

# 1        **Analysis of FRP shear strengthening solutions for reinforced concrete**

## 2        **beams considering debonding failure**

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### 6        **Abstract**

7                          In this paper, a fiber beam model previously developed by the authors for the nonlinear analysis of  
8 strengthened elements, including the effects of shear, is used to predict the response of reinforced  
9 concrete (RC) beams strengthened in shear with fiber reinforced polymers (FRP) sheets. In the  
10 previous version of the model, debonding failure of FRP was not included; hence, its application was  
11 limited to the simulation of wrapped configurations. The model is now extended to account for  
12 debonding failure in order to allow for its application to beams strengthened with U-shaped and side-  
13 bonded configurations. Existing experimental tests on RC beams strengthened in shear by FRP sheets  
14 in both wrapped and U-shaped configurations were numerically simulated. The model reproduces,  
15 with reasonable accuracy, the experimental failure loads, the load-deflection behavior and the strains  
16 in FRP and stirrups with increasing load. The advantages of this proposal are related with the  
17 simplicity and straightforwardness of the beam models to be applied in practical engineering  
18 problems.

19                          Keywords: Strengthening, Shear, FRP laminates, debonding, fiber beam model

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## 21 **INTRODUCTION**

22 There is a lack of worldwide consensus on the evaluation of the contribution of the externally  
23 bonded (EB) FRP reinforcement to the shear strength, in elements strengthened in shear through this  
24 technique (Sas et al. 2009, Pellegrino et al. 2013). One of the main reasons for this might be the  
25 complexity of the resisting mechanisms, not only for the shear strengthening system but also for  
26 reinforced concrete. Another reason might be the difficult evaluation of the laminate debonding  
27 mechanisms, which can be related to the anchorage length of the FRP laminate once the critical shear  
28 crack opens.

29 The existing guidelines (ACI440.2R-08 2008, CNR-DT-200/2004 2004, Concrete Society TR-55  
30 2012, fib Bulletin 14 2001, DAFStb Heft 595 2013) add the contribution of the EB FRP reinforcement  
31 to the shear strength of the unstrengthened element. However, changes in the strut orientation or  
32 additional cracking may change the contribution of the concrete or of the existing transverse  
33 reinforcement to the overall shear strength. The interaction of the FRP shear reinforcement with the  
34 transversal steel or the concrete is considered in some models based on plasticity (Colajani et al.  
35 2005). In addition, a few number of existing formulations consider not only the interaction but also the  
36 laminate debonding (Modifi and Chaallal 2013, Monti and Liotta 2007, Kotynia 2011).

37 The FRP shear strengthening can be performed in different configurations: a) sheets fully  
38 wrapping the cross-section (wrapped); b) sheets or L-shaped laminates bonded on the lateral sides and  
39 the bottom surface of the beam (U-shaped); and c) sheets or laminates bonded in the lateral sides of  
40 the cross-section (side-bonded). The sheets and laminates can be bonded in a continuous or  
41 discontinuous configuration.

42 In the case of U-shaped or side-bonded configurations, the FRP may debond before reaching its  
43 ultimate capacity. Then, the ductility of beams failing in this mode is usually limited (Chen and Teng  
44 2003). To avoid or delay this type of failure, some anchorage devices can be applied (Khalifa and  
45 Nanni 2000). These anchorage devices may consist of rods mounted on the web-flange corner to  
46 anchor the end of the FRP to the compression zone or may consist of steel profiles. However, the use  
47 of bolts or fasteners involves some inconveniences. The anchorage might damage the FRP fibers

48 during installation due to the execution of holes to the FRP, and some stress concentration can appear  
49 at the location of the fasteners (Mofidi et al. 2013).

50 This paper presents numerical studies on the effects of the FRP on the shear strength of RC beams  
51 and the mechanism of debonding failure for U-shaped or side-bonded configurations. A fiber beam  
52 model developed by the authors for the nonlinear analysis of RC and strengthened elements including  
53 the effects of shear (Ferreira et al. 2014a, 2014b, 2013a) is improved to account for debonding failure  
54 of FRP. It is then used to predict the response of RC beams strengthened in shear with FRP sheets in  
55 wrapped and U-shaped configurations. The model is validated through the analysis of some existing  
56 experimental campaigns (Alzate 2012, Mathys 2000, Khalifa and Nanni 2002) on RC beams  
57 strengthened in shear with FRP sheets involving different configurations. An earlier version of the  
58 model, disregarding the debonding failure mechanism of FRP, was previously validated for the  
59 wrapped configuration (Ferreira et al. 2013b). The current model is validated by comparing the  
60 numerical results to those obtained in the experimental program. The model reproduces, with good  
61 accuracy, the experimental failure loads, the load-deflection behavior and the strains in stirrups and  
62 FRP with increasing load until failure. It also reflects the load-sharing between inner transversal steel  
63 reinforcement and EB FRP before and after premature debonding failure.

64 The original contribution of the present work is the consideration of debonding failure mechanism  
65 of FRP in a FEM based on fiber beam approach. This achievement is important due to its simplicity  
66 and computational speed to be applied at true scale structural analysis, making it an attractive tool for  
67 practical engineering.

## 68 **DEBONDING FAILURE CRITERIA OF FRP IN SHEAR STRENGTHENING**

69 As previously mentioned, U-shaped and side-bonded configurations of FRP usually fail due to  
70 debonding after the formation of a critical shear crack (Pellegrino and Modena, 2006). Therefore, a  
71 debonding criteria has to be considered in the numerical model in order to account for this type of  
72 failure. In flexural strengthening, the debonding failure can initiate either at intermediate cracks due to  
73 shear stresses (intermediate crack (IC) debonding), or near the end of the laminate (plate end (PE)  
74 debonding). When debonding occurs, the laminate does not contribute anymore to the cross-section  
75 strength, generally driving to the structural failure, as no equilibrium can be reached between applied

76 forces, reactions and internal forces. For shear strengthening, the debonding failure initiates once the  
 77 shear critical crack opens. Then, the laminate debonds if the FRP bonded length from the shear crack  
 78 to the laminate end is not enough to anchor or transfer the tensile force acting on the FRP. In the side-  
 79 bonded case, debonding can be observed at both sides of the critical shear crack. In the U-shaped case,  
 80 debonding occurs in the upper side of the shear crack.

81 The debonding failure approach implemented in the present model is that proposed by Oller et al.  
 82 (2009). This formulation was originally developed for flexural strengthening to capture IC debonding  
 83 failure and PE debonding. To predict debonding for FRP shear strengthening, the same formulation of  
 84 the plate end debonding can be applied, assuming that for each strip the bonded length ( $L_b$ ) is the FRP  
 85 laminate length shown in Fig. 1. For U-shaped configurations, the bonded length of each strip is the  
 86 bonded length above the critical shear crack. For side-bonded configurations, the bonded length of  
 87 each strip is the minimum length of the laminate above or below the critical shear crack.

88 According to Oller et al. (2009), the maximum transferred force  $F_{\max}$  along the bonded length  $L_b$ ,  
 89 can be expressed as:

$$F_{\max, L_b} = F_0 \begin{cases} \sin\left(\frac{\pi}{2} \frac{L_b}{L_{\text{lim}}}\right) & L_b \leq L_{\text{lim}} \\ 1 & L_b > L_{\text{lim}} \end{cases} \quad (1)$$

$$F_0 = b_L \sqrt{2G_F E_L t_L} \quad (2)$$

$$L_{\text{lim}} = \frac{\pi}{2} \frac{\sqrt{2G_F E_L t_L}}{\tau_{LM}} \quad (3)$$

90 where:  $b_L$  = FRP width;  $t_L$  = FRP thickness;  $E_L$  = laminate modulus of elasticity;  $\tau_{LM}$  = maximum shear  
 91 stress at the interface given by Eq. (4);  $G_F$  = fracture energy or energy by unit area necessary to  
 92 separate the laminate from the support given by Eq. (5). Units are in N and mm

$$\tau_{LM} = C_{\alpha LM} \left( \frac{1}{f_{ctm}} + \frac{1}{f_{cm}} \right)^{-1} \quad (4)$$

$$G_F = C_{\alpha LM}^2 C_F f_{ctm} \quad (5)$$

93 where  $f_{cm}$  = mean value of concrete compressive strength;  $f_{ctm}$  = mean value of concrete tensile

94 strength;  $C_{\tau_{LM}}$  = constant that ranges between 0.37 and 1.56 with a mean value of 0.87 and a standard  
95 deviation of 0.17 according to the shear test database assembled by Oller (2005);  $C_F$  = constant found  
96 to obtain the smallest standard deviation when predicting the experimental maximum force in a single  
97 or double shear test. For the shear test database compiled by Oller (2005),  $C_F$  varies between 0.15 and  
98 0.62, with a mean value of 0.35 and a standard deviation of 0.07.

## 99 **FIBER BEAM MODEL**

### 100 **Fundamentals of the model**

101 The beam model with axial force – shear – bending interaction ( $N$ - $V$ - $M$ ) uses a displacement-based  
102 FE formulation for the nonlinear phased analysis of concrete frame structures. The detailed  
103 formulation and validation of the 1D model with shear critical benchmarks was presented elsewhere  
104 (Ferreira et al. 2014a, 2014b, 2013a). Only a brief description of the fundamentals of the model is  
105 presented here.

106 Fig. 2 presents the general characteristics of the model for the different levels of analysis: element,  
107 section, fiber and material. Regarding the element level, the model is based on the Timoshenko beam  
108 theory with the cross-section discretized into fibers, the longitudinal reinforcement simulated by  
109 means of filaments and transversal reinforcement considered smeared in concrete. At the sectional  
110 level, a shear-sensitive model accounts for the nonlinear force interaction ( $N$ - $V$ - $M$ ). The plane-section  
111 theory, that allows determining the longitudinal strains at each fiber as a function of the generalized  
112 strains of the section, is coupled with a constant shear stress constraint along the cross-section.  
113 Filaments of longitudinal reinforcement are only submitted to axial strains and stresses, following the  
114 plane section theory. Transverse reinforcement (internal steel stirrups and/or EB FRP) is accounted  
115 through its volumetric ratio  $\rho_{st}$  and is submitted to axial stresses  $\sigma_z^{st}$ . Compatibility requirements  
116 impose that the vertical strain  $\varepsilon_z$  in concrete is equal to the strain in the transverse reinforcement. The  
117 computed shear stresses  $\tau_{xz}$  must equate the imposed shear stresses given by the fixed stress constraint  
118  $\tau^*$  of the sectional hypothesis. By guaranteeing these two requirements, the vertical axial strain  $\varepsilon_z$  and  
119 shear strain  $\gamma_{xz}$  of each fiber are outputted. This determination is not linear and an iterative procedure  
120 within the fiber level is needed.

121 Pertaining to the material simulation, a smeared and rotating crack approach is considered for  
 122 concrete. The Hognestad parabola is considered for concrete in compression. Lateral effects of  
 123 softening (Vecchio and Collins 1986) and strength enhancement (Kupfer et al. 1969) factors are  
 124 included. When FRP strengthening is placed by means of a wrapped configuration, the increment of  
 125 both peak strength and ultimate strain of concrete due to the confinement action is considered through  
 126 the model of Spoelstra and Monti (1999). A linear response is assumed for uncracked concrete in  
 127 tension and a tension stiffening curve (Cervenka 1985) is considered for the remaining stresses in the  
 128 cracked stage. Longitudinal and transverse reinforcements (steel and FRP) are under 1D stress-strain  
 129 states determined through linear uniaxial constitutive equations, with kinematic hardening for steel.

130 This model was previously applied to study FRP shear strengthened beams with wrapped  
 131 configurations (Ferreira et al. 2013b). As explained in continuation, this model is extended to account  
 132 for FRP debonding in shear (following the previously presented criteria of Oller et al. 2009) in order to  
 133 expand its application to U-shaped and side-bonded configurations.

