INFORMED SPECULATION AND THE ORGANISATION OF FINANCIAL MARKETS

by

JÜRGEN DENNERT

Thesis submitted for the degree of Doctor of Philosophy
at the London School of Economics

(University of London)

July 1990

ABSTRACT

The thesis deals with several aspects of the impact of informed speculation on financial markets. It consists of four chapters.

Chapter 1 gives a general discussion of the welfare effects of insider trading, and investigates whether insider trading should be prohibited. The following chapters are concerned with more specific questions of the organisation and regulation of financial markets.

Chapter 2 investigates the performance of dealership markets in the presence of informed speculation. It is shown that informed trading creates externalities which might render markets with several competing market makers less liquid than a market with a monopolist specialist.

Chapter 3 deals with a different effect of insider trading on secondary markets. It is argued that insider trading influences the allocation of risk between different classes of investors. The premature resolution of uncertainty due to insider trading makes prices more volatile and more informative. The effects of these two opposed effects on ex-ante investment are ambiguous: both more and less investment may occur.

Chapter 4 investigates a dynamic asset pricing model with informed speculation and noise trading. The properties of the steady state equilibria in a overlapping generations economy with infinite horizon are characterized. It is shown that noise trading leads to feedback effects of the kind that expectations of a high price volatility become self-stabilizing. The effects of asymmetric information and the early release of information are discussed and related to the results of the preceding chapters.

ACKNOWLEDGEMENTS

I owe thanks to Klaus Beucher, Tilman Börgers, Marcia Decastro, Jean-Luc Schneider, Ernst-Ludwig von Thadden, and especially to Martin Hellwig and to my supervisor Ailsa Roell. They helped me as friends and/or advisers through many struggles in the last three years and some of them invested a great amount of work in this thesis. I learnt a lot from them.

I am especially grateful to my parents who gave me the possibilities to do what I wanted.

This thesis was written within the European Doctoral Program in Quantitative Economics. I am very grateful that I was allowed to participate in this program and I would like to thank everybody involved in this program. I would also like to thank Max Steuer, without whom many Economics dissertations at LSE would probably never be submitted, including this one.

At various stages of this dissertation I received financial support from "Deutscher Akademischer Austauschdienst", "European Doctoral Program", "Schweizerischer Nationalfonds zur Förderung der wissenschaftlichen Forschung", "Stiftung Volkswagenwerk", "Wirtschaftswissenschaftliches Zentrum (WWZ) der Universität Basel", and "Studienstiftung des Deutschen Volkes". I am indebted to all of them.

Finally, thanks are due to Air Europe, Alitalia, British Airways, Dan Air and Swissair, who helped to make this a truly European thesis.

There is someone to whom I owe more than thanks, and I would like to dedicate this thesis to her.

CONTENTS

CHAPTER 1

SHOULD INSIDER TRADING BE PROHIBITED?

1. Introduction.

The starting point of most discussions on insider trading is "fairness". Probably to most people it appears rather unjust that some speculators are able to earn profits at the expense of others who just happen to know less about the asset in question. This assessment is quite similar to our attitude towards say a seller of used cars hiding to possible buyers a past accident of the car. Still, there are important economic differences between a market for used cars and financial markets. The market for used cars is a well defined market in which the desires and needs of buyers and sellers are relatively clear. Financial markets, however, perform a very complex task for the whole economy: the allocation of investments among the different sectors of an economy, the allocation of risks between different classes of agents, and consumption smoothing of consumers over time. An analysis of the impact of insider trading on the performance of the financial markets should therefore help us to determine the scope of the fairness argument.

Among the main arguments against insider trading one can distinguish two main lines of reasoning which differ considerably in the definition of behaviour considered to be insider trading. The first, more restrictive argument defines insider trading as a breach of fiduciary duty or at least a breach of trust and confidence. In the modern capitalist economy with its prevalence of the separation of ownership and management, fiduciary duty of the managers is considered to be the basic guaranty of the functioning of this economy by creating the "right" incentive structure. The profits at the expense of their shareholders realized by managers who deal on the basis of information which they have obtained through their position, are considered to arise from an illegitimate breach of the relation of fiduciary duty which links managers to their shareholders.

A second, less restrictive line of reasoning objects to any form of trading on the basis of differentials in information, regardless whether the traders are in a relation of fiduciary duty or not. Apart from the violation of fairness among shareholders with equal rights, it is argued that unrestricted insider trading will lead to a breakdown of capital markets which are unable to perform their role efficiently. Outside investors, afraid of being exploited in the future by better informed traders will not invest in markets characterized by an intensive insider activity. Moreover, it is argued that unrestricted possibilities of insider trading will induce too much costly information acquisition, since private and social benefits of information will in general not coincide.

These arguments have led most industrialized countries to adopt a more or less severe insider legislation, the most notorious exception until now being West Germany. According to the above arguments they differ mainly in the scope of their definition of an insider. However, starting from Manne's influential book on inside trading (Manne(1966)), the academic literature brought forward some arguments against the common firm stance concerning inside trading. Against the fiduciary duty argument, Manne argued that the problem between managers and shareholders is not so much that managers *actively* try to follow strategies designed to harm their shareholders. For him the major drawback of the separation of ownership and control is rather that managers tend to pursue policies which are less dynamic and innovative than the ones an independent entrepreneur would choose. Manne's argument is that the possibility of insider trading just creates the right incentives for managers, i.e., the incentives of an independent entrepreneur. Against the other arguments that unrestricted insider trading leads to a market breakdown and a waste of resources, Manne claims that the prohibition of insider trading prevents prices from reflecting all available information and thus does not allow the capital markets to perform efficiently their role as an allocation mechanism of capital.

Before looking more closely at these arguments[1], it will be useful to give a more precise meaning to the term insider trading. In the public discussion the term is often reserved for "classic" insider trading, i.e., trading of speculators who possess certainty about specific fundamental events which would become public shortly afterwards, and deal on this information. In practice things are a bit more complicated: There are several degrees and manners of inside information, and there

[1] An interesting agnostic viewpoint is to ignore the contents of the debate altogether. Haddock and Macey (1987) follow a public-choice approach to characterize the current insider legislation only in terms of the outcome of an clash of interests between different interest groups.

are also different participants in the business of acquiring and marketing information[1]; it is actually far from obvious how to define "real" insider trading.

On the most elementary level a certain sophistication with the working of the stock markets, the interpretation of balance sheets, the understanding of the impact of new developments in a certain industry etc. is required to transform readily available news into valuable bits of information. Analysts of brokers and banks, but also participants in the industry will possess this sophistication. They will in general both trade on this information and resell it to outside customers. With a certain time lag the results of this news processing will also be available to a general public through the financial newspapers. In general this information will not cause dramatic price changes and profit possibilities; nevertheless it is this information procession which is the basis of the every-day movements at a stock exchange. A whole industry is trying to get news, interpret the news and transmit the news with the only aim to do that a bit faster and more precisely than the others, and to earn a -limited- profit on this advance.

A more sophisticated level of information concerns news, whose impact depend less on a able and thorough interpretation of readily available facts, but will rather lead to unambiguous drastic revaluations of a certain stock. This information contains for example knowledge (or at least the probability) of future changes in the regulatory conditions of a certain market, of new inventions or discoveries, of unexpectedly high profits or losses, future take-overs and the like. This kind of news will be in general only available to the corporate insiders, i.e., to the managers themselves, advising financial institutions and to some extent the analysts performing primary research for brokers and banks. This kind of news differs from the one of the preceding paragraph only in two respects. First, the assessment of its impact does not require a technical analysis, and second, it has more dramatic consequences. I.e., at the same time they are both cheaper and more important, and this from a social and from a private point of view. However, in principle they do not differ from the kind of additional information acquired on a less sophisticated level.

If we consider the major players on stock exchanges we would come to a broad

[1]A nice discussion of these differences can be found in Manne (1966).

classification of information degrees consisting of outside investors - financial industry - corporate insiders. This classification is however rather crude. The knowledge of large institutional investors will in general be comparable to the financial industry. Continental and Japanese banks are at the same time active in primary research and acquire through their direct supervision facilities inside knowledge of firms which is comparable to a corporate insider. It is indeed the bank dominated financial systems, which give a particular good example how fragile the common distinction between "true" inside trading and research is.[1] It is more than unclear how tight the famous Chinese Walls can be in reality. Still another problem is posed by a potential raider in a merger or hostile take-over contest. At the same time, he does research (on the target firm), he is a corporate insider (in relation to his own actions), and in general he is also selling his own managerial abilities to the target firm.

For our discussion it will be furthermore useful to distinguish between the two major roles of capital markets in the investment process. First, they coordinate both directly and indirectly the allocation of new real capital among different firms. The direct way would be through the issue of new shares; a procedure whose importance has significantly declined in recent years. The indirect influence is given by the implicit determination of the necessary rates of returns for internal financing of new investments both of listed and unlisted companies.[2] Second, capital markets organize the reallocation of already existing real capital in a different production context (through hostile or friendly mergers). Our discussion will mainly focus on the paradigm case of new investments, but many of the arguments have general validity.

[1] Kay(1988) distinguishes between insiders and quasi-insiders. In particular in a country like West-Germany there is another, even more severe problem with the activities of banks. Due to the close links to the firms, the resp. dominant bank will in general obtain new firm-specific information earlier than the other creditors. Premature cancellation of a credit has the same effects on the other creditors as insider trading on stock markets on the other shareholders. In our discussion we will concentrate on insider trading on stock markets. The problems arising in connection with insider information and debt have been largely ignored by the literature. However, in particular after the recent EEC-Directive 89/592 (on insider trading), this problem has become the subject of legal and economic treatments in West Germany. See e.g. Franke(1989).

[2] That is, if managers act as they should according to the textbooks; one might shed some doubts on this assumption.

The chapter will be organized as follows. Sections 2, 3 and 4 deal with the three main *economic* arguments listed in the beginning. Section 2 deals with the effects of expected insider trading in the secondary markets on the welfare of investors and ex-ante investment. In section 2 the link between investment activity and insider trading is indirect, because we ask how the *possibility* of insider trading might influence the behaviour of investors. In Section 3 we explore a direct link between investment and insiders. We investigate how insider activity leads to more informative prices and how this influences better investment decisions. We compare these benefits with the costs of this process, the losses inflicted on outsiders and the resources spent on information acquisition. Section 4 discusses the issues related to incentive questions. Section 5 relates the results of the preceding sections with the fairness arguments. We survey the three main forms which this argument takes in legal practice and contrast them with the economic analysis. Finally, we survey in an appendix the additional features of insider trading arising in the take-over process.

2. The Victims of Insider Trading.

2.1 The Redistribution of Wealth

At a first glance, it should be obvious who are the victims of insider trading, namely their trading partners. However, the case is much less clear-cut than might be expected. Of course, there are no problems with the assessment of the manipulative abuse of inside information. There are several possibilities of manipulation for corporate insiders by retaining information, driving prices down by releasing wrong information and the like.[1] There is no question that this manipulation distorts prices, leads to wrong investment decisions of individual investors and to a lower allocation efficiency of capital markets.

When insiders deal on the basis of information which is true but unknown to the market, the identity of the victim is less clear. Every *rational* investor who holds the shares does not have to worry about the insider trading, since it only anticipates events which occur anyhow. If he intends to trade despite the presence of insiders, he should at least expect to be better off than without trading. Hence the only victims which might be harmed by insider trading are those who *must* deal because of some exogenous liquidity shock, i.e., either shareholders who have to sell for some need for liquidity or non-shareholders who receive additional liquidity and want to invest in a risky asset to smooth their consumption path optimally over time.

But a priori it is even unclear how these traders should suffer from insider trading in the future. Future prices may both rise (in the case of good news) and fall (in the case of bad news). Hence, a liquidity trader who arrives in the market shortly after an insider was active might profit or lose (depending on whether he buys or sells and whether the news have been bad or good). Accordingly, if he is risk neutral he should not be worried by the insider trading.

---

[1] See Manne(1966) for a discussion of these practices. Benabou/Laroque(1989) give an analytic treatment of the problematic. They deal in particular with the question whether reputation effects might prevent systematic manipulation of information.

However, a liquidity trader is harmed in a more subtle way which is analogous to the adverse selection effects common from markets like the one for used cars. On most financial markets liquidity is provided by professional traders or market makers, i.e., the usual counterpart of an investor is a professional dealer. If these market makers face an insider risk, they will buy resp. sell shares only with a corresponding discount which compensates them for the potential losses due to information based trading.[1] Thus, already the possibility of insider trading induces additional transaction costs on outside traders, even when the insiders are not present at the moment. This argument is for example developed in King/Roell (1988) who argue that the real harm imposed on outside investors is this additional tax on investments arising through the adverse selection component of the bid-ask-spread.

How important this tax might be, is certainly an empirical question. However, there are also some theoretical problems with the argument. The first concerns the consequences of a prohibition of insider trading via a disclose-or-abstain rule. If such a rule could be strictly enforced, the bid-ask-spread and the transaction costs faced by the liquidity traders would certainly fall. In practice, such a rule will probably only reduce the amount and seriousness of the cases of insider trading. Some insiders will still try to cheat. For practical reasons the regulator will only sue very serious and clear-cut cases of insider trading. However, the effects of a reduction of insider trading (as opposed to a complete elimination) are not entirely clear. Both in strategic models in the spirit of Kyle (1985) in which insiders and market makers take the effect of their own actions on the overall outcome into account, and in competitive models like Grossman/Stiglitz (1980) in which the actions of the individual insider resp. market maker have no effect on prices and allocations, less insider trading may lead to higher or lower spreads depending on the environment.

[1] For a discussion of the insider risk component of the bid-ask-spread see e.g. Glosten/Milgrom (1985), Kyle (1985). The original idea goes back to Bagehot (1971). The effect arises also in standard competitive models with asymmetric information like the one of Grossman/Stiglitz (1980). In these models the uninformed investors, facing insiders and liquidity traders and trying to deduce information from the aggregate order flow play the role of the market makers. The bid-ask-spread arises in the form of price movements due to noisy excess supply/demand. For a more detailed discussion of this and the remainder of this section see chapter 3, section 5.

The second theoretical problem with the bid-ask-spread arises when risk aversion is introduced. If market makers are risk averse, the bid ask-spread will also include a component which compensates market makers for the additional risk they bear when they buy shares (resp. for their suboptimal risk bearing in case they sell). This compensation for risk bearing is obviously the greater the riskier the asset, or put slightly differently, the less information on the asset is available. Hence, liquidity traders might actually have an interest that insiders actively reduce uncertainty over the asset's future returns. These benefits of insider activity are partially offset by the adverse selection effect, but ultimately the benefits of insider trading on the size of the bid-ask-spread may outweigh the costs. If one assumes that the introduction of a disclose-or-abstain rule leads to less information acquisition and less insider activity, such a rule tends to favour most the market makers. We will come back to this point in section 5.

The introduction of risk aversion in the utilities of investors also allows to study an additional effect how outside investors might suffer from insider trading. If outsiders find themselves in a suboptimal portfolio position, i.e., they possess either less or more of the risky asset than they would optimally like to hold, they will want to buy resp. sell shares on the stock market in order to rebalance their portfolio position. Hirshleifer (1971) argued that under these circumstances investors generally have an interest that information is revealed *after* a first round of trade. The argument is essentially an insurance argument. Investors want to be optimally insured against risk before the event occurs; afterwards insurance does not make sense. The link of this reasoning to the problem of insider trading is that insider trading will drive prices up to the true future realization of the risky asset, i.e., outside investors buy resp. sell at high prices if returns are high and at low prices when returns are low. But if prices already reflect most of the information on returns, the remaining profit possibilities for the outsiders are accordingly low. Hence, insiders harm outsiders through their influence on prices and the resulting premature resolution of uncertainty.

To make this argument precise, one must in general control for additional effects of new information, in particular income effects which might offset the negative influence of new information. The argument is for example valid if investors have constant absolute risk aversion.[1]

2.2 The Ex-Ante Effects on Investment

The preceding discussion concentrated exclusively on the direct redistribution of wealth between insiders and outsiders. In a vague sense this might be considered as a contribution to an analysis how well secondary markets perform their role as a device to smooth consumption of economic agents over time. Nothing which is essential to *capital* markets has entered the analysis so far. But ultimately we should be interested in the impact of the performance of secondary markets on original investment. For investors in a new project the link between secondary trading and the original investment should be quite obvious. If the investment project is relatively long lived. i.e., returns can be expected over many periods, the investors are interested in future prices on secondary markets because they have to take into account that at some time they might want to sell their shares. This occurs for instance because of exogenous liquidity needs or because of changing preferences. Therefore, their investment decision will not only depend on the characteristics of the new project, but also on the characteristics of the secondary markets for the shares and in particular on the prices which can be realistically expected on these markets. A general analysis of the link between investment and the liquidity of secondary markets is quite difficult. But even without a formal model the effect of insider trading on initial investment seems quite intuitive as long as all investors are risk neutral. In this case original investors will anticipate that in order to be able to sell their shares later on the secondary markets they have to incur a transaction cost in form of the bid-ask-spread. Accordingly they will only be willing to invest in the risky project if the issue price compensates them for this additional cost of holding the asset. It will become more expensive for firms to finance new projects on the stock exchange and underinvestment will occur (see e.g. Wyplosz(1988)).

[1] For a more detailed treatment of the Hirshleifer effect see e.g. Gale (1985) and Hakansson et.al (1982).

The situation is less obvious if we include risk aversion in the discussion. Risk averse initial investors do not only care about the size of the bid-ask-spread, but also about the volatility of prices on future secondary markets. Given that multi-period rational expectation models with risk averse investors are quite complex, attempts to tackle the problem have assumed an easier deterministic structure. Chapter 3 and 4 of this thesis assume an overlapping generations framework in which each generation lives only for two periods and must sell all the shares in the second period to the next young generation. Ausubel (1989) considers several investment projects and assumes that investors initially cannot diversify and are only later able to diversify on the secondary markets.

Despite certain limitations, the models of chapter 3 and 4 allow some insights concerning the relation between price volatility and investment. From an ex ante viewpoint the rather subtle effect of insider trading on the secondary markets is to shift risk from the future to the present. Insider trading leads to premature resolution of uncertainty and hence in case of an early liquidation of their positions shareholders are confronted with a higher variability of prices if informed traders are active. If risk averse shareholders anticipate this correctly, market value at the date of issue should fall and some investment projects may not take place.

But there is an important opposite effect to this movement. Since share prices should trade on average for a price equivalent to the expected value of future returns minus a risk premium, informed trading resulting in a lower uncertainty in the future should lead to smaller risk premia of the expected prices. Thus, with insider trading in the future, expected share prices should be higher than without informed trading. The anticipation of this will make market value at the date of issue rise again.

Under this perspective the main *ex-ante* effect of insider trading arises through the premature resolution of information which is reflected in prices as a consequence of the insider's trades. Prices become more informative which from an ex-ante viewpoint leads both to higher prices on average and to higher price volatility. The overall outcome of these two effects is unclear and depends very much on the way how trading on secondary markets is organized and modelled. Depending on the chosen specification, underinvestment might occur or not. A particularly intriguing example is given if we model secondary markets in a standard way following Grossman/Stiglitz (1980), i.e., with informed speculators, uninformed speculators and liquidity traders. Depending on the parameter constellations of the model, both less or more investment might be the consequence of changes in the intensity of insider trading. Decisive for the outcome is the amount of additional volatility due to the bid-ask-spread, i.e., how much prices tend to bounce back and forth as reaction to liquidity events.

2.3 Investors with Bounded Rationality

Up to now our arguments relied very strongly on the assumption of hyperrational investors, who have a definite picture of underlying probability distributions and who are able to update them correctly in response to price movements. A slightly different approach to the problem would be to drop the implicit assumption that all the investors are hyper-rational agents who ponder before each trading decision which information could be contained in the fact that they find a counterpart for their transaction, and argue that the arguments against insider trading have a somewhat different outside investor with a more bounded rationality in mind. From this perspective one might defend the insider trading legislation as the protection of investors with bounded forecasting abilities from too well informed speculators.

A discussion of bounded rationality of investors is necessarily somewhat speculative; but it seems unquestionable that most investors form their expectations over the possible returns of risky assets through more or less crude rules of thumbs. To achieve reliable forecasts these rules of thumb require a certain stability of the stochastic processes that underlie price movements, a stability that price volatility due to informed speculation tends to undermine.

One not too implausible view of relatively "rational" uninformed investors would be that their abilities to forecast correctly the information contained in a potential trade are rather restricted, but that they are able to infer (more or less) concisely the information contained in the last observed trades.[1] Such a behaviour will lead

[1] See Hellwig (1982) for a rational expectations model with a lagged updating rule.

the uninformed investors to buy too late in the case of good news and to sell too late in the case of bad news and to find themselves constantly in a suboptimal portfolio position. Another promising way to examine effects of insider trading in a context of bounded rationality might be to include the behaviour of chartists into models of price formation and to investigate in which sense their techniques might lead to overreactions of prices on insider speculation. However, a more detailed discussion of possible interactions between insider trading and possible bounded rationality of investors is beyond the scope of this chapter and we will not further pursue this discussion.

3. Costs and Benefits of Informational Efficiency of Prices.

Modern finance theory has always stressed the importance of informational efficiency of prices (see e.g Fama (1970)). Asset prices should incorporate as fast as possible all available information in order to fulfil their allocative function. What exactly is meant by that allocative function and in which sense more informative prices are able to be better indicators for investment decision, remains somewhat vague. A classic counter-argument of Hirshleifer points out that informative prices might actually be useless from a social point of view. Moreover, since the private incentives for the acquisition of information will be usually redistribution of wealth, i.e., from a social viewpoint not efficiency improving, the resources spent on information are wasted and the overall result of more informative prices on the whole economy are actually negative. Hirshleifer's argument is hence twofold: First he questions the usual presumption that more informative prices are always "better". Second, he opposes the view that information as a good with public-good-character will in general be underproduced and underutilized and points out that actually the opposite may be the case.

Hirshleifer's example considers a pure consumption-insurance world in which ex ante the portfolio positions of all investors are optimally diversified. In this situation there is no efficiency gain from new information: In a pure consumption setting in which everybody is ex-ante in a pareto-optimal portfolio position, information is socially useless. In order to understand this argument, consider the case of full disclosure of information which does not lead to any efficiency gain: In equilibrium the only result of the information is to change prices such that everybody is still satisfied with the endowments he has. The allocation of risks remains unaltered. Exactly the same applies to imperfect information revelation due to insider activity. However, the private incentives to acquire information and to speculate on behalf of the others are given by the profits arising through the redistribution of wealth from outsiders to insiders. Hence it might be profitable to spend resources on the acquisition of new information, although there is no benefit at all for the economy as a whole.

However, the exclusive treatment of capital markets from the insurance viewpoint neglects their role in the process of the allocation and reallocation of real capital, i.e. the decentralized decision about future production. The question then is: How does private information influence investment decisions through the capital market? What are possible efficiency improving effects of informed trading? Despite the vast literature on "efficient" capital markets, there exists unfortunately no real welfare analysis of the allocational role of capital markets. In the sequel we will therefore discuss a very simple example to disentangle allocation effects, distribution effects and costs of information acquisition.

Let us consider a firm that wants to invest in new machines in order to produce a certain good. At the time of the investment decision the outcome of the venture will in general be uncertain; it may depend on the ability of the management and the workforce, parallel developments of competing firms, uncertain prices tomorrow and so on. Let us assume that in order to finance the investment the firm plans a rights issue. As long as managers act in the interest of their shareholders, investment decisions financed through retained earnings can be treated in a similar way. The "real" impact of this investment decision can be considered to be the optimal number of machines to buy.

If the management acts according to the textbook rule and we ignore complications like unanimity of shareholders and incentives of managers, the management will take an investment decision such that the firm's net market value after the issue is maximized. It is at this point that the capital market influences investment decisions. In practice, managers will have to anticipate the stock market's reaction to the investment; if they fail, investors will not invest enough.

It is clear that stock market prices signal to managers things like risk aversion and wealth of potential investors, i.e., potential demand, and hence will influence investment decisions. If there is any role for insiders to play, then it is to make prices contain some fundamental information about the asset itself. It is however less obvious how information transmitted through prices should influence the decision. In the end the managers themselves should be the best informed about the project they are going to undertake. They should also be able to monitor the situation of their competitors as accurately as the capital market. And even if not, a general policy of public disclosures of all firms should be able to achieve all the possible benefits of information.

In practice there may be several problems with such a policy: problems of credibility, too high costs of continuous public disclosures, problems of keeping business secrets. The most severe problem with a policy of constant disclosures is perhaps the high costs which are required if every market participant has to perform the rather complex task of interpreting all the public disclosures of the economy. An alternative mechanism is that corporate insiders and firms specialized in information acquisition, i.e., brokers and banks, deal on the stock market resp. sell the information to speculators who want to deal with it on the stock market and drive the prices to a level where they "reflect" the information. The investing firm would then just have to regard the prices and deduce the necessary information for its own investment. This argument sees the price system as a mechanism to reduce the complexity of the world by transmitting only the relevant information.[1]

What is possible relevant information contained in prices? If share prices of competitors in the same industry (and thus with correlated returns) reflect information about their returns, they determine the cost of capital to the investing firm and thus directly the investment decision through the market value rule. If there exists a futures market for the good produced, futures prices signal which prices can be expected in the future. And finally, although managers are probably in the best position to judge the technological properties of the investment, they are probably not a very objective judge of their own abilities. The share price of the firm will reflect the shareholders' perception of the managers' qualities. This assessment of the management's qualities will influence the market value and thus the investment decision. A very serious practical problem is of course, that the evaluation of the information content of prices is a very complex task; it is hard to imagine that this should always be easier than obtaining the information directly. The futures example might be the most plausible example, because in this case prices are a relatively simple signal. Still, the difficulties of interpreting price signals correctly

[1] The argument is usually attributed to von Hayek (1945).

might shed some doubts on the extent to which managers really infer information from prices.[1]

But let us abstract from these general difficulties and assume that the most efficient way to transmit new information is through informed trading. For our purpose it is enough to acknowledge that information transfer might be socially desirable and that insiders might perform a useful role as information transmitter. But the individual aim of their activity is, of course, speculation: they try to make a profit from superior information at the expense of less well informed traders. Their incentives to acquire information stem merely from the perspective of redistribution of wealth between the uninformed and the informed. They are prepared to spend money for more precise information as long as they expect higher average returns in the future as a recompense.[2]

In contrast to the argument at the beginning of this section, there is now an efficiency effect which could *potentially* offset the costs of information acquisition. Whether this is the case will in general depend less on the insider activity per se than on other factors. How to measure conclusively these efficiency effects appears quite impossible without a satisfactory multi-period general equilibrium model which explicitly links financial to real markets.

One might argue that ultimately we should not be too concerned about financial, but rather about real assets. The outcome will then in particular depend on the

[1]An exception being the take-over-threat inherent in low share prices. There, the information contained in prices concerns the market perception of the manager's abilities and decisions. I shall deal in the appendix with this point that is probably better understood under the viewpoint of the *control* function of prices rather than their informative function in a strict sense.

[2]There are several ways to make this idea formally precise. The best known in the rational expectations literature is probably the Verrechia(1982) version of the Hellwig(1980) model. The information of the insiders is only partially revealed because they can hide behind some exogenously given noise trading. As a function of their characteristics (in the Verrechia model differing risk aversion), their willingness to take speculative positions will vary and thus their willingness to buy information. In equilibrium the least risk averse traders will have acquired information (which is then contained in the equilibrium price), whereas more risk averse traders do not acquire information because it is too costly given the expected extra returns. It is important to see that in any standard model with partially revealing equilibrium prices the extra profits of informed traders depend only on the characteristics of the traders, the costs of information acquisition and in particular on the characteristics of the chosen source of noise. However it does not depend on the results of more informative prices on potential production decisions.

conditions on the market for the real asset. Imagine for example that the supply of machines in question is highly price-inelastic, i.e., changes in the demand for machines will have only a minor impact on the quantity of machines actually sold in equilibrium. In this case a higher precision of the estimate of the cost of capital will have almost no effect on the *real* investment. The opposite would apply to the case of price-elastic supply of machines.

In the case of a small impact of new information on real investment one might be tempted to argue that the new information is a social waste, because real expenses on information acquisition are not compensated through real improvements in investment. However, many additional factors would have to be specified in a welfare analysis. Wealth will not only be redistributed among insiders and outsiders in the financial markets, but also among the shareholders of the investing firm and the shareholders of the firm which produces the machines. Moreover, we also have to take into account that the financial assets play an important role *per se*. The insider trading will in particular determine the allocation of risks in the economy and the possibilities for investors to smooth their consumption path over time. It will certainly be possible to construct simple models in which the real costs of information acquisition are so high and the effects on real investment are so low that this negative trade-off overshadows all the other welfare considerations. But on the whole, such a welfare analysis will always remain ambiguous. It remains ambiguous because the precise nature of the link between financial and real markets is not very well understood. But even if it were, our above argument shows that any general welfare analysis would still depend on certain endogenous characteristics of the markets in question. In our example, the supply of machines might be price elastic or price inelastic (depending on an exogeneous specification of technologies, preferences and the like).

Hence, even in such a simple example the often stated benefits of informative prices are far from obvious. It is possible that the financial industry spends just enough money, it is as possible that huge amounts of money are just wasted for the benefit of some happy few; more cannot be said in this generality. Notice however that, if ever, the argument is much more directed against the cost-intensive financial industry than the corporate insider who obtains information as a cheap by-product of his work.

4. The Incentives of Corporate Insiders.

In the preceding sections we have focused on the effects of insider trading *per se* on shareholders. However, if the insiders are corporate insiders (like the managers of the firms and advising financial institutions) some additional considerations have to be taken into account. Implicitly the preceding discussion assumed managers to behave in the interest of their shareholders and to take optimal investment decisions. Nothing was said about the incentives for the managers to do so.

Manne's (1966) classical argument deals with the issue of the incentives of managers to take optimal decisions in favour of their shareholders. He argues that the perfect remuneration device to make managers act as "entrepreneurs" is to allow them to earn parts of the profits through insider trading. His argument is mainly that this practice puts them in the situation of an independent entrepreneur and makes them more innovative. The position is similar to the motivation of patents: to give an incentive for inventions, one allows the inventor for a certain time to earn excess profits.

A different argument (see e.g. Demsetz (1986)) sees insider profits of managers and majority shareholders as a compensation for the higher firm-specific risk borne by them. Demsetz argues that managers specialize their human capital to the needs to the firm and hence cannot diversify away the firm-specific risk. Similarly, he argues in favour of the insider trading of majority shareholders: Large shareholders imply an "efficient level of monitoring", but incur a high firm-specific risk. According to Demsetz, this makes insider trading desirable in order to encourage specialization of managers and majority shareholding.

Many objections against this position have been raised, like the issue of the overcompensation of managers, moral hazard problems for the managers and the problem of a conflict of interests between managers and shareholders. Moral hazard problems may arise through the obvious temptation to delay, artificially produce or manipulate information. A look on the U.S. stock exchanges before the 1929 depression gives a good impression of the possibilities of large scale price manipulations. Conflicts of interest will occur whenever the trading which is based on inside information raises the cost of business for the firm and hence damages shareholders. This will happen if changes in the firm's share price reveal possible plans of the firm like acquisitions. If insider trading of corporate insiders raises the firm's cost of business through the indirect revelation of business secrets, the behaviour of managers is quite similar to a theft.[1]

The main problem with Manne's argument is that he starts from a rather suggestive aspect of the incentive problem arising through the separation of ownership and control, i.e. the Schumpeterian problem how to make a manager of a big firm act as an innovative entrepreneur rather than as an administrator. However, he does not really deal with the problem whether *in general* interests of managers and financial institutions will coincide with shareholders' interests if one allows for unrestricted insider trading. There is no obvious reason why this should be the case. In fact, no restrictions on inside trading in the future will establish ex ante (i.e. at the date of a new investment) two rather different groups of investors, corporate and financial insiders on the one hand and outsiders on the other hand. These groups differ in the sense that they can expect different rates of returns on their investments; both corporate and financial insiders can expect to bear a smaller risk than outside investors since they can react faster to future news on the asset's returns. Thus the problem is not as usual a problem of asymmetric information on the primary market for new issues, but a problem of expected asymmetric information on the secondary markets in the future.

Under these conditions several scenarios can easily be imagined. There may be a tendency to overinvestment, since insiders will take too risky investment decisions (knowing that the larger part of the risk has to be borne by others). This may also result in a world of underinvestment ex ante, if outside investors take account of this behaviour and demand a premium for the additional risk. Even "reasonable"

[1] See Haddock/Macey (1987). Fishman/Hagerty (1989) construct an example in which shareholders prefer less informative prices, because competing firms can infer business secrets from prices and subsequently compete more effectively with the shareholders' firm. For a discussion of other, related arguments see Ausubel(1989) and Dye(1984).

investment decision may then fail to attract sufficient funding, because outside investors cannot distinguish them from "hyped" investments.

A look on the motivations behind the securities legislation of the "New Deal" in the 1930s shows that these possibilities are not just thought experiments. After the experience of the 1920 with its large number of excessive (and often fraudulent) share issues in which the issuers-entrepreneurs had long sold their participations before bad news became public, the original overinvestment had turned into a large scale underinvestment. Both the "Securities Act of 1933" and the "Securities Exchange Act of 1934" were intended to restore the confidence of the broad public in the stock market.[1]

In practice many firms (at least in the U.S.) have meanwhile imposed restrictions on the trading of their managers and, although Manne considers stock options as too inflexible, they have chosen stock options as an incentive for "entrepreneurial" behaviour.[2]

[1]This restoring of confidence might of course have also other connotations. We will discuss this point further in the next section.
[2]For a survey of current U.S. practices of remuneration see Jensen/Murphy (1990).

5. Fairness, Efficiency and the Confidence in the Markets.

Ever since the "new deal"-laws began to impose restrictions on insider trading, the argument of fairness has played a major role in their justification. Thus, the "Securities Exchange Act of 1934" was motivated by the desire "to assure the maintenance of *fair* and honest markets".

In its most basic form the fairness argument posits that nobody should make profits only on the basis of superior information since this will always be at the expense of somebody else. There are some obvious practical difficulties with this position. A major problem for the jurisdiction has been that usually there are no direct victims of an inside transaction since stock market transactions occur in general via professional intermediaries. This is in sharp contrast to the case of the seller of a damaged car who does not fully inform the buyer. Moreover, it will be often very difficult for a court to define ex post what exactly was superior information and whether the accused really had this information. Starting from the famous "catch-all-clause" rule 10b-5[1] the history of prosecution of insider trading in the U.S. cases gives a rich illustration of these problems.

The practical limitations to sue *every* transaction based on (perhaps only slightly) better information, make it certainly useful to examine possible economic trade-offs arising in the context of insider trading regulation. A better understanding of the economic effects of insider trading could potentially enable us to identify more clearly which forms of insider trading are more damaging and who actually is the victim of insider trading. Accordingly, it should help us both in defining the range of behaviour which we consider as unfair and in being aware of possible allocational and distributive consequences which might be the price to pay for a prohibition of insider trading.

One way to restrict the scope of fairness considerations and link it to economic arguments is the notion of "confidence" in the markets, i.e., to prosecute insider

[1] Issued by the SEC in 1942. On the grounds of rule 10b-5 American courts usually prosecuted insider trading as a violation of the principle of equal treatment of all shareholders.

trading only as far as it undermines the "confidence" in the financial markets. The broad objective of the U.S. securities laws of the 1930s was to restore *public confidence* in the capitalist economy as a whole and the capital markets as their central allocation mechanism for investments in particular. Confidence in capital markets is of course a rather vague term with many different connotations. Its intuitive appeal on the one hand and its missing precision on the other hand seem at the root of much of the confusion around the prohibition of inside trading. What exactly is meant with missing confidence in economic terms? How can confidence be undermined? What economic implications does this have?

Unfortunately the analysis of sections 2 and 3 dealing with the economic implications of trading on the basis of different information, does not give us any conclusive answers. As we have seen, already a precise description of the victims of insider trading, i.e., the ones whose confidence might be shattered, is not without problems. But even if one accepts that liquidity traders suffer most of insider trading (either in the form of higher transaction costs in form of higher bid-ask-spreads or through the premature resolution of uncertainty), I have argued that it is unclear whether a prohibition of insider trading will make liquidity traders really better off.

Also the consequences of missing confidence in the markets are not entirely clear. As we have seen, insider trading might lead to underinvestment, but it also might lead to more and better investment. And finally the fears that unrestricted possibilities of insider trading might render stock exchanges huge casinos in which immense sums are spent on the acquisition of information in order to place bets with better odds might be justified or not. The main problem with all arguments positing negative effects of insider trading is to cope with potential positive effects of new information which potentially enables investors and firms to take better investment decisions. The only tentative conclusion one might draw out of this discussion is that our intuition concerning the "classic" insider, i.e., somebody who by chance gets to know a valuable piece of information which shortly afterwards becomes public anyhow, is in some sense confirmed. His actions certainly do not involve any economic benefits which might be a possible price to pay by the society, if, based on fairness considerations, society decides to prohibit his trading. But of course, it is not these clear-cut cases of insider trading which pose problems.

In the light of this rather shaky assessment, it might seem more promising to focus on the issues of section 4. The discussion there introduces a somewhat different principle of equality which might be justified both on general grounds as on more specific efficiency considerations. Equality in this context is meant to be equality of interests between outside investors and the managers of their wealth, i.e., corporate managers and financial institutions. These parties have concluded at least an implicit contract. This contract presupposes a relation of trust and duty as the basis of fair and equal behaviour between them. It also avoids problems of moral hazard and of an institutionalized inequality between groups of investors with differing return expectations. Apart from possible underinvestment results, this also gives a psychological content to the concept of confidence: nobody likes to deposit money with people whom he distrusts. Hence, a more restrictive way to interpret "fairness" would be to limit prosecution of insider trading to cases involving the breach of fiduciary duty; the mere possession of inside knowledge would not be sufficient. However, if insider trading is deemed illegal only in cases involving a breach of fiduciary duty, it is at least arguable whether one should criminalize insider trading at all. In principle, the prohibition of insider trading could be written in the private contracts between the concerned parties. Any partisan of an insider trading law based on the breach of fiduciary duty would at least have to justify why such contractual arrangements are not feasible or at least not enforceable.[1]

The restriction of insider trading prosecution to cases involving some form of breach of fiduciary duty has been in particular advocated by the U.S. Supreme Court in the 1980s, for instance in the famous cases of Chiarella v. United States and even more pronounced in Dirks v. SEC (the Equity Funding case). The Supreme Court tends to focus in recent decisions on the concept of fiduciary duty rather than on the issue of equality concerning the possession of information: "It is important in this type of case to focus on policing insiders and what they do ...

[1]In West-Germany, this reasoning is one of the main arguments brought forward by the banks against the threat of an insider law. It is argued that self-regulation could solve all the problems of insider trading.

rather than policing information per se and its possession..."[1] The subtle, but in its implications decisive question is, how to define a breach of fiduciary duty. In a narrow sense it requires explicitly that one of the contractual partners is harmed. In a broader sense it covers also the misappropriation of information. The implications for the financial industry (in particular brokers and market makers) are obvious: the misappropriation rule would impose rather severe restrictions on their activities.[2]

At a first glance it seems quite attractive to restrict the prohibition of insider trading to cases involving some form of breach of fiduciary duty, because it might bring its justification on safer grounds from a practical and economic point of view. Still, the price to pay would be that a certain class of market insiders would still be able to earn extra returns which many might judge to be excessive, both judged from the personal effort which was necessary to achieve these profits and the potential social profits linked to the transactions of the insider. Moreover, there are also some internal problems with the concept of fiduciary duty.

First, some institutions like banks might have conflicting fiduciary duties towards different clients. Firm specific informations acquired by the corporate finance department in its function as a financial adviser is supposes to be confidential. The brokerage department owes investors optimal advice. What is the duty of the bank as a whole, if it obtains information that a certain firm will suffer from a large slump of profits in the next quarter?[3] Second, even the concept of a fiduciary duty of the management towards the shareholders is not always unambiguous. Consider the case of a firm who considers its shares to be undervalued and starts a (secret) buy back programme. The old shareholders who do not sell shares to the firm (presumably the majority) profit from this programme, whereas the shareholders selling their shares make losses. Should the managers (or the firm?) be prosecuted

[1] Verdict of Supreme Court in Dirks v. SEC, 103 Supreme Court Review 3265.

[2] The situation in the U.S. is still unclear. Chiarella v. United States rejected the misappropriation theory. In a very similar case (United States v. Newman, 2d Circuit), but this time involving the employee of an investment bank, the same court which had taken the original decision in the Chiarella case ruled against the accused insider on the grounds of the misappropriation theory. Finally, in Foster v. United States the Supreme Court was split on the issue of misappropriation.

[3] The famous "Chinese Walls" are supposed to circumvent this problem by splitting the banks up into several units with different fiduciary duties.

on the grounds of a breach of fiduciary duty?[1] Third, and perhaps most importantly, the fiduciary duty concept not only leaves many forms of insider trading legal, but in some sense even encourages them quite explicitly. Consider the manager of a certain firm, who by means of his privileged position as an industry insider acquires information that a competing firm will make very high profits in the future. Since it is part of his fiduciary duty towards his own shareholders to maximize their profits, he should buy shares of this firm. Doing this he could not be considered as an insider: He does not breach any relation of trust to the target firm's shareholders, because there is none. Moreover, he acts optimally in the interest of his own shareholders, just as they expect him to do.

Thus, regulators face a dilemma. On the one hand there are practical problems to sue *any* transaction based on information differentials and an insufficient understanding of the consequences of such a complete ban for the economy as a whole. On the other hand the prosecution of insider trading only in cases involving a breach of fiduciary seems both too restrictive and may lead to internal contradictions. Given this dilemma one encounters a more restricted form of the equality-principle: the rule of equal access to information. As long as information has been obtained on a way which in principle is open to everybody, it is considered to be legitimate to trade on this information. The "equal access to information"-rule is under many perspectives a typical middle-of-the-road solution. It covers both the "classic" insider and the breach of fiduciary duty. It also covers a simple notion of fairness that nobody should earn money without having worked for it; hence the access to this work should be unrestricted. Moreover, it takes possible economic implications of an insider trading ban into account by ensuring that a certain amount of information still trickles into the market. Implicitly it takes the position that too much or too little information is undesirable, but that there is something like a optimal level of information necessary for the smooth functioning of the economy.

