ESSAYS IN BUSINESS CYCLE MEASUREMENT

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ABSTRACT

This dissertation is concerned with the issue of economic fluctuations; the following related topics are analysed:

- co-integration and the NAIRU hypothesis: the theoretical implications of different classes of models, some implying that the NAIRU is a structural parameter that can only be influenced by supply-side measures, others that the attainable level of unemployment is a function also of demand variables, are first discussed; co-integration techniques (the Engle-Granger and the Johansen procedure) are then used to test the NAIRU hypothesis; the more powerful maximum likelihood method developed by Johansen shows that the unemployment rate is co-integrated with both supply and demand variables only as well as a combination of the two;

- supply versus demand shocks as the driving force of business cycles: using two measures of productivity growth (the Solow residual and the dual residual from the cost function), competing theories of the cycle are tested in a number of OECD countries; the issue of market structure and its relevance to explain economic fluctuations is also addressed; the empirical evidence refutes the "stronger" real business cycle (RBC) hypothesis that denies the role of demand shocks;

- aggregate versus sectoral shocks: their relative importance in the UK economy is evaluated by estimating a vector autoregression (VAR) of the output growth rates of 19 industrial sectors and doing a factor analysis on the innovations; the one-factor model performs quite well when applied to the British data implying that there is an aggregate shock that can account for a high percentage of the fluctuations of output over the cycle;

- the "seasonal cycle" in the UK economy: the quantitative importance of seasonal fluctuations and the existence of a "seasonal cycle" whose main features are very similar to those of the conventional business cycle are documented by running regressions with seasonal dummies and band spectrum regressions; a one-sector, neo-classical model of capital accumulation in which seasonal preferences are explicitly incorporated (the coefficient of risk aversion depending on the season s) is then set up; the model is not rejected by the data, confirming that seasonality is a feature to be explained within the economic model.

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INTRODUCTION

This dissertation is concerned with one of the core issues in macroeconomic theory: the sources of economic fluctuations.

In the last few years there has been renewed interest among economists in seeking an explanation for business cycles. Until the early 70s, there was a general consensus that the shocks driving macroeconomic fluctuations originated on the demand side; the neo-classical synthesis allowed for both Keynesian and monetarist views concerning the determination of aggregate output, and it was thought that such framework, then dominant in textbooks, could capture most of the important aspects of the data. The business cycle problem was considered essentially to have been solved, even though it was realized that the theory had to be refined to bridge the chasm between microeconomic principles and macroeconomic practice. In that conceptual framework there were two blocks, "aggregate demand" and "aggregate supply". Aggregate demand was thought of as explaining the behaviour of the aggregate demand for goods given prices; on the other hand, aggregate supply, including a wage-setting equation (the Phillips curve), a price-setting equation and a relationship between unemployment and output (Okun's law), was regarded as accounting for the behaviour of prices given output. Fluctuations of output in the short run were then attributed mainly to demand shocks moving output and prices in the same direction; conversely, aggregate supply shocks, which include shocks to productivity, dominated output movements in the long run.

However, the economic problems of the 1970s brought to the fore again the search for an explanation of business cycles because the traditional models failed when faced with supply shocks. As a result of this resurgence of interest, a number of theoretical frameworks dealing with the issue of economic fluctuations have been developed.

Much of macroeconomic analysis is now characterized by an equilibrium approach in Hayek's sense. The thrust of it is that a theory of macroeconomic fluctuations does not need to invoke market imperfections, and that the analysis of aggregate behaviour is methodologically similar to the study of microeconomic phenomena.

The general framework is some variant of the following. Economic agents play a dynamic stochastic game of which they know the rules; they have an objective function and choose their strategy in order to maximise it. Assuming that enough is known about each individual problem to analyse it, a theory of the behaviour of aggregate variables is obtained by adding up the decisions of all the players and imposing a solution that makes all these decisions consistent. All sorts of questions can then be addressed, e.g. the rules of the game can be modified to include government policies, and a solution to the new game thus specified can be computed. Many of the proponents of equilibrium theories also make the assumption of perfect competition, and therefore view business cycles as simply the result of individual agents optimising behaviour in a competitive environment.

Early examples of equilibrium business cycle theories are the papers by Lucas (1972) and Barro (1976), which stress the role of nominal shocks in the presence of imperfect information. The prominent role played by aggregate shocks in these models is consistent with the traditional view dating back to Burns and Mitchell (1946) according to which the sources of the cycle, either domestic or external, are mainly aggregate and originate on the demand side. More recent equilibrium models by Black (1982), Kydland and Prescott (1982), and King and Plosser (1983), have suggested that a large fraction of fluctuations in aggregate output may result from such disturbances as technology shocks or taste shocks. In such "real business cycle" (RBC) models, the assumptions that markets are competitive and all information is public are made; they abstract away from monetary phenomena, and have the property that the behaviour of real quantities is determined by real shocks to the economy only. These shocks are often measured by the production function residuals that Solow (1957) first identified. "Real business cycle" models are usually fitted to the data by "calibration", a process in which key parameters are assigned values not directly related to business cycles, like growth observations or panel studies of individual behaviour. Such parameters are then used to construct an artificial economy which, its proponents argue, mimics many of the properties of actual economies, and the claim is made that the model economy is a reasonable representation of the actual economy.

As already remarked, the equilibrium paradigm has been in the ascendant since the economic problems of the 1970s indicated that the solution to the problem of an effective economic management had yet to be found. Given the fact that this conceptual framework is agreed upon by the majority of macroeconomists, we start, in the first chapter of this dissertation, by addressing the question of whether an equilibrium approach can capture the most important aspects of the data; more specifically, we ask whether the sort of equilibrium relationships derived from non-accelerating inflation rate of unemployment (NAIRU) models are consistent with the data. We first examine the theoretical restrictions imposed by different classes of models, some implying that the NAIRU is a structural parameter that can only be influenced by supply-side measures, others that the attainable level of unemployment is a function also of demand variables. We then use co-integration techniques (the Engle-Granger (1987) and the Johansen (1988) procedure) to test the NAIRU hypothesis. The more powerful likelihood method developed by Johansen shows that the unemployment rate is co-integrated with both supply and demand variables only as well as a combination of the two; this implies that the theoretical restrictions derived from NAIRU models are not inconsistent with the data, and that the equilibrium approach can not be rejected.

In the second chapter, we go on to analyse more directly the adequacy of real business cycle models to represent the actual economy by investigating the relative importance of supply versus demand shocks as the driving force of the cycle. As we think that the use of growth and cross-section observations to tie down the parameters of preferences and technology that determine the stochastic behaviour of these models is not entirely appropriate, because different data sets will result in different parameter values, we follow a different strategy, and use two measures of productivity growth (the Solow residual and the dual residual from the cost function) to test competing theories of the cycle in a number of OECD countries. After having discussed also the issue of market structure and its relevance to explain economic fluctuations, we conclude that the empirical evidence refutes the "stronger" real business cycle (RBC) hypothesis that denies the role of demand shocks.

The models surveyed so far ignore the possible role of disaggregate impulses; it has been argued by others that the great diversity in the behaviour of employment across industries or regions suggests that disaggregate factors may be important, and that real economic activity in a particular sector or region may be influenced by factors specific to the sector or region. Long and Plosser (1983) and King and Plosser (1984) allow for the possibility that a fraction of the variations in output growth may result from disaggregate shocks to technology or tastes. Lilien (1982) constructs the time series of the standard deviation of rates of change in employment across eleven sectors for the US economy, and shows that the higher the dispersion in employment growth rates, the higher the unemployment rate.

In the third chapter, we show that it is possible to modify linear general equilibrium models of the business cycle based on the work of Long and Plosser (1983) in order to allow a direct role for disaggregate (industry-specific) factors in the generation of macroeconomic fluctuations. We test the so-called "sectoral shifts hypothesis" by estimating a vector autoregression (VAR) of the output growth rates of nineteen UK industrial sectors, and by carrying out a factor analysis on the innovations; we find that the one-factor model performs well when applied to British data, implying that there is an aggregate shock that can account for a large percentage of the fluctuations of output over the cycle.

All these studies focus exclusively on business cycle fluctuations; however, there is an older tradition (see, e.g., Kuznets (1933), Bursk (1931), and Macaulay (1938)) that investigated also the importance of seasonal patterns in economic activity; the quantitative importance of seasonal fluctuations has been stressed in a recent paper by Barsky and Miron (1989), who find that short-term variation in economic activity is largely affected by seasonal fluctuations, and that those have many of the same characteristics as business cycle fluctuations; they criticize the usual practice of eliminating seasonality from series having seasonal "noise", and point out that their results are challenging to macroeconomists seeking an explanation of aggregate fluctuations.

<u>In the fourth chapter</u>, we also make the point that fluctuations at both seasonal and business cycle frequencies should be regarded as important topics of investigation. By running regressions with seasonal dummies and band spectrum regressions, we first document the existence of a "seasonal cycle" similar to the business cycle. Next, we set up a one-sector, neo-classical model of capital accumulation in which seasonal preferences are explicitly incorporated (the coefficient of risk aversion depending on the season s); we show that the coefficients in the laws of motion of the

variables are a function of the underlying seasonal parameters. The model is not rejected by the data, as our preferred specification for the estimated reduced form equations includes seasonal dummies for both the intercept and the slope coefficient and does not include weather variables, confirming that seasonality is not just a consequence of weather variations but it is a feature to be explained within the economic model.

CHAPTER 1

CO-INTEGRATION AND THE NAIRU HYPOTHESIS

1.1 Introduction

The existence of the non-accelerating-inflation rate of unemployment, or NAIRU, has been one of the theoretical issues more frequently discussed in macroeconomic debates in recent years; in addition, many empirical studies have been conducted and estimates of the natural rate of unemployment have been used as the basis of macroeconomic policy design. In most cases dynamic wage equations have been estimated and the "equilibrium" rate of unemployment has been defined in terms of changes in the rate of wage inflation.

Jenkinson (1988) was the first to use co-integration techniques to test directly the existence of equilibrium relationships implied by NAIRU theories and their claim that aggregate demand does not affect the attainable equilibrium rate of unemployment but simply determines where the economy is in the short run (he refers in particular to the work of Layard and Nickell, 1986). He shows that the unemployment rate is not co-integrated with the supply-side variables which Layard and Nickell use to explain long-run trends in unemployment; secondly, he shows that unemployment is co-integrated with these supply-side variables augmented by some of the demand-side variables used by Layard and Nickell in their model. He then interprets these results as support for the contention that the level of demand has an important impact on the equilibrium the economy attains and that the supply-side factors are not the only ones to enter into the equilibrium relationship.

This paper builds upon Jenkinson's analysis, but differs from it in the empirical strategy adopted and applies some more recently developed econometric techniques in order to show that no such clear-cut conclusions can be drawn. More specifically, we argue that, since the NAIRU is usually associated with a non-linear relationship between the unemployment rate and the supply-side variables, it is more appropriate to test co-integration between the logarithm of the unemployment rate and the supply-side variables; secondly, we use annual (instead of quarterly) data, that are available for a long time period and are more suitable for long-term analysis; thirdly, we show that the empirical results apparently conflicting with models in which the level of demand can not influence the attainable equilibrium of the economy but simply the position of the economy relative to the naturale rate vanish when the statistical procedure developed by Johansen (1989) is used; more generally we point out that it is difficult to draw causal inferences from this kind of long-run analysis: the direction of causality can not be determined unequivocally.

The present paper's organization is as follows. In Section 1.2 we put the use of co-integration analysis to test the

NAIRU hypothesis into context by providing an overview of other established approaches and of the different theoretical implications of the Layard and Nickell model as opposed to a number of other models in which the demand side of the economy can affect long-run equilibrium relationships. In Section 1.3 the main features of co-integrating regressions are introduced. The empirical results obtained using co-integration analysis are discussed in Section 1.4, that contains also a review of Johansen's procedure. Finally, some tentative conclusions are put forth in Section 1.5.

1.2 The NAIRU hypothesis

There are two well established approaches to the estimation of the equilibrium unemployment rate; the first is the equilibrium approach, based on the Lucas supply curve; the other is the Phillips curve.

In the equilibrium approach, unemployment can diverge from its natural rate only in the short-run because of misperceptions; if $p > p^e$, the perceived real wage will be higher than the actual wage and the supply of labour will be higher. Hence

$$U = U^* + \beta(p - p^e) \qquad (1)$$

where U is the actual rate of unemployment, and U* the natural rate. In empirical analysis (e.g. Barro, 1978),

$p - p^e$ is replaced by shocks, like unanticipated money growth, and the natural rate is estimated by setting all shocks equal to 0.

The Phillips curve approach can be summarized in the following way: by substituting for Δw, the rate of change of nominal wages, in an inflation equation, a reduced-form inflation equation is obtained, relating the rate of change of prices to the expected rate of change of prices, the rate of unemployment, and other supply side factors. e.g.

$$\Delta p = \Delta p^e - \gamma(U - f(X)) \tag{2}$$

where X is a vector of supply side variables and $U-f(X)$ is a measure of labour-demand pressure. The natural rate can then be calculated by imposing the restriction $\Delta p=\Delta p^e$. This is the definition of NAIRU, since inflation will accelerate only if $U-f(X)$ is negative and decelerate only if it is positive (we are assuming that $\Delta p^e=\Delta p_{-1}$, since in practice over the ranges of inflation experienced in Western economies, adaptive expectations are a reasonable approximation to rational expectations). The condition that the coefficient of Δp^e in the reduced-form inflation equation, or of Δw^e in an inflation equation, or of Δp^e in a nominal wage equation be equal to 1 is tantamount to saying that there is perfect adjustment. An alternative approach to test the existence of a NAIRU would be to apply co-integration techniques. Even if an individual economic variable is not stationary (e.g. it is I(1)), it is possible that a linear combination of some variables is stationary (I(0)), e.g.

$$z_t = \alpha' x_t \tag{3}$$

where α is the co-integrating vector.

$\alpha' x_t = 0$ may be considered a long-run or "equilibrium" relationship, and z_t is the extent to which the system is out of equilibrium.

Consequently, Jenkinson (1988) argues, if a NAIRU, that can not be affected by demand side variables, existed, the unemployment rate should turn out to be co-integrated only with supply, rather than demand, variables. This can be tested in the way explained by Granger (1983) and Granger and Engle (1987).

The essential steps are:

1) an OLS regression with the variables in levels is run, and co-integration between the variables is tested using Dickey-Fuller (DF), ADF (augmented DF) and CRDW (co-integrating regression Durbin Watson) tests;

2) the residuals are entered in the error correction model (ECM) in place of the level terms and the significance of the error correction term is tested (cointegration between the variables implies

significance of the EC term and viceversa).

The NAIRU hypothesis implies that there is no co-integrating vector between the unemployment rate and a set of demand and supply variables, since attainable equilibrium can not be affected by aggregate demand.

A very well known model of the British economy that has this implication is the Layard-Nickell model, in which the unemployment rate is determined only by an index of mismatch, the replacement ratio, the real price of imports, union power and taxes. This can be contrasted with several classes of models in which demand side variables can affect the long run equilibrium. Some of the approaches recently taken in the literature are briefly reviewed below; the setup of the Layard and Nickell model is then analysed in more detail.

One of the explanations put forward to account for the persistence of deviations from long-run equilibrium values emphasizes the role of insider power, as in the models by Lindbeck and Snower (1988) and Blanchard and Summer (1986).

In their most extreme version, these models are characterized by total hysteresis, that is, the employment rate follows a random walk with drift. The basic idea is that, if an adverse shock hits the economy and people are laid off, the wage pressure at given unemployment will rise as there are fewer workers worried about their jobs. This is especially likely to happen if the "insiders" are organized in unions that fix real wages to ensure the continued employment of the insiders. Hence the effect of the adverse shock that reduces the number of insiders is to decrease employment permanently, since next period's employment will simply be equal to last period's actual employment. Employment is therefore expected to decline, unless there were positive shocks or workers were sufficiently risk averse to select a level of employment much higher than last period's. In less extreme versions, in which the effects of outside unemployment are also taken into account, we only observe "partial hysteresis", and even though the short-run NAIRU can vary it converges to a long-run NAIRU.

Another suitable framework for thinking about the causes of persistence focuses on the way uncertainty affects investment decisions that are costly to reverse later (see Dixit, 1988). A recent line of research exploits the analogies between real and financial investment decisions. An opportunity to make a real investment is a call option on a stock that consists of the capital in place. Making the investment is like exercising the option, and the cost of the investment is the price of the option.

Standard techniques of financial economics can be used to calculate the price of the option, that is, the value of the investment opportunity to the firm, and the optimal rule to exercise the option, that is, the investment criterion. The most important feature of investment decisions in such an uncertain environment is hysteresis, that is, the failure to reverse such decisions when the underlying cause is reversed. A significant factor in investment decisions is the fact that the re-sale value of a machine is often below its purchase price, and hence it would be a costly error for the firm to buy a machine that is not utilized; as a result, an investment will be undertaken only if the firm is reasonably sure that the new capital equipment will be needed. This gap between purchase price and re-sale value of a unit of capital generates multiple equilibria in which equilibrium values depend on the past history of the economy: the tools of option pricing can be used to show that when uncertainty is high firms will adopt a wait-and-see attitude in case things turn out badly ex post. These are models of "optimal inertia", in which a once-for-all change can have permanent effects.

A third line of argument is exemplified by Diamond's (1982) model in which thin markets lead to multiple equilibria and demand management policies are effective since they can shift the economy from a "bad" to a "good" equilibrium and lower the equilibrium unemployment rate. His basic results are due to the introduction of trade frictions; the fictional Walrasian auctioneer is dropped and there are trading externalities: the higher the level of economic activity, the easier it is to trade and the higher is the optimal level of production. Therefore we can have equilibria with high level of trading and production, and other equilibria with low levels of economic activity. In other words, an economy with this type of trade frictions does not have a unique natural rate of unemployment and it can be shifted from one equilibrium to another by demand management policies: "pump-priming" may be effective.

Let us now look at the different implications of the Layard and Nickell model.

We begin by considering the behaviour of firms.

Suppose the economy has a number (n) of identical imperfectly competitive firms, and each firm's production takes place under constant returns to scale.

Hence the production function has the form:

$$Y_i/K_i = g(N_i/K_i) \quad (g' > 0, \quad g'' < 0) \tag{4}$$

The demand for the firm's output depends on its price p_i relative to the aggregate price level p and the level of aggregate demand σ (σ is assumed to depend on world economic activity, government policies and competitiveness, i.e.

$$\sigma = \sigma(Y^*, G, c) \tag{5}$$

Thus we have:

$$Y_i = D(P_i/P,\ \sigma) \quad (D_1 < 0,\ D_2 > 0) \tag{6}$$

Firms set prices as a mark-up on marginal cost and so we obtain:

$$P_i = u(\sigma)wf'(Y_i/K_i) \tag{7}$$

where f is the inverse of g since

$$N_i/K_i = f(Y_i/K_i) \tag{8}$$

implying that

$$f'(Y_i/K_i) = \frac{1}{g'}(N_i/K_i) \tag{9}$$

and $wf'(Y_i/K_i)$ is the marginal cost. Under perfect competition, $u(\sigma)=1$. Conversely, if firms are imperfectly competitive, we have:

$$u(\sigma) = \left(1 - \frac{1}{\eta(\sigma)}\right)^{-1} \tag{10}$$

where $\eta(\sigma)$ is the elasticity of demand.

If there is normal pricing, i.e. if prices are set independently of demand, it is clear that $u'(\sigma)<0$. In the aggregate $p_i=p$, and substituting (6) into (7) we obtain the price equation:

$$p = Y(\sigma)wf'(nD(1,\sigma)/K) \tag{11}$$

Furthermore, combining (8) and (7) we find that:

$$g'(N_i/K_i) = u(\sigma)\,\frac{w}{p}$$

i.e. the MPL is equal to the real wage times the firm's mark-up. Turning now to the wage determination, we may note that wages are affected in the long run by the level of labour demand, the tightness of the labour market as a measure of the unemployment level U, and a group of wage "push" factors Z, including mis-match, replacement ratio, union power, employers' and personal income taxes and so on.

Therefore, in the long run real wages are given by:

$$\frac{w}{p} = \psi(K,u,Z) \qquad (\psi_1 > 0,\ \psi_2 < 0,\ \psi_3 > 0) \qquad (13)$$

Dropping K, introducing p/p^e as an argument of (13) and w/w^e as an argument of (11) and finally considering U as the dependent variable in the labour market, we obtain a three-equation structural model:

labour demand $u = \phi(w/p,\ \sigma)$ (14)

prices $p/w= \gamma(w/w^e,\ \sigma)$ (15)

wages $w/p = \beta(p/p^e,\ u,Z)$ (16)

Eliminating wage and price surprises, this model describing the supply side of the economy can be solved for real wages, unemployment and real demand. In particular, we can derive a reduced-form natural rate equation by combining equations (14) and (16) (using the long-run solutions to these two equations), which determines U conditional on σ:

$$u = \alpha(Z) = \alpha(MM, \rho, p_M, UP, T_L) \tag{17}$$

where MM is an index of mismatch, ρ is the replacement ratio, p_M is the real price of imports, UP is a measure of union power, and T_L stands for taxes.

The approach followed below consists in looking for a co-integrating vector between the unemployment rate and the supply side variables that determine it in the Layard-Nickell model, and for a co-integrating vector between the unemployment rate and the same variables plus some demand side variables, since in the other type of models that we have reviewed the NAIRU can be affected by demand shocks.

It should be noticed, however, that the cointegration test is only for the existence of a long-run NAIRU. It does not rule out persistence effects of the form

$$U^* = \alpha + \beta U_{t-1} \qquad (\beta < 1) \tag{18}$$

that imply a long-run NAIRU equal to $\alpha/(1 - \beta)$. Demand shocks raise U_t and hence have an effect on future NAIRUs although it dies away in time.

This can be easily shown within a model in which firms set prices as a mark-up on expected wages, wage setters set wages as a mark-up on expected prices, and there is a mechanism generating persistence (see R. Layard-C. Bean, 1988). We have:

$$p - w^e = a_0 - a_1U \quad \text{is the price-setting equation} \qquad (19)$$

and

$$w - p^e = b_0 - b_1cU \quad \text{is the wage-setting equation} \qquad (20)$$

The term cU in equation (20) could capture the "ineffectiveness" of the unemployed as job-seekers, since long-term unemployment lowers their morale and actual skills as well as their skills as perceived by the employers and hence reduces the downward pressure on wages of unemployment; or it could reflect the role of insider power in generating persistence: if there is a negative shock, the wage set by the insiders is such as to keep the new, lower level of employment. Thus cU can be approximated by

$$cU = c_0 - c_1U - c_2U_{-1} \qquad (21)$$

The model can be solved for the long-run NAIRU by setting

$$w - w^e = p - p^e = 0 \qquad (22)$$

to give

$$U^* = d_0/(d_1 - d_2) \qquad d_1 > d_2 \tag{23}$$

where

$$d_0 = a_0 + b_0 - c_0b_1; \quad d_1 = a_1 + b_1c_1; \tag{24}$$

$$d_2 = b_1c_2$$

and for the short-term NAIRU:

$$U = d_0 + d_2U_{-1}/d_1 \tag{25}$$

or

$$U = \frac{(d_1 - d_2)U^* + d_2U_{-1}}{d_1} = \alpha + \beta U_{-1} \qquad (\beta < 1) \tag{26}$$

Hence the short-run NAIRU in each period will differ from the long-run NAIRU but will converge towards it.

1.3 Theory of cointegrating regression

First, the basic notion of an integrated series must be explained. The simplest example is a random walk:

$$x_t = x_{t-1} + \epsilon_t \tag{27}$$

where ϵ_t is $IN(0, \sigma_\epsilon^2)$

Thus

$$x_t = \frac{\epsilon_t}{1-L} = \sum_{j=0}^{t-1} \epsilon_{t-j} \qquad \text{if } x_0 = 0 \tag{28}$$

i.e. x_t is the sum of all past innovations and is clearly not stationary.

If we define a stationary series as I(0) (integrated of order 0), then another series is I(k) if $\Delta^k z_t$ is I(0) (in the case of a random walk, $\Delta x_t = \epsilon_t$ is stationary, i.e. x_t is I(1)).[1]

Clearly, if the series is I(k) (k>0), a different distributional theory is required for this non-stationary process. Next, consider a pair of series x_t, y_t each of which is I(1) and without trend in mean. In general, any linear combination of these series is also I(1). However, it is possible that there exists a constant A (the co-integrating parameter), such that

$$z_t = x_t - Ay_t \tag{29}$$

is I(0).

Then x_t, y_t are said to be co-integrated and the relationship

$$x_t = Ay_t \tag{30}$$

might be considered a long-run or "equilibrium" relationship. Economic theory might suggest that the two

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[1] Note that stationarity is in fact not necessary for I(0): the weaker, but more technical requirement is that the series has a spectrum which is finite but non zero at all frequencies.

variables may diverge in the short-run but will be brought together in the long-run and thus (29) measures the extent to which the system is out of equilibrium.

(This notion of co-integration can be generalised and applied to the case in which the variables have trends in their mean and/or x_t is a vector of N time series, each I(d) d>0. Then x_t will be said to be cointegrated cI(d,b) if there exists a co-integrating vector α such that

$$z_t = \alpha' x_t \qquad (31)$$

is I(d-b), b>0).

(29) can be called the "equilibrium error".

Granger (1983) and Granger and Engle (1987) have shown that if x_t, y_t are both I(1) and co-integrated, there always exists a data generating mechanism of the "error correction" form:

$$\Delta x_t = -\rho_1 z_{t-1} + (\Delta x_t,\ \Delta y_t)_{-1} + d(B)\ \epsilon_{1t} \qquad (32)$$

$$\Delta y_t = -\rho_2 z_{t-1} + (\Delta x_t,\ \Delta y_t)_{-1} + d(B)\ \epsilon_{2t}$$

where

$$z_t = x_t - Ay_t$$

Also the reverse is true: data generated by an error correction model must be co-integrated. The reason is the following: if x_t, y_t are I(1) their first differences in equation (32) are I(0), and provided that also the term

z_t is I(0) (i.e. that x_t, y_t are co-integrated) every term in equation (32) will be co-integrated.

A two-step procedure is then suggested by Engle and Granger. First, a prior OLS levels regression is performed to test the hypothesis of co-integration. Then the residuals are entered in the ECM in place of the level terms, thus imposing a restriction on the parameter values of the level terms. Stock (1984) established that the estimator of the co-integrating vector(s) in the first stage regression converges in probability limit to the true value of the parameter faster than the standard OLS estimator - super-consistency. The asymptotic efficiency result concerning the use of the estimated co-integrating vector in place of the true vector (formulated in Theorem 2, Engle-Granger (1987), and the consistency results make this procedure valid and justify the omission of the dynamics from the first stage and the incorporation of the cross-equation restrictions in the second stage.

The advantages of this approach are evident, and several recent applied studies have used it. If the purpose is to isolate the "long-run equilibrium" relations between time series, we can ignore all the difficulties of dynamic specification, and provided that the variables are integrated, we can investigate relationships between them by simple static regressions. Assuming that the sample is reasonably large, it is not necessary to be concerned with the various short-run dynamic components or with simultaneity. The second stage of estimating the dynamic components needs to be undertaken only if a co-integrating relationship is found, and the residuals from the first stage estimation can be used to be entered in the ECM.

