The London School of Economics and Political Science

Essays on Information Asymmetry in Financial Market

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Declaration

I certify that the thesis I have presented for examination for the MPhil/PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).

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Statement of Conjoint Work

I confirm that Chapter 2 "Investment Waves under Cross Learning" is jointly co-authored with Mr.Yao Zeng from Economic Department at Harvard University, and I contributed 50% of this work.

Abstract

I study how asymmetric information affects the financial market in three papers. In the first paper, I study the joint determination of optimal contracts and equilibrium asset prices in an economy with multiple principal-agent pairs. Principals design optimal contracts that provide incentives for agents to acquire costly information. With agency problems, the agents' compensation depends on the accuracy of their forecasts for asset prices and payoffs. Complementarities in information acquisition delegation arise as follows. As more principals hire agents to acquire information, asset prices become less noisy. Consequently, agents are more willing to acquire information because they can forecast asset prices more accurately, thus mitigating agency problems and encouraging other principals to hire agents. This mechanism can explain many interesting phenomena in markets, including multiple equilibria, herding, home bias and idiosyncratic volatility comovement.

In the second paper (co-authored with Yao Zeng from Harvard University), we investigate how firms' cross learning amplifies industry-wide investment waves. Firms' investment opportunities have idiosyncratic shocks as well as a common shock, and firms' asset prices aggregate speculators' private information about these two shocks. In investing, each firm learns from other firms' prices to make better inference about the common shock. Thus, a spiral between firms' higher investment sensitivity to the common shock and speculators' higher weighting on the common shock emerges. This leads to systematic risks in investment waves: higher investment and price comovements as well as their higher comovements with the common shock. Moreover, each firm's cross learning creates a new pecuniary externalities on other firms, because it makes other firms' prices less informative on their idiosyncratic shocks through speculators' endogenous over-weighting on the common shock.

In the third model, we study the effect of introducing an options market on investors' incentive to collect private information in a rational expectation equilibrium model. We show that an options market has two effects on information acquisition: a negative effect, as options

act as substitutes for information, and a positive effect, as informed investors have less need for options and can earn profits from selling them. When the population of informed investors is high because of the low information acquisition cost, the supply for options is larger than the demand, leading to low option prices. Low option prices in turn induce investors to use options instead of information to reduce risk, while informed investors have little opportunity to earn profits from selling options to cover their information acquisition cost. Introducing an options market thus decreases investors' incentive to acquire information, and the prices of the underlying assets become less informative, leading to lower prices and higher volatilities. A dynamic extension of this analysis shows that introducing an options market increases the price reactions to earnings announcements. However, when the information acquisition cost is high, the opposite effects arise. Further analysis shows that our results are robust for more general derivatives. These results provide a potentially unified theory to reconcile the conflicting empirical findings on the options listing of individual stocks in both the U.S. market and international markets.

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Chapter 1

Delegated Information Acquisition and Asset Price

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Abstract: This paper studies the joint determination of optimal contracts and equilibrium asset prices in an economy with multiple principal-agent pairs. Principals design optimal contracts that provide incentives for agents to acquire costly information. With agency problems, the agents' compensation depends on the accuracy of their forecasts for asset prices and payoffs. Complementarities in information acquisition delegation arise as follows. As more principals hire agents to acquire information, asset prices become less noisy. Consequently, agents are more willing to acquire information because they can forecast asset prices more accurately, thus mitigating agency problems and encouraging other principals to hire agents. This mechanism can explain many interesting phenomena in markets, including multiple equilibria, herding, home bias and idiosyncratic volatility comovement.

1.1 Introduction

The asset management industry has experienced tremendous growth with current assets under management comparable to global GDP. Not surprisingly, institutional investors now dominate trading activities in all financial markets.[1] While institutions assist their clients in making investment decisions, agency problems may simultaneously arise. In particular, potential moral hazard emerges when institutions' efforts are largely unobservable, raising the issue of optimal contract design. Given institutions' superior capabilities to acquire information, it is commonplace for clients to delegate information acquisition to them and provide incentives for them through optimal contracting. However, the joint determination of optimal contracts, information acquisition delegation and equilibrium asset pricing has not yet been fully explored in the literature.[2]

This paper contributes to the literature by solving for optimal contracts characterized in a general space and equilibrium asset prices in an economy with multiple principal-agent pairs. I show that the optimal contracts for delegated information acquisition depend on agents' forecasting accuracy for asset prices and payoffs: agents receive high compensation when they produce accurate forecasts. Moreover, I find strategic complementarities in the delegation of information acquisition: the more principals hire agents to acquire information, the more others are willing to do so. As more principals hire agents to acquire information, asset prices become less noisy. As a result, agents are more willing to acquire information because they can forecast asset prices more accurately. Thus, the agency problems are mitigated and other principals are encouraged to hire agents. Such strategic complementarities yield multiple equilibria, and can explain many phenomena, including asset price jumps, herding behaviour, home bias and

[1] French (2008) documents that financial institutions accounted for more than 80% ownership of equities in the U.S. in 2007, compared to 50% in 1980. TheCityUK (2013) estimates the size of assets under management is around \$87 trillion globally, which is equal to global GDP. Meanwhile, Jones and Lipson (2004) reports that institutional trading volume reached 96% of total equity trading volume in NYSE by 2002.

[2] Papers studying optimal contracts without any asset pricing implications include Bhattacharya and Pfleiderer (1985) and Dybvig et al. (2010). Papers studying institutions' impacts on asset pricing without asymmetric information or information acquisition include Vayanos and Woolley (2013) and Basak and Pavlova (2013). The most relevant papers are by Kyle, Ou-Yang and Wei (2011) and Malamud and Petrov (2014). However, they only consider restricted contract space. More importantly, my research has new asset pricing implications, such as strategic complementarities.

idiosyncratic volatility comovement.

The model of this paper features delegated information acquisition, optimal contract design, and equilibrium asset pricing, introducing a two-period economy with one risky asset and one risk-free asset. The risky asset's payoff has two components: the first can be learned by agents and is called fundamental value, while the other cannot be learned and produces residual uncertainty. This economy has a market maker, noisy traders and a mass of principal-agent pairs. The principals are risk neutral while the agents are risk averse. Different principals cannot share agents, and different agents cannot share principals. Before trading, the principals choose whether to hire agents to acquire information regarding fundamental value. When deciding to hire agents, principals design optimal contracts that provide incentives for agents to acquire costly information, after which agents provide forecasts to their corresponding principals. The feasible contracts are general functions of agents' forecasts, the asset price and the payoff. I model agency problems by assuming that agents take hidden actions when acquiring information. When the market opens, the principals submit market orders to the market maker based on agents' forecasts. Having received all orders from the principals and the noisy traders, the market maker then sets the price.

The generality of this model relies on its broad interpretations. The principal-agent pairing can be interpreted as either that between fund managers and in-house analysts, or that between the pension fund trustees/board of directors (within funds) and fund managers. This model can unify both, because the optimal contract problems in the two contexts are essentially equivalent given that agents construct portfolios based on forecasts and principals can directly observe agents' portfolios. Therefore, the assumption regarding who invests is not crucial, and the aforementioned parsimonious model is a natural setting to study information acquisition incentives.

I show that the optimal contracts depend on the agents' forecasting accuracy for the asset price and the payoff. Agents can forecast the asset price and payoff accurately only if they acquire information. Thus, the agents' efforts are related to their forecasting accuracy, which determines their compensation. Specifically, agents receive high compensation when they forecast accurately - in contrast to an economy without agency problems, in which the compensation is constant. As an incentive for accurate forecasting, the bonus decreases with price informativeness and increases with residual uncertainty. When the price becomes more informative or residual uncertainty decreases, it is easier for agents to use information to forecast accurately and then receive high compensation. Consequently, agents are more willing to exert efforts and principals can accordingly provide fewer incentives. These results predict that the bonus is larger for professionals who trade small/growth stocks featuring greater residual uncertainty.

Furthermore, I find that the delegation of information acquisition exhibits strategic complementarities. Price informativeness has two counteractive effects: the first is to lower trading profit; and, the second is to mitigate agency problems. Whereas the first effect leads to standard strategic substitutability due to competition in trading, the strategic complementarities in information acquisition delegation originates from the effect of price informativeness on mitigating agency problems. When more principals hire agents to acquire information, the asset price becomes less noisy. As a result, agents are more willing to acquire information because they can forecast the asset price more accurately, and thus agency problems are mitigated. Clearly, strategic complementarities in information acquisition delegation emerge when price informativeness has a larger impact on mitigating agency problems than that on lowering trading profits. This only occurs when the residual uncertainty is large and compensation must consequently rely largely on agents' forecasts for the asset price. This mechanism causes principals to coordinate information acquisition delegation, therefore introducing the possibility of multiple equilibria. The multiplicity of equilibria may lead to the economy switching between low-information and high-information equilibria without any relation to fundamentals, leading to jumps in asset price and price informativeness.

This model, to my knowledge, is new to the literature to combine optimal contracts characterized in a general space, equilibrium asset pricing and delegated information acquisition. Meanwhile, it shows that the agency problem in information acquisition delegation is a new source for strategic complementarities. In particular, my model yields closed-form solutions for both optimal contracts and equilibrium asset pricing. Although this model is intentionally stylized to focus on information acquisition delegation, it captures realistic institutional features. Moreover, it has a number of implications as follows.

The first implication relates to home bias, a long-standing puzzle.[3] A plausible explanation is that investors have superior information on home assets. However, Van Nieuwerburgh and Veldkamp (2009) argue that investors can easily acquire information about other assets, which could eliminate the information advantage of home investors and mitigate home bias.[4] Although investors can freely acquire information, I show that agency problems lead to home bias: investors tend to acquire more information about assets for which they have an information advantage. I extend the model to consider two groups of principals (A and B) and two risky assets (X and Y); group A (B) is endowed with private information only about asset X (Y). I show that group A has higher incentives to acquire information on asset X relative to asset Y, and vice versa. Group A can use the endowed information to monitor agents, and thus group A's agency problems are less severe when hiring agents to acquire information about asset X relative to asset Y.[5] Consequently, group A is encouraged to hire agents to acquire information and trade more on asset X. This result is in direct contrast to that of the economy without agency problems, in which the decreasing marginal benefit of information discourages group A from acquiring information about asset X. Interpreting group A as home investors on asset X implies that agency problems can explain home bias.

The mechanism above for home bias can also explain industry bias: investors trade more on the assets within their expertise. This prediction is consistent with Massa and Simonov (2006), who document that Swedish investors buy assets highly correlated with their non-financial

[3]Home bias is well documented by Fama and Poterba (1991), Coval and Moskowitz (1999) and Grinblatt and Keloharju (2001). Despite large benefits from international diversification, Fama and Poterba (1991) find that households invest nearly all of their wealth in domestic assets. For example, they find that U.S households invest around 94% of their equity portfolio in the domestic market, while this number is 82% in the UK.

[4]Constraint on international capital flow may explain home bias. However, it is not a major concern currently. In particular, the recent studies (Seasholes and Zhu, 2010 and Coval and Moskowitz, 1999) find that households/fund managers also have a strong home bias in the U.S. market, which suggests this explanation is not satisfactory.

[5]Normally, the principals can use their private information in the subjective evaluation of agents. Even if the private information is not verifiable, some mechanisms, such as reputation concern, could reveal these information.

income. Moreover, because endowed information is more valuable in monitoring agents when the assets have greater residual uncertainty, the home/industry bias is stronger for these assets. This prediction is consistent with Kang and Stulz (1997) and Coval and Moskowitz (1999), who find that the home bias of U.S. fund managers is stronger when they trade small stocks.

The next implication relates to herding, defined as any behavioral similarity caused by interactions amongst individuals (Hirshleifer and Teoh, 2003). I extend the model to assume that each principal can choose to hire his agent to acquire either an exclusive signal or a common signal: the former is only accessible to his agent and is conditionally independent of others, while the latter is accessible to any agent. Under agency problems, I show that principals herd to acquire the common signal when the residual uncertainty is sufficiently large. Herding makes the price sensitive to the common signal itself. Thus, agents are willing to obtain the common signal because this allows them to easily forecast the asset price. In particular, when the residual uncertainty is large, herding emerges because its impact on mitigating agency problems is larger than that on lowering trading profit. This result is in clear contrast to that of the economy without agency problems, in which principals prefer the exclusive signals due to the substitute effect.

Moreover, my model has additional applications. For example, I show that idiosyncratic volatility comovement occurs in a multi-asset extension, in which principals incentivize agents to acquire information on each asset through their forecasting accuracy for the prices of assets with correlated fundamentals. An increase on one asset's idiosyncratic volatility, perhaps due to more noisy traders, discourages information acquisition and consequently leads to higher idiosyncratic volatilities on other correlated assets.

This paper is related to several strands of the literature. First, it is related to literature regarding the optimal contracting in delegated portfolio management, such as Bhattacharya and Pfleiderer (1985), Stoughton (1993), Dybvig, Farnsworth and Carpenter (2010) and Ou-Yang (2003). However, the asset prices play no roles in the aforementioned contracting work. My work on the contracting is most related to Dybvig et al. (2010). They study the optimal contract problem in a complete market, in which the asset price has no informational role;

they find that the optimal compensation involves a benchmark. In contrast to their work, I consider the optimal contracts in general equilibrium and the asset prices play informational roles. I find that the compensation depends on agents' forecasting accuracy for the asset prices and the payoffs.

My paper is also related to recent studies on the institutional investors, such as Basak, Shaprio and Tepla (2006), Basak, Pavlova and Shaprio (2007, 2008), Basak and Makarov (2014), Basak and Pavlova (2013), Dasgupta and Prat (2006, 2008), Dasgupta, Prat and Verardo (2011), Dow and Gorton (1997), He and Krishnamurthy (2012), He and Kondor (2013), Garcia and Vande (2009), Kaniel and Kondor (2013), Buffa, Vayanos and Woolley (2013), Kyle, Ou-Yang and Wei (2011) and Malamud and Petrov (2014). In particular, Buffa, Vayanos and Woolley (2013) study the joint equilibrium determination of optimal contracts and asset prices in a dynamic and multi-asset model. They focus on how the inefficiency of benchmarking arises endogenously and amplifies stock market volatility. However, these authors do not model moral hazard problems in information acquisition. The most relevant works are by Kyle, Ou-Yang and Wei (2011) and Malamud and Petrov (2014). Kyle, Ou-Yang and Wei (2011) consider a moral hazard problem between one principal and one agent in the Kyle (1985) model. They restrict the contract space and solely consider the linear contracts. Furthermore, Malamud and Petrov (2014) also focus on the restricted contract form, which consists of one proportional fee and one option-like incentive fee. My model differs from these papers in the following regard. First, I place no restrictions on the contract space. Second, I find that the agency problems generate strategic complementarities in information acquisition delegation, which is new to this literature.

Last, my paper is related to recent studies on the strategic complementarities, including Dow, Goldstein and Guembel (2011), Froot, Scharfstein and Stein (1992), Garcia and Strobl (2011) and Veldkamp (2006b). Froot, Scharfstein and Stein (1992) find that short-term investors herd to acquire similar information. Because they must liquidate assets before payoffs are realized, the short-term investors can profit on their information only if their information is reflected in future prices by the trades of similarly informed investors. Garcia and

Strobl (2011) find that relative wealth concern can generate complementarities. Because the investors' utilities are negatively affected by others, they tend to hedge others' impacts by following others' information acquisition decision. Dow, Goldstein and Guembel (2011) show that information acquisition complementarities emerge when the asset prices affect the firms' investments. Veldkamp (2006b) finds that when the information production has a scale effect, the selling price of information decreases as more investors buy information. In contrast to their work, the strategic complementarities in my model originates from the effect of price informativeness on mitigating agency problems in delegated information acquisition.

The paper is organized as follows. I introduce the model in Section 2 and solve the optimal contracts in Section 3. Section 4 shows the strategic complementarities and multiple equilibria. Section 5 studies three applications. Section 6 discusses the robustness. In particular, I solve a fully-fledged model with non-linear REE to show that the main results are robust in Section 6. Section 7 concludes.

1.2 Model

1.2.1 Economy

My model is built on Kyle (1985), in which investors submit market orders and a market maker sets the price according to the total order. My model deviates from Kyle (1985) in the following features: there are a mass of investors and each one has trading constraints.[6] Investors in my model trade in a competitive market, and no single individual investor has any price impact.

My economy has a mass of principal-agent pairs. The principals trade the risky asset and have incentives to acquire information for profits. However, these principals are unable to acquire information alone, perhaps because of large information acquisition or opportunity costs. Before trading, principals choose whether to hire agents to acquire information. Because agents' efforts are unobservable, a moral hazard problem arises within each pair. When deciding

[6] The assumptions of a mass of investors in which each one has trading constraints is not new (see Dow, Goldstein and Guembel, 2011, Goldstein, Ozdenoren and Yuan, 2013 and Malamud and Petrov, 2014).

to hire agents, principals design optimal contracts that provide incentives for agents to acquire information. In particular, the population of principals who hire agents is endogenous in my model. My analysis of optimal contracting is similar to that of Dybvig et al. (2010). In particular, I solve optimal contracts without any restriction on the contract space. The optimal contracts will induce agents to make costly efforts and truthfully report signals.

Timeline and Assets. My economy has three periods $t = 0, 1, 2$ and two assets. The first asset is risk-free and the second is risky. The risk-free asset is in zero supply and pays off one unit of consumption good without uncertainty at time $t = 2$. The payoff of the risky asset is denoted by D with two components: V and ϵ. V and ϵ are independent. I call V the fundamental value and ϵ the residual uncertainty. I assume that V depends on equally likely states, h and l, realized at time $t = 2$. V takes V_ω (where $\omega \in \{h, l\}$). Without a loss of generality, I assume that $V_h = \theta$ and $V_l = -\theta$, where $\theta > 0$. The residual uncertainty ϵ is uniformly distributed on $[-M, M]$, where $M > 0$.[7] At time $t = 0$, principals choose whether to delegate information acquisition to agents. When deciding to hire agents, principals write contracts with their agents. The contract is denoted by π. Otherwise, the principal does nothing at time $t = 0$. At time $t = 1$, the market opens and the principals submit market orders.[8] After receiving the total orders, a competitive market maker sets the price. I denote the risky asset's price by P.

Players. There are four types of players. The first type is principals, who choose whether to hire agents, design optimal contracts at $t = 0$, and trade the risky asset at $t = 1$. The second type is agents, who decide whether to accept the contracts and exert costly effort to acquire information about the fundamental value V. The third type is noisy traders, and the last type is a risk-neutral competitive market maker.

There are a mass of principal-agent pairs. Each pair is indexed by $i \in [0, \infty)$. Within each pair i, I denote its principal by principal i and denote its agent by agent i. To simplify the

[7]The assumptions about θ and ϵ are made only to obtain an analytical solution and make the mechanism clear. I will show numerically that the mechanism is robust when θ and ϵ follow more general distributions.

[8]The assumption about market orders is to obtain closed-form solution without losses of any economic insights. In the extension, I allow principals to learn information from the price and then submit limit orders. The numerical results show that the main results are robust.

analysis, I assume that different principals can not share agents, and vice versa. Each pair can be interpreted as one mutual/hedge fund. There can be many interpretations of principal-agent pairs, such as principals as board directors of funds and agents as fund managers/in-house analysts. Moreover, I assume that the total demand from noisy traders is n, which follows a uniform distribution on $[-N, N]$, where $N > 0$.

Agency Problem. Agent i's effort is denoted by $e_i \in \{0, 1\}$. When agent i exerts effort, $e_i = 1$; otherwise, $e_i = 0$. After exerting effort, agent i generates a private signal $s_i \in \{h, l\}$ regarding the risky asset's fundamental value V. I denote the probability with which a signal is correct by

$$\begin{aligned} pe_i + \frac{1}{2}(1 - e_i) &= prob(s_i = h|V = \theta) \\ &= prob(s_i = l|V = -\theta), \end{aligned}$$

where s_i is conditionally independent across agents and $p > \frac{1}{2}$. If agent i shirks, his signal is pure noise. If agent i exerts effort, his signal is informative. If I let $prob(s_i)$ be the unconditional probability of signal s_i, I obtain $prob(s_i = h) = prob(s_i = l) = \frac{1}{2}$. Let $prob^I(V|s_i)$ be the probability of V conditional on signal s_i if agent i exerts effort, and let $prob^U(V|s_i)$ be the probability of V conditional on signal s_i if he shirks. I then have the following:

$$prob^I(V = \theta|s_i = h) = prob^I(V = -\theta|s_i = l) = p, \tag{1.1}$$

$$prob^U(V = \theta|s_i = h) = prob^U(V = -\theta|s_i = l) = \frac{1}{2}. \tag{1.2}$$

To acquire information, each agent bears a utility loss. I assume that all agents have the same CARA utility function $-\exp(-\gamma_a\pi + \gamma_a C)$, where π is compensation, C is information acquisition cost and γ_a is risk aversion.[9] All agents have zero initial wealth. Due to hidden actions, there are moral hazard problems followed by truth telling problems between principals and agents.

[9]When I model agents' utility nesting cost as $-\exp^{-\gamma_a\pi} - C$, the results do not change. In particular, when I consider general HARA utility function for agents, the main results are robust as shown later.

Information Acquisition and Trading. At time $t = 0$, some principals hire agents to acquire information. The population of these principals is denoted by λ, where λ is endogenous. I call these principals informed principals; others are referred to as uninformed principals. While deciding to hire agents, informed principal i writes a contract π_i with agent i. At time $t = 1$, all contracts and λ become public information. Upon receiving report s_i from his agent, informed principal i submits a market order X_i conditional on the report to maximize his utility over final wealth $W_{i,1}$, where $W_{i,1} = W_0 + X_i(D - P) - \pi_i$, and $X_i \in [-1, 1]$. This limited position is due to frictions, such as leverage constraint or limited wealth. Then, uninformed principals submit market order X_U, where $X_U = 0$ due to symmetric distributions of the asset payoff or price. Given the contracts beforehand, the informed principal i's optimization problem in trading is the following:

$$\max_{X_i} E(W_0 + X_i(D - P) - \pi_i | s_i). \tag{1.3}$$

The total orders received by the competitive risk-neutral market maker are

$$X = \int_{i=0}^{\lambda} X_i di + n. \tag{1.4}$$

The market maker sets a price equal to the risky asset's expected payoff conditional on X:

$$P = E(D|X). \tag{1.5}$$

Contracting Problem. With agency problems, principals design optimal contracts π that provide incentives for agents to acquire information at time $t = 0$. In accordance with Dybvig et al. (2010), this type of contract induce agents to exert effort and report the true signals. Because Dybvig et al. (2010) assume that the market is complete, there is no informational role of the price. However, the market is not complete in my model. Moreover, the asset price plays an informational role in monitoring agents because it aggregates information from all principals. The contracts in my model are general functions of agents' reports, the asset price and payoff. The agents either accept or reject the contracts. If agents accept the contracts, they exert costly efforts in information acquisition. After acquiring information, they report their signals to the corresponding principals. The specific contract provided by principal i is a general function $\pi_i(s^R(s_i), P, D)$, where $s^R(s_i)$ is agent i's report conditional on his realized signal s_i.

To formalize my analysis, I consider two problems: the first-best and the agency problem. The first-best problem assumes that each agent's costly effort and signal can be observed by his principal. This problem may not be realistic, but is useful for further comparison. In the agency problem, agents' efforts and signals are unobservable. There is a moral hazard problem followed by a truth telling problem. The revelation principle guarantees that I can focus solely on the contracts that induce agents to truthfully report signals after exerting efforts. The detailed analysis of the two problems follows:

First-best. Principal i chooses $\pi_i(s^R(s_i), P, D)$ at time $t = 0$ and submits demand X_i at time $t = 1$ to maximize his expected utility:

$$\max_{\pi_i(s_i,P,D),X_i(s_i,\pi_i)} \sum_{s_i=\{h,l\}} prob(s_i) \int\int [W_0 + X_i(D-P) - \pi_i(s_i, P, D)] f^I(P, D|s_i) dPdD, \quad (1.6)$$

where $f^I(P, V|s_i)$ is the conditional joint probability density function when agent i acquires information. In the first-best problem, principals design contracts subject to agents' participation constraint,

$$\sum_{s_i=\{h,l\}} prob(s_i) \int\int [-\exp^{-\gamma_a \pi_i(s_i,P,D)+\gamma_a C}] f^I(P, D|s_i) dPdD = -\exp(-\gamma_a W_a), \quad (1.7)$$

where LHS of Equation (1.7) is agent i's expected utility given the premise that he exerts costly effort and reports the true signal. Moreover, W_a is the reserve wealth of agents, which can be interpreted as the agents' outside options.

Agency Problem. In the agency problem, the contract satisfies two type of ICs, including the *Ex Ante IC,* which is the incentive-compatibility of effort exerting

$$\begin{aligned}
&\sum_{s_i=\{h,l\}} prob(s_i) \int\int [-\exp^{-\gamma_a \pi_i(s_i,P,D)+\gamma_a C}] f^I(P,D|s_i)dPdD \\
&\geq \sum_{s_i=\{h,l\}} prob(s_i) \int\int [-\exp^{-\gamma_a \pi_i(s^R(s_i),P,D)}] f^U(P,D|s_i)dPdD,
\end{aligned} \tag{1.8}$$

and the *Ex Post IC,* which is the incentive-compatibility of truth reporting($\forall s_i$ and $s^R(s_i)$: $s \to s$)

$$\int\int [-\exp^{-\gamma_a \pi_i(s_i,P,D)}] f^I(P,D|s_i)dPdD \geq \int\int [-\exp^{-\gamma_a \pi_i(s^R(s_i),P,D)}] f^I(P,D|s_i)dPdD, \tag{1.9}$$

where $f^U(P,V|s_i)$ is the conditional joint probability density function when agent i shirks. The RHS of Equation (1.8) is agent i's expected utility when he shirks. Then, $f^U(P,V|s_i) = f(P,V)$, which is the unconditional joint probability density function. Equation (1.9) induces agents to truthfully report their signals. For any realized signal s_i, the LHS of Equation (1.9) is agent i's utility if he reports the truth signal, whereas RHS of Equation (1.9) is the agents i's utility if he misreports.

Principal i's choice variables are contingent fees $\pi_i(s_i,P,D)$ and a demand schedule $X_i(s_i)$. Each principal i maximizes his utility through simultaneous decisions over trading and optimal contracting. The trading decisions and optimal contracts depend on the population of informed principals. In the equilibrium, the population of informed principals λ renders the expected utility of informed and uninformed principals equal; the difference in utilities between the two types of principals is the expected net benefit of information. I denote the expected net benefit of information by B, where B is the difference between the maximum value of optimization problem in Equation (1.6) and the initial wealth W_0. It is clear that B is difference between the trading profit for informed principals and the expected compensation to agents.

1.2.2 Discussion

Before proceeding, I discuss the assumptions of my model. First, I assume that the principals trade by alone and only agents acquire information. Although this assumption is stylized, my model has broad interpretations. The most direct interpretation is that the principals are fund managers and the agents are in-house analysts. The in-house analysts collect information and report forecasts to fund managers, who trade based on the forecasts. However, the assumption about who invests is not crucial, as is evident if I assume that agents trade instead of principals and that principals can observe or infer agents' contractible portfolios. Because agents construct portfolios based on forecasts, the contracts written upon agents' portfolios, the asset price and the payoff can be transformed into the contracts directly written on agents' forecasts, the asset price and the payoff. In practice, the pension fund trustees/board directors of funds can observe the fund managers' portfolios. Therefore, an alternative interpretation is that the pension fund trustees/board directors of funds, who maximize the households' interests, hire fund managers to simultaneously collect and trade on information. Another interpretation is that the principals are households and the agents are fund managers. Because mutual/hedge funds must disclose their holdings regularly, households could infer the beliefs of fund managers through holding data, although they are noisy(see Kacperczyk, Sialm and Zheng, 2007, Cohen, Polk, Silli, 2010 and Shumway, Szefler and Yuan, 2011). Although households can not choose the management fee, they can use fund flow to provide incentives for fund managers. The fund flow can be viewed as a form of implicit contract.

Furthermore, in accordance with the literature, I assume that the principals are risk-neutral. This assumption simplifies my analysis, while capturing the features of the practice. In practice, principals, such as households or mutual/hedge funds can diversify risks alone. For example, households can allocate money to different assets to diversify risk. In particular, if principals are risk averse, the contracts include a risk-sharing component. However, this risk-sharing component does not overturn my mechanism: an increase in the population of informed principals makes the price more informative and mitigates the agency problems.

The third assumption is that the principals submit market orders and do not learn information from the asset prices. This assumption is not crucial in my model. Introducing learning enables uninformed principals to free ride informed principals by learning information from the price; this affects principals' incentive to acquire information. However, this free-riding problem only affects the strength of the driving force, and will not overturn my mechanism. In particular, this assumption captures my idea in a more complicated dynamic framework, in which there are multiple rounds of trading and principals solely observe current and past prices. It is obvious that such settings will only complicate the model, leading to a loss of tractability, without adding much economic insight. In particular, the numerical results in one extension show that the strategic complementarities are robust when principals can learn information from the asset price.

1.3 Equilibrium

1.3.1 Equilibrium Definition

I formally introduce the equilibrium concept in this section. I focus on symmetric equilibrium with identical contracts. Before trading, principals choose whether to hire agents to acquire information and the population of these principals is endogenous. These principals design optimal contracts that provide incentives for their agents to acquire information and report truthfully. Given these contracts, all principals submit optimal demands when the market opens and a risk-neutral market maker sets the price after receiving the total orders.

DEFINITION **1.3.1.** *A symmetric equilibrium is defined as a collection: a price function P set by a risk-neutral competitive market maker, $P(X) : \mathbb{R} \to \mathbb{R}$; an optimal demand schedule for each principal i, $X_i(s_i) : \mathbb{R} \to \mathbb{R}$; an optimal contract designed by each principal i, $\pi_i(s_i, P, D) : \mathbb{R}^3 \to \mathbb{R}$; and an equilibrium population of principals hiring agents to acquire information, λ. This collection satisfies the following:*

(1) Given the price function solved in Equation (1.5) and the demand schedule solved in Equation (1.3), principal i designs optimal contract $\pi_i(s_i, P, D)$ and the optimal contract problem is equivalent to the problem in Equation (1.6) subject to constraints (1.7), (1.8), and(1.9),

(2) Given contract $\pi_i(s_i, P, D)$, agent i decides whether to accept or reject this contract,

(3) Given the price function in Equation (1.5) and the optimal contract $\pi_i(s_i, P, D)$, prin-

cipal i submits demand X_i to solve Equation (1.3),

(4) A risk-neutral competitive market maker sets the price as the risky asset's expected payoff conditional on total orders. The pricing function is solved in Equation (1.5),

(5) If there exists a positive solution to $B(\lambda) = 0$, an equilibrium with information acquisition is obtained. Otherwise, an equilibrium of no information acquisition is obtained ($\lambda = 0$).

(6) All contracts are identical in this economy.

1.3.2 Equilibrium Characterization

I characterize the equilibrium as one featuring trading strategies and optimal contracting by principals, and a pricing rule by the market maker. I follow a step-by-step approach to illustrate this idea.

Step 1. I first solve for the principals' trading decisions and the market maker's pricing rule given the contracts designed beforehand and the population of informed principals. When the market opens at $t = 1$, the informed principal i submits X_i to maximize $W_0 + X_i(D - P) - \pi_i$, which is his final wealth. Furthermore, uninformed principals submit $X_U = 0$. Because the principals are risk-neutral, there is no hedging demand, and the informed principal i submits $X_i = 1$ after agent i reports $s_i = h$ and submits $X_i = -1$ after agent i reports $s_i = l$. Following the large number theorem, when fundamental value $V = \theta$, the total number of buy orders from informed principals is λp and the total number of sell orders is $\lambda(1 - p)$. Thus the total order received by the market maker is $X = \lambda(2p - 1) + n$. Similarly, the total order received by the market maker is $X = -\lambda(2p - 1) + n$ when $V = -\theta$. Therefore, the total order X is distributed on $[-\lambda(2p - 1) - N, \lambda(2p - 1) + N]$.

Receiving total orders X, the risk-neutral market maker updates his beliefs and sets the price as the risky asset's expected payoff: $P = E(D|X)$. If $-\lambda(2p - 1) + N < \lambda(2p - 1) - N$, the total orders can fully reveal information regarding V and I have $P = V$, which leads to zero trading profits for informed principals. This is impossible because the principals need to pay costs for information. Thus I have the formal lemma regarding the population of informed principals.

LEMMA **1.3.1.** *The population of informed principals satisfies the following:*

$$\lambda < \frac{N}{2p-1}\,. \tag{1.1}$$

This lemma is helpful for further analysis. Then, I have the following lemma regarding price:

LEMMA **1.3.2.** *Given λ and contract $\pi(s,P,D)$, the price follows the rule:*

$$P(X) = \begin{cases} \theta & if \quad N-\lambda(2p-1) < X \le N+\lambda(2p-1)\,, \\ 0 & if \quad -N+\lambda(2p-1) \le X \le N-\lambda(2p-1)\,, \\ -\theta & if \quad -N-\lambda(2p-1) \le X < -N+\lambda(2p-1)\,. \end{cases} \tag{1.2}$$

Lemma 1.3.2 shows that the price increases with the total orders X due to the correlation between the total orders and the fundamental value V. However, with noisy traders, the total orders do not fully reveal V. In particular, the probability that the price equals V is the following:

$$prob(P=V|V) = \frac{\lambda(2p-1)}{N}. \tag{1.3}$$

This probability measures price informativeness. This probability increases with the population of informed principals and the precision of signals, and decreases with the variance of noisy traders' demand.

Step 2. I solve the informed principals' optimal contracts at $t=0$. As Lemma 1.3.2 implies, the asset price is informative regarding V. Thus principals will use the price to monitor agents. The contracting problem is reduced to the optimization problem in Equation (1.6) subject to constraints (1.7), (1.8), and (1.9). Due to risk-neutrality, the principals' trading decisions and contracting problems are independent. Then, the contracting problem can be transferred to the following:

$$\max_{\pi_i(s_i,P,D)} \sum_{s_i=\{h,l\}} prob(s_i) \int\int [-\pi_i(s_i,P,D)] f^I(P,D|s_i) dPdD, \tag{1.4}$$

Equation 1.4 shows that principals minimize expected compensation subject to participant constraint and incentive compatibility. However, if the residual uncertainty is sufficiently small, the asset payoff D is perfectly informative about V and thus there is no role of asset price in the contracting, which is not interesting. To avoid this case, I make the following assumption regarding M:

Assumption **1.3.1.** *M satisfies: $M \geq \theta$.*

From Dybvig et al. (2010), the joint conditional pdf or conditional probability of P and D play important roles in optimal contracts. Thus I characterize the joint conditional pdf or the conditional probability of P and D before I solve the optimal contracts. If agent i exerts effort, signal s_i is informative about V and this indicates that $prob^I(V=\theta|s_i=h) = prob^I(V=-\theta|s_i=l) = p$. Then, I have the following lemma:

Lemma **1.3.3.** *When s_i is informative about V, the conditional pdf is as follows:*

(1) conditional on $s_i = h$,

$$f^I(P=\theta, D|s_i=h) = \begin{cases} \frac{p}{2M}\frac{\lambda(2p-1)}{N} & if \quad -M+\theta \leq D \leq M+\theta \\ 0 & if \quad -M-\theta \leq D < -M+\theta \end{cases} \tag{1.5}$$

$$f^I(P=0, D|s_i=h) = \begin{cases} \frac{p}{2M}\frac{N-\lambda(2p-1)}{N} & if \quad M-\theta \leq D \leq M+\theta \\ \frac{1}{2M}\frac{N-\lambda(2p-1)}{N} & if \quad -M+\theta \leq D < M-\theta \\ \frac{1-p}{2M}\frac{N-\lambda(2p-1)}{N} & if \quad -M-\theta \leq D < -M+\theta \end{cases} \tag{1.6}$$

$$f^I(P=-\theta, D|s_i=h) = \begin{cases} 0 & if \quad M-\theta < D \leq M+\theta \\ \frac{(1-p)}{2M}\frac{\lambda(2p-1)}{N} & if \quad -M-\theta \leq D \leq M-\theta \end{cases} \tag{1.7}$$

(2) conditional on $s_i = l$,

$$f^I(P=\theta, D|s_i=l)=\begin{cases}\frac{(1-p)}{2M}\frac{\lambda(2p-1)}{N} & if \quad -M+\theta\leq D\leq M+\theta\\ 0 & if \quad -M-\theta\leq D<-M+\theta\end{cases} \tag{1.8}$$

$$f^I(P=0, D|s_i=l)=\begin{cases}\frac{1-p}{2M}\frac{N-\lambda(2p-1)}{N} & if \quad M-\theta\leq D\leq M+\theta\\ \frac{1}{2M}\frac{N-\lambda(2p-1)}{N} & if \quad -M+\theta\leq D<M-\theta\\ \frac{p}{2M}\frac{N-\lambda(2p-1)}{N} & if \quad -M-\theta\leq D<-M+\theta\end{cases} \tag{1.9}$$

$$f^I(P=-\theta, D|s_i=l)=\begin{cases}0 & if \quad M-\theta<D\leq M+\theta\\ \frac{p}{2M}\frac{\lambda(2p-1)}{N} & if \quad -M-\theta\leq D\leq M-\theta\end{cases} \tag{1.10}$$

If agent i shirks, signal s_i is uninformative regarding V and this indicates that $prob^U(V=\theta|s_i=h)=prob^U(V=-\theta|s_i=l)=\frac{1}{2}$. Then, I have the following lemma:

Lemma **1.3.4.** *When s_i is uninformative about V, the conditional pdf is as follows:*

$$f^U(P=\theta, D)=\begin{cases}\frac{1}{4M}\frac{\lambda(2p-1)}{N} & if \quad -M+\theta\leq D\leq M+\theta\\ 0 & if \quad -M-\theta\leq D<-M+\theta\end{cases} \tag{1.11}$$

$$f^U(P=0, D)=\begin{cases}\frac{1}{4M}\frac{N-\lambda(2p-1)}{N} & if \quad M-\theta\leq D\leq M+\theta\\ \frac{1}{2M}\frac{N-\lambda(2p-1)}{N} & if \quad -M+\theta\leq D<M-\theta\\ \frac{1}{4M}\frac{N-\lambda(2p-1)}{N} & if \quad -M-\theta\leq D<-M+\theta\end{cases} \tag{1.12}$$

$$f^U(P=-\theta, D)=\begin{cases}0 & if \quad M-\theta<D\leq M+\theta\\ \frac{1}{4M}\frac{\lambda(2p-1)}{N} & if \quad -M-\theta\leq D\leq M-\theta\end{cases} \tag{1.13}$$

Lemma 1.3.3 shows that s_i is correlated with the asset price or the payoff when it is informative. Lemma 1.3.4 shows that s_i is uncorrelated with the asset price or payoff when it is pure noise. Thus agents' efforts are tied to the accuracy of their forecasts for the asset price and the payoff.

To simplify the optimization problems, in accordance with Grossman and Hart (1983) and

Dybvig et al. (2010), I transfer the choice variables. I let:

$$v(s_i, P, D) = \exp[-\gamma_a \pi(s_i, P, D]. \tag{1.14}$$

I can rewrite the contracting problem in a similar form, in which choice variable becomes $v(s_i, P, D)$. Then principal i's contracting problem becomes:

$$\max_{v_i(s_i,P,D)} \sum_{s_i=\{h,l\}} prob(s_i) \int\int \frac{1}{\gamma_a} \log[v_i(s_i, P, D)] f^I(P, D|s_i) dPdD, \tag{1.15}$$

subject to constraints (1.7), (1.8), and (1.9). Then, I use the first-order approach to solve the optimal contracts in both the first-best and the agency problem.

Proposition **1.3.1.** *(First-Best) The optimal contract in the first-best problem is:* $\pi_i(s_i, P, D) = W_a + C$ *.*

Proposition 1.3.1 shows that agents' compensation is constant in the first-best problem. This is slightly different from the previous literature, which assumes that investors are risk-averse and finds that compensation is a proportional fee for risk-sharing purpose. However, I assume that principals are risk-neutral. Therefore, principals do not care about risk and there is no role for risk-sharing. In fact, the compensation is equal to agents' reserve wealth and information acquisition cost. This case is used later for comparative purposes with the agency problem.

Assumption **1.3.2.** *C satisfies:* $C < -\frac{\log 2 + \log(1-p)}{\gamma_a}$ *.*

Assumption 1.3.2 is important because it guarantees that the optimal contract is implementable in the agency problem. I conduct the analysis with agency problems under Assumption 1.3.2.

Proposition **1.3.2.** *(Agency Problem) Given λ, there exists one unique optimal contract in the economy with agency problems. There are two cases regarding optimal contract as follows:*

(1) when $p = 1$, the first-best can be achieved. The optimal contract is the following:

$$\pi_i(s_i, P, D) = \begin{cases} -\infty & if \quad s_i = h \ and \ P = -\theta \\ -\infty & if \quad s_i = h \ and \ D < -M + \theta \\ -\infty & if \quad s_i = l \ and \ P = \theta \\ -\infty & if \quad s_i = l \ and \ D > M - \theta \\ W_a + C & otherwise \end{cases} \tag{1.16}$$

(2) when $p < 1$, the optimal contract is the following:

$$\pi_i(s_i = h, P = \theta, D) = \pi_i(s_i = l, P = -\theta, D) = \frac{\log x}{\gamma_a} \tag{1.17}$$

$$\pi_i(s_i = h, P = -\theta, D) = \pi_i(s_i = l, P = \theta, D) = \frac{\log y}{\gamma_a} \tag{1.18}$$

$$\pi_i(s_i = h, P = 0, D) = \begin{cases} \frac{\log x}{\gamma_a} & if \quad M - \theta < D \leq M + \theta \\ \frac{\log[px+(1-p)y]}{\gamma_a} & if \quad -M + \theta \leq D \leq M - \theta \\ \frac{\log y}{\gamma_a} & if \quad -M - \theta \leq D < -M + \theta \end{cases} \tag{1.19}$$

$$\pi_i(s_i = l, P = 0, D) = \begin{cases} \frac{\log y}{\gamma_a} & if \quad M - \theta < D \leq M + \theta \\ \frac{\log[px+(1-p)y]}{\gamma_a} & if \quad -M + \theta \leq D \leq M - \theta \\ \frac{\log x}{\gamma_a} & if \quad -M - \theta \leq D < -M + \theta \end{cases} \tag{1.20}$$

where x and y are defined in the Appendix. In particular, $x > y$.

Proposition 1.3.2 has several interesting features. First, when the signals acquired by agents are perfectly informative ($p = 1$), the first-best can be achieved through an infinite penalty for incorrect forecasts. Given the finite support of the asset price or the asset payoff, if the asset price or payoff deviates to a large extent from the forecasts, the principals know that the agents are shirking. For example, when agent i acquires information and then reports $s_i = h$, it is impossible that the price is $-\theta$. This infinite penalty achieves the first best.[10] Second, when the signals acquired by agents are not perfectly informative ($p < 1$), the optimal compensation depends on the agents' forecasting accuracy for the asset price and the payoff. For example, when agents report $s_i = h$, agents receive high compensation when the price or payoff is high and low compensation when the price or payoff is low. Agents can forecast the asset price and the payoff accurately if they acquire information. Thus, the forecasting accuracy is related to agents' efforts. This compensation will encourage agents to exert effort and tell the truth. When $p = 1$, the first-best can be achieved, which is not analytically interesting. Thus I focus on the case in which $p < 1$ in the following analysis. I formally state the assumption regarding p as follows:

Assumption **1.3.3.** *p satisfies: $p < 1$.*

1.3.3 Characteristics of Optimal Contract

I show the characteristics of the optimal contract in this section. I focus on how the price informativeness or residual uncertainty affects the compensation.

The bonus, defined by the difference between agents' compensations when they forecast correctly and incorrectly, provides incentives for agents to exert effort. It is given as follows:

Definition **1.3.2.** *The bonus is defined as S_f: $S_f = \frac{\log x - \log y}{\gamma_a}$.*

Because λ measures the price informativeness and M measures the residual uncertainty in the asset's payoff, I show their effects on bonuses as follows:

Proposition **1.3.3.** *Bonus S_f decreases with λ, but increases with M.*

[10] In this basic model, I assume that agents have CARA utilities and do not have limited liability. Thus the infinite penalty can be interpreted as infinite disutilities. For example, if agents have log utilities, $\pi = 0$ provides an infinite penalty for agents. I discuss more general utilities for agents in the following sections.

Proposition 1.3.3 shows that S_f decreases with price informativeness and increases with residual uncertainty. In fact, when the price becomes more informative, agents can forecast the asset price more accurately with information. Agents are therefore more willing to exert efforts. As a result, principals can provide less incentive, which is characterized as a decreased bonus. Similarly, the bonus increases with residual uncertainty. In particular, because both the asset price and the payoff are used in the incentive provision, their effects depend on each other. I then have the following result:

COROLLARY **1.3.1.** *When $\theta = M$, λ has no effect on S_f, that is $\frac{\partial S_f}{\partial \lambda} = 0$ if $\theta = M$.*

When $\theta = M$, the asset payoff is perfectly informative about the fundamental value. Thus, principals solely use the asset payoff in the contracts.

1.4 Agency Problem and Information Acquisition Complementarity

In this section, I show how agency problems in delegated information acquisition affect the financial market. I show that agency problems generate complementarities and multiple equilibria.

1.4.1 First-Best Case

Informed principal i's final wealth $W_{1,i}$ has two components: the first is trading profit, which is $X_i(D-P)$; the second is agents' compensation π_i. Informed principals' expected trading profit is denoted as E_p, where $E_p = E[X_i(D-P)]$. Thus the expected net benefit from information is $B = E[X_i(D-P) - \pi_i]$. Informed principals' expected trading profit is shown as follows:

LEMMA **1.4.1.** *Informed principals' expected trading profit: $E_p = \theta(2p-1)\frac{N-\lambda(2p-1)}{N}$.*

Lemma 1.4.1 shows that informed principals' trading profits decrease with the population of informed principals because of competition in trading. This effect is called the strategic substitute effect. Because compensation is constant in the first-best problem, the net benefit from information decreases with the population of informed principals. The result is shown as follows:

Proposition **1.4.1.** *(First-Best) Information acquisition is a strategic substitute in the first-best problem, that is* $\frac{\partial B}{\partial \lambda} < 0$.

1.4.2 Agency Problem

With agency problems, the compensation depends on the accuracy of agents' forecasts for the asset price and payoff. In particular, the bonus decreases with the population of informed principals, which leads to decreased compensation, and is the source of the strategic complementarity effect. When the residual uncertainty is large, there is a strategic complementarity effect in the information acquisition delegation; otherwise, there is only a strategic substitute effect. When the residual uncertainty is large, the principals rely largely on agents' forecasts for the asset price to incentive them. Thus, price informativeness has a larger impact on mitigating agency problems than lowering trading profit, which generates strategic complementarities. When the residual uncertainty is small, only the substitute effect exists because price informativeness has little impact on mitigating agency problems. The result of information acquisition delegation is shown as follows:

Proposition **1.4.2.** *(Agency Problem) In an economy with agency problems, I have the following:*

(1) for a sufficiently small M*, the information acquisition delegation is a strategic substitute. That is,* $\frac{\partial B}{\partial \lambda} < 0$.

(2) for a sufficiently high M*, there exists* λ^c *satisfying the following: when* $\lambda < \lambda^c$*, the information acquisition delegation is a strategic complement. That is* $\frac{\partial B}{\partial \lambda} > 0$.

1.4.3 Multiplicity of Equilibria

As shown in Grossman and Stiglitz (1980) and Hellwig (1980), one unique equilibrium in information acquisition exists with a strategic substitute effect. However, the strategic complementarities may generate multiple equilibria (Dow, Goldstein and Guembel, 2011, Garcia and Strobl, 2011, Goldstein, Li and Yang, 2013 and Veldkamp, 2006a). Proposition 1.4.2 shows that agency problems produce strategic complementarities when the residual uncertainty is large. Thus, multiple equilibria may emerge in this case. This result is important because it may explain asset price jumps and excess volatilities in the financial market. The equilibrium populations of informed principals in the first-best and agency problem are denoted by λ_{fb} and λ_{sb}, respectively. Because there is a substitute effect in the first-best problem or in the agency problem with low residual uncertainty, tone unique equilibrium exists in both cases, which is shown as follows:

LEMMA **1.4.2.** *There exists one unique equilibrium λ_{fb} regarding information acquisition delegation in the first-best problem.*

LEMMA **1.4.3.** *When M is sufficiently small, there exists one unique equilibrium λ_{fb} regarding information acquisition delegation in the economy with agency problems.*

Because the contract is very complex, I do not characterize all equilibria in the agency problem with large residual uncertainty. However, my goal is to demonstrate the existence of multiple equilibria. In particular, no information acquisition delegation may emerge as one of the equilibria.

PROPOSITION **1.4.3.** *(Agency Problem) When M is sufficiently large, there are three cases regarding information acquisition delegation in the economy with agency problems,*
(1) when $\theta(2p-1) + \log[\exp^{-\gamma_a W_a} - \frac{\exp^{-\gamma_a W_a}(1-\exp^{-\gamma_a C})}{2p-1}] > 0$, all equilibria are with positive population of informed principals, and at at least one equilibrium exists.
(2) when $\theta(2p-1) + \log[\exp^{-\gamma_a W_a} - \frac{\exp^{-\gamma_a W_a}(1-\exp^{-\gamma_a C})}{2p-1}] < 0$ and $\max_{\lambda<\lambda_{fb}} B_{ap}(\lambda) > 0$, there exists at least three equilibria, one of which is $\lambda_{sb} = 0$.

(3) when $\max_{\lambda<\lambda_{fb}} B_{ap}(\lambda) < 0$*, the unique equilibrium is no information acquisition delegation. That is* $\lambda_{sb} = 0$*.*

Proposition 1.4.3 shows that agency problems may generate multiple equilibria. When the information acquisition cost is low (first case), agency problems are not severe and principals have incentives to hire agents. In fact, when the information acquisition cost is high (third case), agency problems are severe and thus no principals have incentives to hire agents. In the second interesting case when information acquisition is neither too high nor too low, agency problems produce multiple equilibria, and non-information is one of these equilibria. When residual uncertainty is high, principals must rely heavily on the asset price in the incentive provision. However, when no principals hire agents to acquire information, the price does not incorporate any information, and the incentive provision from asset price fails. Consequently, agency problems are severe, which deters principals from hiring agents. All results are shown in Figure 4.1.[11]

This proposition has implications for asset price jumps or excess volatilities. With multiple equilibria regarding information acquisition, the economy may switch between non-information equilibrium and high-information equilibria without any relation to fundamentals, leading to jumps in the asset price and informativeness. Because a jump is an extreme form of excess volatilities, the same mechanism can also cause excess volatilities in asset price and informativeness. This result implies that the price informativenesses and institutional ownership are more volatile for small/growth stocks or during recessions, which are usually associated with large residual uncertainties. This result also implies that price jumps and excess volatilities are more likely to occur for small/growth stocks or during recessions, which is consistent with Bennet, Sias and Starks (2003), Campbell, Lettau, Malkiel and Xu (2001), Xu and Malkiel (2003), Ang, Hodrick and Zhang (2006, 2009) and Bekaert, Hodrick and Zhang (2012).

I also examine how agency problems affect asset pricing behavior. I focus on the analysis of price informativeness and return volatility. For price informativeness, because the equilibrium

[11] I set $\theta = 2$, $N = 2$, $p = 0.6$, $W_a = 0$, $C = 0.07$. I also set $M = 5$, $M = 20$ and $M = 200$ for low residual uncertainty, median residual uncertainty and high residual uncertainty cases, respectively.

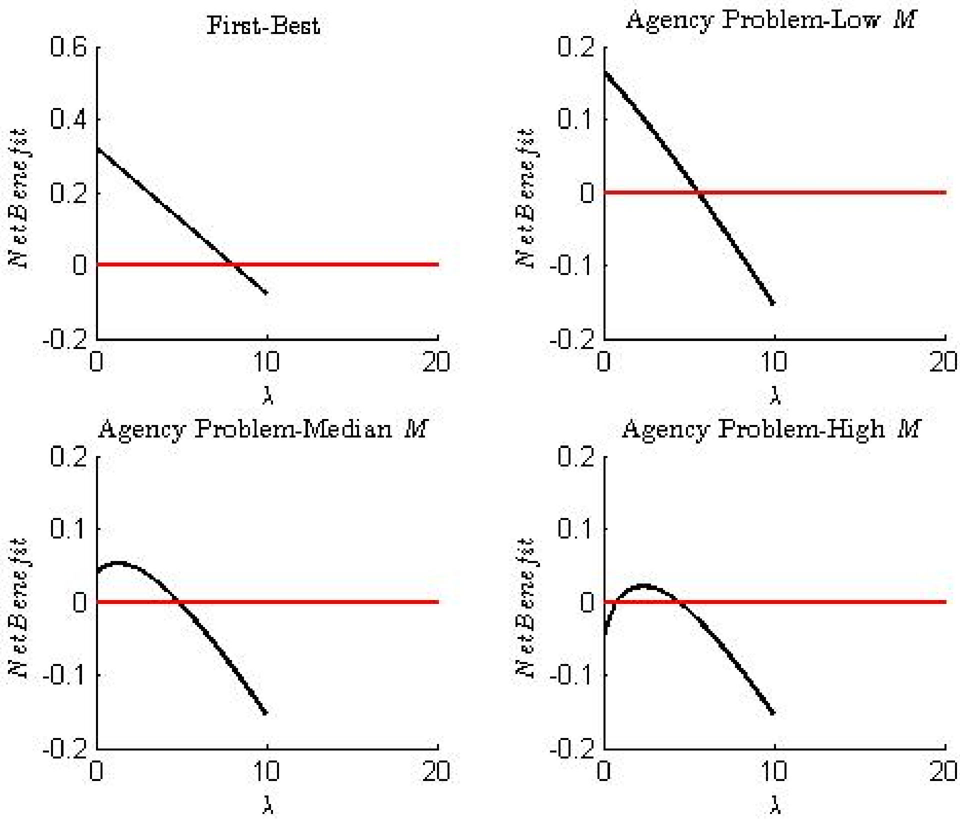

Figure 1.4.1: Information Acquisition Benefit

is not a linear function of fundamental value or noisy traders' demand, the conditional variance $Var(D|P)$ in the conventional literature is not appropriate for my analysis because this measure depends on the price P. In accordance with Malamud and Petrov (2014), I use the price's expected error as price informativeness. When the price is more informative, this expected error is lower:

$$E(|V-P||V) = \frac{\theta[N-\lambda(2p-1)]}{N}. \tag{1.1}$$

For volatility, I calculate the asset return's volatility $Var(V-P)$ as follows:

$$Var(V-P) = \frac{M^2}{3} + \frac{\theta^2[N-\lambda(2p-1)]}{N}. \tag{1.2}$$

When the population of informed principals increases, both the expected error of the price and the volatility decrease. Before proceeding, I know that agency problems negatively affect the net benefit from information, which decreases the prices informativeness. Then, price becomes more sensitive to noisy traders' demand, leading to increased volatility. I denote B_{ap} as the net benefit of information in an economy with agency problems, and denote B_{fb} as the net benefit in the first-best problem. I find the following result:

LEMMA **1.4.4.** *Given λ, the net benefit in the agency problem is lower than the first-best problem. That is $B_{ap} < B_{fb}$.*

I then have the formal result regarding price informativeness and volatility.

PROPOSITION **1.4.4.** *Both price's expected error and volatility are higher in an economy with agency problems than the first-best problem.*

I examine how different parameters affect the population of informed principals. I focus on the case in which M is small because a unique equilibrium exists in this case. When the agents' risk aversion increases, the agency problem becomes more severe and the principals need to provide higher compensation to agents. Thus, I expect that the equilibrium population of informed principals decreases with agents' risk aversion. Furthermore, when M increases, it is more difficult for principals to monitor agents and the agency problem is exacerbated. Thus, the equilibrium population of informed principals decreases with residual uncertainty. These results are shown in the following figures. I note that agents' risk aversion or residual uncertainty does not have any impact on the population of informed principals due to the assumption regarding principals' risk-neutrality. These two figures show that price informativeness is low during recessions, which are associated with large uncertainty.[12]

1.5 Implications

In this section, I extend the basic model in three directions to study its asset pricing implication. First, I show that the agency problems induce principals to herd in terms of acquiring similar

[12] I set $\theta = 2$, $N = 2$, $p = 0.6$, $W_a = 0$, $\gamma_a = 1$, $C = 0.05$ and $M = 20$ for analysis of agents' risk aversion. I set $\theta = 2$, $N = 2$, $p = 0.6$, $W_a = 0$, $\gamma_a = 1$, and $C = 0.075$ for analysis of residual uncertainty M.