However, there are also problems with this position. The emphasis of the equal-access-rule should really lie on the "in principle". In principle market professionals

[1]For a recent discussion of this problem in the light of the EEC Directive see Carbonetti (1990).

and outside investors have the same, unrestricted access to the process of search for new information. But in practice the possibilities of banks and brokers to obtain new information are of course infinitely better. It is hard to conceive how market professionals and outside investors should ever have comparable access to new information. Accordingly, Kay (1988) in a comment on the U.K. legislation on insider trading[2] argues that in practice the principle of equal access to information serves mainly to eliminate corporate insiders in favour of market professionals. Why the favourable treatment of a certain group of participants in the financial market should be fairer towards the other participants and in particular the outside investors is not really obvious in the light of our discussion in section 2.1. The equal-access-rule not only protects market professionals from insider activity but gives them a double advantage over ordinary investors. Neglecting our qualifications of the argument in section 3, their power to exploit their own inside knowledge at the expense of outsiders might be justified as a means to compensate them for their costly research activity, deemed to be beneficial by rendering prices more informative. But they would benefit from a prohibition of insider trading even if they were just lazy and would produce no information at all. Higher uncertainty over future returns leads to higher bid-ask-spreads and hence to higher profits, because the risk aversion component of the bid-ask-spread is rising.

Given that a first best solution is certainly not available, the equal-access-rule appears from the political point of view probably the second best solution, but it still remains unsatisfactory both from an economic and a fairness perspective. From the economic point of view, the rule remains unsatisfactory because a certain group of traders is exempted from prosecution without a convincing economic justification for this exemption. From the fairness perspective, its major drawback seems that it introduces a large "grey" zone of behaviour which might be considered as unfair or not. Often a decision whether a certain information was acquired on a legitimate or illegitimate way will remain a matter of judgement, and this judgement will always be influenced by some subjective qualification of the concept of fairness.

This chapter did not try to solve delicate problems of what we perceive to be fair, but rather attempted to examine economic arguments which might help us in

[2] Mainly the Companies Act of 1980.

determining possible implications of the desire for fairness. It has turned out that our understanding of the economic implications of insider trading is rather shaky and our assessment of the impact of regulations at the least ambiguous. Still, the preceding discussion highlights the relevant issues in any evaluation of the impact of insider trading.

Appendix: Insider Trading and Take-Overs.

Up to now we treated the firm as given and shareholders only decision was whether to participate in a new investment project of this firm. During the production process, however, it may be the case that certain investors desire to buy the whole firm and its productive assets.

This desire can have many origins. Managers of other firms may draw some satisfaction from managing a large conglomerate. Certain firms may feel the desire to diversify their activities and reduce their dependence on a specific activity. Or they may feel that the existing productive assets of the other firm may be used in their own firm in a more profitable way, for instance because of economies of scale and scope, exploitation of existing distribution channels for their own products or the renegotiation of contracts with suppliers and the workforce. Finally, they may feel that the firm is badly managed and that with the existing real assets and workforce much better results could be achieved. In particular this last point has received much attention in recent years. Since internal and debt financing of new investment have made the stock market rather obsolete for the direct determination of the conditions of new issues, the take-over threat has been regarded as an indirect disciplinary device for shareholders.

In the remainder of the appendix we shall only consider effects of insider trading when a take-over actually takes place. But for the disciplinary function of the take-over process the mere threat of a take-over is of course sufficient. A take-over will be the more likely the lower share prices are. In this sense we can speak of a control function of prices. If the stock market feels that the incumbent management is not up to its job, this will depress prices. Together with the implied rising likelihood of a take-over this appears a rather effective mechanism to control the management. The analysts of banks and brokers are probably best placed to to monitor the management's abilities and to distinguish inability from adverse economic shocks which do not fall under the responsability of the managers. Presumambly, the appeal of this argument determines to a large extent the special role which current insider legislation assigns to insider activities of banks and brokers.[1]

Concerning the process of take-overs itself a vast literature discussing the efficiency gains of the current take-over wave has evolved.[2] We will not try to survey this discussion here. For our problem of the regulation of insider trading, it is more interesting, that take-overs allow us to study two peculiar types of insiders more closely. The first group of insiders concerned are the raiders themselves. The second group consists of arbitrageurs or simple speculators, who get to know of a bid before it is publicly announced and who begin to deal in the shares of the target firm. Given the large profit possibilities arising through take-overs, it is not surprising to see that most of the recent spectacular cases of insider trading involved this group.

A.1 The Raider as an Insider

Every bid for a hostile or friendly take-over demands a great deal of preparation and research. At the time of announcing a bid, the raider has in general spent a considerable amount of money to acquire inside information on the situation of the target firm, its asset values, the situation in possible input and output markets and so on. In this sense he should be regarded as an archetypical insider, i.e., as somebody who knows existing information about the firm's possibilities unknown to the broad public and trades according to this information. He takes advantage of the target firm's shareholders and should be prosecuted under any insider trading law which is based on the fairness principle, regardless whether that is defined with respect to the possession of information per se or merely with respect to equal access to information.

However, at the same time the raider fulfils a further role. His information may, at least in principle, serve the efficient reallocation of already existing assets in a more productive context. Furthermore, the expected returns of the firm after

[1] See the discussion of the rule of the equal access to information in section 5.

[2] The evidence seems rather mixed. For empirical surveys see Jensen/Ruback (1983) and Jarrell/Brickley/Netter (1988).

a successful bid will also depend on himself, i.e., on his abilities to develop a successful concept how to integrate the target firm in his own firm and to manage the target firm better than the old management did. Thus, he delivers his own "goodwill", i.e., human capital to the target firm. It is obvious that any raider expects a sufficient compensation both for his physical costs of launching the bid as for his effort.

It is apparently this last aspect which has led the two major countries with explicit take-over legislation, the U.S and the U.K., to allow raiders to purchase secretly a stake in the target firm. Thus, the raider is allowed to recoup at least part of his expenses through trading profits.[1] If at the same time the general insider trading legislation is founded on the principle of equality with respect to information resp. access to it, such a rule produces at least an inconsistency. However, prosecution of insider trading on the grounds of a violation of fiduciary duty would eliminate this contradiction: the raider is in no contractual relation to the target firm, and thus no breach of trust in this sense occurs.

To make matters even more complicated, both countries have also imposed limits on the raider's activities: shareholdings from a certain percentage on have to be disclosed, in a given time only a certain percentage of the shares may be bought without a public announcement, and from a certain point on there even exists the obligation to announce a formal bid.[2] Thus, after having reached this crucial barrier the raider looses most of his informational advantage, i.e his insider status. There is of course still a difference between the announcement to be a large shareholder and the formal obligation of a bid insofar as there still remain uncertainties about the real intentions of the potential raider. The first disclosure mark is a kind of hybrid: it enforces information disclosure, but only partially.[3] It reduces profit possibilities for the raider without completely eliminating them.

[1]For a formal analysis of this process see Kyle/Villa (1988).

[2]In the U.K. for example the City Code on Take-overs and the Rules Governing Substantial Acquisitions of Shares fix the large share holding disclosure limit at 5%, the bidding limit at 30%, and it is forbidden to purchase more than 10% of voting stock in less than 7 days if that results in more than 15% of a company's voting rights.

[3]The U.K. rules actually enforce a step by step disclosure. After having reached the 15%-mark, the purchase of every further full percent point has to be disclosed.

There has been a considerable debate around the height of these maximal toeholds. Given that most empirical investigations indicate that after a bid the raider has to pay on average the whole surplus to the old shareholders, the rule on admissible toe-holds may be very important in the decision to look for potential target firms, developing strategies for them after the take-over and the occurrence of the actual bid.

To justify the disclosure rules with the fairness principle seems quite odd, given that already the concession to raiders to acquire shares secretly is the major violation of this principle. Hence, a priori there should be no reason, why the raider should not reap the full benefit of his action and the disclosure mark should be set at 100%. However, from an economic point of view such a reward system might induce more costly research activity than society wants to encourage.

Therefore, policy makers should ponder at least on the following points. First, a decision should be taken whether take-overs on the whole can be regarded as efficiency improving and not merely as a resource and time wasting means to redistribute wealth. Second, it is not entirely clear whether secret share acquisitions of the raiders tend to favour the "right" take-overs. Kyle/Villa (1988) argue that it rather tends to benefit costly and unnecessary take-overs. Third, also the nature of the raider's investigation may be important. Roell (1987) applies considerations from the patent literature to this problem. If the strategy after the merger can be implemented by many different firms, the maximum toehold should be very low to avoid duplicating of search activity. In the opposite case in which the strategy remains a private good even after the announcement of the bid, i.e., it can only be implemented by the finder, it should be set as high as possible in order to induce as much search as possible.

A.2 The Trading of Other Insiders Before the Bid

Before the official bid is announced, it is almost unavoidable that a substantial number of people and institutions know it in advance. These will include professional advisers like merchant banks and firms who are directly informed by the raider, but also employees of these firms and market professionals without any direct relation to the raider, who get hold of the news through tips, rumours or their own analysis. Under a regime relying on some form of a fairness principle all these people are obviously insiders and should "abstain or disclose". If the breach of fiduciary duty is required, then friendly and hostile insiders have to be treated differently.

As far as friendly insiders are concerned, i.e., insiders who have an arrangement with the raider to resell the shares later to the raider, no breach of duty occurs. They are in no relationship to the target firm, and they do not harm the raider (but rather help him to let the bid go through).[1] As far as hostile insiders are concerned, their trading activity will usually be prosecuted if they are in a fiduciary relationship with the raider or their knowledge is derived from a source with such a relationship.[2]. The reason is that the hostile insider harms the raider by driving up the target firm's share prices faster than it would have been the case otherwise. Thus, he reaps part of the potential profits of the raider.

A recent debate questions the presumption that a hostile raider is necessarily harmful for the take over process. The main argument is[3] that by acquiring blocks of the outstanding stocks, they make the raider's task after the announcement of the bid somewhat easier by eliminating the famous free rider problem (Grossman/Hart (1980)). The free rider problem describes the ambiguous situation of a negligible small shareholder, who has every reason to believe that his own acceptance of the bid has no influence on the success probability of the bid and that he can only gain by keeping his shares. If everybody thinks this way, the bid will certainly not take place. However, if the shares are concentrated in the hands of few large investors, the raider has only to convince these shareholders, i.e., the problem is reduced to a bargaining problem.[4] The remaining small shareholders

[1] A different question is, of course, whether they should be subsumed under the disclosure rules. In the U.K. for example the recent Guinness and Blue Arrow-National Westminster-UBS scandals are the results of very strict rules on "concert parties".

[2] In practice, the extent of prosecution will depend how far the misappropriation theory is adopted. Without the misappropriation theory, the clerk of an advising merchant bank, who finds sensitive take-over news in the waste-paper-basket does *not* commit a crime.

[3] See e.g. Roell (1987)

[4] See e.g. Holmström/Nalebuff(1989) for a formal model in which the free rider problem disap-

will then have an incentive to sell, because in the case of a successful bid they incur the risk of being diluted.

The empirical relevance of the argument is hard to judge. It actually might be in the interest of a raider to overcome this negative externality of the disclosure mark by leaking take-over news to arbitrageurs. He will loose money to them, but at least the chance of a successful bid will rise. The arbitrageurs' activity might be even in the interest of the target firm's shareholders, since they tend to drive prices up to their "right" value. But at least on European markets shareholdings tend to be very much concentrated in the hands of large institutional investors anyhow, and then the need for arbitrageurs is much less apparent.

A.3 The Incentive Effect on Managers

The discussion of incentive problems is quite similar to the one in section 4 in the main text. Therefore we shall only briefly discuss the difference between the raider firm and the raider's managers. On the one hand existing rules allow only the firm to purchase shares of the target firm; its managers are treated as ordinary insiders. On the other hand the main effort of finding an appropriate target firm, developing a new strategy and implementing the bid is not accomplished by the "firm", but by its managers. The neo-classical school would argue in this case that the real incentives have to be set on the level of the managers and not on the level of the firm.[1] But this would almost certainly set the wrong incentives. Since managers do not invest their own money in the bid but the money of their shareholders, there would be an obvious temptation towards purely distributional take-overs and mergers; even without welfare gains the managers could earn enormous extra returns before the announcement of the bid.

pears with a finite number of large shareholders even without bargaining.

[1] Again the best reference for this line of reasoning is Manne (1966).

CHAPTER 2

PRICE COMPETITION BETWEEN MARKET MAKERS IN MARKETS WITH INFORMED SPECULATION

1. Introduction.

In recent years much attention has been devoted to the price formation on stock exchanges which are organized as dealer markets. Most models derive the bid and ask prices on such markets by inventory-theoretic considerations: The two main strands of the literature focus either on the fundamental risk for risk averse market makers who are forced to deviate from their optimal portfolio position (for a survey see Stoll(1985)), or on the adverse selection risk created by the presence of better informed "insiders" (Glosten/Milgrom(1985), Kyle(1985)). Yet very little attention has been spent on a careful modelling of the strategic interaction of all the participants of such markets. In particular the adverse selection literature just imposes zero-profit conditions without explicitly modelling the interaction between competing market makers. While the bidding procedure of Glosten/Milgrom and Kyle might be justifiable in a specialist market in which the specialist observes the market demand before setting prices, the situation of typical dealer markets poses much more severe problems. On these markets competing dealers quote prices and quantities and are obliged by an explicit (London Stock Exchange, New York NASDAQ) or an implicit convention (Foreign Exchange Markets) to stick to their quotations.

In this chapter we will therefore provide an explicit game-theoretic treatment of the effect of adverse selection in such dealer markets. Potential clients on these markets are either "innocent" liquidity traders or well-informed insiders, who trade via competing market makers. The market makers face the problem, that they cannot condition their prices on the respective type of trader. Thus they will incur in average losses on the well informed insiders and will try to compensate this by gains from the liquidity traders (who are forced to trade by some unspecified exogenous reason). They achieve this by setting an ask-price above and a bid-price below the asset's underlying "true" value. The spread between these two prices will depend in general on the probability of informed trading, the transaction volume and the asset's volatility.

This chapter will investigate the impact of competition intensity (more precisely: the number of market makers) on the quality of the market, i.e., the size of the market spread. It turns out that in a market in which market makers have to commit themselves before observing the actual order flow, the number of market makers is an important determinant of the bid-ask-spread.

In our model first market makers quote prices for a certain quantity, and given these prices potential clients determine their optimal trading strategy. Under these circumstances risk-neutral insiders without capacity constraints have an incentive to deal with *every* market maker as much as possible as long as the quoted price is favourable to them given their information. They will choose for every market maker the profit maximizing quantity given the prices of this market maker and independent of the prices of the competing market makers. Assuming that there is only a limited amount of uninformed liquidity trading, this leads to the phenomenon that a growing number of market makers induces relatively more informed trading: On the one hand the market makers compete for the limited amount of liquidity motivated trading, but on the other hand their risk to get informed trading is independent of their competitors.

Thus the individual market maker faces a bidding problem for an asset with an expected value which depends on the number of his competitors. Accordingly, his risk exposure is growing with the number of his competitors and in equilibrium he will balance this by a higher bid-ask-spread. While this seems an intuitive argument, it is however not clear whether it also induces a higher market spread, i.e., a "worse" market. The market makers can vary prices *and* quantities. Hence, the liquidity traders could try to distribute their demand among several market makers, such that their total cost does not become necessarily bigger.

The chapter is organized as follows. Section 2 contains a very simple model with fixed quantities that illustrates the intuition. In this model we can show in particular that the market spread, i.e., the difference between the best bid and the worst ask price is growing with the number of market makers. Moreover, we give conditions under which a monopolist might be preferred by the market. Section 3 presents a general model, that allows for an endogenous choice of both prices and quantities. Section 4 discusses the role of discreteness vs. continuity in the class of such models. The final section discusses the relevance of the model for real markets, in particular the London Stock Exchange. An appendix contains parts of the proofs.

2. Price Competition with Fixed Quantities.

2.1 The Model

We consider a market in which a single risky asset is traded. Its liquidation value $\tilde{v}$ after the market closes is randomly distributed with commonly known distribution function.

There are informed and uninformed traders in the market who can trade this asset only via market makers. We assume that there are n different market makers for the asset at the Stock Exchange. Furthermore we assume, that a regulator has imposed a certain minimum quantity, say $q = 1$. Every market maker sets buying- and selling-prices and is obliged to transact at these prices. We model this as a two-stage-game.

In the first stage every market maker i ($i = 1, .., n$) sets two prices (A_i, B_i). He commits himself to trade in the second period any quantity q with $0 \leq | q | \leq 1$ for the ask-price A_i and the bid-price B_i. At this point we assume, that he will not trade quantities which are bigger than 1.

At the next stage nature draws the true realisation of the asset and the next trader. With probability μ he will be informed about the asset's true value and with probability $1 - \mu$ he will be uninformed. Both types observe all the different prices when they arrive.

If the trader is informed, we assume that he knows the true value of $\tilde{v}$. In this section we will assume that the asset's liquidation value $\tilde{v}$ has only two realizations with equal probability, a "good" one and a "bad" one. We normalise this such that $\tilde{v}$ is given by

$$\tilde{v} = \begin{cases} 1 \quad \text{with prob} \; = \dfrac{1}{2} \\ \\ -1 \quad \text{with prob} \; = \dfrac{1}{2} \end{cases}$$

One could imagine this return structure as arising through a take-over-bid. Everybody knows that a bid will take place, but only the insider knows in advance the value of the bid. The negative lower bid is just a normalization.

The informed trader then determines quantities $x_1,..,x_n$ which he wants to trade with the different market makers, such that his profits given the different prices are maximal. Informed traders are assumed to be risk neutral speculators without capacity constraints, that means their profits from trading with market maker i are given by

$$E_i = (\tilde{v} - A_i)x_i \quad \text{resp.} \quad E_i = (B_i - \tilde{v})x_i \quad ,$$

if they buy shares resp. if they sell shares. Their problem is to maximize this expression for every market maker i and we obtain demand functions $\tilde{x}_i = x_i(\tilde{v}, A_i, B_i)$.

If the trader is uninformed we assume that his demand u is the realization of a random variable $\tilde{u}$ which has only two realizations with equal probability, i.e.,

$$\tilde{u} = \begin{cases} 1 & \text{with prob } = \frac{1}{2} \\ -1 & \text{with prob } = \frac{1}{2} \end{cases} \quad .$$

The problem of the uninformed trader is to determine quantities $u_1,..,u_n$ such that his costs resp. his earnings are minimal resp. maximal, i.e. he minimizes

$$\sum_{i=1}^{n} P_i u_i \quad \text{where} \quad \sum_{i=1}^{n} u_i = u \quad ,$$

where $P_i = A_i$ if he buys and $P_i = B_i$ if he sells. As a result we obtain demand functions $\tilde{u}_i = u_i(\tilde{u}, A_1,..,A_n, B_1,..,B_n)$.

We are now looking for a subgame-perfect Nash-equilibrium in this game, i.e. we solve backwards first for the optimal quantity responses to the proposed prices, and then given these optimal responses of the uninformed traders we look for an Nash- equilibrium in prices at the first stage.

The optimal responses of the traders are in this case very easy to calculate. Since an uninformed trader only wants to trade one unit of the asset and the price schedules are flat, he will simply go to the market maker who charges the best price, i.e. to $\min_i(A_i)$, if he is a buyer and to $\max_i(B_i)$, if he is a seller. If several market makers charge the same best price, we assume that he trades with equal probability with each of them. The following results are independent of the form of the sharing rule.

If the trader is informed, he knows the true realization of $\tilde{v}$. Since he is a risk neutral speculator his optimal trading strategy is then given by

$$\begin{aligned}
x_i &= 1 \quad \text{if } A_i \leq 1 \quad \text{and } \tilde{v} = 1 \\
x_i &= -1 \quad \text{if } B_i \geq -1 \quad \text{and } \tilde{v} = -1 \\
x_i &= 0 \quad \text{otherwise} \quad .
\end{aligned}$$

We note that the uninformed demand from market maker i depends on all prices whereas the insider demand only depends on the price of market maker i. It is this strategic behaviour of the insiders which drives the results of the chapter. The usual assumption that insiders only deal with one market maker, is not tenable in our setting.

Given these optimal responses we can now calculate the Nash-equilibrium in prices of the first stage. In the sequel we will assume that market makers only charge symmetric prices $P_i = A_i = -B_i$, i.e. the strategy space is only one-dimensional. In fact, it is possible to show that the unique equilibrium in prices is a symmetric equilibrium, because of the symmetry assumptions on $\tilde{v}$ and $\tilde{u}$. We will not show this, but assume instead the one-dimensionality of the strategy space.

We assume that the market makers are expected ex-post-profit maximizers, i.e. given the prices of their competitors they maximise

$$E_i = [P_i - E(\tilde{v} \mid \tilde{q} = 1)]Prob(\tilde{q} = 1) + [E(\tilde{v} \mid \tilde{q} = -1) - (-P_i)]Prob(\tilde{q} = -1) \quad ,$$

where $\tilde{q}$ denotes the demand which the market maker expects in the second stage. Define $\phi_k = \frac{1}{k+1}$. We calculate E_i then explicitly as follows:

If $P_i = \min_j(P_j)$ and $P_i < P_j$ for all $j \neq i$ then

$$\begin{aligned}
E_i &= \left((P_i - \mu)\frac{1}{2} + (P_i - \mu)\frac{1}{2}\right) \\
&= \left(\mu(P_i - 1) + (1 - \mu)P_i\right) \quad .
\end{aligned}$$

If $P_i \neq \min_j(P_j)$ then

$$\begin{aligned} E_i &= \left((P_i - 1)\frac{\mu}{2} + (P_i - 1)\frac{\mu}{2}\right) \\ &= \left(\mu(P_i - 1)\right) \quad . \end{aligned}$$

If $P_i = \min_j(P_j)$ but i is tied with k other market makers, then

$$\begin{aligned} E_i &= (P_i - \frac{\mu}{\mu + (1-\mu)\phi_k})\frac{\mu + (1-\mu)\phi_k}{2} + (P_i - \frac{\mu}{\mu + (1-\mu)\phi_k})\frac{\mu + (1-\mu)\phi_k}{2} \\ &= \mu(P_i - 1) + (1-\mu)P_i\phi_k \quad . \end{aligned}$$

This form of the profit function is easy to see. If the market maker charges the best price he will earn in any case P_i. With probability μ the next trader will be informed and he will make losses equal to 1. If there is another market maker who charges a better price there will be no trade with probability $1 - \mu$ because the uninformed trader will not buy or sell from market maker i. If there is any trade, the market maker will make losses $P_i - 1$. The last term comes from the sharing rule if several market makers charge the same best price.

2.2 Existence and Properties of Equilibria

Since market makers now compete in flat price schedules we are in the situation of a pure Bertrand competition. However, in contrast to the standard Bertrand case, the strategic behaviour of the insiders implies

Claim 1:

There exists no equilibrium in pure strategies.

The proof is immediate from the discontinuities arising from the additional terms in the profit function. We only sketch the line of reasoning: Whoever charges the lowest price faces the most favourable potential composition of uninformed and informed demand (i.e. $(1 - \mu) : \mu$). However when both market makers charge the same prices, the probability that the next customer is informed is higher than μ. The reason is that the informed trader will come in any case, whereas the uninformed is indifferent between both market makers. Thus it is always profitable to undercut until $P = \mu$. But if both traders charge these prices their probability to get informed traders is again too high and both are making expected losses. A deviation to $P = 1$ can guarantee then zero-profits. Thus, a pure strategy equilibrium cannot exist.

This non-existence arises formally as in the other well known example of non-existence with Bertrand competition, the Bertrand-Edgeworth case. Non-Existence occurs because of the discontinuity of the payoffs at *every* P with $P = P_1 = .. = P_n$. Even a slight change of prices produces a discontinuous shift in the expected composition of a market maker's clientele. If a market maker slightly undercuts a competitor, the probability that any actual trade comes from an uninformed trader increases sharply. A similar result in a bidding context has been observed in the first-price-auction literature with discrete signals (for the case of credit markets see Broecker (1990); see also footnote 1 of Milgrom (1979)). Since the informational content of the observation that somebody is willing to take one's offer depends discontinuously on prices, undercutting in prices may produce a big change in the conditional expectations of $\tilde{v}$. This source of non-existence of pure strategy equilibria seems to be particular to models of price setting with asymmetric information.

In the first-price auctions with common values the bidders usually differ, because a priori they have obtained different information on the true value of the asset. This information is then interpreted conditional on the fact that at given prices trade occurs. Special to our example is that the information on $\tilde{v}$ is contained in the traded quantities themselves, that is the information is not given exogenously but rather endogenously. We shall later show in Claim 4 that this implies non-existence even in the case in which we allow market makers to compete in arbitrary price schedules, and hence in the case in which a priori the quantities might take any value.

Fortunately there exists at least a mixed strategy equilibrium:

Claim 2:

For every $n > 1$ there exists a finite number of mixed strategy equilibria. A strategy constellation $(\tilde{P}_1, .., \tilde{P}_n)$ is an equilibrium if and only if: There are m active market makers $(m = 2, .., n)$ and $n - m$ inactive market makers (i.e. market makers, who put all weight on $P = 1$). The equilibrium distribution function is the same for every active market maker, it is atomless and defined on the interval $[\mu, 1]$ as

$$F(P) = 1 - \left(\frac{\mu}{1-\mu}\frac{1-P}{P}\right)^{\frac{1}{m-1}} \quad .$$

Proof:
Assume player i plays the pure strategy P_i. If $P_i < \mu$ we have always $E_i < 0$. Thus we only have to show that player i is indifferent between all $P_i \epsilon [\mu, 1]$, if the other players play the above mixed strategy.

Notice first that the probability that market maker i charges the lowest price is given by

$$(1 - F(P_i))^{m-1} \quad .$$

Since the mixed strategies of the players imply that the probability of two players setting the same price is zero, the points of discontinuity are eliminated. The expected profits are therefore given by

$$\begin{aligned} E_i &= [\mu(P_i - 1) + (1-\mu)P_i]\text{Prob}_{\{P_i < P_j \forall j \neq i\}} + \mu(P_i - 1)\text{Prob}_{\{\exists j \neq i | P_i > P_j\}} \\ &= [\mu(P_i - 1) + (1-\mu)P_i](1 - F(P_i))^{m-1} + \mu(P_i - 1)(1 - (1 - F(P_i))^{m-1}) \\ &= P_i - \mu - (1-\mu)P_i(1 - (1 - F(P_i))^{m-1}) \\ &= P_i - \mu - (1-\mu)P_i \frac{P_i - \mu}{(1-\mu)P_i} \\ &= 0 \quad . \end{aligned}$$

Furthermore, it cannot be profitable for a market maker $m+1$ to become active, since

$$(1-F(P))^m < (1-F(P))^{m-1}$$

and thus $E_{m+1} < 0$ for every $P < 1$.

Uniqueness is shown in the appendix.

The above demonstration reflects the force of Bertrand competition. Bertrand competition drives the average gains of every market maker just down to his expected losses, namely μ. The mixed strategy is then a combination of undercutting and loss reduction considerations. Low prices are charged with the hope to undercut, high prices are charged to limit potential losses. This also determines the range of possible prices. Since the market maker with the highest price will only obtain "lemons", there will be an incentive to circumvent losses and to charge $P = 1$. Since every market maker has this option, this immediately implies zero profits. Zero profits on the other hand imply that the lowest price which is charged in equilibrium, will be $P = \mu$, the zero-profit price of the market maker with the best price.

We investigate now the behaviour of the spread for a growing number n of potential market makers. A priori nothing has to happen. Since an equilibrium with m active market makers is an equilibrium for any $n \geq m$ potential market makers, introduction of new potential competitors may leave the actually played equilibrium unchanged. However, the set of equilibria becomes bigger.

This leads us to study the unique equilibrium with all n market makers active. An inspection of the equilibrium distribution function shows that the individual market maker's growing risk exposure induces a growing average spread of the market maker (the distribution functions move monotonically to the right with a growing n). Since he cannot control the quality of his demand, the possibility to trade with "bad" risks, i.e., informed traders, gets the bigger the more market makers are in the market. The reason being that the uninformed demand will only go to the cheapest market maker, whereas the informed trader will try to exploit every possibility to make profits and becomes the dominant player on the market. The only way to compensate for this higher risk is to set higher prices on average.

However, in order to judge the overall impact on the market it is important to investigate the behaviour of the market spread, i.e. the lowest spread which is charged to the uninformed traders. Since quantities are not split up in $|q| < 1$, it is important for the uninformed trader that the best attainable price P (which in this setting would be half of the market spread) is as low as possible.

A priori the behaviour of the random variable $\min_i(\tilde{P}_i)$ is not clear, since we have two opposing effects: on the one hand n is growing, and on the other hand the weight on the prices is shifted to the right. An inspection of the profit function shows however, that the lowest price is growing with n. A growing n and everything else equal would imply that the individual market maker has to do relatively more business with the insiders, which will induce losses. His only possibility to retain non-negative profit is to keep the probability to do profitable business at a given P constant, i.e., independent of the number of his competitors. But this probability is just the probability that the lowest price of his $n-1$ competitors is bigger than P. This is only possible, if the probability that the lowest price of the n competitors is bigger than a given P, is growing with n. Claim 3 gives a formal proof of this argument.

Claim 3:

In average the lowest available spread in the market grows with the number of active market makers. More precisely:

$$E(\min_i(P_i)) \nearrow \frac{\mu}{1-\mu} ln(\frac{1}{\mu}) \quad .$$

Proof: Since in the zero-profit condition

$$0 = [\mu(P_i - 1) + (1-\mu)P_i]\text{Prob}^n(P_i < P_j \forall j \neq i) + \mu(P_i - 1)\text{Prob}^n(\exists j \neq i \mid P_i > P_j)$$

the losses and gains are independent of the number of competitors, this must also be true for the according probabilities in the above expression. Because

$$\text{Prob}^n(P_i < P_j \forall j \neq i) = \text{Prob}^n(P_i < \min_{j \neq i}(P_j)) \quad ,$$

this means, that for every n $\min_{j \neq i}(P_j)$ has the same distribution and thus the same expectation independent of n. But this implies, that the minimum of all n price distributions has a higher expected value than in the according equilibrium with $n-1$ competitors, since the distribution function of $\min_j(P_j)$ is shifted monotonically to the right:

$$\begin{aligned}\mathrm{Prob}^n(\min_j(P_j) < P) &= \mathrm{Prob}^n(P_i < P) + \mathrm{Prob}^n(P_i \geq P)\mathrm{Prob}^n(\min_{j \neq i}(P_j) < P) \\ &= \mathrm{Prob}^n(P_i < P)\mathrm{Prob}^n(\min_{j \neq i}(P_j \geq P) + \mathrm{Prob}^n(\min_{j \neq i}(P_j) < P) \quad .\end{aligned}$$

Since the distribution of $\min_{j \neq i}(P_i)$ is independent of n, and the distribution of $\tilde{P}_i$ moves to the right with a growing n, this expression is obviously falling in n.

The upper limit can be formally calculated as

$$\begin{aligned}E(\min_i(\tilde{P}_i)) &= n \int_{\mu}^{1} P(1 - F(P))^{n-1} df(P) \\ &= \mu + \int_{\mu}^{1} (1 - F(P))^n \, dP \\ &= \mu + (\frac{\mu}{1-\mu})^{\frac{n}{n-1}} \int_{\mu}^{1} (\frac{1-P}{P})^{\frac{n}{n-1}} dP \\ &\nearrow \frac{\mu}{1-\mu} ln(\frac{1}{\mu}) \quad .\end{aligned}$$

The fact that the convergence is monotone in n follows by a straightforward derivation with respect to n.

2.3 The Case of a Monopolist Market Maker

Traditional wisdom would expect that a growing number of market participants should lead to lower prices through growing competition. In our model the higher risk exposure of the individual market maker works against this. Nevertheless we see both forces at work. While it is true that prices get worse with the number of active market makers, these prices are still better than the prices a monopolist would charge, namely P = 1. The Bertrand type price competition implies that the advantages of competition are already fully effective for two market makers (i.e., prices are driven down to the expected value) and for more market makers the risk exposure effect gets dominant.

However, this is not a general result and relies on the fact that in our example the uninformed demand is completely price inelastic (translated in our model by price-independent arrival probabilities). In different examples the risk exposure effect may dominate the competition effect to such an extent that the lowest spread is charged by a monopolistic market maker rather than several competing market makers. We will now develop such an example.

Let us assume that the potential aggregate demand of uninformed traders is highly price elastic, i.e., the number of uninformed traders willing to trade falls drastically with slight price movements. We could think of that as a "thin" market. We translate that in our model by assuming a different probabilistic structure of the arrival process. Since a informed trader does not care about prices, the probability that an informed trader comes next should grow with P. If we assume a very thin market, i.e., most of the noise traders are only willing to trade at prices around 0, the optimal price for the monopolist may be a very low P, because it implies the best expected mixture of informed and uninformed traders. In that case the monopoly profits will be very low. If two market makers want to share this market, their potential gains are so low that they are forced to put most of the weight on high P's to limit potential losses.

The rationale for a monopolistic dealer market would thus be: Whenever the possibilities of the monopolist to exploit the market are already limited to such an extent that the benefits of potential competition become relatively small, the presence of the risk exposure effect may render a monopolistic market the alternative strictly preferred by the "innocent" trader. In this example the limitation is given by an endogenous data of the market, its "thinness". One could, of course, also conceive external regulations which limit the monopolist's possibilities or a combination of both. An example for the latter could be the New York Stock Exchange, where specialists face both regulations and to a certain extent the competition of outside traders.

To be more precise about our example, assume $\tilde{v}$ as before and the following arrival process (where for simplicity we restrict ourselves again to symmetric bid and ask prices): If P is the smallest price offered by a market maker, the probability that an uninformed trader seller arrives next is given by $\frac{A(P)}{2}$ and that an uninformed buyer arrives by $\frac{A(P)}{2}$ as well. Accordingly the probability of an informed speculator arriving is given by $1 - A(P)$, where $A(P)$ is defined for a small $\epsilon > 0$ by

$$A(P) = \begin{cases} 1-\epsilon & \text{if } 0 \le P \le 2\epsilon \\ \dfrac{1-\epsilon}{1-2\epsilon} - \dfrac{1-\epsilon}{1-2\epsilon}P & \text{if } 2\epsilon \le P \le 1 \end{cases} .$$

With an analogous reasoning as before, the expected monopoly profits are calculated as

$$E = P - (1 - A(P)) \quad ,$$

and $P = 2\epsilon$ is the profit maximizing price.

With the techniques of Claim 2 one can easily calculate the mixed strategy equilibrium for two active players, namely

$$F(P) = \begin{cases} \dfrac{P-\epsilon}{(1-\epsilon)P} & \text{if } \epsilon \le P \le 2\epsilon \\ \dfrac{1}{2(1-\epsilon)} & \text{if } 2\epsilon \le P < 1 \\ 1 & \text{if } P = 1 \end{cases} .$$

The probability that in equilibrium both players play $P = 1$, is given by

$$Prob(P_1 = 1, P_2 = 1) = (\frac{1-2\epsilon}{2-2\epsilon})^2 > 2\epsilon \qquad \text{for small } \epsilon \quad .$$

Therefore the lowest average bid-ask-spread will be higher than the monopoly-spread 4ϵ. If there are more than two market makers the spread will become even worse.

3. The General Model.

More natural than the introduction of a regulator who determines the minimum quantity for which market makers are obliged to quote prices, is the assumption that the market makers set prices *and* quantities, or even more generally that they compete in price functions. Therefore we assume now that at the first stage of the game the market makers set price functions $P_i(q)$ and commit themselves to trade any quantity q which arrives in the second period for the price $P_i(q)$. The informed speculator's optimization problem

$$\max_{x_i} E_i = (v - P_i(x_i))x_i$$

resp. the uninformed trader's problem

$$\min \sum_{i=1}^{n} P_i(u_i)u_i \quad \text{where} \quad \sum_{i=1}^{n} u_i = u \quad ,$$

become then obviously much more complicated.

In the sequel we show, that the results of section 2 will not change, if we allow for arbitrary price functions at the first stage. To avoid trivial non-existence of equilibria, we will impose that the quoted price functions are such that the maximization problems of the second stage are solvable. For the sake of simplicity we will furthermore assume symmetric price schedules, i.e., $P(q) = -P(-q)$.

Assume that the n market makers propose price schedules $P_1, .., P_n$ at the first stage. The optimal strategy of the uninformed trader will then be a distribution $\Gamma_{\tilde{u}}(u_1, .., u_n)$, where the arguments are solutions of the maximization problem and the form of $\Gamma_{\tilde{u}}$ is determined by any given sharing rule. For the informed traders we can assume without loss of generality that in case of indifference they trade the smallest profit-maximizing quantities $\tilde{x}_1, .., \tilde{x}_n$. We can now show the generalizations of Claim 1 and Claim 2.

Claim 4:

There exists no equilibrium in pure strategies.

Proof:

The proof proceeds in two steps. We first assume that in equilibrium no mixed equilibrium will be played at the second stage, i.e., for every realization u of $\tilde{u}$ there is exactly one optimal strategy $(u_1, .., u_n)$ of the uninformed players, where $\sum_{i=1}^{n} u_i = u$. For this case we show that any function implying a strategy with $| u_i | < 1$ can be broken by a function which induces $| u_i | = 1$ and finally we apply Claim 1, which gives us non-existence for the latter case. In the second part we show that indeed there is no mixed strategy equilibrium in quantities. For symmetry reasons it is enough to look only at the case $u = 1$ and $v = 1$.

1) Assume that the uninformed trader resp. the informed trader trade u_i resp. x_i with market maker i. Profit maximization of the insider implies

$$(1 - P_i(x_i))x_i \geq (1 - P_i(u_i))u_i \quad .$$

If the inequality were strict, it would be profitable for the market maker to raise $P(x_i)$ until equality is achieved. This reduces the losses from the insiders and leaves the profits from outsiders unchanged. Thus, in equilibrium we always must have

$$(1 - P_i(x_i))x_i = (1 - P_i(u_i))u_i \quad .$$

In particular, for every $q > u_i$ we have

$$(1 - P_i(u_i))u_i \geq (1 - P_i(q))q \quad ,$$

and hence for $i = 1, .., n$

$$P_i(q) > P_i(u_i) \quad \text{for all } q > u_i.$$

Now assume that there is a market maker i with $0 < u_i < 1$ and that $P_i(u_i) = \min_j(P_j(u_j))$. Then it is profitable for market maker i to deviate by setting $P_i(q) = 1$ for $0 \leq q < 1$, and $P_i(1) = P_i(u_i) + \epsilon$, for ϵ sufficiently small. Since for all $j \neq i$ $u_j < 1$ and $P_j(q) > P_j(u_j)$ for all $q > u_j$, the other $n-1$ market makers cannot serve the whole market at the price $P_i(u_i)$, and market maker i will attract the whole informed and uninformed demand. It is easy to see that this will raise profits: Because originally the informed traders were indifferent between playing u_i and x_i, a deviation to $P_i(1) = P_i(u_i)$ will multiply by the same factor both the losses from the informed and the gains from the uninformed traders. Since profits are nonnegative, they will at least not decrease. Thus, in equilibrium only price schedules inducing $\mid u_i \mid = 1$ can be played. But the argument of claim 2 shows that in this case no pure strategy equilibrium exists.

2) To show that there is no mixed strategy equilibrium in quantities, assume that $u_i^1 < u_i^2$ are both profit maximizing solutions of the uninformed trader. We first notice, that the case of $0 = u_i^1$ is excluded by the usual undercutting argument. Thus assume $0 < u_i^1$. Then we have $P_i(u_i^1)u_i^1 = P_i(u_i^2)u_i^2$, because if not, market maker i could eliminate the less favourable case by raising the according price.

Furthermore, profit maximization of the insider implies

$$(1 - P_i(x_i))x_i \geq (1 - P_i(u_i^2))u_i^2 > (1 - P_i(u_i^1))u_i^1 \quad ,$$

and it is profitable to deviate for market maker i: Set $P_i(u_i^2) = 1$ and change $P_i(x_i)$, such that

$$(1 - P_i(x_i))x_i = (1 - P_i(u_i^1))u_i^1 \quad .$$

Hence, the only remaining candidates for an equilibrium are price functions, such that given a realization u there is exactly one optimal strategy $(u_i^1, .., u_i^n)$ for the outsiders.

Now we show, that the mixed strategy equilibrium of section 2 remains an equilibrium in the more general strategy space.

Claim 5:

For every $n > 1$ there exist mixed strategy equilibria. For every $m = 2,..,n$ there is an equilibrium with m active and $n - m$ inactive market makers (i.e. market makers, who put all weight on $P = 1$). The equilibrium distribution function is the same for every active market maker and atomless. It randomizes over the functions

$$P(q) = \begin{cases} P sign(q) & \text{if } \mid q \mid = 1 \\ sign(q) & \text{otherwise} \end{cases} ,$$

where $P \in [\mu, 1]$ is distributed as

$$F(P) = 1 - \left(\frac{\mu}{1-\mu}\frac{1-P}{P}\right)^{\frac{1}{m-1}} .$$

Proof: Assume, that market makers $i = 1, 2, ..m$ play the above strategies and that market maker 1 tries to deviate with a price function $P_1(q)$. This will induce the informed quantities x_1, with

$$(1 - P(x_1))x_1 \geq (1 - P(q_1))q_1 \quad \text{for all } q_1 \quad .$$

If $u = 1$, the uninformed quantities are given by

$$u_1 = \begin{cases} u_{min} & \text{if } (P_1(u_{min})u_{min} + 1 - u_{min}) \leq \min\limits_{i=2,..,n}(P_i) , \\ 0 & \text{otherwise} \end{cases} ,$$

where $u_{min} = \text{argmin}(P_1(u_1)u_1 + 1 - u_1)$. An analogous expression holds, if $u = -1$. Given the assumption of symmetry on the price functions, the profit functions can be expressed in terms of the positive quantities (as in section 2). Market maker

1's profits are given by

$$\begin{aligned}\pi_1 =&\mu(P_1(x_1)-1)x_1+(1-\mu)P_1(u_{\min})u_{\min}\left(1-F(P_1(u_{\min})u_{\min}+1-u_{\min})\right)^{m-1}\\ =&\mu(P_1(x_1)-1)x_1+\mu P_1(u_{\min})u_{\min}\frac{1-(P_1(u_{\min})u_{\min}+1-u_{\min})}{P_1(u_{\min})u_{\min}+1-u_{\min}}\\ \le&-\mu u_{min}+\mu P_1(u_{\min})u_{\min}\frac{1}{P_1(u_{\min})u_{\min}+1-u_{\min}}\\ \le&0\quad ,\end{aligned}$$

because

$$P_1(u_{\min})\le P_1(u_{\min})u_{\min}+1-u_{\min}\quad .$$

With a similar argument it is shown that also for an inactive market maker a deviation is unprofitable.

