H_∞ Controller Design for Networked Predictive Control Systems Based on the Average Dwell-Time Approach

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***Abstract*—This brief focuses on the problem of H_∞ control for a class of networked control systems with time-varying delay in both forward and backward channels. Based on the average dwell-time method, a novel delay-compensation strategy is proposed by appropriately assigning the subsystem or designing the switching signals. Combined with this strategy, an improved predictive controller design approach in which the controller gain varies with the delay is presented to guarantee that the closed-loop system is exponentially stable with an H_∞-norm bound for a class of switching signal in terms of nonlinear matrix inequalities. Furthermore, an iterative algorithm is presented to solve these nonlinear matrix inequalities to obtain a suboptimal minimum disturbance attenuation level. A numerical example illustrates the effectiveness of the proposed method.**

***Index Terms*—Average dwell-time method, H_∞ control, networked control systems (NCSs), predictive control, switched system.**

I. Introduction

NETWORKED control systems (NCSs) is a research area that has emerged in recent years [1]–[8]. A challenging aspect of networked control is that we need to compensate for the negative effects of the network constraints to retain the stability and performance of the system. For this purpose, one technique that has recently been proposed for NCSs is the networked predictive control (NPC) approach, which has been shown to be an effective approach to address this problem [9]–[11]. The main idea is that a sequence of future control predictions is generated at the controller node and transmitted to the actuator node, and then, at the actuator, an algorithm is used to choose the appropriate element from the received control prediction sequence as the actual control input according to the measured network delay. Thus, the effect of the network delay is compensated. The resulting closed-loop system is transformed to a switched system. However, there are three limitations on the work reported in the publications to date. First, system stability has to be subject to arbitrary switching of all subsystems due to the random network delay. Therefore, existing papers are all based on a condition that all subsystems have to possess a common Lyapunov function. This condition is strict because sometimes some values of the network delay are not admissible when considering the system stability of NPC systems. Second, a fixed controller gain is used so that this results in a significant conservative design because the controller gain does not reflect the range of possible delays in the network. Third, the design of the controller gain was not considered.

In this brief, to overcome the first limitation, a novel delay-compensation strategy for NPC systems is proposed by appropriately assigning the subsystem or designing the switching signal. This strategy works, even if there exist unstable subsystems, because they can be omitted in the assigning process. As for the remaining stable subsystems, it is not necessary to have a common Lyapunov function, but the overall system still may be stable under some suitable switching signals. The average dwell-time method is an effective tool for finding such switching signals [12]–[15]. To overcome the second and third limitations, an improved predictive controller scheme is designed in which the controller gain varies with the delay. In contrast with some existing references, which are based on the fixed controller gain approach, these varying feedback controller gains can lead to less conservative results. Based on the average dwell-time technique and this improved predictive controller scheme, the corresponding closed-loop system is exponentially stable with an H_∞-norm bound. Moreover, an iterative algorithm is presented to design the desired controllers with a suboptimal minimum disturbance attenuation level.

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II. Preliminaries and Problem Formulation

The NPC system structure is shown in Fig. 1. It includes two parts, namely, a control prediction generator (CPG) at the controller side and a network delay compensator (NDC) at the actuator side. The plant is modeled as follows:

$$x_{t+1} = Ax_t + Bu_t + Ew_t \quad y_t = Cx_t \quad z_t = Dx_t \tag{1}$$

where $x_t \in R^n$, $u_t \in R^m$, and $y_t \in R^l$ denote the state vector, control input, and control output, respectively; $z_t \in R^p$ is the output to be regulated; $w_t \in R^q$ is the disturbance input belonging to $L_2(0, +\infty)$; and A, B, C, D, and E are known

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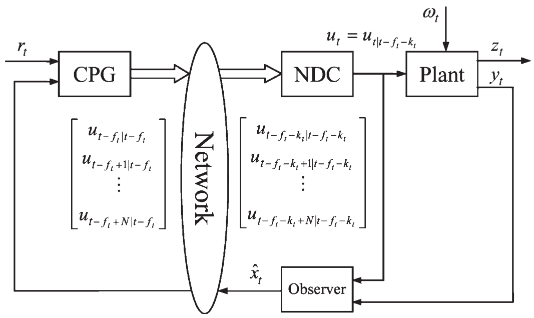

Fig. 1. NPC system structure.

constant matrices with appropriate dimensions. $r_t \in R^n$ is the reference input. Without loss of generality, r_t is assumed to be zero throughout this brief. To measure the network delay, a time-stamp signal is transmitted together with the control predictions. In addition, the following assumptions are made.

