decreases the present value of his future revenues, to the point where this loss can even surpass what he earned on the job. The limited liability constraint is all relative. The shape of the set of mostly non-monetary punishments for bad performance may resemble a short put on the performance measure. The agent can therefore suffer huge losses for low performances, even if low monetary transfers are not permissible.

is well-understood why rewards are optimal with limited liability and risk neutrality, this paper contributes to explaining why punishments may be optimal with risk aversion. While some of the reasons were implicit in Mirrlees (1975), we make them more explicit.

Section 1 presents the model. Section 2 shows that convex contracts and uncapped contracts are dominated. Section 3 compares calls and puts-based compensation. Section 4 studies step contracts. Section 5 concludes.

1.1 The model

Consider the following moral hazard problem. At time 0, an agent chooses his unobservable effort e in the compact set $[0, \bar{e}]$, where $\bar{e}$ is finite, at cost $c\frac{e^2}{2}$. A risk neutral principal designs a compensation contract to induce the agent to exert the finite effort $e^\star \in (0, \bar{e})$, bounded away from zero, at the lowest possible cost.

The contractible performance measure, realized at time 1, is

$$\tilde{\pi} = e + \tilde{\epsilon}$$

The variable $\tilde{\epsilon}$ is realized at time 1, but is unobservable. It is distributed according to the continuous probability density function (p.d.f.) φ, which is symmetric around zero, and the associated cumulative distribution function (c.d.f.) Φ. The mean of $\tilde{\epsilon}$ is normalized at zero and its variance is σ_π^2. Denote the p.d.f. of $\tilde{\pi}$ by ϑ.

The compensation contract, offered to the agent at time -1, maps any realization of $\tilde{\pi}$ into a payment to the agent at time 1. This payment writes as[6]

$$W(\pi) = w + f(\pi)$$

Unless otherwise specified, we only consider compensation profiles such that f is differentiable twice with respect to $\tilde{\pi}$. A function f satisfying these requirements is called an "admissible" contract. Assume that the probability distribution is such that any optimal compensation profile is nondecreasing in the performance measure. For simplicity, assume that the discount rate is zero.

In order to isolate the effect of risk aversion on contract design, we only consider the first and second-order terms of the distribution of payments received by the agent. Our reduced-form

[6]The agent's compensation package consists of two components. A fixed wage, and a variable part. For CEOs, the latter is typically a package consisting of a portfolio of assets with vesting periods ending at time 1 awarded to the agent when the contract is signed, and a formula-based positive bonus paid at time 1. Unless otherwise specified, the agent is not protected by limited liability: should his contract include a short position in a put for example, his overall monetary remuneration would potentially be negative.

model assumes that the agent likes the mean and dislikes the variance of payments. We specify an objective function which is linear in the mean and variance of payments.[7] In addition to being very tractable, this approach has the merit of enabling a sharp focus on risk aversion, to the exclusion of other factors. Formally, we assume that the agent maximizes a mean-variance criterion of the form[8]

$$w + E[f(\tilde{\pi})|e] - \omega var[f(\tilde{\pi})|e] - \frac{c}{2}e^2 \tag{1.1}$$

where ω is the weight attributed to variance in the mean-variance criterion. The parameter ω is therefore a measure of risk aversion. The agent has reservation utility $\bar{U}$, defined as the minimum value of (1.1) in equilibrium such that the agent accepts the contract $W(\pi)$. A given contract $\{w, f\}$ satisfies the participation constraint if and only if

$$w + E[f(\tilde{\pi})|e^\star] - \omega var[f(\tilde{\pi})|e^\star] - c\frac{e^{\star 2}}{2} \geq \bar{U} \tag{1.2}$$

For a given f, define $\underline{w}$ as the value of w which satisfies (1.2) as an equality. For (1.2) to be satisfied, we must have $w \geq \underline{w}$.

Assume that the effort cost c is large enough for the first-order approach to be valid for all given compensation contracts under consideration (this imposes that we consider a bounded set of contracts). This is simply a sufficient condition for the effort choice problem to be always concave when required, even when the compensation profile is convex. This technical requirement guarantees an interior solution to the effort choice problem. For a given f, the necessary and sufficient first-order condition with respect to effort is then

$$E[f'(\tilde{\pi})|e] - \omega \frac{\partial}{\partial e} var[f(\tilde{\pi})|e] - ce = 0 \tag{1.3}$$

The contract f must be such that this equation is verified at $e = e^\star$, so that the equilibrium level of effort is $e^\star$. We call this condition the incentive constraint.

We define the first-best cost of eliciting effort $e^\star$, $C^{FB}(e^\star)$, and the first-best wage $w^\star$, as

$$C^{FB}(e^\star) \equiv \bar{U} + c\frac{e^{\star 2}}{2} \equiv w^\star \tag{1.4}$$

For a given f which satisfies the incentive constraint in (1.3) for $e = e^\star$ and a given $w \geq \underline{w}$ which satisfies the participation constraint in (1.2) for this f, the second-best cost of eliciting effort $e^\star$ is

$$C^{SB}(e^\star) \equiv E[W(\tilde{\pi})|e^\star] = w + E[f(\tilde{\pi})|e^\star]$$

[7] We consider the case of quadratic utility in Chapter 2.

[8] With a linear mean-variance criterion, risk aversion takes the form of aversion to the variance of income, and it is invariant to the level of income. Technically, separating the expectation of payments from their variance greatly increases tractability, as incentives are broken down into two terms.

Using (1.2) and the definition of $\underline{w}$ yields

$$C^{SB}(e^\star) = \bar{U} + c\frac{e^{\star 2}}{2} + \omega var[f(\tilde{\pi})|e^\star] + (w - \underline{w})$$

Given f and w, we define the agency cost of eliciting effort $e^\star$ as

$$AC_{f,w}(e^\star) \equiv C^{SB}(e^\star) - C^{FB}(e^\star) = \omega var[f(\tilde{\pi})|e^\star] + (w - \underline{w}) \tag{1.5}$$

Since $C^{FB}(e^\star)$ is given, the principal's objective of minimizing the expected cost of compensation is equivalent to minimizing the agency cost $AC_{f,w}(e^\star)$ of eliciting a given effort $e^\star$. First, for any given f, this implies setting $w = \underline{w}$, given (1.5) and the fact that we must have $w \geq \underline{w}$ for the participation constraint (1.2) to be satisfied. For any given f which satisfies the incentive constraint in (1.3) for $e = e^\star$, the agency cost of eliciting effort $e^\star$ at the optimum is therefore

$$AC_f(e^\star) = \omega var[f(\tilde{\pi})|e^\star] \tag{1.6}$$

Second, since the agency cost is proportional to the variance of f at the optimum, the problem of the principal may be rewritten as

$$\min_f var[f(\tilde{\pi})|e^\star] \qquad \text{s.t.} \qquad (1.3) \;\; \text{for } e = e^\star \tag{1.7}$$

We say that a given contract f is dominated if and only if the agency cost of implementing $e^\star$ can be reduced while leaving $e^\star$ unchanged. In view of (1.6), a contract f is dominated if and only if there exists another contract which also satisfies the incentive constraint (1.3) for $e = e^\star$ but which is characterized by a lower variance.

1.2 Convex contracts are dominated

We begin by deriving the central results of the paper, by not restricting attention to a particular class of compensation contracts. The first result below is proven in the Appendix. It states that, at the equilibrium level of effort, any optimal contract is such that a marginal increase in effort reduces the variance of the agent's pay.

Lemma 1: Any contract such that

$$\frac{\partial}{\partial e} var[f(\tilde{\pi})|e^\star] > 0$$

is dominated.

The proof is deferred in the Appendix. It is quite instructive. For any given contract in which the variance of compensation conditional on equilibrium effort is locally strictly increasing in

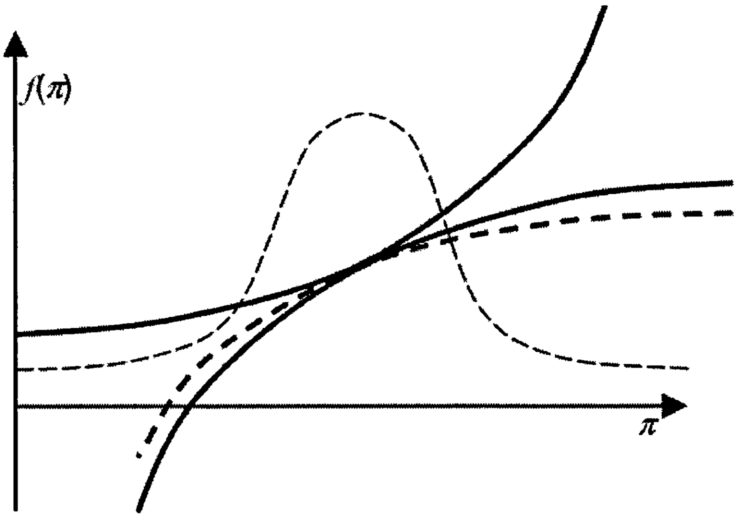

Figure 1.1: A dominated convex contract.

effort, a symmetrical contract with the same average slope and the same variance is constructed. However, this new contract provides more incentives, since a marginal increase in effort at the equilibrium level of effort reduces the variance of compensation - whereas a marginal increase in effort has the opposite effect with the initial contract. The new contract, which necessarily provides excessive incentives, may therefore be flattened. This diminishes the variance of pay, and therefore agency costs. The approach is illustrated in figure 1, where the initial contract is convex, the symmetrical contract is concave, and its flattened transformation is the dotted line. Finally, the fixed wage is adjusted in order to satisfy the participation constraint. With mean-variance preferences, this vertical translation of the contract does not affect either incentives or the variance of pay.

For any f, the fixed wage is set for the participation constraint to bind, and is given by

$$w = \bar{U} + c\frac{e^{\star 2}}{2} + \omega var[f(\tilde{\pi})|e^\star] - E[f(\tilde{\pi}|e^\star)]$$

Thus, under the new contract defined by $f = \frac{1}{\theta}h$, w will increase by

$$\omega\Big(var[\frac{1}{\theta}h(\tilde{\pi})|e^\star] - var[f(\tilde{\pi})|e^\star]\Big) - \Big(E[\frac{1}{\theta}h(\tilde{\pi}|e^\star)] - E[f(\tilde{\pi}|e^\star)]\Big)$$

where $e^\star$ is the equilibrium effort under the contracts $\frac{1}{\theta}h$ and f. From the proof of Lemma 1 in the Appendix, we know that the first term is negative. The impact of the second term is more ambiguous; when $f'' > 0$ and $h'' < 0$ the second term tends to be of a positive sign.[9]

[9]Suppose f is convex. Then, for an unchanged level of effort, h will be below f for any π, so that $E[h(\tilde{\pi}|e^\star)] <$

Unsurprisingly, Lemma 1 renders convex contracts suboptimal.

Theorem 1: It is suboptimal for $W(\pi)$ to be convex in π.

This is due to the simple fact that the variance of compensation conditional on equilibrium effort is locally strictly increasing in effort with a convex contract.

The Mirrlees (1975) result that no optimal contract exists in a certain setting is theoretically powerful but leaves us with little guidance on how to design compensation contracts in practice, all the more that a Mirrlees-type step contract based on extreme punishments for extremely low performance may not be feasible. We not only show that a class of contracts, namely convex contracts, is dominated, but we also show how to construct a contract which improves on any given convex contract. Crucially, such a contract would be concave. We also provide a clear economic intuition to explain why it is suboptimal to make the variance of the pay of a risk averse agent increasing in his effort in equilibrium.

To exploit the agent's aversion to the variance of his pay to provide incentives, the variability of pay should be low for high performances. In particular, Proposition 1 states that pay should be flat for sufficiently high performances.

Proposition 1: For ω bounded away from zero, $W(\pi)$ cannot be infinite for any value of π, and $W(\pi)$ should be constant for all values of π larger than a finite number.

This result implies that concave compensation profiles dominate linear compensation profiles, as long as the agent is risk averse. The intuition is that large variations in pay at the tails of the distribution contribute highly to the variance of pay, while providing relatively low incentives through the pay-performance sensitivity channel (the first term in the incentive constraint (1.3)). In addition, large variation in pay at the right tail of the distribution reduce incentives through their effect on the change in the variance of pay following effort (the second term in the incentive constraint (1.3)). Consequently, Proposition 1 shows that pay should be constant at the right tail of the distribution of performances.

$E[f(\tilde{\pi}|e^{\star})]$. But h does not induce $e^{\star}$. And $\frac{1}{\theta}h$ might be above f on a subset of the domain, so in general we cannot conclude.

1.3 Shorts puts are preferred to long calls

In this section, we restrict attention to options-based contracts, which are the simplest piecewise linear contracts. We start by extending the result of Theorem 1 to any contract which takes the form of a long position in call options, which is convex but not differentiable twice.

Corollary 1: Any contract of the form

$$W(\pi) = w + a \max\{\pi - S, 0\} \tag{1.8}$$

where $a > 0$, is suboptimal.

We show in the proof of Corollary 1 that for a contract of the form (1.8), then $\frac{\partial}{\partial e} var[f(\tilde{\pi})|e] \geq 0$. A direct application of the proof of Lemma 1 then shows that any contract taking the form of a long position in call options is dominated by a contract taking the form of a short position in put options.[10] This is not surprising. With the former, increasing effort increases the variance of pay. With the latter, increasing effort decreases the variance of pay. With a linear contract, a change in effort does not affect the variance of pay.[11]

Even with limited liability, a call-options-based contract can be improved upon. In effect, a long position in call options is not capped, which is suboptimal when the agent is sufficiently risk averse (in the sense that he is not approximately risk neutral), according to Proposition 1. Applying a cap on payments would reduce agency costs. In addition, we have the following existence result.

Proposition 2: Restricting the contract to either take the form of a long position in calls or of a short position in puts, with a given finite strike S, then the optimal contract exists, and takes the form of a short position in put options.

It follows that for any strike S, a short position in puts dominates a long position in calls. This is not surprising: with long calls, increasing effort increases the variance of pay. With a linear contract, a change in effort does not affect the variance of pay. With short positions in

[10]The proof of Lemma 1 describes a contract which dominates any contract characterized by $\frac{\partial}{\partial e} var[f(\tilde{\pi})|e] \geq 0$. In this case, this new contract is by construction a short position in a put.

[11]A long call contract is convex, while a short put contract is concave. In addition, we know from the proof of Theorem 1 that $\frac{\partial}{\partial e} var[f(\tilde{\pi})|e^\star] \geq 0$ if f is convex. We know from Proof 1 that $\frac{\partial}{\partial e} var[f(\tilde{\pi})|e^\star] \leq 0$ if f is concave. Finally, if f linear, then its mirror image h as defined in the Proof of Lemma 1 is defined by $h(\pi) = f(\pi)$ for any π. Since we must also have $\frac{\partial}{\partial e} var[h(\tilde{\pi})|e^\star] = -\frac{\partial}{\partial e} var[f(\tilde{\pi})|e^\star]$, we necessarily have $\frac{\partial}{\partial e} var[f(\tilde{\pi})|e^\star] = 0$ if f is linear.

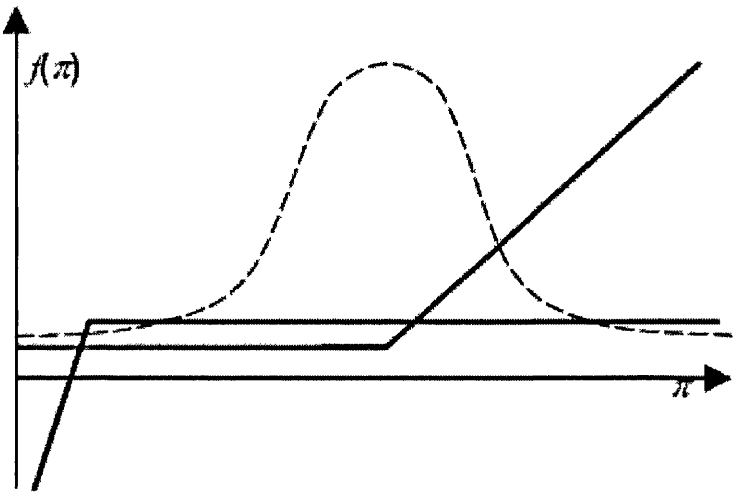

Figure 1.2: ATM long call and OTM short put.

puts, increasing effort decreases the variance of pay. Figure 2 displays a typical at-the-money call option contract, and an out-of-the-money short put contract.

The optimality result that follows highlights the desirability of very low strikes when the performance measure is normally distributed.

Proposition 3: For a given finite S, if we restrict $W(\pi)$ to take one of these two forms:

$$W(\pi) = w + a\max\{\pi - S, 0\} \qquad \text{or} \qquad W(\pi) = w + a\min\{\pi - S, 0\}$$

then the optimal contract exists, and takes the form of the latter.

The strike of the optimal contract is arbitrarily small, while its slope is arbitrarily large. In equilibrium, the agent receives the first-best wage corresponding to effort $e^\star$ almost everywhere, but he is very severely punished for extremely low performances, which are extremely unlikely if adequate effort is exerted. In effect, with a distribution whose likelihood ratio $\frac{\vartheta_e}{\vartheta}$ is arbitrarily small at the left tail of the distribution, it is almost certain that a very low performance is the result of low effort. This means that estimating effort at the left tail of the distribution allows to discriminate almost perfectly between the agent who exerted effort $e^\star$ and the agent who exerted a smaller effort. The former type is approximately insured and receives the first-best wage almost everywhere, whereas the latter type is exposed to a very harsh punishment with a much higher probability. This mechanism almost achieves first-best risk sharing, while also providing adequate incentives.[12]

A result which is counterintuitive at first glance, but consistent with the rest of our analysis, is that more risk aversion is conducive to *less* downside risk protection. Start with a risk neutral agent ($\omega = 0$). Then, any contract which satisfies the participation constraint and the incentive constraint is optimal. This includes convex contracts, which provide protection against downside risk. On the contrary, when the agent is risk averse, a linear contract is not optimal anymore (the proof involves a direct application of Proposition 1), and neither is a convex contract with a floored section at the lower end (simply apply Theorem 1). It is precisely risk aversion that makes concave contracts desirable. But concave contracts clearly offer less downside risk protection. Whereas the form of the contract is inconsequential when $\omega = 0$, as long as it satisfies both the participation constraint and the incentive constraint, all agents with a positive and bounded away from zero ω are given the same short put contract.[13] However, this does not mean that it is not more beneficial for some.

Proposition 4: When $\tilde{\epsilon}$ is normally distributed, the higher ω, the more costly it is to locally deviate from the optimal contract defined in Proposition 3.

This is in line with previous results. We have already shown that risk averse agents should be punished rather than rewarded. Proposition 4 claims that the more risk averse the agent, the costlier it is to attenuate punishments. This generates an interesting prediction. To the extent that top executives are less risk averse than rank-and-file employees[14], the latter will receive more incentives in the form of punishments than the former. Actual compensation practices seem to bear out this prediction: most employees receive fixed wages and are threatened with dismissal in case of a very poor performance, while top executives enjoy more participation to the upside.

All the aforementioned results rely on the symmetry of the probability distribution. This hypothesis was adopted as a normalization: with a linear contract and a symmetrically distributed performance measure, a change in effort leaves the variance of pay unchanged. On

[12]Note that although the resemblance is striking, it differs from the Mirrlees (1975) approximation of the optimal contract, which is a step contract.

[13]Although there is a grey area in an arbitrarily small neighborhood of zero, the mapping from risk aversion to the optimal contract is probably discontinuous at $\omega = 0$.

[14]Because they are typically less wealthy, or because of a selection effect. In the spirit of Knight (1921), Kihlstrom and Laffont (1979) build a model in which more risk averse individuals become workers while the less risk averse become entrepreneurs.

the contrary, with a linear contract and a non-symmetrically distributed performance measure, a change in effort typically changes the variance of pay. However, performance measures are not always symmetrically distributed. For example, the long run distribution of stock prices is approximately lognormal. Fortunately, a lognormal transformation of a lognormally distributed random variable is normally distributed. The curvature of any contract may then be reinterpreted with the tools developed in this paper. In particular, with a lognormal distribution, the contract which is such that the variance of pay is invariant to effort is an affine transformation of the log function, which is concave. A fortiori, giving a CEO a compensation contract convex in the stock price, with restricted stocks and stock-options, makes the variance of his pay an increasing function of his effort. Such a contract would be even more convex as a function of the associated normal variable, so that we know that it is dominated. In short, not only are the results robust to the case of the lognormal distribution, they even argue more strongly for concavity in compensation contracts in this case.

1.4 Step contracts

To take into account a potential lower bound on payments (it would be relevant for an agent protected by limited liability for instance), and determine at which level of performance it is preferable to concentrate incentives, we now study step contracts. A step contract pays off m for $\pi < e^\star + p$, and M for $\pi \geq e^\star + p$, with $M > m$ and a given p. If a step contract is characterized by $p < 0$, then we say that it uses "sticks". Indeed, with such a contract, the absolute value of $m - E[W(\tilde{\pi})]$ is larger than that of $M - E[W(\tilde{\pi})]$, by construction.[15] Otherwise we say that the contract uses "carrots". The next result shows that the "lower bound constraint" is always binding at the optimum.

Proposition 5: With the constraint that $W(\pi) \geq L$ for any π, the optimal contract is characterized by $m = L$.

For contracts which induce the same given level of effort and for which the participation constraint binds, we show in the proof of Proposition 2 that both $\frac{d\Phi(p)}{dm}$ and $\frac{dM}{dm}$ are strictly positive for any value of m, which implies that both p and M are strictly increasing in m at the margin, for any value of m. This in turn implies that the parameters p and M of contracts which satisfy both the participation constraint and the incentive constraint are strictly increasing in m. Denote the parameters of the contract characterized at the optimum (when $m = L$) by $p = 0$

[15]We demonstrate this in passing in the proof of Proposition 2.

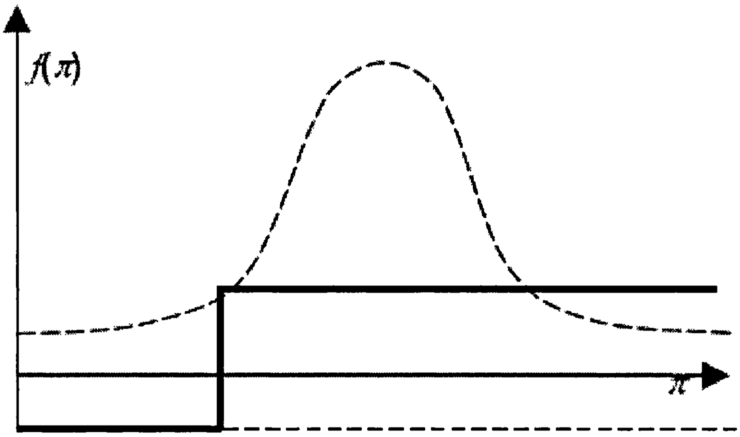

Figure 1.3: An optimal step contract with limited liability.

by $\{0, m_0, M_0\}$.

For a given $L > m_0$, denote the parameters of the optimal contract, which satisfies both the participation constraint and the incentive constraint, by $\{p_{-1}, m_{-1}, M_{-1}\}$. We know from Proposition 2 that $m_{-1} = L < m_0$, so that $p_{-1} < 0$ (by definition of m_0, and because p is increasing in m) and $M_{-1} < M_0$ (by definition of M_0, and because M is increasing in m). In this case, the optimal step contract involves a "punishment" m_{-1} for poor performances, which are relatively unlikely in equilibrium, since $p < 0$. The optimal contract uses sticks.

An intuition for Proposition 2 is that contracts with sticks provide more incentives to agents averse to the variability of their pay. The incentive constraint with step contracts, derived in the proof of Proposition 2 in (1.53), may be rewritten as

$$(M - m)\big[\varphi(p) + \omega\varphi(p)(M - m)\big(1 - 2\Phi(p)\big)\big] = ce^\star \tag{1.9}$$

We know that $\Phi(p) < 0.5$ if and only if $p < 0$, since the probability distribution of $\tilde{\epsilon}$ is symmetric around the mean. In view of (1.9), for given payments m and M, the left-hand side of the incentive constraint is increasing in ω if and only if $p < 0$. That is, the amount of incentives provided is an increasing function of the agent's aversion to the variability of his pay if and only if the contract uses sticks. Whenever the agent is risk averse, this effect is maximized by equating the low payment m to the lower bound L.

In the limit, the optimal contract when L is arbitrarily small is the Mirrlees (1975) approximation of the optimal contract: $m = L$, p tends to minus infinity, and M tends to the first-best wage $w^\star$. In addition, the agency cost is approximately zero with this contract. We prove these two claims when c, $e^\star$, and ω are bounded away from zero and $ce^\star$ is bounded from above in the appendix.

When $L > m_0$, since $m = L$ at the optimum, and both p and M are increasing in m for contracts which satisfy the participation constraint as an equality and the incentive constraint, we know that either the optimal contract uses carrots (i.e., it is characterized by $p > 0$), or it does not satisfy the participation constraint as an equality. We make this argument formal, and show that using sticks when $L > m_0$ involves giving a rent to the agent:

Corollary 2: If $L > m_0$, a contract with $m = L$ and a given $p = \underline{p} < 0$ which satisfies the incentive constraint does not bind the participation constraint.

If $L > m_0$, then using any incentive-compatible contract with sticks rather than carrots results in the agent being paid more than would be needed to keep him at his reservation utility. To avoid this, it may be in the principal's interests to use carrots rather than sticks.

In short, punishments are optimal with unconstrained contracting or with a sufficiently low lower bound on payments, whereas rewards may be optimal with a sufficiently high lower bound on payments. This suggests that carrots or rewards are only used when downward deviations from the first-best wage can only be small, i.e., when the agent cannot be sufficiently punished.[16]

1.5 Conclusion

This paper argues that concavity as a device to motivate effort is too often neglected in compensation contract design. Results suggest that the pay of agents averse to the variability of their compensation should not be convex in the performance measure, and that it should be capped. With a normally distributed performance measure, an optimal contract takes the form of a fixed wage with extremely large punishments for very low outcomes. Lastly, a plausible reason why carrots are used relates to the existence of a binding limited liability constraint. Even though the model is more prescriptive than descriptive, it predicts that well-governed firms which practice efficient contracting will punish bad performances more harshly, and their incentive schemes will be characterized by low observed pay-performance sensitivities. Furthermore, to the extent that top managers are less risk averse than common employees, the later should be incentivized with punishments such as the threat of dismissal, whereas the former may have more upside

[16]For example, the agent may not own much which can be at stake (wealth, a reputation, a potential future career, etc.), and he may have good outside opportunities, so that dismissal is not very costly from his perspective.

participation.

Current stock-based compensation is overwhelmingly either linear, with restricted stocks, or convex, with stock-options. The prevalence of these two instruments, and the virtual absence of any other form of stock-based compensation is puzzling. Given the complexity involved in designing the form of compensation contracts and the results of this paper, why would stock-options emerge as optimal in almost all circumstances? As Hall and Murphy (2003) argued, the most plausible explanation may be the preferential fiscal and accounting treatment granted to stock-options. In other words, the public authorities and the regulators might have skewed pay practices in a suboptimal direction. Alternatively, boards and compensation committees may still be primarily concerned with pay-performance sensitivities, and disregard the incentive effects of the shape of compensation profiles. This also suggests that there is scope for improvements.

Last but not least, by preventing "too large" payments (even for a stellar performance) and "unacceptable" rewards for poor performance, a compensation schedule consisting of a relatively high fixed wage coupled with a punishment for bad performances would help mitigate a backlash against executive compensation. In the current political context, this is a valuable aside.[17]

1.6 Appendix

Proof of Lemma 1:

For any contract such that $\frac{\partial}{\partial e}var[W(\tilde{\pi})|e^\star] > 0$, i.e. such that $\frac{\partial}{\partial e}var[f(\tilde{\pi})|e^\star] > 0$, we need to show that it is possible to design a new contract that induces the same effort $e^\star$ at a lower variance. Indeed, we know from section 1.1 that, given the new contract, w adjusts to satisfy the participation constraint: $w = \underline{w}$, where $\underline{w}$ satisfies the participation constraint (1.2) as an equality (note that a change in w neither affects the incentive constraint in (1.3) nor the variance of compensation). Finally, we know from (1.5) that the new contract with a lower variance is associated with a lower agency cost, and therefore dominates the former.

To this end, in the (π, W) space, consider the contract symmetrical to f with respect to the point $(e^\star, f(e^\star))$: the function defining the new compensation profile is obtained by taking a function symmetrical to f with respect to the horizontal line going through the point $(e^\star, f(e^\star))$, then taking a function symmetrical to this new function with respect to the vertical line going through the same point. Denote the function thus obtained by h.

[17]Regulatory interference appears inevitable if compensation practices do not evolve. The July 23, 2007 *Financial Times* refers to a FT/Harris poll, according to which "Large majorities of people in the US and in Europe want higher taxation for the rich and even pay caps for corporate executives to counter what they believe are unjustified rewards and the negative effects of globalisation."

To start with, assume that effort does not change. Then, we show in Proof 1 later in the appendix that

$$var[h(\tilde{\pi})|e^\star] = var[f(\tilde{\pi})|e^\star]$$
$$E[h'(\tilde{\pi})|e^\star] = E[f'(\tilde{\pi})|e^\star]$$
$$\frac{\partial}{\partial e}var[h(\tilde{\pi})|e^\star] < 0$$

We also show later in the appendix that a sufficient condition for the first-order approach to hold for the contract h if it holds for the contract f is that $E[f''(\pi)] > 0$.

However, effort, as defined in (1.3) by the first-order condition of the agent's problem, will increase under h. Indeed, $e^\star$ solves

$$e^\star = \frac{1}{c}\Big(E[f'(\tilde{\pi})|e^\star] - \omega\frac{\partial}{\partial e}var[f(\tilde{\pi})|e^\star]\Big) \tag{1.10}$$

with $\frac{\partial}{\partial e}var[f(\tilde{\pi})|e^\star] > 0$. The effort $\hat{e}$ induced by h solves

$$\hat{e} = \frac{1}{c}\Big(E[h'(\tilde{\pi})|\hat{e}] - \omega\frac{\partial}{\partial e}var[h(\tilde{\pi})|\hat{e}]\Big) \tag{1.11}$$

But, since $E[h'(\tilde{\pi})|\hat{e}] = E[f'(\tilde{\pi})|e^\star]$ and $\frac{\partial}{\partial e}var[h(\tilde{\pi})|\hat{e}] < 0$, we have $\hat{e} > e^\star$.

For any e and any $\theta > 0$,

$$E\Big[\frac{1}{\theta}h'(\tilde{\pi})|e\Big] = \frac{1}{\theta}E\Big[h'(\tilde{\pi})|e\Big] \tag{1.12}$$
$$\frac{\partial}{\partial e}var\Big[\frac{1}{\theta}h(\tilde{\pi})|e\Big] = \frac{1}{\theta^2}\frac{\partial}{\partial e}var\Big[h(\tilde{\pi})|e\Big] \tag{1.13}$$

It follows from these two equations that the terms $E\Big[\frac{1}{\theta}h'(\tilde{\pi})|e\Big]$ and $\frac{\partial}{\partial e}var\Big[\frac{1}{\theta}h(\tilde{\pi})|e\Big]$ are monotonically decreasing in θ. Furthermore, their first-order derivatives with respect to θ exist and are finite, for $\theta \in (0, \infty)$. Lastly, for $\theta = 1$, effort $\hat{e}$ is induced. Therefore, there exists a $\theta > 1$ such that

$$e^\star = \frac{1}{c}\bigg(E\Big[\frac{1}{\theta}h'(\tilde{\pi})|e^\star\Big] - \omega\frac{\partial}{\partial e}var\Big[\frac{1}{\theta}h(\tilde{\pi})|e^\star\Big]\bigg) \tag{1.14}$$

where $e^\star$ solves (1.10).

The compensation profile $\frac{1}{\theta}h$ induces the same effort as f, but at a lower agency cost. Indeed, the variance of h is equal to the variance of f, so that the variance of $\frac{1}{\theta}h$ is lower than the variance of f. The proof is complete.

Proof 1:

Consider the function g, symmetrical to f with respect to the horizontal line going through the point $(e^\star, f(e^\star))$. Up to an additive constant equal to $2f(e^\star)$, g is the opposite of f: $g(\pi) = -f(\pi) + 2f(e^\star)$. Its variance writes as

$$var[g(\tilde{\pi})] \equiv E\Big[\Big(g(\tilde{\pi}) - E[g(\tilde{\pi})]\Big)^2\Big] = E\Big[\Big(-f(\tilde{\pi}) + 2f(e^\star) + E[f(\tilde{\pi})] - 2f(e^\star)\Big)^2\Big]$$

$$= E\left[\left(f(\tilde{\pi}) - E[f(\tilde{\pi})]\right)^2\right] \equiv var[f(\tilde{\pi})] \tag{1.15}$$

Consider the function h, symmetrical to g with respect to the vertical line going through the point $(e^\star, f(e^\star))$. By definition, it writes as

$$h(\pi) = g\big(\pi - 2(\pi - e^\star)\big) = g(-\pi + 2e^\star)$$

where $\varphi(x)$ is the p.d.f. for a symmetrically distributed variable x of mean 0 and variance σ^2. Because the p.d.f. of $\tilde{\pi}$ is centred around the mean $e^\star$ of $\tilde{\pi}$, we have $\psi(-\pi + 2e^\star) = \psi(\pi)$. The expectation of $h(\tilde{\pi})$ is

$$E[h(\tilde{\pi})] \equiv \int_{-\infty}^{\infty} h(\pi)\psi(\pi)d\pi = \int_{-\infty}^{\infty} g(-\pi + 2e^\star)\psi(\pi)d\pi$$

$$= \int_{-\infty}^{\infty} g(\pi)\psi(-\pi + 2e^\star)d\pi = \int_{-\infty}^{\infty} g(\pi)\psi(\pi)d\pi \equiv E[g(\tilde{\pi})] \tag{1.16}$$

where the second equality uses the definition of g, the third involves a change of variable, and the fourth uses the symmetry of φ unveiled above. Equalities below involve the same steps, plus the fact that $E[g(\tilde{\pi})]$ is a constant.

$$var[h(\tilde{\pi})] \equiv \int_{-\infty}^{\infty} \left(h(\pi) - E[h(\tilde{\pi})]\right)^2 \psi(\pi)d\pi = \int_{-\infty}^{\infty} \left(g\big(-\pi + 2e^\star\big) - E[g(\tilde{\pi})]\right)^2 \psi(\pi)d\pi$$

$$= \int_{-\infty}^{\infty} \left(g(\pi) - E[g(\tilde{\pi})]\right)^2 \psi(-\pi + 2e^\star)d\pi = \int_{-\infty}^{\infty} \left(g(\pi) - E[g(\tilde{\pi})]\right)^2 \psi(\pi)d\pi \equiv var[g(\tilde{\pi})] \tag{1.17}$$

Eventually, combining (2.30) with (2.32),

$$var[h(\tilde{\pi})] = var[g(\tilde{\pi})] = var[f(\tilde{\pi})]$$

The second part of the proof compares $E[h'(\tilde{\pi})]$ to $E[f'(\tilde{\pi})]$. Using the definition of g and remembering that $f(e^\star)$ is a constant, we get $g'(\pi) = -f'(\pi)$, and $E[g'(\tilde{\pi})] = -E[f'(\tilde{\pi})]$. Besides, the derivative of h with respect to π is $h'(\pi) = -g'(-\pi + 2e^\star)$. Going once again through the same steps,

$$E[h'(\tilde{\pi})] \equiv \int_{-\infty}^{\infty} h'(\pi)\psi(\pi)d\pi = \int_{-\infty}^{\infty} -g'(-\pi + 2e^\star)\psi(\pi)d\pi$$

$$= \int_{-\infty}^{\infty} -g'(\pi)\psi(-\pi + 2e^\star)d\pi = \int_{-\infty}^{\infty} -g'(\pi)\psi(\pi)d\pi \equiv -E[g'(\tilde{\pi})] \tag{1.18}$$

Combining these two results, $E[h'(\tilde{\pi})] = -E[g'(\tilde{\pi})] = E[f'(\tilde{\pi})]$.

The third part of the proof shows that the derivatives with respect to effort of the conditional variances of $f(\tilde{\pi})$ and $h(\tilde{\pi})$ have opposite signs. On the one hand, using the definition of g,

$$\frac{\partial}{\partial e} var[g(\tilde{\pi})|e] = \frac{\partial}{\partial e} var[-f(\tilde{\pi}) + 2f(e^\star)|e] = \frac{\partial}{\partial e} var[-f(\tilde{\pi})|e] = -\frac{\partial}{\partial e} var[f(\tilde{\pi})|e] \tag{1.19}$$

On the other hand,

$$\frac{\partial}{\partial e} var[g(\tilde{\pi})|e] = \frac{\partial}{\partial e} \int_{-\infty}^{\infty} \left(g(\pi) - E[g(\tilde{\pi})]\right)^2 \varphi(\epsilon)d\epsilon$$

$$= \int_{-\infty}^{\infty} 2\Big[\frac{\partial}{\partial \pi} g(\pi) - \frac{\partial}{\partial \pi} E[g(\tilde{\pi})]\Big]\Big(g(\pi) - E[g(\tilde{\pi})]\Big)\varphi(\epsilon)d\epsilon$$

Using the definition of h,

$$\frac{\partial}{\partial e} var[h(\tilde{\pi})] = \frac{\partial}{\partial e} \int_{-\infty}^{\infty} \Big(h(\pi) - E[h(\tilde{\pi})]\Big)^2 \varphi(\epsilon)d\epsilon$$

$$= \int_{-\infty}^{\infty} -2\Big[\frac{\partial}{\partial \pi} g(-\pi + 2e^\star) - \frac{\partial}{\partial \pi} E[g(-\tilde{\pi} + 2e^\star)]\Big]\Big(g(-\pi + 2e^\star) - E[g(-\tilde{\pi} + 2e^\star)]\Big)\varphi(\epsilon)d\epsilon$$

$$= \int_{-\infty}^{\infty} -2\Big[\frac{\partial}{\partial \pi} g(\pi) - \frac{\partial}{\partial \pi} E[g(\tilde{\pi})]\Big]\Big(g(\pi) - E[g(\tilde{\pi})]\Big)\varphi(-\epsilon)d\epsilon$$

$$= \int_{-\infty}^{\infty} -2\Big[\frac{\partial}{\partial \pi} g(\pi) - \frac{\partial}{\partial \pi} E[g(\tilde{\pi})]\Big]\Big(g(\pi) - E[g(\tilde{\pi})]\Big)\varphi(\epsilon)d\epsilon$$

$$= -\frac{\partial}{\partial e} var[g(\tilde{\pi})|e] \tag{1.20}$$

Combining (1.19) with (1.20), we have $\frac{\partial}{\partial e} var[h(\tilde{\pi})|e] = -\frac{\partial}{\partial e} var[f(\tilde{\pi})|e]$ for every e.

Proof of Theorem 1:

It is sufficient to show that if f is convex in π, then $\frac{\partial}{\partial e} var[f(\tilde{\pi})|e^\star] \geq 0$. Lemma 1 then gives us the desired result.

To start with,

$$\frac{\partial}{\partial e} var[f(\tilde{\pi})|e^\star] = \int_{-\infty}^{\infty} \frac{\partial}{\partial e}\Big(f(e^\star + \epsilon) - E[f(e^\star + \tilde{\epsilon})]\Big)^2 \varphi(\epsilon)d\epsilon$$

$$= \int_{-\infty}^{\infty} 2\Big(f(e^\star + \epsilon) - E[f(e^\star + \tilde{\epsilon})]\Big)\Big(f'(\pi) - E[f'(\tilde{\pi})]\Big)\varphi(\epsilon)d\epsilon \tag{1.21}$$

The function f being increasing and convex in the performance measure,

$$\frac{\partial f(\pi)}{\partial \epsilon} = \frac{\partial f(\pi)}{\partial \pi}\frac{\partial \pi}{\partial \epsilon} = f'(\pi) > 0$$

$$\frac{\partial f'(\pi)}{\partial \epsilon} = \frac{\partial f'(\pi)}{\partial \pi}\frac{\partial \pi}{\partial \epsilon} = f''(\pi) > 0$$

Therefore,

$$cov(f(\tilde{\pi}), f'(\tilde{\pi})) \geq 0 \tag{1.22}$$

Additionally,

$$cov(f(\tilde{\pi}), f'(\tilde{\pi})) = \int_{-\infty}^{\infty} \Big(f(\pi) - E[f(\tilde{\pi})]\Big)\Big(f'(\pi) - E[f'(\tilde{\pi})]\Big)\varphi(\epsilon)d\epsilon$$

Which is positive because of (1.22). Applying this result to (1.21) completes the proof.

The validity of the first-order approach

Suppose that, for any contract f characterized by $E[f''(\pi)] > 0$,

$$E[f''(\tilde{\pi})|e] + \frac{\partial^2}{\partial e^2} var[f(\tilde{\pi})|e] - c < 0 \tag{1.23}$$

for any nonnegative e. Then the first-order approach is valid for the contract f, since the problem of the agent is concave in effort. In this case, we will show that, for the corresponding contract h defined in the proof of Lemma 1,

$$E[h''(\tilde{\pi})|e] + \frac{\partial^2}{\partial e^2} var[h(\tilde{\pi})|e] - c < 0 \tag{1.24}$$

for any nonnegative e, so that the first-order approach is valid for the contract h.

First, since $h''(\pi) = -f''(-\pi + 2e)$ for any given e,

$$E[h''(\tilde{\pi})|e] \equiv \int_{-\infty}^{\infty} h''(\pi)\psi(\pi|e)d\pi = \int_{-\infty}^{\infty} -f''(-\pi + 2e)\psi(\pi|e)d\pi$$

$$= \int_{-\infty}^{\infty} -f''(\pi)\psi(-\pi + 2e|e)d\pi = -\int_{-\infty}^{\infty} f''(\pi)\psi(\pi|e)d\pi \equiv -E[f''(\tilde{\pi})|e] \tag{1.25}$$

Since $E[f''(\tilde{\pi})|e] > 0$, $E[h''(\tilde{\pi})|e] < 0$.

Second, we now show that, for any given e,

$$\frac{\partial^2}{\partial e^2} var[h(\tilde{\pi})|e] = \frac{\partial^2}{\partial e^2} var[f(\tilde{\pi})|e]$$

We know from the proof of Theorem 1 that, for any h,

$$\frac{\partial}{\partial e} var[h(\tilde{\pi})] = 2cov(h'(\tilde{\pi}), h(\tilde{\pi})) \tag{1.26}$$

Taking the second derivative,

$$\frac{\partial}{\partial e} cov(h'(\tilde{\pi}), h(\tilde{\pi})) = \int_{-\infty}^{\infty} \frac{\partial}{\partial e} \Big\{ \Big(h'(\pi) - E[h'(\pi)]\Big) \Big(h(\pi) - E[h(\tilde{\pi})]\Big) \Big\} \varphi(\epsilon) d\epsilon$$

$$= \int_{-\infty}^{\infty} \left(\Big(h''(\pi) - E[h''(\tilde{\pi})]\Big) \Big(h(\pi) - E[h(\tilde{\pi})]\Big) + \Big(h'(\pi) - E[h'(\tilde{\pi})]\Big)^2 \right) \varphi(\epsilon) d\epsilon \tag{1.27}$$

The second term in the integral above is the same for h as for f, as shown in Proof 1. Substituting for the expression that defines h, the first term in the integral above rewrites as

$$\int_{-\infty}^{\infty} \Big(-f''(-\pi+2e) - E[-f''(-\tilde{\pi}+2e)]\Big) \Big(-f(-\pi+2e) + 2f(e) - E[-f(-\tilde{\pi}+2e) + 2f(e)]\Big) \varphi(\epsilon) d\epsilon$$

After some changes of variables, this becomes

$$\int_{-\infty}^{\infty} \Big(- f''(\pi) - E[-f''(\tilde{\pi})]\Big) \Big(- f(\pi) + 2f(e) - E[-f(\tilde{\pi})] - 2f(e)\Big) \varphi(\epsilon) d\epsilon$$

Rearranging, the first term in the integral in (1.27) is equal to

$$\int_{-\infty}^{\infty} \Big(f''(\pi) - E[f''(\tilde{\pi})]\Big) \Big(f(\pi) - E[f(\tilde{\pi})]\Big) \varphi(\epsilon) d\epsilon$$

All in all,

$$\frac{\partial}{\partial e} cov(h'(\tilde{\pi}), h(\tilde{\pi})) = \frac{\partial}{\partial e} cov(f'(\tilde{\pi}), f(\tilde{\pi}))$$

Using (1.26), we have

$$\frac{\partial^2}{\partial e^2} var[h(\tilde{\pi})|e] = \frac{\partial^2}{\partial e^2} var[f(\tilde{\pi})|e] \tag{1.28}$$

Because of (1.25) and (1.28), for any nonnegative e,

$$E[h''(\tilde{\pi})|e] + \frac{\partial^2}{\partial e^2} var[h(\tilde{\pi})|e] - c < E[f''(\tilde{\pi})|e] + \frac{\partial^2}{\partial e^2} var[f(\tilde{\pi})|e] - c$$

We have assumed in (1.23) that the right-hand side of this inequality is negative for any non-negative e, so that the left-hand-side is negative as well: (1.24) holds for any nonnegative e, and the first-order approach is valid for h.

Proof of Proposition 1:

To simplify notations, normalize w to zero, and let f be defined by $f(\epsilon)$ rather than by $f(\pi)$. This amounts to an horizontal translation, that leaves pay-performance sensitivities and variance unchanged. Assume that the function f is not capped, i.e., it is strictly increasing on $[p, \infty)$ where p is a finite constant. Set $g(\epsilon) = f(\epsilon)$ for $\epsilon \leq k$, and $g(\pi) = f(k)$ for $\epsilon > k$. Assume that $k \geq E[f(\tilde{\epsilon})]$.

The first step demonstrates that the cap $f(k)$ reduces the function's variance.

$$var[g(\tilde{\epsilon})] \equiv \int_{-\infty}^{\infty} \Big(g(\epsilon) - E[g(\tilde{\epsilon})]\Big)^2 \varphi(\epsilon) d\epsilon$$

$$= \int_{-\infty}^{\infty} \Big((g(\epsilon) - E[f(\tilde{\epsilon})]) + (E[f(\tilde{\epsilon})] - E[g(\tilde{\epsilon})])\Big)^2 \varphi(\epsilon) d\epsilon$$

$$= \int_{-\infty}^{k} \Big(f(\epsilon) - E[f(\tilde{\epsilon})]\Big)^2 \varphi(\epsilon) d\epsilon + \int_{k}^{\infty} \Big(f(k) - E[f(\tilde{\epsilon})]\Big)^2 \varphi(\epsilon) d\epsilon$$

$$+ \int_{-\infty}^{\infty} \Big(E[f(\tilde{\epsilon})] - E[g(\tilde{\epsilon})]\Big)^2 \varphi(\epsilon) d\epsilon - 2\Big(E[g(\tilde{\epsilon})] - E[f(\tilde{\epsilon})]\Big) \int_{-\infty}^{k} \Big(f(\epsilon) - E[f(\tilde{\epsilon})]\Big) \varphi(\epsilon) d\epsilon$$

$$-2\Big(E[g(\tilde{\epsilon})] - E[f(\tilde{\epsilon})]\Big) \int_{k}^{\infty} \Big(f(k) - E[f(\tilde{\epsilon})]\Big) \varphi(\epsilon) d\epsilon$$

Using $k \geq E[f(\tilde{\epsilon})]$, the first two integrals added up are smaller than

$$\int_{-\infty}^{\infty} \Big(f(\epsilon) - E[f(\tilde{\epsilon})]\Big)^2 \varphi(\epsilon) d\epsilon$$

which is the variance of $f(\tilde{\epsilon})$. The third integral is equal to

$$\Big(E[f(\tilde{\epsilon})] - E[g(\tilde{\epsilon})]\Big)^2$$

while the last two terms sum up to

$$-2\Big(E[f(\tilde{\epsilon})] - E[g(\tilde{\epsilon})]\Big)^2$$

So

$$var[g(\tilde{\epsilon})] < var[f(\tilde{\epsilon})] - \Big(E[f(\tilde{\epsilon})] - E[g(\tilde{\epsilon})]\Big)^2 < var[f(\tilde{\epsilon})]$$

The variance of $g(\tilde{\epsilon})$ is lower than the variance of $f(\tilde{\epsilon})$.

