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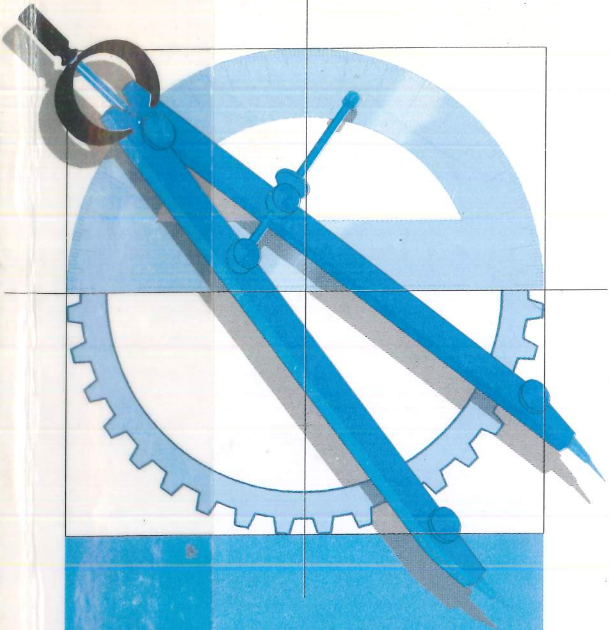

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MASTEC Monograph Series

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Preface

Since 1988 teams of researchers in the United States, Japan and Australia have been involved in a collaborative research project to assess the mental computation ability of their students. The results of this research have been reported elsewhere (McIntosh, Bana & Farrell 1995; McIntosh, Nohda, Reys & Reys 1995). The researchers involved were Professors Robert and Barbara Reys of the University of Missouri - Columbia, Professor Nobuhiko Nohda of the University of Tsukuba and Alistair McIntosh, Jack Bana and Brian Farrell of Edith Cowan University, Perth, Western Australia.

The United States and Australian researchers went on to assess the number sense of the same cohorts of students in their two countries. During this time, a doctoral student at the University of Missouri - Columbia, Der Ching Yang, was conducting research into the number sense of Taiwanese students, adapting some of the test items devised by the research teams. In 1995, while Professors Robert and Barbara Reys were at the University of Göteborg on Fulbright scholarships, a team of researchers including Göran Emanuelsson and Bengt Johansson used many of the items to test the number sense of Swedish students.

This monograph, while focussing mainly on the United States and Australian data, also includes relevant Swedish and Taiwanese data, together with brief discussions kindly contributed by Göran Emanuelsson, Bengt Johansson and Der Ching Yang.

The idea of the development of number sense as a central goal of school mathematics is a recent one, and broad agreement as to its scope is only beginning to emerge. The development of written group tests of number sense is even more embryonic, and indeed not all experts agree that pencil and paper tests, let alone tests including multiple choice items, are an appropriate mode of assessing number sense.

It is hoped that this monograph, which includes full details of the test items, their development and their theoretical basis, as well as full test results and analysis for all relevant items from four countries, will help to move forward the debate in a positive manner.

CHAPTER 1 Background to the Study

Number sense refers to a person's general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful and efficient strategies for managing numerical situations. It results in a view of numbers as meaningful entities and the expectation that mathematical manipulations and outcomes should make sense. Those who view mathematics in this way continually utilise a variety of internal "checks and balances" to judge the reasonableness of numerical outcomes. When an outcome conflicts with the perceived expectation, the person revisits the mathematical situation to view it externally, often through another lens, and attempts to resolve the conflict.

Number sense exhibits itself in various ways as the learner engages in mathematical thinking, including awareness of various levels of accuracy and sensitivity for the reasonableness of calculations. It is characterised by a desire to make sense of numerical situations, by looking for links between new information and previously acquired knowledge, and by an innate drive within the learner to make the forming of these connections a priority (Reys, Barger, Dougherty, Hope, Lembke, Markovits, Parnas, Reehm, Sturdevant, Weber & Bruckheimer, 1991).

Documents calling for reform of school mathematics in many industrialised countries have emphasised the need for students to develop number sense (Cockcroft, 1982; National Council of Teachers of Mathematics, 1989; Japanese Ministry of Education, 1989; Australian Education Council, 1991; Emanuelsson & Johansson, 1996). Although the term "number sense" is relatively new to the language of mathematics curricula, its meaning which emphasises understanding and meaningful learning is commonplace in the literature of mathematics education (Brownell, 1935; Burns, 1994; Hiebert, 1984; Plunkett, 1979; Skemp, 1982).

Like "common sense", number sense is a valued but difficult notion to characterise, and it has stimulated much discussion among mathematics educators (including classroom teachers, curriculum writers, and researchers) and cognitive psychologists. These discussions have included a listing of essential components of number sense (Resnick, 1989; Sowder & Schappelle, 1989; Willis, 1990; Sowder, 1992; McIntosh, Reys & Reys, 1992), descriptions of students displaying number sense or the lack thereof (Howden, 1989; Reys et al., 1991), a theoretical analysis of number sense from a psychological perspective (Greeno, 1991), and discussions of instructional strategies which promote the development of number sense (Brownell, 1945; Kamii, 1989; Reys et al., 1991; Burton, 1993; Burns, 1994).

Various "indicators" of number sense have been hypothesised. These include: well-understood number meanings, existence of and reliance on multiple numerical relationships, recognition of relative magnitude of numbers, awareness of the relative effect of operating on numbers, and use of referents for measures of common objects and situations in their environments (NCTM, 1989). Resnick (1989) offers additional indicators: using the decimal structure of the number system to decompose and recompose numbers to simplify calculations, tending to want to "make sense" of situations involving number and quantity, and using "benchmark" information/data to derive new information (such as using a known number fact to calculate an unknown fact). This latter strategy is commonly used by students to make sense of the basic number facts (Bana & Korbosky, 1995).

Why is the development of number sense important? As Ekenstam (1977) states, "the lack of understanding of what numerals mean must present insuperable barriers to learning mathematics" (p. 317). Students who don't understand that 1.20 is a representation for 1.2, that $^{11}/_{13}$ is less than 1, and that 1000 is ten 100s, must learn and remember a host of rules in order to deal practically with everyday numerical situations. It is unfortunate that students' lack of understanding has not always been obvious, given the emphasis on computational facility where proficiency with rules is often rewarded.

How can instruction be organised to facilitate the development of number sense? Brownell (1945) characterised his theory of meaningful arithmetic as, "instruction which is deliberately planned to teach arithmetical meanings and to make arithmetic sensible to children through its mathematical relationships". Markovits and Sowder (1994) characterise instruction focused on developing number sense as ". . .

instruction designed to provide rich opportunities for exploring numbers, number relationship, and number operations and to discover rules and invent algorithms". In a year-long intervention study in seventh-grade classrooms, they found that experimental units on number magnitude, mental computation and computational estimation produced the effect of students being more likely to elect to use strategies that reflected number sense, and that this was a long-term change.

The notion that mathematics instruction and learning should be based on reflective inquiry and sense-making has long been overshadowed by a quest for high levels of efficiency in computing, the dominant feature of school mathematics curriculum of this century. Evidence suggests that instruction focused on procedural knowledge does not ensure conceptual understanding and/or the mathematical power needed to create, extend, and apply mathematics (Narode, Board & Davenport, 1993). More efficient methods for computing (e.g., an inexpensive calculator) emphasise the need to reconsider the present emphasis on paper/pencil computation methods. If learning these methods does indeed cause a devaluing of understanding and sense-making on the part of the learner and results in only mediocre levels of proficiency, one must question the use of instructional time for such limited and counterproductive results.

While agreement exists that the development of number sense is an important goal for all children, many questions remain unanswered about the routes to achieve this goal. Better information is needed to guide curriculum and instruction efforts in this area. For example, are students developing number sense in the current curriculum oriented toward developing proficiency in standard paper/pencil algorithms? If some students are developing number sense within this environment, what thinking and learning strategies are they employing? If some students are not developing number sense in this environment, what changes in curriculum and instruction would support the development of number sense? Do mathematical tasks such as inventing strategies to estimate and mentally compute utilise and/or support the development of number sense? Do teachers perceive the development of number sense as an important instructional goal? Do they purposefully pursue its development? What type of curricular and instructional approach will best foster the development of number sense?

The research described here sought to advance the discussion of the value and means of developing number sense by gathering evidence regarding the current level of number sense of students aged 8, 10, 12 and 14 in four countries (grades 2, 4, 6 and 8 in the US; grades/years 3, 5, 7 and 9 in Australia; grades 4 and 8 in Sweden; and grades 6 and 8 in Taiwan). In addition, the number sense test scores of students in the USA and Australia were correlated with a measure of mental computation performance to gauge the strength of the relationship of number sense to the ability to compute mentally. What follows is a description of the development of the number sense tests as well as the procedures employed in the study and finally, a discussion of the findings.

Purpose of the study

Why develop and use a number sense test? What value do the results have? It is our belief, supported by some emerging research by Turner (1996), that teachers have an inaccurate view of their students' level of number sense. Although teachers value understanding (this is often how they describe number sense), their instructional techniques and strategies do not necessarily support the development of number sense. We believe that most teachers do not recognise this mismatch between their beliefs and practices. Assessment of number sense provides tangible evidence of students' level of understanding regarding numbers, number magnitude, and effect of operations, that is, their number sense. Our goals include providing teachers with a tool to assess their students' understanding of number and operations and to raise teachers' awareness of the effect of instruction on the development of number sense.

The project aimed in particular to assess the general level of number sense possessed by students in four countries (Australia, Sweden, Taiwan, and the United States) at various age levels (ages 8, 10, 12, 14), to determine whether the level of number sense increases over time (as the level of schooling increases), and to examine the relationship between the level of number sense and the ability to mentally compute.

CHAPTER 2

Design of the Study

Sample

A school district within each country participated in the study and was chosen using similar criteria (mid-sized to large school district near a university). Four schools (three primary/elementary and one high school) were randomly selected from the school districts chosen in Australia, the United States, and Sweden. Three schools (two elementary and one junior high) were randomly selected to participate from the district in Taiwan. Within each primary/elementary school, classes were randomly selected at each of the grade levels. Students in all classes were heterogeneously grouped. In the Australian secondary school where students are streamed on ability, as is the case in most Australian secondary schools, stratified random sampling was used to select three pairs of classes, with each pair at a different level of ability as previously determined by the school. In the American, Swedish and Taiwanese junior high schools, classes were randomly selected to represent all levels of students. Although the number of subjects varied from level to level and country to country, in all cases the number of students taking each item ranged from 115 to 180 students. Table 1 shows the number of students involved in the testing at each age in each country.

Table 1: Numbers of Students in the Study by Age and Country

Country	Age 8	Age 10	Age 12	Age 14
Australia	180	167	168	124
USA	136	139	125	115
Sweden	not tested	170	not tested	154
Taiwan	not tested	not tested	115	119

Number Sense Framework

A framework for examining number sense was developed by McIntosh, Reys and Reys (1992) based on studying and reflecting on the literature associated with number sense, estimation and mental computation. In addition to studying the lists of components of number sense hypothesised by various writers, the literature on students' thinking when asked to estimate and/or mentally calculate was reviewed (Reys, Rybolt, Bestgen & Wyatt, 1982; Levine, 1982; Rubenstein, 1983; Hope & Sherrill, 1987; Sowder & Wheeler, 1989; Case & Sowder, 1990; Markovits & Sowder, 1994). This literature contained some common themes and these were organised into three broad categories (knowledge of and facility with number, knowledge of and facility with operations, and applying knowledge of number and operations to computational settings). The framework (see Appendix A) is an attempt to articulate a structure which clarifies, organises, and interrelates some of the generally agreed-upon components of number sense. For the purpose of developing a number sense test, six strands were identified from the three major categories (number, operation, and application) of the framework. The six strands are described in Appendix B.

The organisational model shown in both the framework and in the six strands is in the formulation stage. The framework and delineation of strands within the framework is useful in facilitating thinking and discussion about the multifaceted notions associated with number sense. It is presented here as a helpful organiser and should not be considered as a fait accompli. On the contrary, it is expected that this and future research will reveal more clearly the various components of number sense.

After formulating the six number sense strands, a search and review of available assessment instruments was conducted to identify items which might provide a "window" into students' thinking as related to the various strands. The work of Markovits and Sowder (1994) and Cramer, Post and Currier (1993) was particularly useful in identifying items which mirrored elements of the framework. In addition to the available instruments, new items were developed to reflect various components of the framework strands. Items were constructed which (a) were on grade level in terms of conceptual knowledge, (b) framed questions in non-routine environments in order to elicit strategy generation based on understanding rather than strategy recall based on familiarity with problem-type, and (c) elicited components of number sense as identified by six strands from the McIntosh, Reys, and Reys framework. The items were compiled into a "number sense item bank" and coded according to number type, number sense strand, and appropriate age level. Examples of items are included in Appendix B.

Early versions of the test items were used in interviews with students in the United States. These interviews provided a means to clarify wording and item format and to probe the thinking of individual students as they responded to items. Items which evoked thoughtful introspection (rather than immediate recall of a rule or known fact) and relied on conceptual understanding, were kept in the item bank. On the other hand, items which could be answered correctly without indication of understanding (explanation rooted in one or more underlying concepts) were removed from the item bank.

The Number Sense Item Bank (NSIB) contains items which we have not rejected, together with all the data we have so far collected using the various items. The item bank is divided into six sections, each section containing items corresponding to one of the six strands described in Appendix B. No claim is made that the item bank represents a comprehensive assessment of number sense. In fact, every time versions of items are used with students and reviewed with teachers, valuable suggestions result that lead to revisions, refinements, and new items. The researchers are continuing to develop additional items to cover gaps revealed by mapping the items onto the framework. Items from the NSIB have been compiled to form group administered number sense tests. The procedures, sample, results, and discussion of this study follow.

Instruments

Four number sense test instruments were constructed and utilised in the research, one for each of four age levels. The Number Sense Tests (NSTs) were designed for group administration and were administered in the last quarter of the school year to students in grades 2, 4, 6 and 8 in the United States, and in grades/years 3, 5, 7 and 9 in Australia (corresponding to ages 8, 10, 12, and 14 in both countries). Variations of the tests (containing both new and common items) were developed and used in grades 4 and 8 in Sweden (ages 10 and 14), and grades 6 and 8 in Taiwan (ages 12 and 14).

The number of items per test varied by grade level and country, ranging from 30 items at age 8 to 45 items at age 14. A few items were included in the tests of one country but not of the other and there were some minor differences in wording. In every country, all items appeared in the students' native language. Draft tests were developed by the researchers and then field tested with students in each of the four countries. Revised tests were then used in the main testing program. Test-retest reliability for the age 12 and 14 USA tests were measured at 0.72 and 0.89 respectively. A complete listing of the items and results for each age-level test for the Australian and United States samples, together with some of the items used in Sweden and Taiwan, can be found in Appendix F.

A mental computation test was also administered to the same populations of students in the USA. and Australia. It was based on an instrument used in an earlier study (Reys, Reys, Nohda & Emori, 1995), contained two parts (orally and visually administered items), and was administered to students the week prior to the NST administration. Data relating to the mental computation testing in the United States and Australia can be found in McIntosh, Nohda, Reys and Reys (1995). Fuller discussions of the Australian results can be found in McIntosh, Bana and Farrell (1995).

Procedures

The NSTs were administered during one class period in the second semester of the school year in Australia, the US and Sweden, and at mid-year in Taiwan. The timing was to ensure comparable ages across the four countries, and to test students after they had covered at least half the year's school program. The protocols for the administration of the NSTs are detailed in Appendix C.

Each test administration was begun by having students work through several practice items in the same format as those in the test itself. All items were read for the 8-year-old students. For all ages, a pacing scheme was used in the administration of the test. Specifically, the instructions were read and students were asked to spend no more than

30 seconds on each item. To encourage students to adhere to the suggested pace, the next item number was announced every 30 seconds. Total administration time was about 30 minutes.

Students wrote responses directly on the test pages. They were not permitted to write any other information. This requirement was designed to prevent the use of any mechanical paper and pencil procedures to arrive at solutions. Such a stipulation was particularly important in items involving estimation.

The NSTs were scored by the researchers by marking each item either correct or incorrect. Means were computed on each test along with other summary statistics (see Appendix E). An item analysis was undertaken for each of the NST items. The detailed item-by-item results are presented in Appendix F. In the case of the Australian and US students who had previously undertaken the mental computation test, comparisons were made between their mental computation and number sense test scores.

CHAPTER 3

Analysis of Results

We begin the analysis by presenting an overview of the results for Australia and the USA. Then we discuss the results for whole number items, concluding the chapter with an analysis of the results for rational number items in Australia and the USA. More specific discussion of the results from Sweden and Taiwan is given in Chapter 4.

Overview

The NST resulted in a wide range of performance. The summaries of the NST results for Australian and US students are shown in Table 2 and Table 3 respectively. An interesting result to note from these summaries was the wide range of ability at all age levels in both countries. The maximum possible score was nearly achieved in each age group. The fact that the mean is approximately fifty percent for each age group indicates that the levels of difficulty of the items were appropriate for this study. It is important not to make any significant comparisons between the countries as the student samples were not representative of the school population of each country.

Table 2: Summary of Australian NST Results

Ages	8	10	12	14
N	180	167	168	124
Max Possible Score	30	35	45	45
Range	3-30	8-30	5-44	8-45
Mean	15.4	18.4	23.6	27.7
SD	5.1	5.1	7.7	9.2
Error of Meas	0.4	0.4	0.6	0.8

Table 3: Summary of US NST Results

Ages	8	10	12	14
N	136	139	125	115
Max Possible Score	30	35	45	45
Range	4-26	5-28	0-41	3-42
Mean	15.1	14.5	16.7	22.0
SD	5.0	4.4	7.2	10.0
Error of Meas	0.4	0.4	0.6	0.9

Almost all the Australian and US students in this study were also administered a mental computation test (MCT). The correlations between scores on the MCT and the NST in each country are shown in Table 4. The results provide evidence that the processes used for each are related. The correlation grows in strength as the age level increases in both countries. The reasonably strong correlations, especially for ages 12 and 14, indicate that mental computation ability may be a good indicator of number sense.

Table 4: Correlation Coefficients Between MCT and NST for US and Australian Samples

Age	8	10	12	14
Australia	0.59	0.65	0.78	0.83
USA	0.64	0.70	0.77	0.88

The results of the NST by number type are contained in Table 5. The Australian and United States results are similar at all ages for whole number. The students in the Australian sample generally scored higher than the US sample, with the difference being most marked for the decimal items.

Table 5: Comparison of USA and Australian Mean Percentages by Number Type

Country	USA	Aus	USA	Aus	USA	Aus	USA	Aus
Ages	8		10		12		14	
Whole number	43	49	48	54	48	60	53	59
Decimals			26	48	29	51	47	71
Fractions	54	60	49	57	24	42	34	50
Percentages					58	84	54	73
Mixed							28	52

Another interesting comparison is that between the Australian girls and boys on the NST. Table 6 shows that in all cases the boys' range is greater (except for age 14); their minimum score is higher (except for age 12); their maximum score is greater; their mean score is greater, and their standard deviation is greater (except for age 8). However, these differences are only significant for the 10-year-olds ($\rho = 0.001$).