Assumption 1: The upper bounds of the time-varying network delays k_t in the forward channel and the feedback channel f_t are not greater than N_1 and N_2, respectively, where N_1 and N_2 are positive integers, i.e., $k_t \in \{0, 1, \ldots, N_1\}$ and $f_t \in \{0, 1, \ldots, N_2\}$, where $t = 0, 1, 2, \ldots$ denotes the sampling instant.

Assumption 2: The numbers of consecutive data dropouts in the forward and feedback channels are less than L_1 and L_2, respectively, both of which are positive integers. It is assumed that the upper bound number of consecutive data dropouts and network delay is equal to $N = N_1 + N_2 + L_1 + L_2$.

III. Predictive Control Strategy

A. Prediction of the Future Control Sequence

Since the system states are normally unavailable, the following state observer is designed:

$$\hat{x}_{t+1} = A\hat{x}_t + Bu_t + L(y_t - C\hat{x}_t) \tag{2}$$

where $\hat{x}_t \in R^n$ and $u_t \in R^m$ are the observed state and the input of the observer, respectively, at time t, and L is the observer gain to be designed later. Note that the above observer is implemented at the plant side, as shown in Fig. 1. This is not a question for the modern "smart" actuator or sensor, which has the capacity to perform some not very complicated calculation, such as the calculation in the NDC and the observer here.

For the system without time delay, the controller is designed using the state feedback control strategy, i.e.,

$$u_t = K_0\hat{x}_t \tag{3}$$

where $K_0 \in R^{m\times n}$ is the control gain matrix to be determined.

When there are time-varying delays and data dropout in the feedback channel, the predictive controller from time $t - f_t + 1$ to t is generated by

$$u_{t-f_t+i|t-f_t} = K_i\hat{x}_{t-f_t}$$

where $i = 1, 2, \ldots, f_t$, $f_t \in \{0, 1, \ldots, N_2 + L_2\}$.

When time-varying delays and data dropout happen in the forward channel, the predictive controllers from $t + 1$ to $t + k_t$ is constructed as

$$u_{t+j|t-f_t} = K_{f_t+j}\hat{x}_{t-f_t}$$

where $j = 1, 2, \ldots, k_t$, $k_t \in \{0, 1, \ldots, N_1 + L_1\}$.

Thus, the state feedback controllers can be given as

$$\begin{aligned} u_t &= u_{t|t-k_t-f_t} = K_{f_t+k_t}\hat{x}_{t-k_t-f_t} \\ &f_t + k_t \in \bar{N} = \{0, 1, \ldots, N\}. \end{aligned} \tag{4}$$

B. Assigning and Compensation of Network Delay

Assuming that the control sequence with network delay $k_t + f_t$ arrives at time t, then

$$U_{t-k_t-f_t|t-k_t-f_t} = \begin{bmatrix} u_{t-k_t-f_t|t-k_t-f_t} \\ u_{t-k_t-f_t+1|t-k_t-f_t} \\ \vdots \\ u_{t-k_t-f_t+N|t-k_t-f_t} \end{bmatrix}.$$

As aforementioned, if the element $u_{t|t-k_t-f_t}$ is chosen as the control input, the impact of the network delay $k_t + f_t$ on the system performance is compensated.

However, from the perspective of system stability, some values of k_t and f_t are not permissible. Therefore, in this situation, it is necessary to modify the network delay. Examining the elements after $u_{t|t-k_t-f_t}$ in the sequence $U_{t-k_t-f_t|t-k_t-f_t}$ enables the control inputs at time $t + i$, $i = 1, 2, \ldots, N - k_t - f_t$, respectively, to be used. This property enables us to assign the network delay we want. As for the elements before $u_{t|t-k_t-f_t}$, they are just discarded.