The second step compares variance effects with g and with f. Let h be a constant equal to $e^\star$, where $e^\star$ is the equilibrium level of effort. Since the equilibrium level of effort does not change, $e^\star$ will remain equal to h.

$$\frac{\partial}{\partial e} var[g(\tilde{\epsilon})] = \frac{\partial}{\partial e}\left\{ \int_{-\infty}^{\infty} \Big(g(\epsilon) - E[g(\tilde{\epsilon})]\Big)^2 \varphi(\epsilon) d\epsilon \right\}$$

$$= \frac{\partial}{\partial e}\left\{ \int_{-\infty}^{k-e^\star+h} \Big(f(\epsilon + e^\star - h) - E[g(\tilde{\epsilon})]\Big)^2 \varphi(\epsilon) d\epsilon \right.$$

$$\left. + \int_{k-e^\star+h}^{\infty} \Big(f(k) - E[g(\tilde{\epsilon})]\Big)^2 \varphi(\epsilon) d\epsilon \right\}$$

$$= \Big(f(k - e^\star + h + e^\star - h) - E[g(\tilde{\epsilon})]\Big)^2 \varphi(k - e^\star + h)$$

$$+2\int_{-\infty}^{k-e^\star+h} \Big(f'(\epsilon + e^\star - h) - \frac{\partial}{\partial e}E[g(\tilde{\epsilon})]\Big)\Big(f(\epsilon + e^\star - h) - E[g(\tilde{\epsilon})]\Big)\varphi(\epsilon) d\epsilon$$

$$-\Big(f(k) - E[g(\tilde{\epsilon})]\Big)^2 \varphi(k - e^\star + h) + 2\int_{k-e^\star+h}^{\infty} \Big(-\frac{\partial}{\partial e}E[g(\tilde{\epsilon})]\Big)\Big(f(k) - E[g(\tilde{\epsilon})]\Big)\varphi(\epsilon) d\epsilon$$

$$= -2\frac{\partial}{\partial e}E[g(\tilde{\epsilon})]\left(\int_{-\infty}^{k} \Big(f(\epsilon) - E[g(\tilde{\epsilon})]\Big)\varphi(\epsilon) d\epsilon + \int_{k}^{\infty} \Big(f(k) - E[g(\tilde{\epsilon})]\Big)\varphi(\epsilon) d\epsilon \right)$$

$$+2\int_{-\infty}^{k} f'(\epsilon)\Big(f(\epsilon) - E[g(\tilde{\epsilon})]\Big)\varphi(\epsilon) d\epsilon$$

$$= 2\int_{-\infty}^{k} f'(\epsilon)\Big(f(\epsilon) - E[g(\tilde{\epsilon})]\Big)\varphi(\epsilon) d\epsilon$$

$$= 2\left(\int_{-\infty}^{k} f'(\epsilon) f(\epsilon)\varphi(\epsilon) d\epsilon - E[g(\tilde{\epsilon})] \int_{-\infty}^{k} f'(\epsilon)\varphi(\epsilon) d\epsilon \right)$$

Letting k tend to infinity, we have

$$\frac{\partial}{\partial e} var[f(\tilde{\epsilon})] = 2\left(\int_{-\infty}^{\infty} f'(\epsilon) f(\epsilon)\varphi(\epsilon) d\epsilon - \int_{-\infty}^{\infty} f(\epsilon)\varphi(\epsilon) d\epsilon \int_{-\infty}^{\infty} f'(\epsilon)\varphi(\epsilon) d\epsilon \right)$$

The difference in variance effects is, up to a multiplicative constant,

$$-\frac{\partial}{\partial e} var[g(\tilde{\epsilon})] + \frac{\partial}{\partial e} var[f(\tilde{\epsilon})] = f(k)[1 - \Phi(k)] \int_{-\infty}^{k} f'(\epsilon)\varphi(\epsilon) d\epsilon$$

$$+ \int_{k}^{\infty} f'(\epsilon) f(\epsilon)\varphi(\epsilon) d\epsilon - \int_{k}^{\infty} f(\epsilon)\varphi(\epsilon) d\epsilon \int_{k}^{\infty} f'(\epsilon)\varphi(\epsilon) d\epsilon$$

Denote this expression by $A(k)$. The variance effect is stronger under g if $A(k)$ is positive.

The third step compares equilibrium efforts under f and g. The change in the average slope is

$$E[g'(\tilde{\epsilon})] - E[f'(\tilde{\epsilon})] = -\int_{k}^{\infty} f'(\epsilon)\varphi(\epsilon) d\epsilon$$

Set

$$B(k) \equiv -\int_k^{\infty} f'(\epsilon)\varphi(\epsilon)d\epsilon + 2\omega A(k) \tag{1.29}$$

Because of (1.3), the equilibrium effort will be higher under g if and only if

$$B(k) > 0 \tag{1.30}$$

As can be easily checked, this expression is zero for an infinite k (this implies that g is identically equal to f):

$$\lim_{k\to\infty} B(k) = 0$$

Besides, $B(k)$ is a sum of products of continuous and differentiable functions. It is therefore continuous and differentiable (using the fact that the product of continuous functions is continuous). Next, we show that

$$\lim_{k\to\infty} B'(k) < 0 \tag{1.31}$$

The derivative of $B(k)$ with respect to k writes as

$$\begin{aligned} B'(k) = \underbrace{f'(k)\varphi(k)}_{T1} + \underbrace{2\omega f'(k)[1-\Phi(k)]\int_{-\infty}^{k} f'(\epsilon)\varphi(\epsilon)d\epsilon}_{T2} \\ \underbrace{-\,2\omega f(k)\varphi(k)\int_{-\infty}^{k} f'(\epsilon)\varphi(\epsilon)d\epsilon}_{T3} + \underbrace{2\omega f(k)[1-\Phi(k)]f'(k)\varphi(k)}_{T4} - \underbrace{2\omega f'(k)f(k)\varphi(k)}_{T5} \\ + \underbrace{2\omega f'(k)\varphi(k)\int_{k}^{\infty} f(\epsilon)\varphi(\epsilon)d\epsilon}_{T6} + \underbrace{2\omega f(k)\varphi(k)\int_{k}^{\infty} f'(\epsilon)\varphi(\epsilon)d\epsilon}_{T7} \end{aligned} \tag{1.32}$$

All terms in this equation (excluding the minus signs), as denoted by $T1, \ldots, T7$, are positive.

First, for k large enough, the term $T5$ is larger than the term $T1$. In effect, since f is strictly increasing on $[p, \infty)$, where p is a finite constant, there exists a large enough k such that $f(k) > \frac{1}{\omega}$, for any strictly positive ω.

Second, for k large enough, the term $T3$ is arbitrarily larger than the term $T2$. To begin with,

$$\frac{1-\Phi(k)}{\varphi(k)} \longrightarrow_{k\to\infty} 1 \tag{1.33}$$

Next, there exists $x \in [k-1, k]$ such that

$$f(k) = f'(x) + f(k-1)$$

For k large enough, $f(k-1)$ is positive and arbitrarily large, while $f'(\epsilon)$ is finite for all ϵ. It follows that as k approaches infinity, $f(k)$ is arbitrarily larger than $f'(k)$.

Third, for k large enough, the term $T3$ is strictly larger than the term $T4$. On the one hand, by assumption $f'(k)$ is finite, and

$$1 - \Phi(k) \longrightarrow_{k\to\infty} 0$$

So that

$$[1 - \Phi(k)]f'(k) \longrightarrow_{k\to\infty} 0$$

On the other hand,

$$\int_{-\infty}^{k} f'(\epsilon)\varphi(\epsilon)d\epsilon \longrightarrow_{k\to\infty} E[f'(\tilde{\epsilon})]$$

But the average pay-performance sensitivity cannot be zero. Suppose this is the case. The function f being nondecreasing, this would imply that $f'(\epsilon) = 0$ for all ϵ. From the proof of Theorem 1 we know that

$$\frac{\partial}{\partial e} var[f(\tilde{\epsilon})] = 2cov(f(\tilde{\epsilon}), f'(\tilde{\epsilon}))$$

A zero derivative generates a zero covariance. Applying this result and $E[f'(\tilde{\epsilon})] = 0$ to (1.3), a zero derivative would result in a zero effort. But $e^\star$ is bounded away from zero. There is a contradiction, which means that $E[f'(\tilde{\epsilon})]$ is strictly positive.

Fourth, for k large enough, the term $T3$ is arbitrarily larger than the term $T6$. For k large enough, $f(k)$ is arbitrarily larger than $f'(k)$, as shown above. In addition,

$$\frac{\int_k^\infty f(\epsilon)\varphi(\epsilon)d\epsilon}{\int_{-\infty}^k f'(\epsilon)\varphi(\epsilon)d\epsilon} \longrightarrow_{k\to\infty} 0$$

Fifth, for k large enough, the term $T3$ is arbitrarily larger than the term $T7$. In effect,

$$\frac{\int_k^\infty f'(\epsilon)\varphi(\epsilon)d\epsilon}{\int_{-\infty}^k f'(\epsilon)\varphi(\epsilon)d\epsilon} \longrightarrow_{k\to\infty} 0$$

Together, the terms $T5$ and $T3$ are larger than the terms $T1$, $T2$, $T4$, $T6$, $T7$, for k large enough. We have proved (1.31). This implies that for any admissible f, there exists a large but finite k such that $B(k)$ is strictly positive. Hence, for all admissible contracts f, there exists a finite cap smaller than k such that the equilibrium effort is strictly higher under g than under f.

It follows that it is optimal for payments to be bounded above. In effect, since f is twice continuously differentiable, there exists an $x \in [0, k]$ such that for any admissible f,

$$f(e^\star + k) = f(e^\star) + f'(e^\star + x)k \tag{1.34}$$

The product of two finite numbers is finite, and so is the sum. All terms in (1.34) being finite, the maximum value of f is finite and bounded.[18]

[18]If $f(k)$ were infinite, the slope would have had to be infinite at some point - since k is finite - which is ruled out by assumption.

For a zero ω, compensation should not necessarily be capped. In this case the absolute value of the second term in (1.29) is zero, so that the absolute value of the first term should be zero for effort to be at least as high under g, which is only achieved with k being infinite.

For ω approaching infinity, compensation should be a constant. In this case the absolute value of the second term in (1.29) tends towards infinity for a given k (such that $A(k)$ is positive and bounded away from zero). To implement the same effort as under f, the function $\frac{1}{\theta}g$ will have to be such that θ is infinite: compensation is approximately constant across performance measures.

We have proved that capping compensation induces the same effort at a lower variance.

The fourth step of the proof identifies sufficient conditions under which altering the compensation contract in such a way does not decrease the agent's welfare, as measured with the mean-variance criterion. Capping pay at $f(k)$ for all $\epsilon \geq k$ reduces expected compensation by

$$\int_k^\infty (f(\epsilon) - f(k))\varphi(\epsilon)d\epsilon$$

Define $C(k)$ as

$$C(k) \equiv \omega var[f(\tilde{\epsilon})] - \omega var[g(\tilde{\epsilon})] - \int_k^\infty (f(\epsilon) - f(k))\varphi(\epsilon)d\epsilon$$

For any k, $C(k)$ is positive if and only if capping pay for all $\epsilon \geq k$ increases the agent's welfare. Since for $k = 0$, g is identically equal to f, we have

$$\lim_{k\to\infty} C(k) = 0$$

Next, we prove that $C'(k)$ is negative when k approaches infinity if the agent is sufficiently risk averse. This implies that there exists a finite k such that the agent's utility is higher with the capped contract g than with the original contract f. Note that

$$\frac{d}{dk}\int_k^\infty (f(\epsilon) - f(k))\varphi(\epsilon)d\epsilon = (f(k) - f(k))\varphi(k) - \int_k^\infty f'(k)\varphi(\epsilon)d\epsilon = -(1 - \Phi(k))f'(k)$$

Differentiating $C(k)$ with respect to k,

$$C'(k) = -\omega\Bigg[\Big(f(k) - \Big(E[f(\tilde{\epsilon})] - \int_k^\infty (f(\epsilon) - f(k))\varphi(\epsilon)d\epsilon\Big)\Big)^2\varphi(k)$$

$$-\Big(f(k) - \Big(E[f(\tilde{\epsilon})] - \int_k^\infty (f(\epsilon) - f(k))\varphi(\epsilon)d\epsilon\Big)\Big)^2\varphi(k)$$

$$\int_{-\infty}^k -2(1 - \Phi(k))f'(k)\Big(f(\epsilon) - E[f(\tilde{\epsilon})] + \int_k^\infty (f(\epsilon) - f(k))\varphi(\epsilon)d\epsilon\Big)\varphi(\epsilon)d\epsilon$$

$$\int_k^\infty -2(1 - \Phi(k))f'(k)\Big(f(k) - E[f(\tilde{\epsilon})] + \int_k^\infty (f(\epsilon) - f(k))\varphi(\epsilon)d\epsilon\Big)\varphi(\epsilon)d\epsilon$$

$$+\int_k^\infty 2f'(k)\Big(f(k)-E[f(\tilde{\epsilon})]+\int_k^\infty (f(\epsilon)-f(k))\varphi(\epsilon)d\epsilon\Big)\varphi(\epsilon)d\epsilon\Bigg] + (1-\Phi(k))f'(k)$$

The first two lines offset each other. The sum of the third and fourth line is zero, because of the definition of g, and because

$$E[g(\tilde{\epsilon})] = E[f(\tilde{\epsilon})] - \int_k^\infty (f(\epsilon)-f(k))\varphi(\epsilon)d\epsilon$$

Eventually,

$$C'(k) = (1-\Phi(k))f'(k) + \left[1-2\omega\Big(f(k)-E[f(\tilde{\epsilon})]+\int_k^\infty (f(\epsilon)-f(k))\varphi(\epsilon)d\epsilon\Big)\right]$$

Since f is increasing, $C'(k)$ is of the same sign as

$$1-2\omega\Big(f(k)-E[f(\tilde{\epsilon})]+\int_k^\infty (f(\epsilon)-f(k))\varphi(\epsilon)d\epsilon\Big)$$

Observe that the term

$$f(k)-E[f(\tilde{\epsilon})]+\int_k^\infty (f(\epsilon)-f(k))\varphi(\epsilon)d\epsilon \tag{1.35}$$

is strictly larger than zero if k is infinite (in this case, the integral is zero, and $f(k) > E[f(\tilde{\epsilon})]$ since f is increasing); it is zero when k is equal to $-\infty$ (this implies that $\lim_{k\to-\infty} C'(k)$ is positive). The derivative of (1.35) with respect to k is

$$f'(k) - \int_k^\infty f'(k)\varphi(\epsilon)d\epsilon = f'(k)\Big(1-(1-\Phi(k))\Big) = f'(k)\Phi(k)$$

which is positive for all k, as f is increasing. If the compensation is not capped, so that

$$\lim_{x\to\infty} f(x) = \infty$$

Then

$$\lim_{k\to\infty} f(k) - E[f(\tilde{\epsilon})] = \infty$$

since $E[f(\tilde{\epsilon})]$ is finite. So for any positive ω bounded away from zero, $C'(k)$ is strictly negative when k approaches infinity.

Putting these results together, $C'(k)$ is increasing on $(-\infty, q)$ and decreasing on (q, ∞), where $q \in (E[f(\tilde{\epsilon})], \infty)$ and q is finite. Furthermore, $C(k)$ tends to zero as k approaches infinity. Hence, there exists a finite k such that setting $f(\pi) = f(k)$ for all $\pi \geq k$ does not violate the agent's participation constraint - that is, the agent's welfare as evaluated with the mean-variance criterion does not decrease.

Proof of Corollary 1:

It is sufficient to show that a contract of the form described in (1.8) is characterized by $\frac{\partial}{\partial e} var[f(\tilde{\pi})|e^\star] \geq 0$. Lemma 1 then gives us the desired result. For a contract of the form described in (1.8),

$$f(\pi) = a \max\{\pi - S, 0\} \tag{1.36}$$

As in the proof of Theorem 1,

$$\frac{\partial}{\partial e} var[f(\tilde{\pi})|e^\star] = \int_{-\infty}^{\infty} 2\big(f(e^\star + \epsilon) - E[f(e^\star + \tilde{\epsilon})]\big)\big(f'(\pi) - E[f'(\tilde{\pi})]\big)\psi(\pi)d\pi = 2cov(f(\tilde{\pi}), f'(\tilde{\pi})) \tag{1.37}$$

where $f'(\pi) = 0$ for $\pi < S$, and $f'(\pi) = a > 0$ for $\pi > S$, so that $E[f'(\tilde{\pi})] > 0$, and $f'(\pi) > E[f'(\tilde{\pi})]$ for $\pi > S$.

Additionally, by definition of the covariance, and for the contract defined in (1.8),

$$\begin{aligned} cov(f(\tilde{\pi}), f'(\tilde{\pi})) = &\int_{-\infty}^{S} \big(f(\pi) - E[f(\tilde{\pi})]\big)\big(f'(\pi) - E[f'(\tilde{\pi})]\big)\psi(\pi)d\pi \\ &+ \int_{S}^{\infty} \big(f(\pi) - E[f(\tilde{\pi})]\big)\big(f'(\pi) - E[f'(\tilde{\pi})]\big)\psi(\pi)d\pi \end{aligned} \tag{1.38}$$

For f of the form (1.36), $f(\pi) = 0$ for $\pi < S$, and $f(\pi) > 0$ for $\pi > S$, so that $E[f(\tilde{\pi})] > 0$. It follows that $f(\pi) < E[f(\tilde{\pi})]$ for $\pi < S$. We are now going to show that both integrals in (1.38) are positive.

First, $f'(\pi) = 0 < E[f'(\tilde{\pi})]$ for $\pi < S$. It follows from this inequality and $f(\pi) < E[f(\tilde{\pi})]$ for $\pi < S$ that the first integral in (1.38) is positive. Second,

$$\int_{S}^{\infty} f(\pi)\psi(\pi)d\pi = E[f(\tilde{\pi})]$$

So that

$$\int_{S}^{\infty} f(\pi)\psi(\pi)d\pi > (1 - \Psi(S))E[f(\tilde{\pi})]$$

which implies that

$$\int_{S}^{\infty} f(\pi)\big(f'(\pi) - E[f'(\tilde{\pi})]\big)\psi(\pi)d\pi > \int_{S}^{\infty} E[f(\tilde{\pi})]\big(f'(\pi) - E[f'(\tilde{\pi})]\big)\psi(\pi)d\pi$$

since $f'(\pi) - E[f'(\tilde{\pi})] = a - E[f'(\tilde{\pi})]$ for $\pi > S$, which is a constant. This shows that the second integral in (1.38) is positive.

Applying these two results to (1.38) yields

$$cov(f(\tilde{\pi}), f'(\tilde{\pi})) \geq 0 \tag{1.39}$$

Equation (1.37) then implies that $\frac{\partial}{\partial e} var[f(\tilde{\pi})|e^\star] \geq 0$ for a contract of the form described in (1.8), and Lemma 1 gives the desired result.

Proof of Proposition 2:

According to Theorem 1, any long call option contract is dominated by a short put option contract. The optimal contract, if it exists, is a short put option contract - as translated vertically by the fixed wage that gives the agent his reservation utility. I show below that only one short put option contract is incentive compatible and satisfies the participation constraint. Therefore, this contract is optimal among the class of contracts consisting of a fixed wage plus a long call or a short put option of strike S.

To this end, I show that given a fixed wage w, there is only one option slope a that satisfies the incentive constraint, and this a exists for any value of w; and given the option slope a, there is only one w that satisfies the participation constraint, and this w exists for any value of a.

First, the incentive constraint is

$$E[f'(\tilde{\pi})|e^{\star}] - \omega\frac{\partial}{\partial e}var[f(\tilde{\pi})|e^{\star}] - ce^{\star^2} = 0 \tag{1.40}$$

The fixed wage w does not enter the incentive constraint. For a short put option with strike S,

$$E[f'(\tilde{\pi})|e^{\star}] = \int_{-\infty}^{S-e^{\star}} a\varphi(\epsilon)d\epsilon = a\Phi(S-e^{\star})$$

Given a finite S and a finite $e^{\star}$, this term is equal to zero for $a = 0$, and tends to infinity as a approaches infinity. Furthermore, this term is increasing in a:

$$\frac{\partial}{\partial a}E[f'(\tilde{\pi})|e^{\star}] = \Phi(S-e^{\star}) > 0$$

We now show that the second term on the left-hand side of (1.48) has the same properties with respect to a. For a short put option with strike S,

$$-\omega\frac{\partial}{\partial e}var[f(\tilde{\pi})|e^{\star}] = -\omega\left[\int_{-\infty}^{S-e}\Big(a(\epsilon-S+e)+w-E[f(\tilde{\pi})]\Big)^2\varphi(\epsilon)d\epsilon+\int_{S-e}^{\infty}\Big(w-E[f(\tilde{\pi})]\Big)^2\varphi(\epsilon)d\epsilon\right]$$

$$= -\omega\Bigg[-\Big(a(S-e-S+e)+w-E[f(\tilde{\pi})]\Big)^2\varphi(S-e)+\Big(w-E[f(\tilde{\pi})]\Big)^2\varphi(S-e)$$

$$+2a\int_{-\infty}^{S-e}\Big(a(\epsilon+e-S)+w-E[f(\tilde{\pi})]\Big)\varphi(\epsilon)d\epsilon$$

$$-\int_{-\infty}^{S-e}2\frac{\partial}{\partial e}E[f(\tilde{\pi})]\Big(a(\epsilon+e-S)+w-E[f(\tilde{\pi})]\Big)\varphi(\epsilon)d\epsilon$$

$$-\int_{S-e}^{\infty}2\frac{\partial}{\partial e}E[f(\tilde{\pi})]\Big(w-E[f(\tilde{\pi})]\Big)\varphi(\epsilon)d\epsilon\Bigg]$$

The first line is zero. The last two lines cancel each other as well: factor out $2\frac{\partial}{\partial e}E[f(\tilde{\pi})]$, then notice that

$$\int_{-\infty}^{S-e}\Big(a(\epsilon+e-S)+w\Big)\varphi(\epsilon)d\epsilon+\int_{S-e}^{\infty}w\varphi(\epsilon)d\epsilon = E[f(\tilde{\pi})] \tag{1.41}$$

Eventually,

$$-\omega\frac{\partial}{\partial e}var[f(\tilde{\pi})|e^\star] = -2\omega a\int_{-\infty}^{S-e}\Big(a(\epsilon+e-S)+w-E[f(\tilde{\pi})]\Big)\varphi(\epsilon)d\epsilon$$

The expression above is equal to zero for $a=0$, and tends to infinity as a approaches infinity. Furthermore, using (1.49), the expression above is positive. It follows that

$$\frac{\partial}{\partial a}\Big\{-\omega\frac{\partial}{\partial e}var[f(\tilde{\pi})|e^\star]\Big\} = -2\omega\int_{-\infty}^{S-e}\Big(2a(\epsilon+e-S)+w-E[f(\tilde{\pi})]\Big)\varphi(\epsilon)d\epsilon > 0$$

To summarize, with a short put of strike S, the expression

$$E[f'(\tilde{\pi})|e^\star] - \omega\frac{\partial}{\partial e}var[f(\tilde{\pi})|e^\star]$$

is zero for $a=0$, tends to infinity as a approaches infinity, and is monotonically increasing in a. Therefore, for any value of w there exists an a such that the incentive constraint (1.48) is satisfied.

Second, the participation constraint is

$$E[f(\tilde{\pi})|e^\star] + w - var[f(\tilde{\pi})|e^\star] - \frac{c}{2}e^{\star^2} \geq \bar{U} \tag{1.42}$$

By definition, w does not have any effect on either $E[f(\tilde{\pi})]$, $var[f(\tilde{\pi})]$, or the effort choice. Given S, f is defined by a and the fact that it's either a long call or a short put. Therefore, for any value of a (and therefore of $E[f(\tilde{\pi})]$ and $var[f(\tilde{\pi})]$), there exists a w that meets the participation constraint, whether f is a long call or a short put.

We have proved that for a given strike S, there exists a slope $a_S^\star$ independent of w that satisfies the incentive constraint (1.48); and that for any a there exists a fixed wage w that satisfies the participation constraint (1.42). Denote the fixed wage that satisfies the participation constraint when $a=a_S^\star$ by $w_S^\star$. The optimal contract exists, and is a short put with slope $a_S^\star$ and associated fixed wage $w_S^\star$.

Proof of Proposition 3:

The mean-variance criterion of an agent exerting effort $e^\star$ is

$$\int_{-\infty}^{S}(w^\star+\varepsilon+a(\pi-S))\psi(\pi|e^\star)d\pi + \int_{S}^{\infty}(w^\star+\varepsilon)\psi(\pi|e^\star)d\pi$$

$$-\omega\Big[\int_{-\infty}^{S}\Big(w^\star+\varepsilon+a(\pi-S)-E[f(\tilde{\pi})]\Big)^2\psi(\pi|e^\star)d\pi + \int_{S}^{\infty}\Big(w^\star+\varepsilon-E[f(\tilde{\pi})]\Big)^2\psi(\pi|e^\star)d\pi\Big]$$

For a given S, the slope a is set to satisfy the incentive constraint, which is

$$\int_{-\infty}^{S}(w^\star+\varepsilon+a(\pi-S))\frac{\partial}{\partial e}\psi(\pi|e^\star)d\pi + \int_{S}^{\infty}(w^\star+\varepsilon)\frac{\partial}{\partial e}\psi(\pi|e^\star)d\pi$$

$$-\omega\left[\int_{-\infty}^{S}\left(w^\star+\varepsilon+a(\pi-S)-E[f(\tilde{\pi})]\right)^2\frac{\partial}{\partial e}\psi(\pi|e^\star)d\pi+\int_{S}^{\infty}\left(w^\star+\varepsilon-E[f(\tilde{\pi})]\right)^2\frac{\partial}{\partial e}\psi(\pi|e^\star)d\pi\right]=ce^\star \tag{1.43}$$

We show in Proof 2 later in the Appendix that, for any given w and S, there is one and only one positive a which satisfies the incentive constraint.

With $\varepsilon = 0$, the mean-variance criterion is lower than $w^\star$ by the following positive amount (which is therefore the shortfall to meet the participation constraint):

$$\int_{-\infty}^{S}-a(\pi-S)\psi(\pi|e^\star)d\pi$$

$$+\omega\left[\int_{-\infty}^{S}\left(w^\star+a(\pi-S)-E[f(\tilde{\pi})]\right)^2\psi(\pi|e^\star)d\pi+\int_{S}^{\infty}\left(w^\star-E[f(\tilde{\pi})]\right)^2\psi(\pi|e^\star)d\pi\right] \tag{1.44}$$

Given the likelihood ratio of the normal distribution, for any arbitrarily large M there exists a S low enough such that

$$\frac{\frac{\partial}{\partial e}\psi(\pi|e^\star)}{\psi(\pi|e^\star)}<-M \tag{1.45}$$

for all $\pi < S$. Or

$$\psi(\pi|e^\star)<-\frac{1}{M}\frac{\partial}{\partial e}\psi(\pi|e^\star)$$

Substituting, (1.44) is smaller than the positive amount

$$-\frac{1}{M}\int_{-\infty}^{S}-a(\pi-S)\frac{\partial}{\partial e}\psi(\pi|e^\star)d\pi$$

$$-\frac{\omega}{M}\left[\int_{-\infty}^{S}\left(w^\star+a(\pi-S)-E[f(\tilde{\pi})]\right)^2\frac{\partial}{\partial e}\psi(\pi|e^\star)d\pi+\int_{S}^{\infty}\left(w^\star-E[f(\tilde{\pi})]\right)^2\frac{\partial}{\partial e}\psi(\pi|e^\star)d\pi\right] \tag{1.46}$$

Using the incentive constraint (1.43), (1.46) is equal to

$$-\frac{1}{M}\int_{-\infty}^{\infty}w^\star\frac{\partial}{\partial e}\psi(\pi|e^\star)d\pi-\frac{1}{M}ce^\star \tag{1.47}$$

For a normal distribution,

$$\frac{\partial}{\partial e}\psi(\pi|e^\star)=\frac{\pi-e^\star}{\sigma^2}\psi(\pi|e^\star)$$

The expression in (1.47) therefore writes as

$$-\frac{1}{M}\int_{-\infty}^{\infty}w^\star\frac{\pi-e^\star}{\sigma^2}\psi(\pi|e^\star)d\pi-\frac{1}{M}ce^\star=-\frac{1}{M}\frac{w^\star}{\sigma^2}\left[E[\tilde{\pi}-e^\star]\right]-\frac{1}{M}ce^\star=-\frac{1}{M}ce^\star$$

Which is given and finite, so that this expression tends to zero as M approaches infinity. Since $w = w^\star + \varepsilon$, and the derivative of the mean-variance criterion with respect to w is equal to one, ε must be strictly positive but may be arbitrarily small for the participation constraint to be satisfied:

$$\varepsilon=\frac{1}{M}ce^\star\longrightarrow_{M\to\infty}0$$

With the contract described in Proposition 1, the second-best cost of eliciting effort $e^\star$ is

$$w^\star + \varepsilon + aE\big[\min\{0, \pi - \underline{S}\}\big] < w^\star + \varepsilon \longrightarrow_{M\to\infty} w^\star$$

where $w^\star$ is by definition the first-best cost of eliciting effort $e^\star$. It follows from (1.5) that the agency cost is approximately zero, and the proof is complete.

Proof 2:

We show that for any given w and S, there is one and only one slope a of a put option contract that satisfies the incentive constraint.

The incentive constraint is

$$E[f'(\tilde{\pi})|e^\star] - \omega\frac{\partial}{\partial e}var[f(\tilde{\pi})|e^\star] - ce^\star = 0 \tag{1.48}$$

where the third term is a constant.

For a short put option with strike S, the first term on the left-hand-side of (1.48) is equal to

$$E[f'(\tilde{\pi})|e^\star] = \int_{-\infty}^{S-e^\star} a\varphi(\epsilon)d\epsilon = a\Phi(S-e^\star)$$

Given a finite S and a finite $e^\star$, this term is equal to zero for $a=0$, and tends to infinity as a approaches infinity. Furthermore, this term is increasing in a:

$$\frac{\partial}{\partial a}E[f'(\tilde{\pi})|e^\star] = \Phi(S-e^\star) > 0$$

We now show that the second term on the left-hand-side of (1.48) has the same properties with respect to a. For a short put option with strike S,

$$-\omega\frac{\partial}{\partial e}var[f(\tilde{\pi})|e^\star] = -\omega\left[\int_{-\infty}^{S-e}\Big(a(\epsilon-S+e)+w-E[f(\tilde{\pi})]\Big)^2\varphi(\epsilon)d\epsilon+\int_{S-e}^{\infty}\Big(w-E[f(\tilde{\pi})]\Big)^2\varphi(\epsilon)d\epsilon\right]$$

$$\begin{aligned}
= -\omega\Bigg[& -\Big(a(S-e-S+e)+w-E[f(\tilde{\pi})]\Big)^2\varphi(S-e) + \Big(w-E[f(\tilde{\pi})]\Big)^2\varphi(S-e) \\
& +2a\int_{-\infty}^{S-e}\Big(a(\epsilon+e-S)+w-E[f(\tilde{\pi})]\Big)\varphi(\epsilon)d\epsilon \\
& -\int_{-\infty}^{S-e}2\frac{\partial}{\partial e}E[f(\tilde{\pi})]\Big(a(\epsilon+e-S)+w-E[f(\tilde{\pi})]\Big)\varphi(\epsilon)d\epsilon \\
& -\int_{S-e}^{\infty}2\frac{\partial}{\partial e}E[f(\tilde{\pi})]\Big(w-E[f(\tilde{\pi})]\Big)\varphi(\epsilon)d\epsilon\Bigg]
\end{aligned}$$

The first line in the expression above is zero. The last two lines cancel each other out, since

$$\int_{-\infty}^{S-e}\Big(a(\epsilon+e-S)+w\Big)\varphi(\epsilon)d\epsilon + \int_{S-e}^{\infty}w\varphi(\epsilon)d\epsilon = E[f(\tilde{\pi})] \tag{1.49}$$

Eventually,

$$-\omega\frac{\partial}{\partial e}var[f(\tilde{\pi})|e^\star] = -2\omega a\int_{-\infty}^{S-e}\Big(a(\epsilon+e-S)+w-E[f(\tilde{\pi})]\Big)\varphi(\epsilon)d\epsilon \qquad (1.50)$$

This expression is equal to zero for $a = 0$, and tends to infinity as a approaches infinity. Furthermore, it is monotonically increasing in a. Indeed, differentiating (1.50) with respect to a,

$$\frac{\partial}{\partial a}\left\{-\omega\frac{\partial}{\partial e}var[f(\tilde{\pi})|e^\star]\right\} = -2\omega\int_{-\infty}^{S-e}(\epsilon+e-S)\varphi(\epsilon)d\epsilon - 2\omega\int_{-\infty}^{S-e}\Big(a(\epsilon+e-S)+w-E[f(\tilde{\pi})]\Big)\varphi(\epsilon)d\epsilon > 0$$

Both integrals in this expression are negative, so that both terms are positive.

To summarize, with a short put of strike S, the expression

$$E[f'(\tilde{\pi})|e^\star] - \omega\frac{\partial}{\partial e}var[f(\tilde{\pi})|e^\star]$$

is zero for $a = 0$, tends to infinity as a approaches infinity, and is monotonically increasing in a. Therefore, for any value of w and S, there exists one and only one a such that the incentive constraint in (1.48) is satisfied.

Proof of Proposition 4:

A short put of strike S is incentive compatible with slope $a^\star$. With an induced fixed wage of w (the level necessary to satisfy the participation constraint), the agency cost is

$$AC(S) = \omega\int_{-\infty}^{\infty}\Big(\min\{a^\star(e^\star+\epsilon-S),0\}+w-E[W(\tilde{\pi})]\Big)^2\varphi(\epsilon)d\epsilon$$

$$= \omega\int_{-\infty}^{S-e^\star}\Big(a^\star(\epsilon-S+e^\star)+w-E[W(\tilde{\pi})]\Big)^2\varphi(\epsilon)d\epsilon + \omega\int_{S-e^\star}^{\infty}\Big(w-E[W(\tilde{\pi})]\Big)^2\varphi(\epsilon)d\epsilon$$

We evaluate the change in agency costs following a marginal change in the strike:

$$\begin{aligned}\frac{d}{dS}AC(S) = \omega\Bigg(&\Big(w-E[W(\tilde{\pi})]\Big)^2\varphi(S-e^\star) - \Big(w-E[W(\tilde{\pi})]\Big)^2\varphi(S-e^\star)\\
&+\int_{-\infty}^{S-e^\star}2\frac{da}{dS}(\epsilon-S+e^\star)\Big(a^\star(\epsilon-S+e^\star)+w-E[W(\tilde{\pi})]\Big)\varphi(\epsilon)d\epsilon\\
&+\int_{-\infty}^{S-e^\star}-2a^\star\Big(a^\star(\epsilon-S+e^\star)+w-E[W(\tilde{\pi})]\Big)\varphi(\epsilon)d\epsilon\\
&-\int_{-\infty}^{S-e^\star}2\frac{d}{dS}E[W(\tilde{\pi})]\Big(a^\star(\epsilon-S+e^\star)+w-E[W(\tilde{\pi})]\Big)\varphi(\epsilon)d\epsilon\\
&-\int_{S-e^\star}^{\infty}2\frac{d}{dS}E[W(\tilde{\pi})]\Big(w-E[W(\tilde{\pi})]\Big)\varphi(\epsilon)d\epsilon\Bigg)\end{aligned}$$

The two terms on the first line offset each other, as do the last two lines. Rearranging,

$$\frac{d}{dS}AC(S) = 2a^\star\omega\Bigg(\frac{da}{dS}\int_{-\infty}^{S-e^\star}(\epsilon-S+e^\star)^2\varphi(\epsilon)d\epsilon$$

$$+\int_{-\infty}^{S-e^\star}\left(-a^\star(\epsilon-S+e^\star)+\left[\frac{da}{dS}(\epsilon-S+e^\star)-1\right]\left[w-E[W(\tilde{\pi})]\right]\right)\varphi(\epsilon)d\epsilon\Big) \tag{1.51}$$

Equation (??) measures the gains from increasing the strike of a short put contract. At the optimal contract (with a strike of $\underline{S}$), we know from the proof of Proposition 3 that increasing the strike results in a rise in agency costs, whose magnitude is proportional to ω.

Proof of Proposition 5:

A step contract is defined by its floor m, its cap M, and the cutoff p. Bearing in mind that

$$E[W(\tilde{\pi})] = m\Phi(p) + M(1-\Phi(p)) \tag{1.52}$$

$$var[W(\tilde{\pi})] = (m - E[W(\tilde{\pi})])^2\Phi(p) + (M - E[W(\tilde{\pi})])^2(1-\Phi(p))$$

We compute derivatives with step contracts:

$$\begin{aligned}
E[W'(\tilde{\pi})] &= \lim_{a\to 0}\int_{p-a}^{p+a}\frac{M-m}{2a}d\Phi(\epsilon) = (M-m)\lim_{a\to 0}\frac{1}{2a}\int_{p-a}^{p+a}d\Phi(\epsilon)\\
&= (M-m)\lim_{a\to 0}\frac{\Phi(p+a)-\Phi(p-a)}{2a} = (M-m)\varphi(p)\\
\frac{\partial}{\partial e}var[W(\tilde{\pi})] &= \frac{\partial}{\partial e}\left\{\int_{-\infty}^{p-e+e^\star}(m-E[W(\tilde{\pi})])^2 d\Phi(\epsilon) + \int_{p-e+e^\star}^{\infty}(M-E[W(\tilde{\pi})])^2 d\Phi(\epsilon)\right\}\\
&= -\varphi(p)(m-E[W(\tilde{\pi})])^2 + \varphi(p)(M-E[W(\tilde{\pi})])^2
\end{aligned}$$

This expression is negative if and only if

$$\left|m - E[W(\tilde{\pi})]\right| > \left|M - E[W(\tilde{\pi})]\right|$$

which is the case if and only if p is lower than zero (because of (1.52) and the fact that, for a symmetrically distributed random variable, $\Phi(0) = 0.5$).

The incentive constraint is

$$(M-m)\varphi(p) - \omega\varphi(p)\left[M^2 - m^2 - 2(M-m)E[W(\tilde{\pi})]\right] = ce^\star$$

Or

$$(M-m)\left[\varphi(p) - \omega\varphi(p)\Big(M + m - 2m\Phi(p) - 2M(1-\Phi(p))\Big)\right] = ce^\star \tag{1.53}$$

We rewrite the incentive constraint in (1.53) as

$$\eta(m, M(m)) \equiv ce^\star$$

Suppose that a given step contract satisfies the incentive constraint. Following a change in m, the adjustment in M required for the incentive constraint to be satisfied, holding p constant, is

$$\frac{dM}{dm} = -\frac{\frac{\partial}{\partial m}\eta(m, M(m))}{\frac{\partial}{\partial M}\eta(m, M(m))} \tag{1.54}$$

The participation constraint is

$$m\Phi(p)+M(1-\Phi(p))-\omega\Big[(m-E[W(\tilde{\pi})])^2\Phi(p)+(M-E[W(\tilde{\pi})])^2(1-\Phi(p))\Big]=\bar{U}+c\frac{e^\star}{2} \quad (1.55)$$

We rewrite (1.55) as

$$\mu(m,[\Phi(p)](m))\equiv\bar{U}+c\frac{e^\star}{2}$$

Suppose that a given step contract satisfies the participation constraint. Following a change in m, the adjustment in p required for the participation constraint to be satisfied, holding M constant, is

$$\frac{d\Phi(p)}{dm}=-\frac{\frac{\partial}{\partial m}\mu(m,[\Phi(p)](m))}{\frac{\partial}{\partial p}\mu(m,[\Phi(p)](m))} \quad (1.56)$$

A given contract which satisfies both the participation constraint and the incentive constraint is dominated if marginally changing m, and adjusting M and p accordingly to satisfy the incentive and participation constraints (so that the agent still participates and still selects the same level of effort $e^\star$), diminishes the expected payments made to the agent (the second-best cost of eliciting effort $e^\star$). We now calculate the sign of the total derivative of expected payments to the agent with respect to m

$$\begin{aligned}\frac{d}{dm}E[W(\tilde{\pi})]&=\Phi(p)+m\frac{d\Phi(p)}{dm}+\frac{dM}{dm}(1-\Phi(p))-M\frac{d\Phi(p)}{dm}\\&=\Phi(p)+(M-m)\Big(-\frac{d\Phi(p)}{dm}\Big)+\frac{dM}{dm}(1-\Phi(p)) \quad (1.57)\end{aligned}$$

Start with the $\frac{d\Phi(p)}{dm}$ term. On the one hand,

$$\begin{aligned}\frac{\partial}{\partial m}\mu(m,[\Phi(p)](m))&=\Phi(p)-\omega\Big[2(1+\Phi(p))(m-E[W(\tilde{\pi})])\Phi(p)+2(1+\Phi(p))(M-E[W(\tilde{\pi})])(1-\Phi(p))\Big]\\&=\Phi(p)-2\omega(1+\Phi(p))\Big[m\Phi(p)-m(\Phi(p))^2-M(1-\Phi(p))\Phi(p)+M(1-\Phi(p))\\&\qquad-m\Phi(p)(1-\Phi(p))-M(1-\Phi(p))^2\Big]=\Phi(p)\end{aligned}$$

On the other hand,

$$\begin{aligned}\frac{\partial}{\partial\Phi(p)}\mu(m,[\Phi(p)](m))&=m-M-2\omega\Big[(M-m)(m-E[W(\tilde{\pi})])\Phi(p)+\Big(m-m\Phi(p)-M(1-\Phi(p))\Big)^2\\&\qquad+(M-m)(M-E[W(\tilde{\pi})])(1-\Phi(p))-\Big(m-m\Phi(p)-M(1-\Phi(p))\Big)^2\Big]\\&=(M-m)\Big[-1-2\omega\Big[m\Phi(p)+M(1-\Phi(p))-E[W(\tilde{\pi})]\Big]\Big]\\&=-(M-m)\end{aligned}$$

So that

$$\frac{d\Phi(p)}{dm}=\frac{\Phi(p)}{M-m}>0 \quad (1.58)$$

Next, consider the $\frac{dM}{dm}$ term On the one hand,

$$\frac{\partial}{\partial m}\eta(m, M(m)) = -\varphi(p) + \omega\varphi(p)\Big[M - m - 2m\Phi(p) - 2M(1-\Phi(p)) - (M-m)(1-2\Phi(p))\Big]$$
$$= -\varphi(p) + \omega\varphi(p)\Big[2m - 2M - 4m\Phi(p) + 4M\Phi(p)\Big]$$

On the other hand,

$$\frac{\partial}{\partial M}\eta(m, M(m)) = \varphi(p) - \omega\varphi(p)\Big[M - m - 2m\Phi(p) - 2M(1-\Phi(p)) + (M-m)(1-2(1-\Phi(p)))\Big]$$
$$= -\varphi(p) + \omega\varphi(p)\Big[2m - 2M - 4m\Phi(p) + 4M\Phi(p)\Big]$$

So that

$$\frac{dM}{dm} = 1 > 0 \tag{1.59}$$

Putting these results together, the change in the second-best cost of a contract following a marginal change in m is

$$\frac{d}{dm}E[W(\tilde{\pi})] = \Phi(p) - (M-m)\frac{\Phi(p)}{M-m} + \frac{dM}{dm}(1-\Phi(p)) = 1 - \Phi(p) > 0 \tag{1.60}$$

which is positive, so any contract with $m > L$ is dominated (because of (1.5)). Therefore, at the optimum, $m = L$.

The Mirrlees approximation of the optimal contract :

We will demonstrate that if L is negative and arbitrarily small, if c, $e^\star$ and ω are bounded away from zero, and if $ce^\star$ is bounded from above, then the Mirrlees approximation of the optimal contract (which is obtained in a setting without any lower bound on payments) is the optimal contract, and the agency cost is approximately zero.

In the first part of the proof, we show in four steps that $m = L$, $M \longrightarrow_{L\to-\infty} w^\star$, $p \longrightarrow_{L\to-\infty} -\infty$.

First, we know from Proposition 2 that the optimal contract is characterized by $m = L$.

Second, since the variance of $W(\tilde{\pi})$ is positive,

$$-\omega\Big[(m - E[W(\tilde{\pi})])^2\Phi(p) + (M - E[W(\tilde{\pi})])^2(1-\Phi(p))\Big] < 0$$

For the participation constraint (1.55) to be satisfied, we therefore need to have

$$m\Phi(p) + M(1-\Phi(p)) > \bar{U} + c\frac{e^\star}{2} = w^\star$$

Since m is arbitrarily small, $0 < \Phi(p) < 1$, and $0 < 1 - \Phi(p) < 1$, this inequality requires that $M > w^\star$. For any given M, let ε be implicitly defined by $M \equiv w^\star + \varepsilon$.

Third, for a negative and arbitrarily small L, given that ω is bounded away from zero and that $M > w^\star$, all terms on the left-hand side of the incentive constraint (1.53) are negligible next to $\varphi(p)L^2\omega$. We must therefore have $\varphi(p)L^2\omega \approx ce^\star$ for the incentive constraint to be satisfied. This rewrites as

$$\varphi(p) \longrightarrow_{L\to-\infty} \frac{ce^\star}{\omega}\frac{1}{L^2} \tag{1.61}$$

Because L is arbitrarily small, and ω is bounded away from zero, the right-hand side of (1.61) tends to zero as L approaches minus infinity. Since $\varphi(p)$ tends to zero as p approaches minus infinity (1.61) is verified if

$$p \longrightarrow_{L\to-\infty} -\infty \tag{1.62}$$

Fourth, the normal distribution is characterized by

$$\frac{\Phi(p)}{\varphi(p)} \longrightarrow_{p\to-\infty} 0 \tag{1.63}$$

Rewrite the participation constraint in (1.55) as

$$L\Phi(p)+(w^\star+\varepsilon)(1-\Phi(p))-\omega\Big[(L-E[W(\tilde{\pi})])^2\Phi(p)+(w^\star+\varepsilon-E[W(\tilde{\pi})])^2(1-\Phi(p))\Big] = w^\star \tag{1.64}$$

Consider the first two terms on the left-hand side of (1.64): Substituting from (1.61) into (1.63):

$$\frac{\Phi(p)}{ce^\star}\omega L^2 \longrightarrow_{p\to-\infty} 0 \tag{1.65}$$

Since ω is bounded away from zero and $ce^\star$ is bounded from above, this implies that

$$E[W(\tilde{\pi})] = L\Phi(p) + (w^\star + \varepsilon)(1 - \Phi(p)) \longrightarrow_{p\to-\infty} w^\star + \varepsilon \tag{1.66}$$

Consider the third term on the left-hand side of (1.64). Since ω is bounded away from zero and $ce^\star$ is bounded from above, (1.65) also implies that

$$\Phi(p)L^2 \longrightarrow_{p\to-\infty} 0$$

So that, using (1.66),

$$(L-E[W(\tilde{\pi})])^2\Phi(p)+(w^\star+\varepsilon-E[W(\tilde{\pi})])^2(1-\Phi(p)) = (L-w^\star-\varepsilon)^2\Phi(p)+(w^\star+\varepsilon-w^\star-\varepsilon)^2(1-\Phi(p)) \longrightarrow_{p\to-\infty} 0 \tag{1.67}$$

Combining (1.66) and (1.67), the participation constraint in (1.64) rewrites as

$$w^\star + \varepsilon \longrightarrow_{p\to-\infty} w^\star \tag{1.68}$$

So that, given (1.62),

$$\varepsilon \longrightarrow_{L\to-\infty} 0 \tag{1.69}$$

and

$$M \longrightarrow_{L \to -\infty} w^\star \tag{1.70}$$

The first part of proof is complete.