#### 134 **Debonding failure of FRP**

135 Fig. 3 summarizes the input hypothesis considered in the sectional model, the output results and  
 136 the criteria for checking FRP debonding failure. The gradients of vertical stresses between the border  
 137 and the shear critical fiber are computed to be compared with the maximum transfer force. The shear  
 138 critical fiber is considered to be located at  $3/4 \cdot h$  ( $h$  in the total height of the cross section), being the  
 139 critical tensile stress  $\sigma_z^{FRP}(z = 3/4 \cdot h)$ . This criteria is a consequence of the basic hypothesis of the  
 140 model, resulting into higher shear strains and higher vertical strains in the more cracked areas (Ferreira  
 141 2013). Since the vertical stress in the border is null, the gradient is equal to the tensile stress in the  
 142 critical fiber. The critical stress in the FRP,  $\sigma_z^{FRP}(z = 3/4 \cdot h)$ , is compared with the maximum vertical  
 143 stress that can be transferred to the FRP,  $\sigma_{max,deb}$ , that corresponds to the maximum transferred force  
 144  $F_{max,Lb}$  (Eq. 6).

$$\sigma_{max,deb} = \frac{F_{max,Lb}}{b_L t_L} \quad (6)$$

145 When the stresses  $\sigma_z^{FRP}$  in the FRP laminate in the critical fiber reach the maximum allowed stress  
 146 that can be transferred by bonding mechanism, the area of the FRP reinforcement of that cross-section

147 is set to zero, and the analysis may continue with redistribution of forces in the remaining steel stirrups  
148 and FRP sheets in other cross sections. The tensional scheme in a fiber for the checking of bond  
149 failure is represented in Fig. 4.

## 150 **VALIDATION OF THE MODEL**

### 151 **Experimental tests by Alzate (2012)**

152 An experimental program on FRP-shear strengthened RC beams was carried out by Alzate (2012)  
153 with the purpose of studying the contribution of FRP to the shear resistance of RC elements. The  
154 beams were simply supported, 4.5 m long and with a rectangular cross section of 0.42 m height and  
155 0.25 m width. A RC beam critical to shear (control beam) was strengthened with different solutions of  
156 FRP in terms of configuration and quantity and tested until failure. From the set of beams tested in the  
157 experimental campaign, the beams strengthened in shear with vertical FRP strips were simulated. Each  
158 beam was submitted to two load tests with different total spans and equal shear spans; only the long  
159 total span configuration was simulated, with a concentrated load applied at a distance of 3 times the  
160 total depth ( $a = 3h = 1.26$  m) from the support. Geometry, internal reinforcement and strengthening  
161 configurations of the specimens are represented in Fig. 5.

162 The beams are reinforced with FRP sheets of 300 mm of width presenting two different  
163 thicknesses - S530 represent unidirectional fibers ( $530 \text{ g/m}^2$ ) with dry fiber thicknesses of 0.293 mm  
164 and S330 represent unidirectional fibers ( $300 \text{ g/m}^2$ ) with dry fiber thicknesses of 0.176 mm - and two  
165 different configurations - wrapped and U-shaped. The names of the tested specimens mean the  
166 following: W90S530 is the beam with wrapped S530 FRP; U90S530 is the beam with U-shaped S530  
167 FRP; W90S300 is the beam with wrapped S300 FRP; and U90S300 is the beam with U-shaped S300  
168 FRP. The fibers of the FRP sheets formed an angle of  $90^\circ$  with respect to the longitudinal axis and the  
169 sheets were spaced at 200 mm from edge to edge.

170 The beams with wrapped FRP strengthening present a ductile shear-bending related failure with  
171 FRP rupture and crushing of concrete near the load application point; in contrast, the beams with U-  
172 shaped configuration presented a brittle shear failure mechanism after FRP debonding (Fig. 6). No  
173 reference to yielding of longitudinal reinforcement is made; however, the load-displacement curve of  
174 the wrapped beams presents a plateau of displacements at peak load (demonstrating the ductility of the

175 response) which is not observed in the case of the U-shaped beams.

176 Experimental data available in Alzate (2012) includes vertical displacements at mid-span  
177 measured by displacement transducers and vertical strains in stirrups and in the FRP sheets monitored  
178 at the shear-span by means of bonded strain gages. The location of the sensors considered in the  
179 validation is represented in Fig. 5.

### 180 *Numerical modelling*

181 The FE mesh used in the numerical simulation is represented in Fig. 7: beam elements with 0.1 m  
182 length, cross-section discretized into fibers of 0.005 m height, longitudinal reinforcement simulated  
183 with steel filaments, both the transversal steel and FRP reinforcement considered smeared with their  
184 respective quantities and material properties. Different specimens of each type were tested (identified  
185 with -a, -b or -c) and also simulated; the only difference between them is the compression strength of  
186 concrete  $f_{cm}$ .

187 The material properties of concrete and FRP considered in the model are listed in Table 1. The  
188 concrete compression strength  $f_{cm}$  was measured in concrete specimens in the lab and is available in  
189 Alzate (2012). The remaining properties were determined by the equations of EC2 (2004), as function  
190 of the experimental value of  $f_{cm}$  (see Fig. 2 for the meanings of the material properties) (see Eqs. (7) to  
191 (9)). For the wrapped configurations, the confinement effect was considered, enhancing the  
192 mechanical properties of concrete. For the U-shaped configuration, confinement effect was neglected.  
193 For the beam with U-shaped configuration (U90), the parameters related to the debonding failure  
194 criteria ( $\tau_{LM}$ ,  $G_F$  and  $t_L$ ) were determined as function of  $f_{cm}$  as shown in Table 1. The characteristics of  
195 the FRP are also listed in Table 1. For steel reinforcement, longitudinal and transversal, the following  
196 properties were considered:  $E_s = 200000$  MPa,  $f_{sy} = 500$ ,  $f_{su} = 580$  MPa,  $\varepsilon_{su} = 0.10$ . Load was applied  
197 incrementally until failure.

$$f_{cm} = 0.30 \cdot \sqrt[3]{f_{ck}^2} \quad \text{if } f_{ck} \leq 60 \text{ N/mm}^2 \quad (7)$$

$$f_{cm} = 2.12 \cdot \ln \left( 1 + \frac{f_{cm}}{10} \right) \quad \text{if } f_{ck} > 60 \text{ N/mm}^2 \quad (8)$$



$$E_c = 22 \cdot \left( \frac{f_{cm}}{10} \right)^{0.3} \quad (9)$$

## 198 ***Results and discussion***

199 The experimental and numerical shear force vs. deflection at mid span are compared in Fig. 8 for  
200 the two beams with different FRP configurations, wrapped (W90) and U-shaped (U90) and for the two  
201 series (S5 means series S530 and S3 means series S300); in case of existence of more than one  
202 specimen of each type they are identified with the letters -a, -b and -c).

203 The numerical results show a good agreement with the experimental response in terms of ultimate  
204 load and along the nonlinear path with increasing load. The summary of computed results at failure  
205 and the comparison with experimental data are presented in Table 2. From the graphics and the table it  
206 is observed that the model is able to predict a correct failure load of the beams with U-shaped FRP  
207 configurations because it accounts for debonding failure. In fact, the W- and U-shaped beams of each  
208 series have exactly the same geometry, reinforcement and quantity of stirrups and FRP; the only  
209 difference is the strengthening configuration. Laminate debonding failure in the U-shaped beam  
210 occurs before FRP reaches its maximum strength; as can be seen in the values of  $\sigma_z^{FRP}$  for the  
211 debonding instant in Table 2. When this value exceeds the maximum stress allowed to be transferred,  
212  $\sigma_{max,deb}$ , the debonding mechanism occurs, setting the FRP area of the cross section to zero. From this  
213 point forward, this FRP element ceases its contribution to the structural response. For all the U-shaped  
214 strengthened beams, the model predicts failure right after debonding occurs, being not able to  
215 redistribute the forces; this is consistent with the experimental observations (Alzate 2012). The beams  
216 with the wrapped configurations fail when FRP reaches the ultimate capacity; hence presenting higher  
217 ultimate load carrying capacities, which is correctly captured by the model.

218 Only converged values are represented in the graphs. The last load step represented related to the  
219 last converged; the next one is when materials failure occurs. The analysis is not able to reach  
220 convergence in the last load step due to vast damage of materials.

221 The computed strains in the transversal reinforcement (inner steel stirrups and EB FRP) with  
222 increasing shear force are compared with the experimental measurements for the beams with different  
223 FRP strengthening configurations (Wrapped and U-shaped). Fig. 9 presents the results of series S530

224 and Fig. 10 of series S300; for location of the sensors see Fig. 5. Only one specimen of each type is  
225 represented; the other specimens presented similar fittings.

226 Despite the difficulty of this comparison, due to the discrete form of the real cracks and the  
227 assumption of smeared cracking by the model, a good consistency between numerical and  
228 experimental results can be observed. The load level for which the stirrups and the FRP reinforcement  
229 start to carry load is well captured by the model. This load level corresponds to the outset of diagonal  
230 cracking. Sensors 1 and 5 located in the bottom of the beam can be more influenced by bending  
231 cracking, and hence, of more difficult comparison. However, in general, it can be observed that the  
232 model is able to capture the overall response of the transverse reinforcement. The load sharing  
233 between external FRP and inner steel stirrups is discussed in detail in the following section.

#### 234 ***Load sharing between external FRP and internal steel stirrups***

235 The computed stresses in the transversal reinforcement (stirrups and FRP) are compared in Fig. 11  
236 for the wrapped and U-shaped configurations; these results are related to the mid-height of the cross  
237 section at the mid shear span. In Fig. 11b it can be observed the load level for which FRP debonding  
238 occurs in the U-shaped beam and the drop of stresses in FRP for onward load levels. After the  
239 occurrence of debonding failure, shear stresses are transferred for the steel stirrups that where already  
240 yielded at this stage leading to its failure and consequent failure of the beam in shear. For the wrapped  
241 configuration (Fig. 11a), the FRP sheets continue to carry load until failure of the FRP; i.e., the load  
242 carrying capacity of the FRP is not limited by the loss of bond. In these graphs, the yielding of stirrups  
243 instant is also marked; it can be noticed that, before this point, FRP and steel stresses are similar; after  
244 yielding, steel cannot increase the load carrying capacity and hence, the FRP increases their stresses  
245 significantly. This is observed in both cases, in wrapped and U-shaped configurations.

246 The computed strains and stresses of the transverse reinforcement (inner steel stirrups and EB  
247 FRP) throughout the cross section located at the mid shear span of the beams with U-shaped  
248 configuration are presented in Figs. 12 and 13, respectively, for the load levels near start of yielding of  
249 transversal steel reinforcement. Bond-slip is not directly considered in the numerical model (only the  
250 debonding failure is considered) resulting into equal strains computed in the vertical direction in the  
251 FRP and steel stirrups. For this reason only one graph is presented in Fig. 12 for each series (S530 and

252 S300).

253 Fig. 13 presents the stresses in the stirrups and in the FRP separately for each series (S530 and  
254 S300). The results correspond to the specimens U90S530-a and U90S300-b.

255 The strains (Fig. 12) and stresses (Fig. 13) presented in these graphs correspond to three load  
256 levels: immediately before yielding, yielding of transversal steel reinforcement located in the shear  
257 critical fiber (located at mid-shear span at  $3/4h$ ) and immediately after yielding. It can be observed  
258 that, yielding of steel reinforcement is spreading from the bottom to the top of the cross section. When  
259 yielding of transversal reinforcement is reached in a fiber, the strains and stresses increase in the FRP  
260 in order to compensate the fact that steel entered in the plastic phase. This can be seen in the change of  
261 the inclination of the diagrams of strains and stresses in FRP in the location of the yielded fiber.

262 The same presentation of results, strains (Fig. 14) and stresses (Fig. 15) along the shear critical  
263 cross section, is performed for the load levels near debonding of FRP. Respectively, the strains and  
264 stresses presented in these graphics correspond to three load levels: immediately before debonding, at  
265 debonding and immediately afterwards. It can be observed how the strains increase significantly when  
266 debonding occurs (Fig. 14a and 14b); and how the FRP ceases to contribute to shear resistance  
267 mechanism at the moment and after occurrence of peeling (Fig. 15b). For this stage of advanced  
268 loading and damage, stirrups are yielded in a large portion of the height of the cross section, as can be  
269 observed in Fig. 15a, and hence, are not capable of increasing stresses, only strains. It can be observed,  
270 that after debonding, the yielding of the stirrups propagates to the top fibers; however, this  
271 transference is very limited, as it immediately reaches the neutral axis. As the stirrups are already  
272 extensively yielded in the moment of FRP debonding, is not possible to transfer the forces carried out  
273 by the FRP to the transversal steel, and consequently, failure occurs right afterwards.