This two-stage estimation has been criticized on the grounds that even in "large" samples finite sample biases may be present in the co-integrating regression, and it has been suggested that all parameters should be estimated simultaneously by non-linear least squares in the final model (see Banerjee et al, 1986).

Before proceeding to test sets of variables for co-integration, it is sensible to establish the properties of the individual series. Various tests have been suggested.

Sargan and Barghava (1983) present a test of the hypothesis that the errors on a regression equation follow a random walk. To test if a series x_t is I(0), the regression

$$x_t = c + u_t \tag{33}$$

is run and the null hypothesis

$$u_t = u_{t-1} + \epsilon_t \qquad \epsilon_t \ IN(0,\sigma_\epsilon^2) \tag{34}$$

is tested against the alternative that the errors follow a stationary first order Markov process, using the

Durbin-Watson (DW) statistic with the critical values calculated by Sargan and Barghava under the unit root null hypothesis.

The DW statistic converges to zero in the case of non-cointegration, but Sargan and Barghava derive the exact significance levels for a bounds test of the hypothesis that the residuals are a random walk. This test has some drawbacks: the critical values for the bounding distribution are far apart, and the test can not be generalized to the case where the first difference of the residuals is I(0) but not serially uncorrelated. However, it has the convenience that the DW statistic is computed in virtually all regression packages; besides, it can be shown to be the uniformly most powerful invariant test. If the null hypothesis is not rejected, the test is repeated on differenced data until H_0 can be rejected.

An alternative procedure has been suggested by Dickey and Fuller (1981): for each series the following regression is estimated:

$$\Delta x_t = \alpha x_{t-1} + v_t \qquad v_t \sim IN\ (0,\ \sigma_v^2) \qquad (35)$$

Under the null hypothesis of non-stationarity, we would expect the value of α to be equal to zero.

A t-test is performed on α using special tables of critical values provided by Dickey and Fuller.[2] To test for higher order autoregressions in the residuals the augmented Dickey-Fuller test (ADF) can be carried out by running the regression:

$$\Delta x_t = \alpha_0 x_{t-1} + \sum_{j=1}^{p} \alpha_j \Delta x_{t-j} + e_t \tag{36}$$

and performing a t-test on α_0 using the critical values presented by Dickey and Fuller.

If the null of a unit root can not be rejected, the data are differenced and the procedure repeated until H_0 can be rejected. To test for co-integration between a pair of series, the co-integrating regression

$$x_t = c + \alpha y_t + u_t \tag{37}$$

is run and a co-integrating regression Durbin-Watson test (CRDW) is performed, the null being that the residuals are I(0). Alternatively, the DF and ADF tests can be used for $\hat{u}_t$.[3] Testing for co-integration between a set of variables

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[2] Note that the test is not invariant to whether the null is a random walk with or without drift.

[3] To derive the test for the case when u_t is serially correlated, a different approach is advocated by Phillips (1987); it is based on the estimation of

$$\sigma^2 = \lim_{T\to\infty} \frac{1}{T} E(S_T^2) = \lim_{T\to\infty} \frac{1}{T} [\sum_{t=1}^{T} E(u_t^2) +$$

$$+ 2 \sum_{i=1}^{T-1} \sum_{t=i+1}^{T} E(u_t u_{t-i})]$$

by using the consistent estimator

is a major issue in this method of analysis. Phillips (1986) presents the limit theory and investigates the properties of OLS when the regressors are I(1) but not co-integrated. He finds that, in a simple linear model in which a variable y_t is regressed against x_t, the slope coefficient converges asymptotically to a random variable and the intercept coefficient tends to infinity at the rate $T^{\frac{1}{2}}$. Moreover, the R^2 also converges to a random variable and hence the usual F statistic tends to infinity, implying that the hypothesis that y_t and x_t are uncorrelated will be rejected on the F test with probability approaching 1 as T tends to infinity. Finally, the DW tends to zero.

Engle and Yoo (1987) point out that the distribution of the t-statistic in the DF test will depend on the number of regressors, and hence tables for each number of regressors need to be calculated by Monte Carlo simulation. For the 5% significance level, and for the case T=200, the critical values for rejection of the non-cointegration hypothesis are as follows:

2 variables: 3.37

3 variables: 3.78

$$S_{Tl}^2 = \frac{1}{T} \left[\sum_{t=1}^{T} u_t^2 + 2 \sum_{i=1}^{l} w_{il} \sum_{t=i+1}^{T} u_t u_{t-i} \right]$$

where $u_t = y_t - y_{t-1}$, l is a function of T to be determined, and $w_{il} = 1 - i/(l+1)$.

4 variables 4.18

5 variables: 4.48

The DF tables give 2.86 for the unit root test with an intercept, and 1.95 for the test with no intercept. Engle and Yoo also estimate significance levels in a representative case for the ADF test, to be performed when the differences of the variables are I(0) but not uncorrelated.

As explained above, if a set of variables are co-integrated, then there always exists an EC formulation of the dynamic model, and viceversa. Hence the residuals from the co-integrating regression can be used as an error correcting variable in the dynamic equation relating x_t to y_t (or viceversa), and if the series are co-integrated the EC term will be statistically significant.[4]

1.4 Empirical results

We have seen that the Layard-Nickell model of the British economy, which basically describes the supply side of the economy considering both the pricing behaviour of imperfectly competitive firms and the process of wage determination, can be solved to derive a reduced-form

[4] As a test of the EC specification of the dynamic model, it is possible to relax the restrictions on the coefficients imposed by the prior co-integrating regression and estimate the unrestricted model in which the levels variables are entered.

natural rate equation. This is the equation to which Jenkinson (1988) refers.

In his paper,he finds that the cointegrating regression Durbin-Watson (CRDW) and the augmented Dickey-Fuller test (ADF) are unable to reject the absence of a NAIRU-type relationship in the data for the UK. Conversely, there exists a cointegrating vector when demand factors are included in the regression: hence the attainable level of unemployment is, he argues, a function of fiscal and monetary policies as well.

Jenkinson uses quarterly data over the period 1954-83, and the dependent variable in his cointegrating regressions is the unemployment rate. However, in most cases, the NAIRU is associated with a non-linear relationship between unemployment and supply-side variables (as shown in a curved Phillips curve); in particular, in the Layard-Nickell model this is shown by the concavity of the function log U.

It seems worthwhile to test the same hypothesis in a cointegrating regression in which log U is the dependent variable, using annual data over a much longer sample period, from 1900 to 1987,[5] (using quarterly data, Nickell finds that log U and supply-side variables are

[5] There are several gaps in the series (see data appendix for details); strictly speaking, co-integration theory applies to a full set of observations.

cointegrated), and also applying the Granger-Engle two-step procedure, given the weak power of the tests for cointegration.

First of all, tests of the order of integration of the single variables have been carried out: DW and ADF tests show that none of the variables are stationary (see Table 3); on the basis of the same tests, they all appear to be I(1) (see Table 4).

Cointegrating regressions of log U against a group of supply-side variables (a mismatch index, the real price of imports, a measure of union density, implicit income tax rate, replacement ratio), and the same supply-side variables plus demand side variables (an index of world trade, government expenditure, exchange rate) have been run. Results are reported in Table 1 (see Engle-Yoo and S.G. Hall for critical values): the CRDW test indicates that log U is cointegrated at the 5% level both with supply-side variables and a combination of supply-side and demand-side variables; on the other hand, the ADF test regressing residuals on past levels and lagged changes rules out the existence of a cointegrating vector in all cases. The results reported in the tables are for regressions including 4 lags; the same conclusions are reached whether or not a time trend is included.

How should these results be interpreted?

It is very well known that an important problem with the use of the CRDW, DF and ADF tests is their lack of power especially for values of the AR root approaching unity. It is likely that the power of the test is low even with 100 observations. Secondly, the exact critical values of the CRDW itself are a function of the data generating process (DGP). Granger and Engle (1985) have computed the critical values under a white noise data generating process; for different DGPs these values are only a benchmark.

Given the likely power of the test against the autoregressive residuals alternative, we also use the error correction representation and the Granger-Engle two-step estimation procedure to provide another test for cointegration. To establish that the joint distribution of log U and a group of other variables is an error correction system, a series of models have been estimated. Following the model building strategy recommended by Granger and Engle, a simple error correction model (ECM) has been estimated first, and then the significance of additional lags of the regressors has been tested. Only the simple ECMs, and the final specification that has been chosen on the basis of this "simple to general" search in the cases in which the error correction term turned out to be significantly less than 0, are reported in Table 2.

The chosen ECMs pass a number of tests, including the LM test for 4th order autocorrelation, and a Chow test for parameter constancy leaving out the observations from 1974 to 1987. It can be seen that the test of the statistical significance of the error correction term indicates that log U is not cointegrated with supply-side variables only, but it is cointegrated with the same supply-side plus demand-side variables.

To summarize, if we take the Engle and Granger two-step approach, that has been followed because of the lack of power of the cointegration tests and the contradictory evidence that they provide in this specific case, we find that, even when the non-linearity of the relationship between unemployment and supply-side variables is taken into account and when annual data for a much longer sample period are used, the unemployment rate is related, in the long run, to demand as well as supply factors.

However, it must be stressed that this methodology does not provide conclusive proof against the NAIRU hypothesis: if there are more than two I(1) variables, then there may be more than one co-integrating vector. It is true that the non-existence of a co-integrating vector with supply-side variables only implies that the NAIRU hypothesis is false; however, the use of the Engle and Granger procedure does not enable us to address the question of the number of co-integrating vectors that may exist between a given set of variables.

Moreover, the existence of a co-integrating vector with supply and demand side variables does not imply that demand matters (it could for instance describe the behaviour of the demand variables). As an economic example, consider an open economy with perfect capital mobility; from the equilibrium relationship between demand and supply, i.e. $\bar{Y} = C(\bar{Y}) + I(r^*) + G + NX(ep^*/p, \bar{Y}, Y^*)$, where $\bar{Y}$ is the long-run level of output, C is consumption, I investment, G government expenditure, NX net exports, r^* the world interest rate, ep^*/p the real exchange rate, and Y^* foreign income, we can derive a function f relating the real exchange rate to G, r^*, $\bar{Y}$ and Y^*: $ep^*/p = f(G, r^*, \bar{Y}, Y^*)$. We would expect to find co-integration between these variables but our function only determines the value of the exchange rate that is needed for demand to be equal to supply. Co-integration simply means that, for a set of variables to have an attainable equilibrium, there must be some causation between them to provide the dynamics of the system. For simplicity, let us consider only two series, x_t and y_t, both I(1), and suppose that there is a constant A such that $z_t = x_t - Ay_t$ is I(0). Given these assumptions, we know that the two series will be generated by an error correction model, in which changes in the variables are driven by the previous value of z_t. A consequence of this model is that either Δx_t or Δy_t (or both) must be caused by z_{t-1}, which, in turn, depends on x_{t-1}, y_{t-1}. Hence, if the two series are co-integrated, either x_{t+1} is caused by y_t or

y_{t+1} by x_t: there must be causation in at least one direction, but such direction can not be determined unambiguously.

Also, as already noted, the tests statistics for co-integration described so far have low power and are based on non-standard distributions.

Consequently, in the remaining of this section we follow Johansen and adopt a maximum likelihood procedure that makes it possible to calculate all the co-integrating vectors and then test for the number of such vectors on the basis of a well-defined distribution. The main steps of Johansen's statistical analysis of co-integration vectors are described below. Let us consider a vector autoregression (VAR):

$$A(L)X_t = \epsilon_t \tag{38}$$

where A(L) is a (k+1) polynomial in L, X_t is px1 and ϵ_t is NIID(0, Ω).

The determinant of A(L) is assumed to have unit roots, implying that

$$A(1) = I - A_1 - A_2 - \ldots\ldots. - A_k$$

that is, the long-run or co-integration matrix, has less than full rank. Its rank r corresponds to the number of co-integrating vectors.

The model can be reformulated as:

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \ldots\ldots\ldots + \Gamma_{k-1} \Delta X_{t-k+1} + \Gamma_k X_{t-k} + \epsilon_t \qquad (40)$$

where

$$\Gamma_i = - I + A_1 + \ldots\ldots\ldots + A_i \qquad i = 1 \text{ to } k \qquad (41)$$

implying that

$$- \Gamma_k = A(1) \qquad (42)$$

The hypothesis that there exist r co-integrating vectors can be expressed as:

$$H_0: \alpha\beta' = A(1) \qquad (43)$$

where α and β are p x r matrices.

The method of estimation is to concentrate the likelihood function with respect to the parameters $\Gamma(i)$, i= 1 to k-1, by regressing:

$$\Delta X_t \text{ on } \Delta X_{t-1}, \Delta X_{t-2} \ldots\ldots \Delta X_{t-k+1} \qquad (44)$$

and

$$X_{t-k} \text{ on } \Delta X_{t-1}, \Delta X_{t-2} \ldots\ldots \Delta X_{t-k+1} \qquad (45)$$

to obtain residuals R_{ot} and R_{kt} respectively, and compute the second moments of all these residuals, denoted S_{oo}, S_{ok} and S_{kk} where

$$S_{ij} = T^{-1} \sum_{t=1}^{T} R_{it} R'_{jt} \qquad \text{for } i,j = 0,k \qquad (46)$$

We finally solve

$$|\lambda S_{kk} - S_{ko} S_{oo}^{-1} S_{ok}| = 0 \tag{47}$$

for the p largest eigenvalues

$$\lambda_1 \geq \lambda_2 \geq \ldots . \geq \lambda_p \tag{48}$$

and the corresponding eigenvectors

$$\hat{\beta} = (\hat{v}_1, \ldots ., \hat{v}_p) \tag{49}$$

normalized by

$$\hat{\beta}' S_{kk} \hat{\beta} = I \tag{50}$$

Tests of the hypothesis of r co-integrating vectors are based on:

$$-2\ln(Q) = -T \sum_{i=r+1}^{p} \ln(1-\lambda_i) \tag{51}$$

which is a test that there are at most r co-integrating vectors, where the λ_i, i= r+1 to p, correspond to the p-r smallest eigenvalues of the diagonal matrix of the eigenvalues in descending order. The quantiles in the distribution of the test statistic are tabulated in Johansen (1988).[6]

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[6] This test statistic is asymptotically distributed as

$$tr\{ \int_0^1 dBB' [\int_0^1 B(u) B(u)' du]^{-1} \int_0^1 BdB' \}$$

where B is a (p-r)-dimensional Brownian motion with covariance matrix I.

Given the eigenvalues and the eigenvectors normalized on the coefficient on the dependent variable being -1, the ML procedure enables us to calculate all co-integrating vectors. (The results of this procedure can be compared with the Engle and Granger co-integrating regressions by choosing each of the variables to be the dependent variable in turn).[7] This statistical analysis is applied here to test again co- integration between the log of the unemployment rate and a set of supply and demand side variables. The eigenvalues and the normalized eigenvectors are presented in Table 5.

The test statistic that under the null there are at most 2 co-integrating vectors of the unemployment rate with supply-side variables only is equal to 34.52.

The null is accepted at the 95% confidence level (the critical value for m=p-r=4 is 38.6; see tables in Johansen, 1988). For the null that there is only one co-integrating vector between unemployment and demand-side variables, the test statistic is equal to 19.98, that is less than the critical value for m=3 (23.8).

Finally, the null that there are 4 cointegrating vectors of the unemployment rate with supply plus demand-side

[7] Hall has done this for an aggregate wage equation, to find that the ML estimator lies within the space of the different OLS estimates of the cointegrating vector.

variables is also accepted at the 95% confidence level: the test statistic is equal to 5.76 (the critical value for m=2 is 12.0).

By dividing the coefficients of all the other variables in the normalized eigenvectors by the coefficient of log U and changing the sign, we obtain the co-integrating vectors reported in Table 6.

To summarize, we are now unable to reject the hypotheses that there are two co-integrating vectors between the unemployment rate and supply-side variables only, that there is one co-integrating vector with demand side variables only, and four co-integrating vectors with a set of demand plus supply side variables. These findings confirm that, by using the Engle and Granger procedure we do not test for all co-integrating vectors, and show that, once the more powerful maximum likelihood procedure developed by Johansen is used, we can not reach any clear-cut conclusions concerning the validity of the theoretical concept of the NAIRU and the role of demand in the long run.

1.5 Conclusions

Jenkinson concludes his paper by saying; "The question of the existence of the sort of equilibrium relationships derived from NAIRU models was strongly questioned, and an alternative model proposed in which the attainable level of unemployment itself was a function of fiscal and monetary policies. The interpretation of the NAIRU as some structural parameter of the economy, whose value can only be influenced by concerted supply-side measures, is therefore challenged".

Our view, as put forward in this paper, is that no such claim can be made and that the data lend themselves to various possible interpretations which can differ in emphasis.

The existence of two co-integrating vectors between the unemployment rate and supply-side variables only indicates that the co-integration properties of the data are not inconsistent with the NAIRU hypothesis; as for the co-integrating vectors including demand variables, they could simply describe the behaviour of the demand variables themselves.

The use of the maximum likelihood procedure that enables us to calculate all co-integrating vectors is therefore not supportive of the contention that the theoretical restrictions imposed on NAIRU models can not be reconciled with the data.

Table 1

Cointegrating regressions

(dependent variable: log U)

Variable.	Parameter estimate.			
constant	8.76	9.65	9.046	5.2
MM	4.42	5.2	4.54	11.91
PM	0.01	0.019	0.01	0.019
UDEN	-5.27	-10.89	-5.32	-4.45
RR				22.31
TY		3.6		0.621
G	-0.00005		-0.00005	
WT	0.025		0.026	
EX	0.049			
CRDW	0.75^{c}	0.85^{c}	0.75^{c}	0.61^{c}
DF	-0.35 (-2.93)	-0.211 (-2.49)	-0.347 (-2.85)	-0.36 (-3.48)
ADF	-0.4 (-1.94)	-0.145 (-1.32)	-0.361 (-1.75)	-0.48 (-3.32)
LM(4)	23.52	37.23	23.52	27.36
sample period	1922-38	1922-38	1922-38	1922-38
	1948-87	1946-87	1948-87	1949-87

c = variables are co-integrated at the 5% level.

t-statistic in parentheses.

constant	5.94	6.58
MM	6.2	7.15
PM	0.014	0.014
UDEN	-1.61	-1.32
RR	16.77	19.86
TY		
G	-0.00005	-0.00006
WT	0.0184	0.0192
EX		-0.20
CRDW	0.89[c]	0.92[c]
DF	-0.51(-4.49)	-0.54(-4.46)
ADF	-0.7(-3.67)	-0.58(-2.85)
LM(4)	25.44	24.96
sample period	1922-38	1922-38
	1949-87	1949-87

Table 2

Error correction models

(dependent variable: Δlog U)

Variable.	Parameter estimate.		
constant			
EC(-1)	-0.424(-4.41)	-0.42(-4.41)	-0.5(-5.31)
ΔlogU(-1)	0.41(3.36)	0.41(3.23)	0.4(3)
ΔPM(-1)	0.014(2.97)	0.014(2.86)	0.017(3.45)
ΔMM(-1)			-1.11(-0.75)
ΔUDEN(-1)			4.36(2.1)
ΔG(-1)			0.00001(0.51)
ΔWT(-1)			-0.018(-2.23)
ΔEX(-1)			0.083(0.99)
CHOW*	0.7937	0.7655	
DW	1.81	1.75	
LM(4)	1.17	1.24	
variables	MM	MM	MM
included in	PM	PM	PM
the corresponding	UDEN	UDEN	UDEN
co-int.regression	WT	WT	WT
	G	G	G
		EX	EX

t-statistic in parentheses.

* leaving out observations from 1974 to 1987.

constant	0.025(0.74)	0.0016(0.058)	
EC(-1)	-0.51(-5.31)	-0.085(-0.98)	-0.56(-5.25)
ΔlogU(-1)	0.37(2.86)	0.044(0.337)	0.53(4.26)
ΔMM(-1)	-0.73(-0.5)	2.12(1.73)	
ΔPM(-1)	0.0167(3.38)	0.0128(2.17)	0.0093(3.02)
ΔUDEN(-1)	4.51(2.17)	0.8(0.334)	
ΔG(-1)	0.00001(0.62)		
ΔWT(-1)	-0.019(-2.26)		
ΔTY(-1)		10.16(2.29)	
ΔMM(-2)			-3.63(-2.78)
ΔRR(-1)			-14.93(-2.77)
variables	MM	MM	MM
included in	PM	PM	PM
the	UDEN	UDEN	UDEN
corresponding	WT		WT
co-int.	G		G
regression		TY	
			RR
DW			1.87
LM(4)			1.75
CHOW*			1.05

constant	0.0068(0.23)	0.028(0.78)	0.021(0.61)
EC(-1)	-0.13(-1.2)	-0.55(-4.57)	-0.48(-4.01)
ΔlogU(-1)	0.16(1.07)	0.4(2.83)	0.35(2.79)
ΔMM(-1)	-0.47(-0.25)	-2.22(-1.35)	-2.41(-1.89)
ΔPM(-1)	0.01(1.78)	0.01(2.03)	0.02(1.85)
ΔUΔEN(-1)	1.28(0.52)	2.95(1.35)	2.31(1.56)
ΔTY(-1)	9.3(2.15)		5.6(1.63)
ΔRR(-1)	-2.67(-0.62)	-6.19(-1.59)	-5.8(-1.33)
ΔG(-1)		0.00001(0.36)	0.0001(0.51)
ΔWT(-1)		-0.14(-1.56)	-0.21(-1.11)
ΔEX(-1)			0.072(0.91)
variables included in the corresponding co-int. regression	MM PM UDEN TY RR	MM PM UDEN RR G WT	MM PM UDEN TY RR G WT EX

Table 3

Testing whether NAIRU variables are I(0)

variable	DW	ADF
log U	0.2063	0.0004(0.059)
MM	0.01357	-0.11(-1.59)
PM	0.0128	0.01865(0.9)
UDEN	0.0154	-0.000009(-0.003)
G	0.0137	0.0099(1.74)
WT	0.0076	0.0359(2.8)
EX	0.0468	-0.01(-1.52)
RR	0.1103	0.00088(0.12)

t-statistic in parentheses.

Note: 4 lags were needed to obtain white noise residuals.

Table 4

Testing whether NAIRU variables are I(1)

variable	DW	ADF
log U	1.6468	-0.003 (-4.26)
MM	1.6115	-0.13 (-4.32)
PM	0.807	-0.027 (-5.17)
UDEN	0.6298	-0.018 (-5.18)
G	0.9226	-0.021 (-3.91)
WT	1.2368	-0.004 (-4.03)
EX	0.8173	-0.14 (-4.39)
RR	1.3888	-0.076 (-3.97)

Note: 4 lags were needed to obtain white noise residuals.

Table 5

supply-side variables only

eigenvalues

0.004310	0.057191	0.233767	0.357951	0.481884	0.561

eigenvectors

variable	mm	pm	uden	rr	ty	log U
mm	1.0	0.00510	-8.23198	-2.39008	5.499	-0.461
pm	910.455	1.0	-171.3686	1269.98	-430.17	-37.45

demand-side variables only

eigenvalues

0.033346	0.130782	0.215304	0.417251

eigenvectors

variable	log U	g	wt	ex
log U	1.0	-0.00168	0.10287	-17.73828

supply plus demand-side variables

eigenvalues

0.008939	0.105161	0.414282	0.528101	0.618136	0.719

eigenvectors

variable	pm	uden	rr	log U	wt	ex
pm	1.0	-402.02	308.821	-41.233	0.0961	14.816
uden	0.0001	1.0	1.127	0.0769	-0.0037	-0.0132
rr	-0.0002	-0.031	1.0	-0.030	0.0016	0.0250
log U	-0.174	-54.312	24.672	1.0	0.0294	-5.905

Table 6

Co-integrating vectors

supply-side variables only

log U = 2.17mm + 0.01pm - 17.89uden - 3.02rr + 11.93ty

log U = 24.31mm + 0.026pm - 4.57uden + 33.91rr - 11.48ty

demand-side variables only

log U = 0.001g - 0.1wt + 17.73ex

supply plus demand-side variables

log U = 0.17pm + 54.31uden - 24.67rr - 0.02wt + 5.9ex

log U = 0.024pm -9.75uden + 7.49rr + 0.0021wt + 0.35ex

log U = -0.00013pm - 14.28uden -16rr + 0.04wt + 0.18ex

log U = -0.007pm -uden + 33.3rr + 0.03wt +0.83ex

(48 obs.)

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DATA APPENDIX

Sample periods for the variables are as follows:

log U (unemployment rate)	1900-1987		
MM (mismatch index)	1922-1987		
PM (import price index)	1900-1913	1919-1987	
UDEN (union density; $\frac{\text{union members}}{\text{total workers}}$)	1900-1913	1920-1987	
RR (replacement ratio)	1920-1938	1949-1987	
G (real government expenditure)	1900-1914	1920-1938	1946-1987
WT (world trade)	1900-1913	1921-1938	1948-1987
EX (real exchange rate)	1900-1914	1919-1987	
TY (income tax rate)	1920-1938	1946-1987	

The main source for the data is "The British Economy Key Statistics, 1900-1970", London and Cambridge Economic Service, 1973. (The series have been updated at the Centre for Economic Performance, London School of Economics, using other sources.)

Note: the mismatch index is constructed as the weighted standard deviation of employment growth rates across nine sectors.

CHAPTER 2

ARE CYCLICAL FLUCTUATIONS IN PRODUCTIVITY DUE MORE TO SUPPLY SHOCKS OR DEMAND SHOCKS? SOME MORE EVIDENCE

2.1 Introduction

The aim of this paper is to provide some more empirical evidence on the hotly debated issue of the main source of economic fluctuations.

For a while, during the sixties, in a period of sustained economic growth, it was even thought that cycles had disappeared, since the Keynesian revolution had brought about a remarkable improvement in economic performance in the Western countries. However, the slowdown and the frequent recessions of industrial economies in the seventies led to a renewed interest in the business cycle as a crucial topic of research.

In the early eighties a new stream of literature devoted to the "real business cycle" (RBC) approach to the analysis of macro-economic fluctuations has emerged (see, for example, Kydland and Prescott (1982), Long and Plosser (1983), and King and Plosser (1984)). This line of research tries to build on the growth theory literature in order to account for fluctuations about potential. It generally makes the assumptions that markets are competitive, all information is public and there are no rigidities.

These modern theorists have adopted the same stochastic approach as Frisch (1933) and Slutzky (1937), which distinguishes between the shocks that cause the variables to differ from their steady state values, and the propagation mechanism that converts serially uncorrelated shocks (or impulses) into serially correlated fluctuations in output. Recent research on economic fluctuations is also characterised by a much better integration between theory and empirical work, since it has been shown that the stochastic behaviour of most economic variables can be accounted for by very simple propagation mechanisms that can be estimated empirically; when shocks disturb relatively simple linear difference equation systems, their dynamics are largely consistent with the main features of business cycles.

There is a widespread consensus about this general theoretical framework, whose development has been made possible by advances in the theory of time series, but the consensus does not extend to the nature of the propagation mechanism and of the disturbances affecting the economy (monetary or real, and if real deriving from changes in tastes or technology). There are two main directions of research: the first is the "real business cycle" (RBC) school, trying to explain business cycles in terms of technological shocks in a competitive environment; the second, in the Keynesian tradition, regards demand shocks as playing a crucial role, but goes beyond previous Keynesian models in that it does not assume slow adjustment of prices and wages but attempts to show how, in the presence of market imperfections, demand shocks can cause fluctuations in economic activity.