Table 6: Comparison of Australian Boys' and Girls' Performances on the NST

Ages(Boys)	8	10	12	14
N	96	83	87	59
Max	30	35	45	45
Range	4-30	9-30	5-44	13-45
Mean	15.5	19.7	24.5	28.9
SD	4.7	5.2	7.9	9.2
Ages(Girls)	**8**	**10**	**12**	**14**
N	84	84	81	65
Max	30	35	45	45
Range	3-25	8-27	7-39	8-44
Mean	15.2	17.3	22.7	26.6
SD	5.5	4.8	7.3	9.0

Whole Number

The descriptions below relate to items involving whole numbers. Almost all the NST items were given to matching cohorts of students in Australia and the USA. Some of the items were also administered to 10-year-olds and 14-year-olds in Sweden, and to 12- and 14-year-olds in Taiwan.

Understanding and Use of the Meaning and Size of Numbers

When asked for the counting number before 600, sixty-three and sixty-six percent of 8-year-olds in the Australian and USA samples respectively stated 599. However, students did not fare as well when asked what came after 47 399. Overall only 16 percent of 8-year-olds and 55 percent of 10-year-olds were successful. It could well be reasoned that the skills of counting on from 399 to 400 and counting back from 600 to 599 are of the same level of difficulty. However, when 399 was part of a much larger number (47 399) even the 10-year-olds struggled with such an item. Thus it appears that the use of large numbers posed considerable difficulty for students, and may relate to their sense of the size of such numbers.

Australian students were asked to arrange the digits 2, 6, 3, 5, and 1 to make the smallest possible number. The item was modified for the American students by replacing the 1 by a second 2. The performances in the two countries are shown in Table 7 and are much higher than for the item involving adding one to 47 399. It seems that many students may know that digits need to be ordered left to right to give the smallest number and vice versa for the largest number, yet they still have conceptual

Table 7: Percentage Scores on Item Asking to Arrange all the Digits 2, 6, 3, 5, 1* to make the Smallest Possible Number

Ages	8	10	12	14
Australia	40	89	93	-
USA	43	63	69	-

* For the USA item the 1 was replaced by a 2

Table 8: Percentage Scores on Item Asking to Arrange all the Digits 2, 6, 3, 5, 1* to make the number nearest to 20 000

Ages	8	10	12	14
Australia	8	43	65	
USA	10	48	57	

* For the USA item the 1 was replaced by a 2

difficulties with such large numbers. This is borne out by the results for the related item shown in Table 8. Here students were asked to use the same digits to make the number nearest to 20 000. Performances are so much lower than for the previous item that they seem to confirm that the scores attained for the item in Table 7 have a rule-bound rather than a conceptual basis.

Understanding and Use of Equivalent Forms and Representations of Numbers

Students in the USA aged 10 and 12 were asked to approximate the three numbers that matched three positions marked on a number line with end-points 0 and 1000, and where the placement of the points corresponded to 100, 500 and 900. The percentages of 10-year-olds correctly identifying the three numbers were 55, 42 and 43 percent respectively. The corresponding percentages of 12-year-olds were 33, 40, and 29 percent. Only about one third of the students were correct on all three numbers. The ranges of acceptable estimates were reduced slightly for the older group. Students seem to experience great difficulty with estimation of numbers for points on the number line.

Understanding the Meaning and Effect of Operations

Table 9 shows the results of a multiple choice item (Item 46, Appendix F) for which the correct response required the recognition that the sum of two 3-digit numbers will be either a 3- or a 4-digit number. The percentages of correct responses are quite low, and suggest that despite the numerous addition computation exercises that these students would have undertaken they do not seem to have a sound concept of what happens when two numbers are added. It would have been interesting to see how the 14-year-olds handled this item. The 14-year-olds were given a similar item in relation to multiplication when they were asked for the number of digits in the product of a pair of two-digit numbers (Item 78, Appendix F). The results were 52, 58, and 69 percent correct for the Swedish, American and Australian samples respectively. However, once again this performance is not high considering the extensive experience that such students would have had in multiplication of whole numbers.

Table 9: Percentages Determining the Size of the Sum of Two 3-Digit Numbers

Ages	10	12	14
Australia	32	52	-
Sweden	21	-	55
USA	20	30	-

Item 6 asked students to supply the missing digits in 431 - 2□□ to make the difference as big as possible. Because of possible misinterpretation of the item, both 00 and 01 were accepted as correct responses. The results in Table 10 show that students at all three levels tested found this item to be very difficult. About a quarter of the students offered 89 or 98 as the correct response.

Table 10: Percentages Correct for Putting One Digit in Each Box so that the Answer Will be as Big as Possible: 431 - 2□□

Ages	8	10	12
Australia	9	23	27
Sweden	-	24	-
USA	7	16	23

Basic properties involving zero and one were tested in two items. For the open sentence 16 x 0 = □, four choices were given for the missing factor. Among 10-year-olds 86 and 87 percent chose the correct response in the US and Australian samples respectively. Almost two thirds of 8-year-olds in the Australian sample were correct. The so-called "zero difficulty" did not seem to be a significant problem. However the most common incorrect response was 16. For the open sentence 15 x □ = 15 the percentages of 10-year-olds choosing the correct missing factor were 81 and 78 for the US and

Australian students respectively, demonstrating that students understood the multiplication property of one. However, only 42 percent of the Australian 8-year-olds responded correctly. The difference in performance by younger children on this item compared to the previous one may be due more to the fact that the missing number is not at the end of the sentence, rather than to differences in conceptual levels between zero and one as operators.

Understanding and Use of Equivalent Expressions

In both Australia and the USA the 8-year-olds were given a diagram with a beam balance showing two bags with 18 and 15 marbles respectively balancing two bags - one with 15 marbles and the other with an unknown number to be identified. The percentages of students who correctly identified the number in the unlabelled bag were 57 and 62 for the USA and Australian samples respectively. These results suggest that about two-fifths of 8-year-olds did not appear to understand the concept of commutativity in addition of whole numbers.

Only two test items, both involving whole numbers, were given to all four age levels in Australia and the US. One item tested understanding of commutativity of multiplication in a real-life context by asking how many apples would be repacked in each of 40 new boxes if they were all previously in 80 boxes with 40 to each box. The results given in Table 11 indicate a steady increase in performance across the four levels. The most common error was the response of 40 apples.

Table 11: Percentages Understanding Commutativity of 80 x 40 in a Real-Life Context

Ages	8	10	12	14
Australia	44	60	82	78
USA	35	40	54	75

Australian and US 12- and 14-year olds were asked to use 93 x 134 = 12 462 to determine the answer to 93 x 135. The results are given in Table 12. To handle this item students needed to be able to extrapolate from a given computation to a related one. It is clear from the results that students in both age groups had considerable difficulty coming to grips with the concepts involved.

Table 12: Using 93 x 134 = 12 462 to Determine the Result of 93 x 135

Ages	12	14
Australia	24	47
USA	14	34

Students aged 10, 12 and 14 were asked to choose the larger number from 145 x 4 and 144 + 146 + 148 + 150 without calculating. The results are shown in Table 13. The small differences across age levels, particularly between 10- and 12-year-olds, is rather surprising.

Table 13: Percentages Correct on Item 48 - Without Calculating the Exact Answer, Which Represents the Larger Amount, 145 x 4 or 146 + 146 + 148 + 150?

Ages	10	12	14
Australia	77	76	85
USA	71	73	89

Computing and Counting

Students of all four age groups in Australia and the USA and in the two age levels in Sweden were asked to estimate how many days they had lived from four given choices. The results given in Table 14 show that students at all levels had great difficulty in distinguishing between 300, 3000, 30 000, and 300 000 days as their approximate age. In fact a high percentage of 8-year-olds thought they were about 300 days old. Obviously this problem is complicated for the students who are used to thinking of their age in years, and many 8-year-olds may not know how many days there are in a year. However, performances again indicate a poor grasp of larger numbers among students at all age levels. There is no consistent pattern of increasing scores across year levels, and most scores are not markedly different from random levels. Such data strongly suggest a high incidence of students guessing rather than reasoning the correct response in this item.

Table 14: Percentages Correct on Item 1 - About How Many Days Have You Lived (300, 3000, 30 000, or 300 000)?

Ages	8	10	12	14
Australia	34	35	38	32
Sweden*	-	28	-	55
USA	19	33	27	47

*Different numbers used in a similar pattern up to millions

Table 15 shows results for an item where students in all four age levels had to estimate the number of triangles in a pictorial set of about 200 by selecting from five choices. None chose 20, and no more than five percent chose 50, but it is evident that they had difficulty discriminating between 100, 200 and 500. Note that choices were modified slightly for Swedish students. It seems that students, in particular the 8-year-olds, have considerable difficulty conceptualising larger numbers, even when visual representations for such numbers are given. There is a marked increase in performance from 8-year-olds to 10-year-olds, but very little progress over the next two-year span.

Table 15: Estimating the Number in a Pictorial Set of 200 Triangles

Response	US 8	US 10	US 12	Aus 8	Aus 10	Aus 12	Swe 10	Swe 14
20	0	0	0	0	0	0	-	-
50	1	4	0	5	3	2	5	3
100	32	23	23	40	34	31	-	-
200*	31	51	54	29	54	62	57	75
500	35	23	19	26	9	5	28	20
1000	-	-	-	-	-	-	10	3
Other	1	0	0	0	0	0	0	0
No Resp .	0	0	2	0	0	0	0	0

* Correct response

Students in both the Australian and Swedish samples had little difficulty in deciding which of the expressions 18 x 17, 16 x 18, and 17 x 19 represented the greatest product. In the USA there was no difference in performance between 12- and 14-year-olds where 84 percent of both groups were correct. There was also no difference between the same two Australian age groups where 89 percent of both groups were correct. However, it is interesting to note the marked drop in performance when students were asked to estimate the product of a similar expression (18 x 19), as shown in Table 16.

Table 16: Percentages Choosing the Best Estimate for 18 x 19 (from 290, 390, 490)

Ages	12	14
Australia	25	55
USA	29	40

The results for two other items involving estimation of products are shown in Table 17. When 10- and 12-year-old students were given a target number of 75 and asked which pair of factors from 4, 18, 50, and 37 would get closest to 75 the performances were reasonable. However, when the target number was raised to 1000, student performances fell dramatically even though the same factors were given to choose from.

Table 17: Percentages Selecting Correct Pairs of Factors from 4, 18, 50, 37 to Get Closest to Target Products

Ages	10	10	12	12
Target Product	75	1000	75	1000
Australia	49	22	75	46
Sweden	-	18	-	-
USA	38	18	61	39

It is clear from the two items in Table 17 and from other results discussed previously that students find it very difficult to make good estimates, and that this difficulty increases markedly for larger numbers.

Rational Number

Students' understanding of rational numbers, especially common fractions and decimals, has been a consistent focus of research (Behr, Harel, Post & Lesh, 1992; Bezuk & Bieck, 1993; Cramer, Post & Currier, 1993; Owens & Super, 1993). Assessing understanding with respect to rational numbers is not an easy task. If it were easy, instruments would have been developed long ago and be widely used today. Most research instruments have included non-traditional items that address different aspects of rational numbers. That is, questions unfamiliar to students are often used to gauge their ability to formulate solution strategies based on conceptual knowledge, rather than recall of a certain problem type. Computation of rational numbers is usually avoided in these assessments because such data are widely available from standardised achievement tests; and prior research has shown that computation performance is not a good predictor of understanding. In fact, computation facility often disguises misconceptions, making the data unusable as a measure of number sense (Carpenter, Coburn, Reys & Wilson, 1978; Carpenter, Corbitt, Kepner, Lindquist & Reys, 1981; Kouba, Brown, Carpenter, Lindquist, Silver & Swafford, 1988; Yang, 1995).

This assessment of rational number concepts is by no means a comprehensive assessment of number sense related to fractions and decimals. However, items from the NST do provide a picture of students' knowledge and conceptions of fractions and decimals. They also reveal surprisingly consistent performance patterns among students in both the United States and Australia. The following discussion of the data from the number sense fraction and decimal items in Australia and the USA is shared to emphasise the importance of gathering this type of data, rather than data on computation ability which is an indicator of the effect of instructional and curricular emphases.

Tables 18 and 19 display items that were used to assess fraction and decimal understanding respectively in the Australian and the USA samples. Whenever identical items were given in both countries, those performances are also reported. In all cases, performance is reported as the percentage of students answering an item correctly. In some cases items were only given in one country, but those items are included here to provide a more complete picture of the variety of items used. The detailed results are in Appendix F. An examination of the results reveal several patterns. Below are a few of our observations.

Performance on items common to different grade levels always increased as students progressed from primary to elementary to junior high school.

This anticipated finding reflects the increasing attention given to fractions and decimals as students progress through school. One of the most dramatic increases in performance was shown by Australian 14-year-old students. There were consistently large increases in performance shown by 14-year-olds over the 12-year-old student performance. The increase was much greater than for any other two consecutive age groups, and also greater than the increases shown by American students in the same age span.

Performance on fraction items was lower than on parallel decimal items for all age groups in both countries.

For example, NST items 32 and 33 are equivalent yet performance on the decimal item was about twice as high as the parallel fraction item. The results for these two items are shown in Table 18.

Table 18: Percentage Scores on Locating Fractions and Decimals on a Number Line

	10-year-olds		12-year-olds	
Items	USA	Aus	USA	Aus
Item 32. Place the numbers 0.1 and 0.8 in their correct positions on this number line: 	57	74	80	86
Item 33. Place the numbers $^{1}/_{10}$ and $^{4}/_{5}$ in their correct positions on this number line: 	19	34	27	56

Items reflecting "typical" mathematics instruction (closely linked to the mathematics curriculum or textbook content) produce the highest performance level.

For example, ordering of fractions (see Appendix F, Item 36), shading a fractional region of a box (see Item 2), or denoting a position on a number line (see Item 35) were among the items with the highest performance levels. On the other hand, estimating with rational numbers (see Item 59), and generalising the effect of an operation on rational numbers (see Items 71 and 80) were difficult.

Items focusing on the density of rational numbers and the notion of "betweenness" were consistently difficult.

For example, about one-fifth and three-fifths of the Australian 12- and 14-year-olds respectively answered Item 57 correctly and their performance was higher than their American counterparts. Similar performance was observed on two items (Items 77 and 80) involving fractions to which only about half of the Australian 10- and 12-year-old students responded correctly.

A closer look at Item 57 shows that the most popular response for American 12- and 14-year-olds and Australian 12-year-olds to the question of how many decimal numbers are between 1.52 and 1.53 was "(a) None", indicating that they don't believe any decimals exist in this range. Only the Australian 14-year-olds had a majority (62 percent) responding correctly to this question.

A related item (58) explored the notion of "betweenness" for fractions. The question asks, "How many different fractions are there between $^2/_5$ and $^3/_5$?" The low percentage of correct response for all students (around 10 percent for 12-year-olds and 22 and 40 percent respectively for American and Australian 14-year-olds) documents that students' notion of the density of fractions is weak. The percentage of students selecting incorrect options provides the basis for many interpretations. Examination of student responses reveals an interesting perspective of students' thinking. Here is an excerpt of an interview of one student who reported that "lots" of fractions exist between $^1/_5$ and $^2/_5$.

I: Can you name fractions that would be between $^1/_5$ and $^2/_5$?

S: $^1/_6$ and $^1/_7$

I: How do you know those come in between $^1/_5$ and $^2/_5$?

S: Because if it's like $^1/_5$ to $^2/_5$, there is $^1/_6$ and $^1/_7$ in between.

I: OK. Could you draw a picture of 1/5 and 2/5. (Student draws number line and marks 1/5 and 2/5.)

1/5 2/5

I: You think 1/6 is in between those two. Can you show me where 1/6 is?

(Student makes additional marks on number line and labels them as shown below)

1/5 1/6 1/7 1/8 1/9 2/2 2/3 2/4 2/5

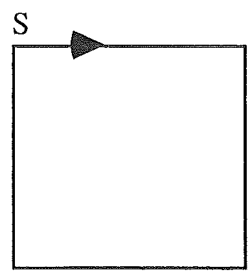
S: I put 1/6 here and make 1/7, it's like measuring, just start going up until you get 1/10 and then 1/9 and then you hit two and you put like two, well like you put two maybe or one, two or something, and then you put two three, and two four and then you get 2/5.

This student could not recall how this idea of fraction ordering had developed. At the least it is a vivid reminder that while her initial answer of "many" might be accepted as correct, the additional probing would lead to the conclusion that some fundamental misconceptions exist.

Items requiring construction of a fraction given a specified condition were difficult. For example, performance on Item 61 (Write a number in the box to make a fraction which represents a number between 2 and 3) ranged from a low of seven percent for American 12-year-olds to a high of 48 percent for Australian 14-year-olds. Likewise, representing a position one-third of the way around a square as represented in Table 19 was difficult for most students. Incorrect responses to this question highlight the lack of a benchmark with which to compare and make judgments about 1/3. In designing the item, we felt that students who used each side of the square to represent 1/4 would only need to be able to understand the relationship between 1/4 and 1/3 to respond correctly to the question. An alternative approach would be to think about the line making up the entire square as a "smooth" one and then to mentally section off the line into three equal segments. In fact, many of the incorrect responses related to using the vertices as "thirds" markers. In other words, think of standing at one corner of the square. What you see from this vantage point is three corners and each might be viewed (incorrectly) as representing a third of the square. Other students reasoned that the mark should go beyond the first corner but not as far as the second (halfway) corner. They then concluded that the $^1/_3$ mark should go halfway down the second side (be halfway between $^1/_4$ and $^1/_2$).

Table 19: Item 42 Testing Concept of Thirds and Quarters

You are going to walk once around a square-shaped field. You start at the corner marked **S** and move in the direction shown by the arrow. Mark with an **X** where you will be after $^1/_3$ of your walk.



Estimation items involving multiplication and division of rational numbers near zero or one revealed a consistent lack of conceptual understanding of either (or both) the numbers and/or the operations.

In particular, the set of four decimal items related to multiplying or dividing by a number near 1 or 0 is interesting, as may be seen in Table 20. Performance was consistently at or below the 50 percent level for the division items and also for the multiplication items, except for the 14-year-old Australian sample. Given the available choices, one would have thought that the item would be relatively easy. However, choosing a correct answer to these questions without calculating calls on an understanding of the numbers involved and also of the effect of the operation - both underlying components of number sense.

Table 20: Percentage Scores on a Sample of Estimation Items Involving Decimals

	10-year-olds		12-year-olds		14-year-olds	
Items	USA	Aus	USA	Aus	USA	Aus
Item 38. Without calculating the exact answer, circle the best estimate for: 29 x 0.98 A. More than 29 B. Less than 29 C. Impossible to tell without working it out	4	28	18	53	54	85
Item 64. Without calculating the exact answer, circle the best estimate for: 87 x 0.09 A. A lot less than 87. B. A little less than 87. C. A little more than 87. D. A lot more than 87.			16	51	50	82
Item 81. Without calculating the exact answer, circle the best estimate for: 54 ÷ 0.09 A. A lot less than 54. B. A little less than 54. C. A little more than 54. D. A lot more than 54.					24	48
Item 84. Without calculating the exact answer, circle the best estimate for: 29 ÷ 0.8 A. Less than 29. B. Equal to 29. C. Greater than 29. D. Impossible to tell without calculating.			20	21	36	49

Fraction and decimal items linked to a number line model were particularly difficult for the younger children (8- and 10-year-olds) surveyed.

This conclusion can be reached from the results for Items 10, 32, 33, 35, 40, 41, and 63 as seen in Appendix F. In fact, as suggested in the earlier excerpt of an interview with a student, much confusion seemed to be associated with the number line.

CHAPTER 4 Discussion of Results

We need to emphasise again that there was never any intention to compare performances between the four countries. In the cases of Sweden and Taiwan there was not a match with the other two countries regarding all the ages tested or the items used. However, the results from all four countries provide some interesting insights into the number sense of students. The following sections discuss aspects of the results from each of the four countries in turn.