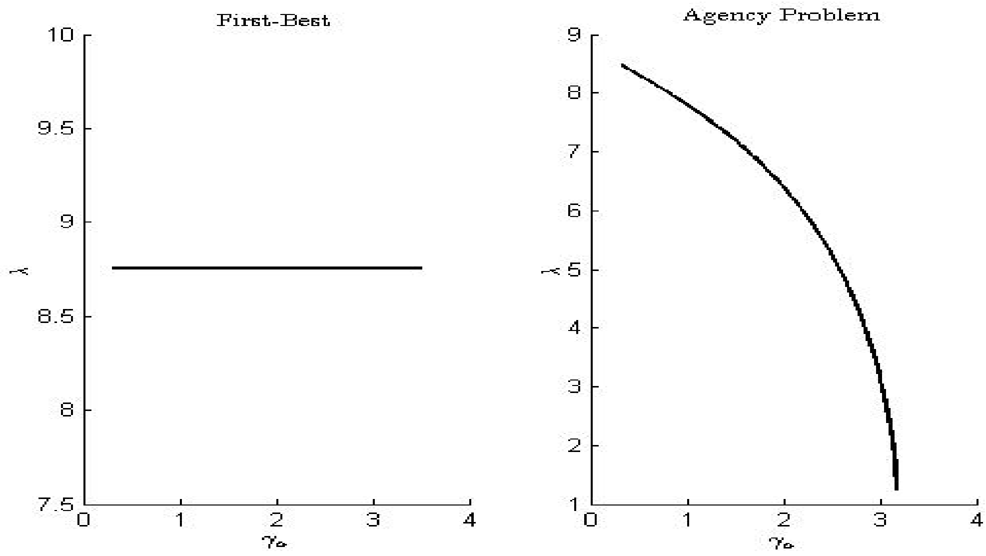

Figure 1.4.2: Population of Informed Principal and Agents' Risk Aversion

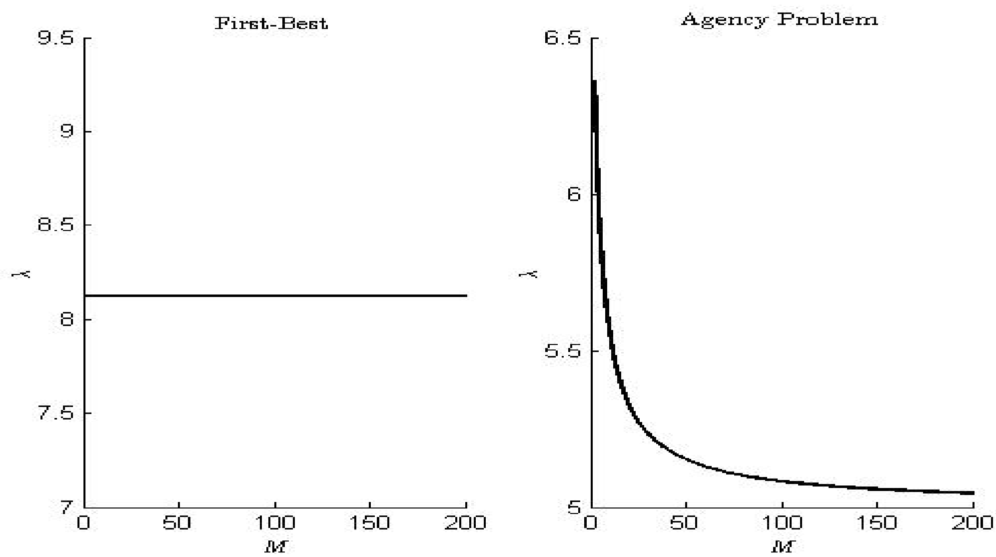

Figure 1.4.3: Population of Informed Principals and Residual Uncertainty

information. This may explain investors' herding behavior in trading. Second, I show that the agency problems encourage principals to acquire disproportionately more information on assets about which they already have an information advantage. This may explain the home/industry bias. Moreover, I shows that the agency problems provide a new and rational explanation for the well-known idiosyncratic volatility comovment.

1.5.1 Herding

In this section, I show that the agency problems induce principals to herd in terms of acquiring similar information. I assume that each principal can choose to hire his agent to acquire either an exclusive signal, which is conditionally independent and can only be acquire by his agent, or a common signal, which can be acquired by any agent. The exclusive signal is $s_i \in \{h, l\}$. The common signal is $s_c \in \{h, l\}$. I assume the probabilities with which these signals are correct are the same ($p > \frac{1}{2}$):

$$\begin{aligned} p &= prob(s_i = h|V = \theta) = prob(s_i = l|V = -\theta) \\ &= prob(s_c = h|V = \theta) = prob(s_c = l|V = -\theta)\,. \end{aligned} \tag{1.1}$$

Then, I have the conditional probability of V as follows:

$$prob^I(V = \theta|s_i = h) = prob^I(V = -\theta|s_i = l) = p, \tag{1.2}$$

$$prob^I(V = \theta|s_c = h) = prob^I(V = -\theta|s_c = l) = p, \tag{1.3}$$

Following the basic model, I assume that if agent i does not exert costly effort, his signal is a pure noise. I denote $prob^U(V|s_i)$ as probability of V conditional on signal s_i if s_i is a pure noise. Furthermore, the information acquisition costs are the same for all signals, which are denoted by C.

I assume that the population of principals who hire agents to acquire s_c is λ, and the population of principals who hire agents to acquire s_i is μ. I follow Garcia and Strobl (2011) to define herding equilibrium as follows:

DEFINITION **1.5.1.** *Herding Equilibrium: one equilibrium is herding equilibrium if* $\mu = 0$ *and* $\lambda > 0$

This definition is following Hirshleifer and Teoh (2003), who define herding as any behavior similarity caused by individuals' interaction. Herding equilibrium occurs only if all informed principals hire agents to acquire the common signal. As argued by Garcia and Strobl (2011), the common signal is less valuable for principals than the exclusive signal because of competition. Thus, without agency problems, herding equilibrium never occur. However, I show that herding equilibrium may emerge in an economy with agency problems through the following mechanism.

There are two groups of informed principals: the first group acquires s_c; the second group acquires s_i. Each principal in the first group is indexed by principal i, where $i \in [0, \lambda]$. And each principal in the second group is indexed by principal j, where $j \in [0, \mu]$. I denote E_p^c and E_p^I as expected trading profits for principals in the first and second group respectively. I denote B_{fb}^c and B_{fb}^I as net benefits of information for different groups in the economy without agency problem. Moreover, I denote B_{ap}^c and B_{ap}^I as net benefit of information for different groups respectively in the economy with agency problems.

For the first group, principal i submits $X_i = 1$ if $s_c = h$, and submits $X_i = -1$ if $s_c = l$. For the second group, principal j submits $X_j = 1$ if $s_j = h$, and submits $X_j = -1$ if $s_j = l$. To simplify the analysis, I only focus on the herding equilibrium. On the herding equilibrium, $\mu = 0$. In this case, if $s_c = h$, the total orders is $X = \lambda + n$. If $s_c = l$, the total orders is $X = -\lambda + n$. Thus, the total orders X is distributed on $[-\lambda - M, \lambda + M]$. Receiving total orders, the market maker sets the price as follows:

LEMMA **1.5.1.** *Given* $\lambda > 0$ *and* $\mu = 0$*, the price follows the rule:*

$$P(X) = \begin{cases} (2p-1)\theta \quad if \;\; N - \lambda < X \leq N + \lambda \\ 0 \quad if \;\; -N + \lambda \leq X \leq N - \lambda \\ -(2p-1)\theta \quad if \;\; -N - \lambda \leq X < -N + \lambda \end{cases} \tag{1.4}$$

To show the existence of a herding equilibrium, I need to calculate expected trading profits for these two groups. Although there is no second group in the herding equilibrium, I also can calculate the expected trading profit for this group assuming one principal j is the marginal principal acquiring an exclusive signal. Then I have the following results:

LEMMA **1.5.2.** *The expected trading profit of principals with the common signal is given by:*

$$E_p^c = (2p-1)\theta \frac{N-\lambda}{N}. \tag{1.5}$$

LEMMA **1.5.3.** *The expected trading profit of principals with an exclusive signal is given by:*

$$E_p^I = (2p-1)\theta \frac{N-(2p-1)^2\lambda}{N}. \tag{1.6}$$

Lemma 1.5.2 and Lemma 1.5.3 shows that the expected trading profit of principals for the second group is higher than the first group. There is a large price impact when principals trade similarly because of having the same information, which makes the total orders more informative about the common signal and decreases the first group's information advantage. Thus, principals have higher incentives to acquire the exclusive signal than the common signal in the economy without agency problems. However, when the residual uncertainty is sufficiently large, principals herd to the common signals in the economy with agency problems. Herding makes the price sensitive to the common signal. Consequently, agents have strong incentives to acquire the common signal as they can easily forecast the asset price with this signal, which mitigates agency problems in acquiring it. Although the exclusive signals can generate more trading profits, agents could not easily forecast asset price with these signals because of their idiosyncratic noises, which worsens the agency problems in acquiring these signals. This mechanism generates the herding equilibrium. I show the formal result as follows:

PROPOSITION **1.5.1.** *Comparing the economy with and without agency problems, I have*
(1) no herding equilibrium occurs in the first-best;
92) when M is small enough, no herding equilibrium occurs in the economy with agency problems; (2) when $-\frac{\log 4p(1-p)}{\gamma_a} < C < (2p-1)\theta - W_a$ and M is large enough, the herding equilibrium

exists in the economy with agency problems.

Proposition 1.5.1 shows that the herding equilibrium occurs when the residual uncertainty is large. This result implies that herding is stronger in small/growth stocks, which have considerable uncertainty. It is consistent with Lakonishok, Shleifer and Vishny (1992) and Wermers (1999), who find that institutional investors have stronger herding behavior in small/growth stocks. Although my model is static, it implies that institutional investors tend to follow the lead of others. When more fund managers trade in one specific stock, others observe this and tend to follow their lead because these followers anticipate that the price will become more informative and the agency problems will be mitigated.

1.5.2 Home/Industry Bias

In this section, I explore the model's implication for the home/industry bias, which is a long-standing puzzle. As documented by Fama and Poterba (1991), Coval and Moskowitz (1999), Grinblatt and Keloharju (2001), Huberman (2001) and Seasholes and Zhu (2010), both households and institutions prefer to trade the assets which are located around their hometowns or home countries. Though it is possible that some behavior biases drive home bias in households, home bias among institutional investors is still puzzling because they are sophisticated investors. Another plausible explanation is that investors have superior information on home assets, Van Nieuwerburgh and Veldkamp (2009) argue that investors can easily acquire information about other assets, which could eliminate home investors' information advantage and mitigate home bias. Even if investors can freely acquire information, I show that home bias still exists with agency problems in information acquisition.

I extend the basic model to consider two groups of principals: the first group has some opportunity to get free information; the second group has no information. The first group is interpreted as home principals based on the conventional belief that investors have an information advantage on home assets. The population of home principals is denoted by ω. Each principal in this group is indexed by i, where $i \in [0, \omega]$. The second group is called foreign principals. Each principal in this group is indexed by j. Any principals can hire agents to acquire information. Furthermore, I assume that principal i in the first group is endowed by a private signal $s_{h,i}$, which takes the form:

$$s_{h,i} = \{V, \emptyset\}. \tag{1.7}$$

The feature of this signal is that $s_{h,i}$ is a pure noise when $s_{h,i} = \emptyset$, and it is perfectly informative if $s_{h,i} = V$. The possibility that $s_{h,i}$ is perfectly informative is denoted by p_h:

$$prob(s_{h,i} = V) = p_h, \tag{1.8}$$

where $0 < p_h < p$. There are two differences between home and foreign principals: the first is that home principals can use their endowed signals in trading; the second difference is that home principals can use their endowed signals in the contracting.[13] Moreover, I assume that the population of home principals hiring agents is λ, and the population of foreign principals hiring agents is μ. Although home principals may know the fundamental value exactly, they also have incentives to acquire information because they have chances to become uninformed. If the endowed signals are informative, home principals only rely on their endowed signals in trading. Otherwise, they have to rely on signals from agents. Thus, the total orders is $X = p_h\omega + (1-p_h)(2p-1)\lambda\omega + (2p-1)\mu + n$ if $V = \theta$. And the total orders is $X = -p_h\omega - (1-p_h)(2p-1)\lambda\omega - (2p-1)\mu + n$ if $V = -\theta$. To simplify the analysis, I let $\eta = p_h\omega + (1-p_h)(2p-1)\lambda\omega + (2p-1)\mu$. Before proceeding, I define two home bias equilibria as follows:

Definition **1.5.2.** *Weak Home Bias Equilibrium: one equilibrium is weak home bias equilibrium if $\lambda > 0$ and $\mu > 0$.*

Definition **1.5.3.** *Strong Home Bias Equilibrium: one equilibrium is strong home bias equi-*

[13] If these signals are not verifiable, there exist some mechanisms inducing principals to reveal their private information, such as imposing an infinite penalty when asset payoff deviates considerably from principals' reports. The infinite penalty can be interpreted as reputation concern. One interpretation of these contracts is the subjective evaluation. Or this type of contract can be interpreted as an implicit contract.

librium if $\lambda > 0$ *and* $\mu = 0$.

Receiving total orders, the market maker sets the price as follows:

$$P(X) = \begin{cases} \theta & if \quad N - \eta < X \leq N + \eta \\ 0 & if \quad -N + \eta \leq X \leq N - \eta \\ -\theta & if \quad -N - \eta \leq X < -N + \eta \end{cases} \tag{1.9}$$

I calculate the expected trading profits for different groups. I denote $E^1_{h,p}$, $E^2_{h,p}$ and $E_{f,p}$ as expected trading profits for home principals who hire agents, home principals who do not hire, and informed foreign principals respectively. They are shown as follows:

$$E_{f,p} = (2p-1)\theta\frac{N-\eta}{N}, \tag{1.10}$$

$$E^1_{h,p} = [p_h + (1-p_h)(2p-1)]\theta\frac{N-\eta}{N}, \tag{1.11}$$

$$E^2_{h,p} = p_h\theta\frac{N-\eta}{N}. \tag{1.12}$$

It is clear that the gain from information for home principals is $(1-p_h)(2p-1)\theta\frac{N-\eta}{N}$, which is lower than the trading profits of informed foreign principals. This is due to the decreasing marginal benefits of information. Thus, without agency problems, home principals have lower incentive than foreign principals to hire agents to acquire information. However, with agency problems, this is not the case. Home principals can use their endowed information in incentive provision, agency problems are not severe for home principals and home principals may have higher incentive to acquire information than foreign principals. The formal results regarding home bias are as follows:

PROPOSITION **1.5.2.** *Comparing the economy with and without agency problems, I have*

(1) neither weak home bias equilibrium nor strong home bias equilibrium occurs in the first-best;

(2) when M *is small enough, neither weak home bias equilibrium nor strong home bias equi-*

librium occurs in the economy with agency problem;

(3) when both M and N are large enough, a strong herding equilibrium exists in the economy with agency problem when $\theta_1 < \theta < \theta_2$.

where θ_1 and θ_2 are defined in the Appendix.

Proposition 1.5.2 shows that home bias occurs when the residual uncertainty is large. This result implies that home bias is stronger when investors trade small/growth stocks. It is consistent with Kang and Stulz (1997) and Coval and Moskowitz (1999). For example, Coval and Moskowitz (1999) find that U.S. fund managers have a stronger home bias when they trade small stocks. It also implies that investors tend to learn more about the assets within their expertise. This prediction is consistent with Massa and Simonov (2006), who find that Swedish investors buy assets highly correlated with their non-financial income.

1.5.3 Idiosyncratic Volatility Comovement

In this section, I explore the model's implication for idiosyncratic volatility comovement, which is documented by recent studies (see Bekaert, Hodrick and Zhang, 2012, Kelly, Lustig and Van Nieuwerburgh, 2013 and Herskovic, Kelly, Lustig and Van Nieuwerburgh, 2013). More importantly, because recent studies (Bansal, Kiku, Shaliastovich and Yaron, 2014, Campbell, Giglio, Polk, Turley, 2014 and Herskovic, Kelly, Lustig and Van Nieuwerburgh, 2013) find that the common factor in idiosyncratic volatilities has significant effects on asset prices, it is important to understand the driving force. In particular, the common factor in idiosyncratic volatilities is not related to the conventional risk factors, and the driving force is still puzzling.

I extend the basic model to consider two risky assets. Each asset is indexed by k, where $k = 1, 2$. Asset k's payoff is denoted by D_k, which has a fundamental value V_k and a residual uncertainty ϵ_k. I assume ϵ_k is uniformly distributed on $[-M_k, M_k]$, where $M_k > 0$. V_k takes θ_k and $-\theta_k$ with equal probability, where $\theta_k > 0$. In particular, I assume that two assets' residual uncertainties are independent of each other, and also are independent of the two fundamentals.

There is a correlation between the two fundamentals as shown:

$$prob(V_2 = \theta_2 | V_1 = \theta_1) = prob(V_2 = -\theta_2 | V_1 = -\theta_1) = q, \tag{1.13}$$

$$prob(V_1 = \theta_1 | V_2 = \theta_2) = prob(V_1 = -\theta_1 | V_2 = -\theta_2) = q. \tag{1.14}$$

The noisy traders' demand in asset k is denoted by n_k following a uniform distribution on $[-N_k, N_k]$, where $N_k > 0$. Noisy traders' demands are independent of other random variables. I assume that each market has one risk-neutral market maker, who sets the price independently from each other. The price of asset k is denoted by P_k. Furthermore, there are two groups of principals: group k can only trade the risky asset k, perhaps due to market segmentation or trading constraints. The population of informed principals in asset k is λ_k. To simplify the analysis, I assume that λ_1 is exogenous, and λ_2 is endogenous. This assumption is reasonable in many circumstances. For example, there are some insiders or home investors, who are endowed with information. The above assumptions are helpful to make the mechanism in my model clear. If the principals can trade both assets, it is possible that there exist other possible effects, which may mitigate or exacerbate my mechanism (see Vayanos and Woolley, 2013 and Cespa and Foucault, 2014). Each informed principal's signal is denoted by $s_{k,i}$. Information structures are the same as the basic model with one risky asset. Then, I have:

$$prob^I(V_k = \theta_k | s_{k,i} = h) = prob^I(V_k = -\theta_k | s_{k,i} = l) = p, \tag{1.15}$$

To avoid the price of asset 1 being fully informative about the fundamental, I have $\lambda_1 < \frac{N_1}{2p-1}$. Although the principals in group 2 can not trade the risky asset 1, they still can write contracts on the prices and payoffs of two assets. Specifically, informed principal i in group 2 design contract $\pi_{2,i}(s^R(s_{2,i}), P_1, P_2, D_1, D_2)$, where $s_{2,i}$ is his agent's signal and $s^R(s_{2,i})$ is the report. The reason why principals in asset 2 write this type of contracts is that two assets' fundamentals are correlated and agents' forecasting accuracy for the price of asset 1 is also related to their effort. I follow the procedure in the basic model to solve the equilibrium prices, optimal contracts, and population of informed principals in group 2. I carry out the numerical

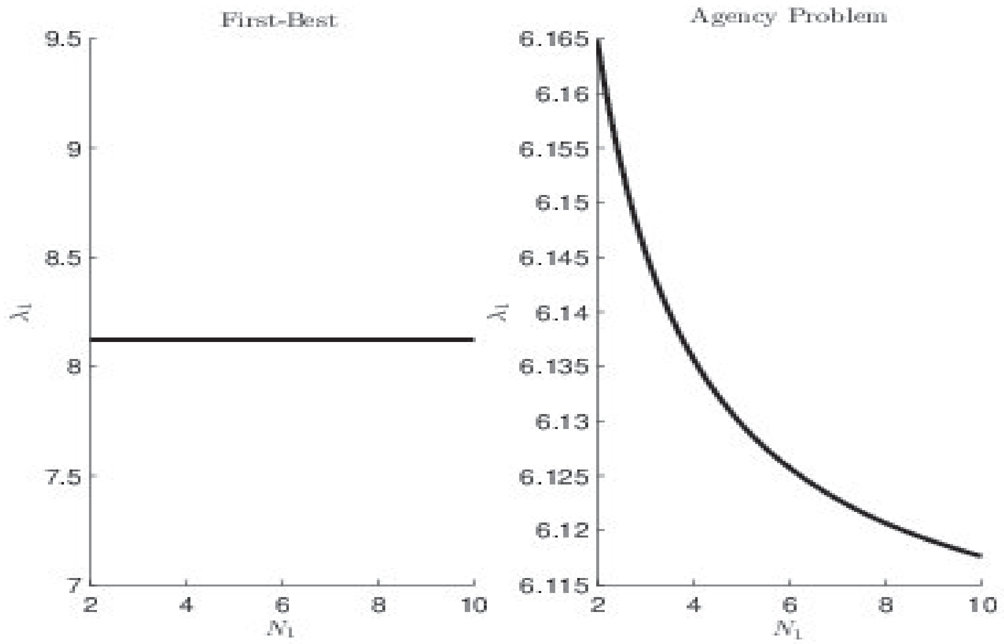

Figure 1.5.1: Noisy Traders in Asset 1 and Population of Informed Principals in Asset 2

studies to show how the agency problems generate the idiosyncratic volatility comovement. Figure 5.1 shows that the population of informed principals in asset 2 decreases with the noisy trades' demand in asset market 1 (N_1) in the economy with agency problems. Because P_1 is also informative to V_1, informed principals in group 2 use the price of asset 1 to monitor their agents. When the noisy traders' demand becomes more volatile in asset 1, P_1 becomes noisier and is more difficult for agents on asset 2 to predict. Consequently, the agents on the asset 2 are less willing to exert effort, which worsen the agency problems and decreases principals' incentives to hire agents on asset 2. This induces the price of asset 2 to become less informative and more sensitive to its noisy traders' demand, leading to increased idiosyncratic volatility (shown in Figure 5.2). Following the same mechanism, when the population of informed principals in group 1 increases, asset 1's price becomes more informative and the principals in group 2 have higher incentives to hire agents (see Figure 5.3). This result is interesting and is related to herding on the industry level(Choi and Sias, 2009).[14]

[14]In Figure 6.1 and Figure 6.2, I set $\theta_1 = \theta_2 = 2$, $M_1 = M_2 = 5$, $N_2 = 2$, $p = 0.6$, $q = 0.8$, $C = 0.75$, $W_a = 0$, $\lambda_1 = 2$. In Figure 6.3, $\theta_1 = \theta_2 = 2$, $M_1 = M_2 = 5$, $N_1 = N_2 = 2$, $p = 0.6$, $q = 0.8$, $C = 0.75$, $W_a = 0$

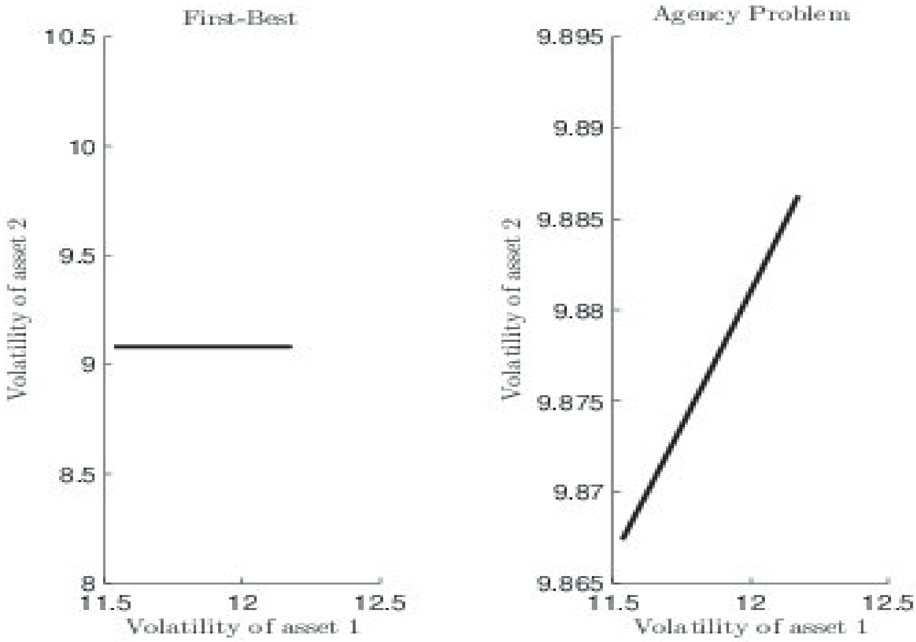

Figure 1.5.2: Idiosyncratic Volatility Comovement

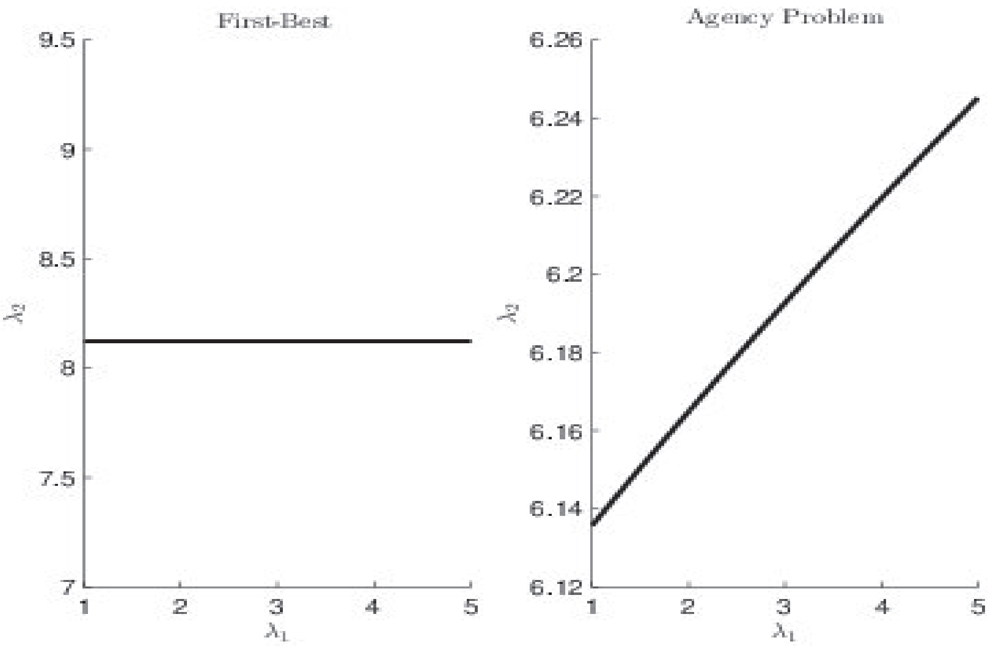

Figure 1.5.3: Population of Informed Principals both Asset Markets

1.6 Generalization

My model assumes: (1) agents have CARA utilities; (2) the fundamental value V takes binary values; (3) principals do not learn information from the asset price. This section relaxes these assumptions and shows that the strategic complementarities are robust.

1.6.1 General Utility Function of Agents

This section shows that my results are robust when agents have a general **hyperbolic absolute risk aversion (HARA)** class of utility functions. The HARA utility function is shown as follows:

$$U(W) = \frac{\gamma}{1-\gamma}[\frac{AW}{\gamma} + K]^{1-\gamma},\ K \geq 0 \tag{1.1}$$

where the utility function is only defined over $\frac{AW}{\gamma} + K > 0$. I know the absolute risk aversion coefficient is given by:

$$-\frac{U''}{U'} = \frac{A\gamma}{AW + K\gamma} \tag{1.2}$$

When $\gamma < 0$, this HARA utility function has an increasing absolute risk aversion, which is implausible. Thus, I only consider the case where $\gamma > 0$. Particularly, this general HARA utility function has several examples which are largely used in finance or economy, such as power utility, negative exponential utility or logarithmic utility.

Assumption **1.6.1.** γ *satisfies:* $\gamma > 0$.

To simplify the analysis, I assume that agents need to incur a utility loss if they exert effort. This utility lose is denoted by C. Thus, the participant constraint and incentive constraints are shown as follows:

$$\sum_{s_i=\{h,l\}} prob(s_i) \int\int \{U[\pi_i(s_i, P, D)] - C\} f^I(P, D|s_i) dPdD = U[W_a], \tag{1.3}$$

where LHS of Equation (1.3) is agent i's expected utility when he exerts costly effort.

Meanwhile, W_a is the reserve wealth of agents.

Ex Ante IC which is the incentive-compatibility of effort constraint

$$\begin{aligned} &\sum_{s_i=\{h,l\}} prob(s_i)\int\int\{U[\pi_i(s_i,P,D)]-C\}f^I(P,D|s_i)dPdD \\ \geq &\sum_{s_i=\{h,l\}} prob(s_i)\int\int[U[\pi_i(s^R(s_i),P,D)]]f^U(P,D|s_i)dPdD \end{aligned} \tag{1.4}$$

Ex Post IC which is incentive-compatibility of truth reporting($\forall s_i$ and $s^R(s_i): s \to s$)

$$\begin{aligned} &\sum_{s_i=\{h,l\}} prob(s_i)\int\int\{U[\pi_i(s_i,P,D)]\}f^I(P,D|s_i)dPdD \\ \geq &\sum_{s_i=\{h,l\}} prob(s_i)\int\int[U[\pi_i(s^R(s_i),P,D)]]f^I(P,D|s_i)dPdD \end{aligned} \tag{1.5}$$

ASSUMPTION **1.6.2.** *I have different cases regarding the information acquisition cost*
Case 1: If $\gamma < 1$, $U(W_a) - \frac{C}{2p-1} > 0$;
Case 2: If $\gamma > 1$, $U(W_a) + \frac{C}{2p-1} < 0$;
Case 3: if $\gamma = 1$ *and* $K = 0$.

This assumption could ensure that the optimal contracts can be implemented and there is interior solution to the contracting for different γ. I show that strategic complementarity effect is robust in the following proposition.

PROPOSITION **1.6.1.** *Under Assumption 1.6.1 and Assumption 1.6.2, when the agents have HARA utility in the economy with agency problems, I have the following results:*
(1)for small enough M*, information acquisition delegation is a strategic substitute. That is ,* $\frac{\partial B}{\partial \lambda} < 0$.
(2) for large enough M*, there exists* λ^{gc} *satisfying: when* $\lambda < \lambda^{gc}$*, information acquisition delegation is a strategic complement. That is* $\frac{\partial B}{\partial \lambda} > 0$.

1.6.2 More General Distribution of V

In this section, I assume that V takes three values in θ, 0 or $-\theta$. In particular, the distribution of V is symmetric. The probability that $V = 0$ is denoted as p_m. The probability of $V = \theta$ or $V = -\theta$ is given by:

$$prob(V = \theta) = prob(V = -\theta) = (1 - p_m)/2, \tag{1.6}$$

After exerting effort, agent i generates a private signal $s_i \in \{h, 0, l\}$ about the risky asset's fundamental value V. The probability with which a signal is correct by

$$prob(s_i = h|V = \theta) = prob(s_i = 0|V = 0) = prob(s_i = l|V = -\theta) = p, \tag{1.7}$$

and

$$prob(s_i = 0|V = \theta) = prob(s_i = 0|V = -\theta) = (1 - p)q, \tag{1.8}$$

and

$$prob(s_i = l|V = \theta) = prob(s_i = h|V = -\theta) = (1 - p)(1 - q), \tag{1.9}$$

and

$$prob(s_i = h|V = 0) = prob(s_i = l|V = -\theta) = (1 - p)/2, \tag{1.10}$$

where s_i is independent across agents and $p \geq \frac{1}{3}$, while $q \geq \frac{1}{2}$. Principal i submits $X_i = 1$ when he receives report $s_i = h$, does nothing when he receives report $s_i = 0$, and submits $X_i = -1$

when he receives report $s_i = l$. Then the market maker sets the price as follows:

$$P(X) = \begin{cases} \theta & if \quad N < X \leq N + \lambda(p - (1-p)(1-q)) \\ \theta \frac{1-p_m}{1+p_m} & if \quad N - \lambda(p - (1-p)(1-q)) < X \leq N \\ 0 & if \quad -N + \lambda(p - (1-p)(1-q)) \leq X \\ & \leq N - \lambda(p - (1-p)(1-q)) \\ -\theta \frac{1-p_m}{1+p_m} & if \quad -N \leq X < -N + \lambda(p - (1-p)(1-q)) \\ -\theta & if \quad -N - \lambda(p - (1-p)(1-q)) \leq X < -N \end{cases} \tag{1.11}$$

It is clear that the price increases with the total orders X. It differs from the binary-state case on the feature that the price takes five values. This difference also shows that the problem will become extremely complicate when I consider more a general distribution of V. I carry out the numerical studies to show that information acquisition complementarities is robust in Figure 6.1, as is the relation between residual uncertainty/agents' risk aversion and price informativeness in Figure 6.2 and Figure 6.3.[15]

1.6.3 Learning

This section shows that my results are robust when principals learn information from the asset price. To obtain analytical solution in the non-linear REE, I modify the information structure and the distribution of residual uncertainty. I assume that the residual uncertainty ϵ follows normal distribution $N(0, \sigma_M^2)$. The private signal acquired by agent i is denoted by s_i. Through the costly effort $e_i \in \{0, e\}$, the joint distributions of his signal s_i and the fundamental value V is a mixture distribution as follows:

$$(b + e_i) f^I(s_i, V) + (1 - b - e_i) f^U(s_i, V), \tag{1.12}$$

[15] I set $\theta = 2$, $N = 2$, $p = 0.6$, $p_m = 0.5$, $q = 0.6$, $W_a = 0$, $C = 0.07$. I also set $M = 5$, $M = 20$ and $M = 200$ for low residual uncertainty, median residual uncertainty and high residual uncertainty cases respectively in Figure 6.1. Then set $\theta = 2$, $N = 2$, $p = 0.6$, $p_m = 0.5$, $q = 0.6$, $W_a = 0$, $gamma_a = 1$, $C = 0.05$ and $M = 20$ for analysis of agents' risk aversion in Figure 6.2. I set $\theta = 2$, $N = 2$, $p = 0.6$, $p_m = 0.5$, $q = 0.6$, $W_a = 0$, $gamma_a = 1$, and $C = 0.075$ for analysis of residual uncertainty M in Figure 6.3.

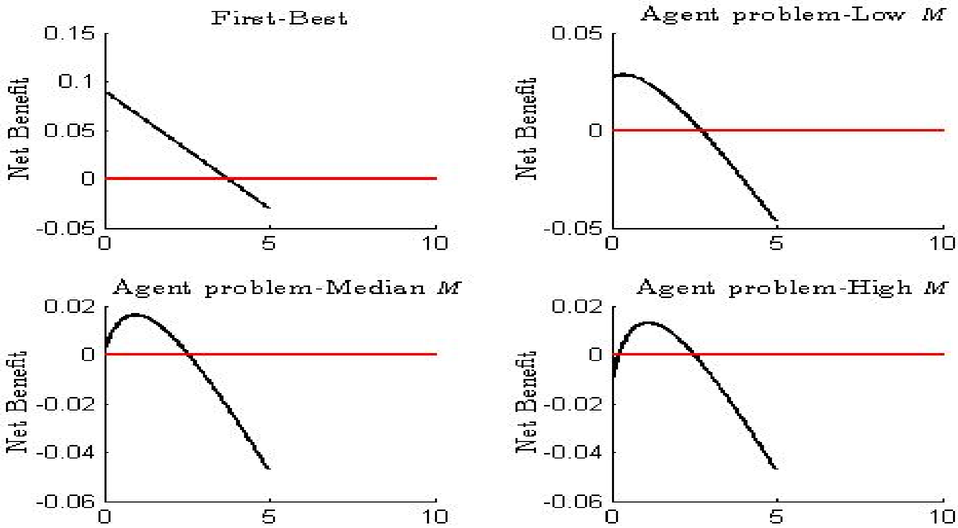

Figure 1.6.1: Information Acquisition Benefit: Triple-State Case

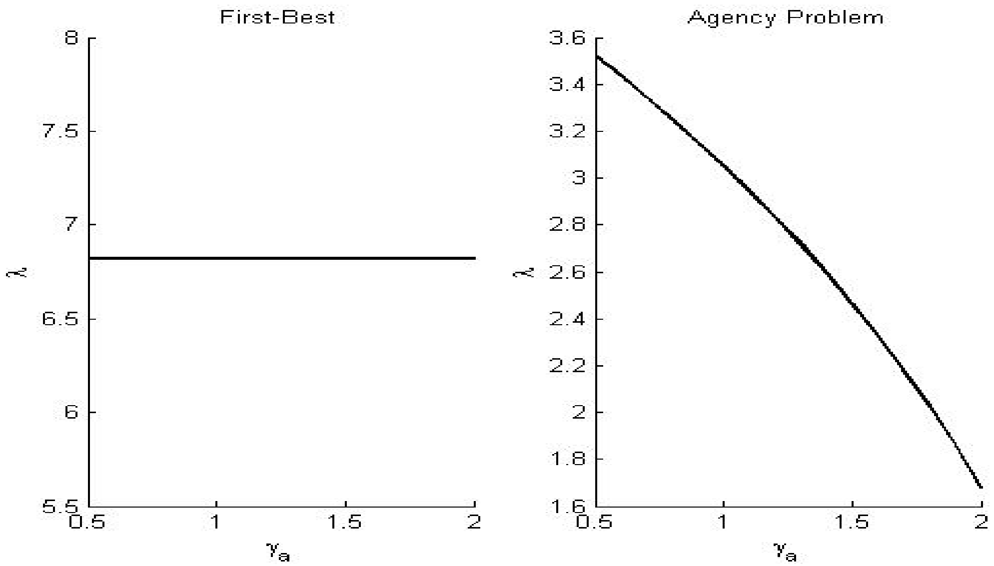

Figure 1.6.2: Population of Informed Principal and Agents' Risk Aversion: Triple-State Case

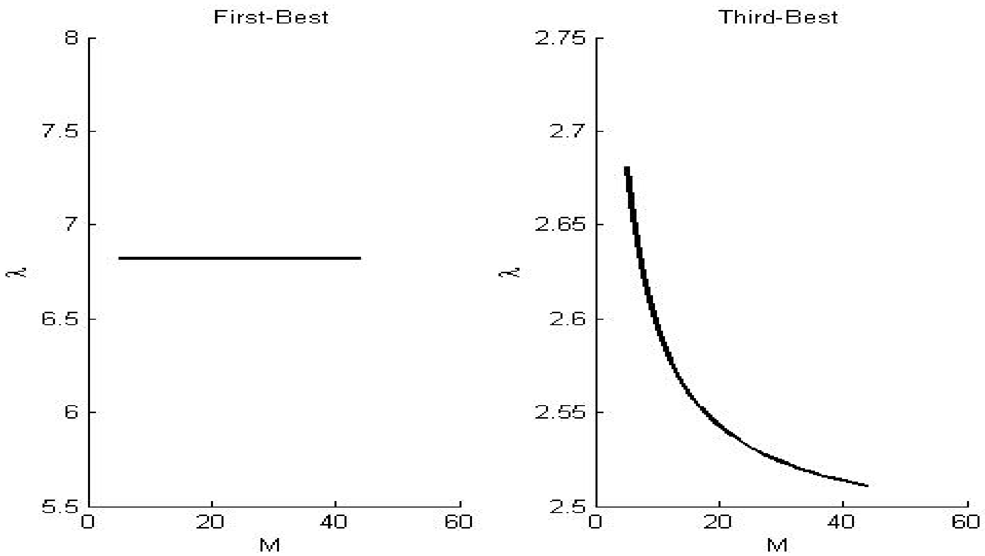

Figure 1.6.3: Population of Informed Principal and Residual Uncertainty: Triple-State Case

where $e > 0$ and $b + e < 1$. Here, f^I is an "informed" distribution and f^U in an "uninformed" distribution. I assume that s_i and V are independent in the uninformed distribution. Moreover, I assume that the probability density of s_i is: $f(s_i) = \frac{1}{2}\frac{1}{\sqrt{2\pi\sigma}}\exp^{-\frac{(s_i-\theta)^2}{2\sigma^2}} + \frac{1}{2}\frac{1}{\sqrt{2\pi\sigma}}\exp^{-\frac{(s_i+\theta)^2}{2\sigma^2}}$. Meanwhile, "informed" joint distribution $f^I(s_i, V) = \frac{1}{2}\frac{1}{\sqrt{2\pi\sigma}}\exp^{-\frac{(s_i-V)^2}{2\sigma^2}}$, while "uninformed" joint distribution $f^U(s_i, V) = \frac{1}{2}f(s_i)$.

One interpretation of the mixture model is that the signals observed by the agents may be informative or not and the agents cannot tell which occurs. In particular, when agents exert efforts, the probabilities that the signals are informative increase. Meanwhile, the mixture model is a simple sufficient condition when I implement the first-order approach to solve the optimal contracts in a general space. Without a loss of generality, I only consider moral hazard problems in information acquisition. This implies that principals could observe the realized signals acquired by agents, but they could not observe whether the agents exert efforts. Moral hazard problems in information acquisitions can be interpreted in many realistic circumstances, such as data collection. Thus, principal i's objective function is as follows:

$$\max_{\pi_i(s_i,P,D),X_i(s_i,\pi_i,P)} \int f(s_i) \int\int [W_0 + X_i(D-P) - \pi_i(s_i,P,D)] f^I(P,D|s_i)dPdDds_i, \quad (1.13)$$

where $X_i(s_i, \pi_i, P)$ is principal i's demand function conditional on the price P and the signal s_i reported by his agent. He maximizes his utility function subject to his agent's participant constraint and incentive compatibility as follows:

PC:

$$\int f(s_i) \int \int [-\exp^{-\gamma_a \pi_i(s_i,P,D)+\gamma_a C}] f^I(P, D|s_i) dPdDds_i = -\exp(-\gamma_a W_a), \tag{1.14}$$

IC:

$$\begin{aligned} &\int f(s_i) \int \int [-\exp^{-\gamma_a \pi_i(s_i,P,D)+\gamma_a C}] f^I(P, D|s_i) dPdDds_i \\ \geq &\int f(s_i) \int \int [-\exp^{-\gamma_a \pi_i(s_i,P,D)+\gamma_a C}] f^U(P, D|s_i) dPdDds_i, \end{aligned} \tag{1.15}$$

where $f^I(P, D|s_i)$ is the conditional probability density given that the agent i exerts effort, and $f^U(P, D|s_i)$ is the conditional probability density given that the agent i shirks.

Now, I assume that there is one continuum of principals and the population of principals hiring agents to acquire information is λ. For the informed principal i, the probability density of s_i conditional on $V = \theta$ is denoted by $\eta_{I,i,h}$, where $\eta_{I,i,h} = \frac{1+b+e}{2}\frac{1}{\sqrt{2\pi}\sigma}\exp^{-\frac{(s-\theta)^2}{2\sigma^2}} + \frac{1-e-b}{2}\frac{1}{\sqrt{2\pi}\sigma}\exp^{-\frac{(s+\theta)^2}{2\sigma^2}}$; the probability density of s_i conditional on $V = -\theta$ is denoted by $\eta_{I,i,l}$, where $\eta_{I,i,l} = \frac{1+b+e}{2}\frac{1}{\sqrt{2\pi}\sigma}\exp^{-\frac{(s+\theta)^2}{2\sigma^2}} + \frac{1-e-b}{2}\frac{1}{\sqrt{2\pi}\sigma}\exp^{-\frac{(s-\theta)^2}{2\sigma^2}}$. For the uninformed principal i,the probability density of s_i conditional on $V = \theta$ is denoted by$\eta_{U,i,h}$, where $\eta_{U,i,h} = \frac{1+b}{2}\frac{1}{\sqrt{2\pi}\sigma}\exp^{-\frac{(s-\theta)^2}{2\sigma^2}} + \frac{1-b}{2}\frac{1}{\sqrt{2\pi}\sigma}\exp^{-\frac{(s+\theta)^2}{2\sigma^2}}$; the probability density of s_i conditional on $V = -\theta$ is denoted by$\eta_{U,i,l}$, where $\eta_{U,i,l} = \frac{1+b}{2}\frac{1}{\sqrt{2\pi}\sigma}\exp^{-\frac{(s+\theta)^2}{2\sigma^2}} + \frac{1-b}{2}\frac{1}{\sqrt{}}$

Due to the assumption about risk-neutral principals, the optimal contracts and asset pricing can be solved separately as our basic model. I solve the model following step-by-step: (1) in the first step, I solve the asset pricing; (2) in the second step, I solve the optimal contract given the population of informed principals; (3)in the third step, I calculate the net benefit of information acquisition to show the strategic complementaries.

Asset Pricing In order to maximize the final wealth, it is necessary to compute the conditional expectation of V for different groups. According to the Bayes's rule, the posterior probability $p_K(s_i, P)$ of state h for principal i of type K after observing s_i and P is given by:

$$p_K(s_i, P) = \frac{f_h(P)\eta_{K,i,h}}{f_h(P)\eta_{K,i,h} + f_l(P)\eta_{K,i,l}} \tag{1.16}$$

where $f_\omega(P)$, $\omega = h, l$ is the probability density of the equilibrium price conditional on the corresponding state of the world. Given the posterior probabilities, principals' demand schedules are shown in the following lemma.

Lemma **1.6.1.** *For any $K = I, U$, there exists a threshold $X_K(P)$ such that the demand schedule for principal i of type K is given by:*

$$X_{K,i} = \begin{cases} 1 & if \quad s_i \geq X_K(P)\,, \\ -1 & if \quad s_i < X_K(P)\,, \end{cases} \tag{1.17}$$

where the threshold $X_K(P)$ is uniquely determined by the condition:

$$p_K(X_K(P), P) = \frac{P + \theta}{2\theta}. \tag{1.18}$$

Having showing the demand schedules, the aggregate demand can be calculated as follows. Conditional on $V = \theta$, the aggregate demand is given by

$$D(P, \theta) = \lambda[1 - (1 + b + e)\Phi(X_I(P) - \theta) - (1 - b - e)\Phi(X_I(P) + \theta)] \tag{1.19}$$

$$+(1 - \lambda)[1 - (1 + b)\Phi(X_U(P) - \theta) - (1 - b)\Phi(X_U(P) + \theta)]; \tag{1.20}$$

conditional on $V = -\theta$, the aggregate demand is given by

$$D(P, -\theta) = \lambda[1 - (1 + b + e)\Phi(X_I(P) + \theta) - (1 - b - e)\Phi(X_I(P) - \theta)] \tag{1.21}$$

$$+(1 - \lambda)[1 - (1 + b)\Phi(X_U(P) + \theta) - (1 - b)\Phi(X_U(P) - \theta)], \tag{1.22}$$

where Φ is the cumulative distribution function for normal distribution $N(0, \sigma^2)$. Consequently, given realized demand from noisy traders, the market clearing condition takes the form as follows:

$$D(P,V) = n \tag{1.23}$$

Thus, the probability density of price P conditional on the value V is denoted as $f_i(P)$. They are calculated as follows:

$$\begin{aligned} f_h(P) = \tfrac{\lambda}{2N}&[(1+b+e)\phi(X_I(P)-\theta) + (1-b-e)\phi(X_I(P)+\theta)]X_I'(P) \\ +\tfrac{(1-\lambda)}{2N}&[(1+b)\phi(X_U(P)-\theta) + (1-b)\phi(X_U(P)+\theta)]X_U'(P) \end{aligned} \tag{1.24}$$

$$\begin{aligned} f_l(P) = \tfrac{\lambda}{2N}&[(1+b+e)\phi(X_I(P)+\theta) + (1-b-e)\phi(X_I(P)-\theta)]X_I'(P) \\ +\tfrac{(1-\lambda)}{2N}&[(1+b)\phi(X_U(P)+\theta) + (1-b)\phi(X_U(P)-\theta)]X_U'(P) \end{aligned} \tag{1.25}$$

where ϕ is the probability density function for the normal distribution $N(0,\sigma^2)$. The key to solve the asset pricing is to solve the thresholds X_I and X_U. I follow Malamud and Petrov (2014) to solve both. I denote the likelihood ratio by:

$$L_K(X) = \eta_{K,i,h}/\eta_{K,i,l}. \tag{1.26}$$

From the Lemma 1.6.1, I have the following condition:

$$L_I(X_I(P)) = L_U(X_U(P)). \tag{1.27}$$

Thus, the relation between $X_U(P)$ and $X_I(P)$ is: $X_U(P) = L_U^{-1}(L_I(X_I(P)))$. It indicates that the solution of X_I can characterize the asset pricing. I have the following result regarding X_I, X_U and the probability density of price P:

PROPOSITION **1.6.2.** *There exists a monotone increasing, absoluately continuous solution $X_I(P)$, $P \in (-\theta,\theta)$ to*

$$2\log L_K(X_I(P)) = \log\frac{P+\theta}{\theta-P}.$$

Meanwhile, $X_I(P)$*,* $X_U(P) = L_U^{-1}(L_K(X_I(P)))$ *and*

$$\begin{array}{c} f_h(P) = \frac{\lambda}{2N}[(1+b+e)\phi(X_I(P)-\theta) + (1-b-e)\phi(X_I(P)+\theta)]X_I'(P) \\ +\frac{(1-\lambda)}{2N}[(1+b)\phi(X_U(P)-\theta) + (1-b)\phi(X_U(P)+\theta)]X_U'(P) \end{array}, \tag{1.28}$$

and

$$\begin{array}{c} f_l(P) = \frac{\lambda}{2N}[(1+b+e)\phi(X_I(P)+\theta) + (1-b-e)\phi(X_I(P)-\theta)]X_I'(P) \\ +\frac{(1-\lambda)}{2N}[(1+b)\phi(X_U(P)+\theta) + (1-b)\phi(X_U(P)-\theta)]X_U'(P) \end{array} \tag{1.29}$$

form a rational expectations equilibrium.

Contracting I use the first-order approach to solve the optimal contracts. First, I let l_1 and l_2 be the Lagrange multipliers on the PC and IC. I can get the expression for optimal compensation as follows:

$$\pi(s_i, P, D) = \frac{\log(l_1 + l_2 - l_2 \frac{\exp^{-\gamma_a C} f^U(P,D|s_i)}{f^I(P,D|s_i)}) + \log(\gamma_a)}{\gamma_a} \tag{1.30}$$

Net Benefit of Information Now I calculate the net benefit of information. Conditional on the fundamental value V and asset price P, the expected trading profit of principal i of type $K = I, U$ is calculated by:

$$\begin{aligned} E_{K,h,P} &= prob(s_i < X_K(P)|V=\theta,P)(P-\theta) + prob(s_i \geq X_K(P)|V=\theta,P)(\theta-P) \\ &= \tfrac{1+b+e}{2}\Phi(X_K-\theta)(P-\theta) + \tfrac{1-b-e}{2}\Phi(X_K+\theta)(P-\theta) \\ &+\tfrac{1+b+e}{2}[1-\Phi(X_K-\theta)](\theta-P) + \tfrac{1-b-e}{2}[1-\Phi(X_K+\theta)](\theta-P), \end{aligned} \tag{1.31}$$

$$\begin{aligned} E_{K,l,P} &= prob(s_i < X_K(P)|V=-\theta,P)(P+\theta) + prob(s_i \geq X_K(P)|V=-\theta,P)(-\theta-P) \\ &= \tfrac{1+b+e}{2}\Phi(X_K+\theta)(P+\theta) + \tfrac{1-b-e}{2}\Phi(X_K-\theta)(P+\theta) \\ &+\tfrac{1+b+e}{2}[1-\Phi(X_K+\theta)](-\theta-P) + \tfrac{1-b-e}{2}[1-\Phi(X_K-\theta)](-\theta-P). \end{aligned} \tag{1.32}$$

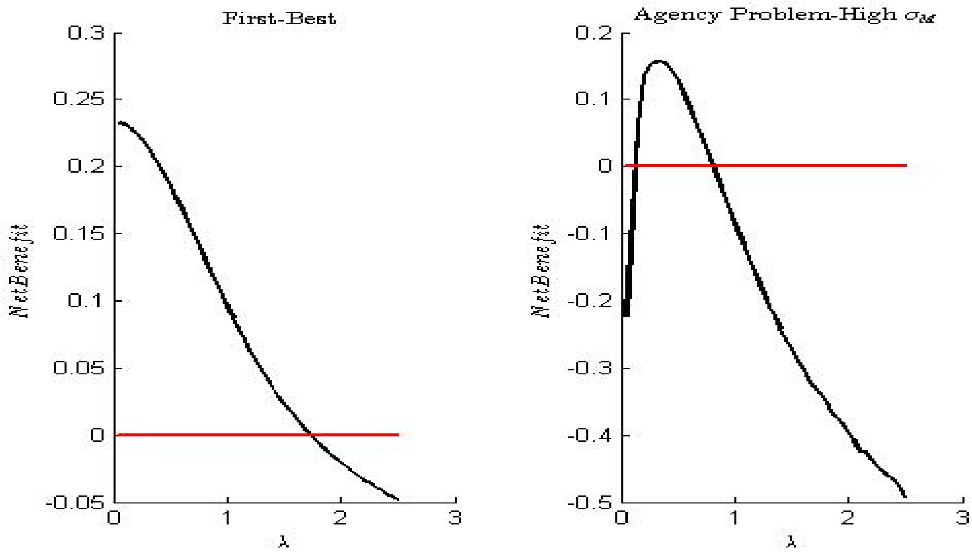

Figure 1.6.4: Information Acquisition Benefit: Learning Case

Then, the expected trading profit for different groups of principals is as follows:

$$E_K = \frac{1}{2}\int E_{K,h,P} f_h(P)dP + \frac{1}{2}\int E_{K,l,P} f_l(P)dP. \tag{1.33}$$

Consequently, the net benefit from information is: $B = E_I - E_U - E(\pi)$. Now, I numerically show that the strategic complementarities are robust when the residual uncertainty has large variance in the following figure.[16]

1.7 Conclusion

I show that optimal contracts depend on the accuracy of agents' forecasts for the asset prices and payoffs. Agents receive high compensation when they produce an accurate forecast. The bonus, as a reward for an accurate forecast, decreases with price informativeness and increases with residual uncertainty of the asset payoffs. When the price becomes more informative or the residual uncertainty decreases, agents can forecast the asset prices or payoffs more accurately with information. Consequently, agents are more willing to exert efforts in acquiring

[16] I set $\theta = 0.5$, $\sigma = 1$, $W_a = 0$, $C = 0.1$, $N = 10$ $\sigma_M = 50$

information. Thus, the principals can decrease the bonus. These results predict that the bonus is larger for professionals, who trade or cover small/growth stocks with larger residual uncertainty or assets with lower institutional ownership.

More importantly, I show that agency problems in delegated information acquisition play important roles in shaping institutional investors' behavior and asset pricing. The novelty of my model is that agency problems generate a strategic complementarities in information acquisition delegation. When more principals hire agents to acquire information, the price becomes less noisy, which make it easier for agents to forecast. Therefore, agents are more willing to exert effort, thereby mitigating agency problems. In turn, other principals are more willing to hire agents. These strategic complementarities lead to multiple equilibria, which have implications for jumps and excess volatilities in asset prices or price informativeness. In particular, multiple equilibria occur when the asset payoff's residual uncertainty is large. This can provide a potential explanation for observed excess volatilities in small/growth stocks or during recessions. My results also predict that price informativeness or institutional ownership tend to have jumps for small/growth stocks. The extensions of this model demonstrate that the agency problems could provide explanations for some phenomena, including idiosyncratic volatility comovement, herding behavior and home/industry bias. Moreover, my model predicts that the herding or home/industry bias is stronger for small/growth stocks.

The driving force for my results is as follows: the price becomes more informative when more principals hire agents to acquire information, which mitigates agency problems. Thus, it is clear that the assumptions about risk-averse principals will not overturn the main mechanisms. However, relaxing these assumptions is interesting. If principals are risk-averse, I expect that the optimal contract will consist of two components: the first is agents' forecasting accuracy, and the second is proportional fee attributable to risk sharing. I leave this extension for further study.