The new line of inquiry based on the idea that macroeconomic fluctuations can be explained without invoking market imperfections probably had its beginning with a paper by Black (1982), who argued that observed fluctuations can be usefully modelled as the realization through time of the set of transactions agreed upon in a complete market Arrow-Debreu economy.

As Prescott concludes in his 1986 paper: "Economists have long been puzzled by the observations that, during peacetime, industrial market economies display recurrent, large fluctuations in output and employment over relatively short time periods.... These observations should not be puzzling, for they are what economic theory predicts." His view is that the behaviour of output, consumption, fixed investment and inventory investment should be studied in Ramsey-like models with technological shocks; such models can explain the joint behaviour of output and its components as dynamic responses to plausible processes for productivity.

Real business cycle models abstract entirely from monetary considerations. In the words of King and Plosser (1984), they see "business cycles as arising from variations in the real opportunities of the private economy, which include shifts in government purchases or tax rates as well as technical and environmental conditions."

To date, attention has been restricted almost exclusively to exogenous shifts in the production technologies of goods, whereas shocks to preferences and to fiscal policy rules of governments have been omitted (many RBC theorists seem to assume that large technology shocks are more likely than large stochastic shocks to tastes).

Since these models have a complete set of markets for contingent claims to future goods, real allocations are Pareto optimal and the central government can not improve welfare; that is one reason why shocks to government spending or tax rates are absent. It is also the case that RBC economists hold the view that equilibrium models that are frictionless can generate fluctuations in output that closely approximate the time series of output in the industrial economies. In the words of Prescott (1986): "Given the people's ability and willingness to inter- and intra-temporally substitute consumption and leisure and given the nature of the changing production set, there would be a puzzle if the American economy did not display the business cycle phenomena. By display the business cycle phenomena, I mean that the amplitudes of fluctuations of the key economic aggregates and their serial correlation properties are close to those predicted by theory".

As already mentioned, a research strategy based on a parsimonious time-series representation of the shocks is adopted: low-order stochastic difference equations and laws of motion for the unobserved shocks are specified, and the covariogram of a given set of variables implied by the model is compared with the sample covariogram. Endogenous sources of dynamics (the propagation mechanism) are then shown to cause patterns of autocorrelations and cross-correlations consistent with the stylized facts known about the cycle.

The endogenous propagation mechanism is assumed to depend on a time-to-build technology (it takes more than one period to build capital goods) or on a specification of preferences in which leisure in the current period is related to leisure in previous periods.

Non-time-separability gives a much higher leisure intertemporal substitution, but it is still not sufficient to produce fluctuations in labour supply, and hence also employment, that closely match the ones observed historically. In order to improve the fit of the model, Hansen (1985) has explored the implications of introducing labour indivisibility into a RBC framework; he finds that the aggregate elasticity of labour supply will be much larger than the elasticities of those whose behaviour is being aggregated.

It has also been been pointed out by critics of RBC models that they fail to satisfy the restrictions on the cross-correlations of asset returns and output (see Mehra and Prescott, 1985).

Moreover, formal methods of estimation and inference do not provide evidence in support of RBC models (see, e.g., Altug, 1985).

To summarize, these models have two really crucial elements: first, they stress the importance of the "propagation mechanism", i.e. of the persistence of the shocks; second, they hold that the driving force of the cycle are supply - as opposed to demand (monetary or fiscal) - shocks.

The emphasis placed on the propagation mechanism is compatible with other equilibrium approaches and even with disequilibrium models; on the other hand, the RBC view according to which the shocks are real is clearly not. As McCallum (1987) points out, we can distinguish between a "weaker" and a"stronger" position: the former does not deny the existence of demand shocks, but emphasizes the fact that supply shocks are quantitatively more important; the latter denies the role of demand shocks altogether. It is the strong hypothesis that really characterizes RBC theories, that otherwise could not be distinguished from other equilibrium models.

In its empirical approach the RBC school has usually resorted to calibration (using growth and cross-section observations to tie down the parameters of preferences and technology) rather than to standard econometric techniques. Empirical investigation has typically proceeded by fitting RBC models to the data and evaluating the extent to which the cycles implied by the models match those exhibited by the data; it has been shown that, for plausible values of the parameters characterizing preferences and technology, the variances of the shocks to technology can be chosen such that equilibrium RBC models imply empirical second moments for a given set of real variables that approximately match the corresponding sample second moments. However, the theory of RBC and its data strategies are quite separate, and the former can be accepted even if a different empirical approach is chosen (see R.E. Manuelli (1986) that makes this point clear).

This is what Matthew Shapiro (1987b) does in his paper: he explains how to test, within a more standard statistical framework and using two measures of productivity growth, the competing theories of the cycle and presents some evidence concerning the U.S. manufacturing sector that could not be reconciled with Keynesian theories or other models in which demand factors have a primary role. This research strategy is certainly less objectionable than one that proceeds by taking the structure of preferences and technology as given and determining the values of the variances of the shocks to technology that are consistent with the observed second moments; if the latter approach is taken, a somewhat arbitrary assessment of the plausibility of the variances and autocovariances of the technology shocks is the only criterion for the acceptance or rejection of the model being evaluated. Here the same method of analysis as Shapiro's is applied to the U.K. and seven other OECD countries.

The paper is organized as follows.

Section 2.2 discusses Shapiro's methodology and provides a brief review of the theoretical issues that arise in measuring productivity growth; Section 2.3 presents the empirical results for the U.K. and seven other OECD countries, as well as analysing the issue of the market structure and its relationship with economic fluctuations and the correct measurement of productivity; Section 2.4 makes some concluding remarks.

2.2 The Methodology

One of the best established facts about business cycles is the procyclical behaviour of productivity. Macroeconomists, however, disagree about the sources of economic fluctuations.

The real business cycle school claims that productivity shocks are the driving force (or source of impulses) of business cycles. In Keynesian or monetarist models, on the other hand, the driving force is aggregate demand shocks, but these models must explain the procyclical movements of productivity, since if firms are always on the labour demand curve positive aggregate demand shocks reduce the marginal product of labour and thus lead to counter-cyclical productivity.

The Keynesian explanation relies on "labour hoarding", i.e. short run "off the production function" behaviour due to contractual commitments that limit the adjustment of labour, costs of adjustment, adverse effect of labour adjustments on morale and so on (see Okun, 1981); Oi (1962) defines labour as a quasi-fixed factor; there is indirect evidence of labour hoarding (short-run elasticity of output with respect to labour greater than 1), but several researchers (e.g. Sims, 1974) think that this just reflects statistical bias. The only direct evidence is a sample survey for the U.S. manufacturing industry by Fay-Medoff (1985) showing that 4% of the hours paid should be classified as hoarded (the questionnaire asks about technically possible percentage reduction in hours paid and actual reduction).

Is it possible to discriminate between these two alternative views of the cycle? Can we answer the question of whether observed fluctuations in productivity are more from supply (real business cycle theories) or demand?

It has been suggested (see M.Shapiro, 1987 and also M. Ohta, 1974) that there are two ways of measuring productivity changes: an output-based and a price-based measure. The first is the method devised by Robert Solow (1957). The derivation is quite simple. Let us consider the following production function with Hicks-neutral technological progress:

$$Y_t = f(N_t, K_t)E_t \tag{1}$$

where Y_t is real value added, N_t is man-hours, K_t is capital and E_t is the productivity shock.

To get real value added, we have to subtract from gross output, which is final sales minus changes in final good inventories, the costs of raw materials and intermediate inputs, all deflated by the respective price indices; failure to do so has important consequences: Bruno (1984) and Bruno and Sachs (1982) suggest that when energy and raw material input prices increase relatively, there is substitution of labour and capital for energy and raw material, so that value added increases faster than gross output and TFP measures based on gross output understate productivity growth.

Let $\dot{e}$, $\dot{y}$, $\dot{k}$ and $\dot{n}$ be the rates of growth of E_t, Y_t, K_t and N_t, and M_k and M_n be the marginal products of K_t and N_t respectively. Then, by taking the derivative of Y_t with respect to time and after some simple algebra we get:

$$\dot{y} = \dot{e} + (M_k K/Y)\dot{k} + (M_n N/Y)\dot{n}. \qquad (2)$$

We then subtract the rates of growth of K_t and N_t, weighted by their elasticities, from the rate of growth of Y_t to obtain a measure of total factor productivity (TFP) growth. Solow also suggested that, under perfect competition in product and labour markets, the marginal product of labour can be replaced by the wage rate, so that the labour elasticity becomes equal to labour' share α, and that, assuming constant returns to scale (CRS), the capital elasticity can be taken to be $1-\alpha$.

In practice, finite differences in the logs of the variables are used, to yield:

$$\Delta\epsilon_t = (\Delta y_t - \Delta k_t) - \alpha_t (\Delta n_t - \Delta k_t) \qquad (3)$$

where α denotes the share of labour and Δ stands for the time derivative of the logarithm of a variable.

In this way, in steady state, we can define the growth residual or "total or multi-factor productivity" (TFP or MFP) growth as the rate of growth of output minus the weighted average of the input growth rates. This is the Divisia Index approach to measuring TFP growth recommended by Jorgenson and Griliches (1967). It is possible to chain small changes and construct a time series index of TFP growth. Denison (1974, 1979) and Kendrick (1973, 1980) use a very similar measure of productivity, the only difference being that they use the same factor share weights for long periods of time instead of different factor shares for each short period.

However, as recognized by Berndt and Fuss (1986), economies are not usually in a steady state of equilibrium: they point out that traditional methods of productivity measurement assume that producers are in long run equilibrium whereas they may be in short run equilibrium; their method is to adjust for variations in capacity utilisation by altering the service price weights of the quasi-fixed inputs arguing that the expected value of the marginal product is the relevant shadow rental price.

Morrison (1985) explains how to derive a measure of capacity utilisation in the presence of adjustment costs for quasi-fixed factors. Let Q^* be the maximum level of output. Then the ratio Q/Q^* can be used as a measure of capacity utilisation and the conventional measure of productivity can be corrected to reflect cyclically varying rates of capacity utilisation.

Other studies do not try to measure utilisation rates directly but use some observable proxies, like the cyclical deviation of profits (see Denison, 1979), unemployment, deviation of employment changes from trend, layoffs, and indices of capacity utilisation based on surveys. Helliwell et al (1985) use the ratio of average unit cost relative to the output price, the ratio of sales to normal output and the ratio of lagged inventory stock to normal output to derive their measure of the utilisation rate.

As to the measurement of capital, usually measures of the gross capital stock are constructed using the perpetual inventory formula:

$$K(t)= K(t-1) + I(t) \qquad (4)$$

where $I(t)$ is gross investment; this approach does not take into account the fact that efficiency of capital decreases over the years. To deal with this sort of problem, the theory of vintage capital has been developed. One good example of this sort of models is Salter (1960). Output is given by a fixed coefficient production function

$$Y = \min(K/a, N/b) \tag{5}$$

where a is the number of machines and b the number of workers required to produce 1 unit of output, with a and b being fixed in the short run; at any given time, the firm will rank vintages in terms of operating costs and use the number of machines required to produce desired output, the lowest cost first. This approach has been taken by Berndt and Fuss (1986), who give a different weight to each vintage of past investment on the basis of its age and the ratio of the real price of energy at the time when the investment was made to the current price; they then construct aggregate capital as the weighted average of the different vintages.

It should then be clear that the Solow residual is deficient as a measure of TFP growth for at least two reasons.

First, one would hope that all variables are correctly measured; for example, the capital stock should be appropriately adjusted for quality changes, deterioration and economic scrapping (in the short run, there is also the capacity utilization question); a proper consideration of this issue would involve the construction of a vintage model whose study goes beyond the aim of this paper.

As for the labour input, this TFP growth measure neglects labour hoarding during recessions; Muellbauer (1986) describes a technique to correct for the fact that observed hours, H, may exceed effective hours H(e): effective hours are defined so that

$$h(e) = h(n) + (H-H(n)/H(n)) - \beta(H(n)/H-H(n)) \qquad (6)$$

where lower case letters denote logarithms, H(n) is a measure of normal hours, and β is a parameter to be estimated. This correction also enables us to control for labour hoarding along the heads dimension, since the term (H-H(n)/H(n)) is procyclical.

Another crucial assumption in deriving the Solow residual, as explained above, is that there are competitive conditions in product and labour markets, assumption that allows us to replace the MPL with the product wage. This point is discussed at length in the next section, where it is shown how it is possible to extend the traditional growth accounting approach to accomodate imperfect competition in product markets.

An alternative way of measuring productivity relies on factor prices: a firm with a CRS production function will have a cost function of the form:

$$C(Y_t, W_t, R_t) = g(W_t, R_t)Y_t/E_t \qquad (7)$$

where R_t is the rental rate of capital and W_t the wage rate.

The marginal cost X_t is given by:

$$X_t = C_y(Y_t, W_t, R_t) = g(W_t, R_t)/E_t \tag{8}$$

and taking logs we get:

$$\log X_t = \log g(W_t, R_t) - \log E_t \tag{9}$$

The percentage change in the marginal cost is given by:

$$(dX_t/dt)/X_t = \{(g_w(.)dW_t/dt)/g(.)\} + \{(g_r(.)dR_t/dt)/g(.)\} -\{(dE_t/dt)/E_t\} \tag{10}$$

or

$$\Delta x_t = \{(g_w)dW_t/dt/g(.)\} + \{(g_r(.)dR_t/dt)/g(.)\} - \Delta\epsilon_t \tag{11}$$

By Shepard's lemma we know that

$$N_t = g_w(.)Y_t/E_t \tag{12}$$

and

$$K_t = g_r(.)Y_t/E_t \tag{13}$$

Substituting into (11) for $g_w(.)$ and $g_r(.)$ and setting the price equal to the marginal cost (i.e. the competitive assumption $\Delta x_t = \Delta p_t$), we get:

$$\Delta p_t = \alpha_t \Delta w_t + (1-\alpha_t)\Delta r_t - \Delta\epsilon_t \tag{14}$$

and finally

$$\Delta\epsilon^P_t = \alpha_t(\Delta w_t - \Delta p_t) + (1 - \alpha_t)(\Delta r_t - \Delta p_t) \tag{15}$$

(Here capital is assumed to be paid its marginal product within the period; under the more realistic assumption that investment decisions have to be made one period in advance and with a Cobb-Douglas production function the dual measure of productivity growth becomes:

$$\Delta\epsilon^P_t = \alpha(\Delta w_t - \Delta p_t) + (1 - \alpha)(\Delta y_t - \Delta k_t)) \tag{16}$$

Under the null hypothesis that measured changes in productivity are true changes,the two measures should be identical. Under the Keynesian alternative, the Solow residual moves independently with aggregate demand: there are cyclical fluctuations in measured productivity because firms hoard labour, not because the true productivity of factors of production changes; consequently, factor prices should not move and the deviation between the two measures should be cyclical.

Consider the following regression of the Solow residual on a constant and the dual residual:

$$\Delta\epsilon_t = \beta + \gamma\Delta\epsilon^P_t + u_t \tag{17}$$

The null hypothesis that measured changes in productivity are true changes can be expressed as: H_0: $\beta = 0$, $\gamma = 1$, ($R^2 = 1$) i.e. the constant term should be 0, the slope coefficient 1 and R^2 should also be equal to 1. The alternative hypothesis that movements in aggregate demand are the driving force of fluctuations in productivity can be tested by including a measure of demand in the regression:

$$\Delta\epsilon_t = \beta + \gamma\Delta\epsilon_t^P + \delta\Delta GNP_t + u_t \tag{18}$$

This equation can be interpreted as the regression of the difference of labour productivity and real wage growth rates on aggregate output growth, since imposing the restriction that the coefficient of $\Delta\epsilon^P$ is 1 leads to the following equation:

$$\Delta\epsilon_t - \Delta\epsilon^P{}_t = \alpha[(\Delta y_t - \Delta n_t) - (\Delta w_t - \Delta p_t)] \tag{19}$$

Under the Keynesian hypothesis, the dependent variable should be procyclical, since, if there is labour hoarding, an increase in aggregate demand should lead to an increase of labour productivity but to no change of the wage.

In his paper, Shapiro, using US annual data from 1950 to 1985, finds that the hypothesis that the slope coefficient is 1 in the first regression can not be rejected for aggregate manufacturing and for most industries; furthermore, the additional regressor (GNP growth) turns out not to be statistically significant. He concludes that these results can not be reconciled with Keynesian theories of the cycle.

2.3 Empirical evidence for the U.K. and seven other OECD countries

The same two equations have been estimated here using UK data[1] for aggregate manufacturing and six industrial sectors (paper, clothing and footwear, timber, textiles, chemicals, food) from 1964 to 1985, and the picture that emerges seems to be quite different. In the regression of the Solow residual on the dual residual (see table 1) the null hypothesis that the slope coefficient is equal to 1 is rejected in 4 cases out of 7, and if the rate of growth of GNP is included as an additional regressor (see table 2) its coefficient turns out to be statistically significant and R^2 increases sharply in 4 cases out of 7.[2] Moreover, in the regression where the restriction $\gamma = 1$ is imposed (see table 5), R^2 indicates explanatory power and the regressor is significant in 4 cases out of 7, showing that there is pro-cyclical deviation between the two measures of productivity growth as predicted by models emphasizing the importance of demand shocks.[3] As already

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[1] For a listing of the data, see data appendix.

[2] If a CES specification for the production function (instead of a Cobb-Douglas) is adopted, R^2 is generally very low, the regressor is not statistically significant and the null hypothesis is rejected.

[3] Tatom (1980) claims that the apparent pro-cyclical behaviour of productivity arises because usually the cyclical variation in the utilization of the capital stock is not taken into account. He shows, for the US, that, accounting for this variation, the pro-cyclical behaviour of productivity and SRIRL disappear; he uses a capacity utilisation variable and a formal test confirms that this reflects variations in the flow of capital services and not other cyclical influences on total factor

mentioned, it is possible to provide alternative estimates of TFP growth embodying a correction for labour hoarding replacing observed hours with effective hours. Thus equations (17) and (18) have been re-estimated using these corrected measures of TFP growth, and the results are virtually the same.

On the basis of this empirical evidence, demand factors seem to play a role in driving economic fluctuations. However, it has to be stressed that these estimates are based only on a few observations. Furthermore, as pointed out by Robert Hall, "for a non competitive firm whose price exceeds its marginal cost, Solow's method is biased": it is an inappropriate measure because labour receives less than its marginal product, and thus α, the share of labour, understates the elasticity of output with respect to labour and $\Delta\epsilon$ is overstated; besides, the bias caused by the assumption of perfect competition implies a pro-cyclical behaviour in measured productivity even in the absence of any correlation between the "true" productivity residual and the cycle: in a boom, when employment grows faster than the capital stock, productivity growth is overstated and conversely, in a slump, it is understated.

Following Hall, we can write marginal cost X as:

$$X = \{W(N-N_{-1}) + R(K-K_{-1}\}/\{(Y-Y_{-1}) - E'Y\} \qquad (20)$$

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productivity, e.g. labour hoarding.

where all letters denote the same variables as before, R is the shadow price of capital if the firm is in short run equilibrium, and E' is the true productivity shock.

Equation (20) can be re-written as

$$y = \{WN/XY\}n + \{RK/XY\}k + E' \qquad ((21)$$

where lower-case letters stand for the rate of growth of the corresponding variable.

The assumption of constant returns to scale (CRS) implies that:

$$\{WN/XY\} = 1 - \{RK/XY\} \qquad (22)$$

and so we get:

$$y - k = \{WN/XY\}(n - k) + E' \qquad (23)$$

In the presence of perfect competition, price equals marginal cost and hence the term {WN/XY} is equal to the share of labour in value added α.

However, if the market is not perfectly competitive, the market price will be given by a mark-up over marginal cost: $p = \mu x$ with $\mu > 1$.

Hence, in general, the valid equation will be of the following form:

$$y - k = \mu\alpha(n - k) + E' \qquad (24)$$

By re-arranging (24), we can obtain equation (25):

$$E = E' + (\mu - 1)\alpha(n - k) \qquad (25)$$

where

$$E = y - k - \alpha(n - k) \qquad (26)$$

is the traditional Solow residual.

If, as it's always done in practice, we use finite differences in the logs of the variables, we get:

$$\Delta\epsilon = \Delta\epsilon' + (\mu-1)\ \alpha(\Delta n-\Delta k) \qquad (27)$$

where Δ stands for the time derivative of the log of a variable, $\Delta\epsilon$ is the Solow residual and $\Delta\epsilon'$ the true productivity shock.

Equation (27) can easily be seen to imply a pro-cyclical behaviour for the Solow residual as long as μ is greater than 1 and the labour-capital ratio varies pro-cyclically.

If the degree of market power was known, we could easily modify the Solow residual in the following way:

$$\Delta\epsilon = (\Delta y - \Delta k) - \mu\alpha(\Delta n-\Delta k) \qquad (28)$$

where μ is the mark-up coefficient

[$\mu = 1/(1-1/\text{elasticity})$].

Hall considers instead the following equation:

$$\Delta y - \Delta k = \mu\alpha(\Delta n - \Delta k) + \Delta\epsilon \qquad (29)$$

and tries to estimate the unknown parameter μ, treating $\Delta\epsilon$

as a constant plus the unobserved random element. OLS can not be applied in this case, since the productivity shift is clearly correlated to changes in employment and output; instrumental variable estimation is required here. What kind of instrument is suitable?

Aggregate output is a suitable instrument only under the identifying assumption that it is not correlated with productivity growth in each sector, i.e. under an assumption that is inconsistent with the real business cycle view; under this view, only truly exogenous variables are legitimate instruments.

As Blanchard notes in his comment on Hall (1986), Hall's model is observationally equivalent to one in which firms are perfectly competitive and productivity shocks are positively correlated among industries and with the rate of growth of aggregate GDP; this positive correlation could account for an estimated mark-up greater than one as found by Hall. In this sense Hall's approach can not really settle the dispute over the primary source of economic fluctuations.

However, several potential instruments are available and this makes it possible to test the implied over-identifying restrictions (Hall's model is just identified): the orthogonality of the residuals from equation (29) and the set of instruments used can be tested by means of a Sargan test; this check of the validity of the instruments improves upon Hall's arbitrary assumption that productivity residuals are uncorrelated with aggregate GDP growth.

The mark-up coefficient has been estimated here (see table 6) using as instruments in various regressions the rate of growth and the lagged rate of growth of GDP, government expenditure, and world trade. As stated before, the rate of growth of GDP is a valid instrument only if this variable is not correlated to productivity growth, assumption that is not testable; the rate of growth of world trade, however, is certainly an appropriate instrument, being a truly exogenous variable. The point estimates of μ obtained using only one instrument are very poor often with implausible values, large standard errors and negative R^2; in most cases the null hypothesis $\mu = 1$ can not be rejected.

The estimates are less imprecise when at least two instruments are included, and the results are not very sensitive to the particular subset chosen in the sense that the null hypothesis of perfect competition can still not be rejected in most cases.

The estimates obtained using the entire set of instruments are also reported in Table 6; a Sargan test of the over-identifying restrictions with a suitable small sample correction shows that the restrictions can not be rejected (see Table 6). The test is straightforward. In general, let

$$y = Xa + u \tag{30}$$

be our equation, where X is a (T x n) matrix of regressors, and let Z be the (m x n) instrument matrix (m > n).

Then the IV estimator $\hat{a}$ can be seen to minimize the following criterion function:

$$Q_0 = u'Z(Z'Z)^{-1}Z'u \tag{31}$$

and the ratio $Q_0/\sigma^2 = Q_1$ is asymptotically distributed as a χ^2 with (m-n) degrees of freedom.

We can use the test in its T x R^2 form: regressing $\hat{u}$ on the Z's we obtain the test statistic Q_1 as the product of number of observations and R^2 from this regression.

The following small sample correction ought to be made:

$$Q_2 = (T-n)/T \times Q_1 \sim \chi^2_{m-n} \tag{32}$$

The values of the test statistic reported in table 6 are all smaller than the critical value of χ^2 with 5 degrees of freedom at the 5% level (11.07) confirming the validity of this set of instruments.

Since productivity shocks could be correlated across industries (i.e. the covariance matrix of the disturbances could be non-diagonal), estimation by three-stage-least-squares (3SLS) has also been carried out (all instruments still being included in the chosen set); in this way it is possible to do a joint test of the null hypothesis that the mark-up coefficient is the same across all industries: a quasi-likelihood ratio test confirms that the null can not be rejected.

The test proposed by Gallant and Jorgenson is A = T(Q(0)-Q(1)), where T is the number of observations, Q(0) is the value of the minimum distance criterion for H_0, and Q(1) its value for the maintained hypothesis; this test is asymptotically χ^2 with degrees of freedom equal to the number of restrictions imposed, that is, the number of parameters in the second model minus the number in the first. In our case, A is equal to 10.64, that is, less than the critical value of χ^2 with 6 degrees of freedom at the 5% level (12.59).

(If a common mark-up coefficient is assumed, its estimated value is 1.23, with a standard error of 0.16). A quasi-likelihood ratio test indicates that the restriction of a common mark-up equal to 1 can not be rejected; A is equal to 8.19, less than the critical value of χ^2 at the 5% level with 7 degrees of freedom (14.07), implying that the null hypothesis of perfect competition can not be rejected in this case either.

Thus, there does not seem to be evidence against perfect competition and of the need to compute alternative measures of total factor productivity that allow for imperfect competition; if this is the case, the empirical evidence presented above is not affected by the bias in the Solow residual due to the incorrect assumption of perfect competition.

The two alternative explanations of the cycle have been tested also using data for the manufacturing sector of seven other OECD countries (Canada, Finland, France, West Germany, Italy, Norway, Sweden). As in the case of the U.K., the estimates of the coefficients are not very precise because not many observations are available, and hence we should be cautious in drawing any conclusions.

Here the null hypothesis (slope coefficient = 1) can not be rejected in 6 cases out of 7; it can be rejected only in the case of Norway. If the rate of growth of GNP is included in the regression, the explanatory power of the equation as indicated by R^2, does not increase much, except in the cases of Italy, Germany and France, where the additional regressor is statistically significant.

Aggregate demand seems to play a role in the cycle in Italy, Norway, Germany and France; in Canada, Finland and Sweden, supply factors appear to be prevalent; the point estimates are quite imprecise.

If the restriction that the coefficient of $\Delta\epsilon^P$ in equation (18) is 1 is imposed, only in 2 cases out of 7 (France and Italy) the regressor is significant indicating a pro-cyclical deviation between the two measures of productivity growth (see table 9).[4]

[4] O. Attanasio-C. Bean (1987) find that the mark-up coefficient is greater than 1 in these countries, but it has a very low t-statistic.

Obviously, there is a potential gain in efficiency from using a SURE estimator if the disturbances in each equation are contemporaneously correlated with the disturbances in other equations.

The SURE estimates are also reported in Table 7 and 8: now the null hypothesis (slope coefficient = 1) can not be rejected in 3 cases (Italy, Finland, and Norway) and the additional regressor is always significant, implying that demand factors play a prominent role. This is confirmed by a likelihood ratio (LR) test for the joint significance of the coefficient of GNP across all seven countries: the LR statistic is equal to 60.1, more than the critical value of χ^2 with 7 degrees of freedom at the 95% confidence level.