Hence, all possible available control sequences (at least 1 and at most $N + 1$) are

$$U_{t-N|t-N}, U_{t-(N-1)|t-(N-1)}, \ldots, U_{t|t}$$

and the corresponding control input candidates are

$$u_{t|t-N}, u_{t|t-(N-1)}, \ldots, u_{t|t} \tag{5}$$

which result in the network delay

$$k_t + f_t = N, N-1, \ldots, 0$$

respectively. The delay range is extended from $0 \sim N_1 + N_2$ to $0 \sim N$; thus, it is called extended network delay. It is worth noting that this delay assignment is not arbitrary because not all $N + 1$ control input candidates are definitely available. Hence, it is called "partly assigning."

Remark 1: Notice that all candidates keep the same form as (4); thus, any one of them can compensate the corresponding network delay. It can be seen that the compensation strategy (4) is a special case of the improved one (5).

IV. H_∞ Control Using a Predictive Controller for NCSs

A. Stability and H_∞ Performance Analysis

According to the controller (4), the observer (2) can be written as

$$\hat{x}_{t+1} = (A - LC)\hat{x}_t + BK_i\hat{x}_{t-i} + LCx_t, \qquad i \in \bar{N}. \tag{6}$$

Thus, the closed-loop form of system (1) can be written as

$$\begin{aligned} x_{t+1} &= Ax_t + Bu_t + Ew_t \\ &= Ax_t + BK_i\hat{x}_{t-i} + Ew_t, \qquad i \in \bar{N}. \end{aligned} \tag{7}$$

The combination of (4), (6), and (7) gives the augmented switched system

$$\bar{x}_{t+1} = \bar{A}_{\sigma(t)}\bar{x}_t + \bar{E}w_t \quad y_t = \bar{C}\bar{x}_t \quad z_t = \bar{D}\bar{x}_t \tag{8}$$

where $\sigma(t)$ is called a switching signal, and

$$\bar{x}_t = \left[x_t^T, x_{t-1}^T, \ldots, x_{t-i}^T, x_{t-i-1}^T, \ldots, x_{t-N}^T, \hat{x}_t^T, \hat{x}_{t-1}^T, \ldots, \hat{x}_{t-i}^T, \ldots, \hat{x}_{t-N}^T\right]^T$$

$$\bar{E} = \begin{bmatrix} E \\ 0_{(2N+1)n\times q} \end{bmatrix} \qquad \bar{C} = [\,C \quad 0_{l\times(2N+1)n}\,]$$

$$\bar{D} = [\,D \quad 0_{p\times(2N+1)n}\,] \quad \bar{A}_i = \begin{bmatrix} \Pi & \Xi_i \\ \Phi & \Omega_i \end{bmatrix}$$

with

$$\Pi = \begin{bmatrix} A & 0_{n\times Nn} \\ I_{Nn} & 0_{Nn\times n} \end{bmatrix}$$

$$\Phi = \begin{bmatrix} LC & 0_{n\times Nn} \\ 0_{Nn\times n} & 0_{Nn\times Nn} \end{bmatrix}$$

$$\Xi_i = \begin{bmatrix} 0_{n\times in} & BK_i & 0_{n\times(N-i)n} \\ 0_{(N+1)n\times in} & 0_{(N+1)n\times n} & 0_{(N+1)n\times(N-i)n} \end{bmatrix}$$

$$\Omega_i = \begin{bmatrix} A-LC & 0_{n\times(i-1)n} & BK_i & 0_{n\times(N-i)n} \\ I_n & I_{(i-1)n} & I_{(N-i)n} & 0_{Nn\times n} \end{bmatrix}.$$

Lemma 1: Given $\gamma_0 > 0$, if there exists a positive definite matrix P such that

$$\begin{bmatrix} \bar{A}_i^T P\bar{A}_i - P + \bar{D}^T\bar{D} & \bar{A}_i^{\,T} P\bar{E} \\ * & \bar{E}^T P\bar{E} - \gamma_0^2 I \end{bmatrix} < 0 \qquad \forall i \in \bar{N} \tag{9}$$

hold, then system (8) satisfies H_∞ control under arbitrary switching.