#### 274 **Experimental tests by Matthys (2000)**

275 The experimental campaign carried out by Matthys (2000) consisted on RC beams strengthened in  
276 shear by means of different configurations of externally bonded FRP reinforcement. The beams were  
277 simply supported, with 4.0 m long and with a rectangular cross section of 0.45 m height and 0.2 m  
278 width, tested under 4-point loading until failure. From the 5 beams tested with different types of FRP  
279 reinforcement, two were considered for simulation in this work, corresponding to the U-shaped

280 configuration: BS4 (continuous FRP) and BS5 (3 discontinuous strips of FRP in each shear span).  
281 Thickness and properties of the FRP are the same for both specimens; the amount of longitudinal  
282 reinforcement (6 rebars of 20 mm of diameter) and inner transversal reinforcement (stirrups of 6 mm  
283 of diameter spaced of 400 mm) are also equal. Experimental data available consists on curves of load  
284 vs. displacements and strains in FRP in different positions in the beams. The geometry and positions  
285 of the strain gauges are represented in Fig. 16.

286 Due to the different ratios of FRP reinforcement, these two beams presented different behaviors:  
287 BS4 presented a ductile response in failure with crushing of concrete in the mid-span and yielding of  
288 longitudinal reinforcement; and BS5 presented a brittle shear failure with FRP rupture and subsequent  
289 debonding.

### 290 *Numerical modelling*

291 The FE mesh used is presented in Fig. 17, with beam elements of 0.1 m length, cross section  
292 discretized into 0.01 m height, longitudinal reinforcement simulated with steel filaments, transverse  
293 reinforcement (steel and FRP) as smeared. The material properties of concrete and FRP, and the  
294 debonding failure criteria parameters are included in Table 1, corresponding to the experimental lab  
295 tests reported in Matthys (2000). No confinement effect was considered due to the FRP. The following  
296 properties were considered for steel (as reported in Mattys 2000): for rebars with 20 mm of diameter  
297 (longitudinal reinforcement)  $E_s = 200000$  MPa,  $f_{sy} = 530$  MPa,  $f_{su} = 620$  MPa,  $\varepsilon_{su} = 0.119$ ; for rebars  
298 with 6 mm of diameter (stirrups)  $E_s = 200000$  MPa,  $f_{sy} = 560$  MPa,  $f_{su} = 590$  MPa,  $\varepsilon_{su} = 0.051$ .

### 299 *Results and discussion*

300 The experimental curves of load vs. displacements are compared with the computed results in Fig.  
301 18 for the two tests. The different ductility on the behavior of these two specimens under increasing  
302 load can be observed: BS4 was not limited by a reduced shear capacity and reached its bending  
303 capacity, probably due to the higher amount of FRP (continuous along the shear span; in contrast,  
304 BS5, with less amount of FRP (discontinuous strips along the shear span) presented a shear failure  
305 with rupture and debonding of FRP at a lower load level. The numerical model is consistent with the  
306 experimental response under increase loading and with the observed failure mechanisms. A summary  
307 of results at failure is included in Table 2.

308 The vertical strains measured in the FRP at different points of the shear span are compared in Fig.  
309 19 with the results of the numerical model, for both specimens. Both graphs present the same scales to  
310 show the small strains observed in BS4, less sensitive to shear and without relevant diagonal cracking,  
311 when compared with BS5, which is a shear critical beam, developing important strains in FRP for  
312 higher load levels. It is observed that for the peak load in BS5, FRP strains were near the ultimate  
313 value; and the tensile stresses are far from the maximum debonding limit (see Table 2).

#### 314 **Experimental tests by Khalifa and Nanni (2002)**

315 Khalifa and Nanni (2002) performed an experimental program of beams with (SW type) and  
316 without (SO type) stirrups strengthened in shear by continuous or discontinuous CFRP U-shaped  
317 sheets with one (90°) or two plies (0-90°). The beams had a rectangular cross section of 150 mm wide  
318 and 305 mm deep. All beams were tested under 4-point loading until failure with different shear spans.  
319 Two different a/d relationships were tested: 3 and 4. Five beams (SO-3-2, SO-3-3, SO-3-4, SO-4-2  
320 and SO-4-3) were considered for simulation in this work. Four 32 mm diameter rebars were used as  
321 longitudinal reinforcement with (two at top and two at bottom face of the cross-section). In series SO,  
322 no stirrups were provided in one side of the beam. Experimental data available consists on curves of  
323 load vs. displacements and strains in FRP for beam SO-3-4. The geometry and positions of the strain  
324 gauges are represented in Fig. 20.

325 All analyzed beams failed due to laminate debonding except for beam SO-4-3 that failed due to  
326 concrete splitting.

#### 327 ***Numerical modelling***

328 The FE mesh used beam elements with 0.1 m of length, cross section discretized into 0.01 m  
329 height, longitudinal reinforcement simulated with steel filaments, transversal reinforcement (steel and  
330 FRP) as smeared. The material properties of concrete and FRP are included in Table 1. No  
331 confinement effect was considered. The following properties were considered for steel (Khalifa and  
332 Nanni, 2002):  $E_s = 200000$  MPa,  $f_{sy} = 460$  MPa,  $f_{su} = 7300$  MPa.

#### 333 ***Results and discussion***

334 Fig. 21 shows the comparison between experimental and numerical results in terms of load vs.  
335 displacements. A good correlation is observed in general, in terms of ultimate load and overall

336 response under increasing loading. A summary of results at failure is included in Table 2. Ultimate  
337 load is overestimated by the numerical model for beam SO-3-4, which can be due to uncertainties on  
338 the properties related to bonding resistance. For all the other beams, the predicted ultimate load is  
339 similar to the experimental values.

340 Since these specimens had no internal transversal steel reinforcement in the critical shear span, the  
341 FRP debonding leads to an immediate loss of equilibrium of internal forces and failure. That is the  
342 reason why the predicted shear load for start of debonding is equal to the ultimate shear load for some  
343 of the beams (SO-3-2, SO-4-2). For the other specimens (SO-3-3, SO3-4, SO-4-3) the model predicted  
344 shear failure previously to debonding of FRP.

345 Fig. 22 compares the FRP strains at different locations along the SO-4-2 beam. The experimental  
346 results show that the FRP strain was zero prior to diagonal crack formation (around 80kN of shear  
347 force) increasing significantly until failure. The maximum local CFRP vertical strain measured at  
348 failure was approximately 0.0045. Given the difficulties on comparing experimental results and  
349 predictions based on smeared cracked approaches, the numerical results fit reasonably within the range  
350 of the experimental values, being able to capture the start of loading of FRP reinforcement and strain  
351 level in failure.

### 352 **Comparison of the numerical FRP stresses at debonding to the guidelines predictions**

353 Table 3 shows the numerical values of the FRP stresses at debonding stage which are compared to  
354 the values given by the existing design guidelines (Fib Bulletin 14 2001, ACI-440.2R-08 2008, CNR-  
355 DT200/2004, TR-55 2012, DafStb Heft 595 2013) to obtain the FRP shear strength contribution. In  
356 addition, it presents the ratio between the numerical and analytical values in brackets. As observed, the  
357 predictions of the Fib Bulletin are closest to the numerical values, although in some cases it  
358 overestimates the FRP stresses. The remaining guidelines are more conservative, underestimating the  
359 stresses in the external reinforcement. For instance, the mean ratio between the numerical and the  
360 analytical predictions for all tests of Alzate (2012) are: 0.97 for the Fib Bulletin 14, 1.56 for the ACI-  
361 440.2R-08, 1.67 for the CNR-DT-200/2004, 1.32 for the TR-55 and 1.36 for the DafStb Heft 595.

### 362 **Study of the efficiency of FRP strengthening system to increase the shear capacity**

363 This part of the paper aims to demonstrate how the model can be used as a tool for studying the

364 efficiency of FRP strengthening solutions for beams critical to shear.

365 The model was already extensively validated for the cases of shear critical RC beams (Ferreira et  
366 al. 2014a, 2014b, 2013a). As an example, the shear critical RC beam taken as the control specimen of  
367 Alzate's experimental campaign is correctly simulated by the model as can be observed in Fig. 23. The  
368 model captures the shear failure mode of the beam as explained in Ferreira et al. (2013b).

369 The model can be used to predict the gain of shear resistance brought by different strengthening  
370 solutions, as exemplified with the U-shaped and wrapped configurations with S530 and S300 FRP  
371 sheets used in the Alzate's (2012) experimental work. Fig. 24 presents the increment of load carrying  
372 capacity brought by the different solutions of FRP strengthening in shear. The strengthening  
373 interventions lead to, not only an increase of shear capacity and consequent load carrying capacity, but  
374 also, an increase of ductility. It also attained a change in the failure mode from brittle shear to bending  
375 with plasticity of the longitudinal reinforcement and ductile response.

376 These results demonstrate how the model can be used to assess the load carrying capacity of RC  
377 beams critical to shear and to predict the gaining of shear resistance achieved from different solutions  
378 of FRP strengthening in terms of shape and quantity. This can lead to a more efficient and rational  
379 design of the strengthening measure.

## 380 **CONCLUSIONS**

381 Debonding failure of FRP in shear strengthened RC beams is studied in this paper by means of a  
382 numerical model based on the fiber beam approach. The previous version of this model was limited to  
383 the application of wrapped configurations as it disregarded this premature type of failure FRP. This  
384 paper describes the enhancement of the model to account for FRP debonding extending its use to sided  
385 and U-shaped configurations of externally bonded reinforcement. Experimental tests available in  
386 literature were numerically simulated. From these analyses the following conclusions are drawn:

387 - The model is able to correctly capture the load-displacement response of the strengthened  
388 beams with wrapped and U-shaped configurations;

389 - The model captured the overall response of the transverse reinforcement (inner steel stirrups  
390 and EB FRP), capturing the debonding of FRP and subsequent failure of the beams for the U-shaped  
391 configurations;

392 - When debonding failure occurs and FRP ceases its contribution to the shear resistance,  
393 stirrups were already extensively yielded and were no longer able to absorb the redistribution of  
394 forces, and failure occurred right after.

395 - When comparing the FRP numerical stresses to the analytical predictions given by the existing  
396 guidelines to obtain the FRP shear strength contribution, it has been observed that most of the existing  
397 guidelines are conservative, assuming stresses between 60% and 75% of the numerical predictions.

398 The model can be used as a tool to study the effects of different strengthening solutions  
399 (configurations, quantities, spacing and thicknesses of FRP) to increase the shear capacity of beams.  
400 The computational and modelling simplicity makes it suitable to real scale practical applications.