2.4 Conclusions

On the whole, the empirical evidence in this paper is quite mixed; no clear-cut picture emerges and the results are not completely supportive of either theory of the cycle; neither supply shocks nor demand shocks alone seem to be able to account for economic fluctuations.

In some countries (Canada, Finland, Sweden) the former seem to dominate the latter; in others (Italy, Norway, West Germany, France) the opposite is true; the SURE estimates and a joint test for all seven countries indicate a more important role for demand factors.

In the U.K., for which not only the manufacturing sector but also some industries have been analysed, models according to which the initiating shocks are from the demand side can more easily be reconciled with the empirical evidence.

The "stronger" RBC hypothesis, that denies the role of demand shocks, seems to be refuted by these findings; on the other hand, it is dubious whether supply (as in the "weaker" RBC hypothesis) or demand shocks are quantitatively more important.

The estimates are not very sensitive to our correction for labour hoarding, and the hypothesis of perfect competition, i.e. of a mark-up coefficient equal to 1, can not be rejected, implying that the traditional TFP growth measure does not have to be adjusted to allow for imperfect competition.

In any case, the results of this paper need to be somewhat qualified: the limited number of observations that are available for the manufacturing sector clearly affects the reliability of the estimates; further research is thus still needed on the source of economic fluctuations.

DATA APPENDIX

For the U.K., the data sources that have been used are the following:

- CSO data bank;
- CENSUS of PRODUCTION;
- EMPLOYMENT GAZETTE;
- ECONOMIC TRENDS annual supplement.

The data are:

Y: real value added (value added deflated by producer price index);

K: net real capital stock;

N: hours of work of all employees;

W: total labour costs divided by N;

G: central government total expenditure;

WT: index of world trade;

GDP: GDP at factor cost;

As for the other OECD countries, data for the manufacturing sector value added at constant and current prices are taken from the OECD National Accounts Statistics, and data for the capital stock from the OECD "Flows and stocks of fixed capital", 1955-80 and 1960-85.

For a description of the other data, including compensation of employees and employment in manufacturing, see the appendix to Berndt and Hesse (1986).

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Table 1.

Equation 17 (U.K.).

	Slope coefficient.	(s.e.)
All manufacturing	0.64[a]	(0.17)
Paper	0.41[a]	(0.16)
Clothing and footwear	1.35	(0.62)
Timber	1.06	(0.20)
Textiles	0.094[a]	(0.17)
Chemicals	1.01	(0.13)
Food	0.42[a]	(0.10)

(standard errors in parentheses).

R^2	DW
0.43	1.77
0.24	2.25
0.19	1.8
0.58	1.15
0.014	2.41
0.75	2.04
0.45	1.81

a stands for "significantly different from 1 at the 5% level".

(Annual data, sample period: 1964-1985).

Estimates using effective hours.

Slope coefficient	(s.e.)
0.65[a]	(0.17)
0.42[a]	(0.17)
1.33	(0.56)
1.08	(0.20)
0.12[a]	(0.17)
1.01	(0.13)
0.38[a]	(0.11)

R^2	DW
0.42	1.78
0.24	2.28
0.23	1.79
0.59	1.17
0.026	2.45
0.75	2.04
0.38	2.02

Table 2.

Equation 18 (U.K.).

	Coeff. of $\Delta\epsilon^P$		Coeff. of ΔGNP	
All manufacturing	0.43[a]	(0.14)	0.93	(0.24)
Paper	0.24[a]	(0.14)	0.95	(0.28)
Clothing and footwear	1.71	(0.49)	1.29	(0.34)
Timber	1.06	(0.21)	-0.05[b]	(0.52)
Textiles	0.097[a]	(0.18)	0.09[b]	(0.61)
Chemicals	0.94	(0.14)	0.34b	(0.29)
Food	0.45[a]	(0.09)	0.30	(0.12)

(s.e. in parentheses).

R^2	DW
0.69	2.0
0.53	2.72
0.54	1.97
0.58	1.15
0.01	2.4
0.77	2.12
0.58	1.84

b stands for "not significantly different from 0 at the 5% level".

Estimates using effective hours.

Coeff. of $\Delta\epsilon^P$		Coeff of ΔGNP	
0.43[a]	(0.14)	0.93	(0.24)
0.25[a]	(0.14)	0.95	(0.29)
1.59	(0.44)	1.27	(0.34)
1.07	(0.21)	-0.47[b]	(0.52)
0.13[a]	(0.18)	0.11[b]	(0.63)
0.94	(0.14)	0.34[b]	(0.28)
0.42[a]	(0.10)	0.28	(0.12)

R^2	DW
0.68	2.03
0.52	2.78
0.55	1.95
0.59	1.17
0.027	2.43
0.77	2.13
0.51	1.94

Table 3.

Equation 17 (U.K.) with a CES production function.

	Slope coefficient.	
All manufacturing	0.25	(0.099)
Paper	0.32	(0.1)
Clothing	0.22	(0.217)
Timber	0.15	(0.151)
Textiles	-0.012	(0.092)
Chemicals	0.33	(0.084)
Food	-0.016	(0.1)

(standard errors in parentheses).

R^2	DW
0.25	1.91
0.35	2.11
0.05	2.1
0.05	0.96
0.0009	2.44
0.45	1.89
0.001	1.35

N.B. The elasticity of substitution is constrained to equal 0.5 _

Table 4.

Equation 18 (U.K.) with a CES production function.

	Coeff. of $\Delta\epsilon^P$		Coeff. of ΔGNP	
All manufacturing	0.1	(0.08)	1.06	(0.3)
Paper	0.18	(0.1)	0.83	(0.3)
Clothing	0.092	(0.2)	1	(0.45)
Timber	0.18	(0.15)	-0.59	(0.8)
Textiles	-0.014	(0.096)	0.073	(0.62)
Chemicals	0.29	(0.1)	0.38	(0.47)
Food	-0.05	(0.11)	0.24	(0.19)

(standard errors in parentheses).

R^2	DW
0.55	2.16
0.54	2.29
0.25	2.19
0.079	1.0
0.0017	2.44
0.47	1.98
0.083	1.41

N.B. The elasticity of substitution is constrained to equal 0.5.

Table 5.

Equation 18 (U.K.) with restriction (coeff. of $\Delta\epsilon^P = 1$).

	Coeff. of ΔGNP.
All manufacturing	0.61 (0.28)
Paper	0.51 (0.42)
Clothing	1.23 (0.33)
Timber	0.038 (0.51)
Textiles	0.74 (0.85)
Chemicals	0.51 (0.25)
Food	0.45 (0.21)

(s.e. in parentheses).

R^2	DW
0.19	1.49
0.073	2.12
0.41	2.11
0.00029	1.19
0.038	1.56
0.18	2.26
0.19	1.77

Table 6.

Equation 29 (U.K.).

Estimated mark-up coeff.

Instruments used:	GDP		world trade (WT)		govt. exp. (GE)	
All manufacturing	3.8	(2.3)	3.13	(4.4)	6.25	(8.8)
Paper	2.25	(0.47)	1.59	(0.93)	2.55	(0.83)
Clothing and footwear	7.5	(15)	0.54	(4.5)	-5.59	(21.4)
Timber	0.61	(0.88)	-13.5	(31)	-1.7	(2.1)
Textiles	1.02	(0.29)	1.58	(1.31)	1.09	(0.41)
Chemicals	2.5	(1.2)	4.13	(16.2)	2.81	(2.11)
Food	1.6	(0.6)	-0.87	(4.41)	2.67	(2.04)

(s.e. in parentheses).

GDP

R^2	DW
-4.7	2.41
0.46	1.18
-22.2	2.06
0.1	1.46
0.36	2.42
-1.37	1.58
0.2	2.02

N.B. In TSP, R^2 can be negative even when there is a constant in the equation.

WT

R^2	DW
-2.89	2.48
0.64	1.8
0.19	2.0
-30.8	2.39
-0.13	1.99
-5.53	1.35
-0.68	1.77

GE

R^2	DW
-15.6	2.24
0.30	1.02
-18.0	1.71
-0.89	2.43
0.33	2.37
-1.93	1.51
-1.54	1.25

Table 6

GDP+lag.GDP+govt.exp.+lag.govt.exp.+WT+lag.WT	SARGAN test
1.48 (0.61)	9.03
2.05 (0.37)	3.15
0.81 (0.58)	8.4
1.06 (0.86)	10.7
1.05 (0.29)	3.99
1.92 (0.79)	1.47
1.07 (0.42)	9.45

R^2	DW
-0.12	2.59
0.55	1.32
0.18	2.18
0.12	1.85
0.35	2.41
-0.48	1.8
0.069	1.35

3SLS estimates.	DW
1.47 (0.67)	2.58
1.95 (0.33)	1.41
1.15 (0.84)	2.37
0.73 (0.77)	1.39
0.93 (0.28)	2.48
1.55 (0.83)	2.0
1.01 (0.54)	1.36

$\chi^2_6 = 10.64$

$\chi^2_7 = 8.19$

Table 7.

Equation 17 (OECD countries).

	Slope coeff.		Sample period	R^2	DW
Canada	1.22	(0.19)	55-83	0.60	1.64
Finland	1.19	(0.25)	60-81	0.53	2.39
France	1.26	(0.2)	55-83	0.58	1.82
West Germany	0.81	(0.19)	59-83	0.43	1.88
Italy	0.8	(0.45)	61-80	0.15	2.04
Norway	0.29[a]	(0.27)	63-83	0.06	1.23
Sweden	0.58	(0.35)	63-83	0.13	2.38

(s.e. in parentheses).

a stands for "significantly different from 1 at the 5% level".

SURE estimates. (sample period 1963-80).

1.15	(0.16)
1.67[a]	(0.22)
1.11	(0.19)
1.20	(0.19)
0.46[a]	(0.25)
0.33[a]	(0.16)
0.50	(0.27)

DW
1.61
1.56
2.17
1.95
2.26
1.19
2.43

log of likelihood function: 277.955

Table 8.

Equation 18 (OECD countries).

	Coeff. of $\Delta\epsilon^P$		Coeff. of ΔGNP	
Canada	0.95	(0.4)	0.25[b]	(0.32)
Finland	1.56	(0.37)	-0.57[b]	(0.41)
France	0.75	(0.3)	0.82	(0.37)
West Germany	-0.44[b]	(0.44)	1.46	(0.48)
Italy	-0.058[b]	(0.31)	1.72	(0.31)
Norway	0.21[b]	(0.32)	0.26[b]	(0.54)
Sweden	0.26[b]	(0.47)	0.66[b]	(0.67)

(s.e. in parentheses).

R^2	DW
0.61	1.51
0.57	2.14
0.64	1.87
0.61	1.73
0.71	1.34
0.07	1.18
0.17	2.6

b stands for "not significantly different from 1 at the 5% level".

SURE estimates.

1.11	(0.085)	0.129	(0.051)
2.88	(0.15)	-0.949	(0.165)
0.63	(0.24)	0.75	(0.29)
0.31	(0.23)	0.83	(0.25)
-0.13	(0.099)	1.27	(0.072)
0.58	(0.055)	0.78	(0.079)
1.79	(0.129)	-2.60	(0.183)

DW

1.72

0.87

2.21

1.88

2.15

1.12

1.68

log of likelihood function: 308.017

Table 9.

Equation 18 (OECD countries) with restriction (coeff. of $\Delta\epsilon^{p}=1$).

	Coeff. of ΔGNP.	
Canada	0.22^{b}	(0.15)
Finland	-0.1^{b}	(0.29)
France	0.58	(0.24)
West Germany	0.24^{b}	(0.21)
Italy	1.21	(0.34)
Norway	-0.41	(0.53)
Sweden	-0.047^{b}	(0.51)

(s.e. in parentheses).

R^2	DW
0.071	1.52
0.0065	2.71
0.18	1.79
0.00057	1.62
0.42	1.28
0.032	1.09
0.00047	2.33

b stands for "not significantly different from 1 at the 5% level".

CHAPTER 3

HOW IMPORTANT ARE SECTORAL SHOCKS AS A DRIVING FORCE OF THE CYCLE?

AN APPLICATION OF FACTOR ANALYSIS TO BRITISH DATA

3.1 Introduction

One of the most important characteristics of business cycles appears to be the tendency of outputs in different sectors to move together. This stylized fact has been traditionally interpreted as being a clear indication that the driving force of economic fluctuations is aggregate shocks. The additional observation that outputs in the various sectors and the general price level move together has also been seen as evidence that cycles need to be explained in terms of aggregate impulses.

Recently, there has been a resurgence of interest in equilibrium business cycle theories. According to these theories, the recurrent fluctuations in output, consumption, investment and other real variables are the natural outcome that emerges from industrial market economies in which consumers and firms solve intertemporal optimisation in an uncertain environment. Moreover, fluctuations in real variables are attributed to exogenous technological and taste shocks that affect the economy at the aggregate level, combined with various sources of endogenous dynamics (e.g. adjustments costs, time-to-build capital goods, non-time separability of preferences).

In the last few years, however, David Lilien (1982) and other economists have put forward an alternative view, that is known as the sectoral shifts hypothesis. In

Lilien's model, sectoral shifts in labour demand can affect both the level and the time path of the aggregate unemployment rate. He constructed a time-series measure of the cross-sectoral dispersion in employment growth rates, and included this dispersion measure in a reduced-form unemployment-rate equation, finding a significant positive relationship between the two. Lilien argues that high values of the dispersion measure should be associated with more sectoral labour reallocation, and since it takes time for workers to find new jobs, some unemployment is unavoidable; slow adjustments of labour to shifts of employment demand between sectors of the economy can explain much of the variance of unemployment over the cycle. In Lilien's view, the natural or frictional rate of unemployment is not constant but varies with the degree of required labour reallocation in the economy. In periods of big shifts of product demand or very rapid technological change large movements of labour across sectors are required and if labour can not instantaneously and costlessly be reallocated unemployment increases. There is a long tradition, going back to the Beveridge Curve, that explains the unemployment rate U in terms of structural imbalances: when mismatching of job and workers is high, the Beveridge Curve, that is the locus of unemployment-vacancy combinations at various levels of demand, shifts outward.

The thrust of Lilien's argument is slightly different. He thinks that a large percentage of the increase in the U rate in the 70s and its cyclical pattern can be attributed to the slow movement of labour out of declining and into expanding sectors of the economy.

In his opinion, these sectoral demand shifts can be considered as changes in the natural rate of unemployment since they are due to the composition of aggregate labour demand rather than its level. He sets up a simple turnover model, in which h, net hiring at the level of the firm, consists of a firm-specific component e, that is distributed among firms with variance $\sigma(t)$, and an aggregate component H:

$$h = H + e \qquad (1)$$

Let L be aggregate layoffs and A aggregate accessions. The following aggregate relationships can be derived:

$$H = A - L \qquad (2)$$

$$L = g(H, \sigma(t)) \qquad 0 > g_1 > -1 \qquad g_2 > 0 \qquad (3)$$

$$A = H + g(H, \sigma(t)) \qquad (4)$$

where the signs of the partial derivatives indicate that an increase in the dispersion measure σ leads to both greater H and A. Finally, Lilien derives a dynamic U equation of the form:

$$U(t) = f(U(t-1), \sigma(t), X(t)) \qquad (5)$$

under the assumptions that the aggregate labour force is constant and that aggregate demand, X(t), affects the duration of unemployment.

The estimated layoff and U equations, in which the observed dispersion of industry employment growth rates is used as a proxy for σ and aggregate demand is measured by unanticipated monetary growth, show that σ can account for most of the U fluctuations in the 70s, when there were shifts in the composition of labour demand, but only for a small fraction of these fluctuations in the 60s.

In another paper by Lilien (1982b), the determinants of stock employment equilibrium are analysed adopting a different but consistent theoretical approach. The basic thrust of the argument is that shifts of product demand create a gap in marginal revenue product of labour among sectors and a temporary increase in U until equality is restored, because decreasing marginal productivity of labour implies that the fall in employment in the declining sector outbalances the rise in the expanding ones.

Lilien's work has been extended by Davis (1987a, 1987b) with similar conclusions: times of high U appear to be times of high dispersion in employment growth rates. Davis emphasizes the role of allocative disturbances and sector-specific human or physical capital; in particular, he finds that fluctuations in the pace of labour reallocation across jobs are the largest component of short-run unemployment rate fluctuations. According to his reallocation-timing hypothesis, the reallocation of specialized resources, like labour, involves costs in the form of foregone output and these costs fluctuate pro-cyclically; thus fluctuations in the average value of production cause fluctuations in the pace of labour reallocation.

Hamilton's paper (1986) is a further example of a model where adverse aggregate disturbances with uneven effects across sectors can cause more fluctuations in unemployment than disturbances of the same magnitude but having even effects; because of time costs of changing sectors, even if the disturbance has little effect on labour's average product, it can result in large fluctuations in the unemployment rate.

The sectoral shift hypothesis has been criticized by Abraham and Katz (1986) who show that, under some conditions that are empirically satisfied, the positive correlation between the dispersion measure and the unemployment rate can be generated by aggregate disturbances as in the traditional business cycle models. This will be the case if sectors' trend growth rate and cyclical sensitivity are negatively correlated, and the change in the unemployment rate and its level are positively correlated. They point out that, if some sectors are more cyclically sensitive than others, dispersion in growth rates might result from movements of aggregate demand. These movements will be associated with increased dispersion in employment growth rates because manufacturing employment is always affected more than service employment by shifts of demand. Their other major criticism is that we should also look at vacancies as a measure of search by firms. According to the sectoral shifts hypothesis, firms should search more intensively when U is high; the empirical evidence, however, is that vacancies are low in times of high U. This suggests that Lilien's results are a case of reverse causality: since different sectors have different income elasticities of demand and different growth rates of employment, movements in aggregate demand generate high employment growth dispersion.

Models attributing fluctuations mainly to intersectoral demand shifts should also explain why reallocation of workers across sectors takes so long. One possible reason is that workers having firm-specific skills are not willing to move to another sector, possibly incurring substantial mobility costs, until they are convinced that the decline of demand in their sector is a permanent rather than cyclical phenomenon.

In some 2-sector models (e.g. Hall, 1975), if there is a contraction in the uncompetitive high-wage sector, workers may prefer to remain unemployed with some probability of getting a job in the high-wage sector instead of moving to the low-wage sector.

Given the fact that employment growth dispersion is only a proxy for labour reallocation, more recent work (see Murphy and Topel, 1987) uses panel data on individuals to analyse this issue, and it appears to confirm that sectoral shifts are not the main determinant of unemployment fluctuations; as U increases, reallocation of workers across sectors seems to decrease.

On the whole, we can say that Lilien's findings do not constitute evidence against the "normal" business cycle hypothesis, i.e. the class of models holding that common disturbances (either from the supply side or the demand side) are the key to the understanding of business cycles.

Furthermore, in their seminal paper on real business cycles (1983), Long and Plosser show that serially uncorrelated and cross-sectionally independent productivity shocks can cause a significant amount of positive cross-sectional correlation (comovement). They investigate the stochastic properties of the system by conducting a simulation and computing the autocovariance matrices and the impulse response functions. Despite the fact that the underlying innovations to each sector are independent of one another, the time series of outputs in the different sectors exhibit a high degree of comovement.

Therefore comovements could result either from aggregate or sectoral shocks: the joint distribution of output growths in a many-shock model can be very similar to the distribution from an aggregate shock model. It is clearly the unexpected part of output growths that most directly reflects the period t shocks; we could then use output innovations to distinguish the effects of cross-sectionally independent shocks from those of aggregate shocks by looking at their correlation matrix: this matrix will have large off-diagonal elements if fluctuations in outputs are mainly due to a few aggregate shocks, and a one-factor or two-factor model will then have a very good fit. Therefore, as the same authors suggest in a later paper (1987), a possible strategy to try to distinguish between aggregate and sectoral disturbances is to do a simple factor analysis on the innovations, even though this statistical procedure has some evident shortcomings.

This methodology is applied here using monthly data from 19 industrial sectors in the United Kingdom for the sample period 1968:1 1987:4.

In the remainder of the paper, Section 3.2 shows that factor analysis is consistent with linear real business cycle models and reviews the fundamentals of this method of analysis; Section 3.3 presents the empirical results;

Section 3.4 summarizes the main findings and draws some conclusions.

3.2 Real business cycle models and factor analysis

The analysis carried out below does not constitute an economic model: a purely statistical model is applied. However a class of macroeconomic models seems to be consistent with the statistical procedure that is used here, and awareness of this fact can be helpful in trying to interpret the results of factor analysis. In particular, it can be easily shown how linear business cycle models of the kind analysed by King and Plosser (1985) can be disaggregated by industry.

The aim of these models is to show how shocks can be propagated over time if there is intertemporal substitution in production. In this class of models, based on the seminal paper by Long and Plosser (1983), real impulses can be propagated over time as a result of economic agents' desire to smooth consumption or production. The equilibrium levels of the variables depend on the previously accumulated capital stock and the current values of the state variables.

If we want to allow for sectoral influences, the decision rule for aggregate output becomes a linear function not only of the capital stock inherited from the previous period and of the aggregate factor X_n (as in the paper by

King and Plosser), but also of an industry-specific factor X_i. That is:

$$Y_t = \gamma k_{t-1} + \beta_N X_{N,t} + \beta_i X_{i,t} \tag{6}$$

where Y_t is a (ix1) vector of output in the various industries, k_{t-1} is a (ix1) vector of capital stocks in the different sectors, $X_{N,t}$ is a scalar (the aggregate factor), $X_{i,t}$ is a (ix1) vector of industry-specific factors, and γ, β_N, and β_i are coefficient matrices.

The law of motion of the capital stock is assumed to be:

$$k_t = BY_t + Ck_{t-1} \tag{7}$$

Substituting (6) into (7) we get:

$$k_t = Ck_{t-1} + B[\gamma k_{t-1} + \beta_N X_{N,t} + \beta_i X_{i,t}] \quad \text{or}$$

$$k_t = [I - (C + B\gamma)L]^{-1} [B\beta_N X_{N,t} + B\beta_i X_{i,t}] \tag{8}$$

Substitution of (8) into (6) gives:

$$\begin{aligned} Y_t &= \beta_N X_{N,t} + \beta_i X_{i,t} + \gamma[I - (C + B\gamma)L]^{-1}[B\beta_{N,t-1} X_{N,t-1} + \\ &\quad + B\beta_i X_{i,t-1}] = \\ &= \beta_N X_{N,t} + \beta_i X_{i,t} + \gamma[I - (C + B\gamma)L]^{-1} B[Y_{t-1} - \gamma k_{t-2}] \end{aligned} \tag{9}$$

and since $k_{t-2} = BY_{t-2} + Ck_{t-3} = (I - CL)^{-1} BY_{t-2}$,

substituting for k_{t-2} into (9) we finally obtain:

$$Y_t = \beta_N X_{N,t} + \beta_i X_{i,t} + \gamma[I-(C + B\gamma)L]^{-1} B[I-\gamma BL(I-CL)^{-1}]Y_{t-1} \tag{10}$$

or $$Y_t = \alpha(L)Y_{t-1} + u_t \tag{11}$$

where $\alpha(L)$ corresponds to the propagation mechanism, and u_t consists of the unobserved components $X_{N,t}$ and $X_{i,t}$, i. e. of aggregate and sectoral shocks.

The dynamics of the system will clearly depend on the interaction of the propagation mechanism and the dynamic behaviour of the factors captured by the disturbance term, and hence disaggregate, as well as aggregate, impulses will play a role. (11) can be estimated and the innovations used to carry out a factor analysis and measure the relative importance of aggregate versus sectoral disturbances. (11) is therefore the theoretical background for the statistical analysis carried out below.

Common factor analysis was invented by Spearman (1904). Its aim is to discover if there are unobservable, hypothetical variables, known as common factors, that contribute to the variance of at least two of the observed variables.

On the other hand, a unique factor is an unobservable, hypothetical variable that contributes to the variance of only one of the observed variables. One unique factor for each observed variable is assumed in the model for common factor analysis. (Notice that factor analysis must be distinguished from component analysis, since a component is an observable linear combination). In general, the observations are normalised so that their expected value equals 0 and their variance equals 1, obtaining the following formula:

$$z_{ig} = \frac{x_{ig}}{\sigma_{x_i}} \tag{12}$$

where $x_{ig} = X_{ig} - \overline{X}_i$ (i=1,2,....n)
(g=1,2,....N)

and σ_{xi} is the standard deviation of the variable. The matrix of simple correlation coefficients has then the following formula:

$$R = ZZ'/N \tag{13}$$

This normalization, that expresses the deviations of the original observations from their arithmetic mean in their standard deviations, is done to make mutual comparison possible. The equation for a common factor model is:

$$z_{ig} = a_{i1} f_{1g} + a_{i2} f_{2g} + \ldots + a_{im} f_{mg} + b_i s_i + c_i e_i \tag{14}$$

where:

-z_{ig} is the value of the ith observation on the gth variable;

-a_{ij} is the regression coefficient of the ith common factor for predicting the jth variable; (j=1,2,....m)

$-f_{jg}$ is the value of the jth observation on the gth common factor;

$-m$ is the number of common factors;

e_i are error factors;

s_i are unique factors.

This can be written in matrix form, considering only the common factors (since specific and error variances are generally not very important in factor analysis), as

$$Z = AF \tag{15}$$

where Z is the nxN matrix of normalised observations, A is the nxm matrix whose elements a_{ij} are known as factor loadings or connection coefficients, and F is the mxN matrix of factors.

It is assumed that the unique factors are uncorrelated with each other and with the common factors.

Substitution of (15) into (13) gives the relationship between R, the correlation matrix of the normalised observations, and A, the matrix of the connection coefficients:

$$R = ZZ'/N = AF(AF)'/N = AFF'A'/N = AA' \tag{16}$$

where the product FF' is 1 since it is assumed for convenience that the factors are uncorrelated with each other and have unit variance.

It can be shown that the vectors a_j are orthogonal because they are proportional to the characteristics vectors v_j of the matrix R, being of the following form:

$$a_{ij} = \frac{v_{ij} \sqrt{\lambda_j}}{\sqrt{\sum_{i=1}^{n} v_{ij}^2}} \tag{17}$$

where λ_j are the characteristic roots of R. Thus the "aspect" vectors a_j are nothing else than scaled characteristic vectors of the symmetric, positive definite matrix R (the term "aspect" is used to denote the column vector with elements a_{ij} or a_{ij}^2 (i=1, 2, ...n) containing the pattern of motion produced by the general causal factor f_j). Factor analysis selects m characteristic vectors f_j out of the n characteristic vectors of the matrix R which can describe the variables in terms of equation (15). Connection coefficients (or factor loadings) are then usually expressed in their squares, so that they can be read as percentages of the normalised total variance of each variable accounted for by the corresponding common factor. The sum of the squares of the factor loadings (also known as connection percentages), or communality, can be read as the percentage of the total variance that is due to all common factors included in the model.

The goodness of fit of the factor model can be assessed by examining the "residual correlation", i.e. the difference between correlations predicted by the model and actual correlations. After the common factors have been extracted, they can be rotated by an orthogonal transformation, that will leave them uncorrelated, whereas in the case of an oblique rotation they become correlated. Since all rotations have the same explanatory power, it is somewhat arbitrary which one is chosen, in the sense that the criterion will not be a statistical one; usually the rotation that provides a better interpretation of the patterns of motion of the variables concerned is selected. For this purpose, the matrix A is rotated about its aspect axis a number of times until the most logical interpretation of the pattern of motion is obtained. The aspect axes in the rotation finally chosen are called final aspects.