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1.8 Appendix

1.8.1 Proofs

This appendix provides all proofs omitted above.

Proof of Lemma 1.3.1. If $\lambda \geq \frac{N}{2p-1}$, market maker will know that $V = \theta$ if $X > -N + \lambda(2p-1)$ and $V = -\theta$ if $X < N - \lambda(2p-1)$. Then market makers will always set $P = V$. If this is the case, informed investors' trading will always equals to zero because $X_i(V - P) = 0$. If the trading profit is zero, investors have no incentive to acquire information. Thus I can conclude that the population of informed investors can not be larger than $\frac{N}{2p-1}$.

□

Proof of Lemma 1.3.2. On the support $[-\lambda(2p-1) - N, \lambda(2p-1) + N]$, the conditional pdf of X follows

$$f(X|V = \theta) = \begin{cases} 1 & if \quad -N + \lambda(2p-1) \leq X \leq N + \lambda(2p-1) \\ 0 & if \quad X < -N + \lambda(2p-1) \end{cases} \tag{1.1}$$

$$f(X|V = -\theta) = \begin{cases} 0 & if \; X > \; N - \lambda(2p-1) \\ 1 & if \quad -N - \lambda(2p-1) \leq X \leq N - \lambda(2p-1) \end{cases} \tag{1.2}$$

Using Bayesian updating, $prob(V = \theta|X) = \frac{\frac{1}{2}f(X|V=\theta)}{\frac{1}{2}f(X|V=\theta)+\frac{1}{2}f(X|V=-\theta)}$. Thus conditional on X, market maker's belief about probability of $V = \theta$ follows:

$$prob(V = \theta|X) = \begin{cases} 1 & if \;\; N - \lambda(2p-1) < X \leq N + \lambda(2p-1) \\ \frac{1}{2} & if \quad -N + \lambda(2p-1) \leq X \leq N - \lambda(2p-1) \\ 1 & if \quad -N - \lambda(2p-1) \leq X \leq -N + \lambda(2p-1) \end{cases} \tag{1.3}$$

Then because $P = prob(V = \theta|X)\theta - [1 - prob(V = \theta|X)]\theta$, I can get the price function.

$\square$

Proof of Lemma 1.3.3 and Lemma 1.3.4. **Step 1** $f^I(P=\theta, D, s_i=h) = f^I(P=\theta, D|s_i = h) \times prob(s_i=h)$.Then

$$
\begin{aligned}
f^I(P=\theta, D, s_i=h) &= prob(P=\theta, s_i=h) \times f^I(D|P=\theta, s_i=h) \\
= prob(V=\theta, s_i=h) &\times prob(N-\lambda(2p-1) \leq n \leq N+\lambda(2p-1)) \times f^I(D|P=\theta, s_i=h) \\
&= \begin{cases} \frac{1}{2M}\frac{p}{2}\ \frac{\lambda(2p-1)}{N} & if \quad -M+\theta \leq D \leq M+\theta \\ 0 & if \quad -M-\theta \leq D < -M+\theta \end{cases}
\end{aligned} \tag{1.4}
$$

Since $prob(s_i=h)=\frac{1}{2}$, I can get t $f^I(P=\theta, D|s_i=h)$ in the Lemma.

Step 2 $f^I(P=-\theta, D, s_i=h) = f^I(P=-\theta, D|s_i=h) \times prob(s_i=h)$. Then

$$
\begin{aligned}
f^I(P=-\theta, D, s_i=h) &= prob(P=-\theta, s_i=h) \times f^I(D|P=-\theta, s_i=h) \\
= prob(V=-\theta, s_i=h) &\times prob(N-\lambda(2p-1) \leq n \leq -N+\lambda(2p-1)) \times f^I(D|P=-\theta, s_i=h) \\
&= \begin{cases} 0 \quad if \quad M-\theta < D \leq M+\theta \\ \frac{1}{2M}\frac{1-p}{2}\ \frac{\lambda(2p-1)}{N} & if \quad -M-\theta \leq D \leq M-\theta \end{cases}
\end{aligned} \tag{1.5}
$$

Since $prob(s_i=h)=\frac{1}{2}$, I can get $f^I(P=-\theta, D|s_i=h)$ in the Lemma.

Step 3 $f^I(P=0,D,s_i=h) = f^I(P=0,D|s_i=h) \times prob(s_i=h)$. Then

$$
\begin{gathered}
f^I(P=0,D,s_i=h) = prob(P=0,s_i=h) \times f^I(D|P=0,s_i=h) \\
= prob(V=\theta,s_i=h) \times prob(-N+\lambda(2p-1) \leq n \leq N-\lambda(2p-1)) \times f^I(D|P=\theta,s_i=h) \\
+prob(V=-\theta,s_i=h) \times prob(-N+\lambda(2p-1) \leq n \leq N-\lambda(2p-1)) \times f^I(D|P=-\theta,s_i=h) \\
= \begin{cases} \frac{p}{4M}\frac{N-\lambda(2p-1)}{N} & if \;\; M-\theta \leq D \leq M+\theta \\ \frac{1}{4M}\frac{N-\lambda(2p-1)}{N} & if \;\; -M+\theta \leq D < M-\theta \\ \frac{1-p}{4M}\frac{N-\lambda(2p-1)}{N} & if \;\; -M-\theta \leq D < -M+\theta \end{cases}
\end{gathered}
\tag{1.6}
$$

Step 4: Then Lemma 1.3.3 and Lemma 1.3.4 can be derived following the same process above. □

Proof of Proposition 1.3.1. I prove this proposition in two steps.

Step 1 (proof of existence and uniquess)

$$
\max_{v_i(s_i,P,D)} \sum_{s_i=\{h,l\}} \sum_{P=\{-\theta,0,\theta\}} \frac{1}{2} \int\int \frac{1}{\gamma_a} \log[v_i(s_i,P,D)] f^I(P,D|s_i) dD, \tag{1.7}
$$

subject to participation constraint:

$$
\sum_{s_i=\{h,l\}} \sum_{P=\{-\theta,0,\theta\}} \frac{1}{2} v_i(s_i,P,D) f^I(P,D|s_i) dD = \exp^{-\gamma_a W_a - \gamma_a C} \tag{1.8}
$$

Then I let $f = \sum_{s_i=\{h,l\}} \sum_{P=\{-\theta,0,\theta\}} \frac{1}{2} \int\int \log[v_i(s_i,P,D)] f^I(P,D|s_i) dD$ and then

$$
\mathcal{D}_1 = \{- \sum_{s_i=\{h,l\}} \sum_{P=\{-\theta,0,\theta\}} \frac{1}{2} v_i(s_i,P,D) f^I(P,D|s_i) dD \geq -\exp^{-\gamma_a W_a - \gamma_a C}\} \tag{1.9}
$$

It is obvious that f is a strictly concave function and $\mathcal{D}_1$ is convex. Then I can conclude that the local maximum of f over $\mathcal{D}_1$ is a global solution to this optimization. This implies that the solution in the first-order approach is the global solution to this problem.

Step 2: (Solution). I denote Lagrange multiplier of by λ_1. Then I can get $v_i(s_i, P, D) = \frac{1}{\gamma_a}\frac{1}{\lambda_1}$ and $\frac{1}{\gamma_a \lambda_1} = \exp^{-\gamma_a W_a - \gamma_a C}$. Then I can conclude that $\pi_i(s_i, P, D) = W_a + C$. □

Proof of Proposition 1.3.2. Step 1 (proof of existence and uniquess in the second-best) The second-best case is proposed by Dybvig et al. (2010) where the principals are able to observe agents' signals, but are not able to observe agents' hidden actions. Thus, there is not misreporting problem. Then I will show that the agency problem in my study is equivalent to this second-best case since the signals or fundamental value V take binary states. Particularly, the IC in the second-best case is:

$$\sum_{s_i=\{h,l\}} \frac{1}{2} \int\int v_i(s_i, P, D)[f^I(P, D|s_i) - \exp^{-\gamma_a C} f^U(P, D|s_i)]dPdD \leq 0 \tag{1.10}$$

Then I let $f = \sum_{s_i=\{h,l\}} \sum_{P=\{-\theta,0,\theta\}} \frac{1}{2} \int\int \log[v_i(s_i, P, D)] f^I(P, D|s_i)dD$ and then

$$\begin{aligned} \mathcal{D}_2 = \{ & - \textstyle\sum_{s_i=\{h,l\}} \sum_{P=\{-\theta,0,\theta\}} \frac{1}{2} v_i(s_i, P, D) f^I(P, D|s_i)dD \geq -\exp^{-\gamma_a W_a - \gamma_a C}; \\ & \sum_{s_i=\{h,l\}} \tfrac{1}{2}\textstyle\int\int v_i(s_i, P, D)[f^I(P, D|s_i) - \exp^{-\gamma_a C} f^U(P, D|s_i)]dPdD \leq 0\} \\ & v_i(s_i, P, D) \geq 0 \end{aligned} \tag{1.11}$$

It is obvious that f is a strictly concave function over D_2, while $\mathcal{D}_2$ is convex. Then I can conclude that the local maximum of f over $\mathcal{D}_2$ is a global solution to this optimization. This implies that the solution in the first-order approach is the global solution to this problem.

Step 2 (case when $p = 1$). The first order condition should be:

$$1 = [\lambda_1 + \lambda_2 - \lambda_2 \frac{\exp^{-\gamma_a C} f(P, D)}{f^I(P, D|s_i)}]\gamma_a v_i(s_i, P, D) \tag{1.12}$$

When $p = 1$, if $\lambda_2 > 0$ I have following cases:

$$\text{when } s_i = h \text{ and } P = -\theta\text{: } \lambda_1 + \lambda_2 - \lambda_2 \frac{\exp^{-\gamma_a C} f(P, D)}{f^I(P, D|s_i)} = -\infty \tag{1.13}$$

when $s_i = h$, $P = 0$ and $-M-\theta \leq D < -M+\theta$: $\lambda_1+\lambda_2-\lambda_2\dfrac{\exp^{-\gamma_a C} f(P,D)}{f^I(P,D|s_i)} = -\infty$ (1.14)

$$\text{when } s_i = l \text{ and } P = \theta\text{: } \lambda_1 + \lambda_2 - \lambda_2\frac{\exp^{-\gamma_a C} f(P,D)}{f^I(P,D|s_i)} = -\infty \tag{1.15}$$

when $s_i = l$, $P = 0$ and $M - \theta \leq D \leq M + \theta$: $\lambda_1 + \lambda_2 - \lambda_2\dfrac{\exp^{-\gamma_a C} f(P,D)}{f^I(P,D|s_i)} = -\infty$ (1.16)

First-order approach will fail here and this indicates that $\lambda_2 = 0$. When $\lambda_2 = 0$, I can conclude that IC will not be binding. I substitute $1 = \lambda_1\gamma_a v_i(s_i, P, D)$ into PC and get $\frac{1}{\lambda_1\gamma_a}$ $= \exp^{-\gamma_a W_a - \gamma_a C}$. Then I can get result shown in the proposition.

Step 3: (case when $p < 1$). I denote Lagrange multiplier of PC by $\lambda_1^{'}$ and Lagrange multiplier of IC by $\lambda_2^{'}$. Then I can get

$$1 = [\lambda_1^{'} + \lambda_2^{'} - \lambda_2^{'}\frac{\exp^{-\gamma_a C} f(P,D)}{f^I(P,D|s_i)}]\gamma_a v_i(s_i, P, D) \tag{1.17}$$

Then I let: $\lambda_1 = \lambda_1^{'}\gamma_a$, $\lambda_2 = \lambda_2^{'}\gamma_a$ and $q = \exp^{-\gamma_a C}$ (where $q < 1$)

From Lemma 1.3.3 and Lemma 1.3.4, I know that:

(1) When $s_i = h$,

$$\frac{1}{v_i(s_i = h, P = \theta, D)} = \lambda_1 + \lambda_2 - \lambda_2\frac{q}{2p} \tag{1.18}$$

$$\frac{1}{v_i(s_i = h, P = -\theta, D)} = \lambda_1 + \lambda_2 - \lambda_2\frac{q}{2(1-p)} \tag{1.19}$$

$$\frac{1}{v_i(s_i = h, P = 0, D)} = \begin{cases} \lambda_1 + \lambda_2 - \lambda_2 \frac{q}{2p} & if \quad M - \theta \leq D \leq M + \theta \\ \lambda_1 + \lambda_2 - \lambda_2 q & if \quad -M + \theta \leq D < M - \theta \\ \lambda_1 + \lambda_2 - \lambda_2 \frac{q}{2(1-p)} & if \quad -M - \theta \leq D < -M + \theta \end{cases} \tag{1.20}$$

(2) When $s_i = l$,

$$\frac{1}{v_i(s_i = l, P = \theta, D)} = \lambda_1 + \lambda_2 - \lambda_2 \frac{q}{2(1-p)} \tag{1.21}$$

$$\frac{1}{v_i(s_i = l, P = -\theta, D)} = \lambda_1 + \lambda_2 - \lambda_2 \frac{q}{2p} \tag{1.22}$$

$$\frac{1}{v_i(s_i = l, P = 0, D)} = \begin{cases} \lambda_1 + \lambda_2 - \lambda_2 \frac{q}{2(1-p)} & if \quad M - \theta \leq D \leq M + \theta \\ \lambda_1 + \lambda_2 - \lambda_2 q & if \quad -M + \theta \leq D < M - \theta \\ \lambda_1 + \lambda_2 - \lambda_2 \frac{q}{2p} & if \quad -M - \theta \leq D < -M + \theta \end{cases} \tag{1.23}$$

To simplify the analysis, I let $x = \lambda_1 + \lambda_2 - \lambda_2 \frac{q}{2p}$ and $y = \lambda_1 + \lambda_2 - \lambda_2 \frac{q}{2(1-p)}$. Then it is clear that I have $\lambda_1 + \lambda_2 - \lambda_2 q = px + (1-p)y$. I substitute $v_i(s_i = l, P = 0, D)$ into PC and IC.

Step 4 (case when $p < 1$). After rearrangment, I have:

$$\begin{aligned} &\tfrac{1}{x} p \tfrac{\lambda(2p-1)}{N} + \tfrac{1}{x} \tfrac{p\theta}{M} \tfrac{N-\lambda(2p-1)}{N} + \tfrac{1}{px+(1-p)y} \tfrac{M-\theta}{M} \tfrac{N-\lambda(2p-1)}{N} \\ &+ \tfrac{1}{y} \tfrac{(1-p)\theta}{M} \tfrac{N-\lambda(2p-1)}{N} + \tfrac{1}{y}(1-p) \tfrac{\lambda(2p-1)}{N} = \exp^{-\gamma_a W_a - \gamma_a C} \end{aligned} \tag{1.24}$$

$$\frac{1}{y} = \frac{1}{x} + \frac{\exp^{-\gamma_a W_a}(1 - \exp^{-\gamma_a C})}{(p - \frac{1}{2})(\frac{\lambda(2p-1)}{N} + \frac{\theta}{M} \frac{N-\lambda(2p-1)}{N})} \tag{1.25}$$

I let $a_1 = \frac{\lambda(2p-1)}{N} + \frac{\theta}{M}\frac{N-\lambda(2p-1)}{N}$, $a_2 = \frac{\exp^{-\gamma_a W_a}(1-\exp^{-\gamma_a C})}{p-0.5}$, then I have $y = \frac{a_1 x}{a_1+a_2 x}$. From the above two equations, I have:

$$\frac{a_1}{x} + \frac{1-a_1}{px+(1-p)\frac{a_1 x}{a_1+a_2 x}} = \exp^{-\gamma_a W_a} - \frac{\exp^{-\gamma_a W_a}(1-\exp^{-\gamma_a C})}{2p-1} \tag{1.26}$$

I let $g(x) = \frac{a_1}{x} + \frac{1-a_1}{px+(1-p)\frac{a_1 x}{a_1+a_2 x}}$. It is obvious that $g(x)$ is a decreasing function of x when $x > 0$. This concludes that there exists unique solution. Let $b = \exp^{-\gamma_a W_a} - \frac{\exp^{-\gamma_a W_a}(1-\exp^{-\gamma_a C})}{2p-1}$. It is obvious that there is one unique positive solution when $b > 0$. I have:

$x = \frac{-[ba_1+(1-p)a_1a_2-a_2]+\sqrt{[ba_1+(1-p)a_1a_2-a_2]^2+4bpa_2a_1}}{2bpa_2}$ and $y = \frac{a_1 x}{a_1+a_2 x}$

Step 5: Now I prove that this second-best is equivalent to the agency problem in my model. I need to prove that agents' utility in truth telling is higher than that when they misreport after receiving informative signals, while agents' utility in truth telling in information acquisition is higher than that when they randomly reports without any information. This is to prove that :

$$\begin{aligned}
&\tfrac{1}{x}p\tfrac{\lambda(2p-1)}{N} + \tfrac{1}{x}\tfrac{p\theta}{M}\tfrac{N-\lambda(2p-1)}{N} + \tfrac{1}{px+(1-p)y}\tfrac{M-\theta}{M}\tfrac{N-\lambda(2p-1)}{N} \\
&\quad + \tfrac{1}{y}\tfrac{(1-p)\theta}{M}\tfrac{N-\lambda(2p-1)}{N} + \tfrac{1}{y}(1-p)\tfrac{\lambda(2p-1)}{N} \\
&\leq \tfrac{1}{y}p\tfrac{\lambda(2p-1)}{N} + \tfrac{1}{y}\tfrac{p\theta}{M}\tfrac{N-\lambda(2p-1)}{N} + \tfrac{1}{px+(1-p)y}\tfrac{M-\theta}{M}\tfrac{N-\lambda(2p-1)}{N} \\
&\quad + \tfrac{1}{x}\tfrac{(1-p)\theta}{M}\tfrac{N-\lambda(2p-1)}{N} + \tfrac{1}{x}(1-p)\tfrac{\lambda(2p-1)}{N}
\end{aligned} \tag{1.27}$$

Since $\frac{1}{y} > \frac{1}{x}$ and $p > \frac{1}{2}$, it is easy to show the above inequality always holds.

□

Proof of Corollary 1.3.1 *and* Proof of Proposition 1.3.3 . First, because $y = \frac{a_1 x}{a_1+a_2 x}$, it is obvious that $S_f = \frac{1}{\gamma_a}\log(1+\frac{\exp^{-\gamma_a W_a}(1-\exp^{-\gamma_a C})}{p-0.5}\frac{x}{\frac{\lambda(2p-1)}{N}+\frac{\theta}{M}\frac{N-\lambda(2p-1)}{N}})$. I let $z = \frac{x}{\frac{\lambda(2p-1)}{N}+\frac{\theta}{M}\frac{N-\lambda(2p-1)}{N}}$.

For S_f, the signs of $\frac{\partial S_f}{\partial \lambda}$and $\frac{\partial S_f}{\partial M}$ depend on $\frac{\partial z}{\partial \lambda}$ and $\frac{\partial z}{\partial M}$ respectively. For the equation $\frac{a_1}{x} + \frac{1-a_1}{px+(1-p)\frac{a_1 x}{a_1+a_2 x}} = b$, the LHS is decreasing with z and decrease with a_1. Because RHS is constant with a_1, then I know that $\frac{\partial z}{\partial a_1} < 0$. Then I have

$$\frac{\partial z}{\partial \lambda} = \frac{\partial z}{\partial a_1}(1 - \frac{\theta}{M})\frac{(2p-1)}{N} < 0 \tag{1.28}$$

$$\frac{\partial z}{\partial M} = -\frac{\partial z}{\partial a_1}\frac{\theta}{M^2}\frac{N - \lambda(2p-1)}{N} > 0 \tag{1.29}$$

From equation, it is clear that when $\theta = M$, I have $\frac{\partial z}{\partial \lambda} = 0$.

$\square$

Proof of Lemma 1.4.1 and Proposition 1.4.1. When $s_i = h$, I know that $X_i = 1$; When $s_i = l$, $X_i = -1$. So I can calculate expected trading profit as follows:

$$\begin{aligned} E_p &= prob(s_i = h)E(D - P|s_i = h) + prob(s_i = l)E(P - P|s_i = l) && (1.30) \\ &= \theta(2p-1)\frac{N - \lambda(2p-1)}{N} && (1.31) \end{aligned}$$

Then it is obvious that $\frac{\partial B}{\partial \lambda} < 0$. $\square$

Proof of Proposition 1.4.2. Let $K = \frac{1}{2}a_1 \log(x) + \frac{1}{2}a_1 \log(y) + (1-a_1)\log[px + (1-p)y]$, I can get $B = \theta(2p-1)\frac{N-\lambda(2p-1)}{N} - K$. Then $\frac{\partial B}{\partial \lambda} = -\frac{\theta(2p-1)^2}{N} - \frac{\partial K}{\partial a_1}\frac{\partial a_1}{\partial \lambda} = -\frac{\theta(2p-1)^2}{N} - \frac{\partial K}{\partial a_1}(1 - \frac{\theta}{M})\frac{\theta(2p-1)}{N}$.

Step 1 (M is small enough) I know that for $M \geqslant \theta$, $\lim_{M \to \theta} \frac{\partial B}{\partial \lambda} = -\frac{\theta(2p-1)^2}{N} < 0$. Because $\frac{\partial B}{\partial \lambda}$ is a continuous function, this implies that there exists a cutoff M^c satisfying $M < M^c$, $\frac{\partial B}{\partial \lambda} < 0$.

Step 2 (M is large enough) I know that for $M = +\infty$, $\lim_{\lambda \to 0} \frac{\partial B}{\partial \lambda} = -\frac{\theta(2p-1)^2}{N} - \lim_{a_1 \to 0} \frac{\partial K}{\partial a_1}\frac{\theta(2p-1)}{N}$

$$\begin{aligned} &\lim_{a_1 \to 0} \tfrac{\partial K}{\partial a_1} = \lim_{a_1 \to 0}\{\tfrac{1}{2}\log(x) + \tfrac{1}{2}\log(y) - \log[px + (1-p)y] \\ &+ \tfrac{1}{2}a_1 \tfrac{1}{x}\tfrac{\partial x}{\partial a_1} + \tfrac{1}{2}a_1 \tfrac{1}{y}\tfrac{\partial y}{\partial a_1} + (1-a_1)\tfrac{1}{px+(1-p)y}[p\tfrac{\partial x}{\partial a_1} + (1-p)\tfrac{\partial y}{\partial a_1}]\} \end{aligned} \tag{1.32}$$

Because $y = \frac{a_1 x}{a_1 + a_2 x}$, I know that $\lim_{a_1 \to 0} x = \frac{1}{bp}$, $\lim_{a_1 \to 0} y = 0$, $\lim_{a_1 \to 0} \frac{\partial x}{\partial a_1} = finite$

$\lim_{a_1\to 0}\frac{\partial y}{\partial a_1} = \lim_{a_1\to 0}(\frac{x}{a_1+a_2x} - \frac{a_1x}{(a_1+a_2x)^2} + \frac{a_1}{a_1+a_2x}\frac{\partial x}{\partial a_1} - \frac{a_1x}{(a_1+a_2x)^2}\frac{\partial x}{\partial a_1}) = finite.$

Thus I can obtain $\lim_{a_1\to 0}\frac{\partial K}{\partial a_1} = -\infty$. Then I can conclude that when M is large enough and λ small enough, $\frac{\partial B}{\partial \lambda} > 0$. This concludes my proof.

□

Proof of Proposition 1.4.3. Step 1. When $\lambda = 0$, I know that $B_{ap}(0) = \theta(2p-1) + \log b$.

Step 2. If $B_{ap}(0) < 0$ and $\max_{\lambda<\lambda_{fb}} B(\lambda) < 0$, the unique equilibrium is no information acquisition equilibrium.

Step 3. If $B_{ap}(0) < 0$ and $\max_{\lambda<\lambda_{fb}} B_{ap}(\lambda) > 0$, I prove that there exist three equilibria. The first one is non-information acquisition equilibrium becasue $B_{ap}(0) < 0$ and $\frac{\partial B_{ap}(0)}{\partial \lambda} > 0$. I let λ^* be the solution to $\max_{\lambda<\lambda_{fb}} B(\lambda)$. Then there exists one solution in $(0, \lambda^*)$. Moreover, because $B_{ap} < B_{fb}$, I know that $B(\lambda_{fb}) < 0$, thus exists one solution in $(\lambda^*, \lambda_{fb})$. Step 4. If $B_{ap}(0) > 0$, because $B_{ap}(\lambda_{fb}) < 0$, then there exists at least one positive solution in $(0, \lambda_{fb})$ □

Proof of Proposition 1.4.4. This result is direct because I know that $B_{ap} < B_{fb}$. □

Proof of Lemma 1.5.1. I know that $E(V|X) = \theta p(V = \theta|X) - \theta p(V = -\theta|X)$. Then because $p(V = \theta|X) = \frac{p(V=\theta,X)}{P(X)}$,then I have:

$$p(V=\theta,X) = \begin{cases} \frac{p}{2} \quad if \quad N-\lambda < X \leq N+\lambda \\ \frac{1}{2} \quad if \quad -N+\lambda \leq X \leq N-\lambda \\ \frac{1-p}{2} \quad if \quad -N-\lambda \leq X < -N+\lambda \end{cases} \tag{1.33}$$

$$p(V=-\theta,X) = \begin{cases} \frac{1-p}{2} \quad if \quad N-\lambda < X \leq N+\lambda \\ \frac{1}{2} \quad if \quad -N+\lambda \leq X \leq N-\lambda \\ \frac{p}{2} \quad if \quad -N-\lambda \leq X < -N+\lambda \end{cases} \tag{1.34}$$

Then I have:

$$p(V = \theta|X) = \begin{cases} p \quad if \ \ N - \lambda < X \leq N + \lambda \\ \frac{1}{2} \quad if \ \ -N + \lambda \leq X \leq N - \lambda \\ 1 - p \ \ if \ \ -N - \lambda \leq X < -N + \lambda \end{cases} \tag{1.35}$$

Thus, I conclude the proof.

□

Proof of Lemma 1.5.2. The proof is shown as follows:

$$\begin{aligned} E_p^c &= p(s_c = h)E(V - P|s_c = h) + p(s_c = l)E(P - V|s_c = l) \\ &= (2p-1)\theta \tfrac{N-\lambda}{N} \end{aligned} \tag{1.36}$$

□

Proof of Lemma 1.5.3. Because

$$\begin{aligned} E(P|s_j = h) &= (2p-1)\theta * prob(s_c = h|s_j = h)\tfrac{\lambda}{N} \\ &-(2p-1)\theta * prob(s_c = l|s_j = h)\tfrac{\lambda}{N} \\ &= \theta \tfrac{\lambda}{N}(2p-1)^3 \end{aligned} \tag{1.37}$$

Following the same logic, I can get $E(P|s_j = l) = -\theta\frac{\lambda}{N}(2p-1)^3$. Then I calculate expected trading profit of investors who acquire private signal as:

$$E_p^I = (2p-1)\theta \tfrac{N-(2p-1)^2\lambda}{N} \tag{1.38}$$

□

Proof of Proposition 1.5.1. Step 1. I prove that no herding equilibrium occurs in the economy without agency problem. Following the analaysis of optimal contract, I know that the payments $\pi = W_a + C$. Then when $\lambda > 0$ and $\mu = 0$ in the herding equilibriu, I will have $E_p^c - W_a - C = 0 > E_p^I - W_a - C$. Because this is impossible, I conclude that herding equilibrium will not occur.

Step 2. I calculate the optimal payment scheme provided by principals who acquire s_c in the herding equilibrium. Because $f^I(P = -(2p-1)\theta, D|s_c = h) = f^I(P = (2p-1)\theta, D|s_c = l) = 0$, the optimal scheme following the proof of Proposition 2.2, I know that

$$\pi(P = -(2p-1)\theta, s_c = h, D) = \pi(P = (2p-1)\theta, s_c = l, D) = -\infty \tag{1.39}$$

Otherweise, $\pi = W_a + C$.

Step 3. I calculate the optimal payment scheme provided by principals who acquire s_i in the herding equilibrium in this step. Before calculation of optimal payment scheme, I calculate pdf of P and D conditional on s_i. To simply the analysis, I only consider the case when M goes to infinity. When M goes to infinity, I know that pdf of P and D conditional on s_i is equivalent to pdf of P conditional on s_i. Then I have the following cases if agents acquire information:

$$prob^I(P|s_i = h) = \begin{cases} \frac{\lambda[p^2+(1-p)^2]}{N} & if\ P = (2p-1)\theta \\ \frac{N-\lambda}{N} & if\ P = 0 \\ \frac{2\lambda p(1-p)}{N} & if\ \ P = -(2p-1)\theta \end{cases} \tag{1.40}$$

$$prob^I(P|s_i = l) = \begin{cases} \frac{2\lambda p(1-p)}{N} & if\ P = (2p-1)\theta \\ \frac{N-\lambda}{N} & if\ P = 0 \\ \frac{\lambda[p^2+(1-p)^2]}{N} & if\ \ P = -(2p-1)\theta \end{cases} \tag{1.41}$$

Then I have the following cases if agents do not acquire information:

$$prob^U(P) = \begin{cases} \frac{\lambda}{2N} & if\ P = (2p-1)\theta \\ \frac{N-\lambda}{N} & if\ P = 0 \\ \frac{\lambda}{2N} & if\ \ P = -(2p-1)\theta \end{cases} \tag{1.42}$$

Following the first-order approach in above proof, I know that

$$\frac{1}{v(s_i = h, P = (2p-1)\theta)} = \frac{1}{v(s_i = l, P = -(2p-1)\theta)} = \lambda_1 + \lambda_2 - \lambda_2 \frac{\exp(-\gamma_a C)}{2[p^2 + (1-p)^2]} \tag{1.43}$$

$$\frac{1}{v(s_i = h, P = -(2p-1)\theta)} = \frac{1}{v(s_i = l, P = (2p-1)\theta)} = \lambda_1 + \lambda_2 - \lambda_2 \frac{\exp(-\gamma_a C)}{4p(1-p)} \tag{1.44}$$

$$\frac{1}{v(s_i = h, P = 0)} = \frac{1}{v(s_i = l, P = 0)} = \lambda_1 + \lambda_2 - \lambda_2 \exp(-\gamma_a C) \tag{1.45}$$

I let $p_1 = p^2 + (1-p)^2$, $x_1 = \lambda_1 + \lambda_2 - \lambda_2 \frac{\exp(-\gamma_a C)}{2p_1}$, $y_1 = \lambda_1 + \lambda_2 - \lambda_2 \frac{\exp(-\gamma_a C)}{2(1-p_1)}$, $a_{11} = \frac{\lambda}{N}$, $a_{21} = \frac{\exp(-\gamma_a W_a)(1-\exp^{-\gamma_a C})}{p_1 - 0.5}$ and $b_1 = \exp^{-\gamma_a W_a} - \frac{\exp(-\gamma_a W_a)(1-\exp^{-\gamma_a C})}{2p_1 - 1}$. When $b_1 < 0$, the solution to solve the optimal contract does not exists.

Step 4. If $\exp^{-\gamma_a W_a} - \frac{\exp(-\gamma_a W_a)(1-\exp^{-\gamma_a C})}{2p_1 - 1} < 0$ and $E_p^c = (2p-1)\theta\frac{N-\lambda}{N} - W_a - C > 0$ for some positive λ, I can get the results in the proposition.

□

Proof of Proposition 1.5.2. Step 1. In the first-best case, it is clear that the optimal payment scheme is constant. That is $\pi = W_a + C$. Because the net befit of information acquisition for home investors is $(1-p_h)(2p-1)\theta\frac{N-\eta}{N} - W_a - C$ and the net benefit of information acquisition for foreign investors is $(2p-1)\theta\frac{N-\eta}{N} - W_a - C$. If $\lambda > 0$, this indicates that

$$(1-p_h)(2p-1)\theta\frac{N-\eta}{N}-W_a-C=0 \tag{1.46}$$

Moreover, this indicates that $(2p-1)\theta\frac{N-\eta}{N}-W_a-C>0$. Thus, it implies that μ should be infinity. This is impossible because price will be fully revealing when μ goes to infinity. Then trading profit will become zero and this violate the assumption that $(1-p_h)(2p-1)\theta\frac{N-\eta}{N}-W_a-C=0$. Therefore, I can conclude that $\lambda=0$. This implies that neither weak herding equilibrium not strong herding equilibrium occur in the first-best case.

Step 2. I only prove that strong herding equilibrium occurs under some condition in the economy with agency problem. Particularly, I try to find the condition under which $\lambda=1$ and $\mu=0$. For the foreign investors, the approach to solve the optimal contract is simila to the proof of Proposition 3.2. I only replace $\lambda(2p-1)$ with η in the proof. When both of M and N go to infinity, I know that net benefit of information acquisition for foreign investors is $B_{f,ap}(0)=(2p-1)\theta+\log[\exp^{-\gamma_a W_a}-\frac{\exp^{-\gamma_a W_a}(1-\exp^{-\gamma_a C})}{2p-1}]$.

Step 3. I take the following steps to solve the optimal contract for the home investors. The conditional pdf of $s_{h,i}$ when s_i is informative is shown as follows:

$$prob^I(s_{h,i}|s_i=h)=\begin{cases}p_h p & if\ s_{h,i}=\theta\\ 1-p_h & if\ s_{h,i}=\emptyset\\ p_h(1-p) & if\ \ s_{h,i}=-\theta\end{cases} \tag{1.47}$$

$$prob^I(s_{h,i}|s_i=l)=\begin{cases}p_h(1-p) & if\ s_{h,i}=\theta\\ 1-p_h & if\ s_{h,i}=\emptyset\\ p_h p & if\ \ s_{h,i}=-\theta\end{cases} \tag{1.48}$$

The conditional pdf of $s_{h,i}$ when s_i is uninformative is shown as follows:

$$prob^U(s_{h,i}) = \begin{cases} \frac{p_h}{2} & if\ s_{h,i} = \theta \\ \frac{1}{2} & if\ s_{h,i} = \emptyset \\ \frac{p_h}{2} & if\ \ s_{h,i} = -\theta \end{cases} \tag{1.49}$$

Following the first-order approach in proof of Proposition, I know that

$$\frac{1}{v(s_i = h, s_{h,i} = \theta)} = \frac{1}{v(s_i = l, s_{h,i} = -\theta)} = \lambda_1 + \lambda_2 - \lambda_2 \frac{\exp(-\gamma_a C)}{2p} \tag{1.50}$$

$$\frac{1}{v(s_i = h, s_{h,i} = -\theta)} = \frac{1}{v(s_i = l, s_{h,i} = \theta)} = \lambda_1 + \lambda_2 - \lambda_2 \frac{\exp(-\gamma_a C)}{2(1-p)} \tag{1.51}$$

$$\frac{1}{v(s_i = h, s_{h,i} = \emptyset)} = \frac{1}{v(s_i = l, s_{h,i} = \emptyset)} = \lambda_1 + \lambda_2 - \lambda_2 \exp(-\gamma_a C) \tag{1.52}$$

This is similar to the proof of Proposition 3.2, I let $x = \lambda_1 + \lambda_2 - \lambda_2 \frac{\exp(-\gamma_a C)}{2p}$, $y = \lambda_1 + \lambda_2 - \lambda_2 \frac{\exp(-\gamma_a C)}{2(1-p)}$, $a_{12} = p_h$, $a_{22} = \frac{\exp(-\gamma_a W_a)(1-\exp^{-\gamma_a C})}{p-0.5}$ and $b_2 = \exp^{-\gamma_a W_a} - \frac{\exp(-\gamma_a W_a)(1-\exp^{-\gamma_a C})}{2p-1}$.

Following proof of Proposition 4.2, I know that the net benefit of informaiton acquisition for home investors is $B_{h,ap}(p_h) = (1-p_h)(2p-1)\theta - K$ (where $K = \frac{1}{2}p_h \log(x) + \frac{1}{2}p_h \log(y) + (1-p_h)\log[px + (1-p)y]$).

When $p_h = 0$, I know that $B_{h,ap}(0) = (2p-1)\theta + \log[\exp^{-\gamma_a W_a} - \frac{\exp^{-\gamma_a W_a}(1-\exp^{-\gamma_a C})}{2p-1}]$. Then I denote the derivative of $B_{h,ap}(p_h)$ with p_h by $\frac{\partial B_{h,ap}}{\partial p_h}$. Then I know that for very small positive ϵ, I know that $B_{h,ap}(\epsilon) = (2p-1)\theta + \log[\exp^{-\gamma_a W_a} - \frac{\exp^{-\gamma_a W_a}(1-\exp^{-\gamma_a C})}{2p-1}] + \epsilon * \frac{\partial B_{h,ap}}{\partial p_h}$. Because $\frac{\partial B_{h,ap}(0)}{\partial p_h}$ is infinity and θ is not in the function $B_{h,ap}$, there exists small enough θ satisfying $(2p-1)\theta + \log[\exp^{-\gamma_a W_a} - \frac{\exp^{-\gamma_a W_a}(1-\exp^{-\gamma_a C})}{2p-1}] = -\epsilon$. In this case, I know that $B_{h,ap}(\epsilon) > 0$ and $B_{h,ap}(0) < 0$. I let $\theta_1 = \frac{-\log[\exp^{-\gamma_a W_a} - \frac{\exp^{-\gamma_a W_a}(1-\exp^{-\gamma_a C})}{2p-1}] - \epsilon}{2p-1}$ and $\theta_2 = \frac{-\log[\exp^{-\gamma_a W_a} - \frac{\exp^{-\gamma_a W_a}(1-\exp^{-\gamma_a C})}{2p-1}]}{2p-1}$. This implies foreign investors never have incentive to acquire information, but home investors have incentive to acquire informtion under the condition:$\theta_1 < \theta < \theta_2$.

$\square$

Proof of Proposition 1.6.1. Following the similar process in the proof of Proposition 1.3.2 and Proposition 1.4.2, I know that

$$\pi_i(s_i = h, P = \theta, D) = \pi_i(s_i = l, P = -\theta, D) = \pi_1 \tag{1.53}$$

$$\pi_i(s_i = h, P = -\theta, D) = \pi_i(s_i = l, P = \theta, D) = \pi_3 \tag{1.54}$$

$$\pi_i(s_i = h, P = 0, D) = \begin{cases} \pi_1 & if \quad M - \theta \le D \le M + \theta \\ \pi_2 & if \quad -M + \theta \le D < M - \theta \\ \pi_3 & if \quad -M - \theta \le D < -M + \theta \end{cases} \tag{1.55}$$

$$\pi_i(s_i = l, P = 0, D) = \begin{cases} \pi_3 & if \quad M - \theta \le D \le M + \theta \\ \pi_2 & if \quad -M + \theta \le D < M - \theta \\ \pi_1 & if \quad -M - \theta \le D < -M + \theta \end{cases} \tag{1.56}$$

As I know that$\pi_1 > \pi_2 > \pi_3$ if all PC and IC in the second-best are binding as shown below. Then it is similar as proof of Proposition 1.3.2, I know that this contract can satisfies ex ante IC and ex post IC. Then I let $U_1 = U(\pi_1)$,$U_2 = U(\pi_2)$ and $U_3 = U(\pi_3)$. Particularly, the PC and IC follows:

$$a_1 p U_1 + (1 - a_1) U_2 + a_1 (1 - p) U_3 = U(W_a) + C \tag{1.57}$$

$$U_1 = U_3 + \frac{C}{a_1(p-0.5)} \tag{1.58}$$

It is clear that $U_2 = U(W_a) - \frac{C}{2p-1} - \frac{a_1}{1-a_1}U_3$.

Now I prove information acquisition complementarity is robust when M is infinite and λ is small enough for different γ as follows.

Now the principals' optimization problem becomes to minimize

$$\min_{\pi_1,\pi_2,\pi_3} a_1 p\pi_1 + (1-a_1)\pi_2 + a_1(1-p)\pi_3 \tag{1.59}$$

This problem can be transferred to:

$$\min_{U_3} G(U_3) \tag{1.60}$$

where $G(U_3) = a_1 p U^{-1}(U_3 + \frac{C}{a_1(p-0.5)}) + (1-a_1)U^{-1}(U(W_a) - \frac{C}{2p-1} - \frac{a_1}{1-a_1}U_3) + a_1(1-p)U^{-1}(U_3)$.

Since a_1 is a linear function of λ. The first-order condition with λ is equivalent to the first-order condition with a_1, I have $\frac{\partial G(U_3)}{\partial U_3} = 0$. It is easy to check that Assumption ass:crra can ensure there exists interior solution to the contracting problem. Particularly, I have the following three cases:

Case 1: If $0 < \gamma < 1$, I know that I U_3 should satisfy: $U_3 > 0$ and $U(W_a) - \frac{C}{2p-1} - \frac{a_1}{1-a_1}U_3 > 0$.

Case 2: If $\gamma > 1$, I know that U_3 should satisfy: $U_3 + \frac{C}{a_1(p-0.5)} < 0$,$U_3 < 0$ and $\frac{U(W_a) - \frac{C}{2p-1}}{1-a_1} - \frac{a_1}{1-a_1}U_3 < 0$.

Case 3: If $\gamma = 1$ and $K = 0$, then I know that $U(W) \approxeq \ln(W)$ (where $\approxeq$ represents liner transformation).

It is obvious that $U^{-}(x)=[(\frac{1-\gamma}{\gamma}x)^{\frac{1}{1-\gamma}}-K]\frac{\gamma}{A}$. I let the solution to this problem to be U_3^*. Then the minimum value of $G(U_3)$ is $G(a_1,U_3^*)$. Now I have effect of a_1 on $G(a_1,U_3^*)$ is:

$$\begin{aligned}\frac{\partial G(a_1,U_3^*)}{\partial a_1} = & pU^{-1}(U_3+\frac{C}{a_1(p-0.5)})-U^{-1}(U(W_a)-\frac{C}{2p-1}-\frac{a_1}{1-a_1}U_3)+(1-p)U^{-1}(U_3)\\ & +\frac{\partial G(a_1,U_3^*)}{\partial U_3^*}\frac{\partial G(U_3^*)}{\partial a_1}-a_1p\frac{\partial[U^{-1}(U_3+\frac{C}{a_1(p-0.5)})]}{\partial(U_3+\frac{C}{a_1(p-0.5)})}\frac{C}{a_1^2(p-0.5)}\\ & +(1-a_1)\frac{\partial[U^{-1}(\frac{U(W_a)-\frac{C}{2p-1}}{1-a_1}-\frac{a_1}{1-a_1}U_3)]}{\partial(\frac{U(W_a)-\frac{C}{2p-1}}{1-a_1}-\frac{a_1}{1-a_1}U_3)}(-\frac{1}{(1-a_1)^2}U_3)\end{aligned} \tag{1.61}$$

For any cases, I know that $\frac{\partial G(a_1,U_3^*)}{\partial U_3^*}\frac{\partial G(U_3^*)}{\partial a_1}=0$. I show information acquisition is complementary case by case.

Case 1($\gamma<1$): I know that: $(1-a_1)\frac{\partial[U^{-1}(\frac{U(W_a)-\frac{C}{2p-1}}{1-a_1}-\frac{a_1}{1-a_1}U_3)]}{\partial(\frac{U(W_a)-\frac{C}{2p-1}}{1-a_1}-\frac{a_1}{1-a_1}U_3)}(-\frac{1}{(1-a_1)^2}U_3)<0$ because $U_3<0$

and $-U^{-1}(U(W_a)-\frac{C}{2p-1}-\frac{a_1}{1-a_1}U_3)+(1-p)U^{-1}(U_3)<0$

and then $pU^{-1}(U_3+\frac{C}{a_1(p-0.5)})-p\frac{\partial[U^{-1}(U_3+\frac{C}{a_1(p-0.5)})]}{\partial(U_3+\frac{C}{a_1(p-0.5)})}\frac{C}{a_1(p-0.5)}=p\frac{\gamma}{A}(\frac{1-\gamma}{\gamma})^{\frac{1}{1-\gamma}}(U_3+\frac{C}{a_1(p-0.5)})^{\frac{1}{1-\gamma}-1}[U_3-\frac{\gamma}{1-\gamma}\frac{C}{a_1(p-0.5)}]-p\frac{\gamma}{A}K<p\frac{\gamma}{A}(\frac{1-\gamma}{\gamma})^{\frac{1}{1-\gamma}}(\frac{C}{a_1(p-0.5)})^{\frac{1}{1-\gamma}-1}[U_3-\frac{\gamma}{1-\gamma}\frac{C}{a_1(p-0.5)}]-p\frac{\gamma}{A}K$

Because $U_3>0$ and $U_3<U(W_a)+C$, I know that $\lim_{a_1\to 0}\frac{\partial G(a_1,U_3^*)}{\partial a_1}=-\infty$. For the net benefit of information acquisition B, I have $\lim_{a_1\to 0}\frac{\partial B}{\partial a_1}=\infty$ for large enough M and small enough λ.

Case 2 ($\gamma>1$) From the proof of , I know that $p(\frac{A\pi_1}{\gamma}+K)^{\gamma}+(1-p)(\frac{A\pi_3}{\gamma}+K)^{\gamma}=(\frac{A\pi_2}{\gamma}+K)^{\gamma}$.

Let $\pi_i^{'}=(\frac{A\pi_1}{\gamma}+K)^{\gamma}$, I know that $\pi_i=[(\pi_i^{'})^{\frac{1}{\gamma}}-K]\frac{\gamma}{A}$, which is a concave function of $\pi_i^{'}$. Then I can have

$pU^{-1}(U_3+\frac{C}{a_1(p-0.5)})-U^{-1}(U(W_a)-\frac{C}{2p-1}-\frac{a_1}{1-a_1}U_3)+(1-p)U^{-1}(U_3)=p\,\pi_1+(1-p)\pi_3-\pi_2<0$

Becasue $U^{-}(x)=[(\frac{1-\gamma}{\gamma}x)^{\frac{1}{1-\gamma}}-K]\frac{\gamma}{A}$, I know that $\frac{\partial[U^{-1}(x)]}{\partial x}=\frac{1}{A}(\frac{1-\gamma}{\gamma}x)^{\frac{1}{1-\gamma}-1}$ and $\frac{\partial^2[U^{-1}(x)]}{\partial x^2}>0$

Then from FOC $\frac{\partial G(U_3)}{\partial a_1} = 0$, I know that

$$a_1 p \frac{\partial [U^{-1}(U_3 + \frac{C}{a_1(p-0.5)})]}{\partial (U_3 + \frac{C}{a_1(p-0.5)})} + (1-a_1) \frac{\partial [U^{-1}(U(W_a) - \frac{C}{2p-1} - \frac{a_1}{1-a_1}U_3)]}{\partial (U(W_a) - \frac{C}{2p-1} - \frac{a_1}{1-a_1}U_3)} + a_1(1-p) \frac{\partial [U^{-1}(U_3)]}{\partial (U_3)} = 0 \tag{1.62}$$

Thus, I have

$p \frac{\partial [U^{-1}(U_3 + \frac{C}{a_1(p-0.5)})]}{\partial (U_3 + \frac{C}{a_1(p-0.5)})} \frac{C}{a_1(p-0.5)} + \frac{\partial [U^{-1}(\frac{U(W_a) - \frac{C}{2p-1}}{1-a_1} - \frac{a_1}{1-a_1}U_3)]}{\partial (\frac{U(W_a) - \frac{C}{2p-1}}{1-a_1} - \frac{a_1}{1-a_1}U_3)} \frac{1}{1-a_1} U_3 = p \frac{\partial [U^{-1}(U_3 + \frac{C}{a_1(p-0.5)})]}{\partial (U_3 + \frac{C}{a_1(p-0.5)})} [\frac{C}{a_1(p-0.5)} - \frac{a_1 U_3}{(1-a_1)^2}] - \frac{a_1 U_3}{(1-a_1)^2}(1-p) \frac{\partial [U^{-1}(U_3)]}{\partial (U_3)}$

Because $[U(W_a) - \frac{C}{2p-1}]\frac{1-a_1}{a_1} < U_3 < -\frac{C}{a_1(p-0.5)}$, I know that $\lim_{a_1 \to 0} \frac{C}{a_1(p-0.5)} - \frac{a_1 U_3}{(1-a_1)^2} = \infty$.

For $\frac{\partial [U^{-1}(x)]}{\partial x} = \frac{1}{A}(\frac{1-\gamma}{\gamma}x)^{\frac{1}{1-\gamma}-1}$, I know that $\lim_{a_1 \to 0} \frac{\partial [U^{-1}(U_3)]}{\partial (U_3)} = 0$ because $U_3 \to -\infty$.

Because $U_3 + \frac{C}{a_1(p-0.5)} > U(W_a) + C$, I have $\frac{\partial [U^{-1}(U_3 + \frac{C}{a_1(p-0.5)})]}{\partial (U_3 + \frac{C}{a_1(p-0.5)})} > \frac{\partial [U^{-1}(x)]}{\partial x}|_{x=U(W_a)+C}$

Thus, I can conclude that

$\lim_{a_1 \to 0} p \frac{\partial [U^{-1}(U_3 + \frac{C}{a_1(p-0.5)})]}{\partial (U_3 + \frac{C}{a_1(p-0.5)})} \frac{C}{a_1(p-0.5)} + \frac{\partial [U^{-1}(\frac{U(W_a) - \frac{C}{2p-1}}{1-a_1} - \frac{a_1}{1-a_1}U_3)]}{\partial (\frac{U(W_a) - \frac{C}{2p-1}}{1-a_1} - \frac{a_1}{1-a_1}U_3)} \frac{1}{1-a_1} U_3 = \infty$

Therefore, it is easy to show that $\lim_{a_1 \to 0} \frac{\partial G(a_1, U_3^*)}{\partial a_1} = -\infty$ and I conclude that $\lim_{a_1 \to 0} \frac{\partial B}{\partial a_1} = \infty$ for large enough M and small enough λ.

Case 3 for$U(W) = \ln(W)$, I directly calculate

$\frac{\partial G(U_3)}{\partial a_1} = a_1 p \exp(U_3 + \frac{C}{a_1(p-0.5)}) + a_1(1-p)\exp(U_3) - a_1 \exp[\frac{U(W_a) - \frac{C}{2p-1}}{1-a_1} - \frac{a_1}{1-a_1}U_3] = 0$

Thus $\exp(\frac{1}{1-a_1}U_3)[p\exp(\frac{C}{a_1(p-0.5)}) + (1-p)] = \exp[\frac{U(W_a) - \frac{C}{2p-1}}{1-a_1}]$

I have $U_3 = U(W_a) - \frac{C}{2p-1} - (1-a_1)\log[p\exp(\frac{C}{a_1(p-0.5)}) + (1-p)]$

Then $\frac{G(a_1, U_3^*)}{\partial a_1} = \exp\{U(W_a) - \frac{C}{2p-1} + a_1 \log[p\exp(\frac{C}{a_1(p-0.5)}) + (1-p)]\}$

Then I let $g(a_1) = a_1 \log[p\exp(\frac{C}{a_1(p-0.5)}) + (1-p)]$

I know that $\lim_{a_1 \to 0} g(a_1) = \frac{a_1^2}{1} \frac{p\exp(\frac{C}{a_1(p-0.5)})}{p\exp(\frac{C}{a_1(p-0.5)}) + (1-p)} \frac{C}{a_1^2(p-0.5)} = \frac{C}{(p-0.5)}$

Then I know that $\frac{\partial g}{\partial a_1} = \frac{1}{a_1}[a_1 \log[p\exp(\frac{C}{a_1(p-0.5)}) + (1-p)] - \frac{C}{(p-0.5)}]$

Thus I have $\lim_{a_1 \to 0} \frac{\partial g}{\partial a_1} = -\infty$

Then I can conclude that $\lim_{a_1 \to 0} \frac{\partial G(a_1, U_3^*)}{\partial a_1} = -\infty$. For the net benefit of information acquisition B, I have $\lim_{a_1 \to 0} \frac{\partial B}{\partial a_1} = \infty$ for large enough M and small enough λ.

$\square$

Proof of Lemma 1.6.1. For principal i, his expected trading profit when he submits 1 is $(2p_K(s_i, P) - 1)\theta - P$, while his expected trading profit when he submits -1 is $P - (2p_K(s_i, P) - 1)\theta$. Because $p_K(s_i, P)$ is increasing with s_i, principal i is indifferent between submitting 1 and -1 when $(2p_K(s_i, P) - 1)\theta - P = 0$. This concludes the proof.

$\square$

Proof of Proposition 1.6.2. First, we have the condition as follows:

$$p_I(X_I, P) = p_U(X_U, P) = \frac{P + \theta}{2\theta} = \frac{1}{1 + \frac{f_l}{f_h}\frac{1}{L_I}}. \tag{1.63}$$

Then we have $\log(\frac{P+\theta}{\theta-P}) = \log(\frac{f_h}{f_l}) + \log(L_I)$. Denote $B(P) = \frac{\partial L_U^{-1}(L_I(X_I))}{\partial P}$. From the expressions of f_h and f_l, we have

$$\begin{aligned}\log(\frac{f_h}{f_l}) &= \log(\frac{\lambda(1+b+e)\phi(X_I-\theta)X_I'+\lambda(1-b-e)\phi(X_I+\theta)X_I'+(1-\lambda)(1+b)\phi(X_U-\theta)BX_I'+(1-\lambda)(1-b)\phi(X_U+\theta)BX_I'}{\lambda(1+b+e)\phi(X_I+\theta)X_I'+\lambda(1-b-e)\phi(X_I-\theta)X_I'+(1-\lambda)(1+b)\phi(X_U+\theta)BX_I'+(1-\lambda)(1-b)\phi(X_U-\theta)BX_I'}) \\ &= \log(\frac{[(1+b+e)\phi(X_I-\theta)+(1-b-e)\phi(X_I+\theta)][\lambda X_I'+\frac{(1-\lambda)(1+b)\phi(X_U-\theta)B+(1-\lambda)(1-b)\phi(X_U+\theta)}{(1+b+e)\phi(X_I-\theta)+(1-b-e)\phi(X_I+\theta)}BX_I']}{[(1+b+e)\phi(X_I+\theta)+(1-b-e)\phi(X_I-\theta)[\lambda X_I'+\frac{(1-\lambda)(1+b)\phi(X_U+\theta)B+(1-\lambda)(1-b)\phi(X_U-\theta)}{(1+b+e)\phi(X_I+\theta)+(1-b-e)\phi(X_I-\theta)}BX_I'}) \end{aligned} \tag{1.64}$$

Because $L_I(X_I) = L_U(X_U)$, we have

$$\begin{aligned}&\frac{(1-\lambda)(1+b)\phi(X_U-\theta)B+(1-\lambda)(1-b)\phi(X_U+\theta)}{(1+b+e)\phi(X_I-\theta)+(1-b-e)\phi(X_I+\theta)} \\ =&\frac{(1-\lambda)(1+b)\phi(X_U+\theta)B+(1-\lambda)(1-b)\phi(X_U-\theta)}{(1+b+e)\phi(X_I+\theta)+(1-b-e)\phi(X_I-\theta)}\end{aligned} \tag{1.65}$$

Thus, $\log(\frac{f_h}{f_l}) = \log(L_I)$

$\square$

Chapter 2

Investment Waves under Cross Learning

Shiyang Huang and **Yao Zeng** (Harvard University)

Abstract: We investigate how firms' cross learning amplifies industry-wide investment waves. Firms' investment opportunities are subject to idiosyncratic shocks as well as a common shock, and firms' asset prices aggregate speculators' private information about the two types of shocks. In investing, each firm learns from other firms' prices (in addition to its own) to make better inference about the common shock. Thus, a spiral between firms' higher investment sensitivity to the common shock and speculators' higher weighting on the common shock emerges. This leads to systematic risks in investment waves: higher investment and price comovements as well as their higher comovements with the common shock. Moreover, each firm's cross learning creates a new pecuniary externality on other firms, because it makes other firms' prices less informative on their idiosyncratic shocks through speculators' endogenous over-weighting on the common shock. This externalities increases in the number of firms, suggesting that more competitive industries may exhibit more inefficient investment waves.