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Table 1. Material properties for concrete, bond and FRP

Tests	Concrete properties				Bond properties		FRP strengthening properties				
	$f_{cm}$ (MPa)	$E_c$ (MPa)	$f_{ctm}$ (MPa)	$\varepsilon_{c,u}$	$G_f$ (MPa.mm)	$\tau_{LM}$ (MPa)	$t_f$ (mm)	$\rho_{FRP}$	$E_{FRP}$ (MPa)	$\varepsilon_{FRP,u}$	$f_{FRP,u}$ (MPa)
U90S5-a	36.95	32560	3.33	0.0035	0.717	2.24	0.293	0.0088	240000	0.0150	4000
U90S5-b	28.01	29970	2.77	0.0035	0.596	1.85	0.293	0.0088	240000	0.0150	4000
U90S3-a	20.50	27290	2.25	0.0035	0.484	1.49	0.176	0.0053	240000	0.0155	3800
U90S3-b	22.58	28090	2.40	0.0035	0.516	1.59	0.176	0.0053	240000	0.0155	3800
U90S3-c	28.01	29970	2.77	0.0035	0.596	1.85	0.176	0.0053	240000	0.0155	3800
W90S5	49.90	34976	3.90	0.0180	No debonding check		0.293	0.0088	240000	0.0150	4000
W90S3-ab	37.00	32575	3.33	0.0130	No debonding check		0.176	0.0053	240000	0.0155	3800
W90S3-b	37.00	32575	3.33	0.0130	No debonding check		0.176	0.0053	240000	0.0155	3800
BS4	38.40	34100	5.72	0.0035	2.557	4.28	0.110	1.1	233000	0.01502	3500
BS5	36.00	33746	3.27	0.0035	2.453	4.10	0.110	0.00014	233000	0.01502	3500
SO-3-2	27.50	25000	2.73	0.0035	1.088	2.72	0.165	0.0009	228000	0.01662	3790
SO-3-3	27.50	25000	2.73	0.0035	0.959	2.55	0.165	0.0013	228000	0.01662	3790
SO-3-4	27.50	25000	2.73	0.0035	1.296	2.97	0.165	0.0022	228000	0.01662	3790
SO-4-2	27.50	25000	2.73	0.0035	1.088	2.72	0.165	0.0009	228000	0.01662	3790
SO-4-3	27.50	25000	2.73	0.0035	1.296	2.97	0.165	0.0022	228000	0.01662	3790

Table 2. Summary of experimental and numerical results at failure

Tests	Experimental data			Numerical results							
	$P_u$ (kN)	$V_u$ (kN)	Failure mode	Failure			Debond failure related results				
				$P_u$ (kN)	$V_u$ (kN)	Failure mode	$P_{u,num}/$ $P_{u,exp}$	$V_{u,deb}$ (kN)	$V_{u,deb}/$ $V_u$	$\sigma_z^{FRP}$ (MPa)	$\sigma_{max,deb}$ (MPa)
U90S5-a	341	247	DS	341	241	PS	1.00	240	0.99	1109	1084
U90S5-b	326	236	DS	315	223	PS	0.97	222	0.99	991	988
U90S3-a	285	207	DS	263	186	PS	0.92	186	1.00	1151	1149
U90S3-c	320	232	DS	311	219	PS	0.97	219	1.00	1332	1275
W90S5	383	276	BS	402	284	BS	1.05	NP	-	-	-
W90S3-ab	432	311	BS	408	289	BS	0.94	NP	-	-	-
W90S3-b	394	284	BS	408	289	BS	1.04	NP	-	-	-
BS4	504	256	B	498	249	B	0.99	NP	-	-	-
BS5	340	174	DS	355	178	DS	1.04	178	1.00	2348	3223
SO-3-2	262	131	DS	260	130	DS	0.99	130	1.00	1710	1710
SO-3-3	266	133	DS	230	115	S-ND	0.86	-	-	-	-
SO-3-4	289	144.5	DS	380	190	S-ND	1.31	-	-	-	-
SO-4-2	255	127.5	DS	270	135	DS	1.06	135	1.00	1710	1710
SO-4-3	310	155	BS	281	140.5	BS	0.91	-	-	-	-

DS= Debonding FRP - Shear

BS=bending-shear

ND= No Debonding

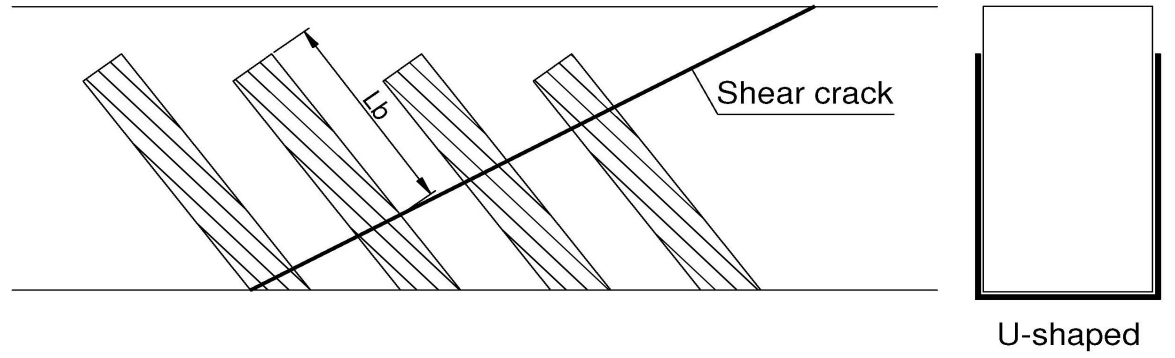
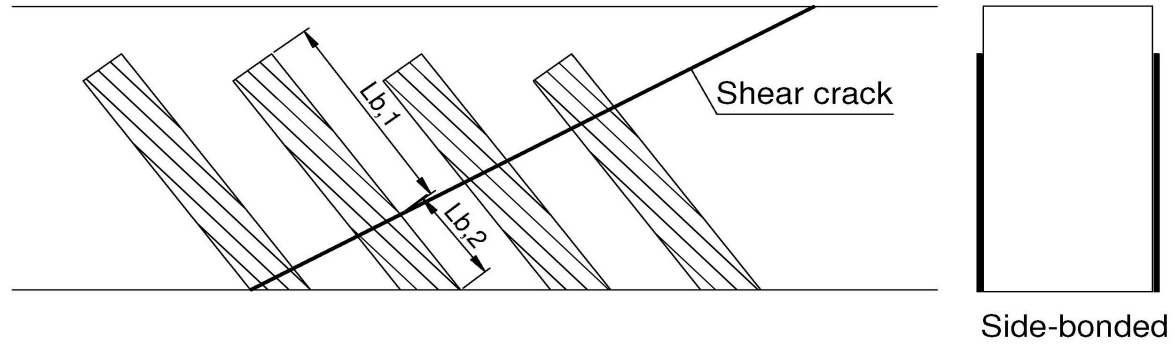
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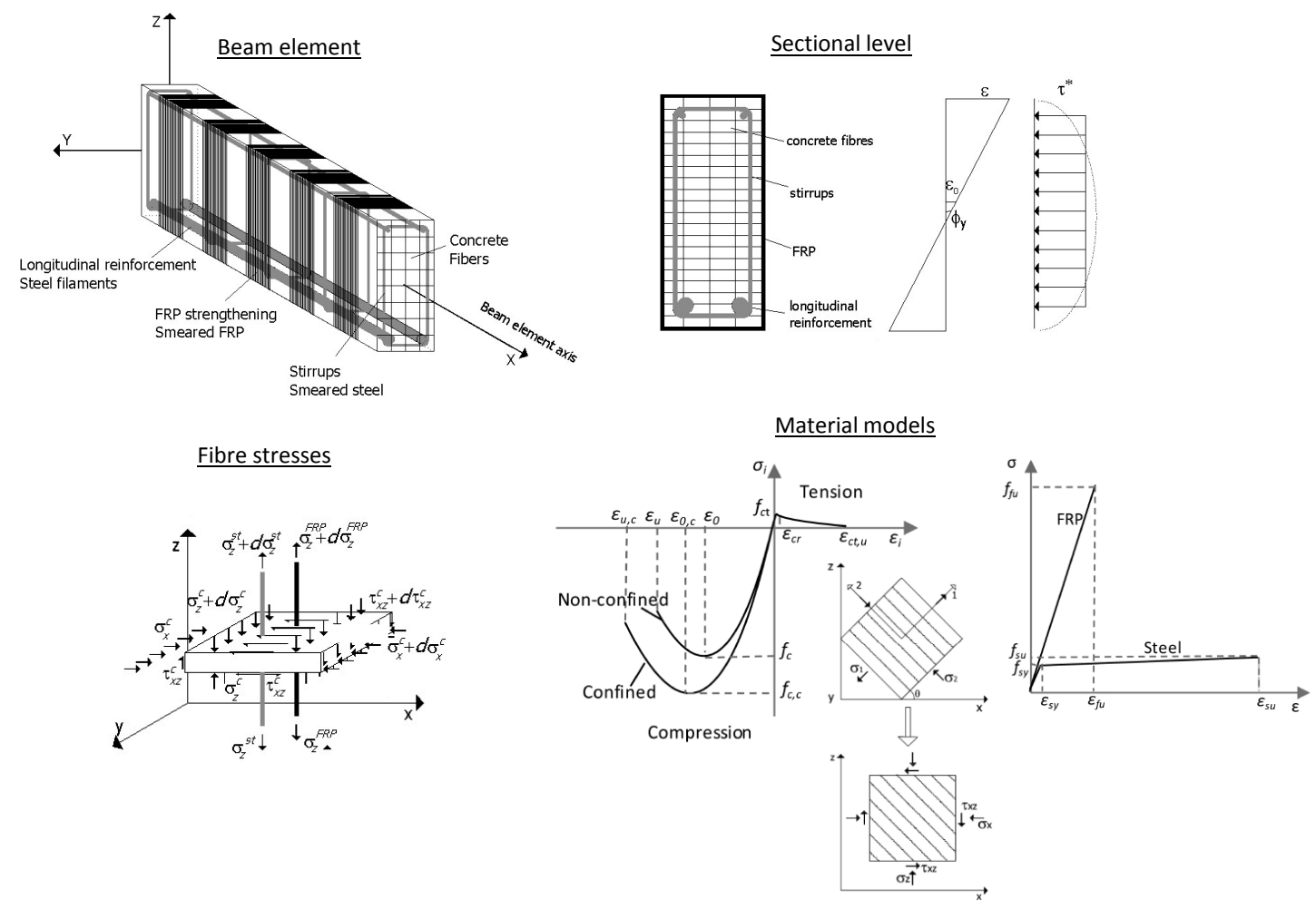
S=shear

Table 3. FRP stresses at debonding. Comparison between numerical and analytical predictions.

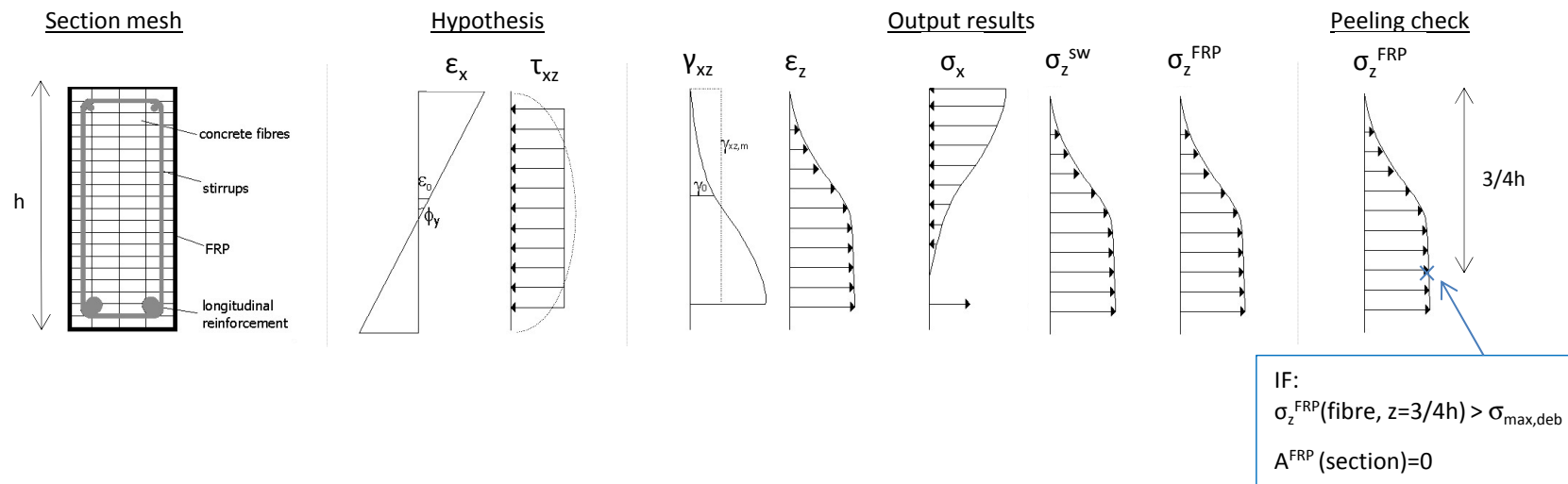
Test	Numerical		Analytical			
	$\sigma_z^{FRP}$ (MPa)	Fib bulletin 14 (2001)	ACI 440.2R- 08 (2008)	CNR- DT200/2004 (2004)	TR-55 (2012)	DafStb Heft 595 (2013)
		$\sigma_{max,deb}$ (MPa)	$\sigma_{max,deb}$ (MPa)	$\sigma_{max,deb}$ (MPa)	$\sigma_{max,deb}$ (MPa)	$\sigma_{max,deb}$ (MPa)
U90S5-a	1109	1103 (1.01)	771 (1.44)	660 (1.68)	826 (1.34)	817 (1.36)
U90S5-b	991	994 (1.00)	641 (1.55)	582 (1.70)	753 (1.32)	728 (1.36)
U90S3-a	1151	1221 (0.94)	681 (1.69)	696 (1.65)	904 (1.27)	852 (1.35)
U90S3-c	1332	1372 (0.97)	838 (1.59)	801 (1.66)	960 (1.39)	970 (1.37)
BS5	2348	3411 (0.69)	932 (2.52)	1216 (1.93)	932 (2.52)	1295 (1.81)
SO-3-2	1710	1381 (1.24)	912 (1.88)	925 (1.85)	912 (1.88)	1044 (1.64)
SO-3-3	-	1101 (-)	912 (-)	906 (-)	912 (-)	1044 (-)
SO-3-4	-	827 (-)	912 (-)	838 (-)	912 (-)	-
SO-4-2	1710	1381 (1.24)	912 (1.88)	925 (1.85)	912 (1.88)	1044 (1.64)
SO-4-3	-	827 (-)	912 (-)	838 (-)	912 (-)	-