The method of analysis described above is subject to an important caveat: in factor analysis all comovements are attributed to the common factors, that are by definition unobservable and are identified with aggregate shocks in our case; since comovements can also be generated by unique factors (in our case, sectoral shocks) that are, however, correlated with each other, this statistical procedure can only determine what the upper limit of the explanatory power of aggregate shocks is.

3.3 Empirical results

The data set consists of monthly (this probably being the suitable time interval to identify sectoral shocks) output growth rates of 19 industrial sectors in the U.K.;[1] the sample period goes from January 1968 to April 1987.
The average pairwise correlation for the unadjusted data, that is a measure of the average correlation of each sector with the others (see annexed tables) shows the extent to which there are comovements across sectors.
It should be noticed that the data exhibit a high degree of seasonality: the R^2 in the regressions of the output growth rate of each industrial grouping on a constant and 11 seasonal dummy variables are generally quite high; hence, to remove this seasonality factor that could account for a large proportion of the comovements, the data have been seasonally adjusted using the moving average (MA) method. The procedure is the following. If y(t) is the series to be adjusted, T is the number of observations and p the periodicity, the MA of y(t) is defined as:

$$(1/2p)*(y(t-p/2)+y(t+p/2)+(1/(p\ \ )*(y(t-p/2+1)+y(t-p/2+2)+ \\ \ldots+y(t+p/2-1)). \qquad (18)$$

We then form the vector F(t), that is, the ratio of the series to its moving average:

[1] They are taken from the CSO (Central Statistical Office) databank.

$$F(t) = y(t)/(MA\ of\ y). \tag{19}$$

The matrix F will be composed of rows each corresponding to a year and having p elements (the total number of elements is obviously T=(T/p)*p). By averaging each column in F we obtain the seasonal factors:

$$C(k) = (1/years)*(F(1,k)+F(2,k)+...) \tag{20}$$

(They are normalized to sum to p).

The seasonally adjusted series is finally computed by dividing the old series through by the seasonal adjustment factors.

The seasonal adjustment causes a drop, in some cases substantial, of the average pairwise correlation, confirming the supposition that similar seasonal patterns are important in trying to explain the observations that there are comovements of output.

The seasonally adjusted data have then been used to estimate the following VAR (vector autoregression):

$$y_t = Ay_{t-1} + By_{t-2} + Cy_{t-12} + u_t \tag{21}$$

where y_t is the vector containing the output growth rates of the 19 sectors and u_t is a vector of disturbance terms.

The R^2 and the percentage standard deviation of the dependent variable for each of the equations are reported in the annexed tables.

Another measure of comovement, i.e. the root mean square of the off-diagonal elements of the correlation matrix of each sector's residuals is also reported.

Principal component and principal factor analysis (the difference being that for the former the prior communality estimates are set equal to 1, whereas for the latter the squared multiple correlations are used as the priors) have then been performed on the residuals from the estimated system, i.e. the innovations. There is a very large eigenvalue (7.27) that has positive loadings on all innovations, the correlation with "other manufacturing" being especially high (0.89); as the characteristic root of an aspect containing the pattern of motion produced by the general causal factor f_j is obtained by adding up the connection percentages (or squares of the connection coefficients as percentage variances of the common variance component) and dividing the total by 100, the presence of a large eigenvalue indicates the important role played by this common factor in explaining the pattern of motion of our set of variables considered as a group; besides, since the sign of the loadings shows in which direction the relevant variable moves with regard to the other variables of the same aspect, the positive loadings tell us that all variables move in the same direction. The final communality estimates range from

0.00024 (mineral oil processing) to 0.80 (other manufacturing).

Principal factor analysis shows that the partial correlation between the innovations controlling the other variables are smaller than the original ones, as we would expect if the common factor model is appropriate for the data; the overall Kaiser's measure of sampling adequacy, that is a summary, for each variable and for all variables together, of how much smaller the partial correlations are than the original correlations, is equal to 0.8, a value that can be considered very good (values below 0.5 are unacceptable), and the individual measures are also very good, only one being below 0.5 (for mineral oil processing the measure is 0.45), the others ranging from 0.59 to 0.94.

The factors loadings are very similar to those in the principal component analysis, because the squared multiple correlations are quite large (with the exception of mineral oil processing and other few sectors). Again, there is a very large eigenvalue (7.0), and the final communality estimates, although being lower than their priors, are in some cases very high. The root mean square of the off-diagonal residual correlations, i.e. the difference between correlations predicted by the principal factor model and actual correlations, are quite low and substantially lower than those between the innovations:

this indicates a very good fit of the model.

When a second factor is also retained in the principal component analysis (the second highest eigenvalue being 1.94), its factor loadings are much lower and negative in 9 cases out of 19, and the final communality estimates increase only slightly. Once again, the partial correlations drop in comparison with the original ones, Kaiser's overall measure of sampling adequacy is 0.8, and the factor pattern is similar to the one observed in the principal component analysis.

The root mean square of the off-diagonal residual correlations decreases slightly and the communality estimates generally increase by a small amount. On the whole, the second factor does not seem to increase significantly the explanatory power of the model.[2] [3]Finally, the weighted residuals (the weight being each sector's share of total manufacturing output) have been used to perform factor analysis, in an attempt to provide some kind of measure of aggregate innovations. If 5 sectors that are clearly not driven by aggregate fluctuations (other transport equipment, man-made fibres,

[2] The plot of the unrotated factor pattern puts the reference axes through the cluster of points representing the innovations, and so the factor pattern has not been rotated.

[3] Even when all 5 eigenvalues greater than 1 are retained, the goodness of fit of the model does not improve.

mineral oil processing, drink and tobacco, coal and coke) are left out, 55% of the variance of this measure is accounted for by the 2-factor model when the priors are the squared multiple correlations, with a Kaiser's measure of sampling adequacy equal to 0.80.

3.4 Conclusions

The evidence presented in this paper seems to suggest that in the U.K. economy there is an aggregate shock, common to all sectors, that is able to account for a high percentage of the fluctuations of output over the cycle.

The one-factor model performs quite well when applied to the British data, as indicated by the residual correlations, the final communality estimates and Kaiser's measure of sampling adequacy.

This certainly does not give much prima facie support to the sectoral shocks view of the cycle, or at least to those theories maintaining that sectoral shocks are more important than aggregate shocks in explaining the cycle.

However, it has to be emphasized once again that this statistical procedure is able only to determine an upper bound of the explanatory power of aggregate shocks, since all comovements are attributed to the common factor that is interpreted as an aggregate disturbance. Therefore, the model is biased towards overestimating the contribution of aggregate impulses, because in the true structure there could be disaggregate shocks that are mutually correlated.

Besides, the common factor is by definition an unobservable, hypothetical variable, that does not have to correspond to any observable aggregate shock. It is thus important to bear this in mind in interpreting the results of this kind of analysis, which has been carried out under the assumption that the common factor could be identified with the aggregate disturbances. In conclusion, even though the one-factor model appears to be quite appropriate for the data being analysed, these results should be taken with caution, since it is possible that the explanatory power of the aggregate factor has been overestimated.

Table 1.

energy and water supply.	-0.21	0.31	4.9	-0.28
metals.	0.37	0.48	8.4	0.27
other minerals, mineral products.	0.48	0.43	4.3	0.42
textiles.	0.50	0.45	4.0	0.39
other manufacturing.	0.50	0.49	3.5	0.41
coal, coke.	0.27	0.33	36.0	0.08
mineral oil processing.	0.28	0.40	4.9	0.32
chemicals.	0.06	0.45	4.2	-0.04
man-made fibres.	0.36	0.48	9.5	0.33
metal goods.	0.42	0.96	21.0	0.22
mechanical engineering.	0.39	0.48	4.6	0.31
electrical and instrumental eng.	0.06	0.52	4.1	-0.09
motor vehicles and parts.	0.37	0.44	10.0	0.30
other transport equipment.	0.15	0.36	5.0	-0.02
food.	0.21	0.52	2.7	-0.12
drink, tobacco.	0.05	0.49	5.4	-0.05
all other manufacturing.	0.39	0.50	4.6	0.33
clothing, footwear, leather.	0.45	0.40	3.9	0.36
paper, printing, publishing.	0.48	0.50	3.4	0.38

column 1 = average pairwise correlation of unadjusted data.

column 2 = R^2 from monthly growth rate VAR.

column 3 = % standard deviation of monthly growth rate of output.

column 4 = average pairwise correlation of adjusted data.

Table 2.

energy, water supply.	0.67	0.26	0.13	0.48
metals.	0.61	0.34	0.30	0.30
other minerals, mineral products.	0.73	0.44	0.57	0.59
textiles.	0.86	0.50	0.76	0.86
other manufacturing.	0.82	0.52	0.81	0.86
coal and coke.	0.14	0.23	0.10	0.39
mineral oil processing.	0.08	0.07	0.0001	0.0001
chemicals.	0.69	0.36	0.35	0.46
man-made fibres.	0.51	0.12	0.02	0.04
metal goods.	0.75	0.47	0.64	0.65
mechanical engineering.	0.82	0.40	0.44	0.48
electrical and industrial eng.	0.87	0.38	0.39	0.43
motor vehicles and parts.	0.54	0.30	0.25	0.25
other transport equipment.	0.49	0.18	0.07	0.07
food.	0.81	0.25	0.16	0.26
drink and tobacco.	0.52	0.20	0.081	0.20
all other manufacturing.	0.82	0.49	0.74	0.82
clothing, paper and footwear.	0.88	0.41	0.53	0.60
paper, printing and publishing.	0.80	0.46	0.58	0.67

column 1 = R^2 from regression of monthly growth rate on a constant and 11 dummies.

column 2 = for each industry, root mean square of correlations with other industries.

column 3 = final communality estimates in the 1-factor model (principal factor analysis).

column 4 = final communality estimates in the 2-factor model (principal factor analysis).

Table 3.

energy, water supply.	0.16	0.07
metals.	0.097	0.09
other minerals, mineral products.	0.070	0.05
textiles.	0.10	0.06
other manufacturing.	0.07	0.047
coal and coke.	0.15	0.081
mineral oil processing.	0.07	0.07
chemicals.	0.11	0.07
man-made fibres.	0.08	0.08
metal goods.	0.08	0.07
mechanical engineering.	0.09	0.07
electrical and instrumental eng.	0.08	0.06
motor vehicles and parts.	0.05	0.05
other transport equipment.	0.09	0.09
food.	0.10	0.06
drink and tobacco.	0.11	0.05
all other manufacturing.	0.08	0.04
clothing, footwear, leather.	0.09	0.07
paper, printing, publishing.	0.09	0.05

column 1 = root mean square of off-diagonal residual correlations in the 1-factor model (principal factor analysis).

column 2 = root mean square of off-diagonal residual correlations in the 2-factor model (principal factor analysis).

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CHAPTER 4

THE SEASONAL CYCLE IN THE U.K. ECONOMY

4.1 Introduction

Recent research on fluctuations of economic variables has focused mainly on business cycle fluctuations, overlooking the possibility that standard macroeconomic variables have strong and quantitavely important seasonal patterns.

An older tradition, that was represented by the NBER approach to the analysis of economic fluctuations, investigated also the seasonality of economic activity; examples of this tradition are studies carried out by Kuznets (1933), Bursk (1931) and Macaulay (1938). This line of research has recently been reopened in a paper by Barsky and Miron (1989), who try to demonstrate that there is a "seasonal business cycle" in the U.S., whose features are strikingly similar to those of the conventional business cycle: most of the stylized facts known about business cycles can be observed also when time series are analysed at the seasonal frequencies.

Below, the quantitative importance of the seasonal fluctuations of a series of macroeconomic variables in the U.K. economy is considered and it is shown to what extent these variables exhibit patterns, at seasonal frequencies, that broadly coincide with those of the business cycle. For each of the major stylized facts, the seasonal and the business cycle are compared to see if they actually have similar qualitative features.

The empirical results are presented in Section 4.2, that sets the scene by discussing briefly some approaches previously taken in the literature to study seasonality in economic models and some of the consequences of ignoring the seasonal components. In Section 4.3, the theory of band spectrum regressions is reviewed and this technique is applied to test for the stability of various economic relationships across seasonal and non-seasonal frequencies.

In Section 4.4, we develop a real business cycle (RBC) model that explicitly includes seasonal fluctuations in the analysis and treats seasonality as one of the features to be explained within an economic model; in particular, it is argued that the allocation of expenditure over the year reflects seasonally-varying parameters in the utility function, and the implications for the time series properties of the variables in the model are analysed; the results from simple regressions with seasonal dummies are shown to be consistent with the predictions of the model. (Similar conclusions are reached when we allow for the possibility of intertemporal substitution of leisure.)

Concluding remarks in Section 4.5 complete the paper.

4.2 Some stylized facts

In many empirical studies, economic time series are analysed only after the "seasonal noise" has been eliminated by resorting to various statistical procedures for obtaining a seasonally adjusted series. However the use of seasonally adjusted data can create some potential problems. Plosser (1979b) shows that forecasts based on the use of adjusted data are less accurate in comparison with those using unadjusted data because the adjustment process can introduce some degree of instability in the stochastic properties of the adjusted data that was not present in the raw data, as the weights employed by the adjustment filter may vary over time. Secondly, if the adjustment procedure is not effective in eliminating seasonality, this can lead to model mis-specification and misleading inferences about the parameters. Finally, spurious dynamic relationships can arise if adjusted and unadjusted data are used in the same model, as often happens since some time series are available in adjusted form, others are not adjusted (e.g. interest rates; see Wallis, 1974).

Another common practice is to remove the seasonal effects by including seasonal intercept dummies in the equations to be estimated, instead of explicitly investigating the economics underlying seasonal variation; this amounts simply to assuming that the functions shift up or down with the season. In many other studies pure time series models, which are constructed without drawing on any theories concerning possible behavioural relationships between variables, are used.

A very simple way of capturing a seasonal effect is by a fourth-order seasonal autoregressive process of the form:

$$Y_t = \phi_4 Y_{t-4} + \epsilon_t \qquad |\phi_4| < 1 \tag{1}$$

and ϵ_t is a white noise. (This is a special case of an AR(4) process, but with the constraint that $\phi_1=\phi_2=\phi_3=0$.) However, unless seasonal movements are felt to be the only predictable feature of the series, such a model will not be appropriate. There are basically three ways of formulating stationary seasonal models making allowance for both seasonal and non-seasonal components. The first is to incorporate a first-order lag to yield:

$$Y_t = \phi_1 Y_{t-1} + \phi_4 Y_{t-4} + \epsilon_t \tag{2}$$

The second is to model the disturbance term as a non-seasonal ARMA(p, q) process, u_t; as a simple example, suppose that u_t is an AR(1) process; the model becomes:

$$(1 - \phi_1 L)(1 - \phi_4 L^4) y_t = \epsilon_t \tag{3}$$

or

$$Y_t = \phi_1 Y_{t-1} + \phi_4 Y_{t-4} + \phi_5 Y_{t-5} + \epsilon_t$$

where $\phi_5 = - \phi_1 \phi_4$

The final possibility is to construct an additive model:

$$Y_t = s_t + u_t \tag{5}$$

where u_t is an ARMA(p,q) process and s_t is a seasonal ARMA process of order $(P,Q)_s$; for instance, if

$$u_t = \frac{\epsilon_t}{1 - \phi_1 L} \tag{6}$$

and

$$s_t = \frac{\gamma_t}{1 - \phi_4 L^4} \tag{7}$$

the model becomes:

$$(1-\phi_1 L)(1-\phi_4 L^4)y_t = \gamma_t + \epsilon_t - \phi_1\gamma_{t-1} - \phi_4\ \epsilon_{t-4} \tag{8}$$

The statistical approach that is used here to take into account both deterministic and stochastic seasonal components is explained below.

Our aim is to estimate seasonal variations in the non-trend component of a time series Y_t. Nelson and Plosser (1982) presented evidence suggesting that the cyclical component of output, etc., should not be modelled as the deviation from a deterministic trend, and that a stochastic trend formulation is preferable; they carry out Dickey-Fuller tests to distinguish between DS (difference stationary) and TS (trend stationary) representation of the series and conclude that the hypothesis that there is a unit root in the AR polynomial can not be rejected and thus that an ARIMA representation with a unit root is appropriate for ln Y_t. Several other studies by Clark

(1986), Evans (1986), Stock and Watson (1986), Watson (1986), Campbell and Mankiw (1986) also find that this specification is the one suggested by the empirical data. Hence the detrended series that will be used here are the log growth rates of the variables.

Let y_t denote the detrended series. A model incorporating both stochastic and deterministic seasonality is the following:

$$y_t = \sum_{s=1}^{4} \beta_s d_{t,s} + C(L)u_t \tag{9}$$

where $d_{t,s}$ is a seasonal dummy for quarter (the data are quarterly), β_s is the corresponding coefficient, and C(L) a polynomial in the lag operator that satisfies the usual condition

$$\sum_{i=0}^{\infty} c_i^2 < \infty$$

Clearly, deterministic seasonality is picked up by the seasonal dummy coefficients, whereas the polynomial C(L) accounts for indeterministic (or stochastic) seasonality.

Below only deterministic seasonality is analysed, to see if the series exhibit regular peaks and troughs, and the estimated equation is

$$y_t = \sum_{s=1}^{4} \beta_s d_{t,s} + e_t$$

where e_t is the stochastic component of the series. The estimation method is ordinary least squares (OLS), since OLS estimates of the seasonal dummy coefficients are consistent and asymptotically efficient. (The technique described by Newey and West (1987) to obtain consistent estimates of the standard errors is used to allow for the possibility that there is autocorrelation in the error term.) The sample period is 1955:2 to 1985:4.[1] For each series the following statistics are presented (see table 1):

1) the standard deviation of the fitted values of the regression, which is a measure of the standard deviation of the deterministic seasonality of the dependent variable;

2) the standard error of the regression, which is an estimate of the standard deviation of the business cycle and stochastic seasonality component of the dependent variable;

3) the R^2 of the regression, measuring the percentage of the variation of the dependent variable that can be attributed to deterministic seasonality.

Table 2 contains, for each series, the difference between the mean of the dependent variable and each dummy

[1] see appendix 2 for a description of the data.

coefficient: this can be interpreted as the average percentage deviation of each variable from its trend in that quarter, since it is the difference between the average growth rate of the variable in that quarter and the overall growth rate. The first three statistics, reported in table 1, are a measure of the quantitative importance of seasonal fluctuations; the entries in table 2 provide information about the seasonal patterns of the variables.

The estimated standard deviation of the deterministic seasonal component in the log growth rate of real GDP is 4.25%; business cycle and stochastic seasonal components account for 2.18% of the fluctuations of the log growth rate of GDP; deterministic seasonality accounts for 74% of the deviation from trend of this variable. All the series have deterministic seasonal components, and they are quantitavely important especially for consumption (11.9), nominal interest rates (15.6), unemployment (13.9), retail sales (13.1) and fixed investment (10.2). The percentage of the variation due to deterministic seasonality is particularly high for consumption of services (excluding rents and rates), consumption, retail sales and GDP. R^2 for the consumption of durables is almost 0. Business cycle and/or stochastic seasonal components matter particularly for consumption of durables, unemployment and interest rates. Variations in government expenditure on goods and services, as well as taxes, appear to be due to a large extent to deterministic seasonality. Only a small percentage of fluctuations in M1 and M3 is explained by seasonal dummies; this percentage is even smaller for retail prices. As for the labour market, employment, working population and unemployment exhibit deterministic seasonal components, whereas labour costs , wages and normal weekly hours do not (R^2 is almost 0).

Considering now the seasonal patterns of the variables, it can be noticed that output is well below trend in the first quarter and above trend in the others, especially in the last quarter. Consumption exhibits a similar seasonal pattern, but its dummy coefficients are larger; consumption of durables, instead, appears to be above trend in the first quarter and below trend in the others, and for consumption the deviation from trend is negative both in the first and the last quarter. Government expenditure has a peak in the third quarter and is above trend also in the first. M3 and nominal interest rates also reach their peaks in the last quarter; the same is true for fixed investment and retail sales. Most of these series show a decline from the fourth quarter to the first quarter. As for the labour market, wages and labour costs have a peak in the first quarter, like unemployment and total employees in employment; employment in IOP (Index of Production) industries, instead, has a trough in the first quarter. Total hours (average hours times employment) do not increase substantially from the third to the fourth quarter, in spite of the fact that output peaks in that quarter.

Tables 3 and 4 show the corresponding entries for each variable when the regression are run for two sub-samples of the 1955-85 period. It turns out that ,on the whole, there are no significant differences in the quantitative importance of the seasonal fluctuations and in the seasonal patterns of the variables between the two sub-periods (there are, of course, a few exceptions, like wages and labour costs: the entries of table 3b for the sub- period 73:01 85:04 are remarkably lower than the corresponding ones for the other sub-period or the entire sample period).

Let us now examine in turn each of the stylized facts of the conventional business cycle and see if the same characteristics are displayed by the "seasonal cycle".

1) <u>The behaviour of quantity variables</u>

We have seen above that, even though there are some exceptions, the series considered have very similar seasonal patterns, most of them declining in the last quarter and increasing sharply in the first quarter of the year. Output comovements, or the fact that the series exhibit high conformity, in Mitchell's terminology, or high coherence, in modern time series language, is one of the regularities according to Lucas (1977), of the business cycle. It appears that positive correlation between various quantity variables is a feature of the seasonal cycle, as well as of the business cycle.

2) <u>Production smoothing</u>

It is usually argued that firms hold inventories to smooth production in the presence of random demand shocks. However it is often found in empirical studies that the variance of output is bigger than the variance of sales. This can be accounted for if there are decreasing marginal costs of production or costs shocks, rather than demand shocks, are the major source of inventory fluctuations, or if demand shocks exhibit positive serial correlation; this would cause production counter-smoothing: a positive demand shock would make firms revise upwards their expected demand and future output would be more variable than future sales.

In the case of the U.K., the seasonal patterns of output and retail sales are very similar (they differ only in the third quarter); inventories peak in the third quarter, when output is only slightly above trend, and there is still a positive change in the last quarter when the largest expansion in output and sales occurs.

This seems to suggest that, in spite of the anticipated nature of seasonal fluctuations, production smoothing does not occur, and firms are happy to meet the increase in demand in the last quarter by increasing their output; this is puzzling within the standard neo-classical model with convex costs of production, but is consistent with flat marginal cost; as suggested by Hall (1986), the fact that marginal cost does not increase rapidly with output can be due to excess capacity; he argues that the majority of U.S. industries are non-competitive: they achieve equilibrium with their firms operating along flat parts of their marginal cost schedules, and this is more likely in Chamberlinian competition than in perfect competition.

3) <u>Comovements of nominal and real variables</u>

As pointed out by Lucas (1977), the fact that monetary aggregates and velocity measures are procyclical is another of the regularities of business cycles.

To date, there are four classes of models dealing with the interaction between real and nominal variables:

i) real business cycle models;

ii) imperfect information equilibrium business cycle models;

iii) models with preset nominal prices plus rational expectations (RE);

iv) financial/credit theories of the cycle.

In real business cycle models, money is passive and correlated with output only because a positive shock leads to an increase in the demand for transactions services;

that is confirmed by the fact that there appears to be a much larger correlation between output and inside money than between output and outside money.

In the second class of models, that rely on monetary misperceptions, a monetary expansion leads to an increase in the perceived relative local price and thus to a change in agents' decisions.

In the third approach, even in the presence of RE, long-term labour contracts lead to sticky nominal wages and hence to a potential role for monetary policy.

In the fourth class of models, shocks to credit markets, whether due to monetary policy or to other sources, have effects on real output; since money and credit are correlated, there appears to be a relation between money and economic activity.

The entries of table 2 show that M3 and output are highly correlated: they both peak in the fourth quarter, have a trough in the first quarter, and are above trend in the other two quarters. A plot of real GDP against M1 and M3 (see Figure 2 and 3; all variables are in growth rates) clearly shows this high correlation at the seasonal frequencies. It seems plausible that M3 increases in the last quarter to accommodate the increase in spending and output: hence, money seems to be endogenous at the seasonal frequencies. Clearly, if we think that the correlation between money and output has the same cause, at the seasonal and business cycle frequencies, this finding would be inconsistent with Lucas-type models: it could not reflect misperceptions since the seasonal movements of these variables can largely be anticipated. The relationship between the seasonal components of output and nominal money can be analysed by regressing the former against the latter (both in log growth rates) and using the seasonal dummies as the only instrument (This is equivalent to regressing the seasonal pattern of one variable against the seasonal pattern of the other, but it has the advantage that the estimated coefficient and its standard error take into account the sample size).[2] The estimated regression coefficient is 0.48 (with a standard error of 0.11) when M1 is the independent variable, and 0.58 (with a standard error of 0.11) when M3 is the independent variable.

4) Prices and output

The entries of table 1 show that the standard deviation of the deterministic seasonal component, the standard deviation of the business cycle plus stochastic seasonal component and the percentage of variation due to deterministic seasonality are all lower for prices than

[2] An alternative procedure is to analyse the coherence function, that gives the correlation coefficient between the two series by frequency, and the gain function, giving the regression coefficient by frequency.

for output: the amplitude of the fluctuations is smaller for prices at the seasonal, as well as the business cycle, frequencies.

In a recent paper, Bils (1987) has examined the cyclical behaviour of price/marginal cost margins, and found that short-run marginal cost is pro-cyclical and output price does not respond to movements of marginal cost; consequently, price/marginal cost margins are anti-cyclical. This is consistent with market- clearing models in which the elasticity of goods demand behaves procyclically.

5) Cyclical behaviour of labour productivity

One of the best established facts about business cycles is the procyclical behaviour of labour productivity. This is consistent with real business cycle theories in which the driving force is supply shocks; on the other hand, models in which fluctuations of economic variables are due to aggregate demand shocks should lead to a countercyclical behaviour of labour productivity. The Keynesian explanation for this observed phenomenon relies on "labour hoarding": firms are off the labour demand curve, because of costs of adjustments or contractual commitments.[3] There

[3] See also J.J. Rotemberg-L.H. Summers: "Labour hoarding, inflexible prices and procyclical productivity", NBER Working Paper no. 2591, May 1988.
They suggest that important aspects of productivity behaviour, in particular its procyclicality, can be

is indirect evidence of labour hoarding: the elasticity of output with respect to labour input is often found to be greater than 1 in empirical studies, but this could be due to statistical bias (Sims, 1974) or variation in the utilization of capital services (Lucas, 1970). A sample survey by Fay and Medoff (1985), however, shows that 4% of the hours paid should be classified as hoarded.

A plot of real GDP growth and total hours growth against time (see figure 4 and 5) shows clearly that labour productivity is procyclical, since all peaks in total hours correspond to magnified peaks in GDP. Furthermore, a regression of the log growth rate of output against the log growth rate of total hours using the seasonal dummies as instruments gives a coefficient of 2.78 (with a standard error of 0.53).

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captured by a model combining a plausible degree of price rigidity with costs of adjusting capacity and labour hoarding. This procyclical behaviour would also arise in the case of increasing returns and market power analysed by Hall.