Using the definition of the agency cost in (1.5), we know from (1.66) that the agency cost with the proposed contract is equal to

$$L\Phi(p) + (w^\star + \varepsilon)(1 - \Phi(p)) - w^\star \longrightarrow_{L \to -\infty} \varepsilon$$

Using (1.69), the agency cost is approximately zero for an arbitrarily small L, and the proof is complete.

Sketch of proof of Corollary 2:

The optimal contract being characterized by $m = L$, Φ being strictly increasing, and both $\frac{d\Phi(p)}{dm}$ and $\frac{dM}{dm}$ being strictly positive (as shown in the proof of Proposition 2), an optimal contract which satisfies the participation constraint as an equality and the incentive constraint will be characterized in equilibrium by a given $p = \underline{p} < 0$ if and only if $L < m_0$. Denote the L and M associated to this given $\underline{p}$ by $\underline{L}$ and $\underline{M}$, respectively.

Suppose that $L = \bar{L} > m_0$. Consider the contract with $p = \underline{p}$ and $m = \bar{L}$. We determine the value of M such that is satisfies the incentive constraint, and we denote it by $\bar{M}$. The incentive constraint (1.53), derived in the proof of Proposition 2, may be rewritten as

$$(M - m)\Big[\varphi(p) + \omega\varphi(p)(M - m)\big(1 - 2\Phi(p)\big)\Big] = ce^\star \tag{1.71}$$

In order for two given contracts characterized by the same value of p to satisfy (1.71), they need to be characterized by the same value of $M - m$, so that we must have

$$\bar{M} = \bar{L} + (\underline{M} - \underline{L}) \tag{1.72}$$

The contract characterized by $\{\underline{p}, \bar{L}, \bar{M}\}$ is obtained from the contract characterized by $\{\underline{p}, \underline{L}, \underline{M}\}$, with an upward translation of both payments m and M by the same amount $\bar{L} - \underline{L}$ (by construction in the case of m, and because of (1.72) in the case of M). The former contract is therefore characterized by a higher expected payment, and by the same variance of pay as the latter. Because of this, the fact that the same level of effort is induced under both contracts, and the fact that the latter contract satisfies the participation constraint, the participation constraint in (1.2) cannot be binding under the former contract.

Chapter 2

Are convex contracts optimal for prudent and risk averse CEOs?

Compensation contracts which are concave in the performance measure are well-structured to provide incentives to risk averse agents, since the covariance between the pay-performance sensitivity and marginal utility is then positive. In particular, convex contracts are suboptimal for agents with quadratic utility. Nevertheless, convex contracts, which protect against downside risk, give prudent agents a higher expected utility for a given expected payoff. The optimal structure of compensation trades off these two forces. For example, with CRRA preferences and a normally distributed performance measure, the prudence effect dominates the risk aversion effect for values of relative risk aversion smaller than one. However, with a lognormally distributed stock price, the form of observed CEO contracts suggests that CEOs are more prudent than is assumed with CRRA preferences. In effect, the typical CEO contract is best explained by assuming a low and decreasing relative risk aversion. The model also shows that the positive skewness of the stock price distribution largely drives the concavity of the optimal contract.

" (...) if managers valued stock options at their Black-Scholes value, the optimal granting policy would be to grant an infinite number of options at an infinite exercise price. The absurdity of this result underscores the need to introduce managerial risk aversion into any analysis of executive stock-option valuations and incentives." - Brian J. Hall and Kevin J. Murphy, 2000.

A major justification for contingent CEO pay is to align the interests of the CEO with those of his firm. This leaves open the question of the optimal form of incentives: is it preferable to use punishments or rewards? Concave contracts or convex contracts? The literature is inconclusive on this point. This paper therefore starts by analyzing how the form of compensation contracts interacts with the form of the utility function to affect incentives, the valuation of implied transfer rules, and contractual efficiency. Then it focuses on the determinants of the form of CEOs' contingent pay as a solution to moral hazard. It identifies the structure of preferences and the preference parameters implied by observed CEO compensation contracts. The preferences of CEOs which best fit the data feature prudence and decreasing relative risk aversion. Provided that preferences are thus inferred from the data, the moral hazard model explains reasonably well the form of observed contracts.

Empirical evidence suggests that CEO pay is convex in the stock price. This is mostly attributable to the widespread use of stock-options, which now account for 51 percent of total pay of the median S&P500 CEO, according to Murphy (2002). Unfortunately, the convexity of most observed compensation profiles with respect to the appropriate performance measure (the stock price in the case of CEOs), is not satisfactorily explained by standard models of efficient contracting, such as those of Jenter (2002), Hall and Murphy (2002), or Dittmann and Maug (2007).[1] By contrast, this paper shows how and why convex contracts can be optimal in the standard contracting model with moral hazard. It is noteworthy that I do not need any of the assumptions generally used in the literature to generate convex contracts, namely risk neutrality, limited liability, loss aversion, moral hazard in risk, and taxation.[2] This paper emphasizes the

[1]Using numerical simulations of a CRRA-lognormal model, Jenter (2002) compares granting call options to granting stocks, and concludes that stocks are more efficient. Dittmann and Maug (2007) take a CRRA-lognormal model to the data, and conclude that U.S. CEOs should receive stocks and a *short* position in call options - which is equivalent to giving them a long position in stocks and a short position in puts. In a similar model, Hall and Murphy (2002) show that the optimal strike of stock-options is zero, thereby rejecting the convexity of stock-options in favor of the linearity of restricted stocks.

[2]First, risk neutrality and feasibility constraints (including limited liability) are crucial in Innes (1990), as well as in the baseline moral hazard model in Tirole (2005). With a limited liability constraint and risk neutrality, Innes (1990) and to a lesser extent Lambert and Larcker (2004) obtain out-of-the-money option-like contracts. Second, my paper shows that neither loss aversion (a kink in the utility function) nor risk-seeking behaviors (a partly convex utility function) are required for convex contracts to be optimal. The model of executive

importance of a convex marginal utility, also called prudence.[3] Prudent agents save more if their future income is risky, and they are averse to downside risk. The main contribution of the paper is to identify and separate the effects of risk aversion and prudence on compensation design: whereas the former is conducive to concave contracts and punishments, the latter is conducive to convex contracts and rewards. In a calibrated CRRA-lognormal model, option-like contracts which are so prevalent in executive pay are only optimal for very low values of relative risk aversion. However, the effect of prudence on compensation design dominates the effect of risk aversion for values of relative risk aversion smaller than one. The model therefore predicts that agents with high relative risk aversion will primarily be incentivized with punishments for poor performances, while agents with low relative risk aversion will primarily be incentivized with upward participation and rewards for good performances.

These results may be explained by the two effects arising from individual preferences which determine the curvature of optimal compensation contracts in models of moral hazard. The first is the impact of increasing effort on risk exposure, already recognized in Jenter (2002) and the first chapter of this thesis. When the agent is risk averse, a contract in which steep slopes correspond to low payments is well structured to maximize incentives. In effect, steep changes

compensation of de Meza and Webb (2007) features the former, while the model of Dittmann, Maug and Spalt (2008) include both. Instead, my framework nests loss aversion, but also allows for a smooth utility function. Third, I postulate that managerial effort affects the mean of returns, not its variance. Many papers, including Hirshleifer and Suh (1992), show that the curvature of the compensation profile affects the manager's attitude toward the variance of the performance measure. DeFusco, Johnson, and Zorn (1990) find that the implied volatility of a firm's stock price tends to increase after stock-options are granted. In Feltham and Wu (2001), stock-options become preferred to stocks when managerial effort affects the risk of the firm. However, Ross (2004) shows that a risk averse agent whose pay contract is convex in a given variable may still be averse to the volatility of this variable. Given this result, a fixed payment would be more effective at mitigating risk-avoiding behaviors on the part of the CEO (not to mention that it would also be less costly), a point already made by Hemmer, Kim and Verrecchia (2000). Fourth, tax considerations and accounting rules may also favor stock-options, as argued in Hall and Murphy (2003) and Jensen and Murphy (2004). Apart from the limited liability constraint, I do not consider these factors. I also assume that the efficient contractual paradigm is valid: in particular, I do not take the managerial power view of Bebchuk and Fried (2004), which argues that remuneration instruments are vehicles for rent extraction by entrenched executives.

[3] An agent is risk averse if his marginal utility is decreasing in payments. He is prudent if his marginal utility is convex in payments. This is a reasonable assumption. First, prudence is implied by decreasing absolute risk aversion. Second, prudence implies a preference for distributions with a positive skewness, with a small loss (relative to the expected payoff) with high probability and a large gain with low probability. Such a preference has been extensively documented - see for example Kraus and Litzenberger (1976). Third, prudent agents have a precautionary saving motive, which has been validated empirically. See, for instance, Browning and Lusardi (1996) and Gourinchas and Parker (2001). Fourth, Scott and Horvath (1980) show that prudence is necessary for marginal utility to be positive for all wealth levels.

in pay are then concentrated on regions where the agent has a high marginal utility, i.e., where he is most sensitive to changes in his pay. This suggests the utilization of concave contracts and punishments.

The second is the effect of prudence, which can also be interpreted as aversion to downside risk. This is because a positive third derivative of the utility function reflects a decreasing concavity. It ensues that a prudent agent does not heavily discount upward variations from a given payment, but he strongly discounts downward variations. This suggests the utilization of convex contracts and rewards, which protect against downside risk. It is notable that decreasing absolute (or relative) risk aversion is not necessary to obtain this result.[4]

In brief, risk aversion, captured by a negative second derivative of the utility function, is conducive to concave contracts for incentives-related reasons, whereas prudence, captured by a positive third derivative, is conducive to convex contracts for valuation-related reasons. The analysis starts by only considering a quadratic utility function, which is risk averse and not prudent. This restriction is not innocuous, since a dominance argument then shows that the optimal contract cannot be convex, or cannot take the form of rewards.[5] However, we then demonstrate that a prudent agent values a convex payoff profile more highly than a symmetrical concave profile with equal expected payoff. Far from being negligible, prudence needs to be considered, lest convex contracts be suboptimal.

To obtain these results, the paper develops a new method to evaluate the effects of different compensation contracts. The challenge consists in separating the impact of prudence from the impact of risk aversion (with CARA and CRRA utility functions, prudence is intrinsically linked with risk aversion). The traditional approach, which maps the parameters of incentive-compatible and individually rational contracts into agency costs, is inevitably uni-dimensional, and therefore not appropriate for this purpose. Most notably, Mirrlees (1975 and 1999) shows that under certain conditions, paying the agent a fixed wage associated with a strong punishment for a very low performance is approximately optimal. However, we do not impose the hypotheses needed in Mirrlees (a likelihood ratio which tends to minus infinity as the performance measure approaches minus infinity, and a utility function with unbounded support). In addition, the Mirrlees contract is not observed in practice, perhaps because it is not robust to limited liability and would be difficult to enforce. Another widely-used result is the Holmstrom (1979) condition,

[4]This can easily be checked by considering a CARA utility function, which has constant absolute risk aversion but is prudent.

[5]The same result holds with mean-variance preferences, which capture pure risk aversion, or aversion to fluctuations in wealth. In a simple and general moral hazard setup with a symmetrically distributed probability density function and a performance additive in effort and noise, the first chapter of this thesis shows that convex contracts are suboptimal for an agent with mean-variance preferences.

which describes the optimal contract as a function of the agent's marginal utility and the likelihood ratio of performances. One of its limitations is that it does not allow to disentangle the effect of risk aversion from the effect of prudence, since both affect marginal utility. To overcome these limitations, the second section of this paper uses a different and complementary approach, in which contracts characterized by the same cost are compared in terms of the incentives they generate, and the expected utility they are associated with in equilibrium. Using a symmetric probability density function (with respect to the mean), symmetric contracts (with respect to a point) with the same expected payoff and the same average slope but opposite curvatures are compared. In this setting, only the nonlinearity of the utility function may explain that two symmetric contracts deliver different incentives, and are associated with a different expected utility. More precisely, I show that risk aversion matters for relative incentives, and prudence matters for relative expected utility. All results in this part of the paper are obtained in a very general setting, with minimal hypotheses. They clearly identify the advantages associated with convexity (respectively rewards) and concavity (respectively punishments).[6]

These results are then applied to the issue of CEO pay. Given the lognormal distribution of long-run stock returns, the purpose is to establish the nature of CEO preferences implied by observed contracts.[7] Dittmann and Maug (2007) derive the nonlinear optimal contract with a lognormally distributed stock price, but they assume CRRA preferences. Our approach differs: we determine what the observed contract reveals about the form of the utility function of CEOs, assuming that the moral hazard model is appropriate.

With CRRA preferences, risk aversion becomes negligible relative to prudence as the coefficient of relative risk aversion γ approaches zero. In line with the implications of the previous results of the paper, I show that the degree of concavity of the optimal contract is an increasing function of γ. This explains why the already referenced recent literature, which uses relatively high values of γ, dismisses stock-options as suboptimal. We nevertheless face a CEO compensation puzzle: in the same way that the value of γ implied by the observed equity premium is very high in a calibrated CRRA-lognormal model, the value of γ implied by typical CEO contracts is very low. However, relaxing the hypothesis of constant relative risk aversion, and letting CEOs have decreasing relative risk aversion, enables the model to better match the data. This is simply because it makes CEOs relatively more prudent and less risk averse than with

[6]In the same vein, Lambert, Larcker and Verrecchia (1991) assess separately the value of a compensation contract from the perspective of the manager, and the incentives it generates. Hall and Murphy (2002) numerically compare the executive's valuation and the incentives associated with different compensation packages characterized by the same cost.

[7]Hemmer, Kim, and Verrechia (2000) solve a similar problem of optimal contracting, but they postulate that the stock price follows a gamma distribution.

CRRA preferences. When CEOs have low and decreasing relative risk aversion, the optimal contract looks quite similar to the typical CEO contract, and switching to the former does not generate economically significant savings. Thus, our separation of the effects of risk aversion and prudence not only delivers economic intuitions and illuminates the tradeoff behind technical optimality results, but it also provides us with some guidance to match preferences parameters to the data.

Finally, the paper shows that the form of the optimal contract strongly depends on the skewness of the stock price distribution. With limited liability, stock-options-based contracts would be optimal for a normally distributed stock price and a CEO with log utility. But the optimal contract for the same CEO would be concave if the stock price were lognormally distributed instead. The fact that Hemmer, Kim, and Verrechia (2000) obtain a similar result in a different setting suggests that this is a robust prediction of models of moral hazard.

The first section presents the model and some basic results, including the benchmark case of a risk averse agent who is not prudent. The second section turns to agents who are both risk averse and prudent over the whole domain, and disentangles the effects of risk aversion and prudence on the effectiveness of incentives and the valuation of different contracts. The third section calibrates both a CRRA-lognormal model and a HARA-lognormal model to a representative CEO, and identifies the preferences implied by observed contracts. The fourth section concludes.

2.1 The model

We use a standard single period principal-agent model in the spirit of Holmstrom (1979). A risk neutral principal wants to implement a given level of effort. He makes a take-it-or-leave-it contract offer[8] to the agent before the period starts. We only consider the first step of optimal contracting in Grossman and Hart (1983), which consists in minimizing the agency cost of implementing a given effort $e^{\star}$.[9]

Technology. By exerting a nonobservable effort e at the beginning of the period, the agent

[8]The relative efficiency of different contracts depends on the structure of compensation. It is unrelated to the distribution of bargaining power, which only affects the level of compensation. Formally, the model can accommodate any allocation of bargaining power by adjusting the reservation utility of the agent. This does not affect any of the results.

[9]This is achieved by characterizing the contract which induces the given level of effort at the minimum cost. Any optimal contract being a solution to the first step problem for a given level of effort, a contract which violates our results cannot be optimal.

displaces the mean of the distribution of the contractible performance measure $\tilde{\pi}$, which decomposes as $\tilde{\pi} = e + \tilde{\epsilon}$ and is realized at the end of the period. The random variable $\tilde{\epsilon}$ is distributed according to the probability density function φ, which is symmetric around the mean of zero, and the continuous c.d.f. Φ. Its support can be unbounded, unless otherwise stated. Given effort $e^{\star}$, the p.d.f. of $\tilde{\pi}$ is ϑ. We assume that the monotone likelihood ratio property holds.[10]

Preferences. The agent's welfare is separable in effort and wealth. The cost of effort is $\psi(e)$, where ψ is an increasing and convex function mapping $[0, \infty)$ to $[0, \infty)$; ψ satisfies the Inada conditions $\lim_{e \to 0} \psi'(e) = 0$ and $\lim_{e \to \infty} \psi'(e) = \infty$. The initial wealth of the agent is normalized at zero. The agent maximizes his expected welfare, which is equal to the expected utility of his end-of-period wealth minus the effort cost. His utility function, u, is three times differentiable, and increasing in end-of-period wealth W. Marginal utility is bounded above everywhere, except possibly in the limit, as W approaches minus infinity. An agent is globally risk averse if and only if $u''(x) \leq 0$ for any x on the domain. An agent is globally prudent if and only if $u'''(x) \geq 0$ for any x on the domain. Unless otherwise specified, we assume that the agent is globally strictly risk averse ($u'' < 0$) and globally prudent ($u''' \geq 0$). A Taylor expansion of the utility function u around any given payment W^0 highlights these two factors:

$$u(W) \approx u(W^0) + u'(W^0)(W - W^0) + \frac{1}{2}u''(W^0)(W - W^0)^2 + \frac{1}{6}u'''(W^0)(W - W^0)^3$$

Let $\bar{U} \geq u[0]$ denote the agent's reservation utility.

Compensation contracts. The principal designs a contract that makes end-of-period payments to the agent contingent on the performance measure $\tilde{\pi}$. This compensation contract is defined by the transfer function $W(\pi)$, which maps the support of $\tilde{\pi}$ into $(-\infty, \infty)$. When contracts are evaluated at the equilibrium effort $e^{\star}$, we work for notational convenience with payments as a function of ϵ rather than π.

The analysis will often involve step contracts. A step contract is a triplet $\{q, \underline{w}, \bar{w}\}$. It pays $\underline{w}$ for $\epsilon \in [-\infty, q]$, and $\bar{w}$ for $\epsilon \in [q, \infty]$. Its wedge is $\hat{w} \equiv \bar{w} - \underline{w}$. Letting the cutoff q be strictly positive, a *punishment contract*, denoted by P, is a step contract that pays a "punishment" $\underline{w}_P$ for $\epsilon \in [-\infty, -q]$, and a wage $\bar{w}_P$ for $\epsilon \in [-q, \infty]$, while a *reward contract*, denoted by R, is a

[10]Given that the performance measure is additive in effort and noise, assuming that

$$\left(\varphi_{\pi}(\pi - e)\right)^2 > \varphi(\pi - e)\varphi_{\pi\pi}(\pi - e)$$

for every π ensures the monotone likelihood ratio property: a higher effort increases the likelihood of a high performance (see the appendix for details). It guarantees that the compensation profile be increasing in the performance on the whole domain.

step contract that pays a wage $\underline{w}_R$ for $\epsilon \in [-\infty, q]$, and a "reward" $\bar{w}_R$ for $\epsilon \in [q, \infty]$. Because q is positive and $\Phi(0) = 0.5$ with a symmetric p.d.f., a punishment or a reward (depending on the contract) occur relatively rarely in equilibrium, with probability less than one half. That is why the punishment contract can be viewed as a fixed wage associated with an occasional punishment, and the reward contract can be viewed as a fixed wage associated with an occasional reward.

Constraints on contracting. (not applicable in section 2, see the footnotes below) To be accepted, the principal's offer must satisfy the agent's participation constraint at the equilibrium effort:

$$\int_{-\infty}^{\infty} u[W(\pi)]\varphi(\epsilon)d\epsilon = \bar{U} + \psi(e^\star) \tag{2.1}$$

If it does, the contract is said to be individually rational.[11]

Using the first-order approach (see the next paragraph) and Leibniz's theorem, the incentive constraint, as evaluated at the equilibrium effort, is

$$\int_{-\infty}^{\infty} W'(\pi)u'[W(\pi)]\varphi(\epsilon)d\epsilon = \psi'(e^\star) \tag{2.2}$$

A contract that satisfies this constraint is said to be incentive-compatible.[12]

The first-order approach. This paper uses the first-order approach, which allows to replace a continuum of incentive constraints by the local incentive constraint at the equilibrium level of effort if the maximization problem of the agent is concave in effort. I show in the appendix that the first-order approach is always valid with step contracts in the setting used, as long as marginal utility is decreasing. The first-order approach is also always valid with concave contracts, as the second derivative of the agent's objective function is then negative:

$$E\Big[W''(\tilde{\pi})u'[W(\tilde{\pi})] + (W'(\tilde{\pi}))^2 u''[W(\tilde{\pi})]\Big] - \psi''(e^\star) < 0$$

since $W''(\tilde{\pi}) < 0$, $u'[W(\tilde{\pi})] > 0$, $u''[W(\tilde{\pi})] < 0$, and $\psi''(e^\star) > 0$.

Only convex contracts are potentially problematic. However, for any given convex contract W such that $E[W''(\tilde{\pi})u'[W(\tilde{\pi})]$ is a finite constant, there exists a sufficiently convex cost function such that the first-order approach is valid. Put differently, the set of convex contracts for which the first-order approach is valid can be enlarged as needed by increasing $\psi''(e^\star)$, i.e., by increasing the convexity of the cost function. Since in this paper we consider given convex contracts, it is always possible to ensure that the first-order approach holds in each case. Admittedly, this

[11] We replace this constraint with an expected cost constraint in section 2 of the paper.

[12] The equilibrium effort which solves this equation may not be $e^\star$ in section 2 of the paper.

approach would not be appropriate for selecting the most efficient contract in an unbounded set of convex contracts.

Agency costs. Denote by $W^\star$ the first-best payment that corresponds to effort $e^\star$. That is, the agent is compensated for his reservation utility plus the effort cost:

$$u[W^\star] \equiv \bar{U} + \psi(e^\star) \tag{2.3}$$

As by assumption $\bar{U} \geq u[0]$, the utility function is increasing and the effort cost is nonnegative, we have

$$W^\star \geq 0 \tag{2.4}$$

The agency cost corresponds to the cost of inducing effort $e^\star$ for a given compensation contract W, beyond compensation for effort and for the reservation utility. Denote the agency cost by $AC_{u,W}$: it is a function of the utility function $u(W)$ and of the compensation contract $W(\pi)$. By definition, the expected cost of compensation with a contract W is the agency cost associated with it plus the first-best cost:

$$E[W(\tilde{\pi})] \equiv W^\star + AC_{u,W} \tag{2.5}$$

Definition: An *optimal contract* minimizes the agency cost, given $e^\star$ and the agent's preferences. Any contract with a nonpositive agency cost is optimal.

The problem of the principal is to minimize the agency cost of inducing effort $e^\star$. Note that in a given setup, the optimal contract is not necessarily unique. In particular, several contracts can in certain settings achieve the required effort $e^\star$ at a zero agency cost.

To start with, suppose that the preferences of the agent are such that his marginal utility is constant. Let the principal offer any feasible contract W that satisfies both the agent's participation constraint and his incentive constraint.[13] Because the first derivative of a risk neutral agent's utility is constant and all higher-order derivatives are zero,

$$u(W(\pi)) = u(W^\star) + u'(W^\star)\big(W(\pi) - W^\star\big)$$

Taking expectations,

$$E[u(W(\tilde{\pi}))] = u(W^\star) + u'(W^\star)\big(E[W(\tilde{\pi})] - W^\star\big)$$

Besides, using (2.1) and (2.3),

$$E[u(W(\tilde{\pi}))] = u(W^\star)$$

[13]As previously indicated, the effort level to be elicited is given, and set at $e^\star$. It is not necessarily the first-best effort, which explains why the (risk-neutral) agent is not necessarily the residual claimant.

So that, marginal utility being positive,

$$E[W(\tilde{\pi})] = W^{\star}$$

Substituting into (2.5), the agency cost of any contract satisfying both constraints (2.1) and (2.2) when the agent is risk neutral is zero.

Now consider the case of an agent whose marginal utility is decreasing. We are going to derive a measure of the agency cost of any contract W. We know that if u is concave, there exists a v_W in-between W^0 and W such that[14]

$$u(W) = u(W^0) + u'(W^0)(W - W^0) + \frac{1}{2}u''(v_W)(W - W^0)^2 \tag{2.6}$$

Setting $W^0 = W^{\star}$ in (2.6), taking expectations and rearranging, we get

$$E[W(\tilde{\pi})] - W^{\star} = \frac{1}{u'(W^{\star})}\left(E[u(W(\tilde{\pi})] - u(W^{\star}) - \frac{1}{2}E\Big[u''(v_W)(W(\tilde{\pi}) - W^{\star})^2\Big]\right) \tag{2.7}$$

Where $E[u(W(\tilde{\pi})] = u(W^{\star})$ because of (2.1) and (2.3). Moreover, the left-hand side of (2.7) being equal to the agency cost, we can write:

$$AC_{u,W} = -\frac{1}{2u'(W^{\star})}E\Big[u''(v_W)(W(\tilde{\pi}) - W^{\star})^2\Big] \tag{2.8}$$

which is positive if the utility function is concave, and where v_W is increasing in W. Plugging in (2.7) yields

$$AC_{u,W} = \frac{1}{u'(W^{\star})}\int_{-\infty}^{\infty}\Big[\Big(u(W^{\star}) + u'(W^{\star})[W(\pi) - W^{\star}]\Big) - u[W(\pi)]\Big]\varphi(\epsilon)d\epsilon \tag{2.9}$$

The agency cost is the sum over all states of nature, as weighted by their respective probabilities in equilibrium, of the difference between the valuation of compensation in this state by a risk neutral and a risk averse agent.

As a benchmark case, assume that the second derivative of u is constant, equal to k (where k is negative), on the whole domain. Using (2.8), it appears that the agency cost is then proportional to the sum over all possible states of nature, as weighted by their respective probabilities in equilibrium, of the squared difference between the state-contingent pay $W(\pi)$ and the first-best payment $W^{\star}$:

$$AC_{u,W} = \kappa\int_{-\infty}^{\infty}(W(\pi) - W^{\star})^2\varphi(\epsilon)d\epsilon \tag{2.10}$$

where κ is a positive constant equal to $-\frac{1}{2}\frac{k}{u'(W^{\star})}$. Notice that unless $E[W(\tilde{\pi})] = W^{\star}$, this expression is *not* proportional to the variance of $W(\tilde{\pi})$.

[14]This is an application of the mean value theorem, and is proved for example in Simon and Blume (1994), p.828.

Finally, we identify instances where the agency cost can easily be eliminated, even though the utility function of the agent is concave. The idea is to concentrate payments in the interval where the agent is risk neutral, which is only possible if he is risk neutral around the first-best payment. Divide the range of payments into three intervals: $(-\infty, W_1)$, (W_1, W_2), and (W_2, ∞), where $W_1 < W^\star < W_2$. Then

Claim 1:

- If $W_2 - W_1$ is large enough and if marginal utility is constant on (W_1, W_2), then an optimal contract exists, and induces $e^\star$ at zero agency cost.
- If marginal utility is constant on $(-\infty, W_2)$, then an optimal contract exists, and induces $e^\star$ at zero agency cost.
- If marginal utility is constant on (W_1, ∞), then an optimal contract exists, and induces $e^\star$ at zero agency cost.

The proof is in the appendix. In these cases, individually rational and incentive-compatible contracts can be constructed such that the agent's marginal utility is constant over the range of all possible payments he may receive in equilibrium, so that the agency cost is zero.

Optimal contracting with risk aversion. We now study optimal contract design with quadratic utility,[15] which captures risk aversion and excludes prudence: by construction, quadratic utility is the only concave utility function with $u''' = 0$. This constitutes the benchmark case. In this setting, we show that convex contracts and step contracts with rewards are suboptimal. Roughly speaking, agents who are exclusively risk averse should not be offered upside participation.

Although the expected utility with a quadratic utility function is fully described by the equilibrium mean and variance of pay, a quadratic utility function is not linear in the mean and variance of pay. In this sense, the results obtained below technically differ from the results obtained in chapter 1, notwithstanding the similar economic intuition.

[15] I construct a utility function with a negative and constant second derivative in the appendix. Quadratic utility and the mean-variance criterion provide a useful benchmark, although they are questionable: the agent is just as averse to upward variations as to downward variations, which does not concur with either economic intuition or empirical and experimental studies. This unsatisfactory symmetry does not matter only if payoffs are linear in a symmetrically distributed random variable.

Proposition 1a (benchmark case): If u is quadratic, any reward contract with $\bar{w}$ inferior to an arbitrarily large constant is dominated by a punishment contract.

Risk aversion makes it more efficient to offer punishments rather than rewards for incentive purposes. A risk averse agent with a constant second derivative discounts as heavily downward and upward deviations from the optimal risk sharing rule. But for the same deviation with respect to the first-best payment, a punishment offers more incentives than a symmetric reward. The intuition is that the variance of the agent's pay is decreasing in effort when punishments are used, whereas it is increasing in effort when rewards are used. Punishments can therefore be smaller and still be incentive-compatible. This diminishes the discount applied to the transfer rule, and therefore reduces the agency cost.

We now consider concave and convex contracts. For technical reasons, the support of $\tilde{\epsilon}$ cannot be unbounded in the following proposition, although the bounds can be arbitrarily large. See the first chapter of this thesis for a proof of the same result as in proposition 1b with an agent who maximizes a linear mean-variance criterion, in which case the support of $\tilde{\epsilon}$ can be unbounded. Proposition 1b below exactly mirrors proposition 1a. This is because concave contracts share essential properties with punishments, as do convex contracts with rewards.

Proposition 1b (benchmark case): If u is quadratic, any compensation contract convex in the performance measure is dominated by a concave contract.

Risk aversion makes it more efficient to offer concave contracts rather than convex contracts. The intuition is as in the case with step contracts.[16]

We also show in Claim 2 in the appendix that payments to an agent whose utility function is capped should be capped. Notice that CEO compensation contracts are typically not capped.

[16]Quadratic utility implies mean-variance analysis. For any convex contract W, the proof of proposition 1b constructs a concave contract V with the same pay-performance sensitivity, the same expected payoff, and the same variance as the original contract W. But with a concave contract, the covariance between the pay-performance sensitivity and marginal utility turns positive, which delivers further incentives for effort. The proof proceeds by flattening the contract V until it elicits the same effort as W. The same effort is hence induced at the cost of a smaller deviation from $W^{\star}$, and therefore at a lower agency cost, as defined in (2.10). This demonstrates that W is suboptimal.

2.2 Disentangling the effects of risk aversion and prudence

If a positive and decreasing marginal utility were sufficient to describe individual preferences, the convexity of most observed managerial contracts would be puzzling. In effect, we have seen that it is inefficient to offer convex contracts to agents who have quadratic utility (in the preceding section) or whose objective function is linear in the mean and variance of payments (in the preceding chapter of the thesis). However, there are good reasons to believe that agents are both risk averse and prudent. In this case, results of the previous section and of the previous chapter do not apply. This section identifies the channel through which prudence potentially re-establishes the optimality of convex contracts.

Since risk aversion and prudence pull loosely speaking the curvature of the optimal contract in opposite directions, it is impossible to get an optimality result in the general case (the next sections present optimality results in the CARA-normal and CRRA-lognormal cases). To get around this obstacle, this section disentangles the interaction between risk aversion and the contract curvature on the one hand, and between prudence and the contract curvature on the other hand. In a sense to be defined below, risk aversion ensures that concave contracts or punishment contracts generate more incentives, while prudence ensures that convex contracts or reward contracts are associated with a higher expected utility.

As in the quadratic utility case, risk averse agents have a decreasing marginal utility, which tends to render concave contracts and punishments more apt at providing incentives. However, with prudence, which implies aversion to downside risk,[17] downward deviations from $W^\star$ are more costly in terms of agency costs than upward deviations of the same magnitude. Valuation considerations alone therefore suggest the utilization of convex contracts and rewards. The following propositions formalize this tradeoff.

In the usual approach used elsewhere in this paper, the agent's expected utility is constant across admissible contracts. In the approach used in this section, the expected payment from the principal is constant across contracts that have different effects on the agent. For a given cost of the contract for the principal, we separately evaluate the agent's valuation of the transfer rule

[17]The case with u'' negative and decreasing is quite uncommon - especially because it represents an agent more averse to upward variations than to downward variations in his wealth. Furthermore, it cannot be resolved in general, because the sign of $E[u'(V(\tilde{\pi}))] - E[u'(W(\tilde{\pi}))]$ is indeterminate when neither u nor W are specified - the linear marginal utility implied by a constant u'' was crucial to obtain an equality between these two terms in the benchmark case. Let us just mention that u' is still decreasing, which implies that concave contracts tend to provide more incentives than convex contracts for the same pay-performance sensitivity; and the fact that u'' is decreasing makes downward deviations from $W^\star$ less costly in terms of agency costs than upward deviations - which makes punishments more cost-effective than rewards: convex contracts are suboptimal.

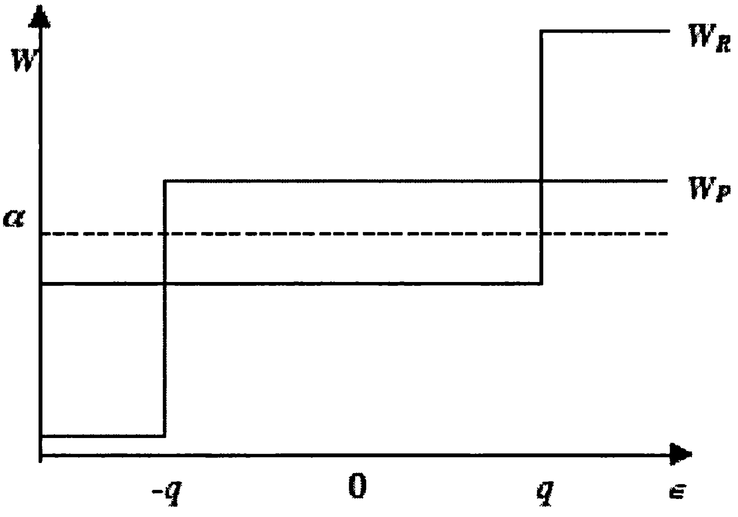

Figure 2.1: Comparing a punishment contract to a reward contract

implied by the contract, and the effort incentives it generates. In other words, we do not require contracts to be individually rational or to implement the level of effort e^*. Instead, we separate the impact of the contract's curvature on the agent's valuation of the transfer rule and on his effort. In doing so we emphasize the tradeoff between downside risk protection provided by convex contracts and additional effort inducements provided by concave contracts. The former is valuable if the agent is averse to downside risk, whereas the latter comes into play with risk averse agents.

To start with, we compare any punishment contract W_P to a reward contract W_R, which is symmetrical to W_P with respect to the point $(0, E[W_P])$ in the (ϵ, W) plane. This has the following implications. First, the respective cutoffs of the contracts, $\epsilon = -q$ and $\epsilon = q$, are equidistant from $\epsilon = 0$. Second, both contracts have the same wedge:

$$\bar{w}_P - \underline{w}_P = \bar{w}_R - \underline{w}_R \equiv \hat{w} \tag{2.11}$$

Third, since the p.d.f. is symmetric around $\epsilon = 0$, both step contracts have the same expected payoff in equilibrium:

$$E[W_P] = \Phi(-q)\underline{w}_P + (1 - \Phi(-q))\bar{w}_P = \Phi(q)\underline{w}_R + (1 - \Phi(q))\bar{w}_R = E[W_R] \equiv \alpha \tag{2.12}$$

This implies that these two contracts are as costly to the principal.

Along the same lines, we compare any given convex contract W_E to a concave contract W_A

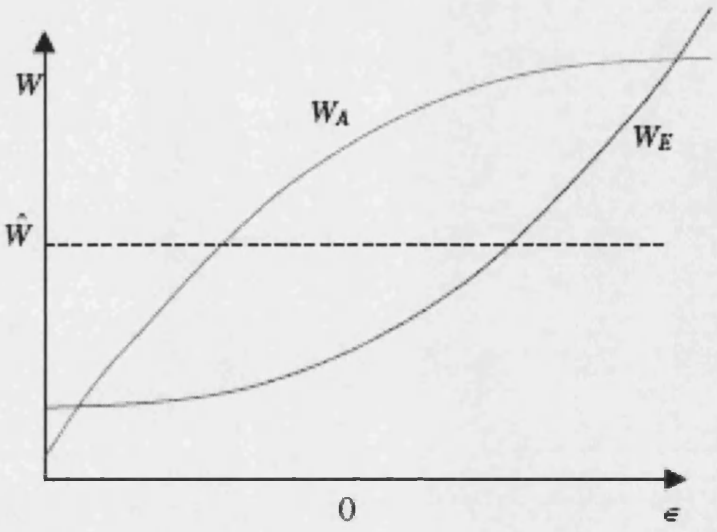

Figure 2.2: Comparing a concave contract to a convex contract.

defined by

$$W_A(\pi) = 2\hat{W} - W_E(-\pi + 2e^\star) \tag{2.13}$$

and conversely. Set $\hat{W}$ such that

$$E[W_i(\pi)|e^\star] = \hat{W} \tag{2.14}$$

for $i = A, E$. This implies that

$$E[W_A(\pi)|e^\star] = E[W_E(\pi)|e^\star]$$

These two contracts, which share the same average slope, would implement the same effort if the agent were risk neutral.

The agent's valuation of a contract and the incentives it delivers are a function of three factors: the contract itself, the probability distribution, and the utility function. With a linear utility, two contracts with the same expected payoff would be characterized by the same expected utility, and two contracts with the same average slope would provide the same incentives. With a symmetric p.d.f., symmetrical contracts with the same expected payment and the same average slope will be valued differently and induce different effort levels only if the utility function is nonlinear.

We begin by comparing the agent's valuation of the structure of payments of any two step contracts satisfying the relations described above, i.e., characterized by the same expected cost, the same wedge $\hat{w}$, and opposite cutoffs. We momentarily ignore effort to focus only on risk sharing. The first proposition shows that the agent's valuation of such symmetrical transfer rules is independent of the concavity of his utility function. What matters is the convexity of his marginal utility.

Proposition 2a: Suppose that $e = e^{\star}$. An agent with preferences characterized by $u''' = 0$ derives the same expected utility from this reward contract and from the corresponding punishment contract. An agent with preferences characterized by a constant and positive u''' derives a higher expected utility from a reward contract than from the corresponding punishment contract. An agent with preferences characterized by a variable u''' derives a higher expected utility from a reward contract than from the corresponding punishment contract if $u^{(2i-1)}$ is positive for all $i \geq 2$.

Notice that this last condition is satisfied by CARA and CRRA functions.[18] This is the first central result of the paper. Agents who are prudent (with $u''' > 0$) are averse to downside risk, i.e., they apply a heavier discount to downward variations than to upward variations from a given payoff level. When agents are prudent, risk sharing considerations therefore favor rewards rather than punishments. The same result holds when we compare any convex contract to the corresponding concave contract described above.

Proposition 2b: Suppose that $e = e^{\star}$. For any given convex contract W_E, an agent with preferences characterized by $u''' = 0$ derives the same expected utility from this transfer rule and from the corresponding concave transfer rule W_A defined in (2.13). An agent with preferences characterized by a constant and positive u''' derives a higher expected utility from W_E than from W_A. An agent with preferences characterized by a variable u''' derives a higher expected utility from W_E than from W_A if $u^{(2i-1)}$ is positive for all $i \geq 2$.

The intuition is the same as in proposition 2a. By definition, prudent agents have a preference for positive skewness. Ultimately, convexity give prudent agents a higher expected utility simply because the skewness of payments is positive with a convex contract, and negative with a concave contract.

Now turn to effort inducement, by again comparing two step contracts satisfying the rela-

[18]More generally, Scott and Horvath (1980) derive positive preferences for odd-order central moments, and negative preferences for even order central moments as necessary consequences of a positive and decreasing marginal utility - with the requirement that agents have consistent preferences, in the sense that the sign of their preferences is independent of the wealth level.

tions described above.

Proposition 3a: If $u'' < 0$, a given punishment contract delivers more incentives than the corresponding reward contract. If $u'' = 0$, both contracts deliver equal incentives.

The same result holds for concave versus convex contracts.

Proposition 3b: If the agent's preferences are characterized by $u'' < 0$ and $u'''' = 0$, any given convex contract W_E delivers strictly less incentives than the corresponding concave contract W_A defined in (2.13). If the agent's preferences are such that $u'' < 0$ and $u^{(2i)}$ is nonpositive for all $i \geq 2$, a convex contract W_E delivers strictly less incentives than the corresponding concave contract W_A.

This is the second central result of the paper. For risk averse agents, downward variations from the first-best payment generate more incentives than symmetrical upward variations of the same magnitude. This result holds regardless of the sign of u'''. For a given contract, the extent of effort incentives depends on the concavity of the utility function.[19] Notice that the last conditions in propositions 2b and 3b are satisfied by both CARA and CRRA utility functions.[20] In particular, the fact that aversion to kurtosis (in payments) also makes concave contracts more apt at providing incentives has an intuitive interpretation. A concave contract results in a relatively low probability of very high payments, and a relatively high probability of very low payments: it shifts the density away from the right tail of the distribution of payments and toward the left tail. It follows that with a contract concave in a symmetrically distributed performance measure, increasing effort (which shifts density away from low outcomes and toward higher outcomes) reduces the kurtosis of the equilibrium payment distribution - which the agent values if he is averse to kurtosis.

Propositions 2a and 2b on the one hand, and 3a and 3b on the other hand, highlight the essential tradeoff in compensation contract design. Protecting prudent agents against downside

[19]It is worth underlining that although prudence increases effort (in the sense that, everything else being equal, an agent with a positive u''' exerts more effort than an agent with a zero u''', as will be shown later), it does not per se increases the relative incentives provided by either convex or concave contracts. Only the fourth and higher-order even derivatives matter in this respect (this is apparent in (2.53) in the appendix).

[20]More generally, they are satisfied by HARA utility functions, as defined by

$$-\frac{u''(x)}{u'(x)} = \frac{1}{ax+b} > 0$$

as long as a is nonnegative (a negative a would imply an increasing absolute risk aversion).

risk by offering rewards rather than punishments must be traded off against the property of punishments to concentrate incentives on regions where the agent is most sensitive to changes in his pay. Risk aversion does not result in any tradeoff in this setting: with a constant and negative u'', punishments dominate rewards and concavity dominate convexity, as stated in propositions 2a and 2b. It is aversion to downside risk, captured by a positive u''', that potentially reinstates the relative efficiency of rewards and convexity.

In general, we cannot determine which effect will dominate. We can expect them to interact along the range of payments, and the optimal contract to trade off these two forces. Nevertheless, optimality results can be obtained, but they require more assumptions on the utility function and on the probability distribution of performances. In the next section, we characterize optimal contract in a widely used setting.

2.3 Optimal contracts for CEOs

2.3.1 CRRA preferences and the CEO compensation puzzle

The CRRA-lognormal setting is commonly used in models of executive compensation. First, the CRRA utility function exhibits desirable properties, such as risk aversion, prudence, and decreasing absolute risk aversion. Second, with compounded returns, it is reasonable to assume that the long-term distribution of stock prices is lognormal.[21] We use this setting, and impose the constraint that payments are nonnegative. With this limited liability constraint, a Mirrlees-type contract which approximates the first-best with extreme punishments is not feasible. However, the optimal contract in this setting can be obtained in closed form using the Holmstrom (1979) approach, even with a limited liability constraint, as we show in the Appendix.

Calibrating the model is straightforward: the parameters of the distribution of stock prices can be estimated. The only unknown, in the utility function, is the coefficient of relative risk aversion γ. Picking the correct value is all the more difficult that estimates of γ vary widely: experiments yield values close to 1, while values around 50 are required to generate the historical equity risk premium of the U.S. market in a standard asset pricing model.[22] Dittmann and Maug

[21] A stock price process which follows a geometric Brownian motion is lognormally distributed in the long run.

[22] The literature on the equity risk premium was originated by Mehra and Prescott (1985). Arrow (1971) argues that relative risk aversion should be close to 1, for theoretical reasons. Kydland and Prescott (1982) need a relative risk aversion between 1 and 2 to replicate the observed fluctuations in consumption and investment. Finally, Campbell, Lo and MacKinlay (1997), as well as Ait-Sahalia and Lo (2000) summarize estimates of relative risk aversion obtained in the macroeconomic literature. In experiments, Harrison, Lau and Rutstrom (2007) obtain an average value of relative risk aversion of 0.67, while Bombardini and Trebbi (2007) obtain an average value of 1. The latter also review the experimental literature.

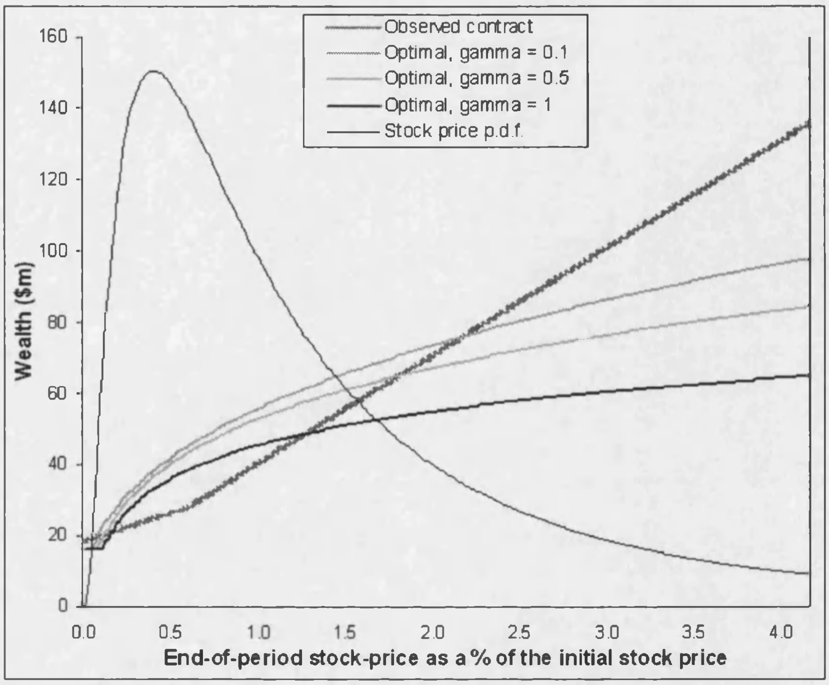

Figure 2.3: Optimal compensation contracts for different coefficients of RRA.