2.1 Introduction

Industry-wide investment waves are frequently observed in history, especially after the arrival of major technology or financial innovations involving high uncertainty.[1] However, existing theories are often silent on one of their defining features: high systematic risks associated with many firms.[2] Specifically, in investment waves, a firm's real investment and asset price co-move greatly with other firms' investments and prices (see Rhodes-Kropf, Robinson and Viswanathan, 2005, Pastor and Veronesi, 2009, Hoberg and Phillips, 2010, Bhattacharyya and Purnanandam, 2011, Patton and Verardo, 2012, Greenwood and Hanson, 2013, for recent empirical documents). Also, both primary and secondary financial market participants overweight some industry-wide common news while underweight their corresponding idiosyncratic news in making investment decisions (see Peng, Xiong, and Bollerslev, 2007 and more broadly Rhodes-Kropf, Robinson and Viswanathan, 2005, Hoberg and Phillips, 2010, and Bhattacharyya and Purnanandam, 2011). An even more surprising fact recently documented is that more competitive industries exhibit more inefficient investment waves with higher systematic risks (Hoberg and Phillips, 2010, Greenwood and Hanson, 2013). Our paper provides a new rational theory that helps unify these facts of industry-wide investment waves that seem jointly puzzling otherwise.

Our mechanism to generate industry-wide investment waves highlights firms' cross learning, which means that firms learn from other firms' asset prices (in addition to their own asset prices) in making investment decisions, a natural fact well documented empirically but

[1] The most typical examples include the "railway mania" of the UK in the 1840s, the rapid development of automobiles and radio in the 1920s, and most recently the surge of the Internet in the 1990s, among many others. In addition to technological progress, other notable examples include major financial innovations like asset-backed securities (ABS) and credit default swaps (CDS), as well as the Mississippi Scheme and the South Sea Bubble, in which market structures experienced dramatic changes.

[2] The most popular explanation of investment waves comes from the literature of bubbles (see Brunnermeier and Oehmke, 2013, Xiong, 2013, for surveys of various models and evidence). These theories have focused on the over-investment or over-valuation of one single firm, and have often referred to behavioral aspects. The modern literature of macro-finance (see Brunnermeier, Eisenbach and Sannikov, 2013, for an extensive survey) also generates various forms of over-investment, over-borrowing, and over-lending, by highlighting agency or financial frictions. Also see He and Kondor (2013) for a most recent treatment of two-sided pecuniary externality in generating inefficient investment cycles. This literature focuses more on the macroeconomic implications of over-investment, such as fire sales and financial crises, rather than on the microeconomic anatomy of multi-firm investment waves as we tend to emphasize.

overlooked in the theoretical literature.[3] It has been also explicitly identified by recent empirical work (Foucault and Fresard, 2014, Ozoguz and Rebello, 2013). Our model builds on the burgeoning literature that highlights the feedback from secondary market asset prices to primary market investment decisions (see Bond, Edmans and Goldstein, 2012, for an extensive survey on the theoretical literature). Specifically, since secondary market participants may have incremental information that is unavailable to firms and primary market participants, firms or their capital providers may learn from the asset prices in the secondary markets for making investment decisions, and this in turn affects the asset prices in the secondary markets (see Chen, Goldstein, and Jiang, 2007, Edmans, Goldstein, and Jiang, 2012, for empirical evidence). The feedback literature, however, has not explored the multi-firm context and the cross-learning mechanism we emphasize which generate industry-wide investment waves.

Towards our goal, we extend the classical feedback framework to admit multiple firms with two fundamentally different types of shocks and cross learning. We highlight that investment opportunities in an industry or an economy are generally correlated, so that firms have the incentives to learn from each other's asset prices. To fix idea, Figure 1 depicts the typical landscape of a classical feedback story without this consideration, even if it can literally accommodate many firms. Although these firms can take advantage of their respective feedback channel for making better investment decisions, they are essentially separated in segmented economies and others' asset prices are irrelevant. Thus, they can be modeled by a representative firm. This is also the reason that why the existing feedback models usually feature one single firm or one single asset.[4]

[3] In the seminal field survey by Graham and Harvey (2001), CFOs of firms report that they tend to rely on other firms' prices in making capital budgeting decisions, and this in turn affects CEOs' investment decisions. As far as we know, this point has not been formally taken into account in existing corporate finance models.

[4] One exception is Subrahmanyam and Titman (2013), in which a private firm learns from the stock price of another public firm to make investment decision. The private firm's investment affects the profitability of the public firm through competition, which further generates interesting macroeconomic implications. But the public firm does not invest by itself and the private firm also does not have its own asset price. Hence, their model still features the standard feedback channel as shown in Figure 1. Their formal model also admits two private firms, which introduces an additional externality in terms of investment complementarity that amplifies their feedback effect. But as the authors have claimed, the introduction of two private firms is inessential for most of their results.

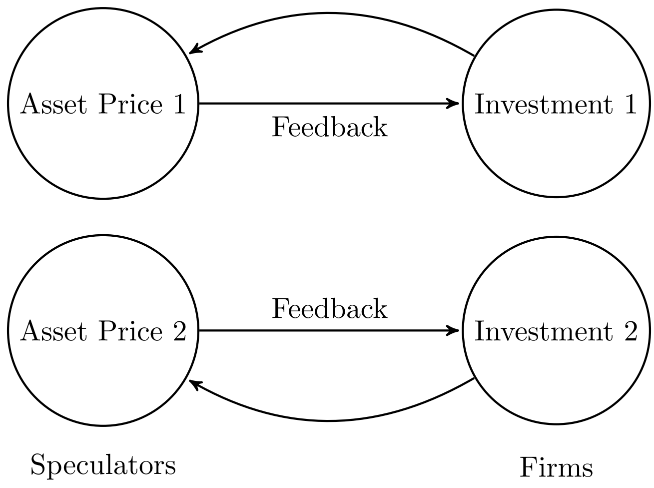

Figure 2.1.1: Self-Feedback Benchmark

Instead, the novelty of our work is to develop a tractable model, admitting two-way cross learning of firms from other firms' asset prices (in addition to their own), and to identify a new pecuniary externality involved. We explicitly model correlated investment opportunities by incorporating two fundamentally different shocks that necessitate firms' cross learning. When the fundamental of each firm's asset is subject to both a common productivity shock (industry shock)[5] and an idiosyncratic shock (firm-specific shock), other firms' asset prices are informative about the common shock for the firm in question. Thus, the firm in question uses other firms' asset prices (and its own) to know more about the common shock for making better investment decisions; similarly to other firms. Such cross learning makes firms' investments more sensitive to the common shock, encouraging secondary market speculators to weight information about the common shock more in trading. This in turn makes firms' asset prices more informative about the common shock, further encouraging firms to cross learn and thus resulting in an even higher investment sensitivity to the common shock. As a consequence, even a tiny common shock can be amplified significantly. This mechanism is reminiscent of the classic signal extraction problem and the resulting rational herding highlighted by Scharfstein and Stein (1990), Froot, Scharfstein and Stein (1992) and many others, while we explicitly

[5]In a broader sense, our common shock can also be interpreted as a shock to the entire economy. Hence, our model speaks to not only industry-wide investment waves but also more broadly economy-wide investment waves.

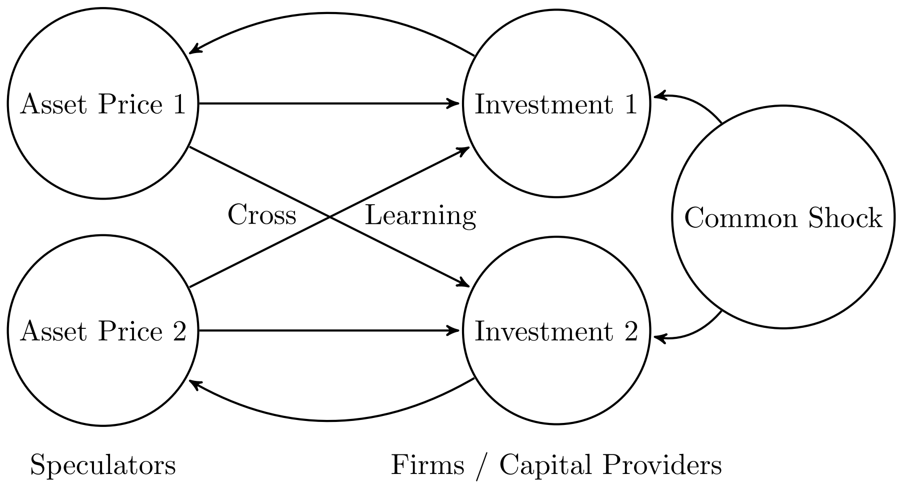

Figure 2.1.2: Firm / Capital Provider Cross Learning

consider the feedback from financial markets to the real economy and do not rely on any forms of short-termism. Moreover, in our framework, when one firm makes use of other firms' asset prices, it does not internalize a negative pecuniary externality that those prices become less informative about other firms' idiosyncratic shocks, because of the secondary market speculators' endogenous over-weighting on the common shock. This externality leads to higher investment inefficiency. Interestingly, the new pecuniary externality takes effect through the informativeness rather than the level of prices. Figure 2 illustrates the idea of cross learning and contrasts it to the standard feedback framework. Empirically, firms' cross learning has been documented by recent studies like Foucault and Fresard (2014)[6] and Ozoguz and Rebello (2013) and the magnitude is shown to be considerable, serving as a foundation for our theory.

The predictions of our model are consistent with many empirical regularities on investment waves. Compared to a benchmark in which firms are unable to learn from others' asset prices, cross learning generates a higher weight of the speculators on the information of the common shock in trading (as documented by Peng, Xiong, and Bollerslev, 2007 and more

[6]For the purpose of developing empirical hypotheses, Foucault and Fresard (2014) build a suggestive model, featuring one-way learning: one focal firm may learn from its peer firm's price while not the other way around, and the peer firm does not invest. That model lays out a nice foundation for their empirical analysis. However, it generates neither inefficient multi-firm investment waves nor comovements with the common productivity shock as emphasized in our paper. The setup and mechanisms of their model are also completely different from ours.

broadly by Rhodes-Kropf, Robinson and Viswanathan, 2005, Hoberg and Phillips, 2010, and Bhattacharyya and Purnanandam, 2011) and the firms' higher investment sensitivity to the common shock.[7] We further show that under cross learning, a firm's investment and price co-move more greatly with 1) other firms' investments and prices, and with 2) the common productivity shock as well, fitting in line with the evidence in Rhodes-Kropf, Robinson and Viswanathan (2005), Pastor and Veronesi (2009), Hoberg and Phillips (2010), Bhattacharyya and Purnanandam (2011) and Patton and Verardo (2012). We interpret these patterns as higher systematic risks in industry-wide investment waves. Compared to alternative theories, cross learning is further consistent with Maksimovic, Phillips and Yang (2013)'s message that investment waves are more significant among public firms than among private firms, by highlighting the indispensable role of explicit prices in public financial markets. To the best of our knowledge, our work is the first to establish the existence and significance of firms' two-way cross-learning effect on both investments, prices, and systematic risks.

Along this line, we investigate many circumstances in which the changes of economic conditions generate higher systematic risks in investment waves via the cross-learning mechanism, which are otherwise puzzling. First, an increasing uncertainty on the common productivity shock, most typically induced by the introduction of major technological innovations, leads to stronger weighting on the information of the common shock and higher systematic risks. This is consistent with the empirical facts in Brunnermeier and Nagel (2004) and Pastor and Veronesi (2006, 2009) and the more broadly documented evidence in the bubble literature (Brunnermeier and Oehmke, 2013, Xiong, 2013). Our new mechanism contributes to the existing rational learning mechanisms (Pastor and Veronesi, 2009, Johnson, 2007), by featuring both multi-firm investment waves and inefficiency. Second, an improvement of the firms' knowledge on the common productivity shock leads to higher systematic risks, consistent with the facts in Greenwood and Nagel (2009). Last, lower market liquidity or higher variance of idiosyncratic noisy supply also leads to higher systematic risks. These empirical regularities have

[7]Complementary to the theoretical literature that highlights investors' attention allocation to the common shock (see Peng and Xiong, 2006, Veldkamp, 2006, Veldkamp and Wolfers, 2007), our work speaks to its endogenous origin from firms' cross learning as well as its feedback into firms' investment decisions.

been frequently ascribed to separate behavioral accounts in the past literature, while our work provides a consistent rational explanation.

Our framework allows for a clear welfare analysis, offering a new perspective to look at the relationship between inefficient investment waves and industrial competition. Due to the unaligned interests of firms and speculators in feedback and the new pecuniary externality associated with cross learning, the investment waves are inefficient. In particular, we show that as the number of firms in an industry increases, cross learning becomes stronger, leading to a more severe pecuniary externality. This suggests a rationale for the puzzling facts identified in Hoberg and Phillips (2010) and Greenwood and Hanson (2013) that more competitive industries exhibit more predictable financial and real boom-bust cycles as well as greater market and real inefficiencies. According to Hoberg and Phillips (2010), no single existing theory can accommodate their findings. Our cross-learning mechanism with the new pecuniary externality implies that more competitive industries may exhibit more over-weighting on the common shock, more under-weighting on the idiosyncratic shocks, and more inefficient investment waves with higher systematic risks, consistent with Hoberg and Phillips (2010) and Greenwood and Hanson (2013)'s messages. Ozoguz and Rebello (2013) have also explicitly identified that firms in more competitive industries have a higher investment sensitivity to stock prices of their peers, which supports our predictions.

Fundamentally, the amplification effect of cross learning stems from a series of endogenous strategic complementarities and a spiral that are absent in existing literature. At the beginning, the dependence of investment on asset price results in an endogenous complementarity between each firm's investment sensitivity on the common shock and speculators' weighting on information about the common shock. When multiple firms' cross learning is introduced, a new spiral comes out. Cross learning first makes different firms' investments more correlated with the common shock as well as with each other. As a result, speculators find it more profitable to put a higher weight on information about the common shock. Since asset prices thus become relatively more informative about the common shock, firms' investment sensitivities on the common shock increase even more. This spiral further generates two new complementarities in our multi-firm setting. The first is among speculators' weights on the information about the common shock in each asset market, and the second is among different firms' relative investment sensitivities to the common shock. The interaction of the spiral and these endogenous complementarities is again seen in Figure 2, which constructs a strong amplification effect from a fundamental shock to systematic risks. In contrast to the existing literature involving complementarities in financial markets (see Veldkamp, 2011, for an extensive review), our mechanism does not rely on any exogenous complementarities (for example, higher-order beliefs or coordination in actions) but a well documented fact that firms learn from own and other firms' asset prices. Given that trading in financial markets usually exhibit natural strategic substitutability, our endogenous complementarities and spiral are of more significance.

Related Literature. Our work contributes to the literature of rational models on investment booms and busts.[8] Early literature has focused on the role of industrial organizations (for example, Reinganum, 1989, Jovanovic and McDonald, 1994) or self-fulfilling expectations (for example, Shleifer, 1986) in generating investment waves, but financial markets are generally absent in these classic papers. The modern literature has been paying increasing attention to the role of learning in financial markets.[9] In the rational learning model of Rhodes-Kropf and Viswanathan (2004), which shares a similar signal extraction problem with ours, managers cannot distinguish between common misvaluation and possible idiosyncratic synergies, leading to merger and acquisition waves. Pastor and Veronesi (2009) propose a more explicit learning model, in which the uncertain productivity of a new technology is subject to learning. Learning and the ensuing technology adoption makes the uncertainty from idiosyncratic to systematic, generating investment waves. In this spirit, Johnson (2007) argues that firms

[8]This literature is more broadly related to the bubble and modern macro-finance literatures mainly based on behavioral or belief aspects and agency or financial frictions, as discussed above. The focuses of those literatures are however different from ours. Our model is not intended as a general dynamic theory of booms and busts either.

[9]Other notable rational models on investment waves include DeMarzo, Kaniel, and Kremer (2007, 2008), featuring endogenous relative wealth concerns from the non-tradability of future endowments, Carlson, Fisher and Giammarino (2004) and Aguerrevere (2009), featuring investments as real options, and Ozdenoren and Yuan (2013), featuring contractual externalities given the average performance contractable and the resulting excessive risk-taking.

may learn about uncertain investment opportunities in the form of experimenting, which also generates investment waves. Our contribution to this literature is three-fold. First, our model features multiple firms and their cross learning explicitly, which allows us to study industry-wide investment waves directly rather than to look at them from the perspective of single-firm investment cycles. Second, we cast the microstructure of public asset markets explicitly by an adapted Kyle (1985) model, ensuring us to reflect the indispensable role of public financial markets as suggested by Maksimovic, Phillips and Yang (2013). Lastly, our model identifies a new externality regarding the use of information about common shocks and idiosyncratic shocks in making inefficient investment decisions.

Our framework also contributes to the burgeoning feedback literature as surveyed by Bond, Edmans and Goldstein (2012). Among existing models, closely related are Goldstein, Ozdenoren and Yuan (2013), Goldstein and Yang (2014a,b), and Sockin and Xiong (2014a,b), all of which highlight the feedback from (secondary market) speculators' information aggregation to (primary market) capital providers' scale-varying real investment decisions. Technically, these papers have also employed a log-normal framework. Our contribution is to investigate multi-firm feedback by introducing a tractable two-way cross-learning framework with two types of shocks, which generates the new pecuniary externality and various implications that are absent in existing models.

Identifying the externality associated with cross learning contributes to the large pecuniary externality literature.[10] The classical pecuniary externality takes effect through the level of prices: agents do not internalize the impacts of their actions on equilibrium price levels, leading to a welfare loss under various frictions. In our framework, instead, firms that make real investment decisions do not fully internalize the impacts of cross learning on equilibrium price informativeness. This leads to a "tragedy of the commons" regarding the use of the price system as an information source under multi-firm cross learning. In this sense, our pecuniary externality is reminiscent of the learning externality in the early dynamic learning and herding literature (for example, Vives, 1997) that an agent, when responding to his private information,

[10]See Stiglitz (1982), Greenwald and Stiglitz (1986), Geanakoplos and Polemarchakis (1985), and Farhi and Werning (2013), He and Kondor (2013), Davila (2014) for recent theoretical developments.

does not take into account the benefit of increased informativeness of public information in the future.[11]

Our work is also related to the literature on the interaction across asset markets or fundamentals, in particular the models that highlight learning. This literature has focused on speculators' learning rather than firms' cross learning as we model. Cespa and Foucault (2014) consider the contagion of illiquidity across segmented markets by introducing a concept of cross-asset learning. By cross-asset learning, speculators trading in one market can potentially learn from the asset price in another market, which generates propagation. Goldstein and Yang (2014c) model an environment in which different speculators are informed of different fundamentals affecting one single asset. Trading on information about the two fundamentals exhibits complementarity, suggesting that greater diversity of information improves price informativeness. Our model complements to those papers by focusing on the implications of firm cross learning on both real investments and asset prices, in contrast to their exchange economy setting that focuses on trading.

Finally, our framework is broadly related to a large macroeconomic literature focusing on dispersed information, in particular on the different roles of private and public information. Closely related are Angeletos, Lorenzoni and Pavan (2012) on the role of beauty contest in generating investment exuberance and Amador and Weill (2010) on the crowding-out effect of exogenous public information provision to the use of private information.[12] Compared to Angeletos, Lorenzoni and Pavan (2012) highlighting information spillover from real activities to financial markets, our cross-learning framework with more detailed financial market structures arguments it with the opposite learning channel explicitly. Importantly, by modeling cross learning with two types of shocks, we are able to generate new strategic complementarities and the new spiral towards the common shock, which are consistent with empirical evidence. Our

[11] The recent study of Vives (2014) further combines the classical pecuniary externality (through the level of prices) and the learning externality associated with exogenous public information in an industrial competition context. This is different from our new pecuniary externality through endogenous price informativeness on the two shocks. Its focus is also on the strategic interaction in product markets instead of our endogenous cross learning in financial markets.

[12] Amador and Weill (2010) also relies on the earlier idea in Vives (1993) that the more informative prices are, the less agents rely on private information, with the consequence that less information will be incorporated into prices.

externality is also different from theirs that features beauty contest in signaling and higher-order uncertainties. Complementary to Amador and Weill (2010), the externality in our model stems from a different mechanism and suggests a new crowding-out effect: endogenous over-weighting of the common shock crowds out the use of information about the idiosyncratic shocks. Neither of these two papers has distinguished between common productivity shock and idiosyncratic productivity shocks.

The rest of the paper is organized as follows. Section 3.2 lays out the model, featuring correlated investment opportunities and cross learning. Section 2.3 characterizes the cross-learning equilibrium and benchmarks it to the self-feedback case. Section 2.4 investigates important implications of cross learning with a focus on systematic risks in investment waves. Section 2.5 explores the externality and the relationship between investment inefficiency and competition. Section 2.6 discusses some extensions of the model. All proofs are delegated to Appendix unless otherwise noted.

2.2 Model

2.2.1 Economy

The model extends the feedback framework of Goldstein, Ozdenoren and Yuan (2013) for a different focus on capital provider cross learning.[13] We consider a continuum of 1 of firms, $i \in [0, 1)$, each having an asset traded in a secondary market. Each firm i's corresponding asset market is occupied by a mass 1 of informed risk-neutral speculators, respectively. We index speculators for firm i by a couple (i, j), with $j \in [0, 1)$.[14] Each firm i's corresponding secondary market is occupied by noise traders. Each firm also has an exclusive capital provider i in a primary market who decides how much capital to provide to the firm for investment purpose.

[13]For related papers building on this framework or sharing a similar mathematical foundation in modeling, see Sockin and Xiong (2014a,b) and Goldstein and Yang (2014a,b). These papers do not consider fundamentally different productivity shocks or multiple firms' cross learning as we do.

[14]Since the speculators do not have a diversification motive, our results are unaffected if we assume that they can trade all assets. In other words, market segmentation in terms of trading plays no roles in our model.

There are three dates, $t = 0, 1, 2$. At date 0, the speculators trade in their corresponding asset market with their private information, and the asset price aggregates their information. At date 1, the capital providers observe the asset prices of both their own firm and all the other firms. Having observed all the prices and received their private information, the capital providers decide the amount of capital to provide for their corresponding firms and the firms undertake investment accordingly. All the cash flows are realized at date 2.

2.2.2 Capital Providers and Investment

All the firms in the economy have an identical linear production technology: $Q(I_i) = AF_iI_i$, where I_i is the amount of capital provided by capital provider i to firm i, and A and F_i are two stochastic productivity shocks. Specifically, shock A captures an industry-wide common productivity shock, and shock F_i captures the idiosyncratic productivity shock for firm i only.[15] Denote by a and f_i the natural logs of these shocks, and assume that they are normal and mutually independent:

$$a \sim N(0, 1/\tau_a)\,, \text{ and } f_i \sim N(0, 1/\tau_f)\,,$$

where τ_a and τ_f are positive and $i \in [0, 1)$.

The introduction of multiple firms and the two fundamentally different productivity shocks plays an important role in necessitating firms' cross learning. Specifically, if the investment opportunities are uncorrelated, cross learning makes no sense. On the other hand, however, if the investment opportunities are perfectly correlated, all asset prices become identical and thus there is no need to learn from other's prices as well. To flesh our cross-learning mechanism out, we abstract away from possible industrial organization of the firms' product market.

At date 1, all the capital providers choose the amount of capital I_i simultaneously in their respective primary markets. Capital provider i captures a proportion $\kappa \in (0, 1)$ of the output $Q(I_i)$ by providing I_i, which incurs a private quadratic adjustment cost, $C(I_i) = \frac{1}{2}cI_i^2$. Thus,

[15] In what follows, we omit the term productivity for brevity at times when there is no confusion.

capital provider i's problem at $t = 1$ is

$$\max_{I_i} \mathbb{E}\left[\kappa A F_i I_i - \frac{1}{2} c I_i^2 | \Gamma_i\right], \tag{2.1}$$

where Γ_i is the information set of capital provider i at $t = 1$. It consists of the price of firm i's own asset, P_i, and those of all the other firms' assets, denoted by the set $\{P_{-i}\}$ for brevity,[16] formed at date 0 as endogenous public signals, as well as their private signals about the (log) productivity shocks a and f_i. Specifically, we assume that each capital provider i gets a private noisy and independent signal $s_{a,i}$ about the (log) common productivity shock a with precision τ_s, and another private noisy and independent signal $s_{f,i}$ about its own (log) idiosyncratic productivity shock f_i with precision τ_f:

$$s_{a,i} = a + \varepsilon_{a,i}\,, \text{ where } \varepsilon_{a,i} \sim N(0, 1/\tau_{sa})\,, \text{ and}$$

$$s_{f,i} = f_i + \varepsilon_{f,i}\,, \text{ where } \varepsilon_{f,i} \sim N(0, 1/\tau_{sf})\,.$$

That is, for capital provider i, the information set is $\Gamma_i = \{P_i, \{P_{-i}\}, s_{a,i}, s_{f,i}\}$.

Different from existing literature, one major novelty of our setup is to allow capital providers to learn from other firms' asset prices as well as own firms' prices, which we formally call cross learning. As will be highlighted later, although the capital providers only care about their own firms, they use the prices of other firms' assets for making better investment decisions.

2.2.3 Speculators and Secondary Market Trading

At date 0, the remaining cash flow $(1-\kappa)Q(I_i)$, as an asset, is traded in a separate competitive secondary market for each firm i. For firm i, denote the price of this asset by P_i. To focus on capital providers' cross learning, we do not consider any possible monetary transfers from the secondary market to the firm, but highlight the information revealed in the secondary market

[16] As will be elaborated later, we focus on symmetric equilibria in which the firm in question i always puts the same weight on each of other firms' asset prices in cross learning. Thus, it is unnecessary for us to distinguish between those asset prices in analyzing cross learning.

trading.[17] In the asset market of firm i, each speculator (i, j) has two private and independent signals about the common shock and the respective idiosyncratic shock. Specifically, the first signal is about the common shock:

$$x_{ij} = a + \varepsilon_{x,ij}, \text{ where } \varepsilon_{x,ij} \sim N(0, 1/\tau_x)\,,$$

and the second signal is about the firm-specific idiosyncratic shock:

$$y_{ij} = f_i + \varepsilon_{y,ij}, \text{where } \varepsilon_{y,ij} \sim N(0, 1/\tau_y)\,.$$

Thus, the information set of speculator (i, j) is $\Gamma_{ij} = \{x_{ij}, y_{ij}\}$.[18]

Based on their private information, the speculators submit limited orders in a similar manner of Kyle (1985), with an additional constraint that each speculator can buy or sell up to a unit of the asset.[19] Formally, the speculators maximize their expected trading profit, taking the asset price as given.[20] Their problems at $t = 0$ are

$$\max_{d_{ij} \in [-1,1]} d_{ij} \mathbb{E}\left[(1-\kappa) A F_i I_i - P_i \middle| \, \Gamma_{ij}\right], \tag{2.2}$$

where d_{ij} is speculator (i, j)'s demand. The aggregate demand from the speculators in market i is given by $D_i = \int_0^1 d_{ij} \mathrm{d}j$.

We assume that the noisy supply in asset market i takes the following form:

$$\Delta(\zeta, \xi_i, P_i) = 1 - 2\Phi(\zeta + \xi_i - \lambda \log P_i)\,,$$

[17] Hence, this asset can be interpreted as either equity of the firm or a derivative on the return from the firm's investment. See a more detailed justification of this point in Goldstein, Ozdenoren and Yuan (2013).

[18] The fact that the speculators' information set does not consist of the asset prices is not essential. For any firm i, even if its speculators can learn from its asset price P_i, as long as they do not cross learn from other firms' asset prices $\{P_{-i}\}$, all of our results are unaffected.

[19] As discussed in Goldstein, Ozdenoren and Yuan (2013), the specific size of this position limit is inessential for the results, as long as speculators cannot take unlimited positions, otherwise the prices will be fully revealing. This constraint can be easily justified by their capital or borrowing constraints.

[20] The asset price can be viewed as set by a unmodeled market maker, as that in Kyle (1985).

where

$$\zeta \sim N(0, \tau_\zeta^{-1})\,, \text{ and } \xi_i \sim N(0, \tau_\xi^{-1})\,.$$

We elaborate the noisy supply. $\Phi(\cdot)$ denotes the cumulative standard normal distribution function. The first shock ζ captures a common noisy supply shock that can be viewed as industry-wide sentiment or industry-wide fund flow. The second shock ξ_i captures the idiosyncratic noisy supply shock in market i that can be viewed as styled trading or uninformed investors' unobserved preferences. The presence of a common noisy supply not only makes our framework more general, but more importantly prevents the aggregate price from fully revealing the common productivity shock. Both noisy supply shocks ζ and ξ_i are independent and also independent of other shocks in the economy. Meanwhile, λ in the noisy supply function captures price elasticity and can be viewed as market liquidity. When λ is high, the demand from speculators can be easily absorbed and thus their aggregate demand has little impact on the asset prices.

Finally, in equilibrium, the prices will clear each asset market by equalizing the aggregate speculator demand to the noisy supply in each asset market i:

$$D_i = \Delta(\zeta, \xi_i, P_i)\,. \tag{2.3}$$

2.2.4 Discussion

Before proceeding, we discuss some important differences of our settings from the past literature in the feedback literature, in particular, Goldstein, Ozdenoren and Yuan (2013), Foucault and Fresard (2014), and the contemporaneous study by Goldstein and Yang (2014a,b). There are, of course, more differences between our work and the existing feedback literature than what we discuss below, but the following ones are crucial for our mechanism and thus help stand out our contribution.

First, to lay out a foundation for characterizing cross learning, our model features a continuum of many firms. To accommodate multiple firms and two-way cross learning imposes new technical challenges in the sense of finding closed form solutions. To this end, our model provides a tractable approach not only suited for our purpose but potentially useful for future work in other directions.

Second, built upon the multiple-firm setup, our economy features two fundamentally different productivity shocks: one is common to all firms while the other is firm-specific. In most previous literature, there is only one productivity shock. One exception is Goldstein and Yang (2014a,b) who consider two shocks on the cash flow. However, their two shocks are fundamentally symmetric. Specifically, their two shocks differ in an exogenous informational sense that the capital providers perfectly observe one but not the other. Instead, our model allows us to explicitly recover how cross learning affects the endogenous sensitivities of firms' investment on the specific common shock and the idiosyncratic shocks. The contrast between the two fundamentally different shocks plays an important role in generating investment waves as well as delivering welfare implications on inefficient investment waves and competition.

Third, to highlight the interaction of the two fundamentally different shocks under cross learning, our model does not feature any public information of the speculators as often seen in the literature. Our efficiency implications come endogenously from a new pecuniary externality absent in previous literature that focuses on coordination failure or higher-order beliefs.

Finally, in contrast to the hypotheses development in Foucault and Fresard (2014), our framework features fully two-way cross learning instead of one-way learning by a focal firm from its peer firm. The one-way learning channel in Foucault and Fresard (2014) gives clear predictions on how the peer firm's stock price may affect the focal firm's investment, but the peer firm itself does not invest or learn. Our framework with two-way cross learning as well as more detailed real and financial market structures captures the new strategic complementarities and spiral toward the common productivity shock. The fundamental difference between the common shock and the idiosyncratic shocks matters only when the fully two-way cross learning is introduced. This eventually generates industry-wide inefficient investment waves consistent with empirical regularities.

2.3 Cross-Learning Equilibrium

2.3.1 Equilibrium Definition

We formally introduce the equilibrium concept. We focus on symmetric linear equilibria that are standard in the literature. Specifically, the speculators in market i long one share of the corresponding asset when $\phi_i x_{ij} + y_{ij} > \mu_i$, and short one share otherwise, where ϕ_i and μ_i are two constants that will be determined in equilibrium. Since agents are risk neutral and firms are symmetric in our framework, symmetry further implies that $\phi_i = \phi$ and $\mu_i = \mu$, which mean that all the speculators use symmetric trading strategies in all asset markets, and the information contents of all asset prices are also symmetric.

DEFINITION **2.3.1.** *A (symmetric) cross-learning equilibrium is defined as a collection of a price function for each firm i,* $P_i(a, f_i, \zeta, \xi_i)$*:* $\mathbb{R}^4 \to \mathbb{R}$*, an investment policy for each capital provider i,* $I_i(s_{a,i}, s_{f,i}, P_i, \{P_{-i}\})$*:* $\mathbb{R}^2 \times \mathbb{R}^\infty \to \mathbb{R}$*, and a linear monotone trading strategy for each speculator* (i, j)*,* $d_{ij}(x_{ij}, y_{ij}) = \boldsymbol{1}(\phi_i x_{ij} + y_{ij} > \mu_i) - \boldsymbol{1}(\phi_i x_{ij} + y_{ij} \leqslant \mu_i)$*, such that*

i) each capital provider i's investment policy $I_i(s_{a,i}, s_{f,i}, P_i, \{P_{-i}\})$ *solves problem (2.1),*

ii) each speculator (i, j)*'s trading strategy* $d_{ij}(x_{ij}, y_{ij})$ *is identical and solves problem (2.2), and*

iii) market clearing condition (2.3) is satisfied for each market i.

2.3.2 Equilibrium Characterization

We characterize the equilibrium, featuring the capital providers' cross learning. The equilibrium is hard to solve and involves many fixed-point problems, so we follow a step-by-step approach.

Step 1. We first solve for the price functions, which helps characterize the information contents of prices from the capital providers' perspective. We have the following lemma:

LEMMA **2.3.1.** *The speculators' trading leads to the following equilibrium price of each asset* *i:*

$$P_i = \exp\left(\frac{\phi_i}{\lambda\sqrt{\tau_x^{-1}\phi_i^2+\tau_y^{-1}}}a + \frac{1}{\lambda\sqrt{\tau_x^{-1}\phi_i^2+\tau_y^{-1}}}f_i + \frac{\zeta+\xi_i}{\lambda} - \frac{\mu_i}{\lambda\sqrt{\tau_x^{-1}\phi_i^2+\tau_y^{-1}}}\right). \tag{2.1}$$

Hence, from any capital provider i's perspective, the price for its own firm i's asset is equivalent to the following signal in predicting the common shock a:

$$z_a(P_i) = \frac{\lambda\sqrt{\tau_x^{-1}\phi_i^2+\tau_y^{-1}}\log P_i + \mu_i}{\phi_i} = a + \frac{1}{\phi_i}f_i + \frac{\sqrt{\tau_x^{-1}\phi_i^2+\tau_y^{-1}}}{\phi_i}(\zeta+\xi_i), \tag{2.2}$$

and is equivalent to the following signal in predicting the corresponding idiosyncratic shock f_i:

$$z_f(P_i) = \lambda\sqrt{\tau_x^{-1}\phi_i^2+\tau_y^{-1}}\log P_i + \mu_i = f_i + \phi_i a + \sqrt{\tau_x^{-1}\phi_i^2+\tau_y^{-1}}(\zeta+\xi_i). \tag{2.3}$$

Lemma 2.3.1 not only helps specify the information contents of a firm's asset price to its own capital provider, but also hints those to other firms' capital providers. Thus, it suggests the presence of capital providers' cross learning when feasible. The next step formulates the idea.

Step 2. We then characterize the informational consequences of cross learning. Specifically, we show that, when cross learning is feasible, that is, capital provider i's information set includes both P_i and $\{P_{-i}\}$, the capital provider relies on the aggregate price as well as the own asset price (in addition to their own private signals) in inferring the two productivity shocks. We impose the symmetry conditions $\phi_i = \phi$ and $\mu_i = \mu$ to conditions (2.1), (2.2) and (2.3) now as we focus on symmetric equilibria, and we also define the aggregate price as

$$\overline{P} = \int_0^1 P_i \mathrm{d}i\,.$$ [21]

Lemma **2.3.2.** *For capital provider i, when her information set includes both P_i and $\{P_{-i}\}$,*

[21] The fact that the asset prices are equally weighted in calculating the aggregate price is inessential to their information contents. Our results carry through even if we choose arbitrarily positive weights.

these asset prices are informationally equivalent to the following two signals:

i) a signal based on the aggregate price $\overline{P}$:

$$z_a(\overline{P}) = a + \frac{\sqrt{\tau_x^{-1}\phi^2 + \tau_y^{-1}}}{\phi}\zeta \tag{2.4}$$

for predicting the common shock a, with the precision

$$\tau_{\overline{pa}} = \frac{\tau_x \tau_y \tau_\zeta \phi^2}{\tau_x + \tau_y \phi^2}, \tag{2.5}$$

which is increasing in ϕ, and

ii) a signal based on the own asset price P_i as well as the aggregate price $\overline{P}$:

$$z_{f,i}(\overline{P}) = z_f(P_i) - \phi z_a(\overline{P}) = f_i + \sqrt{\tau_x^{-1}\phi^2 + \tau_y^{-1}}\xi_i \tag{2.6}$$

for predicting the corresponding idiosyncratic shock f_i, with the precision

$$\tau_{\overline{pf}} = \frac{\tau_x \tau_y \tau_\xi}{\tau_x + \tau_y \phi^2}, \tag{2.7}$$

which is decreasing in ϕ.

Along with Lemma 2.3.1, Lemma 2.3.2 implies that cross learning changes the feedback channel in which a capital provider uses asset prices to infer the two productivity shocks: she now uses the aggregate price $\overline{P}$ to infer the common shock a and still uses the own price P_i to infer the idiosyncratic shock f_i. Intuitively, for capital provider i, other firms' asset prices $\{P_{-i}\}$ are uninformative on the idiosyncratic shock f_i but informative on the common shock a. Hence, when other firms' asset prices are observable, which is natural in reality, the capital provider of the firm in question uses them to make better inference about the common shock. In particular, in a symmetric equilibrium, the aggregate price is sufficient for this purpose, as all asset prices are symmetric. By the law of large numbers, $\overline{P}$ only aggregates information about the common shock a: the information about idiosyncratic shocks and about the idiosyncratic noisy supply shocks all gets wiped out, while the presence of the common noisy supply shock still prevents the aggregate price from fully revealing. This makes $z_a(\overline{P})$, as characterized in (2.4), the most informative signal about the common shock a the capital provider can get. Moreover, knowing $z_a(\overline{P})$, the capital provider also eliminates the information about the common shock and about the common noisy supply shock when she uses her own price P_i to infer the idiosyncratic shock f_i, as characterized in (2.6).

Step 3. We then solve for the capital providers' optimal investment policy under cross learning. This indicates the real consequences of cross learning. Lemma 2.3.2 implies that, under cross learning, capital provider i uses the new signal $z_a(\overline{P})$ and her private signal $s_{a,i}$ to infer the common shock a, and the new signal $z_{f,i}(\overline{P})$ and the private signal $s_{f,i}$ to infer the idiosyncratic shock f_i. Thus, we have the following lemma.

LEMMA **2.3.3.** *Observing $s_{a,i}, s_{f,i}, P_i$ and $\{P_{-i}\}$, capital provider i's optimal investment policy is*

$$I_i = \frac{\kappa}{c}\exp\left[\frac{\tau_{sa}s_{a,i} + \tau_{\overline{pa}}z_a(\overline{P})}{\tau_a + \tau_{sa} + \tau_{\overline{pa}}} + \frac{1}{2(\tau_a + \tau_{sa} + \tau_{\overline{pa}})} + \frac{\tau_f s_{f,i} + \tau_{\overline{pf}}z_{f,i}(\overline{P})}{\tau_f + \tau_{sf} + \tau_{\overline{pf}}} + \frac{1}{2(\tau_f + \tau_{sf} + \tau_{\overline{pf}})}\right]. \tag{2.8}$$

The investment policy is intuitive. On the one hand, the optimal amount of investment is higher when the share κ of capital provider is higher while lower when the investment cost c is higher. On the other hand, the capital providers infer the two productivity shocks a and f_i independently but simultaneously in making investment decisions, reflected in the first and third terms in the parenthesis. In particular, the capital providers find it optimal to learn from both the own asset prices as well as other firms' prices, which are summarized in the two new signals $z_a(\overline{P})$ and $z_{f,i}(\overline{P})$. This fits quite in line with the recent empirical facts about firms' and capital providers' cross learning behavior (Foucault and Fresard, 2014, Ozoguz and Rebello, 2013).

According to Lemma 2.3.3, we propose the following intuitive concept of investment sensitivity to capture how the capital providers' investment decision responds to the two productivity shocks under cross learning.

Definition **2.3.2.** *For capital providers, the investment sensitivity to the common productivity shock and that to the idiosyncratic productivity shock are defined as:*

$$S_a(\tau_{\overline{pa}}) = \frac{\tau_{sa} + \tau_{\overline{pa}}}{\tau_a + \tau_{sa} + \tau_{\overline{pa}}}\ , \ \text{ and } S_f(\tau_{\overline{pf}}) = \frac{\tau_{sf} + \tau_{\overline{pf}}}{\tau_f + \tau_{sf} + \tau_{\overline{pf}}}\ ,$$

respectively. We call S_a the common investment sensitivity and S_f the idiosyncratic investment sensitivity henceforth.

We highlight that, the investment sensitivity depends on not only the capital providers' private signals about the corresponding shock, but also the new endogenous price signals coming from cross learning as characterized in Lemma 2.3.2. In particular, these two notions of investment sensitivity are increasing functions of $\tau_{\overline{pa}}$ and $\tau_{\overline{pf}}$, respectively, which are in turn affected by the speculators' trading strategy. Hence, by Lemma 2.3.2, we have the following straightforward lemma that bridges the capital providers' investment sensitivity and the speculators' weight ϕ on the signal of the common productivity shock.

Lemma **2.3.4.** *The common investment sensitivity $S_a(\tau_{\overline{pa}})$ is increasing in ϕ while the idiosyncratic investment sensitivity $S_f(\tau_{\overline{pf}})$ is decreasing in ϕ.*

Lemma 2.3.4 is helpful because it offers an intuitive look at the real consequences of learning from asset prices in the economy with two fundamentally different shocks. When speculators' weight ϕ is higher, they put more weight on the information about the common shock, and thus asset prices become more informative about the common shock while less informative about the idiosyncratic shocks. This in turn leads to a more sensitive investment policy in response to the common shock while a less sensitive one in response to the idiosyncratic shock.

Step 4. We finally close the model by solving for the speculators' equilibrium trading strategy, characterized by the weight ϕ and the constant μ. This also pins down other equilibrium outcomes since they are all functions of ϕ.

For speculator (i, j), her expected profit of trading given her available information is

$$\mathbb{E}\left[(1-\kappa)AF_iI_i - P_i | x_{ij}, y_{ij}\right]\ , \tag{2.9}$$

in which I_i and P_i have been characterized by conditions (2.8) and (2.1) respectively.

It is easy to show that, speculators' expected profit (2.9) of trading asset i can be expressed as

$$\mathbb{E}\left[(1-\kappa)AF_iI_i - P_i|x_{ij}, y_{ij}\right] = \frac{\kappa(1-\kappa)}{c}\exp\left(\alpha_0 + \alpha_1 x_{ij} + \alpha_2 y_{ij}\right) - \exp\left(\gamma_0 + \gamma_1 x_{ij} + \gamma_2 y_{ij}\right) \,,$$

where $\alpha_0, \alpha_1, \alpha_2, \gamma_0, \gamma_1$, and γ_2 are all functions of ϕ:

$$\begin{aligned}
\alpha_1 &= (S_a + 1)\frac{\tau_x}{\tau_a + \tau_x}\,, \\
\alpha_2 &= (S_f + 1)\frac{\tau_y}{\tau_f + \tau_y}\,, \\
\gamma_1 &= \frac{\phi}{\lambda\sqrt{\tau_x + \tau_y\phi^2}}\frac{\tau_x}{\tau_a + \tau_x}\,, \\
\gamma_2 &= \frac{1}{\lambda\sqrt{\tau_x + \tau_y\phi^2}}\frac{\tau_y}{\tau_f + \tau_y}\,.
\end{aligned}$$

By definition, in a symmetric cross-learning equilibrium with cross learning, we have

$$\phi = \frac{\alpha_1 - \gamma_1}{\alpha_2 - \gamma_2}\,.$$

Plugging in $\alpha_1, \alpha_2, \gamma_1$ and γ_2 yields

$$\phi = \frac{\left(S_a + 1 - \frac{\phi}{\lambda\sqrt{\tau_x+\tau_y\phi^2}}\right)\frac{\tau_x}{\tau_a+\tau_x}}{\left(S_f + 1 - \frac{1}{\lambda\sqrt{\tau_x+\tau_y\phi^2}}\right)\frac{\tau_y}{\tau_f+\tau_y}}\,. \tag{2.10}$$

Analyzing this equation by further plugging in S_a and S_f, which are both functions of ϕ, we reach a unique cross-learning equilibrium, formally characterized by the following proposition.

PROPOSITION **2.3.1.** *For a high enough noisy supply elasticity λ, there exists a cross-learning equilibrium in which the speculators put a positive weight $\phi > 0$ on the signal of the common productivity shock. For a high enough information precision τ_y (of the speculators' signal on the idiosyncratic shock), the equilibrium is unique.*

To establish the existence of a unique equilibrium is essential for our further analysis regarding investment waves, as it allows us to investigate that how changes in economic environment affect investments and prices through the cross-learning mechanism. When ϕ is higher, the speculators put more weight on the information about the common shock in trading, encouraging all the capital providers to respond to the common shock more sensitively through cross learning, which in turn leads to an even higher ϕ. This new spiral gives rise to many implications in line with the empirical phenomena regarding industry-wide investment waves as we explore later.

The conditions to guarantee a unique cross-learning equilibrium are not only standard in the feedback literature (see Goldstein, Ozdenoren and Yuan, 2013, among many others) but empirically plausible. A relatively high noisy supply elasticity λ implies that markets are liquid enough. A relatively high information precision τ_y of the speculators' signal on the idiosyncratic shock suggests that asset market participants understand their target firms better than the whole industry. These two conditions are in particular appropriate when we focus on the contexts leading to investment waves: relatively liquid markets and relatively more uncertain macroeconomic news.[22,23]

2.3.3 Self-Feedback Benchmark

Having established the existence and uniqueness of a cross-learning equilibrium, we benchmark the cross-learning equilibrium to the corresponding self-feedback equilibrium in a comparable economy. This self-feedback benchmark helps understand how the presence of cross learning affects the capital providers' investment policy and the speculators' trading strategy, in contrast to the counterfactual where cross learning is absent. In demonstrating these effects, we again focus on the difference of the speculators' weight ϕ on the signal of the common productivity shock in the two respective equilibria, as all equilibrium outcomes are functions of this weight.

[22]In the appendix, we explore other sufficient conditions that guarantee a unique cross-learning equilibrium. Our results regarding investment waves and investment inefficiency survive under other sets of sufficient conditions.

[23]We have numerically shown that these conditions are not restrictive. Even for reasonably small λ and τ_y, our model still features a unique cross-learning equilibrium. These numerical results are reported in Section 2.5.

We still consider unique symmetric equilibria and denote by ϕ' the speculators' weight on the signal of the common productivity shock in the self-feedback benchmark.

Formally, the only difference of the benchmark economy is that, each capital provider i observes its own asset price P_i but not other firms' asset prices $\{P_{-i}\}$. That is, capital provider i's information set is $\Gamma_i = \{P_i, s_{a,i}, s_{f,i}\}$. We have

$$P_i = \exp\left(\frac{\phi'}{\lambda\sqrt{\tau_x^{-1}(\phi')^2 + \tau_y^{-1}}}a + \frac{1}{\lambda\sqrt{\tau_x^{-1}(\phi')^2 + \tau_y^{-1}}}f_i + \frac{\zeta + \xi_i}{\lambda} - \frac{\mu_i}{\lambda\sqrt{\tau_x^{-1}(\phi')^2 + \tau_y^{-1}}}\right),$$

which is equivalent to the following two signals

$$z_a(P_i) = \frac{\lambda\sqrt{\tau_x^{-1}(\phi')^2 + \tau_y^{-1}}\log P_i + \mu_i}{\phi'} = a + \frac{1}{\phi'}f_i + \frac{\sqrt{\tau_x^{-1}(\phi')^2 + \tau_y^{-1}}}{\phi'}(\zeta + \xi_i)$$

in predicting the common shock a and

$$z_f(P_i) = \lambda\sqrt{\tau_x^{-1}(\phi')^2 + \tau_y^{-1}}\log P_i + \mu_i = f_i + \phi' a + \sqrt{\tau_x^{-1}(\phi')^2 + \tau_y^{-1}}(\zeta + \xi_i)$$

in predicting the corresponding idiosyncratic shock f_i. The precisions of $z_a(P_i)$ and $z_f(P_i)$ are denoted as τ_{pa} and τ_{pf} where

$$\tau_{pa} = \frac{1}{\frac{1}{(\phi')^2}\tau_f^{-1} + \frac{\tau_x^{-1}(\phi')^2+\tau_y^{-1}}{(\phi')^2}(\tau_\zeta^{-1} + \tau_\xi^{-1})},$$

and

$$\tau_{pf} = \frac{1}{(\phi')^2\tau_a^{-1} + (\tau_x^{-1}(\phi')^2 + \tau_y^{-1})(\tau_\zeta^{-1} + \tau_\xi^{-1})}.$$

Following the same definition of investment sensitivity and the same analysis for the capital providers' investment policy and the speculators' trading strategy, we have

$$
\begin{aligned}
S'_a &= \frac{\tau_{sa}+\tau_{pa}}{\tau_a+\tau_{sa}+\tau_{pa}} + \frac{\tau_{pf}}{\tau_f+\tau_{sf}+\tau_{pf}}\phi' \,, \\
S'_f &= \frac{\tau_{sf}+\tau_{pf}}{\tau_f+\tau_{sf}+\tau_{pf}} + \frac{\tau_{pa}}{\tau_a+\tau_{sa}+\tau_{pa}}\frac{1}{\phi'} \,, \\
\alpha'_1 &= \left(S'_a+1\right)\frac{\tau_x}{\tau_a+\tau_x} \,,
\end{aligned}
$$

$$
\begin{aligned}
\alpha'_2 &= \left(S'_f+1\right)\frac{\tau_y}{\tau_f+\tau_y} \,, \\
\gamma'_1 &= \frac{\phi'}{\lambda\sqrt{\tau_x+\tau_y(\phi')^2}}\frac{\tau_x}{\tau_a+\tau_x} \,, \\
\gamma'_2 &= \frac{1}{\lambda\sqrt{\tau_x+\tau_y(\phi')^2}}\frac{\tau_y}{\tau_f+\tau_y} \,.
\end{aligned}
$$

In the self-feedback equilibrium, we also have

$$
\phi' = \frac{\alpha'_1-\gamma'_1}{\alpha'_2-\gamma'_2}
$$

to pin down the speculators' weight on the information of the common shock. Plugging in $\alpha'_1, \alpha'_2, \gamma'_1$ and γ'_2 yields

$$
\phi' = \frac{\left(S'_a+1-\frac{\phi'}{\lambda\sqrt{\tau_x+\tau_y(\phi')^2}}\right)\frac{\tau_x}{\tau_a+\tau_x}}{\left(S'_f+1-\frac{1}{\lambda\sqrt{\tau_x+\tau_y(\phi')^2}}\right)\frac{\tau_y}{\tau_f+\tau_y}} \,. \tag{2.11}
$$

Therefore, we have the following proposition regarding the comparison between the cross-learning equilibrium and the corresponding self-feedback benchmark. We focus on comparable cases in which a self-feedback equilibrium and its corresponding cross-learning equilibrium are both unique.

Proposition **2.3.2.** *For a high enough noisy supply elasticity λ, a low enough idiosyncratic noisy supply shock precision τ_ξ, and a high enough information precision τ_y (of the speculators' signal on the idiosyncratic shock), there exists a unique self-feedback equilibrium in which speculators put a positive weight $\phi' > 0$ on the signal of the common productivity shock. In* *particular, $\phi' < \phi$, where ϕ is the speculators' weight on the signal of the common productivity shock in the corresponding cross-learning equilibrium.*

The comparison between a cross-learning equilibrium and its corresponding self-feedback equilibrium implies that, the presence of cross-learning may encourage the speculators to put a higher weight ϕ on the information about the common productivity shock. We also have the following straightforward corollary regarding the information precisions of the endogenous price signals and the capital providers' investment sensitivities, all of which are functions of ϕ.

COROLLARY **2.3.1.** *Compared to its corresponding self-feedback equilibrium, a cross-learning equilibrium features a higher ratio of the asset price information precision in predicting the common shock to that in prediction the idiosyncratic shock, i.e., $\tau_{\overline{pa}}/\tau_{\overline{pf}} > \tau_{pa}/\tau_{pf}$, and a higher ratio of the investment sensitivity to the common shock to that to the idiosyncratic shock, i.e., $S_a/S_f > S'_a/S'_f$.*

The results in Proposition 2.3.2 and Corollary 2.3.1 uncover the informational and real consequences of cross learning in equilibrium. Intuitively, when the capital providers are able to cross learn from each other's asset prices (in addition to their own firms' prices), they indeed do so in equilibrium as other firms' asset prices help them better infer the common shock. This makes firms' investments relatively more correlated with the common shock as well as with each other. Thus, the speculators find it more profitable to put more weight on the information about the common shock. This further makes asset prices becoming relatively more informative about the common shock in guiding investment decisions, and thus the capital providers respond to the common shock even more sensitively in investing. This spiral is absent in existing feedback models, and it indeeds plays an important role in amplifying industry-wide investment waves as we fully explore in the next section.

2.4 Systematic Risks in Investment Waves

The most important implications of cross learning are on the systematic risks in industry-wide investment waves. This comes from the endogenous spiral between the capital providers' investment sensitivity to the common shock and the speculators' weighting on the information about the common shock, as shown in Section 2.3. In our multi-firm setting, this spiral further leads to two new endogenous strategic complementarities. The new spiral and strategic complementarities help generate empirical implications of systematic risks in many relevant economic environments that seem jointly puzzling otherwise.

2.4.1 Impacts of Speculators' Weight on Systematic Risks

It is instructive to first investigate the impacts of the speculators' weight ϕ (on the information of the common shock) on systematic risks, taking the weight as given. Along the way, we also introduce our measures of systematic risks in investment waves.

Definition **2.4.1.** *The correlation coefficients between the investments of two firms and between the asset prices of two firms are defined as:*

$$\beta_I = \frac{Cov(\log I_i, \log I_j)}{\sqrt{Var(\log I_i)}\sqrt{Var(\log I_j)}}\,,\ \ \textit{and}\ \beta_P = \frac{Cov(\log P_i, \log P_j)}{\sqrt{Var(\log P_i)}\sqrt{Var(\log P_j)}}\,,$$

respectively. We call β_I the investment beta and β_P the price beta henceforth.

We take the investment beta β_I and the price beta β_P as two major measures of systematic risks in investment waves, on both the real and financial aspects, respectively. Typically, stronger investment waves are associated with a higher β_I and a higher β_P. However, as the recent study by Hong and Sraer (2013) argues, some investment waves only exhibit a higher investment beta β_I but not a higher price beta β_P. Hence, it is helpful to us to distinguish between these two betas in characterizing different types of investment waves.

We have the following intuitive result on the impacts of the speculators' weight ϕ on the two betas. When the speculators put a higher weight on the information of the common shock, the capital providers' investment sensitivities to the common shock increases, which makes their investments more correlated. Moreover, this in turn encourages the speculators to put a higher weight on the common productivity shock, which results in a higher correlation between asset prices. With the comparison between a cross-learning equilibrium and its corresponding self-feedback equilibrium in Section 2.3, these predictions shed lights on the empirical regularities in papers such as Rhodes-Kropf, Robinson and Viswanathan (2005), Pastor and Veronesi (2006, 2009), Hoberg and Phillips (2010), Bhattacharyya and Purnanandam (2011) and Patton and Verardo (2012).

LEMMA **2.4.1.** *Both the investment beta β_I and the price beta β_P are increasing in ϕ when $\phi > 0$.*

Similarly, we also look at the correlations between investment and the two productivity shocks, respectively. As a complement to Definition 2.3.2 of investment sensitivity and the associated Lemma 2.3.4, the following definition shoots a closer look at the equilibrium investments' correlation with the two shocks.

DEFINITION **2.4.2.** *The correlation coefficient between investment and the common productivity shock and that between investment and the idiosyncratic productivity shock are defined as:*

$$\beta_A = \frac{Cov(\log I_i, \log A)}{\sqrt{Var(\log I_i)}\sqrt{Var(\log A)}}\,, \ \ \textit{and}\ \beta_F = \frac{Cov(\log I_i, \log F_i)}{\sqrt{Var(\log I_i)}\sqrt{Var(\log F_i)}}\,,$$

respectively. We call β_A the common investment correlation and β_F the idiosyncratic investment correlation henceforth.

Intuitively, when the speculators put a higher weight on the information of the common shock, both investments and prices become more correlated with the common productivity shock instead of the idiosyncratic shocks. This is because the asset prices become more informative in predicting the common shock but less informative in predicting the idiosyncratic shock.