It is interesting that the proofs of both Claim 4 and Claim 5 rely on a particular effect: The market makers always have an incentive to bid for the *whole* market. They quote competitive prices for large quantities, rather than trying to share the market between them.

4. Discrete versus Continuous Models.

Although the pure Bertrand competition leads to an intuitive result even in the generalized framework of section 3, the role of the discreteness of the example for the non-existence result is not entirely clear. To investigate this question a bit further, we propose to introduce in the general model continuous random variables $\tilde{v}$ and $\tilde{u}$.

Assume that the liquidation value $\tilde{v}$ has a continuous distribution function with support $(-\infty, \infty)$, mean v_0 and variance s_v^2. Let the uninformed demand u be the realization of a random variable $\tilde{u}$, with continuous distribution function, support $(-\infty, \infty)$, mean 0 and variance s_u^2.

This model has a relation to the first-price-auction literature for divisible assets. There are, however, two crucial differences to the standard specification of such an auction (e.g. Wilson(1979)). First, we assume profit-maximizing behaviour of the traders at the second stage rather than a market-clearing-rule. Second, we introduce a cost-term in the market makers' profit function, namely the expected losses arising through insider trading. Both the expected gains from outsiders and the expected losses from insiders depend on the whole price function. For continuous random variables $\tilde{u}$ and $\tilde{v}$ this price dependence becomes non-trivial, and the auction-theoretic approach to reduce the problem to a pointwise maximization problem in $\mathbb{R}$ does not work any more.

A priori it is difficult to see how to treat the problem in the rather large set of all functions. In a first, heuristic step we will therefore reduce the strategy space of the market makers to a more tractable function space, the space of all linear functions.

Claim 6:

Let $n > 2$ and $0 < \mu < 1$. Assume furthermore that the market makers compete in linear price schedules $P_i(q) = a_i + b_i q$.
Then there exists an unique equilibrium in this strategy space. It is symmetric and given by

$$P(q) = v_0 + \frac{1}{2}\sqrt{\frac{\mu}{1-\mu}}\sqrt{\frac{n^3}{n-2}\frac{s_v}{s_u}}q \quad .$$

The equilibrium profits of the market makers are positive and are given by

$$E = \sqrt{\mu(1-\mu)}\sqrt{\frac{1}{n^3(n-2)}s_v s_u} \quad .$$

The spread for a given quantity $| q |$ is thus given by

$$S(| q |) = \sqrt{\frac{\mu}{1-\mu}}\sqrt{\frac{n^3}{n-2}\frac{s_v}{s_u}} | q | \quad .$$

The calculations are straightforward and are contained in the appendix. The result is analogous to Kyle(1989), who considers a game in which insiders, market makers (called uninformed speculators) and uninformed noise traders draw simultaneously. Fixing the strategy space in the first stage has the same consequences as the Nash-assumption in a simultaneous play. The equilibrium has the intuitive properties familiar from Kyle's work: the spread is growing with the asset's volatility and falling with the volatility of expected turnover.

The linear strategies create a somewhat puzzling result concerning the number of market makers: Although the spread of the individual market maker becomes larger (i.e. his price schedule becomes steeper), the market spread is falling with the number of competitors. Since the price schedules are linear functions, an uninformed trader with demand u will always have an incentive to split up his demand between the market makers to reduce his total costs: in equilibrium his demand is given by

$$u_i(\tilde{u}) = \frac{\tilde{u}}{n} \quad \text{for all } i = 1,..,n \quad .$$

Thus, the costs of a noise trader with demand u (i.e., the market spread faced by him) is given by

$$\begin{aligned} C(u) &= v_0 u + b(\frac{u}{n})u \\ &= v_u + \frac{1}{2}\sqrt{\frac{\mu}{1-\mu}}\sqrt{\frac{n}{n-2}\frac{s_v}{s_u}}u^2 \quad , \end{aligned}$$

and this expression is falling in n. Competition in linear price functions is less brutal than Bertrand competition, because it does not immediately reduce profits to zero. The stronger competition becomes (i.e. the more competitors are in the market), the more surplus has to be given up by the market makers. They can only partially compensate the adverse selection effect by widening their spreads. It is this effect, which makes the uninformed traders better off with more competition.

Nevertheless it is quite difficult to interpret this result, since the importance of the linearity assumption is unclear. The linearity creates somehow artificially positive profits of the market makers, and these positive profits in turn will be diminished by a growing intensity of competition. However, it is open what happens in equilibrium if we allow for a larger strategy space. It remains to be solved in the general case, how to treat an auction for shares in the absence of the usual market clearing rule with strategic behaviour of both insiders and uninformed traders at the second stage.

5.Discussion.

This chapter tries to give an explicit model of competition in financial markets with asymmetric information. It replaces the usual zero-profit condition imposed on the market makers with a price setting mechanism of several utility maximizing auctioneers. We have seen that in a world with asymmetric information among traders and the auctioneers (i.e., the market makers) the strategic interaction between the price setters may create severe problems of non-existence of pure strategy equilibria. This phenomenon arises because the information content of traded quantities depends directly on the quoted prices. For a given quantity, changes in prices induce a revision in the forecasts of the future realization of the risky asset conditional on the observation that this quantity is traded. We have shown both in the special case of Claim 1 with fixed quantities and in the general case of Claim 4 with arbitrary quantities that this leads to the non-existence of pure strategy equilibria. However, a unique mixed strategy equilibrium always exists. It has the property that an increase in the number of market makers induces a higher market spread, because the risk exposure of the individual market maker increases.

Behind our analysis of the quality of a market lies an implicit assumption concerning the nature of liquidity traders and insiders. Insiders in our model are best seen as speculators who have acquired on a more or less illegal way precise information about the risky asset. This chapter takes the approach that one should not be concerned by the welfare of these speculators, but only by the welfare of the traders without access to the private information.

The implication of our model is then that a Stock Exchange deciding to organize trading via market makers with the obligation to quote bid- and ask- prices at *every* time, should limit the number of market makers to one or at most to two market makers. The argument is, that a bidding contest between risk neutral market makers implies zero profits (i.e. more precisely, zero excess returns), and that the incentive to bid always for the whole market will induce a higher risk the more market makers are present.

However, one could argue, that assuming risk neutrality biases the case in favour of our proposition. The attempts to derive results concerning the market structure from the assumption of risk averse market makers (Ho and Stoll (1983), Grossman and Miller (1988)), seem to indicate that the spread should fall with the number of market makers. Ho and Stoll consider imbalances in the market market makers inventories arising through trade and argue that competition prevents spreads to be set by the market makers with the less favourable inventory position. Grossman and Miller use the argument that a higher number of market makers will lead to a better allocation of the fundamental risk (which plays no role in a risk neutral setting), and thus to lower spreads.

From a regulator's point of view one could oppose to these claims that the same goal could be achieved by designating a single market maker with a large capital basis and low risk aversion. Given the negative externalities of competition arising through insider trading, this should still be the best solution.

Moreover, from a descriptive point of view the robustness of these models is unclear. They neither incorporate the risk exposure effect in their models nor do they model the interaction between market makers as a strategic game. To reach more precise conclusions about the market structure of markets with designated market makers, a treatment of the risk exposure *and* the risk diversification effect should therefore be a natural extension of our game.

This will in particular imply a careful treatment of the obligation to deal with competing market makers. On the one hand, the inter dealer trade is the basic means to redistribute fundamental risk. On the other hand, market makers should be reasonably treated as 'quasi-insiders". Their constant presence on the market gives them certainly an informational advantage over outside investors. Accordingly, since market makers can identify their competitors as potentially better informed traders, there is an obvious incentive to discriminate against them. The outcome of the interplay of fundamental risk diversification and insider risk exposure will then crucially depend on the possibilities to redistribute large quantities profitably among the market makers on the hand, and to quote only prices for small quantities to avoid the insider risk on the other hand. Dealing with a competing market maker always involves two possibilities: the redistribution of fundamental risk in a profitable way for both dealers and the danger that the other market maker just uploads his informed trading on the other market makers.

The subtle strategy choices of the market makers were demonstrated in late 1988 by the developments on the London Stock Exchange. Some dealers had given up the practice of quoting prices for relatively large quantities. Instead, they only quoted prices for the minimum quantity (for which they are able to offer a narrower spread, since both fundamental and insider risk are falling). Implicitly they agreed to stick to the quoted prices for much bigger quantities as long as the clients were not professional or semi- professional traders; the latter could not trade at all or only for less favourable prices. Yet, given the market rule that market makers are only obliged to deal with each other up to the minimum quantity quoted by them, they were not able any more to redistribute the fundamental risk. Meanwhile, the obligation to deal with competing market makers has been abolished completely. Reducing fundamental and insider risk at the same time seems to be impossible.

This example shows that a richer strategic analysis incorporating risk aversion and the problems of inter dealer trade is certainly the most interesting extension of our approach.

Appendix.

1)Proof of uniqueness of Claim 2:

The proof proceeds in two steps. Step 1 shows that no mixed strategy equilibrium with atoms is possible. Step 2 establishes that any atomless mixed strategy equilibrium is the one given in Claim 2.

Step 1: Assume first that market maker i puts in equilibrium positive weight α on a price $\bar{P}$ with $1 > \bar{P} > \mu$. First we consider the case that the other market makers do not charge prices in an interval $[\bar{P}, \bar{P} + \epsilon]$, but charge prices above. Then, it would be profitable for the market maker to raise $\bar{P}$. On the other hand, there must exist a constant $\epsilon > 0$ such that the other market makers do not charge prices in the interval $[\bar{P}, \bar{P} + \epsilon]$. If not, they should exploit the discontinuity of the pay-offs at $\bar{P}$ and undercut. But if the other market makers only charge prices below $\bar{P}$ it cannot be optimal to put positive weight on a loss making strategy.

If $\bar{P} = \mu$ then there exists for every weight α a positive number $\bar{\epsilon}$, such that the other players cannot charge profitably prices $P_j \in [\mu, \mu + \bar{\epsilon}]$, because for a $P_j \in [\mu, \mu + \bar{\epsilon}]$

$$E_j < P_j - \mu - (1-\mu)P_j Prob(P_i < P_j) < (\mu+\bar{\epsilon})(1-\alpha(1-\mu)) - \mu < 0 \quad \text{for small } \bar{\epsilon} \quad .$$

Thus it is always profitable to raise $\bar{P}$ from μ to $\mu + \bar{\epsilon}$.

Step 2: We show now that every equilibrium with $2 \leq m \leq n$ active market makers and no atoms in $[\mu, 1)$ has the given form. Denote by $F_1, .., F_m$ the distribution functions in equilibrium. The proof proceeds in three parts. We first show that every market maker makes zero profits in equilibrium. Second, we show that every symmetric equilibrium (i.e., $F_i = F_j$ for all $i \neq j$) is the one of Claim 2. Finally, we show that every equilibrium must be symmetric.

a) If equilibrium profits of market maker i are given by E_i, we know from the definition of a mixed strategy equilibrium that for every P which in equilibrium is played by market maker i

$$(1.1) \qquad E_i = P - \mu - (1-\mu)P\left(1 - \prod_{j \neq i}(1 - F_j(P))\right) \quad .$$

To show that $E_i = 0$ for every market maker, we have to show that for every market maker $F_i(P) < 1$ for every $P < 1$, i.e., that each market maker plays $P = 1$ in equilibrium. Therefore assume that there is a $\bar{P}_j < 1$ such that for some market maker j $F_j(\bar{P}_j) = 1$. If j is chosen such that $\bar{P}_j$ is the lowest of all prices fulfilling this condition for some market maker, then there will be a market maker i $(i \neq j)$ and a P with $1 > P \geq \bar{P}_j$ such that

$$\begin{aligned} E_i &= P - \mu - (1-\mu)P\big(1-0\big) \\ &= \mu(P-1) \quad . \end{aligned}$$

Thus, equilibrium profits of market maker i would be negative which is impossible in equilibrium.

b) Assume that the equilibrium is symmetric, i.e.,

$$F(P) = F_1(P) = .. = F_m(P) \quad \text{for all } P \in [\mu, 1] \quad .$$

Since equilibrium profits are zero, we know from (1.1) that for all P which are played in equilibrium

$$0 = P - \mu - (1-\mu)P\big(1 - (1 - F(P))^{m-1}\big) \quad .$$

Solving this equation for F and using the continuity of F we obtain precisely the solution of Claim 2 for all $P \in [\mu, 1]$.

c) Thus, it only remains to show that every equilibrium is symmetric. Assume the contrary and define

$$a := \sup\{P \mid F_i(P) \neq F_j(P) \text{ for some } i \text{ and for at least one } j \neq i\} \quad .$$

Clearly, $\mu < a \leq 1$ and $F_i(a) = F_j(a)$ for all $j \neq i$. Moreover, the continuity of the F's implies that there is a positive number ϵ such that for every $P \in [a-\epsilon, a]$ $F_i(P) \neq F_j(P)$ for some i and for at least one $j \neq i$.

The idea of the proof is now roughly as follows. With (1.1) we show that for prices slightly below a some market makers cannot play strictly increasing F's. But if some market makers do not put any weight on these prices, it cannot be optimal for *any* market maker to charge these prices in equilibrium. The rest of the proof makes this idea precise.

We first show that there exists a $\epsilon > 0$, such that in $[a-\epsilon, a]$ there are k ($k \geq 1$) market makers with F_i constant and $m-k$ market makers with F_i strictly increasing. If all F_i were increasing, then equation (1.1) gives for all $P \in [a-\epsilon, a]$

$$(1.2) \qquad 0 = P - \mu - (1-\mu)P\big(1 - \prod_{j \neq i}(1 - F_j(P))\big) \text{ for all } i = 1, .., m \quad .$$

Pairwise subtraction of equation (1.2) and subsequent division implies that $F_i(P) = F_j(P)$ for all $i, j = 1, .., m$, which is impossible by the definition of a.

Thus, assume wlog. that the k market makers with constant F_i are numbered as $i = 1, .., k$. The definition of a mixed strategy equilibrium and (1.2) imply then for every $P \in [a-\epsilon, a)$

$$(1.3) \qquad 0 \geq P - \mu - (1-\mu)P\big(1 - (1 - F_1(a))^{k-1} \prod_{j=k+1}^{m} (1 - F_j(P))\big)$$

and for $i = k+1, .., m$

$$(1.4) \qquad 0 = P - \mu - (1-\mu)P\big(1 - (1 - F_1(a))^{k} \prod_{j=k+1, j \neq i}^{m} (1 - F_j(P))\big) \quad .$$

But by construction a is chosen such that for all $P \geq a$ $F_i(P) = F_j(P)$ for all $i \neq j$. Since F_{k+1} is increasing and F_1 constant in $[a-\epsilon, a]$, we obtain for $P \in [a-\epsilon, a)$

$$1 - F_{k+1}(P) > 1 - F_1(a) \quad ,$$

and thus from (1.3)

$$0 > P - \mu - (1-\mu)P\big(1 - (1 - F_1(a))^{k} \prod_{j=k+2}^{m} (1 - F_j(P))\big) \quad ,$$

which is a contradiction to (1.4). Hence, a is not well defined, i.e., every mixed strategy equilibrium must be symmetric.

2) Proof of Claim 6:

The proof consists of three parts. We first solve the second stage of the game and derive the market makers' profit functions. Part 2 constructs the equilibrium and part 3 shows the uniqueness of the equilibrium.

Part1: Given the linear price strategies and the resulting quadratic profit functions, it is easy to calculate the optimal responses in period 2. The informed trader will choose as a response to prices

$$P_i(q) = a_i + b_i q$$

the strategy

$$x_i = (v - a_i)/2b_i,$$

which is the solution to his first order condition

$$v - a_i - 2b_i x_i = 0 \quad .$$

The first order conditions of the uninformed are

$$2b_i u_i - 2b_n (u - \sum_{j=1}^{n-1} u_j) = a_n - a_i \quad , \text{for } i = 1..,n-1 \quad .$$

The general solution to this linear problem is given by

$$u_i = A_i + B_i u \quad ,$$

where

$$A_i = \frac{1}{2} \frac{\sum_{j \neq i} (\prod_{k \neq j,i} b_k)(a_j - a_i)}{\sum_{j=1}^{n} \prod_{k \neq j} b_k}$$

and

$$B_i = \frac{\prod_{k \neq i} b_k}{\sum_{j=1}^{n} \prod_{k \neq j} b_k} \quad .$$

(To see this, just put the expressions in the first order condition.)

Denote by $\tilde{u}_i$ the strategy $u_i(\tilde{u}, a_1, .., a_n, b_1, .., b_n)$. In contrast to the insider demand the uninformed demand is a function of all prices. Knowing these responses we can now treat the price competition in the first step. The market maker tries to anticipate the random demand in the next period given all the prices of the other market makers. The future demand will be given by

$$\tilde{q}_i(P_1, .., P_n) \quad = \quad \begin{cases} x_i(\tilde{v}, P_i) & \text{with prob. } \mu \\ u_i(\tilde{u}, P_1, .., P_n) & \text{with prob. } (1-\mu) \end{cases} \quad .$$

We assume that the market makers are risk neutral expected ex-post-profit maximizers. Given the prices of their competitors they will choose a_i and b_i to maximise

$$E_i = \mu \ E_v((P_i(\tilde{x}_i) - \tilde{v})\tilde{x}_i) + (1-\mu)E_u((P_i(\tilde{u}_i) - v_0)\tilde{u}_i) \quad .$$

The expectations are formed with respect to the subscript random variable. Formally the expression can be derived by using Bayes' law. Intuitively the formula is clear. With probability μ the next trader is informed and in this case the market maker will earn $P_i(\tilde{x}_i)\tilde{x}_i$ and lose $\tilde{v}\tilde{x}_i = x^{-1}(\tilde{x}_i)\tilde{x}_i$ to the informed trader. With probability $(1-\mu)$ the trading contains no information and then his losses will just be $v_0\tilde{u}_i$.

Since the strategies of the traders are linear, the arguments of the integrals are quadratic and this enables us to express the profit functions as simple functions in the variances of the asset and the noise. The first integral can be written as

$$
\begin{aligned}
E_v\left((a_i + b_i\frac{\tilde{v}-a_i}{2b_i} - \tilde{v})\frac{\tilde{v}-a_i}{2b_i}\right) &= -\frac{1}{4b_i}E_v\left((\tilde{v}-a_i)^2\right) \\
&= -\frac{1}{4b_i}E_v\left((\tilde{v}-v_0)^2 + (v_0-a_i)^2 + 2(\tilde{v}-v_0)(v_0-a_i)\right) \\
&= -\frac{1}{4b_i}s_v^2 - \frac{1}{4b_i}(v_0-a_i)^2 \quad .
\end{aligned}
$$

Because the mean of $\tilde{u}$ is zero, the second integral is simply equal to

$$(a_i + b_i A_i - v_0)A_i + b_i B_i^2 s_u^2 \quad ,$$

and combining these two expressions gives

$$E_i = -\mu\frac{1}{4b_i}(v_0-a_i)^2 + (1-\mu)(a_i + b_i A_i - v_0)A_i - \mu\frac{1}{4b_i}s_v^2 + (1-\mu)b_i B_i^2 s_u^2 \quad .$$

Having obtained this simple analytical expression for the profit functions, we are now able to show the existence result for a Nash-equilibrium in prices.

Part 2: We first show that $a_i = v_0$ for all market makers. If all the market makers are playing $a_i = v_0$ a deviation can never be profitable regardless of the b_i played, since then

$$-\frac{1}{4b_i}(a_i - v_0)^2 < 0 \quad \text{and}$$

$$(a_i + b_i A_i - v_0)A_i < \frac{1}{2}(a_i - v_0)A_i < 0 \quad .$$

To see the last inequality notice that

$$b_i A_i^2 < \frac{1}{2}(v_0 - a_i)A_i \quad .$$

We obtain that in equilibrium both terms in the profit function arising from the intercept are equal to zero.

We construct now the symmetric equilibrium in the slopes b_i. Assume wlog. that the market makers $i = 2, .., n$ are playing the strategy

$$P(q) = v_0 + bq$$

and that market maker 1 plays the strategy

$$P_1(q) = v_0 + aq \quad .$$

The profit function for market maker 1 is then given by

$$E_i = -\frac{\mu}{4}\frac{s_v^2}{a} + (1-\mu)\frac{ab^2}{((n-1)a+b)^2}s_u^2 \quad .$$

Derivation with respect to a gives

$$\frac{\partial E_i}{\partial a} = \frac{\mu}{4}\frac{s_v^2}{a^2} + (1-\mu)\frac{b^2((n-1)a+b) - 2(n-1)ab^2}{((n-1)a+b)^3}s_u^2 \quad .$$

For b to be an equilibrium the profits of market maker 1 have to be maximal at $a = b$, i.e.

$$\begin{aligned}\frac{\partial E_i}{\partial a}_{|a=b} &= \frac{\mu}{4}\frac{s_v^2}{b^2} + (1-\mu)\frac{nb^3 - 2(n-1)b^3}{n^3 b^3}s_u^2 \\ &= \frac{\mu}{4}\frac{s_v^2}{b^2} + (1-\mu)\frac{2-n}{n^3}s_u^2 \\ &= 0 \quad .\end{aligned}$$

(Notice that for $n = 2$ there is always an incentive to overcut. Together with the proof that every equilibrium must be symmetric (see below), this establishes non-existence for $n = 2$.)

Solving the above equation for b gives the desired expression if $n > 2$. Furthermore, a second derivation gives

$$\frac{\partial^2 E_i}{\partial a^2} = -\frac{\mu}{2}\frac{s_v^2}{a^3} + (1-\mu)\frac{b^2(n-1)^2(2a-4b)}{((n-1)a+b)^4}s_u^2 \quad ,$$

which shows that $a = b$ is actually the maximum.

Part 3: Assume that an equilibrium is given by $((\bar{a}_1, \bar{b}_1), ..(\bar{a}_n, \bar{b}_n))$. We show that always $\bar{a}_i = v_0$ and $\bar{b}_1 = ... = \bar{b}_n$.

To see the first assume wlog. that $\bar{a}_1$ is the smallest and $\bar{a}_n$ the biggest intercept. If $\bar{a}_1 = \bar{a}_n$ the claim is obvious. Assume therefore that $\bar{a}_1 < \bar{a}_n$. If no profitable deviation from $\bar{a}_n$ shall be possible we must have

$$(\bar{a}_n + \bar{b}_n \bar{A}_n - v_0)\bar{A}_n > 0 \quad ,$$

(if not a deviation to v_0 is always profitable) and since $\bar{A}_n < 0$ this gives

$$(\bar{a}_n + \bar{b}_n \bar{A}_n - v_0)) < 0 \quad .$$

On the other hand

$$(\bar{a}_n + \bar{b}_n \bar{A}_n - v_0) > \bar{a}_n + \frac{\bar{a}_1 - \bar{a}_n}{2} - v_0 \quad ,$$

and we get

$$0 > \frac{\bar{a}_1 + \bar{a}_n}{2} - v_0 \quad .$$

If no profitable deviation from $\bar{a}_1$ shall be possible we must have

$$(\bar{a}_1 + \bar{b}_1 \bar{A}_1 - v_0)\bar{A}_1 > 0$$

and since $\bar{A}_1 > 0$ this gives with a similar reasoning as before

$$0 < (\bar{a}_1 + \bar{b}_1 \bar{A}_1 - v_0) < \bar{a}_1 + \frac{\bar{a}_n - \bar{a}_1}{2} - v_0 \quad ,$$

and hence the contradiction

$$0 < \frac{\bar{a}_n + \bar{a}_1}{2} - v_0 \quad .$$

To show the second part observe first that $\bar{b}_i = 0$ is impossible since then the profits are $-\infty$. It follows that every $\bar{b}_i$ is an interior solution of

$$\frac{\partial E_i}{\partial b_i}_{|b_i = \bar{b}_i} (\bar{b}_1, ., b_i, ., \bar{b}_n) = 0 \quad .$$

Define now $P_i = \prod_{j \neq i} b_j$ and $\sum = \sum_{i=1}^{n} P_i$. Calculation of the derivatives gives

$$\begin{aligned}\frac{\partial E_i}{\partial b_i}_{|b_i = \bar{b}_i} &= \frac{\mu}{4}\frac{s_v^2}{\bar{b}_i^2} + (1-\mu)\frac{P_i^2(2P_i - \sum)}{(\sum)^3} s_u^2 \\ &= 0 \qquad \text{for all i.}\end{aligned}$$

Rearranging this expression we obtain

$$\frac{\mu}{4(1-\mu)}\frac{s_v^2}{s_u^2}(\textstyle\sum)^3(\prod_{j=1}^{n} \bar{b}_j) = \sum -2P_i \quad ,$$

which implies that for all i, that $P_i = const$(independent of i). By dividing the different P_i's we get the desired result.

CHAPTER 3

INSIDER TRADING AND THE ALLOCATION OF RISKS

1. Introduction.

A major issue in recent debates on the prosecution of insider trading has been the conflict between the broad concept of violation of a general equity rule of shareholders and the more restricted concept of breach of fiduciary duty. Under the first the mere possession of inside information and its subsequent exploitation is sufficient for prosecution. Insider trading is conceived as a breach of an elementary principle of fairness among shareholders which includes in particular equality with respect to information. Under the second concept trading on the grounds of different information per se is not regarded as sufficient for prosecution. Instead it is required that the inside information in question has been obtained through a breach of fiduciary duty (e.g. between a manager and his shareholders).

The general fairness rule has been criticized mainly for practical reasons. First it is often hard to define who really was the victim of an insider transaction, since many stock market transactions occur via professional intermediaries and not directly between two private parties. It is also often very difficult for a court to define ex post what exactly was superior information and whether the accused really had this information or whether he had just a good intuition. Moreover, thought to its logical end, an absolute prohibition of trading on undisclosed information would eliminate any trading incentive except for liquidity reasons and hence stock prices might (at least without a very costly constant disclosure policy) become very poor indicators of recent developments.

This paper sets out to investigate whether apart from fairness considerations the principle of equality of shareholders with respect to information can also be supported by additional arguments relating to economic efficiency. Therefore, it is useful to remember that any justification of the prosecution of insider trading relies to some extent on the fear that with unrestricted insider trading the capital markets would break down. Investors will lose their confidence that markets are run in a fair and honest way and stay away from these markets. However, it is not entirely clear how this might occur. There are several ways to undermine investors' confidence and precipitate the breakdown of markets. One possibility would be moral hazard on the part of managers and advising financial or legal professionals, which could lead to delayed, artificial or manipulated release of information. Both corporate and financial insiders can expect higher rate of returns than outside investors; this clash of interests might produce a misallocation of resources or underinvestment through adverse selection effects. Such an erosion of confidence through moral hazard and adverse selection would give a rationale for the prosecution of insider trading based on the concept of fiduciary duty.

An argument relating equity among shareholders to the breakdown of markets is much broader in its scope. It implies that the mere presence of expected information differentials on secondary markets deters the financing of new investments. Confidence in markets and its economic consequences are not related to undisclosed information of traders with whom one is linked through a relation of trust, but rather to information differentials *per se*. This chapter argues that such an effect might arise through changes in the allocation of risk due to insider activity. However, we will also see that, although insider trading influences investment activity, the direction of this influence is ambiguous.

2. An Outline of the Argument.

We noted already that the general fairness argument is an argument concerning trading on secondary markets. In modelling these markets we are faced with the problem that in a world without gains from trade, i.e., a world in which trading can only be generated by differences in information, there is no scope for insiders to profit at the expense of outsiders. In fact, the no-trade theorem of Milgrom/Stokey (1982) states that an outsider should always anticipate that at a price for which the insider is willing to trade, it must be profitable for the insider and hence unprofitable for himself. Since there are no gains from trade he will consequently refrain from any trade and the insider cannot inflict any harm on him. In order to overcome this problem one has to model a trading process with some form of gains from trade, which makes people willing to trade even in the presence of informational asymmetries.

One way to do this is to consider a world with risk averse investors who differ in their endowments. Hirshleifer (1971) argued that under these circumstances investors generally have an interest that information is revealed *after* a first round of trade. The argument is essentially an insurance argument. Investors want to be optimally insured against risk before the event occurs; afterwards insurance does not make sense. Applying this reasoning to our problem, insider trading will drive prices up to the true future realization of the risky asset, i.e., outside investors buy resp. sell at high prices if returns are high and at low prices when returns are low. But if prices already reflect most of the information on returns, the remaining profit possibilities of the outsiders are accordingly low. Hence, insiders harm outsiders through their influence on prices and the resulting premature resolution of uncertainty. To make this argument precise, one must in general control for additional income effects of new information.

A second way consists in assuming that investors face sometimes liquidity constraints of some form and just *have to* sell. Intuitively one would expect that this class of people should suffer most from insider trading. One possibility to model such a constraint is the assumption of a finite lifetime, which forces investors to liquidate their positions when they are old.

As a simple means to model this idea, we will study a three-period-world with overlapping generations in which investment is longer lived than the original investors. At date 0 entrepreneurs invest in a risky venture and finance this investment by issuing shares to the first young generation. Hence, young investors buy the risky assets at date 0 and in the absence of bequests they try to sell their shares to the next generation of young investors at date 1 before dying. This creates the need for a secondary market for this share at which both generations have an interest in trading. At date 2 the risky asset pays dividends to the second generation and is liquidated. We assume that at date 1 there are insiders in the secondary market and investigate the effect of their trading activity on both investor generations and initial investment.

The overlapping generations framework allows us to study both the effects of the Hirshleifer effect and the effect on liquidity traders. In principle, for the young buyers of generation 1, insider trading reduces their profit possibilities, because insiders drive prices up to the asset's true value. Given the capital stock of the economy, outsiders of generation 1 would prefer insiders to abstain from trading. Moreover, we are able to study the impact of insider trading on the first investor generation which finances the investment. We can study the effect of expected insider trading on both this generation and the initial investment, i.e., the capital stock of the economy.

The amount of ex-ante investment is influenced by insider trading through the prices at date 1. From an ex ante viewpoint the effect of insider trading on initial investors is to render future prices more risky. That is, the problem for investors in a new share issue will be that in case of an early liquidation of their positions they will face a higher variability of prices if informed traders are active. If risk averse shareholders anticipate this higher liquidation risk correctly, they will only be willing to hold the asset if the original issuer compensates them for the risk, i.e., the initial issue price must fall and as a consequence the investment might even not be undertaken. At a first glance this additional negative effect of insider trading on the cost of new capital seems to make the case against insider trading very strong. But while this first effect, i.e., that insider trading makes holding the asset riskier, might be expected, there is also a countereffect to this. Since share prices should be on average equivalent to the expected value of future returns minus a risk premium, the lower uncertainty concerning the future returns as a result of insider trading should lead to smaller risk premia of the prices on future secondary markets. Thus, with insider trading in the future, the expected mean of share prices should be higher than without informed trading. The anticipation of this second effect will induce a rise of the initial issue prices. A priori the overall outcome of the two opposed effects is unclear. The point we shall focus on is that insider trading changes the relation of expected mean returns and risk of the risky asset for different classes of investors in a different way. Shifting the risk from the final realization of the asset to the interim prices is not a neutral operation. Depending on how we model the markets, investors might appreciate lower uncertainty more than they dislike higher price volatility and vice versa.

In this paper we will investigate in detail how a given market structure will influence the overall effect of insider trading. Section 3 presents the basic model and studies the impact of premature arrival of new information on the allocation of risks among different investor generation. In particular, we explore the consequences of an unequal willingness to bear risk on the part of the different generations of investors. Section 4 considers several models in which market frictions give insiders an informational advantage over outsiders. After the study of a model with outside investors who behave "irrationally", we deal with models with exogenous and endogenous noise preventing prices from being fully revealing. It will turn out that depending on the interaction among information revelation and the chosen market frictions the effects on initial investment are ambiguous. Both less and more investment may occur if insider trading is expected in the future. Section 5 discusses in detail the welfare analysis of the most interesting of these models, the specification of an asset market with exogenous noise along the lines of Grossman/Stiglitz (1980). Section 6 concludes.

A final word concerns the particular way insider trading is modelled in this paper. Throughout the paper all the investors, including the insiders, are assumed to be risk averse. Furthermore, every investor and even the insider is small in comparison to the market as a whole and therefore acts competitively, i.e., as a price taker. This obviously conflicts with the popular image of an insider as a large trader who acquires important information shortly before it becomes public anyhow (and is hence assumed to be risk neutral and to act strategically). Instead, in this paper I want to focus on the impact of information differentials as such, regardless of the nature of the information and the way it has been acquired. Even then it is of course arguable whether a framework with price taking can ever be regarded as an adequate modelling of insider trading. (See Kyle (1989) and Gale/Hellwig(1988)). A more thorough discussion of the implications of a model with strategically acting insiders is beyond the scope of this paper.

The basic trade-off analyzed in the sequel seems *per se* to be independent of insiders being modelled as price takers rather than taking the price effects of their actions into account when submitting market or limit orders (demand functions). It relies, however, essentially on three ingredients: That all investors submit demand functions (rather than compete in prices as in the last chapter), that all investors are risk averse, and that prices are separating in the sense that they are at least somewhat responsive to information. Separating equilibria in strategic games are, of course, quite different from the equilibria in a setting with price taking agents. In this paper we only focus on the welfare implications of prices which move with the inside information and neglect additional inefficiencies arising through the exercise of oligopolistic power and the insider's incentive constraints. (See Gale/Hellwig(1988), Laffont/Maskin(1990).) However, issues linked to risk neutrality, pooling equilibria (in which prices are independent of information) and competition in prices can meaningfully only be analyzed in a strategic setting.

3. The Basic Model.

Let us consider a 3-period model with two investor generations and an original entrepreneur generation. The first investor generation (called generation 0), represented by the continuum $[0,1]$, is active at date 0 and wants to sell all its assets at date 1 before dying. The second investor generation (called generation 1), which again is represented by $[0,1]$ is born at date 1 and can trade at date 1 with the first generation. At date 2 all assets are liquidated, the second generation consumes and dies.

The economy involves two assets, a riskless asset with a net interest rate 0, serving as a numeraire, and a risky asset, i.e. shares of a firm. At the beginning of date 0 the shares of the firm are owned by the old entrepreneurs who die at date 0 and have no other choice than selling the shares to the young investors of generation 0. The going concern value of the firm at the beginning of period 0 is normalized to 0. The firm however has the option to invest in a risky project with returns after two periods. For simplicity we model the investment decision as a simple Yes/No decision: an investment of c per share will result in future per share returns $\tilde{u}$ at date 2. The decision rule for the firm is assumed to take the very simple form:

$$\begin{array}{c} \text{Invest iff } P_0 \geq c \\ \text{Do not invest iff } P_0 < c \quad , \end{array}$$

where P_0 denotes the market value of a share after the issue, i.e., the price which balances generation 0's per capita demand for the share with the per capita supply $\bar{x}$. If the investment takes place, the stock exchange opens again at date 1, the old generation 0 submits its price inelastic supply of shares, the young generation 1 submits its demand and we denote the market clearing equilibrium price at period 1 by P_1. At the beginning of period 2, the firm pays $\tilde{u}$ and is liquidated.

Generations 0 and 1 are assumed to be expected utility maximizers with constant absolute risk aversions a_0 and a_1, i.e., they choose an optimal quantity X of the risky asset in order to maximize

$$E(U_i(X)) = 1 - E(exp(-a_i\tilde{W}_i(X))) \qquad i = 0,1 \quad ,$$

where $\tilde{W}_i$ denotes the end-of-life wealth. Their initial wealth is W units of the riskless asset. Hence, $\tilde{W}_i$ is given by

$$\tilde{W}_0(X) = W + (\tilde{P}_1 - P_0)X \quad \text{resp.} \quad \tilde{W}_1(X) = W + (\tilde{u} - P_1)X \quad .$$

To make the model tractable we will assume that $\tilde{u}$ is a normally distributed random variable with two independent components

$$\tilde{u} \sim \tilde{\theta} + \tilde{\epsilon}, \quad \text{where } \tilde{\theta} \sim \mathrm{N}(E(\tilde{\theta}), \sigma_\theta^2), \quad \tilde{\epsilon} \sim \mathrm{N}(0, \sigma_\epsilon^2).$$

Investors at date 0 are assumed to have no further information about $\tilde{u}$ than the a priori distribution. Whereas for simplicity we assume that at the date of investment no informational asymmetries exist, we shall later assume that at date 1 before the opening of the stock exchange information about the true realization of the systematic component of $\tilde{u}$ (i.e., $\tilde{\theta}$) becomes known to at least a part of generation 1 (the insiders).

But before dealing in more detail with date 1, let us assume for the moment that the equilibrium price $\tilde{P}_1$ seen from period 0 is normally distributed. (In our subsequent examples $\tilde{P}_1$ will always be normally distributed. We will not further mention this point.) Then by well known techniques (see e.g. Grossman/Stiglitz(1980)), solving the investors' maximization problem will lead to demand functions

$$X(P_0) = \frac{E(\tilde{P}_1) - P_0}{a_0 \operatorname{var}(\tilde{P}_1)},$$

and hence to the market value

$$P_0 = E(\tilde{P}_1) - a_0 Var(\tilde{P}_1)\bar{x} \quad . \tag{3.1}$$

Equation (3.1) is the basic equation of the whole chapter. In the remainder of this chapter we will investigate the behaviour of P_0 as a function of $\tilde{P}_1$ for different price formation models at date 1.

3.1 No Information versus Full Revelation of Information

Before proceeding to the analysis of insider trading we shall deal with the benchmark case of the comparison of no disclosure with full disclosure of the available information. If at date 1 investors have no additional information about $\tilde{u}$ apart from the a priori distribution, P_1 will be deterministic and given by

$$\begin{aligned} P_1 &= E(\tilde{u}) - a_1 Var(\tilde{u})\bar{x} \\ &= E(\tilde{\theta}) - a_1(\sigma_\theta^2 + \sigma_\epsilon^2)\bar{x} \quad . \end{aligned}$$

Substituting this in (3.1) gives

$$P_0 = E(\tilde{\theta}) - a_1(\sigma_\theta^2 + \sigma_\epsilon^2)\bar{x} \quad . \tag{3.2}$$

If, however, the realization θ of $\tilde{\theta}$ becomes publicly known already at date 1, then $\tilde{P}_1$ will be given by

$$\tilde{P}_1 = \tilde{\theta} - a_1\sigma_\epsilon^2\bar{x} \quad ,$$

and thus

$$P_0 = E(\tilde{\theta}) - a_1\sigma_\epsilon^2\bar{x} - a_0\sigma_\theta^2\bar{x} \quad . \tag{3.3}$$

A comparison of (3.3) and (3.2) shows the basic effect of an early release of information. For the original investor generation early disclosure will change both risk and average return of the investment project: The additional information will render $\tilde{P}_1$ more volatile, but on the other hand raises its mean. The premature arrival of information shifts risk from the future to the present and compensates for this risk shifting through higher expected returns. If the distribution of risk aversion over time is identical ($a_0 = a_1$), these two effects will just cancel out. If generation 1 is more risk averse ($a_1 > a_0$), shifting risk to generation 0 will raise the initial market value since this generation is more willing to bear the risk. In the opposite case $a_1 < a_0$, initial market value will fall since generation 0 would prefer the additional risk at date 1 to be borne by generation 1.

In our framework full disclosure of information is just an extreme form of insider trading, i.e., every investor of generation 1 is an insider. If only a part of generation

1 were informed about the realization of $\tilde{\theta}$, the risk shifting effect could in principle be lower. Note however that with our assumptions on the information structure prices in a competitive equilibrium will always fully reveal the information of the insiders.[1] If outsiders correctly update their expectations of $\tilde{\theta}$ as a function of the price, insider trading will always end up in the same situation as if information were publicly revealed.

Even if outsiders did not deduce all the information from prices, the above phenomenon due to a different willingness to bear risk would not completely disappear. If $a_1 \gg a_0$, insider trading at date 1 always leads to a higher market value at date 0, and accordingly for $a_1 \ll a_0$ market value at date 0 is always lower. In section 4 we will study the effects of non-fully revealing prices. In order to isolate these effects we assume in the following sections that $a_0 = a_1 = a$.

3.2 First Observations on Welfare

To conclude this section and to look at the risk-shifting-effect from another perspective we calculate the certainty equivalents of the investors' utilities under the two regimes. Therefore we neglect any possible effects on investment and assume that the investment costs are low enough such that the investment is undertaken in any case. We compare the expected utilities of the investors in the two bench-mark cases.

Generation 1 suffers from the premature arrival of inside information. This "Hirshleifer"-effect is easily seen by calculating their ex ante utilities when inside information arrives:

$$\begin{aligned} E(U(\tilde{W}_2)) &= 1 - exp(-a_1 W) E(exp(-a_1(\tilde{\theta} + \tilde{\epsilon} - \tilde{P}_1)\bar{x})) \\ &= 1 - exp(-a_1 W) exp(-\frac{a_1^2 \bar{x}^2}{2} \sigma_\epsilon^2) \quad , \end{aligned}$$

[1] Assume the contrary that for two different realizations of $\tilde{\theta}$ the corresponding equilibrium prices are equal. At this price the demand of outsiders will be identical whereas the insider demand differs as a function of $\tilde{\theta}$. Thus, the price cannot be an equilibrium price for both realizations.

and comparing it to the ex ante utility without premature arrival of information:

$$\begin{aligned} E(U(\tilde{W}_2)) &= 1 - exp(-a_1 W)E(exp(-a_1(\tilde{\theta} + \tilde{\epsilon} - P_1)\bar{x})) \\ &= 1 - exp(-a_1 W)exp(-\frac{a_1^2 \bar{x}^2}{2}(\sigma_\theta^2 + \sigma_\epsilon^2)) \quad . \end{aligned}$$

The investors of generation 0 prefer the early arrival of new information. In equilibrium, risk sharing between the entrepreneurs and generation 0 implies that the compensation for bearing risk rises more than the risk to be borne. We will investigate this phenomenon in more detail in section 5.3 of this chapter and in section 3.3 of chapter 4.