(2007) use values of γ between 0.5 and 10, and conclude that the optimal contract is concave for an overwhelming majority of CEOs (except for very low performances, because of limited liability). This is puzzling, since current managerial compensation heavily uses stock-options, and is generally convex in the stock price.[23]

A legitimate question is whether there is any value of γ that approximately generates the compensation contract of a typical CEO in a calibrated CRRA-lognormal model of efficient contracting. To answer it, I apply Dittmann and Maug's approach to their representative CEO, with a limited liability constraint imposing nonnegative payments (more details can be found in the appendix). As shown in figure 3, the optimal contract is indeed concave for plausible values of relative risk aversion. However, optimal contracts become less concave as γ diminishes. This is because, with CRRA utility, risk aversion becomes negligible relative to prudence as

[23]According to Dittmann and Maug (2007), taking into account bonus payments and salary changes makes observed compensation contracts even more convex. Furthermore, in his review of the literature, Murphy (1999) states that "virtually all of the sensitivity of pay to corporate performance for the typical CEO is attributable to the explicit rather than the implicit part of the CEO's contract. (...) pay-performance sensitivities are driven primary by stock options and stock ownership." Regarding dismissals, Dittmann and Maug (2007) convincingly argue that taking them into account hardly modifies the shape of observed compensation contracts, a view corroborated by Murphy (1999).

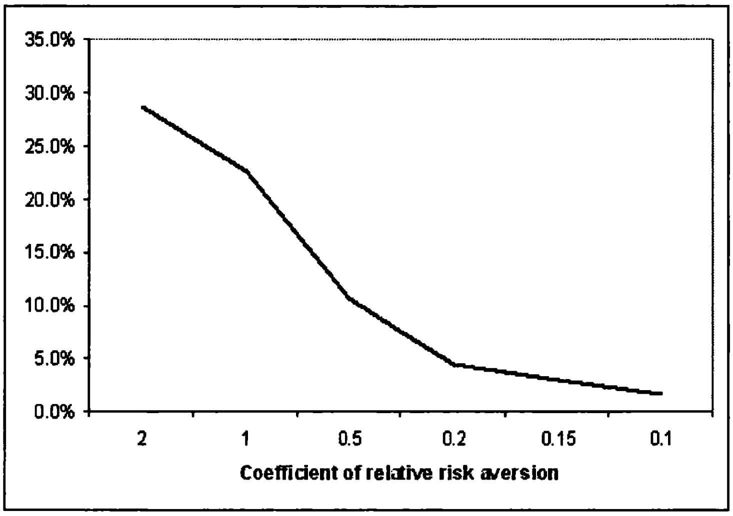

Figure 2.4: Savings associated with a switch to the optimal contract, as a proportion of the current cost of CEO pay.

the coefficient of relative risk aversion γ approaches zero. More precisely, at $u(x)$, the index of absolute risk aversion is $\frac{\gamma}{x}$, while the index of absolute prudence is $\frac{\gamma+1}{x}$.[24]

As shown in figure 4, the potential savings associated with a switch to the optimal contract are decreasing in the coefficient of relative risk aversion γ, and become so low as to be economically insignificant for sufficiently low values of γ. With $\gamma = 0.1$, switching to the optimal contract would reduce the expected cost of CEO pay by only 1.6 percent. Similarly, the concavity of the optimal contract is increasing in γ, so that it is not very pronounced when γ is sufficiently small. Together, these results demonstrate that the moral hazard model fits the data for low values of γ, since the observed contract is then quite efficient. Benefits associated with switching to the optimal contact are small for low values of relative risk aversion. Only if it is assumed that CEOs have a high relative risk aversion are observed contracts are quite inefficient, or does the moral hazard model fail to explain the data.[25]

With CRRA utility, a very low coefficient of relative risk aversion is needed for observed

[24]The index of absolute risk aversion is the relevant measure of risk aversion to a risk such as contingent pay, which is additive in wealth. The index of absolute prudence, as introduced by Kimball (1990), is defined at every x as $-\frac{u'''(x)}{u''(x)}$.

[25]Arguably, this could be because all contracts which satisfy the participation constraint and the incentive constraint are approximately optimal for low coefficients of risk aversion. This is not the case. For example, for $\gamma = 0.1$, a stock-option-like contract which satisfies the aforementioned constraints of optimal contracting may be derived, and it costs 7 percent more to the firm than the observed contract.

CEO contracts to be approximately efficient in a model with moral hazard. This is the CEO compensation puzzle. Part of it may be explained by a selection effect: relative to the average individual, entrepreneurs and CEOs are arguably either less risk averse, or tend to underestimate risk.

2.3.2 The skewness of the stock price distribution

Admittedly, since optimal contracts are concave for plausible values of relative risk aversion, it is tempting to conclude that risk aversion matters more than prudence in compensation contract design. Nevertheless, it is unclear what the convexity or concavity of a contract mean when the p.d.f. is not symmetric around the mean.[26] Crucially, this is the case of the lognormal distribution. Fortunately, we know that a logarithmic transformation of a lognormally distributed variable is normally distributed. Operating such a transformation enables us to plot a given contract as a function of a normal variable, which is symmetrically distributed and therefore fits in our model. This transformation "convexifies" any contract. For instance, a contract which is linear as a function of a lognormal variable is convex as a function of the corresponding normal variable.[27]

More generally, a given contract which is contingent on a symmetrically distributed random variable (think about $ln(\varepsilon)$, which has zero skewness if $\tilde{\varepsilon}$ is lognormally distributed) becomes more concave as it is plotted against a convex transformation of this random variable (think about $\tilde{\varepsilon}$, which is lognormally distributed with positive skewness). This is because applying a convex transformation to any symmetrically distributed random variable increases its skewness, as demonstrated in the appendix. But any function of this variable is concavified when plotted against a convex transformation of this variable. It ensues that, all else equal, as the stock price distribution becomes more positively skewed, any given contract (including the optimal contract) becomes more concave.[28]

[26]The approach of this paper relies on symmetry, and is not directly applicable when the p.d.f. of the performance measure is not symmetric.

[27]If this is unclear, consider a contract which is linear when a random variable, say $\tilde{\varepsilon}$, is lognormally distributed. For instance, assume that in the (ε, W) plane, the contract goes through the points $(0,0)$, $(1,1)$ and (e,e). By definition, the variable $ln(\tilde{\varepsilon})$ is normally distributed. The same contract in the $(ln(\varepsilon), W)$ plane goes through the points $(-\infty, 0)$, $(0,1)$ and $(1,e)$. It is identical to the exponential function, which is convex. Note that on the relevant intervals, it is equivalent to represent the W contract in the $(ln(\varepsilon), W)$ plane, or the $\exp\{W\}$ contract in the (ε, W) plane. Either way, this transformation convexifies the form of the contract. Conversely, a contract which is linear when plotted against a normally distributed random variable plots as an affine transformation of the logarithm function against a lognormally distributed random variable.

[28]This observation calls for a reinterpretation of the result of Kim, Hemmer and Verrecchia (2000): increasing skewness may not diminish the "desirability" of convexity because it shrinks downside risk, but simply because

As explained in the appendix, with the constraint that payments be nonnegative, the optimal contract in a CRRA-lognormal setting takes this form (by definition, $\tilde{\pi}$ is *lognormally* distributed):

$$W(\pi) = \begin{cases} (\alpha_0 + \alpha_1 ln(\pi))^{\frac{1}{\gamma}} - W_0 exp\{r_f T\} & \text{if } \pi \geq \bar{\pi} \\ 0 & \text{if } \pi < \bar{\pi} \end{cases}$$

where α_1 is positive, and $\bar{\pi} \equiv \exp\{\frac{(W_0 exp\{r_f T\})^{\gamma} - \alpha_0}{\alpha_1}\}$. Even though the optimal contract becomes more concave as γ increases, it is only capped in the limit, as γ approaches infinity. Otherwise, imposing a cap on payments prevents the optimal contract from being implemented. The same contract as a function of a *normally* distributed random variable $\tilde{x}$, with $x \equiv ln(\pi)$, is

$$W(\pi) = \begin{cases} (\alpha_0 + \alpha_1 x)^{\frac{1}{\gamma}} - W_0 exp\{r_f T\} & \text{if } x \geq \bar{x} \\ 0 & \text{if } x < \bar{x} \end{cases}$$

where α_1 is positive, and $\bar{x} \equiv \frac{(W_0 exp\{r_f T\})^{\gamma} - \alpha_0}{\alpha_1}$. Given relative risk aversion γ, the curvature of the optimal contract looks very different when plotted as a function of x:

Proposition 6: As a function of a normally distributed random variable, and on the region where the limited liability constraint is not binding, the optimal contract is concave if and only if $\gamma > 1$, convex if and only if $\gamma < 1$, and linear for a log utility.

It is remarkable that this result holds for any values of the parameters other than relative risk aversion. The intuition is that for any wealth level, the ratio of the index of absolute prudence to the index of absolute risk aversion, $\frac{\gamma+1}{\gamma}$, is decreasing in γ. As proposition 6 indicates, prudence is stronger than risk aversion for values of relative risk aversion less than 1, and conversely for values of relative risk aversion greater than 1. For a log utility, the two effects cancel out and the optimal contract is linear. Note that where the limited liability constraint is not binding, the optimal contract with $\gamma = 1$ in figure 3 has the form of an affine transformation of the log function, but is linear when plotted in the plane of figure 5. That is, a contract exclusively based on stock-options would be optimal for a CEO with log utility if it were contingent on a normally distributed performance measure.[29]

As displayed in figure 5, for a range of plausible preference parameters, the optimal contracts represented as a function of the normalized stock price distribution with zero skewness look very

it alters the properties of the plane on which the contract is plotted.

[29] A stock price which follows a Brownian motion is lognormally distributed in the long run. Note that because of the central limit theorem, this property of compounded returns does not rely on the normality of instantaneous returns, but only on the axiom of independent increments.

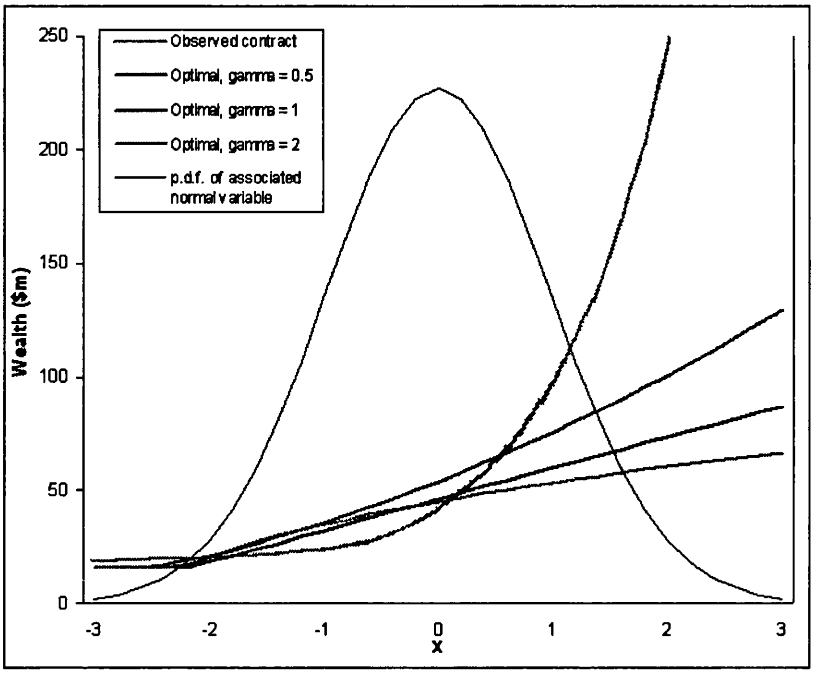

Figure 2.5: The observed contract and the optimal contracts for different coefficients of RRA as a function of the normalized stock price.

much like stock-options. This finding has one major implication: it is mainly the positive skewness of the stock price which drives the concavity of the optimal contract when the stock price is assumed to be lognormally distributed. Acknowledging this reconciles the seemingly contradictory results of Dittmann and Maug (2007) and Jenter (2002) who rejected stock-options on the one hand, and Hemmer, Kim, and Verrecchia (2000) who showed in a setting with a gamma distribution that they could be optimal on the other hand. The apparent inconsistency stems from the fact that a gamma distribution exhibits a smaller skewness than the lognormal distribution for most parameter values: while the latter has a skewness of 5.99 in the case of the representative CEO of Dittmann and Maug, a gamma distribution with values for k ranging from 2 to 9 would have a skewness in-between $\frac{2}{3}$ and $\sqrt{2}$.

Nevertheless, the fact that contracts featuring stocks and stock-options are convex as a function of a variable whose distribution is approximately lognormal means that they are even more convex when plotted against the corresponding normally distributed variable, as shown in figure 5. Understanding that skewness strongly determines the curvature of the optimal contract does not change the fact that pay is contracted based on a lognormally distributed stock price. The CEO compensation puzzle remains.

2.3.3 Introducing decreasing relative risk aversion

Given the stock price probability distribution, what follows determines the structure and parameters of preferences which best fit the data. In the same way that the equity premium puzzle can be partly solved by disentangling between risk aversion and the intertemporal elasticity of substitution with Epstein-Zin preferences, the CEO compensation puzzle may be partly solved by disentangling between risk aversion and prudence. We will show that the moral hazard model of efficient contracting is quite successful at explaining the form of CEO pay if CEOs are less risk averse and more prudent than is assumed with CRRA preferences.

When considered in view of propositions *2b* and *3b*, the result that the optimal contract is "too concave" when CEOs have CRRA utility suggest that CEOs are relatively less risk averse and more prudent than is assumed by preferences with constant relative risk aversion. In other words, the concavity of their utility function seems to be relatively lower for high payments and relatively higher for low payments than with a CRRA utility. With CRRA utility, decreasing the coefficient of relative risk aversion results in a uniformly lower concavity. Instead, the utility function which fits the data would have a relative risk aversion decreasing with wealth.[30] If relative risk aversion is indeed decreasing with wealth, calibrating a misspecified model with CRRA preferences to the data will yield a very low implied coefficient of RRA for wealthy individuals (including CEOs), and a very high implied coefficient of RRA for poor individuals. This would in turn explain why the coefficient of relative risk aversion which best fits observed CEO contracts is surprisingly low in this setting.

We now show that relaxing the assumption that relative risk aversion is constant markedly improves the performance of the moral hazard model for CEO pay.

A utility function with harmonic absolute risk aversion (HARA) can be calibrated to be more prudent and less risk averse than CRRA. A HARA utility function is defined by the parameters a and b, where $0 < b < 1$ to ensure prudence and decreasing absolute risk aversion.[31] It takes

[30] These is some evidence that relative risk aversion is decreasing with wealth. Arrow (1965) show that the wealth elasticity of investments at the risk-free rate is smaller than unity with DRRA. The empirical evidence points in this direction: Cohn, Lewellen, Lease, and Schlarbaum (1975) and Kessler and Wolff (1991) show that the proportion of risky assets in household portfolios is strongly increasing in wealth, a finding corroborated by Levy (1994) in an experimental study, and Siegel and Hoban (1982) for wealthy households - although in a similar study, Blume and Friend (1975) cannot reject the hypothesis of constant relative risk aversion. Lastly, in a survey of income and consumption at the household level, Ogaki and Zhang (2001) also conclude in favor of DRRA.

[31] For technical reasons, it is impossible to solve for the optimal contract with CARA utility, which is why b must be strictly positive. However, the robust fact that risk premia for additive risks are decreasing with wealth indicates that individuals have a decreasing absolute risk aversion.

the form:

$$u(W) = \left(a + \frac{W}{b}\right)^{1-b}$$

Its index of relative risk aversion is

$$R(W) = W\left(a + \frac{W}{b}\right)^{-1}$$

Relative risk aversion is decreasing in wealth if a is negative, whereas it is constant if $a = 0$. To ensure that relative risk aversion is positive, we must have $W > -ab$. For a given value of b, this condition bounds the value of a below in order for the lower bound for wealth not to be too high. It is unlikely that there exists preference parameters of a simple utility function which precisely generate the optimality of the observed contract. Our approach therefore consists in identifying and characterizing the preference parameters a and b which best fit the data, in the sense that they minimize the suboptimality of the typical CEO contract - which is measured by the potential savings associated with a switch to the optimal contract identified by the moral hazard model.

This involves two steps. First, for every eligible couple $\{a, b\}$, we set the contract parameters α_0 and α_1 such that the optimal contract (denoted by O) is individually rational and incentive-compatible, in the sense that it gives the CEO the same expected utility and provides as much incentives as the observed contract (denoted by D):

$$E\big[u(W_O(\tilde{\pi}))\big] = E\big[u(W_D(\tilde{\pi}))\big]$$

$$E\big[W_O'(\tilde{\pi})u'(W_O(\tilde{\pi}))\big] = E\big[W_D'(\tilde{\pi})u'(W_D(\tilde{\pi}))\big]$$

Numerical results were obtained by a trial-and-error process and were subsequently checked with Monte Carlo simulations.

Second, for every eligible couple $\{a, b\}$, we compute the difference between the agency cost of the observed contract and the agency cost of the optimal contract whose parameters were determined in the first step. This is equivalent to minimizing the difference between the expected payoff of both contracts. Since the optimal contract minimizes agency costs, the couple $\{a, b\}$ which minimizes this difference (i.e., minimizes potential savings) is the one that best fits the data. In short, for incentive-compatible and individually rational contracts, the preference parameters $\{a, b\}$ which minimize the suboptimality of the observed contract are given by

$$\min_{a,b} \Big\{E[W_D(\tilde{\pi})] - E[W_O(\tilde{\pi})]\Big\}$$

Potential savings are reported in figure 6.

Holding a constant, potential savings are an increasing function of the coefficient of relative risk aversion. We already obtained this result in the CRRA case with $a = 0$, but it seems to

		b		
		0.25	0.50	0.75
a	-10	NA	9.18%	17.16%
	-5	3.30%	10.88%	16.40%
	-1	5.53%	10.82%	15.80%
	0	**5.61%**	**10.81%**	**15.60%**
	1	5.66%	10.75%	15.42%
	10	5.76%	10.39%	14.30%

Figure 2.6: Savings generated by the optimal contract relative to the observed contract.

hold more generally for any value of a. The suboptimality of the observed contract is therefore minimized for very low values of b.

For low values of b, savings are an increasing function of a. For example, for $b = 0.5$, savings amount to 10.8% of the cost of the observed contract under the hypothesis of constant relative risk aversion ($a = 0$), against 9.2% for $a = -10$. Having a decreasing relative risk aversion (a negative a) reduces the inefficiency of the observed contract. As shown in figure 7, it also makes the optimal contract less concave than with constant relative risk aversion, thus reducing the difference in curvature between the optimal contract and the observed contract. As shown in figure 8, pay is then relatively lower for low performances, and relatively higher for high performances, while the pay-performance sensitivity is not declining as steeply as with constant relative risk aversion. In sum, having preferences with decreasing relative risk aversion ($a < 0$) in addition to decreasing absolute risk aversion ($b > 0$) allows the model to better match the data. With lower values of b, the same qualitative results hold, and they are even more economically significant: for $b = 0.25$, assuming that $a = -5$ instead of relying on the hypothesis of constant relative risk aversion reduces potential savings by 40%. However, the model performs poorly for higher values of b, including for $b = 0.75$. In particular, potential savings remain big for any

	Change in final wealth following a deviation of the stock price with respect to the mean by			
	-2 std dev	-1 std dev	+1 std dev	+2 std dev
Observed contract	**-51%**	**-42%**	**131%**	**482%**
Optimal contract, b=0.25, a=-5	**-56%**	**-42%**	**91%**	**262%**
Optimal contract, b=0.25, a=0	-60%	-35%	46%	108%
Optimal contract, b=0.25, a=10	-61%	-33%	38%	82%
Optimal contract, b=0.5, a=-10	-60%	-42%	65%	154%
Optimal contract, b=0.5, a=0	-61%	-34%	40%	87%
Optimal contract, b=0.5, a=10	-62%	-33%	37%	79%
Optimal contract, b=0.75, a=-10	-60%	-33%	36%	76%
Optimal contract, b=0.75, a=0	-62%	-33%	35%	72%
Optimal contract, b=0.75, a=10	-62%	-33%	34%	70%

Figure 2.7: The shape of the observed contract and of the optimal contracts for different preference parameters.

value of a.[32]

These results suggest that the relative risk aversion of CEOs is low and decreasing with wealth: CEOs are more prudent than is assumed with CRRA preferences. In effect, the preference parameters which minimize the suboptimality of the observed contract are a small b combined with a negative a (whose magnitude is here bounded below for technical reasons). Since a CRRA utility function does not seem to accurately reflect the preferences of CEOs, assuming CRRA preferences will lead either to a rejection of the moral hazard model of efficient contracting for CEO compensation, or to the conclusion that observed CEO contracts are inefficient, or to the CEO compensation puzzle.

If agents actually have decreasing relative risk aversion, then they will only have the low relative risk aversion required for significant upward participation (carrots rather than sticks) to be optimal when they are sufficiently wealthy. Postulating decreasing relative risk aversion therefore also contributes to explaining why stock-options and other convex incentive mechanisms are confined to well-paid employees, and why the estimated relative risk aversion of wealthy

[32]The optimal contract with a negative a is less concave than the one with a positive a, but only slightly. Moreover, the former lies below the latter, and is consequently characterized by a lower expected cost, thus generating lower savings relative to the observed contract. In this case, stipulating decreasing relative risk aversion does not make a big difference.

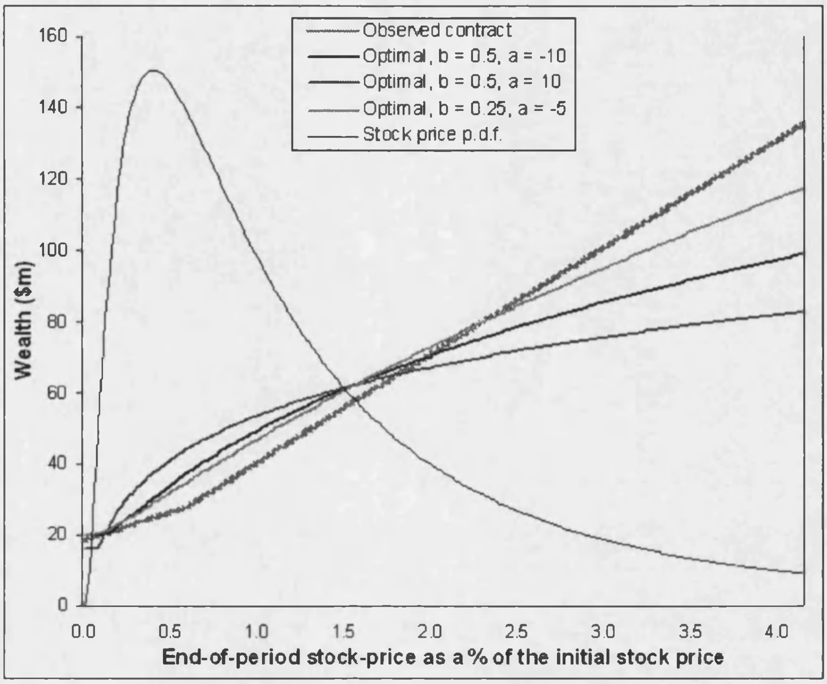

Figure 2.8: Optimal compensation contracts for different preference parameters.

CEOs is quite low.[33] This gives a joint prediction of the model and of the decreasing relative risk aversion hypothesis: the degree of convexity of the CEO contract should be an increasing function of his or her wealth.

2.4 Conclusion

This paper has shown that designing a compensation contract to induce a given level of effort involves a tradeoff between the valuation and the effectiveness of incentives. Whereas efficient risk sharing leads to protection against downside risk and upside participation for prudent agents, incentive considerations lead to contracts with a relatively flat wage for most outcomes coupled with strong punishments for poor performances. Risk aversion alone makes convex contracts suboptimal, while prudence potentially makes some convex contracts optimal. Given the form of managerial compensation contracts, and the paucity of punishments for failure, it seems that prudence, rather than risk aversion, is key to understanding the curvature of CEO pay. This

[33] It is not implausibly low. In a model of consumption behavior over the life cycle which incorporates precautionary savings (and therefore prudence), Gourinchas and Parker (2002) estimate that the coefficient of relative risk aversion of households is between 0.5 and 1.4. With decreasing relative risk aversion, the relative risk aversion of very wealthy individuals may very well be slightly below this range of values.

suggests a reason why studies which focus exclusively on risk aversion, such as the first chapter of this thesis, fail to explain commonly used CEO compensation contracts. In this context, the paper has shown that calibrating the relative weight of prudence and risk aversion in a utility function is crucial. In particular, a CRRA utility function is relatively too risk averse and not prudent enough to account for commonly used CEO contracts. Our analysis suggests that CEOs have a low and decreasing relative risk aversion as well as a decreasing absolute risk aversion.

The model generates a number of predictions. First, efficient incentive schemes will typically be characterized by observed low pay-performance sensitivities: in equilibrium, the agent is approximately insured, but still adequately incentivized. The model notably predicts that for a given level of incentives, compensation schemes such as stock-options-based ones which use small and moderate rewards will be characterized by a higher average pay-performance sensitivity. Second, the convexity of managerial pay should be a decreasing function of the skewness of the stock price distribution. Third, with decreasing relative risk aversion, the convexity of the CEO's pay should be an increasing function of his or her wealth. Fourth, the lower the agent's relative risk aversion, the more convex should be his or her compensation contract.

This implies that convex contracts which concentrate variations in pay on the right tail of performances should be offered to agents whose relative risk aversion is relatively small and whose wealth is relatively large. Concave contracts which concentrate variations in pay on the left tail (on the subset of performances where the limited liability constraint is not binding) should be offered to agents whose relative risk aversion is relatively large and whose wealth is relatively low. To the extent that CEOs and entrepreneurs are less risk averse and more wealthy than rank-and-file employees, it makes sense that the former be incentivized with convex stock-options, and the latter with punishments such as dismissal. Perhaps the preponderance of stock-options in the booming late 1990s and early 2000s was predicated on the fact that CEOs were then very wealthy, and on the assumption that they have a very low risk aversion, especially if they are entrenched. If this is accurate, then the model predicts that compensation practices will switch from stock-options to restricted stocks and other "less convex" instruments as the economic prospects deteriorate.

2.5 Appendix

2.5.1 The relative efficiency of different contracts in a CARA-normal setting

Since the advantages of concavity (respectively punishments) depend on risk aversion, and the advantages of convexity (respectively rewards) depend on prudence, no optimality or dominance result can be obtained with global risk aversion combined with global prudence. But we can be more specific. At the cost of specifying the agent's utility function and the probability distribution, this section measures the agency costs of individually rational and incentive-compatible step contracts and short put contracts in a CARA-normal framework, and identifies quasi-optimal contracts within these two classes of contracts.[34]

Agency costs measure (in)efficiency. With step contracts, we measure the efficiency of concentrating incentives at any given point of the performance measure distribution. Given that rewards attenuate downside risk (see proposition 2a) and punishments are more effective at providing incentives (see proposition 3a), it is a priori unclear which type of contract is more efficient. However, it turns out that the latter effect dominates: except at the tails of the distribution, punishments tend to be more efficient. Next, we study short put contracts. They are the simplest concave contracts, so that the first-order approach is always valid, and they can easily be constructed with put options. Furthermore, they span a wide range of contracts: at the one end, a linear contract; at the other end, a contract resembling the Mirrlees mechanism. Mapping the strikes of options into agency costs allows to determine whether it is preferable to have small incentives along a large subinterval of performances, or strong incentives along a small subinterval. Long call contracts are discussed in footnote 25.

Step contracts

We start with step contracts. Assume that the probability distribution of the noise is normal and the agent has CARA utility. He is therefore both globally risk averse and globally prudent. Figure 12 reports the results of a numerical simulation in which the standard deviation of $\tilde{\epsilon}$ is 1, and the coefficient of absolute risk aversion is 1. It displays the agency cost, as a percentage of

[34] In the widely used CARA-normal framework nested in our model, the curvature is not a trivial issue. It is well known that we can use mean-variance analysis when the contract is linear (intuitively, the level of effort does not affect the variance of compensation, and the agent is as exposed to downside risk as to upside risk). But we know from the first chapter of this thesis that with a mean-variance criterion, the optimal contract cannot be convex, and a linear contract is dominated by the same contract to which a cap is applied, which renders it concave. Therefore, in a CARA-normal framework the optimal contract cannot be linear.

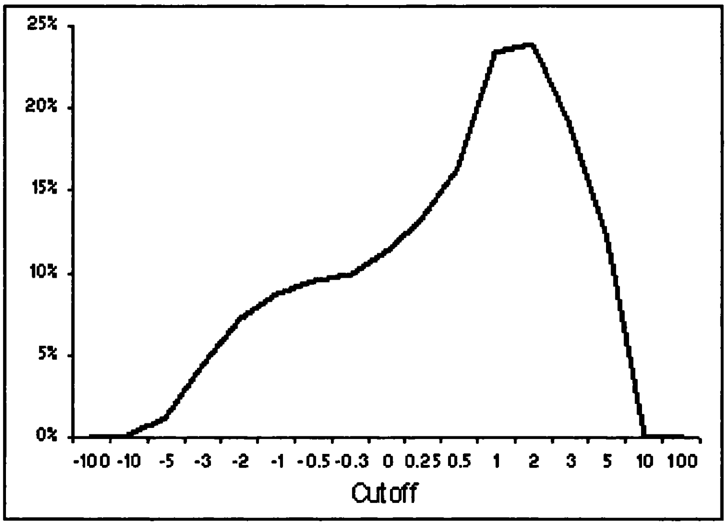

Figure 2.9: Agency cost of step contracts as a percentage of the first-best cost: CARA-normal.

the first-best cost, of an incentive-compatible and individually rational contract, as a function of the cutoff q (further details on the method used can be found below). These results are robust: the shape of agency costs as a function of the cutoff is exactly identical for other values of the coefficient of absolute risk aversion. The implications of this fact are discussed in the last paragraph of this section of the paper.

We observe that (i) the cost of the contract converges toward a lower bound, the first-best cost, for a low enough cutoff q: the absolute value of the likelihood ratio is so high at extreme outcomes that these outcomes are almost perfectly informative. (ii) The agency cost is then increasing in q. The major force at play in the interval comprising intermediate values of q is the greater effectiveness of punishments at motivating effort, as highlighted in propositions 1a, 1b, 3a, and 3b. The agency cost is highest for $q \approx 2$ (at $q = 2$, the agent earns the reward around 5% of the time in equilibrium): "small" rewards are inefficient. (iii) The agency cost then decline, and quickly converge toward its lower bound of zero as q becomes large enough, for the same reason as in (i).[35] The downside risk is mitigated by the fact that payments are bounded below with a step contract. It is nevertheless present, which explains why the agency cost is slightly higher at $q = -10$ than at $q = 10$, which can only be attributed to downside

[35]Consider the following thought experiment. In the absence of asymmetric incentives and without aversion to downside risk (or prudence), but with an agent who is not risk neutral (this is inconsistent with the previous postulates, but is only for the sake of the argument), the agency costs of step contracts would only reflect variations of the likelihood ratio, and be symmetric around the origin.

risk aversion.[36] Thus, the convergence towards the first-best cost happens on both tails of the distribution, but it is faster to the right. This is attributable to the lingering effect of downside risk aversion on the left tail. Yet, the difference vanishes altogether for significantly higher absolute values of q: the likelihood ratio effect then becomes predominant, as observed in (i) and (iii).

To summarize, around the middle of the distribution, (small) punishments beat (small) rewards. The agency cost represent 7.2% of the first-best cost of the contract for $q = -2$, against 23.9% for $q = 2$. This is due to the greater incentive power of punishments. As we approach the tails of the distribution, (big) rewards beat (big) punishments. The agency cost represents an infinitesimal fraction of the first-best cost of the contract for $q = 10$, against 0.1% for $q = -10$. This is due to aversion to downside risk. At the tails of the distribution, both (extreme) punishments and (extreme) rewards are approximately optimal. This is due to the asymptotic properties of the likelihood ratio. The result below proves that concentrating transfers at either tail of the distribution is quasi-optimal.

Proposition 4: In a CARA-normal setting, the agency cost of an incentive-compatible and individually rational step contract converges toward zero as its cutoff q approaches plus or minus infinity.

The institutional lesson is that limited liability and caps on payments prevent the superior informativeness of extreme performances from being exploited optimally. Qualitatively similar results are obtained in a CRRA-lognormal setting. They are presented below.

Short put contracts

For step contracts whose cutoff falls in the range of relatively likely performances, punishments are more efficient than rewards. Since punishments and concave contracts share the same essential properties, we now study the efficiency of short puts with the same approach.[37] The

[36]Likelihood ratios are exactly symmetrical, and the greater incentive power of punishments pushes the agency cost in the other direction.

[37]The first-order approach is not guaranteed to work with long calls, especially for high strike prices. Existence problems and multiple equilibria issues erupt, which is why results for call contracts with positive strikes are not reported. With a long call contract, raising the slope of the contract may either augment or diminish the equilibrium effort (on the one hand it increases the slope, but on the other hand it reduces marginal utility at every performance) and increases expected utility at the equilibrium effort; raising w reduces the equilibrium effort (by decreasing marginal utility) and increases expected utility. There may be zero, one or more equilibria, some of which may be invalid. However, it is worth pointing out that as expected, an incentive-compatible

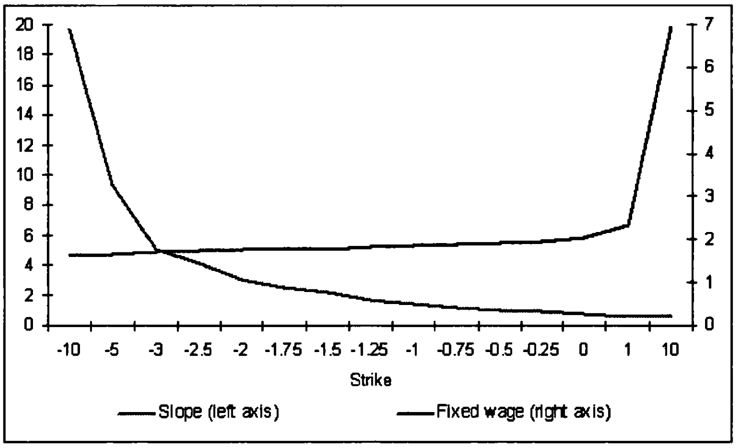

Figure 2.10: Parameters of incentive-compatible and individually rational short puts.

principal offers the agent a fixed wage w and s short positions in a put (s may also be viewed as the slope of the contract on the relevant interval), with strike k as a function of ϵ. These contracts are piecewise linear and concave.

Figures 13 and 14 report the second set of results. I use the same approach as with step contracts, which consists in comparing the agency costs of incentive-compatible and individually rational contracts, as obtained in a numerical simulation with a coefficient of absolute risk aversion of 1 and a standard deviation of 1. For every given strike k, the unique couple (s, w) that satisfies both the incentive constraint and the participation constraint is computed by iterations.[38] The contract parameters are displayed in figure 13. The agency cost of the contract is then reported in figure 14, as a percentage of the first-best cost. The lower it is, the more efficient the contract is.

One striking lesson is that the agency cost is a non-monotonic function of the strike of a short put contract. First, short put contracts with a very low strike are approximately optimal. This

and individually rational long call with a very low strike (for which the first-order approach is therefore in all likelihood valid) approximates a linear contract, and so incurs the same agency cost as the symmetric short put (with a very high strike), or 7.8%. An at-the-money long call contract (with strike $k = 0$) is characterized by an agency cost of 23.7%, a result nevertheless dependent on the validity of the first-order approach.

[38]This couple is unique. In effect, with any short put contract, raising s augments the equilibrium effort (by increasing both the slope and marginal utility at every performance) but reduces expected utility at the equilibrium effort; raising w has the opposite effect of reducing the equilibrium effort (by decreasing marginal utility) and of increasing expected utility.

Figure 2.11: Agency cost of short puts as a percentage of the first-best cost.

is attributable to the greater incentive power of punishments, combined with a likelihood ratio of high magnitude which guarantees that agents who exert high effort are approximately insured (in particular, the downside risk is literally negligible at the equilibrium effort, given the asymptotic properties of the likelihood ratio). This suggests that the optimal short put contract may have a very low strike and a steep slope - this conjecture is made formal in the proposition below.

Proposition 5: In a CARA-normal setting, the agency cost of an incentive-compatible and individually rational short put contract converges toward zero as its strike approaches minus infinity.

Second, the spike in agency costs for negative strikes not too distant from zero indicates that aversion to downside risk is a powerful force in this region. A smaller strike entails a steeper slope to maintain incentive-compatibility; highly discounted very low payments then occur with a non-negligible probability. Third, a positive and significant pay-performance sensitivity around and to the right of the expected performance (at $\epsilon = 0$) in equilibrium is undesirable, for incentives-related reasons. Fourth, capping compensation reduces efficiency: for a sufficiently high strike, a linear contract dominates a short put contract - the linear contract being a short put contract with an infinite strike. This is due to the likelihood ratio effect already discussed.[39]

[39]The suboptimality of capping a linear compensation contract (at a relatively high level) contrasts with the fact that uncapped contracts are dominated (by specific capped contracts) when the agent has mean-variance

A final note on contracting with CARA utility: the optimal contract does not depend on either the agent's wealth or his risk aversion. First, since CARA preferences do not exhibit any wealth effect, it is known that there are no benefits to wealth-based contracting when the agent has CARA utility. This means that the intensity of incentives provision (the level of effort to be induced) should not be a function of the agent's wealth. Second, with CARA preferences, the optimal structure of payments is invariant to the coefficient of absolute risk aversion. Although not reproduced here, agency costs as a function of the cutoffs of step contracts have the same shape as with $\alpha = 1$ for any value of the coefficient of absolute risk aversion α.[40] This means that the relative cost of concentrating incentives at any point of the performance measure distribution is independent of the value of α. The intuition is that with a CARA utility function with a given α, the index of absolute risk aversion is α, and so is the index of absolute prudence.[41] This implies that all agents with CARA preferences, whatever their risk aversion and whatever their wealth, should be given the same contract - with the same level of incentives and the same structure of payments. This result does not hold with CRRA utility.

2.5.2 Proofs and discussions

How the MLRP is obtained

If $\tilde{\pi} = e + \tilde{\epsilon}$, then $\vartheta(\pi|e) = \varphi(\pi - e)$, and $\vartheta_e(\pi|e) = -\varphi_\pi(\pi - e)$.

The MLRP is

$$\frac{d}{d\pi}\left\{\frac{\vartheta_e(\pi|e)}{\vartheta(\pi|e)}\right\} \geq 0 \qquad \forall\pi$$

Which can be rewritten as

$$\frac{d}{d\pi}\left\{\frac{-\varphi_\pi(\pi - e)}{\varphi(\pi - e)}\right\} \geq 0 \qquad \forall\pi$$

The p.d.f. of $\tilde{\pi}$ therefore satisfies the MLRP if and only if

$$\Big(\varphi_\pi(\pi - e)\Big)^2 > \varphi(\pi - e)\varphi_{\pi\pi}(\pi - e) \qquad \forall\pi \tag{2.15}$$

Since the left-hand side reaches its minimum at $\epsilon = 0$, this condition is more likely to be satisfied when the cumulative distribution function of $\tilde{\epsilon}$ is weakly convex to the left of the mean and weakly concave to the right, and when the corresponding p.d.f. is concave around the mean. In such a case, $\varphi_{\pi\pi}$ is negative when $\varphi(\pi - e)$ is large, and positive values of $\varphi_{\pi\pi}$ are weighted by

preferences, as in the first chapter of this thesis.

[40] More precisely, the ratio of agency costs associated with any two given cutoffs is invariant to the value of α, where the parameters of incentive-compatible and individually rational step contracts are re-calculated for any value of α.

[41] The index of absolute prudence, as introduced by Kimball (1990), is defined at every x as $-\frac{u'''(x)}{u''(x)}$.

low values of $\varphi(\pi - e)$. The condition (2.15) is satisfied by the normal distribution as long as the variance is not infinite.

Proof of claim 1:

Let Θ be the c.d.f. of $\tilde{\pi}$. The local incentive constraint (2.2) can also be written in the following form:

$$\int_{-\infty}^{\infty} u[W(\pi)]\frac{\partial}{\partial e}\vartheta(\pi)d\pi = \psi'(e^{\star})$$

Consider the compensation contract defined by $W(\pi) = \underline{W}$ for $\pi \in (-\infty, q)$ and $W(\pi) = \bar{W}$ for $\pi \in [q, \infty)$. The corresponding incentive constraint is

$$\int_{-\infty}^{q} u[\underline{W}]\frac{\partial}{\partial e}\vartheta(e^{\star}+\epsilon)d\epsilon + \int_{q}^{\infty} u[\bar{W}]\frac{\partial}{\partial e}\vartheta(e^{\star}+\epsilon)d\epsilon = \psi'(e^{\star}) \tag{2.16}$$

where we use the fact that $d\pi = d(e+\epsilon) = d\epsilon$. The participation constraint is

$$\int_{-\infty}^{q} u[\underline{W}]\vartheta(e^{\star}+\epsilon)d\epsilon + \int_{q}^{\infty} u[\bar{W}]\vartheta(e^{\star}+\epsilon)d\epsilon = \bar{U} + \psi(e^{\star}) \tag{2.17}$$

If $\underline{W} \geq W_1$ and $\bar{W} \leq W_2$, then, marginal utility being constant,

$$u[\underline{W}] = u[W^{\star}] + u'[W^{\star}](\underline{W} - W^{\star}) \qquad \text{and} \qquad u[\bar{W}] = u[W^{\star}] + u'[W^{\star}](\bar{W} - W^{\star}) \tag{2.18}$$

Plugging into (2.9), the agency cost of this contract is zero, which implies that q, $\underline{W}$ and $\bar{W}$ can be set so that

$$\Theta(q)\underline{W} + (1-\Theta(q))\bar{W} = W^{\star} \tag{2.19}$$

Plugging the terms in (2.18) into the participation constraint, (2.17) is now

$$u[W^{\star}] + u'[W^{\star}]\Big(\Theta(q)\underline{W} + (1-\Theta(q))\bar{W} - W^{\star}\Big) = \bar{U} + \psi(e^{\star}) \tag{2.20}$$

Comparing with (2.3), the participation constraint is satisfied and binding if and only if (2.19) holds. Take the payments $\underline{W}$ and $\bar{W}$ as given. Since by definition $\Theta(q)$ is an increasing and continuous function equal to zero for the smallest possible value of q and to one for the highest possible value of q, and since $W_1 < W^{\star} < W_2$, for any admissible value of $\underline{W}$ and $\bar{W}$, there exists a q that satisfies the participation constraint.

Finally, the payments must be adjusted to satisfy the incentive constraint. Given $\underline{W} < W^{\star}$ and q, there exists $\bar{W}$ such that the incentive constraint (2.16) is satisfied. Alternatively, given $\bar{W} > W^{\star}$ and q, there exists $\underline{W}$ such that the incentive constraint (2.16) is satisfied. In the first case, this necessitates that W_2 be sufficiently larger than $W^{\star}$. In the second case, this necessitates that W_1 be sufficiently smaller than $W^{\star}$. One way or another, the wedge between W_1 and W_2 must be "large enough".

The first point in claim 1 is proved. The proofs of the following two points follow by respectively setting $W_1 = -\infty$ and $W_2 = \infty$.

A utility function with a constant second derivative

Consider the function $u(x) = -x^2$ on the interval $(-1, 0)$. This interval has the power of the continuum, and hence of the set of all real numbers (which means that there exists a one-to-one correspondence between these two sets). We are going to extend it to (a, b), where $(a, b) \in R^2$. Denote the new function by U. For $\lambda \in [0, 1]$, it is defined by

$$\lim_{y \to \lambda a + (1-\lambda) b} U'(y) = \lim_{x \to -\lambda} u'(x)$$

This equation establishes a one-to-one correspondence between y and λ, and between λ and x. It therefore establishes a one-to-one correspondence between x and y. Given a and b, we can express the argument y of U as a function of λ: $U'(y) = u'(-\lambda)$. Importantly, $U'(y)$ is positive on (a, b). Let $a = -b$, so that $U'(0) = u'(-0.5) = 1$. The function U is also characterized by

$$U''(y) \equiv u''(x) \equiv -2 \qquad \forall y \in (a, b), x \in (-1, 0)$$

$$U^n(y) \equiv 0 \qquad \forall y \in (a, b), n \geq 3$$

Set $U(0) = 0$. For $y \in (0, b)$, use the fundamental theorem of calculus to get:

$$U(y) = \int_0^y U'(z) dz$$

Likewise for $y \in (-b, 0)$,

$$U(y) = -\int_y^0 U'(z) dz$$

Finally, let b be arbitrarily large.

The first-order approach with step contracts

The incentive constraint[42] is

$$\lim_{a \to 0} \int_{q-a}^{q+a} s u'[W(\epsilon)] \varphi(\epsilon) d\epsilon = \psi'(e^\star)$$

where $s = \frac{\bar{w} - \underline{w}}{2a}$. Or

$$\varphi(-q) \hat{w} \int_{\underline{w}_P}^{\bar{w}_P} u'[W] dW = \psi'(e^\star) \tag{2.21}$$

The left-hand side of the incentive constraint (2.21) is independent of e. Since the effort cost $\psi(e)$ is convex, only one e satisfies (2.21) given the parameters of the contract (which are in turn adjusted so that the e in question is $e^\star$). Moreover, the agent's optimization problem

[42] A more detailed derivation is available in the proof of proposition 3a.

does not have a corner solution: at $e = 0$, the agent increases his expected utility by marginally increasing his effort, since $\psi'(0) = 0$; as e approaches infinity, the agent increases his expected utility by marginally decreasing his effort, since $\lim_{e\to\infty} \psi'(e) = \infty$ and the marginal utility of monetary transfers is bounded above. As a consequence, the first-order approach is valid if and only if the second-order condition is verified at $e^\star$, i.e., if the second derivative of the agent's objective function at $e^\star$ is negative.

It writes as

$$E\Big[W''(\tilde{\epsilon})u'[W(\tilde{\epsilon})]\Big] + E\Big[(W'(\tilde{\epsilon}))^2 u''[W(\tilde{\epsilon})]\Big] - \psi''(e^\star) \tag{2.22}$$

Start with the first term. Special treatment is needed, since W'' is undefined for a step contract. But we know that a step contract is obtained as the limit of a floored and capped contract with a linear slope centered around the cutoff q, as the slope $\frac{\bar{w}-\underline{w}}{2a}$ approaches infinity. This latter contract is convex for low performances around the low kink of the contract at $q-a$, and convex for high performances around the high kink at $q+a$. Furthermore, the curvature is exactly symmetric: $W''(q-a) = -W''(q+a) \equiv W''(q) > 0$. For $0 < x \leq \frac{\underline{w}+\bar{w}}{2}$,

$$E\Big[W''(\tilde{\epsilon})u'[W(\tilde{\epsilon})]\Big] \approx \lim_{a\to 0}\Big\{P(q-a < \tilde{\epsilon} < q+a)$$

$$\Big(P(\tilde{\epsilon} < q | q-a < \tilde{\epsilon} < q+a)W''(q-a)\int_{\underline{w}}^{\underline{w}+x} u'[W]dW + P(\tilde{\epsilon} > q | q-a < \tilde{\epsilon} < q+a)W''(q+a)\int_{\bar{w}-x}^{\bar{w}} u'[W]dW\Big)\Big\}$$

$$\approx \lim_{a\to 0}\Big\{\varphi(q)2a\frac{1}{2}W''(q)(u[\underline{w}+x] - u[\underline{w}] - u[\bar{w}] + u[\bar{w}+x])\Big\}$$

With a positive and decreasing marginal utility, this term is positive.