LEMMA **2.4.2.** *The common investment correlation β_A is increasing in ϕ while the idiosyncratic* *investment correlation β_F is decreasing in ϕ when $\phi > 0$.*

In what follows, we focus on the investment beta β_I and the price beta β_P in exploring the full equilibrium dynamics, highlighting the speculators' endogenous weight and equilibrium systematic risks under cross learning. The investigation on the common investment correlation β_A and the idiosyncratic investment correlation β_F yields the same insights.

2.4.2 Endogenous Cross Learning and Systematic Risks

Having established the impacts of the speculators' weight ϕ (on the information about the common shock) on systematic risks, we turn to one of the most interesting parts of the paper, which investigates how the changes of economic environments affect equilibrium systematic risks through the cross-learning mechanism. This unifies several empirical regularities that are otherwise hard to reconcile without taking the capital providers' cross learning into account. Mathematically, we perform formal comparative statics of the equilibrium betas with respect to exogenous parameters. We elaborate the first comparative statics (with respect to the common uncertainty) in more detail to explore the underlying mechanism, and the other comparative statics will follow the same intuition.

Common Uncertainty

We first focus on the effects of common uncertainty, which is captured by the prior precision τ_a of the common productivity shock. We view the change of common uncertainty as an important case, because a majority of industry-wide and economy-wide investment waves is associated with an increasing common uncertainty at the first place. The most typical driver for an increasing common uncertainty is the arrival of all-purpose technology or financial innovations, as documented in Brunnermeier and Nagel (2004), Pastor and Veronesi (2006, 2009), and more broadly the literature of bubbles. Our predictions help deliver a new perspective to look at the impacts of innovations and the accompanying increasing common uncertainty on the systematic risks in investment waves, highlighting the cross-learning mechanism.

We use the following assumption (only valid in this subsection on common uncertainly) to flesh out the cross-learning mechanism.

ASSUMPTION **2.4.1.** *The ratios τ_{sa}/τ_a (of the capital providers' signal precision on the common shock and the prior precision on the common shock) and τ_x/τ_a (of the speculators' signal precision on the common shock and the prior precision on the common shock) are kept as constants when τ_a changes.*

Assumption 2.4.1 not only helps shut down a direct information channel that confounds the cross-learning mechanism (only in this case about common uncertainty) but also captures the reality better. By keeping the two ratios constant, both the capital providers and the speculators do not find their private information more valuable in predicting the common productivity shock. This is actually closer to the reality that, when the common uncertainty increases, no market participant naturally has an advantage in resolving the common uncertainty. In this case, our cross-learning mechanism plays an important amplification role that is impossible otherwise. Assumption 2.4.1 is also completely benign; our results are only stronger without it.

LEMMA **2.4.3.** *Increasing the common uncertainty leads to a higher weight of the speculators on the information about the common shock. Specifically, the speculators' weight ϕ is decreasing in τ_a.*

From Lemma 2.4.3, we understand that an increasing in the common uncertainty leads to a stronger cross-learning spiral towards the common shock, despite that both the capital providers and speculators experience equally increasing uncertainty in their private information on the common shock. The following proposition further establishes the impacts on the equilibrium systematic risks.

PROPOSITION **2.4.1.** *Increasing the common uncertainty leads to both a higher investment beta and a higher price beta in equilibrium. Specifically, β_I and β_P are both decreasing in τ_a. We* *further decompose the effects into two negative components (the same for β_I and β_P):*

$$\frac{d\beta(\tau_a,\phi)}{d\tau_a} = \underbrace{\frac{\partial\beta(\tau_a,\phi)}{\partial\tau_a}}_{\textit{Mechanical Effect} <0} + \underbrace{\frac{\partial\beta(\tau_a,\phi)}{\partial\phi}\frac{\partial\phi}{\partial\tau_a}}_{\textit{Cross-Learning Effect} <0} < 0\,.$$

Proposition 2.4.1 indicates two effects contributing to the higher systematic risks associated with an increasing common uncertainty. The first is a mechanical effect that does not depend on the endogenous interaction between the capital providers and the speculators under cross learning. Intuitively, when the common uncertainty increases, speculators' investment sensitivity to the common shock increases as well. This immediately results in a higher correlation among firms' investments and prices. Figure 2.4.1 illustrates this mechanical effect in a two-firm example.

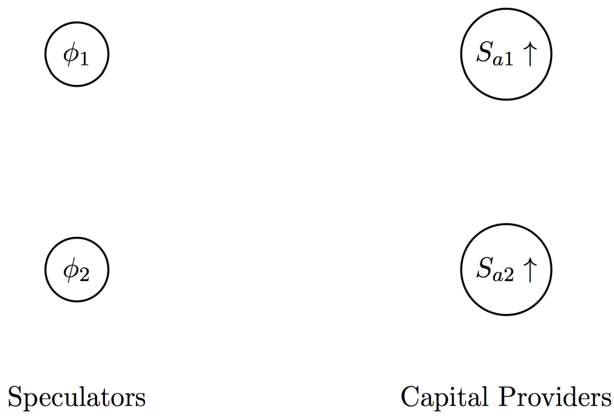

Figure 2.4.1: Mechanical Effect on Systematic Risks

The second effect, the cross-learning effect, is more interesting and only at play in our multi-firm cross-learning framework with two types of shocks. It reflects the new spiral between the capital providers' investment sensitivity to the common shock and the speculators' weight on the signal of the common shock. Interestingly, it takes place even when only some (not all) firms in the economy perceive the increasing common uncertainty.[24] Figure 2.4.2 illustrates this cross-learning effect in a two-firm example. Suppose, without loss of generality, firm 1's

[24] Technically, this requires some non-meaure-zero firms to perceive the increasing common shock.

capital provider perceives the increasing common uncertainty. As in the upper-left panel, firm 1's investment sensitivity to the common shock S_{a1} first increases (along with a decreasing investment sensitivity to its idiosyncratic shock), leading to a higher weight ϕ_1 on the information of the common shock by its speculators. Then, as in the upper-right panel, a higher ϕ_1 results in an even higher S_{a1} since firm 1 learns from its own price. More importantly, because of cross learning, firm 2's investment sensitivity to the common shock S_{a2} also increases, since firm 2 finds firm 1's price more informative about the common shock and thus understands the common shock better. It then naturally leads to a higher weight ϕ_2 on the information of the common shock by firm 2's speculators, as in the lower-left panel. Finally, the increase of ϕ_2 results in even higher S_{a1} and S_{a2} by cross learning, as in the lower-right panel. The entire process suggests two new strategic complementarities only under cross learning: the first is among speculators' weights on the information about the common shock in each market, and the second is among different firms' relative investment sensitivities to the common shock. With the two strategic complementarities, the spiral goes on and on and eventually pushes the economy to a new equilibrium with much higher systematic risks.

Our predictions on systematic risks after an increasing common uncertainty are consistent with the literature (Brunnermeier and Nagel, 2004, Pastor and Veronesi, 2006, 2009) that documents the increasing systematic risks after major technological innovations, as these innovations often come with industry-wide uncertain market prospects. In particular, the cross-learning effect sheds lights on the huge magnitude of systematic risks in these investment waves that are often ascribed to behavioral biases (see Brunnermeier and Oehmke, 2013, Xiong, 2013, for surveys).

Capital Providers' Access to Information

We then turn to the capital providers' access to private information, captured by the two precisions τ_{sa} and τ_{sf} regarding the two productivity shocks, respectively. Again, we have the following lemma pertaining to the speculators' endogenous weight.

LEMMA **2.4.4.** *Increasing the capital providers' information precision on the common shock* *leads to a higher weight of the speculators on the information about the common shock, while increasing the capital providers' information precision on the idiosyncratic shock leads to a lower weight. Specifically, the speculators' weight ϕ is increasing in τ_{sa} while decreasing in τ_{sf}.*

Lemma 2.4.4 prescribes that, when the capital providers have better information on the common shock, the equilibrium cross-learning spiral towards the common shock is also stronger; while better information on the idiosyncratic shock pushes the cross-learning spiral towards the idiosyncratic shocks. This further leads to the following proposition. Similar to Proposition 2.4.1, we have the mechanical effect and the cross-learning effect, both in the same direction.

PROPOSITION **2.4.2.** *For the capital providers' access to private information, we have the following results.*

i) Increasing the precision on the common shock leads to a higher investment beta when the precision is not large, and always a higher price beta; specifically, β_I is increasing in τ_{sa} when $\tau_{sa} > \tau_a + \tau_x \tau_\zeta$ and β_P is always increasing in τ_{sa}.

ii) Increasing the precision on the idiosyncratic shock leads to both a lower investment beta and a lower price beta; specifically, β_I and β_P are always decreasing in τ_{sf}.

The predictions here are broadly supported by the empirical evidence in Greenwood and Nagel (2009). It suggests that younger and more confident capital providers, who tend to have better knowledge about the industry-wide common shock compared to that on their idiosyncratic shocks, tilt their investments more towards the common shock, leading to higher investment and price correlations. Greenwood and Nagel (2009) admit that the magnitude of systematic risks they have observed is obviously larger than any existing rational models can accommodate and thus refer to behavioral explanations. In this sense, our predictions provide a new angle to investigate such effects from a rational perspective, highlighting the potential of strong cross learning.

Liquidity Trading

We also investigate the effects of liquidity trading, captured by the market liquidity λ and the two precisions of noisy supplies τ_ζ and τ_ξ. Similarly, we have the following intuitive lemma on the speculators' endogenous weight.

LEMMA **2.4.5.** *For liquidity trading, a higher weight of the speculators on the information about the common shock results from a lower market liquidity, a lower variance of common noisy supply, or a higher variance of idiosyncratic noisy supply. Specifically, the speculators' weight ϕ is decreasing in λ, increasing in τ_ζ, and decreasing in τ_ξ.*

The predictions along the three dimensions are all intuitive. When the market liquidity is higher, it is easier for the noisy traders to absorb speculators' demand, so that the cross-learning spiral towards the common shock is weaker. When the variance of the common noisy supply is lower, speculators are more likely to trade upon the common productivity shock. In contrast, when the variance of the idiosyncratic noisy supply is lower, speculators are less likely to trade upon the common shock, which results in a weaker spiral towards the common shock,

These predictions are further reflected in the following proposition, speaking to the overall effects of liquidity trading on investment waves. Again, similar to Proposition 2.4.1, we have the mechanical effect and the cross-learning effect in the same direction.

PROPOSITION **2.4.3.** *For liquidity trading, we have the following results.*

i) A higher investment beta β_I results from a lower market liquidity, a lower variance of common noisy supply, or a higher variance of idiosyncratic noisy supply. Specifically, β_I is decreasing in λ, increasing in τ_ζ, and decreasing in τ_ξ.

ii) A higher price beta β_P results from a lower market liquidity, or a higher variance of idiosyncratic noisy supply. Specifically, β_P is decreasing in λ and decreasing in τ_ξ.

2.5 Investment Inefficiency and Competition

An important question is that how firms' cross learning affects real investment efficiency. On the positive side, cross learning allows capital providers to take advantage of more information that would not be available if they were not able to observe their own and other firms' asset prices. However, the interests between capital providers and speculators in learning the two types of shocks are not perfectly aligned. More importantly, each firm's cross learning further creates a new pecuniary externality on other firms. These frictions result in investment inefficiency. In particular, the pecuniary externality associated with cross learning increases in the number of firms, suggesting that more competitive industries may exhibit more inefficient investment waves.

In evaluating that how these frictions affect investment efficiency, we proceed by two steps. First, we evaluate the overall investment efficiency and show that any cross-learning equilibrium always features investment inefficiency. Then we characterize the new pecuniary externality induced by cross learning to better understand the origin of such inefficiency. By doing this, we particularly underscore the implications of competition on inefficient investment waves through the new pecuniary externality.

2.5.1 Overall Investment Efficiency

Formally, we define investment efficiency by the ex-ante expected net benefit of the total investments by all the firms, given that capital providers may learn from all publicly available asset prices:

Definition **2.5.1.** *The investment efficiency of the economy is defined as*

$$R = \int_0^1 R_i di\,,$$

where

$$R_i = \mathbb{E}\left[\mathbb{E}\left[AF_iI_i - \frac{c}{2}I_i^2|\Gamma_i\right]\right]$$

denotes each firm i's ex-ante expected net benefit of investment, given its capital provider's information set under cross learning: $\Gamma_i = \{P_i, \{P_{-i}\}, s_{a,i}, s_{f,i}\}$.

We have the following proposition indicating the universal presence of investment inefficiency in a cross-learning equilibrium. We focus on the cases in which a unique cross-learning equilibrium is guaranteed.

PROPOSITION **2.5.1.** *There always exists a unique optimal weight* $\phi^* \geqslant 0$ *of the speculators on the signal of the common shock that maximizes investment efficiency. In particular, for a high enough noisy supply elasticity* λ *and a high enough information precision* τ_y *(of the speculators' signal on the idiosyncratic shock), the optimal weight is always smaller than that in the corresponding cross-learning equilibrium, i.e.,* $\phi^* < \phi$.

Proposition 2.5.1 indicates that when the speculators' signal on the idiosyncratic shock is relatively more precise, they tend to put an inefficiently high weight on the other signal about the common shock. This makes capital providers to respond to the common shock inefficiently too sensitively, leading to inefficient investment waves. This particular inefficiency fits quite in line with what we have observed in typical investment waves (for example, Rhodes-Kropf, Robinson and Viswanathan, 2005, Peng, Xiong, and Bollerslev, 2007, Hoberg and Phillips, 2010, Bhattacharyya and Purnanandam, 2011) that both primary and secondary market investors pay inefficiently too much attention to common shocks or noisy macroeconomics news while ignore informative idiosyncratic news.[25]

To better understand the impacts of cross learning on investment efficiency and potentially shed lights on corrective policies, we perform comparative statistics of investment efficiency with respect to several economic parameters. Again, we focus on unique cross-learning equilibria by assuming that the noisy supply elasticity λ and the information precision τ_y (of speculators' signal on the idiosyncratic shock) are high enough.

[25]Our framework is in fact general enough to admit the opposite case: when the speculators' signal on the common shock is relatively more precise, they tend to put an inefficiently too high weight on the signal about the idiosyncratic shocks, also leading to generic investment inefficiency. This case is empirically less plausible, but we still explore the theoretical possibilities in the appendix.

Proposition **2.5.2.** *In a cross-learning equilibrium, investment efficiency is higher when the market liquidity is higher, or the precision of idiosyncratic noisy supply is higher, or the capital providers' information precision on the idiosyncratic productivity shock is higher. Specifically, R is increasing in λ, τ_ξ, and τ_{sf}.*

The comparative statics regarding the investment efficiency are intuitive. First, a higher market liquidity has a corrective effect on the investment efficiency. That is, in a deeper asset market, the speculators' trading positions can be more easily absorbed. Specifically, when an asset market is more liquid or deeper, it becomes harder for the same amount of informed trading to impact the asset price. This is in particular beneficial when cross learning is strong after the arrival of major innovations or other common news involving high uncertainty, because the inefficient impact from speculators' overuse of information about the common shock can be better absorbed.

Importantly, this corrective effect on real investment efficiency helps justify recent regulatory concerns and practices by the SEC in limiting informed speculators' trading positions but at the same time lifting the participation barrier to less informed market makers and retail investors. These two are hard to be reconciled as approaches to correct investors' irrationality or to sidestep limits to arbitrage. In this sense, our cross-learning mechanism does a better job in delivering policy implications than typical models featuring bubbles.

Second, increasing investment efficiency in an economy with cross learning calls for a better use of information about the idiosyncratic shocks in the economy. Any policies on financial disclosure or government communication failing to keep this point in mind may end up crowding out the idiosyncratic news and resulting in investment inefficiency. This policy implication fits broadly in line with the recent studies that speak to the dark side of financial disclosures or central bank communications (Di Maggio and Pagano, 2013, Kurlat and Veldkamp, 2013). Theoretically, the endogenous overuse of information on the common shock due to multi-firm cross learning results in an inefficient crowding-out effect on the use of information on the idiosyncratic shock. Thus, it also complements the idea on the crowding-out effect of public information provision on the use of private information (see Amador and Weill, 2010).

2.5.2 Competition and Cross Learning

To help better understand the origin of investment inefficiency, we perform a theoretical exercise to further identify a new pecuniary externality induced by cross learning. In particular, in doing so, we extend our baseline model to admit finite number of firms. This allows us not only to underscore the efficiency change associated with different extent of cross learning but to investigate the relationship between competition and inefficient investment waves, which has been a well documented puzzle in recent empirical literature (see Hoberg and Phillips, 2010, Greenwood and Hanson, 2013, among others).

We first outline the extended cross-learning framework. A major challenge in identifying the externality associated with cross learning is to deal with the information endowment effect. Specifically, when the actual number of firms increase, the total amount of information in the economy also increases, leading to an efficiency gain to each firm. This information endowment effect confounds the identification of externalities and thus needs to be controlled properly. To achieve this goal, our extended cross-learning framework still features a continuum of 1 of firms being able to learn from all asset prices. However, we assume that the speculators do not fully internalize capital providers' cross learning. Concretely, they believe that each firm only observes and learns from as many as $n \geqslant 1$ asset prices, including its own price. This setting delivers an equilibrium weight (of the speculators on the information about the common shock) identical to that in a corresponding economy with n finite firms operating and the speculators fully internalizing their cross learning, while keeps the total amount of information endowment invariant with n. Hence, we are able to stand out the externality associated with cross learning as the number of firms increases.

We rigorously formulate the idea above as follows. We divide all the firms into $n \geqslant 1$ groups, a continuum of $1/n$ of firms in each group. The firms still observe and learn from all the asset prices as in the baseline model, regardless of the grouping. However, the speculators do not fully internalize firms' cross learning as before. Specifically, let $i \in [0, 1/n)$ denote one firm in the first group. The speculators believe that for any i, the n firms in the set

$\{i + k/n | 0 \leqslant k \leqslant n-1, k \in \mathbb{Z}\}$ learn only from the asset prices of each other but not from the asset prices of other firms outside the set. Figure 5 offers an illustration of the case when $n = 3$, in which the speculators believe that the three red firms (i, $i + 1/3$, and $i + 2/3$) cross learn only from each other and the three blue firms (i', $i' + 1/3$, and $i' + 2/3$) cross learn only from each other, similar for other firm triples.

This setting has several advantages, both economically and technically. First, it casts industry competition in a straightforward way. Since the speculators are risk neutral, it looks to them as if there are exactly n firms operating in the economy. Thus, the speculators' weight in equilibrium is identical to that in a corresponding economy with exactly n firms and the speculators fully internalizing their cross learning. Second, it helps identify the pecuniary externality induced by cross learning while keeps the total information endowment fixed. Especially, the efficiency change associated with cross learning takes place only through the speculators' endogenous weighting over the two types of shocks, making it possible to distinguish that from firms' actual information endowment. Last, this setting offers a smooth transition between the baseline model with full cross learning (as n goes to infinity) and the self-feedback benchmark (as n equals to 1). This not only makes our analysis analytically tractable but helps unify all the results and intuitions.

We acknowledge again that we are abstracting away from any possible industrial organization of the firms' product markets. Rather, we make use of the number of firms as a proxy for competition, which we believe is the most relevant measure.[26] This allows us to underscore the cross-learning mechanism by highlighting it as the only interaction among firms. In this sense, our model serves as a benchmark for further research that may take more aspects of industrial competition along with firms' cross learning into account.[27]

We proceed to characterize the equilibrium in the extended framework and the corre-

[26]To use the number of firms to proxy competition is common in the literature, especially when information is a focus (see Vives, 2010, for a survey).

[27]For example, Peress (2010) offers an interesting analysis on the impacts of monopolistic competition in product markets on stock market efficiency, but does not consider feedback to real investments or cross learning as we do. He does not consider the implications on investment waves as well.

sponding investment efficiency. We still consider symmetric equilibria, and denote by ϕ_n the speculators' weight on the signal of the common shock, when the speculators believe that each firm only learns from as many as n asset prices, including its own. For convenience, we call the associated equilibrium an n-learning equilibrium.

Formally, each capital provider i still observes its own asset price P_i and all other firms' asset prices $\{P_{-i}\}$. Same as before, the information content of any asset price is characterized by

$$P_i = \exp\left(\frac{\phi_n}{\lambda\sqrt{\tau_x^{-1}\phi_n^2 + \tau_y^{-1}}}a + \frac{1}{\lambda\sqrt{\tau_x^{-1}\phi_n^2 + \tau_y^{-1}}}f_i + \frac{\zeta + \xi_i}{\lambda} - \frac{\mu_i}{\lambda\sqrt{\tau_x^{-1}\phi_n^2 + \tau_y^{-1}}}\right),$$

equivalent to a signal

$$z_n\left(P_i\right) = \phi_n a + f_i + \sqrt{\tau_x^{-1}\phi_n^2 + \tau_y^{-1}}\left(\zeta + \xi_i\right).$$

However, in an n-learning equilibrium, the speculators believe that each capital provider only learns from its own price as well as the other $n-1$ firms' asset prices. Specifically, from the speculators' perspective, due to symmetry, each capital provider i has four signals: the own private signals $s_{a,i}$ and $s_{f,i}$, the signal $z_n(P_i)$ from its own asset price, and another signal $z_n(P_{-i})$ coming from the other $n-1$ asset prices:[28]

$$z_n\left(P_{-i}\right) = \phi_n a + \frac{\sum_{l\neq i} f_l}{n-1} + \sqrt{\tau_x^{-1}\phi_n^2 + \tau_y^{-1}}\left(\zeta + \frac{\sum_{l\neq i}\xi_l}{n-1}\right).$$

From the speculators' perspective, capital provider i uses these four signals to infer the sum of the two (log) productivity shocks, $a + f_i$, in making investment decisions. Concretely, the speculators believe that capital provider i updates beliefs as

$$\mathbb{E}[a + f_i|\Gamma_i] = \mathbf{z}'\mathrm{Var}(\mathbf{z})^{-1}\mathrm{Cov}(a + f_i, \mathbf{z}), \tag{2.12}$$

[28] When $n = 1$, only the first three signals are relevant and the n-learning equilibrium degenerates to a self-feedback equilibrium.

where $\mathbf{z} = [s_{a,i}, s_{f,i}, z_n(P_i), z_n(P_{-i})]'$. As a consequence, the speculators' perceived investment sensitivities S_{an} to the common shock and S_{fn} to the idiosyncratic shocks are read off from the conditional expectation (2.12). Following the same approach as before in solving for the speculators' optimal weight in trading, we finally get

$$\phi_n = \frac{\left(S_{an} + 1 - \frac{\phi_n}{\lambda\sqrt{\tau_x + \tau_y \phi_n^2}}\right)\frac{\tau_x}{\tau_a + \tau_x}}{\left(S_{fn} + 1 - \frac{1}{\lambda\sqrt{\tau_x + \tau_y \phi_n^2}}\right)\frac{\tau_y}{\tau_f + \tau_y}}. \tag{2.13}$$

Clearly, the equilibrium condition (2.13), with the investment sensitivities prescribed by condition (2.12), is equivalent to that in a corresponding economy with n firms operating and speculators fully internalizing their cross learning, so is the equilibrium weight ϕ_n, while the expression of investment efficiency can be shown to be the same as that in Definition 2.5.1. Therefore, we have the following proposition regarding the equilibrium weight ϕ_n and investment efficiency R_n in an n-learning equilibrium. We still focus on comparable cases in which all n-learning equilibria are unique.

Proposition **2.5.3.** *For a high enough noisy supply elasticity λ, a low enough idiosyncratic noisy supply shock precision τ_ξ, and a high enough information precision τ_y (of the speculators' signal on the idiosyncratic shock),*

i) for all $n \geqslant 1$, there exists a unique n-learning equilibrium in which the speculators put a positive weight $\phi_n > 0$ on the signal of the common productivity shock,

ii) for all $n > 1$, $\phi^* < \phi'(= \phi_1) < \phi_n < \phi$, in particular, ϕ_n is increasing in n, where ϕ, ϕ' are the equilibrium weights in the baseline cross-learning equilibrium and in the self-feedback equilibrium, respectively, and ϕ^* is the optimal weight that maximizes investment efficiency, and

iii) for all $n \geqslant 1$, $R < R_n < R^*$, in particular, R_n is decreasing in n, where R is the investment efficiency in the baseline cross-learning equilibrium and R^* is the optimal investment efficiency.

Proposition 2.5.3 offers a clear identification of the externality and efficiency loss associated with cross learning. Under the parameters we are interested, when the number of firms increases, cross learning makes the speculators to put an increasing weight ϕ_n on the signal of the common shock. Along with the established results in Section 2.4, this suggests stronger investment waves with higher systematic risks. Moreover, since the information endowment is controlled, this leads to a decreasing investment efficiency, associated with an increasing extent of cross learning. The key to understand this is a new externality through the speculators' weighting over the two types of shocks in response to the capital providers' cross learning. When each capital provider learns from other firms' asset prices, she only cares about her own investment decision and wants to use other firms' asset prices for better inferring the common shock. This makes her investment more sensitive to the common shock, which in turn encourages the speculators to put a higher weight on the signal of the common shock. However, she does not internalize the endogenous cost on other firms' investment decisions, because her cross learning makes asset prices endogenously less informative on other firms' idiosyncratic productivity shocks, through the speculators' endogenous response in terms of weighting the two shocks. When there are more firms in the economy, the speculators respond more heavily to the capital providers' cross learning and each asset price is also used by more firms, which implies a stronger externality not being internalized by each capital provider in cross learning.

We highlight the externality we have identified as a new pecuniary externality, taking effect through the informativeness of prices instead of price levels. In the classical pecuniary externality literature (see Stiglitz, 1982, Greenwald and Stiglitz, 1986, and Geanakoplos and Polemarchakis, 1985, and for recent theoretical developments see Farhi and Werning, 2013, He and Kondor, 2013, and Davila, 2014 for a comprehensive treatment), agents do not internalize the impacts of their actions on equilibrium price levels, leading to a welfare loss under various frictions. In particular, the classical pecuniary externality generates welfare transfers across agents through the levels of prices. In our framework, instead, the capital providers do not fully internalize the impacts of cross learning on equilibrium price informativeness. This leads to a typical "tragedy of the commons" regarding the use of the price system as an information source under multi-firm cross learning. This tragedy-of-the-commons observation is absent in classical single-firm feedback models. In this sense, our pecuniary externality is also reminiscent of the notion of learning externality in the earlier dynamic learning and herding literature (for example, Vives, 1997) that an agent, when responding to private information, does not take into account the benefit of increased informativeness of public information in the future. This literature, however, does not explicitly consider the roles of financial markets and in particular the feedback from market prices to investments as we do.

Along with the results in Section 2.4, the new pecuniary externality associated with cross learning offers a new perspective to investigate the puzzling fact that more competitive industries exhibit more inefficient investment waves with higher systematic risks. This fact has been recently documented in Hoberg and Phillips (2010) and shown to be robust after many relevant controls. As they suggest, however, no single theory in the literature can accommodate their findings. More recently, Greenwood and Hanson (2013) find a similar pattern in the cargo ship industry that also applies to other industries. They estimate a behavioral theory in which firms over-extrapolate exogenous demand shocks and partially neglect the endogenous investment responses of their competitors. Our fully rational cross-learning framework helps reconcile these facts by explicitly identifying the pecuniary externality associated with competition and its impacts on real investment efficiency. Relatedly, Ozoguz and Rebello (2013) have empirically identified that firms in more competitive industries adapt investments more sensitively to stock prices of their peers, which supports our theory.

It is worth noting that, when $n = 1$, that is, the speculators believe that there is only one firm operating, the economy still features investment inefficiency. Under the parameters we are interested, this benchmark investment inefficiency comes from the fact that the capital providers find the information about their idiosyncratic shocks more valuable whereas the speculators still find it profitable to put a considerable weight on the signal of the common shock in trading. This conflict of interests between capital providers (or firms) and speculators is generally present in the feedback literature in different forms (see the survey by Bond, Edmans and Goldstein, 2012), and Goldstein and Yang (2014a) formally identify it as the mismatch channel of feedback. Thus, the contribution of our work is first to extend the mismatch channel to a multi-firm feedback framework with two fundamentally different types of shocks, and then more importantly, to identify the new pecuniary externality associated with cross learning that is absent in classical feedback models.

Although our framework allows for an analytical characterization, we also offer numerical examples to help illustrate the pecuniary externality and efficiency loss associated with different extent of cross learning. We set $\tau_a = \tau_f = \tau_{sa} = \tau_{sf} = \tau_x = \tau_\zeta = 1$, $\tau_y = 10$, $\tau_\xi = 0.1$, $\lambda = 2$, $\kappa = 1$, and $c = 0.5$. The left panel of Figure 2.5.2 depicts the equilibrium weight ϕ_n as n increases as the blue solid line. When n becomes larger, the weight gradually approaches that in the baseline cross-learning equilibrium, as depicted by the red dashed line. The right panel of Figure 2.5.2 depicts the log of the efficiency loss due to cross learning, measured by $\log(R^*/R_n)$. As shown in the blue solid line, the efficiency loss associated with cross learning increases in n, suggesting a more severe pecuniary externality as competition becomes stronger. In the baseline cross-learning equilibrium, the pecuniary externality is the strongest, as depicted by the red dashed line. These results are robust to a very wide range of parameters once λ and τ_y are relatively large while τ_ξ is relatively small, which are empirically relevant as we discussed above.

Admittedly, our identification of the pecuniary externality associated with cross learning does not attempt to offer a comprehensive evaluation of the merits of competition. Relatedly, the investment efficiency R_n in an n-learning equilibrium cannot be interpreted as a direct measure of the investment efficiency in an actual competitive industry with n firms. Our point is focused, however, to suggest a new perspective to look at the relationship between inefficient investment waves and competition, a puzzling fact well documented recently and hard to be reconciled with existing theories. We admit that, despite the fact that competition increases the extent of cross learning, with new adverse implications for investment efficiency, it may well remain desirable when all other social benefits and costs of competition are taken into account.

2.6 Discussion

Our cross learning framework focuses on the systematic risks and investment inefficiency in investment waves, which we believe are less understood in the literature. It is also natural to rely on our framework to shed lights on some other commonly observed phenomena and to add new insights. This section discusses two directions.

2.6.1 Over-investment under Cross Learning

Investment waves usually exhibit both high systematic risks (second moment) and over-investment (first moment). Although the latter has been well addressed in the literature, our framework is easily adaptable to generate so. Especially, our cross learning framework offers a new perspective to explain why over-investment happens more often in technologies or industries that are more sensitive to common shocks.

We keep all the settings in our baseline model except for introducing two different investment technologies. Specifically, each firm i now has two mutually exclusive projects, one only subject to the common shock A while the other only subject to the idiosyncratic shock F_i. We call the former common project and the latter idiosyncratic project. Introducing the two types of projects is a parsimonious way to model the cross-section of different technologies or industries according to their different sensitivity to the common shock. For simplicity, here we only allow each firm to allocate a fixed amount of money between the two projects. Hence, each capital provider's problem is:

$$\max_{I_i \in [0,1]} \left[A I_i + F_i (1 - I_i) | \Gamma_i \right].$$

We again highlight cross learning: $\Gamma_i = \{P_i, \{P_{-i}\}, s_{a,i}, s_{f,i}\}$. This adapted setting is in the similar spirit of Dow, Goldstein and Guembel (2011) but enriches it with both cross learning and the firm's debate between the common project and the idiosyncratic project.

Following the same equilibrium concept as our baseline model, one can show that a cross-learning equilibrium features over-investment in the common project while under-investment in the idiosyncratic project, compared to the first best. The intuition is the same as before. When the capital providers are able to cross learn, the speculators again find it more profitable to put a higher weight on the information about the common shock. This makes the prices more informative about the common shock and thus encourages the capital providers to invest more on the common project while less on their idiosyncratic projects.

Complementary to the existing literature about over-investment, our cross-learning mechanism has two new implications. First, over-investment is more likely to happen in technologies or industries that are more sensitive to common shocks, which is reflected by the common project in our stylized model. This fits quite in line with the recent episodes of over-investment in the IT industry and in housing markets. Second, which is perhaps more subtle and interesting, over-investment in the common project is always accompanied by under-investment in the idiosyncratic projects at the same time. This suggests that over-investment does not necessarily imply an inefficiently large economy scale but rather an inefficient composition of various economic activities.

The comparative statics of the adapted model also offer predictions consistent with the reality. For example, when the common project has a higher ex-ante expected productivity, cross learning is stronger and thus the equilibrium features a higher level of over-investment in the common shock. Dow, Goldstein and Guembel (2011) and more recently Fajgelbaum, Schaal, and Taschereau-Dumouchel (2014) provide full-fledged models to demonstrate such a relationship between investment and information provision. They have similar predictions on how beliefs of productivity affect investment decisions. These papers, though featuring self-feedback and speaking to the level of investment directly, do not consider cross learning and the two types of shocks as we do.

2.6.2 Industry Momentum under Cross Learning

The contemporaneous study by Sockin and Xiong (2014b) uses a feedback model to generate return momentum in a housing cycle context. Although our model does not aim to provide a general dynamic account for investment waves, the introduction of multiple firms and the two types of shocks also help shed lights on the understanding of momentum by further establishing a channel between cross learning and industry momentum.

Industry momentum, first identified by Moskowitz and Grinblatt (1999), suggests that industry portfolios also exhibit considerable momentum, and it even accounts for much of the individual stock momentum. As discussed by Moskowitz and Grinblatt (1999), individual stock momentum may be explained by a number of behavioral theories focusing on investors' information barrier or risk appetite. But there have been no formal theories that directly speak to the existence and the magnitude of industry momentum. Our framework can potentially offer a consistent rational theory for both individual stock momentum and industry momentum, highlighting firms' investment activities and their cross learning instead.

In our benchmark three-period model, the standard definition of overall individual momentum is

$$M_i = \mathrm{Cov}\left(\log(AF_iI_i) - \log P_i, \log P_i\right) \,,$$

and industry momentum can be defined as

$$\overline{M} = \mathrm{Cov}\left(\log \int_0^1 AF_iI_i \mathrm{d}i - \log \overline{P}, \log \overline{P}\right) \,.$$

It is straightforward to show that both individual stock momentum and industry momentum exist in equilibrium, and their magnitudes increase in the speculators' weight ϕ on the information about the common shock. Specifically, M_i and $\overline{M}$ are always positive when the noisy supply elasticity λ and the information precision τ_y (of speculators' signal on the idiosyncratic shock) are large enough, and they increase in ϕ. Intuitively, when the speculators put a higher weight on the common shock, the asset prices become more informative about the common shock and thus firms' investment also becomes more sensitive to the common shock. Therefore, the common shock plays a more important role in determining both the asset prices and the eventual cash flows of the firms, implying both a stronger individual stock momentum and a stronger industry momentum. Moreover, according to the results regarding the relationship between cross learning and competition in Section 2.5, both individual stock momentum and industry momentum may be stronger in more competitive industries, also due to a stronger cross learning effect.

It is worth highlighting that our mechanism to generate individual momentum and industry momentum is fundamentally different from the prevailing explanations that highlight overconfidence (Daniel, Hirshleifer, and Subrahmanyam, 1998), sentiment (Barberis, Shleifer, and Vishny, 1998), or slow information diffusion (Hong and Stein, 1999). In those models, investors generally ignore some information content revealed by asset prices. In contrast, in our model, the capital providers' rational cross learning from all available asset prices plays a central role.

2.7 Conclusion

Firms and capital providers' cross learning behavior is not only empirically important but also theoretically relevant for commonly observed investment waves. We have developed a tractable model to admit cross learning and delivered a series of predictions regarding investment waves. We have illustrated that investment waves comes from new strategic complementarities and a spiral that coordinate capital providers' investment sensitivity and speculators' weight in trading towards the common productivity shock. However, cross learning may lead to higher investment inefficiency, because capital providers do not internalize the new externality that other firms' asset prices become less informative on their idiosyncratic productivity shocks. In more competitive industries, cross learning tends to be stronger, potentially leading to more inefficient investment waves with higher systematic risks. Hence, appropriate policy interventions are called for to correct the inefficiency in industry-investment waves.

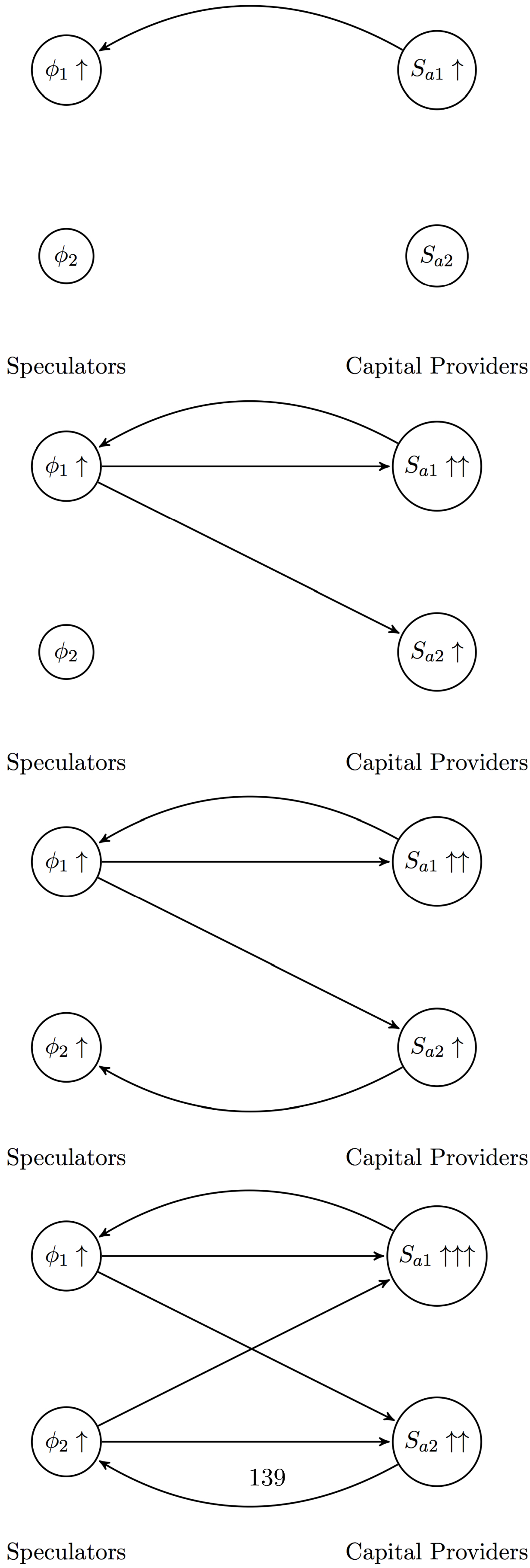

Figure 2.4.2: Learning Effect on Systematic Risks

Order: Upper-Left, Upper-Right, Lower-Left, Lower-Right

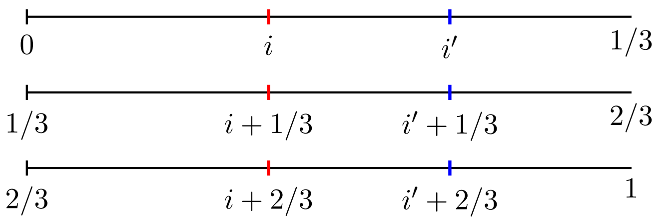

Figure 2.5.1: Extended Cross-Learning Framework

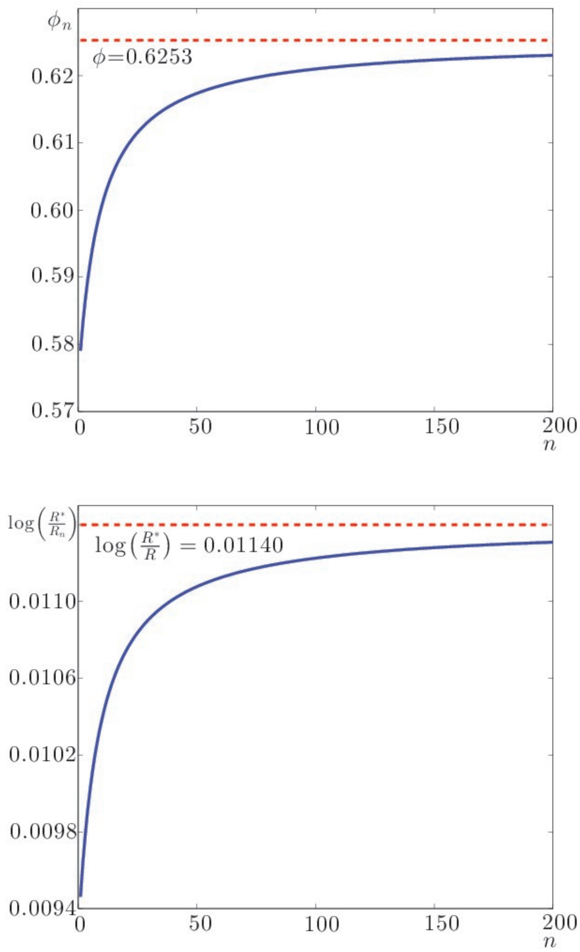

Figure 2.5.2: Competition on Cross Learning and Efficiency Loss

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2.8 Appendix

2.8.1 Proofs

This appendix provides all proofs omitted above with auxiliary results.

Proof of Lemma 2.3.1. According to the equilibrium definition, any speculator (i, j) longs one share of asset i when $\phi_i x_{ij} + y_{ij} > \mu_i$ and shorts one share otherwise. Equivalently, speculator (i, j) longs one share of asset i when

$$\frac{\phi_i x_{ij} + y_{ij}}{\sqrt{\tau_x^{-1}\phi_i^2 + \tau_y^{-1}}} > \frac{\mu_i - (\phi_i a + f_i)}{\sqrt{\tau_x^{-1}\phi_i^2 + \tau_y^{-1}}},$$

and shorts one share otherwise. Thus, in asset market i, all speculators' aggregate demand is

$$D_i = 1 - 2\Phi\left(\frac{\mu_i - (\phi_i a + f_i)}{\sqrt{\tau_x^{-1}\phi_i^2 + \tau_y^{-1}}}\right).$$

Hence, in equilibrium, market clearing implies

$$1 - 2\Phi\left(\frac{\mu_i - (\phi_i a + f_i)}{\sqrt{\tau_x^{-1}\phi_i^2 + \tau_y^{-1}}}\right) = 1 - 2\Phi(\zeta + \xi_i - \lambda \log P_i),$$

which further implies that the equilibrium price in asset market i is

$$P_i = \exp\left(\frac{\phi_i}{\lambda\sqrt{\tau_x^{-1}\phi_i^2 + \tau_y^{-1}}}a + \frac{1}{\lambda\sqrt{\tau_x^{-1}\phi_i^2 + \tau_y^{-1}}}f_i + \frac{\zeta + \xi_i}{\lambda} - \frac{\mu_i}{\lambda\sqrt{\tau_x^{-1}\phi_i^2 + \tau_y^{-1}}}\right).$$

This concludes the proof. □

Proof of Lemma 2.3.2. In a symmetric equilibrium, the capital providers put a same weight ϕ on the information of the common shock in any asset market i. Thus, by Lemma 2.3.1, for asset price P_i, its equivalent signal in predicting the common shock a becomes

$$z_a(P_i) = a + \frac{1}{\phi} f_i + \frac{\sqrt{\tau_x^{-1}\phi^2 + \tau_y^{-1}}}{\phi}(\zeta + \xi_i)\,.$$

Since f_i and ξ_i are both i.i.d. and have zero means, the aggregate price $\overline{P}$ is equivalent to the following signal

$$z_a(\overline{P}) = \int_0^1 \left(a + \frac{1}{\phi} f_i + \frac{\sqrt{\tau_x^{-1}\phi^2 + \tau_y^{-1}}}{\phi}(\zeta + \xi_i) \right) \mathrm{d}i = a + \frac{\sqrt{\tau_x^{-1}\phi^2 + \tau_y^{-1}}}{\phi}\zeta$$

in predicting the common shock a. It immediately follows the construction of the other signal $z_{f,i}(\overline{P})$ in predicting the idiosyncratic shock.

Finally, it is easy to verify that any combination of the asset prices $\{P_i, i \in [0,1]\}$ cannot be more informative in predicting the two productivity shocks. This concludes the proof. □

PROOF OF LEMMA 2.3.3. From the capital providers' problem (2.1), the first order condition is

$$\begin{aligned} I_i &= \frac{\kappa}{c}\mathbb{E}[\exp(a + f_i)|\Gamma_i] \\ &= \frac{\kappa}{c}\exp\left(\mathbb{E}[a + f_i|\Gamma_i] + \frac{1}{2}\mathrm{Var}[a + f_i|\Gamma_i]\right)\,. \end{aligned}$$

By Lemma 2.3.2, we know that $s_{a,i}$ and $z_a(\overline{P})$ are only informative about the common shock a and $s_{f,i}$ and $z_{f,i}(\overline{P})$ are only informative about the idiosyncratic shock f_i. Applying Bayesian updating immediately leads to the following optimal investment policy

$$I_i = \frac{\kappa}{c}\exp\left[\frac{\tau_{sa}s_{a,i} + \tau_{\overline{pa}}z_a(\overline{P})}{\tau_a + \tau_{sa} + \tau_{\overline{pa}}} + \frac{1}{2(\tau_a + \tau_{sa} + \tau_{\overline{pa}})} + \frac{\tau_f s_{f,i} + \tau_{\overline{pf}}z_{f,i}(\overline{P})}{\tau_f + \tau_{sf} + \tau_{\overline{pf}}} + \frac{1}{2(\tau_f + \tau_{sf} + \tau_{\overline{pf}})}\right].$$

This concludes the proof. □

PROOF OF LEMMA 2.3.4. This is a direct application of Lemma 2.3.2 to Definition 2.3.2. □

Proof of Proposition 2.3.1. We proceed step by step.

Step 1: Proof of the existence of the solution.

Following the equilibrium condition (2.10) for the cross-learning case, let

$$g(\phi) = \phi - \frac{\alpha_1 - \gamma_1}{\alpha_2 - \gamma_2} = \phi - \frac{\frac{\tau_{sa}+\tau_{\overline{pa}}}{\tau_a+\tau_{sa}+\tau_{\overline{pa}}} + 1 - \frac{\phi}{\lambda\sqrt{\tau_x^{-1}\phi^2+\tau_y^{-1}}}}{\frac{\tau_{sf}+\tau_{\overline{pf}}}{\tau_f+\tau_{sf}+\tau_{\overline{pf}}} + 1 - \frac{1}{\lambda\sqrt{\tau_x^{-1}\phi^2+\tau_y^{-1}}}} \frac{\frac{\tau_x}{\tau_a+\tau_x}}{\frac{\tau_y}{\tau_f+\tau_y}},$$

where $\tau_{\overline{pa}}$ is given by (2.5) and $\tau_{\overline{pf}}$ is given by (2.7), both being function of ϕ. It is easy to check that $\lim_{\phi\to-\infty} g(\phi) < 0$ and $\lim_{\phi\to+\infty} g(\phi) > 0$ by the following two equations:

$$\lim_{\phi\to-\infty} \frac{\frac{\tau_{sa}+\tau_{\overline{pa}}}{\tau_a+\tau_{sa}+\tau_{\overline{pa}}} + 1 - \frac{\phi}{\lambda\sqrt{\tau_x^{-1}\phi^2+\tau_y^{-1}}}}{\frac{\tau_{sf}+\tau_{\overline{pf}}}{\tau_f+\tau_{sf}+\tau_{\overline{pf}}} + 1 - \frac{1}{\lambda\sqrt{\tau_x^{-1}\phi^2+\tau_y^{-1}}}} \frac{\frac{\tau_x}{\tau_a+\tau_x}}{\frac{\tau_y}{\tau_f+\tau_y}} = \frac{\frac{\tau_x}{\tau_a+\tau_x}}{\frac{\tau_y}{\tau_f+\tau_y}} \frac{\frac{\tau_{sa}+\tau_\zeta\tau_x}{\tau_a+\tau_{sa}+\tau_\zeta\tau_x} + 1 - \frac{\sqrt{\tau_x}}{\lambda}}{\frac{\tau_{sf}}{\tau_f+\tau_{sf}} + 1},$$

and

$$\lim_{\phi\to+\infty} \frac{\frac{\tau_{sa}+\tau_{\overline{pa}}}{\tau_a+\tau_{sa}+\tau_{\overline{pa}}} + 1 - \frac{\phi}{\lambda\sqrt{\tau_x^{-1}\phi^2+\tau_y^{-1}}}}{\frac{\tau_{sf}+\tau_{\overline{pf}}}{\tau_f+\tau_{sf}+\tau_{\overline{pf}}} + 1 - \frac{1}{\lambda\sqrt{\tau_x^{-1}\phi^2+\tau_y^{-1}}}} \frac{\frac{\tau_x}{\tau_a+\tau_x}}{\frac{\tau_y}{\tau_f+\tau_y}} = \frac{\frac{\tau_x}{\tau_a+\tau_x}}{\frac{\tau_y}{\tau_f+\tau_y}} \frac{\frac{\tau_{sa}+\tau_\zeta\tau_x}{\tau_a+\tau_{sa}+\tau_\zeta\tau_x} + 1 - \frac{\sqrt{\tau_x}}{\lambda}}{\frac{\tau_{sf}}{\tau_f+\tau_{sf}} + 1}.$$

The analysis above indicates that there always exists a solution of ϕ to the equilibrium condition (2.10), i.e., $g(\phi) = 0$, by the intermediate value theorem. Especially, when $\lambda > 1/\sqrt{\tau_x^{-1}}$, we know that

$$f(0) = -\frac{\frac{\tau_{sa}+\tau_{\overline{pa}}}{\tau_a+\tau_{sa}+\tau_{\overline{pa}}} + 1}{\frac{\tau_{sf}+\tau_{\overline{pf}}}{\tau_f+\tau_{sf}+\tau_{\overline{pf}}} + 1 - \frac{1}{\lambda\sqrt{\tau_x^{-1}}}} \frac{\frac{\tau_x}{\tau_a+\tau_x}}{\frac{\tau_y}{\tau_f+\tau_y}} < 0\,.$$

We conclude that there always exists a positive solution $\phi > 0$ as long as λ is large enough.

Step 2: Proof of the uniqueness of the solution when τ_f is large enough.

By simple algebra, the equilibrium condition (2.10) is re-expressed as

$$\left(\frac{\tau_{sa}+\tau_{\overline{pa}}}{\tau_a+\tau_{sa}+\tau_{\overline{pa}}} + 1 - \frac{\phi}{\lambda\sqrt{\tau_x^{-1}\phi^2+\tau_y^{-1}}}\right)\frac{\tau_x}{\tau_a+\tau_x} = \phi\left(\frac{\tau_{sf}+\tau_{\overline{pf}}}{\tau_f+\tau_{sf}+\tau_{\overline{pf}}} + 1 - \frac{1}{\lambda\sqrt{\tau_x^{-1}\phi^2+\tau_y^{-1}}}\right)\frac{\tau_y}{\tau_f+\tau_y}\,. \tag{2.1}$$

Applying Taylor expansion to the terms in equation (2.1) with respect to τ_y^{-1} yields:

$$\frac{\tau_{sa}+\tau_{\overline{pa}}}{\tau_a+\tau_{sa}+\tau_{\overline{pa}}} = -\frac{\tau_a}{\tau_a+\tau_{sa}+\tau_{\overline{pa}}} - \frac{\tau_a}{(\tau_a+\tau_{sa}+\tau_{\overline{pa}})^2}\frac{\tau_\zeta}{\tau_x^{-2}\phi^2}\,\tau_y^{-1} + o(\tau_y^{-1})\,,$$

$$\frac{\tau_{sf}+\tau_{\overline{pf}}}{\tau_f+\tau_{sf}+\tau_{\overline{pf}}} = 1-\frac{\tau_f}{\tau_f+\tau_{sf}+\tau_{\overline{pf}}} - \frac{\tau_f}{(\tau_f+\tau_{sf}+\tau_{\overline{pf}})^2}\frac{\tau_\xi}{\tau_x^{-2}\phi^4}\,\tau_y^{-1} + o(\tau_y^{-1})\,,$$

$$\frac{\phi}{\lambda\sqrt{\tau_x^{-1}\phi^2+\tau_y^{-1}}} = \frac{1}{\lambda\sqrt{\tau_x^{-1}}} - \frac{1}{2\lambda\tau_x^{-\frac{3}{2}}\phi^2}\tau_y^{-1} + o(\tau_y^{-1})\,,$$

and

$$\frac{1}{\lambda\sqrt{\tau_x^{-1}\phi^2+\tau_y^{-1}}} = \frac{1}{\lambda\phi\sqrt{\tau_x^{-1}}} - \frac{1}{2\lambda\tau_x^{-\frac{3}{2}}\phi^3}\tau_y^{-1} + o(\tau_y^{-1})\,.$$

Plugging them back into equation (2.1), we have:

$$\begin{aligned}
&\tfrac{\tau_x}{\tau_a+\tau_x}\tfrac{\tau_f+\tau_y}{\tau_y}\left[2-\tfrac{\tau_a}{\tau_a+\tau_{sa}+\tau_{\overline{pa}}}-\tfrac{\tau_a}{(\tau_a+\tau_{sa}+\tau_{\overline{pa}})^2}\tfrac{\tau_\zeta}{\tau_x^{-2}\phi^2}\,\tau_y^{-1}-\tfrac{1}{\lambda\sqrt{\tau_x^{-1}}}+\tfrac{1}{2\lambda\tau_x^{-\frac{3}{2}}\phi^2}\tau_y^{-1}+o(\tau_y^{-1})\right]\\
=\quad&\phi\left[2-\tfrac{\tau_f}{\tau_f+\tau_{sf}+\tau_{\overline{pf}}}-\tfrac{\tau_f}{(\tau_f+\tau_{sf}+\tau_{\overline{pf}})^2}\tfrac{\tau_\xi}{\tau_x^{-2}\phi^4}\,\tau_y^{-1}-\tfrac{1}{\lambda\phi\sqrt{\tau_x^{-1}}}+\tfrac{1}{2\lambda\tau_x^{-\frac{3}{2}}\phi^3}\tau_y^{-1}+o(\tau_y^{-1})\right]\,,
\end{aligned}$$

which becomes a cubic equation of ϕ when τ_y goes to infinity:

$$\left(\frac{\tau_{sa}+\tau_x\tau_\zeta}{\tau_a+\tau_{sa}+\tau_x\tau_\zeta}+1-\frac{\sqrt{\tau_x}}{\lambda}\right)\frac{\tau_x}{\tau_a+\tau_x} = \frac{\tau_{sf}\phi^3+\tau_x\tau_\xi\phi}{\tau_f\phi^2+\tau_{sf}\phi^2+\tau_x\tau_\xi}+\phi-\frac{\sqrt{\tau_x}}{\lambda}\,. \tag{2.2}$$

Note that, the left hand side of equation (2.2) does not depends on ϕ. Denote by $h(\phi)$ the right hand side of (2.2), and its first order derivative with respect to ϕ is given by

$$\frac{\partial h(\phi)}{\partial\phi} = 1-\frac{\tau_f\phi^2}{\tau_f\phi^2+\tau_{sf}\phi^2+\tau_x\tau_\xi}+1-\frac{2\tau_f\tau_x\tau_\xi\phi^2}{(\tau_f\phi^2+\tau_{sf}\phi^2+\tau_x\tau_\xi)^2} > 0\,,$$

which indicates that the right hand side of equation (2.2) is increasing in ϕ and thus we have a unique solution to equation (2.2). Therefore, since $g(\phi)$ is a continuous function of τ_y, there always exists one unique solution to $g(\phi)=0$, i.e., equation (2.1), when τ_y is large enough. This concludes the proof. □

PROOF OF PROPOSITION 2.3.2. This proof is similar to the proof of Proposition 2.3.1. In the benchmark case without cross learning, the equilibrium condition (2.11) is re-expressed as

$$\begin{aligned}&\left(\frac{\tau_{sa}+\tau_{pa}}{\tau_a+\tau_{sa}+\tau_{pa}}+\frac{\tau_{pf}}{\tau_f+\tau_{sf}+\tau_{pf}}\phi'+1-\frac{\phi}{\lambda\sqrt{\tau_x^{-1}\phi^2+\tau_y^{-1}}}\right)\frac{\tau_x}{\tau_a+\tau_x}\\ = \;&\phi\left(\frac{\tau_{sf}+\tau_{pf}}{\tau_f+\tau_{sf}+\tau_{pf}}+\frac{\tau_{pa}}{\tau_a+\tau_{sa}+\tau_{pa}}\frac{1}{\phi'}+1-\frac{1}{\lambda\sqrt{\tau_x^{-1}\phi^2+\tau_y^{-1}}}\right)\frac{\tau_y}{\tau_f+\tau_y}\,,\end{aligned}$$

which further reduces to

$$\left(\frac{\tau_{sa}}{\tau_a+\tau_{sa}}+1-\frac{\sqrt{\tau_x}}{\lambda}\right)\frac{\tau_x}{\tau_a+\tau_x}=\phi\left(\frac{\tau_{sf}}{\tau_f+\tau_{sf}}+1-\frac{\sqrt{\tau_x}}{\lambda\phi}\right)\,, \tag{2.3}$$

when τ_y goes to infinity, τ_ξ goes to zero and $\lambda > \sqrt{\tau_x}$. Since the right hand side of equation (2.3) is increasing inf ϕ, we know that there must exist one unique solution to equation (2.3).