Turning to the welfare of generation 0, we note that with insider trading generation 0's utility is given by

$$E(U(\tilde{W}_1)) = 1 - exp(W)exp(-\frac{a^2 \bar{x}^2}{2}\frac{\sigma_\theta^2}{2}) \quad ,$$

and without insider trading it is just equal to

$$E(U(\tilde{W}_1)) = 1 - exp(W) \quad .$$

Finally, profits of the entrepreneurs who undertake the risky project will depend on whether the initial market value rises or falls. We have seen that depending on the differences of risk aversion between generation 0 and 1, this might both fall and rise: If generation 0 is less risk averse than generation 1, the profits of the entrepreneur generation will rise and vice versa.

Now consider a planner who can raise lump sum taxes from some investors and pay lump sum transfers to others. These payments occur in form of the riskless asset and cannot be conditioned on any information events. Assume furthermore that generation 0 investors are less risk averse than generation 1 (i.e., initial market value is higher with insiders than without them). Then the central planner could legalize insider trading and still make everybody better off through the following redistribution: Tax both the entrepreneur generation and generation 0 such that their ex-ante utilities correspond to the case without insider trading (resp. are slightly above it). Pay the revenue of these taxes as a lump sum transfer to generation 1 and generation 1 will be better off than in the old regime: Calculating the certainty equivalents of the above utilities one obtains that the per capita revenues from taxes expressed in units of riskless bonds are

$$a_1 \sigma_\theta^2 \bar{x}^2 - a_0 \sigma_\theta^2 \bar{x}^2 + \frac{a_0}{2} \sigma_\theta^2 \bar{x}^2$$

and the required per capita wealth to compensate generation 1 for their losses from insider trading is

$$\frac{a_1}{2} \sigma_\theta^2 \bar{x}^2 \quad ,$$

and revenues are clearly higher if $a_0 < a_1$. Consequently, if the first investor generation is less risk-averse, it is better for the whole economy if this generation bears all the risk. In the opposite case, when generation 1 is less risk averse, it would be better that this generation bears more of the risk, and hence the case of no information disclosure would be prefered by the economy.

The drawback of our redistribution scheme is that there is no reason at all why we should prefer one generation at the expense of the other generation. Since there are victims and beneficiaries of the inflow of new information, both worlds cannot be pareto-ranked. Unambiguous welfare implications in our model can only be obtained, if a rise resp. fall of the initial market value influences directly the investment. If the investment does not take place because of a decrease of P_0, then everybody is strictly worse off. Vice versa, if an increase in the initial market value allows an investment project to go ahead, this makes everybody better off.

4. Insider Trading with Partially Revealing Prices.

In the last section we studied the "pure" effect of new information on stock markets: Both the volatility of prices and the size of the risk premia are changed. However, since prices were fully revealing, the role of "informed trading" in a rational expectations equilibrium was merely to reveal the information to the market. Thus, we have been unable to capture the notion of insiders making profits at the expense of their direct trading partners and cannot really judge the impact of asymmetric information. Outside buyers could infer all the inside information from prices and hence insiders and outsiders were identical. In order to generate a difference between insiders and outsiders in a model with all investors acting as price takers one has to introduce further frictions in the information structure. In this section we will study three different ways to introduce such a friction. We will first study the case in which outsiders do not infer the relevant information from prices although in principle this is possible for them. Next we deal with an economy in which prices cannot be fully revealing because exogenously given noisy demand for the risky asset prevents a unambiguous relation between information and prices. Lastly we treat an example in which the same occurs because there is too much information to be unambiguously reflected in prices. In all three cases there are incentives to trade even if the original endowments were pareto optimal before the trading round. In the first case it is the "stupidity" of outsiders. In the second case it is the incentive of rational investors to exploit the noise traders. Finally, in the third case new information changes the optimal portfolio position for every investor in a different way; the new information itself creates a reason to trade.

In this section we will study how the pure effect of the inflow of new information interacts with the chosen frictions in the market. We investigate the overall impact of the this interaction on price volatility and ex-ante investment. The results will turn out to be very sensitive with respect to the chosen specification of the informational structure.

4.1 Insider Trading with "Stupid" Outsiders

Assume again that at date 1 the realization θ of $\tilde{\theta}$ is already determined. Only a part of generation 1 however obtains some information about this realization. As in Grossman/Stiglitz(1980), we will assume a very simple information structure: a fixed proportion λ of generation 1 knows the true value θ before the stock exchange opens. The remaining proportion $1-\lambda$ of generation 1 only knows the a priori distribution of $\tilde{u}$. A more realistic assumption would be a world in which different investors receive independent signals with different precision (as modelled by Hellwig(1980)). This more complicated assumption would not change our qualitative results, which essentially depend only on the pricing equation (3.1).

In order to give the insiders an informational advantage, we will assume in this section that uninformed investors at date 1 are stupid. They are stupid in the sense that, although insiders trade according to their information and in principle this information could be deduced from prices, they will determine their demand only on the basis of their a priori expectations. Exploiting the usual well known properties of the exponential utility and normally distributed returns (see again Grossman/Stiglitz(1980) we obtain the following expressions for informed and uninformed per capita demand:

$$X_I(P_1) = \frac{\theta - P_1}{a\sigma_\epsilon^2} \quad ,$$
$$X_U(P_1) = \frac{E(\tilde{\theta}) - P_1}{a(\sigma_\theta^2 + \sigma_\epsilon^2)} \quad .$$

In equilibrium P_1 must clear the market, i.e., the mean per capita demand of generation 1 has to be equal to the inelastic supply of shares of the old generation $\bar{x}$:

$$\lambda X_I(P_1) + (1-\lambda)X_U(P_1) = \bar{x} \quad ,$$

and thus

$$\begin{aligned}
\tilde{P}_1 &= \alpha_1 E(\tilde{\theta}) + \alpha_2 \tilde{\theta} - a\alpha_3 \bar{x} \quad \text{where} \\
\alpha_1 &= \alpha_3 \frac{1-\lambda}{\sigma_\theta^2 + \sigma_\epsilon^2} \quad , \\
\alpha_2 &= \alpha_3 \frac{\lambda}{\sigma_\epsilon^2} \quad , \\
\alpha_3 &= \frac{1}{\frac{\lambda}{\sigma_\epsilon^2} + \frac{1-\lambda}{(\sigma_\theta^2 + \sigma_\epsilon^2)}} \quad .
\end{aligned}$$

Using $E_0(E_1(\tilde{\theta})) = E_0(\tilde{\theta})$ and $\alpha_1 + \alpha_2 = 1$, substitution in (3.1) gives

$$\begin{aligned}
P_0 &= (\alpha_1 + \alpha_2)E(\tilde{\theta}) - a\alpha_3 \bar{x} - a\alpha_2^2 \sigma_\theta^2 \bar{x} \\
&= E(\tilde{\theta}) - a(\alpha_3 + \alpha_2^2 \sigma_\theta^2)\bar{x} \quad . \qquad (4.4)
\end{aligned}$$

Equation (4.4) illustrates two general features of the class of models which we investigate. First, since insider trading per se only shifts risk over time but cannot alter the fundamental return expectations, its impact on the initial market value will only appear through the risk premium component. Second, it shows that the risk premium is composed of two opposed components.

The first component $(a\alpha_3 \bar{x})$ reflects the beneficial impact of insider trading. Since insiders demand lower risk premia their orders will on average drive up prices at date 1. When the market consists of more and more insiders relative to outsiders, the aggregate risk premium of the market will approach insiders' risk premia. That is, a rising proportion of insiders λ will always lead to lower $a\alpha_3 \bar{x}$. In a loose sense one could attribute this to the beneficial effect of competition between the insiders. The source of their extra returns, i.e., the difference between the risk premium of the market and the risk premium which they know to be justified, disappears slowly as more insiders compete for it. The second part $(a\alpha_2^2 \sigma_\theta^2 \bar{x})$ contains the harmful aspect of insider trading. It reflects the additional risk created by the extra volatility of P_1. This risk component will typically rise with growing λ. Since more insiders means more insider orders, mean per capita demand will become more and more volatile (as a function of the realization θ) and thus also the equilibrium price $\tilde{P}_1$.

Comparing the overall outcome of these two effects we first observe as in section 3 that the two extreme cases $\lambda = 0$ (no insiders) and $\lambda = 1$ (full disclosure) are equivalent. Given that we assume now $a_0 = a_1 = a$, the two effects just balance out. For the case of asymmetric information, i.e., $0 < \lambda < 1$, we collect the results in Claim 1 (see also figure 1).

Claim 1:

If $0 < \lambda < 1$ the initial market value P_0 is always higher than in the case $\lambda = 0$ and $\lambda = 1$.

P_0 is a concave function of λ, i.e., the initial market value first increases in λ, and then decreases in λ.

Proof: See appendix.

Since outsiders at date 1 do not react on the incoming information, price risk rises initially less than risk premia fall. Formally this is reflected in the fact that risk aversion falls with λ, whereas price variance rises with λ^2: Risk effects are second order effects whereas mean effects are of first order.

4.2 Insider Trading in a Noisy Rational Expectations Equilibrium

In this and the following section we shall drop the assumption that outside investors do not infer information from prices. Instead we will look at the other extreme and assume that investors are perfectly rational Bayesian individuals, who calculate the distribution of $\tilde{\theta}$ conditional on the equilibrium price and thus on the inside information contained in it. To avoid that investors can deduce the information completely from prices, we have to impose some mechanism which renders prices only partially revealing. First we shall treat the case in which prices do not reflect unambiguously all information because of the presence of some exogenously given noise in the supply of shares.

To keep matters simple we remain in the Grossman/Stiglitz(1980)-framework of the last section. Thus we obtain the following expressions for informed and uninformed per capita demand:

$$\begin{aligned}
X_I(P_1) &= \frac{\theta - P_1}{a\sigma_\epsilon^2} \quad , \\
X_U(P_1) &= \frac{E(\tilde{\theta} \mid P_1) - P_1}{a(\mathrm{var}(\tilde{\theta} \mid P_1) + \sigma_\epsilon^2)} \quad .
\end{aligned}$$

In equilibrium P_1 must clear the market, i.e., the mean per capita demand of generation 1 has now to be equal to the inelastic supply of shares of the old generation $\bar{x}$ plus the random noise supply $\tilde{x}$:

$$\lambda X_I(P_1) + (1-\lambda) X_U(P_1) = \bar{x} + \tilde{x} \quad ,$$

where we assume that $\tilde{x} \sim \mathrm{N}(0, \sigma_x^2)$.

Defining

$$w_\lambda = \tilde{\theta} - a\frac{\sigma_\epsilon^2}{\lambda}\tilde{x}$$

the linear equilibrium price can then be written as (see Grossman/Stiglitz(1980))

$$\begin{aligned}
\tilde{P}_1 &= \alpha_1 E(\tilde{\theta}) + \alpha_2 w_\lambda - a\alpha_3 \bar{x} \quad \text{with} \\
\alpha_1 &= 1 - \alpha_2 \\
\alpha_2 &= \alpha_3 \left(\frac{\lambda}{\sigma_\epsilon^2} + \frac{1-\lambda}{\sigma_{\tilde{u}|w_\lambda}^2} \frac{\sigma_\theta^2}{var(w_\lambda)}\right)) \\
\alpha_3 &= \frac{1}{\frac{\lambda}{\sigma_\epsilon^2} + \frac{1-\lambda}{\sigma_{\tilde{u}|w_\lambda}^2}} \quad ,
\end{aligned}$$

where the conditional variance $\sigma_{\tilde{u}|w_\lambda}^2$ is given by

$$\begin{aligned}
\sigma_{\tilde{u}|w_\lambda}^2 &= \sigma_{\tilde{u}|P}^2 \\
&= \sigma_\theta^2 + \sigma_\epsilon^2 - \frac{(\sigma_\theta^2)^2}{var(w_\lambda)} \quad .
\end{aligned}$$

For our later purposes it is useful to rewrite α_2 and α_3 in the following form (which follows after some algebra):

$$\begin{aligned}
\alpha_2 &= \lambda \frac{\lambda\sigma_\theta^2 + a^2(\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2}{\lambda^2\sigma_\theta^2 + a^2(\lambda\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2} \quad , \\
\alpha_3 &= \sigma_\epsilon^2 \frac{\lambda^2\sigma_\theta^2 + a^2(\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2}{\lambda^2\sigma_\theta^2 + a^2(\lambda\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2} \quad .
\end{aligned}$$

Observe[1] that

$$\alpha_2(0) = 0, \quad \frac{\alpha_2\sigma_\epsilon^2}{\lambda}_{|\lambda=0} = \alpha_3(0) = \sigma_\epsilon^2 + \sigma_\theta^2,$$

and

$$\alpha_2(1)\sigma_\epsilon^2 = \alpha_3(1) = \sigma_\epsilon^2 \quad .$$

Substituting the equilibrium price in (3.1) gives

$$P_0 = E(\tilde{\theta}) - a(\alpha_3 + \alpha_2^2\sigma_\theta^2 + a^2\frac{(\alpha_2\sigma_\epsilon^2)^2}{\lambda^2}\sigma_x^2)\bar{x} \quad . \tag{4.5}$$

The risk premium component of (4.5) resembles closely the one of (4.4). The first part stems again from the reduced risk premia at date 1, but will now be lower than in the last section since outside investors will use instead of σ_θ^2 the conditional variance $\sigma_{\tilde{u}|P}^2$. The second term reflects as before the higher risk today created by the additional variance of $\tilde{P}_1$, but will in general be higher than in the last section. Since outsiders learn from prices and change their demand accordingly, prices will contain more of the true realization of $\tilde{\theta}$. The third part is new. It stems from the additional risk for the initial investors created by the noise traders. Noise traders create a "bid-ask-spread" around the average estimation of the asset value. Accordingly, the third term reflects the additional price volatility created by this "bid-ask-spread", i.e., the random bouncing back of transaction prices between the price in the case of excess noise demand and the price in the case of excess noise demand/supply. The term bid-ask-spread is here somewhat loosely adopted from the literature on the microstructure of financial markets. Since we consider aggregate order flows rather than assuming that liquidity traders arrive sequentially, it should be interpreted as an aggregate or average bid-ask-spread. In section 5 we will give a more detailed discussion of it; we show in particular that the bid-ask-spread can be decomposed in a risk bearing component and an adverse selection component. The risk bearing component has to be paid by the noise traders in order to compensate the counterpart of a trade for the additional risk he has to bear. The adverse selection component is similar to the one encountered

[1]Note that α_1, α_2 and α_3 are always functions of λ. However, we will usually drop the argument and put it only when necessary.

in chapter 2. It is the amount a liquidity trader has to pay because the uniformed speculators suspect him to be an insider.

The behaviour of the three terms as a function of the proportion of insiders at date 1 takes again different directions. As a consequence of rising competitive pressure among insiders, the first term is as in the last section falling in λ. The second term is again rising in λ, since with more insiders prices convey more information. Both movements will be much sharper than in in the case with "stupid" outsiders. Since outsiders infer the information on $\tilde{u}$ from prices and adapt their demand accordingly, prices at date 1 will reflect the true realization of $\tilde{\theta}$ more concisely than in the section 4.1. But since the second term still reflects a risk effect (i.e., a second order effect), the first term should at least for small λ dominate the second term.

Hence, the overall outcome will be determined by the third term, reflecting the interaction between noise trading and informed speculation. To understand the direction of this term, note first that the presence of noise trading at date 1 adds from an ex-ante view an additional liquidation risk for the initial investors. The size of this additional risk is not independent of the information on $\tilde{u}$. Investors' demand at date 1 will be the less price elastic the riskier the asset becomes. If there is no information on $\tilde{u}$ available at date 1 already minor changes in noisy supply will imply major changes in equilibrium prices, because the risk bearing component of the bid-ask-spread is relatively large. If information is fully disclosed, demand will become more price-elastic because the risk bearing component of the bid-ask-spread will be lower. Hence, seen from date 0 equilibrium prices will be more volatile without premature arrival of information than with fully disclosed information. Insider trading at date 1 will always have a tendency to raise initial market values since it eliminates the noise induced additional risk. An inspection of (4.5) for the cases of no resp. full disclosure shows this effect clearly. For $\lambda = 0$ we have

$$\begin{aligned} P_0 &= E(\tilde{\theta}) - a(\sigma_\theta^2 + \sigma_\epsilon^2 + \operatorname{var}(\tilde{P}_1))\bar{x} \\ &= E(\tilde{\theta}) - a\big(\sigma_\theta^2 + \sigma_\epsilon^2 + a^2(\sigma_\theta^2 + \sigma_\epsilon^2)^2\sigma_x^2\big)\bar{x} \quad , \end{aligned}$$

whereas for $\lambda = 1$

$$\begin{aligned} P_0 &= E(\tilde{\theta}) - a(\sigma_\epsilon^2 + \mathrm{var}(\tilde{P}_1))\bar{x} \\ &= E(\tilde{\theta}) - a(\sigma_\theta^2 + \sigma_\epsilon^2 + a^2(\sigma_\epsilon^2)^2\sigma_x^2)\bar{x} \quad . \end{aligned}$$

Clearly the initial market value in the full disclosure case is higher than in the case of no disclosure. The behaviour for intermediate values of λ is unclear. Intuitively, one could expect that in the case of very high noise induced risk (high σ_x^2), the positive, risk-reducing effect created by the third term should dominate the overall outcome. However, for low σ_x^2 one has to take a further effect into account. If σ_x^2 is very low, the behaviour of the bid-ask-spread will be determined by the reaction of the adverse selection component on shifts in λ. It turns out that at least for sufficiently low λ, the adverse selection component of the bid-ask-spread is increasing with λ to such an extent that the induced higher price volatility leads to a decrease of the initial market value.

Claim 2:

For σ_x^2 and λ sufficiently small, P_0 is decreasing in λ.

For σ_x^2 sufficiently small and λ sufficiently large, P_0 is increasing in λ.

For σ_x^2 sufficiently large, P_0 is increasing in λ.

Proof: See appendix.

For σ_x^2 sufficiently small, initial market value will fall for small proportions of informed traders, and only for relatively large λ the initial market value will rise above the initial market value level of the case without insider trading. Accordingly, an increase in λ will first create a decrease in investment, and only with λ sufficiently high the benefits of the new information, i.e., the reduction of the noise induced price risk due to noise traders, leads to more investment.

For a full intuitive understanding of this result, a detailed analysis of the properties of the "bid-ask-spread", i.e., the price impact of noisy excess demand and supply, is required. The properties of the bid-ask-spread will also determine the welfare results. Given that this analysis turns out to be quite complicated, we defer it to section 5.

4.3 Insider Trading in a Partially Revealing REE without Noisy Supply

We have seen already that in order to give a role to insider trading in a rational expectations equilibrium prices have to be partially revealing. One way to make prices partially revealing is to introduce in the model some unspecified random noise trading. The purpose of this section is to show that the trade offs developed so far are independent of this specification, but depend merely on the partially revealing nature of prices. Therefore, we will study an example in the spirit of the second main strand of constructing partially revealing equilibria, i.e., equilibria in which prices cannot reveal all information because the space of pay-off relevant informations has higher dimension than the price space. (For a survey of partially revealing REE see e.g. Jordan/Radner(1982).) The example claims no generality and is constructed in order to stay as closely to our previous discussion as possible. It preserves the basic trade-off between risk premium and price volatility effect, but it will turn out that this trade-off takes a rather extreme form.

We deal with the same time structure as before and also generation 0 is assumed to behave as before. However, we will consider a different utility for generation 1. Assume that random returns at period 2 consist of two components $\tilde{\theta}^1$ and $\tilde{\theta}^2$, where the $\tilde{\theta}^i$ are i.i.d. and

$$\tilde{\theta}^i \sim N(E(\tilde{\theta}), \sigma_\theta^2) \quad i = 1, 2 \quad .$$

Furthermore let the individual members of generation 1 differ in their relative appreciations of $\tilde{\theta}^1$ and $\tilde{\theta}^2$, characterized by some parameter $\alpha \in [0, 1]$. Let the date 2 wealth $\tilde{W}_2^\alpha$ of individual α be given by

$$\tilde{W}_2^\alpha = W + \alpha\tilde{\theta}^1 + (1 - \alpha)\tilde{\theta}^2 + \tilde{\epsilon}_2 - P_1 \quad .$$

The α are assumed to be distributed according to some given distribution on [0,1] with mean $\frac{1}{2}$ and density f.[1] The economic interpretation of the α is obviously difficult; they are chosen not for realism but rather for obtaining a closed form solution of a partially revealing REE without noise supply. A possible story could run along the lines that returns are partially paid out through direct dividend payments and partially through the buying back of stock by the firm. If these two modes are taxed differently and investors do not face the same tax rates, the α^i would stand for these different tax rates.

Again a proportion λ of generation 1 is informed. To avoid any interference between the fact of being an insider and the preferences, we assume that the distribution of the α over the informed and uninformed investors is the same. Before the stock exchange opens at date 1 insiders learn the true realization of $\tilde{\theta}^1$ and $\tilde{\theta}^2$, whereas outsiders only know the a priori distributions. Outsiders are Bayesian, i.e., outside investor α calculates the distribution of

$$\alpha\tilde{\theta}^1 + (1-\alpha)\tilde{\theta}^2 + \tilde{\epsilon}_2$$

conditional on the equilibrium price P_1. Mean per capita demand is now given by

$$X_I(P_1) = \int_0^1 \frac{\alpha\theta^1 + (1-\alpha)\theta^2 - P_1}{a\sigma_\epsilon^2} df(\alpha) \quad ,$$

$$X_U(P_1) = \int_0^1 \frac{E(\alpha\tilde{\theta}^1 + (1-\alpha)\tilde{\theta}^2 \mid P_1) - P_1}{a(\text{var}(\alpha\tilde{\theta}^1 + (1-\alpha)\tilde{\theta}^2 \mid P_1) + \sigma_\epsilon^2)} df(\alpha) \quad .$$

In equilibrium P_1 must again clear the market, i.e., the mean per capita demand of generation 1 has to be equal to the inelastic supply of shares of the old generation $\bar{x}$:

$$\lambda X_I(P_1) + (1-\lambda)X_U(P_1) = \bar{x} \quad .$$

We show now that a linear rational expectations equilibrium price (distribution) can be calculated as follows.

[1] The use of the α is similar to an example of Laffont(1985). The assumption on the mean of the α is inessential. It serves merely to guarantee prices which are symmetric in $\tilde{\theta}^1$ and $\tilde{\theta}^2$. Without mentioning it in the sequel we assume furthermore that the α are distributed such that the following integrals exist.

Claim 3:

There is a rational expectations equilibrium of the following form:

$$\text{For } \lambda = 0: \quad P_1 = E(\tilde{\theta}) - a\rho_0 \bar{x} \quad ,$$

$$\text{for } \lambda > 0: \quad \tilde{P}_1 = \frac{1}{2}\tilde{\theta}^1 + \frac{1}{2}\tilde{\theta}^2 - a\rho_\lambda \bar{x} \quad .$$

For $\lambda = 0$ ρ_0 is given by

$$\rho_0 = \frac{1}{\int\limits_0^1 \frac{1}{((\alpha)^2+(1-\alpha)^2)\sigma_\theta^2+\sigma_\epsilon^2} df(\alpha)} \quad ,$$

and for $\lambda > 0$

$$\rho_\lambda = \frac{1}{\frac{\lambda}{\sigma_\epsilon^2} + (1-\lambda)\int\limits_0^1 \frac{1}{((\alpha)^2+(1-\alpha)^2-0.5)\sigma_\theta^2+\sigma_\epsilon^2} df(\alpha)} \quad .$$

Proof: To derive the equilibrium, we first solve the equilibrium condition for $\tilde{P}_1$ and obtain

$$\begin{aligned}
\tilde{P}_1 = & \frac{(1-\lambda)\int\limits_0^1 \frac{E(\alpha\tilde{\theta}^1+(1-\alpha)\tilde{\theta}^2|\tilde{P}_1)}{\mathrm{var}(\alpha\tilde{\theta}^1+(1-\alpha)\tilde{\theta}^2|\tilde{P}_1)+\sigma_\epsilon^2} df(\alpha)}{\frac{\lambda}{\sigma_\epsilon^2} + (1-\lambda)\int\limits_0^1 \frac{1}{\mathrm{var}(\alpha\tilde{\theta}^1+(1-\alpha)\tilde{\theta}^2|\tilde{P}_1)+\sigma_\epsilon^2} df(\alpha)} \\
& + \frac{\frac{\lambda}{\sigma_\epsilon^2}}{\frac{\lambda}{\sigma_\epsilon^2} + (1-\lambda)\int\limits_0^1 \frac{1}{\mathrm{var}(\alpha\tilde{\theta}^1+(1-\alpha)\tilde{\theta}^2|\tilde{P}_1)+\sigma_\epsilon^2} df(\alpha)} \left(\frac{\tilde{\theta}^1}{2} + \frac{\tilde{\theta}^2}{2}\right) \\
& - \frac{a}{\frac{\lambda}{\sigma_\epsilon^2} + (1-\lambda)\int\limits_0^1 \frac{1}{\mathrm{var}(\alpha\tilde{\theta}^1+(1-\alpha)\tilde{\theta}^2|\tilde{P}_1)+\sigma_\epsilon^2} df(\alpha)} \bar{x} \quad .
\end{aligned}$$

For $\lambda = 0$ we can drop the conditioning on $\tilde{P}_1$ and the claim follows.

For $\lambda > 0$ we set

$$\tilde{P}_1 = \gamma\tilde{\theta}^1 + \gamma\tilde{\theta}^2 + \delta \quad .$$

Since the random variables are normally distributed we obtain the linear regression

$$E(\alpha\tilde{\theta}^1 + (1-\alpha)\tilde{\theta}^2 \mid \tilde{P}_1) = E(\tilde{\theta}) + \frac{\gamma\sigma_\theta^2}{2\gamma^2\sigma_\theta^2}(\tilde{P}_1 - E(\tilde{P}_1))$$
$$= \frac{\tilde{\theta}^1}{2} + \frac{\tilde{\theta}^2}{2} \quad ,$$

and the conditional variance is given by

$$\operatorname{var}(\alpha\tilde{\theta}^1 + (1-\alpha)\tilde{\theta}^2 \mid \tilde{P}_1) = \big((\alpha)^2 + (1-\alpha)^2\big)\sigma_\theta^2 - \frac{\sigma_\theta^2}{2} \quad .$$

Substituting in the price equation and comparison of the coefficients gives

$$\delta = \frac{a}{\frac{\lambda}{\sigma_\epsilon^2} + (1-\lambda)\int_0^1 \frac{1}{((\alpha)^2+(1-\alpha)^2-0.5)\sigma_\theta^2+\sigma_\epsilon^2}df(\alpha)}\bar{x}$$
$$= a\rho_\lambda\bar{x} \quad ,$$

and

$$\gamma = \frac{\frac{\lambda}{2\sigma_\epsilon^2} + \frac{(1-\lambda)}{2}\int_0^1 \frac{1}{((\alpha)^2+(1-\alpha)^2-0.5)\sigma_\theta^2+\sigma_\epsilon^2}df(\alpha)}{\frac{\lambda}{\sigma_\epsilon^2} + (1-\lambda)\int_0^1 \frac{1}{((\alpha)^2+(1-\alpha)^2-0.5)\sigma_\theta^2+\sigma_\epsilon^2}df(\alpha)}$$
$$= \frac{1}{2} \quad .$$

This gives the desired form of the equilibrium.

In order to determine the initial market value P_0 we substitute the equilibrium price in equation (3.1) and obtain

$$\text{For } \lambda = 0: \quad P_0 = E(\tilde{\theta}) - a\rho_0\bar{x} \quad ,$$
$$\text{for } \lambda > 0: \quad P_0 = E(\tilde{\theta}) - a(\rho_\lambda + \frac{\sigma_\theta^2}{2})\bar{x} \quad . \tag{4.6}$$

Comparison of (4.6) with (4.5) and (3.2) shows that the structure of the initial market value equation is similar. The effect of insider trading is contained in the risk premium, and the risk premium consists again of the two opposed factors reflecting the higher price volatility of $\tilde{P}_1$ and the lower risk premia of date 1 prices. The difference between (4.5) and (4.6) is that in contrast to the case with exogenous noise trading, $\tilde{P}_1$ reflects immediately all the information, and this regardless of the proportion of insiders. A higher proportion of insiders does not render the prices more informative. Its only effect is to raise the expected value of $\tilde{P}_1$ through the competition effect. Thus, the mere presence of a few insiders creates a discontinuity in both the variance and the mean of prices at date 1 and might even lead to a discontinuous change of initial market values. However, initial market values will always be higher with insider trading than without it (see also figure 3).

Claim 4:

The initial market value P_0 is an increasing function of λ. In particular, for $\lambda > 0$ the initial market value P_0 will always be higher than in the case without information.

Proof: For the second statement, we show in the appendix that for all $\lambda > 0$

$$\rho_\lambda + \frac{\sigma_\theta^2}{2} < \rho_0 \quad .$$

For $\lambda > 0$, a derivation with respect to λ shows that the market risk premium $a\rho_\lambda \bar{x}$ is decreasing in λ. Hence, P_0 is increasing in λ.

Accordingly, price volatility rises less fast than the *average* risk premia demanded by generation 2 fall. A rising number of insiders will even strengthen this effect through the competition effect and induces the initial market value to rise in λ. The inflow of new information is beneficial, because it eliminates risk. Moreover, there are no trading frictions due to noise trading as in the last model. Accordingly, investment levels are unambiguously increasing with the intensity of informed trading. We show in the appendix that insider trading not only induces higher investment levels, but also leads to a higher welfare (defined as the sum of the certainty equivalents of investors' expected utilities) of the economy as a whole.

These observations highlight also a general difference between partially revealing REE models with and without exogenous noise. In the model of this section all the available information is immediately incorporated in the prices (independent of the number of insiders), whereas in a noisy model this takes only place gradually. In both models new information is ultimately valuable. Thus, after information is fully reflected in the prices, the market risk premium at date 1 has always fallen stronger than price volatility has risen and the initial market value is accordingly higher. Only if the new information is not yet fully included in the price, price volatility might rise stronger than the risk premia fall.[1]

[1]The implications of the model of this section concerning the investment levels resemble closely the results of Ausubel (1989), who considers a different partially revealing model without exogenous noise trading. A detailed comparison of the two models is however difficult, since Ausubel's model does not allow for explicit solutions.

5. Welfare and the Bid-Ask-Spread.

In this section we will further investigate the example of section 4.2 with exogenous noise trading. We have seen that the effects of insider trading in this model depend on a rather complex interaction between the impact of premature arrival of information and the properties of the term which we dubbed "bid-ask-spread". We shall investigate in this section the properties of this bid-ask-spread more closely. In 5.1 we justify the terminology. 5.2 studies the effects of shifts in the intensity of insider trading on the bid-ask-spread. Finally, in 5.3 we study the welfare implications of these changes in the bid-ask-spread.

5.1 Transaction Costs and the Bid-Ask-Spread

Consider a trader who for some unspecified liquidity reasons wants to trade the amount x of a risky asset. The asset will pay $\tilde{u}$ in the next period. Denote the price on the stock exchange for a quantity x by $\tilde{P} = P(\tilde{u}, x)$, where $\tilde{P}$ may depend on $\tilde{u}$ if we allow for premature resolution of uncertainty. The expected trading costs of the liquidity trader are then given by

$$TC = E_{\tilde{u}}(x(\tilde{P} - \tilde{u})) \quad ,$$

where the subscript indicates that the expectation is formed with respect to $\tilde{u}$. Assuming that x is the realization of a random variable $\tilde{x}$, which represents the distribution of possible liquidity trades, the average expected trading costs for liquidity traders are given by

$$ATC = E_{\tilde{x},\tilde{u}}(\tilde{x}(\tilde{P} - \tilde{u})) \quad .$$

Now reconsider the example of chapter 2 on the price competition between market makers. There we set $E(\tilde{u}) = 0$ and $\tilde{x}$ takes the values 1 and -1 with equal probability $\frac{1}{2}$. As long as no informed traders have been active before the liquidity trader arrives, $\tilde{P}$ is independent of the true future realization of $\tilde{u}$ and the transactions costs can be calculated as

$$ATC = \frac{1}{2}(P(1) - 0) - \frac{1}{2}(P(-1) - 0)$$
$$= \frac{P(1) - P(-1)}{2} \quad .$$

Hence, average transaction costs are just given by the half of the bid-ask-spread, defined as the difference between the ask price $P(1)$ and the bid price $P(-1)$.

Our discussion of the bid-ask-spread in chapter 2 took place in a static one-shot game. However, from a more dynamic perspective it will be very important for a liquidity trader at exactly which time he arrives on the market. Assume for instance in our example of chapter 2 that a liquidity trader arrives shortly after an insider has traded. If the market is transparent in the sense that market makers have to announce their previous trades, every market maker knows at this moment the true realization of the asset: Since liquidity traders trade only with one market maker whereas insiders trade with every market maker, the observation that every market makers has traded reveals the insider's information. Thus, the bid-ask-spread faced by the liquidity trader in this case will be 0. The example shows that depending on the chosen dynamic specification, the link between the bid-ask-spread at a certain period and the average expected trading costs of the liquidity traders might be quite complex. The average transaction costs will certainly be determined by the average bid-ask-spread, but the precise nature of this relation depends on the specific dynamics.

In section 4.2 we have considered the trading of insiders, market makers (i.e., the uninformed speculators) and liquidity traders in a competitive world. In this setting, the dynamics of the arrival process are eliminated by assuming that all traders arrive together on the market and submit their orders simultaneously. Accordingly, one cannot speak any more of a bid-ask-spread faced by a *single* trader, since prices react only on aggregate market demand. However, as before we can meaningfully define the average transaction costs of the liquidity traders arising from the price impact of noisy excess demand supply as

$$ATC = E_{\tilde{x},\tilde{u}}(\tilde{x}(\tilde{P} - \tilde{u})) \quad .$$

If the distribution of the liquidity demand $\tilde{x}$ is symmetric around 0, we can rewrite this as

$$\begin{aligned} ATC &= \frac{1}{2}E_{\tilde{x}\geq 0,\tilde{u}}(\tilde{x}(P(\tilde{u},\tilde{x}) - \tilde{u})) - \frac{1}{2}E_{\tilde{x}\geq 0,\tilde{u}}(\tilde{x}(P(\tilde{u},-\tilde{x}) - \tilde{u})) \\ &= \frac{E_{\tilde{x}\geq 0,\tilde{u}}(\tilde{x}(P(\tilde{u},\tilde{x}) - (P(\tilde{u},-\tilde{x}))}{2} \quad . \end{aligned}$$

In analogy to our discussion in the case with sequential arrival of traders one might call this last expression half of the "average" bid-ask-spread faced by the net liquidity demand $\tilde{x}$. The bid-ask-spread in the competitive setting is precisely the amount to which prices move in order to allow the market to absorb a noisy demand/supply shock. The reader should however bear in mind that the term bid-ask-spread remains only an analogy. There is no model which would give a clear link between the bid-ask-spread faced by traders on real markets and this "average" bid-ask-spread of a competitive model with simultaneous arrival of traders. Given that the sources of transaction costs in the competitive setting are exactly the same as the ones we usually associate with the bid-ask-spread of real markets, the use of the term here is quite suggestive. Indeed, we will later see that the "bid-ask-spread" in a competitive model can be decomposed as the one in markets with sequential arrival of traders in two parts: it consists of a risk bearing component and an adverse selection component. The latter arises as in strategic models, because the market can only imperfectly distinguish between a demand/supply shock due to noise trading and additional demand/supply due to new information.

5.2 The Size of the Bid-Ask-Spread

5.2.1 The Competitive Case

In this section we shall investigate the size of the bid-ask-spread in the Grossman/Stiglitz model of section 3.2 of the last chapter. For convenience we just recapitulate the basic assumptions of the model. The future returns of the risky asset are given by

$$\tilde{u} \sim \tilde{\theta} + \tilde{\epsilon}, \quad \text{where } \tilde{\theta} \sim \mathrm{N}(E(\tilde{\theta}), \sigma_\theta^2), \quad \tilde{\epsilon} \sim \mathrm{N}(0, \sigma_\epsilon^2).$$

Investors have constant absolute risk aversion a. A fixed proportion λ of the investors (the insiders) learns the true realization θ of $\tilde{\theta}$ before the stock exchange opens. The remaining proportion $1-\lambda$ of generation 1 only knows the a priori distribution of $\tilde{u}$. There is a given supply of the shares, denoted by $\bar{x}$, and additionally a random noise supply $\tilde{x} \sim \mathrm{N}(0, \sigma_x^2)$.

The linear equilibrium price can then be written as

$$\begin{aligned}
\tilde{P}_1 &= \alpha_1 E(\tilde{\theta}) + \alpha_2 (\tilde{\theta} - a\frac{\sigma_\epsilon^2}{\lambda}\tilde{x}) - a\alpha_3 \bar{x} \quad \text{where} \\
\alpha_1 &= 1 - \alpha_2 \\
\alpha_2 &= \lambda \frac{\lambda\sigma_\theta^2 + a^2(\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2}{\lambda^2\sigma_\theta^2 + a^2(\lambda\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2} \\
\alpha_3 &= \sigma_\epsilon^2 \frac{\lambda^2\sigma_\theta^2 + a^2(\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2}{\lambda^2\sigma_\theta^2 + a^2(\lambda\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2} \quad .
\end{aligned}$$

Recall that

$$\alpha_2(0) = 0, \quad \frac{\alpha_2\sigma_\epsilon^2}{\lambda}_{|\lambda=0} = \alpha_3(0) = \sigma_\epsilon^2 + \sigma_\theta^2, \quad \text{and } \alpha_2(1)\sigma_\epsilon^2 = \alpha_3(1) = \sigma_\epsilon^2 \quad .$$

This formulation shows that α_3 can be interpreted as the average risk perceived by the market. It is nothing else than the weighted average of the risk estimates of the two classes of investors. The coefficients α_1, α_2 and $a(\frac{\alpha_2}{\lambda}\sigma_\epsilon^2 - \alpha_3)$ are weighted averages of the coefficients of the (linear) estimations of $\tilde{u}$'s future realization.

Now calculate the average expected trading costs of the liquidity traders as

$$\begin{aligned}
ATC(\lambda) &= E(\tilde{x}(\tilde{P} - \tilde{u})) \\
&= a(\frac{\alpha_2}{\lambda}\sigma_\epsilon^2 - a_3)\sigma_x^2 + a\alpha_3\sigma_x^2 \quad .
\end{aligned}$$

This decomposition allows us to distinguish the two components of the transaction costs.[1] The second part ($a\alpha_3$) is the risk bearing cost which would also be present without asymmetric information, and which has to be paid in order to compensate the counterpart of a trade for the additional risk he has to bear. The first component is analogous to the adverse selection component of the bid-ask-spread which we encountered in chapter 2. It is the amount a liquidity trader has to pay

[1] For a detailed treatment of the two components of the bid-ask-spread in the more general framework of the Hellwig (1980) model see Admati/Pfleiderer(1990).

because the uninformed speculators suspect him to be an insider. We characterize the behaviour of the two components in the following proposition. The size of the bid-ask-spread also helps us to determine the behaviour of the price volatility at date 1 as a function of λ, as we show in the last part of Claim 5.

Claim 5:

Let $\lambda \in [0,1]$.

a) The risk bearing component of the bid-ask-spread is decreasing in λ: $\frac{\partial}{\partial \lambda}(\alpha_3) < 0$.

b) For λ sufficiently low the adverse selection component is increasing in λ: $\frac{\partial}{\partial \lambda}(\frac{\alpha_2 \sigma_\epsilon^2}{\lambda} - a_3) > 0$.

For λ sufficiently large the adverse selection component is decreasing in λ: $\frac{\partial}{\partial \lambda}(\frac{\alpha_2 \sigma_\epsilon^2}{\lambda} - a_3) < 0$.

c) For σ_x^2 and λ sufficiently low the bid-ask-spread is increasing in λ: $\frac{\partial}{\partial \lambda}(ATC(\lambda)) > 0$.

For σ_x^2 sufficiently low and λ sufficiently large the bid-ask-spread is decreasing in λ: $\frac{\partial}{\partial \lambda}(ATC(\lambda)) < 0$.

For σ_x^2 sufficiently large and for every λ the bid-ask-spread is decreasing in λ: $\frac{\partial}{\partial \lambda}(ATC(\lambda)) < 0$.

d)The bid-ask-spread with full information is lower than the one without information: $ATC(0) > ATC(1)$.

e) For σ_x^2 sufficiently small, var$(\tilde{P}_1)$ is increasing in λ for sufficiently small λ and decreasing for sufficiently large λ: $\frac{\partial}{\partial \lambda}(\alpha_2^2 \sigma_\theta^2 + a^2(\frac{\alpha_2 \sigma_\epsilon^2}{\lambda})^2 \sigma_x^2) > (<) 0$. However, var$(\tilde{P}_1)$ is higher for $\lambda = 1$ than for $\lambda = 0$.

For σ_x^2 sufficiently large, var$(\tilde{P}_1)$ is monotonically decreasing in λ: $\frac{\partial}{\partial \lambda}(\alpha_2^2 \sigma_\theta^2 + a^2(\frac{\alpha_2 \sigma_\epsilon^2}{\lambda})^2 \sigma_x^2) < 0$. In particular, for σ_x^2 sufficiently large, var$(\tilde{P}_1)$ is lower for $\lambda = 1$ than for $\lambda = 0$.

Proof: See appendix.

Statements a) and d) are quite intuitive. Prices become the more informative the more insiders are active in the market. Accordingly, holding the asset becomes less risky, risk premia fall and therefore also the risk bearing component of the bid-ask-spread. The same effect makes liquidity traders prefer the situation in which every investor is informed (which is equivalent to a public disclosure of information) to the situation without any informed speculators. In both cases the adverse selection cost is obviously zero, but in the case of $\lambda = 1$ the risk bearing costs are much lower. Ultimately, liquidity traders are always interested in a very high insider activity, because they profit from the induced reduction of the risk bearing costs.

In order to understand the behaviour of the transaction costs in the most interesting case, i.e., for low levels of λ, we have to study more closely the behaviour of the adverse selection component of the bid-ask-spread. The adverse selection costs first increase and then decrease with λ. For low levels of noise trading this effect will dominate the reduction of the risk bearing costs, for high levels of noise trading the reduction of risk becomes dominant. It turns out that results b) and c) depend partially on the risk aversion of the agents and partially on the competitive setting of the Grossman/Stiglitz model. This will become clearer if we compare the previous results with the properties of the bid-ask-spread in a strategic setting with risk neutral market makers. Therefore, we will study in the next section a strategic model of a market with insiders, uninformed speculators and liquidity traders. This will help us in the more detailed discussion of the determinants of the adverse selection costs.