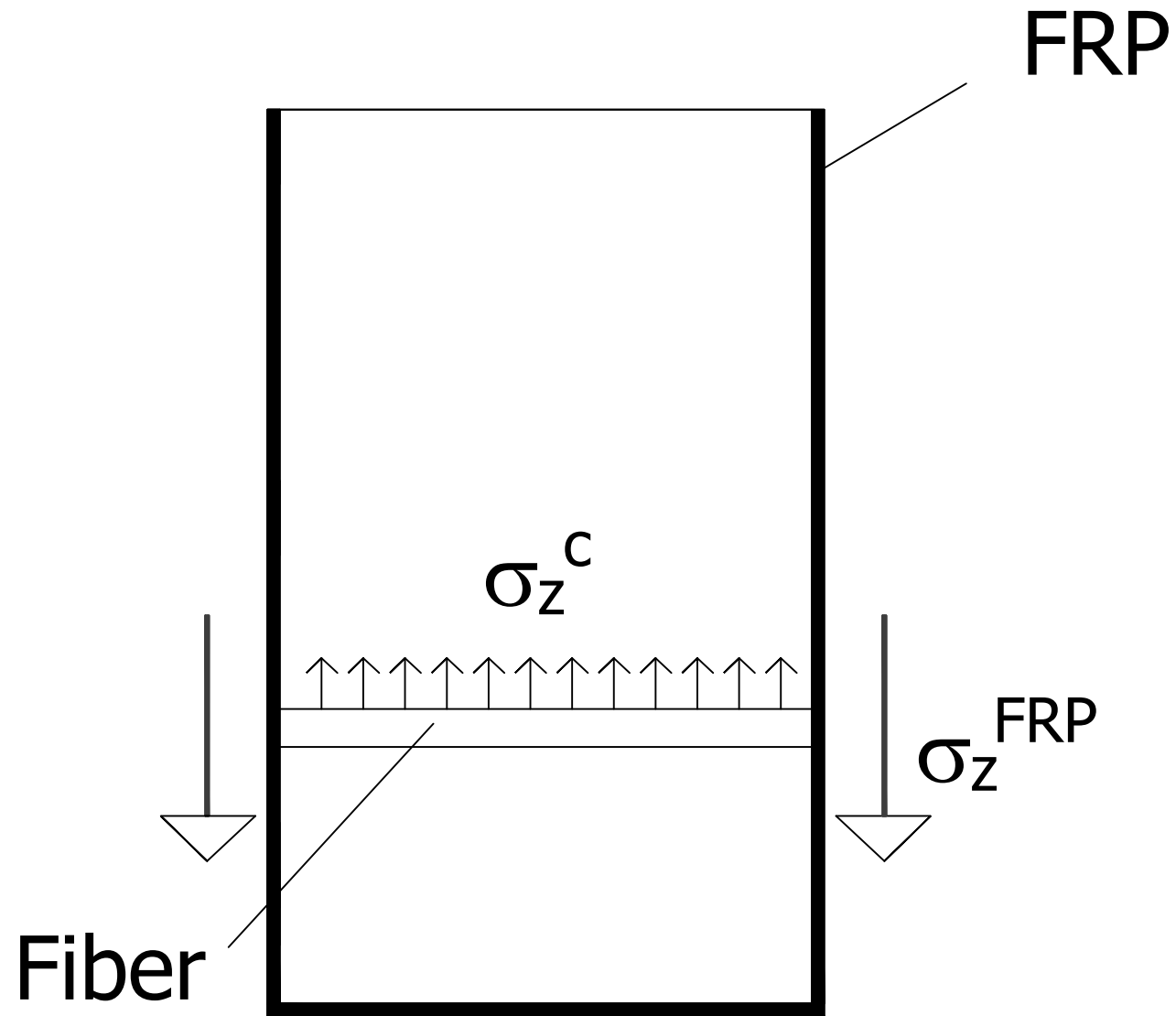
\*in brackets numerical to analytical ratios







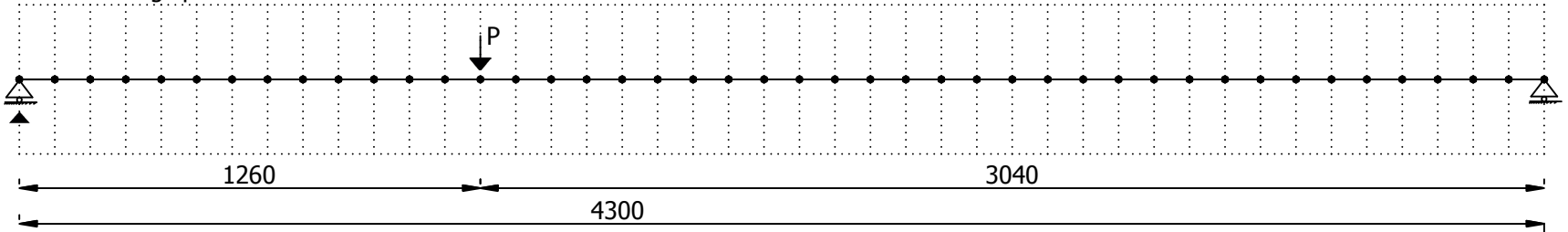




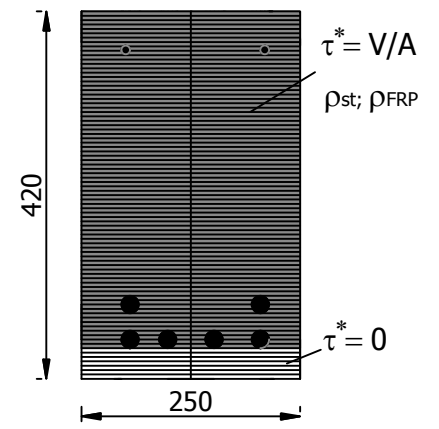


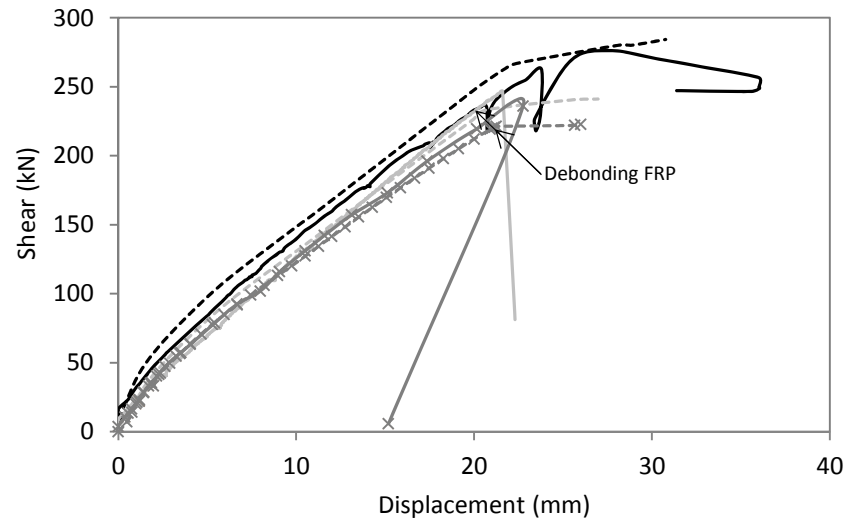


Test 1 - Long span

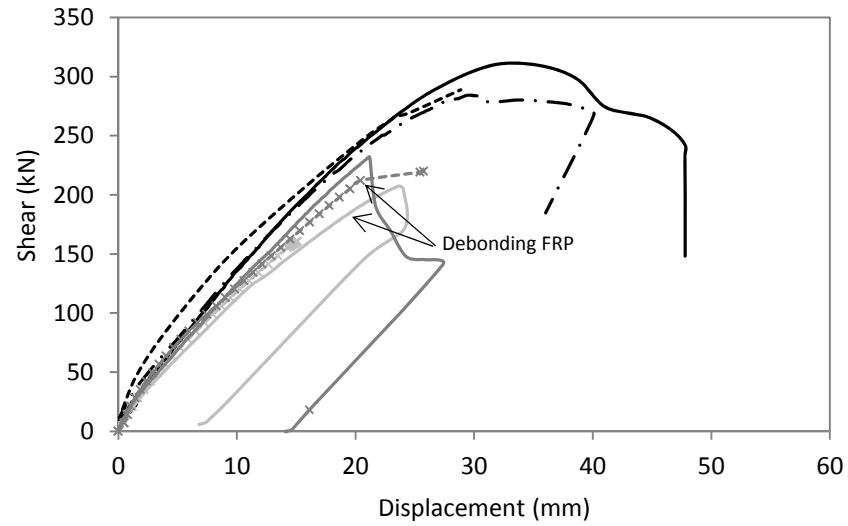


- Legend:
- Nodes
  - Elements
  - ▬ Shear resistant fibers
  - ▭ Non shear resistant fibers