Proof: Choose a common Lyapunov function as

$$V_t = \bar{x}_t P^T \bar{x}_t \tag{10}$$

where P is a positive definite matrix satisfying matrix inequalities (9). Along the trajectory of system (8), calculating the difference of Lyapunov function candidate (10), it is easy to establish the above result. ■

Definition 2 [15]: For $\alpha > 0$ and $\gamma_0 > 0$, system (8) is said to satisfy weighted H_∞ control, if under a zero initial condition, it holds that

$$\sum_{t=0}^{+\infty} e^{-\alpha t} z_t^T z_t \le \gamma_0^2 \sum_{t=0}^{+\infty} w_t^T w_t. \tag{11}$$

Assumption 3: Given $\gamma_0 > 0$, we can design the controller gains $K_{N_1+N_2}, \ldots, K_N$ to make subsystems $\{\bar{A}_{N_1+N_2}, \bar{A}_{N_1+N_2+1}, \ldots, \bar{A}_N\}$ have a common Lyapunov function so that H_∞ control is satisfied for every subsystem. Namely, the matrix inequalities

$$\begin{bmatrix} \bar{A}_i^T P\bar{A}_i - P + \bar{D}^T\bar{D} & \bar{A}_i^T P\bar{E} \\ * & \bar{E}^T P\bar{E} - \gamma_0^2 I \end{bmatrix} < 0, \quad \forall i \in \{N_1+N_2, \ldots, N\}$$

have solution $P > 0$.

We can classify all subsystems of (8) into the following four categories:

Ψ_1: $\{\bar{A}_{N_1+N_2}, \bar{A}_{N_1+N_2+1}, \ldots, \bar{A}_N\}$;
Ψ_2: Part of $\{\bar{A}_0, \bar{A}_1, \ldots, \bar{A}_{N_1+N_2-1}\}$ that can be stabilized and have a common Lyapunov function with Ψ_1;
Ψ_3: Part of $\{\bar{A}_0, \bar{A}_1, \ldots, \bar{A}_{N_1+N_2-1}\}$ that can be stabilized but have no common Lyapunov function with Ψ_1;
Ψ_4: Part of $\{\bar{A}_0, \bar{A}_1, \ldots, \bar{A}_{N_1+N_2-1}\}$ that cannot be stabilized.

We use ψ_1, ψ_2, ψ_3, and ψ_4 to denote the subscript sets to which the subsystems correspond to Ψ_1, Ψ_2, Ψ_3 and Ψ_4, respectively.

We are now in the position to give the main result.

Theorem 1: Given $\alpha > 0$, $\gamma_0 > 0$, if there exist matrices $\bar{P}_i > 0$ such that

$$\Pi = \begin{bmatrix} \bar{A}_i^T \bar{P}_i \bar{A}_i - e^{-\alpha}\bar{P}_i + \bar{D}^T\bar{D} & \bar{A}_i^T \bar{P}_i \bar{E} \\ * & \bar{E}^T \bar{P}_i \bar{E} - \gamma_0^2 I \end{bmatrix} > 0 \tag{12}$$

hold $\forall i \in M = \psi_1 \cup \psi_2 \cup \psi_3$, then system (8) satisfies H_∞ control for any switching signal $\sigma(t) : [0, +\infty) \to M$ satisfying the average dwell time

$$\tau_a \ge \tau_a^* = \frac{\ln \mu}{\alpha} \tag{13}$$

where

$$\bar{P}_i = \begin{cases} P_{N_1+N_2}, & i \in \psi_1 \cup \psi_2 \\ P_i, & i \in \psi_3 \end{cases}$$

and $\mu \ge 1$ satisfies

$$\bar{P}_i \le \mu \bar{P}_j \qquad \forall i, j \in M. \tag{14}$$

Proof: Choose a Lyapunov function candidate as

$$V_t = \bar{x}_t^T \bar{P}_{\sigma(t)} \bar{x}_t \tag{15}$$

where $\sigma(t) \in M$, $\bar{P}_i$ are positive definite matrices satisfying matrix inequalities (12).