Table 1: Log Growth Rates (sample period: 55:02 85:04)

Variables	S.D. of Dummies	S.E. of Regr.	R^2
MGDP	4.25	2.18	0.74
CON	5.82	2.14	0.86
CONSE	8.53	2.22	0.93
CONDU	11.9	11.6	0.078
FIX	10.2	5.72	0.69
K	0.21	0.21	0.026
TOE	6.34	3.95	0.62
SUBS	1.3	1.11	0.28
GOVCU	6.79	4.31	0.60
NAFS	2.75	2.78	0.0013
VX	4.99	4.50	0.20
MVACU	4.97	4.81	0.087
IMPV	4.24	3.96	0.14
INV	3.1	5.72	0.14
SAL	13.1	2.02	0.97
RUE	13.9	11.4	0.34
EE	0.63	0.5	0.38
EEP	0.85	0.73	0.27
NWH	0.21	0.21	0.029
TH1	0.84	0.71	0.29
TH2	0.63	0.51	0.36
WP	0.5	0.38	0.44
PRI	1.58	1.51	0.11
PMF	3.48	3.33	0.10
TBR	15.6	15.6	0.027
LCPU	25.7	25.7	0.025
WSPU	25.6	25.6	0.026
AWE	1.96	1.73	0.24
M1	3.31	3.07	0.16
M3	2.98	2.82	0.13

Table 2: Log Growth Rates (sample period: 55:02 85:04)

Variables	Q 1	Q 2	Q 3	Q 4
MGDP	-6.21	1.49	0.99	3.39
CON	-9.0	4.1	1.0	3.9
CONSE	-2.7	7.5	6.6	-12.1
CONDU	5.8	-1.8	-1.7	-2.2
FIX	-12.1	-1.2	4.9	8.9
K	0.05	-0.05	-0.01	0.01
TOE	-8.0	3.66	-0.25	4.7
SUBS	5.64	-9.9	-2.73	7.1
GOVCU	4.5	-8.0	4.8	-1.3
NAFS	0.147	0.006	0.06	-0.12
VX	-2.69	2.72	-1.66	1.62
MVACU	1.3	1.3	-0.4	-2.17
IMPV	0.76	1.56	0.26	-2.64
INV	-0.292	1.718	2.202	1.2
SAL	-19.0	3.80	-0.1	16.0
RUE	9.5	-12.4	-1.0	4.2
EE	0.54	0.48	0.19	-0.16
EEP	-0.67	3.1	0.49	0.1
NWH	0.001	-0.037	0.06	-0.12
TH1	-0.67	-0.08	0.54	0.08
TH2	-0.54	0.43	0.25	0.18
WP	-0.37	0.1	0.51	-0.2
PRI	-0.03	0.813	-0.7	-0.13
PMF	1.4	-0.87	-1.2	0.9
TBR	-2.04	-3.0	2.5	2.6
LCPU	7.2	-2.6	-2.2	-2.2
WSPU	7.4	-2.6	-2.07	-2.3
AWE	-1.65	0.7	0.1	0.6
M1	-1.3	-1.0	2.2	0.04
M3	-1.8	0.2	0.4	1.2

Table 3A : Log Growth Rates (sample period: 55:-2 72:04)

Variables	S.D. of Dummies	S.E. of Regr.	R^2
MGDP	4.44	1.7	0.85
CON	6.34	1.65	0.93
CONDU	11.0	9.94	0.22
CONSE	9.02	1.40	0.97
FIX	9.80	5.41	0.70
K	0.14	0.14	0.036
TOE	5.78	2.58	0.80
SUBS	13.0	10.6	0.36
GOVCU	6.15	4.26	0.54
NAFS	2.86	2.92	0.0018
VX	5.29	4.85	0.19
MVACU	4.36	4.16	0.12
IMPV	4.39	4.04	0.19
INV	2.8	4.38	0.20
SAL	12.4	1.41	0.98
RUE	16.8	12.1	0.50
EE	0.54	0.45	0.33
EEP	0.71	0.55	0.42
NWH	0.25	0.26	0.025
TH1	0.70	0.53	0.45
TH2	0.56	0.47	0.33
WP	0.43	0.38	0.25
PRI	0.91	0.81	0.23
PMF	1.78	1.73	0.095
TBR	15.6	15.7	0.028
LCPU	33.8	33.8	0.044
WSPU	33.8	33.7	0.045
AWE	1.65	1.22	0.47
M1	3.45	3.08	0.26
M3	3.09	3.0	0.13

Table 3B: Log Growth Rates (sample period 73:01 85:04)

Variables	S.D. of Dummies	S.E. of Regr.	R^2
MGDP	4.0	1.91	0.78
CON	5.09	1.74	0.88
CONDU	13.1	9.24	0.52
CONSE	7.88	2.35	0.91
FIX	10.7	5.05	0.79
K	0.23	0.22	0.098
TOE	7.0	4.73	0.56
SUBS	12.9	11.3	0.26
GOVCU	7.57	3.73	0.77
NAFS	2.61	2.69	0.0008
VX	4.6	3.57	0.43
MVACU	5.59	5.37	0.13
IMPV	4.06	3.76	0.19
INV	3.19	6.92	0.12
SAL	14.0	1.94	0.98
RUE	8.56	7.42	0.29
EE	0.73	0.53	0.49
EEP	0.93	0.82	0.26
NWH	0.11	0.11	0.13
TH1	0.95	0.85	0.25
TH2	0.73	0.55	0.45
WP	0.59	0.30	0.74
PRI	1.8	1.71	0.14
PMF	4.56	4.23	0.18
TBR	15.8	16.0	0.032
LCPU	2.42	2.48	0.011
WSPU	2.38	2.42	0.021
AWE	2.11	1.91	0.23
M1	3.0	2.72	0.23
M3	2.89	2.67	0.21

Table 4A: Log Growth Rates (sample period: 55:02 72:04)

Variables	Q 1	Q 2	Q 3	Q 4
MGDP	-6.77	3.13	-2.0	3.43
CON	-10.21	5.62	-0.0026	4.04
CONDU	1.05	4.8	-8.71	2.94
CONSE	-3.93	8.65	7.67	-12.55
FIX	-13.91	2.1	2.97	8.1
K	-0.003	-0.035	-0.005	-0.042
TOE	-7.66	2.51	-1.55	6.3
SUBS	3.9	-9.5	-4.7	10.6
GOVCU	2.3	-6.8	5.2	-0.5
NAFS	2.0	0.007	-0.068	-0.118
VX	-1.32	1.9	2.97	2.65
MVACU	1.8	0.17	0.68	-2.39
IMPV	1.5	0.8	1.1	-3.1
INV	-0.012	1.55	1.98	3.49
SAL	-19.24	4.26	1.117	14.96
RUE	12.83	-17.07	-4.4	9.33
EE	-0.368	0.392	0.193	-0.239
EEP	-0.79	-0.085	0.369	0.289
NWH	-0.005	-0.03	0.07	-0.04
TH1	-0.81	0.054	0.43	0.246
TH2	-0.379	0.374	2.05	-0.278
WP	-0.21	-0.006	0.34	-0.139
PRI	0.015	0.546	-0.668	1.0
PMF	0.37	-0.35	-0.68	0.62
TBR	-1.43	-0.29	3.5	2.0
LCPU	12.7	-4.11	-3.9	-3.8
WSPU	12.8	-4.24	-3.9	-3.9
AWE	-1.705	0.63	-0.29	1.26
M1	-1.217	-0.46	2.93	-1.36
M3	-1.914	0.17	1.13	0.42

Table 4B: Log Growth Rates (sample period: 73:01 85:04)

Variables	Q 1	Q 2	Q 3	Q 4
MGDP	-5.48	-0.067	2.85	3.31
CON	-7.98	2.22	2.34	3.84
CONDU	11.14	-9.87	7.34	-8.76
CONSE	-1.6	7.61	5.23	-11.75
FIX	-12.1	-5.38	7.29	11.41
K	0.122	-0.066	-0.018	-0.039
TOE	-8.73	5.11	1.41	2.23
SUBS	7.78	-10.3	0.06	2.48
GOVCU	6.7	-9.58	4.5	-2.23
NAFS	0.104	0.025	-0.025	-0.105
VX	-4.516	3.9	0.27	0.334
MVACU	0.72	2.97	-1.82	-1.85
IMPV	0.08	2.77	-0.9	-1.95
INV	-0.58	1.97	2.44	3.9
SAL	-20.6	3.0	1.19	17.7
RUE	5.1	-5.96	3.77	-2.88
EE	-0.769	0.62	0.22	-0.074
EEP	-0.65	0.176	0.643	-0.18
NWH	0.0047	-0.067	0.043	0.0235
TH1	-0.64	0.11	0.686	-0.15
TH2	-0.764	0.547	0.264	0.314
WP	-0.579	0.18	0.73	-0.335
PRI	-0.088	1.1	-0.68	-0.41
PMF	2.7	-1.69	-2.1	1.15
TBR	-2.51	-2.79	1.37	3.94
LCPU	0.209	-0.42	0.23	-0.01
WSPU	0.35	-0.51	0.29	-0.13
AWE	-1.5	1.08	0.79	-0.18
M1	-1.42	-1.4	1.48	1.56
M3	-1.73	2.0	-0.27	2.0

4.3 Band spectrum regression

It is possible to set up the classical linear regression model in the frequency domain by applying a finite Fourier transform to the dependent and independent variables, thus obtaining a series of observations that are indexed by frequency rather than by time. The transformed observations can then be used to carry out "spectral regressions".

There are several possible applications of these models to economic problems. Firstly, if the disturbances are serially correlated in the time domain, they will be approximately uncorrelated in the frequency domain. However, the effect of the transformation into the frequency domain is to produce a vector of disturbances that are heteroscedastic. Nevertheless, as Engle and Gardner (1976) and Nicholls and Pagan (1977) show, there are theoretical and pratical advantages to working in the frequency domain in this case. The other major reasons for using the transformed data is to carry out "band spectrum regressions", i.e. omitting some frequencies. This can be useful in dealing with errors-in-variables, since they tend to be a serious problem especially at high frequencies, and so their effects on the OLS estimator can be reduced by dropping these frequencies. Furthermore, if the observations exhibit a strong seasonal pattern, it may be useful to specify a model that does not include the seasonal frequencies, and a test of the exclusion can be performed. Consider the linear regression model:

$$y = XB + u \tag{11}$$

where y is an nx1 vector of observations on the dependent variable, X is an nxk matrix of observations on the independent variables, and u is an nx1 vector of disturbance terms with 0 mean and constant variance σ^2. Let us pre-multiply each term in (11) by a matrix Z, with dimensions nxn, whose elements are defined as follows:

$$z_{ts} = n^{-\frac{1}{2}} \qquad t = 1 \tag{12}$$

$$z_{ts} = (2/n)^{\frac{1}{2}} \cos[\pi * t(s-1)/n] \quad t=2,4,6,....n-2 \text{ or } n-1$$

$$z_{ts} = (2/n)^{\frac{1}{2}} \sin[\pi * (t-1)(s-1)/n] \quad t=3,5,7,...n-1 \text{ or } n$$

$$z_{ts} = n^{\frac{1}{2}}(-1)^{s+1} \quad t=n \text{ if } n \text{ even} \quad s=1,.....n$$

to obtain

$$\tilde{y} = \tilde{X}\, B + \tilde{u} \tag{13}$$

where $\tilde{y} = Zy$, $\tilde{X} = XZ$, $\tilde{u} = Zu$. This transformation leads to a frequency domain interpretation of the linear regression model. The disturbance term in (13) will still be white noise if it was white noise in (11) since Z is an orthogonal matrix and therefore

$$E(\tilde{u}\,\tilde{u}') = E(Zuu'Z') = \sigma^2 ZZ' = \sigma^2 I \tag{14}$$

and the OLS estimator of B is also unchanged since

$$b = (\tilde{X}'\,\tilde{X})^{-1}\,\tilde{X}'\tilde{y} = (X'Z'ZX)^{-1}\,X'Z'Zy = (X'X)^{-1}\,X'y \tag{15}$$

In this context, we may consider leaving some frequencies out of the regression in model (13) to test whether the parameter estimates are stable across frequencies. A finite sample test of the exclusion, i.e. of the hypothesis that different frequencies satisfy the same model, is derived in Engle (1974). The test is a Chow-Fisher test which is computed by calculating two regressions[4], one restricting the coefficients to be the

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[4] Since the transformed observations are real, standard regression packages can be used to carry out the calculations; the finite Fourier transform in real terms described above is the one suggested by Harvey (1978); Engle (1974) takes a finite Fourier transform in complex terms, by multiplying each term in (11) by a matrix W of Fourier elements defined as $w_{jk}=(1/\surd(T))*e^{ijk/T}$ where $i=\surd(-1)$ to get

$$\tilde{y} = \tilde{X}B + \tilde{u}$$

where $\tilde{y} = Wy$, $\tilde{X} = WX$, $\tilde{u} = Wu$. and the variables are complex. In order to carry out band spectrum regressions, Engle then suggests first to pre-multiply $\tilde{y}$ and $\tilde{X}$ by a TxT matrix A which has zeros every where except in positions on the leading diagonal corresponding to the included frequencies (this recovers certain of the observations), and then to transform everything back into the time domain by means of an inverse Fourier transform to yield the real sets of observations:

$$y^* = W'A\tilde{y} = W'AWy$$

$$X^* = W'A\tilde{X} = W'AWX$$

Standard regression packages can be used to regress y* on X*, even though some adjustment has to be made because the number of degrees of freedom is equal to T'-k (where T' is the number of included frequencies) whereas the regression package will assume T-k degrees of freedom. The real finite Fourier transform appears to be computationally same across frequencies, the other allowing them to vary. The statistic

$$[(v'v-u'u)/c-d)]/[(u'u)/d] \tag{16}$$

(where v'v is the RSS in the restricted regression having c degrees of freedom and u'u is the RSS in the unrestricted regression with d degrees of freedom) is distributed as F with c-d and d degrees of freedom.

The following reduced-form equations have been estimated over the sample period 1955:1 1985:4 (for equ. (22) and (23) the sample period is 1963:2 1983:2):

mgdp = a+b*th1   where th1=awh*eep (17)

mgdp = a+b*th2   where th2=awh*ee (18)

mgdp = a+b*sal+c*inv (19)

mgdp = a+b*pri (20)

con = a+b*mgdp (21)

mgdp = a+b*m1 (22)

mgdp = a+b*m3 (23)

(see appendix 2 for the definition of the variables.)

To test whether the relationship between the variables are

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much more attractive.

stable across frequencies, the same equations were estimated for three different frequency bands: seasonal, low and high. For a time series with p observations for period of interest, the seasonal frequencies are defined to be

$$2m\pi/p, \; m = 1,2,\ldots L[p]$$

where L[p] is the largest integer less than or equal to p/2. For example, with a monthly time series, the seasonal frequencies are $\pi/6$, $\pi/3$, $\pi/2$, $2\pi/3$, $5\pi/6$ and π. With quarterly data the seasonal harmonics at which seasonal variance can be expected are $\pi/2$ and π. The exclusion of these frequencies is tantamount to regressing the data for each variable on four seasonal dummies.

A more flexible procedure is followed here: a band is excluded around each of the frequencies as follows:

Table 5

Frequency bands as fraction of cycle

Variables:	mgdp, th1, th2, sal, inv, pri, con (124 observations).
total:	0.0-0.5
low:	0.0-0.241
seasonal:	0.241-0.265, 0.491-0.5
high:	0.265-0.491
Variables:	m1, m3 (82 observations).
total:	0.0-0.5
low:	0.0-0.243
seasonal:	0.243-0.268, 0.487-0.5
high:	0.268-0.487

Table 5 can be obtained in the following way. Frequencies are given by $\lambda_j = 2\pi_j/n$, where j is the number of observations per unit time and n the total number of observations; we consider 0.5 instead of π as unit time and exclude a band around each of the seasonal frequencies; we then define low frequency the components for which the period is more than one year (i. e. the cycle is completed in more than one year), and high frequency those completing more cycles in one year (a sinusoidal with period $(2\pi)/\lambda_j$ executes j complete cycles in the span of the data).

A Chow-Fisher test that compares the RSS in the regression excluding the seasonal frequencies with the RSS of the low and high frequencies regression has been performed and the results are given in the table below:

Table 6

CHOW-FISHER tests

equ. (17)	$F(2,109) = 0.5$
equ. (18)	$F(2,109) = 0.88$
equ. (19)	$F(3,107) = 0.068$
equ. (20)	$F(2,109) = 1.39$
equ. (21)	$F(2,109) = 0.44$
equ. (22)	$F(2,72) = 0.33$
equ. (23)	$F(2,72) = 0.316$

As can be seen, the null hypothesis that the estimated relationships are stable across frequencies (i.e. that the coefficients are the same) can not be rejected. We can also compare the estimates of the same equations when only the seasonals are used: i.e. we can use the seasonal components only to estimate a separate behavioural model. A wider band around each of the seasonal frequencies is considered for this purpose, as the following table shows:

Table 7

Frequency bands as fraction of cycle

Variables:	mgdp, th1, th2, sal, inv, pri, con (124 observations).
total:	0.0-0.5
low:	0.0-0.16
seasonal:	0.16-0.345, 0.426-0.5
high:	0.345-0.426
Variables:	m1, m3 (82 observations).
total:	0.0-0.5
low:	0.0-0.181
seasonal:	0.181-0.304, 0.426-0.5
high:	0.304-0.426

The estimated coefficients are quite similar for total hours, and bigger for m1, m3 and pri at the seasonal frequencies; consumption responds more to income and income more to inventories at the non-seasonal frequencies.

Table 8

Estimated coefficients (s.e).

dep. variable		low	high	seas.	non-seas.
equ.(17)	th1	-0.14(0.0076)	-0.17(0.8)	-0.17(0.02)	-0.14(0.0077)
equ.(18)	th2	-0.20(0.019)	-0.31(0.022)	-0.20(0.044)	-0.20(0.019)
equ.(19)	sal	1.95(0.16)	2.0(0.11)	1.97(0.12)	1.95(0.16)
	inv	0.85(1.92)	-6.44(2.75)	-0.21(1.4)	0.85(2.01)
equ.(20)	pri	1.03(0.08)	1.41(0.06)	1.46(0.14)	1.03(0.09)
equ.(21)	mgdp	1.32(0.11)	1.42(0.058)	1.11(0.072)	1.32(0.11)
equ.(22)	m1	0.415(0.045)	0.35(0.035)	0.47(0.1)	0.41(0.032)
equ.(23)	m3	0.175(0.019)	0.154(0.015)	0.20(0.044)	0.175(0.013)

On the whole, these empirical findings are strikingly similar to those of Barsky and Miron, confirming that business cycles and seasonal cycles have almost identical characteristics, even though they differ in the fact that seasonal patterns can be anticipated, whereas the shocks driving the business cycle presumably can not.

4.4 The model

We have pointed out above that previous studies have dealt with seasonality either by trying to remove its effects or by using time series models that ignore possible behavioural relationships between the variables.

Our view is that seasonality should be treated as a feature to be explained within the economic model and that there is no compelling reason why the same parameters or functional forms should apply to different seasons. In the model of this section we explicitly include a seasonal parameter in the consumption function. What is the rationale for this choice?

It is possible to argue that the observed high expenditure in the fourth quarter is partly explained by weather considerations (winter requiring, e.g., more heating), but the same should be true in the first quarter (the lowest spending period). Moreover, other studies show that the components for which expenditure is particularly high in the fourth quarter compared with the first are alcoholic drink and tobacco, clothing and footwear, and other goods.

At least for the first and third of these, allocation is clearly a matter of choice on the part of the consumer and therefore indicates that the consumer derives different satisfaction from expenditure in the fourth quarter of the year, and that purchases in the various seasons are treated as different commodities. Besides, there is the additional argument that changes in the labour supply have undoubtedly a seasonal pattern.

We analyze seasonal fluctuations by using a one-sector, neoclassical model of capital accumulation. We begin by considering the preferences, technology and endowments of the economy under study.

<u>Preferences.</u> We consider a representative infinitely-lived household with preferences over goods and leisure represented by

(2.1) $$U = \log [\Sigma^{\infty}_{t=0} \beta^t 1/(1-\sigma_s) C_{t,s}^{1-\sigma s}] + v(L), \beta < 1, 0 < \sigma_s$$

where $C_{t,s}$ is consumption, $L_{t,s}$ is leisure in period t and corresponding season s (the same time subscripts t for time and s for season are used for all variables), and $L = \{ L_{t,s}, L_{t+1,s'}, \ldots L_{\infty,s''} \}$ where $s' = s(t+1)$, $s'' = s(\infty)$ and s is a mapping from the set of integers to the set {1,2,3,4} where

(2.2) s = 1 when t/4 is an integer, s = 2 when (t+1)/4 is an integer, s = 3 when (t+2)/4 is an integer, s = 4 when (t+3)/4 is an integer.

The parameter σ_s can be interpreted as the coefficient of relative risk aversion and its inverse is the constant intertemporal elasticity of substitution of consumption for this utility function in season s. We model the seasonal cycle as being due to seasonally varying preferences instead of seasonal production shocks because it can not be easily explained why firms do not smooth their production given the fact that they can anticipate the seasonal fluctuations. It is known that we can filter out the seasonal pattern if seasonal effects are additive. As long as firms can identify the deterministic seasonal component and separate it from the random component, production can be smoothed out for the anticipated fluctuations. Why might firms not be able to smooth out seasonal fluctuations? It may be due to a non-linear relation between the seasonal part and the random part which can not be separated out. This can be captured by our set up. To simplify the analysis we assume at first that v = 0; the implication of a changing labour supply are analyzed later on.

Production possibilities. There is only one final good in this economy and it is produced according to a constant returns to scale Cobb-Douglas production technology given by

(2.3) $\qquad Y_{t,s} = A_t\, K_{t,s}^{\alpha}\, (X_t\, N_{t,s})^{1-\alpha}$,

where $K_{t,s}$ is the predetermined capital stock (chosen at t-1), X_t is the exogenous technical progress, $N_{t,s}$ is the labour input, A_t is the temporary change in total factor productivity and $Y_{t,s}$ is the output at time t.

Capital evolution. Since the commodity can be either consumed or invested, the capital stock evolves according to

(2.4) $\qquad K_{t+1,s'} = (1-\delta)\, K_{t,s} + I_{t,s}$,

where s' = s(t+1), $I_{t,s}$ is gross investment and δ is the rate of depreciation of capital. The production function (2.3) and the accumulation equation (2.4) imply that the rates of growth of output, consumption, capital and investment per capita all move closely with the exogenous technical progress X_t. Let us assume $X_t = X_{t-1}\exp(e_t)$.

In each period, the household faces two constraints: (i) total time allocated to work and leisure must not exceed the endowment N*, $L_{t,s} + N_{t,s} \leq N^*$. (ii) output can not exceed expenditure (we assume that government expenditure is zero and the economy is close).

(2.5) $\qquad Y_{t,s} \leq C_{t,s} + I_{t,s}$.

Since our set up satisfies the conditions of the second welfare theorem[5], the allocation of resources achieved by a decentralized competitive equilibrium would be the same as that chosen by a central planner who maximizes the utility of the representative economic agent in the model. So we solve the problem by deriving the intertemporal conditions that are satisfied on the optimal path that would be chosen by a central planner who maximizes the utility function (2.1) subject to the constraints (2.3), (2.4) and (2.5) with respect to $K_{t,s}$, $C_{t,s}$ conditional on the information at time 0 and the given k_0. The standard method of analyzing real business cycle models is to transform the economy into a stationary one where dynamics are more amenable to analysis. So we deflate all variables by X_t, and use lower-case letters to denote the deflated variables with corresponding upper-case letters. First we transform the utility function into

$$(2.6) \qquad U^* = \Sigma^{\infty}_{t=0}\ \beta^t\ 1/(1-\sigma_s)\ c_{t,s}^{1-\sigma s}\ X_t^{1-\sigma s},\ \beta < 1,\ 0 < \sigma_s$$
$$= \Sigma^{\infty}_{t=0}\ \beta_{s,t}\ 1/(1-\sigma_s)\ c_{t,s}^{1-\sigma s},$$

where $\beta_{t,s} = \beta^t\ X_t^{1-\sigma s}$ and $U = \log U^*$. Second, the capital accumulation equation becomes

$$(2.7) \qquad \exp(e_{t+1})\ k_{t+1,s'} = (1-\delta)\ k_{t,s} + i_{t,s}.$$

Combining the constraints, we form the Lagrangian

[5] The basic reference is Arrow and Hahn (1971). See also Prescott and Lucas (1972).

(2.8) $L = E_0 \{\Sigma^{\infty}_{t=0} \beta_{s,t}\ 1/(1-\sigma_s)\ c_{t,s}^{1-\sigma s}$
$+ \Sigma^{\infty}_{t=0} \Gamma_t\ [\ A_t\ k_{t,s}^{\alpha} - c_{t,s} - \exp(e_{t+1})\ k_{t+1,s'} + (1-\delta)$
$k_{t,s}\ ]\}.$

where Γ_t is the Lagrange multiplier at time t; for convenience, we discount it with $\beta_{t,s}$, to obtain as the current value of the shadow price of capital $v_{t,s} = \Gamma_t/\beta_{s,t}$. Maximization of (2.8) with respect to $k_{t+1,s'}$ and $c_{t,s}$, t = 1,2... yields the following first order conditions:

(2.9) $c_{t,s}^{-\sigma s} = v_{t,s}$

(2.10) $E_0\ \{\ v_{t+1,s'}\ [1-\delta+A_{t+1}\ \alpha\ k_{t+1,s'}^{\alpha-1}\ ] - v_{t,s}\ \exp(e_{t+1})\} = 0$

(2.11) $A_t\ k_{t,s}^{\alpha} - c_{t,s} - \exp(e_{t+1})\ k_{t+1,s'} + (1-\delta)\ k_{t,s} = 0$

(2.12) $\lim_{t\to\infty} \beta_{t,s}\ v_{t,s}\ k_{t+1,s'} = 0$

where (2.9), (2.10) and (2.12) must hold for all t = 1,2,..∞ and (2.12) is the transversality condition.