The second term in (2.22) is

$$E\Big[(W'(\tilde{\epsilon}))^2 u''[W(\tilde{\epsilon})]\Big] = \int_{q-a}^{q+a} \frac{\hat{w}^2}{(2a)^2} u''[W(\epsilon)]\varphi(\epsilon)d\epsilon$$

$$= \hat{w}^2\varphi(q)(u'[\underline{w}] - u'[\bar{w}])\lim_{a\to 0}\frac{1}{2a}$$

With a decreasing marginal utility, this expression is negative. The second derivative of the agent's objective function evaluated at $e^\star$ is

$$\varphi(q)W''(q)(u[\underline{w}+x] - u[\underline{w}] - u[\bar{w}] + u[\bar{w}+x])\lim_{a\to 0}\{a\} + \hat{w}^2\varphi(q)(u'[\underline{w}] - u'[\bar{w}])\lim_{a\to 0}\frac{1}{2a} - \psi''(e^\star)$$

Since marginal utility is decreasing, the first term is positive, and the second term is negative. However, the former becomes arbitrarily small as a approaches zero, whereas the absolute value of the latter explodes. Effort cost being convex, the last term is negative. The second derivative of the agent's objective function is always negative at $e^\star$, and the first-order approach is valid.

Proof of proposition 1a:

Assume that the second derivative of u is constant on the whole domain. Since $\bar{w}_R$ is bounded, we can construct a quadratic utility function whose marginal utility is positive for the set of eligible payments - even when $\tilde{\epsilon}$ has an unbounded support.

Given the reward contract W, defined by $\{q, \underline{w}_R, \bar{w}_R\}$ and $q > 0$ that satisfies the participation constraint of the agent and induces him to exert effort $e^\star$, consider the contract V defined by

$$\{-q, \underline{w}_P, \bar{w}_P\} \equiv \{-q, 2E[W(\tilde{\pi})] - \bar{w}_R, 2E[W(\tilde{\pi})] - \underline{w}_R\}$$

By construction, V is a punishment contract.

Rewrite the agency cost in (2.9) for a utility function with a constant second derivative, i.e., of the form $u(x) = kx^2 + hx + a$, where $k < 0$, $h > 0$:

$$AC_{u,W} = \int_{-\infty}^{\infty} \Big[\Big(kW^{\star^2} + hW^\star + a + (kW^\star + h)[W(\pi) - W^\star] \Big) - kW(\pi)^2 - hW(\pi) - a \Big] \varphi(\epsilon) d\epsilon \quad (2.23)$$

Taking expectations term by term:

$$AC_{u,W} = kW^{\star^2} + hW^\star + a + (kW^\star + h)\Big[E[W(\tilde{\pi})] - W^\star\Big] - kE[W(\tilde{\pi})^2] - hE[W(\tilde{\pi})] - a \quad (2.24)$$

Substituting for the expression of the second moment about the origin:

$$AC_{u,W} = kW^{\star^2} + kW^\star\Big[E[W(\tilde{\pi})] - W^\star\Big] - k\Big[var[W(\tilde{\pi})] + (E[W(\tilde{\pi})])^2\Big] \quad (2.25)$$

Notice that if $k = 0$, there is no agency cost, as expected. In this equation, the two terms that differ across contracts are the expectation and the variance of compensation. With quadratic utility, two contracts with the same expectation and the same variance have the same agency cost. Should one satisfy the participation constraint of the agent, then the other satisfies it too, as the expected quadratic utility of any random variable x only involves the expected value of x and its variance:

$$E[u(\tilde{x})] = E[k\tilde{x}^2 + h\tilde{x} + a] = kE[\tilde{x}^2] + hE[\tilde{x}] + a = kvar[\tilde{x}] + k(E[\tilde{x}])^2 + hE[\tilde{x}] + a$$

Bearing that in mind, first note that the expected payment of V is equal to the expected payment of W:

$$E[V(\pi)] = \Phi(-q)\underline{w}_P + (1 - \Phi(-q))\bar{w}_P = (1 - \Phi(q))(2E[W(\tilde{\pi})] - \bar{w}_R) + \Phi(q)(2E[W(\tilde{\pi})] - \underline{w}_R)$$

$$= -(1 - \Phi(q))\bar{w}_R - \Phi(q)\underline{w}_R + 2E[W(\tilde{\pi})] = E[W(\tilde{\pi})]$$

where the symmetry of the probability distribution was used. Second, the variance of V is equal to the variance of W:

$$var[V(\tilde{\pi})] = \Phi(-q)(\underline{w}_P - E[V(\tilde{\pi})])^2 + (1 - \Phi(-q))(\bar{w}_P - E[V(\tilde{\pi})])^2$$

$$= (1-\Phi(q))(2E[W(\tilde{\pi})] - \bar{w}_R - E[V(\tilde{\pi})])^2 + \Phi(q)(2E[W(\tilde{\pi})] - \underline{w}_R - E[V(\tilde{\pi})])^2$$

$$= (1-\Phi(q))(E[W(\tilde{\pi})] - \bar{w}_R)^2 + \Phi(q)(E[W(\tilde{\pi})] - \underline{w}_R)^2 = var[W(\tilde{\pi})]$$

Now consider the incentives provided by W and V. The left-hand side of the incentive constraint with a reward contract is

$$\int_{-\infty}^{\infty} W'(\pi)u'[W(\pi)]\varphi(\epsilon)d\epsilon = \lim_{a\to 0}\int_{q-a}^{q+a} \frac{1}{2a}(\bar{w}_R - \underline{w}_R)u'[W(\pi)]\varphi(\epsilon)d\epsilon$$

$$\approx \varphi(q)\hat{w}\int_{\underline{w}_R}^{\bar{w}_R} u'[W]dW \tag{2.26}$$

The left-hand side of the incentive constraint with a punishment contract is likewise

$$\int_{-\infty}^{\infty} V'(\pi)u'[V(\pi)]\varphi(\epsilon)d\epsilon \approx \varphi(-q)\hat{w}\int_{\underline{w}_P}^{\bar{w}_P} u'[W]dW \tag{2.27}$$

The facts that $\Phi(0) = 0.5$ and $q > 0$, together with the fact that both contracts have the same expected payment, imply that

$$\bar{w}_P + \underline{w}_P < \bar{w}_R + \underline{w}_R$$

In turn, this inequality, when coupled with decreasing marginal utility and the fact that both contracts have the same wedge between payments, implies that the left-hand side of the incentive constraint of a punishment contract is higher than the left-hand side of the incentive constraint of a reward contract. The wedge between $\bar{w}_P$ and $\underline{w}_P$ is too large: the punishment contract induces an effort higher than $e^{\star}$.

Maintaining the cutoff $-q$ fixed, we raise $\underline{w}_P$ while simultaneously reducing $\bar{w}_P$ in order for the expected payment to remain unchanged, until the wedge between the two payments is sufficiently reduced for the punishment contract to be incentive-compatible. For any unit increase in $\underline{w}_P$, $\bar{w}_P$ changes by

$$d\bar{w}_P = -\frac{\Phi(-q)}{1-\Phi(-q)} < 0 \tag{2.28}$$

Given this condition, the increase in $\underline{w}_P$ is determined for (2.27) to be equal to (2.26). There exists a unique solution to this problem. In effect, the change in the left-hand side of the incentive constraint is

$$d\Big\{\varphi(-q)\hat{w}\int_{\underline{w}_P}^{\bar{w}_P} u'[W]dW\Big\} = \Big[-u'[\underline{w}_P]d\underline{w}_P + u'[\bar{w}_P]d\bar{w}_P + d\hat{w}(u[\bar{w}_P] - u[\underline{w}_P])\Big]\varphi(-q)$$

Marginal utility and $d\underline{w}_P$ being positive, the first term is negative. As $d\bar{w}_P$ is negative, the second term is negative as well. The third term is also negative: utility is increasing, and the wedge between payments shrinks. That is, the left-hand side of the incentive constraint is reduced by an increase in $\underline{w}_P$ and the corresponding decrease in $\bar{w}_P$. What is more, with

$\underline{w}_P = \bar{w}_P$, it is equal to zero. Therefore, there exists only one couple $\{\underline{w}_P, \bar{w}_P\}$ that preserves the expected payment and induces effort $e^\star$.

At this point, W and V implement the same effort at the same agency cost (since they share the same expected payment, see (2.5)). However, reducing the wedge between $\underline{w}_P$ and $\bar{w}_P$ while maintaining the expected payment fixed has reduced the variance of V (this is trivial), which relaxes the participation constraint. The last stage of the proof consists in reducing both $\underline{w}_P$ and $\bar{w}_P$ in order for the left-hand side of the incentive constraint to remain unchanged, until the participation constraint binds. Downward translations in $\underline{w}_P$ and $\bar{w}_P$ must be such that

$$d\Big\{\varphi(-q)\hat{w}\int_{\underline{w}_P}^{\bar{w}_P} u'[W]dW\Big\} = 0$$

Or

$$\frac{d\bar{w}_P}{d\underline{w}_P} = \frac{u'(\underline{w}_P) + u[\bar{w}_P] - u[\underline{w}_P]}{u'(\bar{w}_P) + u[\bar{w}_P] - u[\underline{w}_P]}$$

The right-hand side of this equation is positive. Hence, both payments must diminish. Since marginal utility is decreasing, we must have $d\bar{w}_P < d\underline{w}_P$. The "fixed wage" $\bar{w}_P$ must be reduced more than the "punishment" $\underline{w}_P$. Reducing both $\underline{w}_P$ and $\bar{w}_P$ obviously reduces the expected payment, and therefore the agency cost. The new contract, which the agent accepts, induces $e^\star$ at a lower agency cost than W. It therefore dominates W.

Proof of proposition 1b:

Assume that the second derivative of u is constant on the whole domain.

Given the convex contract W that satisfies the participation constraint of the agent and induces him to exert effort $e^\star$, consider the contract

$$V_\delta(\pi) \equiv -\delta W(-\pi + 2e^\star) + w \tag{2.29}$$

with δ strictly positive. Since $W(\pi)$ is convex in π, $\delta W(-\pi + 2e^\star)$ is convex in π, and $V_\delta(\pi)$ is concave in π.

In equation (2.25), the two terms that differ across contracts are the expectation and the variance of compensation. With quadratic utility, two contracts with the same expectation and the same variance have the same agency cost. If one satisfies the participation constraint, then the other does too, as the expected quadratic utility of a random variable $\tilde{x}$ is only a function of the variance and the expectation of x:

$$E[u(\tilde{x})] = E[k\tilde{x}^2 + h\tilde{x} + a] = kE[\tilde{x}^2] + hE[\tilde{x}] + a = kvar[\tilde{x}] + k(E[\tilde{x}])^2 + hE[\tilde{x}] + a$$

For any value of w, the functions $V(\pi)$ and $W(\pi)$ have the same variance for $\delta = 1$. In effect, consider the function Y, symmetric to W with respect to the horizontal line going through the point $(e^\star, W(e^\star))$:

$$Y(\pi) = -W(\pi) + 2W(e^\star)$$

Its variance writes as

$$var[Y(\tilde{\pi})] \equiv E\Big[\Big(Y(\tilde{\pi}) - E[Y(\tilde{\pi})]\Big)^2\Big] = E\Big[\Big(- W(\tilde{\pi}) + 2W(e^\star) + E[W(\tilde{\pi})] - 2W(e^\star)\Big)^2\Big]$$
$$= E\Big[\Big(W(\tilde{\pi}) - E[W(\tilde{\pi})]\Big)^2\Big] \equiv var[W(\tilde{\pi})] \tag{2.30}$$

Consider the function $V_{\delta=1}$, symmetric to Y with respect to the vertical line going through the point $(e^\star, W(e^\star))$. By definition, it writes as

$$V_{\delta=1}(\pi) = Y\Big(\pi - 2\big(\pi - e^\star\big)\Big) = Y(-\pi + 2e^\star)$$

The expectation of $V_{\delta=1}(\pi)$ is

$$E[V_{\delta=1}(\tilde{\pi})] \equiv \int_{-\infty}^{\infty} V_{\delta=1}(\pi)\vartheta(\pi)d\pi = \int_{-\infty}^{\infty} Y(-\pi + 2E[\tilde{\pi}])\vartheta(\pi)d\pi$$
$$= \int_{-\infty}^{\infty} Y(\pi)\varphi(-\pi + 2E[\tilde{\pi}])d\pi = \int_{-\infty}^{\infty} Y(\pi)\vartheta(\pi)d\pi \equiv E[Y(\tilde{\pi})] \tag{2.31}$$

where the second equality uses the definition of g, the third involves a change of variable (and the fact that the variable of integration is immaterial), and the fourth uses the symmetry of φ unveiled above. Equalities below involve the same steps, plus the fact that $E[Y(\tilde{\pi})]$ is a constant.

$$var[V_{\delta=1}(\tilde{\pi})] \equiv \int_{-\infty}^{\infty} \Big(V_{\delta=1}(\pi) - E[V_{\delta=1}(\tilde{\pi})]\Big)^2 \vartheta(\pi)d\pi = \int_{-\infty}^{\infty} \Big(Y\big(-\pi+2e^\star\big) - E[Y(\tilde{\pi})]\Big)^2 \vartheta(\pi)d\pi$$
$$= \int_{-\infty}^{\infty} \Big(Y(\pi) - E[Y(\tilde{\pi})]\Big)^2 \varphi(-\pi+2e^\star)d\pi = \int_{-\infty}^{\infty} \Big(Y(\pi) - E[Y(\tilde{\pi})]\Big)^2 \vartheta(\pi)d\pi \equiv var[Y(\tilde{\pi})] \tag{2.32}$$

Eventually, combining (2.30) with (2.32),

$$var[V_{\delta=1}(\tilde{\pi})] = var[Y(\tilde{\pi})] = var[W(\tilde{\pi})] \tag{2.33}$$

Set w such that $E[W(\tilde{\pi})] = E[V(\tilde{\pi})]$. Then W and V have the same agency cost, and they both satisfy the participation constraint.

Now consider the incentives these two contracts deliver. For any contract f,

$$E\big[f'(\tilde{\pi})u'[f(\tilde{\pi})]\big] = cov\big(f'(\tilde{\pi}), u'[f(\tilde{\pi})]\big) + E[f'(\tilde{\pi})]E[u'[f(\tilde{\pi})]] \tag{2.34}$$

First, compare the sign of the first term in the equation above for $f = W$ and $f = V$. Combine

$$\frac{\partial}{\partial \pi}W'(\pi) = W''(\pi) > 0 \qquad \text{and} \qquad \frac{\partial}{\partial \pi}u'[W(\pi)] = W'(\pi)u''[W(\pi)] < 0$$

To get

$$cov(W'(\tilde{\pi}), u'[W(\tilde{\pi})]) < 0 \tag{2.35}$$

Then combine

$$\frac{\partial}{\partial \pi}V'(\pi) = V''(\pi) < 0 \qquad \text{and} \qquad \frac{\partial}{\partial \pi}u'[V(\pi)] = V'(\pi)]u''[V(\pi)] < 0$$

To get

$$cov(V'(\tilde{\pi}), u'[V(\tilde{\pi})]) > 0 \tag{2.36}$$

Second, compare both components of the second term in (2.34) for $f = W$ and $f = V_{\delta=1}$. On the one hand, using the function Y as defined above, remembering that $W(e^{\star})$ is a constant, we get $Y'(\pi) = -W'(\pi)$, and $E[Y'(\tilde{\pi})] = -E[W'(\tilde{\pi})]$. The derivative of $V_{\delta=1}$ with respect to π is $V'_{\delta=1}(\pi) = -Y'(-\pi + 2e^{\star})$ Going once again through the same steps,

$$E[V'_{\delta=1}(\tilde{\pi})] \equiv \int_{-\infty}^{\infty} V'_{\delta=1}(\pi)\vartheta(\pi)d\pi = \int_{-\infty}^{\infty} -Y'(-\pi + 2E[\tilde{\pi}])\vartheta(\pi)d\pi$$

$$= \int_{-\infty}^{\infty} -Y'(\pi)\varphi(-\pi + 2E[\tilde{\pi}])d\pi = \int_{-\infty}^{\infty} -Y'(\pi)\vartheta(\pi)d\pi \equiv -E[Y'(\tilde{\pi})] \tag{2.37}$$

Combining these two results,

$$E[V'_{\delta=1}(\tilde{\pi})] = -E[Y'(\tilde{\pi})] = E[W'(\tilde{\pi})] \tag{2.38}$$

On the other hand, since the second derivative of the utility function is constant, its first derivative is of the form $u'(x) = 2kx + h$, where k and h are constants. Because agency costs are identical under W and V, we have from (2.5):

$$E[W(\tilde{\pi})] = E[V(\tilde{\pi})]$$

Substituting for the expression of V,

$$E[W(\tilde{\pi})] = E[-\delta W(-\tilde{\pi} + 2e^{\star}) + w]$$

Multiplying both sides by $2k$ then adding h, for $\delta = 1$,

$$E[2kW(\tilde{\pi}) + h] = E[2k(-W(-\tilde{\pi} + 2e^{\star}) + w) + h]$$

Identifying the form of the utility function,

$$E[u'(W(\tilde{\pi}))] = E[u'(-W(-\tilde{\pi} + 2e^{\star}) + w)]$$

That is,

$$E[u'(W(\tilde{\pi})] = E[u'(V(\tilde{\pi})] \tag{2.39}$$

Using (2.38) and (2.39), it appears that the second term on the right-hand side in (2.34) is identical for W and V. Using (2.35) and (2.36), the contract V creates more incentives than W: V elicits a higher effort than W, at the same agency cost.

The proof proceeds by showing that there exists a $\delta \in (0,1)$ such that V induces the same effort as W at a lower agency cost.

First, incentives delivered by V, $E\big[V'(\tilde{\pi})u'[V(\tilde{\pi})]\big]$, are zero for $\delta = 0$, are monotonically increasing in δ, and are greater than incentives delivered by W for $\delta = 1$, as shown above. The first claim is trivial: with $\delta = 0$, $V(\pi) = w$ for all π, so that $V'(\pi)$ is identically zero. As for the second claim:

$$\begin{aligned}
&\frac{\partial}{\partial\delta}E\Big[V'(\tilde{\pi})u'[V(\tilde{\pi})]\Big]\\
&= \frac{\partial}{\partial\delta}E\Big[\delta W'(-\tilde{\pi}+2e^\star)u'[-\delta W(-\tilde{\pi}+2e^\star)+w]\Big]\\
&= E\Big[\frac{\partial}{\partial\delta}\Big\{\delta W'(-\tilde{\pi}+2e^\star)u'[-\delta W(-\tilde{\pi}+2e^\star)+w]\Big\}\Big]\\
&= E\Big[W'(-\tilde{\pi}+2e^\star)u'[-\delta W(-\tilde{\pi}+2e^\star)+w]+\delta W'(-\tilde{\pi}+2e^\star)(-W(-\tilde{\pi}+2e^\star))u''[-\delta W(-\tilde{\pi}+2e^\star)+w]\Big]
\end{aligned} \tag{2.40}$$

Because W and u are increasing, the first term is positive. Because the second derivative of the utility function is constant and negative, the second term is of the same sign as

$$E\Big[W'(-\tilde{\pi}+2e^\star)W(-\tilde{\pi}+2e^\star)\Big] = cov\Big(W'(-\tilde{\pi}+2e^\star), W(-\tilde{\pi}+2e^\star)\Big)+E[W'(-\tilde{\pi}+2e^\star)]E[W(-\tilde{\pi}+2e^\star)]$$

The covariance is positive, as W is increasing and convex:

$$\frac{\partial}{\partial\pi}W(-\pi+2e^\star) > 0 \qquad \text{and} \qquad \frac{\partial}{\partial\pi}W'(-\pi+2e^\star) > 0$$

Because W is increasing, $E[W'(-\tilde{\pi}+2e^\star)]$ is positive. Combining (2.4) with a nonnegative agency cost, $E[(W(-\tilde{\pi}+2e^\star)]$ is positive. The second term in (2.40) is therefore positive.

To summarize, we have shown that

$$E\Big[V'(\tilde{\pi})u'[V(\tilde{\pi})]\Big] = \begin{cases} 0 & \text{if } \delta = 0 \\ E\Big[W'(\tilde{\pi})u'[W(\tilde{\pi})]\Big] & \text{if } \delta = 1 \end{cases}$$

and that the term $E\Big[V'(\tilde{\pi})u'[V(\tilde{\pi})]\Big]$ is monotonically increasing in δ on the whole domain. It follows that there exists a δ smaller than one that implements the same effort $e^\star$ as the contract W.

Second, the agency cost of V is equal to the agency cost of W for $\delta = 1$, as previously proved. Furthermore, using the definition of V and the measure of agency costs in the quadratic utility case in (2.10),

$$AC_{u,V} = \kappa\int_{-\infty}^{\infty}\Big(-\delta W(-\pi+2e^\star)+w-W^\star\Big)^2\varphi(\epsilon)d\epsilon$$

Taking the derivative with respect to δ,

$$\frac{\partial}{\partial\delta}AC_{u,V} = 2\kappa E\Big[\Big(-W(-\tilde{\pi}+2e^\star)+\frac{\partial w}{\partial\delta}\Big)\Big(-\delta W(-\tilde{\pi}+2e^\star)+w-W^\star\Big)\Big] \tag{2.41}$$

Dropping the positive constant 2κ for notational convenience and rearranging,

$$\frac{\partial}{\partial\delta}AC_{u,V} = \Big(E\Big[-W(-\tilde{\pi}+2e^\star)\Big]+\frac{\partial w}{\partial\delta}\Big)E\Big[-\delta W(-\tilde{\pi}+2e^\star)+w-W^\star\Big]+cov\Big(W(-\tilde{\pi}+2e^\star),\delta W(-\tilde{\pi}+2e^\star)\Big) \tag{2.42}$$

The "fixed wage" w adjusts so that the participation constraint (2.1) holds. Applying the implicit function theorem to the participation constraint with the contract V,

$$\frac{\partial w}{\partial\delta} = -\frac{E\Big[-W(-\tilde{\pi}+2e^\star)u'[-\delta W(-\tilde{\pi}+2e^\star)+w]\Big]}{E\Big[u'[-\delta W(-\tilde{\pi}+2e^\star)+w]\Big]} \tag{2.43}$$

Plugging into (2.42),

$$\frac{\partial}{\partial\delta}AC_{u,V} = E\big[-\delta W(-\tilde{\pi}+2e^\star)+w-W^\star\big]\Big(-E[W(-\tilde{\pi}+2e^\star)]+\frac{cov\Big(W(-\tilde{\pi}+2e^\star),u'[-\delta W(-\tilde{\pi}+2e^\star)+w]\Big)}{E\Big[u'[-\delta W(-\tilde{\pi}+2e^\star)+w]\Big]}$$

$$+E[W(-\tilde{\pi}+2e^\star)]\Big)+\delta cov\Big(W(-\tilde{\pi}+2e^\star),W(-\tilde{\pi}+2e^\star)\Big) \tag{2.44}$$

With quadratic utility, marginal utility is linear. Removing constants out of the covariance,

$$\frac{\partial}{\partial\delta}AC_{u,V} = -2k\delta cov\Big(W(-\tilde{\pi}+2e^\star),W(-\tilde{\pi}+2e^\star)\Big)\frac{E\big[-\delta W(-\tilde{\pi}+2e^\star)+w-W^\star\big]}{2kE\big[-\delta W(-\tilde{\pi}+2e^\star)+w\big]}$$

$$+\delta cov\Big(W(-\tilde{\pi}+2e^\star),W(-\tilde{\pi}+2e^\star)\Big)$$

As the agency cost is nonnegative,

$$E\big[-\delta W(-\tilde{\pi}+2e^\star)+w-W^\star\big]\geq 0$$

With (2.4), we know that $W^\star \geq 0$, which yields

$$\frac{E\big[-\delta W(-\tilde{\pi}+2e^\star)+w-W^\star\big]}{E\big[-\delta W(-\tilde{\pi}+2e^\star)+w\big]}\leq 1$$

It immediately follows that the derivative of the agency cost as a function of δ is positive.

As a consequence, a concave contract V defined in (2.29) with $\delta < 1$ implements $e^\star$ at a lower agency cost than the convex contract W. For any convex contract W, there exists a concave contract V which dominates W.

Capped utility function and capped payments:

Very risk averse agents may not value payments in excess of a threshold. In this case their compensation should be bounded above.

Claim 2: If there exists W_4 such that $u(W_5) = u(W_4)$, for any $W_5 > W_4$, then $W(\tilde{\pi}) \leq W_4$ for all $W(\tilde{\pi})$.

Increasing transfers above the threshold W_4 is worthless to the agent, and it does not have any incentive value. This implies that the transfer function to agents whose utility function is capped should be capped.

Proof:

Compare any contract $\hat{W}$ characterized by $\hat{W}(\pi) > W_4$ on a subinterval $[p, \infty)$ to one where $W(\pi) = W_4$ on the same subinterval. On the one hand, the latter is less costly to the principal. On the other hand, the agent is indifferent between these two contracts, and they generate the same effort. In effect, the participation constraint is unaffected:

$$\int_p^\infty u[\hat{W}(\pi)]\varphi(\epsilon)d\epsilon = \int_p^\infty u[W_4]\varphi(\epsilon)d\epsilon$$

The incentive constraint is unaffected as well, as $u'[\hat{W}(\pi)] = 0$ for $\hat{W}(\pi) \geq W_4$:

$$\int_p^\infty \hat{W}'(\pi)u'[\hat{W}(\pi)]\varphi(\epsilon)d\epsilon = 0$$

Therefore, any contract with $\hat{W}(\tilde{\pi}) > W_4$ on a subinterval is dominated by a contract where $W(\pi) = W_4$ on this subinterval.

Proof of proposition 2a:

As a normalization, define $\hat{w}$ to be the wedge necessary to induce a risk neutral agent to exert adequate effort. A risk neutral agent has a constant marginal utility, say $u'(W^\star)$. His incentive constraint writes as

$$\hat{w}(q)u'(W^\star)\varphi(q) = \psi'(e^\star)$$

For any q, $\hat{w}$ solves the equation above:

$$\hat{w}(q) = \frac{1}{\varphi(q)} \frac{\psi'(e^\star)}{u'(W^\star)} \tag{2.45}$$

Denote the expected utility provided by a contract with punishments by EU_P, and by EU_R for a contract with rewards. By definition,

$$EU_P = \Phi(-q)u[\underline{w}_P] + (1 - \Phi(-q))u[\bar{w}_P]$$

$$EU_R = \Phi(q)u[\underline{w}_R] + (1 - \Phi(q))u[\bar{w}_R]$$

Using (2.45) and (2.12),

$$EU_P = \Phi(-q)u\left[\alpha - \frac{1 - \Phi(-q)}{\varphi(-q)} \frac{\psi'(e^\star)}{u'(W^\star)}\right] + (1 - \Phi(-q))u\left[\alpha + \frac{\Phi(-q)}{\varphi(-q)} \frac{\psi'(e^\star)}{u'(W^\star)}\right]$$

$$EU_R = \Phi(q)u\left[\alpha - \frac{1 - \Phi(q)}{\varphi(q)} \frac{\psi'(e^\star)}{u'(W^\star)}\right] + (1 - \Phi(q))u\left[\alpha + \frac{\Phi(q)}{\varphi(q)} \frac{\psi'(e^\star)}{u'(W^\star)}\right]$$

Rewriting,

$$EU_P = au[\alpha - \beta b] + bu[\alpha + \beta a]$$

$$EU_R = bu[\alpha - \beta a] + au[\alpha + \beta b]$$

where $b > a > 0$, and $\beta > 0$. We use a third order Taylor expansion around α.

$$u(x) = u(\alpha) + u'(\alpha)(x-\alpha) + \frac{1}{2}u''(\alpha)(x-\alpha)^2 + \frac{1}{6}u'''(y)(x-\alpha)^3$$

We know that there exists $y_-^b \in [\alpha - \beta b, \alpha]$, $y_+^a \in [\alpha, \alpha + \beta a]$, $y_-^a \in [\alpha - \beta a, \alpha]$, $y_+^b \in [\alpha, \alpha + \beta b]$ such that (this involves a standard application of the mean value theorem).

$$u(\alpha - \beta b) = u(\alpha) + u'(\alpha)(-\beta b) + \frac{1}{2}u''(\alpha)(-\beta b)^2 + \frac{1}{6}u'''(y_-^b)(-\beta b)^3$$

$$u(\alpha + \beta a) = u(\alpha) + u'(\alpha)(\beta a) + \frac{1}{2}u''(\alpha)(\beta a)^2 + \frac{1}{6}u'''(y_+^a)(\beta a)^3$$

$$u(\alpha - \beta a) = u(\alpha) + u'(\alpha)(-\beta a) + \frac{1}{2}u''(\alpha)(-\beta a)^2 + \frac{1}{6}u'''(y_-^a)(-\beta a)^3$$

$$u(\alpha + \beta b) = u(\alpha) + u'(\alpha)(\beta b) + \frac{1}{2}u''(\alpha)(\beta b)^2 + \frac{1}{6}u'''(y_+^b)(\beta b)^3$$

Substituting,

$$EU_P = (a+b)u[\alpha] + u'[\alpha](-a\beta b + b\beta a) + \frac{1}{2}u''[\alpha]\Big(a(-\beta b)^2 + b(\beta a)^2\Big)$$
$$+\frac{1}{6}au'''(y_-^b)(-\beta b)^3 + \frac{1}{6}bu'''(y_+^a)(\beta a)^3$$

$$EU_R = (a+b)u[\alpha] + u'[\alpha](-b\beta a + a\beta b) + \frac{1}{2}u''[\alpha]\Big(b(-\beta a)^2 + a(\beta b)^2\Big)$$
$$+\frac{1}{6}bu'''(y_-^a)(-\beta a)^3 + \frac{1}{6}au'''(y_+^b)(\beta b)^3$$

First, if u'' is a constant, so that $u''' = 0$, then $EU_P = EU_R$. Second, if u''' is a positive constant, then mobilize $b > a > 0$ to get $EU_P < EU_R$. Third, if $u^{(4)} \neq 0$, then $EU_P < EU_R$ if and only if

$$-u'''(y_-^b)b^2 + u'''(y_+^a)a^2 < -u'''(y_-^a)a^2 + u'''(y_+^b)b^2$$

Equivalently,

$$\Big(u'''(y_+^a) + u'''(y_-^a)\Big)a^2 < \Big(u'''(y_-^b) + u'''(y_+^b)\Big)b^2$$

This condition will be satisfied if $u^{(i)}$ is positive for odd $i \geq 3$. In effect, if u is continuously differentiable an infinite number of times, then Taylor's theorem gives

$$EU_P = \sum_{i=0}^{\infty}\frac{1}{(2i)!}u^{(2i)}(\alpha)\Big(a(-\beta b)^{2i} + b(\beta a)^{2i}\Big) + \sum_{i=1}^{\infty}\frac{1}{(2i-1)!}u^{(2i-1)}(\alpha)\Big(a(-\beta b)^{2i-1} + b(\beta a)^{2i-1}\Big)$$

$$EU_R = \sum_{i=0}^{\infty}\frac{1}{(2i)!}u^{(2i)}(\alpha)\Big(b(-\beta a)^{2i} + a(\beta b)^{2i}\Big) + \sum_{i=1}^{\infty}\frac{1}{(2i-1)!}u^{(2i-1)}(\alpha)\Big(b(-\beta a)^{2i-1} + a(\beta b)^{2i-1}\Big)$$

As can be easily checked, all even order terms in EU_P are equal to those in EU_R; on the contrary, if $u^{(2i-1)}$ is positive (respectively negative) for all $i \geq 2$, then all odd order terms are larger (respectively smaller) in EU_R, except for $i = 1$ for which there is equality. Consequently, if $u^{(2i-1)}$ is positive for all $i \geq 2$, then EU_R is larger than EU_P.

Proof of proposition 2b:

Denote a concave contract by W_A, and its expected utility by EU_A. Denote a convex contract by W_E, and its expected utility by EU_E. By definition,

$$EU_A \equiv \int_{-\infty}^{\infty} u[W_A(\pi)]\varphi(\epsilon)d\epsilon$$

$$EU_E \equiv \int_{-\infty}^{\infty} u[W_E(\pi)]\varphi(\epsilon)d\epsilon$$

We henceforth adopt the same approach as in the proof of proposition 2a, except that we integrate the agent's utility over the whole range of payments, instead of only two payments.

Using a Taylor expansion around $\hat{W}$ for a concave contract, there exists a function $y(W)$ such that

$$\begin{aligned} EU_A = \int_{-\infty}^{\infty} \Big[& u(\hat{W}) + u'[\hat{W}](W_A(\pi) - \hat{W}) + \frac{1}{2}u''[\hat{W}](W_A(\pi) - \hat{W})^2 \\ & + \frac{1}{6}u'''[y(W)](W_A(\pi) - \hat{W})^3 \Big] \varphi(\epsilon)d\epsilon \\ = u(\hat{W}) + u'[\hat{W}]\Big(\int_{-\infty}^{\infty} & W_A(\pi)\varphi(\epsilon)d\epsilon - \hat{W}\Big) + \frac{1}{2}u''[\hat{W}]\int_{-\infty}^{\infty} \Big(W_A(\pi) - \hat{W}\Big)^2 \varphi(\epsilon)d\epsilon \\ & + \frac{1}{6}u'''[y(W)]\int_{-\infty}^{\infty} \Big(W_A(\pi) - \hat{W}\Big)^3 \varphi(\epsilon)d\epsilon \end{aligned}$$

where the second term is zero because of (2.14). Similarly for a convex contract defined by $W_E = 2\hat{W} - W_A(-\pi + 2e^\star)$,

$$\begin{aligned} EU_E = u(\hat{W}) & + \frac{1}{2}u''[\hat{W}]\int_{-\infty}^{\infty} \Big(\hat{W} - W_A(-\pi + 2e^\star)\Big)^2 \varphi(\epsilon)d\epsilon \\ & + \frac{1}{6}u'''[y(W)]\int_{-\infty}^{\infty} \Big(\hat{W} - W_A(-\pi + 2e^\star)\Big)^3 \varphi(\epsilon)d\epsilon \end{aligned}$$

Adding up all higher order terms and using Taylor's theorem,

$$\begin{aligned} EU_A = & \sum_{i=0}^{\infty} \frac{1}{(2i)!}u^{(2i)}(\hat{W})\int_{-\infty}^{\infty} \Big(W_A(\pi) - \hat{W}\Big)^{2i} \varphi(\epsilon)d\epsilon \\ & + \sum_{i=1}^{\infty} \frac{1}{(2i-1)!}u^{(2i-1)}(\hat{W})\int_{-\infty}^{\infty} \Big(W_A(\pi) - \hat{W}\Big)^{2i-1} \varphi(\epsilon)d\epsilon \end{aligned}$$

$$EU_E = \sum_{i=0}^{\infty} \frac{1}{(2i)!}u^{(2i)}(\hat{W})\int_{-\infty}^{\infty} \Big(\hat{W} - W_A(-\pi + 2e^\star)\Big)^{2i} \varphi(\epsilon)d\epsilon$$

$$+\sum_{i=1}^{\infty}\frac{1}{(2i-1)!}u^{(2i-1)}(\hat{W})\int_{-\infty}^{\infty}\Big(\hat{W}-W_A(-\pi+2e^{\star})\Big)^{2i-1}\varphi(\epsilon)d\epsilon$$

All even order terms in EU_E are equal to those in EU_A since for any integer $i \geq 0$,

$$\int_{-\infty}^{\infty}\Big(W_E(\pi)-\hat{W}\Big)^{2i}\varphi(\epsilon)d\epsilon=\int_{-\infty}^{\infty}\Big(\hat{W}-W_A(-\pi+2e^{\star})\Big)^{2i}\varphi(\epsilon)d\epsilon$$

$$=\int_{-\infty}^{\infty}\Big(\hat{W}-W_A(\pi)\Big)^{2i}\varphi(\epsilon)d\epsilon=\int_{-\infty}^{\infty}\Big(W_A(\pi)-\hat{W}\Big)^{2i}\varphi(\epsilon)d\epsilon \qquad (2.46)$$

where the second equality follows from $E[\tilde{\pi}] = e^{\star}$ and the fact that the probability distribution is symmetric around the mean. The first central moment is zero, as previously observed. The signs of higher odd-order central moments are still left to determine. I prove below, in the last part of the proof, that their sign is positive for a convex W. The proof that they are negative for a concave W follows the exact same lines. Combining this result with the condition that $u^{(2i+1)}$ is positive for $i \geq 1$ implies that $EU_E \geq EU_A$, which proves proposition 2b. Also, it immediately follows that if u''' is a positive constant (so that all higher-order derivatives are zero), then $EU_E \geq EU_A$. Lastly, if $u''' = 0$, we have $EU_E = EU_A$.

Consider any increasing and convex function f with argument π. Denote its mean by m_1. We are going to replicate the function f as the limit of a sequence of piecewise linear functions g_j. Set up a linear function g_0 with any positive slope such that $g_0(e^{\star}) = m_1$. Call the intersection between f and m_1 a "node", denote it by n_1, and set $\hat{\pi}_1 \equiv f^{-1}(m_1)$. Clearly, since m_1 is the mean of f,

$$\int_{-\infty}^{\hat{\pi}_1}\Big(m_1-f(\pi)\Big)\vartheta(\pi)d\pi=\int_{\hat{\pi}_1}^{\infty}\Big(f(\pi)-m_1\Big)\vartheta(\pi)d\pi$$

Next, set up a piecewise linear function g_1 defined by its origin n_1 and its slopes s_1^- and s_1^+ such that

$$g_1(\hat{\pi}_1)=m_1$$

$$\int_{-\infty}^{\hat{\pi}_1}\Big(m_1-s_1^-(\pi-\hat{\pi}_1)\Big)\vartheta(\pi)d\pi=\int_{-\infty}^{\hat{\pi}_1}f(\pi)\vartheta(\pi)d\pi$$

$$\int_{\hat{\pi}_1}^{\infty}\Big(m_1+s_1^+(\pi-\hat{\pi}_1)\Big)\vartheta(\pi)d\pi=\int_{\hat{\pi}_1}^{\infty}f(\pi)\vartheta(\pi)d\pi$$

These last two equalities imply that the function g_1 has the same mean m_1 as f. The transformation of f into g_1 is mean-preserving.

This is illustrated in figure 15. We now repeat this procedure on each sub-interval.

Denote the mean of f on $(-\infty, \hat{\pi}_1)$ by m_2^-:

$$\int_{-\infty}^{\hat{\pi}_1}f(\pi)\vartheta(\pi)d\pi\equiv m_2^-$$

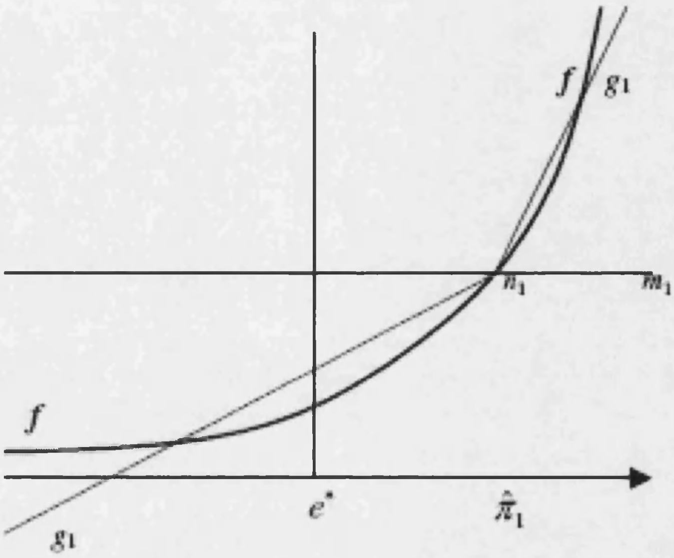

Figure 2.12: A mean-preserving transformation: step 1.

Denote the mean of f on $(\hat{\pi}_1, \infty)$ by m_2^+:

$$\int_{\hat{\pi}_1}^{\infty} f(\pi)\vartheta(\pi)d\pi \equiv m_2^+$$

Denote the intersections between f and m_2^- by n_2^-, and set $\hat{\pi}_2^- \equiv f^{-1}(m_2^-)$ (where $\hat{\pi}_2^- \in (-\infty, \hat{\pi}_1)$). Denote the intersections between f and m_2^+ by n_2^+, and set $\hat{\pi}_2^+ \equiv f^{-1}(m_2^+)$ (where $\hat{\pi}_2^+ \in (\hat{\pi}_1, \infty)$). Clearly, since f has a mean of m_2^- on $(-\infty, \hat{\pi}_1)$, and of m_2^+ on $(\hat{\pi}_1, \infty)$,

$$\int_{-\infty}^{\hat{\pi}_2^-} \left(m_2^- - f(\pi)\right)\vartheta(\pi)d\pi = \int_{\hat{\pi}_2^-}^{\hat{\pi}_1} \left(f(\pi) - m_2^-\right)\vartheta(\pi)d\pi$$

$$\int_{\hat{\pi}_1}^{\hat{\pi}_2^+} \left(m_2^+ - f(\pi)\right)\vartheta(\pi)d\pi = \int_{\hat{\pi}_2^+}^{\infty} \left(f(\pi) - m_2^+\right)\vartheta(\pi)d\pi$$

Next, at each of the nodes n_2^- and n_2^+, set up a piecewise linear function g_2 defined by its origin - either n_2^- or n_2^+ - and its slopes - respectively s_2^{--}, s_2^{-+}, s_2^{+-}, and s_2^{++} - such that, for the node of coordinates $(\hat{\pi}_2^+, m_2^+)$,

$$g_2(\hat{\pi}_2^+) = m_2^+$$

$$\int_{\hat{\pi}_1}^{\hat{\pi}_2^+} \left(m_2^+ - s_2^{+-}(\pi - \hat{\pi}_2^+)\right)\vartheta(\pi)d\pi = \int_{\hat{\pi}_1}^{\hat{\pi}_2^+} f(\pi)\vartheta(\pi)d\pi$$

$$\int_{\hat{\pi}_2^+}^{\infty} \left(m_2^+ + s_2^{++}(\pi - \hat{\pi}_2^+)\right)\vartheta(\pi)d\pi = \int_{\hat{\pi}_2^+}^{\infty} f(\pi)\vartheta(\pi)d\pi$$

And likewise from the node of coordinates $(\hat{\pi}_2^-, m_2^-)$. To summarize, on the interval $(-\infty, \hat{\pi}_1)$ both g_1 and g_2 have a mean of m_2^-; on the interval $(\hat{\pi}_1, \infty)$ both g_1 and g_2 have a mean of m_2^+.

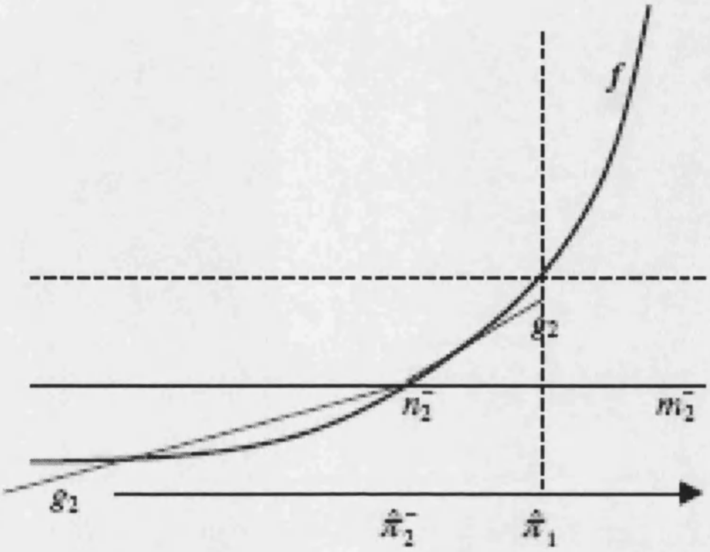

Figure 2.13: A mean-preserving transformation: step 2.

This second step on the interval $(-\infty, \hat{\pi}_1)$ is illustrated in figure 16.

Repeat the same procedure on the four intervals $(-\infty, \hat{\pi}_2^-]$, $[\hat{\pi}_2^-, \hat{\pi}_1]$, $[\hat{\pi}_1, \hat{\pi}_2^+]$, and $[\hat{\pi}_2^+, \infty)$, and so on.

Eventually, for any π,

$$\lim_{j \to \infty} g_j(\pi) = f(\pi)$$

Departing from a linear function g_0, we constructed f step by step, where each step involved a mean-preserving transformation of g_j into g_{j+1}, for $j \geq 0$. That is, for any j,

$$E[g_j(\pi)] = E[f(\pi)] = m_1$$

Notice that g_j is a discontinuous, piecewise linear function. Transforming g_j into g_{j+1} approximates the convex function f more closely.

We are now going to show that for a convex f, each step of this transformation increases the i-th central moment, where $i \geq 2$.

Consider the interval $(\underline{\pi}, \bar{\pi})$, on which g_j has slope s_j and

$$\int_{\underline{\pi}}^{\bar{\pi}} g_j(\pi)\vartheta(\pi)d\pi \equiv m_{j+1}$$

Since by construction we must also have

$$\int_{\underline{\pi}}^{\bar{\pi}} f(\pi)\vartheta(\pi)d\pi \equiv m_{j+1}$$

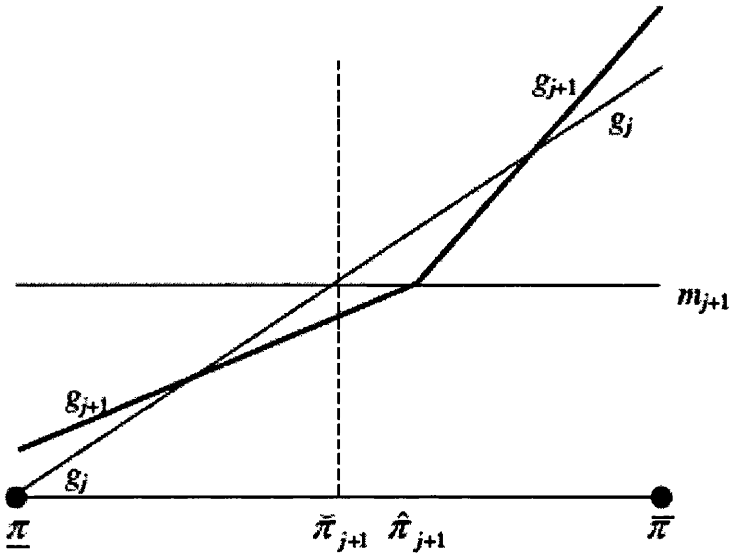

Figure 2.14: Change in odd-order moments.

We know that g_{j+1} on $(\underline{\pi}, \bar{\pi})$ is such that

$$g_{j+1}(\hat{\pi}^+_{j+1}) = m_{j+1}$$

and

$$\int_{\underline{\pi}}^{\hat{\pi}_{j+1}} \Big(m_{j+1} - s^-_{j+1}(\pi - \hat{\pi}_{j+1})\Big)\vartheta(\pi)d\pi = \int_{\hat{\pi}_{j+1}}^{\bar{\pi}} \Big(s^+_{j+1}(\hat{\pi}_{j+1} - \pi) - m_{j+1}\Big)\vartheta(\pi)d\pi$$

where, because f is convex, $\breve{\pi}_{j+1} \equiv g_j^{-1}(m_{j+1}) < g_{j+1}^{-1}(m_{j+1}) \equiv \hat{\pi}_{j+1}$. This is illustrated in figure 17.