Along the trajectory of system (8), for a Lyapunov function candidate (15), we have

$$V_{t+1} - e^{-\alpha}V_t + \Gamma_t = [\,\bar{x}_t^T \quad w_t^T\,]\,\Pi \begin{bmatrix} \bar{x}_t \\ w_t \end{bmatrix}$$

which then leads to the following based on (12):

$$V_{t+1} \le e^{-\alpha(t+1)}V_0 - \sum_{l=0}^{t} e^{-\alpha(t-l)}\Gamma_l$$

where $\Gamma_t = z_t^T z_t - \gamma_0^2 \omega_t^T \omega_t$.

For any time interval $[0, t)$, let $0 = t_0 < t_1 < \cdots < t_q = t_{N_{\sigma(t)}}$ denote the switching time instants of $\sigma(t)$. Thus

$$V_t \le e^{-\alpha(t-t_q)}V_{t_q} - \sum_{l=t_q}^{t-1} e^{-\alpha(t-l-1)}\Gamma_l.$$

Since $V_{t_i} \le \mu V_{t_i^-}$ holds for every switching time instant t_i from (14), we obtain by induction that

$$V_t \le e^{-\alpha t + N_\sigma(0,t)\ln\mu}V_0 - \sum_{l=0}^{t-1} \mu^{N_\sigma(l+1,t)} e^{-\alpha(t-l-1)}\Gamma_l. \tag{16}$$

When $w = 0$, we get from (16), under the condition $N_\sigma(0,t) \le (t/\tau_a)$, that $V_t \le e^{-(\alpha-(\ln\mu/\tau_a))t}V_0$. Exponential stability directly follows from this inequality.

In the following, we show that the closed-loop system satisfies the H_∞ performance bound. Under zero initial conditions, multiplying $\mu^{-N_\sigma(0,t)}$ on both sides of (16) yields

$$0 \leq -\sum_{l=1}^{t} \mu^{-N_\sigma(0,l)} e^{-\alpha(t-l)} \Gamma_{l-1}. \quad (17)$$

Then, similar to the proof of Theorem 2 in [14], the result is easily obtained. The proof is completed. ■

Remark 2: When $\mu = 1$, $\bar{P}_i = \bar{P}_j$ holds for any $i, j \in M$, which means a common Lyapunov function exists for all subsystems. For this case, from (17), the normal L_2-gain directly follows.

B. Design of an H_∞ Controller

In this section, we extend Theorem 1 to design the H_∞ controller gains K_i for system (8).

Define matrices

$$B_1 = \begin{bmatrix} B \\ 0_{(2N+1)n\times m} \end{bmatrix} \quad B_2 = \begin{bmatrix} 0_{(N+1)n\times m} \\ B \\ 0_{Nn\times m} \end{bmatrix}$$

$$\tilde{I} = \begin{bmatrix} 0_{(N+1)n\times n} \\ I_n \\ 0_{Nn\times n} \end{bmatrix} \qquad \tilde{C} = [\, C \quad 0_{p\times Nn} \quad -C \quad 0_{p\times Nn} \,]$$

$$I_0 = [\, 0_{n\times(N+1)n} \quad I_n \quad 0_{n\times Nn} \,]$$
$$I_1 = [\, 0_{n\times(N+2)n} \quad I_n \quad 0_{n\times(N-1)n} \,]$$
$$\cdots$$
$$I_i = [\, 0_{n\times(N+i+1)n} \quad I_n \quad 0_{n\times(N-i)n} \,]$$
$$\tilde{A}_i = \begin{bmatrix} \Pi & 0_{(N+1)n\times(N+1)n} \\ 0_{(N+1)n\times(N+1)n} & \Pi \end{bmatrix}$$

where Π is defined in Theorem 1.

Then, $\bar{A}_i$ can be written as

$$\bar{A}_i = \tilde{A} + B_1 K_i I_i + \tilde{I} L \tilde{C} + B_2 K_i I_i. \quad (18)$$

From (18), inequalities (12) are equivalent to the following matrix inequalities:

$$\begin{bmatrix} -e^{-\alpha}\bar{P}_i + \bar{D}^T\bar{D} & 0 & \bar{A}_i^T \\ * & -\gamma_0^2 I & \bar{E}^T \\ * & * & -\bar{P}_i^{-1} \end{bmatrix} < 0, \qquad i \in M. \quad (19)$$

In view of the above discussion, we can now obtain the following theorem.