5.2.2 The Strategic Case

We shall analyze a modification of Kyle's (1985) model. There are n informed traders who know the true realization of the systematic component $\tilde{\theta}$. In contrast to Kyle we allow the informed speculators to have constant absolute risk aversion $a \geq 0$. Each insider submits a profit maximizing market order $\tilde{q}_i = q_i(\tilde{\theta})$

(i=1,..,n). Noise traders submit the realization x of $\tilde{x}$. The market makers observe the realization of the aggregate order flow

$$\tilde{Q} = \sum_{i=1}^{n} \tilde{q}_i + \tilde{x} \quad .$$

Following Kyle we assume that the market as a whole has infinite risk bearing capacity and that market makers set prices such that the market is efficient, i.e.,

$$P(Q) = E(\tilde{\theta} + \tilde{\epsilon} \mid \tilde{Q} = Q) \quad .$$

Insiders future wealth given the realization θ is

$$\tilde{W} = \left(\theta + \tilde{\epsilon} - P(q_i + \sum_{j \neq i} q_j + \tilde{x})\right) q_i \quad .$$

There is a certain problem here how to treat $\tilde{x}$ and $\tilde{\epsilon}$, because the uncertainty concerning $\tilde{x}$ is resolved much earlier. Moreover, by submitting demand schedules as in Kyle (1989) the insider could condition his demand on the true realization of $\tilde{x}$ and thus avoid any risk concerning the realization of the noise demand. Mainly for the technical convenience of being able to deal with the mathematically simpler case of market orders we *define* their maximization problem as the problem to find a q such that

$$E_{\tilde{\epsilon}}\left(U(E_{\tilde{x}}(\tilde{W}))\right) = E_{\tilde{\epsilon}}\left(exp(-a\left(\theta + \tilde{\epsilon} - E_{\tilde{x}}(P(q_i + \sum_{j \neq i} q_j + \tilde{x}))\right) q_i)\right) \quad ,$$

is maximal. The qualitative results would remain unchanged if we allowed insiders to submit demand functions. We solve now for a symmetric linear equilibrium in prices and quantities.

Claim 6:

There exists exactly one symmetric and linear equilibrium of the model. It is given by

$$\tilde{q}_i = \frac{\tilde{\theta} - E(\tilde{\theta})}{(n+1)b + a\sigma_\epsilon^2} \quad \text{for } i = 1, .., n$$
$$P(Q) = E(\tilde{\theta}) + bQ \quad ,$$

where b is determined as the unique positive solution of the equation

$$b((n+1)b + a\sigma_\epsilon^2)^2 \sigma_x^2 = n(b + a\sigma_\epsilon^2)\sigma_\theta^2 \quad .$$

Proof: The proof is a straightforward extension of Kyle's proof and is contained in the appendix.

As in the last section we calculate the average expected trading costs of the liquidity traders and obtain

$$\begin{aligned} ATC(n) &= E(\tilde{x}(\tilde{P} - \tilde{u})) \\ &= b\sigma_x^2 \quad . \end{aligned}$$

Since market makers are assumed to have infinite risk bearing capacity, transaction costs consist only of adverse selection costs. The next claim characterizes these costs.

Claim 7:

If $a\sigma_\epsilon^2$ is sufficiently large and n sufficiently low, then b is an increasing function of n; for sufficiently large n or sufficiently small $a\sigma_\epsilon^2$, b is always decreasing in n.

Proof: See appendix.

Consequently, as long as the informed traders' risk aversion (resp. the risk itself) is not too low, the adverse selection component of the bid-ask-spread behaves as in the competitive case. First it increases with the intensity of informed speculation, and only after a certain point more insider trading reduces the bid-ask-spread again. Only for a low risk aversion (resp. a low residual risk) the bid-ask-spread is highest for a single insider and decreases if the number of informed traders increases. In the case with risk neutral informed speculators where b is given by

$$b = \sqrt{\frac{n}{(n+1)^2} \frac{\sigma_\theta^2}{\sigma_x^2}} \quad ,$$

which is clearly decreasing in n, this has already been shown by Admati/Pfleiderer (1989).

5.2.3 Comparison of the Two Cases

The basic observation for the strategic case is that *in equilibrium* insiders always have an interest to mimic the liquidity traders: given that they can anticipate that equilibrium prices are efficient *and* given that prices are linear in quantities, their profits from taking speculative positions are maximal when they are as indistinguishable as possible for the market makers. This is seen most clearly in the case without risk aversion and a single insider ($a = 0$ and $n = 1$). Calculation of the equilibrium quantities gives

$$\tilde{q} = \sqrt{\frac{\sigma_x^2}{\sigma_\theta^2}}(\tilde{\theta} - E(\tilde{\theta}))$$

and thus

$$\mathrm{var}(\tilde{q}) = \sigma_x^2 \quad .$$

In equilibrium the distributions of the informed trader's quantities and the liquidity trader's quantities are exactly the same. If the number of insiders increases, the weight in the equilibrium distribution of the aggregate quantities of insiders is shifted to the right, i.e., on average the aggregate amount of informed trading will be higher. The reason for that is quite similar to the well known phenomenon in standard Cournot games that an increase in the number of competitors raises the equilibrium supply. The variance of the aggregate demand is given by

$$\mathrm{var}(\sum_{i=1}^{n} \tilde{q}_i) = \sqrt{n}\sigma_x^2 \quad .$$

Hence, it becomes easier for the market makers to separate liquidity demand from informed demand and to offer better bid-ask-spreads to the liquidity traders.

With risk aversion the situation is slightly different. Risk aversion reduces the willingness of insiders to take speculative positions. This limited risk bearing capacity shifts in equilibrium the weight in the distribution of the informed demand to the left of the distribution of liquidity demand:

$$\begin{aligned}\operatorname{var}(\tilde{q}) &= \frac{1}{(2b + a\sigma_\epsilon^2)^2}\sigma_\theta^2 \\ &= \frac{b}{b + a\sigma_\epsilon^2}\sigma_x^2 \\ &< \sigma_x^2 \quad .\end{aligned}$$

An increase in the number of informed traders shifts again the distribution of the aggregate informed demand to the right and this shift creates exactly the effect which is responsible for the behaviour of the bid-ask-spread in Claim 7. As long as the distribution is not shifted sufficiently to the right (i.e., for sufficiently low n and sufficiently high influence of the risk aversion effect), the two distributions move actually closer together. Thus, it becomes harder for the market makers to distinguish between informed demand and liquidity demand and he has to charge higher bid-ask-spreads for the liquidity traders. Ultimately, if the number of insiders becomes large enough, the aggregate informed demand becomes so high that it becomes easier to distinguish between noise traders and insiders. Accordingly, bid-ask-spreads will become lower, if enough insiders are active.

A very similar reason is responsible for the behaviour of the bid-ask-spread in the competitive setting. In the competitive setting, informed speculators have no market power. Given that every insider knows that he has no influence on the aggregate demand and hence on prices, there is no incentive for him to try to mimic the liquidity traders, since the informativeness of prices is independent of his own actions. Consequently, the weight of the distribution of the informed quantities will lie to the left of the one of the noise traders as long as the proportion of insiders λ is sufficiently low. A raise in the proportion of insiders increases the variance of the aggregate informed quantities and hence moves the two distributions closer together. Insiders and noise traders are harder to separate and the bid-ask-spread faced by the liquidity traders becomes higher. If the amount of informed speculation becomes sufficiently large, it becomes again easier for the uninformed speculators to distinguish noise trading from informed trading, because noise traders trade on average much lower quantities than the insiders as a whole. Hence, for high λ the bid-ask-spread is falling.

This last description explains our statement that the behaviour of the bid-ask-spread in the competitive framework is partially due to price-taking and partially due to risk aversion. It is due to price taking because this assumption guarantees that for a low number of insiders the aggregate informed demand will be on average lower than the liquidity demand. It is due to risk aversion in the sense that in a strategic setting in which insiders can actively try to become undistinguishable from noise traders, the result only persists if their risk aversion is sufficiently high in order to prevent them from taking too large speculative positions.

5.3 More Observations on Welfare.

Since liquidity demand is not really specified in the standard rational expectations models in the spirit of Grossman/Stiglitz (1980) and Hellwig (1980), it is hard to draw any specific welfare conclusions for the noise traders. Behind the notion of a liquidity trader stands the idea that agents use capital markets to smooth their consumption over their life time. Young investors tend to borrow, middle aged investors tend to save and old investors will dissave and consume. Stochastic liquidity trading may then exist both on an aggregate level and an individual level. Individually, agents might face unanticipated borrowing or lending constraints. On an aggregate level liquidity trading will depend on the stochastic population dynamics. Given the absence of a dynamic model which allows us to integrate this dynamic savings behaviour of economic agents and the performance of the financial markets at a given date, it is not clear how to measure exactly the welfare properties with exogenous noise. In our model noise traders can reasonably only be interpreted as punters. Reflecting this dilemma the literature has taken three different ways to measure the welfare of liquidity traders. One strand (e.g., Admati/Pfleiderer (1989, 1990)) considers only the size of the transaction costs. A second line assumes noise traders to be risk averse (e.g., Leland (1990)). Finally, some authors integrate the rational investors and the liquidity traders by assuming that the noise occurs in the form of stochastic changes in the endowments of the informed and uninformed speculators (e.g., Verrechia (1982)).

In two senses the magnitude of the bid-ask-spread might be a good indicator for possible welfare implications for liquidity traders. First, given that there are many ways to specify the ex-ante utilities of noise traders it is in some sense the most neutral indicator of what the noise traders stand to loose. Second, we have shown in section 4.3 that the size of the bid-ask-spread is also a good indicator for the possible ex-ante influence of price volatility. Recall that the initial market value of an investment was given by the equation

$$P_0 - E(\tilde{\theta}_2) = -a(\alpha_3 + \alpha_2^2\sigma_\theta^2 + a^2\frac{(\alpha_2\sigma_\epsilon^2)^2}{\lambda^2}\sigma_x^2)\bar{x} \quad .$$

Our analysis in chapter 3 has shown that the first and the second term on the right hand side (reflecting the lower risk premia and the higher price volatility due to the inflow of new information) roughly balance each other. Decisive whether investment ultimately increases or decreases is the behaviour of the third term on the right hand side which reflects the price volatility created by the bid-ask-spread. A higher bid-ask-spread will induce higher price volatility and in this sense the size of the bid-ask-spread is a good indicator also for the initial investment.

The common procedure in welfare comparisons of different regimes in models of the Grossman/Stiglitz type is to compare the aggregate welfare under these regimes. Aggregate welfare is defined as the weighted sum of the certainty equivalents of the utilities of the different investor classes. However, this procedure has three disadvantages. We have already mentioned the first one, namely that it is unclear how to measure exactly the utility of liquidity traders. Second, as we have observed in section 3.2, the calculation of the aggregate welfare does not make sense if changes in the secondary markets influence ex-ante investment. If the outcome of a change in regulation is less investment, it becomes meaningless that the aggregate welfare on the secondary markets increases conditional on the investment taking place. Without the risky asset the secondary market for this asset just does not exist. The third point concerns the fact that different regimes can in general not be pareto-ranked: Some investors will lose, others will gain. Hence, a political decision has to be taken in favour of which class of investors we want to redistribute. This is a bit odd since we do not exactly know for what reasons the liquidity traders trade. For instance, noise traders might just be speculating "opinion traders", and there is no reason why we should redistribute wealth in favour of them. Moreover, even if the redistribution issue could be decided unambiguously, the redistribution schemes would in general require much more information on the part of the central planner than can be reasonably expected.

For these reasons we will restrict ourselves in the sequel to characterize the utility gains and losses of the different classes of investors and ignore the issue of aggregate welfare. We will first assume that shifts in the intensity of insider trading do not influence the capital stock $\bar{x}$, i.e., all our calculations treat the capital stock as fixed. We have already analyzed the behaviour of the bid-ask-spread as a function of the amount of informed speculation in the market. Therefore, we will now study the utility of informed and uninformed speculators. At the end of this section, we will briefly study the welfare of the initial investors of generation 0.

We first calculate the utilities of the informed speculators. All our calculations follow closely the techniques of Grossman/Stiglitz (1980)[1]. Also, some of our comparative statics results have already been obtained in their paper. Insider's wealth is given by

$$\tilde{W}_I = \frac{(\tilde{u} - \tilde{P})^2}{a(\sigma_\theta^2 + \sigma_\epsilon^2)} \quad .$$

Integrating with respect to $\tilde{\epsilon}$ we obtain

$$E(U(\tilde{W}_I) = 1 - E\left(exp(\frac{\alpha_1^2 \sigma_\theta^2 + (\frac{\alpha_2 \sigma_\epsilon^2}{\lambda})^2 \sigma_x^2}{2\sigma_\epsilon^2} \tilde{Z}) \right) \quad ,$$

[1]To simplify the formulas we set the coefficient of risk aversion $a = 1$ and the initial endowment of the riskless asset $W = 0$.

where

$$\tilde{Z} = \frac{\alpha_1(\tilde{\theta} - E(\tilde{\theta})) + \frac{\alpha_2\sigma_\epsilon^2}{\lambda}\tilde{x} + \alpha_3\bar{x}}{\sqrt{\alpha_1^2\sigma_\theta^2 + (\frac{\alpha_2\sigma_\epsilon^2}{\lambda})^2\sigma_x^2}}$$

is distributed as χ^2. Exploiting the properties of the moment generating functions of the χ^2-distribution, we obtain

$$E(U(\tilde{W}_I)) = 1 - \sqrt{\frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \alpha_1^2\sigma_\theta^2 + (\frac{\alpha_2\sigma_\epsilon^2}{\lambda})^2\sigma_x^2}} exp(-\frac{1}{2}\frac{(\alpha_3\bar{x})^2}{\sigma_\epsilon^2 + \alpha_1^2\sigma_\theta^2 + (\frac{\alpha_2\sigma_\epsilon^2}{\lambda})^2\sigma_x^2}).$$

Furthermore, Grossman/Stiglitz (1980) have shown that

$$E(U(\tilde{W}_U)) = 1 - \sqrt{\frac{\text{var}(\tilde{u} \mid w_\lambda)}{\sigma_\epsilon^2}}\Big(E(U(\tilde{W}_I)) - 1\Big) \quad .$$

We can decompose the utility in two different components. The first is contained in the factor in front of the exponential function and reflects the profits from noise traders; it would arise even if the capital stock of the economy were zero. The second, namely the profits from holding on average the quantity $\bar{x}$ is contained in the argument of the exponential function; the profits stem from risk sharing and amount to roughly half of the risk premia times the capital stock $\bar{x}$ (plus an additional factor stemming from the noisy transaction prices which distort the optimal risk sharing between generations).

We begin by comparing the situation with full revelation and without information.

> Claim 8:
>
> The expected utility of both insiders and outsiders is higher in the case without information ($\lambda = 0$) than in the case with full revelation ($\lambda = 1$).

Proof: See appendix.

There are two intuitive reasons for this result. The first is contained in the argument of the exponential function and has to do with what we dubbed "Hirshleifer"-effect in the last chapter. Since we assumed investors to start from a zero endowment they suffer from a premature arrival of information, because the asset become less risky. But investors prefer holding a more risky asset because in equilibrium their compensation for the higher risk from the sellers increases stronger than the risk. The size of this effect depends strongly on the size of the capital stock; in particular it would disappear if we set $\bar{x} = 0$.

The second reason stems from the "pure" effect of insider trading and is contained in the factor in front of the exponential function. Also this factor is higher in the case with full information. Since everybody is informed there is nothing to gain on the additional information. However, the risk bearing component of the bid-ask-spread decreases because the asset becomes less risky after the information arrival. Accordingly, investors' profits from the risk bearing service offered to the liquidity traders fall. With a continuity argument we can conclude that also for sufficiently high levels of λ both informed and uninformed speculators will be worse off than in the situation without information. Liquidity traders will face lower transaction costs because of the substantial decrease in the risk bearing component of the bid-ask-spread.

Therefore, we turn now to the more interesting case in which only a small part of investors is informed, i.e., to the case of sufficiently low λ. We concentrate first on the "pure" insider effect and ignore the additional influence from the exponential term.

Claim 9:

Let $\bar{x} = 0$.

If σ_x^2 is sufficiently large the utility of every informed and every uninformed speculator is monotonically decreasing in λ.

If σ_x^2 is sufficiently small, the utility of every informed and every uninformed speculator is increasing in λ for small λ.

Proof: See appendix.

The reasons behind this result[1] are as follows. The income of both insiders and outsiders include the compensation for risk bearing which liquidity traders have to

[1]Implicit in the formulation of the result is that we treat λ as a kind of natural constant. If we compare two regimes which differ with respect to λ we perform comparative statics on two

pay. The higher the volume of a liquidity motivated trade, the higher the profits from risk bearing become. If on average the traded quantities in the market are large (characterized by a high σ_x^2), this income from risk bearing becomes the dominant source of income for both insiders and outsiders. Since more informed speculation reduces uncertainty, the profits from risk bearing are reduced as well and both insiders and outsiders suffer.

If σ_x^2 is relatively small, this source of income is less important and insiders profit mainly from their inside knowledge. We have shown in the last section that at least for small λ an increase in λ makes it more difficult to distinguish insiders from liquidity traders. Hence, the aggregate profits of the insiders should rise. Claim 9 shows that even the profits per insider are rising for small λ, but there is no intuitive reason why this should necessarily be the case. Even less obvious is the result for the outsiders. The outsiders are not really hurt by more insider trading, since they can widen their bid-ask-spreads. Ultimately, the liquidity traders have to pay for the increase in informed speculation. But a priori it is hard to see how outsiders might profit from more insider trading. Apparently, outside investors (which one might also consider as market makers) are able to widen their bid-ask-spreads more than would be necessary to compensate them for the higher insider risk.[1]

If we introduce additionally the effects arising from the "Hirshleifer effect"-term by allowing for $\bar{x} > 0$, the results become ambiguous. Although in general the argument of the exponential function tends to decrease (and hence also utility),

independent, different worlds in which we ignore that an individual who was an insider in the first world has a lower utility if he is an outsider in the second world. Since the *levels* of insider utility are always higher than the ones of the outsiders, every outsider would prefer to be an insider. A more sophisticated analysis would treat λ as an endogenous variable, for example determined by the cost of information acquisition. If the costs of information acquisition were equal for every investor, net utilities of insiders (i.e., net of expected costs) in equilibrium would be equal to outsider utility and the issue of utility losses of insiders who become outsiders would then disappear (as in Grossman/Stiglitz (1980)).

[1] The terminology is suggestive, but in a setting where agents act as price takers not entirely true. Uninformed speculators cannot actively influence the "average" bid-ask-spread. The fact that the "average" bid-ask-spread becomes larger for a higher λ is an equilibrium phenomenon. Notice however, that a very similar phenomenon occurred in the strategic setting with positive profits of chapter 2. Inspection of the market makers' profits in Claim 6 shows that they are increasing in the arrival probability μ of informed speculators as long as μ is sufficiently small. In equilibrium market makers *set* a higher bid-ask-spread.

this is not true for all parameter values. But even in parameter regions in which it decreases, the overall effect on utility is not clear. Consider for instance the case of low λ. In the proof of Claim 9 we have shown that $\alpha_1^2\sigma_\theta^2 + (\frac{\alpha_2\sigma_\epsilon^2}{\lambda})^2\sigma_x^2$ increases in λ for low λ, and that the risk premium component α_3 is always decreasing in λ. Hence, the risk reduction decreases the argument of the "Hirshleifer"-component. But the overall effect of the increase in the speculative utility component and the decrease in the "Hirshleifer"-component will depend how strong the latter effect may become. Both the level of the capital stock $\bar{x}$ and the extent to which ex-ante investment responds to shifts in λ are important. We will not pursue this discussion here, because without a more general formalization of the initial investment stage, utility gains and losses are hard to judge. We will discuss this point in more detail in section 5.5 of the next chapter.

Finally we turn to the welfare of the initial investors in the model of chapter 3. The gross revenues of the entrepreneurs are given by the initial market value of the risky asset, i.e., their revenues will fall for low λ and they will ultimately rise above the level without informed speculation if enough informed speculators are active. For the initial investors the situation is different. Since the entrepreneurs have to share the risk with them, the initial investors are actually interested in projects which are as risky as possible. The discount on risk which the entrepreneurs have to offer to the initial investors in order to induce them to invest, becomes the higher the riskier the project is. Accordingly, the utility of the initial investors increases if and only if the price volatility increases. Calculating the certainty equivalent of his utility we obtain

$$\begin{aligned} E(U(\tilde{W}) &= E(\tilde{P}_1 - P_0)\bar{x} - \frac{a}{2}\text{var}(\tilde{P}_1 - P_0)\bar{x} \\ &= \frac{a}{2}\text{var}(\tilde{P}_1)\bar{x} \quad , \end{aligned}$$

which is clearly increasing with the variance of future prices. We characterized price volatility at date 1 in Claim 5. Accordingly, generation 1 will incur welfare losses, if the early release of information reduces the overall risk of the economy (σ_x^2 large), and he tends to gain if this risk reducing effect is small (σ_x^2 small). Only if the lower initial market value prevents the entrepreneurs from undertaking the investment, generation 0 would be worse off with a higher price volatility at date 1.

To conclude this section, we summarize our results on the welfare gains and losses created by an increase in the number of informed speculators on secondary markets. For a sufficiently low number of insiders an increase in λ leads to less investment, lower revenues for the entrepreneurs and higher transaction costs for liquidity traders, if the average liquidity volume is not too large, i.e., σ_x^2 is small. Neglecting effects arising from the size of the available capital stock $\bar{x}$, initial investors, insiders and uninformed speculators (resp. market makers) tend to gain. If the number of insiders is sufficiently high, the risk reducing effect of new information becomes dominant. This induces higher investment, increasing revenues of the entrepreneurs and lower transaction costs for the liquidity traders. Initial investors', insiders' and uninformed speculators' welfare tends to decrease.

6. Conclusions.

We have shown in a very simple and stylized model how expected differences in information among shareholders may influence investment ex ante. We have argued that the indirect effect of insider activity on secondary markets is to shift risk forward and to alter allocation of risk among the investors. On the one hand insider trading raises the risk of holding an asset through a higher price volatility; on the other hand it also raises the expected returns via lower risk premia in the future. Through this effect insider trading might lead to the "breakdown" of markets: With insider trading expected in the future certain investments might not be undertaken today because of a lower market value of the investment. However, we have also shown that "more" insider trading need not necessarily lead to lower initial market values. Depending on the specific environment, more insiders can also lead to a higher investment activity ex ante. Premature resolution of uncertainty is not a neutral phenomenon: lower uncertainty might be appreciated less or more than lower price volatility.

The examples of this chapter show that for the judgement of the effects of insider trading one has to be very careful in dealing with the interaction between the value of new information per se and the market frictions which create the informational advantage. For instance, in the examples of section 4.2 and 4.3 the full revelation of information induced higher investment because it reduced the overall risk to be borne by the economy. However, the behaviour of price volatility, risk premia and initial investment in the presence of asymmetric information depended strongly on the respective specification of the market frictions. In the example of section 4.3 the beneficial effect dominated any effects arising from asymmetric information, whereas in the example with exogenous noise trading of section 4.2 and section 5 the effects of the asymmetric information between insiders and outsiders dominated in certain parameter ranges the risk reducing effects of the new information and led to lower investment.

At this point the reader might wonder, in which sense actual observed regulations of insider trading correspond to shifts in the variable λ of our model. Discussions on the prohibition of insider trading often turn around the introduction of a "disclose-or-abstain" rule, i.e., a rule which in our model should automatically lead to $\lambda = 0$ or $\lambda = 1$. But in practice, such a rule will only increase the expected costs of insider trading. The observed intensity of insider trading will depend strongly on the precise definition of insider information and the zeal with which the regulator enforces the law. Hence, even the introduction of a "disclose-or-abstain"-rule will be at least implicitly a decision on the desired λ. Moreover, the "disclose-or-abstain"-rule is not the only observed rule to regulate insider trading. Competing rules are the "equal-access-to-information"-rule and rules which prohibit insider trading only if some breach of fiduciary duty is involved. In practice, the choice of one of these rules is equivalent to choosing a certain level of insider trading. Under the "equal-access'-rule, market insiders like market makers, brokers etc. are typically exempted from the prohibition of certain forms of insider trading. This should lead to a higher λ than a "disclose-or-abstain"-rule. An even higher λ should be expected under the fiduciary-duty-rule, which basically regards only corporate insiders. In our model a regulator would need extraordinarily detailed information to take any sensible decision among these different rules.

A final word regards the limitations of our model. The three-period framework is very useful to highlight the basic effect of the premature arrival of information, i.e., that it shifts risk over time and alters the allocation of risks among different classes of investors. However, there are also several drawbacks of our model. First, one might be tempted to suspect that our results are mainly driven by the assumption of an economy with a finite number of trading dates. In an overlapping generations model with infinite horizon the benefits of new information might disappear because *each* generation faces now risky prices in the next period. Second, the existence of only two trading dates forces us to treat traders at date 0 and date 1 asymmetrically in the welfare analysis; a symmetric treatment of generations would certainly be desirable. Third, the investment structure of our model is rather rudimentary. In particular in the (perhaps most interesting) case of section 5, we have seen that the capital stock interacts in a rather subtle way with redistributional effects. In order to understand these effects better, one would need a richer investment structure. To deal with these issues the next chapter will consider an infinite horizon version of the model with exogenous noise trading.

Appendix.

1. Proof of Claim 1

To show that the risk component of P_0 is always lower (and hence P_0 always higher) than in the case without information resp. full disclosure we simply calculate the derivatives. We shall see that

$$\frac{\partial}{\partial\lambda}(\alpha_3 + \alpha_2^2\sigma_\theta^2) < 0 \quad \text{for } \lambda = 0,$$

$$\frac{\partial}{\partial\lambda}(\alpha_3 + \alpha_2^2\sigma_\theta^2) > 0 \quad \text{for } \lambda = 1 \quad \text{and}$$

$$\frac{\partial^2}{\partial\lambda^2}(\alpha_3 + \alpha_2^2\sigma_\theta^2) \text{ changes sign at most once.}$$

Therefore, define

$$x = \frac{1}{\sigma_\epsilon^2} \quad \text{and} \quad y = \frac{1}{\sigma_\theta^2 + \sigma_\epsilon^2} \quad ,$$

and observe $y < x$. We obtain

$$\frac{\partial}{\partial\lambda}(\alpha_3 + \alpha_2^2\sigma_\theta^2) = \frac{y-x}{(\lambda x + (1-\lambda)y)^2}\left(1 - \frac{2\lambda x}{y} - \frac{2\lambda x}{y}\frac{\lambda(y-x)}{\lambda x + (1-\lambda)y}\right) \quad ,$$

and thus

$$\begin{aligned}\frac{\partial}{\partial\lambda}(\alpha_3 + \alpha_2^2\sigma_\theta^2) &= \frac{y-x}{y^2}\\ &< 0 \quad \text{for } \lambda = 0\end{aligned}$$

and

$$\begin{aligned}\frac{\partial}{\partial\lambda}(\alpha_3 + \alpha_2^2\sigma_\theta^2) &= \frac{y-x}{x^2}\left(1 - \frac{2x}{y} - \frac{2}{y}(y-x)\right)\\ &= (y-x)\left(-\frac{1}{x^2}\right) > 0 \quad \text{for } \lambda = 1 \quad .\end{aligned}$$

A further derivation with respect to λ gives

$$\frac{\partial^2}{\partial\lambda^2}(\alpha_3 + \alpha_2^2\sigma_\theta^2) = 2\frac{x-y}{(\lambda x + (1-\lambda)y)^4}\left(((1-\lambda)y - \lambda x)(x-y) + xy\right) \quad ,$$

which only for large λ may become negative.

2. Proof of Claim 2

The comparative statics of the Grossman/Stiglitz model is quite tedious and estimates over the whole range of possible λ's are difficult. Therefore, for σ_x^2 sufficiently small we will restrict our attention to the behaviour of the initial market value for λ close to 0 resp. λ close to 1. More precisely, we will show that

$$\frac{\partial}{\partial\lambda}(\alpha_3 + \alpha_2^2\sigma_\theta^2 + a^2\alpha_2^2\frac{(\sigma_\epsilon^2)^2}{\lambda^2}\sigma_x^2)_{|\lambda=0} > 0 \quad ,$$

and

$$\frac{\partial}{\partial\lambda}(\alpha_3 + \alpha_2^2\sigma_\theta^2 + a^2\alpha_2^2\frac{(\sigma_\epsilon^2)^2}{\lambda^2}\sigma_x^2)_{|\lambda=1} < 0,$$

if σ_x^2 is sufficiently small. For intermediate values of λ in principle anything could happen. By continuity we only know from (4.5) that for very high λ the initial market value has to be higher than in the case without any insiders. However, it seems reasonable to conjecture that P_0 as a function of λ takes the shape of figure 2, i.e., the graph takes a U-shaped form.

For the proof of the previous inequalities we first recall the expressions for α_2 and α_3:

$$\alpha_2 = \lambda\frac{\lambda\sigma_\theta^2 + a^2(\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2}{\lambda^2\sigma_\theta^2 + a^2(\lambda\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2} \quad ,$$

and

$$\alpha_3 = \sigma_\epsilon^2\frac{\lambda^2\sigma_\theta^2 + a^2(\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2}{\lambda^2\sigma_\theta^2 + a^2(\lambda\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2} \quad .$$

We first calculate the derivatives of the three terms in the above expression.

$$\begin{aligned}
&\frac{\partial}{\partial\lambda}\alpha_3 \\
&= \sigma_\epsilon^2\frac{2\lambda\sigma_\theta^2(\lambda^2\sigma_\theta^2 + a^2(\lambda\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2) - (\lambda^2\sigma_\theta^2 + a^2(\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2)(2\lambda\sigma_\theta^2 + a^2\sigma_\theta^2\sigma_\epsilon^2\sigma_x^2)}{(\lambda^2\sigma_\theta^2 + a^2(\lambda\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2)^2} \\
&= \frac{(\lambda^2 - 2\lambda)a^2(\sigma_\theta^2)^2\sigma_\epsilon^2\sigma_x^2 - a^4(\sigma_\theta^2)^2(\sigma_\epsilon^2)^2(\sigma_x^2)^2 - a^4\sigma_\theta^2(\sigma_\epsilon^2)^3(\sigma_x^2)^2}{(\lambda^2\sigma_\theta^2 + a^2(\lambda\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2)^2} \\
&< 0 \quad ,
\end{aligned}$$

$$\frac{\partial}{\partial\lambda}(\alpha_2) = a^2(\sigma_\epsilon^2)^2\sigma_x^2\frac{(1-\lambda)(2\lambda\sigma_\theta^2 + a^2\sigma_\theta^2\sigma_\epsilon^2\sigma_x^2)) + \lambda^2\sigma_\theta^2 + (\lambda\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2}{(\lambda^2\sigma_\theta^2 + a^2(\lambda\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2)^2} \quad ,$$

$$\begin{aligned}
&\frac{\partial}{\partial\lambda}(\frac{\alpha_2}{\lambda}) \\
&= \frac{\sigma_\theta^2(\lambda^2\sigma_\theta^2 + a^2(\lambda\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2) - (\lambda\sigma_\theta^2 + a^2(\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2)(2\lambda\sigma_\theta^2 + a^2\sigma_\theta^2\sigma_\epsilon^2\sigma_x^2)}{(\lambda^2\sigma_\theta^2 + a^2(\lambda\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2)^2} \\
&= \frac{a^2\sigma_\theta^2(\sigma_\epsilon^2)^2\sigma_x^2 - \lambda\sigma_\theta^2 - a^2(\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2(2\lambda\sigma_\theta^2 + a^2\sigma_\theta^2\sigma_\epsilon^2\sigma_x^2)}{(\lambda^2\sigma_\theta^2 + a^2(\lambda\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2)^2} \quad .
\end{aligned}$$

We first observe that

$$\frac{\partial\alpha_3}{\partial\lambda}_{|\lambda=0} = -\frac{\sigma_\theta^2(\sigma_\theta^2 + \sigma_\epsilon^2)}{\sigma_\epsilon^2} \quad .$$

Moreover, we have that $\alpha_2(0) = 0$ and hence

$$\frac{\partial}{\partial\lambda}(\alpha_2^2)_{|\lambda=0} = 2\alpha_2(0)\frac{\partial}{\partial\lambda}(\alpha_2)_{|\lambda=0} = 0 \quad .$$

Finally we calculate

$$\begin{aligned}
\frac{\partial}{\partial\lambda}(a^2\alpha_2^2\frac{(\sigma_\epsilon^2)^2}{\lambda^2}\sigma_x^2) &= (2\frac{\alpha_2}{\lambda}\frac{\partial}{\partial\lambda}(\frac{\alpha_2}{\lambda}))_{|\lambda=0}(a^2(\sigma_\epsilon^2)^2\sigma_x^2) \\
&= 2\frac{\sigma_\theta^2 + \sigma_\epsilon^2}{\sigma_\epsilon^2}\sigma_\theta^2(1 - a^2(\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_x^2) \quad .
\end{aligned}$$

Combining these three expressions shows the inequality in the case $\lambda = 0$.

Similar calculations show the inequality in the case $\lambda = 1$ (see also the calculations in the proof of Claim 5).

For the last proposition of Claim 2 it is sufficient to show that $\text{var}(\tilde{P}_1)$ is monotonically decreasing in λ. This is shown in the proof of Claim 5.

3. Proof of Claim 4

For the comparative statics of equation (4.6) observe that

$$(\alpha)^2 + (1-\alpha)^2 - 0.5 > 0 \quad \text{for all } \alpha \in [0,1]$$

and thus

$$\frac{\partial \rho_\lambda}{\partial \lambda} < 0 \quad \text{for all } \lambda \in [0,1] \quad .$$

To see that the initial market value in the case of no insider trading is lower than in the case with insider trading we show that

$$\rho_{\lambda|\lambda=0} + \frac{\sigma_\theta^2}{2} < \rho_0 \quad .$$

This follows in particular if we can show that for any $g \in C([0,1])$ and every $c \in [0, \max(g))$ we have

$$\frac{1}{\int\limits_0^1 \frac{1}{g(\alpha)-c} df(\alpha)} + c \leq \frac{1}{\int\limits_0^1 \frac{1}{g(\alpha)} df(\alpha)} \quad .$$

But it is easily seen that this is an implication of Hõlder's inequality. First observe that the inequality is trivially fulfilled if $c = 0$. Derivation of the left hand side with respect to c gives

$$\frac{\partial}{\partial c}\left(\frac{1}{\int\limits_0^1 \frac{1}{g(\alpha)-c} df(\alpha)} + c\right) = -\frac{1}{\left(\int\limits_0^1 \frac{1}{g(\alpha)-c} df(\alpha)\right)^2}\left(\int\limits_0^1 \frac{1}{(g(\alpha)-c)^2} df(\alpha)\right) + 1 \leq 0 \quad ,$$

where the last inequality is just Hõlder's inequality. Since the right hand side is independent of c this shows the claim.

4. Welfare analysis of section 4.3

Consider first the outsiders of generation 1: we have

$$\begin{aligned}\tilde{W}_2^\alpha &= W + (\alpha\tilde{\theta}_2^1 + (1-\alpha)\tilde{\theta}_2^2 + \tilde{\epsilon}_2 - \tilde{P}_1)X_U(\tilde{P}_1)\\ &= W + \big((\alpha - .5)(\tilde{\theta}_2^1 - \tilde{\theta}_2^2) + \tilde{\epsilon}_2 + a\bar{x}\rho_\lambda\big)\frac{a\bar{x}\rho_\lambda}{a((\alpha^2 + (1-\alpha)^2 - .5)\sigma_\theta^2 + \sigma_\epsilon^2)} \quad .\end{aligned}$$

Since $\tilde{W}$ is normally distributed, we know that

$$E(exp(-a\tilde{W})) = exp(-aE(\tilde{W}) + \frac{a^2}{2}\mathrm{var}(\tilde{W})) \quad ,$$

and since

$$\begin{aligned}&E(\tilde{W}_2^\alpha) - \frac{a}{2}\mathrm{var}(\tilde{W}_2^\alpha)\\ &= \frac{(a\rho_\lambda\bar{x})^2}{a((\alpha^2 + (1-\alpha)^2 - .5)\sigma_\theta^2 + \sigma_\epsilon^2)} - \frac{(2(\alpha - .5)^2\sigma_\theta^2 + \sigma_\epsilon^2)(a\rho_\lambda\bar{x})^2}{a^2((\alpha^2 + (1-\alpha)^2 - .5)\sigma_\theta^2 + \sigma_\epsilon^2)^2}\\ &= \frac{(a\rho_\lambda\bar{x})^2}{a((\alpha^2 + (1-\alpha)^2 - .5)\sigma_\theta^2 + \sigma_\epsilon^2)}\Big(1 - \frac{(\alpha^2 + (1-\alpha)^2 - .5)\sigma_\theta^2 + \sigma_\epsilon^2}{2((\alpha^2 + (1-\alpha)^2 - .5)\sigma_\theta^2 + \sigma_\epsilon^2)}\Big)\\ &= \frac{(a\rho_\lambda\bar{x})^2}{2a((\alpha^2 + (1-\alpha)^2 - .5)\sigma_\theta^2 + \sigma_\epsilon^2)} \quad ,\end{aligned}$$

where we used

$$2(\alpha - .5)^2 = \alpha^2 + (1-\alpha)^2 - .5 \quad ,$$

we finally obtain

$$E(U(\tilde{W}^\alpha)) = 1 - exp(-aW)exp(-a^2\bar{x}^2\frac{\rho_\lambda^2}{(\alpha^2 + (1-\alpha)^2 - .5)\sigma_\theta^2 + \sigma_\epsilon^2}) \quad .$$

For the insiders observe that

$$\begin{aligned}\tilde{W}_2^\alpha &= W + (\alpha\tilde{\theta}_2^1 + (1-\alpha)\tilde{\theta}_2^2 + \tilde{\epsilon}_2 - \tilde{P}_1)X_I(\tilde{P}_1)\\ &= W + \frac{1}{a\sigma_\epsilon^2}\big((\alpha - .5)(\tilde{\theta}_2^1 - \tilde{\theta}_2^2) + a\bar{x}\rho_\lambda\big)^2 + \tilde{\epsilon}_2\frac{1}{a\sigma_\epsilon^2}\big((\alpha - .5)(\tilde{\theta}_2^1 - \tilde{\theta}_2^2) + a\bar{x}\rho_\lambda\big) \quad .\end{aligned}$$

Integrating first over $\tilde{\epsilon}$ and exploiting the normality of $\tilde{\epsilon}$ one obtains

$$E_{\theta,\tilde{\epsilon}}(exp(-a\tilde{W}_2^\alpha)) = exp(-aW)E_\theta(exp(-\frac{a}{2}\frac{1}{a\sigma_\epsilon^2}\big((\alpha - .5)(\tilde{\theta}_2^1 - \tilde{\theta}_2^2) + a\bar{x}\rho_\lambda\big)^2 \quad .$$

Now rewrite the argument of the last expression as

$$\frac{a}{2}\frac{1}{a\sigma_\epsilon^2}\left((\alpha-.5)(\tilde{\theta}_2^1-\tilde{\theta}_2^2)+a\bar{x}\rho_\lambda\right)^2=\frac{(\alpha-.5)^2\sigma_\theta^2}{\sigma_\epsilon^2}\left(\frac{\tilde{\theta}_2^1-\tilde{\theta}_2^2}{\sqrt{2\sigma_\theta^2}}+\frac{a\bar{x}\rho_\lambda}{\sqrt{2\sigma_\theta^2}(\alpha-.5)}\right)^2 \quad ,$$

observe that the term in the right bracket has a noncentral χ^2- distribution and exploiting the properties of the moment generating functions of the χ^2-distributions we get

$$E(U(\tilde{W}_2^\alpha))=1-exp(-aW)\cdot$$
$$\sqrt{\frac{\sigma_\epsilon^2}{(\alpha^2+(1-\alpha)^2-.5)\sigma_\theta^2+\sigma_\epsilon^2}}exp\left(-\frac{1}{2}\frac{a^2\bar{x}^2\rho_\lambda^2}{(\alpha^2+(1-\alpha)^2-.5)\sigma_\theta^2+\sigma_\epsilon^2}\right) .$$

Note that insiders are only better off with their inside information, if σ_ϵ^2 is small.

Finally we calculate the ex ante utility of generation 0 investors. Their future wealth is given by

$$\tilde{W}_1=W+(\tilde{P}_1-P_0)\bar{x} \quad ,$$

and since $\tilde{P}_1$ is normally distributed

$$\begin{aligned}
E(exp(-a\tilde{W}))&=exp(-aE(\tilde{W})+\frac{a^2}{2}\text{var}(\tilde{W}))\\
&=exp(-a^2\frac{\sigma_\theta^2}{2}\bar{x}^2+\frac{a^2}{2}\frac{\sigma_\theta^2}{2}\bar{x}^2)\\
&=exp(-\frac{a^2\bar{x}^2}{2}\frac{\sigma_\theta^2}{2}) \quad .
\end{aligned}$$

As in section 3 we consider now a planner who can raise lump sum taxes from some investors and pay lump sum transfers to others. Again, these payments occur in form of the riskless asset and cannot be conditioned on any information events. However, in this example the central planner must be able to pay personalized transfers (i.e., as a function of α) in order to be able to make everybody better off. Per capita income from taxes is now

$$a(\rho_0-\rho_\lambda-\frac{\sigma_\theta^2}{4})\bar{x}^2 \quad ,$$

and the required transfer to outsider α is

$$a\left(\frac{\rho_0^2}{2((\alpha^2+(1-\alpha)^2)\sigma_\theta^2+\sigma_\epsilon^2)}-\frac{\rho_\lambda^2}{2((\alpha^2+(1-\alpha)^2-.5)\sigma_\theta^2+\sigma_\epsilon^2)}\right)\bar{x}^2 \quad .$$

Integrating over α and substituting the definitions of ρ_0 and ρ_λ, one sees that the per capita revenue exceeds the per capita expenses

$$\int_0^1 a\left(\frac{\rho_0^2}{2((\alpha^2+(1-\alpha)^2)\sigma_\theta^2+\sigma_\epsilon^2)}-\frac{\rho_\lambda^2}{2((\alpha^2+(1-\alpha)^2-.5)\sigma_\theta^2+\sigma_\epsilon^2)}\right)\bar{x}^2\,df(\alpha) \quad .$$

Hence, at least under certain conditions appropriate redistribution policies might make everybody in the economy better off if insider trading is legal rather than prohibited.