Aspects of the Australian Study

One aspect of the results for Australian students was considered in detail. This was as follows: What can be said about the number sense of the 'average' Australian student?

It was decided to use, as a definition of 'average', those students whose overall results on the NST placed them in the middle quintile of their age group. If more than 75 percent of these students gave a correct answer to an item, this item was considered to be within the capabilities of the average student at that age. If 25 percent or less of these students gave a correct answer to an item, this item was considered not to be within the capabilities of the average student at that age. However, if the proportion correct was between 25 and 75 percent then these students were not considered for either category. While these boundaries are arbitrary, they enable some cautious generalisations to be made.

It must also be remembered that only a subset of the total bank of questions was given to any particular age-group. Thus there are questions given to younger age-groups which most of the older age group could have answered successfully, and questions given to older age-groups, which few of the younger age-group could have answered.

In endeavouring to answer the question about the number sense of average Australian students the results of students at age 10 and 14 only were considered. The 10-year-olds were seen as representing the primary or elementary school students, while the 14-year-olds were seen as representing the secondary school students in the sample.

Using the above criteria, the 'average' 10-year-olds showed that they could:

- arrange 5 digits to make the largest number possible;
- write a whole number one more than a given 5-digit number;
- place 0.1, 0.5 and 0.8 on a number line in relation to zero and one;
- express a shaded part of a whole as a simple fraction;
- name a subset of a set as a simple fraction;
- compare rates if the relationship is very simple;
- indicate the results of multiplying by zero and one; and
- understand the relationship between repeated addition and multiplication.

Using the above criteria, the 'average' 10-year-olds showed that they could not:

- place $^1/_{10}$ and $^4/_5$ on a number line;
- mark $^1/_3$ relative to $^1/_4$ on a number line;
- place 0.05 on a number line; and
- estimate products around 1000.

One might summarise these results as follows:

The average Australian 10-year-old has a reasonable understanding of notation and place value of whole numbers and numbers with one, but not two, places of decimals. Understanding of fractions is limited to representations of simple fractions as parts of a whole and subsets of a set, but not as points on a number line.

Using the above criteria, the 'average' 14-year-olds showed that they could:

- estimate a shaded portion of a rectangle as a decimal;
- place 2- and 3-decimal numerals on a number line;
- say how many 10-cent coins in an amount of money involving dollars and cents;
- name the largest of several fractions all with the same numerator;
- recognise fractions close to one half;

- compare rates if the relationship is simple;
- use commutativity of whole numbers;
- appreciate the effect of multiplying by a decimal less than one;
- recognise the equivalence of dividing by 2 and multiplying by 0.5; and
- recognise a reasonable estimate for the product of decimals.

Using the above criteria, the 'average' 14-year-olds showed that they could not:

- name fractions between $^2/_5$ and $^3/_5$;
- recognise with confidence relationships between $^2/_5$ and other fractions and representations;
- recognise fractions between $^3/_4$ and 1; and
- choose a best estimate of their age in days.

One might summarise these results as follows:

The average Australian 14-year-old has a good understanding of notation and place value and representations of whole numbers and up to at least two places of decimals. Understanding of simple fractions is limited and does not include awareness of the density of fractions, a secure understanding of common fraction notation or an awareness of the relative size of any but the simplest fractions. There is an awareness of the effect of multiplying by decimals less than one, and of relationships between multiplication and division involving decimals. Quantitative estimation is generally weak.

Aspects of the United States Study

The US student NST results are considered here in the same manner as for the Australian results in the previous section. Thus the "average students" were defined as those in the middle quintile; and these were considered capable of handling a particular item if 75 percent or more were successful, and not able to handle the item if only 25 percent or less were correct. While these definitions of average successful and unsuccessful students are somewhat arbitrary we believe they do allow for some useful generalisations.

Only the performance of 10-year-olds representing elementary school samples and the 14-year-olds representing the secondary school samples in the USA were considered here. Thus we followed the same procedure as for the Australian students.

Using the above criteria, the average 10-year-olds showed that they could:

- estimate the largest of three similar product expressions;
- express a shaded part of a whole as a simple fraction;
- name a subset of a set as a simple fraction;
- indicate the result of multiplying a whole number by one;
- indicate the result of multiplying by zero; and
- understand the relationship between repeated addition and multiplication.

Using the above criteria, the average 10-year-olds showed that they could not:

- place $^1/_{10}$ and $^4/_5$ on a number line;
- write a fraction between $^1/_2$ and 1;
- write the fraction nearest to $^1/_2$, given the numbers 3, 4, 9, 12;
- place 0.05 on a number line;
- select a point for 2.19 from given points on the number line;
- estimate whole number products around 1000; and
- estimate products of whole numbers and decimals such as 29 x 0.98.

We might summarise these results as follows:

The average US 10-year-old has a reasonable understanding of the operations with whole numbers and the multiplication properties of one and zero. Understanding of fractions is limited to representations of simple fractions as parts of a whole and subsets of a set, but not as points on a number line; and decimals are not well understood.

Using the above criteria, the average 14-year-olds showed that they could:

- understand the relationship between repeated addition and multiplication;
- place 0.5, 0.05 and 0.005 on a number line;
- name the largest of several fractions all with the same numerator;
- compare rates if the relationship is simple;
- use commutativity of whole numbers; and
- understand an increase of 50% and 100% of an amount.

Using the above criteria, the average 14-year-olds showed that they could not:

- say how many dimes in an amount of money involving dollars and cents;
- say how many decimals there are between 1.52 and 1.53;
- name fractions between $^2/_5$ and $^3/_5$;
- recognise fractions between $^3/_4$ and 1;
- estimate improper fractions on the number line;
- understand the relationship between multiplication and division using fractions or large numbers;
- determine the percentage increase when a score of 40 is raised to 50; and
- calculate an average speed.

One might summarise these results as follows:

The average US 14-year-old has a good understanding of the relationship between multiplication and addition but not of that between multiplication and division of large numbers or fractions. There is some understanding of simple rates and percentages. Understanding of simple fractions is limited and does not include awareness of the density of fractions, a secure understanding of common fraction notation or an awareness of the relative size of any but the simplest fractions. There is an understanding of decimal notation but no appreciation of the density of decimals.

Aspects of the Swedish Study[1]

Background

During the first decades of the Swedish compulsory schooling (1840-1880), primary education in mathematics was mainly directed at advancing skill with standard algorithms and applications on different types of problems (Johansson & Emanuelsson, 1994). At the end of the nineteenth century the value of understanding mathematical concepts and ideas was emphasised. In addition to the four basic operations, emphasis was placed on different ways of thinking, stressing the importance of basic number sense and being able to understand and use the relations between numbers (Johansson & Wistedt, 1991). This trend continued and deepened through the first half of the twentieth century. With the eventual entry of pocket calculators into Swedish society (and schools) during the middle of the 1970s a new discussion about non-algorithmic basic skills (and number sense) emerged. Ekenstam & Greger (1982) concluded that computation with the four rules of arithmetic did not seem to contribute to increasing the children's conceptual under-standing to any great extent (p. 42).

What should be the content of primary mathematics education if the four computational algorithms are de-emphasised? What do our children know about arithmetic outside the standard algorithms? These questions were the focus of a

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[1] Contributed by Göran Emanuelsson and Bengt Johansson

seminar during a five month period of Fulbright Visiting Professors Barbara and Robert Reys at Göteborg University during the spring of 1995, which also coincided with a two-week visit by Alistair McIntosh of Edith Cowan University. Together we reviewed and discussed the literature about number sense and connections of this knowledge to Swedish studies and the new national curriculum in mathematics (Emanuelsson & Johansson, 1996).

During the seminars tests on number sense developed by McIntosh, Reys & Reys (1992) were revised and extended. The tests were given to a Swedish student sample in grades 4 and 8 (10-year-olds and 14-year-olds) and the results were analysed and discussed. Results from this testing are included in Appendix F and some results are discussed elsewhere in the monograph.

This collaborative effort increased our consciousness about the importance of number sense and its relation to other parts of the Swedish national curriculum. We became convinced that there is much to be done to stimulate interest and engagement in the discussion of number sense among Swedish teachers. In order to invite and involve teachers in the discussion and study of children's development of number sense we prepared a series of articles for the journal *Nämnaren* (volume 22). The articles, published in Swedish, described different aspects of our work (Reys, Reys, Emanuelsson, Johansson et al., 1995a, 1995b; Reys, Reys & Emanuelsson, 1995). A summary of each article in the series is described here together with a report on its early impact.

What is *Nämnaren*?

Nämnaren is a Swedish journal for mathematics education. It is published in the Department of Didactics, Göteborg University. It has been issued four times a year since 1974. Target groups are teachers, teacher educators, and researchers working with education and developmental work in mathematics education, K–12. Its objectives are to:

- refine, publish, and comment on descriptions and results from teachers' approved experiences, from investigations and research studies in mathematics education;
- follow the international growth and development of mathematics education; and

- make contributions to the development of professional language and professional approaches in the field of mathematics education in Sweden.

Nämnaren is used in developmental work, inservice education, teacher education, master education, and in the development of syllabi and text-books for teacher education in mathematics. To facilitate its use a database is being developed where one can find information about volume, year, pages, title, author, abstract, references and key words. A short presentation of the content in the latest issue and a link to the database is given on the Internet at http://didserv.did.gu.se/matemati/senaste.htm

Nämnaren Series on Number Sense

A series of articles was developed to:

- provoke discussion and thinking about a topic stressed in the new Swedish curriculum and for which teachers seem to lack ideas and instruments for improving their instruction; and
- promote collaboration between researchers and practitioners. More specifically, to learn if a series of articles in a professional journal can stimulate communication and exchange of knowledge and ideas between researchers and practitioners, and whether people who are reading the articles . . . influence one another's thought and professional activities (Kilpatrick, 1991, p. 21).

The series consisted of three articles, all published within a year. A brief description of the process and some comments on the content in the three articles follows.

What is number sense?

The first article in *Nämnaren* 22(2) announced the series and provided background information regarding number sense. A sample of test items (18 items for 10-year-olds and 18 items for 14-year-olds) were included in the article (Reys, Reys, Emanuelsson, Johansson et al., 1995a, pp. 23-29). The items were chosen to provide a balance across the framework and identification of six strands of number sense (Reys et al., 1996):

- Understanding of the meaning and size of numbers;

- Understanding and use of equivalent representations of numbers;
- Understanding the meaning and effect of operations;
- Understanding and use of equivalent expressions;
- Computing and counting strategies; and
- Measurement benchmarks.

The items were included in the journal as a "pull-out" supplement, one section for grade 4 and another for grade 8. Guidelines and suggestions for using the test items were also provided. In the narrative of the article we described the 'history' of the tests. More specifically, we described the background and the construction of the tests and discussed the six major strands around which the instrument was constructed. We also described the process used to collect data from about 300 Swedish pupils using the number sense items as well as our observations in giving the test.

From observations and discussions with the Swedish pupils and their teachers we knew that the students were stimulated and challenged by the items. The teachers found the character of the test items somewhat unusual and thought that many of their students had little experience with these types of test items. We pointed out that the items were designed to stimulate reflection and thinking rather than skill in performing long and tedious calculations. We also discussed how traditional tests often focus on arithmetic skills and related applications. We questioned if such tests assess children's understanding of number and operations (e.g. different representations of numbers and relations within and between such representations (Emanuelsson, 1995)). We pointed out that there is evidence that students with excellent results on traditional paper-and-pencil tests may also show surprising weakness in number sense (Ekenstam & Greger, 1982; Sowder, 1992; Yang, 1995).

Within the article we invited readers to choose and try out items from the tests, to relate their own experiences of number sense to the strands and the items, to investigate the reactions of their pupils, and then compare their class results and analysis with the results and discussions we would present in the next issue of

Nämnaren. We pointed out that the development of good assessment tools should be in harmony with the development of good instruction. We invited readers to collaborate with us, welcoming their reactions, suggestions, and questions.

Swedish Student Performance on Number Sense.

In the second article in *Nämnaren* 22(3) we pointed out that almost every document describing improvement in mathematics education stresses the importance of children acquiring number sense (Reys, Reys, Emanuelsson, Johansson et al., 1995b). We admitted many questions remain to be answered regarding how to attain the goals we seem to agree on. There is a need for better tools to implement curricula and educational efforts. We raised questions, such as, Do you consider the development of number sense an important goal for instruction? What are the thinking and learning strategies of children developing number sense? What type of curriculum and what types of learning activities give the best effect on the development of number sense? By research and developmental work we suggested that answers to these important questions could be developed. As an example, we shared results of two aspects of the Swedish study – performance of Swedish 10-year-olds and 14-year-olds on the number sense test and beliefs regarding number sense of the teachers of the students tested. We also shared results from data collected using similar items in the United States and Australia.

Results of a Teacher Survey on Number Sense.

In connection with testing the fourth and eighth grade (10- and 14-year-old) students, teachers of the participating classes answered a questionnaire after having studied the number sense items. From the survey we found that number sense (as it was defined by the tests) was considered as being very important but that teachers did not spend much time on activities of this kind. At the end of the survey the teachers were asked to rank the items in terms of the level of difficulty for the actual grade. Their judgment showed a very high correspondence with student performance, especially in grade 4. Comparisons with data collected using similar items in other countries were also made. The article included a brief discussion of the common findings across countries related to the study including:

- Teachers in each country considered number sense important;

- Test items were regarded as new or different compared to current instruction and traditional curriculum;
- Agreement that much more attention to questions and problems of this type in mathematics classes is needed;
- There was a wide range in responses to the tests in every country;
- Boys had better results than girls but the differences were not always significant; and
- Performances of Swedish students were typically between the performance of students in Australia (highest performance) and the USA.

Meaningful Numbers.

The purpose of the third article in *Nämnaren* 22(3) was to reflect on the results of the test and to give some response to promote reflection on the importance and the role of the teacher in developing number sense (Reys, Reys & Emanuelsson, 1995). We discussed possible reasons for the results which included:

- shortage of relevant activities on number sense;
- instruction is dominated in focus and time by paper/pencil algorithms; and
- rote learning, with students practising mathematics in isolation or in situations devoid of conceptual understanding.

Once again we pointed out that number sense is not new but an important area for teaching and learning mathematics. In its simplest form good number sense means sense-making of mathematics. A person with good number sense has a flexible and rich knowledge of mathematics in terms of being able to use what has already been learned about numbers and develop different ways of looking at representations, non-routine situations and problems. We discussed the importance of the teacher's role in the development of number sense including:

- ***To promote the development of sense–making.*** Students often look toward the teacher as "the one who knows all the answers". One role in our work to support students' development is to help them understand that meaningful mathematical competence should be constructed in a continuous process. The mathematics must be individually internalised. To create meaning is to explore and invent associations, relations and pattern making, and test one's own statements.

- ***To create classroom settings where why (the meaning) is as important as what (the answer) or how (the method).*** We wrote that reflection is a process which is closely connected to good number sense. This competence includes ability and insight to control the reasonableness of an answer considering requirement and context. Every teacher in mathematics should aim for a classroom climate where reflection and evaluation are important elements in the work. The question "Is the answer reasonable?" ought to be used more frequently. It would be preferable to encourage students to justify their answers by describing their ways of thinking.

- ***To present activities that challenge and engage students to re-invent conceptions from different perspectives.*** One of the main reasons students do not develop number sense seems to be that they have not been invited to or engaged in mathematical sense making. There could be many reasons – for example, a shortage of activities and material that promote investigations in number sense; the first years of primary education in mathematics have been dominated by teaching facts and procedures; one believes that concepts are learned when algorithms are performed, yet fundamental relationships involving numbers and operations may remain mysterious for many children.

- ***To promote students' reflection on their own learning.*** All over the world the curricula in mathematics emphasise that learning mathematics is about searching for meaning, relevance and ideas to create and to justify new ways of thinking in different contexts. We meet many children using formulae and procedures with high accuracy in schools today. They are successful in school as long as they reproduce what they have learnt. This is what we mostly assess with written tests (Sowder, 1992).

To summarise, we emphasised that the development of number sense is an individual, lifelong, and complex process. The greater our knowledge of number sense and its importance, the more likely we will make a conscious effort to internalise number sense in our thinking and promote similar experience for our students. This was the primary purpose we had in mind in developing the series of articles in *Nämnaren*. We ended with an open invitation to all the readers to give us feedback, to contribute to the creation of activities, and to collaborate and exchange ideas for the development of this important area.

Some Results from the Series of Articles

The articles about number sense have been well received by the readers of *Nämnaren*. We got feedback and reactions from teachers, teacher educators, and people responsible for local and national development and research in mathematics education. A few samples of the feedback follow:

- Teachers have requested copies of the entire test (we published selected items in the pullout) to use in their classes or in school, among other things as a part of the assessment and evaluation every teacher is bound to execute.
- As part of an inservice teacher training program, teachers used the *Nämnaren* pullouts in all classes, and also made versions for use in other grades.
- We received reports that the number sense articles have been used in teacher training colleges throughout Sweden. One student group at our university has constructed a similar test for grade 10.
- The number sense articles have influenced the non-statutory guidance to the new Swedish curriculum of mathematics (Emanuelsson & Johansson 1996) and supplementary material financed by the National Agency for Education (Emanuelsson, Johansson et al., 1996). Our study has influenced work on the recently published materials for formative assessment in grade 2 and grade 7, and on the national summative test in mathematics for grade 5.
- The positive experiences and reactions to the number sense articles have encouraged us to edit articles about a similar Norwegian study, focused on decimal number sense, in the same way inviting our readers to analyse test items and give feedback (Brekke & Støren, 1995; Brekke, 1995; Brekke, 1996).
- The number sense articles have also been noticed by other researchers at Göteborg University, working with individual differences in cognitive functions (Gustafsson

& Undheim, 1996). During 1996, Swedish versions of the number sense tests have been given to 800 students in each of the grades 4, 6 and 8. The purpose is to test a model for hierarchical description of intelligence. Besides the number sense test the students will be given a test of the mother tongue, a spatial test, an inductive test, and a traditional summative classroom tests in mathematics. The design suggests interesting possibilities to compare the result of the number sense test with the outcome of the other instruments.

Summary and Discussion

From the evidence we have gathered, the series of articles has influenced many teachers' reflections on what mathematical knowledge and skills children should develop. There are indications that the *Nämnaren* series has stimulated and engaged readers from different professions with interest in and responsibility for the field of mathematics education. Curricula, syllabi, supplementary material, and research studies are often published as complete packages with no possibilities of participation or contribution in a dialogue with the authors. It is too often the exception that a scientific article has an enclosed concrete description of the activities and methods that have generated the empirical data.

These shortcomings make it very difficult for a teacher to value the scientific work – for example, relevance and validity from the teacher's point of view (Kilpatrick, 1995). In our opinion the kind of dialogue and openness characterising the number sense articles is necessary if teachers should be given such opportunities. We presented a project in progress and invited the readers to participate on their own terms.

We hope we have demonstrated respect for the teachers' ideas and concerns in order to create a dialogue between all those involved in influencing mathematics education (Firsov, 1995). We think that our effort has been a positive step forward in utilising a professional journal to bring research and practice a little closer. This experience has already encouraged us to begin exploring additional ways of creating interaction and dialogue among mathematics teachers in Sweden with *Nämnaren*. Perhaps sharing our experience will encourage similar efforts in other countries.

Aspects of the Taiwanese Study[1]

Number sense has received considerable attention in the United States, Australia, and other countries, yet number sense is not mentioned in the national mathematics curriculum in Taiwan. The phrase is rarely heard in discussions among mathematics educators, in mathematics classrooms or found in publications in Taiwan.