On the other hand, when τ_ξ goes to zero (and when τ_y goes to infinity and $\lambda > \sqrt{\tau_x}$), equation (2.2) in the case with cross learning becomes

$$\left(\frac{\tau_{sa}+\tau_{\overline{pa}}}{\tau_a+\tau_{sa}+\tau_{\overline{pa}}}+1-\frac{\sqrt{\tau_x}}{\lambda}\right)\frac{\tau_x}{\tau_a+\tau_x}=\phi\left(\frac{\tau_{sf}}{\tau_f+\tau_{sf}}+1-\frac{\sqrt{\tau_x}}{\lambda\phi}\right)\,. \tag{2.4}$$

We compare between the two equations (2.3) and (2.4). It is clear that their right hand sides are the same, while the left hand side of (2.3) (for the benchmark case without cross learning) is smaller than the left hand side of (2.4) (for the case with cross learning). Thanks to the continuity with respect to τ_y and τ_ξ of the two equilibrium conditions (2.11) and (2.10) in the two cases, we conclude that the equilibrium ϕ' in the benchmark case without cross learning is always lower than the equilibrium ϕ is the problem with cross learning, as long as $\lambda > \sqrt{\tau_x}$, τ_y is large enough, and τ_ξ is small enough. □

In the following proofs, we will frequently refer to the two notions of investment sensitivity defined in Definition 2.3.2, i.e., common investment sensitivity S_a and idiosyncratic investment sensitivity S_f. They are both functions of ϕ in equilibrium.

Proof of Lemma 2.4.1. We first consider the investment beta β_I. Recall the investment policy (2.8), we have

$$\log I_i = \frac{\tau_{sa} s_{a,i} + \tau_{\overline{pa}} z_a(\overline{P})}{\tau_a + \tau_{sa} + \tau_{\overline{pa}}} + \frac{1}{2(\tau_a + \tau_{sa} + \tau_{\overline{pa}})} + \frac{\tau_f s_{f,i} + \tau_{\overline{pf}} z_{f,i}(\overline{P})}{\tau_f + \tau_{sf} + \tau_{\overline{pf}}} + \frac{1}{2(\tau_f + \tau_{sf} + \tau_{\overline{pf}})} .$$

Following the definition of β_I and after some algebra, we reach

$$\beta_I = \frac{S_a/\tau_a - \frac{\tau_{sa}}{(\tau_a + \tau_{sa} + \tau_{\overline{pa}})^2}}{S_a/\tau_a + S_f/\tau_f} .$$

To simplify the analysis, let $g_1 = S_a/\tau_a$, $g_2 = \tau_{sa}/\left(\tau_a + \tau_{sa} + \tau_{\overline{pa}}\right)^2$, and $g_3 = S_f/\tau_f$. By Lemma 2.3.4, it is straightforward that g_1 is increasing in ϕ and both g_2 and g_3 are decreasing in ϕ. Thus, as $\phi > 0$, we also have that g_1 is increasing in ϕ^2 and both g_2 and g_3 are decreasing in ϕ^2. Furthermore, we have

$$\begin{aligned} \frac{\partial \beta_I}{\partial \phi^2} &= \frac{g_1' - g_2'}{g_1 + g_3} - \frac{(g_1 - g_2)(g_1' + g_3')}{(g_1 + g_3)^2} \\ &= \frac{[(g_1' - g_2')g_1 - (g_1 - g_2)g_1'] + (g_1' - g_2')g_3 - (g_1 - g_2)g_3'}{(g_1 + g_3)^2} , \end{aligned}$$

where g_1', g_2' and g_3' stands for the first order derivative with respect to ϕ^2.

Since we know that $g_1' - g_2' > g_1'$ (due to the fact that g_2 is decreasing in ϕ^2) and $g_1 > g_1 - g_2$, we have $(g_1' - g_2')g_1 - (g_1 - g_2)g_1' > 0$. Meanwhile, we have $g_1' > 0$, $g_2' < 0$, and $g_3 > 0$, so that $(g_1' - g_2')g_3 > 0$. Lastly, since $g_1 > g_2$ and $g_3' < 0$, we also know that $(g_1 - g_2)g_3' < 0$. Therefore, we conclude that $\partial \beta_I / \partial \phi^2 > 0$, i.e., β_I is an increasing function of ϕ when $\phi > 0$.

We then consider the price beta β_P. Recall the pricing function (2.1), we have

$$\log P_i = \frac{\phi}{\lambda\sqrt{\tau_x^{-1}\phi^2 + \tau_y^{-1}}} a + \frac{1}{\lambda\sqrt{\tau_x^{-1}\phi^2 + \tau_y^{-1}}} f_i + \frac{\zeta + \xi_i}{\lambda} - \frac{\mu}{\lambda\sqrt{\tau_x^{-1}\phi^2 + \tau_y^{-1}}} .$$

Following the definition of β_P and after some algebra, we reach that

$$\beta_P = \frac{\frac{\phi^2}{\tau_x^{-1}\phi^2+\tau_y^{-1}}\frac{1}{\tau_a}+\frac{1}{\tau_\zeta}}{\frac{\phi^2}{\tau_x^{-1}\phi^2+\tau_y^{-1}}\frac{1}{\tau_a}+\frac{1}{\tau_\zeta}+\frac{1}{\tau_x^{-1}\phi^2+\tau_y^{-1}}\frac{1}{\tau_f}+\frac{1}{\tau_\xi}}\,.$$

To simplify, let

$$h_1 = \frac{\phi^2}{\tau_x^{-1}\phi^2+\tau_y^{-1}}\frac{1}{\tau_a}+\frac{1}{\tau_\zeta} \tag{2.5}$$

and

$$h_2 = \frac{1}{\tau_x^{-1}\phi^2+\tau_y^{-1}}\frac{1}{\tau_f}+\frac{1}{\tau_\xi}\,. \tag{2.6}$$

It is straightforward that h_1 is increasing in ϕ and h_2 is decreasing in ϕ. Hence, we have

$$\frac{\partial\beta_P}{\partial\phi^2} = \frac{-g_1\frac{\partial g_2}{\partial\phi^2}}{(g_1+g_2)^2} > 0\,,$$

which indicates that β_P is an increasing function of ϕ when $\phi > 0$. □

Proof of Lemma 2.4.2. We first consider common investment correlation β_A. Recall the investment policy (2.8) and the definition of β_A, we have

$$\begin{aligned}\beta_A &= \frac{\mathrm{Cov}(\log I_i, \log A)}{\sqrt{\mathrm{Var}(\log I_i)}\sqrt{\mathrm{Var}(\log A)}} \\ &= \frac{\frac{\tau_{sa}+\tau_{\overline{pa}}}{\tau_a+\tau_{sa}+\tau_{\overline{pa}}}}{\sqrt{\tau_a}\sqrt{\frac{\tau_{sa}+\tau_{\overline{pa}}}{\tau_a+\tau_{sa}+\tau_{\overline{pa}}}\frac{1}{\tau_a}+\frac{\tau_f+\tau_{\overline{pf}}}{\tau_f+\tau_{sf}+\tau_{\overline{pf}}}\frac{1}{\tau_f}}} = \frac{S_a}{\sqrt{\tau_a}\sqrt{S_a/\tau_a+S_f/\tau_f}}\,.\end{aligned}$$

It is convenient for us to consider instead

$$\frac{1}{\tau_a\beta_A^2} = \frac{S_a/\tau_a+S_f/\tau_f}{S_a^2} = \frac{1}{\tau_a S_a}+\frac{S_f}{\tau_f S_a^2}\,.$$

By Lemma 2.3.4, since S_a is increasing in ϕ^2 and S_f is decreasing in ϕ^2 when $\phi > 0$, it is straightforward that $1/\tau_a\beta_A^2$ is decreasing in ϕ^2. This indicates that β_A is an increasing function of ϕ when $\phi > 0$.

We then consider the idiosyncratic investment correlation β_F. Again, recall the investment policy (2.8) and the definition of β_F, we have

$$\begin{aligned}\beta_F &= \frac{\mathrm{Cov}(\log I_i, \log F_i)}{\sqrt{\mathrm{Var}(\log I_i)}\sqrt{\mathrm{Var}(\log F_i)}} \\ &= \frac{\frac{\tau_f+\tau_{\overline{pf}}}{\tau_f+\tau_{sf}+\tau_{\overline{pf}}}}{\sqrt{\tau_f}\sqrt{\frac{\tau_{sa}+\tau_{\overline{pa}}}{\tau_a+\tau_{sa}+\tau_{\overline{pa}}}\frac{1}{\tau_a}+\frac{\tau_f+\tau_{\overline{pf}}}{\tau_f+\tau_{sf}+\tau_{\overline{pf}}}\frac{1}{\tau_f}}} = \frac{S_f}{\sqrt{\tau_f}\sqrt{S_a/\tau_a+S_f/\tau_f}}\,.\end{aligned}$$

Similarly, it is convenient for us to consider instead

$$\frac{1}{\tau_f\beta_F^2} = \frac{S_a/\tau_a+S_f/\tau_f}{S_f^2} = \frac{S_a}{\tau_a S_f^2} + \frac{1}{\tau_f S_f}\,,$$

which is decreasing in ϕ^2, again by Lemma 2.3.4. Hence, we conclude that β_F is an increasing function of ϕ as well, when $\phi > 0$. □

Proof of Lemma 2.4.3. Following Assumption 2.4.1, we keep the ratios τ_{sa}/τ_a and τ_x/τ_a constant when consider the changes of τ_a. We also focus on the case when τ_y and λ are large enough so that a unique solution of ϕ is guaranteed. Specifically, the reduced equilibrium condition (2.2) (in the proof of Proposition 2.3.1) is re-expressed as

$$\frac{\tau_{sa}/\tau_a+\tau_x\tau_\zeta/\tau_a}{\tau_a(1+\tau_{sa}/\tau_a+\tau_x\tau_\zeta/\tau_a)}\frac{\tau_x/\tau_a}{1+\tau_x/\tau_a}+\frac{\tau_x/\tau_a}{1+\tau_x/\tau_a}+\frac{\sqrt{\frac{\tau_x}{\tau_a}\tau_a}}{\lambda}\frac{1}{1+\tau_x/\tau_a} = \frac{\tau_{sf}\phi^3+\frac{\tau_x}{\tau_a}\tau_\xi\phi\tau_a}{\tau_f\phi^2+\tau_{sf}\phi^2+\frac{\tau_x}{\tau_a}\tau_\xi\tau_a}+\phi\,. \tag{2.7}$$

When λ is high enough, the first order derivative of the left hand side of equation (2.7) with respect to τ_a is

$$-\frac{\tau_{sa}/\tau_a+\tau_x\tau_\zeta/\tau_a}{\tau_a^2(1+\tau_{sa}/\tau_a+\tau_x\tau_\zeta/\tau_a)}\frac{\tau_x/\tau_a}{1+\tau_x/\tau_a}+\frac{\sqrt{\tau_x/\tau_a}}{\lambda}\frac{1}{1+\tau_x/\tau_a}\frac{1}{2\sqrt{\tau_a}} < 0\,.$$

And it is straightforward that the right hand side of (2.7) is an increasing function of τ_a. Thus, when τ_a increases, the left hand side of (2.7) decreases, which further calls for a decreasing ϕ to make the right hand side of (2.7) to decrease as well. This concludes the proof. □

Proof of Proposition 2.4.1. Again, following Assumption 2.4.1, we keep the ratios τ_{sa}/τ_a and τ_x/τ_a constant when consider the changes of τ_a. We still focus on the case when τ_y and λ are large enough so that a unique solution of ϕ is guaranteed.

We first consider the investment beta

$$\beta_I = \frac{S_a - g}{S_a + \tau_a S_f/\tau_f}\,,$$

where

$$g = \left(\frac{\tau_{sa}}{\tau_a + \tau_{sa} + \tau_{\overline{pa}}}\right)^2 \frac{\tau_a}{\tau_{sa}}\,. \tag{2.8}$$

It is instructive for us to decompose the total effects of the changing of τ_a on β_I into two parts: the mechanical effect and the cross-learning effect:

$$\frac{d\beta_I(\tau_a,\phi)}{d\tau_a} = \frac{\partial\beta_I(\tau_a,\phi)}{\partial\tau_a} + \frac{\partial\beta_I(\tau_a,\phi)}{\partial\phi}\frac{\partial\phi}{\partial\tau_a}\,.$$

The sign of the second term, the cross-learning effect, is straightforward by Lemma 2.4.1 and Lemma 2.4.3. Specifically, Lemma 2.4.1 indicates that $\partial\beta_I(\tau_a,\phi)/\partial\phi > 0$ and Lemma 2.4.3 indicates that $\partial\phi/\partial\tau_a < 0$, so that the cross-learning effect is negative in this case.

For the first term, the mechanical effect, since we keep τ_{sa}/τ_a and τ_x/τ_a constant and $\tau_{\overline{pa}} = \tau_x\tau_\varsigma$ when τ_y goes to infinity, we know that S_a and g are constant in this case. On the other hand, S_f is increasing in τ_x given $\tau_{\overline{pf}} = \tau_x\tau_\xi/\phi^2$ and thus is also increasing in τ_a given that τ_x/τ_a is constant. This indicates that $\tau_a S_f/\tau_f$ is increasing in τ_a. Hence, we know that $\partial\beta_I(\tau_a,\phi)/\partial\tau_a < 0$, i.e., the mechanical effect is negative as well.

Taking the two effects together, we know that the total effect is also negative, i.e., $d\beta_I(\tau_a,\phi)/d\tau_a < 0$.

We then consider the price beta

$$\beta_P = \frac{\frac{\phi^2}{\tau_x^{-1}\phi^2+\tau_y^{-1}}\frac{1}{\tau_a} + \frac{1}{\tau_\zeta}}{\frac{\phi^2}{\tau_x^{-1}\phi^2+\tau_y^{-1}}\frac{1}{\tau_a} + \frac{1}{\tau_\zeta} + \frac{1}{\tau_x^{-1}\phi^2+\tau_y^{-1}}\frac{1}{\tau_f} + \frac{1}{\tau_\xi}} = \frac{h_1}{h_1+h_2}\,,$$

where h_1 and h_2 are already defined in (2.5) and (2.6) (in the proof of Lemma 2.4.1).

Again, we decompose the total effects of the changing of τ_a on β_P into two parts: the mechanical effect and the cross-learning effect:

$$\frac{d\beta_P(\tau_a,\phi)}{d\tau_a} = \frac{\partial\beta_P(\tau_a,\phi)}{\partial\tau_a} + \frac{\partial\beta_P(\tau_a,\phi)}{\partial\phi}\frac{\partial\phi}{\partial\tau_a}\,.$$

Keep in mind that we keep τ_{sa}/τ_a and τ_x/τ_a constant in this case. First, it is straightforward to see that the mechanical effect $\partial\beta_P(\tau_a,\phi)/\partial\tau_a$ is negative, because β_P is an increasing function of h_1 that is in turn decreasing in τ_a at the same time. Furthermore, Lemma 2.4.1 indicates that $\partial\beta_P(\tau_a,\phi)/\partial\phi > 0$ and Lemma 2.4.3 indicates that $\partial\phi/\partial\tau_a < 0$, which together imply that the cross-learning effect is negative as well. Therefore, we conclude that the total effect is also negative, i.e., $d\beta_P(\tau_a,\phi)/d\tau_a < 0$. □

PROOF OF LEMMA 2.4.4. The proof is similar to the proof of Lemma 2.4.3. We again focus on the case when τ_y and λ are large enough so that a unique solution of ϕ is guaranteed. In this case, we recall the reduced equilibrium condition (2.2) (in the proof of Proposition 2.3.1):

$$\left(\frac{\tau_{sa}+\tau_x\tau_\zeta}{\tau_a+\tau_{sa}+\tau_x\tau_\zeta}+1-\frac{\sqrt{\tau_x}}{\lambda}\right)\frac{\tau_x}{\tau_a+\tau_x} = \frac{\tau_{sf}\phi^3+\tau_x\tau_\xi\phi}{\tau_f\phi^2+\tau_{sf}\phi^2+\tau_x\tau_\xi}+\phi-\frac{\sqrt{\tau_x}}{\lambda}\,.$$

It is clear that the left hand side of (2.2) is increasing in τ_{sa}. Hence, when τ_{sa} increase, the right hand side of (2.2) increases as well. On the other hand, we have already known that the right hand side of (2.2) is increasing in ϕ. Hence, in equilibrium, ϕ increases. This indicates that ϕ is an increasing function of τ_{sa}.

The analysis is similar for τ_{sf}. The right hand side of (2.2) is increasing in τ_{sf}, while the left hand side is independent of τ_{sf}. Thus, when τ_{sf} increase, ϕ decreases to ensure a constant right hand side of (2.2). This indicates that ϕ is a decreasing function of τ_{sf}. □

PROOF OF PROPOSITION 2.4.2. We first consider the comparative statics with respect to τ_{sa}.

For the investment beta β_I, we have

$$\beta_I = 1 - \frac{S_f/\tau_f + g/\tau_a}{S_a/\tau_a + S_f/\tau_f}\,,$$

where g is already defined in (2.8) (in the proof of Proposition 2.4.1).

We also decompose the total effects of the changing of τ_{sa} on β_I into two parts: the mechanical effect and the cross-learning effect:

$$\frac{d\beta_I(\tau_{sa},\phi)}{d\tau_{sa}} = \frac{\partial\beta_I(\tau_{sa},\phi)}{\partial\tau_{sa}} + \frac{\partial\beta_I(\tau_{sa},\phi)}{\partial\phi}\frac{\partial\phi}{\partial\tau_{sa}}\,.$$

Clearly, Lemma 2.4.1 and Lemma 2.4.4 indicate that the second term, the cross-learning effect, is positive. For the first term, the mechanical effect, we first know that S_a is increasing in τ_{sa}, given ϕ fixed. Also, it is easy to show that g/τ_a is increasing in τ_{sa} (given ϕ fixed) when $\tau_{sa} < \tau_a + \tau_x\tau_\zeta$, and decreasing in τ_{sa} (also given ϕ fixed) when $\tau_{sa} > \tau_a + \tau_x\tau_\zeta$. Hence, we conclude that when $\tau_{sa} > \tau_a + \tau_x\tau_\zeta$, the mechanical effect is positive, and thus total effect $d\beta_I(\tau_{sa},\phi)/d\tau_{sa}$ is positive as well. When $\tau_{sa} < \tau_a + \tau_x\tau_\zeta$, the mechanical effect is negative and thus the total effect is ambiguous.

For the price beta β_P, there is only cross-learning effect but no mechanical effect. Hence, by Lemma 2.4.1 and Lemma 2.4.4 we have that

$$\frac{d\beta_P(\tau_{sa},\phi)}{d\tau_{sa}} = \frac{\partial\beta_P(\tau_{sa},\phi)}{\partial\phi}\frac{\partial\phi}{\partial\tau_{sa}} > 0\,.$$

We then consider the comparative statics with respect to τ_{sf} in a similar manner. For the investment beta β_I, we have

$$\beta_I = \frac{S_a/\tau_a - g/\tau_a}{S_a/\tau_a + S_f/\tau_f}\,.$$

By the similar decomposition and again by Lemma 2.4.1 and Lemma 2.4.4, we know that both the mechanical effect and the cross-learning effect in this case are negative. So that the total effect is also negative:

$$\frac{d\beta_I(\tau_{sf},\phi)}{d\tau_{sf}}=\frac{\partial\beta_I(\tau_{sf},\phi)}{\partial\tau_{sf}}+\frac{\partial\beta_I(\tau_{sf},\phi)}{\partial\phi}\frac{\partial\phi}{\partial\tau_{sf}}<0\,.$$

For the price β_P, again, there is only cross-learning effect but no mechanical effect. Hence, by Lemma 2.4.1 and Lemma 2.4.4 we have that

$$\frac{d\beta_P(\tau_{sf},\phi)}{d\tau_{sf}}=\frac{\partial\beta_P(\tau_{sf},\phi)}{\partial\phi}\frac{\partial\phi}{\partial\tau_{sf}}<0\,.$$

This concludes the proof. □

PROOF OF LEMMA 2.4.5. We focus on the case when τ_y and λ are large enough so that a unique solution of ϕ is guaranteed. We first consider the effect of λ. Recall the re-expressed reduced equilibrium condition (2.7) (in the proof of Lemma 2.4.3):

$$\frac{\tau_{sa}/\tau_a+\tau_x\tau_\zeta/\tau_a}{\tau_a(1+\tau_{sa}/\tau_a+\tau_x\tau_\zeta/\tau_a)}\frac{\tau_x/\tau_a}{1+\tau_x/\tau_a}+\frac{\tau_x/\tau_a}{1+\tau_x/\tau_a}+\frac{\sqrt{\frac{\tau_x}{\tau_a}\tau_a}}{\lambda}\frac{1}{1+\tau_x/\tau_a}=\frac{\tau_{sf}\phi^3+\frac{\tau_x}{\tau_a}\tau_\xi\phi\tau_a}{\tau_f\phi^2+\tau_{sf}\phi^2+\frac{\tau_x}{\tau_a}\tau_\xi\tau_a}+\phi\,.$$

It is clear that the left hand side of (2.7) is decreasing in λ while the right hand side is independent of λ. Hence, when λ increases, ϕ decreases in equilibrium.

We then consider the effects of τ_ζ and τ_ξ. Recall the reduced equilibrium condition (2.2) (in the proof of Proposition 2.3.1):

$$\left(\frac{\tau_{sa}+\tau_x\tau_\zeta}{\tau_a+\tau_{sa}+\tau_x\tau_\zeta}+1-\frac{\sqrt{\tau_x}}{\lambda}\right)\frac{\tau_x}{\tau_a+\tau_x}=\frac{\tau_{sf}\phi^3+\tau_x\tau_\xi\phi}{\tau_f\phi^2+\tau_{sf}\phi^2+\tau_x\tau_\xi}+\phi-\frac{\sqrt{\tau_x}}{\lambda}\,.$$

On the one hand, the left hand side of (2.2) is increasing in τ_ζ, while the right hand side is independent of τ_ζ, so that ϕ is increasing in τ_ζ in equilibrium. On the other hand, the right hand side of (2.2) is increasing in τ_ξ while the left hand side is independent of τ_ξ, so that ϕ is decreasing in τ_ξ in equilibrium. □

Proof of Proposition 2.4.3. We first consider the comparative statics with respect to λ. Since λ has no mechanical effect on ether β_I or β_P, we focus on the cross-learning effect along. By Lemma 2.4.1 and Lemma 2.4.5, we know that both β_P and β_I are decreasing in λ.

We then consider the comparative statics with respect to τ_ζ. For the investment beta β_I, we have

$$\beta_I = \frac{S_a - g}{S_a + \tau_a S_f / \tau_f},$$

where g is defined in (2.8) (in the proof of Proposition 2.4.1).

Again, we decompose the total effects of the changing of τ_ζ on β_I into two parts: the mechanical effect and the cross-learning effect:

$$\frac{d\beta_I(\tau_\zeta, \phi)}{d\tau_\zeta} = \frac{\partial \beta_I(\tau_\zeta, \phi)}{\partial \tau_\zeta} + \frac{\partial \beta_I(\tau_\zeta, \phi)}{\partial \phi} \frac{\partial \phi}{\partial \tau_\zeta}.$$

By Lemma 2.4.1 and Lemma 2.4.5, we know that the cross-learning effect is positive. For the mechanical effect, when ϕ is fixed, it is easy to show that $\partial \beta_I(\tau_\zeta, \phi)/\partial \tau_{\overline{pa}} > 0$. Since we know that $\partial \tau_{\overline{pa}}/\partial \tau_\zeta > 0$, we get that the mechanical effect is also positive. Hence, the total effect $d\beta_I(\tau_\zeta, \phi)/d\tau_\zeta$ is positive.

However, the total effect on the price beta β_P is ambiguous in this case. We have

$$\beta_P = \frac{h_1}{h_1 + h_2},$$

where h_1 and h_2 are already defined in (2.5) and (2.6) (in the proof of Lemma 2.4.1). Decomposition gives

$$\frac{d\beta_P(\tau_\zeta, \phi)}{d\tau_\zeta} = \frac{\partial \beta_P(\tau_\zeta, \phi)}{\partial \tau_\zeta} + \frac{\partial \beta_P(\tau_\zeta, \phi)}{\partial \phi} \frac{\partial \phi}{\partial \tau_\zeta}.$$

Lemma 2.4.1 and Lemma 2.4.5 give a positive cross-learning effect, i.e., the second term. However, it is easy to show that the first term, the mechanical effect, is negative. Hence, the total effect is ambiguous and will be determined by other model parameters.

We finally consider the comparative statics with respect to τ_ξ. Similarly, we follow the decomposition above. For the investment beta β_I, by Lemma 2.4.1 and Lemma 2.4.5, we know that the cross-learning effect is negative. For the mechanical effect, when ϕ is fixed, it is easy to show that $\partial\beta_I(\tau_\xi,\phi)/\partial\tau_{\overline{pf}} > 0$. Since we know that $\partial\tau_{\overline{pf}}/\partial\tau_\xi > 0$, we get that the mechanical effect is also negative. Hence, the total effect $d\beta_I(\tau_\xi,\phi)/d\tau_\xi$ is negative. Following similar arguments and again by Lemma 2.4.1 and Lemma 2.4.5, we know that both the mechanical effect and the cross-learning effect on the price beta β_P are also negative, so that the total effect on β_P is negative as well. $\square$

Proof of Proposition 2.5.1. Following the definition of real investment efficiency, we know that

$$R = \int_0^1 R_i di\,,$$

where

$$R_i = \mathbb{E}\left[AF_iI_i - \frac{c}{2}I_i^2\right] = \frac{\kappa(2-\kappa)}{2c}\mathbb{E}\left[AF_i\mathbb{E}\left[AF_i|\Gamma_i\right]\right]\,,$$

and

$$\mathbb{E}[AF_i|\Gamma_i] = \exp\left[\frac{\tau_{sa}s_{a,i} + \tau_{\overline{pa}}z_a(\overline{P})}{\tau_a+\tau_{sa}+\tau_{\overline{pa}}} + \frac{1}{2(\tau_a+\tau_{sa}+\tau_{\overline{pa}})} + \frac{\tau_f s_{f,i} + \tau_{\overline{pf}}z_{f,i}(\overline{P})}{\tau_f+\tau_{sf}+\tau_{\overline{pf}}} + \frac{1}{2(\tau_f+\tau_{sf}+\tau_{\overline{pf}})}\right].$$

Since κ and c are constant, without loss of generality, we set $\kappa = 1$ and $c = 0.5$ to ease the exposition. After some tedious algebra, the investment efficiency R is re-expressed in a much simpler and more intuitive form:

$$R = \exp\left(\frac{1+S_a}{\tau_a} + \frac{1+S_f}{\tau_f}\right)\,. \tag{2.9}$$

We solve for the socially optimal ϕ^* that maximizes R. Taking the first order condition gives

$$\frac{\partial \log R}{\partial(\phi^2)} = \frac{\tau_x^2\tau_\zeta\tau_y}{(\tau_a+\tau_{sa}+\tau_{\overline{pa}})^2}\frac{1}{(\tau_x+\tau_y\phi^2)^2} - \frac{\tau_x\tau_y^2\tau_\xi}{(\tau_f+\tau_{sf}+\tau_{\overline{pf}})^2}\frac{1}{(\tau_x+\tau_y\phi^2)^2} = 0\,, \tag{2.10}$$

which reduces to

$$\frac{(\tau_a + \tau_{sa} + \tau_{\overline{pa}})^2}{(\tau_f + \tau_{sf} + \tau_{\overline{pf}})^2} = \frac{\tau_x \tau_\zeta}{\tau_y \tau_\xi}\,. \tag{2.11}$$

Since $\tau_{\overline{pa}}$ is increasing in ϕ^2 while $\tau_{\overline{pf}}$ is decreasing in ϕ^2, we know that the left hand side of (2.11) is increasing in ϕ^2. Therefore, there is a unique non-negative solution of ϕ^*.

We further compare between the socially optimal weight ϕ^* and the weight ϕ in the cross-learning equilibrium, focusing on the case in which τ_y and λ are large enough so that there is always a unique positive solution of ϕ. We re-express the first order condition (2.10) as

$$\frac{\partial \log R}{\partial(\phi^2)} = \frac{\tau_x \tau_y^2}{(\tau_a + \tau_{sa} + \tau_{\overline{pa}})^2}\left[\frac{\tau_x \tau_\zeta}{\tau_y} - \frac{(\tau_a + \tau_{sa} + \tau_{\overline{pa}})^2 \tau_\xi}{(\tau_f + \tau_{sf} + \tau_{\overline{pf}})^2}\right] = 0\,. \tag{2.12}$$

When τ_y goes to infinity, we know that $\tau_x \tau_\zeta / \tau_y$ goes to 0, and we also have

$$\left(\frac{\tau_a + \tau_{sa} + \tau_{\overline{pa}}}{\tau_f + \tau_{sf} + \tau_{\overline{pf}}}\right)^2 = \left(\frac{\tau_a + \tau_{sa} + \tau_x \tau_\zeta}{\tau_f + \tau_{sf} + \tau_x \tau_\zeta/\phi^2}\right)^2 > 0\,.$$

Hence, when τ_y and λ are large enough, the left hand side of (2.12) is always negative. Therefore, we conclude that the cross-learning equilibrium ϕ is always larger than the socially optimal ϕ^* when τ_y and λ are large enough. $\square$

Proof of Proposition 2.5.2. Recall the expression of investment efficiency (2.9):

$$R = \exp\left(\frac{1 + S_a}{\tau_a} + \frac{1 + S_f}{\tau_f}\right)\,.$$

We again focus on the case when τ_y and λ are large enough so that a unique solution of ϕ is guaranteed. We first consider the comparative statics with respect to λ. Lemma 2.3.4 implies that $\partial S_f / \partial \phi < 0$ and Lemma 2.4.5 implies that $\partial \phi / \partial \lambda < 0$. Since there is no direct effect of λ on S_f, we know that S_f is increasing in λ in equilibrium. Moreover, because the effect of ϕ on S_a is negligible when τ_y is sufficiently large, we eventually know that that R is an increasing function of λ.

We then consider τ_ξ. Similarly, Lemma 2.3.4 implies that $\partial S_f/\partial\phi < 0$ and Lemma 2.4.5 implies that $\partial\phi/\partial\tau_\xi < 0$, so that S_f, and thus R is increasing in τ_ξ in equilibrium.

We finally consider τ_{sf}. It is clear that $\partial S_f/\partial\tau_{sf} > 0$, i.e., the mechanical effect is positive. For the cross-learning effect, Lemma 2.3.4 implies that $\partial S_f/\partial\phi < 0$ and Lemma 2.4.4 implies that $\partial\phi/\partial\tau_\xi < 0$. Hence, the total effect is positive as well, i.e., S_f is increasing in τ_{sf} in equilibrium. Since the effect of ϕ on S_a is negligible when τ_y is sufficiently large, we eventually know that that R is an increasing function of τ_{sf}. $\square$

Proof of Proposition 2.5.3. Part i) is straightforward following the proofs of Proposition 2.3.1 and Proposition 2.3.2. For part ii), we make use of the conditional expectation (2.12). Specifically, we have

$$\mathrm{Var}(\mathbf{z}) = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \sigma_{24} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \sigma_{34} \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44} \end{bmatrix},$$

where

$$\begin{aligned}
\sigma_{11} &= \tau_a^{-1} + \tau_{sa}^{-1}, \\
\sigma_{12} &= 0, \\
\sigma_{13} &= \phi_n \tau_a^{-1}, \\
\sigma_{14} &= \phi_n \tau_a^{-1}, \\
\sigma_{22} &= \tau_f^{-1} + \tau_{sf}^{-1}, \\
\sigma_{23} &= \tau_f^{-1}, \\
\sigma_{24} &= 0, \\
\sigma_{33} &= \tau_f^{-1} + \phi_n^2\tau_a^{-1} + (\tau_x^{-1}\phi_n^2 + \tau_y^{-1})(\tau_\zeta^{-1} + \tau_\xi^{-1}), \\
\sigma_{34} &= \phi_n^2\tau_a^{-1} + (\tau_x^{-1}\phi_n^2 + \tau_y^{-1})\tau_\zeta^{-1},
\end{aligned}$$

$$\sigma_{44} = \frac{\tau_f^{-1}}{n-1} + \phi_n^2 \tau_a^{-1} + (\tau_x^{-1}\phi_n^2 + \tau_y^{-1}) \left(\tau_\zeta^{-1} + \frac{\tau_\xi^{-1}}{n-1} \right),$$

and

$$\mathrm{Cov}(\mathbf{z}, a + f_i) = [\tau_a^{-1}, \tau_f^{-1}, \phi_n \tau_a^{-1} + \tau_f^{-1}, \phi_n \tau_a^{-1}] .$$

By condition (2.12), we get the expressions of S_{an} and S_{fn} after some tedious algebra and plug them into the equilibrium condition. Denote by RHS the right hand side of the equilibrium condition (2.13) and we get

$$\frac{\partial \lim_{\tau_y \to \infty} RHS(\phi_n, n)}{\partial n} = C_1 C_2 C_3 C_4 C_5 (C_6 + C_7 + C_8)^2 , \tag{2.13}$$

where

$$\begin{aligned}
C_1 &= \phi_n^2(\tau_f + \tau_{sf})\tau_\zeta + \tau_x \tau_\zeta \tau_\xi - \phi_n(\tau_a + \tau_{sa} + \tau_x \tau_\zeta)\tau_\xi , \\
C_2 &= \phi_n^2 \tau_a(\tau_f + 2\tau_{sf})\tau_\zeta + 2\tau_a \tau_x \tau_\zeta \tau_\xi + \phi_n \tau_f(\tau_a + 2(\tau_{sa} + \tau_x \tau_\zeta))\tau_\xi , \\
C_3 &= \phi_n^2 \tau_f + \tau_x \tau_\xi , \\
C_4 &= \phi_n^2 \tau_f \tau_x^2 \tau_\xi , \\
C_5 &= \tau_a + \tau_x , \\
C_6 &= \phi_n^3 \tau_f^2 \tau_x \tau_\zeta \tau_\xi + \phi_n \tau_f \tau_x^2 \tau_\zeta \tau_\xi^2 + 2(\tau_a + \tau_{sa})\tau_x^2 \tau_\zeta \tau_\xi^2 , \\
C_7 &= \phi_n^4 \tau_f(\tau_f + 2\tau_{sf})((\tau_a + \tau_{sa})\tau_\zeta + n(\tau_a + \tau_{sa} + \tau_x \tau_\zeta)\tau_\xi) , \\
C_8 &= \phi_n^2 \tau_x \tau_\xi((\tau_a + \tau_{sa})(3\tau_f + 2\tau_{sf})\tau_\zeta + ((2n-1)\tau_f + 2\tau_{sf})(\tau_a + \tau_{sa} + \tau_x \tau_\zeta)\tau_\xi) .
\end{aligned}$$

Note that, only the first term C_1 has a negative component. However, when τ_ξ is small enough, C_1 is always strictly positive, so is the entire derivative (2.13). It implies that when τ_y is large enough and τ_ξ is small enough, the equilibrium ϕ_n is increasing in n. Also, the proof of Proposition 2.5.1 directly implies that $\phi'(= \phi_1) > \phi^*$, so that $\phi_n > \phi^*$ for all $n \geqslant 1$.

Finally, for part iii), by the proof of Proposition 2.5.1, in particular condition (2.9), we know that

$$R_n = \exp\left(\frac{1 + S_a(\phi_n)}{\tau_a} + \frac{1 + S_f(\phi_n)}{\tau_f}\right),$$

where S_a and S_f are the capital providers' investment sensitivities in the baseline cross-learning case pinned down by however the equilibrium weight in the corresponding n-learning equilibrium. By Lemma 2.3.4, it follows that $R < R_n < R^*$ for all $n \geqslant 1$. This concludes the proof. $\square$

Chapter 3

How Options Affect Information Acquisition and Asset Pricing

Shiyang Huang

Abstract: We study the effect of introducing an options market on investors' incentive to collect private information in a rational expectation equilibrium model. We show that an options market has two effects on information acquisition: a negative effect, as options act as substitutes for information, and a positive effect, as informed investors have less need for options and can earn profits from selling them. When the population of informed investors is high because of the low information acquisition cost, the supply for options is larger than the demand, leading to low option prices. Low option prices in turn induce investors to use options instead of information to reduce risk, while informed investors have little opportunity to earn profits from selling options to cover their information acquisition cost. Introducing an options market thus decreases investors' incentive to acquire information, and the prices of the underlying assets become less informative, leading to lower prices and higher volatilities. A dynamic extension of this analysis shows that introducing an options market increases the price reactions to earnings announcements. However, when the information acquisition cost is high, the opposite effects arise. Further analysis shows that our results are robust for

more general derivatives. These results provide a potentially unified theory to reconcile the conflicting empirical findings on the options listing of individual stocks in both the U.S. market and international markets.

3.1 Introduction

As one of the largest derivative markets, the options market has experienced tremendous growth (see Figure 3.1).[1] Further, the effect of options listing on the underlying asset market is a hot topic in policy, industry and academia, and it has become extremely important since the financial crisis of 2007-08.[2] Although many empirical studies address this issue, findings regarding options listing around the world are conflicting. As one example, in the U.S. market, the effects of options listing on the underlying individual stocks over the last 30 years are completely different from such effects 30 years ago.[3] Indeed, for the period before 1980, previous empirical studies find that options listing increased underlying stock prices, decreased volatilities, and decreased price reactions to earnings announcements (Conrad, 1999, Detemple and Jorion, 1990 and Skinner, 1989). However, for the period after 1980, recent studies find the opposite effects (Sorescu, 2000 and Mayhew and Mihov, 2000). There are no plausible explanations for these conflicting findings. Hence, they remain puzzling.

To our knowledge, few theoretical studies examine the effects of derivatives on their underlying assets with endogenous information acquisition. Among the few studies, Cao (1999) and Massa (2002) show that introducing derivatives, including options, increases the underlying asset's price, decreases volatility and decreases price reactions to earnings announcements. They thus provide explanations to the empirical findings regarding options listing in U.S. before 1980, but offer little guidance on the effects after 1980.

In this paper, we exam the effect of an options market on investors' incentive to collect private information in a rational expectation equilibrium model. Following the canonical frameworks of Grossman and Stiglitz (1980) and Hellwig (1980), we have one risky asset and one

[1] Data Source: *SELECT SEC AND MARKET DATA FISCAL 2013.* Data is about all sales of options listed on exchange and excludes options on indexes

[2] After credit crunch of 2007-08, *The Dodd-Frank Wall Street Reform and Consumer Protection Act* , which was signed into federal law on July 21, 2010, is considered to bring most significant changes to financial regulation on derivative markets including interest rate option and currency option. Then there is a hot debate on whether Dodd-Frank is enough to prevent systemic risk.

[3] Although most empirical studies are about U.S. market, there are some evidences to show that option listing decreases price and increases volatility of underlying individual stocks in developed markets, such as Germany (Heer et al., 1997), while it increases price and decreases volatility in developing markets, such as India (Nair, 2008).

Figure 3.1.1: Trading Activity in the U.S. Options Market (in millions of dollars)

risk-free asset in our economy. We then introduce an options market that includes a set of call and put options on the risky asset. Investors choose whether to acquire private information before trading. We compare investors' information acquisition decisions before and after the options market opens. More important, we examine the effect of the options market on the underlying asset through its effect on information acquisition.

We find that introducing an options market has two effects on information acquisition. First, options act as substitutes for private information because both options and private information are valuable in reducing risk.[4] Investors can hence choose whether to acquire information or use options. Thus, introducing an options market negatively affects investors' incentive to acquire information. This first effect is a substitution effect. Second, options are valuable for investors with imprecise information because such investors face high uncertainty. Informed investors therefore have less need for options than uninformed investors. In an equilibrium where the net supply of options is zero, informed investors earn profits by selling options to uninformed investors. Thus, introducing an options market positively affects investors' incentive to acquire information. This second effect is a profit-making effect. The effect of an options market on information acquisition depends on these two effects. When the

[4]The intuition can be shown from the Black-Scholes model, which shows that the option price increases with underlying asset value's volatility

information acquisition cost is low, the population of informed investors is high, which leads to a lower demand than supply for options. Consequently, the option prices are low, affording informed investors little opportunity to cover the information acquisition cost by selling options. Meanwhile, investors can use cheap options instead of private information to reduce their risk. Therefore, the two effects work in conjunction to decrease investors' incentive to acquire information. The price of the underlying asset then becomes less informative, resulting in a lower price and higher volatility. With less precise private information, investors must rely more on public information, which generates greater price reactions to earnings announcements. By contrast, when the information acquisition cost is sufficiently high, the population of informed investors is low, which leads to a higher demand than supply for options. Consequently, option prices are high, offering informed investors large opportunity to cover the information acquisition cost by selling options. Therefore, the profit-making effect exerts a larger counteracting force against the substitution effect, leading to opposite effects on information acquisition and asset pricing. Moreover, we show that our mechanism is robust for other derivatives, such as straddles.

We also find that the effect of an options market on information acquisition depends on the precision of public information. When public information is precise, the population of informed investors is low before the options market opens. Consequently, the demand for options is larger than the supply, affording informed investors large opportunity to earn profits. Thus, introducing an options market increases investors' incentive to collect private information, increases the price of the underlying asset, decreases volatility, and decreases price reactions to earnings announcements. When public information is imprecise, the population of informed investors is high, leading to low option prices. Consequently, the opposite effects on the underlying asset arise.

Moreover, we show that the introduction of additional trading rounds has similar effects to the introduction of an options market. Brennan and Cao (1996) argues that additional trading rounds, which can be interpreted as after-hour or round-the-clock trading, can improve the welfare of all investors because both additional trading rounds and derivatives markets increase risk-sharing opportunities. However, their effect on information acquisition is unclear. Following Brennan and Cao (1996), we extend our model to consider multiple rounds of trading, where each round provides a new public information. Complementing to their study, we find that additional trading rounds produce asymmetric benefits for different groups, which leads to non-monotonic effects on information acquisition. Because risk sharing occurs between different groups, the relative benefits depend on the competition within each group. For example, when the population of informed investors is high, the competition within the group of informed investors is high. Consequently, the benefit from more risk-sharing opportunities is lower for informed investors than for uninformed investors, which reduces the marginal benefit of information. Thus, when the population of informed investors is high because of the low information acquisition cost, introducing additional trading rounds decreases investors' incentive to acquire information, lowering the asset price and increasing volatility. When the information acquisition cost is high, the opposite effects arise.

Our results indicate that the effects of options listing on the underlying assets depend on the information acquisition cost. Our results therefore provide a unified explanation for the conflicting findings regarding the effects of options listing on underlying individual stocks in the U.S. market and international markets. For example, before 1980 when information acquisition cost is conventionally believed to have been high, our results are consistent with the findings in U.S. that options listing increased underlying stock prices (Branch and Finnerty, 1981, Conrad, 1999 and Detemple and Jorion, 1990), decreased volatilities (Hayes and Tennenhaum, 1979, Skinner, 1989, Conrad, 1999, Ho, 1993 and Damodaran, 1991), and decreased price reactions to earnings announcements (Jennings and Starks, 1986, Skinner, 1990, Damodaran, 1991 and Ho, 1993).[5] After 1980 when information acquisition cost is conventionally believed to be low, our results are consistent with the opposite empirical findings that options listings decrease underlying stock prices (Detemple and Jorion, 1990, Sorescu, 2000 and Mayhew and Mihov, 2000), increase volatilities(Freund et al., 1994, Bollen, 1998 and Mayhew and Mihov, 2000), and increase price reactions to earnings announcements (Mendenhall and Fehrs, 1999).

[5]Information acquisition cost is lower after 1980 than that before 1980 because the technology is developed and it is easier for investors to search for information.

Meanwhile, our results could also explain the empirical findings in international markets. For example, according to the conventional belief, the information acquisition cost is high in emerging markets, but low in developed markets. Our results are consistent with existing empirical findings: options listings increase the underlying stock prices and decrease volatilities in emerging markets, such as India (Nair, 2008), but decrease the underlying stock prices and increase volatilities in some developed markets, such as Germany (Heer et al., 1997).

Related Literature Our study is related to several strands of literature. First, this study is associated with theoretical studies on the effects of derivatives on underlying assets, such as those by Grossman (1988), Biais and Hillion (1994), Huang and Wang (1997), Cao (1999) and Massa (2002). Cao (1999) and Massa (2002) are the most relevant to the present study, as they examine the effects of derivatives on information acquisition. Both authors find that introducing derivatives increases the prices of underlying assets, decreases price volatilities and decreases the price reactions to earnings announcements. The derivative examined in Massa (2002) conveys new information, which leads to increased price informativeness. By contrast, the derivatives examined in Cao (1999) and our study do not convey any additional information by themselves. However, Cao (1999) finds only a profit-making effect for the examined derivatives. Specifically, the author considers two groups of investors: inactive investors, who are unable to acquire information, and active investors, who determine the precision of private information. The author concludes that introducing derivatives increases the information precisions for active investors. However, in his model, the inactive investors are not able to acquire information, which hinders the substitution effect. Thus, derivatives have monotonic effects on information acquisition. In contrast to Cao (1999) and Massa (2002), we find that derivatives have two effects on information acquisitions: substitution effect and profit-making effect. More important, we find that the effects of derivatives on information acquisition and the underlying asset depend on the information acquisition cost and the precision of public information.

Meanwhile, our approach takes a first step to model an explicit options market in an economy with information asymmetry. AlthoughCao (1999) studies the effects of derivatives on their underlying assets, the derivatives in the proposed model take reduced forms and they are interpreted as straddles. The most relevant paper to ours is by Cao and Ou-Yang (2009), who also model a set of call and put options. However, the authors only conduct the analysis in an economy with heterogeneous beliefs without any implications for information acquisition.

Our work is also related to the large strand of literature on financial innovation (Allen and Gale, 1994, Brock, Hommes and Wagener, 2009, Dow, 1998, Dieckmann, 2011, Duffie and Rohi, 1995, Simsek, 2013a,b, Weyl, 2007 and Chabakauri, Yuan, Zachariadis, 2014). However, most studies in this literature stream examine the impact of financial innovations without information asymmetry. For example, Brock, Hommes and Wagener (2009), Simsek (2013a) and Simsek (2013b) emphasize the destabilizing effect of financial innovations due to heterogeneous beliefs. The most relevant paper to ours in this body of literature is by Dow (1998), who proposes a hedge-more/bet-more effect in an economy with asymmetric information. The author finds that a new asset induces risk averse arbitrageurs to hedge their positions in the preexisting security, which affects the old market's liquidity. This hedge-more/bet-more effect may have a negative effect on all investors' welfare. However, we show that options do not have a direct effect on the underlying asset, which confirms the findings by Chabakauri, Yuan, Zachariadis (2014). Moreover, we find that options affect the underlying assets through their effects on information acquisition.

The reminder of the paper is organized as follows. We introduce the model setup in Section 2 and solve a model without an options market. In Section 3, we study the effects of an options market on information acquisition and the underlying asset in a static model. Section 4 then extends the static model to a dynamic model. Section 5 discusses more general derivative. Section 6 concludes and discusses our empirical predictions.

3.2 Model

Based on the canonical frameworks with one risky asset and one risk-free asset by Grossman and Stiglitz (1980) and Hellwig (1980), we introduce an options market. Our goal is to compare the equilibrium population of informed investors and asset pricing before and after an options market is introduced into the economy. Before we solve the equilibrium in the economy with the options market, we solve the equilibrium in the economy without the options market in this section.

3.2.1 Timeline and assets

There are two periods in our economy, $T = 0, 1$. There is one risk-free asset and one risky asset. The risk-free asset is in zero supply, and it pays off one unit of a consumption good without uncertainty. The risky asset pays off D and has a positive supply of $\overline{X}$, where $D \sim N(\overline{D}, \frac{1}{h})$.

There is an options market in our economy, and the underlying asset is the risky asset. Following Cao and Ou-Yang (2009), we assume that the options market consists of a set of call and put options. The strike price of one specific option is denoted by G. The call option with strike price G then has a payoff as $(D - G)^+$, whereas the put option with strike price G has a payoff as $(G - D)^+$. The net supply of each option is zero. Because of the put-call parity, we can only consider call options with positive strike prices and put options with negative strike prices to simplify our analysis.[6] We assume that informed investor i's demand for risky asset is X_i, that his demand for call options with strike prices G to $G + dG$ is $X_{i,CG}$, and that his demand for put options with strike prices G to $G+dG$ is $X_{i,PG}$. Moreover, we assume that the uninformed investors' demand for risky asset is X_U, that their demand for call options with strike prices G to $G + dG$ is $X_{U,CG}$, and that their demand for put options with strike prices G to $G+dG$ is $X_{U,PG}$. The price of the risky asset is denoted by P. The price of a call option with strike price G is P_{CG} and the price of a put option with strike price G is P_{PG}. Our model differs from that of Cao and Ou-Yang (2009) in that we introduce an options market into an economy with asymmetric information, whereas they focus on the heterogeneous beliefs.

[6] If we introduce options with all strike prices, our results are robust because call options with negative strike prices are redundant because they can be replicated by put options with negative strike prices and stock.

3.2.2 Investors and information acquisition

There is one continuum of investors. The investors' utility function over the final wealth at $T = 1$ follows a standard CARA utility function with risk-averse coefficient γ:

$$- \exp(-\gamma W_1) \,, \tag{3.1}$$

where W_1 is the wealth at $T = 1$ and is equal to $W_0 + X_i(D - P)$. Each investor is indexed by i, where $i \in [0, 1]$. Without a loss of generality, we assume that all investors have zero endowment of the risky asset, and that they have the same initial wealth W_0. The market opens at $T = 0$. For informed investor i, he or she has a private signal about the risky asset's payoff before trading at $T = 0$:

$$S_i = D + \epsilon_i \,, \tag{3.2}$$

where ϵ_i follows normal distribution $N(0, \frac{1}{s})$ and is independent of cross investors ($corr(\epsilon_i, \epsilon_j) = 0$ for $i \neq j$). We assume that the precision of private signals that investors acquire is the same. Fruther, investors can only acquire one private signal. If the investors choose to acquire the private signals, then they need to pay a cost C, which is called then information acquisition cost. The population of informed investors is denoted by ω, which is endogenous in our economy. At $T = 1$, the payoff is realised and all investors consume their total wealth.

In addition to these investors,some noisy traders exist in the market. We assume that the total demand from noisy traders is n, which follows normal distribution $N(0, \frac{1}{q})$.

3.2.3 Information acquisition without an options market

We first derive the equilibrium given the population of informed investors ω and then solve the equilibrium ω. This section shows that the equilibrium ω decreases with the information acquisition cost. In the analysis that follows, we compare the equilibrium ω in the economy with and without an options market. This comparison demonstrates the effect of an options market on the underlying asset pricing through the information acquisition channel.

All of the investors submit their demand conditional on their information sets, and the equilibrium price clears the market. Informed investor i's information set is $\mathcal{F}_i = \{S_i, P\}$, whereas uninformed investors' information set is $\mathcal{F}_U = \{P\}$. As shown by Grossman and Stiglitz (1980) and Hellwig (1980), the following linear equilibrium exists:

$$P = \overline{D} - \frac{\gamma \overline{X}}{B} + \frac{(\omega s + \frac{\omega^2 s^2 q}{\gamma^2})(D - \overline{D} + \frac{\gamma}{\omega s} n)}{B} , \tag{3.3}$$

Informed investor i's demand:

$$X_i = \frac{E(D|\mathcal{F}_i) - P}{\gamma Var(D|\mathcal{F}_i)} , \tag{3.4}$$

Uninformed investors' demand:

$$X_U = \frac{E(D|\mathcal{F}_U) - P}{\gamma Var(D|\mathcal{F}_U)} , \tag{3.5}$$

where

$$B = h + \omega s + \frac{\omega^2 s^2 q}{\gamma^2} , \tag{3.6}$$

$$E(D|\mathcal{F}_i) = \overline{D} + \frac{s(S_i - \overline{D}) + \frac{\omega^2 s^2 q}{\gamma^2}(D - \overline{D} + \frac{\gamma}{\omega s} n)}{h + s + \frac{\omega^2 s^2 q}{\gamma^2}} \quad \text{and} \quad Var(D|\mathcal{F}_i)^{-1} = h + s + \frac{\omega^2 s^2 q}{\gamma^2} , \tag{3.7}$$

$$E(D|\mathcal{F}_U) = \overline{D} + \frac{\frac{\omega^2 s^2 q}{\gamma^2}(D - \overline{D} + \frac{\gamma}{\omega s} n)}{h + \frac{\omega^2 s^2 q}{\gamma^2}} \quad \text{and} \quad Var(D|\mathcal{F}_U)^{-1} = h + \frac{\omega^2 s^2 q}{\gamma^2} . \tag{3.8}$$

We substitute the investors' demand into their final wealth and the expected utility of informed investors/uninformed investors is given by (where U_I is the informed investors' expected utility, and U_U is the uninformed investors' expected utility):

$$U_I = -\exp[-\gamma W_0 - \frac{\overline{X}^2}{2\gamma B^2 Var(D-P)}] \times \frac{1}{\sqrt{Var(D-P)B_i}} , \tag{3.9}$$

$$U_U = -\exp[-\gamma W_0 - \frac{\overline{X}^2}{2\gamma B^2 Var(D-P)}] \times \frac{1}{\sqrt{Var(D-P)B_U}} , \tag{3.10}$$

where

$$B_i = h + s + \frac{\omega^2 s^2 q}{\gamma^2} \quad \text{and} \quad B_U = h + \frac{\omega^2 s^2 q}{\gamma^2}\,. \tag{3.11}$$

The informed investors' utility is clearly higher than the uninformed investors' utility. Thus, the informed investors gain from private information. In the equilibrium, the population of informed investors ω should render the gain from private information and the cost C equal. Then, we define the gain from information acquisition G as: [7]

Definition **3.2.1.** *The Gain from information acquisition is* $G = (U_U/U_I)^2$

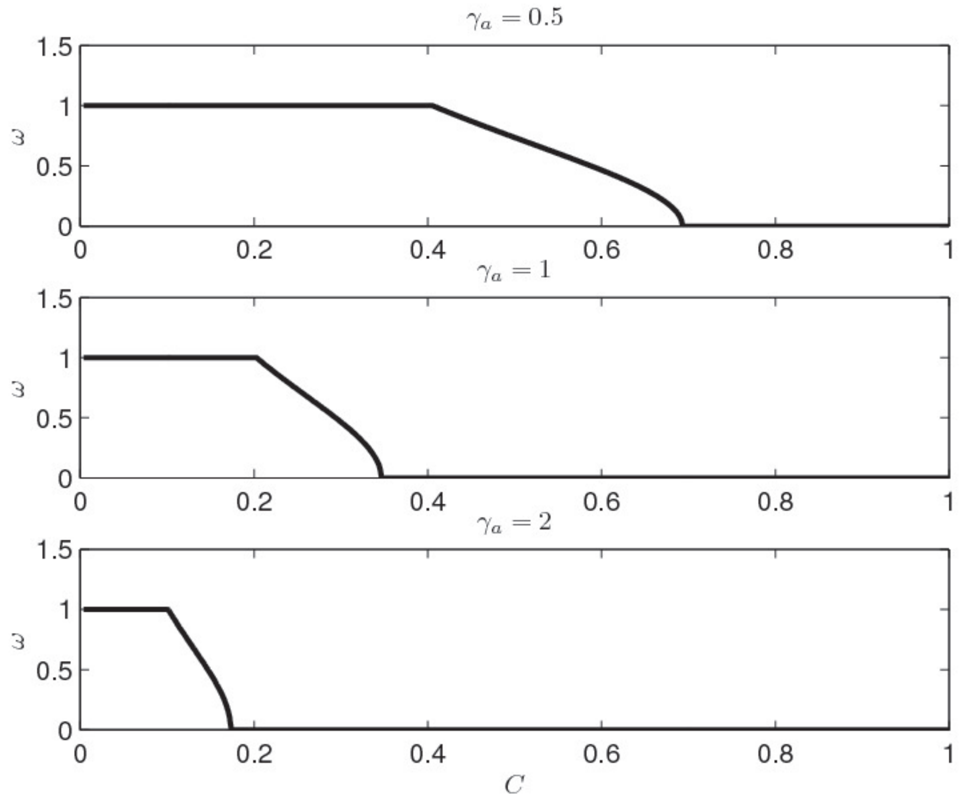

Figure 3.2.1: The Relationship between the Population of Informed Investors and the Acquisition Cost

The gain from information acquisition is $\frac{B_i}{B_U}$. We can therefore show the results regarding ω^* as follows (see Figure 2).