5. Proof of Claim 5

The claim follows with appropriate derivations of α_2 and α_3.

a)

$$\begin{aligned}
&\frac{\partial}{\partial\lambda}\alpha_3 \\
&=\sigma_\epsilon^2\frac{2\lambda\sigma_\theta^2(\lambda^2\sigma_\theta^2+a^2(\lambda\sigma_\theta^2+\sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2)-(\lambda^2\sigma_\theta^2+a^2(\sigma_\theta^2+\sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2)(2\lambda\sigma_\theta^2+a^2\sigma_\theta^2\sigma_\epsilon^2\sigma_x^2)}{(\lambda^2\sigma_\theta^2+a^2(\lambda\sigma_\theta^2+\sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2)^2} \\
&=\frac{(\lambda^2-2\lambda)a^2(\sigma_\theta^2)^2\sigma_\epsilon^2\sigma_x^2-a^4(\sigma_\theta^2)^2(\sigma_\epsilon^2)^2(\sigma_x^2)^2-a^4\sigma_\theta^2(\sigma_\epsilon^2)^3(\sigma_x^2)^2}{(\lambda^2\sigma_\theta^2+a^2(\lambda\sigma_\theta^2+\sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2)^2} \\
&<0 \quad .
\end{aligned}$$

b)

$$\frac{\alpha_2}{\lambda}\sigma_\epsilon^2-\alpha_3=\sigma_\epsilon^2\sigma_\theta^2\frac{\lambda-\lambda^2}{\lambda^2\sigma_\theta^2+a^2(\lambda\sigma_\theta^2+\sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2} \quad .$$

Hence

$$\begin{aligned}
&\frac{\partial}{\partial\lambda}\left(\frac{\alpha_2}{\lambda}\sigma_\epsilon^2-\alpha_3\right) \\
&=\sigma_\epsilon^2\sigma_\theta^2\frac{(1-2\lambda)(\lambda^2\sigma_\theta^2+a^2(\lambda\sigma_\theta^2+\sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2)-(\lambda-\lambda^2)(2\lambda\sigma_\theta^2+a^2\sigma_\theta^2\sigma_\epsilon^2\sigma_x^2)}{(\lambda^2\sigma_\theta^2+a^2(\lambda\sigma_\theta^2+\sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2)^2} \quad .
\end{aligned}$$

For $\lambda = 0$ we get

$$\frac{\partial}{\partial\lambda}(\frac{\alpha_2}{\lambda}\sigma_\epsilon^2 - \alpha_3)_{|\lambda=0} = \sigma_\epsilon^2\sigma_\theta^2\frac{a^2\sigma_\epsilon^2\sigma_\epsilon^2\sigma_x^2}{(a^2\sigma_\epsilon^2\sigma_\epsilon^2\sigma_x^2)^2} \quad ,$$

and for $\lambda = 1$

$$\frac{\partial}{\partial\lambda}(\frac{\alpha_2}{\lambda}\sigma_\epsilon^2 - \alpha_3)_{|\lambda=1} = -\sigma_\epsilon^2\sigma_\theta^2\frac{(\sigma_\theta^2 + a^2(\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2)}{(\sigma_\theta^2 + a^2(\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2)^2} \quad .$$

Thus, for λ sufficiently small, the adverse selection component is increasing in λ, and for λ sufficiently large it is decreasing in λ.

c)

$$\begin{aligned}
&\frac{\partial}{\partial\lambda}(\frac{\alpha_2}{\lambda}) \\
&= \frac{\sigma_\theta^2(\lambda^2\sigma_\theta^2 + a^2(\lambda\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2) - (\lambda\sigma_\theta^2 + a^2(\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2)(2\lambda\sigma_\theta^2 + a^2\sigma_\theta^2\sigma_\epsilon^2\sigma_x^2)}{(\lambda^2\sigma_\theta^2 + a^2(\lambda\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2)^2} \\
&= \frac{a^2\sigma_\theta^2(\sigma_\epsilon^2)^2\sigma_x^2 - \lambda^2(\sigma_\theta^2)^2 - a^2(\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2(2\lambda\sigma_\theta^2 + a^2\sigma_\theta^2\sigma_\epsilon^2\sigma_x^2)}{(\lambda^2\sigma_\theta^2 + a^2(\lambda\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2)^2} \quad .
\end{aligned}$$

For $\lambda = 0$ this becomes

$$\frac{\partial}{\partial\lambda}(\frac{\alpha_2}{\lambda}\sigma_\epsilon^2)_{|\lambda=0} = \sigma_\epsilon^2\frac{a^2\sigma_\theta^2(\sigma_\epsilon^2)^2\sigma_x^2 - a^2\sigma_\theta^2(\sigma_\epsilon^2)^3(\sigma_x^2)^2}{(\lambda^2\sigma_\theta^2 + a^2(\lambda\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2)^2} \quad ,$$

which is positive for σ_x^2 sufficiently small. By continuity this is true for small λ. If $\lambda = 1$ and hence for all large λ the expression is clearly negative. The same is true for all λ, if σ_x^2 is sufficiently large.

d) For $\lambda = 0$

$$\frac{\alpha_2}{\lambda}_{|\lambda=0} = \frac{\sigma_\theta^2 + \sigma_\epsilon^2}{\sigma_\epsilon^2} > 1 \quad ,$$

and for $\lambda = 1$

$$\frac{\alpha_2}{\lambda}_{|\lambda=1} = \frac{\sigma_\theta^2 + \sigma_\epsilon^2}{\sigma_\theta^2 + \sigma_\epsilon^2} = 1 \quad .$$

e) We calculate

$$\frac{\partial}{\partial\lambda}(\alpha_2^2\sigma_\theta^2 + (\frac{\alpha_2\sigma_\epsilon^2}{\lambda})^2\sigma_x^2) = 2\frac{\alpha_2}{\lambda}\{\alpha_2 + (\frac{\partial}{\partial\lambda}(\frac{\alpha_2}{\lambda}))(\lambda^2\sigma_\theta^2 + (\sigma_\epsilon^2)^2\sigma_x^2)\} \quad .$$

From c) we know that $\frac{\partial}{\partial\lambda}(\frac{\alpha_2}{\lambda})$ is decreasing in λ, if σ_x^2 is sufficiently large. Since $0 \leq \alpha_2 \leq 1$, this shows that the price variance is monotonically decreasing in λ, if σ_x^2 is sufficiently large.

If σ_x^2 is sufficiently small, we know from c) that for small λ $\frac{\partial}{\partial\lambda}(\frac{\alpha_2}{\lambda})$ is increasing in λ. Hence, the price volatility is increasing as well.

If σ_x^2 sufficiently small and λ sufficiently large, set $\lambda = 1$ and show that the derivative becomes negative. From c) and because $\alpha_2(1) = 1$ we have for $\lambda = 1$

$$\begin{aligned}
&\alpha_2 + (\frac{\partial}{\partial\lambda}(\frac{\alpha_2}{\lambda}))(\lambda^2\sigma_\theta^2 + (\sigma_\epsilon^2)^2\sigma_x^2) \\
&= \sigma_\theta^2\{1 - \frac{\Big((\sigma_\epsilon^2)^2\sigma_x^2 + \sigma_\theta^2 + 2\sigma_\theta^2\sigma_\epsilon^2\sigma_x^2 + (\sigma_\theta^2 + \sigma_\epsilon^2)(\sigma_\epsilon^2\sigma_x^2)^2\Big)(\sigma_\theta^2 + \sigma_\epsilon^2\sigma_x^2)}{(\sigma_\theta^2 + (\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2)^2}\} \quad .
\end{aligned}$$

Estimating this expression gives the desired inequality.

6. Proof of Claim 6

If insider i buys/sells the quantity q_i his future wealth is given by

$$\tilde{W} = E_{\tilde{x}}\left(\tilde{\theta} + \tilde{\epsilon} - P(q_i + \sum_{j\neq i} q_j + \tilde{x})\right)q_i \quad .$$

Assume that $P(Q) = c + bQ$ and consider the quantities of the competing insiders as given. Because of the normality assumptions the maximization problem is then equivalent to

$$\max_{q_i} \quad E(\tilde{W}) - \frac{a}{2}\mathrm{var}(\tilde{W}) \quad .$$

Solving the first order condition gives

$$q_i = \frac{\theta - c - b\sum_{j\neq i} q_j}{2b + a\sigma_\epsilon^2} \quad ,$$

and the second order condition

$$2b + a\sigma_\epsilon^2 > 0 \quad .$$

Requiring symmetry gives

$$nq_i = n\frac{\theta - c}{2b + a\sigma_\epsilon^2} - n\frac{(n-1)bq_i}{2b + a\sigma_\epsilon^2} \quad ,$$

and thus

$$nq_i = n\frac{\theta - c}{(n+1)b + a\sigma_\epsilon^2} \quad \text{for } i = 1, .., n \quad .$$

Market makers observe the realization Q of $\tilde{Q} = n\tilde{q}_i + \tilde{x}$ and set prices such that

$$P(Q) = E(\tilde{\theta} + \tilde{\epsilon} \mid \tilde{Q} = Q) \quad .$$

Normality makes the regression linear and hence

$$c = E(\tilde{\theta})$$

and

$$b = \frac{\frac{n}{(n+1)b + a\sigma_\epsilon^2}\sigma_\theta^2}{\left(\frac{n}{(n+1)b + a\sigma_\epsilon^2}\right)^2 \sigma_\theta^2 + \sigma_x^2} \quad .$$

Before rearranging to obtain the desired expression of Claim 6, we note that any solution $b < 0$ must fulfil $b < -\frac{a}{n+1}\sigma_\epsilon^2$.

In order to show uniqueness rearrange the above expression and note that b is given is given as the roots of the polynomial

$$\begin{aligned} F(b) =& (n+1)^2\sigma_x^2 b^3 + 2(n+1)a\sigma_\epsilon^2\sigma_x^2 b^2 \\ &+ (a^2(\sigma_\epsilon^2)^2\sigma_x^2 - n\sigma_\theta^2)b - na\sigma_\epsilon^2\sigma_\theta^2 \quad . \end{aligned}$$

Clearly, $F(0) \leq 0$ and calculating the second derivative we see that the positive b are in the convex part of the polynomial:

$$\begin{aligned} \frac{d^2}{db^2}F(b) &= 6(n+1)^2\sigma_x^2 b + 4(n+1)a\sigma_\epsilon^2\sigma_x^2 \\ &> 0 \quad \text{for } b > 0. \end{aligned}$$

Hence there is only one positive root.

Since the second order condition allows also for $-\frac{a}{n+1}\sigma_\epsilon^2 > b > -\frac{a}{2}\sigma_\epsilon^2$, we have to check that there is no root in this domain. For $a = 0$, this is trivial. Thus consider the case $a > 0$. In the hypothetical case $\sigma_\theta^2 = 0$, there are only two roots of the polynomial, i.e., $b = 0$ and $b = -\frac{a}{n+1}\sigma_\epsilon^2$. Since the latter b is a double root, we have in particular that with $\sigma_\theta^2 = 0$ also $F(b) < 0$ for all $b < -\frac{a}{n+1}\sigma_\epsilon^2$. For positive σ_θ^2 we can estimate in the relevant domain of b

$$-n\sigma_\theta^2 b - na\sigma_\epsilon^2\sigma_\theta^2 < -na\sigma_\epsilon^2\sigma_\theta^2(-\frac{1}{2}+1) < 0 \quad .$$

Accordingly, $F(b) < 0$ for all $-\frac{a}{n+1}\sigma_\epsilon^2 > b > -\frac{a}{2}\sigma_\epsilon^2$, which establishes the uniqueness of the equilibrium.

Proof of Claim 7

Divide F by $(n+1)^2$ to obtain the polynomial Q:

$$\begin{aligned} Q(b) =& \sigma_x^2 b^3 + \frac{2}{(n+1)} a\sigma_\epsilon^2\sigma_x^2 b^2 \\ &+ \frac{1}{(n+1)^2}(a^2(\sigma_\epsilon^2)^2\sigma_x^2 - n\sigma_\theta^2)b - \frac{n}{(n+1)^2}a\sigma_\epsilon^2\sigma_\theta^2 \quad . \end{aligned}$$

The equilibrium b is still given as the unique positive root of Q. Furthermore, $Q(0) = -\frac{n}{(n+1)^2}a\sigma_\epsilon^2\sigma_\theta^2$ is decreasing in n, and

$$\frac{d}{db}Q(b) = 3\sigma_x^2 b^2 + \frac{4}{(n+1)}a\sigma_\epsilon^2\sigma_x^2 b + \frac{1}{(n+1)^2}(a^2(\sigma_\epsilon^2)^2\sigma_x^2 - n\sigma_\theta^2) \quad .$$

At least for those n with

$$a^2(\sigma_\epsilon^2)^2\sigma_x^2 - n\sigma_\theta^2 > 0 \quad ,$$

the first derivative of Q is decreasing in n for every $b \geq 0$. Hence, the positive unit root of Q is increasing in n. This establishes the first part of the claim. The second part follows from a continuity argument and the observation that in the case $a = 0$, b is a decreasing function of n.

7. Proof of Claim 8

We first compare the arguments of the exponential functions and show that they are lower in the case with full information. This amounts to showing that

$$\frac{(\sigma_\theta^2 + \sigma_\epsilon^2)^2}{\sigma_\epsilon^2 + \sigma_\theta^2 + (\sigma_\theta^2 + \sigma_\epsilon^2)^2 \sigma_x^2} - \frac{(\sigma_\epsilon^2)^2}{\sigma_\epsilon^2 + (\sigma_\epsilon^2)^2 \sigma_x^2} > 0 \quad .$$

To show this, observe that the expression is equal to 0 for $\sigma_\theta^2 = 0$, and calculate the derivative with respect to σ_θ^2.

We next show that the "pure" information effect is lower in the case with full revelation. For the insiders this is clear since

$$\sigma_\theta^2 + (\sigma_\epsilon^2 + \sigma_\theta^2)^2 \sigma_x^2 > (\sigma_\epsilon^2)^2 \sigma_x^2 \quad .$$

For the outsiders we have to show that

$$\frac{\sigma_\theta^2 + \sigma_\epsilon^2}{\sigma_\epsilon^2} \frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \sigma_\theta^2 + (\sigma_\theta^2 + \sigma_\epsilon^2)^2 \sigma_x^2} < \frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + (\sigma_\epsilon^2)^2 \sigma_x^2} \quad .$$

This follows by rewriting the inequality as

$$\frac{1}{1 + (\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_x^2} < \frac{1}{1 + \sigma_\epsilon^2 \sigma_x^2} \quad .$$

8. Proof of Claim 9

The first part of the claim follows by calculating

$$\frac{\partial}{\partial \lambda}(\alpha_1^2 \sigma_\theta^2 + (\frac{\alpha_2}{\lambda})^2 (\sigma_\epsilon^2)^2 \sigma_x^2) = (-2(1 - \alpha_2)\sigma_\theta^2 + \frac{\alpha_2}{\lambda}(\sigma_\epsilon^2)^2 \sigma_x^2) \frac{\partial}{\partial \lambda} \frac{\alpha_2}{\lambda} \quad .$$

For σ_x^2 sufficiently large, the first term on the right hand side is clearly positive and the derivative on the right hand side becomes negative according to Claim 5.

In order to show that the "pure insider effect" is decreasing in λ for λ and σ_x^2 sufficiently small, we consider the derivative of $\alpha_1^2\sigma_\theta^2 + (\frac{a_2}{\lambda})^2(\sigma_\epsilon^2)^2\sigma_x^2$ with respect to λ at $\lambda = 0$. We show that this expression is increasing in λ for small σ_θ^2, and thus expected utility is increasing as well:

$$\frac{\partial}{\partial\lambda}(\alpha_1^2\sigma_\theta^2 + (\frac{a_2}{\lambda})^2(\sigma_\epsilon^2)^2\sigma_x^2)_{|\lambda=0} = 2\sigma_\theta^2\frac{\sigma_\theta^2+\sigma_\epsilon^2}{\sigma_\epsilon^2}\frac{1-(1+a^2\sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2}{a^2(\sigma_\epsilon^2)^2\sigma_x^2} \quad ,$$

which is positive for σ_x^2 small.

For the uninformed speculators we calculate the additional factor as

$$\frac{var(\tilde{u} \mid w_\lambda)}{\sigma_\epsilon^2} = \frac{\lambda^2\sigma_\theta^2 + (\sigma_\theta^2+\sigma_\epsilon^2)\sigma_\epsilon^2\sigma_x^2}{\lambda^2\sigma_\theta^2 + (\sigma_\epsilon^2)^2\sigma_x^2} \quad ,$$

and a derivation of this expression with respect to λ shows

$$\frac{\partial}{\partial\lambda}\left(\frac{var(\tilde{u} \mid w_\lambda)}{\sigma_\epsilon^2}\right)_{|\lambda=0} = 0 \quad .$$

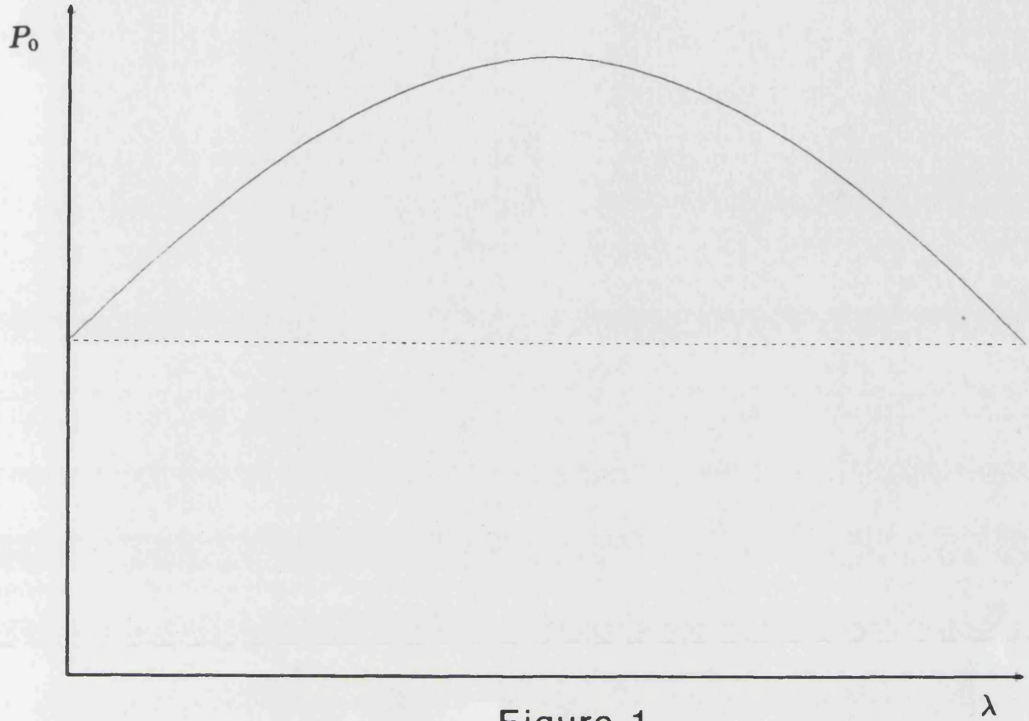

Figure 1

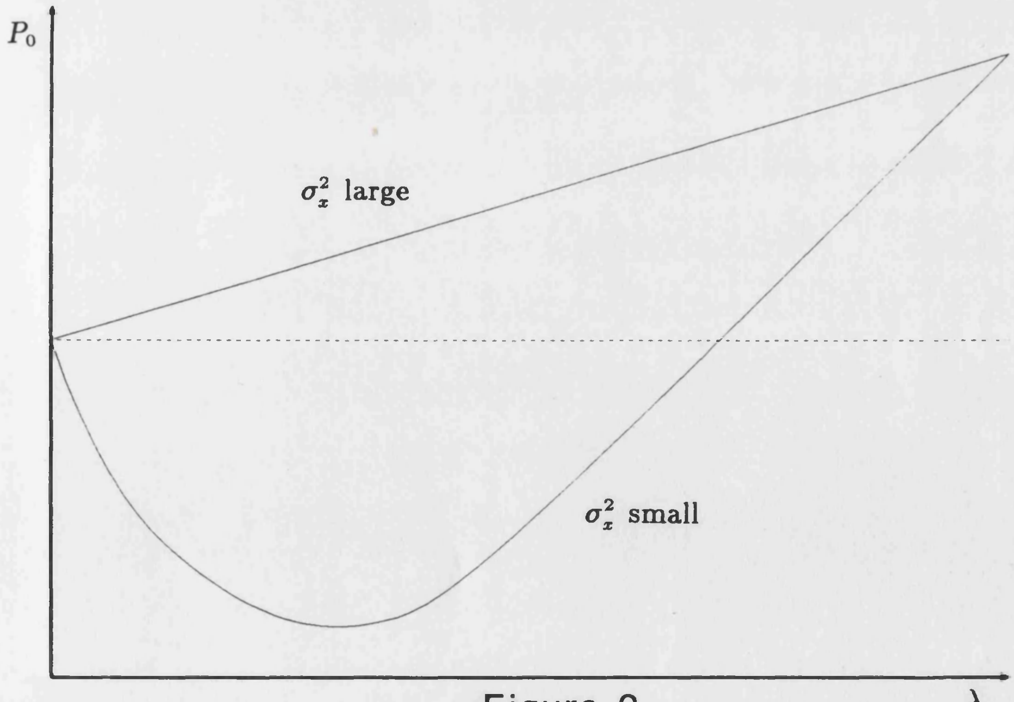

Figure 2

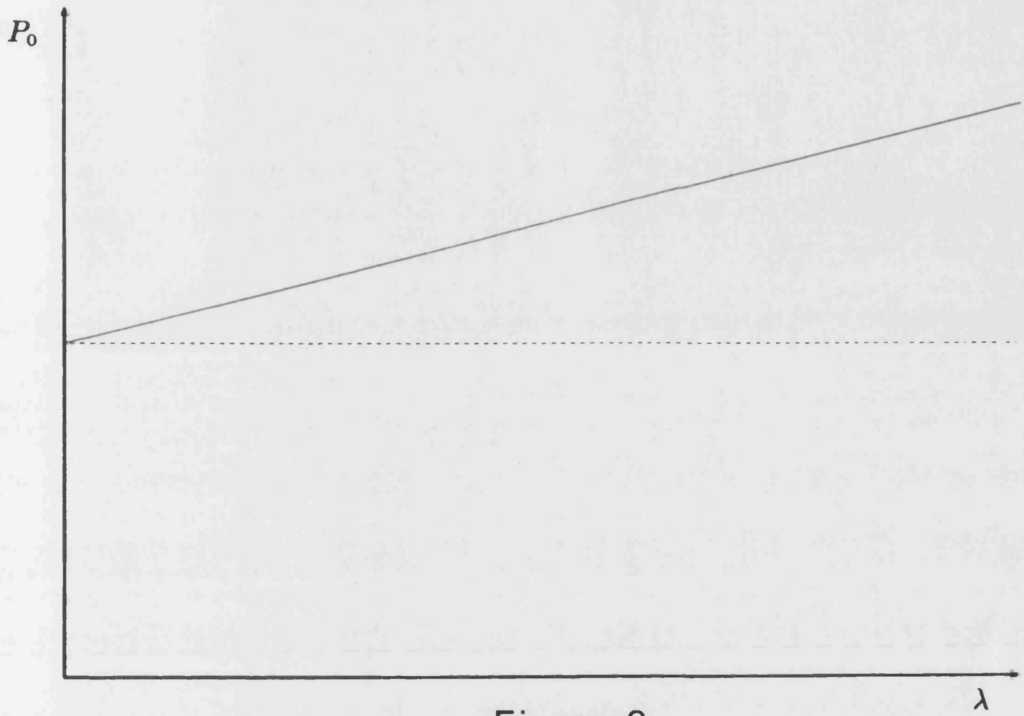

Figure 3

CHAPTER 4

A DYNAMIC ASSET PRICING MODEL WITH INFORMED SPECULATION

1. Introduction.

In this chapter we present a model of dynamic asset pricing in markets with noise trading and asymmetric information. We will study an overlapping generations economy with infinite horizon, and with one risky and one riskfree asset. At each trading date the inelastic supply of "old" individuals is bought by risk averse "young" investors. Some of these investors are assumed to be informed about the future dividend returns of the risky asset. Moreover, at each trading date there are noisy demand and supply shocks which are not further specified.

We can show that under certain conditions this economy possesses (multiple) steady state equilibria. Our analysis of these equilibria focuses on two main points. First, we study the impact of noise trading on the behaviour of price volatility and risk premia. We show that the presence of noise trading allows for feedback effects of the kind that expectations of a high price volatility become self-stabilizing. Expectations of a high future price volatility induce a high perceived risk of holding the asset at present. Accordingly, already minor noisy demand shocks may produce a high present price volatility, since prices have to move a lot in order to let the market absorb the extra demand. We show in particular, that there are equilibria in which this feedback effect prevents the convergence of the share prices towards the risky asset's fundamental value even if the investors' risk aversion becomes arbitrarily small.

Second, the analysis of the effects of the premature arrival of information and asymmetric information allows us to overcome some defects of the three-period-model of the last chapter. In particular, we can separate the analysis of the effects of new information from the analysis of asymmetric information.

We show that in an economy with zero interest rate, investors are indifferent between the premature arrival of new information and no information at all. The decrease of dividend risk is exactly compensated by the increase of price risk. If the interest rate is positive, the early release of information may decrease the risk to be borne by the economy. Since future returns are discounted with the interest rate whereas future risk is not, the early release of information may in certain equilibria be beneficial.

In contrast to the analysis of the noisy rational expectations equilibrium of the last chapter, the existence of an equilibrium in which the new information per se is neutral allows us to study the effect of asymmetric information in isolation. The analysis focuses again on the properties of the bid-ask-spread. Even with neutral information we can show that shifts in the intensity of insider trading may lead under certain circumstances to higher price volatility, higher risk premia and less investment, but that it may also induce the contrary. Moreover, we show that the welfare implications of insider trading are ambiguous.

The chapter is organized as follows. Section 2 presents the model, and shows that under certain assumptions there are exactly two steady state equilibria. We characterize all other equilibria and study the stability of the steady states. Furthermore, we investigate how price volatility and risk premia in the different equilibria depend on the amount of noise trading and the risk aversion of the investors. Section 3 studies the impact of the early release of information and asymmetric information on price volatility, risk premia and welfare. We close this section with a detailed analysis of the behaviour of investment levels. Section 4 discusses the implications and limitations of the model.

2. The Model.

We consider an overlapping generations model with two-period lived agents. To concentrate on the portfolio choice of the young generation we drop any additional decision like first period consumption and labour supply.[1] The economy has an infinite number of periods, indexed by $t = .., -1, 0, 1, ...$ At every date t a new generation t is born which can invest in a riskless and a risky asset. At date $t+1$ the dividends of the risky asset are paid out to generation t. Generation t sells all the assets to the new generation $t+1$ and leaves the market. Each generation is represented by the interval $[0,1]$ and every investor of a generation has constant absolute risk aversion a. For each unit of the riskless asset investors obtain R units in the next period, where we assume that $R \geq 1$. In period t the risky asset pays per capita the realization of a random variable $\tilde{u}_t$ where the $\tilde{u}_t$'s are assumed to be i.i.d. and distributed as in chapter 3, i.e.,

$$\tilde{u}_t \sim \tilde{\theta}_t + \tilde{\epsilon}_t, \quad \text{where } \tilde{\theta}_t \sim \mathrm{N}(E(\tilde{\theta}), \sigma_\theta^2), \quad \tilde{\epsilon}_t \sim \mathrm{N}(0, \sigma_\epsilon^2).$$

We assume that a proportion λ of the young investor generation t knows at date t the true realization of the systematic component $\tilde{\theta}_{t+1}$ of $\tilde{u}_{t+1}$. There is a fixed stock per capita $\bar{x}$ of the risky asset in each period. Moreover, we assume that in each period there is an additional noisy supply $\tilde{x}_t$, where the $\tilde{x}_t$ are i.i.d., and distributed as $\tilde{x}_t \sim \mathrm{N}(0, \sigma_x^2)$.[2]

Each generation is assumed to maximize its expected utility, i.e., given a price P_t generation t chooses a quantity X_t of the risky asset in order to maximize

$$E(U(X)) = 1 - E(exp(-a\tilde{W}_{t+1}(X))) \quad ,$$

[1]In the context of financial markets similar models have been used by DeLong et. al. (1988) and Pagano (1989). The overlapping generations structure might be alternatively interpreted as an economy with myopic investors.

[2]For a general equilibrium analysis the normality assumptions are of course an impossible assumption and are only imposed for tractability reasons. Normally distributed noisy supply plus no restrictions on short sales means that there are some agents who offer arbitrarily large bets on the outcome of the productive activity. But since these bets have to be honoured in any case, this implies that these noise traders have potentially infinite wealth. The analogous problem (negative consumption) arises for the rational investors.

where $\tilde{W}_{t+1}$ denotes the end-of-life wealth. Their initial wealth is W units of the riskless asset. Hence, $\tilde{W}_{t+1}$ is given by

$$\tilde{W}_{t+1}(X) = RW + (\tilde{u}_{t+1} + \tilde{P}_{t+1} - RP_t)X \quad .$$

We will look for rational expectations equilibria in normally distributed prices, i.e., a sequence of price distributions $\{.., \tilde{P}_{t-1}, \tilde{P}_t, \tilde{P}_{t+1}, ..\}$, which at each date t fulfil the following conditions:

(i) $\tilde{P}_t$ is a function of $(\tilde{\theta}_{t+1}, \tilde{x}_t)$, such that $\tilde{u}_{t+1}$ and $\tilde{P}_t$ are jointly normal.
(ii) Each investor maximizes his utility taking P_t as given and knowing the true joint distributions of $(\tilde{u}_{t+1}, \tilde{P}_t)$ and $(\tilde{u}_{t+2}, \tilde{P}_{t+1})$.
(iii) $\tilde{P}_t(\tilde{\theta}_{t+1}, \tilde{x}_t)$ clears the market at date t, given the realizations of $\tilde{\theta}_{t+1}$ and $\tilde{x}_t$.

Assumption (ii) is a kind of "double" rational expectations hypothesis. All investors need for their maximization problem an expectation on the joint distribution of $(\tilde{\theta}_{t+2}, \tilde{P}_{t+1})$. Moreover, the uninformed investors at date t also need an expectation concerning the joint distribution of $(\tilde{\theta}_{t+1}, \tilde{P}_t)$ in order to deduce information on $\tilde{\theta}_{t+1}$ from prices at date t. We assume that in equilibrium both expectations are self-fulfilling. Assumption (i) is necessary to keep the model tractable. It implies the normality of $(\tilde{u}_{t+1}, \tilde{P}_t)$, as well as the pairwise independence of the $\tilde{P}_t$. Thus, any equilibrium can be characterized by a sequence of price means and price variances $\{.., (E(\tilde{P}_{t-1}), \text{var}(\tilde{P}_{t-1})), (E(\tilde{P}_t), \text{var}(\tilde{P}_t)), (E(\tilde{P}_{t+1}), \text{var}(\tilde{P}_{t+1})), ...\}$.

2.1 Existence

In order to calculate equilibria of our economy, we first note that formally investors' decision problem in each period is the same as in the Grossman/Stiglitz (1980) setting of chapter 3. However, the returns on the risky asset are now given by $\tilde{u}_{t+1} + \tilde{P}_{t+1}$ rather than only by $\tilde{u}$. Accordingly, the per capita demands at date t are given by

$$X_I(P_t) = \frac{\theta_{t+1} + E(\tilde{P}_{t+1}) - RP_t}{a(\sigma_\epsilon^2 + \text{var}(\tilde{P}_{t+1}))} \quad ,$$

$$X_U(P_t) = \frac{E(\tilde{u}_{t+1} + \tilde{P}_{t+1} \mid \tilde{P}_t = P_t) - RP_t}{a\text{var}(\tilde{u}_{t+1} + \tilde{P}_{t+1} \mid \tilde{P}_t = P_t)} \quad .$$

In equilibrium P_t must clear the market, i.e., mean per capita demand of generation t has to be equal to the inelastic supply of shares of the old generation $\bar{x}$ plus the random noise supply $\tilde{x}$. Observing that instead of the residual risk σ_ϵ^2 the investors face now the residual risk $\sigma_\epsilon^2 + \mathrm{var}(\tilde{P}_{t+1})$ we obtain with a similar reasoning as in section 4.2 of chapter 3 that equilibria fulfilling (i)-(iii) are given by the following recursion formula

$$\tilde{P}_t = \frac{1}{R}\left(\alpha_1 E(\tilde{\theta}) + E(\tilde{P}_{t+1}) + \alpha_2(\tilde{\theta}_{t+1} - a\frac{\sigma_\epsilon^2 + \mathrm{var}(P_{t+1})}{\lambda}\tilde{x}_t) - a\alpha_3\bar{x}\right) \tag{2.1}$$

where

$$\alpha_1 = 1 - \alpha_2$$

$$\alpha_2 = \lambda\frac{\lambda\sigma_\theta^2 + a^2(\sigma_\theta^2 + \sigma_\epsilon^2 + \mathrm{var}(P_{t+1}))(\sigma_\epsilon^2 + \mathrm{var}(P_{t+1}))\sigma_x^2}{\lambda^2\sigma_\theta^2 + a^2(\lambda\sigma_\theta^2 + \sigma_\epsilon^2 + \mathrm{var}(P_{t+1}))(\sigma_\epsilon^2 + \mathrm{var}(P_{t+1}))\sigma_x^2}$$

$$\alpha_3 = (\sigma_\epsilon^2 + \mathrm{var}(P_{t+1}))\frac{\lambda^2\sigma_\theta^2 + a^2(\sigma_\theta^2 + \sigma_\epsilon^2 + \mathrm{var}(P_{t+1}))(\sigma_\epsilon^2 + \mathrm{var}(P_{t+1}))\sigma_x^2}{\lambda^2\sigma_\theta^2 + a^2(\lambda\sigma_\theta^2 + \sigma_\epsilon^2 + \mathrm{var}(P_{t+1}))(\sigma_\epsilon^2 + \mathrm{var}(P_{t+1}))\sigma_x^2} \quad .$$

α_1, α_2 and α_3 are always functions of λ and $\mathrm{var}(\tilde{P}_{t+1})$. However, we will usually drop the argument and put it only when necessary. Observe that

$$\alpha_2(0, \mathrm{var}(P_{t+1})) = 0 \quad ,$$

$$\frac{\alpha_2(\lambda, \mathrm{var}(P_{t+1}))(\sigma_\epsilon^2 + \mathrm{var}(P_{t+1}))}{\lambda}_{|\lambda=0} = \alpha_3(0, \mathrm{var}(P_{t+1})) = \sigma_\epsilon^2 + \sigma_\theta^2 + \mathrm{var}(P_{t+1}) \quad ,$$

$$\frac{\alpha_2(\lambda, \mathrm{var}(P_{t+1}))(\sigma_\epsilon^2 + \mathrm{var}(P_{t+1}))}{\lambda}_{|\lambda=1} = \alpha_3(1, \mathrm{var}(P_{t+1})) = \sigma_\epsilon^2 + \mathrm{var}(P_{t+1}) \quad .$$

In a first attempt to solve the existence problem we will restrict our attention to steady state equilibria. We define the economy to be in a steady state, if the $\tilde{P}_t$ are identically distributed, i.e., if there is a $\tilde{P}$ such that

$$\tilde{P}_t \sim \tilde{P} \sim \mathrm{N}(E(\tilde{P}), \sigma_P^2) \quad \text{for all } t.$$

Inspection of (2.1) shows that we can deal with the problem of finding a steady state mean and a steady state variance separately. We shall first deal with the problem of finding a steady state variance. Calculation of the price variance in equation (2.1) gives as the equation for the steady state variance

$$(2.2) \qquad \sigma_P^2 = \frac{1}{R^2}\left(\alpha_2^2\sigma_\theta^2 + a^2\left(\frac{\alpha_2}{\lambda}\right)^2(\sigma_\epsilon^2 + \sigma_P^2)^2\sigma_x^2\right) \quad .$$

Substituting the expressions for α_2 in (2.2), one sees that solving this equation for σ_P^2 is equivalent to solving for the roots of a polynomial of order 6. Therefore, we will deal first with the two bench-mark cases of $\lambda = 0$ and $\lambda = 1$ in which the equation reduces to a second order equation.

Claim 1:

Let $\lambda = 0, 1$.

If $\sigma_x^2 = 0$, there is a unique steady state variance, i.e.,

$$\sigma_P^2 = \lambda\frac{\sigma_\theta^2}{R^2} \quad .$$

If $\sigma_x^2 > 0$ we have to distinguish two cases:

If $\lambda\sigma_\theta^2 + (1-\lambda)R^2\sigma_\theta^2 + R^2\sigma_\epsilon^2 > \frac{R^4}{4a^2\sigma_x^2}$ there is no steady solution.

If $\lambda\sigma_\theta^2 + (1-\lambda)R^2\sigma_\theta^2 + R^2\sigma_\epsilon^2 \leq \frac{R^4}{4a^2\sigma_x^2}$ there are two steady state solutions. They are characterized by the price variances

$$\bar{\sigma}_P^2(0) = \sqrt{\frac{R^4}{4a^4(\sigma_x^2)^2} - \frac{R^2(\sigma_\theta^2 + \sigma_\epsilon^2)}{a^2\sigma_x^2}} + \frac{R^2}{2a^2\sigma_x^2} - \sigma_\theta^2 - \sigma_\epsilon^2 \quad ,$$

$$\bar{\sigma}_P^2(1) = \sqrt{\frac{R^4}{4a^4(\sigma_x^2)^2} - \frac{\sigma_\theta^2 + R^2\sigma_\epsilon^2}{a^2\sigma_x^2}} + \frac{R^2}{2a^2\sigma_x^2} - \sigma_\epsilon^2 \quad ,$$

resp.

$$\underline{\sigma}_P^2(0) = -\sqrt{\frac{R^4}{4a^4(\sigma_x^2)^2} - \frac{R^2(\sigma_\theta^2 + \sigma_\epsilon^2)}{a^2\sigma_x^2}} + \frac{R^2}{2a^2\sigma_x^2} - \sigma_\theta^2 - \sigma_\epsilon^2 \quad ,$$

$$\underline{\sigma}_P^2(1) = -\sqrt{\frac{R^4}{4a^4(\sigma_x^2)^2} - \frac{\sigma_\theta^2 + R^2\sigma_\epsilon^2}{a^2\sigma_x^2}} + \frac{R^2}{2a^2\sigma_x^2} - \sigma_\epsilon^2 \quad .$$

Proof: (2.2) becomes for $\lambda = 0$ resp $\lambda = 1$

$$\sigma_P^2(0) = \frac{1}{R^2}\left(a^2(\sigma_\theta^2 + \sigma_\epsilon^2 + \sigma_P^2(0))^2\sigma_x^2\right) \quad ,$$

resp.

$$\sigma_P^2(1) = \frac{1}{R^2}\left(\sigma_\theta^2 + a^2(\sigma_\epsilon^2 + \sigma_P^2(1))^2\sigma_x^2\right) \quad .$$

Solving these quadratic equations gives $\bar{\sigma}_P^2$ and $\underline{\sigma}_P^2$.

The two steady state variances highlight a general feature of the introduction of exogenous noise trading in a model of dynamic asset pricing: the presence of noise traders creates a very peculiar feedback effect in the economy. If the investors at date t expect a very variable price P_{t+1}, holding the asset at date t becomes very risky. Accordingly, the risk premia at date t become very large. This in turn will create a very high volatility of P_t itself. In the high volatility equilibrium $\bar{\sigma}_P^2$ the beliefs of a very high price volatility become self-fulfilling. Price volatility is basically created by the expectations of price volatility. Note that this phenomenon arises although the $\tilde{P}_t$ are pairwise independent.

Feedback effects of this sort cannot occur in the models of Pagano (1989) and DeLong et. al. (1988), because of the special form which the noise takes in their models. Price volatility in their models is basically given by the "real" variance of the asset plus the noise variance. The most striking feature of our feedback effect is that low levels of noise trading or risk aversion may even strengthen it. In the high volatility equilibrium $\bar{\sigma}_P^2$ a low risk aversion or a low variance of noise supply lead to a very high price variance; price volatility is nearly independent of the "real" return characteristics of the asset. Investors expect such a high volatility in the future that even small noisy demand or supply shocks lead to major price movements, and hence to a high price volatility today.

While in the high volatility equilibrium $\bar{\sigma}_P^2$ volatility expectations dominate the intrinsic risk characteristics of the asset, the low volatility equilibrium $\underline{\sigma}_P^2$ represents the opposite extreme. $\underline{\sigma}_P^2$ is the lowest steady state variance such that both the risk of the asset and the feedback effect are taken into account. This will become clearer in the stability analysis of 2.2. Heuristically, it is easily seen by the following thought experiment in the case $\lambda = 1$ (see also figure 1). The lowest possible candidate for the price variance would be $\frac{\sigma_\theta^2}{R^2}$. But because of the feedback effect this cannot be an equilibrium: the lowest possible price variance would be $\frac{\sigma_\theta^2}{R^2} + a^2(\sigma_\epsilon^2 + \frac{\sigma_\theta^2}{R^2})^2\sigma_x^2$. The iteration of this argument will lead to the steady state price variance $\underline{\sigma}_P^2$.

The difference between the two equilibria becomes also clear by looking at the behaviour of the equilibria for $\sigma_x^2 \to 0$. For $\sigma_x^2 \to 0$ we have obviously that $\bar{\sigma}_P^2 \to \infty$: because the feedback effect disappears, variance expectations have to become higher and higher to become selffulfilling. For $\underline{\sigma}_P^2$ one sees (for instance with l'Hopital's rule) that $\underline{\sigma}_P^2 \to \lambda\frac{\sigma_\theta^2}{R^2}$: the feedback effect disappears and only the intrinsic risk of the asset is taken into account.