A review of textbooks and observations in classrooms in Taiwan indicates that mathematics education in Taiwan emphasises and rewards exact answers. This attention to exact answers may produce high computational performance, but the extent to which these processes transfer to number sense is unknown. McIntosh, Reys, & Reys (1992) state that "students highly skilled at paper/pencil computations (often the gauge by which success in mathematics is measured) may or may not be developing number sense" (p. 3). The research described in this section was designed to explore the extent of that relationship.

Purpose

The purpose of this research was to:

- examine the relationship between the computational skills of Taiwanese students and their level of number sense;
- investigate the general level of number sense possessed by students in grades 6 and 8 (ages 12 and 14) in Taiwan; and
- identify the difference in the level of number sense associated with the grade level of the students.

Instruments

A twenty-item Written Computation Test (WCT) was constructed by the researcher for 12-year-olds and 14-year-olds. All of the test items were displayed in the native

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1 Contributed by Der Ching Yang

language of Taiwanese students. The WCT items were open ended and consistent with the Taiwanese national mathematics curriculum.

A forty-item Taiwan Number Sense Test (TNST) was also constructed by the researcher. The TNST drew heavily on items used in the Australian and United States testing described elsewhere in this monograph and also included some items which were used by other researchers (Carpenter, Corbitt, Kepner, Lindquist & Reys, 1980; Markovits & Sowder, 1994; Reys, Reys, Emanuelsson, Johansson, Maerker & Rosen, 1995). The same test was used for both grades. The first 20 items of the TNST were developed to parallel the WCT Items. That is, the same numbers were involved but the formats of the related items were different.

Discussion of Results

This research provides strong evidence that Taiwanese students perform at very different levels on written computation and number sense tests. Basically, the number sense performance level for both 12-year-old and 14-year-old students in Taiwan was weaker than their written computation performance.

When students were asked to use paper-and-pencil to find an exact answer, the performance level was significantly higher than when they were asked parallel questions involving number sense. More specifically, it seems that Taiwanese students highly skilled in paper-and-pencil computation are not equally skilled in their use of non-computational methods which rely on number sense to solve similar problems.

Some examples show a sharp difference between students' written computation performance and number sense ability. For example, Table 21 shows the percentages of the same cohorts of students giving each response on two parallel items, one each from the WCT and the TNST respectively. Whereas 61 percent of the 12-year-old students and 63 percent of the 14-year-old students could correctly calculate $^{12}/_{13} + ^{7}/_{8}$, only 25 percent and 38 percent respectively of these same students gave 2 as the best estimate for the computation.

Table 21: Results of Two Parallel Fraction Items from the TNST and WCT

TNST Item:			WCT Item:		
Without calculating an exact answer, circle the best estimate for $^{12}/_{13} + ^{7}/_{8}$			Calculate $^{12}/_{13} + ^{7}/_{8}$		
TNST Results (Percentages):			WCT Results (Percentages):		
	Age 12	Age 14		Age 12	Age 14
A. 1	10	20	Correct	61	63
B. 2	25	38	Incorrect	39	37
C. 19	36	14			
D. 21	16	12			
E. I don't know	10	16			
F. No response	3	0			

Table 22 shows the results of another pair of parallel items from the WCT and the TNST. Whereas 61 percent of the 12-year-old students and 71 percent of the 14-year-old students could correctly calculate 534.6 x 0.545, only 11 percent and 25 percent respectively of these same students could place the decimal point correctly in the answer by inspection.

Table 22: Results of Two Parallel Decimal Items from the TNST and WCT

TNST Item:			WCT Item:		
The following multiplication problem has been carried out except for placing the decimal point. Place the decimal point using estimation: 534.6 x 0.545 = 291357			Calculate 534.6 x 0.545		
TNST Results (Percentages):			WCT Results (Percentages):		
	Age 12	Age 14		Age 12	Age 14
A. 29.1357	87	66	Correct	61	71
B. 291.357	11	25	Incorrect	39	29
C. Other answers	2	9			

In both cases student performance on the WCT item was significantly higher than on the parallel TNST item. This result indicates that many students could find an exact answer by using paper-and-pencil methods, but they lacked understanding of the procedures that they were using.

The results also indicated that student understanding of the "density" of numbers was markedly higher for decimals than for fractions. For example, Table 23 shows that 32 percent of the 12-year-old students and 78 percent of the 14-year-old students were aware that there were 'lots' of decimals between 1.42 and 1.43, but only 11 percent and 35 percent respectively of these same students were aware that there were 'lots' of fractions between $^2/_5$ and $^3/_5$.

Table 23: Response Percentages on Items Involving "Density" of Fractions and Decimals

TNST Decimal Item:			TNST Fraction Item:		
How many different decimals are there between 1.42 and 1.43?			How many different fractions are there between $^2/_5$ and $^3/_5$?		
TNST Decimal Results:			TNST Fraction Results:		
	Age 12	Age 14		Age 12	Age 14
A. None	36	5	A. None	60	40
B. One	10	5	B. One	8	4
C. A few	5	3	C. A few	6	3
D. Lots	32	78	D. Lots	11	35
E. No response	17	9	E. No response	15	18

The overall impression given by the results is that Taiwanese students perform well at pencil and paper calculations involving exact answers, but that the acquisition of this competence does not necessarily produce number sense.

Implications

This study has involved research into number sense in four countries through a series of collaborative efforts initiated by Professors Bob and Barbara Reys at the University of Missouri - Columbia. It has involved three studies, one in the United States and Australia, one in Sweden, and one in Taiwan; each of which has had a different emphasis. The study in the United States and Australia was broadly based, involving some 1100 students in eight schools and seeking to arrive at a general picture of number sense among populations of students aged 8, 10, 12 and 14 years. The Swedish study put particular emphasis on the involvement of teachers and the professional development activity initiated through the mathematics education journal *Nämnaren*. The Taiwanese study focussed particularly on probing the misunderstandings which existed despite apparent computational facility.

In spite of these differences of approach and intent the three studies shared one major element: all three sought to assess number sense through pencil-and-paper testing. This has both strengths and weaknesses. There is a growing revolt, both in the United States (where perhaps the worst excesses of multiple choice testing have flourished) and elsewhere, against pencil-and-paper testing as a means of assessing students. At the same time, while electronic means are being looked at, a paper-and-pencil approach is the only way currently available for gathering a lot of information quickly and cheaply.

There is no doubt that on many occasions when the researchers looked at the answers given by individual students, or the overall responses of a cohort of students to an item, their first response was: "I wish we could probe that further through interviews". When some interviews with American students were conducted, their responses were illuminating. For example, a student responded that there were 'lots' of fractions between $^1/_5$ and $^2/_5$; but when invited in an oral interview to name some, the student offered $^1/_6$ and $^1/_7$ as examples.

It may well be that an individual student's performance can only partially be assessed by group pencil-and-paper testing. However the purpose of these studies was not to focus on individual results, but to see what could be gathered about the state of number sense over wider populations. We believe that the data gathered does allow us to state some conclusions with confidence, while also pointing to many areas which would prove interesting to probe further whether by pencil-and-paper tests or other means. We have considerable confidence because in so many cases the data provided similar evidence from two or more countries.

However more research is needed into the extent to which pencil-and-paper tests assess number sense, and into extending assessment mechanisms so as to gather richer information which can be of direct use to teachers as well as researchers and administrators.

It does appear that whatever we are testing (what we have called 'number sense') does develop with age. Almost every item which was given to more than one age group had a higher percentage of correct answers amongst the older cohorts (one exception was Item 1 which asked students to select the number of days they had lived). Whether this improvement happens because of school, in spite of school, or regardless of school is a matter which merits further study. If we accept that schooling does have some effect, then it would be valuable to discover which aspects of schooling, or which pedagogical approaches have a major beneficial influence on the growth of number sense.

All the studies suggest, and the Taiwanese study pinpoints most effectively, the fact that number sense does not necessarily grow hand in hand with, or develop as a consequence of, the development of technical expertise in formal written computation. The emphasis on developing standard written algorithms for dealing with whole numbers, decimals and fractions, which still pervades almost all schools at the ages tested, does not appear to bring with it a practical understanding of place value, an ability to estimate quantity or an instinctive and true feeling for the nature of fractions or decimals. When we have discussed these data with teachers they agree that what is lacking is important, and they also agree that their teaching has not traditionally focussed on these aspects. We have since tried to provide some practical help in this regard by publishing four books which focus on activities to develop number sense at different age levels (McIntosh, Reys & Reys, 1997). Certainly more curriculum development, case studies and action research are needed to develop effective practices.

Since no students in any age group responded to more than 45 items, it is inevitable that the data is thinly spread over the whole arena of what constitutes number sense. There must be many details, and some broad areas of territory, which are not adequately covered by the items given in this study. Nevertheless we believe that the process followed in this study of working from our original number sense framework, to a delineation of six strands, and from these to the development of items, gives some degree of confidence that there are no major omissions. The main omission from the American/Australian study was that of measurement benchmarks (Strand 6) which we had initially considered to be outside our definition of number sense. However, the Swedish research team thought otherwise and persuaded us that our original description was too narrow.

The fact that in the American/ Australian study we tested the mental computation ability and the number sense of the same cohorts of students meant that we were able to look for the correlation between the two abilities. The results suggest that the two abilities are indeed linked, particularly after the age of 12. There are at least two implications which arise from this: first, it would appear that one way to develop number sense is to develop mental computation ability, giving it greater prominence at the expense of formal written computation. Second, more emphasis should be given to the assessment of mental computation when assessing students' mathematical ability whether at school, local or national level. This is of particular significance in Australia in view of the current moves to assess numeracy at state and federal level. How mental computation should be assessed is discussed in a previous monograph (McIntosh, Bana & Farrell, 1995). We argue there that mental computation can be assessed by group pencil-and-paper tests, provided that students write only the answer, have about 20 seconds for each item and, preferably that the calculations are read out orally and not seen or written down by the students.

Finally, we have been heartened and excited by the degree to which teachers have participated with enthusiasm in all the studies, have accepted the area of investigation as an important and relevant aspect of their work, and have been eager to have the results, to discuss them and to seek ways of working on the implications of the results in their classrooms. This was shown very clearly in the Swedish study. The development of number sense appears to be potentially a very fruitful area of collaboration between teachers, researchers and curriculum developers.

References

Australian Education Council. (1991). *A national statement on mathematics for Australian schools.* Melbourne: Curriculum Corporation.

Bana, J. & Korbosky, R. (1995) *Children's knowledge and understanding of basic number facts.* Perth: MASTEC, Edith Cowan University.

Behr, M.J., Harel, G., Post, T. & Lesh, R. (1992). Rational number, ratio and proportion. In Douglas A. Grouws (Ed.) *Handbook of research on mathematics teaching and learning* (pp. 115-126). New York: Macmillian.

Bezuk, N.S. & Bieck, M. (1993). Current research on rational numbers and common fractions: Summary and implications for teachers. In Douglas T. Owens (Ed.) *Research ideas for the classroom: Middle grades mathematics* (pp. 118-136). Reston, VA: National Council of Teachers of Mathematics.

Brekke, G. (1995). Oppfatninger av desimaltall. [Conceptions of decimal numbers]. *Nämnaren, 22(*4), 27-34.

Brekke, G. (1996). Regning med decimaltall. [Calculation with decimal numbers]. *Nämnaren, 23(*1), 16-20.

Brekke, G. & Støren, H. (1995). Kvalitet i matematikundervisningen. [Quality in mathematics instruction]. *Nämnaren, 22(*3), 10-14.

Brownell, W.A. (1935). Psychological considerations in the learning and the teaching of arithmetic. In *The teaching of arithmetic: The tenth yearbook of the National Council of Teachers of Mathematics.* New York: Teachers College, Columbia University. 1-31.

Brownell, W.A. (1945). When is arithmetic meaningful? *Journal of Educational Research, 38,* 481-498.

Burns, M. (1994). Arithmetic: The last holdout. *Phi Delta Kappan,* (February), 471-476.

Burton, G. (1993). *Number sense and operations.* Reston, VA: National Council of Teachers of Mathematics.

Carpenter, T.P., Coburn, T.G., Reys, R.E. & Wilson, J.W. (1978). *Results from the first mathematics assessment of the national assessment of educational progress.* Reston, VA: National Council of Teachers of Mathematics.

Carpenter, T.P., Corbitt, M.K., Kepner, H.S., Lindquist, M.M. & Reys, R.E. (1981). *Results from the second mathematics assessment of the national assessment of educational progress,* Reston. VA: National Council of Teachers of Mathematics.

Case, R. & Sowder, J.T. (1990). The development of computational estimation: A neo-Piagetian analysis. *Cognition and Instruction, 7*, 79-104.

Cockcroft, W.H. (1982). *Mathematics Counts*. London: HMSO.

Cramer, K., Post, T. & Currier, S. (1993). Learning and teaching ratio and proportion: Research implications. In Douglas T. Owens (Ed.) *Research ideas for the classroom: Middle grades mathematics* (pp. 159-178). Reston, VA: National Council of Teachers of Mathematics.

Ekenstam, A. (1977). On children's quantitative understanding of numbers. *Educational Studies in Mathematics, 8*, 317-332.

Ekenstam, A. & Greger, K. (1982). Non-algorithmic basic skills. *Journal für Matematik-Didaktik, 3*(1), 19-44.

Emanuelsson, G. (1995). Språk, symboler och uttrycksformer. [Language, symbols and forms of expression]. *Nämnaren 22*(2), 2-3.

Emanuelsson, G. & Johansson, B. (1996). *Kommentar till kursplan och betygskriterier I matematik, Lpo 94.* [Non-statutory guidance to the new Swedish curriculum of mathematics and the new marking system]. Stockholm: Liber Distribution.

Emanuelsson, G., Johansson, B., Reys, B.J. & Reys, R.E. (1996). *Assessing the development of number sense: Using a journal to engage Swedish teachers in developmental work.* Paper presented at the Research Pre-session of the National Council of Teachers of Mathematics Annual Meeting in San Diego, California, April, 1996.

Firsov, V. (1995). Mathematics education as theoretical knowledge. *Nordic Studies in Mathematics Education 3*(4), 7-19.

Greeno, J. G. (1991). Number sense as situated knowing in a conceptual domain. *Journal for Research in Mathematics Education, 22*(3), 170-218.

Gustafsson, J.E., & Undheim, J.O. (1996). Individual differences in cognitive functions. In D. C. Berliner & R. C. Calfee (Eds.), *Handbook of educational psychology* (pp. 186-242). New York: Macmillan.

Hiebert, J. (1984). Children's mathematics learning: The struggle to link form and understanding. *The Elementary School Journal, 84*, 496-513.

Hope, J.A., & Sherrill, J.M. (1987). Characteristics of unskilled and skilled mental calculators. *Journal for Research in Mathematics Education, 18*(2), 98-111.

Howden, H. (1989). Teaching number sense. *The Arithmetic Teacher, 36*(6), 6-11.

Japanese Ministry of Education. (1989). *Curriculum of mathematics for the elementary school.* Tokyo: Printing Bureau.

Johansson, B., & Emanuelsson, G. (1994). "Begrundelseproblemet" i den elementära matema-tikundervisningen i Sverige. [Justifications of primary mathematics education in Sweden]. In G. Emanuelsson, B. Johansson, B. Rosén & R. Ryding (Eds.), *Dokumentation av den 8:e Matematikbiennalen.* Institutionen för ämnesdidaktik, Göteborg Universitet, pp. 11:3-11:6.

Johansson, B., & Wistedt, I. (1991). Tal och räkning – ett historiskt perspektiv. [Number and arithmetic – a historical perspective]. In G. Emanuelsson, B. Johansson, & R. Ryding (Eds.), *Tal och räkning 1* (pp. 28-44). Lund: Studentlitteratur.

Kamii, C. (1989). *Young children continue to reinvent arithmetic, second grade: Implications of Piaget's theory.* New York: Teachers College Press.

Kilpatrick, J. (1991). Scattering, storing, shaping: Journals in Mathematics Education. *Nämnaren 18(3/4),* 16-23.

Kilpatrick, J. (1995). Staking claims. *Nordic Studies in Mathematics Education 3*(4), 21-42.

Kouba, V.L, Brown, C.A., Carpenter, T.P., Lindquist, M.M., Silver, E.A. & Swafford, J.O. (1988). Results of the fourth NAEP assessment of mathematics: Number, operations and word problems. *Arithmetic Teacher, 35*(8), 14-19.

Levine, D. R. (1982). Strategy use and estimation ability of college students. *Journal for Research in Mathematics Education, 13*(5), 350-359.

Markovits, Z. & Sowder, J. (1994). Developing number sense: An intervention study in grade 7. *Journal for Research in Mathematics Education, 25*(1), 4-29.

McIntosh, A., Bana, J. & Farrell, B. (1995). *Mental computation in school mathematics: Preference, attitude and performance of students in years 3, 5, 7 and 9.* Perth: MASTEC, Edith Cowan University.

McIntosh, A.J., Nohda, N., Reys, B.J. & Reys R.E. (1995). Mental computation performance in Australia, Japan and the United States. *Educational Studies in Mathematics,* 29, 237-258.

McIntosh, A.J., Reys, B.J. & Reys, R.E. (1992). A proposed framework for examining number sense. *For the Learning of Mathematics 12*(3), 2-8.

McIntosh, A. Reys, B. & Reys, R. (1997). *Number SENSE grades 1-2.* Palo Alto: Dale Seymour Publications.

McIntosh, A. Reys, B. & Reys, R. (1997). *Number SENSE grades 3-4.* Palo Alto: Dale Seymour Publications.

McIntosh, A. Reys, B., Reys, R. & Hope, J. (1997). *Number SENSE grades 4-6.* Palo Alto: Dale Seymour Publications.

McIntosh, A. Reys, B. & Reys, R. (1997). *Number SENSE grades 6-8.* Palo Alto: Dale Seymour Publications.

Narode, R., Board, J. & Davenport, L. (1993). Algorithms supplant understanding: Case studies of primary students' strategies for double-digit addition and subtraction. In J. Rossi Becker & B.J. Pence (Eds.) *Proceedings of the Fifteenth Annual Meeting, North American Chapter of the International Group for the Psychology of Mathematics Education,* Vol. I.

National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: NCTM.

Owens, D.T. & Super, D.B. (1993). Teaching and learning decimal fractions. In Douglas T. Owens (Ed.) *Research ideas for the classroom: Middle grades mathematics* (pp. 137-158). Reston, VA: National Council of Teachers of Mathematics.

Plunkett, S. (1979). Decomposition and all that rot. *Mathematics in School 8*(3), 2-5.

Resnick, L. B. (1989). Defining, assessing and teaching number sense. In J. Sowder & B. Schappelle (Eds.), *Establishing foundations for research on number sense and related topics: Report of a conference.* San Diego, CA: San Diego State University, Center for Research in Mathematics and Science Education.

Reys, B.J., Barger, R., Dougherty, B., Hope, J., Lembke, L., Markovits, Z., Parnas, A., Reehm, S., Sturdevant, R., Weber, M., & Bruckheimer, M. (1991). *Developing number sense in the middle grades.* Reston. VA: NCTM.

Reys, B.J., Reys, R.E., & Emanuelsson, G. (1995). Meningsfulla tal. [Meaningful numbers]. *Nämnaren 22*(4), 8-12.

Reys, B.J., Reys, R.E., Emanuelsson, G., Johansson, B., et al. (1995a). Vad är god taluppfattning? [What is number sense?] *Nämnaren 22*(2), 23-29.