Theorem 2: If there exist positive definite matrices $\bar{P}_i > 0$, $i \in M$, such that $\forall i, j \in M$ matrix inequalities (19) hold, then system (8) with the controllers (4) is exponentially stable with the H_∞-norm bound γ_0.

Remark 3: The conditions for the H_∞ controller analysis problem in Theorem 1 are difficult to solve because $\bar{A}_i$ contain K_i and L. In Theorem 2, we separate K_i and L from $\bar{A}_i$ to obtain the controller gain K_i and L by solving a set of LMIs.

It is noted that the conditions in Theorem 2 do not meet the LMI conditions because of the terms $\bar{P}_i$ and $\bar{P}_i^{-1}$. The problem can be solved based on the method proposed in [16]

By replacing the term $\bar{P}_i^{-1}$ in (19) by W_i, we get

$$\begin{bmatrix} -e^{-\alpha}\bar{P}_i + \bar{D}^T\bar{D} & 0 & \bar{A}_i^T \\ * & -\gamma_0^2 I & \bar{E}^T \\ * & * & -W_i \end{bmatrix} < 0, \qquad i \in M. \quad (20)$$

Then, the minimization problem involving the LMI constraints can be formulated as follows:

$$\text{minimize trace}(\bar{P}_i W_i)$$
$$\text{subject to (20)},$$
$$\begin{bmatrix} \bar{P}_i & I \\ I & W_i \end{bmatrix} \geq 0, \qquad i \in M. \quad (21)$$

C. Algorithm

The following iterative algorithm is presented, where (19) is a stopping criterion, since it is numerically very difficult in practice to obtain the optimal solution, and thus, only the suboptimal disturbance attenuation level γ_0 can be obtained within a specified number of iterations.

1) Choose a sufficiently large initial γ_0 such that there exists a feasible solution to the LMI conditions in (20) and (21).
2) Find a feasible set $(\bar{P}_i, W_i, K_i, L\ (i \in M))$ satisfying LMIs in (20) and (21). Set $k = 0$.
3) Solve the following LMI problem for the variables $\bar{P}_i, W_i$:

   $$\text{minimize trace} \sum (\bar{P}_i^k W_i + \bar{P}_i {W_i}^k), \quad i \in M.$$
   subject to LMIs in (20) and (21).

4) If condition (19) is satisfied, then return to Step 2 after decreasing γ_0 to some extent. If conditions (20) and (21) are not satisfied within a specified number of iterations, then exit. Otherwise, set $k = k + 1$, $\bar{P}_i^{k+1} = \bar{P}_i$, ${W_i}^{k+1} = W_i$, and go to Step 3.

Remark 4: It is also noticed that the use of the cone-complementary algorithm has some disadvantages; for example, it requires more computing time and is memory consuming. Therefore, other alternative methods to solve the nonlinear matrix inequalities are worth exploring further, for instance, the method in [17].

V. Simulation

Consider NPC system (1) with the following system matrices:

$$A = \begin{bmatrix} 1.01 & 0.2710 & -0.4880 \\ 0.4820 & 0.1 & 0.24 \\ 0.0020 & 0.3681 & 0.7070 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 5 \\ 3 & -2 \\ 5 & 4 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 3 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0.02 & 0 & 0.03 \\ 0.04 & 0.01 & 0.01 \end{bmatrix} \quad E = 0.01I$$

and the disturbance input, i.e.,

$$w_t = \begin{cases} [1 \quad 1 \quad 1]^T, & 0 < t \leq 2 \\ 0, & t > 2 \end{cases}.$$

This example is similar to the one in [11], where the disturbance effect is not considered.

It is assumed that the upper bounds of the network delays k_t in the forward channel and the feedback channel f_t are all not greater than 1, and the numbers of consecutive data dropouts in the forward and feedback channels are all not greater than 1. That is to say, $N = 4$.