In the general case $0 < \lambda < 1$ existence of steady state equilibria would have to be shown with a general contraction argument. Since for our comparative statics analysis we will concentrate on the cases in which λ is close to 0 or 1, we will not show this here. However, for λ sufficiently close to 0 or 1 and $\sigma_x^2 > 0$[1] we obtain

<u>Claim 1':</u>

Let λ sufficiently close to 0 or 1 and $\sigma_x^2 > 0$.

If $\sigma_\theta^2 + \sigma_\epsilon^2 < \frac{R^2}{4a^2\sigma_x^2}$ there are exactly two steady state solutions for the price variances. There is a low volatility equilibrium $\underline{\sigma}_P^2(\lambda)$ with

$$\underline{\sigma}_P^2(\lambda) \to \underline{\sigma}_P^2(0) \quad (\underline{\sigma}_P^2(1)) \text{ for } \lambda \to 0 \quad (1) \quad ,$$

and similarly a high volatility equilibrium $\bar{\sigma}_P^2(\lambda)$ with

$$\bar{\sigma}_P^2(\lambda) \to \bar{\sigma}_P^2(0) \quad (\bar{\sigma}_P^2(1)) \text{ for } \lambda \to 0 \quad (1) \quad .$$

[1]For $\sigma_x^2 = 0$ the case $\lambda > 0$ is equivalent to $\lambda = 1$, because prices are fully revealing.

Proof: Rewriting equation (2.2) as a polynomial in λ, and noting that α_2 and $\frac{\alpha_1}{\lambda}$ are continuous functions of λ, the claim follows by continuity.

We turn now to the price means in a steady state equilibrium. Inspection of (2.1) shows that for $R = 1$ there cannot be a steady state equilibrium. For $R > 1$ we show Claim 2.

Claim 2:

Let $R > 1$ and λ sufficiently close to 0 or 1.

If $\sigma_x^2 > 0$ and if $\sigma_\theta^2 + \sigma_\epsilon^2 < \frac{R^2}{4a^2\sigma_x^2}$, then there are are exactly two steady state equilibria given by the form

$$\tilde{P}_t = \frac{1}{R}\left(\frac{R}{R-1}E(\tilde{\theta}) - \frac{R}{R-1}a\alpha_3\bar{x} + \alpha_2(\tilde{\theta}_{t+1} - E(\tilde{\theta})) - a\frac{\alpha_2}{\lambda}(\sigma_\epsilon^2 + \sigma_P^2)\tilde{x}_t\right).$$

In the first equilibrium

$$\sigma_P^2 = \underline{\sigma}_P^2(\lambda), \quad \alpha_2 = \alpha_2(\lambda, \underline{\sigma}_P^2(\lambda)), \quad \alpha_3 = \alpha_3(\lambda, \underline{\sigma}_P^2(\lambda))$$

and in the second equilibrium

$$\sigma_P^2 = \bar{\sigma}_P^2(\lambda), \quad \alpha_2 = \alpha_2(\lambda, \bar{\sigma}_P^2(\lambda)), \quad \alpha_3 = \alpha_3(\lambda, \bar{\sigma}_P^2(\lambda)) \quad .$$

If $\sigma_x^2 = 0$ there is a unique equilibrium of the above form in which $\sigma_P^2(0) = 0$ and $\sigma_P^2(\lambda) = \frac{\sigma_\theta^2}{R^2}$ for all $\lambda > 0$.

Proof: For the variances the claim follows from Claim 1'. For the means, form the expectations on both sides of equation (2.1), replace $E(\tilde{P}_t)$ and $E(\tilde{P}_{t+1})$ by $E(\tilde{P})$ and solve for $E(\tilde{P})$.

2.2 Multiple Equilibria and Stability

As with all overlapping generations models, our model allows for a plethora of equilibria. In the following we will therefore investigate these equilibria and compare them to the steady state equilibria of the last section. Again we separate the discussion of equilibria price means and price variances. We will first deal with the price means.

Choose any equilibrium with the equilibrium variances

$$\{..,\mathrm{var}(\tilde{P}_{t-1}),\mathrm{var}(\tilde{P}_t),\mathrm{var}(\tilde{P}_{t+1}),...\} \quad .$$

Assume furthermore that for a given t and a given constant c, the price mean at period t is given by $E(\tilde{P}_t) = c$. Solving the recursion formula (2.1), we obtain the equilibrium means at the other dates:

$$E(\tilde{P}_{t+i}) = R^i c - \sum_{j=0}^{i-1} R^j E(\tilde{\theta}) + \sum_{j=0}^{i-1} R^j \rho(t+j) \quad ,$$

$$E(\tilde{P}_{t-i}) = \frac{1}{R^i} c + \sum_{j=1}^{i} \frac{1}{R^j} E(\tilde{\theta}) - \sum_{j=1}^{i} \frac{1}{R^j} \rho(t-j) \quad i = 1,2,.. \quad ,$$

where $\rho(t+j)$ indicates the risk premium of the prices at date $t+j$.

Even if the price variances are in one of the steady state equilibria, there is still an infinity of equilibria with "bubbles" in the price mean. This is a consequence of the myopic behaviour of the investors (see e.g. Tirole (1982)). For $R > 1$ the bubbles are exploding: Denoting by $\bar{c}$ the steady state mean, the above recursion formula implies

$$E(\tilde{P}_{t+i}) - \bar{c} = R^i (c - \bar{c}) \quad .$$

Since our main focus later will be on price volatility and risk premia, the existence of these bubbles is not too disturbing for most of the comparative statics and the welfare analysis of section 3.

In section 3 we will mainly be concerned with the behaviour of equilibrium price variances. Fortunately, it turns out that Claim 2 characterizes essentially all equilibrium price variances. This is clear for the case $\sigma_x^2 = 0$ in which there is only a unique equilibrium. For the case $\sigma_x^2 > 0$ we will first restrict the analysis to the cases $\lambda = 0$ and $\lambda = 1$ and characterize all equilibrium price variances. We show that the high volatility steady state is "stable" in the sense that the variances of all other non-steady state equilibria converge towards $\bar{\sigma}_P^2$ for $t \to \infty$ (see also figure 2).

Claim 3:

Let $\lambda = 0, 1$ and $\sigma_x^2 > 0$.

If $\lambda\sigma_\theta^2 + (1-\lambda)R^2\sigma_\theta^2 + R^2\sigma_\epsilon^2 > \frac{R^4}{4a^2\sigma_x^2}$ there is no equilibrium.

If $\lambda\sigma_\theta^2 + (1-\lambda)R^2\sigma_\theta^2 + R^2\sigma_\epsilon^2 \leq \frac{R^4}{4a^2\sigma_x^2}$ there is an infinity of equilibria. The variances $\{.., \text{var}(\tilde{P}_{t-1}), \text{var}(\tilde{P}_t), \text{var}(\tilde{P}_{t+1}), ...\}$ of any equilibrium have the following properties:

$$\inf_t(\text{var}(\tilde{P}_t)) \geq \underline{\sigma}_P^2 \quad .$$

If $\inf_t(\text{var}(\tilde{P}_t)) > \underline{\sigma}_P^2$ then

$$\text{var}(\tilde{P}_t) \to \bar{\sigma}_P^2 \text{ for } t \to \infty \quad .$$

Proof: The proof is basically a contraction argument. Define the operator $T^\lambda : I\!R^+ \to I\!R^+$ by

$$T^\lambda(x) = \frac{1}{R^2}\left((\alpha_2(\lambda,x))^2\sigma_\theta^2 + \left(\frac{\alpha_2(\lambda,x)}{\lambda}\right)^2(\sigma_\epsilon^2 + x)^2\sigma_x^2\right) \quad ,$$

where $\alpha_2(\lambda, x)$ is defined as in (2.1) by replacing σ_P^2 by x. Observe that the variances of any equilibrium are defined by the recursion

$$\text{var}(\tilde{P}_t) = T^\lambda(\text{var}(\tilde{P}_{t+1})) \quad t = .., -1, 0, 1, ..$$

In order to show that there is an infinity of such equilibrium paths, one estimates that for every $x > \underline{\sigma}_P^2$

$$T^\lambda(x) > \underline{\sigma}_P^2 \quad .$$

Hence, any given initial value $x > \underline{\sigma}_P^2$ at some date t defines together with the recursion formula an equilibrium path.

The lower bound for possible equilibria is obtained by showing that there is a $\kappa < 1$ such that for all $0 < x < \underline{\sigma}_P^2$

$$\underline{\sigma}_P^2 - T^\lambda(x) < \kappa(\underline{\sigma}_P^2 - x) \quad .$$

The latter inequality implies that any possible equilibrium path with a price variance $\text{var}(\tilde{P}_t) < \underline{\sigma}_P^2$ for a date t must have negative variances at some date $t + i$. This is a contradiction.

The proof of the convergence result consists in showing that for each $\delta > 1$ there is a $\kappa(\delta) > 1$ such that for all $x > \delta\underline{\sigma}_P^2$

$$| T^\lambda(x) - \bar{\sigma}_P^2 | > \kappa(\delta) | x - \bar{\sigma}_P^2 | \quad .$$

The details are left for the appendix.

To obtain the analogous result in the general case $0 < \lambda < 1$ would be technically quite difficult. Since for the comparative statics in section 3 it is sufficient to look at λ close to 0 or 1, we will not pursue this question here. However, it is possible to show that with the above proof plus a continuity argument an analogous result can be obtained for λ sufficiently close to 0 resp. 1.

On the basis of Claim 3 one would be tempted to concentrate on the high volatility equilibrium paths with $\sigma_P^2 = \bar{\sigma}_P^2$ as the unique stable equilibrium.[1] However, there is also a reason to prefer the low volatility steady state. The dynamics of Claim 3 have assumed that all agents know the true model. A somewhat weaker assumption would be that agents do not know the model, but form optimal forecasts of the variables they are interested in. We would like to study the dynamics of a given learning rule if investors start with expectations on $\tilde{P}_{t+1}$ off the equilibrium. Off the equilibrium paths there are of course innumerable possibilities for beliefs. Given that we restrict our equilibria to normally distributed price functions, we will assume that all investors know this and have only normally distributed beliefs. In general, there will also be a a considerable number of plausible learning mechanisms. These mechanisms may involve very complicated calculations and their

[1]That is, stable in the variances. The means are of course highly unstable.

convergence in general is far from clear (see e.g. Bray (1982)). We will not spell out a particular learning mechanism but rather follow a cruder approach which is similar to the one used by Lucas (1978).

Consider the following learning mechanism in the cases $\lambda = 0,1$. Assume that at each date t investors start with some expectations over $\tilde{P}_{t+1}$ characterized by $\tilde{P}^{(1)}_{t+1} \sim \mathrm{N}(E(\tilde{P}^{(1)}, \sigma_P^{2(1)})$. Given these beliefs investors calculate at each t their demands and markets clear, i.e., we obtain a "temporary" equilibrium with price distributions $\tilde{P}_t$ which clear the market at each t. Now assume that investors replace their initial beliefs $\tilde{P}^{(1)}$ by the observed temporary equilibrium distributions, i.e., $\tilde{P}^{(2)}_{t+1} \sim \tilde{P}_{t+1}$ and iterate this argument. If this process converges against an equilibrium we call this equilibrium stable under learning (resp. locally stable if it only converges for initial beliefs which are not too distant from the equilibrium).

It turns out that under this "learning" rule, there is only one locally stable equilibrium, namely the unstable one of Claim 3. Given that the contraction argument of Claim 3 is shown independently of the fact whether we are on or off the equilibrium path this is not really surprising. Stability under learning requires exactly the opposite direction of convergence as the stability of Claim 3.

Claim 4:

Let $R > 1$, $\lambda = 0,1$ and $\sigma_x^2 > 0$.

If $\lambda\sigma_\theta^2 + (1-\lambda)R^2\sigma_\theta^2 + R^2\sigma_\epsilon^2 \leq \frac{R^4}{4a^2\sigma_x^2}$, then there is exactly one equilibrium which is locally stable under the above learning rule.
It is given by

$$\tilde{P}_t = \frac{1}{R}\Bigg(\frac{R}{R-1}E(\tilde{\theta}) - \frac{R}{R-1}a((1-\lambda)\sigma_\theta^2 + \sigma_\epsilon^2 + \underline{\sigma}_P^2)\bar{x} \\ + \lambda(\tilde{\theta}_{t+1} - E(\tilde{\theta})) - a((1-\lambda)\sigma_\theta^2 + \sigma_\epsilon^2 + \underline{\sigma}_P^2)(\tilde{x}_t - \bar{x})\Bigg) \quad .$$

Proof: We will only sketch the proof. For the equilibrium variances we use the properties of the operator T^λ established in Claim 3. If the investors start with a variance expectation $\sigma_P^{2(1)} < \bar{\sigma}_P^2$, the temporary equilibrium variances will be closer to $\bar{\sigma}_P^2$ than $\sigma_P^{2(1)}$. This follows directly from the proof of Claim 3. If the initial belief $\sigma_P^{2(1)} > \bar{\sigma}_P^2$, then the learning process will not converge.

For the equilibrium mean, we use the observation that all price bubbles are exploding. Hence, if investors start with initial beliefs with mean $E(\tilde{P}^{(1)})$ which are different from the steady state mean, this will lead to a temporary equilibrium mean $E(\tilde{P}^{(2)})$ which is closer to the steady state mean.

Again, with a continuity one would be able to extend the scope of Claim 4 to all λ with λ sufficiently close to 0 or 1. The above "learning" mechanism is of course rather crude. The aim of Claim 4 is merely to stress that there is no clear selection device which would make us prefer one of the two equilibria with steady state variances over the other.

3. Comparative Statics.

Even if we restricted comparisons of equilibria only to steady state equilibria, the existence of two steady state equilibria renders comparative statics somewhat dubious. Changes in λ might switch the economy from the low volatility equilibrium to the high volatility equilibrium and vice versa. Nevertheless, we will assume in the sequel that changes in λ will not lead to switches from the low volatility equilibrium to the one with high volatility. Our main justification for this is that the stability analysis of the last section indicates that we can concentrate at least on one of the two steady state equilibria. For a full analysis of changes in regulation an additional investigation of dynamic adjustment processes would certainly be desirable. Assume for instance that the economy is in a particular steady state with a given intensity of insider trading and that at some date t an unanticipated change in insider regulation occurs which leads to a new level of insider activity. After this change the economy finds itself off the new steady state level. Then we might consider three possible cases. First, the economy might immediately jump to the new steady state we have selected. This is the view we will take in the sequel. Second, we might assume that the investors do not know the true distribution of the equilibrium prices and adapt their beliefs only slowly over time. The specification of such a procedure would involve learning of a similar kind as discussed in the last section and would presumably favour convergence towards the low volatility equilibrium. Lastly, we could assume that the economy is always in a rational expectations equilibrium, but that some endogenous variables only adjust slowly over time to a new steady state level. A candidate for such a variable would be the capital stock of the economy (if appropriately endogenized). If adjustments of the capital occur only slowly because of the presence of adjustment costs, one could study the convergence of the resulting equilibrium path. Given the stability analysis of the last section such a procedure would probably tend to converge towards the high volatility equilibrium.

In the remainder of this section we will discuss the impact on price volatility, risk premia, bid-ask-spreads and investment of changes in λ for each of the two classes of equilibria. We will compare the low price volatility equilibria with the equilibria in the three-period-world of chapter 3, sections 4.2 and 5. In particular, we will distinguish between effects due to new information per se, and effects due to asymmetric information. The comparative statics of the high volatility equilibria are mainly dictated by the effects arising from the selffulfilling beliefs.

3.1 Volatility and Risk Premia: Full Revelation vs. No Information

We begin with the two bench-mark cases of no information and full information. If $R = 1$ then price volatility in the case with full information includes the additional variability of the information on $\tilde{\theta}_{t+1}$, and is exactly σ_θ^2 higher than in the case without information. However, the risk of holding the asset remains constant. Instead of facing the intrinsic risk concerning $\tilde{\theta}_{t+1}$, investors face the higher price risk, which arises because prices at date $t+1$ move now with $\tilde{\theta}_{t+2}$. Accordingly, although the price variance increases in the case full revelation of information, the variance of the *returns* at date $t+1$ conditional on the information on $\tilde{\theta}_{t+1}$ is the same in both cases. Without information the variance of returns is given by

$$\alpha_3 = \sigma_\theta^2 + \sigma_\epsilon^2 + \sigma_P^2 = +(-)\sqrt{\frac{1}{4a^4(\sigma_x^2)^2} - \frac{\sigma_\theta^2 + \sigma_\epsilon^2}{a^2\sigma_x^2}} + \frac{1}{2a^2\sigma_x^2} \quad ,$$

and with full information it is just identical:

$$\alpha_3 = \sigma_\epsilon^2 + \sigma_P^2 = +(-)\sqrt{\frac{1}{4a^4(\sigma_x^2)^2} - \frac{\sigma_\theta^2 + \sigma_\epsilon^2}{a^2\sigma_x^2}} + \frac{1}{2a^2\sigma_x^2} \quad .$$

We turn now to the case $R > 1$. If the interest rate is positive, the premature resolution of uncertainty changes the risk to be borne by the economy. The variance $\underline{\sigma}_P^2$ rises by less than σ_θ^2, whereas the variance $\bar{\sigma}_P^2$ rises by more than σ_θ^2. From Claim 1 we know that for $\underline{\sigma}_P^2$

$$\begin{aligned}\underline{\sigma}_P^2(1) - \underline{\sigma}_P^2(0) &= \sigma_\theta^2 + \sqrt{\frac{R^4}{4a^4(\sigma_x^2)^2} - \frac{R^2(\sigma_\theta^2 + \sigma_\epsilon^2)}{a^2\sigma_x^2}} - \sqrt{\frac{R^4}{4a^4(\sigma_x^2)^2} - \frac{R^2\sigma_\epsilon^2 + \sigma_\theta^2}{a^2\sigma_x^2}} \\ &< \sigma_\theta^2 \quad .\end{aligned}$$

An analogous formula holds for $\bar{\sigma}_P^2$. For large σ_x^2, $\underline{\sigma}_P^2$ might even be lower in the case with full information. This is similar to the three-period case of the last chapter. The heuristic reason is that for large σ_x^2 the noise induced risk $\sigma_\theta^2\sigma_x^2$ is larger than the direct risk σ_θ^2.

As a consequence of the behaviour of the price volatilities, the risk premia in the low variance equilibrium fall, whereas the risk premia in the high volatility equilibrium rise. Without information the conditional variances are given by

$$\alpha_3 = (\sigma_\theta^2 + \sigma_\epsilon^2 + \sigma_P^2) = +(-)\sqrt{\frac{R^4}{4a^4(\sigma_x^2)^2} - \frac{R^2(\sigma_\theta^2 + \sigma_\epsilon^2)}{a^2\sigma_x^2}} + \frac{R^2}{2a^2\sigma_x^2} \quad ,$$

and with full information they become

$$\alpha_3 = (\sigma_\epsilon^2 + \sigma_P^2) = +(-)\sqrt{\frac{R^4}{4a^4(\sigma_x^2)^2} - \frac{R^2\sigma_\epsilon^2 + \sigma_\theta^2}{a^2\sigma_x^2}} + \frac{R^2}{2a^2\sigma_x^2} \quad .$$

In order to understand this result, it is useful to decompose the steady state equation (2.2). For $\lambda = 0$ we can write

$$R^2\sigma_P^2 = a^2(\sigma_\theta^2 + \sigma_\epsilon^2)^2\sigma_x^2 + 2a^2(\sigma_\theta^2 + \sigma_\epsilon^2)\sigma_P^2\sigma_x^2 + a^2(\sigma_P^2)^2\sigma_x^2 \quad ,$$

and for $\lambda = 1$

$$R^2\sigma_P^2 = \sigma_\theta^2 + a^2(\sigma_\epsilon^2)^2\sigma_x^2 + 2a^2\sigma_\theta^2\sigma_P^2\sigma_x^2 + a^2(\sigma_P^2)^2\sigma_x^2 \quad .$$

The first two terms on the right hand side represent the "intrinsic" price risk, i.e., the price risk which is unavoidable given that information is reflected in the prices and prices move with the risk premium demanded from noise traders. The two remaining terms arise through the feedback effect between price risk and noise trading. Observe that even if the intrinsic risk were zero, i.e., if the risky asset were effectively riskfree, there would still be a high volatility price equilibrium given by $\bar{\sigma}_P^2 = \frac{1}{R^2a^2\sigma_x^2}$. The difference between $\underline{\sigma}_P^2$ and $\bar{\sigma}_P^2$ is now simply that in the case of $\underline{\sigma}_P^2$ the price risk behaves essentially as the intrinsic risk, i.e., lower intrinsic risk leads to lower price risk, and that in the case of $\bar{\sigma}_P^2$ the opposite applies, i.e., lower intrinsic risk leads to higher price risk. The latter is due to the fact that the only factor which imposes some limits on the selffulfilling belief is the intrinsic risk characteristics of the asset. The higher the intrinsic risk of the asset the lower the equilibrium volatility, because equilibrium volatility is more and more dominated by the dividend risk, and the feedback effect linked to the expectations becomes lower.

Hence, the effects of changes in information on price risk can be explained by the effects on the "intrinsic" price risk. It is easy to see why the intrinsic risk of holding the asset in the low variance equilibrium becomes lower with full information: Prices discount only future returns with the interest rate, but not the risk. Seen from date t the price $\tilde{P}_{t+1}$ becomes more variable, because the realization of $\tilde{\theta}_{t+2}$ is announced. But the returns of period $t+2$ are discounted with R and accordingly the price variance rises by less than σ_θ^2 (in the extreme case $\sigma_x^2 = 0$ to $\frac{\sigma_\theta^2}{R^2}$). Therefore, premature arrival of information reduces the intrinsic risk to be borne in every period. Accordingly, the price risk in the low volatility case will decrease and the price risk in the high volatility case will increase.

In this sense, the analysis of the three-period case in section 5 of chapter 3 should be best understood as the extreme case of an economy in which everybody anticipates that interest rates at date 2 become infinity (and hence prices 0). The premature release of information can only have a real effect on the risk to be borne by the economy, if the interest rates are positive. The static models of asymmetric information in the spirit of Grossman/Stiglitz (1980) or Hellwig (1980) are just a very extreme case of this risk reducing effect of new information by setting implicitly the interest rate in subsequent periods to infinity.

3.2 Volatility and Risk Premia with Asymmetric Information

The advantage of the infinite horizon model is that it allows us to disentangle the effects of new information and the effects of insider trading. Therefore, we will first study the case with zero interest rate, when the new information per se has no effects. We state the results in Claim 5.

Claim 5:

Let $R = 1$.

For λ sufficiently small the price volatility $\underline{\sigma}_P^2(\lambda)$ (resp. $\bar{\sigma}_P^2(\lambda)$) and the risk premium $a\alpha_3(\lambda, \underline{\sigma}_P^2(\lambda))$ (resp. $a\alpha_3(\lambda, \bar{\sigma}_P^2(\lambda))$) are increasing (resp. decreasing) in λ.

For λ sufficiently large the price volatility $\underline{\sigma}_P^2(\lambda)$ (resp. $\bar{\sigma}_P^2(\lambda)$) and the risk premium $a\alpha_3(\lambda, \underline{\sigma}_P^2(\lambda))$ (resp. $a\alpha_3(\lambda, \bar{\sigma}_P^2(\lambda))$) are decreasing (resp. increasing) in λ.

Proof: See appendix.

In order to characterize the behaviour of price volatility fully, one would need explicit solutions of equation (2.2) also for intermediate values of λ. However, in all numerical solutions price volatility and risk premia were either concave or convex in λ, i.e., they were first increasing (decreasing) up to a certain maximum (minimum) and then decreasing (increasing).

To understand this result heuristically it is useful to rewrite the steady state equation (2.2) slightly:

$$(2.2) \qquad \sigma_P^2 = \frac{1}{R^2}\left(\alpha_2^2\sigma_\theta^2 + a^2\left(\left(\frac{\alpha_2}{\lambda}(\sigma_\epsilon^2 + \sigma_P^2) - \alpha_3\right) + \alpha_3\right)^2\sigma_x^2\right) \quad .$$

Now assume a change in λ and, hypothetically, hold σ_P^2 fix to study the effects of this change on the "intrinsic" risk. As we have shown in chapter 3, a rise in the intensity of insider trading has three effects which are linked to the real characteristics of the asset. First, prices will incorporate more of the information and hence vary stronger with $\tilde{\theta}_{t+1}$ (as expressed in the term $\alpha_2^2\sigma_\theta^2$). Second, the adverse selection component of the bid-ask-spread (i.e., the term $\frac{\alpha_2}{\lambda}(\sigma_\epsilon^2 + \sigma_P^2) - \alpha_3$) will change. We have shown in chapter 3 (section 5) that for low λ it will rise and for high λ it will decline. Finally, since more information on future returns becomes known, the direct risk of holding the asset decreases and the risk-aversion component (i.e., α_3) of the bid-ask-spread decreases. In chapter 3 we have explained why for sufficiently low noise levels and for small λ the adverse selection effect dominates the risk bearing effect. The same effect happens here: The intrinsic price volatility increases because the volatility component due to the adverse selection component of the bid-ask-spread increases. The opposite effect takes place for high λ: both the adverse selection and the risk bearing component decrease and dominate the volatility increase due to the fact that more information is reflected in prices.

This discussion is only heuristic, because holding σ_P^2 in the above formula constant can only be considered as a thought experiment. But the proof of Claim 5 shows that in the case $0 < \lambda < 1$ something very similar to the case $\lambda = 0, 1$ happens, i.e., the behaviour of $\underline{\sigma}_P^2(\lambda)$ and $\bar{\sigma}_P^2(\lambda)$ is mainly dictated by the behaviour of the intrinsic risk. Accordingly, $\underline{\sigma}_P^2(\lambda)$ resp. $\bar{\sigma}_P^2(\lambda)$ increase resp. decrease for low λ and vive versa for high λ.

The behaviour of the risk premia differs from the results of chapter 3, section 5. For low λ, both the price volatility $\underline{\sigma}_P^2(\lambda)$ *and* the risk premia are rising with λ. The reason is that for low λ the rise of the price volatility is mainly due to the adverse selection effect, whereas there is no risk reduction from the inflow of new information.

The preceding discussion of the low volatility equilibrium gives a good illustration how the results in the three-period world of chapter 3 depend both on asymmetric information and the risk reducing effect of new information. Without the risk reduction from new information ($R = 1$), price volatility and risk premia depend only on the intensity of insider trading. Price volatility goes up because more information is reflected in prices *and* because the adverse-selection-component of the bid-ask-spread goes up. Risk premia increase because the price volatility increases.

If we introduce additionally a real effect of information by allowing $R > 1$, the comparative statics of the low volatility equilibrium becomes more similar to the one of the three-period model. However, in order to let the effects of new information dominate the effects of asymmetric information, interest rates would have to be unreasonably high. We restrict our attention to the low volatility equilibrium and small λ and leave it to the reader to derive analogous results for λ close to 1 and the high volatility equilibrium.

Claim 6:

Let $R > 1$.

For $R < \sqrt{2}$ and λ sufficiently small, the price volatility $\underline{\sigma}_P^2(\lambda)$ and the risk premium $a\alpha_3(\lambda, \underline{\sigma}_P^2(\lambda))$ are increasing in λ.

For $R > \sqrt{2}$ and λ sufficiently small, the risk premium $a\alpha_3(\lambda, \underline{\sigma}_P^2(\lambda))$ is decreasing in λ. The price volatility $\underline{\sigma}_P^2(\lambda)$ is increasing in λ if σ_x^2 small, and decreasing in λ if σ_x^2 large.

Proof: See appendix.

For low λ and a positive interest rate the arrival of new information reduces the intrinsic risk of holding the asset and hence the corresponding part of the risk-bearing component of the bid-ask-spread. If the interest rate *and* the average quantity of noise trading are sufficiently large, this will dominate the increase of the adverse-selection component of the bid-ask-spread and decrease price volatility. The risk premia always decrease in the case of a sufficiently high reduction of intrinsic risk.

3.3 Welfare Comparisons: Full Revelation vs. No Information

As in the last chapter, the welfare effects of changes in λ depend very much on the specification of the model. For the noise traders it is hard to find any convincing welfare measure, given that in this model we can only interpret them as punters. The most neutral indicator is probably their average transaction costs, i.e., the size of the bid-ask-spread. The transaction costs can be calculated[1] as

$$ATC(\lambda, \sigma_P^2(\lambda)) = \frac{1}{R}\frac{\alpha_2(\lambda, \sigma_P^2(\lambda))}{\lambda}(\sigma_\epsilon^2 + \sigma_P^2(\lambda))\sigma_x^2 \quad .$$

The behaviour of the transaction costs follows immediately from our discussion of the last section. In the cases with resp. without information the bid-ask-spread

[1] For notational convenience we set in this and the following sections the coefficient of risk aversion $a = 1$ and the initial endowment of the riskless asset $W = 0$.

consists only of the risk bearing component because there is no adverse selection problem. Accordingly, the size of the bid-ask-spread will depend only on the increase resp. decrease of the risk to be borne by the economy. For $R = 1$ the bid-ask-spreads do not differ with or without information. For $R > 1$, bid-ask-spreads in the low volatility equilibrium are lower with full information, and in the high volatility equilibrium they are higher: For $\sigma_P^2 = \underline{\sigma}_P^2$ we have

$$\begin{aligned}
ATC(0, \underline{\sigma}_P^2(0)) &= \frac{1}{R}(\sigma_\theta^2 + \sigma_\epsilon^2 + \underline{\sigma}_P^2(0))\sigma_x^2 \\
&= -\frac{1}{R}\left(\sqrt{\frac{R^4}{4(\sigma_x^2)^2} - \frac{R^2(\sigma_\theta^2 + \sigma_\epsilon^2)}{\sigma_x^2}} + \frac{R^2}{2\sigma_x^2}\right)\sigma_x^2 \\
&\geq -\frac{1}{R}\left(\sqrt{\frac{R^4}{4(\sigma_x^2)^2} - \frac{R^2\sigma_\epsilon^2 + \sigma_\theta^2}{\sigma_x^2}} + \frac{R^2}{2\sigma_x^2}\right)\sigma_x^2 \\
&= \frac{1}{R}(\sigma_\epsilon^2 + \underline{\sigma}_P^2(1))\sigma_x^2 \\
&= ATC(1, \underline{\sigma}_P^2(1)) \quad ,
\end{aligned}$$

and for $\sigma_P^2 = \bar{\sigma}_P^2$

$$\begin{aligned}
ATC(0, \bar{\sigma}_P^2(0)) &= \frac{1}{R}(\sigma_\theta^2 + \sigma_\epsilon^2 + \bar{\sigma}_P^2(0))\sigma_x^2 \\
&= \frac{1}{R}\left(\sqrt{\frac{R^4}{4(\sigma_x^2)^2} - \frac{R^2(\sigma_\theta^2 + \sigma_\epsilon^2)}{\sigma_x^2}} + \frac{R^2}{2\sigma_x^2}\right)\sigma_x^2 \\
&\leq \frac{1}{R}\left(\sqrt{\frac{R^4}{4(\sigma_x^2)^2} - \frac{R^2\sigma_\epsilon^2 + \sigma_\theta^2}{\sigma_x^2}} + \frac{R^2}{2\sigma_x^2}\right)\sigma_x^2 \\
&= \frac{1}{R}(\sigma_\epsilon^2 + \bar{\sigma}_P^2(1))\sigma_x^2 \\
&= ATC(1, \bar{\sigma}_P^2(1)) \quad .
\end{aligned}$$

Our next step will be to calculate the welfare of the informed and uninformed speculators. With the techniques of Grossman/Stiglitz (1980) we obtain in a similar way[1] as in section 5 of chapter 3 that the expected utility of the informed is given

[1] Note that the residual risk is now given by $\sigma_\epsilon^2 + \sigma_P^2$ rather than by σ_ϵ^2. Also note that for the calculation of the expected utilities it is irrelevant whether the price *means* are in a steady state equilibrium or whether price means follow a bubble.

by

$$E(U_I) = 1 - \sqrt{\frac{\sigma_\epsilon^2 + \sigma_P^2}{\sigma_\epsilon^2 + \sigma_P^2 + \alpha_1^2\sigma_\theta^2 + (\frac{\alpha_2(\sigma_\epsilon^2+\sigma_P^2)}{\lambda})^2\sigma_x^2}}\cdot \exp\left(-\frac{1}{2}\frac{(\alpha_3\bar{x})^2}{\sigma_\epsilon^2 + \sigma_P^2 + \alpha_1^2\sigma_\theta^2 + (\frac{\alpha_2(\sigma_\epsilon^2+\sigma_P^2)}{\lambda})^2\sigma_x^2}\right),$$

and for the uninformed investors

$$E(U_U) = 1 - \sqrt{\frac{\mathrm{var}(\tilde{\theta} \mid w_\lambda) + \sigma_\epsilon^2 + \sigma_P^2}{\sigma_\epsilon^2 + \sigma_P^2}}\sqrt{\frac{\sigma_\epsilon^2 + \sigma_P^2}{\sigma_\epsilon^2 + \sigma_P^2 + \alpha_1^2\sigma_\theta^2 + (\frac{\alpha_2(\sigma_\epsilon^2+\sigma_P^2)}{\lambda})^2\sigma_x^2}}\cdot \exp\left(-\frac{1}{2}\frac{(\alpha_3\bar{x})^2}{\sigma_\epsilon^2 + \sigma_P^2 + \alpha_1^2\sigma_\theta^2 + (\frac{\alpha_2(\sigma_\epsilon^2+\sigma_P^2)}{\lambda})^2\sigma_x^2}\right)$$

Again we can decompose the utility in two different components. The first is contained in the factor in front of the exponential function and reflects the profits from noise traders; it would arise even if the capital stock of the economy were zero. The second, namely the profits from holding on average the quantity $\bar{x}$ is contained in the argument of the exponential function; the profits stem from risk sharing and amount to roughly half of the risk premium (plus a factor factor stemming from the noisy transaction prices which distort the optimal risk sharing between generations). We compare now the case of no information with the case of full revelation of information.

Claim 7:

$E(U_I)$ is always lower for $\lambda = 1$ than for $\lambda = 0$.

For $R = 1$, $E(U_U)$ is the same for $\lambda = 0$ and for $\lambda = 1$.

For $R > 1$, $E(U_U)$ is lower for $\lambda = 1$ than for $\lambda = 0$, if the low volatility equilibrium is played.

For $R > 1$, $E(U_U)$ is higher for $\lambda = 1$ than for $\lambda = 0$, if the high volatility equilibrium is played.

Proof: See appendix.

It is obvious that the welfare of insiders is lower, when they lose their informational advantage, i.e., in the case with full revelation of information. More interesting is the case of the uninformed speculators. If $R = 1$, we have already seen that information does not change the distribution of returns. Accordingly, the welfare of the outsiders does not depend on the fact whether they do know the information or not. If $R > 1$, new information reduces the intrinsic risk of the asset. We have already shown that in the low volatility equilibrium this implies that investors bear a lower risk in the case with full information. Thus both components of the utility are increasing (and hence utility decreasing). The direct profits from noise traders decline, because the risk-bearing component of the bid-ask-spread decreases. The second component decreases because of the "Hirshleifer"-effect already encountered in the welfare analysis of chapter 3, section 5.

The opposite applies to the high volatility equilibrium. We have shown that a reduction of the intrinsic risk increases price risk to such an extent that holding the asset with full information is more risky in the case with full information than in the case without information. Accordingly, both components of the utility are lower (and hence utility higher) in the case with information than in the case without information.

3.4 Welfare Comparisons: Asymmetric Information

We will see that for the rational speculators the welfare effects depend on several effects and the overall outcome depends strongly on the parameters. In order not to overburden this chapter we will restrict ourselves to the welfare comparisons for the noise traders and the uninformed speculators in the low volatility equilibrium, which is more interesting for a comparison with the three-period-case.

We begin again by discussing the transaction costs of noise traders, i.e., the size of the bid-ask-spread. With asymmetric information, the bid-ask-spread is composed of the adverse selection component and the risk bearing component. For $0 < \lambda < 1$

we can write

$$ATC(\lambda, \sigma_P^2(\lambda)) = \frac{1}{R}\left(\frac{\alpha_2(\lambda, \sigma_P^2(\lambda))}{\lambda}(\sigma_\epsilon^2 + \sigma_P^2(\lambda)) - \alpha_3(\lambda, \sigma_P^2(\lambda))\right)\sigma_x^2$$
$$+ \frac{1}{R}\alpha_3(\lambda, \sigma_P^2(\lambda))\sigma_x^2 \quad .$$

The properties of both components have been shown as a by-product in the proof of Claim 5. For $R = 1$, both components increase with λ for low λ and decrease with λ for high λ. Accordingly, the bid-ask-spread is increasing for low λ, decreasing for high λ and has its maximum value for some intermediate value of λ. For $R > 1$ and sufficiently large, we have seen in Claim 6 that the reduction of intrinsic risk due to the inflow of new information may decrease price volatility if σ_x^2 is sufficiently large. Since price volatility is the sum of the volatility due to the information contained in the prices and the volatility due to the bid-ask-spread, we can conclude that at least for small λ and for R and σ_x^2 sufficiently high, an increase in λ leads to a lower bid-ask-spread and hence to lower transaction costs for the noise traders. Combining the calculations of the proofs of claim 5 and of claim 5 of chapter 3, one can even show that the bid-ask-spread decreases monotonically with the intensity of insider trading, if R and σ_x^2 are sufficiently high. This is just a reaffirmation of our observation that for R sufficiently large, the welfare analysis of the low volatility equilibrium is essentially the same as in the three-period model.

Concerning the utility of informed and uninformed speculators, the reader should by now be familiar with the different effects. The profits from trading with noise traders will go up if the bid-ask-spread increases. The profits from holding on average the capital stock $\bar{x}$ are determined by the "Hirshleifer"-effect. These profits decline, if the risk to be borne decreases. The impact of shifts in λ on the utility of outsiders and insiders follows then from the analysis of the preceding sections. If R is close to 1, both the bid-ask-spread and the riskiness of holding the asset increase in λ for low λ, and decrease in λ for high λ. Accordingly, the utilities first increase and ultimately decrease. In particular, there is an intermediate level $\bar{\lambda}$ of insider trading at which the utility of both insiders and outsiders is maximal. If the effect of the inflow of new information becomes stronger, i.e., for a sufficiently high R, the welfare analysis becomes again similar to the one in section 5 of chapter 3 (in particular Claim 8 and Claim 9). The riskiness of holding the asset decreases with an increase in λ, and accordingly profits from this source decrease. Profits from noise traders depend on the reaction of the bid-ask-spread to a shift in λ. If σ_x^2 is sufficiently large, they will decrease with λ. Hence, the utility of both insiders and outsiders will decrease with λ. If σ_x^2 is sufficiently low, the bid-ask-spread may increase due to a strong increase of the adverse-selection component and the profits from trading with the noise traders increase for both insiders and outsiders. The overall effect on utility will again depend on the relation between the two components of the utility.

We leave it to the reader to make these assertions precise. The proofs follow by combining the proofs of Claims 5, 6 and 7 of this chapter with the proofs of Claims 8 and 9 of chapter 3.

3.5 Welfare with an Endogenous Capital Stock

The purpose of this last section is to improve our understanding of the two sources of profits for the rational investors. We focus especially on the profits arising from holding the asset as such, which are contained in the argument of the exponential function in the expected utilities.

Therefore, we will first assume that there is no noise trading in the market, i.e., $\sigma_x^2 = 0$. Assume furthermore that $R > 1$. From Claim 2 we know that in this case there is exactly one steady state equilibrium given by

$$P_t = \frac{1}{R-1}(E(\tilde{\theta}) - (\sigma_\theta^2 + \sigma_\epsilon^2)\bar{x})$$

for $\lambda = 0$ and

$$\tilde{P}_t = \frac{1}{R(R-1)}E(\tilde{\theta}) + \frac{1}{R}\tilde{\theta}_{t+1} - \frac{1}{R-1}(\sigma_\epsilon^2 + \frac{\sigma_\theta^2}{R^2})\bar{x}$$

for $\lambda > 0$. Calculating as before[1] the expected utilities, we have for $\lambda = 0$

$$E(U_U) = 1 - exp\left(-\frac{1}{2}(\sigma_\theta^2 + \sigma_\epsilon^2)^2\bar{x}^2\right)$$

[1] Observe that $\alpha_3(0) = \sigma_\theta^2 + \sigma_\epsilon^2$ and $\alpha_3(1) = \sigma_\epsilon^2 + \frac{\sigma_\theta^2}{R^2}$.

and for $\lambda > 0$

$$E(U_U) = 1 - exp\left(-\frac{1}{2}(\sigma_\epsilon^2 + \frac{\sigma_\theta^2}{R^2})^2 \bar{x}^2\right) \quad .$$

Although the risk to be borne by each investor is lower with full information, his expected utility from holding the asset decreases. The riskier an asset, the more it is preferred by the investors. At a first glance this seems surprising. One might suspect that if shareholders know that they have to sell their stock in the next period and the returns of the stock become less variable, this should raise their utility. However, in an economy beginning at $-\infty$ with a given capital stock this is not true. Each generation just shifts the burden of the additional risk backwards. Because in equilibrium the compensation for bearing risk grows faster than the risk, it is effectively generation $-\infty$ which bears the additional risk. In an insurance context, this implication of the "Hirshleifer"-effect is not too disturbing. However, in the context of capital markets it looks like a somewhat artificial effect. It regards the capital stock as something which at $-\infty$ has simply appeared. In practice however, capital has to be installed at some date. Therefore, we will now modify our economy slightly and consider an initial investment stage.

Assume that the economy begins at some date 0, when an entrepreneur can decide to invest in a new venture.[1] He can choose a level of infinitely lived capital $\bar{x}$ by paying a fixed unit cost such that his profit is maximal. We assume that he has no own funds and must finance the investment by issuing shares on the stock market at date 0. We assume furthermore that the economy is immediately in the steady state equilibrium. For simplicity suppose that the entrepreneur is risk neutral. Thus, his maximization problem is

$$\max_{\bar{x}}\{E(\tilde{P}_0)\bar{x} - c\bar{x}\} \quad .$$

Substituting from Claim 2 for $E(\tilde{P}_0)$ and after a derivation with respect to $\bar{x}$ we

[1] Alternatively, we could endogenize the capital stock in a more traditional way as in Pagano (1989). In Pagano's model the "firm" decides each period about the optimal capital stock following a rule of net market value maximization and a steady state capital stock can then be calculated. Here, this approach would produce the same results. We deal with an initial investment stage only for technical convenience.

obtain

$$\bar{x} = \frac{E(\tilde{\theta}) - (R-1)c}{2\alpha_3(\lambda)} \quad .$$

This gives us a quite intuitive result. A reduction of the asset's riskiness leads to more investment, and an increase of the riskiness depresses investment. Substituting the optimal capital stock in the utilities of investors we see that utility with and without without information is now identical and given by

$$E(U_U) = 1 - exp\Big(-\frac{1}{8}(E(\tilde{\theta}) - (R-1)c)^2\Big) \quad .$$

The increase in investment due to a lower risk just compensates the welfare losses of the investors in the case with full revelation of information.