Reys, B., Reys, R., Emanuelsson, G., Johansson, B., et al. (1995b). Svenska elevers tal-uppfattning. [Swedish student performance on number sense]. *Nämnaren 22*(3), 34-40.

Reys, B.J., Reys, R.E., McIntosh, A., Emanuelsson, G., Johansson, B., & Yang, D.C. (1996). *Assessing number sense of students in Australia, Sweden, Taiwan and the United States.* (Manuscript submitted for publication.)

Reys, B. J., Reys, R. E., Nohda, N. & Emori, H. (1995). Mental computation performances and strategy use of Japanese students in grades 2, 4, 6 and 8. *Journal for Research in Mathematics Education, 26*(4), 304-326.

Reys, R.E., Rybolt, J.F., Bestgen, B.J., & Wyatt, J.W. (1982). Processes used by good computational estimators. *Journal for Research in Mathematics Education, 13*(3), 183-201.

Rubenstein, R.N. (1983). Mathematical variables related to computational estimation. *Dissertation Abstracts International, 44*, 695A. (University Microfilms No. 83-06, 935).

Skemp, R. R. (1982). Understanding the symbolism of mathematics [Special Issue]. *Visible Language, 16*(3).

Sowder, J.T. & Schappelle, B.P. (Eds.). (1989). *Establishing foundations for research on number sense and related topics: Report of a conference.* San Diego, CA: San Diego State University, Center for Research in Mathematics and Science Education.

Sowder, J.T. (1992). Estimation and number sense. In D. A. Grouws (Ed), *Handbook of research on mathematics teaching and learning* (pp. 371-389). New York: Macmillan.

Sowder, J. T., & Wheeler, M. M. (1989). The development of concepts and strategies used in computational estimation. *Journal for Research in Mathematics Education. 20*(2), 130-146.

Turner, B. (1996). *A Sixth grade teacher's beliefs about her students' number sense.* Unpublished paper. Department of Curriculum and Instruction, University of Missouri.

Willis, S. (Ed). (1990). *Being numerate: What counts?* Hawthorn, Victoria: Australian Council for Educational Research.

Yang, D. C. (1995). *Number sense performance and strategies possessed by sixth and eighth grade students in Taiwan.* Unpublished doctoral dissertation, University of Missouri: Columbia.

APPENDIX A

Number Sense Framework

The original framework for analysing number sense is illustrated below. An overall definition of number sense is followed by three columns, in which the components of number sense are analysed in greater detail moving from left to right. For a full description and analysis see McIntosh, Reys and Reys (1992).

Number Sense: A propensity for and an ability to use numbers and quantitative methods as a means of communicating, processing and interpreting information. It results in an expectation that numbers are useful and that mathematics has a certain regularity (makes sense).

- 1. Knowledge of and facility with NUMBERS
  - 1.1 Sense of orderliness of numbers
    - 1.1.1 Place value
    - 1.1.2 Relationship between number types
    - 1.1.3 Ordering numbers within and among number types
  - 1.2 Multiple representations for numbers
    - 1.2.1 Graphical/symbolic
    - 1.2.2 Equivalent numerical forms (including decomposition/ recomposition)
    - 1.2.3 Comparison to benchmarks
  - 1.3 Sense of relative and absolute magnitude of numbers
    - 1.3.1 Comparing to physical referent
    - 1.3.2 Comparing to mathematical referent
  - 1.4 System of benchmarks
    - 1.4.1 Mathematical
    - 1.4.2 Personal
- 2. Knowledge of and facility with OPERATIONS
  - 2.1 Understanding the effect of operations
    - 2.1.1 Operating on whole numbers
    - 2.1.2 Operating on fractions/decimals
  - 2.2 Understanding mathematical properties
    - 2.2.1 Commutativity
    - 2.2.2 Associativity
    - 2.2.3 Distributivity
    - 2.2.4 Identities
    - 2.2.5 Inverses
  - 2.3 Understanding the relationship between operations
    - 2.3.1 Addition/Multiplication
    - 2.3.2 Subtraction/Division
    - 2.3.3 Addition/Subtraction
    - 2.3.4 Multiplication/Division
- 3. Applying knowledge of and facility with numbers and operations to COMPUTATIONAL SETTINGS
  - 3.1 Understanding the relationship between problem context and the necessary computation
    - 3.1.1 Recognise data as exact or approximate
    - 3.1.2 Awareness that solutions may be exact or approximate
  - 3.2 Awareness that multiple strategies exist
    - 3.2.1 Ability to create and/or invent strategies.
    - 3.2.2 Ability to apply different strategies
    - 3.2.3 Ability to select an efficient strategy
  - 3.3 Inclination to utilise an efficient representation and /or method
    - 3.3.1 Facility with various methods (mental, calculator, paper/pencil)
    - 3.3.2 Facility choosing efficient number(s)
  - 3.4 Inclination to review data and result for sensibility
    - 3.4.1 Recognise reasonableness of data
    - 3.4.2 Recognise reasonableness of calculation

APPENDIX B

Number Sense Strands

Appendix B shows a later analysis of number sense than that contained in Appendix A. The six strands have developed from the three components of number sense - namely numbers, operations and computational settings, contained in the left hand column of Appendix A: strands 1 and 2 relate to number, strands 3 and 4 to operations and strands 5 and 6 to computational settings.

This analysis proved more practical as a working definition in the formulation and classification of number sense items for the tests and for the later development of the number sense item bank. Sample items are given after the description of each strand below.

1. Understanding of the meaning and size of numbers (Number Concepts).

Understanding of the base ten number system (whole numbers, fractions, and decimals), including patterns and place value which provide clues to the meaning/size of a number (e.g., $^5/_6$ is a fraction less than one, it is close to one because of the relationship between the numerator and denominator, or 1000 is a large number if you are referring to the population of a school but a small number if you are referring to the population of a town). Could involve relating and/or comparing numbers to standard or personal benchmarks. Includes comparing the relative size of numbers within a single representational form.

Sample item (Age 14)

How many different fractions are there between $^2/_5$ and $^3/_5$?	A. None. Why? ______________ B. One (Name it: __________) C . A few (Name two: __________) D. Lots (Name two: __________)

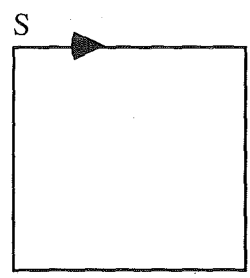
Sample item (Age 10)

Oscar is born today. In what year will he be 100 years old?	_____

2. Understanding and use of equivalent forms and representations of numbers (Multiple Representations).

Recognition that numbers take many different numerical and representational forms (e.g., fraction as a decimal, a whole number in expanded form, or a decimal on a number line) and can be thought about and manipulated in many ways to benefit a particular purpose. Ability to identify and/or reformulate numbers to produce an equivalent form. Use of decomposition and recomposition to reformulate numbers for ease in processing. Relating and/or comparing size of number(s) to a physical referent (e.g. collection of items, shaded region, or position on a number line). Includes crossing among various representational forms.

Sample item (Ages 10-14)

You are going to walk once around a square-shaped field. You start at the corner marked S and move in the direction shown by the arrow. Mark with an X where you will be after $1/3$ of your walk.	

Sample item (Age 14)

Circle <u>all</u> the statements that are <u>true</u> about the number $2/5$.	A. It is greater than $1/2$ B. It is the same as 2.5 C. It is equivalent to 0.4 D. It is greater than $1/3$ E. It is less than $1/4$

3. Understanding the meaning and effect of operations (Effect of Operations).

Understanding the meaning and effect of an operation either generally or as it relates to a certain set of numbers (e.g., division means breaking a number into a specified number of equivalent subgroups, or multiplying by a number less than 1 produces a product less than the other factor). Includes judging the reasonableness of a result based on understanding the numbers and operations being employed.

Sample item (Age 10)

When a 3-digit number is added to a 3-digit number, the result is:	A. Always a 3-digit number. B. Always a 4-digit number. C. Always a 5-digit number. D. Either a 3, 4, or 5-digit number. E. Either a 3 or 4-digit number

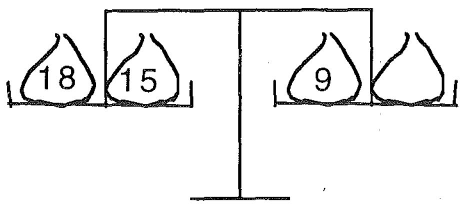
Sample item (Age 14)

Without calculating the exact answer, circle the best estimate for: 87 x 0.09	A. a lot less than 87 B. a little less than 87 C. a little more than 87 D. a lot more than 87

4. Understanding and use of equivalent expressions (Equivalent Expressions)

Translation of expressions to equivalent forms. Generally used to re-evaluate and/or more efficiently process computation. Includes understanding and use of arithmetic properties (commutativity, associativity, distributivity) to simplify expressions and develop solution strategies (e.g., use of distributive property to multiply 7 x 52).

Sample item (Age 10)

 Barbara has balanced some bags of marbles. The numbers show how many marbles are in the bags. How many marbles are in the unmarked bag? Circle your answer.	A. 6 B. 9 C. 15 D. 24 E. 42

Sample item (Age 14)

93 x 134 is equal to 12462. How much larger than 12462 is 93 x 135?	A. 92 B. 93 C. 134 D. 135 E. Impossible to tell

5. Computing and Counting Strategies

Applying various number sense components previously described in the formulation and implementation of a solution process to a counting or computational (estimation, mental computation, paper/pencil, calculator) situation (e.g., Is 29 x 38 more or less than 400?, or How many birds are in the sky?).

Sample item (Ages 8-14)

About how many days have you lived? Circle your answer.	A. 300 B. 3 000 C. 30 000 D. 300 000

6. Measurement Benchmarks

Applying various number sense components previously described in the formulation and implementation of a solution process to a measuring situation. Requires an understanding and use of standard, non-standard and/or personal benchmark units of measure (e.g., a textbook weighs about a kilogram or 5 oranges make a pound, or the angle is slightly less than a right angle so it must be about 85 degrees). Involves measuring attributes such as mass, length, capacity, volume, time, and angles.

Sample item (Age 10)

What is the weight of a man who is 180 cm tall? Ring your answer.	A. 10 kg B. 40 kg C. 80 kg D. 180 kg

APPENDIX C

Test Administration Protocol

Instructions for Number Sense Test Administrator - Years 3, 5, 7, 9 (Ages 8, 10, 12, 14)

1. Introduce yourself to the class:

- Say, "I am ------ from Edith Cowan University. Last term I gave you a test in mental maths The same test has been given to children in Japan and the USA. Today I have another short test for you on number. This will also be given in Japan and the USA. The results are not for your school marks, but try your hardest on each question."

2. Personal details:

- Have students clear their desk and get a pen/pencil ready.
- Give out test papers face down, then ask students to turn the paper over and have them write in their name and other details as required.

3. Preparation for test:

- Tell students: "Today I want you to do the maths problems mentally. That is, do all the calculations (working) in your head. Only write the answer or circle the right answer. Don't do any other writing. In many questions you will be asked to estimate rather than calculate the answer exactly. Be sure to follow those directions. I will read each question while you follow me. Then I'll give you half a minute - 30 seconds - to do it, before asking you to go on to the next question.

4. Practice questions:

- Put OHT of cover page on OHP and say: "The left side of the page has the questions and the right side is where you show your answers. I now want you to try the first practice question. I'll read it for you while you follow on your sheet, and when I finish reading you'll have half a minute - 30 seconds - to do it". Read the question out loud. Allow half a minute. Indicate and justify the correct answer and the need to circle the matching letter rather than the whole answer.

- Say: "Now we'll try the second practice question". Read the question out loud, allow half a minute, then indicate and explain the correct response and how this was to be recorded.

- Say: "There are 30/35/45 questions in this test and they are all set out like these two. I'll read each question and then allow half a minute - 30 seconds. This should be plenty of time for each question. If you make a mistake cross it out and try again. Don't forget to only write the answer - no other writing is allowed. Are there any questions about the test?" Answer as appropriate.

5. The test:

- Say: "Now we are ready to start. Turn to the next page. Question 1 says . . .". Read the question out loud, making sure to emphasise underscored words, and allow 30 seconds. Then say: "Question 2 . . ."; and so on until the test is complete.

APPENDIX D

Extract of 10-Year-Olds' Test

Number Sense Group Test - Year 5

Name: ______________________ **School:** ______________________

(first) (last)

Year: _____ **Sex:** ______ **Teacher:** ______________________

Practice Questions:

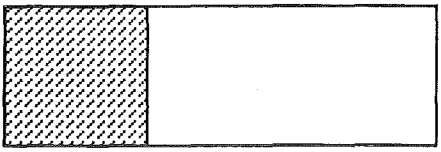
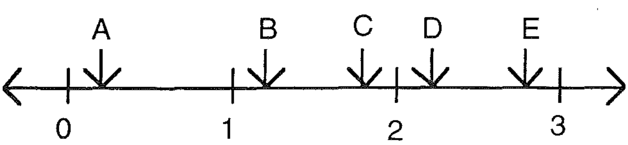
1.	Without counting exactly, about how many children are there in your class? (Circle the nearest answer.)	A 3 B 30 C 300 D 3000
2.	What number goes in the box to make this sentence true? 30 + ☐ = 50	_______________

DO NOT turn over the page until you are told.

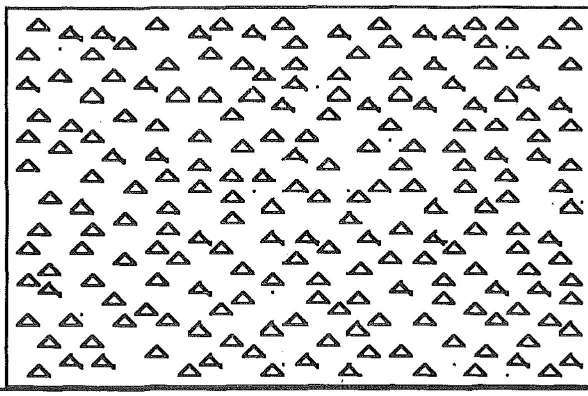
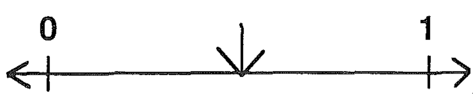
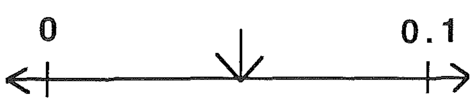
DO NOT write anything except your answer.

There are 35 questions. You will have 30 seconds for each question.

NUMBER SENSE GROUP TEST - YEAR 5

QUESTION	ANSWER
1. For a long time Jane has been putting only 10 cent coins in her piggy bank. Last night she opened it and counted her money. She had \$46.70. How many 10 cent coins were in the bank?	____________________
2. <u>About</u> how much of this box is shaded? Give your answer as a <u>fraction</u>. 	____________________
3. Place the numbers 0.1 and 0.8 in their correct positions on this number line.	
4. Place the numbers $\frac{1}{10}$ and $\frac{4}{5}$ in their correct positions on this number line.	
5. $\frac{3}{4}$ is a fraction between $\frac{1}{2}$ and 1. Write another fraction between $\frac{1}{2}$ and 1.	____________________
 6. Which point on the number line above best represents 2.19?	____________________

7.	Circle the fraction which represents the largest amount:	A $\frac{5}{6}$ B $\frac{5}{7}$ C $\frac{5}{8}$ D $\frac{5}{9}$
8.	Put two of the numbers **4, 9, 12** in the boxes to make a fraction as close as possible to $\frac{1}{2}$.	$\frac{\square}{\square}$
9.	<u>About</u> how many days have you lived? (Circle the nearest answer.)	A 300 B 3000 C 30 000 D 300 000
10.	<u>Without calculating the exact answer</u>, circle the best estimate for: 24 x 0.98	A More than 24 B Less than 24 C Impossible to tell without working it out
11.	The police department is counting the number of cars on a certain highway. The counting meter now reads: | 4 | 7 | 3 | 9 | 9 | What will it read after one more car passes by?	| | | | | |

12. If I have \$378 in my savings account and withdraw all my money, how many 10-dollar notes would the bank be willing to give me?	________________
13. <u>About</u> how many triangles are there here? (Circle the nearest answer.) 	A 20 B 50 C 100 D 200 E 500
14. <u>Estimate</u> the <u>decimal</u> shown by the arrows on these number lines:  	________________
15.  	________________

APPENDIX E

Summaries of Results

The summaries presented below only provide data on the Australian and US results for the Mental Computation Test (MCT) and the NST. Detailed results of the NST for all four countries are presented in Appendix F.

Summary of Australian MCT Results

Ages	8	10	12	14
N	163	163	163	152
Max Poss Score	30	30	40	40
Range	0-29	0-29	2-40	12-40
Mean	12.3	13.6	26.6	30.7
SD	6.2	6.7	9.1	6.9
Error of Meas	0.49	0.52	0.72	0.56

Summary of Australian NST Results

Ages	8	10	12	14
N	180	167	168	124
Max Poss Score	30	35	45	45
Range	3-30	8-30	5-44	8-45
Mean	15.4	18.4	23.6	27.7
SD	5.1	5.1	7.7	9.2
Error of Meas	0.4	0.4	0.6	0.8

Summary of US MCT Results

Ages	8	10	12	14
N	137	141	125	119
Max Poss Score	30	30	40	40
Range	1-29	0-22	0-38	2-40
Mean	12.2	9.2	16.3	23.2
SD	5.8	4.5	9.0	11.6
Error	0.49	0.52	0.72	0.56

Summary of US NST Results

Ages	8	10	12	14
N	136	139	125	115
Max Poss Score	30	35	45	45
Range	4-26	5-28	0-41	3-42
Mean	15.1	14.5	16.7	22.0
SD	5.0	4.4	7.2	10.0
Error	0.4	0.4	0.6	0.9

Summary of NST Mean Percentages for Each Number Sense Strand

	8-year-olds		10-year-olds		12-year-olds		14-year-olds	
Strand	USA	Aus	USA	Aus	USA	Aus	USA	Aus
1	48	54	36	57	35	56	36	56
2	51	58	42	57	34	57	45	69
3	33	40	43	44	32	48	47	66
4	46	53	65	70	38	50	49	58
5	46	54	37	43	43	55	40	55

APPENDIX F | Detailed Results

Item 1 (Counting and Computation, Whole Numbers)

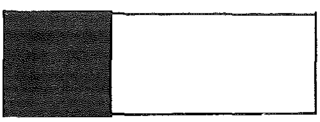
About how many days have you lived? (Circle the nearest answer.)	A	300
	B	3000
	C	30 000
	D	300 000

Variations in Question: Swe used different numbers in a similar pattern to millions.

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #[#]	1	9	11	13	1	9	11	13	1	1
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
A	46	6	7	0	31	7	1	1	8	0
B*	19	33	27	47	34	35	38	32	28	55
C	21	39	44	32	24	37	48	54	25	23
D	13	20	19	18	11	21	13	13	27	14
E									11	7
No Response	1	0	0	2	0	0	0	0	1	1
Quintiles % correct										
1st	33	44	44	70	47	64	76	84	59	61
2nd	37	43	32	57	36	29	48	36	27	65
3rd	7	29	20	44	31	36	18	21	24	61
4th	19	21	24	57	25	26	27	16	24	48
5th	0	26	17	9	33	21	18	4	9	40

Item 2 (Multiple Representations, Fractions)

About how much of this box is shaded? Give your answer as a fraction.