We focus on the action above m_{j+1} - the proof for the part under m_{j+1} is exactly symmetric, and therefore omitted. By construction, the mean-preserving transformation of g_j into g_{j+1} is such that

$$\int_{\breve{\pi}_{j+1}}^{\bar{\pi}} \Big(s_j(\pi - \breve{\pi}_{j+1}) - m_{j+1}\Big)\vartheta(\pi)d\pi = \int_{\hat{\pi}_{j+1}}^{\bar{\pi}} \Big(s^+_{j+1}(\pi - \hat{\pi}_{j+1}) - m_{j+1}\Big)\vartheta(\pi)d\pi \qquad (2.47)$$

Based on this equality - which implies that $s^+_{j+1} > s_j$, we must show that, for any integer $i \geq 2$,

$$\int_{\breve{\pi}_{j+1}}^{\bar{\pi}} \Big(s_j(\pi - \breve{\pi}_{j+1}) - m_{j+1}\Big)^i\vartheta(\pi)d\pi < \int_{\hat{\pi}_{j+1}}^{\bar{\pi}} \Big(s^+_{j+1}(\pi - \hat{\pi}_{j+1}) - m_{j+1}\Big)^i\vartheta(\pi)d\pi \qquad (2.48)$$

I prove this result heuristically. The function g_j is turned into g_{j+1} by conducting a series of incremental changes, each of which is mean-preserving by construction. Where π is higher than $\breve{\pi}_{j+1}$ and such that $g_{j+1}(\pi) < g_j(\pi)$, the area in-between $g_j(\pi)$ and $max\{m_{j+1}, g_{j+1}(\pi)\}$ is successively transferred to exactly cover the area in-between $g_{j+1}(\pi)$ and $g_j(\pi)$, where $g_{j+1}(\pi) >$

$g_j(\pi)$. This shift transforms g_j into g_{j+1}. This transformation is mean-preserving by construction, but we shall determine its impact on the i-th central moment, for $i \geq 2$.

Given a function g (prior to the first step, $g \equiv g_j$, and after the last step, $g \equiv g_{j+1}$), reduce g at x by a small $A(x)$ in a neighborhood of x of length u, where x is such that $g_j(x) > g_{j+1}(x)$, and increase g at x' by a small $A(x')$ in a neighborhood of x' of length u, where x' is such that $g_j(x') < g_{j+1}(x')$. This implies that $x' > x$, and $g(x') > g(x)$. Repeat this procedure as needed to transform g_j into g_{j+1}. At any point during this transformation, the mean of the function g is

$$\int_{\tilde{\pi}_{j+1}}^{\overline{\pi}} \Big(g(\pi) - m_{j+1}\Big)\vartheta(\pi)d\pi \tag{2.49}$$

Given $dg(x) = -A(x)$ and $dg(x') = A(x')$, applying the first difference operator to this expression gives

$$d\int_{\tilde{\pi}_{j+1}}^{\overline{\pi}} \Big(g(\pi) - m_{j+1}\Big)\vartheta(\pi)d\pi = \int_{\tilde{\pi}_{j+1}}^{\overline{\pi}} \Big(dg(\pi) - m_{j+1}\Big)\vartheta(\pi)d\pi \approx -A(x)\varphi(x)u + A(x')\varphi(x')u$$

The change in the mean is the sum of a loss of $A(x)\varphi(x)u$ and of a gain of $A(x')\varphi(x')u$. The i-th central moment about m_{j+1} of the function g, $i \geq 2$, is

$$\int_{\tilde{\pi}_{j+1}}^{\overline{\pi}} \Big(g(\pi) - m_{j+1}\Big)^i\vartheta(\pi)d\pi \tag{2.50}$$

Similarly applying the first difference operator to this expression,

$$d\int_{\tilde{\pi}_{j+1}}^{\overline{\pi}} \Big(g(\pi)-m_{j+1}\Big)^i\vartheta(\pi)d\pi = \int_{\tilde{\pi}_{j+1}}^{\overline{\pi}} d\Big(g(\pi)-m_{j+1}\Big)^i\vartheta(\pi)d\pi = \int_{\tilde{\pi}_{j+1}}^{\overline{\pi}} dg(\pi)\; i\Big(g(\pi)-m_{j+1}\Big)^{i-1}\vartheta(\pi)d\pi$$
$$\approx -A(x)i(g(x) - m_{j+1})^{i-1}\varphi(x)u + A(x')i(g(x') - m_{j+1})^{i-1}\varphi(x')u \tag{2.51}$$

The change in the i-th central moment is the sum of a loss of $A(x)i(g(x) - m_{j+1})^{i-1}\varphi(x)u$ and of a gain of $A(x')i(g(x') - m_{j+1})^{i-1}\varphi(x')u$. The mean-preserving condition which relates $A(x')$ to $A(x)$ is

$$\frac{A(x')\varphi(x')}{A(x)\varphi(x)} = 1$$

This implies that

$$\frac{A(x')(g(x') - m_{j+1})^{i-1}\varphi(x')}{A(x)(g(x) - m_{j+1})^{i-1}\varphi(x)} > 1$$

for any $i \geq 0$, because $g(x') > g(x) > m_{j+1}$. This means that the expression in (2.51) is positive: the i-th central moment, $i \geq 2$, is increased by this mean-preserving transformation.

Below m_{j+1}, the converse holds for symmetrical reasons, and the absolute value of the (negative for an odd-order moment, positive for an even-order moment) i-th central moment about the mean, $i \geq 2$, is decreased by the mean-preserving transformation. It follows that the net effect of the transformation on odd-order central moments is positive.[43]

[43] As we have (2.46), we are not concerned about the effect on even-order moments.

All in all, with a convex f, each transformation of g_j into g_{j+1}, for $j \geq 0$, preserves the mean but increases any i-th central moment of odd-order greater or equal to 3. Since g_0 is linear, any odd-order i-th central moment about the mean of g_0 is zero. Starting from g_0 and combining all these steps leaves the mean unchanged and increases any i-th central moment, $i = 3, 5, \ldots$. But an infinite number of such steps yields the function f, whose i-th central moment, $i = 3, 5, \ldots$, is therefore positive.

Proof of proposition 3a:

The left-hand side of the incentive constraint with a given punishment contract denoted by P is

$$IC_P = \int_{-\infty}^{\infty} W'(\pi)u'[W(\pi)]\varphi(\epsilon)d\epsilon = \lim_{a\to 0}\int_{-q-a}^{-q+a} \frac{1}{2a}(\bar{w}_P - \underline{w}_P)u'[W(\pi)]\varphi(\epsilon)d\epsilon$$

$$\approx (\bar{w}_P - \underline{w}_P)\lim_{a\to 0}\left\{\frac{\Phi(-q+a) - \Phi(-q-a)}{2a}\Big[u[W(\pi)]\Big]_{-q-a}^{-q+a}\right\} = \varphi(-q)\hat{w}\int_{\underline{w}_P}^{\bar{w}_P} u'[W]dW \quad (2.52)$$

where we used the fact that φ becomes approximately flat as a becomes arbitrarily small.

The left-hand side of the incentive constraint with the symmetrical reward contract denoted by R is likewise

$$IC_R = \int_{-\infty}^{\infty} W'(\pi)u'[W(\pi)]\varphi(\epsilon)d\epsilon \approx \varphi(q)\hat{w}\int_{\underline{w}_R}^{\bar{w}_R} u'[W]dW$$

Necessarily, we must have $\bar{w}_P > \underline{w}_R$, otherwise one of the participation constraints would be violated. The difference in incentives provided by these two contracts is

$$IC_P - IC_R = \varphi(q)\hat{w}\Big[\int_{\underline{w}_P}^{\bar{w}_P} u'[W]dW - \int_{\underline{w}_R}^{\bar{w}_R} u'[W]dW\Big]$$

$$= \varphi(q)\hat{w}\Big[\int_{\underline{w}_P}^{\underline{w}_R} u'[W]dW - \int_{\bar{w}_P}^{\bar{w}_R} u'[W]dW\Big]$$

By construction, $\underline{w}_R - \underline{w}_P = \bar{w}_R - \bar{w}_P$. The result that

$$IC_P > IC_R$$

follows from $\bar{w}_P > \underline{w}_R$, and the fact that marginal utility is decreasing.

With a constant marginal utility, both contracts provide as much incentives, so that the level of effort is identical under both contracts.

Proof of proposition 3b:

The left-hand side of the incentive constraint is

$$E\Big[W'(\tilde{\pi})u'[W(\tilde{\pi})]\Big] = cov\Big(W'(\tilde{\pi}), u'[W(\tilde{\pi})]\Big) + E[W'(\tilde{\pi})]E[u'[W(\tilde{\pi})]]$$

Assume that W is concave (respectively convex). Then W' is decreasing in π (resp. increasing), while u' is decreasing and W' is positive, so that the covariance term is positive (resp. negative). As already shown in the proof of proposition 2b, $E[W'(\tilde{\pi})]$ is the same for both contracts W_A and W_E.

We now show that $E[u'[W(\tilde{\pi})]]$ is larger (respectively identical) with a concave contract W_A than with the corresponding convex contract W_E if $u^{(2i)}[\hat{W}] < 0$ (resp. $u^{(2i)}[\hat{W}] = 0$), for $i = 2, 3, \ldots$:

$$E[u'[W(\tilde{\pi})]] = u'[\hat{W}] + u''[\hat{W}]E[W(\pi) - \hat{W}] + \frac{1}{2}u'''[\hat{W}]E[W(\pi) - \hat{W}]^2 + \frac{1}{6}u''''[\hat{W}]E[W(\pi) - \hat{W}]^3 + \ldots \tag{2.53}$$

where $\hat{W} \equiv E[W(\pi)]$. First, $E[W(\pi) - \hat{W}] = 0$. Second, by construction, the terms $u^{(i)}[\hat{W}]$, for $i = 1, 2, \ldots$ are invariant across contracts. Third, the even order terms $E[W(\pi) - \hat{W}]^{(2i)}$, for $i = 1, 2, \ldots$, are the same whether with a convex contract W_E or with the corresponding concave contract W_A, as already shown in the proof of proposition 2b. Fourth, the odd order terms $E[W(\pi) - \hat{W}]^{(2i-1)}$, for $i = 2, 3, \ldots$, are positive with a convex contract, and negative with a concave contract, as shown in the proof of proposition 2b. Putting all these elements together, $E[u'[W(\tilde{\pi})]]$ is larger with a concave contract than with the corresponding convex contract if $u^{(2i)}[\hat{W}] < 0$, for $i \geq 2$. If $u^{(2i)}[\hat{W}] = 0$, for all $i \geq 2$, then the term $E[u'[W(\tilde{\pi})]]$ is the same for a concave contract and the corresponding convex contract.

All things considered, the left-hand side of the incentive constraint is higher with a concave contract than with a convex contract if the utility function verifies $u^{(2i)} \leq 0$, for $i \geq 2$.

Step contracts in a CARA-normal setting

Using the notations already defined in section 1, the participation constraint is

$$\Phi(q)(-exp\{-\alpha \underline{w}\}) + (1 - \Phi(q))(-exp\{-\alpha \bar{w}\}) = \bar{U} + \psi(e^{\star})$$

Or

$$(-exp\{-\alpha \underline{w}\})\Big[1 + (1 - \Phi(q))exp\{-\alpha \hat{w}\}\Big] = \bar{U} + \psi(e^{\star}) \tag{2.54}$$

The incentive constraint is

$$\lim_{a \to 0} \int_{q-a}^{q+a} \alpha s \; exp\{-\alpha W(\epsilon)\}\varphi(\epsilon)d\epsilon = \psi'(e^{\star})$$

where $s = \frac{\bar{w} - \underline{w}}{2a}$. In the interval $(q - a, q + a)$ under consideration, as a approaches zero, then, loosely speaking, W follows a Bernoulli distribution with probability $1/2$, since φ is approximately flat on an arbitrarily small interval.[44] Using this insight, the incentive constraint rewrites

[44] More precisely, ε follows a Bernoulli distribution, with $w = \underline{w} + \hat{w}\varepsilon$.

as

$$\alpha\hat{w}\varphi(q)\int_{\underline{w}}^{\bar{w}} exp\{-\alpha W\}dW = \psi'(e^{\star})$$

Or

$$\hat{w}\Big[-exp\{-\alpha\bar{w}\} + exp\{-\alpha\underline{w}\}\Big]\varphi(q) = \psi'(e^{\star}) \tag{2.55}$$

For any given q, and for admissible values of $\bar{U}$ and $e^{\star}$, the solution to the system of two equations (2.54) and (2.55) in two unknowns $\underline{w}$ and $\bar{w}$ exists and is unique.[45]

Proof of proposition 4:

The ratio of the left-hand side of (2.54) over the right-hand side is equal to one, and similarly for (2.55). We therefore have

$$\frac{-exp\{-\alpha\underline{w}\}\Big[\Phi(q) + (1-\Phi(q))exp\{-\alpha\hat{w}\}\Big]}{\hat{w}exp\{-\alpha\underline{w}\}[1+exp\{-\alpha\hat{w}\}]\varphi(q)} = \frac{\bar{U}+\psi(e^{\star})}{\psi'(e^{\star})}$$

Or

$$\frac{\Phi(q) + (1-\Phi(q))exp\{-\alpha\hat{w}\}}{\hat{w}[1+exp\{-\alpha\hat{w}\}]\varphi(q)} = -\frac{\bar{U}+\psi(e^{\star})}{\psi'(e^{\star})} \tag{2.56}$$

As q approaches $-\infty$, $1-\Phi(q)$ is approximately equal to 1. The equilibrium condition (2.56) is

$$\frac{\Phi(q) + exp\{-\alpha\hat{w}\}}{1+exp\{-\alpha\hat{w}\}} \rightarrow_{q\rightarrow-\infty} -\frac{\bar{U}+\psi(e^{\star})}{\psi'(e^{\star})}\hat{w}\varphi(q) \tag{2.57}$$

where $\Phi(q)$ is approximately zero. Rearranging this equation,

$$\frac{exp\{-\alpha\hat{w}\}}{1+exp\{-\alpha\hat{w}\}}\frac{1}{\bar{U}+\psi(e^{\star})} \rightarrow_{q\rightarrow-\infty} -\frac{\hat{w}\varphi(q)}{\psi'(e^{\star})} \tag{2.58}$$

Equating to the incentive constraint (2.55) and performing some algebra yields

$$-exp\{-\alpha\bar{w}\} \rightarrow_{q\rightarrow-\infty} \bar{U}+\psi(e^{\star}) \tag{2.59}$$

Using the definition of $W^{\star}$ in (2.3), the only solution to this equation is

$$\bar{w} \rightarrow_{q\rightarrow-\infty} W^{\star}$$

[45]The left-hand side of the participation constraint is increasing in $\underline{w}$ and in $\bar{w}$. It is equal to minus infinity for $\underline{w} = -\infty$ and $\bar{w} = -\infty$, and to zero for $\underline{w} = \infty$ and $\bar{w} = \infty$. The left-hand side of the incentive constraint is decreasing in $\underline{w}$ and increasing in $\bar{w}$. It is equal to 0 for $\underline{w} = \bar{w}$, and to infinity for a finite $\bar{w}$ and $\underline{w} = -\infty$. If the LHS of the incentive constraint is too low, and so is the LHS of the participation constraint, then raise $\bar{w}$. If the LHS of the incentive constraint is too low, and the LHS of the participation constraint is too high, then diminish $\underline{w}$. If the LHS of the incentive constraint is too high, and the LHS of the participation constraint is too low, then raise $\underline{w}$. If the LHS of the incentive constraint is too high, and so is the LHS of the participation constraint, then diminish $\bar{w}$.

The cost of the contract is

$$\Phi(q)\underline{w} + (1-\Phi(q))\bar{w}$$

It is bounded above by $\bar{w}$. Therefore, the agency cost is approximately zero.

Now consider the case where q approaches infinity. The equilibrium condition (2.56) is

$$\frac{1}{\hat{w}}\frac{1+(1-\Phi(q))exp\{-\alpha\hat{w}\}}{\varphi(q)(1+exp\{-\alpha\hat{w}\})} = -\frac{\bar{U}+\psi(e^\star)}{\psi'(e^\star)} > 0 \tag{2.60}$$

Dividing both the numerator and the denominator of the second fraction by $\varphi(q)$,

$$\frac{1}{\hat{w}}\frac{\frac{1}{\varphi(q)}+\frac{1-\Phi(q)}{\varphi(q)}exp\{-\alpha\hat{w}\}}{1+exp\{-\alpha\hat{w}\}} = -\frac{\bar{U}+\psi(e^\star)}{\psi'(e^\star)} \tag{2.61}$$

As the ratio $\frac{1-\Phi(q)}{\varphi(q)}$ approaches zero for q sufficiently large, and $\varphi(q)$ approaches zero as q tends to infinity, this implies

$$\hat{w} \rightarrow_{q\rightarrow\infty} \infty$$

Using this fact, (2.61) becomes

$$\frac{1}{\hat{w}}\left[\frac{1}{\varphi(q)}+\frac{1-\Phi(q)}{\varphi(q)}\frac{exp\{-\alpha\hat{w}\}}{1+exp\{-\alpha\hat{w}\}}\right] = -\frac{\bar{U}+\psi(e^\star)}{\psi'(e^\star)} \tag{2.62}$$

Or

$$\frac{1}{\hat{w}}\left[\frac{1}{1-\Phi(q)}+\frac{exp\{-\alpha\hat{w}\}}{1+exp\{-\alpha\hat{w}\}}\right] = -\frac{\varphi(q)}{1-\Phi(q)}\frac{\bar{U}+\psi(e^\star)}{\psi'(e^\star)} \tag{2.63}$$

The fraction of exponentials approaches zero as $\hat{w}$ tends to infinity. Rearranging,

$$(1-\Phi(q))\hat{w} = -\frac{1-\Phi(q)}{\varphi(q)}\frac{\psi'(e^\star)}{\bar{U}+\psi(e^\star)} \tag{2.64}$$

But for the normal distribution, the ratio $\frac{1-\Phi(q)}{\varphi(q)}$ approaches zero for q sufficiently large. Therefore,

$$(1-\Phi(q))\hat{w} \rightarrow_{q\rightarrow\infty} 0$$

The cost of the contract is then

$$\underline{w} + (1-\Phi(q))\hat{w} \approx \underline{w}$$

Finally, satisfying the participation constraint requires that $\underline{w} < W^\star$. Thus, the expected cost of the contract is approximately equal to the first-best cost, and the agency cost is approximately zero.

CARA-normal simulation with puts and calls:

In a CARA-normal setting, we are going to compare the relative incentive effects and relative agent's valuation of option contracts featuring either short puts or long calls that are as costly to the principal, as in section 2. A contract is defined by its strike k and its slope s, which are

Short Put

k	-10	-1	0	0	1	10
s	20	1	1	2	1	1
w	2.0	2.1	2.4	2.8	3.1	12.0
Cost to the principal	2	2	2	2	2	2
Agent's valuation	-0.27	-0.14	-0.17	-0.47	-0.21	-0.22
Effort incentives	2.71	0.04	0.13	0.88	0.20	0.22

Long Call

k	-10	-1	0	0	1	10
s	1	1	1	2	1	5
w	-8.0	0.9	1.6	1.2	1.9	2.0
Cost to the principal	2	2	2	2	2	2
Agent's valuation	-0.22	-0.18	-0.15	-0.20	-0.14	-0.14
Effort incentives	0.22	0.12	0.05	0.10	0.02	0.00

Figure 2.15: Valuation and incentives of option contracts.

exogenously set. Given these values, a coefficient of absolute risk aversion of 1 and a standard deviation of 1, the fixed wage w is calculated by Mathematica in order to equate the cost of any contract to 2. For each combination of k and s, and the associated w, figure 18 reports the agent's valuation of the contract at the equilibrium effort (the left-hand side of the participation constraint), and the effort incentives created by this contract (the left-hand side of the incentive constraint).

The results are consistent with propositions 2b and 3b. For example, compare two contracts with the same expected cost: a short put contract with a strike of 1 and a slope of 1 to a long call contract with a strike of -1 and a slope of 1 (both are in red in figure 18). As expected, the long call contract yields a higher expected utility, but the short put contract elicits greater effort. Also note that a short put contract with a slope of 1, a high strike (10) and a fixed wage of 12 is approximately valued as a symmetric long call contract with a slope of 1, a low strike (-10) and a fixed wage of 8. This is because both are approximations of a linear contract of slope 1 that cuts the y axis at 2. They constitute in this sense the benchmark case. Finally, long call contracts with high strikes seem quite inefficient, as they provide extremely low incentives. However, the opposite holds for symmetric short put contracts. Indeed, for a short put contract with k low enough, multiplying both the strike and the slope by a constant larger than 1 only negligibly affects the agent's valuation of the contract, but provides much higher incentives.

Proof of proposition 5:

From (2.3), the first-best payment $W^\star$ is such that

$$-exp\{-\alpha W^\star\} = \bar{U} + \psi(e^\star) \tag{2.65}$$

At the second-best, the pay schedule must satisfy both the participation constraint and the incentive constraint:

$$\int_{-\infty}^{k} -exp\{-\alpha(w + s(\epsilon - k))\}\varphi(\epsilon)d\epsilon + \int_{k}^{\infty} -exp\{-\alpha w\}\varphi(\epsilon)d\epsilon = \bar{U} + \psi(e^\star) \tag{2.66}$$

$$s\alpha \int_{-\infty}^{k} exp\{-\alpha(w + s(\epsilon - k))\}\varphi(\epsilon)d\epsilon = \psi'(e^\star) \tag{2.67}$$

The left-hand side of the incentive constraint is the average pay-performance sensitivity multiplied by the expected marginal utility conditional on ϵ being lower than the strike k. These two equations rewrite as

$$-exp\{-\alpha w\}\left[\int_{-\infty}^{k} exp\{-\alpha s(\epsilon - k)\}\varphi(\epsilon)d\epsilon + 1 - \Phi(k)\right] = \bar{U} + \psi(e^\star) \tag{2.68}$$

$$s\alpha\ exp\{-\alpha w\} \int_{-\infty}^{k} exp\{-\alpha s(\epsilon - k)\}\varphi(\epsilon)d\epsilon = \psi'(e^\star) \tag{2.69}$$

Call the integral in both equations A. It is equal to

$$\begin{aligned}
A = \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{k} exp\{-\alpha s(\epsilon - k)\}exp\left\{-\frac{\epsilon^2}{2\sigma^2}\right\}d\epsilon &= \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{k} exp\left\{-\frac{2\alpha s(\epsilon - k)\sigma^2 + \epsilon^2}{2\sigma^2}\right\}d\epsilon \\
&= \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{k} exp\left\{-\frac{(\epsilon - (-\alpha s\sigma^2))^2 - 2\alpha sk\sigma^2 - \alpha^2 s^2\sigma^4}{2\sigma^2}\right\}d\epsilon \\
&= \frac{1}{\sqrt{2\pi}\sigma}exp\left\{\alpha sk + 0.5\alpha^2 s^2\sigma^2\right\}\int_{-\infty}^{k} exp\left\{-\frac{(\epsilon - (-\alpha s\sigma^2))^2}{2\sigma^2}\right\}d\epsilon \\
&= \Phi(k + \alpha s\sigma^2)exp\{\alpha sk + 0.5\alpha^2 s^2\sigma^2\}
\end{aligned}$$

We perform a change of variable to express A as a function of the standard normal c.d.f., F. Let

$$y \equiv \frac{\epsilon - (-\alpha s\sigma^2)}{\sigma}$$

So that

$$dy = \frac{d\epsilon}{\sigma}$$

And

$$b \equiv \frac{k - (-\alpha s\sigma^2)}{\sigma}$$

Then

$$A = exp\left\{\alpha sk + 0.5\alpha^2 s^2\sigma^2\right\}\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{b} exp\left\{-\frac{y^2}{2}\right\}\sigma dy$$

$$= \sigma \; exp\left\{ \alpha s k + 0.5\alpha^2 s^2 \sigma^2 \right\} F(b)$$

The ratio of the left-hand side of (2.68) over the right-hand side is equal to one, and similarly for (2.69). We therefore have

$$\frac{-exp\{-\alpha w\}\left[A + 1 - \Phi(k)\right]}{s\alpha \; exp\{-\alpha w\} A} = \frac{\bar{U} + \psi(e^\star)}{\psi'(e^\star)}$$

where the right-hand side of this equation is a constant, in the sense that it is invariant across admissible contracts. Rearranging,

$$\frac{1}{s\alpha}\left(1 + \frac{1 - \Phi(k)}{A}\right) = -\frac{\bar{U} + \psi(e^\star)}{\psi'(e^\star)} \tag{2.70}$$

Let the strike k of the short put be arbitrarily small. Then, for the contract to be incentive compatible and satisfy (2.69), the slope s needs to be arbitrarily large. This contract features extremely large punishments for extremely bad performances - which occur very rarely. Furthermore, as s becomes arbitrarily large, $\frac{1-\Phi(k)}{A}$ must become arbitrarily large as well for (2.70) to be satisfied. Since $1 - \Phi(k)$ approaches 1 as k becomes arbitrarily small, this mean that A must approach zero. As k is arbitrarily small and s arbitrarily large, the participation constraint (2.68) is therefore approximately

$$-exp\{-\alpha w\} \approx \bar{U} + \psi(e^\star) \tag{2.71}$$

Comparing with (2.65), the fixed wage is approximately equal to the first-best wage $W^\star$. At the equilibrium effort, the optimal risk sharing rule (full insurance of the agent) is almost in place, but not completely, the agent will therefore be paid a fixed wage w higher than $W^\star$. The first-best is arbitrarily closely approximated, but not fully attained, with this short put contract. Since the cost of a short put contract with an arbitrarily small k is lower than the cost of its fixed wage w, and since w is arbitrarily close to $W^\star$, the agency cost is arbitrarily small.

Numerical simulations in a CRRA-lognormal setting

We conduct the same numerical simulations as in section 3 with step contracts, but in a CRRA-lognormal setting. By definition, $ln(\tilde{\epsilon})$ is normally distributed. Its mean is normalized at 0 and its variance is set at 1. Figures 19 and 20 report the agency costs of individually rational and incentive compatible step contracts as a function of their cutoff. As can be seen in figure 15, with a coefficient of relative risk aversion γ equal to 2, the results are qualitatively extremely similar to those obtained with a CARA utility.[46]

[46] Only one detail stands out: the slow convergence to the first-best cost on the "rewards" side. This is because $\exp\{10\} \approx 22,000$. The cutoff must therefore be extremely high for asymptotic properties of the likelihood ratio to come into play.

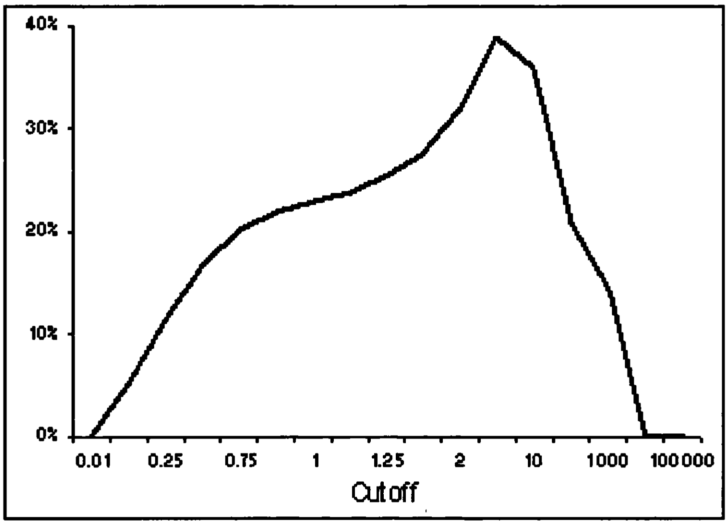

Figure 2.16: Agency cost of step contracts as a % of the first-best cost: CRRA-lognormal, $\gamma = 2$.

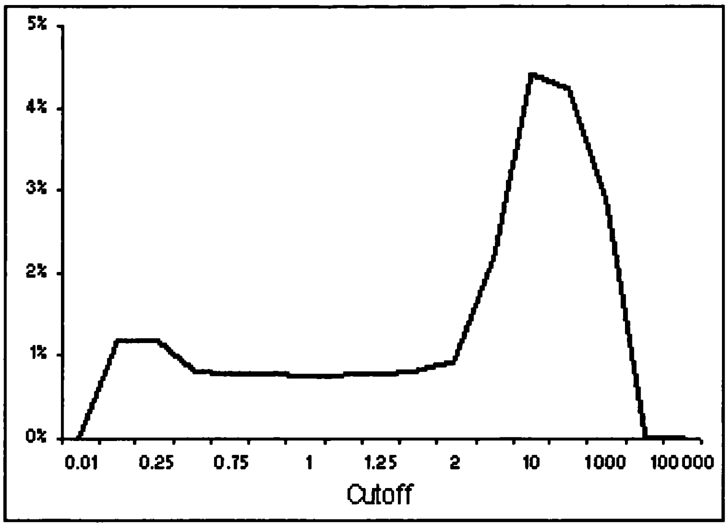

Figure 2.17: Agency cost of step contracts as a % of the first-best cost: CRRA-lognormal, $\gamma = 1$.

However, as explained in section 4, as γ diminishes the prudence effect becomes relatively stronger than the risk aversion effect. Hence the bump in agency costs in the low cutoffs region when $\gamma = 1$, which can be seen in figure 16. These results suggest that punishments are all the more efficient relative to rewards that agents have a high relative risk aversion.

Optimal contracts in a CRRA-lognormal model

The Holmstrom condition below describes the optimal contract W when the principal is risk-neutral and the first-order approach applies.

$$\frac{1}{u'(W(\pi))} = \lambda + \mu \frac{\vartheta_e(\pi)}{\vartheta(\pi)} \tag{2.72}$$

Dittmann and Maug (2007) study a model in which an agent with a CRRA utility function controls at a cost the mean of the performance measure distribution, which is lognormally distributed. They show that with nonnegative transfers, the optimal contract takes the form:

$$W(\pi) = \begin{cases} (\alpha_0 + \alpha_1 ln(\pi))^{\frac{1}{\gamma}} - W_0 exp\{r_f T\} & \text{if } \pi \geq \bar{\pi} \\ 0 & \text{if } \pi < \bar{\pi} \end{cases}$$

where $\bar{\pi} \equiv \exp\{\frac{(W_0 exp\{r_f T\})^{\gamma} - \alpha_0}{\alpha_1}\}$, α_0 and α_1 are two constants which are determined to satisfy the participation constraint and the incentive constraint, γ is the coefficient of relative risk aversion, W_0 is the initial wealth of the CEO, r_f is the risk-free interest rate on an annual basis, and T is the length of the period.

According to Dittmann and Maug, the representative CEO has a fixed wage of $\$1.2m$, is endowed with 0.42% of his company's equity and 0.50% in stock-options, and has an initial wealth unrelated to his company of $9.1m$. The market value of equity of his company is $\$3.7bn$, the options' strike amounts to 63% of the time 0 value of the stocks, the time period is 8.5 years, the risk-free rate is 6.6%, and the dividend yield is 2.3%. We use the same data, notably in order to facilitate comparability.

We apply their criterion for the validity of the first-order approach in this setting. While more details are available in their paper, it basically requires that the optimal contract be more concave than the observed contract. This will be verified in all cases.

Convex transformations and skewness

Assume that the random variable $\tilde{\varepsilon}$ is symmetrically distributed. Its skewness is equal to

$$\frac{E\big[(\tilde{\varepsilon} - E[\tilde{\varepsilon}])^3\big]}{E\big[(\tilde{\varepsilon} - E[\tilde{\varepsilon}])^2\big]^{\frac{3}{2}}}$$

Since $\tilde{\varepsilon}$ is symmetrically distributed, the numerator is zero, and therefore the skewness is zero.

The skewness of $f(\tilde{\varepsilon})$ is

$$\frac{E\left[(f(\tilde{\varepsilon}) - E[f(\tilde{\varepsilon})])^3\right]}{E\left[(f(\tilde{\varepsilon}) - E[f(\tilde{\varepsilon})])^2\right]^{\frac{3}{2}}} \tag{2.73}$$

The denominator in (2.73) is positive. In addition, if f is convex, the proof of proposition 2b shows that the numerator in (2.73) is positive (simply identify $f(\varepsilon)$ with $W_E(\pi)$ and $E[f(\tilde{\varepsilon})]$ with $\hat{W} = E[W_E(\tilde{\pi})]$). We have shown that a convex transformation of any symmetrically distributed random variable has a positive skewness. As a corollary, since the skewness of any symmetrically distributed random variable is zero, a convex transformation of this variable increases its skewness.

Proof of proposition 7:

I need to show that the left-hand side of the incentive constraint (2.2) is larger if a zero-mean risk is added to compensation.

The LHS of (2.2) with a zero-mean risk is

$$\begin{aligned} E_{\tilde{\pi}}\Big[E_{\tilde{x}}\Big[W'(\tilde{\pi})u'[W(\tilde{\pi}) + \tilde{x}]\Big]\Big] &= E_{\tilde{\pi}}\Big[W'(\tilde{\pi})E_{\tilde{x}}\Big[u'[W(\tilde{\pi}) + \tilde{x}]\Big]\Big] \\ > E_{\tilde{\pi}}\Big[W'(\tilde{\pi})u'[W(\tilde{\pi}) + E_{\tilde{x}}[\tilde{x}]]\Big] &= E_{\tilde{\pi}}\Big[W'(\tilde{\pi})u'[W(\tilde{\pi})]\Big] \end{aligned}$$

which is the LHS of (2.2) without a zero-mean risk. The inequality follows from Jensen inequality and the fact that u' is convex.

Proof of corollary 1:

Given any contract W, the effect of adding a pure (zero-mean) bounded noise that alters probabilities in the $[W^N, \infty)$ interval is twofold.

First, a direct application of proposition 7 shows that it will increase incentives.

Second, it will increase the agent's valuation of the contract. Suppose that $\tilde{x}$ is distributed on $[-c, \infty)$, with $E[\tilde{x}] = 0$. Then, with ϕ being the c.d.f. of the original contract W conditional on the equilibrium effort $e^{\star}$, adding $\tilde{x}$ to the contract W increases the expected utility of the agent:

$$\int_{-\infty}^{W^N+c} u[W]d\phi(W) + \int_{W^N+c}^{\infty} E_{\tilde{x}}u[W + \tilde{x}]d\phi(W)$$

$$> \int_{-\infty}^{W^N+c} u[W]d\phi(W) + \int_{W^N+c}^{\infty} u[W + E_{\tilde{x}}[\tilde{x}]]d\phi(W) = \int_{-\infty}^{\infty} u[W]d\phi(W)$$

The utility function u being convex on the relevant interval, inequality follows from Jensen inequality.

Thus, both the participation constraint and the incentive constraint are relaxed.

Chapter 3

The optimal timing of compensation with managerial short-termism

This paper introduces a new continuous-time principal-agent model to determine the optimal timing of stock-based incentives, when the effects of managerial actions materialize with a lag and are only progressively understood by shareholders. Whereas early contingent compensation hedges the manager against the accumulation of exogenous shocks, the fact that shareholders receive signals with increasing precision suggest that contingent compensation should be postponed. The optimal timing of compensation trades-off these two forces. The model then emphasizes the tradeoff between providing incentives and preventing short-termism. In effect, the optimal timing rule obtained is not always robust to managerial short-termism: it might induce a manager to manipulate early signals of performance, or to pick a project with a shorter time horizon. To circumvent either type of short-termism, incentives are lower-powered, and the compensation date is postponed - unless noise dominates, shocks are multiplicative, and the manager's motivation for short-termism is to reduce the variability of his compensation, in which case it may be preferable to pay him even earlier.

The recent travails of major companies have reignited the debate on the timing of executive compensation. There is growing concern that executives are remunerated before the full effects of their actions are revealed, which decouples their pay from their contribution and generates perverse incentives. This paper specifies why and when CEOs and other top executives should be paid earlier rather than later. It analyzes the power and timing of efficient and robust stock-based incentive schemes. To this end, it introduces a new principal-agent model, which is unique in combining three essential elements of CEO pay: (i) an output realized some time after the managerial action, (ii) information asymmetries between the manager and investors, and (iii) a stock price process continuously set by rational investors on which the manager's pay may be contingent.[1]

Uncertainty alone cannot explain the widespread practice of delaying the contingent payment of managers. In effect, consider a standard moral hazard problem in which the manager of a firm controls the probability distribution of its future revenues through an observable but uncontractible action. In equilibrium, it is possible to reward the manager without exposing him to any risk, by making his compensation contingent on the share price established after he took his action, but before uncertainty is realized. The "market monitoring" of Holmstrom and Tirole (1993) suffices. On the contrary, delaying a manager's compensation exposes him to more risk, as developments occur over which he does not have any control, but which will nevertheless be incorporated into the share price.[2]

If this were the whole story, we would not observe delayed compensation. Are there some countervailing effects? The main argument against immediate managerial compensation, first

[1]In most discrete time agency models (see the seminal papers of Mirrlees (1975), Holmstrom (1979), Grossman and Hart (1983)), the agent exerts effort at the beginning of the period, and a contractible output is realized at the end of the period. By contrast, in the more recent agency literature in continuous time (see for example Holmstrom and Milgrom (1987), de Marzo and Sannikov (2006), Sannikov (2008), Williams (2004)), the agent exerts effort continuously, and a contractible output is realized continuously and instantaneously. This technical difference matters. Whereas the performance of a salesman can often be measured at the end of each day, the performance of a firm (and hence of its CEO) which undertook a given project is only fully measurable at the end of the project's time horizon.

[2]With informed investors with rational expectations, establishing a market to trade a claim on the firm's future revenues enables to make an observable but not contractible action contractible. This property extends to other settings. Without information asymmetries, if competitive markets for contingent claims whose payoff depends on the action can be set up, observability can readily be converted into contractibility. The price of the claim must simply be informative about the agent's action. The existence of a one-to-one mapping is all that is needed. The action can then be inferred from the price of the tradable claims, so that the agent's pay can be made contingent on this price. In frictionless markets, this device is costless. In brief, the first-best outcome in moral hazard problems can in general still be reached if the agent's action is observable but not contractible.

articulated by Narayanan (1985), states that managers have private information.[3] In the model used in this paper, it takes time for investors to correctly interpret the manager's action. In effect, they initially lack the adequate knowledge or information. They will therefore be prone to misjudging the consequences of the manager's action. Only with the passage of time will their assessment get more precise. Since the manager's private information will only be incorporated progressively in the share price process, a manager with early compensation may be led to take an inappropriate action, or could be considerably exposed to the risk of mispricing.

This paper formalizes the tradeoff at the core of delayed compensation. On the one hand, the manager's private information[4] is progressively revealed over a period of time. On the other hand, noise accumulates in the performance measure as time passes. From the manager's ex ante perspective, the first risk fades with time, whereas the second grows. The optimal timing rule minimizes the risk of the manager's pay, by balancing the accumulation of noise against the risk of mispricing emanating from information asymmetries. It can be summarized by distinguishing between four cases. If information asymmetries outweigh noise, compensation should be contingent on the end-of-period share price, so that the manager is thoroughly protected from investors' mistaken assessments. If noise outweighs information asymmetries, compensation should be contingent on the initial date's share price, so that the manager is thoroughly protected from noise. If information asymmetries are eliminated at an increasing rate, the manager should be compensated either based on the initial or the end-of-period share price. If information asymmetries are eliminated at a decreasing rate, the manager should be compensated at the time when the rate in question is equal to the rate at which noise accumulates.

The model is then expanded by taking into consideration the perverse incentives that this timing of compensation may create. In particular, a risk averse manager may be tempted to either artificially improve early signals of performance so as to maximize the stock price at the compensation date, or to opt for a project with a shorter time horizon but a lower expected payoff so as to reduce the variability of his remuneration. We call the latter case managerial short-termism of the second type. The former case, which we call managerial short-termism of

[3] A recent paper by Edmans, Gabaix, Sadzik, and Sannikov (2009) also has predictions on the optimal timing of pay in a continuous time model. However, deferred pay is then optimal not because of progressively resolved information asymmetries, but because of consumption smoothing, and to prevent the manager from artificially shifting stock returns from one period to another.

[4] Managerial private information is exogenous in the model. Nevertheless, it might be possible for managers to reveal part or all of it to the market. However, managers can in practice withhold information; and they might not be credible when revealing soft information. They may be more credible and more willing to convey information if their interests are more congruent with investors'. Such a communication game is a natural extension of the model.

the first-type, also encompasses the type of short-termism which consists in inefficiently trading off a higher short-term payoff for a lower long-term payoff.[5] In the words of Krugman (2002), "a system that lavishly rewards executives for success tempts those executives, who control much of the information available to outsiders, to fabricate the appearance of success." Rational investors are not duped in equilibrium, and the manager does not derive any rent, but the outcome may still be inefficient.

Remarkably, the same remedies apply to both types of short-termism. First, short-termism can be alleviated with lower-powered incentives. Thus, there is a conflict between mitigating the effect of private benefits of control (or equivalently providing incentives for effort), which can be achieved by a rise in the pay-performance sensitivity, and preventing short-termism, which calls for a lessened pay-performance sensitivity. The optimal pay-performance sensitivity will generally balance these two conflicting objectives. In cases where the short-termism problem is very severe, the paper also shows that it is optimal to provide muted incentives. These results shed light on the positive correlation between the propensity to misreport earnings and the level of the pay-performance sensitivity empirically identified by Burns and Kedia (2006), which is precisely what the model would predict with regards to the first type of short-termism.

Second, short-termism can often be alleviated by postponing the evaluation and payment date. This is the case with the first type of short-termism, or with the second type in a model with additive shocks, or with multiplicative shocks and relatively strong informational asymmetries. Regarding the first type of short-termism, this explains the empirical finding of Richardson and Waegelein (2002) that long-term performance plans are associated with lower levels of manipulation of short-term earnings. Regarding the second type of short-termism, the model's predictions are consistent with the empirical results of Larcker (1983), Kumar and Sopariwala (1992), Kole (1997), and Richardson and Waegelein (2003).

However, with short-termism of the second type, multiplicative shocks, and sufficiently strong noise, the model shows that it is optimal to have an *earlier* evaluation and payment date. The intuition is that with multiplicative shocks, noise magnifies information asymmetries. The commonly-held belief that the possibility of managerial short-termism demands that the manager be compensated later than would otherwise have been the case is thus dependent upon technological assumptions which may or may not be verified.

[5] As far as investors are concerned, the (discounted) sum of payoffs matters. All that is really needed is not a sequence of payoffs, but early signals based on which the CEO is paid - this role may be played by the short-term payoff. This will in turn induce the CEO to produce early signals of high quality if his pay depends on it, to the detriment of the final outcome - the (discounted) sum of all payoffs. This is precisely the short-termism of the first type we described.

In an extension of the model, the pay-performance sensitivity process may continuously depend on the stock price process. We then show that it is optimal for it to be increasing whenever the rate at which information asymmetries are resolved at a given time is larger than the rate at which noise accumulates. Depending on the production technology, the optimal pay-performance sensitivity process may therefore be highly non-monotonic; but it will never be equal to zero, even on the least informative subintervals. In addition, it is notable that the aforementioned alterations to the incentive scheme which tackle potential short-termism (whether of the first or of the second type) hold whether compensation is concentrated at a single date, or is contingent upon the whole path of the stock price.

To obtain these results, the paper develops a stylized continuous time model in which the manager's pay may depend on a share price process set by a representative rational investor. This investor determines the share price according to his information at a given time, and he does not make any systematic error when assessing the manager's action.[6] On the one hand, the final payoff progressively deviates from its ex ante value, because of the accumulation of noise. This alone makes the share price a more and more noisy estimate of the manager's action. On the other hand, the investor progressively learns the value of the payoff with accumulated noise at a given time, by continuously receiving signals whose precision is increasing with time. This learning process captures the diminution of uncertainty attributable to information asymmetries as time passes and the investor adjusts his assessment. The model distinguishes between the first-best, when there are no agency problems, the second-best with agency problems but when the manager cannot game the compensation system, and the third-best, when he can by being short-termist.

The paper is related to the multitask literature. Admittedly, the manager has only one task: selecting the firm's project. However, when projects differ on dimensions other than their expected payoff and their private cost to the manager, it is shown that giving the manager strong incentives on one dimension only may be inefficient - a result reminiscent of Holmstrom and Milgrom (1991). In the theoretical literature on managerial short-termism, myopic behavior can be a response to early termination of projects with low first-period returns by uninformed financiers, as in von Thadden (1995). The manager is thus led to prefer a safer short-term project with a lower expected payoff. Short-termism can also emerge when managers can manipulate investors' expectations by increasing the short-term payoff at the expense of the long-term payoff, as in Narayanan (1985) and Stein (1989).

The first section presents the baseline model and derives the optimal timing of compensation

[6]This paper assumes that rational investors have the same beliefs and information. By contrast, in Bolton, Scheinkman and Xiong (2006), a speculative component in the stock price prompts managerial short-termism.

at the second-best. The second section introduces opportunities for two types of managerial short-termism, and highlights how it could modify the second-best optimal power and timing of compensation. The third section describes optimal pay-performance sensitivity processes, and studies the implications of multiplicative shocks. The fourth section concludes.

3.1 Optimal timing of compensation

3.1.1 The model

Consider a firm initially owned by a founder and run by a manager. The sequence of events is the following: at $t = -2$, the founder offers a contract to the manager. If he accepts, the manager then chooses between two projects and invests in the chosen project at $t = -1$. At $t = 0$, the founder sells the firm to an investor. The firm's stock is publicly traded from time 0 to time T. At time T, it entitles its owner to the project's payoff.

Technology

For simplicity, the discount factor is zero. There are two projects: efficient and inefficient. At time T, the payoff of the efficient project is equal to $\bar{A}+\sigma^N B_T^N$, where B_t^N is a Brownian motion[7], with common knowledge diffusion process $\sigma^N < K$, where K is a positive finite constant. The inefficient project confers nonpecuniary private benefits of control worth P (in monetary terms) to the manager, but its payoff is only equal to $\underline{A} + \sigma^N B_T^N$, with $\underline{A} < \bar{A}$.

At time 0, the firm issues a stock, which is publicly traded from time 0 to time T, with an endogenously determined price S_t at time t. At time T, its payoff is equal to the payoff of the selected project, $A + \sigma^N B_T^N$, where A is $\bar{A}$ if the efficient project is chosen by the manager at time -1, and $\underline{A}$ otherwise.

Agents and objective functions

The objective of the manager is to maximize the expected utility of his end-of-period wealth plus any private benefits of control p (either $p = 0$ or $p = P$). Assume that he has CARA utility with coefficient of absolute risk aversion γ. For a payment W and a private benefit of control p, his utility is therefore

$$u(W,p) = -exp\{-\gamma(W+p)\}$$

[7] It is therefore characterized by the standard properties of Brownian motions. In particular, it is equal to 0 at $t = 0$, and it writes as $B_t^N = \int_0^t dB_s^N$, for any $t \geq 0$. For any $s > t$, the term $B_s^N - B_t^N$ is normally distributed with mean 0 and variance $s - t$.

The risk neutral founder is the initial owner of the firm. His objective is to maximize the time 0 stock price of the firm (which is the price at which he will sell his stake) minus the compensation cost (he is paying the manager, with whom he entered a contract).

There is one risk neutral, unconstrained, and competitive representative investor, who is the only agent allowed to trade the stock. At any given time from time 0 to time T, this investor therefore sets the stock price according to his information.

Contracting

At time -2, the founder offers a compensation contract to the manager. The manager accepts the job if and only if his expected utility with the contract exceeds his reservation utility $\bar{U}$. Define his reservation wage as $\bar{W}$, where $-exp\{-\gamma\bar{W}\} \equiv \bar{U}$.

For simplicity and tractability, we assume that the manager's pay can depend linearly on the stock price process $\{S_t\}_{0\leq t\leq T}$. We also assume that pay may only be contingent on the stock price at one predetermined point in time, τ, where $\tau \in [0, T]$. The nonnegative parameter b is the sensitivity of pay to performance at this time. Contract parameters are common knowledge. In the baseline model, the manager's contingent pay writes as

$$w + bS_\tau$$

where w is the fixed wage. The compensation contract may therefore be described as $\{w, \tau, b\}$.