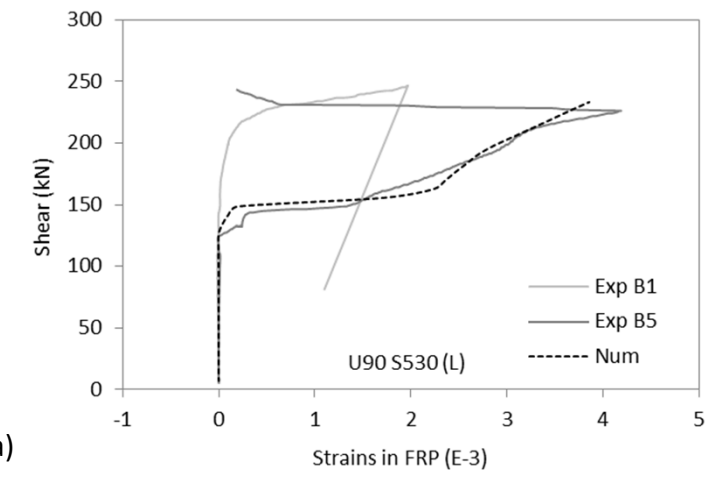
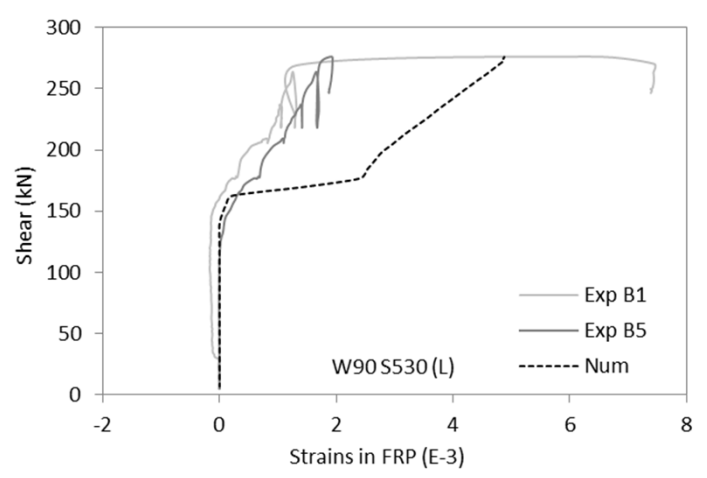
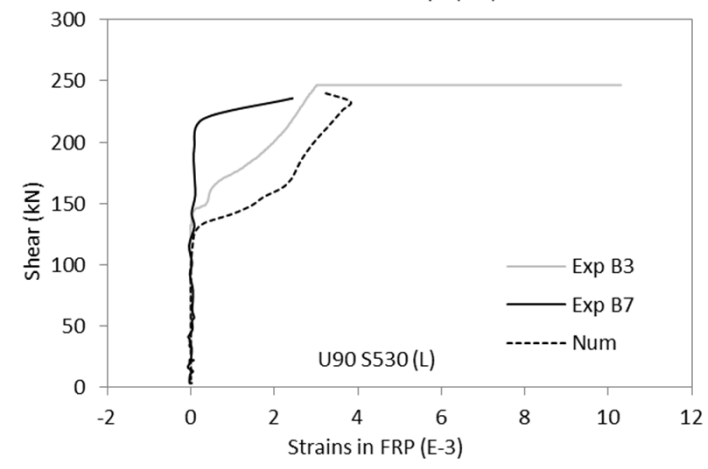
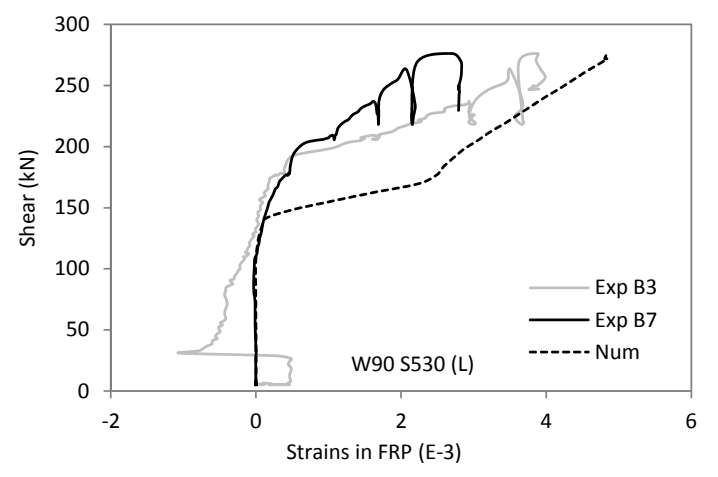
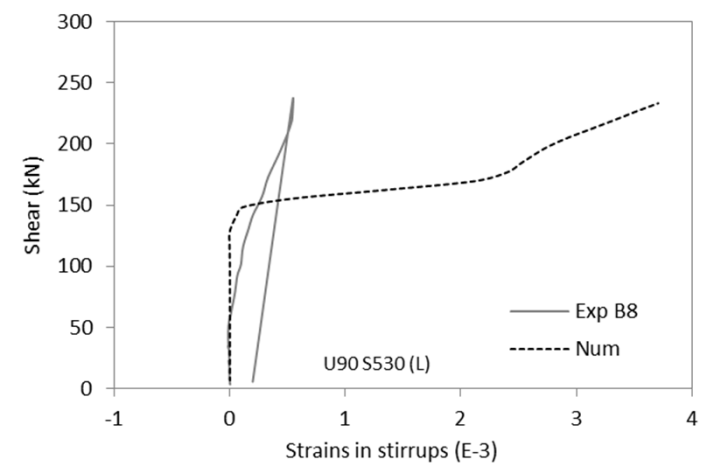
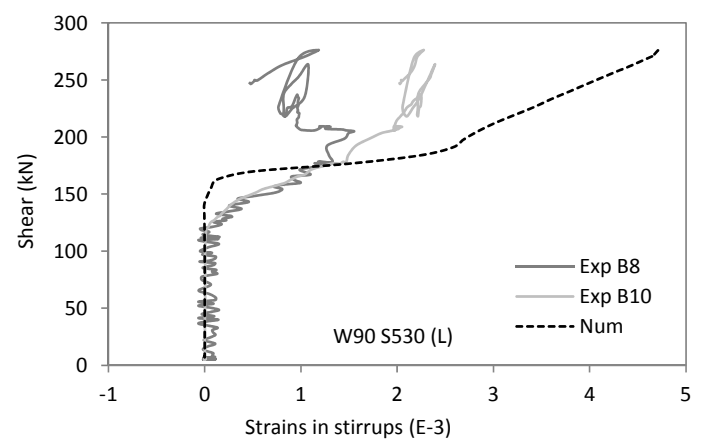




a)



b)

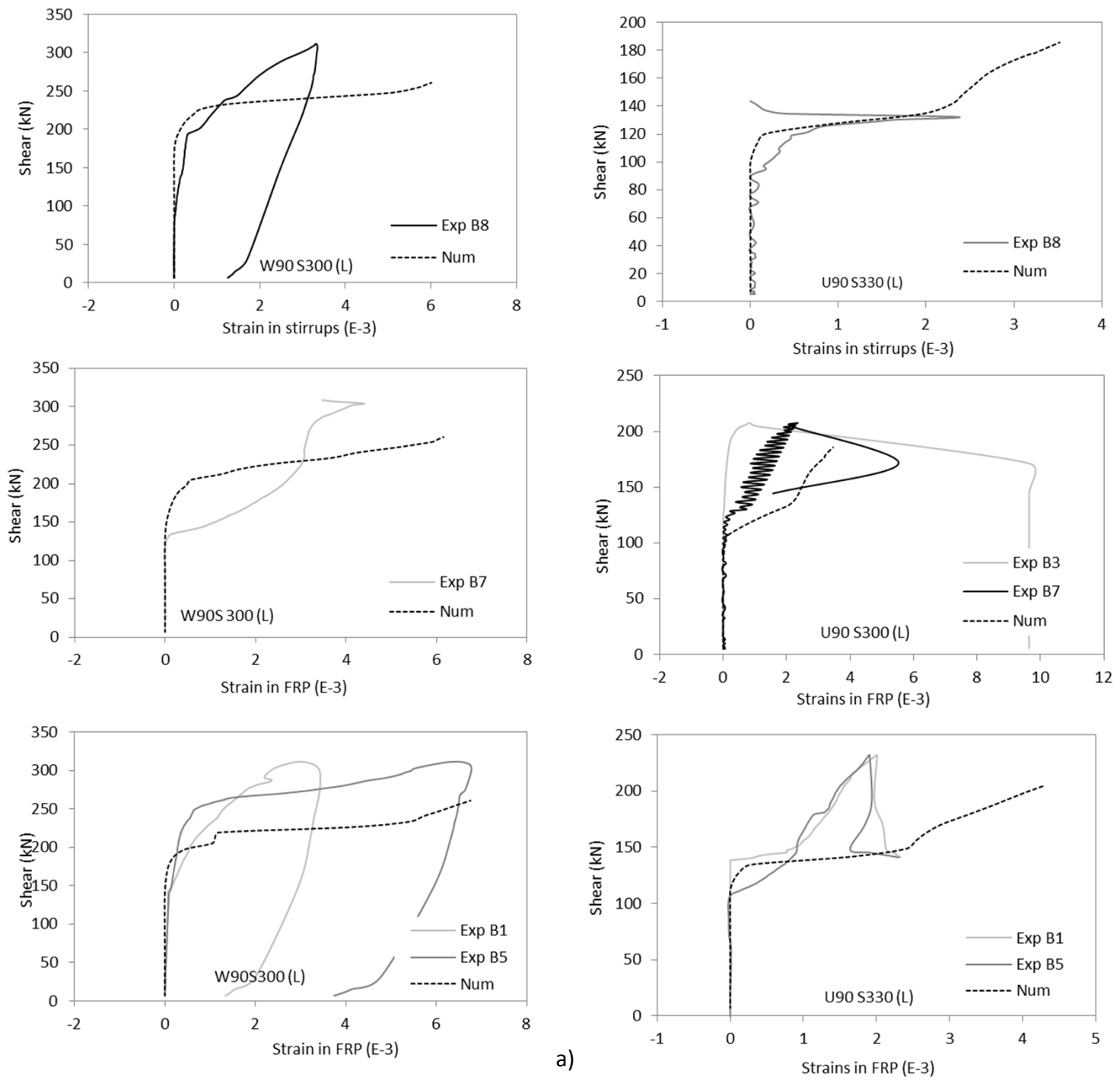


a)

b)

Figure 10

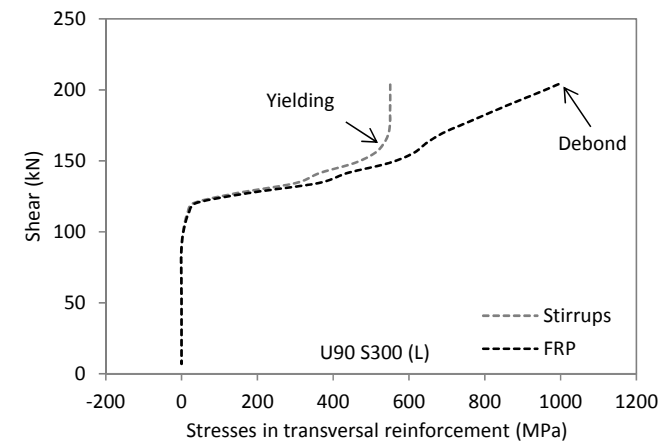
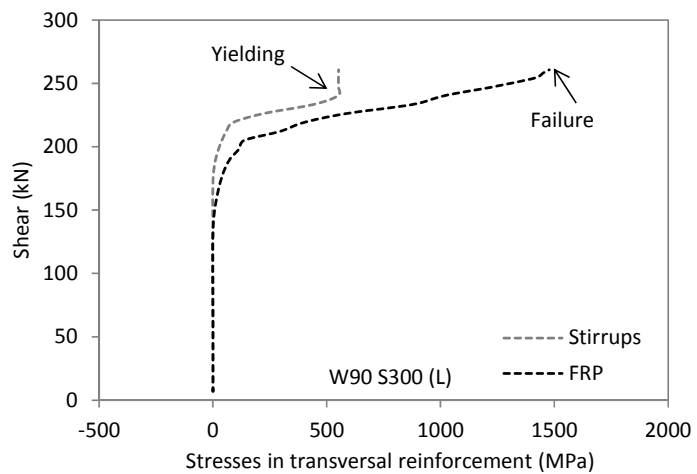
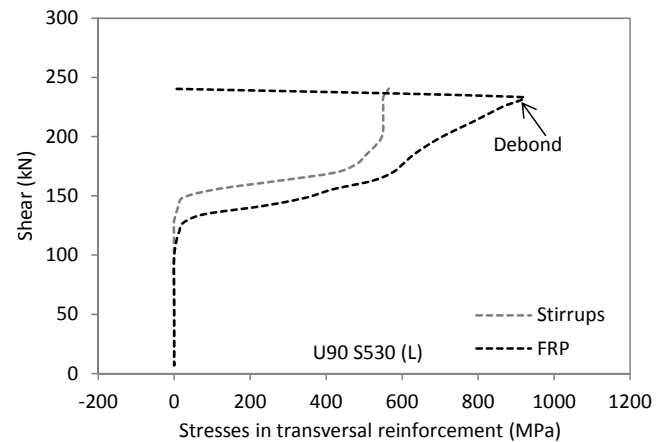
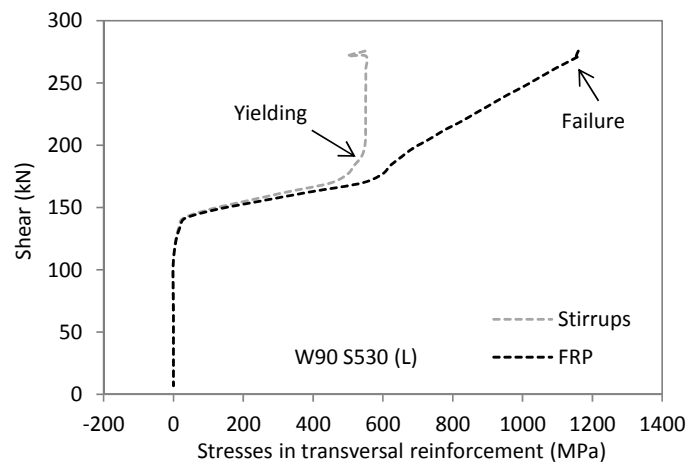
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a)