Variations in Question: 'Give your answer as a fraction" not included in USA or Swe

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #	2	2			2	2			2	34
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
1/3*	53	77			66	90				
0.2 - 0.4*									61	94
1 - 5 cm									12	0
Misc. Incorrect	38	20							17	6
No Response	9	4							10	0
Quintiles % correct										
1st	78	100			92	100				
2nd	59	86			75	100				
3rd	63	75			69	91				
4th	37	68			58	79				
5th	29	56			33	79				

[#] Indicates the question number for each item in each country's NST.

* Indicates the correct response for each item.

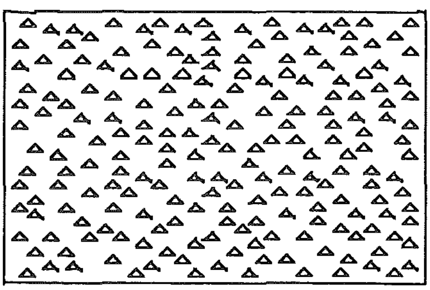
Item 3 (Number Concepts, Fractions)

Tom cuts a cake into four equal pieces and eats two of them. What fraction of the whole cake is left?	________________

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #	3				10				3	
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
1/2 or 1/4*	58				61				77	
1/2	12									
2/4	46									
2	16									
4/2	10									
Misc. Incorrect	10								22	
No Response	5								2	
Quintiles % correct										
1st	96				94				94	
2nd	70				69				88	
3rd	63				67				85	
4th	48				50				77	
5th	14				22				38	

Item 4 (Counting and Computation, Whole Numbers)

<u>About</u> how many triangles are there here? (Circle the nearest answer.)



A 20

B 50

C 100

D 200

E 500

Variations in Question: Swe A 50, B 200, C 500, D 1000.

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #	4	13	22		4	13	22		4	2
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
A	0	0	0		0	0	0		5	3
B	1	4	0		5	3	2		*57	*75
C	32	23	23		40	34	31		28	20
D*	31	51	54		29	54	62		10	3
E	35	23	19		26	9	5			
Misc. Incorrect	1									
No Response	0	0	2		0	0	0		0	0
Quintiles % correct										
1st	37	59	44		56	64	62		77	87
2nd	37	54	68		28	56	61		56	58
3rd	37	54	68		33	52	71		65	81
4th	22	50	52		19	47	67		47	87
5th	21	37	38		11	55	50		41	63

Item 5 (Effect of Operations, Whole Numbers)

The digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Put one digit in each box so that the answer will be as big as possible.	4□□ - 231 = ?

Variations in Question: A digit can be used only once (USA). Swe: First sentence omitted.

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #	5	20	29		5	20	29		6	
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
98*	37	52	53		24	29	39		61	
99	9	5			41	42	39			
Misc. Incorrect	51	41	44						27	
No Response	4	2	2						12	
Quintiles % correct										
1st	67	85	68		92	85	88		85	
2nd	44	61	72		66	88	91		82	
3rd	37	64	44		72	76	76		68	
4th	26	54	52		55	71	59		53	
5th	7	22	29		42	36	71		18	

Item 6 (Effect of Operations, Whole Numbers)

Put one digit in each box so that the answer will be as big as possible	431 - 2□□ = ?

Variations in Question: A digit can be used only once (USA). Swe: First sentence omitted.

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #	6	21	30		6	21	30		7	
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
00 or 01*	8	16	23		11	38	54		24	
98	21	18								
99	6	7								
Misc Incorrect	60	55	75						61	
No Response	4	4	2						15	
Quintiles % correct										
1st	26	37	52		14	70	82		65	
2nd	0	21	36		9	47	70		18	
3rd	4	7	8		20	27	53		21	
4th	0	11	16		6	32	52		12	
5th	4	4	0		6	9	18		6	

Item 7 (Number Concepts, Whole Numbers)

Here are five digits: **2, 6, 3, 5, 1.** Arrange <u>all</u> these digits to make the smallest number possible.	________________

Variations in Question: USA 2, 6, 3, 5, 2

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #	7	28	34		7			
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
12356*	43	63	69		40	89	93	
Misc. Incorrect	46	32	23		60	11	7	
No Response	12	6	7		0	0	0	
Quintiles % correct								
1st	81	82	84		72	97	97	
2nd	48	86	80		47	94	100	
3rd	33	46	68		39	94	94	
4th	19	61	68		39	97	97	
5th	32	37	46		3	61	79	

Item 8 (Number Concepts, Whole Numbers)

Here are five digits, **2, 6, 3, 5, 1.** Arrange them to make the number nearest to 20 000.	________________

Variations in Question: 2, 6, 3, 5, 2 (USA 10 & 12: USA 8 as above).

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #	8	29	35		8	29	35		8	3
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
21356*	10	48	57		8	43	65		12	42
Misc. Incorrect	74	43	35		92	57	35		83	54
No Response	15	9	8		0	0	0		5	5
(Quintiles % correct)										
1st	33	89	96		19	64	94		24	65
2nd	11	46	64		11	59	76		15	29
3rd	7	46	56		3	42	65		15	29
4th	0	36	40		3	38	58		9	52
5th	0	22	25		3	12	35		0	37

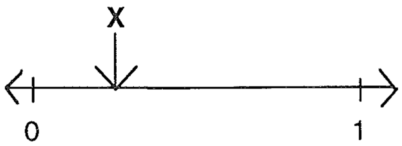
Item 9 (Multiple Representations, Fractions)

Shade $\frac{3}{4}$ of this rectangle.	

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #	9							
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Correct	82							
Incorrect	16							
No Response	2							
Quintiles % correct								
1st	96							
2nd	93							
3rd	81							
4th	74							
5th	57							

Item 10 (Multiple Representations, Fractions)

What fraction matches the letter **X** on this number line?
(Circle the correct answer.)



A $\frac{1}{2}$

B $\frac{1}{3}$

C $\frac{1}{4}$

D $\frac{1}{5}$

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #	10				11			
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
A	25				18			
B	25				27			
C*	24				42			
D	18				8			
Misc. Incorrect	0				0			
No Response	9				5			
Quintiles % correct								
1st	41				58			
2nd	19				56			
3rd	30				36			
4th	4				22			
5th	25				39			

Item 11 (Number Concepts, Whole Numbers)

The police department is counting the number of cars on a certain highway. The counting meter now reads:

| 4 | 7 | 3 | 9 | 9 |

What will it read after one more car passes by?

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #	11	11			3	11			10	
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
4\7\4\0\0*	12	36			20	71			65	
5\7\3\9\9	6									
4\7\3\10\10	2									
5\8\4\10\10	9	12								
4\7\3\9\10	20	8			9	1				
Misc. Incorrect	41	42							23	
No Response	10	2							12	
Quintiles % correct										
1st	41	67			50	97			100	
2nd	11	43			33	85			91	
3rd	7	32			11	82			65	
4th	0	32			6	74			50	
5th	0	4			0	18			21	

Item 12 (Number Concepts, Whole Numbers)

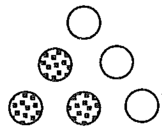
The police department is counting the number of cars on a certain highway. The counting meter now reads:

| 4 | 7 | 3 | 9 | 9 |

What will it read after ten more cars pass by?

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #	12								11	
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
4\7\4\0\9*	4								47	
4\17\13\19\19	7									
4\7\3\9\19	4									
Misc Incorrect	68								39	
No Response	16								14	
Quintiles % correct										
1st	19								85	
2nd	0								68	
3rd	0								35	
4th	0								35	
5th	0								9	

Item 13 (Multiple Representations, Fractions)



Circle the fraction which shows how much has been shaded.

A. $\frac{2}{4}$

B. $\frac{2}{6}$

C. $\frac{4}{6}$

D. $\frac{4}{2}$

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #	13	34			13	34		
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
A	15	5			24	2		
B	2	4			7	2		
C*	76	85			62	93		
D	7	6			7	2		
Misc Incorrect	0	0			0	0		
No Response	0	0			0	0		
Quintiles % correct								
1st	93	89			97	100		
2nd	93	89			64	94		
3rd	74	82			64	91		
4th	81	86			42	91		
5th	43	70			42	88		

Item 14 (Number Concepts, Whole Numbers)

Circle any piles of shoes which can be put into pairs with no shoes left over.

A 7 shoes

B 34 shoes

C 63 shoes

D 10 shoes

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #	14				14			
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
A	7				3			
B	48				42			
C	10				6			
D	83				77			
B & D*	38				34			
Incorrect	60							
No Response	3							
Quintiles % correct								
1st	63				67			
2nd	52				39			
3rd	48				33			
4th	19				22			
5th	7				8			

Item 15 (Number Concepts, Whole Numbers)

There are ten children in a line at the classroom door. Jenny is sixth in line. How many children are ahead of her?	________

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #	15				15			
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
5*	77				63			
4	15							
Misc Incorrect	7							
No Response	1							
Quintiles % correct								
1st	85				67			
2nd	81				75			
3rd	74				67			
4th	78				58			
5th	68				47			

Item 16 (Number Concepts, Whole Numbers)

There are ten children in a line at the classroom door. Peter is fourth in line. How many children are behind him?	________

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #	16				16			
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
6*	66				71			
3	14							
Misc Incorrect	17							
No Response	3							
Quintiles % correct								
1st	89				89			
2nd	89				86			
3rd	63				72			
4th	56				64			
5th	32				42			

Item 17 (Number Concepts, Whole Numbers)

Sally's grandfather is more than 59 years old and less than 72 years old. Write down three different ages he could be.	______ or ______ or ______

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #	17				17			
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
0 Correct	10							
1 Correct	7				7			
2 Correct	9				13			
3 Correct*	68				81			
No Response	5				0			
Quintiles % correct								
1st	89				100			
2nd	89				92			
3rd	85				94			
4th	70				64			
5th	29				53			

Item 18 (Number Concepts, Fractions)

How many quarters make a dollar?	____________

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #	18							
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
4*	87							
Incorrect	10							
No Response	4							
Quintiles % correct								
1st	100							
2nd	96							
3rd	96							
4th	85							
5th	57							

Item 19 (Number Concepts, Whole Numbers)

How many dimes make a dollar?	________

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #	19							
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
10*	72							
Incorrect	23							
No Response	5							
Quintiles % correct								
1st	100							
2nd	93							
3rd	85							
4th	56							
5th	46							

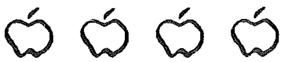
Item 20 (Number Concepts, Whole Numbers)

When counting, what is the number that comes before 600?	________

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #	20				20			
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
599*	63				66			
500	19							
Misc. Incorrect	15							
No Response	4							
Quintiles % correct								
1st	100				92			
2nd	70				78			
3rd	78				72			
4th	41				61			
5th	25				25			

Item 21 (Counting and Computation, Fractions)

Peter took half of the apples from a bag. Here are Peter's apples:



Ben took all the others from the bag.
How many apples were there in the bag to start with?

____

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #	21				21			
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
8*	59				58			
4	24							
2	1							
Misc Incorrect	15							
No Response	1							
Quintiles % correct								
1st	85				97			
2nd	74				67			
3rd	67				64			
4th	48				42			
5th	21				22			

Item 22 (Effect of Operations, Whole Numbers)

Five bugs have fifteen spots each.
Which of these tells us how many spots there are altogether? (Circle your answer.)

A 5 + 15

B 15 + 15 + 15 + 15 + 15

C 15 + 5

D 5 + 5 + 5 + 5 + 5

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #	22				22				14	
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
A	18				14				13	
B*	56				74				65	
C	11				4				5	
D	12				9				8	
Misc Incorrect	0				0				1	
No Response	4				0				9	
Quintiles % correct										
1st	85				100				91	
2nd	74				81				68	
3rd	44				75				62	
4th	44				58				68	
5th	32				44				38	

Item 23 (Counting and Computation, Whole Numbers)

A school has 610 children. If 98 children are away on a trip, <u>about</u> how many are still at school? (Circle your answer.)	A 400 B 500 C 600 D 700

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #	23				23			
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
A	26				21			
B*	58				67			
C	4				8			
D	7				4			
No Response	4				0			
Quintiles % correct								
1st	93				83			
2nd	85				78			
3rd	56				75			
4th	41				67			
5th	21				31			

Item 24 (Counting and Computation, Whole Numbers)

Whitney has ten dollars. She has six dollars less than Rebecca. How much does Rebecca have?	____________

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #	24				24			
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
$16.00*	54				67			
4	16							
Misc. Incorrect	26							
No Response	4							
Quintiles % correct								
1st	96				83			
2nd	63				89			
3rd	70				75			
4th	22				56			
5th	18				33			

Item 25 (Counting and Computation, Fractions)

A watermelon is cut into quarters. Then each quarter is cut in half.
How many pieces of watermelon are there now?

Circle your answer.

A 2

B 4

C 6

D 8

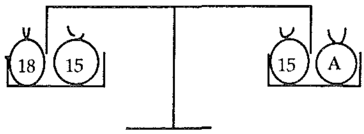
	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #	25				25			
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
A	7				3			
B	24				12			
C	12				17			
D*	54				67			
No Response	3				1			
Quintiles % correct								
1st	78				89			
2nd	44				64			
3rd	59				89			
4th	44				50			
5th	43				44			

Item 26 (Multiple Representations, Fractions)

Shade in three quarters of this rectangle.

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #	26							
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Correct	50							
Incorrect	43							
No Response	7							
Quintiles % correct								
1st	89							
2nd	56							
3rd	30							
4th	41							
5th	36							

Item 27 (Equivalent Expressions, Whole Numbers)



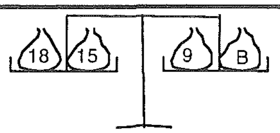
Jim has balanced some bags of marbles. The numbers show how many marbles are in each bag.
How many marbles are in the bag marked A?
(Circle the correct answer.)

A 3
B 15
C 18
D 33

Variations in Question: Swe & USA: Fifth option '0' - option A. Swe: No letter A in bag. 'How many marbles are in the unmarked bag?'

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #	27				28				17	
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
A	7				7				1	
B	4				13				1	
C	15				18				* 89	
D*	57				62				2	
E	15				0				4	
No Response	1								4	
Quintiles % correct										
1st	67				81				100	
2nd	78				81				97	
3rd	44				61				88	
4th	52				42				88	
5th	46				44				71	

Item 28 (Equivalent Expressions, Whole Numbers)



Barbara has balanced some bags of marbles. The numbers show how many marbles are in the bags. How many marbles are in the bag marked B?

A 6
B 9
C 15
D 24
E 42

Variations in Question: Swe: Bag not labelled B. 'How many . . . in the unmarked bag?'

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #	28								18	
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
A	9								2	
B	36								5	
C	14								8	
D*	23								77	
E	12								5	
No Response	5								4	
Quintiles % correct										
1st	44								97	
2nd	33								85	
3rd	22								77	
4th	11								85	
5th	4								38	

Item 29 (Counting and Computation, Fractions)

Rustin had $5 and gets change in quarters. How many quarters will he get?	A 4 B 5 C 9 D 20 E 25

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #	29							
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
A	10							
B	15							
C	9							
D*	45							
E	18							
No Response	3							
Quintiles % correct								
1st	78							
2nd	56							
3rd	52							
4th	26							
5th	14							

Item 30 (Equivalent Expressions, Whole Numbers)

The farmer has stored all his apples in 80 boxes with 40 apples in each box. He now needs to repack them all into 40 new boxes. How many apples will there be in each new box?	A. 2 B. 40 C. 80 D. 120

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #	30	35	43	38	30	35	43	38
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
A	2	12	5	4	9	9	4	5
B	45	33	23	14	38	20	7	6
C*	35	40	54	75	44	60	82	78
D	13	13	13	5	8	11	7	11
No Response	4	2	5	1	1	0	0	0
Quintiles % correct								
1st	52	54	84	100	72	82	97	100
2nd	30	36	52	78	47	59	76	88
3rd	26	50	48	96	47	55	85	83
4th	52	36	48	57	25	62	73	64
5th	18	23	38	44	28	42	79	56

Item 31 (Number Concepts, Decimals)

For a long time Jane has been putting only 10 cent coins in her piggy bank. Last night she opened it and counted her money. She had $46.70. How many 10 cent coins were in the bank?	____________

Variations in Question: dimes (USA)

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #		1	20			1	20	
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
467*		6	35			44	72	
4670		9	0					
Misc. Incorrect		61	57					
No Response		24	8					
Quintiles % correct								
1st		26	68			91	94	
2nd		0	60			62	88	
3rd		4	28			33	76	
4th		0	12			21	79	
5th		0	4			12	24	

Item 32 (Multiple Representations, Decimals)

Place the numbers 0.1 and 0.8 in their correct positions on this number line:

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #		3	18			3	18	
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
YY*		57	80			74	86	
NY		2	1			2	2	
YN		7	9			7	6	
NN		25	7			17	6	
No Response		9	4			0	0	
Quintiles % correct								
1st		89	96			100	97	
2nd		54	92			88	97	
3rd		61	88			82	91	
4th		54	68			50	73	
5th		30	54			48	74	

Item 33 (Multiple Representations, Fractions)

Place the numbers $\frac{1}{10}$ and $\frac{4}{5}$ in their correct positions on this number line:

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #		4	19			4	19	
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
YY*		19	27			34	56	
NY		1	5			1	4	
YN		7	14			18	16	
NN		65	48			47	24	
No Response		8	5			0	0	
Quintiles % correct								
1st		37	68			64	88	
2nd		21	24			53	76	
3rd		21	28			15	62	
4th		14	12			24	33	
5th		0	4			15	21	

Item 34 (Number Concepts, Fractions)

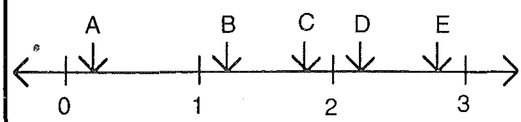
$\frac{3}{4}$ is a fraction between $\frac{1}{2}$ and 1.

Write two other fractions, between $\frac{1}{2}$ and 1.

_________ and _________

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #		5	21					
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
0 Correct		58	27					
1 Correct		18	23					
2 Correct*		11	31					
Misc. Incorrect		0	0					
No Response		14	17					
Quintiles % correct								
1st		25	92					
2nd		11	20					
3rd		7	24					
4th		4	16					
5th		7	0					

Item 35 (Multiple Representations, Decimals)



Which letter on the number line above best represents 2.19?

____

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #		6	2	2		6	2	2	24	4
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
A		2	0	0		1	0	0	2	0
B		1	1	2		5	1	0	4	0
C		4	2	0		5	1	2	14	3
D*		20	34	73		47	79	97	35	84
E		64	62	25		35	18	2	44	12
Misc. Incorrect		6	1	0		0	0	0	0	0
No Response		3	1	0		7	1	0	2	1
Quintiles % correct										
1st		30	68	96		76	94	100	44	90
2nd		21	28	96		53	97	96	24	90
3rd		18	28	74		45	76	96	38	77
4th		21	16	57		41	85	100	27	84
5th		7	29	44		21	44	92	41	80

Item 36 (Number Concepts, Fractions)

Circle the fraction which represents the largest amount:

A $\frac{5}{6}$ B $\frac{5}{7}$

C $\frac{5}{8}$ D $\frac{5}{9}$

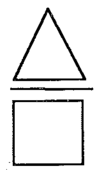
	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #		7	4	4		7	4	4		5
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
A*		29	74	85		42	79	98		92
B		1	1	1		1	2	1		0
C		0	3	2		1	3	0		1
D		69	22	11		57	17	2		6
No Response		1	0	1		0	0	0		1
Quintiles % correct										
1st		67	100	100		73	97	100		90
2nd		32	76	91		50	94	100		97
3rd		32	80	87		42	94	100		97
4th		7	68	87		29	76	96		90
5th		7	46	61		15	32	92		87

Item 37 (Number Concepts, Fractions)

Use two of the numbers below

3, 4, 9, 12

to make a fraction as close as possible to $\frac{1}{2}$.