Since the manager is risk averse and the founder is risk neutral, the first-best risk sharing rule consists in paying the manager a fixed wage, so that $w = \bar{W}$ and $b = 0$. Deviating from this rule by making the manager's pay variable for incentive reasons will be costly. For any given compensation contract $\{w, \tau, b\}$, let the agency cost be the difference between the manager's expected pay if he chooses $A = \bar{A}$ in equilibrium, which is $E_{-1}\left[w + bS_\tau | A = \bar{A}\right]$, and his reservation wage $\bar{W}$.[8]

We make some additional assumptions. Firstly, parameters are such that the firm is expected, ex ante (at $t = -2$), to operate at a profit: $\bar{A}$ minus the second-best expected payment to the manager required to induce him to invest in the efficient project is nonnegative. Secondly, in the baseline model, this quantity is larger than $\underline{A}$ minus the expected payment to the manager required to induce him to invest in the inefficient project with private benefits of control, which is nevertheless also nonnegative. This implies that, in the baseline model, parameters are such

[8] E_{-1} denotes the conditional expectation operator at $t = -1$. The information of the founder is admittedly the same at $t = -2$ and $t = -1$: the manager chooses in which project to invest at $t = -1$, but the founder already knows whether or not he will invest in the efficient project. Given the parameters of the contract that he offers at $t = -2$, the founder knows which action the manager will take at $t = -1$, so there is no arrival of new information at $t = -1$ in equilibrium.

that it is always in the founder's interests to incur the agency cost and induce the manager to select the efficient project. We need some additional notations to formalize these assumptions, which are made formal in equation (3.4) below. Given that the manager chooses the efficient project, the objective function of the founder, who pays the manager's compensation and offers the contract $\{w, \tau, b\}$, is

$$E_{-1}\left[S_0 - (w + bS_\tau)|A = \bar{A}\right] \tag{3.1}$$

Thirdly, we assume that the (risk-neutral) founder commits himself at $t = -1$ to selling the firm at $t = 0$ at the prevailing market price.[9] Fourthly, a liquidation of the firm at any time between $t = 0$ and $t = T$ is infinitely costly. This captures the notion that investment is irreversible.[10]

The information structure

The Brownian motion B_t^N is not observed by any agent, at any time. Only the payoff $A + \sigma^N B_T^N$ is publicly observed at time T.

The manager knows $\underline{A}$, $\bar{A}$, and his choice of A. The founder knows $\underline{A}$ and $\bar{A}$, but he cannot credibly communicate this information to the investor (although he does not observe the manager's choice of A, he may infer it from the parameters of the contract). The representative investor neither knows $\underline{A}$, $\bar{A}$, nor the manager's choice of A. However, he progressively learns the value of the project's conditional expected payoff $A + \sigma^N B_t^N$: for all $t \in [0, T]$, he observes a noncontractible signal v_t at time t. His initial information is therefore crystallized in the signal v_0, which may be viewed as his prior.[11]

Assume that the time 0 signal, v_0, and the time t signal, v_t, are respectively

$$v_0 = A - \int_0^T \sigma_s^U dB_s^U \tag{3.2}$$

$$v_t = A + \sigma^N B_t^N - \int_t^T \sigma_s^U dB_s^U \tag{3.3}$$

where B_t^U is another Brownian motion, uncorrelated with B_t^N, with time-varying, deterministic, and common knowledge diffusion process $\sigma_t^U < K$ for all t, where K is a positive finite constant.

[9]This assumption may be microfounded by assuming that the founder is an insider, and wants to protect himself against charges of insider trading. This can be achieved by placing at $t = -1$ a sell order at the market price prevailing at $t = 0$.

[10]It is possible that, ex-post, at $t = 0$, the stock price is lower than the expected payment to the manager, so that the firm is expected to operate at a loss. Since liquidation is infinitely costly, the firm must keep on operating from time 0 to time T. This implies that no profitable renegotiation is possible, and any possible (expected) losses are incurred by the firm's founder.

[11]Since neither the manager nor the founder do anything after time -1, it is inconsequential whether or not they observe the process v_t at any time t. Likewise, since the representative investor only acts from $t = 0$, his information prior to $t = 0$ is irrelevant.

For simplicity and tractability, we assume that the diffusion process σ_t^U is either constant, or monotonically increasing or decreasing with t. The Brownian motion B_t^U is fully realized before time 0, and never observed by any agent at any time (however, the signal v_t is observed by the representative investor at time t).

The dynamics of the σ_t^U process determine whether information asymmetries are resolved at the beginning or at the end of the time interval $[0, T]$. They are mostly resolved at the beginning if σ_t^U is decreasing in t, and mainly towards the end if σ_t^U is increasing in t. By contrast, the overall magnitude of σ_t^U is the extent of information asymmetries: the less private information the manager possesses relative to the investor, the smaller the overall magnitude of σ_t^U. If σ_t^U were identically zero, there would not be any information asymmetries, and the passage of time would only add noise to the signal process v_t.

This representation of the signal process has the advantage of being very tractable, and also has three desirable properties. First, this process is consistent with rational expectations, since the representative investor does not make any systematic mistake when assessing the stock value. Second, the variance between the signal v_t and the conditional expected payoff $A + \sigma^N B_t^N$ is decreasing in t, which is consistent with the progressive attenuation of the mispricing due to information asymmetries. Third, at any time t, v_t is a sufficient statistic for the project's payoff $A + \sigma^N B_T^N$ (this result is proven in the Appendix). Thus, v_0 may be viewed as an uninformative prior over the project's payoff.

Lastly, as will be clear later, the conditions that the firm is expected to operate at a profit, and that the founder optimally incurs the agency cost to induce the manager to undertake the efficient project in the baseline model, reduce to assuming the following:

$$\bar{A} - \bar{W} - \frac{\gamma}{2}\Big(\frac{P}{\bar{A} - \underline{A}}\Big)^2 \Big(\sigma^{N^2}\tau + \int_\tau^T \sigma_t^{U^2} dt\Big) > \underline{A} - \bar{W} \geq 0 \tag{3.4}$$

The stock price

The discount factor is zero, the representative investor is risk neutral and a sufficient statistic for his information at time t is v_t. For any $t \in [0, T]$ and $j \in [t, T]$, we therefore have in equilibrium:

$$S_t = E_t[S_j | v_t] \tag{3.5}$$

At time T, the payoff $A + \sigma^N B_T^N$ of the stock is known, so

$$S_T = A + \sigma^N B_T^N \tag{3.6}$$

Setting $j = T$ in (3.5) and substituting,

$$S_t = E_t[A + \sigma^N B_T^N | v_t]$$

At any time $t \in [0, T]$, the market clearing price S_t, at which the net demand from the representative investor equals the net supply of the stock, is equal to the conditional expectation of the project's payoff. Using (3.3),

$$S_t = E_t\Big[v_t + \int_t^T \sigma_s^U dB_s^U | v_t\Big] = v_t \tag{3.7}$$

Applying this equality to (3.2) and (3.3) respectively yields the time 0 and time t stock prices:

$$S_0 = A - \int_0^T \sigma_s^U dB_s^U \tag{3.8}$$

$$S_t = A + \sigma^N B_t^N - \int_t^T \sigma_s^U dB_s^U \tag{3.9}$$

Combining (3.8) and (3.9) gives another expression for S_t:

$$S_t = S_0 + \sigma^N B_t^N + \int_0^t \sigma_s^U dB_s^U \tag{3.10}$$

For $t = T$,

$$S_T = S_0 + \sigma^N B_T^N + \int_0^T \sigma_s^U dB_s^U$$

Finally, the dynamics of S_t may be derived from either (3.9) or (3.10):

$$dS_t = \sigma^N dB_t^N + \sigma_t^U dB_t^U \tag{3.11}$$

This deserves clarification. The investor does not know the value of A, and at time t he does not observe any of the different terms on the right-hand-side of (3.9). Instead, at any time t he observes v_t, and sets the stock price S_t according to this information. Equation (3.9) shows how information asymmetries fade away as t approaches T. It is also how the manager, who knows the value of A, perceives the evolution of the stock price process. Loosely speaking, this method of unwinding a Brownian motion allows to use the Brownian motion B_t^U to *reduce* uncertainty, from the investor's perspective.

The tradeoff is apparent: as time passes, there is more noise in the stock price process, but less information asymmetries. In other words, the higher t is, the more S_t differs from A because of the accumulated noise, but the less S_t differs from A because of information asymmetries. A manager compensated at time τ is thus exposed to two sources of risk: the accumulated noise from time 0 to time τ on the one hand, and the remaining information asymmetries at time τ on the other hand (the higher τ, the less information asymmetries remain). The longer the time horizon of the project, the more the manager is exposed to risk. In this sense, longer-term projects are more costly in terms of agency costs.[12] This will be crucial in the next section.

3.1.2 The optimal contract

The founder designs the compensation contract $\{w, \tau, b\}$ so as to maximize his objective function in (3.1) conditional on the manager accepting the contract and choosing the efficient project. If these two constraints are met, the objective function of the founder rewrites as

$$E_{-1}\left[\bar{A} - (w + bS_\tau)|A = \bar{A}\right] = \bar{A} - \left(w + bE_{-1}[S_\tau|A = \bar{A}]\right) \tag{3.12}$$

The value of $\bar{A}$ being given, the objective of the founder reduces to minimizing the expected cost of the manager's pay, subject to the participation constraint and the incentive constraint defined below. We begin by setting b and w so as to bind these two constraints. Then we optimize over the compensation date, τ.

In a CARA-normal framework with a contract linear in the outcome, the expected utility of pay is fully described by the expected pay and the variance of pay. The variance of the manager's pay given his information at time -1, which includes the value of A and contract parameters, is obtained from (3.9):

$$var_{-1}[bS_\tau|A] = b^2\left(\sigma^{N^2}\tau + \int_\tau^T \sigma_t^{U^2} dt\right) \tag{3.13}$$

This expression is independent of A.

The manager will therefore opt for the efficient project if and only if his expected pay (including private benefits of control) is higher with the efficient project than with the inefficient project. The incentive constraint is

$$w + b\bar{A} \geq w + b\underline{A} + P \tag{3.14}$$

Or, since it is binding in equilibrium[13], it gives the second-best optimal pay-performance sensitivity, denoted by $b^\star$:

$$b^\star = \frac{P}{\bar{A} - \underline{A}} \tag{3.15}$$

[12]Shleifer and Vishny (1990) provide another reason, related to financial arbitrage, as to why long-term projects are prone to greater mispricing than short-term projects.

[13]If it is not binding, then make it binding by decreasing b, and adjust w so that the manager's expected pay is unchanged. The founder is therefore indifferent between these two schemes since they are characterized by the same expected cost. But the variance of the manager's pay is lower with the new scheme. It follows that w can be reduced until the manager reaches his reservation utility, which reduces the expected cost of the pay scheme. This shows that a nonbinding incentive constraint cannot be optimal.

The fixed wage w is set to bind the participation constraint. Using the certainty equivalent approach, the participation constraint is[14]

$$w + bE_{-1}[S_\tau | A = \bar{A}] - \frac{\gamma}{2} b^2 var_{-1}[S_\tau | A = \bar{A}] \geq \bar{W} \tag{3.16}$$

This gives the equilibrium value of the fixed wage at the second-best, denoted by $w^\star$:

$$w^\star = \bar{W} - b^\star \bar{A} + \frac{\gamma}{2} b^2 \Big(\sigma^{N^2} \tau + \int_\tau^T \sigma_t^{U^2} dt \Big) \tag{3.17}$$

In equilibrium, the expected cost of the contract $\{w^\star, \tau, b^\star\}$ is equal to

$$w^\star + E_{-1}[b^\star S_\tau | A = \bar{A}]$$

Substituting for the value of $w^\star$ from (3.17), the expected cost of compensation is

$$\bar{W} + \frac{\gamma}{2} b^{\star 2} \Big(\sigma^{N^2} \tau + \int_\tau^T \sigma_t^{U^2} dt \Big) \tag{3.18}$$

It decomposes as the reservation wage plus the agency cost, which is proportional to the variance of the manager's pay. The objective of the founder consists in minimizing the expected cost of the contract. Since the fixed wage $w^\star$ and the pay-performance sensitivity $b^\star$ are already set, this reduces to choosing the compensation date τ in order to minimize the variance of the manager's pay, which is proportional to the (ex-ante) variance of the time τ stock price.

The agency cost can be eliminated in three special cases. If there are no information asymmetries ($\sigma_t^U \equiv 0$), full hedging against B_t^N is possible: compensation is then contingent on S_0 ($\tau = 0$), and the manager does not carry any risk. In the absence of noise ($\sigma^N = 0$), it is costless to wait for information asymmetries to unwind: compensation is then contingent on S_T ($\tau = T$), and the manager does not carry any risk either. Lastly, if the manager is risk neutral ($\gamma = 0$), the timing of payments does not matter. If one of these three cases does not apply, the problem is nontrivial.

In view of (3.18), the problem of the founder can be written as

$$\min_\tau \frac{\gamma}{2} b^{\star 2} \Big(\sigma^{N^2} \tau + \int_\tau^T \sigma_t^{U^2} dt \Big) \qquad \text{where} \qquad b^\star = \frac{P}{\bar{A} - \underline{A}} \tag{3.19}$$

Then we have the following result:

Proposition 1: The second-best optimal timing of compensation, $\tau^\star$, is as follows:

- If $\sigma_t^U > \sigma^N$ for all t, then $\tau^\star = T$.

[14]It should be clear from this equation that, in equilibrium, the expected pay (the first two terms on the left-hand-side of (3.16)) is larger than the certainty equivalent (the left-hand-side of (3.16)).

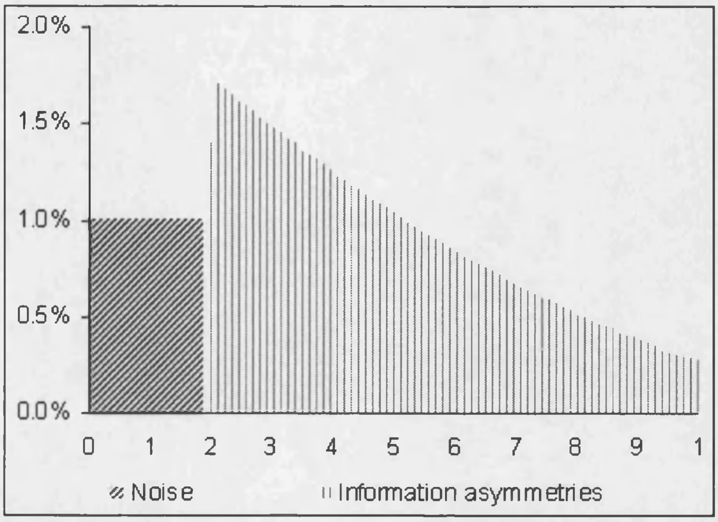

Figure 3.1: A suboptimal timing: the manager is paid at time 2.

- If $\sigma_t^U < \sigma^N$ for all t, then $\tau^\star = 0$.
- If none of the above applies and σ_t^U is increasing in t, $\tau^\star = 0$ if $\int_0^T {\sigma_t^U}^2 dt < {\sigma^N}^2 T$, and $\tau^\star = T$ if $\int_0^T {\sigma_t^U}^2 dt > {\sigma^N}^2 T$.
- If none of the above applies and σ_t^U is decreasing in t, then $\tau^\star$ is implicitly defined by $\sigma_{\tau^\star}^U = \sigma^N$.

The proof is in the Appendix. We call $\{w^\star, \tau^\star, b^\star\}$ the second-best optimal contract parameters. Given the complexity of the model, it is remarkable that the rate at which noise accumulates and the rate at which information asymmetries are resolved fully determine the optimal timing of compensation. Paying the manager early hedges him against the noise which accumulates in the stock price. Paying the manager late ensures that most of his private information has been incorporated in the stock price, and so hedges him against mispricing attributable to information asymmetries. As a rule, the stronger the noise is relative to information asymmetries, the earlier the manager should be paid. In view of (3.6), it is noteworthy that paying the manager at time T is equivalent to using an accounting-based measure of performance, the payoff of the project, rather a stock-based measure of performance.

Figures 1 and 2 illustrate these arguments. Suppose that $\sigma^N = 10\%$, and the process σ^U decreases linearly from a level of 15% at time 0, to a level of 5% at time 10. In figure 1, the manager is paid based on the time 2 stock price. The sum of the two areas in figure 1 gives the

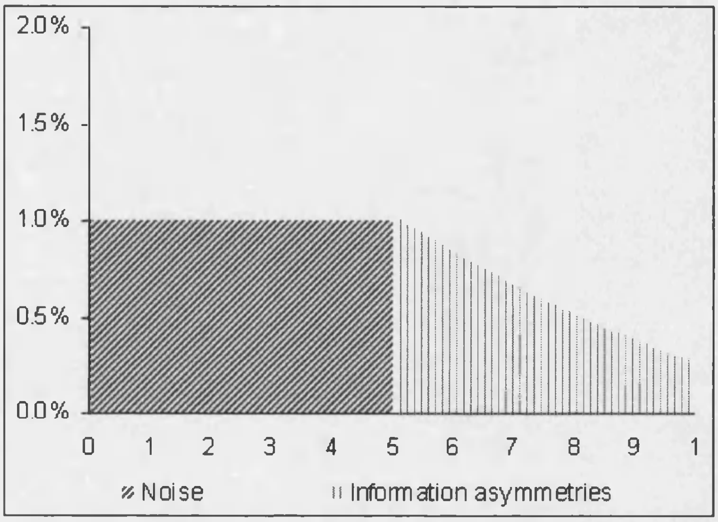

Figure 3.2: An optimal timing: the manager is paid at time 5.

variance of the time 2 stock price on which the pay of the manager is based, from the manager's ex ante perspective. This is precisely the criterion which is minimized at the optimum, as can be seen in (3.19). The diagonally hashed area is the part of the stock price variance attributable to noise, which accumulates from time 0 to time 2, while the vertically hashed area is the part of the stock price variance due to information asymmetries which are not resolved yet at time 2. Clearly, it is possible to reduce the variance of the manager's pay by paying him based on the time 5 stock price rather than the time 2 stock price, as shown in figure 2. More noise will then accumulate, which increases the variance of his pay, and more information asymmetries will be resolved, which decreases the variance of his pay. But the second effect will dominate, since $\sigma_t^U > \sigma^N$ from $t = 2$ to $t = 5$.

3.2 Optimal timing with moral hazard and short-termism

The model is now extended to allow for managerial short-termism. We explore both possible types of short-termism in turn, and we emphasize how the second-best outcome of the previous section is generally not robust to enlarging the action set of the manager. We then show how the optimal timing and power of compensation should be altered under third-best efficiency, which allows for short-termism.

3.2.1 Short-termism of the first type

Assume that the manager can either invest in one of the two projects already described, or in an inefficient short-term project with expected payoff equal to $\underline{A}$ and time horizon T. This inefficient project does not confer any private benefits of control, but a manager who selects it can manipulate the signal process at a cost, so that it indicates that he has invested in a very high payoff project. In this context, the manager may be tempted to incur some costs so as to enjoy the high pay associated with a big expected payoff in the future.[15]

Formally, the time T payoff of the inefficient short-term project is $\underline{A} + \sigma^N B_T^N$. A manager who chooses this project incurs a monetary cost $\frac{c}{2}(s+\epsilon)^2$ to manipulate the signal process up to a given time s, where $c > 0$ and $\epsilon \geq 0$. Manipulating the signal process up to time s ensures that the signal process v_t is centered at $A = \hat{A} > \bar{A}$ from time 0 to time s, and drops at $A = \underline{A}$ at time s:[16]

$$v_t = \hat{A} + \sigma^N B_t^N - \int_t^T \sigma_z^U dB_z^U$$

for $t \leq s$, and

$$v_t = \underline{A} + \sigma^N B_t^N - \int_t^T \sigma_z^U dB_z^U$$

for $t > s$. We assume that the difference $\hat{A} - \bar{A}$, as well as c and ϵ, are common knowledge, so that the investor is able to assess ex ante whether it is in the manager's interests to invest in the inefficient short-term project given contract parameters.

Suppose for the moment that the investor believes that the manager opts for the efficient project. If the manager is compensated at time τ, the condition that must be satisfied for this belief to be rational is

$$w + b\bar{A} \geq w + b\hat{A} - \frac{c}{2}(\tau + \epsilon)^2$$

The left-hand side is the expected pay of a manager who invests in the efficient project, while the right-hand side is the expected pay net of costs of a manager who invests in the inefficient short-term project and manipulates the signal process up to the time τ when he is compensated. Since both projects are associated with the same variance of pay, it can be omitted in the condition

[15]Of course, the investor will not be misled in equilibrium. But should it be in the manager's interests to manipulate the signal process, he will do it, which will be anticipated by investors. The manager will therefore not derive any rent in equilibrium, and the outcome will be inefficient. It should be noted that in these circumstances, the manager would be worse off not manipulating the signal process.

[16]This jump would in theory make it possible to set up a mechanism whereby the manager is punished in the event when such a jump is observed. However, we rule this out in our model, since we want to capture the manipulation of signals, which in practice never end so abruptly as to be contractible - not to mention the fact that jumps in a stock price process may occur for other unrelated reasons.

above. This condition is rewritten as

$$\tau > \sqrt{2b\frac{\hat{A}-\bar{A}}{c}} - \epsilon \equiv \underline{\tau} \tag{3.20}$$

If this condition is satisfied, the manager will not indulge in short-termism of the first type, and the investor's expectation that he will invest in the efficient project will be correct.[17]

However, if the optimal timing of compensation $\tau^\star$ defined in Proposition 1 is smaller than $\underline{\tau}$, a manager compensated at time $\tau^\star$ will select the inefficient short-term project rather than the efficient project: the second-best outcome described in the previous section is not feasible anymore. The inequality in (3.20) highlights the three possible responses to short-termism. First and foremost, it may be optimal to deviate from the optimal timing rule by postponing compensation and choosing $\tau = \underline{\tau}$, to ensure that the efficient project is chosen rather than the short-term inefficient project. Notice that when noise is sufficiently small relative to information asymmetries, the optimal timing rule yields $\tau = T$. In this case, the condition in (3.20) will be satisfied at the second-best optimal timing rule for a larger set of parameter values. Second, it would be desirable to reduce the pay-performance sensitivity b, which fuels short-termism. This is admittedly impossible in this setting, since any diminution in b would result in the manager opting for the other inefficient project, which provides private benefits of control. A third response consists in making it more costly to manipulate the signal process, by raising c or ϵ. Increased auditing, or intense scrutiny from independent equity research analysts, could help.

We now show that when the short-termism problem is very severe, it is optimal to provide muted incentives.

Proposition 2: When $\hat{A}$ is sufficiently high, or when c and ϵ are sufficiently low, it is optimal to set $b = 0$.

For some parameter values, it may be too costly to provide incentives for the manager to pick the efficient project rather than the inefficient short-term project. The next best option (third-best) is then to induce him to take the inefficient project with private benefits of control. This is achieved by giving him only a fixed wage, which ensures that he will prefer this project to the inefficient short-term project.[18]

[17] Although the proof is omitted, it is easy to see that investing in the efficient project is a dominant strategy for the manager if this condition is satisfied, while investing in the inefficient short-term project is a dominant strategy if this condition is *not* satisfied.

[18] It is always better to induce the manager to take the inefficient project with private benefits of control

3.2.2 Short-termism of the second type

Now assume that the manager can either invest in the inefficient project with private benefits of control already described in the baseline model, or invest in a project with a "time horizon" of r, $0 \leq r \leq T$ and expected payoff $\bar{A}_r$. As we explain below, the time horizon only denotes the time at which information asymmetries are fully resolved – the payoff is still realized at time T, exactly as before.

To introduce a potential conflict of interest between the founder and the manager, we need two elements. First, we assume that the expected payoff $\bar{A}_r$ is increasing in the time horizon r (the inefficient project with private benefits of control is still characterized by an expected payoff of $\underline{A}$). For simplicity, assume a linear relationship: for a given $\mu > 0$ and any $r \in [0, T]$,

$$\bar{A}_r = \bar{A} - \mu(T - r) \tag{3.21}$$

Second, from the manager's ex ante perspective, the variability of the stock price due to information asymmetries is increasing in the time horizon r of the project. Formally, if the manager chooses a project with time horizon r and expected payoff $\bar{A}_r$, we assume that the signal process is described by:

$$v_t = \bar{A}_r + \sigma^N B_t^N - \int_t^r \sigma_s^U dB_s^U \tag{3.22}$$

for $t \leq r$, and

$$v_t = \bar{A}_r + \sigma^N B_t^N \tag{3.23}$$

for $t > r$ (if the manager opts for the inefficient project with private benefits of control, the signal process is still described in (3.2) and (3.3), as in the preceding section.[19]) The manager effectively chooses when information asymmetries vanish. Notice that the project with $r = T$ is the efficient project of the baseline model. Following the same steps as in section 1, we get

$$S_t = v_t$$

for any $t \in [0, T]$.

In the baseline model, the only alternative to the inefficient project with private benefits of control is the efficient project. Now, as an alternative to the inefficient project with private benefits of control, the manager may choose among a set of projects with different expected returns and time horizons (where the time horizon determines the time at which information

rather than the inefficient short-term project, since their final payoffs have the same distribution, but the former provides private benefits of control - which relaxes the participation constraint, and enables a diminution of the fixed wage.

[19]Equivalently, the signal he receives in this case is generated by the signal process described in (3.22) and (3.23), except that $\bar{A}_r$ is replaced by $\underline{A}$ and r is replaced by T.

asymmetries are fully resolved). At one end of the spectrum (a low r), some projects have relatively low expected returns but are less plagued by information asymmetries. One example is for the manager to reinvest in existing lines of business. At the other end of the spectrum (a high r), some projects have higher expected returns but are characterized by strong information asymmetries. One example is for the manager to launch a brand new project involving lots of changes which are hard to apprehend for outside investors.

We are now going to analyze the manager's problem, given a compensation contract. First, we show in the Appendix that, conditional on the manager not choosing the inefficient project with private benefits of control, $r \geq \tau$ at the optimum: the manager optimally chooses a time horizon r higher than the time τ at which he is paid. Intuitively, as long as $r < \tau$, the manager's expected compensation is an increasing function of r, while his exposure to noise is independent of r, and he is not exposed at all to any variance of pay emanating from information asymmetries. Second, given parameters $\{w, \tau, b\}$ of the compensation contract, the manager's certainty equivalent wealth of choosing the project with a time horizon of $r \geq \tau$ is

$$CE(r) = w + b\bar{A}_r - \frac{\gamma}{2}b^2\left({\sigma^N}^2\tau + \int_\tau^r {\sigma_s^U}^2 ds\right) \tag{3.24}$$

where $\bar{A}_r$ is defined in (3.21). Denote by $r^\star$ the time horizon that maximizes the manager's certainty equivalent in (3.24), given b and the compensation time τ. The manager clearly faces a tradeoff. On the one hand, shortening the project's time horizon r diminishes the expected payoff of the project, which diminishes his expected pay. On the other hand, shortening the project's time horizon r reduces the variability of the signal process at any time t, which reduces the variance of his stock-based pay.

We say that there is no short-termism problem if and only if, given the second-best optimal timing $\tau^\star$, $r^\star = T$. Contract parameters are then as in the previous section: the efficient timing of compensation is robust to short-termism of the second type. In particular, this is always the case when the second-best optimal timing is $\tau^\star = T$, since we know that $r^\star \geq \tau$.

The problem of a manager being given a compensation contract not necessarily robust to short-termism is solved in the Appendix. For the manager to select a project with a long time horizon r, superior expected returns must compensate him for the extra risk borne. More precisely, the manager tends to optimally select $r^\star < T$ when μ and the compensation time τ are low, and when his risk aversion γ and the pay-performance sensitivity b are high. Indeed, (i) the higher μ is, the more a longer time horizon project is valuable in terms of expected payoffs, (ii) the higher the compensation date τ, the less information asymmetries remain to be eliminated by shortening the project's time horizon r. However, (iii) the higher the manager's risk aversion γ, the more expected pay he is willing to forgo to reduce the variance of his pay, and

(iv) the higher b, the more the manager's pay is variable, and the more risky his remuneration is (remember that he may decrease his risk by shortening the project's time horizon τ).[20] It follows that short-termism of the second-type can be prevented by altering the values of τ and b,[21] at the cost of a deviation from the second-best optimal contract parameters $\{w^\star, \tau^\star, b^\star\}$. The following propositions makes these results formal.

Just as with short-termism of the first-type, we begin by showing that for certain parameter values, implementing the efficient project is so costly that it is optimal not to provide any incentives to the manager.

Proposition 3: When $\tau^\star \neq T$, $\mu > \frac{\bar{A}-\underline{A}}{T}$, σ^N and σ_t^U are bounded away from zero for all t, and γ is sufficiently high, then it is optimal to set $b = 0$.

First, when shocks are substantial, it may be too costly to make the pay of a very risk averse manager sufficiently sensitive to the stock price so that he picks the efficient project with a long horizon rather a less risky project with a shorter time horizon. Second, when $\mu > \frac{\bar{A}-\underline{A}}{T}$, the expected payoff of a project with a time horizon of $\tau = 0$ is even lower than $\underline{A}$. Proposition 3 implies that, with a very risk averse manager and a sufficiently high μ, it is more efficient to induce the manager to pick the inefficient project with private benefits of control, whose payoff is $\underline{A}$. This is achieved by giving him only a fixed wage.

For any set of parameter values, the following result holds:

Proposition 4: With potential short-termism of the second type, the pay-performance sensitivity b is lower than $b^\star$, and the optimal compensation date τ is lower than $\tau^\star$.

First, the model predicts that short-termism of the second type tends to lower the pay-performance sensitivity. Second, the manager tends to be paid later than at the second-best. With potential short-termism of the second-type, it is *never* optimal for the founder to pay the manager earlier than $\tau^\star$. This result is all the more remarkable that it applies to all four cases distinguished in Proposition 1. On the contrary, it is sometimes optimal for the founder to postpone the compensation date, depending on parameter values. Postponing compensation is all the more valuable that μ is high, so that short-termism of the second-type greatly lowers

[20]For a small b, the manager mostly cares about his expected remuneration, not about the variance of his remuneration – and conversely.

[21]Other options not modelled in the paper include: shrinking the project set available to the manager, hiring a manager will low risk aversion, and altering the values of the parameters μ, σ^N, σ^U.

the firm's expected value, and that γ, σ^N, σ_t^U and $b^\star$ are low, so that it is not very costly to compensate the manager for bearing extra risk.

Information asymmetries are directly responsible for short-termism of the second type. However, the best cure against both moral hazard and short-termism may paradoxically be to reduce the amount of noise which accumulates in the performance measure. Indeed, in the baseline model, more information asymmetries result in a more delayed compensation (a higher $\tau^\star$), while more noise results in earlier compensation. The latter is more problematic because it renders the prevention of short-termism more difficult. If there were no noise, the second-best optimal timing of compensation would be $\tau^\star = T$, and short-termism would not occur under the second-best optimal contract parameters. Diminishing σ^N is therefore doubly worthwhile in the presence of potential short-termism. Not only does it lower the agency cost, but it also mitigates the short-termism problem. The best remedy for short-termism of the second type could thus consist in filtering the noise out of the stock price process, so as to obtain a precise performance measure (relative to the manager's choice of A) at time T. This could be achieved with relative performance evaluation and the hedging of risks at the corporate level.

3.3 Extensions

3.3.1 Continuous remuneration

We now relax the constraint that contingent remuneration be concentrated at a single date. At any time t, $0 \leq t \leq T$, the manager earns $b_t S_t$. A setting with continuous pay enables us to analyze the optimal dynamics of the pay-performance sensitivity. This section begins by deriving qualitative results in the general case at the second-best, i.e., in the absence of short-termism. Then it reports quantitative results in a numerical example. Finally, it discusses how the second-best pay-performance sensitivity process should be altered when short-termism of the first or second type is an issue.

At the second-best, the problem of the principal is to minimize agency costs while ensuring that the manager accepts the contract (the participation constraint) and chooses the efficient project (the incentive constraint). As in section 1, the optimal contract therefore solves:

$$\min_{\{b_t\}} var_{-1}\left[\int_0^T b_t S_t dt\right]$$

$$\text{s.t.} \quad w + E_{-1}\left[\int_0^T b_t S_t dt\right] - \frac{\gamma}{2} var_{-1}\left[\int_0^T b_t S_t dt\right] \geq \bar{W} \quad \text{and} \quad \int_0^T b_t \bar{A} dt \geq \int_0^T b_t \underline{A} dt + P \tag{3.25}$$

In the same way as in the first section, the fixed wage adjusts to satisfy the participation constraint as an equality. Since it does not affect the variance of compensation, it does not affect agency costs. The project choice does not affect the variance of compensation either, which is therefore omitted from the incentive constraint. Calculations are relegated to the Appendix, where we derive this intuitive but important result:

Proposition 5: At any point t, if $\sigma_t^U > \sigma^N$, then b_t is locally increasing in t at the second-best optimum.

At any point in time, the pay-performance sensitivity process is increasing (respectively decreasing) if the stock price becomes more (resp. less) informative of the value of A. A corollary is that the pay-performance sensitivity will only be constant throughout time if $\sigma_t^U = \sigma^N$ for all t. If information asymmetries are always stronger than noise, so that $\sigma_t^U > \sigma^N$ for any t (note that this corresponds to the first case of the optimal timing rule of Proposition 1), then the pay-performance sensitivity monotonically rises with time. Conversely, if noise is always stronger than information asymmetries, the pay-performance sensitivity process monotonically declines with time.

Proposition 5 gives us the qualitative relationship at any time t between the pay-performance sensitivity b_t on the one hand, and the rate at which information asymmetries are resolved (σ_t^U) relative to the rate at which noise accumulates (σ^N) on the other hand. In any subinterval, if information asymmetries are resolved faster than noise accumulates, it is more efficient for later payments to be more sensitive to performance than earlier payments. This being said, a bang-bang solution which would consist in making pay contingent on performance only on a relatively more informative subset of $[0, T]$ is not optimal. In effect, we show in the proof of Proposition 5 that, at the second-best optimum,

$$b_t = \frac{\lambda}{2} \frac{1}{\sigma^{N2} t + \int_t^T \sigma_s^{U2} ds} \tag{3.26}$$

where λ is a constant. Equation (3.26) implies that b_t is never zero for any t, since both σ^N and σ_t^U are bounded from above. The intuition is that spreading the pay-performance sensitivity on the whole time interval makes "diversification" possible. In effect, suppose that all the pay-performance sensitivity is concentrated around, say, time t, where $0 < t < T$, then the manager is exclusively - but strongly - exposed to B^N from time 0 to time t, and to B^U from time t to T. This means that he does not have any exposure to B^N from time t to T, or to B^U from time 0 to t.[22]

[22]An heuristic argument shows that it is suboptimal for the pay-performance sensitivity (PPS) to be zero on

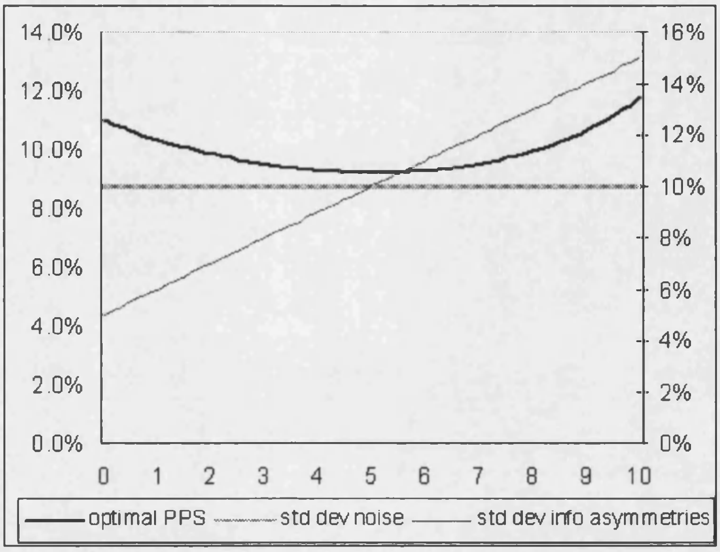

Figure 3.3: The optimal pay-performance sensitivity process.

In order to illustrate how the optimal pay-performance sensitivity process $\{b_t\}$ quantitatively depends on the $\{\sigma_t^U\}$ process, we now use a numerical example to derive the former for different forms of the latter. In all cases, the parameter values are $P = 1$, $\bar{A} = 2$, $\underline{A} = 1$, $T = 10$, and $\sigma^N = 10\%$. For these parameter values, figures 3 to 9 depict the optimal pay-performance sensitivity process $\{b_t\}$ for different $\{\sigma_t^U\}$ processes.[23]

If information asymmetries are mostly resolved late (that is, if σ_t^U is increasing in t), then the pay-performance sensitivity will tend to be decreasing in t for small t, and increasing in t for times close to T. That is, the pay-performance sensitivity will be high both around time 0 and time T, and low in the interim. This is displayed in figures 3 and 6 (note that this corresponds

any subinterval, as long as σ_t^U and σ^N are not infinite. Suppose that the PPS b_t is positive from, say, time k to time T, and is zero elsewhere. For the sake of the argument, assume that b_k is positive and bounded away from zero. Reduce the PPS by an arbitrarily small amount ε on the interval $[k, k+dt]$, and shift it to the interval $[k-dt, k]$. Assuming that σ_t^U is a process with a first derivative bounded from above and from below, the change in variance is equal to

$$\left([b_k-\varepsilon]^2-b_k^2\right)\left[{\sigma^N}^2 dt+{\sigma_k^U}^2 dt\right]+\varepsilon^2\left[{\sigma^N}^2 dt+{\sigma_{k-dt}^U}^2 dt\right] = -\varepsilon(2b_k-\varepsilon)\left[{\sigma^N}^2 dt+{\sigma_k^U}^2 dt\right]+\varepsilon^2\left[{\sigma^N}^2 dt+{\sigma_{k-dt}^U}^2 dt\right]$$

The first term is negative and of order ε, whether the second term is of order ε^2. Since ε is arbitrarily small, the second term is negligible in front of the first, and the change in variance is negative. The economic intuition is that the increase in variance as the weight b_t on a subinterval is increased from zero to a small amount is approximately zero.

[23]For any t, b_t is as in (3.26), where the value of λ is derived so that the incentive constraint in (3.25) is satisfied.

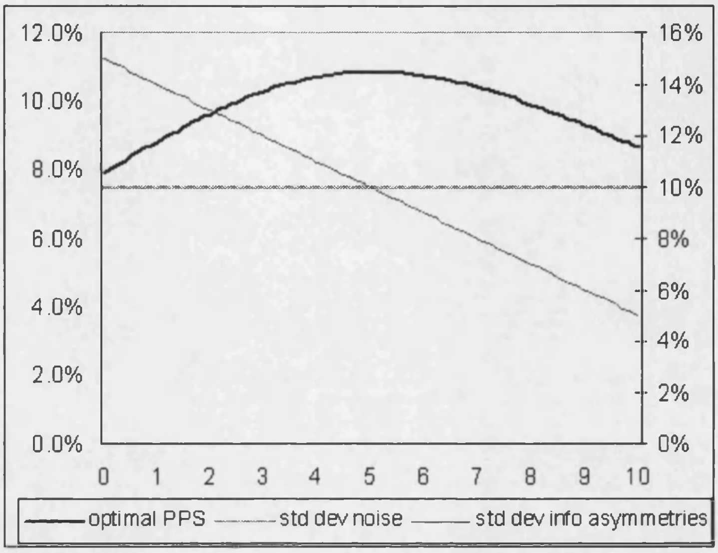

Figure 3.4: The optimal pay-performance sensitivity process.

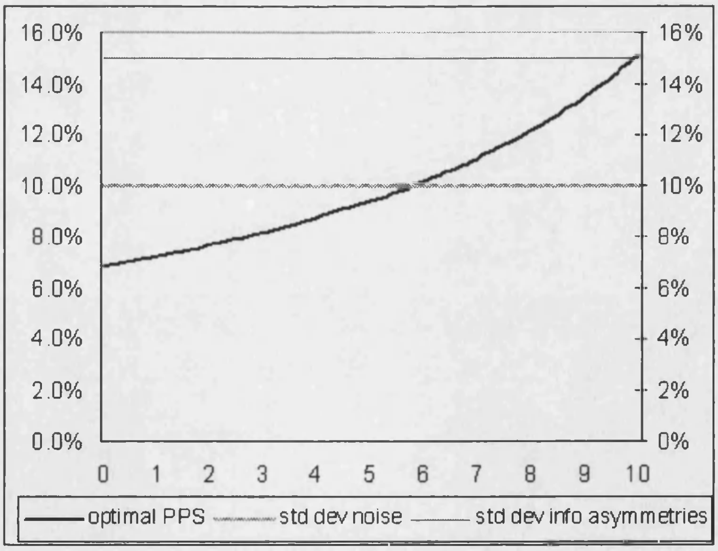

Figure 3.5: The optimal pay-performance sensitivity process.

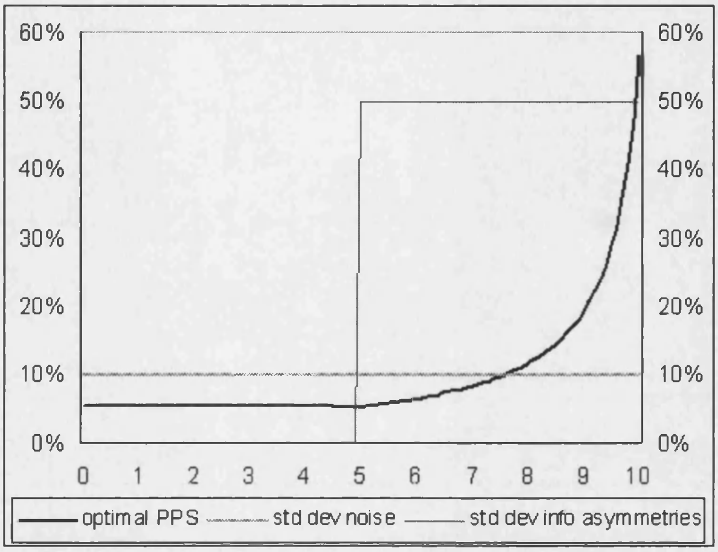

Figure 3.6: The optimal pay-performance sensitivity process.

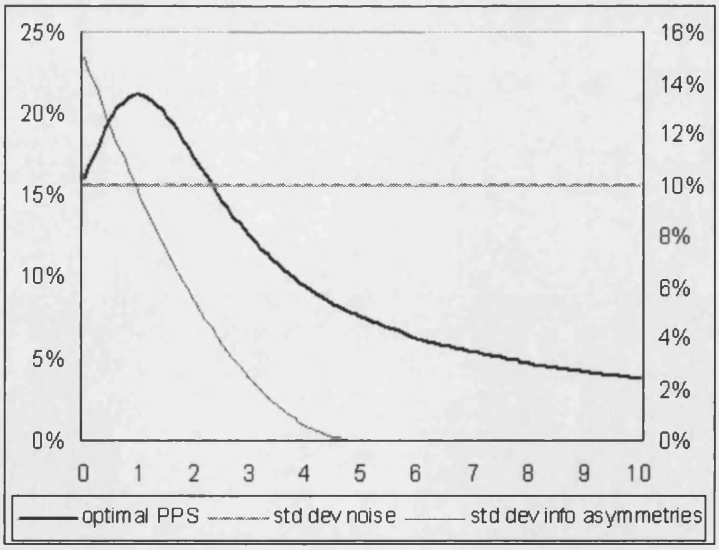

Figure 3.7: The optimal pay-performance sensitivity process.

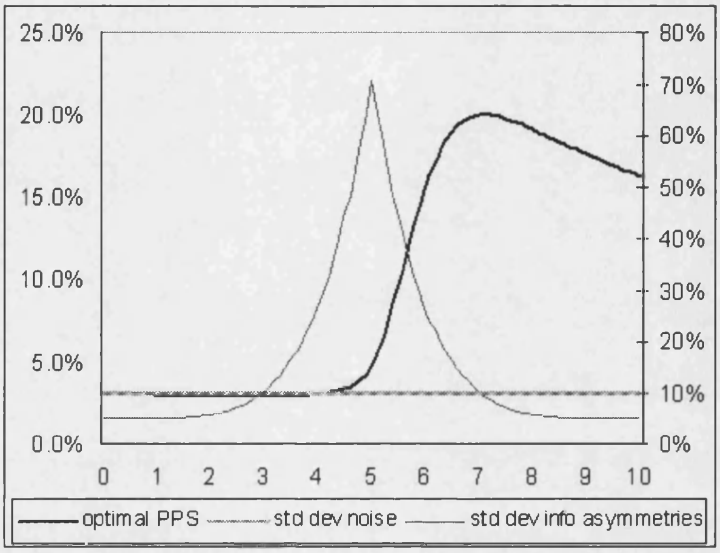

Figure 3.8: The optimal pay-performance sensitivity process.

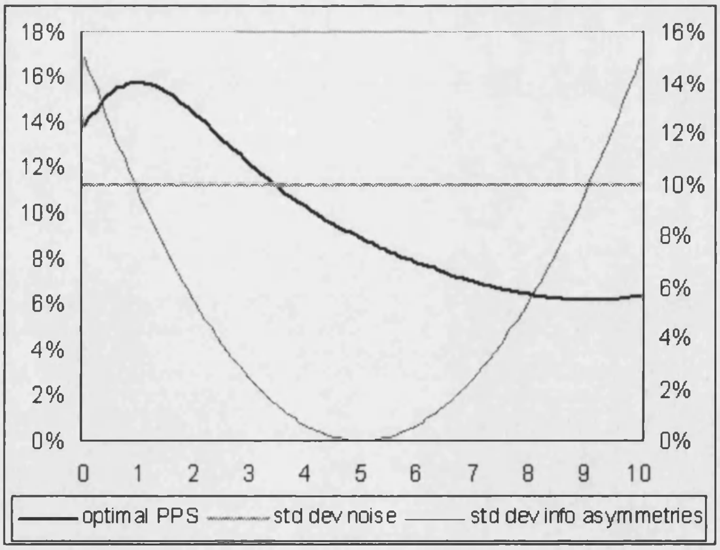

Figure 3.9: The optimal pay-performance sensitivity process.

to the third case of the optimal timing rule of Proposition 1).

If information asymmetries are mostly resolved early (that is, if σ_t^U is decreasing in t), then the pay-performance sensitivity will tend to be increasing in t for small t, and decreasing in t for times close to T. That is, the pay-performance sensitivity will be low at time 0 and at time T, and will reach a maximum at a time t in-between times 0 and T. This is displayed in figures 4 and 7 (note that this corresponds to the fourth case of the optimal timing rule of Proposition 1).

Finally, figure 8 displays the highly non-monotonic optimal pay-performance sensitivity process when information asymmetries are mostly resolved half-way through the project's life. In figure 9, half of information asymmetries is resolved quite early, and half is resolved quite late. In this case, it is optimal to pay the manager rather early, since almost half of information asymmetries are already resolved, while the accumulated noise is still quite small.

We now analyze modifications to the pay-performance sensitivity process caused by the potential for short-termism. For a given pay-performance sensitivity process $\{b_t\}$, the expected pay of a manager who invests in the efficient project is

$$w + \int_0^T b_t \bar{A} dt \tag{3.27}$$

With short-termism of the first type, and given that the investor believes that the efficient project is undertaken, the expected pay net of costs of a manager who invests in the short-term inefficient project and manipulates the signal process until time m is

$$w + \int_0^m b_t \hat{A} dt + \int_m^T b_t \underline{A} dt - \frac{c}{2}(m+\epsilon)^2 \tag{3.28}$$

With short-termism of the first type, the variance of pay is invariant to the project choice, and therefore irrelevant to determine the decision of the manager. Obviously, if (3.28) is larger than (3.27) for any $m \in (0,T]$, then the managerial contract is such that the manager will invest in the inefficient short-term project rather than in the efficient project, and the investor's beliefs are invalidated. In this case, discouraging investment in the short-term inefficient project involves a modification of the contract.