[7] We set h=1,s=1 and q=1 in the Figure 2

PROPOSITION **3.2.1.** *In the equilibrium without an options market, the population of informed investors renders the gain from information acquisition and its cost equal. Then, there are three cases:*

Case 1: If $C \geq C_{d1}$*, the equilibrium population of informed investors is* $\omega^* = 0$*.*

Case 2: If $C_{d2} < C < C_{d1}$*, the equilibrium population of informed investors is* $\omega^* \in (0, 1)$*.*

Case 3: If $C \leq C_{d2}$*, the equilibrium population of informed investors is* $\omega^* = 1$*.*

where C_{d1} *and* C_{d2} *are defined as in the Appendix.*

COROLLARY **3.2.1.** *When* $C_{d2} < C < C_{d1}$*,* ω^* *is monotonically decreasing with the information acquisition cost* C*.*

Based on the equilibrium population of informed investors, we examine the effect of introducing an options market on ω and the underlying asset in the following sections. Because we have corner solutions in Case 1 and Case 3, we focus on the Case 2 to conduct the study.

3.3 Introduction of an Option Market

In this section, we analyze the effects an options market on investors' information acquisition decisions and the underlying asset. We study the role of the information acquisition cost in the effects. We first solve a static model with an option markets, and then we extend this static model to a dynamic model in next section. We demonstrate the robustness of the results.

After an options market in introduced, the investors' information sets differ from before the options market is introduced. For informed investor i, his or her information set is $\mathcal{F}_i = \{S_i, P, P_{CG}, P_{PG}\}$, whereas uninformed investors' information set is $\mathcal{F}_U = \{P, P_{CG}, P_{PG}\}$. Our conjecture is that the underlying risky asset's price is a linear function of fundamental payoff D and the noisy traders' demand n. The partially revealing rational expectations equilibrium regarding P, P_{CG}, P_{PG} and the investors' demands is described in the following proposition.

PROPOSITION **3.3.1.** *There exists one equilibrium in* $T = 0$*. Equilibrium* P *and* P_G *are given* *by:*

$$P = \overline{D} - \frac{\gamma \overline{X}}{B} + \frac{(\omega s + \frac{\omega^2 s^2 q}{\gamma^2})(D - \overline{D} + \frac{\gamma}{\omega s} n)}{B}\,, \tag{3.1}$$

$$P_{CG} = (P-G)N(\sqrt{B}(P-G)) + \frac{1}{\sqrt{B}}\exp(-\frac{B(P-G)^2}{2})\,, \qquad \textit{where } G \geqslant 0\,, \tag{3.2}$$

$$P_{PG} = (G-P)N(\sqrt{B}(G-P)) + \frac{1}{\sqrt{B}}\exp(-\frac{B(G-P)^2}{2})\,, \qquad \textit{where } G < 0\,, \tag{3.3}$$

Informed investor i's demands for risky asset is:

$$X_i = \frac{E(D|\mathcal{F}_i) - P}{\gamma Var(D|\mathcal{F}_i)} - \frac{(B - B_i)}{\gamma} P\,, \tag{3.4}$$

Informed investor i's demands for options is:

$$X_{i,CG} = \frac{1}{\gamma}(B - B_i) \quad \textit{and} \quad X_{i,PG} = \frac{1}{\gamma}(B - B_i)\,, \tag{3.5}$$

Uninformed investor's demand for risky asset is:

$$X_U = \frac{E(D|\mathcal{F}_U) - P}{\gamma Var(D|\mathcal{F}_U)} - \frac{(B - B_U)}{\gamma} P\,, \tag{3.6}$$

Uninformed investor j's demands for options is:

$$X_{U,CG} = \frac{1}{\gamma}(B - B_U) \quad \textit{and} \quad X_{U,PG} = \frac{1}{\gamma}(B - B_U)\,, \tag{3.7}$$

where $B = h + \omega s + \frac{\omega^2 s^2 q}{\gamma^2}$, $B_i = h + s + \frac{\omega^2 s^2 q}{\gamma^2}$ *and* $B_U = h + \frac{\omega^2 s^2 q}{\gamma^2}$.

Several interesting features of Proposition 3.3.1 are notable. First, the option prices are functions of the price of the underlying asset, and they do not convey any additional information, in contrast to the derivative in Massa (2002), which carries additional information by itself. Because options do not carry additional information, we can isolate the effect proposed by Massa (2002) based on this feature. Second, B_U and B_i represent information precisions of information for uninformed and informed investors respectively, whereas B is the precision of the aggregate information. Following the intuition that the value of options depends on investors' conditional volatility regarding the underlying asset's payoff, informed investors' demand for options is lower than uninformed investors's demand. In the equilibrium where the net supply of each option is zero, informed investors are on the short side of options. Thus, introducing an options market provides an opportunity for informed investors to profit from selling options. Following the same mechanism, the third feature is that the aggregate option prices decrease with the precision of aggregate information B, which is shown in the following Lemma 3.3.1. The analysis implies that options have a similar effect to information in reducing risk. [8]

Lemma **3.3.1.** *The aggregate price of options is* $\int_0^\infty P_{CG} dG + \int_{-\infty}^0 P_{PG} dG = \frac{1}{2}(\frac{1}{B} + P^2)$.

In line with Grossman and Stiglitz (1980), the equilibrium population of informed investors renders the expected utility of informed and uninformed investors equal. Before we perform the comparisons, we must show that there is a unique equilibrium in information acquisition with an options market. Otherwise, showing the effects of options would be difficult. To demonstrate the existence of a unique equilibrium, we only need to show that the gain from information decreases with the population of informed investors ω, which can be shown as follows.[9]

Lemma **3.3.2.** *The gain from information* G *with an option market is* $\exp(\frac{s}{B})$.

The gain from information clearly decreases with the population of informed investors, which implies that a unique solution exists for information acquisition. However, whether the equilibrium population of informed investors in the economy with options is higher than that without options is unclear. Because the information acquisition cost is constant, the equilibrium population of informed investors is higher in the economy with options if $\exp(\frac{s}{B})$ is higher than $\frac{B_i}{B_U}$, and vice versa. Figure 3.3 shows that when the population of informed investors is zero, $\exp(\frac{s}{B})$ is higher $\frac{B_i}{B_U}$. This result indicates that when the population of

[8]The aggregate payoff of options is: $D^2 = 2\int_0^\infty (D-G)^+ dG + 2\int_{-\infty}^0 (G-D)^+ dG$.

[9]We set h=1,s=1, q=1 and $\gamma = 0.5$ in the Figure 3

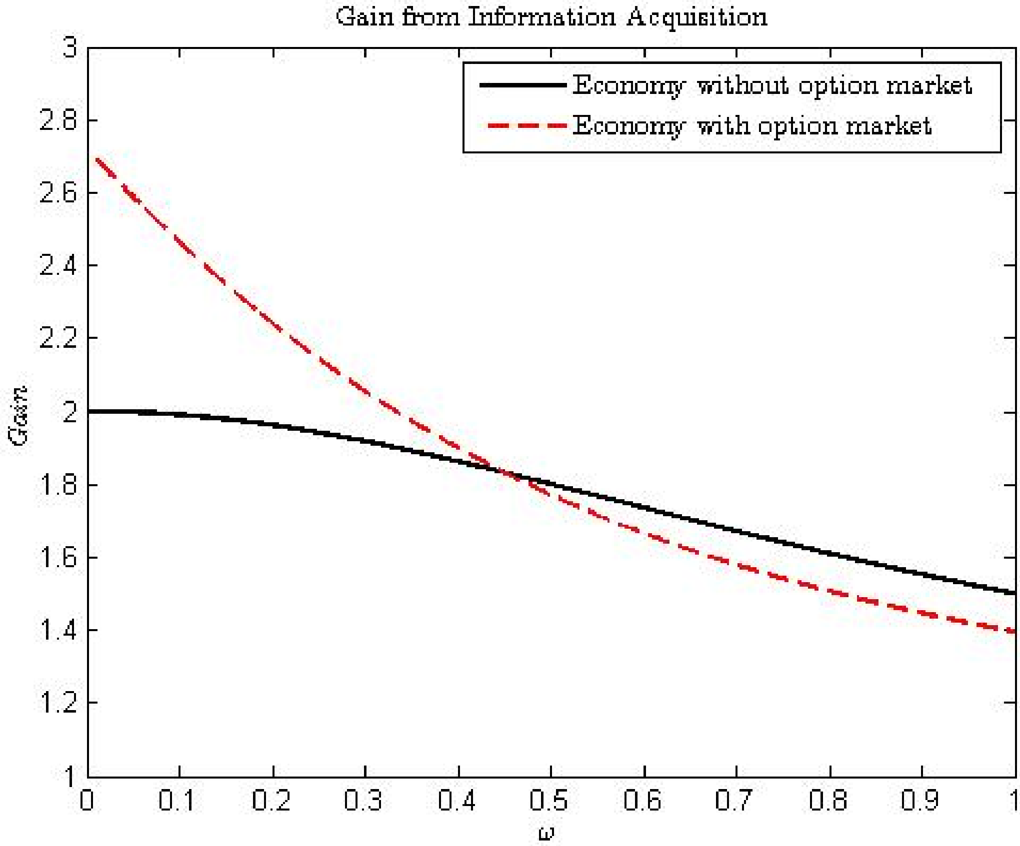

Figure 3.3.1: Gain from Information Acquisition: Effect of an Options Market

informed investors is close to 0, the gain from information is higher in the economy with options than in the economy without options. Thus, introducing an options market increases investors' incentive to acquire information. When the population of informed investors is 1, $\exp(\frac{s}{B})$ is smaller than $\frac{B_i}{B_U}$. This result indicates that when the population of informed investors is close to 1, the gain from information is lower in the economy with options than in the economy without options. Thus, introducing an option market decreases investors' incentive to acquire information.[10] Because the population of informed investors depends on the information acquisition cost, we obtain the following formal results with regard to the effect of options on information acquisition.

Proposition 3.3.2. *When $C \in (C_{d2}, C_{d1})$, cutoffs C_3 and C_4 exists, which satisfies the*

[10]The analysis here uses the relations: $\frac{x}{1+x} < \ln(1+x) < x$ for $x > 0$

following conditions:

(1) when $C > C_4$, introducing an options market increases the population of informed investors.

(2) when $C < C_3$, introducing an options market decreases the population of informed investors.

where C_3 and C_4 are defined as in the Appendix and $C_3 < C_4$.

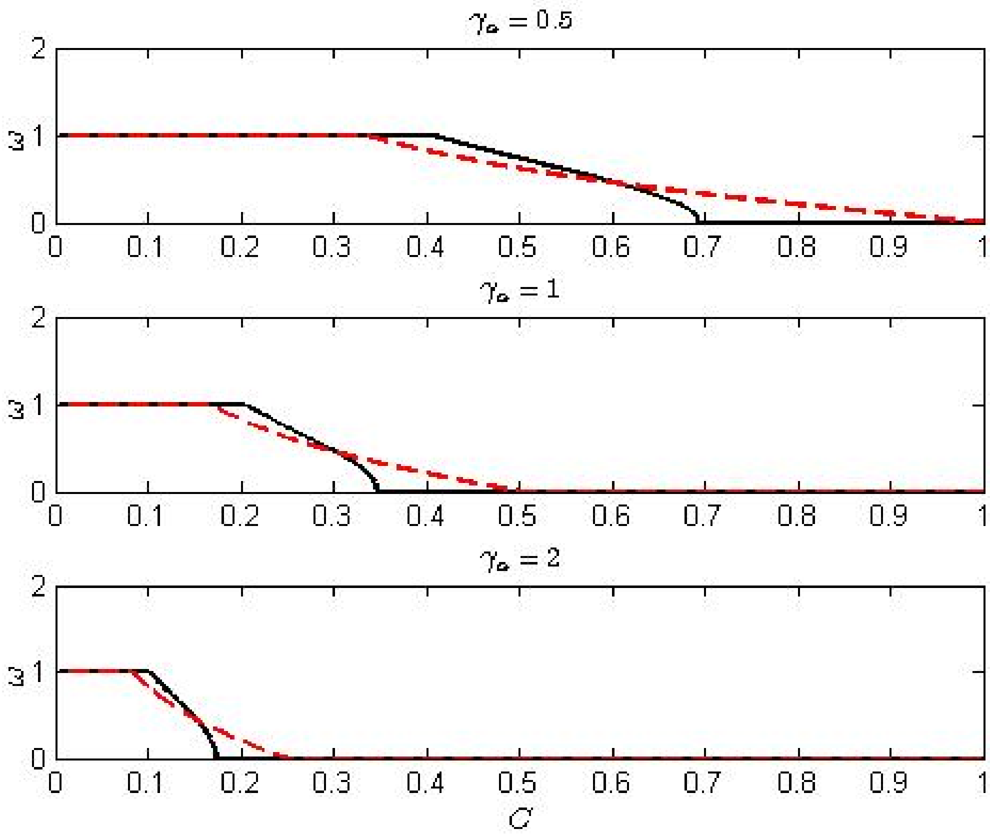

Figure 3.3.2: The Relationship Between Population of Informed Investors and Acquisition Cost: Effect of Option Market

Proposition 3.3.2(see Figure 4) shows that introducing an options market increases investors' incentive to acquire information when the information acquisition cost is high and decreases investors' incentive to acquire information when the information acquisition cost is low.[11] From the Proposition 3.3.1, we know that the supply of options is higher than the demand when the population of informed investors is high. Meanwhile, as shown in Lemma 3.3.1, the aggregate option prices tend to be low. The selling profits of informed investors in the options markets clearly depend on both the demand per supplier and the option prices. Thus, the profits from selling options are low for informed investors, and they will not cover the information acquisition cost when the demand is too low. Moreover, investors could use cheap options instead of information to hedge their portfolios risk, which implies that introducing an options market decreases investors' incentive to acquire information. By contrast, the supply of options is lower than then demand when the population of informed investors is low, leading to high aggregate option prices. In particular, when the demand per supplier is sufficiently large, information investors' profits from selling options will cover the information acquisition cost, which implies that introducing an options market increases investors' incentive to acquire information. Given its effect on information acquisition, the options market has a direct effect on the price informativeness. To show the effect of options on price informativeness, we define price informative as follows.

DEFINITION **3.3.1.** *The price informativeness I: $I = \frac{1}{Var(D|P)}$.*

Because the option prices do not convey any additional information, the above definition of I captures all of the information that is conveyed by the market. Thus we can conveniently show the effect of options on price informativeness. Price informativeness is clearly $\frac{\omega^2 s^2 q}{\gamma^2}$, and it increases with the population of informed investors. Because introducing an options market affects the population of informed investors, we obtain the following formal results regarding price informativeness.

PROPOSITION **3.3.3.** *When $C \in (C_{d2}, C_{d1})$,*

(1) when $C > C_4$, introducing an options market increases price informativeness I.

(2) when $C < C_3$, introducing an options market decreases price informativeness I.

where C_3 and C_4 are defined as in the Appendix and $C_3 < C_4$.

[11] We set h=1,s=1, q=1 in the Figure 4

In addition to the effect of options on information acquisition, we also examine their effects on the price and volatility of the underlying asset. Because uncertainty exists regarding the asset payoff, the price is discounted. The expected difference between the asset payoff and price is called the cost of capital: $E(D-P)$. The cost of capital $E(D-P)$ decreases with the expected asset price. Thus the result for expected asset price is equivalent to the analysis on the cost of capital. The expected asset price is given by:

$$E(P) = \overline{D} - \frac{\gamma \overline{X}}{B}, \tag{3.8}$$

and the volatility $V(D-P)$ is given by:

$$Var(D-P) = \frac{1}{B} + \frac{\omega s + \gamma^2 q^{-1}}{B^2}. \tag{3.9}$$

The expected asset price clearly increases with B, and the volatility decreases with B. Thus, we have the following results:

PROPOSITION **3.3.4.** *When* $C \in (C_{d2}, C_{d1})$*,*
(1) when $C > C_4$*, introducing an options market increases the expected asset price and decreases the price change volatility.*
(2) when $C < C_3$*, introducing an options market decreases the expected asset price and increases the price change volatility.*

These results arise from the effect of options on information acquisition. When the information acquisition cost is high, introducing an options market increases the population of informed investors. Because informed investors have information that is precise, they trade more aggressively, and they are more willing to absorb noisy supply, which indicates that the demand for the underlying asset increases, along with a higher expected price level. The price then becomes less sensitive to noisy supply because increased price informativeness. Consequently, the non-fundamental volatility decreases, leading to decreased total volatility. When the information acquisition cost is low, a similar mechanism generates the opposite results. In contrast to the findings of Cao (1999) and Massa (2002), these results imply that the information acquisition cost plays an important role in the effect of options on asset pricing.

We also examine the effect of the information acquisition cost on the trading volume of options. The trading volume in the options market is calculated as follows:

LEMMA **3.3.3.** *The trading volume in option market* $V_O = \frac{1}{\gamma}(\int_0^\omega |(B_i - B)|di + (1-\omega)|B - B_U|) = \frac{2\omega(1-\omega)s}{\gamma}$.

Because the short side comes from informed investors and long side comes from uninformed investors in the options market, intuitively, the trading volume is zero when all investors are informed or uninformed. When the information acquisition cost increases from zero, the population of uninformed investors increases, which enhances the trading between the different groups. When the information acquisition cost is sufficiently high, the population of uninformed investors is high, which makes total total trading volume vanish.

PROPOSITION **3.3.5.** *The trading volume in the options market exhibits a hump shape as a function of the information acquisition cost:* V_o *decreases with the information acquisition cost when* C *is higher than* C_M*, and increases with the information acquisition cost when* C *is lower than* C_M*,*

where C_M *is defined as in the Appendix.*

3.4 Dynamic Model with an Options Market

This section aims to demonstrate the robustness of the results in Section 3 are robust. Moreover, the dynamic model is helpful for studying the effect of an options market on the price reaction to public information.

3.4.1 Dynamic model without an Options Market

To model dynamic trading, following Brennan and Cao (1996), we assume that there is an approximate continuous trading time from $T = 0$ to $T = 1$. The trading ends at t, where t is between $T = 0$ and $T = 1$. To make the model tractable, we assume that investors can trade only once in a small time interval z, which means that trading occurs only in the time intervals $[0, z), [z, 2z), [3z, 4z)....[(K-1)z, Kz)$, where K is the largest integer satisfying $Kz \leq t$. [12] We index the interval $[(j-1)z, jz)$ by trading round j. Before each trading round j, a public signal is released. The public signal before trading round j is

$$S_{c,j} = D + \epsilon_{c,j} \, , \tag{3.1}$$

where $j = 1, 2....K$ and $\epsilon_{c,j}$ follows normal distribution $N(0, \frac{1}{c_j z})$. $\epsilon_{c,j}$ is independent from trading rounds and is independent from noise in investors' private signal. We assume that z tends to be zero throughout our analysis. This assumption guarantees that the public information flow is sufficiently smooth and that the price change volatility tends to be zero between two consecutive trading rounds when the time interval is close to zero. We further assume that the precision of the aggregate public information until trading round j is F_j, where $F_j = \sum_{k=1}^{k=j} c_k z$. Therefore, F_K is the aggregate precision of public information in this dynamic model. Furthermore, to simplify the analysis, we assume that there are no additional noisy traders after the initial trading round. [13]

Investors submit their demand schedules conditional on their information sets. The information set for informed investor i in trading session j is $\mathcal{F}_{i,j} = \{S_i, S_{c,k}, P_k, \;\; k = 1, 2....j\}$, whereas the information set for uninformed investors in trading session j is $\mathcal{F}_{U,j} = \{S_{c,k}, P_k, k = 1, 2....j\}$. We assume that investor i submits optimal demand schedule $X_{i,j}$ in trading round j. Our conjecture is that the risky asset's price function is a linear function of fundamental payoff D, the noisy traders' demand n, and public signals. The partially revealing rational expectations equilibrium is described in the following proposition.

Proposition **3.4.1.** *Given the population of informed investors ω, one partially revealing*

[12] Here, z is similar to dt in continuous-time model and it captures the feature of fast trading in practice. When z approximates to be zero, this model converges to the continuous-time model in Brennan and Cao (1996)

[13] Without additional noise traders, it is possible that there are two equilibria (Brennan and Cao, 1996). As argued by Brennan and Cao (1996), one of the equilibria is fully revealing equilibrium and investors make portfolio choices neither conditional on price nor private information, which is not appealing.

rational expectations equilibrium exists, where investors' demand schedule, investors' beliefs and equilibrium prices are given by:

$$P_j = \overline{D} - \frac{\overline{X}}{B_j} + \frac{(\omega s + \frac{\omega^2 s^2 q}{\gamma^2})(D - \overline{D} + \frac{\gamma}{\omega s} n) + \sum_{k=1}^{j} c_k z (S_{c,k} - \overline{D})}{B_j}\,, \tag{3.2}$$

where $j = 1, 2....K, K+1, ...L$.

Informed investors' demand is:

$$X_{i,j} = \frac{E(D|\mathcal{F}_{i,j}) - P_j}{\gamma Var(D|\mathcal{F}_{i,j})}\,, \tag{3.3}$$

where

$$E(D|\mathcal{F}_{i,j}) = \overline{D} + \frac{s(D - \overline{D} + \epsilon_i) + \frac{\omega^2 s^2 q}{\gamma^2}(D - \overline{D} + \frac{\gamma}{\omega s} n) + \sum_{k=1}^{j} c_k z (S_{c,k} - \overline{D})}{h + s + \frac{\omega^2 s^2 q}{\gamma^2} + \sum_{k=1}^{j} c_k z}\,, \tag{3.4}$$

$$Var(D|\mathcal{F}_{i,j}) = \frac{1}{h + s + \frac{\omega^2 s^2 q}{\gamma^2} + \sum_{k=1}^{j} c_k z}\,, \tag{3.5}$$

Uninformed investors' demand is:

$$X_{U,j} = \frac{E(D|\mathcal{F}_{U,j}) - P_j}{\gamma Var(D|\mathcal{F}_{U,j})}\,, \tag{3.6}$$

where

$$E(D|\mathcal{F}_{U,j}) = \overline{D} + \frac{\frac{\omega^2 s^2 q}{\gamma^2}(D - \overline{D} + \frac{\gamma}{\omega s} n) + \sum_{k=1}^{j} c_k z (S_{c,k} - \overline{D})}{h + \frac{\omega^2 s^2 q}{\gamma^2} + \sum_{k=1}^{j} c_k z}\,, \tag{3.7}$$

$$Var(D|\mathcal{F}_{U,j}) = \frac{1}{h + \frac{\omega^2 s^2 q}{\gamma^2} + \sum_{k=1}^{j} c_k z}\,, \tag{3.8}$$

and

$$B_j = h + \omega s + \frac{\omega^2 s^2 q}{\gamma^2} + \sum_{k=1}^{j} c_k z\,. \tag{3.9}$$

Proposition 3.4.1 shows that the prices only reveal information through $D - \overline{D} + \frac{\gamma}{\omega s} n$, and that the investors behave myopically because there are no additional noisy traders. The expected utilities of informed and uninformed investors are shown in the following Lemma

3.4.1.

Lemma **3.4.1.** *The expected utility of informed investors in the economy with K trading rounds is given by*

$$U_I = -\frac{1}{\sqrt{Var(D-P_1)B_{i,1}}}\exp[-\gamma W_0 + \gamma C - \frac{\gamma \overline{X}^2}{2B_0^2 Var(D-P_0)}] \times \prod_{j=2}^{j=K} \frac{1}{\sqrt{1+\frac{c_j z (B_j - B_{i,j})^2}{B_{i,j-1}B_j^2}}}, \tag{3.10}$$

and the expected utility of uninformed investors is given by

$$U_U = -\frac{1}{\sqrt{Var(D-P_1)B_{U,1}}}\exp[-\gamma W_0 - \frac{\gamma \overline{X}^2}{2B_0^2 Var(D-P_0)}] \times \prod_{j=2}^{j=K} \frac{1}{\sqrt{1+\frac{c_j z (B_j - B_{U,j})^2}{B_{U,j-1}B_j^2}}}. \tag{3.11}$$

where $B_{i,j} = h + s + \frac{\omega^2 s^2 q}{\gamma^2} + \sum_{k=1}^{j} c_k z$, $B_{U,j} = h + \frac{\omega^2 s^2 q}{\gamma^2} + \sum_{k=1}^{j} c_k z$ and B_j is defined as above.

The gain from information G is obviously $\frac{B_{i,1}}{B_{U,1}} \prod_{j=2}^{j=N} \frac{1+\frac{c_j z (B_j - B_{i,j})^2}{B_{i,j-1}B_j^2}}{1+\frac{c_j z (B_j - B_{U,j})^2}{B_{U,j-1}B_j^2}}$, where $\frac{1+\frac{c_j z (B_j - B_{i,j})^2}{B_{i,j-1}B_j^2}}{1+\frac{c_j z (B_j - B_{U,j})^2}{B_{U,j-1}B_j^2}}$ is the additional gain generated by trading round j. To provide a further comparison, we must show that there is a unique equilibrium of information acquisition. Thus, we must show whether the gain from information decreases with the population of informed investors. We obtain the following result regarding information acquisition:

Proposition **3.4.2.** *The gain from information G decreases with the population of informed investors ω.*

Proposition 3.4.2 shows that a unique equilibrium exists. Further, the equilibrium population of informed investors decreases with the information acquisition cost, which is shown below.

Corollary **3.4.1.** *In an economy with K trading rounds, the population of informed investors renders the gain from information and the information acquisition cost equal. There are three cases:*

Case 1: If $C \geq C_1$, the equilibrium population of informed investors $\omega^* = 0$.

Case 2: If $C_2 < C < C_1$, the equilibrium population of informed investors $\omega^* \in (0, 1)$.

Case 3: If $C \leq C_2$,the equilibrium population of informed investors $\omega^* = 1$.

where C_1 and C_2 are defined as in the Appendix.

Corollary **3.4.2.** *When $C_2 < C < C_1$, ω^* decreases with the information acquisition cost C.*

Given the information acquisition cost C, we show how the precision of public information F_K affects the equilibrium population of informed investors ω in the following results.

Corollary **3.4.3.** *In an economy with K trading rounds, the population of informed investors renders the gain from information and the information acquisition cost equal. There are three cases:*

Case 1: If $F_K \geq F_1$, the equilibrium population of informed investors $\omega^* = 0$.

Case 2: If $F_2 < F_K < F_1$, the equilibrium population of informed investors $\omega^* \in (0, 1)$.

Case 3: If $F_K \leq F_2$,the equilibrium population of informed investors $\omega^* = 1$.

where F_1 and F_2 are defined in the Appendix.

Corollary **3.4.4.** *When $F_2 < F_K < F_1$, ω^* decreases with the public information precision F_k.*

Corollary 3.4.3 intuitively indicates that investors' incentive to acquire private information decreases with the precision of public information owing to the decreasing marginal benefit of information. Because there are corner solutions in Case 1 and Case 3, we focus on the Case 2 to perform the analysis regarding the public information.

3.4.2 Dynamic model with an options market

In this section, we solve a dynamic model with an options market to demonstrate that the robustness of the results regarding the effects of options on information acquisition and the underlying asset from the static model. Furthermore, we show the effect of an options market on price reactions to public information.

We introduce an options market, that consists of a section of call and put options, as in Section 3. Let P_k be the risky asset's price in trading round k, $P_{CG,k}$ be the price for a call option with strike price G, and $P_{PG,k}$ be the price for a put option with strike price G. The information set for For informed investor i in trading round j is $\mathcal{F}_{i,j} = \{S_i, S_{c,k}, P_k, P_{CG,k}, P_{PG,k},$ $k = 1, 2....j\}$, whereas the information set for uninformed investors in trading round j is $\mathcal{F}_{U,j} = \{S_{c,k}, P_k, P_{CG,k}, P_{PG,k},\ \ k = 1, 2....j\}$. We assume that investor i submits optimal demand schedule $X_{i,j}$ for the risky asset, $X_{i,CG,j}$ for call option with strike price G, and $X_{i,PG,j}$ for put option with strike price G in trading round j. Our conjecture is that the underlying asset's price function is a linear function of fundamental payoff D, the noisy trader n, and public signals. The partially revealing rational expectations equilibrium is described in the following proposition.

Proposition **3.4.3.** *Given the population of informed investors ω, one partially revealing rational expectations equilibrium exists, where investors' demand schedule, investors' beliefs and equilibrium prices are given by:*

$$P_j = \overline{D} - \frac{\gamma \overline{X}}{B_j} + \frac{(\omega s + \frac{\omega^2 s^2 q}{\gamma^2})(D - \overline{D} + \frac{\gamma}{\omega s} n) + \sum_{k=1}^{j} c_k z (S_{c,k} - \overline{D})}{B_j}\,, \tag{3.12}$$

$$P_{CG,j} = (P_j - G) N(\sqrt{B_j}(P_j - G)) + \frac{1}{\sqrt{B_j}} \exp(-\frac{B_j (P_j - G)^2}{2}) \qquad \textit{where } G \geqslant 0\,, \tag{3.13}$$

$$P_{PG,j} = (G - P_j) N(\sqrt{B_j}(G - P_j)) + \frac{1}{\sqrt{B_j}} \exp(-\frac{B_j (G - P_j)^2}{2}) \qquad \textit{where } G < 0\,, \tag{3.14}$$

where $j = 1, 2....K$.

Informed investors' demand for the underlying asset is:

$$X_{i,j} = \frac{E(D|\mathcal{F}_{i,j}) - P_j}{\gamma Var(D|\mathcal{F}_{i,j})} - \frac{(B_j - B_{i,j})}{\gamma} P_j \,, \tag{3.15}$$

Informed investors' demand for options is

$$X_{i,CG,j} = \frac{1}{2\gamma}(B_j - B_{i,j}) \quad \text{and} \quad X_{i,PG,j} = \frac{1}{2\gamma}(B_j - B_{i,j}) \,, \tag{3.16}$$

Uninformed investors' demand for the underlying asset is:

$$X_{U,j} = \frac{E(D|\mathcal{F}_{U,j}) - P_j}{\gamma Var(D|\mathcal{F}_{U,j})} - \frac{(B_j - B_{U,j})}{\gamma} P_j \,, \tag{3.17}$$

Uninformed investors' demand for options is

$$X_{U,CG,j} = \frac{1}{2\gamma}(B_j - B_{U,j}) \quad \text{and} \quad X_{U,PG,j} = \frac{1}{2\gamma}(B_j - B_{U,j}) \,. \tag{3.18}$$

where $B_j = \omega B_{i,j} + (1-\omega) B_{U,j}$, $B_{i,j} = h + s + \frac{\omega^2 s^2 q}{\gamma^2} + \sum_{k=1}^{j} c_k z$ *and* $B_{U,j} = h + \frac{\omega^2 s^2 q}{\gamma^2} + \sum_{k=1}^{j} c_k z$

Proposition 3.4.3 shows that investors' optimal demands for the underlying asset and options are similar to that found in the static model. An interesting finding is that option prices only depend on the price of the underlying asset, and that they do not convey any additional information. Regarding investors' utility, we obtain the following lemma

Lemma **3.4.2.** *Informed investor i's expected utility in trading round j is*

$$EV_{i,j} = -\frac{1}{\sqrt{\frac{B_j}{B_{i,j}}}} E\{\exp[-\gamma W_{i,j} - \frac{[E(D|\mathcal{F}_{i,j+1}) - P_{j+1}]^2}{2\gamma Var(D|\mathcal{F}_{i,j+1})} + \frac{B_j - B_{i,j}}{2} \frac{1}{B_j}]\} \,, \tag{3.19}$$

and uninformed investors' expected utility in trading round j is:

$$EV_{U,j} = -\frac{1}{\sqrt{\frac{B_j}{B_{U,j}}}} E\{\exp[-\gamma W_{U,j} - \frac{[E(D|\mathcal{F}_{U,j+1}) - P_{j+1}]^2}{2\gamma Var(D|\mathcal{F}_{U,j+1})} + \frac{B_j - B_{U,j}}{2}\frac{1}{B_j}]\}\,. \tag{3.20}$$

Lemma 3.4.2 shows that the gain from information is $\exp(\frac{s}{B})$, where $B = h + \omega s + \frac{\omega^2 s^2 q}{\gamma^2}$. Further, this gain decreases with the population of informed investors. Thus, a unique interior solution exists to render the gain from information and the cost equal. To conduct the analysis over the expected asset price and price change volatility, we know that the expected price is given by

$$E(P_j) = \overline{D} - \frac{\gamma \overline{X}}{B_j}\,, \tag{3.21}$$

The price change volatility is given by

$$Var(P_{j+1} - P_j) = \frac{1}{B_j} - \frac{1}{B_{j+1}} + \frac{c_{j+1}^2}{B_j^2 B_{j+1}^2}(\omega s + \frac{\gamma^2}{q}), \tag{3.22}$$

and the price informativeness is given by

$$I = \frac{1}{Var(D|P_j)}\,. \tag{3.23}$$

We demonstrate the robustness of the effects of options on information acquisition and the underlying asset in the dynamic model as follows.

PROPOSITION **3.4.4.** *When $C \in (C_2, C_1)$, cutoffs C_3 and C_4 exists, which satisfies the following condition:*

(1) When $C > C_4$, introducing an options market increases the population of informed investors, increases price informativeness, increases the expected asset price, and decreases price change volatility.

(2) When $C < C_3$, introducing an options market decreases the population of informed investors, decreases price informativeness, decreases the expected asset price, and increases

price change volatility.

where C_3 and C_4 are defined as in the Appendix and $C_3 < C_4$.

Findings regarding price reactions to earnings announcements in the U.S. market are also conflicting, as previous studies suggest that options listing decreased earnings announcements' surprise before 1980 (Jennings and Starks, 1986, Skinner, 1990, Damodaran, 1991 and Ho, 1993), but increases earnings announcements' surprise after 1980 (Mendenhall and Fehrs, 1999). The proxy for the price reactions to public information in trading round j is $\frac{c_j z}{B_j}$, which decreases with B_j. When investors have more information about fundamental value, intuitively, the surprise to earnings announcements would be smaller. The result presented below shows that the effect of options listing on the price reactions to public information depends on the information acquisition cost. This result also differs from that by Cao (1999), who concludes that options listing decreases price reactions to public information.

PROPOSITION **3.4.5.** *When $C \in (C_2, C_1)$,*

(1) When $C > C_4$, introducing an options market decreases price reactions to public information.

(2) When $C < C_3$, introducing an options market increases price reactions to public information.

where C_3 and C_4 are defined as in the Appendix and $C_3 < C_4$.

As shown in Corollary 3.4.3, the precision of public information also affects investors' incentive to acquire information. Thus, we can expect the effect of an options market on information acquisition and asset prices to depend on the precision of public information. According to Proposition 3.4.6, the effects of options in the economy with precise public information are similar to those with high information acquisition costs, whereas the effects of options in an economy with imprecise public information are similar to those with low information acquisition costs. The population of informed investors is low when public information is precise before the introduction of options, which leads to a high demand for options and high option prices. Introducing an options market then provides informed investors with an opportunity to earn profits from selling options to cover the information acquisition cost, which increases investors' incentive to acquire information. When public information is imprecise, the population of informed investors is high, leading to low option prices. Thus, investors can use cheap options instead of information to reduce risk, which lowers investors' incentives to acquire information.

PROPOSITION **3.4.6.** *When $F_K \in (F_2, F_1)$,*

(1) When $F_K > F_4$, introducing an options market increases investors' incentive to acquire information, increases price informativeness, increases the expected asset price and decreases price change volatility.

(2) When $F_K < F_3$, introducing an options market decreases investors' incentive to acquire information, decreases price informativeness, decreases the expected asset price and increases price change volatility.

where F_3 and F_4 are defined as in the Appendix and $F_3 < F_4$.

3.4.3 Effect of additional trading rounds

As argued by Brennan and Cao (1996), additional trading rounds have a similar effect to derivatives in improving investors' welfare. However, whether additional trading rounds have similar effects to an option market in affecting investors' incentive to acquire information is unclear. This issue is important because it has important implications on the after-hour or round-the-clock trading, which is associated high operational costs. This section formally addresses this question.

We assume that introducing additional trading rounds increases trading time from t to s, where $s > t$. The increase in trading time from t to s can be interpreted as after-hour or round-the-clock trading. Additional trading time increases the number of time intervals and the last time interval is $[(L-1)z, Lz)$, where L is the largest integer satisfying $Lz \leq s$.[14] Before

[14]This modeling about after-hour or round-the-clock trading is similar to Brennan and Cao (1996). The only

each trading round, a public signal is released. The public signal before trading round j is

$$S_{c,j} = D + \epsilon_{c,j} \,, \tag{3.24}$$

where $j = 1, 2....K, K+1, ...L$ and $\epsilon_{c,j}$ follows normal distribution $N(0, \frac{1}{c_j z})$. $\epsilon_{c,j}$ is independent cross trading sessions and is independent of noise in investors' private signal. The analysis follows the dynamic model without options. The expected utilities of informed investors and uninformed investors in the economy with additional trading rounds are given by:

The expected utility of informed investors is:

$$U_I = -\frac{1}{\sqrt{Var(D-P_1)B_{i,1}}} \exp[-\gamma W_0 + \gamma C - \frac{\gamma \overline{X}^2}{2B_0^2 Var(D-P_0)}] \times \prod_{j=2}^{j=L} \frac{1}{\sqrt{1+\frac{c_j z (B_j - B_{i,j})^2}{B_{i,j-1} B_j^2}}} \,, \tag{3.25}$$

and the expected utility of uninformed investors is:

$$U_U = -\frac{1}{\sqrt{Var(D-P_1)B_{U,1}}} \exp[-\gamma W_0 - \frac{\gamma \overline{X}^2}{2B_0^2 Var(D-P_0)}] \times \prod_{j=2}^{j=L} \frac{1}{\sqrt{1+\frac{c_j z (B_j - B_{U,j})^2}{B_{U,j-1} B_j^2}}} \,, \tag{3.26}$$

where $B_{i,j} = h + s + \frac{\omega^2 s^2 q}{\gamma^2} + \sum_{k=1}^{j} c_k z$, $B_{U,j} = h + \frac{\omega^2 s^2 q}{\gamma^2} + \sum_{k=1}^{j} c_k z$ and B_j is defined as above

It is obvious that gain Δ_G from additional trading rounds is $\frac{B_{i,1}}{B_{U,1}} \prod_{j=K+1}^{j=L} \frac{1+\frac{c_j z (B_j - B_{i,j})^2}{B_{i,j-1} B_j^2}}{1+\frac{c_j z (B_j - B_{U,j})^2}{B_{U,j-1} B_j^2}}$. If Δ_G is larger than 1, additional trading rounds provide a greater benefit to informed investors than uninformed ones. Then, investors have a greater incentive to acquire information after the introduction of additional trading rounds. If Δ_G is less than 1, additional trading rounds decrease investors' incentive to acquire information. Proposition 3.4.7 shows that additional trading opportunities encourage more investors to acquire information when the information

difference is that we assume there are approximate continuous trading times, while they assume discrete trading sessions.

cost is high, but discourage investors from acquiring information when this cost is low. Although additional trading opportunities improve the welfare of all investors (Brennan and Cao, 1996) owing to more risk-sharing opportunities, the benefits are asymmetric for different groups. When the population of informed investors is high because of low information acquisition costs, this additional benefit is low for informed investors because of the high competition within this group, whereas the benefit is high for uninformed investors. Thus, investors' incentive to acquire information is diminished and the equilibrium population of informed investors is reduced. When the cost is high, the opposite effect arises.

PROPOSITION **3.4.7.** *When $C \in (C_2, C_1)$,*

(1) When $C > C_4$, introducing additional trading rounds increases the population of informed investors, increases price informativeness, increases the expected asset price, decreases price change volatility and decrease price reactions to public information.

(2) When $C < C_3$, introducing additional trading rounds decreases the population of informed investors, decreases price informativeness, decreases expected asset price, increases price change volatility and increases price reactions to public information.

where C_3 and C_4 are defined in the Appendix and $C_3 < C_4$.

3.5 Discussion

The previous sections focus on the analysis of an options market. Considering some general derivatives is also interesting. Thus, this section provides a further analysis of the derivatives that are modeled by Cao (1999) and shows that our main mechanism is robust to the use of derivatives other than options.

Following Cao (1999), we assume that a derivative asset's payoff is a function of D and P. The specific function is denoted by $g(|D - P|)$, where $g(\cdot)$ is a monotonic function. We assume that informed investor i's demand for this derivative is X_{Gi}, and uninformed investors' demand for this derivative is X_{GU}. Moreover, the equilibrium price of this derivative is denoted by P_G. Following Cao (1999), we obtain the following results regarding investors' demand and equilibrium prices:

$$P = \overline{D} - \frac{\gamma \overline{X}}{B} + \frac{(\omega s + \frac{\omega^2 s^2 q}{\gamma^2})(D - \overline{D} + \frac{\gamma}{\omega s} n)}{B}, \tag{3.27}$$

Informed investor i's demand is:

$$X_i = \frac{E(D|\mathcal{F}_i) - P}{\gamma Var(D|\mathcal{F}_i)}, \tag{3.28}$$

Uninformed investors' demand is:

$$X_U = \frac{E(D|\mathcal{F}_U) - P}{\gamma Var(D|\mathcal{F}_U)}, \tag{3.29}$$

Informed investors' demand for the derivative satisfies:

$$\int_0^{+\infty} (g(y) - P_G) \exp[-B_i y^2/2 - \gamma X_{Gi} g(y)] dy = 0, \tag{3.30}$$

Uninformed investors' demand for the derivative satisfies:

$$\int_0^{+\infty} (g(y) - P_G) \exp[-B_U y^2/2 - \gamma X_{GU} g(y)] dy = 0, \tag{3.31}$$

The market clearing condition is:

$$\omega X_{Gi} + (1 - \omega) X_{GU} = 0 \tag{3.32}$$

Then, the expected utility of informed investors is:

$$U_I^G = U_I \sqrt{\frac{2}{\pi B_i}} \int_0^{+\infty} \exp[-B_I y^2/2 - \gamma (X_{Gi} - P_G) g(y)] dy, \tag{3.33}$$

$$U_U^G = U_U \sqrt{\frac{2}{\pi B_U}} \int_0^{+\infty} \exp[-B_U y^2/2 - \gamma(X_{GU} - P_G)g(y)]dy, \tag{3.34}$$

We know the gain from information with this derivative is $G = (U_U^G/U_I^G)^2$. Because obtaining analytical solutions is difficult, we rely on numerical studies. In the numerical studies, we consider two special cases for $g(\cdot)$: first, $g(y) = y$; second, $g(y) = y^2$. In particular, we compare the gain from information in the economy with and without this derivative, given the population of informed investors. The results are illustrated in Figure 5 and Figure 6.[15] The gain from information is clearly larger in the economy with derivatives than in that without derivatives when the population of informed investors is small. The opposite results is obtained when the population of informed investors is large.

3.6 Conclusions

This paper examines the effect of introducing an options market on investors' incentive to acquire private information and the pricing behaviour of the underlying asset. As a novel finding, this paper demonstrates that introducing an options market increases investors' incentive to acquire private information when the information acquisition cost is high, but decreases their incentive to acquire private information when the cost is low. Consequently, when the information acquisition cost is high, an options market increases the underlying asset's price informativeness, increases the expected asset price, decreases price volatility and decreases market responses to earnings announcements. By contrast, when the information acquisition cost is low, the opposite effects arise. These results can provide a potentially unified theory for the conflicting findings on the effect of options listing in the U.S. market and international markets.

[15] The detailed proof can be found in Cao (1999). We set h=1,s=1, q=1 and $\gamma = 0.5$ in the Figure 5 and Figure 6

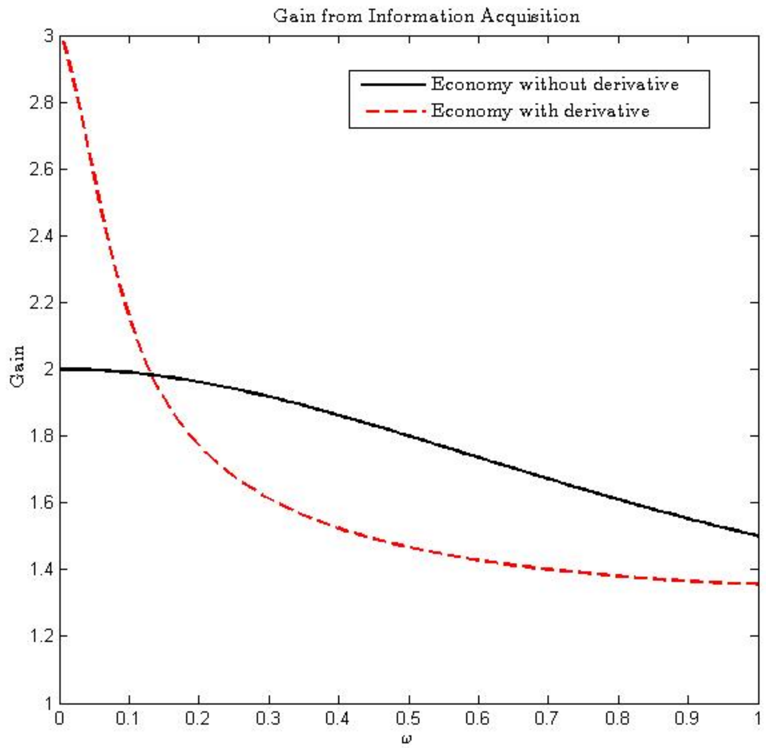

Figure 3.5.1: Gain from Information Acquisition: $g(y) = y$

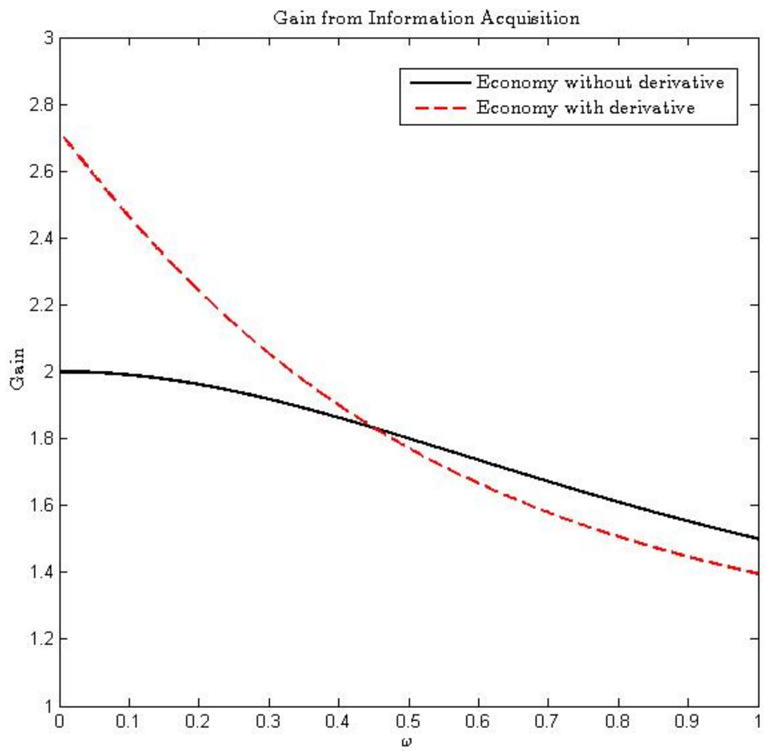

Figure 3.5.2: Gain from Information Acquisition: $g(y) = y^2$

Moreover, this paper also provides some innovative predictions: First, although we do not formally study the effect of options listing on market liquidity, this paper predicts that an options market increases the liquidity of the underlying asset market in an illiquid market and decreases liquidity in a liquid market. When the population of informed investors is high, price is less sensitive to noisy supply, reducing the price impact of the noisy supply. Thus, the market has high liquidity. As shown above, a large population of informed investors leads to a higher supply for options than demand, which is associated with low option prices and low profits from selling options. Introducing an options market decreases investors' incentive to acquire information, which results in lower price informativeness. Consequently, the price impact of the noisy supply increases and market liquidity deteriorates. Opposite effect of options listing arises in illiquid market with a small population of informed investors. This is consistent with the findings by Fedenia and Grammatikos (1992). Second, options listing has stabilizing effect (increasing price informativeness, raising asset price, decreasing price volatility and market reactions to earnings announcements) when the public information is precise, but has destabilization effect (decreasing price informativeness, decreasing asset price, increasing price volatility and market responses to earnings announcements) when the public information is imprecise. third, introducing an options market and implementing after-hour or round-the-clock trading have stabilizing effects (improving informational efficiency, decreasing price volatility) on the underlying assets with high information acquisition costs, such as small firms and firms with low analyst coverage; Fourth, introducing an options market and implementing after-hour or round-the-clock trading have destabilization effects (harming informational efficiency, increasing price volatility) on the underlying assets with low information acquisition costs, such as large or well-known firms and firms with high analyst coverage.

Although previous theoretical studies on derivatives find that introducing derivatives increase asset prices and decrease price volatilities (Cao, 1999 and Massa, 2002), these studies can not reconcile the findings: options listing increases asset prices, decreases price volatility and decreases price reactions to earning announcements in U.S. market before 1980, but yields the opposite effects after 1980. Further, these studies can not explain the findings: options listing tends to have stabilizing effects in emerging markets, such as India, but have destabilization effects in some developed markets, such as Germany. Our results not only explain these conflicting facts regarding the effects of options listing, but also shed new light on debates about whether a derivative market has (de)stabilizing effects on the underlying asset market.

Because we aim to obtain tractable solutions, our model assumes that there are no additional noisy traders after the initial trading round in the dynamic model. However, extending our model to consider time-varying noisy traders may provide an interesting future research avenue. Such as extension would also be useful for studying market liquidity in a general dynamic model. In addition, future research may study the effect of other financial innovations on investors' incentive to acquire information, as in the study by Simsek (2013a,b). More important, we notice that the options have no direct impact on underlying assets because of the assumptions of CARA utility and normal distributions. Although this feature helps to elucidate the effects of options on information acquisition, relaxing these assumptions and analyzing the effects derivatives on asset prices under general utility functions may provide a fruitful research avenue. We leave all of these to further studies.

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3.7 Appendix

This appendix provides all proofs omitted above.

Proof of Proposition 3.2.1.

$$EV_i = -\exp[-\gamma W_0 - \frac{\overline{X}^2}{2\gamma B^2 Var(D-P)}] \times \frac{1}{\sqrt{Var(D-P)B_i}} \tag{3.1}$$

$$EV_U = -\exp[-\gamma W_0 - \frac{\overline{X}^2}{2\gamma B^2 Var(D-P)}] \times \frac{1}{\sqrt{Var(D-P)B_U}} \tag{3.2}$$

The Gain G in the case without derivative security is $\frac{B_i}{B_U}$. In the equilibrium, investors should break even the gain from information acquisition and cost. If $G(0) \leq \exp(2\gamma C)$, the equilibrium fraction of informed investors $\omega^* = 0$. If $G(0) > \exp(2\gamma C) > G(1)$, the equilibrium fraction of informed investors $\omega^* \in (0,1)$ which satisfies $G(\omega^*) = \exp(2\gamma C)$. If $\exp(2\gamma C) \leq G(1)$,the equilibrium fraction of informed investors $\omega^* = 1$.. Therefore, we can get the lemma. And C_{d1} satisfy $G(0) = \exp(2\gamma C_{d1})$ and C_{d2} satisfy $G(1) = \exp(2\gamma C_{d2})$. Since G is a decreasing function of ω, it is obvious that ω^* is a decreasing function with C when $C_{d2} < C < C_{d1}$

□

Proof of Proposition 3.3.1 and Proof of Lemma 3.3.1. To prove that price function and demands are in the equilibrium, we should prove that the market is clearing in the equilibrium and Euler condition holds for the demand of different assets. Given the informed investors and uninformed investors' demand of the risky assset and options, we have the following market clearing condition:

for the stock, we have

$$\omega X_i + (1-\omega)X_U + n = \overline{X} \tag{3.3}$$

It is clear that the price in the proposition clears the market of the risky asset.

for the options, we have

$$\frac{\omega}{\gamma}(B - B_i) + (1 - \omega)(B - B_U) = 0 \tag{3.4}$$

Since $B = h + \omega s + \frac{\omega^2 s^2 q}{\gamma^2} + c$, it is clear that the option market is clearing.

Next we will show the Euler condition holds for the demand of different assets. For informed investor i's final wealth is given by:

$$W_{i,1} = W_{i,0} + X_i(D - P) + \int_0^\infty X_{i,CG}[(D - G)^+ - P_{CG}]dG + \int_{-\infty}^0 X_{i,PG}[(G - D)^+ - P_{PG}]dG \tag{3.5}$$

Given the equilibrium $X_{i,CG}$ and $X_{i,PG}$, we firstly prove that the proposed demand of risky asset satisfies the first order condition for investors' optimization problem.