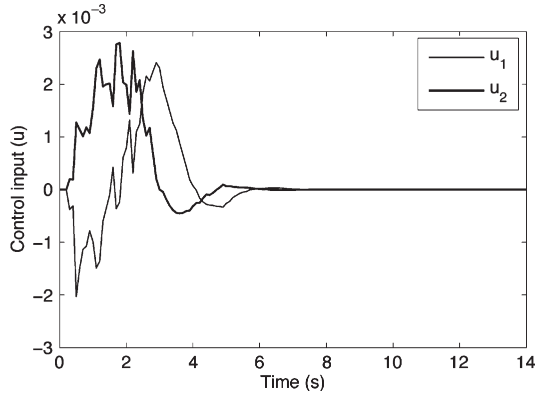

Fig. 2. Control inputs of system (1).

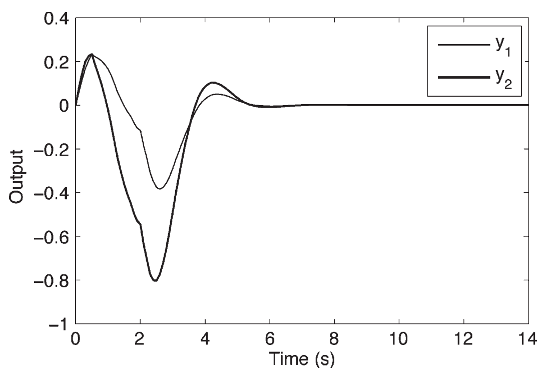

Fig. 3. Outputs of system (1).

Choosing $\alpha = 0.1$ and $\mu = 1.05$, in all subsystems, $\bar{A}_2$, $\bar{A}_3$, and $\bar{A}_4$ have common P; and $\bar{A}_0$ or $\bar{A}_1$ has no common P with $\bar{A}_2$, $\bar{A}_3$, and $\bar{A}_4$ and satisfy (19). Thus, $\sigma(t) : [0, +\infty) \to M = \{0, 1, 2\}$. From (13), we get $\tau_a = 0.5 \geq \tau_a^* = 0.4879$. By applying the algorithm to this example, the maximum iteration number is chosen to be 100. By applying the proposed algorithm in Section IV-C to this example, where the iteration number is chosen to be 100 and MATLAB/LMI Toolbox is used to solve the LMI problem, we obtain

$$K_0 = \begin{bmatrix} 0.0054 & -0.0099 & -0.0520 \\ -0.0202 & -0.0070 & 0.0448 \end{bmatrix}$$
$$K_1 = \begin{bmatrix} 0.0007 & -0.0146 & -0.0333 \\ -0.0202 & 0.0054 & 0.0340 \end{bmatrix}$$
$$K_2 = \begin{bmatrix} -0.0056 & -0.0122 & -0.0228 \\ -0.0144 & 0.0052 & 0.0290 \end{bmatrix}$$
$$K_3 = \begin{bmatrix} -0.0097 & -0.0105 & -0.0158 \\ -0.0104 & 0.0057 & 0.0259 \end{bmatrix}$$
$$K_4 = \begin{bmatrix} -0.0125 & -0.0098 & -0.0116 \\ -0.0078 & 0.0071 & 0.0267 \end{bmatrix}$$
$$L = \begin{bmatrix} -0.2477 & 0.2676 \\ -0.0055 & 0.0829 \\ 0.2157 & -0.0534 \end{bmatrix}$$

and the H_∞ performance bound $\gamma_0 = 0.02$.

The control input and output trajectory of the system are shown in Figs. 2 and 3, respectively. It is easy to see that the system is stable and has acceptable performance.

Compared with our previous paper [11], the approach in this brief is a significant improvement for the following three reasons: First, the varying controller gains are easy to design, whereas the fixed controller gain in [11] needs to be selected in advance. Second, the proposed approach can solve the four-step delay and higher delay problem, whereas employing the method in [11] can address only the three-step delay problem. Third, by using the method in this brief, it only takes 6 s to reach a stable response, whereas in [11], it takes approximately 100 s to reach a stable response.