With short-termism of the second type, given a pay-performance sensitivity process b_t, the certainty equivalent wealth of a manager choosing a project with time-horizon r is

$$CE(r) = \int_0^T \Big(b_t \bar{A}_r - \frac{\gamma}{2} b_t^2 {\sigma^N}^2 t\Big) dt - \frac{\gamma}{2}\int_0^r b_t^2 \Big(\int_t^r {\sigma_s^U}^2 ds\Big) dt$$

If short-termism of the first or second type is a problem given the second-best optimal pay-performance sensitivity process, then this process may be altered in two ways to ensure that the manager does not choose a short-term project. First, the whole pay-performance sensitivity process may be adjusted downwards. With short-termism of the first type, this leaves the cost of manipulation unchanged, but reduces its benefits. With short-termism of the second type, this reduces the variance of pay more than the expected pay. Second, compensation may be postponed, by concentrating pay-performance sensitivities towards the end of the project's life. With short-termism of the first type, the manager must then manipulate the signal process for a long time to benefit from it, while the cost of manipulation is quadratic in time. With short-termism of the second-type, this potentially eliminates any variance of pay attributable to information asymmetries if $r^\star < T$, which in turn may make it optimal for the manager to raise the time horizon $r^\star$. Once again, the same measures which contribute to mitigating short-termism of the first type also contribute to mitigating short-termism of the second type. The following Propositions make these results formal.

Denote by $\{b_t'\}$ the second-best optimal pay-performance sensitivity process (when there are no opportunities for short-termism). Then short-termism of the first type may be overcome in two ways.

Proposition 6a: First, there exists a $k \in (0,1)$ such that, for every $t \in [0,T]$, setting $b_t = kb_t'$ ensures that there does not exist an $m \in [0,T]$ such that (3.28) is larger than (3.27). Second, when T is sufficiently large, there exists a transformation which is such that $b_t < b_t'$ for t lower than a given $q \in (0,T)$, $b_t > b_t'$ for t larger than a given $q \in (0,T)$, and $\int_0^T b_t dt = \int_0^T b_t' dt$, which ensures that, with the process $\{b_t\}$, there does not exist an $m \in [0,T]$ such that (3.28) is larger than (3.27).

Similarly, short-termism of the second type may be overcome in two ways.

Proposition 6b: First, for $k \in (0,1)$, setting $b_t = kb_t'$ ensures that the manager chooses an $r^\star$ larger than the $r^\star$ corresponding to the $\{b_t'\}$ process. Second, if σ_t^U is increasing in t, given the $r^\star$ corresponding to the $\{b_t'\}$ process, setting $b_t = 0$ for all $t < q$, where $q \in (r^\star, T)$ and is sufficiently large, and $b_t = \frac{1}{T-q} \int_0^T b_s' ds$ for all $t > q$ ensures that the manager chooses an $r^\star$ larger than the $r^\star$ corresponding to the $\{b_t'\}$ process.

Lastly, we show with two numerical examples that the two types of short-termism tend to become problematic in different circumstances.

Consider a setting with the same parameters values as in the numerical example above, and $\hat{A} = 5$, $c = 4$, and $\epsilon = 0.5$. Then short-termism of the first type imposes a modification to the optimal pay-performance sensitivity process in the case displayed in figure 7,[24] (lest the manager invests in the inefficient short-term project and manipulates the signal process for 1.1 years). On the contrary, the optimal pay-performance sensitivity process displayed in figure 6,[25] needs not be altered. In the former case, noise is relatively strong for most of the time interval, so that the pay-performance sensitivity is high early on, and then decreasing. This makes an initial manipulation of the signal process especially appealing. In short, the optimal second-best pay-performance sensitivity process is all the less robust to short-termism of the first type that information asymmetries are resolved very early and noise is relatively strong for the rest of the time interval.

However, short-termism of the second type tends to be problematic when short-termism of the first type is not, and conversely. Thus, with the same parameters as above, $\gamma = 5$ and $\mu = 10\%$, the optimal time horizon from the manager's perspective with the optimal pay-performance sensitivity process derived in the absence of short-termism is unchanged in the aforementioned case displayed in figure 7, but falls to 5.2 years in the case displayed in figure 6. In the latter case, information asymmetries are relatively strong and mostly resolved at the end of the time interval. Since choosing a project with a shorter time horizon, say r, eliminates the risk emanating from information asymmetries which are in principle realized from time r to time T, it is precisely appealing when they are strong in this time interval. In short, short-termism of the second type tends to be more problematic when information asymmetries are resolved late and are then strong relative to noise.

3.3.2 Multiplicative shocks

Although a model with multiplicative shocks soon becomes untractable[26], it delivers some interesting and counterintuitive lessons regarding short-termism of the second type. The model is derived along the same lines as in section 1, except that shocks are multiplicative instead of being additive.

Let the time T payoff have an unconditional mean of A (where $A = \bar{A}$ if the efficient project is undertaken, and $A = \underline{A}$ otherwise), and be such that the accumulation of noise follows a geometric Brownian motion. For any t in $[0, T]$, define the process π_t by

$$\pi_0 = A$$

$$d\pi_t = \sigma^N \pi_t dB_t^N$$

[24]When $\sigma_t^U = 0.006(5-t)^2$ for $t \in [0,5]$, and $\sigma_t^U = 0$ for $t \in [5,10]$.

[25]When $\sigma_t^U = 0$ for $t \in [0,5]$, and $\sigma_t^U = 50\%$ for $t \in [5,10]$

[26]Notably because projects with different expected payoffs also have different variances. This renders the incentive constraint very convoluted.

For a realization B_T^N of the Brownian motion, the time T payoff is therefore equal to

$$\pi_T = A\,exp\bigg\{-\frac{1}{2}\sigma^{N^2}T + \sigma^N B_T^N\bigg\}$$

Let the signal v_t, received by the investor at time t, be

$$v_t = A\,exp\bigg\{-\frac{1}{2}\Big(\sigma^{N^2}t - \int_t^T \sigma_s^{U^2}ds\Big) + \sigma^N B_t^N - \int_t^T \sigma_s^U dB_s^U\bigg\}$$

As in section 1, two conditions must be satisfied by the share price process. First, in the absence of arbitrage opportunities, the share price at time T must be equal to the payoff of the project:

$$S_T = A\,exp\bigg\{-\frac{1}{2}\sigma^{N^2}T + \sigma^N B_T^N\bigg\} \tag{3.29}$$

Second, because of the assumption of risk-neutrality and of a zero discount factor, the time t share price must be equal to the investor's conditional expectation as of time t of the final payoff:

$$S_t = E_t[S_T|v_t]$$

As in section 1, this implies that

$$S_t = v_t$$

The share price process therefore writes as

$$S_t = A\,exp\bigg\{-\frac{1}{2}\Big(\sigma^{N^2}t - \int_t^T \sigma_s^{U^2}ds\Big) + \sigma^N B_t^N - \int_t^T \sigma_s^U dB_s^U\bigg\} \tag{3.30}$$

This immediately gives the time 0 share price:

$$S_0 = A\,exp\bigg\{\frac{1}{2}\int_0^T \sigma_s^{U^2}ds - \int_0^T \sigma_s^U dB_s^U\bigg\}$$

Combining this expression with (3.30), the time t share price also writes as

$$S_t = S_0\,exp\bigg\{-\frac{1}{2}\Big(\sigma^{N^2}t + \int_0^t \sigma_s^{U^2}ds\Big) + \sigma^N B_t^N + \int_0^t \sigma_s^U dB_s^U\bigg\} \tag{3.31}$$

Either (3.30) or (3.31) gives the dynamics of S_t:

$$dS_t = \sigma^N S_t dB_t^N + \sigma_t^U S_t dB_t^U \tag{3.32}$$

The share price follows a multidimensional geometric Brownian motion with zero drift.

The optimal timing rule at the second-best is as in section 1. However, multiplicative shocks have other implications for addressing short-termism of the second type. Assume that the manager may select the time horizon r of the project, such that the expected payoff as a function of r is given by (3.21), as in the previous section, and such that the investor receives a signal v_t at time t:

$$v_t = A\,exp\bigg\{-\frac{1}{2}\Big(\sigma^{N^2}t - \int_t^r \sigma_s^{U^2}ds\Big) + \sigma^N B_t^N - \int_t^r \sigma_s^U dB_s^U\bigg\}$$

for $t \leq r$ and

$$v_t = A\, exp\bigg\{ -\frac{1}{2}{\sigma^N}^2 t + \sigma^N B_t^N \bigg\}$$

for $t > r$. Then we obtain the following result.

Proposition 7: When the manager may select the time horizon r of the project whose expected payoff is given by (3.21), then, for any time horizon $r^\star$, a sufficiently high value of σ^N induces the manager to choose an even lower value of $r^\star$. In addition, when σ^N is sufficiently large, increasing the compensation date τ results in a diminution of $r^\star$.

Consequently, when noise is strong and shocks are multiplicative, inducing the manager to pick a project with a long time horizon may best be achieved by paying him even earlier than at the second-best optimal timing rule. This is due to the structure of uncertainty. With additive shocks, the impact of information asymmetries on the variance of pay is independent of the amount of noise. With multiplicative shocks, the impact of a given amount of information asymmetries on the variance of pay is an increasing function of the noise which accumulates in the stock price until time τ. For a given r, a later compensation time τ increases the amount of noise, but reduces the amount of information asymmetries. If noise is sufficiently strong, it may then be worthwhile for the manager to reduce the amount of information asymmetries by diminishing r, an outcome which the principal prefers to avoid. When noise is strong, it may therefore be preferable to reduce the length over which noise builds up in the share price by lowering the compensation date τ.

3.4 Conclusion

The compensation of the manager should be contingent on the stock price process at the time which minimizes the variance of his pay from his ex ante perspective. Whereas information asymmetries between the manager and shareholders favor late contingent compensation, the fact that noise progressively accumulates in the performance measure favors early contingent compensation.

If the optimal timing rule derived with this framework is not robust to short-termism, the power of the incentive scheme should be lowered, and the compensation date should in most cases (but not always) be postponed. Even though it is necessary to provide incentives for the manager to opt for a project with high expected payoffs, it may even be optimal to provide muted incentives when the short-termism problem is very severe. These results hold for both types of short-termism. Finally, as long as investors are not naive, stock-based compensation remains optimal: even when the manager can manipulate the signals received by investors, it is generally not optimal to pay him based on the accounting-based measure of performance realized at the end of the project's life.

The model generates cross-sectional predictions. Firms whose managers do not have a lot of private information (think about firms with recurring investments, firms in traditional industries that investors easily comprehend) will pay managers early. Firms where information asymmetries are strong (think about firms with new projects, investment banks with opaque strategies, and firms in ascending industries that investors do not fully understand yet) will delay compensation more. Besides the industrial sector, other potential proxies for information asymmetries include the amount of R&D expenditures, the firm size, the firm age, the tenure of the current CEO, the market-to-book ratio (possibly with a nonlinear relationship), the volatility of profits, and the distribution of market shares (industries with a few leaders are typically more stable). Moreover, the model shows that firms which manage to produce an almost noise-free performance measure, whether because their profits are not significantly affected by noise, or because they can filter it out, will delay compensation more. This will mainly depend on the industrial sector of the firm.

Regarding short-termism, firms in which managers may easily manipulate signals of performance or choose the riskiness and time horizon of projects will tend to pay managers later, and use lower pay-performance sensitivities. Although more research would be needed on this dimension, this presumably includes firms subject to strong information asymmetries between management and investors. If this conjecture is correct, then more information asymmetries result in a latter compensation date not only for optimal hedging reasons, but also in order to prevent short-termism.

3.5 Appendix

The signal v_t is a sufficient statistic for $A + \sigma^N B_T^N$ at time t

The signal v_t, received at time t, is

$$v_t = A + \sigma^N B_T^N + \sigma^N (B_t^N - B_T^N) + \int_t^T \sigma_s^U dB_s^U$$

Similarly, the signal v_{t-u}, received at time $t-u$, for $0 < u \leq t$, is

$$v_{t-u} = A + \sigma^N B_T^N + \sigma^N (B_{t-u}^N - B_T^N) + \int_{t-u}^T \sigma_s^U dB_s^U$$

Therefore,

$$Pr(A + \sigma^N B_T^N | v_t, v_{t-u}) =$$

$$Pr\Big(A+\sigma^N B_T^N \Big| A+\sigma^N B_T^N+\sigma^N(B_t^N-B_T^N)+\int_t^T \sigma_s^U dB_s^U, A+\sigma^N B_T^N+\sigma^N(B_{t-u}^N-B_T^N)+\int_{t-u}^T \sigma_s^U dB_s^U\Big)$$

$$= Pr\Big(A+\sigma^N B_T^N \Big| A+\sigma^N B_T^N+N_t, A+\sigma^N B_T^N+N_t+\sigma^N(B_{t-u}^N-B_t^N)+\int_{t-u}^t \sigma_s^U dB_s^U\Big) = Pr(A+\sigma^N B_T^N|v_t)$$

where

$$N_t \equiv \sigma^N(B_{t-u}^N - B_T^N) + \int_{t-u}^T \sigma_s^U dB_s^U$$

This shows that v_t is a sufficient statistic for $A + \sigma^N B_T^N$.

Proof of Proposition 1:

First, consider the case where $\sigma_t^U > \sigma^N$ for all t. Then increasing any given compensation time $\tau \neq T$ by dt results in a change in the agency cost, as defined in (3.19), of

$$\frac{\gamma}{2}b^2(\sigma^{N^2}dt - \sigma_\tau^{U^2}dt)$$

which is negative, since $\sigma_t^U > \sigma^N$ for any t. Increasing the compensation time τ decreases the agency cost. This shows that the given compensation time $\tau \neq T$ was not optimal. Applying the same reasoning to any compensation $\tau \in [0, T)$ shows that the agency cost is monotonically decreasing in the compensation time τ. Therefore, at the optimum, $\tau = T$.

Second, the result that $\tau^\star = 0$ for the case where $\sigma_t^U < \sigma^N$ for all t is proved symmetrically.

Third, suppose that σ_t^U is increasing in t, and none of the former two cases apply. Define r by $\sigma_r^U \equiv \sigma^N$. Setting $\tau > r$ results in a lower agency cost than with $\tau = r$, since $\sigma_t^U > \sigma^N$ for any $t > r$ (the reasoning is as in the first case above). Setting $\tau < r$ results in a lower agency cost than with $\tau = r$, since $\sigma_t^U < \sigma^N$ for any $t < r$ (the reasoning is as in the second case above). The optimum is therefore given by a corner solution: either $\tau = 0$, or $\tau = T$. The agency cost is then respectively:

$$\frac{\gamma}{2}var_{-1}[bS_0|A] = \frac{\gamma}{2}b^2\int_0^T \sigma_t^{U^2}dt \tag{3.33}$$

$$\frac{\gamma}{2}var_{-1}[bS_T|A] = \frac{\gamma}{2}b^2\sigma^{N^2}T \tag{3.34}$$

Since the optimal timing of pay $\tau^\star$ minimizes the agency cost, it follows that $\tau^\star = 0$ if ${\sigma^N}^2 T > \int_0^T {\sigma_t^U}^2 dt$, and $\tau^\star = T$ if ${\sigma^N}^2 T < \int_0^T {\sigma_t^U}^2 dt$.

Fourth, suppose that σ_t^U is decreasing in t, and none of the first two cases apply - which implies that there exists a s, $0 \leq s \leq T$ such that $\sigma_s^U = \sigma^N$. Define r by $\sigma_r^U \equiv \sigma^N$. Setting $\tau > r$ results in a higher agency cost than with $\tau = r$, since $\sigma_t^U < \sigma^N$ for any $t > r$ (the reasoning is as in the second case above). Setting $\tau < r$ results in a higher agency cost than with $\tau = r$, since $\sigma_t^U > \sigma^N$ for any $t < r$ (the reasoning is as in the first case above). The optimum is therefore characterized by an interior solution:

$$\sigma_{\tau^\star}^U = \sigma^N \tag{3.35}$$

Proof of Proposition 2:

To ensure that the efficient project is preferred to the inefficient short-term project and to the inefficient project with private benefits of control, both (3.15) and (3.20) must hold, which requires that

$$\tau \geq \underline{\tau} = \sqrt{2\frac{\hat{A}-\bar{A}}{\bar{A}-\underline{A}}\frac{P}{c}} - \epsilon \tag{3.36}$$

For $\hat{A}$ sufficiently high, or c and ϵ sufficiently low, $\underline{\tau} > T$. It is consequently impossible to set contract parameters such that (3.36) hold, i.e., such that the manager chooses the efficient project. The choice is therefore between the inefficient project with private benefits of control, and the inefficient short-term project. Since both are characterized by the same expected payoff $\underline{A}$, the principal is indifferent on this dimension. Since they are also characterized by the same variance, it is impossible to set contract parameters b and τ so as to induce the manager to choose one rather the other. Finally, since the inefficient project with private benefits of control provides private benefits of control and the other does not, the manager will always choose the former, for any value of b and τ. Because the risk averse manager's pay does not need to be variable, the principal will set $b = 0$. In addition, to ensure that the participation constraint is satisfied, he will set the fixed wage at $w = \bar{W} - P$. With this contract, the manager will pick the inefficient project with private benefits of control.

Proof that $r \geq \tau$ with short-termism of the second type:

We show that a manager will never choose a project with a time horizon r less than τ. Indeed, if $r < \tau$, the certainty equivalent of the manager is

$$CE(r) = b\bar{A}_r - \frac{\gamma}{2}b^2{\sigma^N}^2\tau$$

If $r \geq \tau$, the certainty equivalent is

$$CE(r) = b\bar{A}_r - \frac{\gamma}{2}b^2\left({\sigma^N}^2\tau + \int_\tau^r {\sigma_s^U}^2 ds\right) \tag{3.37}$$

So that

$$CE(\tau) = b\bar{A}_\tau - \frac{\gamma}{2}b^2{\sigma^N}^2\tau \tag{3.38}$$

When $r < \tau$, then $\bar{A}_r < \bar{A}_\tau$ (since $\bar{A}_r$ is increasing in r) and $CE(r) < CE(\tau)$.

The manager's problem with short-termism of the second type

We investigate what happens when, given the second-best optimal timing $\tau^\star$, $r^\star$ is smaller than T, so that there is potentially a short-termism problem, and the second-best outcome described in the first section may not be feasible anymore.

The first-order condition of the manager's problem, which is obtained by taking the derivative of the certainty equivalent (3.24) with respect to the project's horizon r (with $r \geq \tau$), is

$$b\mu - \frac{\gamma}{2} b^2 {\sigma_r^U}^2 = 0 \tag{3.39}$$

Rearranging,

$$\sigma_r^U = \sqrt{\frac{2\mu}{\gamma b}} \tag{3.40}$$

The project horizon r thus obtained in this first-order condition maximizes the certainty equivalent $CE(r)$ if σ_t^U is increasing in t, and minimizes it if σ_t^U is decreasing in t.

If σ_t^U is increasing in t, the manager's certainty equivalent is concave in r, and the solution is interior, except in two cases. First, if the first-order condition (3.39) gives an r smaller than τ, this implies that $CE(r)$ is decreasing in r on $[\tau, T]$, so that $r^\star = \tau$ (since we know that $r^\star \geq \tau$). Second, if (3.39) gives an r larger than T, this implies that $CE(r)$ is increasing in r on $[0, T]$, so that $r^\star = T$, and there is no short-termism problem. Otherwise, the solution is interior, and the optimal time horizon $r^\star$ is increasing in μ, decreasing in γ, in b,[27] and in the amplitude of σ^U, as is apparent from (3.40).

If σ_t^U is decreasing in t, the manager's certainty equivalent is convex in r, and we have a corner solution. The manager chooses $r = T$ (respectively $r = \tau$) if the inequality below is verified (respectively is not verified):

$$b(\bar{A}_T - \bar{A}_\tau) > \frac{\gamma}{2} b^2 \int_\tau^T {\sigma_s^U}^2 ds$$

Substituting for the value of $\bar{A}_\tau$,

$$b\mu(T - \tau) > \frac{\gamma}{2} b^2 \int_\tau^T {\sigma_s^U}^2 ds \tag{3.41}$$

The manager will be all the more likely to choose $r = T$ that μ is high, that τ is high, that γ is low, that b is low, and that σ_t^U is low in the interval $[\tau, T]$ (the latter is all the more likely that in the case under investigation, σ_t^U is decreasing in t).

Proof of Proposition 3:

[27]In the first-order condition (3.39), the first term is proportional to b, while the second term is proportional to b^2. For b in $[0, \infty)$, maximizing the left-hand side of the equation (3.39) over b is achieved with b in the neighborhood of zero (but different from zero), in order for b to be arbitrarily larger than b^2.

We consider the cases where $\tau^\star \neq T$. The certainty equivalent wealth of a manager compensated at time τ who chooses the project with a time horizon of r ($r \geq \tau$) is

$$CE(r) = w + b\bar{A}_r - \frac{\gamma}{2}b^2\left(\sigma^{N^2}\tau + \int_\tau^r \sigma_s^{U^2} ds\right) \tag{3.42}$$

Assume that $b > 0$, σ^N and σ_t^U are bounded away from zero, and γ is sufficiently high. Then the certainty equivalent wealth of the manager in (3.42) is maximized by choosing $r^\star = \tau$. In equilibrium,

$$CE(r^\star) = w + b\bar{A}_\tau - \frac{\gamma}{2}b^2\sigma^{N^2}\tau \tag{3.43}$$

The objective function of the founder as a function of τ, given the optimal response of the manager ($r = \tau$) is

$$\bar{A}_\tau - \bar{W} - \frac{\gamma}{2}b^2\sigma^{N^2}\tau$$

We show by contradiction that in this case, setting $\tau > 0$ is suboptimal. Given this optimal response of the manager, a marginal change in the compensation time τ of $d\tau < 0$ has two effects for the founder. First, the expected payoff of the project changes by $\mu d\tau$, which is negative (since $d\tau < 0$). Second, the agency cost of compensation changes by $\frac{\gamma}{2}b^2\sigma^{N^2}d\tau$, which is negative. Since γ is sufficiently high, the second effect dominates the first one, and it is in the interests of the founder to marginally reduce τ. Since this result does not depend on τ, it holds for any $\tau \in (0, T]$. Therefore, since $\frac{\gamma}{2}b^2\sigma^{N^2} > \mu$ (which follows from the fact that γ is sufficiently large), then at the optimum $\tau = 0$. We have shown that, for any value of $b \neq 0$, the objective function of the founder is maximized by setting $\tau = 0$.

We now show that, for $\tau = 0$, the objective function of the founder is maximized by setting $b = 0$. In particular, we will show that the objective function of the founder is maximized by inducing the manager to select the inefficient project with private benefits of control rather than the project with $r = 0$. With $\tau = 0$, we know that $r = 0$. But since $\mu > \frac{\bar{A}-\underline{A}}{T}$, $\bar{A}_0 < \underline{A}$: the expected payoff of the project with $r = 0$ is lower than the expected payoff of the inefficient project with private benefits of control.

For $\tau = 0$ and a given b, the certainty equivalent wealth of a manager opting for the project with $r = 0$ is

$$CE(r = 0) = w + b\bar{A}_0 \tag{3.44}$$

For $\tau = 0$ and a given b, the certainty equivalent wealth of a manager opting for the inefficient project with private benefits of control is

$$w + b\underline{A} + P \tag{3.45}$$

Inducing the manager to pick the former rather than the latter is achieved for

$$b < \frac{P}{\bar{A}_0 - \underline{A}} \tag{3.46}$$

Since $\bar{A}_0 < \underline{A}$, b would need to be negative, which is impossible. For any contract parameters, the manager therefore had rather select the inefficient project with private benefits of control.

If the manager opts for the inefficient project with private benefits of control, the objective function of the principal is, for a given b and a given τ:

$$(1-b)\underline{A} - w \tag{3.47}$$

where the fixed wage w adjusts to satisfy the participation constraint in (3.16), so that (3.47) rewrites as

$$\underline{A} - \bar{W} + P - \frac{\gamma}{2} b^2 \left({\sigma^N}^2 \tau + \int_\tau^r {\sigma_s^U}^2 ds \right) \tag{3.48}$$

This expression is minimized by setting $b = 0$. This is incentive compatible, since we already noted that the manager selects the inefficient project with private benefits of control for any contract parameters (as long as the participation constraint is satisfied).

Proof of Proposition 4:

For parameter values such that it is optimal to set $b = 0$ (see Proposition 3), we have $b = 0 < b^\star$, and the value of τ is irrelevant. We now consider parameter values such that it is optimal to set a value for b sufficiently high that the manager is prevented from investing in the inefficient project with private benefits of control.

We first tackle the pay-performance sensitivity. Remember that the pay-performance sensitivity $b^\star$ with second-best efficiency is set so that the manager is indifferent between the efficient project with time horizon T and the inefficient project with private benefits of control:

$$b^\star = \frac{P}{\bar{A}_T - \underline{A}} \tag{3.49}$$

For this value of b, the manager is then indifferent between the efficient project with a time horizon of $r = T$ and the inefficient project with private benefits of control. But a manager who chooses a time horizon of $r^\star < T$ derives a higher expected utility from choosing $r = r^\star$ rather than $r = T$. It follows that for $b = b^\star$, he strictly prefers the project with a time horizon of $r^\star$ to the inefficient project with private benefits of control. Therefore, the pay-performance sensitivity b may be decreased while still ensuring that the manager does not opt for the inefficient project with private benefits of control.

We now show that the optimal timing of pay at the third-best is greater than or equal to the second-best optimal timing of pay $\tau^\star$. If, for given values of b and τ, the manager chooses a project with a time horizon of r, the objective function of the principal in (3.12) is

$$\bar{A}_r - b\bar{A}_r - w = \bar{A}_r - b\bar{A}_r - \bar{W} + b\bar{A}_r - \frac{\gamma}{2}b^2\Big({\sigma^N}^2\tau + \int_\tau^r {\sigma_t^U}^2 dt\Big)$$
$$= \bar{A}_r - \bar{W} - \frac{\gamma}{2}b^2\Big({\sigma^N}^2\tau + \int_\tau^r {\sigma_t^U}^2 dt\Big)$$

where the first equality follows from the participation constraint in (3.16). We know that, for any τ, $r \geq \tau$.

If $\sigma_t^U > \sigma^N$ for all t, $\tau^\star = T$, so that $r = T$ when $\tau = \tau^\star$ (since $r \geq \tau$): the second-best outcome is feasible at the third-best, with $b = b^\star$ and $\tau = \tau^\star$. We have shown that, in the first case of proposition 1, it is never optimal to pay the manager earlier than $\tau^\star$.

If $\sigma_t^U < \sigma^N$ for all t, $\tau^\star = 0$, so that τ at the third-best is necessarily such that $\tau \geq \tau^\star$. We have shown that, in the second case of proposition 1, it is never optimal to pay the manager earlier than $\tau^\star$, and it may be optimal to pay him later than $\tau^\star$.

If σ_t^U is increasing in t and we have both $\sigma_0^U < \sigma^N$ and $\sigma_T^U > \sigma^N$ (this corresponds to the third case of proposition 1), so we know that either $\tau^\star = 0$, or $\tau^\star = T$. In the latter case, setting $\tau = \tau^\star = T$ induces $r = T$, since $r \geq \tau$: the second-best outcome is feasible at the third-best, with $b = b^\star$ and $\tau = \tau^\star$. In the former case, τ at the third-best cannot be lower than $\tau^\star = 0$. In this case, we also know that the time horizon r selected by the manager's problem is given by (3.40); in particular, $r = 0$ if $\sigma_0^U > \sqrt{\frac{2\mu}{\gamma b}}$. In this case, with $\tau = 0$ and $r = 0$, the objective function of the founder (from (3.12)) is

$$\bar{A}_0 - \bar{W} \tag{3.50}$$

Marginally increasing τ by Δt increases r by the same amount (since $r \geq \tau$ and $CE(r)$ is decreasing in r on $[\tau, T]$). With $\tau = \Delta t$ and therefore $r = \Delta t$, the objective function of the founder (from (3.12)) is

$$\bar{A}_{\Delta t} - \bar{W} - \frac{\gamma}{2}b^2{\sigma^N}^2\Delta t$$

Comparing with (3.50), it is optimal to increase the compensation date by Δt if and only if

$$\mu > \frac{\gamma}{2}b^2{\sigma^N}^2 \tag{3.51}$$

We have shown that, in the third case of proposition 1, it is never optimal to pay the manager earlier than $\tau^\star$, and it is sometimes optimal to pay him later than $\tau^\star$, when $\tau^\star = 0$ (notably for parameter values which satisfy equation (3.51) above).

If σ_t^U is decreasing in t and we have both $\sigma_0^U > \sigma^N$ and $\sigma_T^U < \sigma^N$ (this corresponds to the fourth case of proposition 1), we set

$$F(\tau) \equiv b\mu(T-\tau) - \frac{\gamma}{2}b^2 \int_\tau^T {\sigma_s^U}^2 ds > 0 \tag{3.52}$$

We know from (3.41) that the manager chooses $r = T$ if $F(\tau) \geq 0$. Differentiating the left-hand side of (3.52) with respect to τ gives

$$-b\mu + \frac{\gamma}{2}b^2 {\sigma_\tau^U}^2 \equiv G(\tau) \tag{3.53}$$

Since σ_t^U is decreasing in t, $F(\tau)$ is concave in (τ). If (3.52) is satisfied at $\tau = \tau^\star$, then $r^\star = T$ when $\tau = \tau^\star$: it is not optimal to pay the manager earlier than $\tau^\star$. Below, we analyse two cases in turn.

First, if at the compensation time $\tau = \tau^\star$, we have $G(\tau) < 0$:

$${\sigma_\tau^U}^2 < \frac{2\mu}{\gamma b} \tag{3.54}$$

Then the manager opts for $r^\star = T$, so that no changes to the second-best optimal contract parameters are needed. We show this result by contradiction. Assume that the manager opts for $r^\star = \tau^\star$. From (3.41), this implies that

$$\frac{\int_{\tau^\star}^T {\sigma_t^U}^2 dt}{T - \tau^\star} \geq \frac{2\mu}{\gamma b} \tag{3.55}$$

But because σ_t^U is decreasing in t,

$$\frac{\int_{\tau^\star}^T {\sigma_t^U}^2 dt}{T - \tau^\star} < \frac{{\sigma_{\tau^\star}^U}^2 (T - \tau^\star)}{T - \tau^\star} = {\sigma_{\tau^\star}^U}^2 \tag{3.56}$$

Combining (3.54), (3.55) and (3.56) yields

$$\frac{2\mu}{\gamma b} \leq \frac{\int_{\tau^\star}^T {\sigma_t^U}^2 dt}{T - \tau^\star} < {\sigma_{\tau^\star}^U}^2 < \frac{2\mu}{\gamma b}$$

Which is impossible, so that $r^\star = T$ when $\tau = \tau^\star$ and $G(\tau) < 0$. In this case, it is not optimal to pay the manager earlier than $\tau^\star$.

Second, if at the compensation time $\tau = \tau^\star$, we have $G(\tau) > 0$:

$${\sigma_\tau^U}^2 \geq \frac{2\mu}{\gamma b} \tag{3.57}$$

If (3.52) is not satisfied at $\tau = \tau^\star$, then $r^\star = \tau$ when $\tau = \tau^\star$. Since $G(\tau^\star) > 0$, we know that $F(\tau)$ is increasing in τ in a neighbourhood of $\tau^\star$, so that sufficiently raising τ may result in $r^\star = T$ (of course, it is also possible that $F(\tau) < 0$ for any $\tau \in [0, T]$, in which case raising τ is useless). Any marginal increase in τ increases agency costs by $\frac{\gamma}{2}b^2(\sigma^N - \sigma_\tau^U)dt$, which is positive since $\sigma_t^U < \sigma^N$ when σ_t^U is decreasing and $\tau > \tau^\star$. We are therefore looking for the smallest value of τ such that the manager selects $r^\star = T$ rather than $r^\star = \tau$. Define $\bar{\tau}$ by

$$b\mu(T - \bar{\tau}) = \frac{\gamma}{2}b^2 \int_{\bar{\tau}}^T {\sigma_s^U}^2 ds \tag{3.58}$$

By construction, $\bar{\tau}$ is the minimum value of the compensation time τ such that $r^\star = T$. For a given value of b and $\tau = \bar{\tau}$, the manager chooses a project with a time horizon of T so that the objective function of the principal in (3.12) is

$$\bar{A}_T - \bar{W} - \frac{\gamma}{2}b^2\Big({\sigma^N}^2\bar{\tau} + \int_{\bar{\tau}}^T {\sigma_t^U}^2 dt\Big)$$

For a given value of b and $\tau = \tau^\star$, the manager chooses a project with a time horizon of $\tau^\star$, so that the objective function of the principal in (3.12) is

$$\bar{A}_{\tau^\star} - \bar{W} - \frac{\gamma}{2}b^2{\sigma^N}^2\tau^\star$$

Setting $\tau = \bar{\tau} > \tau^\star$ dominates $\tau = \tau^\star$ if and only if (substituting for the values of $\bar{A}_t$):

$$\mu(T - \tau^\star) > \frac{\gamma}{2}b^2\Big({\sigma^N}^2(\bar{\tau} - \tau^\star) + \int_{\bar{\tau}}^T {\sigma_t^U}^2 dt\Big)$$

For these parameter values, it is optimal to set $\tau = \bar{\tau} > \tau^\star$.

Proof of Proposition 5:

The problem of the principal can be rewritten as

$$\min_{\{b_t\}} \int_0^T b_t^2\Big({\sigma^N}^2 t + \int_t^T {\sigma_s^U}^2 ds\Big)dt \quad \text{s.t.} \quad \int_0^T b_t dt = \frac{P}{\bar{A} - \underline{A}} \tag{3.59}$$

If $\sigma_t^U > \sigma^N$ for all t or $\sigma_t^U < \sigma^N$ for all t, then the corner solution is to concentrate all pay-performance sensitivity respectively at time T, or at time 0. Otherwise, the first-order condition is

$$\int_0^T 2b_t\Big({\sigma^N}^2 t + \int_t^T {\sigma_s^U}^2 ds\Big)dt - \lambda T = 0$$

where λ is the Lagrange multiplier associated with the incentive constraint. Rewriting,

$$\int_0^T \Big(b_t\Big({\sigma^N}^2 t + \int_t^T {\sigma_s^U}^2 ds\Big) - \frac{\lambda}{2}\Big)dt = 0$$

That is,

$$b_t = \frac{\lambda}{2}\frac{1}{{\sigma^N}^2 t + \int_t^T {\sigma_s^U}^2 ds} \tag{3.60}$$

where λ is a constant determined so that the incentive constraint is satisfied as an equality. The pay-performance sensitivity b_t is increasing in t if and only if

$$\frac{\lambda}{2}\frac{1}{{\sigma^N}^2 - {\sigma_t^U}^2} > 0$$

Or

$${\sigma^N}^2 < {\sigma_t^U}^2$$

Proof of Proposition 6a:

Suppose that given the second-best optimal pay-performance sensitivity process, b'_t, (3.28) is larger than (3.27).

First, with the pay-performance sensitivity process $b_t \equiv kb'_t$ for all t, (3.27) and (3.28) rewrite respectively as

$$w + \int_0^T kb'_t\bar{A}dt \tag{3.61}$$

$$w + \int_0^m kb'_t\hat{A}dt + \int_m^T kb'_t\underline{A}dt - \frac{c}{2}(m+\epsilon)^2 \tag{3.62}$$

There exists a k sufficiently low that (3.62) is smaller than (3.61) for any $m \in [0,T]$, i.e., such that the inequality

$$k\Big(\int_0^m b'_t\hat{A}dt + \int_m^T b'_t\underline{A}dt + \int_0^T b'_t\bar{A}dt\Big) < \frac{c}{2}(m+\epsilon)^2$$

is satisfied for any $m \in [0,T]$. The first part of the proof is complete.

Second, consider the pay-performance sensitivity process defined by

$$b_t = 0 \qquad \text{for} \qquad t < q$$

$$b_t = \frac{1}{T-q}\int_0^T b'_t dt \qquad \text{for} \qquad t > q$$

Then (3.27) and (3.28) rewrite respectively as

$$w + \int_q^T \frac{1}{T-q}\Big(\int_0^T b'_s ds\Big)\bar{A}dt \tag{3.63}$$

and

$$w + \int_q^T \frac{1}{T-q}\Big(\int_0^T b'_s ds\Big)\underline{A}dt - \frac{c}{2}(m+\epsilon)^2 \tag{3.64}$$

for $m < q$, or

$$w + \int_q^m \frac{1}{T-q}\Big(\int_0^T b'_s ds\Big)\hat{A}dt + \int_m^T \frac{1}{T-q}\Big(\int_0^T b'_s ds\Big)\underline{A}dt - \frac{c}{2}(m+\epsilon)^2 \tag{3.65}$$

for $m > q$. In the former case, the fact that $\bar{A} > \underline{A}$ implies that (3.27) is larger than (3.28), since. In the latter case, when $m > q$, (3.27) is larger than (3.28) if and only if

$$\frac{c}{2}(m+\epsilon)^2 > \frac{1}{T-q}\int_0^T b'_s ds\Big((m-q)\hat{A} + (T-m)\underline{A} - (T-q)\bar{A}\Big) \tag{3.66}$$

But

$$\frac{c}{2}(m+\epsilon)^2 > \frac{c}{2}(q+\epsilon)^2 > (T-q)(\hat{A}-\bar{A})\frac{1}{T-q}\int_0^T b'_s ds > \frac{1}{T-q}\int_0^T b'_s ds\Big((m-q)\hat{A}+(T-m)\underline{A}-(T-q)\bar{A}\Big) \tag{3.67}$$

The second inequality is satisfied for T and q large enough, since then

$$\frac{c}{2}(q+\epsilon)^2 > (\hat{A}-\bar{A})\int_0^T b'_s ds$$

This implies that (3.66) is satisfied, and (3.27) is larger than (3.28) for T and q large enough. The second part of the proof is complete.

Proof of Proposition 6b:

Given a compensation schedule $\{b_t\}_{0\leq t\leq T}$, the expected utility of a manager who chooses an efficient project with time-horizon r is

$$E\left[-exp\left\{-\gamma\int_0^T b_t S_t dt\right\}\right] = E\left[-exp\left\{-\gamma\left(\int_0^T b_t(\bar{A}_r+\sigma^N B_t^N)dt+\int_0^r b_t\left(\int_t^r \sigma_s^U dB_s^U\right)dt\right)\right\}\right]$$
$$= -exp\left\{-\gamma\left(\int_0^T \left(b_t\bar{A}_r - \frac{\gamma}{2}b_t^2{\sigma^N}^2 t\right)dt - \frac{\gamma}{2}\int_0^r b_t^2\left(\int_t^r {\sigma_s^U}^2 ds\right)dt\right)\right\}$$

Maximizing the expression above is equivalent to maximizing the certainty equivalent wealth.

Given any pay-performance sensitivity process $\{b_t\}$, the certainty equivalent wealth of a manager opting for a project with time-horizon r is

$$CE(r) = \int_0^T \left(b_t\bar{A}_r - \frac{\gamma}{2}b_t^2{\sigma^N}^2 t\right)dt - \frac{\gamma}{2}\int_0^r b_t^2\left(\int_t^r {\sigma_s^U}^2 ds\right)dt$$

Consider the pay-performance sensitivity process $\{b_t'\}$. We prove both parts of the Proposition sequentially, by considering in turn the case in which σ_t^U is increasing in t, and then the case in which σ_t^U is decreasing in t.

If σ_t^U is increasing in t, $CE(r)$ is concave in r, and the optimal time horizon r solves the first-order condition

$$\mu\int_0^T b_t' dt - \frac{\gamma}{2}{\sigma_{r^\star}^U}^2\int_0^{r^\star} {b_t'}^2 dt = 0 \tag{3.68}$$

The equilibrium time horizon $r^\star$ is zero if the r which solves this equation is less than zero, and $r^\star = T$ if the optimal r which solves this equation is larger than T.

First, for $k \in (0,1)$, setting $b_t = kb_t'$ in the first-order condition (3.68) and dividing both sides by k yields

$$\mu\int_0^T b_t' dt - k\frac{\gamma}{2}{\sigma_{r^\star}^U}^2\int_0^{r^\star} {b_t'}^2 dt = 0 \tag{3.69}$$

Since $k \in (0,1)$, the second term must increase for the equality to be satisfied. This is achieved by an increase in $r^\star$. Since $b_t \geq 0$, an increase in $r^\star$ increases the value of the integral. Moreover, since σ_t^U is increasing in t, increasing $r^\star$ also increases the value of $\sigma_{r^\star}^U$.

Second, setting $b_t = 0$ for all $t < q$ (where $q > r^\star$ by assumption) and $b_t = \frac{1}{T-q}\int_0^T b_s' ds$ for all $t > q$ makes the second term in (3.68) equal to zero. Equality can only be maintained if $r^\star$ is increased.

If σ_t^U is decreasing in t, $CE(r)$ is convex in r, and the manager chooses $r^\star = T$ if

$$\int_0^T b_t'\mu T dt - \frac{\gamma}{2}\int_0^T {b_t'}^2\left(\int_t^T {\sigma_s^U}^2 ds\right)dt > 0 \tag{3.70}$$

otherwise he chooses $r = 0$. For $k \in (0,1)$, setting $b_t = kb'_t$ in the left-hand side of (3.70) and dividing through by k yields

$$\int_0^T b'_t \mu T dt - k\frac{\gamma}{2}\int_0^T {b'_t}^2 \left(\int_t^T {\sigma_s^U}^2 ds \right) dt \tag{3.71}$$

which is larger than the left-hand side of (3.70), since $k < 1$. Setting $b_t = kb'_t$ therefore ensures that the manager chooses $r^\star = T$ for a larger set of parameter values.

Proof of Proposition 7:

A manager compensated at time τ optimally chooses the project with time horizon $r^\star$. If $r^\star < \tau$, his certainty equivalent is

$$CE(r^\star) = b\bar{A}_{r^\star} - \frac{\gamma}{2} b^2 \bar{A}_{r^\star}^2 var\left[exp\left\{ -\frac{1}{2}{\sigma^N}^2 \tau + \sigma^N B^N \right\} \right]$$

The variance term is equal to

$$var[exp\{\sigma^N B^N\}] = E\left[\left(exp\{\sigma^N B^N\} \right)^2 \right] - \left(E\left[exp\{\sigma^N B^N\} \right] \right)^2 = E\left[exp\{2\sigma^N B^N\} \right]$$
$$-\left(exp\{\frac{1}{2}{\sigma^N}^2 \tau\} \right)^2 = exp\{2{\sigma^N}^2\tau\} - exp\{{\sigma^N}^2\tau\} = exp\{{\sigma^N}^2\tau\}\left[exp\{{\sigma^N}^2\tau\} - 1 \right]$$

Thus, the certainty equivalent is

$$CE(r^\star) = b\bar{A}_{r^\star} - \frac{\gamma}{2} b^2 \bar{A}_{r^\star}^2 \left[exp\{{\sigma^N}^2\tau\} - 1 \right]$$

If $r^\star \geq \tau$, the certainty equivalent of the manager is

$$CE(r^\star) = b\bar{A}_{r^\star} - \frac{\gamma}{2} b^2 \bar{A}_{r^\star}^2 var\left[exp\left\{ -\frac{1}{2}{\sigma^N}^2 \tau + \sigma^N B^N - \frac{1}{2}\int_\tau^{r^\star} {\sigma_s^U}^2 ds + \int_\tau^{r^\star} \sigma_s^U dB_s^U \right\} \right]$$

The variance term is

$$exp\left\{ -{\sigma^N}^2\tau - \int_\tau^{r^\star} {\sigma_s^U}^2 ds \right\} var\left[exp\left\{ \sigma^N B^N + \int_\tau^{r^\star} \sigma_s^U dB_s^U \right\} \right]$$
$$= exp\left\{ -{\sigma^N}^2\tau - \int_\tau^{r^\star} {\sigma_s^U}^2 ds \right\} \left(exp\left\{ 2{\sigma^N}^2\tau + 2\int_\tau^{r^\star} {\sigma_s^U}^2 ds \right\} - exp\left\{ {\sigma^N}^2\tau + \int_\tau^{r^\star} {\sigma_s^U}^2 ds \right\} \right)$$
$$= exp\left\{ {\sigma^N}^2\tau + \int_\tau^{r^\star} {\sigma_s^U}^2 ds \right\} - 1$$

Thus, the certainty equivalent is

$$CE(r^\star) = b\bar{A}_{r^\star} - \frac{\gamma}{2} b^2 \bar{A}_{r^\star}^2 \left(exp\left\{ {\sigma^N}^2\tau + \int_\tau^{r^\star} {\sigma_s^U}^2 ds \right\} - 1 \right) \tag{3.72}$$

For a given τ and a given $r^\star$, marginally increasing $r^\star$ increases the manager's certainty equivalent by:

$$b\mu(1+\gamma b) - \gamma b^2 \, exp\left\{ {\sigma^N}^2\tau + \int_\tau^{r^\star} {\sigma_t^U}^2 dt \right\} \left[\mu + \frac{1}{2}\bar{A}_{r^\star}^2 {\sigma_{r^\star}^U}^2 I_{r^\star \geq \tau} \right] \tag{3.73}$$

where $I_{r^\star \geq \tau}$ is a dummy variable equal to 1 if $r^\star \geq \tau$, and 0 otherwise. For a given $\tau > 0$, and a given $r^\star$, then the expression above is negative when σ^N is sufficiently large, so that the manager chooses a lower $r^\star$. The first part of the Proposition is proven.

The first derivative of (3.73) with respect to τ is

$$-\gamma b^2({\sigma^N}^2 - {\sigma_\tau^U}^2)exp\left\{{\sigma^N}^2\tau + \int_\tau^{r^\star} {\sigma_t^U}^2 dt\right\}\left[\mu + \frac{1}{2}\bar{A}_{r^\star}^2 {\sigma_{r^\star}^U}^2 I_{r^\star \geq \tau}\right] \tag{3.74}$$

This expression is negative if and only if $\sigma^N > \sigma_\tau^U$. In this case, increasing τ reduces the value of the first derivative of the certainty equivalent with respect to r (when this first derivative is positive for a given time horizon $r^\star$, the manager increases $r^\star$). On the one hand, the first derivative of the certainty equivalent with respect to r in (3.73) is negative for some parameter values. On the other hand, if $\sigma^N > \sigma_\tau^U$, then increasing τ makes it more likely that the first derivative in (3.73) is negative, in the sense that it is negative for a larger set of parameter values. Therefore, if $\sigma^N > \sigma_\tau^U$, either increasing τ does not change the sign of (3.73), or it makes it negative. In the latter case, increasing τ lowers $r^\star$.

We now summarize the results. If parameter values are such that (3.73) is positive for given values of τ and $r^\star$, then, if $\sigma^N < \sigma_\tau^U$, increasing τ will always increase $r^\star$; but if $\sigma^N > \sigma_\tau^U$, increasing τ may lower $r^\star$. If parameter values are such that (3.73) is negative for given values of τ and $r^\star$ (this will be the case if σ^N is sufficiently large) then, if $\sigma^N < \sigma_\tau^U$, increasing τ may increase $r^\star$; but if $\sigma^N > \sigma_\tau^U$, increasing τ will always lower $r^\star$. Overall, when σ^N is sufficiently high, increasing τ results in a diminution of $r^\star$, and conversely. The second part of the Proposition is proven.

Chapter 4

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