Due to $D^2 = 2\int_0^\infty (D - G)^+ dG + 2\int_{-\infty}^0 (G - D)^+ dG$, we have

$$W_{i,1} = W_{i,0} + X_i(D - P) + \frac{B - B_i}{\gamma}\frac{D^2}{2} - \frac{B - B_i}{\gamma}(\int_0^\infty P_{CG}dG + \int_{-\infty}^0 P_{PG}]dG) \tag{3.6}$$

Informed investors maximize expected utility

$$\begin{gathered} -E\{\exp(-\gamma W_{i,1})|\mathcal{F}_i\} \\ = -E\{\exp[-\gamma(W_{i,0} + X_i(D - P) + \tfrac{B-B_i}{\gamma}\tfrac{D^2}{2} - \tfrac{B-B_i}{\gamma}(\int_0^\infty P_{CG}dG + \int_{-\infty}^0 P_{PG}]dG))]|\mathcal{F}_i\} \\ = -\frac{1}{\sqrt{1+(B-B_i)\frac{1}{B_i}}}\exp[-\gamma W_{i,0} + (B - B_i)(\int_0^\infty P_{CG}dG + \int_{-\infty}^0 P_{PG}]dG))] + \gamma X_i P - \gamma X_i E(D|\mathcal{F}_i) \\ -\tfrac{B-B_i}{2}E^2(D|\mathcal{F}_i) + \tfrac{1}{2}(\gamma X_i + (B - B_i)E(D|\mathcal{F}_i))^2 \tfrac{1}{B_i}\tfrac{1}{1+(B-B_i)\frac{1}{B_i}}] \end{gathered} \tag{3.7}$$

FOC, we have: $\gamma P - \gamma E(D|\mathcal{F}_i) + (\gamma X_i + (B - B_i)E(D|\mathcal{F}_i))\frac{\gamma}{B} = 0 \Rightarrow X_i = \frac{B_i}{\gamma}(E(D|\mathcal{F}_i) - P) - \frac{1}{\gamma}(B - B_i)P$

This proves that the proposed demand of risky asset satisfies the first order condition for investors' optimization problem. Now we show that proposed demands and prices for the options satisfy the Euler conditions. This means that we need to prove that:

$$E[((D-G)^{+}-P_{CG})\exp(-\gamma W_{i,1})|\mathcal{F}_i]=0 \qquad E[((G-D)^{+}-P_{PG})\exp(-\gamma W_{i,1})|\mathcal{F}_i]=0 \tag{3.8}$$

Since $\int_0^{+\infty}(P-G)N(\sqrt{B}(P-G))dG+\int_0^{+\infty}\frac{1}{\sqrt{B}}\exp(-\frac{B(P-G)^2}{2})dG+\int_\infty^0(G-P)N(\sqrt{B}(G-P))dG+\int_0^{+\infty}\frac{1}{\sqrt{B}}\exp(-\frac{B(G-P)^2}{2})dG$

$=\int_0^{+\infty}(P-G)N(\sqrt{B}(P-G))dG+\int_\infty^0(G-P)N(\sqrt{B}(G-P))dG+\frac{1}{B}$

$=\int_0^{+\infty}(P-G)\int_{-\infty}^{\sqrt{B}(P-G)}\frac{1}{\sqrt{2\pi}}\exp(-\frac{1}{2}x^2)dxdG$

$+\int_{-\infty}^0(G-P)\int_{-\infty}^{\sqrt{B}(G-P)}\frac{1}{\sqrt{2\pi}}\exp(-\frac{1}{2}x^2)dxdG+\frac{1}{B}$

$=\int_{-\infty}^{P\sqrt{B}}\frac{1}{\sqrt{2\pi}}\int_0^{P-\frac{x}{\sqrt{B}}}(P-G)dG\exp(-\frac{1}{2}x^2)dx+\int_{-\infty}^{-P\sqrt{B}}\frac{1}{\sqrt{2\pi}}\int_{P+\frac{x}{\sqrt{B}}}^0(G-P)dG\exp(-\frac{1}{2}x^2)dx+\frac{1}{B}$

$=\int_{-\infty}^{P\sqrt{B}}\frac{1}{\sqrt{2\pi}}(\frac{P^2}{2}-\frac{1}{2B}x^2)\exp(-\frac{1}{2}x^2)dx+\int_{-\infty}^{-P\sqrt{B}}\frac{1}{\sqrt{2\pi}}(\frac{P^2}{2}-\frac{1}{2B}x^2)\exp(-\frac{1}{2}x^2)dx+\frac{1}{B}$

$=\frac{P^2}{2}N(P\sqrt{B})+\frac{P^2}{2}N(-P\sqrt{B})-\frac{1}{2B}\int_{-\infty}^{P\sqrt{B}}\frac{1}{\sqrt{2\pi}}x^2\exp(-\frac{1}{2}x^2)dx$

$-\frac{1}{2B}\int_{-\infty}^{-P\sqrt{B}}\frac{1}{\sqrt{2\pi}}x^2\exp(-\frac{1}{2}x^2)dx+\frac{1}{B}$

$=\frac{P^2}{2}+\frac{1}{2B}$

This indicates that $\int\limits_0^\infty P_{CG}dG+\int\limits_{-\infty}^0 P_{PG}]dG=\frac{1}{2}(\frac{1}{B}+P^2)$. Then we put $\int\limits_0^\infty P_{CG}dG+\int\limits_{-\infty}^0 P_{PG}]dG=\frac{1}{2}(\frac{1}{B}+P^2)$ into final wealth, we have:

$$\begin{aligned}W_{i,1}&=W_{i,0}+[\tfrac{B_i}{\gamma}(E(D|\mathcal{F}_i)-P)-\tfrac{1}{\gamma}(B-B_i)P](D-P)+\tfrac{B-B_i}{\gamma}\tfrac{D^2}{2}-\tfrac{B-B_i}{\gamma}\tfrac{1}{2}(\tfrac{1}{B}+P^2)\\&=W_{i,0}+\tfrac{B_i}{\gamma}(E(D|\mathcal{F}_i)-P)(D-P)+\tfrac{B-B_i}{\gamma}\tfrac{D^2}{2}-\tfrac{B-B_i}{\gamma}\tfrac{1}{2}(\tfrac{1}{B}+P^2)\\&=W_{i,0}+\tfrac{B_i}{\gamma}(E(D|\mathcal{F}_i)-P)(D-P)+\tfrac{B-B_i}{2\gamma}[(D-P)^2-\tfrac{1}{B}]\end{aligned} \tag{3.9}$$

For the Euler Equation $E[((D-G)^{+}-P_{CG})\exp(-\gamma W_{i,1})|\mathcal{F}_i]=0$, we have $E[\exp(-\gamma W_{i,1})|\mathcal{F}_i]=-\frac{1}{\sqrt{\frac{B}{B_i}}}\exp[-\gamma W_{i,0}+\frac{B-B_i}{2B}-\frac{B_i}{2}(E(D|\mathcal{F}_i)-P)^2]$. Let $x=D-P$, $\mu=E(D|\mathcal{F}_i)-P$, $B_i=Var(D-P|\mathcal{F}_i)$

$E[(D-G)^{+}\exp(-\gamma W_{i,1})|\mathcal{F}_i] = \int_{G-P}^{+\infty}[x-(G-P)]\exp[-\gamma W_{i,0}+\frac{B-B_i}{2B}]\frac{\sqrt{B_i}}{\sqrt{2\pi}}\exp(-B_i\mu x - \frac{B-B_i}{2}x^2 - \frac{B_i(x-\mu)^2}{2})dx$

$= \int_{G-P}^{+\infty}[x-(G-P)]\exp[-\gamma W_{i,0}+\frac{B-B_i}{2B}-\frac{B_i}{2}(E(D|\mathcal{F}_i)-P)^2]\frac{\sqrt{B_i}}{\sqrt{2\pi}}\exp(-\frac{B}{2}x^2)dx$

$=\exp[-\gamma W_{i,0}+\frac{B-B_i}{2B}-\frac{B_i}{2}(E(D|\mathcal{F}_i)-P)^2][\int_{G-P}^{+\infty}x\frac{\sqrt{B_i}}{\sqrt{2\pi}}\exp(-\frac{B}{2}x^2)dx+\int_{G-P}^{+\infty}(P-G)\frac{\sqrt{B_i}}{\sqrt{2\pi}}\exp(-\frac{B}{2}x^2)dx]$

$= \exp[-\gamma W_{i,0}+\frac{B-B_i}{2B}-\frac{B_i}{2}(E(D|\mathcal{F}_i)-P)^2][(P-G)\frac{1}{\sqrt{\frac{B}{B_i}}}N(\sqrt{B}(P-G))+\frac{\sqrt{B_i}}{B}\exp(-\frac{B(P-G)^2}{2})]$

(where $\int_{G-P}^{+\infty}(P-G)\frac{\sqrt{B_i}}{\sqrt{2\pi}}\exp(-\frac{B}{2}x^2)dx = \int_{\sqrt{B}(G-P)}^{+\infty}(P-G)\frac{\sqrt{B_i}}{\sqrt{2\pi}\sqrt{B}}\exp(-\frac{y^2}{2})dy = (P-G)\frac{1}{\sqrt{\frac{B}{B_i}}}[1-N(\sqrt{B}(G-P))] = (P-G)\frac{1}{\sqrt{\frac{B}{B_i}}}N(\sqrt{B}(P-G))$

$\int_{G-P}^{+\infty}x\frac{\sqrt{B_i}}{\sqrt{2\pi}}\exp(-\frac{B}{2}x^2)dx = \int_{\sqrt{B}(G-P)}^{+\infty}y\frac{\sqrt{B_i}}{\sqrt{2\pi}B}\exp(-\frac{y^2}{2})dy = \frac{\sqrt{B_i}}{B}\exp(-\frac{B(G-P)^2}{2}))$

From the Euler Condition, we have $P_{CG} = (P-G)N(\sqrt{B}(P-G)) + \frac{1}{\sqrt{B}}\exp(-\frac{B(P-G)^2}{2})$. This verifies the proposed the price function in the proposition. Following the similar procedure, it is obvious that the price function of put option takes the form in the propositions. Following the similar procedure, we can prove that uninformed investors' demand functions of risky asset and options take the forms in the proposition. □

Proof of Lemma 3.3.2. In the case with derivative security, for the informed investors' utility, we put $X_i = \frac{E(D|\mathcal{F}_i)-P}{Var(D|\mathcal{F}_i)}$ and $X_{i,G} = \frac{1}{2\gamma}(\frac{1}{P_G} - \frac{1}{Var(D|\mathcal{F}_i)})$ into

$$-\frac{1}{\sqrt{1+2\gamma X_{i,G}Var(D|\mathcal{F}_i)}}\exp[-\gamma(W_{i,0}-X_{i,G}P_G) + \frac{Var(D|\mathcal{F}_i)[\gamma X_d - \frac{E(D|\mathcal{F}_i)-P}{Var(D|\mathcal{F}_i)}]^2}{2(1+2\gamma X_{d,G}Var(D|\mathcal{F}_i))} - \frac{1}{2}\frac{(E(D|\mathcal{F}_i)-P)^2}{Var(D|\mathcal{F}_i)}] \tag{3.10}$$

$\Leftrightarrow$

$$-\frac{1}{\sqrt{\frac{Var(D-P)}{P_G}}}\exp[-\gamma(W_{i,0}+\frac{(1-\omega)s}{2B}-\frac{\overline{X}^2}{2\gamma B^2 Var(D-P)}) \tag{3.11}$$

we follow the same calculation, we can get the uninformed investors' utility as

$$-\frac{1}{\sqrt{\frac{Var(D-P)}{P_G}}}\exp[-\gamma(W_{i,0}-\frac{\omega s}{2B})--\frac{\overline{X}^2}{2\gamma B^2 Var(D-P)}] \tag{3.12}$$

Therefore, the gain G in the case with derivative security is $\exp(\frac{s}{B})$

□

Proof of Proposition 3.3.2. Whether the introduction of derivative security increase the fraction of informed investors depends on $\exp(\frac{s}{B})-\frac{B_i}{B_U}$. We can transform $\exp(\frac{s}{B})-\frac{B_i}{B_U}$ into $\frac{s}{B}-\ln(\frac{B_i}{B_U})$. From the proof of Proposition 3.4.2, we know that $\sum_{j=2}^{T}(\frac{1}{B_{j-1}}-\frac{1}{B_j})\frac{(B_j-B_{i,j})^2/B_{j-1}}{1+(B_{i,j-1}-B_{j-1})/B_{j-1}}$ $=\int_{\frac{1}{B_T}}^{\frac{1}{B_1}}\frac{(1-\omega)^2s^2t}{1+(1-\omega)st}dt=(1-\omega)s(\frac{1}{B_1}-\frac{1}{B_T})-[\ln(1+\frac{(1-\omega)s}{B_1})-\ln(1+\frac{(1-\omega)s}{B_T})]$

when $B_T\to\infty$, we have $\sum_{j=2}^{\infty}(\frac{1}{B_{j-1}}-\frac{1}{B_j})\frac{(B_j-B_{i,j})^2/B_{j-1}}{1+(B_{i,j-1}-B_{j-1})/B_{j-1}}=\frac{(1-\omega)s}{B_1}-\ln(1+\frac{(1-\omega)s}{B_1})$

Furthermore, $\sum_{j=2}^{\infty}(\frac{1}{B_{j-1}}-\frac{1}{B_j})\frac{(B_j-B_{U,j})^2/B_{j-1}}{1+(B_{i,j-1}-B_{j-1})/B_{j-1}}=\frac{-\omega s}{B_1}-\ln(1+\frac{-\omega s}{B_1})$

It is obvious that

$$\begin{gathered}\frac{s}{B_1}-\ln(\frac{B_i}{B_U})\\ =\sum_{j=2}^{\infty}(\frac{1}{B_{j-1}}-\frac{1}{B_j})\frac{(B_j-B_{i,j})^2/B_{j-1}}{1+(B_{i,j-1}-B_{j-1})/B_{j-1}}-\sum_{j=2}^{\infty}(\frac{1}{B_{j-1}}-\frac{1}{B_j})\frac{(B_j-B_{i,j})^2/B_{j-1}}{1+(B_{i,j-1}-B_{j-1})/B_{j-1}}\end{gathered} \tag{3.13}$$

If $\frac{(B_j-B_{i,j})^2/B_{j-1}}{1+(B_{i,j-1}-B_{j-1})/B_{j-1}}>\frac{(B_j-B_{i,j})^2/B_{j-1}}{1+(B_{i,j-1}-B_{j-1})/B_{j-1}}$, then $\exp(\frac{s}{B})-\frac{B_i}{B_U}>0$ and introduction of derivative security will increase the fraction of informed investors.

If $\frac{(B_j-B_{i,j})^2/B_{j-1}}{1+(B_{i,j-1}-B_{j-1})/B_{j-1}}<\frac{(B_j-B_{i,j})^2/B_{j-1}}{1+(B_{i,j-1}-B_{j-1})/B_{j-1}}$, then $\exp(\frac{s}{B})-\frac{B_i}{B_U}<0$ and introduction of derivative security will decrease the fraction of informed investors. Following the proof the Proposition 3.4.7, we can get results in this proposition. □

Proof of Proposition 3.3.3. Price informativeness $I=\frac{\omega^2s^2q}{\gamma^2}$. It is obvious that I is an increasing function of *omega*. Following Proposition 3.3.2, this proposition can be derived directly. $\square$

Proof of Proposition 3.3.4. (a)The expected asset price is $\overline{D} - \frac{\overline{X}}{B_j}$. Since $B = h + \omega s + \frac{\omega^2 s^2}{\gamma^2} q + c$, thus B is an increasing function of ω and then we can conclude that expected asset price is also an increasing function of ω. (b) The market response to public information is $\frac{c}{B}$ which is a decreasing function of ω. (c) The price change volatility is

$$Var(D - P) = \frac{1}{B} + \frac{\omega s + \gamma^2 q^{-1}}{B^2} \tag{3.14}$$

The derivative of $Var(D - P)$ with ω is:

$$-\frac{s + \frac{2\omega s^2}{\gamma^2} q}{B^2} + \frac{s}{B^2} - \frac{2(\omega s + \gamma^2 q^{-1})}{B^3}(s + \frac{2\omega s^2}{\gamma^2} q) < 0 \tag{3.15}$$

So we can conclude that first derivative of $Var(D - P)$ with ω is negative and thus the price change volatility is a decreasing function of ω. Therefore, we can get the results in the proposition. $\square$

Proof of Proposition 3.3.5 and Lemma 3.3.3 . As shown in the analysis, $V_o = \int_0^\omega |(B_i - B)|di + (1 - \omega)|B - B_U| = 2\omega(1 - \omega)s$. When $\omega \leq \frac{1}{2}$, V_o is an increasing function of ω. As proved in Lemma 4.1, in (C_{d2}, C_{d1}), the equilibrium fraction of

informed investors is a decreasing function of information acquisition cost and there is unique corresponding information acquisition cost C_M which induces the fraction of informed investors to be $\frac{1}{2}$. This means that when $C > C_M$, V_o is a decreasing function of C; when $C < C_M$, V_o is a increasing function of C. This completes the proof. $\square$

Proof of Proposition 3.4.1 and Lemma 3.4.1 . We use backward induction to prove the linear price function and investors' demand. This means that we firstly prove that the P_j, $X_{i,j}$ and $X_{U,j}$ follows the proposition, and then we prove that P_{j-1}, $X_{i,j-1}$ and $X_{U,j-1}$ follows the proposition. In the economy of T trading sessions (where $T = N$ or $N+M$), informed investor i's final wealth $W_{i,F} = W_0 + X_{i,1}(P_2 - P_1) + X_{i,2}(P_3 - P_2) + X_{i,3}(P_4 - P_3) + \ldots X_{i,T}(D - P_T)$ and liquidity suppliers' final wealth $W_{U,F} = W_0 + X_{U,1}(P_2 - P_1) + X_{U,2}(P_3 - P_2) + X_{U,3}(P_4 - P_3) + \ldots X_{U,T}(D - P_T)$. We also have the dynamic of investors' wealth as: $W_{i,j} = W_{i,j-1} + X_{i,j-1}(P_j - P_{j-1})$ and $W_{U,j} = W_{U,j-1} + X_{U,j-1}(P_j - P_{j-1})$.

At trading round T, informed investor i's information set $\mathcal{F}_{s,T} = \{s_i, s_{c,k}, P_k \;\; k = 1, 2 \ldots T\}$ and the conditional distribution of D in their beliefs are

$$\begin{aligned} E(D|\mathcal{F}_{i,T}) &= \overline{D} + \frac{s(D - \overline{D} + \epsilon_i) + \frac{\omega^2 s^2}{\gamma^2}(D - \overline{D} + \frac{\gamma}{\omega s} n) + \sum_{k=1}^{T} c_k (s_{c,k} - \overline{D})}{h + s + \frac{\omega^2 s^2}{\gamma^2} + \sum_{k=1}^{T} c_k}; &(3.16) \\ Var(D|\mathcal{F}_{i,T}) &= \frac{1}{h + s + \frac{\omega^2 s^2}{\gamma^2} + \sum_{k=1}^{T} c_k} &(3.17) \end{aligned}$$

They try to maximize their utility over the final wealth:

$$EV_{i,T} = \max_{X_{i,T}} - \exp[-\gamma\{W_{i,T} + X_{i,T}(D - P_T)\}] \tag{3.18}$$

So informed investor i's optimal demand is:$X_{i,T} = \frac{E(D|\mathcal{F}_{i,T}) - P_T}{\gamma Var(D|\mathcal{F}_{i,T})}$. We substitute $X_{i,T}$ into the above equation, we have liquidity demanders' equivalent utility is:

$$EV_{i,T} = -\exp[-\gamma\{W_{i,T-1} + \frac{[E(D|\mathcal{F}_{i,T}) - P_T]^2}{2\gamma Var(D|\mathcal{F}_{i,T})}\}] \tag{3.19}$$

Uninformed investors' information set $\mathcal{F}_{U,T} = \{s_{c,k}, P_k \;\; k = 1, 2 \ldots T\}$ and the conditional distribution of D in their beliefs are

$$\begin{aligned} E(D|\mathcal{F}_{U,T}) &= \overline{D} + \frac{\frac{\omega^2 s^2}{\gamma^2}(D - \overline{D} + \frac{\gamma}{\omega s} n) + \sum_{k=1}^{T} c_k (s_{c,k} - \overline{D})}{h + s + \frac{\omega^2 s^2}{\gamma^2} + \sum_{k=1}^{T} c_k}; &(3.20) \\ Var(D|\mathcal{F}_{U,T}) &= \frac{1}{h + \frac{\omega^2 s^2}{\gamma^2} + \sum_{k=1}^{T} c_k} &(3.21) \end{aligned}$$

They try to maximize their utility over the final wealth:

$$EV_{U,T} = \max_{X_{U,T}} - \exp[-\gamma\{W_{D,T} + X_{U,T}(D - P_T)\}] \tag{3.22}$$

So uninformed investors' optimal demand is:$X_{U,T} = \frac{E(D|\mathcal{F}_{U,T})-P_T}{\gamma Var(D|\mathcal{F}_{U,T})}$. We substitute $X_{U,T}$ into the above equation, we have uninformed investors' equivalent utility is:

$$EV_{U,T} = -\exp[-\gamma\{W_{U,T-1} + \frac{[E(D|\mathcal{F}_{U,T}) - P_T]^2}{2\gamma Var(D|\mathcal{F}_{U,T})}\}] \tag{3.23}$$

In the market clearing condition: $\int\limits_{i=0}^{\omega} X_{i,T}di + (1-\omega)X_{U,T} + n = \overline{X}$. We can get the price function as the description in the proposition.

Now we turn to the trading round $T-1$. Given the price and optimal demands in trading round $T-1$. For the informed investors, they maximize utility

$$EV_{i,T-1} = -\max_{X_{i,T-1}} \exp[-\gamma\{W_{i,T-2} + X_{i,T-1}(P_T - P_{T-1}) + \frac{[E(D|\mathcal{F}_{i,T}) - P_T]^2}{2\gamma Var(D|\mathcal{F}_{i,T})}\}] \tag{3.24}$$

Let $B_j = h+\omega s+\frac{\omega^2 s^2}{\gamma^2}+\sum_{k=1}^{j} c_k$, $K_{i,j} = h+s+\frac{\omega^2 s^2}{\gamma^2}+\sum_{k=1}^{j} c_k$ and $K_{U,j} = h+\frac{\omega^2 s^2}{\gamma^2}+\sum_{k=1}^{j} c_k$

In the conjecture price, $P_T = \frac{B_{T-1}P_{T-1}+c_T s_{c,T}}{B_T} \Rightarrow P_T - P_{T-1} = \frac{c_T(s_{c,T}-P_{T-1})}{B_T}$

$E(D|\mathcal{F}_{i,T}) = \frac{B_{i,T-1}E(D|\mathcal{F}_{i,T-1})+c_T s_{c,T}}{B_{i,T}}$

$\Rightarrow E(D|\mathcal{F}_{i,T}) - P_T = \frac{B_{i,T-1}E(D|\mathcal{F}_{i,T-1})+P_T B_T - B_{T-1}P_{T-1}}{B_{i,T}} - P_T$

$= (\frac{B_T}{B_{i,T}} - 1)(P_T - P_{T-1}) + \frac{B_{i,T-1}}{B_{i,T}}[E(D|\mathcal{F}_{i,T-1}) - P_{T-1}]$

So in informed investor i's belief:$E(P_T - P_{T-1}|\mathcal{F}_{i,T-1}) = \frac{c_T(E(D|\mathcal{F}_{i,T-1})-P_{T-1})}{B_T}$and $Var(P_T - P_{T-1}|\mathcal{F}_{i,T-1}) = \frac{c_T^2(\frac{1}{B_{i,T-1}}+\frac{1}{c_T})}{B_T^2} = \frac{c_T B_{i,T}}{B_{i,T-1}B_T^2}$

$$
\begin{aligned}
&-\max_{X_{i,T-1}} \exp[-\gamma\{W_{i,T-2}+X_{i,T-1}(P_T-P_{T-1})\\
&+\frac{B_{i,T}(\frac{B_T}{B_{i,T}}-1)^2(P_T-P_{T-1})^2}{2\gamma}\\
&+\frac{B_{i,T}(\frac{B_T}{B_{i,T}}-1)(P_T-P_{T-1})\frac{B_{i,T-1}}{B_{i,T}}[E(D|\mathcal{F}_{i,T-1})-P_{T-1}]}{\gamma}\\
&+\frac{B_{i,T}\frac{B_{i,T-1}^2}{B_{i,T}^2}[E(D|\mathcal{F}_{i,T-1})-P_{T-1}]^2}{2\gamma}\}]\\
&\Leftrightarrow -\max_{X_{d,\tau_{T-2}}} \frac{1}{\sqrt{1+B_{i,T}((\frac{B_T}{B_{i,T}}-1)^2\frac{c_T B_{i,T}}{B_{i,T-1}B_T^2}}}\exp[-\gamma\{W_{i,T-2}\\
&+(X_{i,T-1}+\frac{(\frac{B_T}{B_{i,T}}-1)B_{i,T-1}[E(D|\mathcal{F}_{i,T-1})-P_{T-1}]}{\gamma})\frac{c_T(E(D|\mathcal{F}_{i,T-1})-P_{T-1})}{B_T}\\
&+\frac{B_{i,T}\frac{B_{i,T-1}^2}{B_{i,T}^2}[E(D|\mathcal{F}_{i,T-1})-P_{T-1}]^2}{2\gamma}\\
&-\frac{1}{2}\frac{\begin{matrix}[\gamma X_{i,T-1}+(\frac{B_T}{B_{i,T}}-1)B_{i,T-1}[E(D|\mathcal{F}_{i,T-1})-P_{T-1}]\\ +B_{i,T}(\frac{B_T}{B_{i,T}}-1)^2\frac{c_T(E(D|\mathcal{F}_{i,T-1})-P_{T-1})}{B_T}]^2\end{matrix}}{\gamma(1+B_{i,T}(\frac{B_T}{B_{i,T}}-1)^2\frac{c_T B_{i,T}}{B_{i,T-1}B_T^2})}\\
&*\frac{c_T B_{i,T}}{B_{i,T-1}B_T^2}\}]
\end{aligned}
\tag{3.25}
$$

FOC, we have:

$$
\begin{aligned}
&\frac{c_T(E(D|\mathcal{F}_{i,T-1})-P_{T-1})}{B_T}-\frac{\frac{c_T B_{i,T}}{B_{i,T-1}B_T^2}}{1+B_{i,T}(\frac{B_T}{B_{i,T}}-1)^2\frac{c_T B_{i,T}}{B_{i,T-1}B_T^2}}\\
&\times\{[\gamma(X_{i,T-1}+(\frac{B_T}{B_{i,T}}-1)B_{i,T-1}[E(D|\mathcal{F}_{i,T-1})-P_{T-1}]\\
&+B_{i,T}(\frac{B_T}{B_{i,T}}-1)^2\frac{c_T(E(D|\mathcal{F}_{i,T-1})-P_{T-1})}{B_T}]\}\\
&=0
\end{aligned}
\tag{3.26}
$$

$\Rightarrow \gamma(X_{i,T-1}+(\frac{B_T}{B_{i,T}}-1)B_{i,T-1}[E(D|\mathcal{F}_{i,T-1})-P_{T-1}]+B_{i,T}(\frac{B_T}{B_{i,T}}-1)^2\frac{c_T(E(D|\mathcal{F}_{i,T-1})-P_{T-1})}{B_T}]$

$=\frac{(E(D|\mathcal{F}_{i,T-1})-P_{T-1})B_{i,T-1}B_T}{B_{i,T}}(1+B_{i,T}(\frac{B_T}{B_{i,T}}-1)^2\frac{c_T B_{i,T}}{B_{i,T-1}B_T^2})$

$\Rightarrow X_{i,T-2}=\frac{(E(D|\mathcal{F}_{i,T-1})-P_{T-1})B_{i,T-1}}{\gamma}$ Since $Var(D|\mathcal{F}_{i,T-1})=\frac{1}{B_{i,T-1}}$. We have proved that $X_{i,T-2}$ is the same as the proposition.

Then we put $X_{i,T-1}$ into the utility function, we have:

$$EV_{i,T-1} = \begin{array}{c} -\frac{1}{\sqrt{1+\frac{c_T(B_T-B_{i,T})^2}{B_{i,T-1}B_T^2}}} \\ \times \exp[-\gamma\{W_{i,T-2}+\frac{[E(D|\mathcal{F}_{i,T-1})-P_{T-1}]^2}{2\gamma Var(D|\mathcal{F}_{i,T-1})}\}] \end{array} \tag{3.27}$$

For the uninformed investors' demands, we can follow the same methodology and just replace $B_{i,j}$ with $B_{U,j}$. And we have the uninformed investors' demand $X_{U,T-1} = \frac{(E(D|\mathcal{F}_{U,T-1})-P_{T-1})B_{U,T-1}}{\gamma}$ Since $Var(D|\mathcal{F}_{U,T-1}) = \frac{1}{B_{U,T-1}}$. We have proved that $X_{s,\tau_{T-2}}$ is the same as the proposition. The market clearing condition, we can get the price function $P_{1,\tau_{T-2}}$ as the proposition. We can have liquidity suppliers' expected utility is

$$EV_{U,T-1} = -\frac{1}{\sqrt{1+\frac{c_T(B_T-B_{U,T})^2}{B_{U,T-1}B_T^2}}}\exp[-\gamma\{W_{U,T-2}+\frac{[E(D|\mathcal{F}_{U,T-1})-P_{T-1}]^2}{2\gamma Var(D|\mathcal{F}_{U,T-1})}\}] \tag{3.28}$$

Proceeding recursively, we can get the price functions and demands as the propositions. This complete the proof of the proposition.

For the lemma, we proceed recursively and can get that:

$$EV_i = \begin{array}{c} -\exp[-\gamma\{W_0+\frac{[\ E(D|\mathcal{F}_{i,1})\ -P_1]^2}{2\gamma Var(D|\mathcal{F}_{i,1})}\}] \\ \times\prod_{j=1}^{j=T}\frac{1}{\sqrt{1+\frac{c_j(B_j-B_{i,j})^2}{B_{i,j-1}B_j^2}}} \end{array} = \begin{array}{c} -\exp[-\gamma W_0-\frac{\gamma\overline{X}^2}{2B_0^2Var(D-P_0)}] \\ \times\frac{1}{\sqrt{Var(D-P_1)B_{i,1}}}\prod_{j=1}^{j=T}\frac{1}{\sqrt{1+\frac{c_j(B_j-B_{i,j})^2}{B_{i,j-1}B_j^2}}} \end{array} \tag{3.29}$$

$$EV_U = \begin{array}{c} -\exp[-\gamma\{W_0+\frac{[\ E(D|\mathcal{F}_{U,1})\ -P]^2}{2\alpha Var(D|\mathcal{F}_{U,1})}\}] \\ \times\prod_{j=1}^{j=T}\frac{1}{\sqrt{1+\frac{c_j(B_j-B_{U,j})^2}{B_{U,j-1}B_j^2}}} \end{array} = \begin{array}{c} -\exp[-\gamma W_0-\frac{\gamma\overline{X}^2}{2B_0^2Var(D-P_0)}] \\ \times\frac{1}{\sqrt{Var(D-P_1)B_{U,1}}}\prod_{j=1}^{j=T}\frac{1}{\sqrt{1+\frac{c_j(B_j-B_{U,j})^2}{B_{U,j-1}B_j^2}}} \end{array} \tag{3.30}$$

□

Proof of Proposition 3.4.2 .

$$\begin{aligned}\ln(G) &= \ln(1+\frac{s}{h+\frac{\omega^2 s^2}{\gamma^2}+c_1 z}) + \sum_{j=2}^{N}\ln(1+\frac{c_j z(B_j-B_{i,j})^2}{B_{i,j-1}B_j^2}) - \sum_{j=2}^{N}\ln(1+\frac{c_j z(B_j-B_{U,j})^2}{B_{U,j-1}B_j^2}) \\ &= \ln(1+\frac{s}{h+\frac{\omega^2 s^2}{\gamma^2}+c_1 z}) + \sum_{j=2}^{N}\frac{c_j z(B_j-B_{i,j})^2}{B_{i,j-1}B_j^2} - \sum_{j=2}^{N}\frac{c_j z(B_j-B_{U,j})^2}{B_{U,j-1}B_j^2}\end{aligned} \tag{3.31}$$

In the above equation, we have

$$\begin{aligned}\frac{c_j z(B_j-B_{i,j})^2}{B_{i,j-1}B_j^2} &= (\frac{1}{B_{j-1}}-\frac{1}{B_j})\frac{(B_j-B_{i,j})^2/B_{j-1}}{1+(B_{i,j-1}-B_{j-1})/B_{j-1}}\frac{B_{j-1}}{B_j} = \\ = (\frac{1}{B_{j-1}}-\frac{1}{B_j})\frac{(B_j-B_{i,j})^2/B_{j-1}}{1+(B_{i,j-1}-B_{j-1})/B_{j-1}} &- (\frac{1}{B_{j-1}}-\frac{1}{B_j})^2\frac{(B_j-B_{i,j})^2}{1+(B_{i,j-1}-B_{j-1})/B_{j-1}} \\ &= (\frac{1}{B_{j-1}}-\frac{1}{B_j})\frac{(B_j-B_{i,j})^2/B_{j-1}}{1+(B_{i,j-1}-B_{j-1})/B_{j-1}}\end{aligned} \tag{3.32}$$

Therefore,

$$\begin{aligned}&\sum_{j=2}^{T}(\frac{1}{B_{j-1}}-\frac{1}{B_j})\frac{(B_j-B_{i,j})^2/B_{j-1}}{1+(B_{i,j-1}-B_{j-1})/B_{j-1}} \\ &= \int_{\frac{1}{B_T}}^{\frac{1}{B_1}}\frac{(1-\omega)^2 s^2 t}{1+(1-\omega)st}dt \\ &= (1-\omega)s(\frac{1}{B_1}-\frac{1}{B_T}) - [\ln(1+\frac{(1-\omega)s}{B_1}) - \ln(1+\frac{(1-\omega)s}{B_T})]\end{aligned} \tag{3.33}$$

Similarly, we will have

$$\sum_{j=2}^{T}(\frac{1}{B_{j-1}}-\frac{1}{B_j})\frac{(B_j-B_{U,j})^2/B_{j-1}}{1+(B_{U,j-1}-B_{j-1})/B_{j-1}} = \int_{\frac{1}{B_T}}^{\frac{1}{B_1}}\frac{\omega^2 s^2 t}{1-\omega st}dt = -\omega s(\frac{1}{B_1}-\frac{1}{B_T}) - [\ln(1-\frac{\omega s}{B_1}) - \ln(1-\frac{\omega s}{B_T})]$$

Therefore, $\ln(G) = s(\frac{1}{B_1}-\frac{1}{B_T}) + \ln(1+\frac{s}{B_{U,T}})$ and it is obvious that $\ln(G)$ is a decreasing function of ω

□

Proof of Corollary 3.4.1 . In the equilibrium, investors should break even the gain from information acquisition and cost. If $G(0) \leq \exp(2\gamma C)$, the equilibrium fraction of informed investors $\omega^* = 0$. If $G(0) > \exp(2\gamma C) > G(1)$, the equilibrium fraction of informed investors $\omega^* \in (0,1)$ which satisfies $G(\omega^*) = \exp(2\gamma C)$. If $\exp(2\gamma C) \leq G(1)$,the equilibrium fraction of informed investors $\omega^* = 1.$. Therefore, we can get the corollary. And C_1 satisfy $G(0) = \exp(2\gamma C_1)$ and C_2 satisfy $G(1) = \exp(2\gamma C_2)$. Since G is a decreasing function of ω, it is obvious that ω^* is a decreasing function with C when $C_2 < C < C_1$ □

Proof of Corollary 3.4.3 . In the equilibrium, investors should break even the gain from information acquisition and cost. If $G(0) \leq \exp(2\gamma C)$, the equilibrium fraction of informed investors $\omega^* = 0$. If $G(0) > \exp(2\gamma C) > G(1)$, the equilibrium fraction of informed investors $\omega^* \in (0,1)$ which satisfies $G(\omega^*) = \exp(2\gamma C)$. If $\exp(2\gamma C) \leq G(1)$,the equilibrium fraction of informed investors $\omega^* = 1.$. Therefore, we can get the corollary. And F_1 satisfy $G(0) = \exp(2\gamma C)$ and F_2 satisfy $G(1) = \exp(2\gamma C)$. Since G is a decreasing function of ω, it is obvious that ω^* is a decreasing function with F_k when $F_2 < F_k < F_1$ □

Proof of Proposition 3.4.3 and Lemma 3.4.2 . There are several steps to prove that the proposition holds.

Step 1: Following the similar procedure in the proof of Proposition 4.1, we know that $\int\limits_0^\infty P_{CG,j}dG + \int\limits_{-\infty}^0 P_{PG,j}dG = \frac{1}{2}(\frac{1}{B_j} + P_j^2)$

Step 2: We want to prove that the expected utility of informed investors and uninformed are as shown in the Lemma 5.1 given the proposed equilibrium in the Proposition 5.1. Given the equilibrium in Proposition 5.1, we have

$$\begin{aligned} W_{i,j+1} = W_{i,j} + [\tfrac{B_{i,j}}{\gamma}(E(D|\mathcal{F}_{i,j}) - P_j) - \tfrac{1}{\gamma}(B_j - B_{i,j})P_j](P_{j+1} - P_j) \\ + \tfrac{B_j - B_{i,j}}{2\gamma}(\tfrac{1}{B_{j+1}} + P_{j+1}^2 - \tfrac{1}{B_j} - P_j^2) \\ = W_{i,j} + \tfrac{B_{i,j}}{\gamma}(E(D|\mathcal{F}_{i,j}) - P_j)(P_{j+1} - P_j) + \tfrac{B_j - B_{i,j}}{2\gamma}(P_{j+1} - P_j)^2 \\ + \tfrac{B_j - B_{i,j}}{2\gamma}(\tfrac{1}{B_{j+1}} - \tfrac{1}{B_j}) \end{aligned} \tag{3.34}$$

where $B_{T+1} = +\infty$ (because final payoff is realized and investors have infinite information precision) and $P_{T+1} = D$

We can use backward induction to prove that

$$EV_{i,j} = -\frac{1}{\sqrt{\frac{B_j}{B_{i,j}}}} \exp[-\gamma W_{i,j} - \frac{[E(D|\mathcal{F}_{i,j}) - P_j]^2}{2Var(D|\mathcal{F}_{i,j})} + \frac{B_j - B_{i,j}}{2}\frac{1}{B_j}] \tag{3.35}$$

For last period, this is true following the proof of Proposition 4.1. Now we assume that this holds for period $j+1$, for period j, we have:

$$EV_{i,j} = -\frac{1}{\sqrt{\frac{B_{j+1}}{B_{i,j+1}}}} E\{\exp[-\gamma W_{i,j+1} - \frac{[E(D|\mathcal{F}_{i,j+1}) - P_{j+1}]^2}{2Var(D|\mathcal{F}_{i,j+1})} + \frac{B_{j+1} - B_{i,j+1}}{2}\frac{1}{B_{j+1}}]\} \tag{3.36}$$

As the proof the Proposition 3.1, we have: $E(D|\mathcal{F}_{i,j+1}) - P_{j+1} = (\frac{B_{j+1}}{B_{i,j+1}} - 1)(P_{j+1} - P_j) + \frac{B_{i,j}}{B_{i,j+1}}[E(D|\mathcal{F}_{i,j}) - P_j]$, $E(P_{j+1} - P_j|\mathcal{F}_{i,j}) = \frac{c_{J+1}(E(D|\mathcal{F}_{i,j}) - P_j)}{B_{j+1}}$ and $Var(P_{j+1} - P_j|\mathcal{F}_{i,j}) = \frac{c_{j+1}B_{i,j+1}}{B_{i,j}B_{j+1}^2}$

So we substitute them into $EV_{i,j}$, we have:

$$EV_{i,j} = -\frac{1}{\sqrt{\frac{B_j}{B_{i,j}}}} E\{\exp[-\gamma W_{i,j} - \frac{[E(D|\mathcal{F}_{i,j+1}) - P_{j+1}]^2}{2\gamma Var(D|\mathcal{F}_{i,j+1})} + \frac{B_j - B_{i,j}}{2}\frac{1}{B_j}]\} \tag{3.37}$$

The ex-ante expected utility for

$$EV_i = -\exp[-\gamma W_0 - \frac{\overline{X}^2}{2\gamma B^2 Var(D-P)}] \times \frac{1}{\sqrt{Var(D-P)B_i}} \tag{3.38}$$

$$EV_U = -\exp[-\gamma W_0 - \frac{\overline{X}^2}{2\gamma B^2 Var(D-P)}] \times \frac{1}{\sqrt{Var(D-P)B_U}} \tag{3.39}$$

Step 3: to simplify the analysis, we want to prove that final wealth $W_{i,F} = W_{i,j} + \frac{B_{i,j}}{\gamma}(E(D|\mathcal{F}_{i,j}) - P_j)(D - P_j) + \frac{B_j - B_{i,j}}{2\gamma}[(D - P_j)^2 - \frac{1}{B_j}]$

Here, we use backward induction to prove that. For the final period T, this is true following the proof of Proposition 4.1. We assume that this is true for period $j+1$. Then we would like to prove this is true for period j. $W_{i,F} = W_{i,j} + \frac{B_{i,j}}{\gamma}(E(D|\mathcal{F}_{i,j}) - P_j)(P_{j+1} - P_j) + \frac{B_j - B_{i,j}}{2\gamma}[(P_{j+1} -$

$P_j)^2 + \frac{1}{B_{j+1}} - \frac{1}{B_j}] + \frac{B_{i,j+1}}{\gamma}(E(D|\mathcal{F}_{i,j+1}) - P_{j+1})(D - P_{j+1}) + \frac{B_{j+1}-B_{i,j+1}}{2\gamma}[(D - P_{j+1})^2 - \frac{1}{B_{j+1}}]$

(Since $E(D|\mathcal{F}_{i,j+1}) - P_{j+1} = (\frac{B_{j+1}}{B_{i,j+1}} - 1)(P_{j+1} - P_j) + \frac{B_{i,j}}{B_{i,j+1}}[E(D|\mathcal{F}_{i,j}) - P_j]$)

$= W_{i,j} + \frac{B_{i,j}}{\gamma}(E(D|\mathcal{F}_{i,j}) - P_j)(P_{j+1} - P_j) + \frac{B_j - B_{i,j}}{2\gamma}(P_{j+1} - P_j)^2 + \frac{B_{i,j+1}}{\gamma}[(\frac{B_{j+1}}{B_{i,j+1}} - 1)(P_{j+1} - P_j) + \frac{B_{i,j}}{B_{i,j+1}}[E(D|\mathcal{F}_{i,j}) - P_j]](D - P_{j+1}) + \frac{B_{j+1}-B_{i,j+1}}{2\gamma}[(D - P_{j+1})^2 - \frac{B_j - B_{i,j}}{2\gamma B_j}$

$= W_{i,j} + \frac{B_{i,j}}{\gamma}(E(D|\mathcal{F}_{i,j}) - P_j)(D - P_j) + \frac{B_j - B_{i,j}}{2\gamma}[(P_{j+1} - P_j)^2 + 2(P_{j+1} - P_j)(D - P_{j+1}) + (D - P_{j+1})^2] - \frac{B_j - B_{i,j}}{2\gamma B_j}$

$= W_{i,j} + \frac{B_{i,j}}{\gamma}(E(D|\mathcal{F}_{i,j}) - P_j)(D - P_j) + \frac{B_j - B_{i,j}}{2\gamma}[(D - P_j)^2 - \frac{1}{B_j}]$

Step 4: we now prove that Euler conditions hold for every period. We also use backward induction to prove it. Euler conditions hold for the final period following the proof of Proposition 4.1. We assume that Euler conditions is true for the period $j+1$. This indicates that the proposed demands and price functions take the forms in the proposition.

We need to prove that

$$E[(P_{j+1} - P_j)\exp(-\gamma W_{i,j+1})|\mathcal{F}_{i,j}] = 0 \tag{3.40}$$

$$E[(P_{CG,j+1} - P_{CG,j})\exp(-\gamma W_{i,j+1})|\mathcal{F}_{i,j}] = 0 \tag{3.41}$$

$$E[(P_{PG,j+1} - P_{PG,j})\exp(-\gamma W_{i,j+1})|\mathcal{F}_{i,j}] = 0 \tag{3.42}$$

Following Cao and Ou-Yang (2009), we have

$$P_{j+1}\exp(-\gamma W_{i,j+1}) = E[D\exp(-\gamma W_{i,F})|\mathcal{F}_{i,j}] \qquad \exp(-\gamma W_{i,j+1}) = E[\exp(-\gamma W_{i,F})|\mathcal{F}_{i,j}] \tag{3.43}$$

$$P_{CG,j+1}\exp(-\gamma W_{i,j+1}) = E[(D-G)^+\exp(-\gamma W_{i,F})|\mathcal{F}_{i,j}] \tag{3.44}$$

$$P_{PG,j+1}\exp(-\gamma W_{i,j+1}) = E[(G-D)^+\exp(-\gamma W_{i,F})|\mathcal{F}_{i,j}] \tag{3.45}$$

This means that we need to prove that:

$$\begin{aligned}E[P_j \exp(-\gamma W_{i,j+1})|\mathcal{F}_{i,j}] = E[P_{j+1} \exp(-\gamma W_{i,j+1})|\mathcal{F}_{i,j}]\\ = E[D \exp(-\gamma W_{i,F})|\mathcal{F}_{i,j}]\\ = E[D \exp(-\gamma(W_{i,j} + \tfrac{B_{i,j}}{\gamma}(E(D|\mathcal{F}_{i,j}) - P_j)(D - P_j)\\ + \tfrac{B_j - B_{i,j}}{2\gamma}[(D - P_j)^2 - \tfrac{1}{B_j}]))|\mathcal{F}_{i,j}]\end{aligned} \tag{3.46}$$

and

$$\begin{aligned}E[P_{CG,j} \exp(-\gamma W_{i,j+1})|\mathcal{F}_{i,j}] = E[P_{CG,j+1} \exp(-\gamma W_{i,j+1})|\mathcal{F}_{i,j}]\\ = E[(D - G)^+ \exp(-\gamma W_{i,F})\mathcal{F}_{i,j}]\\ = E[(D - G)^+ \exp(-\gamma(W_{i,j} + \tfrac{B_{i,j}}{\gamma}(E(D|\mathcal{F}_{i,j}) - P_j)(D - P_j)\\ + \tfrac{B_j - B_{i,j}}{2\gamma}[(D - P_j)^2 - \tfrac{1}{B_j}]))|\mathcal{F}_{i,j}]\end{aligned} \tag{3.47}$$

and

$$\begin{aligned}E[P_{PG,j} \exp(-\gamma W_{i,j+1})|\mathcal{F}_{i,j}] = E[P_{PG,j+1} \exp(-\gamma W_{i,j+1})|\mathcal{F}_{i,j}]\\ = E[(G - D)^+ \exp(-\gamma W_{i,F})\mathcal{F}_{i,j}]\\ = E[(G - D)^+ \exp(-\gamma(W_{i,j} + \tfrac{B_{i,j}}{\gamma}(E(D|\mathcal{F}_{i,j}) - P_j)(D - P_j)\\ + \tfrac{B_j - B_{i,j}}{2\gamma}[(D - P_j)^2 - \tfrac{1}{B_j}]))|\mathcal{F}_{i,j}]\end{aligned} \tag{3.48}$$

The above three equations take similar forms in the proof the Proposition 4.1. Following the similar procedures, the above equations hold for period j. This completes the proof. □

Proof of Proposition 3.4.4 . Following proof or Proposition 3.2, the gain of information acquisition without options is $\ln(G) = s(\frac{1}{B_1} - \frac{1}{B_T}) + \ln(1 + \frac{s}{B_{U,T}})$. When there are infinite trading periods, the aggregate information precision of public information goes to infinity. That is $B_T \to \infty$ and $B_{U,T} \to \infty$, then $\ln(G) \to \frac{s}{B_1}$ which is the gain of information acquisition with options. This indicates that the gain of information acquisition with options is equivalent to the gain of information acquisition with infinite trading periods (This is consistent with the argument in Brennan and Cao (1996)).

Following proof of Proposition 3.3, we know that the gain of information acquisition from additional one trading period for informed investors is higher than uninformed investors when information acquisition cost C is higher than C_4 and the gain of information acquisition from additional one trading period for uninformed investors is higher than informed investors when information acquisition cost C is smaller than C_3. Following the same logic in Proposition 3.3, we complete the proof of this proposition. $\square$

Proof of Proposition 3.4.5 . Price reaction to public information in trading session j is $\frac{c_j z}{B_j}$ which is decreasing function of *omega*. Then this result can be directly derived. $\square$

Proof of Proposition 3.4.6 . Step 1: we characterize F_3 and F_4.

when $\omega > \frac{1}{2}$, since$\frac{c_j z(B_j - B_{i,j})^2}{B_{i,j-1}B_j^2} / (\frac{c_j z(B_j - B_{U,j})^2}{B_{U,j-1}B_j^2}) = \frac{(1-\omega)^2 B_{U,j-1}}{\omega^2 B_{i,j-1}} < 1$, then $\prod_{j=K+1}^{j=L} \frac{1+\frac{c_j z(B_j - B_{i,j})^2}{B_{i,j-1}B_j^2}}{1+\frac{c_j z(B_j - B_{U,j})^2}{B_{U,j-1}B_j^2}} <$ 1

when $\omega < \frac{1}{1+\sqrt{1+\frac{s}{h}}}$, since $\frac{(1-\omega)^2}{\omega^2} > 1 + \frac{s}{h} > \frac{B_{i,j-1}}{B_{U,j-1}}$, then $\prod_{j=K+1}^{j=L} \frac{1+\frac{c_j (B_j - B_{i,j})^2}{B_{i,j-1}B_j^2}}{1+\frac{c_j (B_j - B_{U,j})^2}{B_{U,j-1}B_j^2}} > 1$.

Since G is a decreasing function of ω, there exist F_3 which satisfies $G(\frac{1}{2}, F_3) = \exp(2\gamma C)$ and F_4 which satisfies $G(\frac{1}{1+\sqrt{1+\frac{s}{h}}}, F_4) = \exp(2\gamma C)$. When $F_K > F_4$, ω^* is smaller than $\frac{1}{1+\sqrt{1+\frac{s}{h}}}$ and thus $\prod_{j=K+1}^{j=L} \frac{1+\frac{c_j (B_j - B_{i,j})^2}{B_{i,j-1}B_j^2}}{1+\frac{c_j (B_j - B_{U,j})^2}{B_{U,j-1}B_j^2}} > 1$. Since $G(\omega^*, F_K) = \exp(2\gamma C)$ in the case with K trading sessions, $G(\omega^*) * \prod_{j=K+1}^{j=L} \frac{1+\frac{c_j (B_j - B_{i,j})^2}{B_{i,j-1}B_j^2}}{1+\frac{c_j (B_j - B_{U,j})^2}{B_{U,j-1}B_j^2}} > \exp(2\gamma C)$. It is obvious that the equilibrium fraction of informed investors in the case with additional trading sessions is higher than ω^*.

When $F_K < F_3$, we can get the opposite conclusion following the similar logic.

Step 2: we study option market's effects on asset pricing for different public information precision (a)The expected asset price is $\overline{D} - \frac{\overline{X}}{B_j}$. Since $B_j = h + \omega s + \frac{\omega^2 s^2}{\gamma^2} + \sum_{k=1}^{j} c_k$, thus B_j is an increasing function of ω and then we can conclude that expected asset price is also an increasing function of ω.

(b) The market response to public information is $\frac{c_k}{B_j}$ which is a decreasing function of ω.

(c) The price change volatility is

$$\begin{gathered}Var(P_{j+1}-P_j)=\frac{c_{j+1}^2}{B_{j+1}^2}Var(s_{c,j+1}-P_j)\\=\frac{c_{j+1}^2}{B_{j+1}^2}[\frac{1}{c_{j+1}}+Var(D-P_j)]=\frac{1}{B_j}-\frac{1}{B_{j+1}}\\+\frac{c_{j+1}^2}{B_j^2B_{j+1}^2}(\omega s+\frac{\gamma^2}{q}).\end{gathered} \tag{3.49}$$

The derivative of $Var(P_{j+1}-P_j)$ with ω is:

$$\begin{gathered}-\frac{c_{j+1}}{B_j^2B_{j+1}}(s+2\frac{\omega s^2}{\gamma^2})-\frac{c_{j+1}}{B_jB_{j+1}^2}(s+2\frac{\omega s^2}{\gamma^2})+\frac{c_{j+1}^2s}{B_j^2B_{j+1}^2}\\-\frac{2c_{j+1}^2}{B_j^3B_{j+1}^2}(\omega s+\frac{\gamma^2}{q})(s+2\frac{\omega s^2}{\gamma^2})-\frac{2c_{j+1}^2}{B_j^2B_{j+1}^3}(\omega s+\frac{\gamma^2}{q})(s+2\frac{\omega s^2}{\gamma^2}).\end{gathered} \tag{3.50}$$

since

$$-\frac{c_{j+1}s}{B_j^2B_{j+1}}-\frac{c_{j+1}s}{B_jB_{j+1}^2}+\frac{c_{j+1}^2s}{B_j^2B_{j+1}^2}<0 \tag{3.51}$$

So we can conclude that first derivative of $Var(P_{j+1}-P_j)$ with ω is negative and thus the price change volatility is a decreasing function of ω.

Therefore, we can get the results in the proposition.

$\square$

Proof of Proposition 3.4.7 . There are several steps:

Step 1: we characterize C_3 and C_4 and effects of additional trading opportunities on information acquisition. When $\omega>\frac{1}{2}$, since $\frac{c_jz(B_j-B_{i,j})^2}{B_{i,j-1}B_j^2}/(\frac{c_jz(B_j-B_{U,j})^2}{B_{U,j-1}B_j^2})=\frac{(1-\omega)^2B_{U,j-1}}{\omega^2B_{i,j-1}}<1$, then

$$\prod_{j=K+1}^{j=L}\frac{1+\frac{c_jz(B_j-B_{i,j})^2}{B_{i,j-1}B_j^2}}{1+\frac{c_jz(B_j-B_{U,j})^2}{B_{U,j-1}B_j^2}}<1$$

when $\omega<\frac{1}{1+\sqrt{1+\frac{s}{h}}}$, since $\frac{(1-\omega)^2}{\omega^2}>1+\frac{s}{h}>\frac{B_{i,j-1}}{B_{U,j-1}}$, then $\prod_{j=K+1}^{j=L}\frac{1+\frac{c_j(B_j-B_{i,j})^2}{B_{i,j-1}B_j^2}}{1+\frac{c_j(B_j-B_{U,j})^2}{B_{U,j-1}B_j^2}}>1$.

Since G is a decreasing function of ω, there exist C_3 which satisfies $G(\frac{1}{2}) = \exp(2\gamma C_3)$ and C_4 which satisfies $G(\frac{1}{1+\sqrt{1+\frac{s}{h}}}) = \exp(2\gamma C_4)$. When $C > C_4$, ω^* is smaller than $\frac{1}{1+\sqrt{1+\frac{s}{h}}}$ and thus $\prod_{j=K+1}^{j=L} \frac{1+\frac{c_j(B_j-B_{i,j})^2}{B_{i,j-1}B_j^2}}{1+\frac{c_j(B_j-B_{U,j})^2}{B_{U,j-1}B_j^2}} > 1$. Since $G(\omega^*) = \exp(2\gamma C)$ in the case with K trading sessions, $G(\omega^*) * \prod_{j=K+1}^{j=L} \frac{1+\frac{c_j(B_j-B_{i,j})^2}{B_{i,j-1}B_j^2}}{1+\frac{c_j(B_j-B_{U,j})^2}{B_{U,j-1}B_j^2}} > \exp(2\gamma C)$. It is obvious that the equilibrium fraction of informed investors in the case with additional trading sessions is higher than ω^*.

When $C < C_3$, we can get the opposite conclusion following the similar logic.

Step 2: we study effects on additional trading opportunities on asset pricing. (a)The expected asset price is $\overline{D} - \frac{\overline{X}}{B_j}$. Since $B_j = h + \omega s + \frac{\omega^2 s^2}{\gamma^2} + \sum_{k=1}^{j} c_k$, thus B_j is an increasing function of ω and then we can conclude that expected asset price is also an increasing function of ω.

(b) The market response to public information is $\frac{c_k}{B_j}$ which is a decreasing function of ω.

(c) The price change volatility is

$$\begin{aligned} Var(P_{j+1} - P_j) &= \frac{c_{j+1}^2}{B_{j+1}^2} Var(s_{c,j+1} - P_j) \\ = \frac{c_{j+1}^2}{B_{j+1}^2}[\frac{1}{c_{j+1}} + Var(D - P_j)] &= \frac{1}{B_j} - \frac{1}{B_{j+1}} \\ &+ \frac{c_{j+1}^2}{B_j^2 B_{j+1}^2}(\omega s + \frac{\gamma^2}{q}). \end{aligned} \tag{3.52}$$

The derivative of $Var(P_{j+1} - P_j)$ with ω is:

$$\begin{aligned} &-\frac{c_{j+1}}{B_j^2 B_{j+1}}(s + 2\frac{\omega s^2}{\gamma^2}) - \frac{c_{j+1}}{B_j B_{j+1}^2}(s + 2\frac{\omega s^2}{\gamma^2}) + \frac{c_{j+1}^2 s}{B_j^2 B_{j+1}^2} \\ &- \frac{2c_{j+1}^2}{B_j^3 B_{j+1}^2}(\omega s + \frac{\gamma^2}{q})(s + 2\frac{\omega s^2}{\gamma^2}) - \frac{2c_{j+1}^2}{B_j^2 B_{j+1}^3}(\omega s + \frac{\gamma^2}{q})(s + 2\frac{\omega s^2}{\gamma^2}). \end{aligned} \tag{3.53}$$

since

$$-\frac{c_{j+1}s}{B_j^2 B_{j+1}} - \frac{c_{j+1}s}{B_j B_{j+1}^2} + \frac{c_{j+1}^2 s}{B_j^2 B_{j+1}^2} < 0 \tag{3.54}$$

So we can conclude that first derivative of $Var(P_{j+1} - P_j)$ with ω is negative and thus the price change volatility is a decreasing function of ω.

Therefore, we can get the results in the proposition.

$\square$