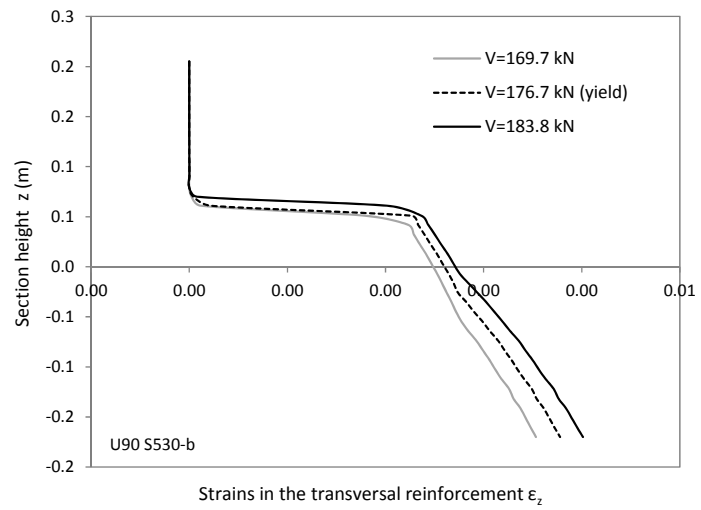
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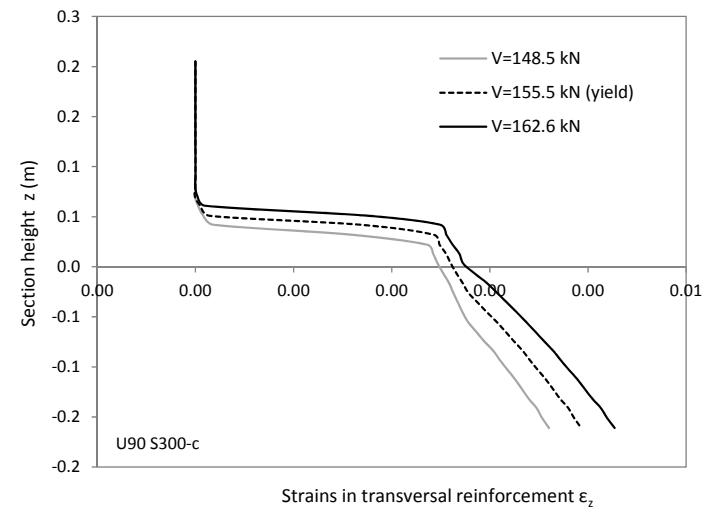


a)

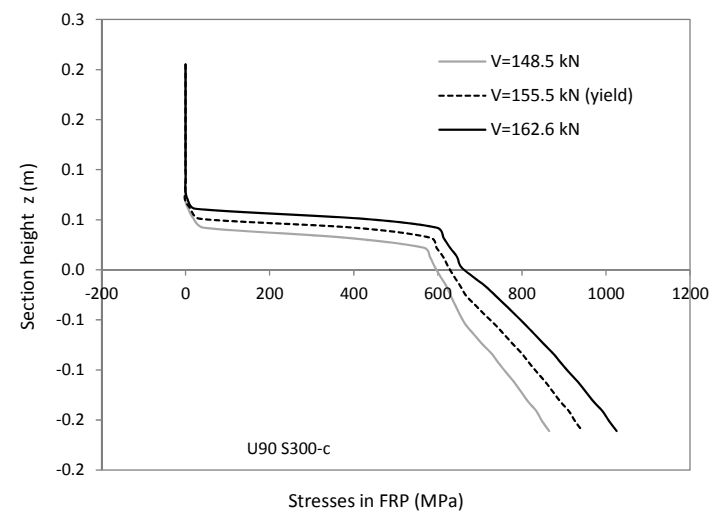
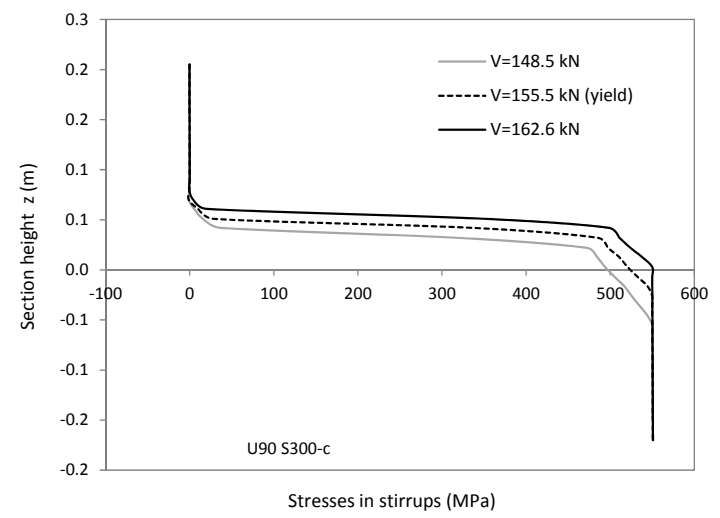
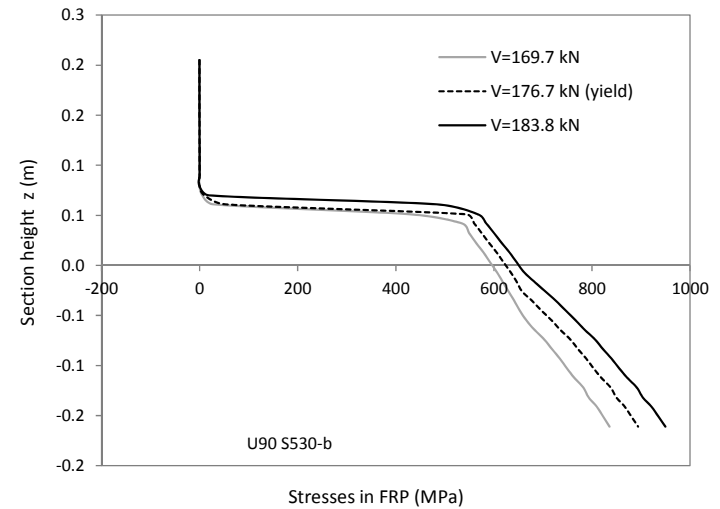
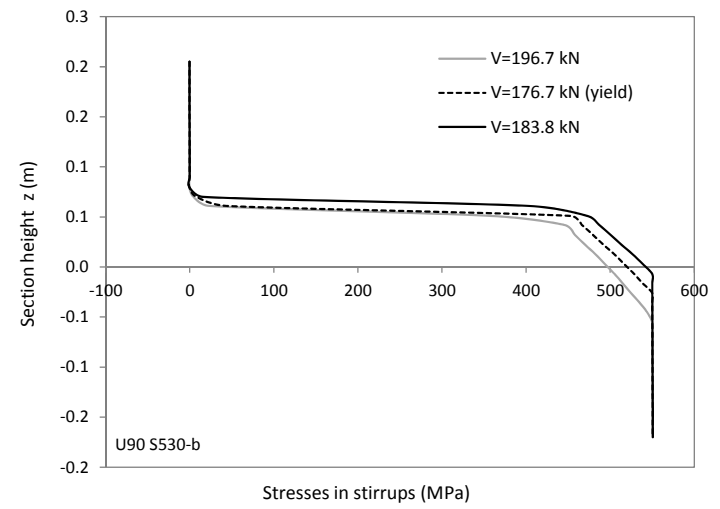
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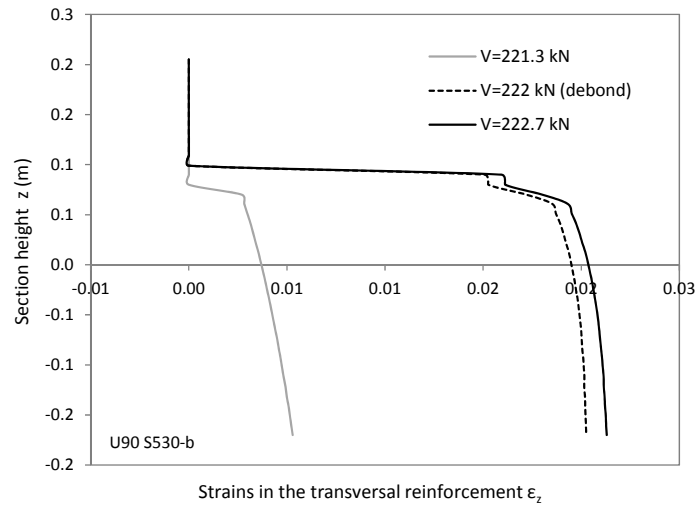


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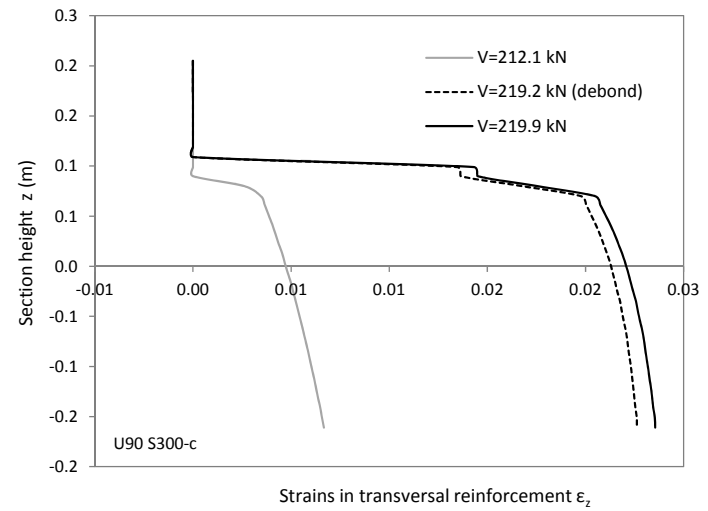