However, this is not a general result and depends strongly on the extent to which investment reacts on shifts in the riskiness of the asset. Assume for instance that the cost function is not linear, but given by

$$C(\bar{x}) = c\bar{x}^2 \quad .$$

The optimal capital stock becomes then

$$\bar{x} = \frac{E(\tilde{\theta})}{2(\alpha_3(\lambda) + (R-1)c)} \quad .$$

Investment still reacts positively to a decrease in the asset's riskiness, but less strongly. In particular, the increase in investment does not compensate any more for the utility decrease due to the lower risk. For $\lambda = 0$ utility is given by

$$E(U_U) = 1 - exp\Big(-\frac{1}{8}\frac{(\sigma_\theta^2 + \sigma_\epsilon^2)^2}{(\sigma_\theta^2 + \sigma_\epsilon^2 + (R-1)c)^2}\Big)$$

and for $\lambda > 0$

$$E(U_U) = 1 - exp\Big(-\frac{1}{8}\frac{(\sigma_\epsilon^2 + \frac{\sigma_\theta^2}{R^2})^2}{(\sigma_\epsilon^2 + \frac{\sigma_\theta^2}{R} + (R-1)c)^2}\Big) \quad .$$

We show in the appendix that in the latter case utility is lower.

Returning with these insights to the welfare analysis of sections 3.3 and 3.4, our assessment of the welfare effects of insider trading becomes even more ambiguous. If we assume linear investment costs and substitute the optimal capital stock in the formula for the utility of uninformed investors, we obtain

$$E(U_U) = 1 - \sqrt{\frac{\mathrm{var}(\tilde{\theta} \mid w_\lambda) + \sigma_\epsilon^2 + \sigma_P^2}{\sigma_\epsilon^2 + \sigma_P^2}} \sqrt{\frac{\sigma_\epsilon^2 + \sigma_P^2}{\sigma_\epsilon^2 + \sigma_P^2 + \alpha_1^2\sigma_\theta^2 + (\frac{\alpha_2(\sigma_\epsilon^2+\sigma_P^2)}{\lambda})^2\sigma_x^2}} \cdot exp\left(-\frac{1}{8}\frac{(E(\tilde{u}) - (R-1)c)^2}{\sigma_\epsilon^2 + \sigma_P^2 + \alpha_1^2\sigma_\theta^2 + (\frac{\alpha_2(\sigma_\epsilon^2+\sigma_P^2)}{\lambda})^2\sigma_x^2}\right)$$

This reverses the analysis of the "Hirshleifer" component of the last two sections. As an example consider the case of small R and low λ. An increase in λ increases price volatility and the riskiness of the stock. Hence, investment decreases and using the results of Claim 5 one shows that that the utility from the "Hirshleifer"-component decreases. The extent of this decrease depends on the size of the capital stock. If $E(\tilde{\theta}) - (R-1)c$ is small, the increase in the other component of the expected utility will dominate the utility decrease due to the lower capital stock, and vice versa.

Finally, we could as before neutralize the effect of changes in risk on the investment level by choosing an appropriate cost function. In this case the welfare analysis of sections 4.2 and 4.3 would remain basically unchanged. Hence, the reaction of the investor's welfare on shifts in the intensity of insider trading, will both depend on the *level* of the capital stock and on the *responsiveness* of investment to changes in the riskiness of the asset. A clear-cut assessment of the impact of insider trading on welfare is impossible.

4. Conclusion.

The results from the analysis of our model are threefold. The first result regards the impact of noise trading on equity prices. The existence of noise introduces a feedback effect in the economy which may render beliefs of a high price volatility selffulfilling. Especially the high volatility equilibrium of our model is an intriguing demonstration of the impact of this feedback. The high volatility equilibrium provides another example how the existence of noise trading may lead to higher price volatility and higher risk than might be justified by the dividend returns on the asset. That noise trading may increase risk premia is not really surprising. The "fads" of Schiller (1984) (see also Schiller (1990)) have already been formalized in models with noise trading by Campbell/Kyle (1988) and DeLong et.al. (1988). The special feature of our model is that price volatility and risk premia do not diminish if investors' risk aversion or the level of noise trading decrease. Rather the opposite happens: Lower risk aversion or less noise trading render the feedback effect linked to expectations even stronger and lead to an increase in the risk premia and excess returns. This effect is strongly opposed to the common presumption that a decrease in risk aversion should bring equity prices closer to their fundamental value.

The second result concerns the impact of new information on prices, i.e., the comparison between the situation with no information on future returns and full disclosure of all information on the future returns. We have shown that in a infinite horizon model, the disclosure of information has almost no effect on the risk to be borne by the economy as long as interest rates are sufficiently close to 0. This result differs from the results obtained in the static specifications of models with asymmetric information as in Grossman/Stiglitz (1980) or Hellwig (1980) in which the premature resolution of uncertainty reduces the direct risk of holding the asset. In a infinite horizon model this reduction of uncertainty concerning the dividend risk is exactly compensated by the increase in price risk.

However, if the interest rate becomes larger, full disclosure has an effect on the allocation of risk in the economy, because returns are discounted with the interest rate, whereas the variances are not. We have shown that the effect on price volatility and risk premia is ambiguous; they may increase or decrease. Whether we regard the case when the early release of information has a real impact on the perceived riskiness of the asset as more relevant, depends of course on our interpretation of the length of the trading intervals. The longer the distance between two successive trading dates in our model, the higher should be the interest rate and the stronger should be the impact of new information. In our model, information which is released shortly before the realization of the relevant variable, has no effect on the economy, information which is released long before the realization changes the risk of holding the asset.

The ambiguity of the impact of the release of new information in the case of a positive interest rate might also give an additional explanation, why firms do not circumvent any problems with asymmetric information and insider trading by releasing information as soon as it becomes available. Usual arguments concern the costs of such a disclosure policy, credibility problems and the like. In the infinite horizon world of our model, firms face the additional problem that they cannot judge the impact of the release of information if they do not know which type of equilibrium is played. If the high volatility equilibrium is played, full information will increase price volatility and risk premia. Moreover, in both equilibria the impact of new information on shareholder's welfare is not clear.

Our third result concerns the impact of asymmetric information and insider trading on prices. The effect of asymmetric information on the ex-ante distributions of prices works mainly through what we termed "bid-ask-spread", i.e., the amount to which prices change as a response to noisy supply or demand shocks to the asset market. From an ex-ante perspective, these shocks create an extra volatility of prices through the price movements due to the bid-ask-spread. The size of the bid-ask-spread depends on the intensity of insider trading; a change in the amount of insider trading changes both the adverse selection and the risk bearing component of the bid-ask-spread. We have shown that the overall effect of these changes is ambiguous, it depends both on the played equilibrium and the parameters of the model. The final assessment is even further complicated by the interaction between the effects of asymmetric information and the possible risk reduction due to the inflow of new information.

In this sense, our results on asymmetric information can be considered as an extension of the conclusions of Hart/Kreps (1986) concerning the stabilizing effect of speculation. Hart and Kreps conclude that "it is hard to say much about the effects of speculation on price stability" and "just as hard to say anything about the welfare implications". In our model it is impossible to treat the impact of manipulation per se, because the random demand shocks to the economy are independent of prices. But regarding the effects of *informed* speculation, we might summarize our results as Hart and Kreps. It is hard to draw definite conclusions on the impact of different levels of informed speculation on price stability and welfare.[1]

Thus, a possible regulator of our economy would be in a rather uncomfortable position. Deciding on possible restrictions on insider trading, he would not only have to know in which equilibrium the economy finds itself, but also all the relevant parameter values of the respective equilibrium to find out whether a reduction of insider trading would lead to more or less stable prices and to lower or higher welfare. The analysis of this chapter might be also useful for other regulatory issues involving informational externalities. As an example consider the recent proposals on "sunshine trading". Sunshine trading was proposed as a remedy for potential negative interactions between program trading and market crashes. It was argued (see Genotte/Leland (1990), Admati/Pfleiderer (1990); see also Roell (1989)) that the preannouncement of future trades by program traders (regarded as noise traders) would decrease price volatility, the transaction costs of noise traders and the probability of market crashes. However, these assertions rely to a large extent on the fact that this preannouncement can be credibly transmitted to the *whole* market; something which for example under the current regulations of the New York Stock Exchange cannot to be taken for granted. If the announcements

[1] A related observation has been made by Stein (1987). Stein studies a model in which an additional group of speculators enters a market. He analyzes the trade-off between enhanced risk-bearing capacity of the larger market, and shifts in the informational asymmetries if the additional group has a different information level than the old market. His analysis of the informational externalities is similar to the analysis of shifts in λ in our model.

reach only part of the market, an additional effect has to be taken into account. The investors who are aware of the amount of noise trading acquire indirectly the status of insiders. To them prices convey all the inside information and they can submit the same orders as the original insiders. Hence, if preannouncement only reaches part of the market, its effects are quite similar to the one of shifts in λ investigated in this chapter; in particular, the same ambiguous trade-offs as before will arise.

A final word concerns the robustness of our results under different specifications. Wang (1990) considers a modification of the model of Campbell/Kyle (1988) in which he introduces asymmetric information in a similar way as in Grossman/Stiglitz (1980). He studies a certain class of equilibria and reports that in these equilibria price volatility and risk premia are monotonically decreasing with the intensity of insider trading. In our model this case can only occur in the low volatility equilibrium. Even then, the risk reduction effect of new information must be strong and has to dominate completely the effects of asymmetric information, i.e., interest rates and the variance of noise trading have to be very high. Both Campbell/Kyle and Wang use a continuous-time framework which makes the results difficult to compare. The decisive difference is that both papers study an economy with infinitely lived rational investors. A detailed comparison of the case with myopic investor behaviour and the case with infinite planning horizon is certainly necessary.

However, in some sense both approaches might be considered complementary. If one starts from the assumption that real markets are characterized to some extent by noise trading in the form of opinion and/or liquidity trading, then any rational investor should have two things in mind. First, he should try to speculate and exploit the noisy price movements. For the study of this effect a model with infinitely lived agents is much better suited than a overlapping generations model with two generations. Although the young generation also speculates in the above sense, a model with finitely lived agents cannot fully capture the effect that investors can defer trading over time. Infinitely lived agents can wait with sell orders until a demand shock leads to high prices and with buy orders until a supply shock leads to low prices.

But a rational investor should also take a second point into account. He should realize that at some point in the future he might become a liquidity trader himself, i.e., that some exogenous shock might force him to liquidate his positions. This reasoning will make him directly interested in the liquidity of the market, i.e., the prices he can achieve at times when he *has to* leave the market. This second aspect can only be captured by the overlapping generations model. In this sense, an overlapping generations model might also be the more appropriate way to deal with the problem of insider trading from an ex-ante perspective. Intuitively, one would expect that informed speculators should especially harm those traders, who cannot choose their market timing but rather have a real need to trade.

4. Appendix.

1. Proof of Claim 3:

We show all three parts of the claim only for the case $\lambda = 0$. The proofs for $\lambda = 1$ are analogous.

a) We have to show that $T^0(x) > \underline{\sigma}_P^2$ for every $x > \underline{\sigma}_P^2$. Therefore, choose a $x = \underline{\sigma}_P^2 + \epsilon$ with $\epsilon > 0$. Using that $\alpha_2(0,x) = 0$ and $\frac{\alpha_2(0,x)x}{\lambda} = \sigma_\theta^2 + \sigma_\epsilon^2 + x$ for all x, we calculate

$$\begin{aligned}
T^0(x) - \underline{\sigma}_P^2 &= \frac{a^2\sigma_x^2}{R^2}(\sigma_\theta^2 + \sigma_\epsilon^2 + \epsilon + \underline{\sigma}_P^2)^2 - \underline{\sigma}_P^2 \\
&= \frac{a^2\sigma_x^2}{R^2}\{2(\sigma_\theta^2 + \sigma_\epsilon^2 + \underline{\sigma}_P^2)\epsilon + \epsilon^2\} \\
&> \epsilon(1 - \sqrt{1 - \frac{4a^2\sigma_x^2}{R^2}(\sigma_\theta^2 + \sigma_\epsilon^2)}) \\
&> 0 \quad .
\end{aligned}$$

b) Choose a $x = \underline{\sigma}_P^2 - \epsilon$ with $\underline{\sigma}_P^2 > \epsilon > 0$. We have to show that there is a $\kappa < 1$ such that

$$\underline{\sigma}_P^2 - T^0(x) < \kappa\epsilon \quad .$$

With similar calculations as before, we obtain

$$\begin{aligned}
\underline{\sigma}_P^2 - T^0(x) &= \underline{\sigma}_P^2 - \frac{a^2\sigma_x^2}{R^2}(\sigma_\theta^2 + \sigma_\epsilon^2 - \epsilon + \underline{\sigma}_P^2)^2 \\
&= \frac{a^2\sigma_x^2}{R^2}\{2(\sigma_\theta^2 + \sigma_\epsilon^2 + \underline{\sigma}_P^2)\epsilon - \epsilon^2\} \\
&< \epsilon(1 - \sqrt{1 - \frac{4a^2\sigma_x^2}{R^2}(\sigma_\theta^2 + \sigma_\epsilon^2)}) \\
&< \kappa\epsilon \quad ,
\end{aligned}$$

c) Let $\delta > 1$, and choose a $x = \bar{\sigma}_P^2 + \epsilon$ with $x > \delta\underline{\sigma}_P^2$. We have to show that there is a $\kappa(\delta) > 1$ such that

$$| T^0(\bar{\sigma}_P^2 + \epsilon) - \bar{\sigma}_P^2 | > \kappa(\delta) | \epsilon | \quad .$$

As before we calculate

$$
\begin{aligned}
| T^0(\bar{\sigma}_P^2+\epsilon)-\bar{\sigma}_P^2 | &= | \frac{a^2\sigma_x^2}{R^2}(\sigma_\theta^2+\sigma_\epsilon^2+\epsilon+\bar{\sigma}_P^2)^2-\bar{\sigma}_P^2 | \\
&= | \frac{a^2\sigma_x^2}{R^2}\{2(\sigma_\theta^2+\sigma_\epsilon^2+\bar{\sigma}_P^2)\epsilon+\epsilon^2\} | \\
&\geq | \epsilon | \{(1+\sqrt{1-\frac{4a^2\sigma_x^2}{R^2}(\sigma_\theta^2+\sigma_\epsilon^2)})-\frac{a^2\sigma_x^2}{R^2} | \epsilon |\} \\
&> \kappa(\delta) | \epsilon | \quad ,
\end{aligned}
$$

for a suitable $\kappa(\delta) > 1$. The last inequality follows because $x > \delta\underline{\sigma}_P^2$ and hence for a suitable $\bar{\delta} < 1$

$$
\begin{aligned}
\epsilon &> \bar{\delta}(\underline{\sigma}_P^2-\bar{\sigma}_P^2) \\
&= -\bar{\delta}\frac{R^2}{a^2\sigma_x^2}\sqrt{1-\frac{4a^2\sigma_x^2}{R^2}(\sigma_\theta^2+\sigma_\epsilon^2)} \quad .
\end{aligned}
$$

2. Proof of Claim 5:

We know from the steady state equation (2.2) that for $\sigma_P^2 = \underline{\sigma}_P^2(\lambda), \bar{\sigma}_P^2(\lambda)$

$$
\sigma_P^2 = \frac{1}{R^2}(\alpha_2^2\sigma_\theta^2+(\frac{\alpha_2}{\lambda})^2(\sigma_\epsilon^2+\sigma_P^2)^2\sigma_x^2) \quad .
$$

Calculating the derivatives with respect to λ on both sides and evaluating at $\lambda = 0$ we obtain

$$
\text{(A1)} \quad \frac{\partial}{\partial\lambda}(\sigma_P^2)_{|\lambda=0} = \frac{1}{R^2}\{2\alpha_2\sigma_P^2+2a^2\frac{\alpha_2}{\lambda}(\sigma_\epsilon^2+\sigma_P^2)\sigma_x^2\}\frac{\partial}{\partial\lambda}(\frac{\alpha_2}{\lambda}(\sigma_\epsilon^2+\sigma_P^2))_{|\lambda=0} \quad .
$$

Observe first that $\alpha_2(0) = 0$. In order to calculate the derivative on the second side, decompose again in the adverse selection component and the risk bearing component:

$$
\frac{\partial}{\partial\lambda}(\frac{\alpha_2}{\lambda}(\sigma_\epsilon^2+\sigma_P^2))_{|\lambda=0} = \frac{\partial}{\partial\lambda}(\frac{\alpha_2}{\lambda}(\sigma_\epsilon^2+\sigma_P^2)-\alpha_3)_{|\lambda=0}+\frac{\partial}{\partial\lambda}(\alpha_3)_{|\lambda=0} \quad .
$$

For notational convenience define

$$
y = \sigma_\epsilon^2+\sigma_P^2 \quad .
$$

As in the proof of Claim 5 of chapter 3 we calculate both derivatives on the right hand side.

(A2) $$\frac{\partial}{\partial\lambda}\Big(\frac{\alpha_2}{\lambda}(\sigma_\epsilon^2+\sigma_P^2)-\alpha_3\Big)_{|\lambda=0}=\frac{\sigma_\theta^2}{y\sigma_x^2}\quad,$$

$$\begin{aligned}\frac{\partial}{\partial\lambda}(\alpha_3)_{|\lambda=0} &= \frac{\partial}{\partial\lambda}(\sigma_P^2)\frac{\sigma_\theta^2+y}{y}_{|\lambda=0}+\\ &\frac{1}{y^3}\{(\sigma_\theta^2+2y)y^2\frac{\partial}{\partial\lambda}(\sigma_P^2)-(\sigma_\theta^2+y)y(\sigma_\theta^2 y+2y\frac{\partial}{\partial\lambda}(\sigma_P^2))\}_{|\lambda=0}\\ &= \frac{\partial}{\partial\lambda}(\sigma_P^2)\frac{\sigma_\theta^2+y}{y}_{|\lambda=0}-\frac{1}{y}\{\sigma_\theta^2\frac{\partial}{\partial\lambda}(\sigma_P^2)+(\sigma_\theta^2+y)\sigma_\theta^2\}_{|\lambda=0}\end{aligned}$$

(A3) $$=\frac{\partial}{\partial\lambda}(\sigma_P^2)_{|\lambda=0}-\frac{(\sigma_\theta^2+y)\sigma_\theta^2}{y}_{|\lambda=0}\quad.$$

Adding both terms gives

(A4) $$\frac{\partial}{\partial\lambda}\Big(\frac{\alpha_2}{\lambda}(\sigma_\epsilon^2+\sigma_P^2)\Big)_{|\lambda=0}=\frac{\partial}{\partial\lambda}(\sigma_P^2)_{|\lambda=0}+\{\frac{\sigma_\theta^2}{y\sigma_x^2}\{1-(\sigma_\theta^2+y)\sigma_x^2\}_{|\lambda=0}\quad.$$

Substituting (A4) in (A1) gives

$$\frac{R^2}{a^2}\frac{\partial}{\partial\lambda}(\sigma_P^2)_{|\lambda=0}=2(\sigma_\theta^2+y)\{\frac{\partial}{\partial\lambda}(\sigma_P^2)\sigma_\theta^2\sigma_x^2+\frac{\sigma_\theta^2}{y}(1-(\sigma_\theta^2+y)\sigma_x^2)\}_{|\lambda=0}\quad.$$

Solving this equation for $\frac{\partial}{\partial\lambda}(\sigma_P^2)$ gives

(A5) $$\frac{\partial}{\partial\lambda}(\sigma_P^2)_{|\lambda=0}=a^2\sigma_\theta^2\frac{\sigma_\theta^2+y}{y}\frac{2-2(\sigma_\theta^2+y)\sigma_x^2}{R^2-2a^2(\sigma_\theta^2+y)\sigma_x^2}_{|\lambda=0}\quad.$$

Now observe that

$$(R^2-2a^2(\sigma_\theta^2+y)\sigma_x^2)_{|\lambda=0}=R^2(1-\frac{2a^2}{R^2}\sigma_x^2(\sigma_\theta^2+\sigma_\epsilon^2+\sigma_P^2))_{|\lambda=0}$$

(A6) $$= \begin{cases}-R^2\Big(\sqrt{1-\frac{4a^2\sigma_x^2}{R^2}}\Big) & \text{for } \sigma_P^2=\bar{\sigma}_P^2(\lambda)\\ R^2\Big(\sqrt{1-\frac{4a^2\sigma_x^2}{R^2}}\Big) & \text{for } \sigma_P^2=\underline{\sigma}_P^2(\lambda)\quad.\end{cases}$$

We want to show that

$$\frac{\partial}{\partial\lambda}(\underline{\sigma}_P^2(\lambda))_{|\lambda=0} > 0$$

and

$$\frac{\partial}{\partial\lambda}(\underline{\sigma}_P^2(\lambda))_{|\lambda=0} < 0 \quad .$$

Both inequalities follow from

$$(2 - 2a^2(\sigma_\theta^2 + y)\sigma_x^2)_{|\lambda=0} = 2 - R^2 + R^2(1 - \frac{2a^2\sigma_x^2}{R^2}(\sigma_\theta^2 + \sigma_\epsilon^2 + \sigma_P^2))_{|\lambda=0}$$

$$(A7) \qquad = \begin{cases} 2 - R^2 - R^2(\sqrt{1 - \frac{4a^2\sigma_x^2}{R^2}}) & \text{for } \sigma_P^2 = \bar{\sigma}_P^2(\lambda) \\ 2 - R^2 + R^2(\sqrt{1 - \frac{4a^2\sigma_x^2}{R^2}}) & \text{for } \sigma_P^2 = \underline{\sigma}_P^2(\lambda) \quad . \end{cases}$$

If $R = 1$ we have in both cases that

$$(2 - 2a^2(\sigma_\theta^2 + y)\sigma_x^2)_{|\lambda=0} > 0 \quad .$$

To calculate the risk premia, we substitute (A5) in (A3) to obtain

$$(A8) \qquad \frac{\partial}{\partial\lambda}(\alpha_3)_{|\lambda=0} = a^2\sigma_\theta^2\frac{\sigma_\theta^2 + y}{y}\frac{2 - R^2}{R^2 - 2a^2(\sigma_\theta^2 + y)\sigma_x^2}_{|\lambda=0} \quad .$$

$R = 1$ implies $2 - R^2 > 0$ and hence risk premia increase in the low volatility equilibrium and decrease in the high volatility equilibrium.

In the case $\lambda = 1$ the calculations are analogous.

<u>3. Proof of Claim 6:</u>

We know from (A7) in the proof of Claim 5 that

$$\begin{aligned}(2 - 2a^2(\sigma_\theta^2 + y)\sigma_x^2)_{|\lambda=0} &> 2 - R^2 \\ &> 0 \quad \text{for } R < \sqrt{2} \quad ,\end{aligned}$$

and hence price volatility always increases for $R < \sqrt{2}$. For $R > \sqrt{2}$ set $\sigma_x^2 = 0$ in (A7). Then

$$\begin{aligned}(2 - 2a^2(\sigma_\theta^2 + y)\sigma_x^2)_{|\lambda=0} &= 2 - R^2 + R^2 \\ &> 0 \quad .\end{aligned}$$

By continuity price volatility increases for all σ_x^2 sufficiently small. Finally, set $\sigma_x^2 = \frac{R^2}{4a^2}$ in (A7) and

$$\begin{aligned}(2 - 2a^2(\sigma_\theta^2 + y)\sigma_x^2)_{|\lambda=0} &= 2 - R^2 \\ &< 0 \quad .\end{aligned}$$

By continuity price volatility decreases for all σ_x^2 sufficiently high.

Risk premia are falling for $R > \sqrt{2}$, because of (A8) in the proof of Claim 5.

4. Proof of Claim 7:

For $\lambda = 0$ we have

$$\begin{aligned}E(U_U) = 1 - &\sqrt{\frac{\sigma_\theta^2 + \sigma_\epsilon^2 + \sigma_P^2(0)}{\sigma_\theta^2 + \sigma_\epsilon^2 + \sigma_P^2(0) + (\sigma_\theta^2 + \sigma_\epsilon^2 + \sigma_P^2(0))^2\sigma_x^2}} \cdot \\ &exp\left(-\frac{1}{2}\frac{(\sigma_\theta^2 + \sigma_\epsilon^2 + \sigma_P^2(0))^2\bar{x}^2}{\sigma_\theta^2 + \sigma_\epsilon^2 + \sigma_P^2(0) + (\sigma_\theta^2 + \sigma_\epsilon^2 + \sigma_P^2(0))^2\sigma_x^2}\right) \quad ,\end{aligned}$$

and for $\lambda = 1$

$$\begin{aligned}E(U_U) = 1 - &\sqrt{\frac{\sigma_\epsilon^2 + \sigma_P^2(1)}{\sigma_\epsilon^2 + \sigma_P^2(1) + (\sigma_\epsilon^2 + \sigma_P^2(1))^2\sigma_x^2}} \cdot \\ &exp\left(-\frac{1}{2}\frac{(\sigma_\epsilon^2 + \sigma_P^2(1))^2\bar{x}^2}{\sigma_\epsilon^2 + \sigma_P^2(1) + (\sigma_\epsilon^2 + \sigma_P^2(1))^2\sigma_x^2}\right) \quad .\end{aligned}$$

Since for $R = 1$ and $\sigma_P^2 = \underline{\sigma}_P^2, \bar{\sigma}_P^2$

$$\sigma_P^2(1) = \sigma_P^2(0) + \sigma_\theta^2 \quad ,$$

both expressions are equal.

For $R > 1$ one has to show that

$$\frac{1}{1+(\sigma_\theta^2+\sigma_\epsilon^2+\underline{\sigma}_P^2(0))\sigma_x^2} < \frac{1}{1+(\sigma_\epsilon^2+\underline{\sigma}_P^2(1))\sigma_x^2} \quad ,$$
$$\frac{\sigma_\theta^2+\sigma_\epsilon^2+\underline{\sigma}_P^2(0)}{1+(\sigma_\theta^2+\sigma_\epsilon^2+\underline{\sigma}_P^2(0))\sigma_x^2} > \frac{\sigma_\epsilon^2+\underline{\sigma}_P^2(1)}{1+(\sigma_\epsilon^2+\underline{\sigma}_P^2(1))\sigma_x^2} \quad ,$$
$$\frac{1}{1+(\sigma_\theta^2+\sigma_\epsilon^2+\bar{\sigma}_P^2(0))\sigma_x^2} > \frac{1}{1+(\sigma_\epsilon^2+\bar{\sigma}_P^2(1))\sigma_x^2} \quad ,$$
$$\frac{\sigma_\theta^2+\sigma_\epsilon^2+\bar{\sigma}_P^2(0)}{1+(\sigma_\theta^2+\sigma_\epsilon^2+\bar{\sigma}_P^2(0))\sigma_x^2} < \frac{\sigma_\epsilon^2+\bar{\sigma}_P^2(1)}{1+(\sigma_\epsilon^2+\bar{\sigma}_P^2(1))\sigma_x^2} \quad .$$

All inequalities follow by a derivation with respect to σ_θ^2.

5. Welfare comparison of section 3.5:

We want to show that

$$exp\left(-\frac{1}{8}\frac{(\sigma_\theta^2+\sigma_\epsilon^2)^2}{(\sigma_\theta^2+\sigma_\epsilon^2+(R-1)c)^2}\right) < exp\left(-\frac{1}{8}\frac{(\sigma_\epsilon^2+\frac{\sigma_\theta^2}{R^2})^2}{(\sigma_\epsilon^2+\frac{\sigma_\theta^2}{R}+(R-1)c)^2}\right) \quad .$$

This follows from the observation that

$$\frac{d}{dR}\frac{f(R)}{f(R)+(R-1)c} = c\big((R-1)\frac{d}{dR}f(R)-f(R)\big) \quad ,$$

and by setting $f(R)=\frac{\sigma_\theta^2}{R}$.

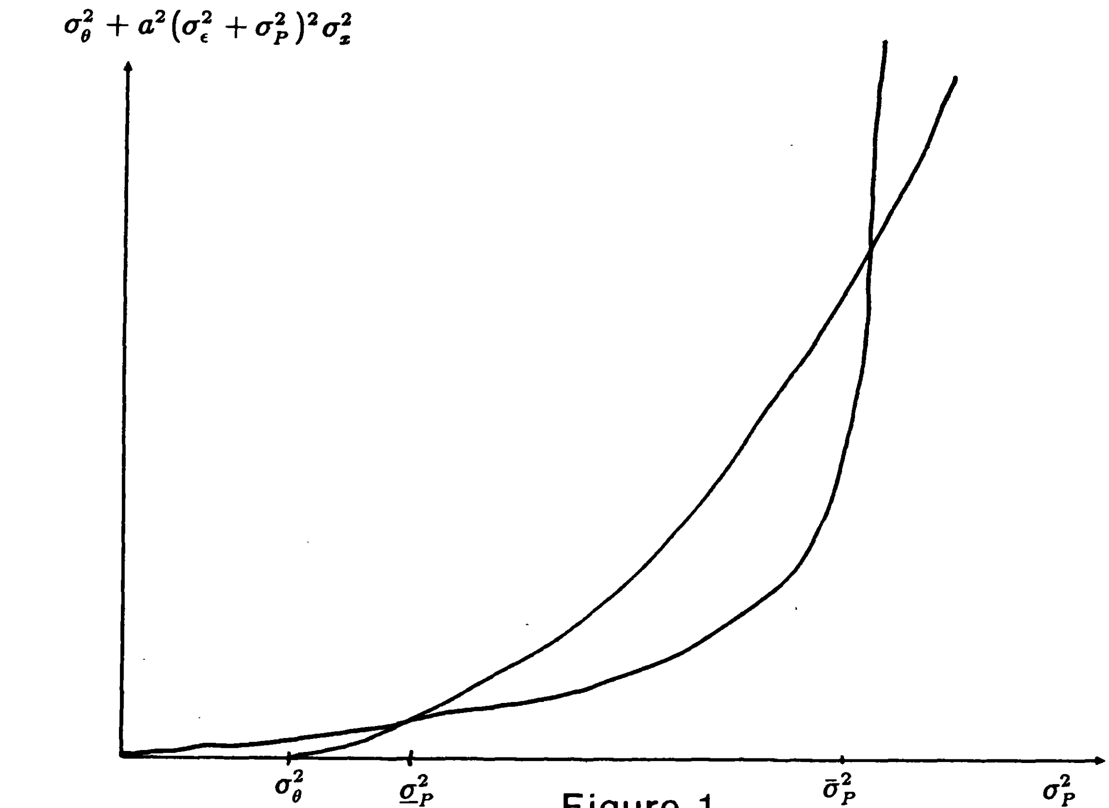

Figure 1

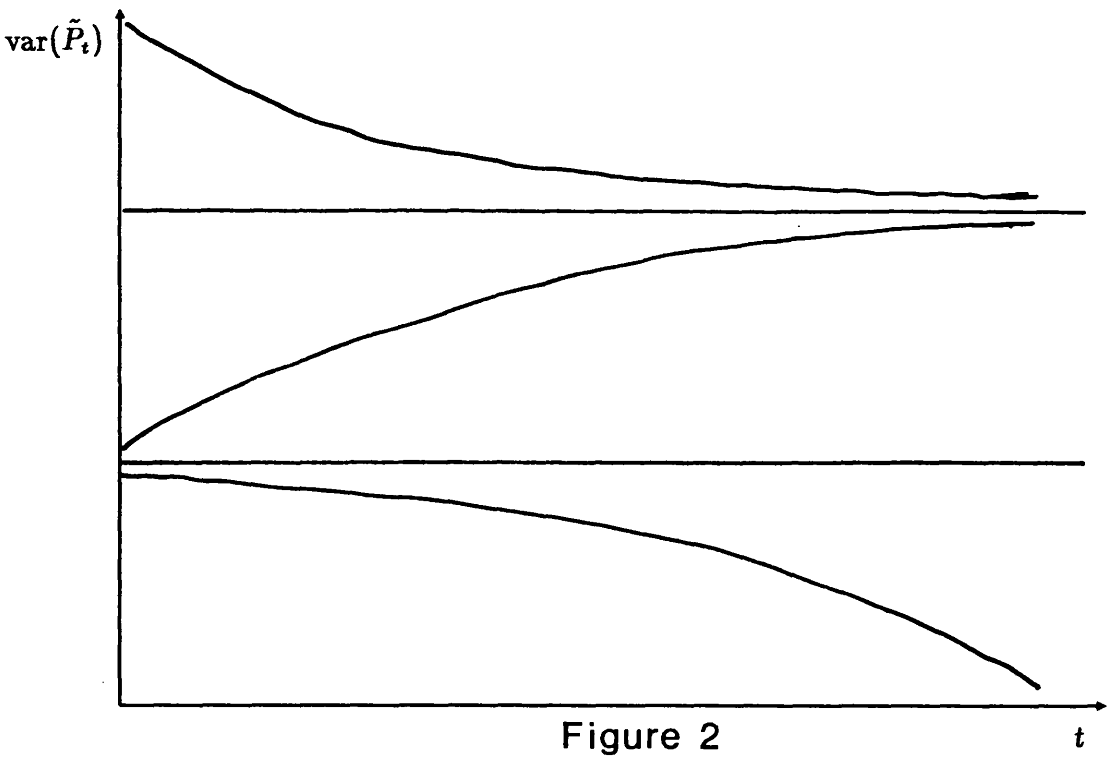

Figure 2

References

Admati, A. (1985): *A Noisy Rational Expectations Equilibrium for Multi Asset Securities Markets.* Econometrica 53, 629-657

Admati, A. and P. Pfleiderer(1987): *Viable Allocations of Information in Financial Markets.* Journal of Economic Theory 43, 76-115

Admati, A. and P. Pfleiderer(1989): *A Theory of Intraday Patterns: Volume and Price Variability* Review of Financial Studies 1, 3-40

Admati, A. and P. Pfleiderer(1990): *Sunshine Trading: The Effects of Preannouncement on Traders' Welfare and on Price Variability.* mimeo, Stanford University

Ausubel L. (1989): *Insider Trading in a Rational Expectations Economy.* mimeo, Northwestern University

Bagehot, W. (1971): *The Only Game in Town.* Financial Analysts Journal

Benabou, R. and G. Laroque (1989): *Using Privileged Information to Manipulate Markets: Insiders, Gurus and Credibility.* mimeo, INSEE, Paris

Bray, M. (1982): *Learning, Estimation and the Stability of Rational Expectations.* Journal of Economic Theory 26, 318-339

Bray, M. (1989): *Rational Expectations, Information and Asset Markets.* in: Hahn, F.(ed.): *The Economics of Missing Markets, Information and Games.* Clarendon Press, Oxford

Broecker, T. (1990): *Credit-worthiness Tests and Interbank Competition.* Econometrica 58, 429-452

Campbell, J.Y. and A. Kyle (1988): *Smart Money, Noise Trading and Stock Price Behaviour.* NBER technical report 71

Carbonetti, F. (1990): *Repurchasing of own Securities and Insider Dealing.* mimeo, IMI Rome

DeLong, J.B, A.Shleifer, L.H.Summers and R.H.Waldman (1988): *The Economic Consequences of Noise Traders.* NBER working paper 2395

Demsetz, H. (1986): *Corporate Control, Insider Trading, and Rates of Return.* American Economic Review, Papers and Proceedings 76, 313-316

Dye, R. (1984): *Insider Trading and Incentives.* Journal of Business 57, 295-313

Fama, E. (1970): *Efficient Capital Markets: A Review of Theory and Empirical Work.* Journal of Finance 25, 383-417

Franke, G. (1989): *Inside Information in Bank Lending and the European Insider Directive.* mimeo, Universität Konstanz

Fishman, M. and K.Hagerty(1989): *Insider Trading and the Efficiency of Stock Prices.* mimeo, Northwestern University

Holmström, B. and B. Nalebuff (1989): *To the Raider Goes the Surplus?* mimeo, Yale University

Gale, D. (1982): *Money in Equilibrium.* Cambridge University Press

Gale, D. and M. Hellwig (1988): *Informed Speculation in Large Markets.* mimeo, Universität Basel

Genotte, G. and H. Leland (1990): *Market Liquidity, Hedging and Crashes.* American Economic Review (forthcoming)

Glosten, L. and P. Milgrom (1985): *Bid, Ask and Transaction Prices in a Specialist Market with Heterogeneously Informed Traders.* Journal of Financial Economics 14, 71-100

Grossman, S. (1977): *The Existence of Futures Markets, Noisy Rational Expectations, and Informational Externalities.* Review of Economic Studies 44, 431-449

Grossman, S. and O. Hart (1980): *Takeover Bids, the Free-rider Problem and the Theory of the Corporation.* Bell Journal of Economics 11, 42-63

Grossman, S. and M. Miller (1988): *Liquidity and Market Structure.* Journal of Finance 43, 617-637

Grossman, S. and J. Stiglitz (1980): *On the Impossibility of Informational Efficient Markets.* American Economic Review 70, 393-408

Haddock, D. and J. Macey (1987): *Regulation on Demand: a Private Interest Model, with an Application to Insider Trading Regulation.* Journal of Law and Economics 30, 311-350

Hakansson, N., G. Kunkel and J. Ohlson (1982): *Sufficient and Necessary Conditions for Information to Have Social Value in Pure Exchange.* Journal of Finance 37, 1169-1181

Hart O. and D. Kreps (1986): *Price Destabilizing Speculation.* Journal of Political Economy 94, 927-952

von Hayek, F. (1945): *The Use of Knowledge in Society.* American Economic Review 35, 519-530

Hellwig, M. (1980): *On the Aggregation of Information in Competitive Markets.* Journal of Economic Theory 22, 477-498

Hellwig, M. (1982): *Rational Expectations Equilibria with Conditioning on Past Prices: a Mean-Variance Example.* Journal of Economic Theory 26, 279-312

Hirshleifer, J. (1971): *The Private and Social Value of Information and the Reward to Incentive Activity.* American Economic Review 61

Ho, T. and H. Stoll (1983): *The Dynamics of Dealer Markets under Competition.* Journal of Finance 38, 1053-1074

Jarrell, G.A., J.A. Brickley and J.M. Netter (1988): *The Market for Corporate Control: The Evidence since 1980.* Journal of Economic Perspectives 2, 49-68

Jensen, M. and K. Murphy (1990): *Performance Pay and Top-management Incentives.* Journal of Political Economy 98, 225-264

Jensen, M. and R. Ruback (1983): *The Market for Corporate Control: The Scientific Evidence.* Journal of Financial Economics 11, 5-50

Jordan, S.J. and R. Radner (1982) *Rational Expectations in Microeconomic Models: An Overview.* Journal of Economic Theory 26, 201-223

Kay, J. (1988): *Comment of King/Roell: Insider Trading.* Economic Policy 6, 187-189

King, M. and A.Roell (1988): *Insider Trading.* Economic Policy 6, 165-187

Kyle, A. (1985): *Continuous Auctions and Insider Trading.* Econometrica 53, 1315-1335

Kyle, A. (1989): *Informed Speculation with Imperfect Competition.* Review of Economic Studies 56, 317-356

Kyle, A. and J.L. Vila (1988): *Noise Trading and Take-overs.* mimeo, New York University

Laffont, J.J. (1985): *On the Welfare Analysis of Rational Expectations Equilibria with Asymmetric Information.* Econometrica 53, 1-30

Laffont, J.J. and E. Maskin (1990): *The Efficient Market Hypothesis and Insider Trading on the Stock Market.* Journal of Political Economy 98, 70-93

Leland, H. (1990): *Insider Trading: Should it be Prohibited?* mimeo, Laboratoire d'Econometrie, Paris

Lucas, R.J. (1978): *Asset Prices in an Exchange Economy.* Econometrica 46, 1429-1445

Manne, H.G. (1966): *Insider Trading and the Stock Market.* The Free Press, New York

Manove, M. (1989): *The Harm from Insider Trading and Informed Speculation.* Quarterly Journal of Economics 104, 823-846

Milgrom, P. (1979): *A Convergence Theorem for Competitive Bidding with Differential Information.* Econometrica 47, 679-688

Milgrom, P. and N. Stokey (1982): *Information, Trade and Common Knowledge.* Journal of Economic Theory 26, 17-27

Pagano, M. (1989): *Endogenous Market Thickness and Stock Price Volatility.* Review of Economic Studies 56, 269-288

Roell, A. (1987a): *Rules and the Quality of Markets.* mimeo, LSE

Roell, A. (1987b): *The Regulation of Take-overs.* FMG discussion paper, LSE

Roell, A. (1989): *Dual Capacity Trading and the Quality of the Market.* mimeo, Tilburg University

Shiller, R.J. (1984): *Stock Prices and Social Dynamics.* Brookings Papers on Economic Activity, 457-498

Shiller, R.J. (1990): *Market Volatility.* MIT Press, Cambridge, Mass.

Stein, J. (1987): *Informational Externalities and Welfare-reducing Speculation.* Journal of Political Economy 95, 1123-1145

Stoll, H.R. (1985): *Alternative Views of Market Making.* in: Amihud, Y. et al. (ed.): *Market Making and the Changing Structures of the Securities Industries.* Lexington, Mass.

Tirole, J. (1982): *On the Possibility of Speculation under Rational Expectations.* Econometrica 53, 1163-1181

Verrechia, R. (1982): *Information Acquisition in a Noisy Rational Expectations Economy.* Econometrica 50, 1415-1430

Voss, J. (1984): *Das US-Amerikanische Insiderkonzept.* Bankrechtliche Sonderveröffentlichungen des Instituts für Bankwirtschaft und Bankrecht der Universität zu Köln

Wilson, R. (1979): *Auction for Shares.* Quarterly Journal of Economics 93, 675-689

Wang, J. (1989): *Asset Prices, Stock Returns, Price Volatility, Risk Premium and Trading Strategies under Asymmetric Information.* mimeo, Wharton School

Wyplosz, C. (1988): *Comment of King/Roell: Insider Trading.* Economic Policy 6, 187-189