Using the standard argument of perfect foresight or rational expectation hypothesis, we can say that the sequential capital market, labour market and good market equilibria also support the optimal competitive equilibrium. Therefore the sequences $\{c_{t,s}\}$, $\{k_{t,s}\}$, $\{r_{t,s}\}$ and $\{w_{t,s}\}$[6] t = 1,2...∞ will be the sequences of equilibrium market clearing quantities and prices. The dynamics of the

[6] where $r_{t,s} = A_t\ \alpha\ K_{t,s}^{\alpha-1}\ (X_t\ N_{t,s})^{1-\alpha}$ and $w_{t,s} = A_t\ (1-\alpha)\ K_{t,s}^{\alpha}\ (X_t)^{1-\alpha}\ N_{t,s}^{-\alpha}$.

system can be derived from the first order conditions (2.9), (2.10) and (2.11). They can be reduced to a non-linear system of first order difference equations in k and v or a second order equation in k only. The boundary conditions of the system are the transversality condition and initial capital stock k_0. We simplify the dynamic analysis by approximating the first order conditions in terms of the percentage deviation from the mean growth path, so we can express (2.9), (2.10) and (2.11) as

(2.9') $$- \sigma_s \log c_{t,s} = \log v_{t,s}$$

(2.10') $$E_0\, E_{t+1} \{ \log v_{t+1,s'} + \Phi \log A_{t+1} + \Phi\, (\alpha-1) \log k_{t+1,s'} - \log v_{t,s} - e_{t+1} \} = 0$$

(2.11') $$\log A_t + \alpha \log k_{t,s} - q_c \log c_{t,s} - (1-q_c)\, \phi \log k_{t+1,s'} + (1-q_c)\, (\phi-1) \log k_{t,s} = 0.$$

where Φ is the ratio of the mean of the marginal return from capital r to the mean of the gross return $1-\delta+r$, q_c is consumption's mean share of output and ϕ is the inverse of the mean gross investment capital ratio, I_t/K_{t+1}. Combining (2.9'), (2.11') and (2.10') we obtain the following second order difference equation:

(2.13) $$E_{t+1} \{ \log k_{t+2,s''} + \Omega_{s',1} \log k_{t+1,s'} + \Omega_{s',2} \log k_{t,s} - \Omega_{s',3} \log A_{t+1} - \Omega_{s',4} \log A_t - \Omega_{s',5}\, e_{t+1} \} = 0.$$

where s" = s(t+2), s' = s(t+1), and

$$\Omega_{s',1} = \{\Phi(\alpha-1)q_c + \sigma_s(1-q_c)[\sigma_{s'}/\sigma_s - \phi(\sigma_{s'}/\sigma_s+1) - \alpha\sigma_{s'}/\sigma_s]\}* 1/[\sigma_{s'}\phi(1-q_c)] \tag{2.14}$$

$$\Omega_{s',2} = (\sigma_s/\sigma_{s'})*[(1-q_c)(\phi-1)+\alpha]/[\phi(1-q_c)] \tag{2.15}$$

$$\Omega_{s',3} = -\ \Phi\ [q_c/(1-q_c)]/[\phi\sigma_{s'}] \tag{2.16}$$

$$\Omega_{s',4} = -\ (\sigma_s/\sigma_{s'})/[\phi(1-q_c)] \tag{2.17}$$

$$\Omega_{s',5} = q_c/(1-q_c)]/(\phi\sigma_{s'}) \tag{2.18}$$

Equation (2.13) can be factorized to get:

$$(1-\mu_{s',1}\ L)(1-\mu_{s',2}\ L)\ E_{t+1}\ \log k_{t+2,s''} - E_{t+1}\{\Omega_{s',3}\log A_{t+1} + \Omega_{s',4}\log A_t + \Omega_{s',5}\ e_{t+1}\} = 0. \tag{2.19}$$

where L is the lag operator, $\mu_{s',1}, \mu_{s',2}$ are the roots of the quadratic equation $1 + \Omega_{s',1}L + \Omega_{s',2}L^2$ and $\mu_{s',1} < 1 < \mu_{s',2}$. We reduce (2.19) to

$$E_{t+1}\ (1-\mu_{s',1}\ L)\ \log k_{t+2,s''} = E_{t+1}\{\ \Omega_{s',3}\log A_{t+1} + \Omega_{s',4}\log A_t + \Omega_{s',5}\ e_{t+1}\ \}/(1-\mu_{s',2}L). \tag{2.20}$$

From the transversality condition (2.12), we know that there is a specific value of the initial shadow price $v_{t,s}$ which prevents the system from moving along an explosive path. Therefore we pick the stable root to solve backward and the unstable root to solve forward. Readers interested in the details should read Blanchard & Kahn (1980)'s paper. However, there may arise some time inconsistencies in deriving the solution function from (2.20). The standard argument to get the solution is that the household conjectures a solution k_{t+2} which is a function of k_{t+1} and a_{t+1}, and uses it to solve for k_{t+1} conditional on k_t and a_t. If the solution function k_{t+1} is different from the function k_{t+2}, then the household updates his conjecture about k_{t+2}, replacing it with k_{t+1}. He keeps iterating until the conjecture is close enough to the solution just found. This argument does not hold here because next period preference parameters are no longer the same as current period preference parameters. It is inappropriate to replace the conjecture about next period with the solution obtained in this period. But as long as the preference structure is deterministic, the way to find the solution is still the same with only some modifications. The household can only compare the conjecture about $k_{t+2,s}$. with the one about $k_{t-2,s}$ having the same preference parameter. This means that he has to find simultaneously the solution functions $k_{t,s}$ for all s in each iteration. However, our solution omits this complication.

Assume $E_{t-1}[e_t] = 0$ (it can be a constant) and $Var[e_t] = V_e^2$ for $t = 1,2,...\infty$. and let $a_t = \log A_t$, and $a_{t+1} = B(L)\ a_t + u_{t+1}$, with $E_{t-1}[u_t] = 0$ and $Var[u_t] = V_u^2$. This kind of decomposition of the productivity shock is valid because any univariate ARIMA time series can be decomposed into a stationary component and a random walk. Let us simplify the solution by assuming $B(L) = \rho$; after some manipulations[7], we get

(2.21) $\quad E_{t+1} \log k_{t+2,s''} - \mu_{s',1} \log k_{t+1,s'} = b_{s'} a_{t+1},$

and $\quad E_{t+1} \log k_{t+2,s''} = \log k_{t+2,s''}$. Therefore we get

(2.22) $\quad \log k_{t+2,s''} = \mu_{s',1} \log k_{t+1,s'} + b_{s'} a_{t+1}.$

Furthermore, we can express $\log K_{t+1,s'}$, $\log Y_{t+1,s'}$[8] as

$$\log K_{t+1,s'} = \mu_{s,1} \log K_{t,s} + (1-\mu_{s,1}) \log X_t + b_s a_t + e_t \tag{2.23}$$

$$\log Y_{t+1,s'} = \mu_{s,1} \log Y_{t,s} + (1-\mu_{s,1}) \log X_t + (b_s+L^{-1}-\mu_{s,1}) a_t + e_t. \tag{2.24}$$

Since $\log c_{t,s} = \pi_{s,ck} \log k_{t,s} + \pi_{s,ca} a_{t,s}$[9] and $\log i_{t,s} = \pi_{s,ik} \log k_{t,s} + \pi_{s,ia} a_{t,s}$[10], after some tedious algebra we finally

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[7] Note that $(1-\mu_{s',2})^{-1} = -L^{-1}\mu_{s',2}^{-1} (1-\mu_{s',2}^{-1}L^{-1})^{-1}$, where $(1-\mu_{s',2}^{-1}L^{-1})^{-1} = \Sigma^{\infty}_{j=0} \mu_{s',2}^{-j}L^{-j}$ if $\mu_{s,2} > 1$. Therefore (2.20) can be expressed as

$$E_{t+1} \{ (1-\mu_{s',1} L) \log k_{t+2,s''} - \Omega_{s',4}/\mu_{s',2} \log A_{t+1} + \Omega_{s',3} \Sigma^{\infty}_{j=0} \mu_{s',2}^{-(j+1)}\log A_{t+j+1} +$$
$$\Omega_{s',4} \Sigma^{\infty}_{j=1} \mu_{s',2}^{-(j+1)}\log A_{t+j} + \Omega_{s',5} \Sigma^{\infty}_{j=0} \mu_{s',2}^{-(j+1)} e_{t+j+1} \} = 0$$

and it reduces to (2.22) since $E_t[e_{t+j}] = 0$ for $j > 0$, implying that the last term equals zero.

[8] since $\log y_{t+1} = \alpha \log k_{t+1} + a_{t+1}$.

[9] From (2.11) & (2.22) we get $\pi_{s,ck} = (1-q_c)/q_c [\phi(1-\mu_{s,1})-1] + \alpha/q_c$ and $\pi_{s,ca} = [1-(1-q_c)\phi b_s]/q_c$.

[10] From $\log i_{t,s} = \log k_{t+1,s'} + (\phi-1)\log k_{t,s}$ we get $\pi_{s,ik} = \mu_{s,1}+(\phi-1)$ and $\pi_{s,ia} = b_s$.

obtain:

$$(2.25)\qquad \log C_{t+1,s'} = \kappa_{s,c1} \log C_{t,s} + \kappa_{s,c2} \log X_t + \kappa_{s,c3}\, a_t + e_t$$

$$(2.26)\qquad \log I_{t+1,s'} = \kappa_{s,i1} \log I_{t,s} + \kappa_{s,i2} \log X_t + \kappa_{s,i3}\, a_t + e_t$$

where $\kappa_{s,c1} = \mu_{s,1}\ \pi_{s',ck}/\pi_{s,ck}$, $\kappa_{s,c2} = (1-\mu_{s,1}\pi_{s',ck}/\pi_{s,ck})$ and $\kappa_{s,c3} = (\pi_{s,ck}b_s+\pi_{s,ca}L^{-1}-\mu_{s,1}\pi_{s,ca}\pi_{s',ck}/\pi_{s,ck})$. Similarly $\kappa_{s,i1} = \mu_{s,1}\ \pi_{s',ik}/\pi_{s,ik}$, $\kappa_{s,i2} = (1-\mu_{s,1}\pi_{s',ik}/\pi_{s,ik})$ and $\kappa_{s,i3} = (\pi_{s,ik}b_s+\pi_{s,ia}L^{-1}-\mu_{s,1}\pi_{s,ia}\pi_{s',ik}/\pi_{s,ik})$. One can notice that consumption $C_{t+1,s'}$ and investment $I_{t+1,s'}$ are co-integrated with co-integrating factor 1 if there is no seasonal variation in the consumption behaviour.

We have focused so far on seasonally varying consumption preferences, but people also follow a seasonal pattern in taking time off for holidays. When people can substitute leisure intertemporally, seasonally varying preferences may explain more of the seasonal fluctuations of output, consumption and employment than seasonally varying preferences over consumption only. Besides, the inclusion of a seasonal pattern for leisure also imposes more cross equation restrictions, hence providing an additional way to test the underlying model.

In the rest of this section, we set $\sigma_s = \sigma$ for all s and specify v(L) as

$$(2.27)\qquad v(L) = \Sigma^{\infty}_{t=0}\ \beta^t\ L_{t,s}^{\ 1-\lambda_s}/(1-\lambda_s)$$

where λ_s is the intertemporal elasticity of labour supply at season s, $0 < \lambda_s$ and $L_{t,s} = N* - N_{t,s}$. We model utility U as (2.1) since the long run growth of consumption will have no effect on the intertemporal substitution of the labour supply. By maximizing U subject to:

$$(2.28) \qquad A_t k_{t,s}{}^{\alpha} N_{t,s}{}^{1-\alpha} - c_{t,s} - \exp(e_{t+1}) k_{t+1,s'} + (1-\delta) k_{t,s} = 0,$$

we obtain the following five first order conditions,

$$(2.29) \qquad U^{*-1}\, \beta^*_t\, c_t^{-\sigma} = \Gamma_{t,s}$$

$$(2.30) \qquad (N* - N_{t,s})^{-\lambda_s}\, \beta^t = \Gamma_{t,s}\, A_t\, (1-\alpha)\, k_t^{\alpha}\, n_{t,s}{}^{-\alpha}$$

$$(2.31) \qquad E_0 \{ \Gamma_{t+1,s'}\, [1-\delta+A_{t+1}\, \alpha\, k_{t+1,s'}{}^{\alpha-1}\, n_{t,s}{}^{1-\alpha}] - \Gamma_{t,s} \exp(e_{t+1}) \} = 0$$

$$(2.32) \qquad A_t\, k_{t,s}{}^{\alpha}\, n_{t,s}{}^{1-\alpha} - c_{t,s} - \exp(e_{t+1})\, k_{t+1,s'} + (1-\delta)\, k_{t,s} = 0$$

$$(2.33) \qquad \lim_{t\to\infty} \beta^*_t\, \Gamma_{t,s}\, k_{t+1,s'} = 0$$

where U^* equals $X^0\, c^{1-\sigma}/(1-\sigma)\, \Sigma^{\infty}_{t=0}\, \beta^t \exp[(\sigma-1)\mu t]$, with $\mu = E(e_t)$, $\beta^*_t = \beta^t\, X_t^{1-\sigma}$ and $\Gamma_{t,s}$ is the shadow price of the capital. U^* is non-stationary but $U^*\, X_t^{-(\sigma-1)}$ is stationary. Furthermore, given these conditional first order conditions, we can set up a first order dynamic system in $\hat{k}_{t,s}$ and $\hat{N}_{t,s}$, which are the deviations from steady state of the corresponding variables (^ denotes the deviation from the steady state):

$$(2.34)\qquad \begin{bmatrix} \hat{k}_{t+1,s'} \\ \hat{N}_{t+1,s'} \end{bmatrix} = P_s \begin{bmatrix} \hat{k}_{t,s} \\ \hat{N}_{t,s} \end{bmatrix} + Q_s\hat{A}_{t+1} + R_s\hat{A}_t$$

where P_s is a 2x2 matrix and R_s & Q_s are 2x1 vectors. One can find how the elements of P_s, Q_s and R_s relate with the structural parameters in the appendix 1. Using the Equivalent Certainty Method as in King, Plosser & Rebello (1988), we first decompose P_s as $D_s\mu_s D_s^{-1}$, where D_s is the matrix of eigenvectors of P_s and μ_s is a diagonal matrix with the eigenvalues of P_s on the diagonal. Since one of the eigenvalues is greater than β^*_t, the system will be on the explosive path for an arbitrary $\hat{N}_{0,s'}$. There is only one specific value of $\hat{N}_{0,s'}$ which satisfies the transversality condition (2.33). Intuitively, when an unanticipated shock hits the economy, the labour supply jumps instantaneously to a new dynamic path and adjusts with the capital stock towards the steady state. In general, the adjustment path of capital $k_{t,s}$ depends on the entire sequence $\{\hat{A}_t\}^{\infty}_{t=0}$. The time path of efficient capital accumulation can be expressed as

$$(2.35)\qquad \hat{k}_{t+1,s'} = \mu_{s,1}\,\hat{k}_{t,s} + \psi_{s,1}\,\hat{A}_t + \psi_{s,2}\,\Sigma^{\infty}_{j=0}\,\mu_{s,2}^{-j}\,\hat{A}_{t+j+1}.$$

where $\mu_{s,1}$ and $\mu_{s,2}$ are the eigenvalues of P_s which $\mu_{s,1} < \mu_{s,2}$, $\psi_{s,1}$ and $\psi_{s,2}$ are complicated functions of the underlying parameters of preferences and technology. The solution is more or less the same as in the case above with consumption only in the utility function. If we make the same assumption concerning A_t as above and apply the expectation operator E_t to (2.35), we get:

$$(2.36) \qquad \hat{k}_{t+1,s'} = \mu_{s,1}\,\hat{k}_{t,s} + [\psi_{s,1} + \psi_{s,2}\rho/(1-\rho\mu_{s,1}^{-1})]\,\hat{A}_t.$$

Using (A.10) and taking the deviations from the steady state, we get the following equation:

$$(2.37) \qquad (1-q_c)\phi\log\hat{k}_{t+1,s'} = (\alpha+\phi-1)(1-q_c)\log\hat{k}_{t,s} + [(1-\alpha)+\alpha q_c - q_c\lambda'_s]\log\hat{N}_{t,s} + [1-q_c/\sigma]\log\hat{A}_t \;.$$

Combining (2.36) and (2.37) together, we get $\hat{N}_{t+1,s'} = \mu_{s,1}\,\hat{N}_{t,s} + \varsigma_{t+1,s'}$, where $\varsigma_{t+1,s'}$ is a seasonal MA(1). Similarly for $\hat{y}_{t+1,s'}$, by using the production identity $\hat{y}_{t+1,s'} = \alpha\hat{k}_{t+1,s'} + (1-\alpha)\hat{N}_{t+1,s'} + \hat{A}_{t+1,s'}$, we get $\hat{y}_{t+1,s'} = \mu_{s,1}\,\hat{y}_{t,s} + \xi_{t+1,s'}$, where $\xi_{t+1,s'}$ is also a seasonal MA(1).

To see if the data give support to our model, we have run some simple regressions using seasonal dummies and weather variables. Let x_t be the log of the variable under consideration. We estimate, using GDP, consumption, fixed investment, government expenditure and employment in turn, the following equations:

(a) $\quad \Delta x_t = \alpha_0 + \alpha_1\,\Delta x_{t-1} + u_t$

(b) $\quad \Delta x_t = \sum_{s=1}^{4} \alpha_{0,s}\,d_s + \alpha_1\,\Delta x_{t-1} + u_t$

(c) $$\Delta x_t = \alpha_0 + \sum_{s=1}^{4} \alpha_{1,s} d_s \Delta x_{t-1} + u_t$$

(d) $$\Delta x_t = \sum_{s=1}^{4} \alpha_{0,s} d_s + \sum_{s=1}^{4} \alpha_{1,s} d_s \Delta x_{t-1} + u_t$$

where d_s is a seasonal dummy for quarter s and $\Delta = 1-L$ (L is the lag operator).

The estimation results are reported in Table 9. A likelihood ratio test can be used to test for the significance of the seasonal dummies, to see if seasonal factors affect the intercept and the slope of the function as predicted by our model; the test is computed as $LR = 2(L_1 - L_0)$, where L_1 is the value of the log of the likelihood function for the maximum of the unconstrained model and L_0 is the value when the constraints are imposed; the LR statistic is distributed asymptotically as a χ^2 with degrees of freedom equal to the number of constraints.

Starting with output, it can be noticed that LR tests indicate that equation (d) performs better than equation (b) (since the LR statistic is equal to 13.096 and the critical value of χ^2 with 3 degrees of freedom at the 95% confidence level is 7.815) and equation (a) (the LR statistic is equal to 173.204 and the critical value of χ^2 with 6 degrees of freedom is 12.592); hence the specification to be preferred includes seasonal dummies affecting the intercept and the slope coefficient as implied by our model.

On the basis of LR tests, equation (d) has to be chosen also for employment (the LR statistic is equal to 18.2 implying rejection of the restrictions in equation (b)), and for consumption (the corresponding LR statistic is equal to 14.8); only in the case of fixed investment and government expenditure the restrictions implied by equation (b) as opposed to equation (d) can not be rejected, and therefore only the intercept appears to be affected by seasonal factors (the LR statistic for fixed investment is equal to 6.9; in the case of government expenditure equation (b) is to be preferred to equation (d) since the LR statistic is equal to 7.6).

In most cases the seasonal dummies are individually significant; as for the seasonal patterns of the variables, they have already been examined in the previous sections.

To show that these seasonal patterns reflect seasonally-varying parameters in the underlying behavioural functions rather than simply be due to weather changes, we have re-estimated for all variables the chosen specification adding three weather variables: mean daily temperature, sunshine and rainfall (see Table 10). None of these variables turn out to be significantly different from 0, and the seasonal dummies are generally still significant (the exception is represented by the intercept dummies for output, government expenditure and employment).

As we are dealing with quarterly data, we might expect to find fourth-order autocorrelation in the disturbance term (i.e. $u_t = \phi_4 u_{t-4} + \epsilon_t$). The appropriate test is then a modified Durbin-Watson statistic as proposed by Wallis (see Wallis, 1972):

$$d_4 = \frac{\sum_{t=5}^{n} (e_t - e_{t-4})^2}{\sum_{t=1}^{n} e_t^2}$$

where the e's are the usual OLS residuals. Our preferred specifications pass this test, implying that we are unable to reject the null hypothesis

H_0: $\phi_4 = 0$

These findings confirm that the seasonal dummies are not simply picking up the effects of weather changes, and give further support to our hypothesis that allocation across seasons is not just a consequence of weather variations but also a matter of choice.

4.5 Conclusions

In this paper we have argued that seasonal fluctuations should be studied in their own right.

We have first documented the quantitative importance of seasonal fluctuations in the U.K. economy and shown that there is a "seasonal cycle" whose main features are very similar to those of the conventional business cycle.

We have then analysed seasonal fluctuations by setting up a one-sector, neo-classical model of capital accumulation in which seasonal preferences are explicitly incorporated, since the coefficient of risk aversion σ_s depends on the seasons; after having deflated all variables by the exogenous rate of technical progress to transform the economy into a stationary one, we have derived the dynamics of the system and shown that the AR(1) coefficients of the variables in the model are functions of the underlying seasonal parameters (this turns out to be the case also in the extended model where the labour supply is not constant). This clearly indicates that the common practice of assuming that the seasonal component is only additive and of filtering it out is not an appropriate way of dealing with seasonality and may lead to spurious results; as the time series properties of the variables are affected by the seasonal component, seasonality should be modelled and the economics underlying seasonal variation should be explicitly investigated.

An appropriate treatment of seasonality is therefore necessary for the interpretation of the data, and it can be rationalized by noticing that allocation of expenditure over each year appears to be a matter of choice on the part of the consumer.

Future work should consider seasonality a feature to be explained within the economic model.

Appendix 1

The first order conditions are the following (A1-A5):

(A1) $$U^{*-1}\, \beta^{*}_{t}\, c_t^{-\sigma} = \Gamma_{t,s}$$

(A2) $$(N^* - N_{t,s})^{-\lambda s}\, \beta^{t} = \Gamma_{t,s}\, A_t\, (1-\alpha)\, k_t^{\alpha}\, n_{t,s}^{-\alpha}$$

(A3) $$E_0\, \{\, \Gamma_{t+1,s'}\, [1-\delta+A_{t+1}\, \alpha\, k_{t+1,s'}^{\alpha-1}\, n_{t,s}^{1-\alpha}] - \Gamma_{t,s}\, \exp(e_{t+1})\, \} = 0$$

(A4) $$A_t\, k_{t,s}^{\alpha}\, n_{t,s}^{1-\alpha} - c_{t,s} - \exp(e_{t+1})\, k_{t+1,s'} + (1-\delta)\, k_{t,s} = 0$$

(A5) $$\lim_{t\to\infty} \beta^{*}_{t}\, \Gamma_{t,s}\, k_{t+1,s'} = 0.$$

From (A1) and (A2), we can get

(A6) $$c_t^{-\sigma} = \frac{U^{*}\, X_t^{-(\sigma-1)}\, (N^* - N_{t,s})^{-\lambda s}}{A_t\, (1-\alpha)\, k_{t,s}^{\alpha}\, N_{t,s}^{-\alpha}}\, .$$

We use the approximation $\log(N^*-N_{t,s}) \approx \log(N^* - N) + N/(N^*-N)(\log N_{t,s} - \log N)$ where N is the steady state labour supply and take logs of (A6), to obtain (A7):

(A7) $$\log c_{t,s} = \{\, b_0 + \log A_t + \alpha \log k_{t,s} + (\lambda'_s - \alpha)\log N_{t,s} - \log U^{*} + (\sigma-1)\log X_{t,s}\, \}/\sigma.$$

where b_0 is a constant and $\lambda'_s = N/(N^*-N)\lambda_s$. By substituting (A6) into (A1), and using (A3) we get

(A8) $$[\Phi(\alpha-1)+\alpha]\log k_{t+1,s'} + [\Phi(1-\alpha)-\lambda'_{s'}-\alpha] \log N_{t+1,s'} = b_1$$
$$\alpha \log k_{t,s} -(\alpha+\lambda'_s) - (\Phi+1)\log A_{t+1} + \log A_t - e_{t+1},$$

where b_1 is a constant. From (A4), we have

(A9) $$\log A_t + \alpha \log k_{t,s} - (1-\alpha) \log N_{t,s} - q_c \log c_{t,s} - (1-q_c) \phi \log k_{t+1,s'} + (1-q_c) (\phi-1) \log k_{t,s} = 0.$$

Hence we finally have:

(A10) $$(1-q_c)\phi \log k_{t+1,s'} = (\alpha+\phi-1)(1-q_c)\log k_{t,s} + [(1-\alpha)+\alpha q_c-q_c\lambda'_s]\log N_{t,s} + [1-q_c/\sigma)]\log A_t + q_c \log(U^* X_t^{-(\sigma-1)}).$$

So we can eliminate the constant by subtracting all variables from their "steady state" values ($\hat{k}_{t,s}$ and $\hat{N}_{t,s}$ denote the deviations from the steady state values of the corresponding variables). Therefore we can write the system of difference equations as (2.34) where

$$P_s = \begin{bmatrix} (1-q_c)\phi & 0 \\ \Phi(\alpha-1)+\alpha & \Phi(1-\alpha)-\lambda'_s-\alpha \end{bmatrix}^{-1} \begin{bmatrix} (\alpha+\phi-1)(1-q_c) & (1-\alpha)+q_c(\alpha-\lambda'_s) \\ \alpha & -(\alpha+\lambda'_s) \end{bmatrix}$$

$$Q_s = \begin{bmatrix} 1-q_c & 0 \\ \Phi(\alpha-1)+\alpha & \Phi(1-\alpha)-\lambda'_s-\alpha \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -(\Phi+1) \end{bmatrix}$$

$$R_s = \begin{bmatrix} 1-q_c & 0 \\ \Phi(\alpha-1)+\alpha & \Phi(1-\alpha)-\lambda'_s-\alpha \end{bmatrix}^{-1} \begin{bmatrix} 1-q_c/\sigma \\ 1 \end{bmatrix}$$

Appendix 2

Description of the data series

MGDP: Gross Domestic Product at market prices, million, constant (1980) prices. Source: Central Statistical Office Data Bank (CSODB).

CON: total consumers' expenditure at constant (1980) prices. Source: Economic Trends Annual Supplement (ETAS).

CONDU: consumers' expenditure on durable goods at constant (1980) prices. Source: ETAS.

CONSE: consumers' expenditure on other services (excluding rents and rates) at constant (1980) prices. Source: ETAS.

FIX: total fixed investment at constant (1980) prices. Source: ETAS.

K: capital stock, million, 1980 prices. Interpolated from annual figures using a depreciation rate implied by the known end of the year figures. Source: unpublished data from the CSO.

TOE: general government taxes on expenditure, million, constant (1980) prices. Source:CSODB.

SUBS: general government subsidies, million, constant (1980) prices. Source: CSODB.

GOVCU: general government expenditure on goods and services, million, constant (1980) prices. The CSODB, (code ABKF), gives quarterly data back to 1961 Q1 and then annual. The quarterly figures for 1955-1960 are interpolated from the annual figures.

NAFS: adjusted fiscal stance. Data interpolated from annual data. Source: unpublished paper from Paul Kong.

VX: exports of goods and services at market prices, million, constant (1980) prices. Source: CSODB.

MVACU: imports of goods and services, million, current prices. Source: CSODB.

IMPV: imports of goods and services, million, constant prices. Source: CSODB.

INV: this variable is constructed as STO/SAL.
-STO: stock changes (all industries). Source: ETAS.
-SAL: value of retail sales. Source: ETAS.

RUE: male unemployment rate. The number unemployed are on a consistent pre-1982 definition. For before 1982 sources are the British Labour

Statistics Historical Abstract (BLSHA), British Labour Statistics Yearbook (BLSYB) and Department of Employment Gazette (DEG). Post 1982 the source is successive issues of the Unemployment Unit Bulletin. Data are for end quarter.

EEP: employees in employment, male and female, in index of production industries. The basis for this series is the EGHS series for "production and construction industries (SIC 1980)" in EGHS. A consistent series for the years before 1977 was produced using overlapping figures from the EGHS and DEG.

EE: total employees in employment, male and female, SIC 1980. Source: as for EEP.

AWH: average hours worked, full-time manual men (21 years and over). Data interpolated from annual observations. Source: DEG.

TH1: this variable is constructed as AWH*EEP.

TH2: this variable is constructed as AWH*EE.

WP: working population. Source: ETAS and DEG.