VI. Conclusion

The problem of H_∞ control for a class of NPC systems has been studied in this brief. An improved compensation strategy based on the average dwell-time method was combined with a novel controller design approach to make the corresponding closed-loop system exponentially stable with the H_∞ performance bound. In contrast with some existing methods, which are based on the fixed controller gain and common Lyapunov function, these varying feedback controller gains and multiple Lyapunov function matrices can lead to less conservative results.

References

[1] G. C. Walsh and H. Ye, "Scheduling of networked control systems," *IEEE Control Syst. Mag.*, vol. 21, no. 1, pp. 57–65, Feb. 2001.
[2] A. V. Savkin, "Analysis and synthesis of networked control systems: Topological entropy, observability, robustness and optimal control," *Automatica*, vol. 42, no. 1, pp. 51–62, Jan. 2006.
[3] L. A. Montestruque and P. Antsaklis, "Stability of model-based networked control systems with time-varying transmission times," *IEEE Trans. Autom. Control*, vol. 49, no. 9, pp. 1562–1572, Sep. 2004.
[4] Z. H. Mao, B. Jiang, and P. Shi, "Protocol and fault detection design for nonlinear networked control systems," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 56, no. 3, pp. 255–259, Mar. 2009.
[5] W. Zhang, M. S. Branicky, and S. M. Philipsm, "Stability of networked control systems," *IEEE Control Syst. Mag.*, vol. 21, no. 1, pp. 84–99, Feb. 2001.
[6] X. M. Tang and J. S. Yu, "Feedback scheduling of model-based networked control systems with flexible workload," *Int. J. Autom. Comput.*, vol. 5, no. 4, pp. 389–394, Oct. 2008.
[7] D. Yue, Q. L. Han, and P. Chen, "State feedback controller design of network control systems," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 51, no. 11, pp. 640–644, Nov. 2004.
[8] D. Yue, Q. L. Han, and J. Lam, "Network-based robust H_∞ control of systems with uncertainty," *Automatica*, vol. 41, no. 6, pp. 999–1007, Jun. 2005.
[9] Y.-B. Zhao, G.-P. Liu, and D. Rees, "Networked predictive control systems based on Hammerstein model," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 55, no. 5, pp. 469–473, May 2008.
[10] G.-P. Liu, Y. Xia, D. Rees, and W. Hu, "Design and stability criteria of networked predictive control systems with random network delay in the feedback channel," *IEEE Trans. Syst., Man, Cybern. C, Appl. Rev.*, vol. 37, no. 2, pp. 173–184, Mar. 2007.
[11] G.-P. Liu, Y. Xia, J. Chen, D. Rees, and W. Hu, "Networked predictive control of systems with random network delays in both forward and feedback channels," *IEEE Trans. Ind. Electron.*, vol. 54, no. 3, pp. 1282–1297, Jun. 2007.
[12] J. P. Hespanha and A. S. Morse, "Stability of switched systems with average dwell-time," in *Proc. 38th IEEE Conf. Decision Control*, Dec. 1999, pp. 2655–2660.
[13] D. Liberzon, *Switching in Systems and Control*. Boston, MA: Birkhauser, 2003.
[14] X. M. Sun, J. Zhao, and D. J. Hill, "Stability and L_2-gain analysis for switched delay systems: A delay-dependent method," *Automatica*, vol. 42, no. 10, pp. 1769–1774, 2006.
[15] G. S. Zhai, B. Hu, K. Yasuda, and A. Michel, "Disturbance attenuation properties of time-controlled switched systems," *J. Franklin Inst.*, vol. 338, no. 7, pp. 765–779, Nov. 2001.
[16] E. I. Ghaoui, F. Oustry, and M. A. Rami, "A cone complementarity linearization algorithm for static output-feedback and related problems," *IEEE Trans. Autom. Control*, vol. 42, pp. 1171–1176, Aug. 1997.
[17] H. D. Tuan, P. Apkarian, and T. Q. Nguyen, "Robust and reduced order filtering: New LMI-based characterizations and methods," *IEEE Trans. Signal Process.*, vol. 49, no. 12, pp. 2875–2984, Dec. 2001.