	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #		8	5	5						6
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
4/9*		4	22	48						54
3/9 or 4/12		6	23	23						
3/4 or 9/12		71	30	16						
Misc. Incorrect		13	11	7						43
No Response		5	13	6						3
Quintiles % correct										
1st		15	76	83						55
2nd		0	24	83						58
3rd		4	4	44						58
4th		4	4	22						52
5th		0	0	9						47

Item 38 (Effect of Operations, Decimals)

Without calculating the exact answer, circle the best estimate for:

29 x 0.98

A more than 29

B less than 29

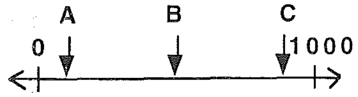
C impossible to tell without working it out

Variations in Question: Swe: Item 24 x 0.98, Responses - A little less than 24, B much less than 24, C little more than 24, D much more than 24, E impossible to tell without working it out. Tai: 36 x 0.96. Responses - A more than 36, B less than 36, C equal to 36, D can't tell.

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14	Tai 12	Tai 14
Item #		10	13	17		10	13	17		8		
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14	Tai 12	Tai 14
A		59	58	37		53	40	10		*79	28	16
B*		4	18	54		28	53	85		7	72	84
C		33	23	8		19	7	6		8	0	0
D										2	0	0
E										3		
No Response		4	2	1		0	0	0		1	0	0
Quintiles % correct												
1st		0	48	96		55	94	100		87		
2nd		7	20	87		29	79	96		81		
3rd		4	8	44		24	59	92		84		
4th		7	4	35		15	27	76		77		
5th		4	8	9		15	6	60		63		

Item 39 (Multiple Representations, Whole Numbers)

Estimate the number shown by each arrow:



A_______________

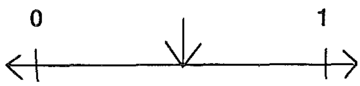
B_______________

C_______________

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #		12	38					
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
A (100)		55	33					
B (500)		42	40					
C (900)		43	29					
No Response								

Item 40 (Multiple Representations, Decimals)

Estimate the decimal shown by the arrow on the number line:



____

Variations in Question: Swe: Arrow on 0.75

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #		14	23	30		14	23	30	29	9
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
0.4 - 0.6 (Swe 0.6 - 0.9)*		29	67	89		65	80	98	47	96
Incorrect		47	23	6					41	4
No Response		23	11	5					12	0
Quintiles % correct										
1st		56	92	100		100	97	100	62	94
2nd		29	76	100		76	88	100	65	97
3rd		43	60	96		79	88	96	47	100
4th		29	72	83		48	71	96	38	97
5th		11	33	65		24	53	96	24	93

Item 41 (Multiple Representations, Decimals)

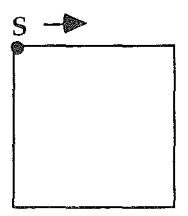
Estimate the decimal shown by the arrow on the number line:

______

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #		15	24	31		15	24	31		10
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
0.04 - 0.06*		15	21	63		26	65	90		94
Misc. Incorrect		68	60	30						5
No Response		26	19	7						1
Quintiles % correct										
1st		15	76	96		61	100	100		97
2nd		7	16	87		38	82	100		90
3rd		0	4	83		24	65	92		94
4th		4	4	44		3	67	92		94
5th		4	4	4		3	15	68		97

Item 42 (Multiple Representations, Fractions)

You are going to walk once around a square-shaped field. You start at the corner marked S and move in the direction shown by the arrow. Mark with an X where you will be after $^1/_3$ of your walk.



	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #		16	25			16	25		30	11
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
In range*		36	25			25	51		26	81
Right edge						19	19			
Out of range		63	73							
top									4	1
1st corner									6	2
2nd corner									10	0
bottom									7	5
3rd corner									37	8
left									1	1
Misc. Incorrect									2	0
No Response		1	2						8	2
Quintiles % correct										
1st		63	48			42	88		50	84
2nd		39	20			44	61		38	81
3rd		25	36			18	53		21	71
4th		39	8			6	27		15	94
5th		11	13			15	26		6	77

Item 43 (Equivalent Expressions, Whole Numbers)

Without calculating the exact answer, circle the largest product.	A. 18 x 17 B. 16 x 18 C. 17 x 19

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #		17	26			17	26	
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
A		11	10			10	10	
B		3	4			1	1	
C*		84	84			89	89	
Misc. Incorrect		1	0			0	0	
No Response		1	2			0	0	
Quintiles % correct								
1st		93	96			94	97	
2nd		86	88			94	88	
3rd		82	92			85	100	
4th		86	72			82	76	
5th		67	71			91	82	

Item 44 (Effect of Operations, Percentages)

Mary had $426 and spent 90 percent of the money on clothes. Without calculating an exact answer, circle the best estimate for how much she spent	A slightly less than $426 B much less than $426 C slightly more than $426 D impossible to tell without calculating

Variations in Question: Swe: 95%.

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #		18	27	25					31	
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
A*		38	52	65					38	
B		37	23	23					28	
C		10	8	4					8	
D		13	14	6					19	
No Response		1	3	2					7	
Misc. Incorrect		1	0	0					1	
Quintiles % correct										
1st		59	92	96					65	
2nd		57	60	83					41	
3rd		21	48	74					38	
4th		43	44	52					27	
5th		11	17	22					21	

Item 45 (Effect of Operations, Decimals)

Which is greater?	
	A 29 ÷ 0.8
	B 29 x 0.8
	C 29 + 0.8
	D Impossible to tell without calculating

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #		19	28					
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
A*		9	6					
B		75	78					
C		9	7					
D		5	7					
Misc. Incorrect		0	0					
No Response		3	2					
Quintiles % correct		4	16					
1st		0	0					
2nd		11	0					
3rd		11	8					
4th		19	4					
5th								

Item 46 (Effect of Operations, Whole Numbers)

When a 3-digit number is added to a 3-digit number the result is:	
	A always a 3-digit number
	B always a 4-digit number
	C always a 5-digit number
	D either a 3, 4 or 5-digit number
	E either a 3 or 4 digit number

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #		22	31			22	31		32	18
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
A		15	8			16	10		25	8
B		4	4			3	5		9	5
C		9	7			3	4		10	7
D		44	48			43	29		24	25
E*		20	30			32	52		21	55
No Response		9	3			3	0		1	0
Misc. Incorrect		0	1			0	0		10	0
Quintiles % correct										
1st		22	60			52	79		56	65
2nd		21	44			44	73		21	71
3rd		32	24			30	47		12	55
4th		18	12			24	48		9	39
5th		7	8			9	15		6	43

Item 47 (Counting and Computation, Whole Numbers)

Barb is a fifth grader at my school. She says that she is 30 000 days old. Is that possible? Say why.	A. Yes B. No C. Maybe Tell why. ________________ ________________

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #		23						
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
A		33						
B*		38						
C		27						
No Response		1						
Quintiles % correct								
1st		50						
2nd		25						
3rd		46						
4th		21						
5th		43						

Item 48 (Equivalent Expressions, Whole Numbers)

<u>Without calculating</u>, circle the expression which represents the larger amount.	A. 145 x 4 B. 144 + 146 + 148 + 150

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #		24	32	33		24	32	33
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
A		29	24	9		23	24	15
B*		71	73	89		77	76	85
No Response		0	3	3		0	0	0
Quintiles % correct								
1st		71	84	96		91	91	100
2nd		68	84	100		85	94	96
3rd		82	80	87		85	76	88
4th		57	64	74		68	70	80
5th		70	50	87		55	47	60

Item 49 (Counting and Computation, Whole Numbers)

Without calculating the exact answers, circle the best estimate for: 145 x 4	A. Greater than 4500 B. Less than 4500 C. Impossible to tell without calculating

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #		25						
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
A		19						
B*		68						
C		12						
No Response		1						
Quintiles % correct								
1st		75						
2nd		82						
3rd		57						
4th		64						
5th		57						

Item 50 (Counting and Computation, Whole Numbers)

Without calculating the exact answers, circle the best estimate for: 18 x 19	A. 290 B. 390 C. 490

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #		26	33			26	33	
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
A		55	42			66	36	
B*		29	40			25	55	
C		14	15			8	6	
No Response		2	2			1	3	
Quintiles % correct								
1st		29	64			30	74	
2nd		21	32			29	48	
3rd		36	48			15	56	
4th		29	36			24	52	
5th		27	21			27	47	

Item 51 (Counting and Computation, Decimals)

Ten bottles of juice cost $7.95 at one store. I can get 5 bottles for $4.15 at a second store. Where is the juice cheaper - at the first or second store?	A. First store B. Second store Tell how you decided: ____________________ ____________________

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #		27				27		
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
A*		53				74		
B		45				26		
No Response		1				0		
Quintiles % correct								
1st		79				85		
2nd		64				76		
3rd		57				85		
4th		46				82		
5th		17				42		

Item 52 (Counting and Computation, Whole Numbers)

Which two numbers multiplied together give an answer closest to the target number? **4 18 50 37** Target Number 75	_______ and _______

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #		30	36			30	36	
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
18 x 4*		38	61			49	75	
Incorrect		58	38			51	25	
No Response		4	1			0	0	
Quintiles % correct								
1st		70	88			82	94	
2nd		46	72			71	85	
3rd		25	84			45	82	
4th		29	32			32	70	
5th		22	29			15	44	

Item 53 (Counting and Computation, Whole Numbers)

Which two numbers multiplied together give an answer closest to the target number?

4 18 50 37

Target Number: 1000

_______ and _______

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #		31	37			31	37		35	
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
18 x 50*		18	39			22	46		18	
18 x 37						0	8			
Incorrect		79	59			78	46		78	
No Response		3	2						4	
Quintiles % correct										
1st		33	72			52	79		32	
2nd		25	40			32	55		21	
3rd		7	40			12	53		24	
4th		18	20			12	27		15	
5th		7	21			3	15		0	

Item 54 (Equivalent Expressions, Whole Numbers)

16 x 0 = ☐

The number in the box....

A. ...must be 16

B. ...must be 160

C. ...must be 0

D. ...could be any number

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #		32			26	32		
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
A		11			32	9		
B		1			0	2		
C*		86			65	87		
D		2			3	1		
No Response		1			0	1		
Quintiles % correct								
1st		89			83	97		
2nd		96			81	100		
3rd		82			53	88		
4th		86			67	79		
5th		67			42	73		

Item 55 (Equivalent Expressions, Whole Numbers)

$$15 \times \square = 15$$

The number in the box....

A. ...must be 0

B. ...must be $\frac{1}{5}$

C. ...must be 1

D. ...must be 15

E. ...could be any number

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #		33			27	33		
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
A		8			28	15		
B		2			6	1		
C*		81			42	78		
D		4			11	2		
E		5			9	2		
No Response		0			4	2		
Quintiles % correct								
1st		93			78	97		
2nd		93			58	94		
3rd		82			33	82		
4th		79			36	71		
5th		50			6	48		

Item 56 (Number Concepts, Decimals)

Scott ran 100 metres in 14.52 seconds. Kelly took 2 tenths of a second longer. How long did it take Kelly to run 100 metres?

Circle your answer.

A 34.52 seconds

B 16.52 seconds

C 14.72 seconds

D 14.54 seconds

E 14.50 seconds

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #			1	1			1	1
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
A			2	0			1	2
B			7	4			14	5
C*			38	57			54	70
D			49	37			28	21
E			3	2			3	2
No Response			1	1				
Quintiles % correct								
1st			76	96			65	100
2nd			36	74			67	88
3rd			32	52			53	67
4th			28	44			52	60
5th			17	22			35	36

Item 57 (Number Concepts, Decimals)

How many different decimals are there between 1.52 and 1.53? Circle your answer and then fill in the blank.	A None. Why? ________________ B One. What is it? ________________ C A few. Give two: _______ and _______ D Lots. Give two: ______ and ______

Variations in Question: Circle one of the following (Aus). Swe: In C & D 'Give two in decimal form'. Tai: 'between 1.42 & 1.43', and no examples asked for.

	USA 8	USA 10	USA 12	USA 14	Aus 10	Aus 12	Aus 14	Swe 14	Tai 12	Tai 14
Item #			3	3		3	3			
Response %	USA 8	USA 10	USA 12	USA 14	Aus 10	Aus 12	Aus 14	Swe 14	Tai 12	Tai 14
A			55	45		42	23	23	36	5
B			23	8		33	11	14	10	5
C			9	0		6	2	2	5	3
D & example*			3	40		16	62	51		
D without example			2	4		1	1	6	* 32	* 78
No Response			8	3		2	1	4	17	9
Quintiles % correct										
1st			16	96		32	96	68		
2nd			0	70		15	92	58		
3rd			0	22		9	54	45		
4th			0	13		3	28	42		
5th			0	0		3	12	40		

Item 58 (Number Concepts, Fractions)

How many different fractions are there between $\frac{2}{5}$ and $\frac{3}{5}$? Circle your answer and then fill in the blanks.	A None. Why? ________________ B One. What is it? ________________ C A few. Give two: _______ and _______ D Lots. Give two: ______ and ______

Variations in Question: Circle the correct answer (Aus). Swe: In C & D, 'Give two examples in fraction form'. Tai: no examples asked for.

	USA 8	USA 10	USA 12	USA 14	Aus 10	Aus 12	Aus 14	Swe 14	Tai 12	Tai 14
Item #			6	6		6	6			
Response %	USA 8	USA 10	USA 12	USA 14	Aus 10	Aus 12	Aus 14	Swe 14	Tai 12	Tai 14
A			57	53		57	29	47	60	40
B			14	9		29	21	14	8	4
C			8	5		4	7	5	6	3
D & example*			1	7		7	40	12		
D without example			11	15		2	2	13	*11	* 35
No Response			10	11		1	1	8	15	18
Quintiles % correct										
1st			4	30		24	92	26		
2nd			0	4		0	52	26		
3rd			0	0		0	25	0		
4th			0	0		0	20	0		
5th			0	0		12	8	7		

Item 59 (Multiple Representations, Fractions)

Circle all the statements that are <u>true</u> about the number $\frac{2}{5}$.

A It is greater than $\frac{1}{2}$

B It is the same as 2.5

C It is equivalent to 0.4

D It is greater than $\frac{1}{3}$

Variations in Question: Swe: E: It is less than $\frac{1}{4}$
Comments: Swe: Presumably C = C but not D, and D = D but not C. For ages 12/14

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #			7	7			7	7		21
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
A			1	10			8	4		3
B			52	20			25	17		5
C (not D)			4	28			34	34		36
D (not C)			15	10			17	20		11
E			-	-			-	-		7
C & D*			5	31			31	37		23
Misc. Incorrect			19	N/A						14
No Response			9	0						2
Quintiles % correct										
1st			8	65			68	76		26
2nd			4	43			30	68		23
3rd			4	26			29	21		19
4th			8	13			18	20		29
5th			0	9			9	0		20

Item 60 (Multiple Representations, Decimals)

Circle the decimal which best represents the amount of the box shaded.

A 0.018

B 0.15

C 0.4

D 0.801

E 0.52

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #			8	8			8	8
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
A			8	10			6	10
B*			29	59			51	74
C			52	24			29	9
D			2	5			3	4
E			4	2			10	2
No Response			4	1			1	1
Quintiles % correct								
1st			56	96			85	100
2nd			36	70			70	96
3rd			24	61			41	71
4th			16	39			36	64
5th			13	30			24	40

Item 61 (Number Concepts, Fractions)

Write a number in the box to make a fraction which represents a number between 2 and 3.	$\frac{\square}{8}$

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #			9	11			9	11		23
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Correct			7	35			25	48		33
Incorrect			82	60			75	52		56
No Response			12	5			0	0		11
Quintiles % correct										
1st			32	78			65	96		45
2nd			0	61			36	80		42
3rd			0	22			21	42		19
4th			0	9			3	8		32
5th			0	4			0	12		27

Item 62 (Equivalent Expressions, Decimals)

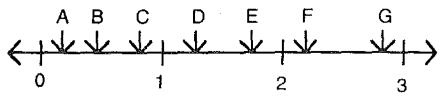
0.5 x 840 is the same as:	A 840 ÷ 2 B 5 x 840 C 5 x 8400 D 840 ÷ 5 E 0.50 x 84

Variations in Question: Option E 0.05 x 84 (USA)

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #			10	12			10	12		7
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
A*			15	37			32	65		48
B			52	22			18	2		2
C			7	15			14	19		30
D			10	11			7	6		5
E			15	12			30	9		12
No Response			2	3			0	0		1
Misc. Incorrect										1
Quintiles % correct										
1st			40	78			59	96		52
2nd			20	57			42	88		61
3rd			4	30			29	75		39
4th			8	22			21	44		45
5th			0	0			6	20		43

Item 63 (Multiple Representations, Fractions)

In the fraction $\frac{5}{8}$, 5 is the numerator and 8 is the denominator.



Which letter in the number line above names a fraction where the numerator is slightly more than the denominator?

Variations in Question: Swe and Tai: Introductory sentence not included.

	USA 10	USA 12	USA 14	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14	Tai 12	Tai 14
Item #		12	14		12	14		24		
Response %	USA 10	USA 12	USA 14	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14	Tai 12	Tai 14
A		15	10		6	7		6		
B		7	3		2	5		1		
C		11	10		11	6		7		
D*		15	30		14	37		19	* 14	* 32
E		6	11		10	7		17		
F		13	10		34	20		16		
G		14	6		13	12		8		
Misc. Incorrect		8	1					11	25	44
No Response		11	19					16	61	24
Quintiles % correct										
1st		36	70		35	76		19		
2nd		16	44		12	40		26		
3rd		8	22		6	38		19		
4th		12	13		6	16		16		
5th		0	4		12	16		13		

Item 64 (Effect of Operations, Decimals)

Without calculating the exact answer, circle the best estimate for:

87 x 0.09

A a lot less than 87
B a little less than 87
C a little more than 87
D a lot more than 87

Variations in Question: USA: An exact .

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #			14	21			14	21		25
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
A*			16	50			51	82		65
B			18	17			12	8		16
C			28	20			30	8		10
D			35	8			7	2		8
No Response			2	4			0	0		1
Quintiles % correct										
1st			44	87			97	92		77
2nd			8	78			70	100		58
3rd			4	48			53	92		68
4th			8	35			27	76		58
5th			17	4			6	52		63

Item 65 (Effect of Operations, Fractions)

Without calculating, which total is more than 1? (Circle the correct answer.)	A $\frac{2}{5} + \frac{3}{7}$ B $\frac{1}{2} + \frac{4}{9}$ C $\frac{3}{8} + \frac{2}{11}$ D $\frac{4}{7} + \frac{1}{2}$

Variations in Question: Without calculating (not USA). Tai: A $\frac{5}{11} + \frac{3}{7}$, B $\frac{7}{15} + \frac{5}{12}$, C $\frac{1}{2} + \frac{4}{9}$, D $\frac{5}{9} + \frac{8}{15}$

	USA 10	USA 12	USA 14	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14	Tai 12	Tai 14
Item #		15	23		15	23		26		
Response %	USA 10	USA 12	USA 14	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14	Tai 12	Tai 14
A		15	10		13	12		12	10	9
B		22	14		26	19		21	22	17
C		32	19		26	7		11	16	20
D*		27	53		36	61		46	52	54
No Response		5	4		0	1		8	0	0
Misc. Incorrect		0	0		0	0		2	0	0
Quintiles % correct										
1st		60	91		65	96		58		
2nd		8	83		48	64		55		
3rd		24	61		24	71		29		
4th		36	22		21	48		42		
5th		4	9		21	28		47		

Item 66 (Equivalent Expressions, Fractions)

Write '*is greater than*', '*is equal to*' or '*is less than*' to make this a true statement:	$5 \times 7\frac{1}{2}$ ________ $35 + \frac{1}{2}$

Variations in Question: Swe $5 \times 7\frac{1}{2}$ is A: less than $35 + \frac{1}{2}$, B: equal to $35 + \frac{1}{2}$, C: greater than $35 + \frac{1}{2}$, D: can't tell without working it out.