PRI: general index of retail prices. Source: ETAS.

PMF: Prices of materials and fuels purchased by manufacturing, 1908=100. Source: ETAS and Monthly Digest of Statistics (MDS).

TBR: average discount rate on 91 day treasure bills. Figures for mid quarter. Source: MDS, Financial Statistics (FS).

LCPU: labour costs per unit of output. Source: DE (unpublished).

WSPU: wages and salaries per unit of output. Source: DE (unpublished).

NWH: normal weekly hours, full-time manual men (21 years and over). Source: DEG and New Earnings Survey (NES).

M1: Source: CSODB. The sample period for this variable is 1963:02 1983:03.

M3: Source: CSODB. Sample period: 1963:02 1983:03.

TEMP: mean daily air temperature at the sea level (England and Wales); data are monthly; quarterly observations are obtained by taking the average. Source: CSO Annual Abstract of Statistics (CSOAAS).

SUN: mean daily sunshine. Quarterly observations are obtained by taking the average. Source: CSOAAS.

RAIN: rainfall. Quarterly observations are obtained by taking the average. Source: CSOAAS.

Note:

-for INV, the estimated equation is:

$$INV = b1*d1 + b2*d2 + b3*d3 + b4*d4 \;;$$

-for some variables, to convert figures at current prices into corresponding figures at constant prices, the series FJAK (purchasing power of the pound) from ETAS has been used; it is based on movements in consumers' expenditure deflator from 1948 to 1962 and on the general index of retail prices for 1962 onwards (average 1980=100).

Table 9

(s.e. in parentheses)

MGDP.

Coefficients	equ(a)	equ(b)	equ(c)	equ(d)
α_0	.008 (.003)		.022 (.081)	
α_1	-.450 (.080)	-.400 (.080)		
$\alpha_{0,1}$		-.030 (.004)		-.060 (.010)
$\alpha_{0,2}$		-.0005 (.005)		-.020 (.010)
$\alpha_{0,3}$		.025 (.004)		.020 (.004)
$\alpha_{0,4}$		.047 (.003)		.040 (.004)
$\alpha_{1,1}$			-1.730 (.140)	.190 (.250)
$\alpha_{1,2}$			-.086 (.100)	-.750 (.160)
$\alpha_{1,3}$			-.400 (.140)	-.480 (.110)
$\alpha_{1,4}$			-.390 (.210)	-.120 (.180)
R^2	.20	.78	.63	.80
DW	2.22	2.07	1.99	2.08
DW4	-	-	-	2.01
Log Likelihood Function (LF)	226.2	306.3	273.4	312.8

GOVCU.

Coefficients	equ(a)	equ(b)	equ(c)	equ(d)
α_0	.042 (.005)		.042 (.006)	
α_1	-.610 (.071)	.076 (.007)		
$\alpha_{0,1}$		-.029 (.007)		.007 (.008)
$\alpha_{0,2}$		.055 (.008)		-.020 (.014)
$\alpha_{0,3}$		.040 (.009)		.040 (.010)
$\alpha_{0,4}$		-.350 (.009)		.023 (.010)
$\alpha_{1,1}$			.330 (.290)	-.130 (.270)
$\alpha_{1,2}$			-1.130 (.110)	-.480 (.170)
$\alpha_{1,3}$			-.600 (.130)	-.680 (.160)
$\alpha_{1,4}$			-.300 (.100)	-.120 (.130)
R^2	.38	.65	.57	.67
DW	1.9	1.9	1.82	1.96
DW4	-	1.99	-	-
Log Likelihood Function (LF)	184.8	220.5	207.1	224.1

EEP.

Coefficients	equ(a)	equ(b)	equ(c)	equ(d)
α_0	-.0017 (.0007)		-.0029 (.0006)	
α_1	.511 (.078)	.730 (.062)		
$\alpha_{0,1}$		-.009 (.001)		-.009 (.001)
$\alpha_{0,2}$		.005 (.001)		-.004 (.001)
$\alpha_{0,3}$		.003 (.001)		.003 (.001)
$\alpha_{0,4}$		-.003 (.001)		-.004 (.001)
$\alpha_{1,1}$			.750 (.120)	.590 (.090)
$\alpha_{1,2}$			.120 (.100)	.590 (.120)
$\alpha_{1,3}$			.450 (.170)	.690 (.120)
$\alpha_{1,4}$			1.22 (.190)	1.20 (.140)
R^2	.26	.66	.4	.71
DW	1.94	2.29	1.99	2.21
DW4	-	-	-	2.05
Log Likelihood Function (LF)	427.0	475.2	440.4	484.3

FIX.

Coefficients	equ(a)	equ(b)	equ(c)	equ(d)
α_0	-.026 (.009)		.042 (.009)	
α_1	-.160 (.090)	.022 (.92)		
$\alpha_{0,1}$		-.11 (.014)		-.120 (.024)
$\alpha_{0,2}$		.013 (.014)		-.010 (.019)
$\alpha_{0,3}$		.067 (.010)		.069 (.010)
$\alpha_{0,4}$		-.022 (.010)		-.091 (.016)
$\alpha_{1,1}$			-1.04 (.12)	.150 (.180)
$\alpha_{1,2}$			.160 (.120)	-.200 (.150)
$\alpha_{1,3}$			-.010 (.230)	-.120 (.180)
$\alpha_{1,4}$			.860 (.170)	.390 (.200)
R^2	.025	.69	.53	.71
DW	2.07	2.00	2.1	1.99
DW4	-	2.03	-	-
Log Likelihood Function (LF)	108.2	179.4	153.4	182.9

CON.

Coefficients	equ(a)	equ(b)	equ(c)	equ(d)
α_0	.035 (.005)		.038 (.006)	
α_1	-.510 (.078)	-.190 (.090)		
$\alpha_{0,1}$		-.056 (.007)		-.072 (.014)
$\alpha_{0,2}$		.052 (.007)		.033 (.011)
$\alpha_{0,3}$		.045 (.007)		.064 (.011)
$\alpha_{0,4}$		.069 (.007)		.054 (.006)
$\alpha_{1,1}$			-1.57 (.110)	.054 (.220)
$\alpha_{1,2}$			-.400 (.100)	-.450 (.150)
$\alpha_{1,3}$			-.120 (.100)	-.470 (.160)
$\alpha_{1,4}$			.610 (.170)	.270 (.170)
R^2	.26	.86	.81	.88
DW	2.11	1.84	2.04	1.88
DW4	-	-	-	1.97
Log Likelihood Function (LF)	192.2	299.2	277.7	306.6

Table 10:

Dependent Variable Variable	MGDP	GOVCU	EEP	FIX	CON
Coefficients					
$\alpha_{0,1}$	-.12 (.05)	.014 (.130)	-.018 (.010)	-.45 (.14)	-.07 (.05)
$\alpha_{0,2}$	-.1 (.06)	-.098 (.130)	-.008 (.016)	-.44 (.18)	.026 (.07)
$\alpha_{0,3}$	-.075 (.07)	-.031 (.150)	-.011 (.018)	-.45 (.21)	.054 (.08)
$\alpha_{0,4}$	-.034 (.05)	-.031 (.120)	-.014 (.014)	-.27 (.18)	.05 (.06)
$\alpha_{1,1}$	.10 (.25)	-.350 (.086)	.059 (.090)	.012 (.090)	.07 (.22)
$\alpha_{1,2}$	-.78 (.16)		.570 (.120)		-.43 (.15)
$\alpha_{1,3}$	-.45 (.11)		.710 (.120)		-.46 (.15)
$\alpha_{1,4}$	-.10 (.18)		1.28 (.140)		.26 (.17)
Temperature	.002 (.001)	.002 (.002)	.0003 (.0003)	.008 (.037)	.0002 (.0013)
Sunshine	-.001 (.004)	-.008 (.009)	-.0001 (.001)	.006 (.120)	.00044 (.00044)
Rain	-.00017 (.00011)	-.0003 (.0002)	-.00003 (.00003)	.00001 (.00003)	-.000077 (.00011)
R^2	.81	.60	.71	.71	.88
DW	2.07	1.86	2.21	2.01	1.89
DW4	1.91	1.88	2.02	1.93	1.85
Log Likelihood Function (LF)	316.0	221.8	485.8	182.9	307.0

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Figure 1: The Seasonal Fluctuations of GNP

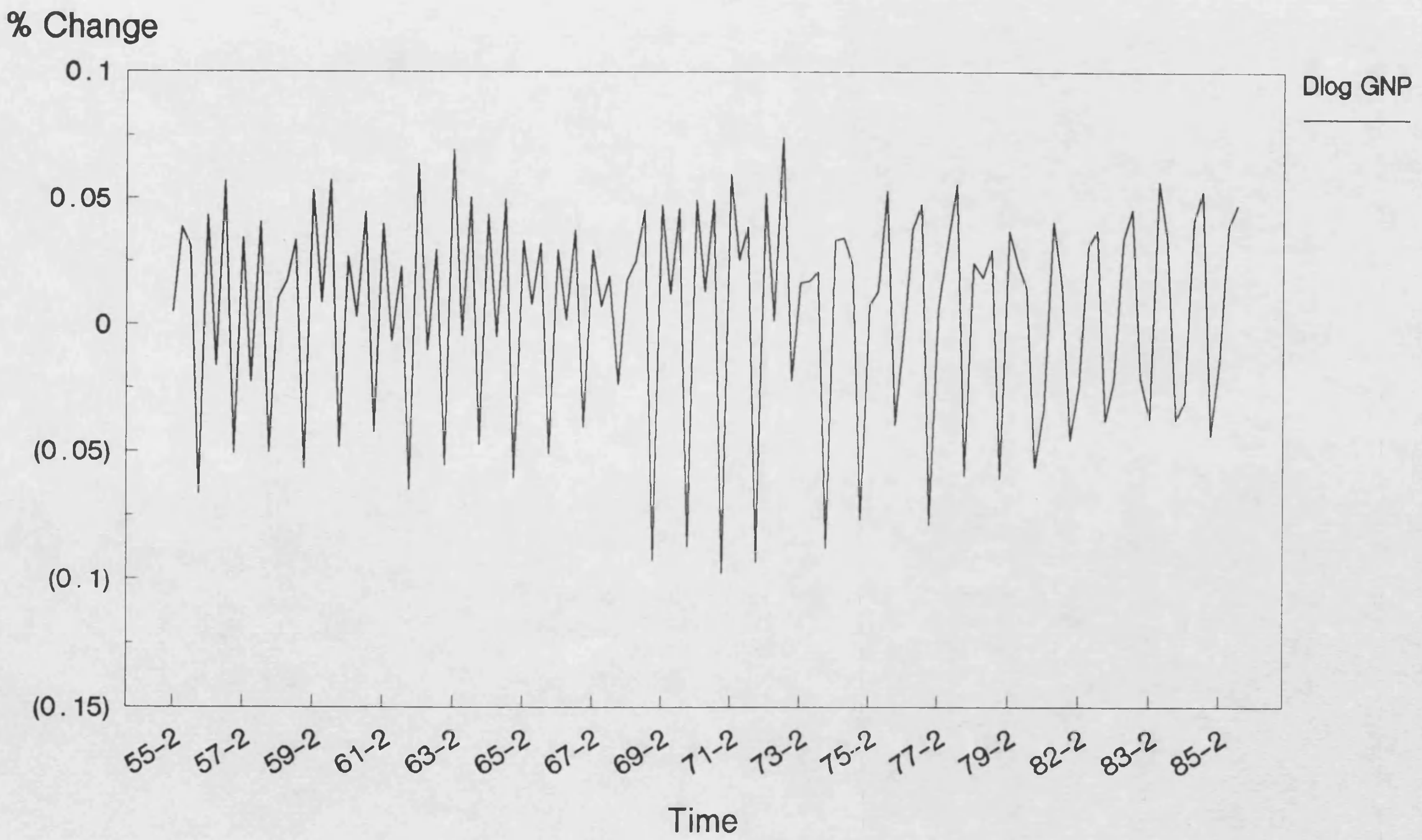


Figure 2: The Seasonal Fluctuations of GNP and M1

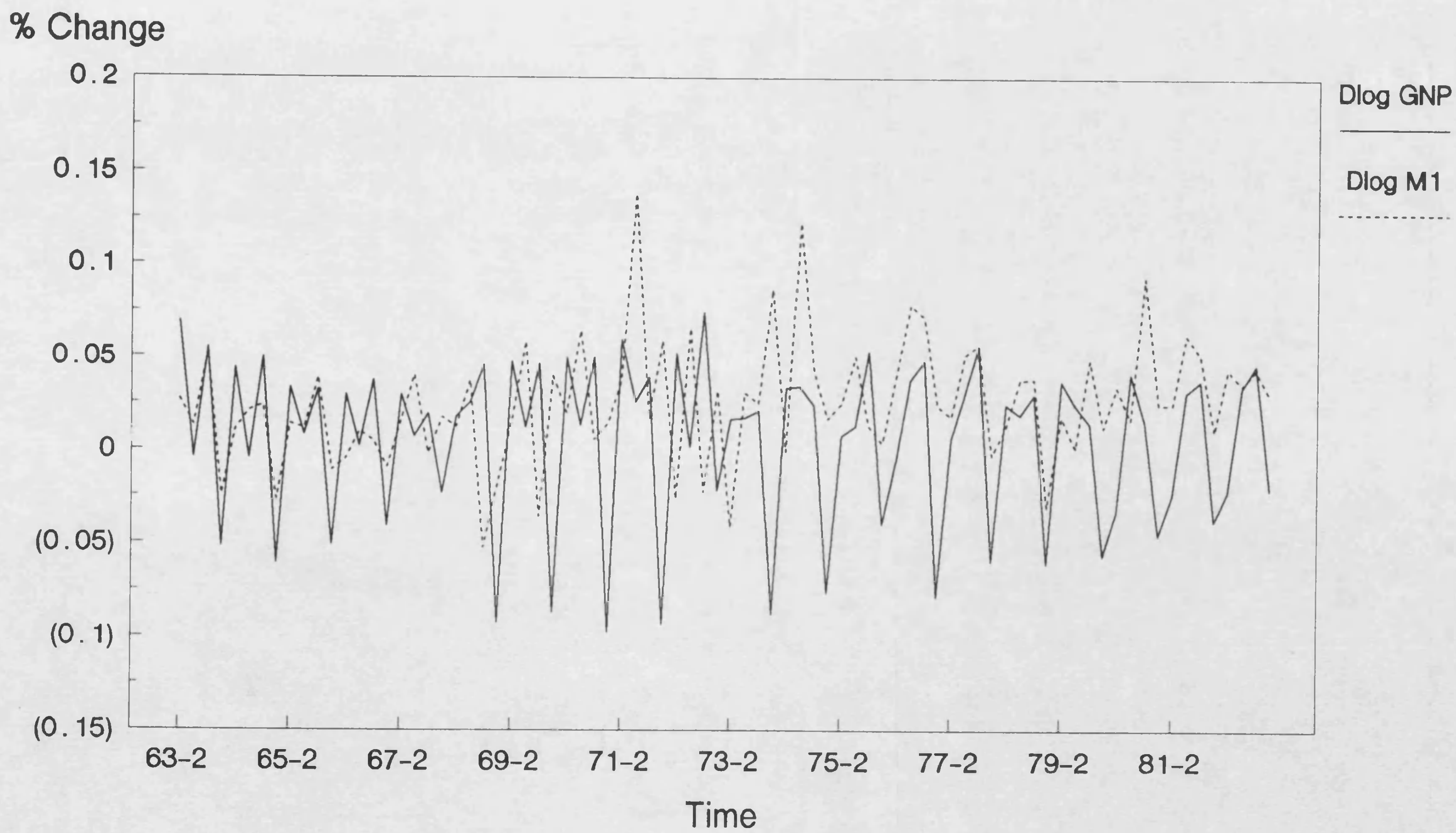


Figure 3: The Seasonal Fluctuations of GNP and M3

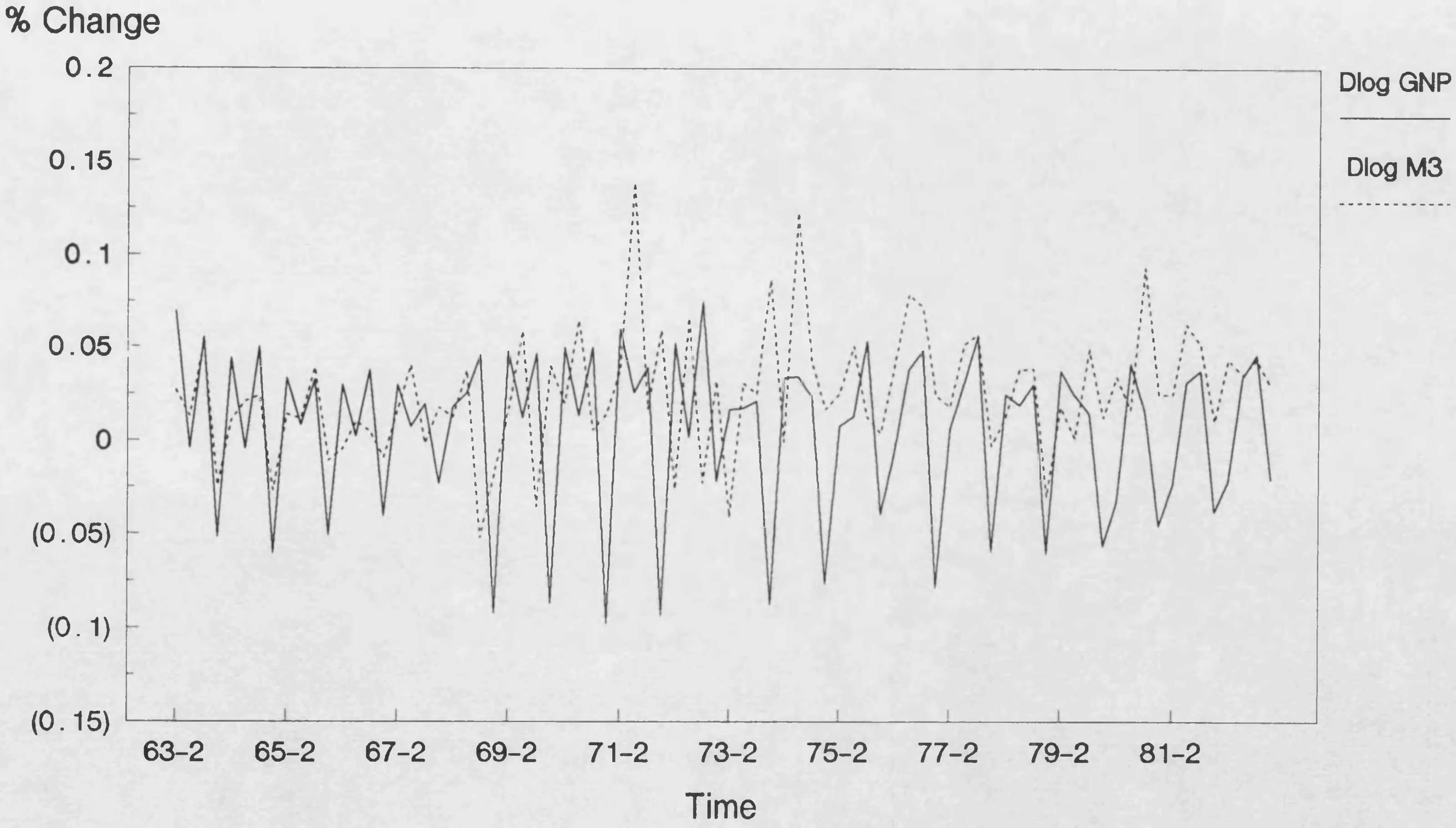
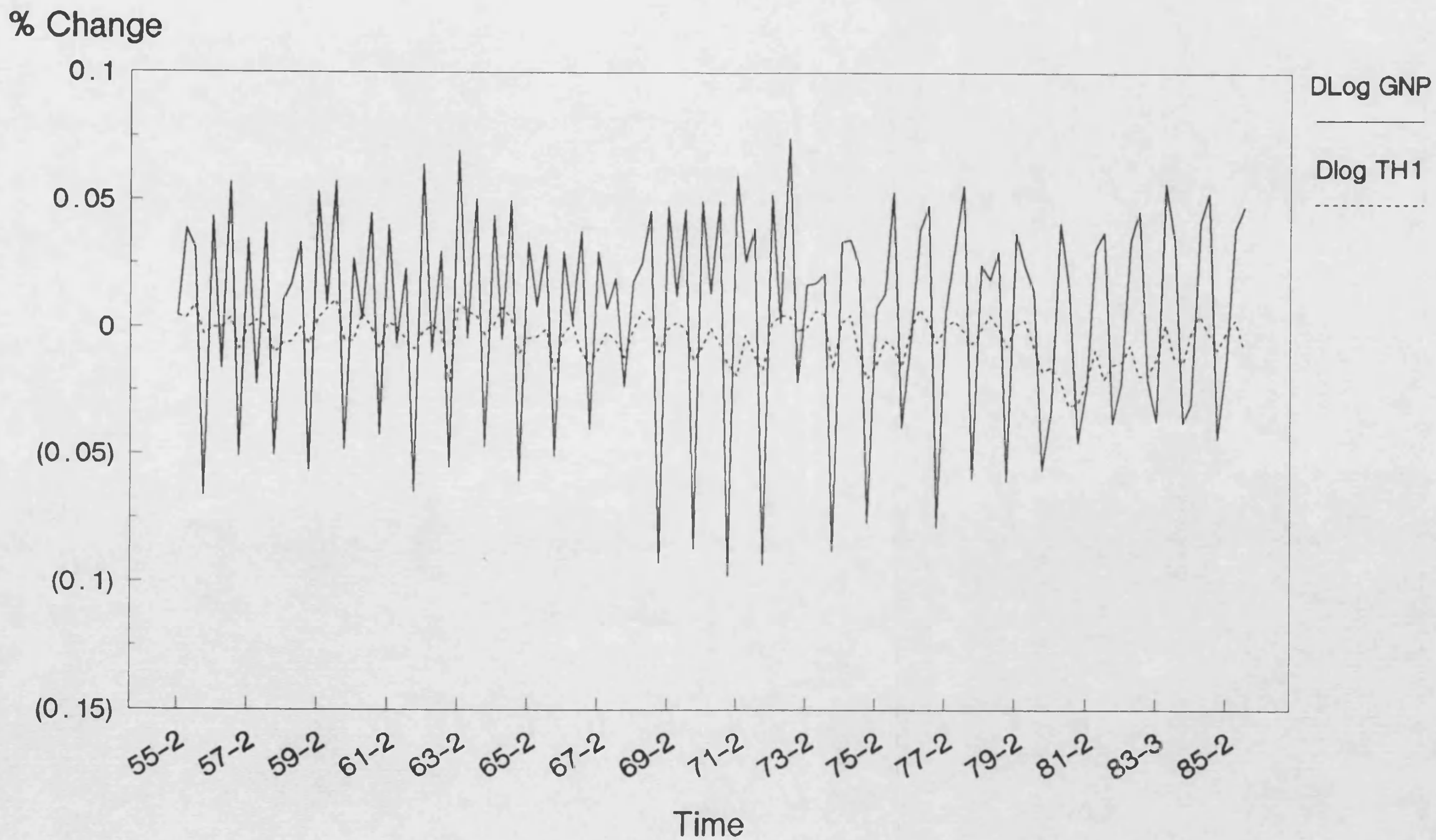
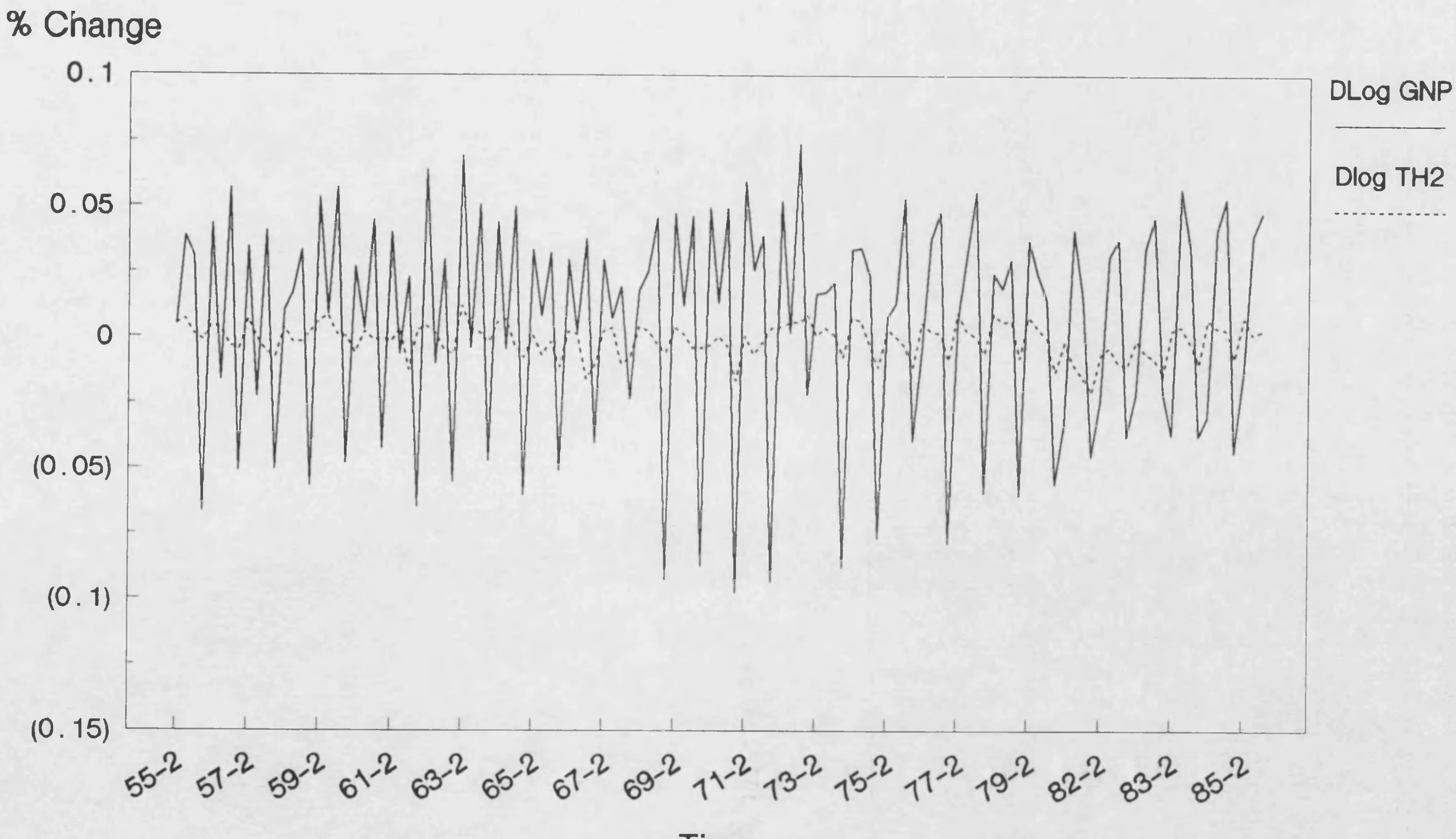


Figure 4: The Seasonal Fluctuations of GNP and Employment



TH1 = AWH*EEP

Figure 5: The Seasonal Fluctuations of GNP and Employment



TH2 = AWH*EE