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #			16	24			16	24		27
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
A										5
B										47
C*										34
D										8
Greater*			46	50			44	57		
Less			23	15						
Equal			29	35						
No Response			2	0						5
Quintiles % correct										
1st			60	83			79	96		29
2nd			48	65			45	84		42
3rd			36	39			41	38		32
4th			52	44			33	56		32
5th			33	17			21	12		37

Item 67 (Effect of Operations, Decimals)

Without calculating, decide which one of these answers is reasonable, and circle it:	A	45 x 1.05 = 39.65
	B	4.5 x 6.5 = 292.5
	C	87 x 1.076 = 93.61
	D	589 x 0.95 = 595.45

Variations in Question: Three of these answers are definitely wrong (USA 14) ...incorrect (USA 12). Decide which one is reasonable and circle it (Aus)

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #			17	28			17	28		28
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
A			19	20			9	9		12
B			27	17			18	9		7
C*			23	43			42	69		63
D			26	17			31	14		15
No Response			6	3			0	0		3
Misc. Incorrect			0	0			0	0		1
Quintiles % correct										
1st			40	87			68	88		68
2nd			32	30			64	92		71
3rd			4	48			26	79		71
4th			32	26			36	48		42
5th			4	22			18	36		63

Item 68 (Equivalent Expressions, Mixed)

Circle the number which can be put in both boxes to make this sentence true:	A	0
	B	0.1
243 x ☐ = ☐ x 24.3	C	1
	D	10

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #			39	34			39	34
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
A*			13	43			21	40
B			37	34			43	40
C			12	6			14	9
D			17	15			18	11
No Response			16	3			3	0
Misc. Incorrect			5	0			1	0
Quintiles % correct								
1st			24	96			50	88
2nd			20	74			15	60
3rd			12	26			24	42
4th			4	13			9	8
5th			4	4			9	4

Item 69 (Equivalent Expressions, Whole Numbers)

93 x 134 is equal to 12462. Use this to write the answer to the following: 93 x 135	

Variations in Question: Swe How much larger is 12462 than 93 × 135? A: 92, B: 93, C: 134, D: 135, E: none of these.

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #			40	35			40	35		30
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
A										7
B*										40
C										7
D										20
E										20
12555*			14	34			24	47		
Incorrect			68	53			76	53		1
No Response			19	13			0	0		5
Quintiles % correct										
1st			32	65			65	80		32
2nd			12	57			36	56		42
3rd			12	30			18	58		52
4th			8	13			3	32		42
5th			4	4			0	8		33

Item 70 (Counting and Computation, Whole Numbers)

93 x 134 is equal to 12462. Use this to find the answer to the following: 12462 ÷ 930	______________

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #			41	36			41	36
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
13.4*			7	30			29	54
Incorrect			64	49			71	46
No Response			30	22			0	0
Quintiles % correct								
1st			28	78			65	80
2nd			4	52			42	56
3rd			0	17			26	75
4th			0	0			9	48
5th			0	0			0	12

Item 71 (Effect of Operations, Fractions)

Circle the number you can put in the box to make this sentence true: $\frac{1}{2} \times \square = \frac{3}{6}$	A $\frac{2}{4}$ B $\frac{2}{3}$ C 1 D 3

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #			42	37			42	37		31
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
A			19	14			8	10		16
B			21	28			28	13		22
C*			16	28			28	46		21
D			43	27			34	31		33
No Response			1	4			2	0		7
Misc. Incorrect			0	0			0	0		1
Quintiles % correct										
1st			44	78			65	88		26
2nd			12	39			27	68		39
3rd			8	13			29	46		7
4th			12	4			18	24		26
5th			4	4			0	4		10

Item 72 (Effect of Operations, Percentages)

A tank holds 1000 fish. If I increase the number by 50%, how many fish will there be now in the tank? (Circle the correct answer.)	A 500 B 1050 C 1500 D 2000

Variations in Question: If the number of fishes increases by 50 percent, how many fish are in the tank? (USA)

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #			44	39			44	39		32
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
A			12	9			5	1		0
B			29	7			7	3		1
C*			52	75			82	94		90
D			3	10			6	2		7
No Response			3	0			0	0		2
Quintiles % correct										
1st			88	96			94	100		87
2nd			60	83			91	100		97
3rd			64	83			88	96		94
4th			36	61			79	92		87
5th			13	52			59	84		83

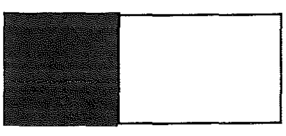
Item 73 (Effect of Operations, Percentages)

Dale had $150. She spent 100% of it. How much money did she have left? (Circle the correct answer.)	A $0 B $50 C $100 D $150 E $250 F $300

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #			45	40			45	40		33
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
A*			64	91			85	98		97
B			30	4			11	1		2
C			2	3			1	0		0
D			2	3			2	1		1
E			0	0			0	0		0
F			0	0			0	0		0
No Response			2	0			0	0		0
Quintiles % correct										
1st			100	100			97	96		100
2nd			88	96			91	100		97
3rd			68	100			94	100		97
4th			32	91			88	100		100
5th			29	70			56	96		90

Item 74 (Multiple Representations, Decimals)

Circle the decimal which best represents the amount of the box shaded.



A 0.018

B 0.15

C 0.4

D 0.801

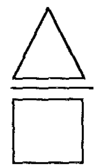
E 0.52

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #				9				9
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
A				4				1
B				20				7
C*				57				79
D				4				6
E				15				7
No Response				1				0
Quintiles % correct								
1st				100				100
2nd				87				100
3rd				52				83
4th				35				72
5th				13				40

Item 75 (Number Concepts, Fractions)

In the fraction $\frac{1}{8}$, the numerator is 1.

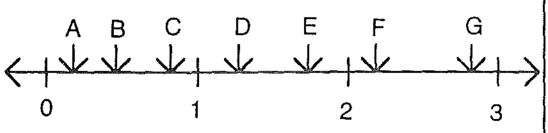
Fill in the boxes to make a fraction between 0 and $\frac{1}{10}$ whose numerator is <u>not</u> 1.



	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #				10				10		35
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Correct				17				35		34
Incorrect				65				65		48
No Response				18				0		18
Quintiles % correct										
1st				61				96		39
2nd				9				40		42
3rd				4				29		23
4th				9				12		36
5th				0				0		30

Item 76 (Multiple Representations, Fractions)

In the fraction $\frac{5}{8}$, 5 is the numerator and 8 is the denominator.



Which letter in the number line above names a fraction where the numerator is nearly twice the denominator?

____

Variations in Question: Swe: No introductory sentence. Also '...is a little less than twice...'.
Tai: No introductory sentence.

	USA 10	USA 12	USA 14	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14	Tai 12	Tai 14
Item #			15			15		36		
Response %	USA 10	USA 12	USA 14	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14	Tai 12	Tai 14
A			4			2		3		
B			5			4		13		
C			4			6		8		
D			1			4		16		
E*			20			35		20		
F			15			15		12		
G			31			28		3		
Correct									* 3	* 20
Misc. Incorrect			2			0		5	21	47
No Response			18			6		20	76	33
Quintiles % correct										
1st			44			60		19		
2nd			26			28		23		
3rd			9			33		26		
4th			13			24		19		
5th			9			28		13		

Item 77 (Number Concepts, Fractions)

Circle all fractions listed here which are greater than $\frac{3}{4}$ but less than 1.	$\frac{2}{3}$ $\frac{5}{8}$ $\frac{4}{5}$ $\frac{7}{10}$ $\frac{4}{3}$

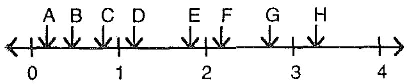
	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #				16				16		37
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
2/3				38				42		
5/8				17				22		
4/5				61				67		
7/10				24				23		
4/3				15				7		
4/5 only*				30				31		20
No Response				6						75
Misc. Incorrect				64						5
Quintiles % correct										
1st				57				68		16
2nd				57				48		29
3rd				17				21		23
4th				9				12		26
5th				9				4		7

Item 78 (Effect of Operations, Whole Numbers)

When a 2-digit number is multiplied by a 2-digit number, the result is: (Circle the correct answer.)	A	Always a 3-digit number
	B	Always a 4-digit number
	C	Either a 3- or 4-digit number
	D	Sometimes a 5-digit number

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #				18				18		39
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
A				10				19		23
B				10				2		10
C*				58				69		52
D				18				10		11
No Response				4				0		3
Misc. Incorrect				0				0		1
Quintiles % correct										
1st				74				88		55
2nd				70				72		55
3rd				44				63		39
4th				52				60		52
5th				52				60		60

Item 79 (Effect of Operations, Fractions)



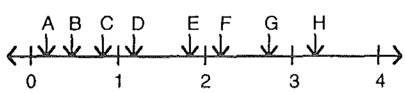
On the number line above, which letter best represents the following:

D x G

____

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #				19				19		40
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
A				1				0		0
B				1				0		1
C				2				0		1
D				2				0		1
E				6				6		8
F				2				9		9
G				1				5		5
H*				48				68		34
Misc. Incorrect				20				0		12
No Response				17				12		30
Quintiles % correct										
1st				83				88		52
2nd				52				60		26
3rd				57				75		39
4th				35				56		16
5th				13				60		40

Item 80 (Effect of Operations, Fractions)



On the number line above, which letter best represents the following:

E ÷ F

____

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #				20				20		41
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
A				6				17		18
B				15				16		5
C*				12				27		7
D				10				21		6
E				3				3		2
F				4				2		2
G				6				1		6
H				4				1		8
Misc. Incorrect				17				0		40
No Response				23				12		6
Quintiles % correct										
1st				26				72		10
2nd				13				36		7
3rd				13				17		3
4th				9				8		7
5th				0				4		10

Item 81 (Effect of Operations, Decimals)

<u>Without calculating the exact answer</u>, circle the best estimate for: 54 ÷ 0.09	A a lot less than 54 B a little less than 54 C a little more than 54 D a lot more than 54

Variations in Question: Swe: 56, not 54. Tai: 72÷0.025, responses as for others

	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14	Tai 12	Tai 14
Item #			22				22		42		
Response %	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14	Tai 12	Tai 14
A			36				30		35	30	14
B			16				7		26	17	10
C			22				15		13	17	20
D*			24				48		20	33	56
No Response			2				0		6	3	0
Quintiles % correct											
1st			48				80		29		
2nd			35				64		39		
3rd			26				58		10		
4th			0				24		13		
5th			9				12		10		

Item 82 (Equivalent Expressions, Whole Numbers)

Write '*is greater than*', '*is equal to*' or '*is less than*' to make this a true statement:	456 ÷ 8 ________ 456 x $\frac{1}{8}$

Variations in Question: Swe Circle the correct statement. 456 ÷ 8 is, A: less than 456 × $\frac{1}{8}$, B: equal to 456 × $\frac{1}{8}$, C: greater than 456 × $\frac{1}{8}$.

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #				26				26		45
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
A										28
B*										46
C										20
Greater than				20						
Lesser than				32						
Equal to*				45				48		
No Response				2						6
Quintiles % correct										
1st				100				64		45
2nd				61				72		65
3rd				17				58		39
4th				30				32		36
5th				13				12		47

Item 83 (Equivalent Expressions, Whole Numbers)

A four digit number is represented by ####. If #### ÷ 30 > 40, then which of these statements is true?	A. 30 x 40 > #### B. 30 x 40 < #### C. 30 x #### < 40 D. 40 x #### < 30

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #				27				27
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
A				24				21
B*				43				44
C				17				25
D				10				10
No Response				5				0
Quintiles % correct								
1st				70				72
2nd				35				52
3rd				44				58
4th				35				40
5th				30				0

Item 84 (Effect of Operations, Decimals)

<u>Without calculating the exact answer</u>, circle the best estimate for: 29 ÷ 0.8	A less than 29 B equal to 29 C greater than 29 D impossible to tell without calculating

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #				29		19	28	29		44
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
A				55		64	73	49		51
B				3		4	2	1		4
C*				36		20	21	49		37
D				5		11	4	1		4
No Response				2		1	0	0		5
Quintiles % correct										
1st				57		3	18	92		45
2nd				44		18	21	68		52
3rd				30		21	21	63		16
4th				22		29	27	16		39
5th				26		30	21	8		33

Item 85 (Multiple Representations, Decimals)

Estimate the decimal shown by the arrow on the number line:

____

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #				32				32
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
0.005*				64				90
Incorrect				26				
No Response				10				
Quintiles % correct								
1st				96				100
2nd				87				100
3rd				83				96
4th				48				88
5th				4				68

Item 86 (Multiple Representations, Mixed)

Put these numbers in order, starting with the smallest on the top row.

0.595 $\frac{3}{5}$ 61% 0.3 30.5%

____

____

____

____

____

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #				41				41
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Correct				28				52
Incorrect				66				48
No Response				6				0
Quintiles % correct								
1st				70				88
2nd				44				84
3rd				26				58
4th				0				20
5th				0				8

Item 87 (Counting and Computation, Percentages)

A student increased his exam score from 40 to 50. What percentage increase is this? Circle your answer.	A 10% B 25% C 50% D 90%

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #				42				42
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
A				58				34
B				5				5
C				19				23
D*				15				38
No Response				3				0
Quintiles % correct								
1st				44				88
2nd				13				48
3rd				9				33
4th				4				12
5th				4				8

Item 88 (Counting and Computation, Whole Numbers)

A cat eats 600 g of fish in 4 days. How many grams will the cat eat in 6 days?	A 400 g B 600 g C 800 g D 900 g E 1000 g

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #				43				43
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
A				1				1
B				1				1
C				27				25
D*				63				69
E				6				5
No Response				2				0
Quintiles % correct								
1st				83				92
2nd				83				76
3rd				83				75
4th				30				52
5th				35				48

Item 89 (Counting and Computation, Whole Numbers)

A trip took 5 hours travelling at an average speed of 80 kilometres per hour. The return trip took 4 hours. What was the average speed for the round trip?

____

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #				44				44
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Correct				25				59
Incorrect				59				
No Response				15				
Quintiles % correct								
1st				70				96
2nd				39				80
3rd				9				67
4th				4				44
5th				4				8

Item 90 (Counting and Computation, Percentages)

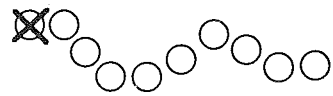
Last week a diary cost $4.50. This week there is 10% off the cost of all diaries. What is the cost of the diary this week?

____

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #				45				45
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
$4.05*				34				61
Correct				53				
No Response				13				
Quintiles % correct								
1st				83				100
2nd				57				80
3rd				26				63
4th				4				40
5th				0				24

Item 91 (Number Concepts, Whole Numbers)

There is a cross on the first circle. Put a cross on the seventh circle.



	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #					12			
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Correct					82			
Incorrect					18			
Quintiles % correct								
1st					92			
2nd					81			
3rd					78			
4th					83			
5th					78			

Item 92 (Number Concepts, Whole Numbers)

Thirty-four is the same as 34.

Four hundred and three is the same as: ____________

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #					18			
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Correct					74			
Incorrect					26			
Quintiles % correct								
1st					100			
2nd					81			
3rd					86			
4th					78			
5th					28			

Item 93 (Number Concepts, Whole Numbers)

Thirty-four is the same as 34. Six thousand and ninety-two is the same as:	______________

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #					19			
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Correct					43			
Incorrect					57			
Quintiles % correct								
1st					86			
2nd					58			
3rd					31			
4th					33			
5th					6			

Item 94 (Effect of Operations, Whole Numbers)

Rustin had $50 and then spends $29. He is given $24 in change. Which sum could he do to check if this is the right change? (Circle the correct answer)	A. 29 + 24 B. 24 + 50 C. 50 + 24 D. 50 + 29

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #					29			
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
A*					31			
B					16			
C					22			
D					29			
No Response					2			
Quintiles % correct								
1st					53			
2nd					31			
3rd					33			
4th					22			
5th					14			

Item 95 (Multiple Representations, Fractions)

Shade in three quarters $\left(\frac{3}{4}\right)$ of this shape.

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #					9			
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Correct					61			
Incorrect								
Quintiles % correct								
1st					97			
2nd					81			
3rd					64			
4th					39			
5th					22			

Item 96 (Number Concepts, Fractions)

$\frac{3}{4}$ is a fraction between $\frac{1}{2}$ and 1.

Name another fraction between $\frac{1}{2}$ and 1.

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #						5	21			38
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Correct						47	56			62
Incorrect						53	44			23
No Response						0	0			14
Quintiles % correct										
1st						82	82			77
2nd						59	73			58
3rd						52	56			61
4th						21	45			58
5th						24	24			57

Item 97 (Counting and Computation, Fractions)

Put two of the numbers

4, 9, 12

in the boxes to make a fraction as close as possible to $\frac{1}{2}$.

$\frac{\square}{\square}$

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #						8	5	5
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
4 and 9*						65	72	78
Incorrect						35	28	22
Quintiles % correct								
1st						94	91	92
2nd						59	91	96
3rd						64	76	92
4th						56	61	68
5th						55	41	44

Item 98 (Number Concepts, Whole Numbers)

If I have $378 in my savings account and withdraw all my money, how many 10-dollar notes would the bank be willing to give me?

____

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #						12	38		38	
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
37*						51	70		53	
Incorrect						49	30		34	
No Response						0	0		14	
Quintiles % correct										
1st						88	91		97	
2nd						59	79		71	
3rd						36	76		53	
4th						53	58		38	
5th						18	44		6	

Item 99 (Effect of Operations, Decimals)

Mary had $426 and spent 0.9 of it on clothes. Without calculating the exact answer, circle the best estimate for how much she spent.

A. slightly less than $426

B. much less than $426

C. slightly more than $426

D. impossible to tell without calculating

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #						18	27	25
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
A*						50	56	71
B						40	36	23
C						2	1	2
D						7	7	4
Quintiles % correct								
1st						70	76	88
2nd						56	64	92
3rd						48	56	63
4th						41	48	44
5th						36	35	68

Item 100 (Equivalent Expressions, Whole Numbers)

Jim bought 3 sleeping bags at $98 each. How could he work out how much he spent?

(Circle the correct answer.)

A 3 lots of $100, take $1

B 3 lots of $100, take $2

C 3 lots of $100, take $3

D 3 lots of $100, take $6

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
Item #						23			39	
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14	Swe 10	Swe 14
A						8			12	
B						25			31	
C						13			14	
D*						53			33	
No Response						1			10	
Quintiles % correct										
1st						85			59	
2nd						65			32	
3rd						58			24	
4th						32			21	
5th						24			29	

Item 101 (Effect of Operations, Whole Numbers)

Without calculating the exact answer, circle the best estimate for: 45 x 105	A 4000 B 4600 C 5200

	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
Item #						25		
Response %	USA 8	USA 10	USA 12	USA 14	Aus 8	Aus 10	Aus 12	Aus 14
A						22		
B*						60		
C						19		
Quintiles % correct								
1st						73		
2nd						68		
3rd						73		
4th						56		
5th						+30		

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