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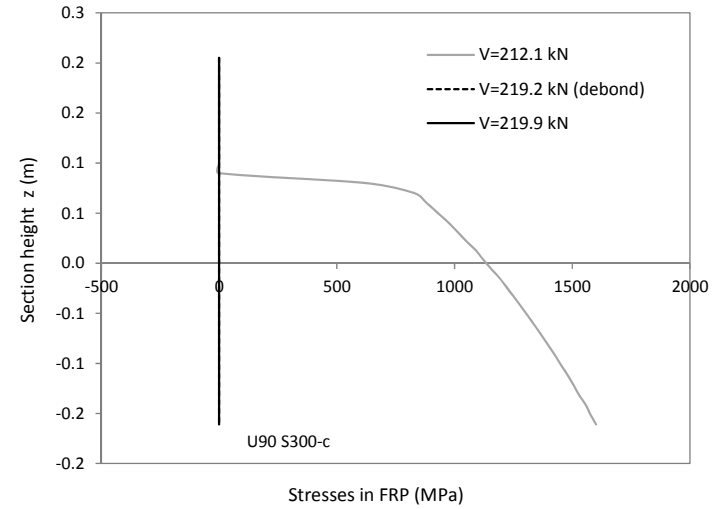
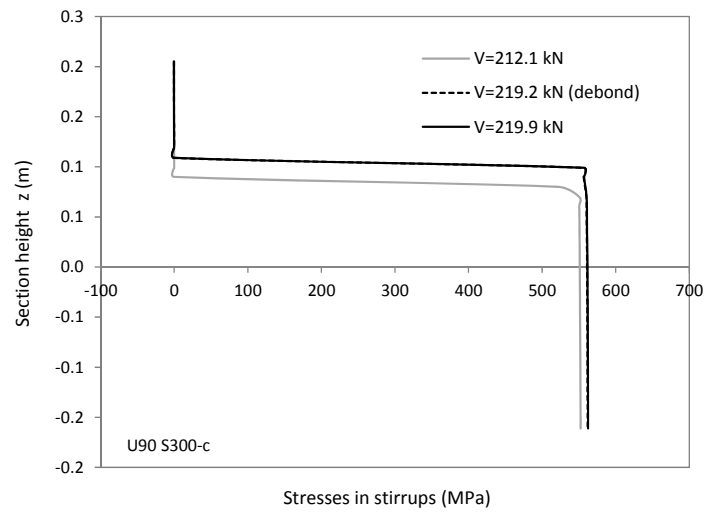
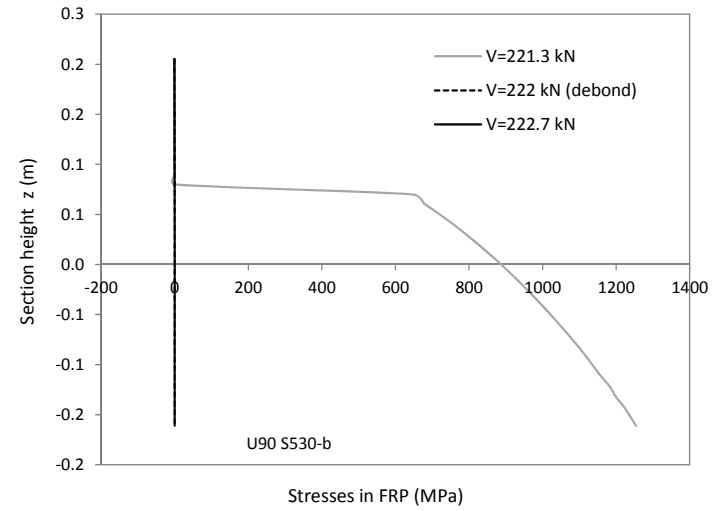
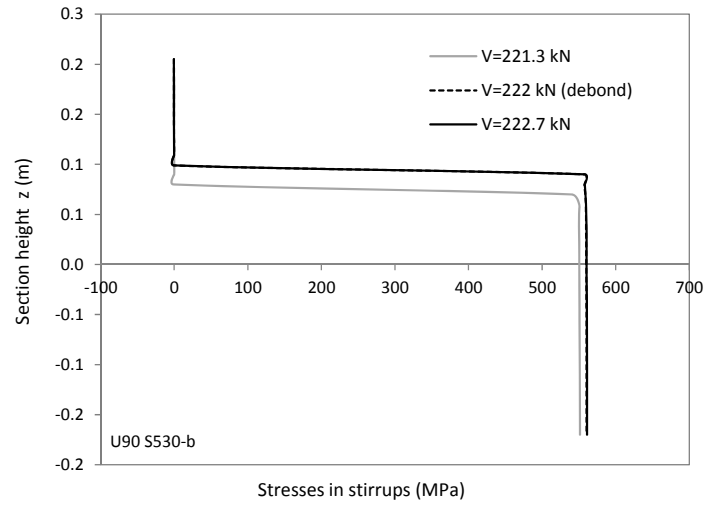
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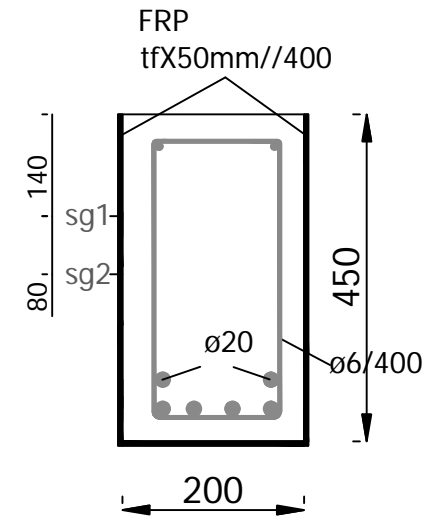
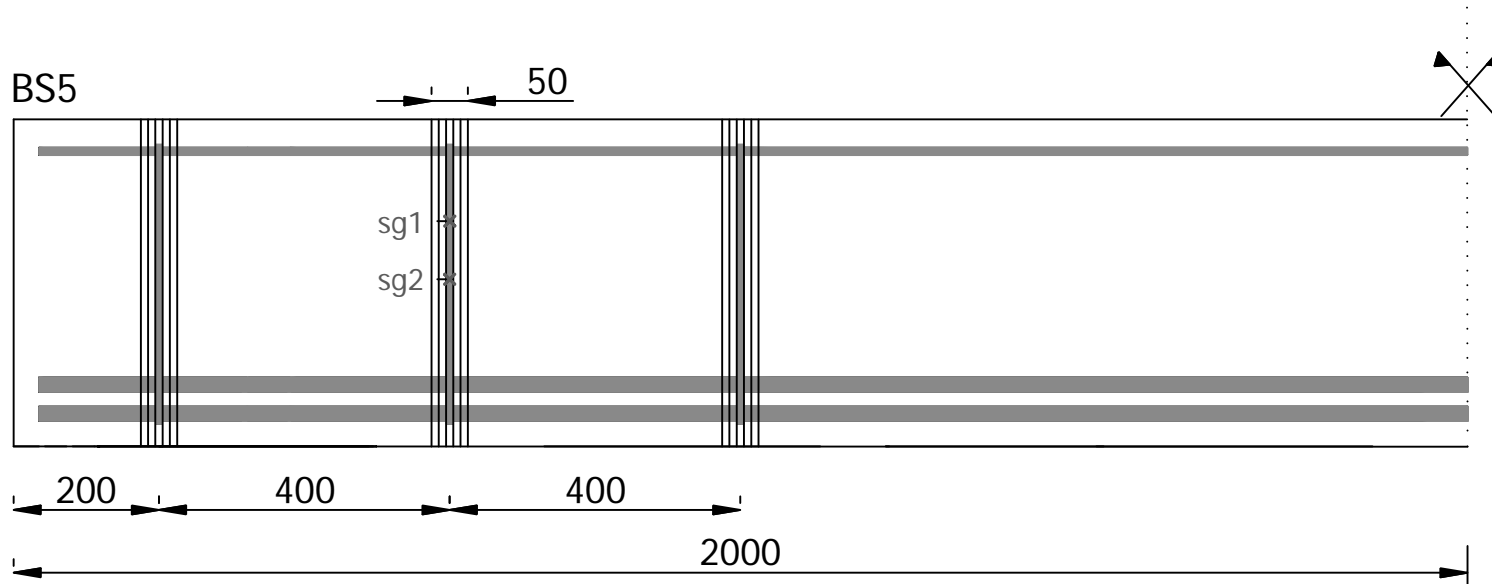
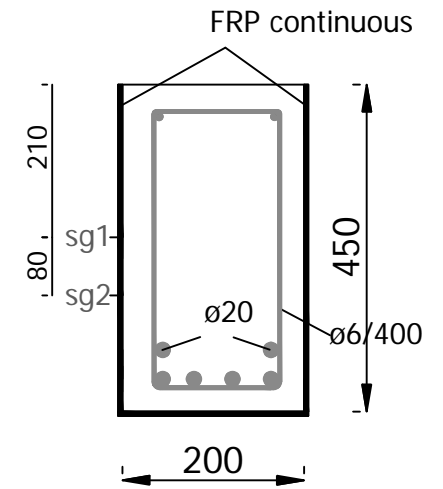
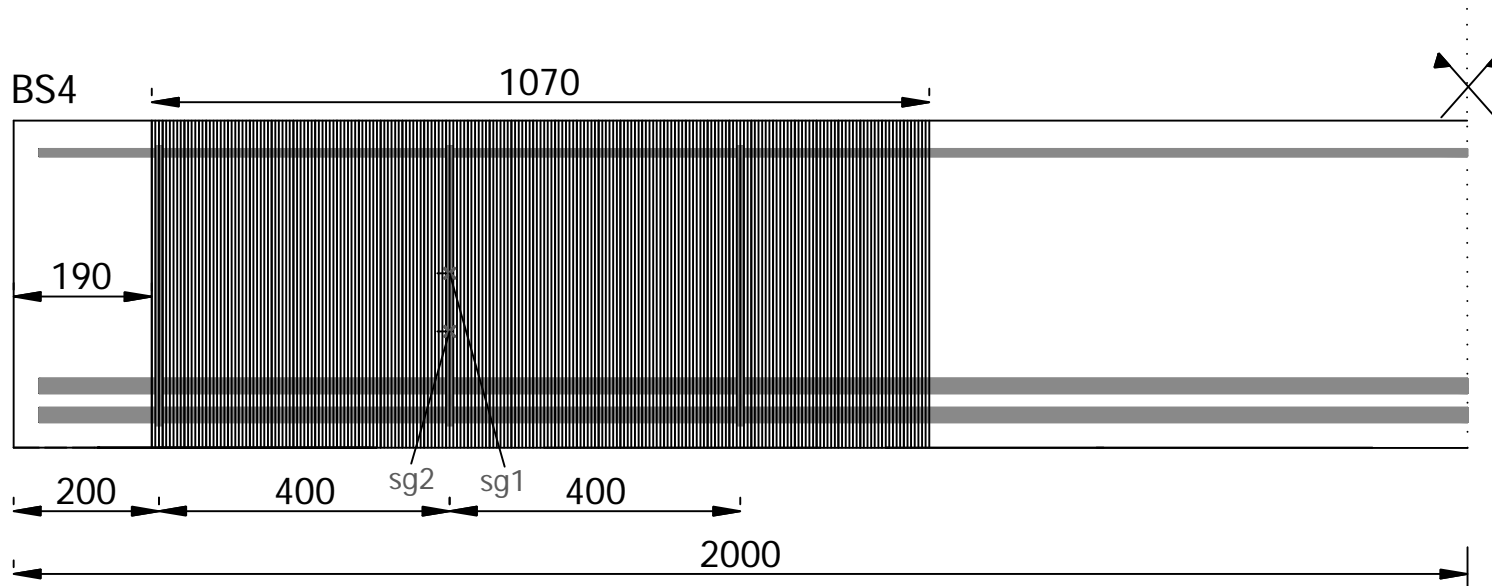


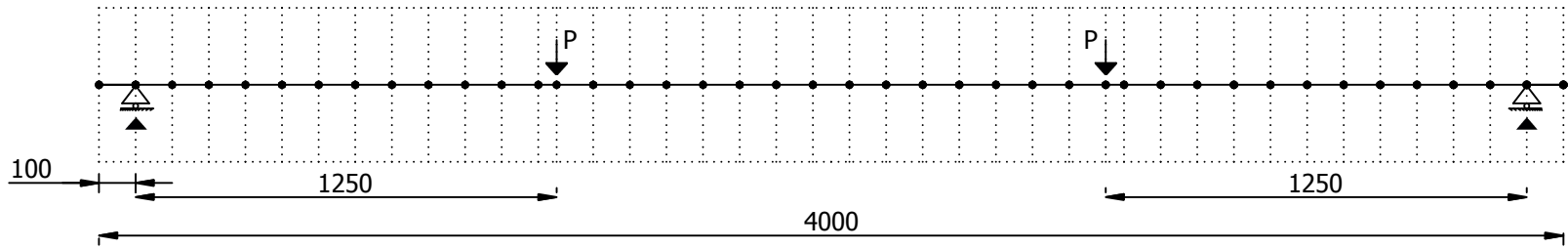
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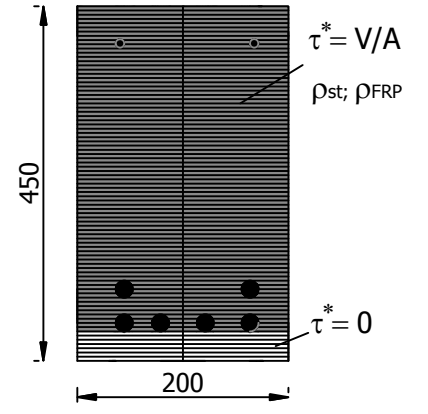
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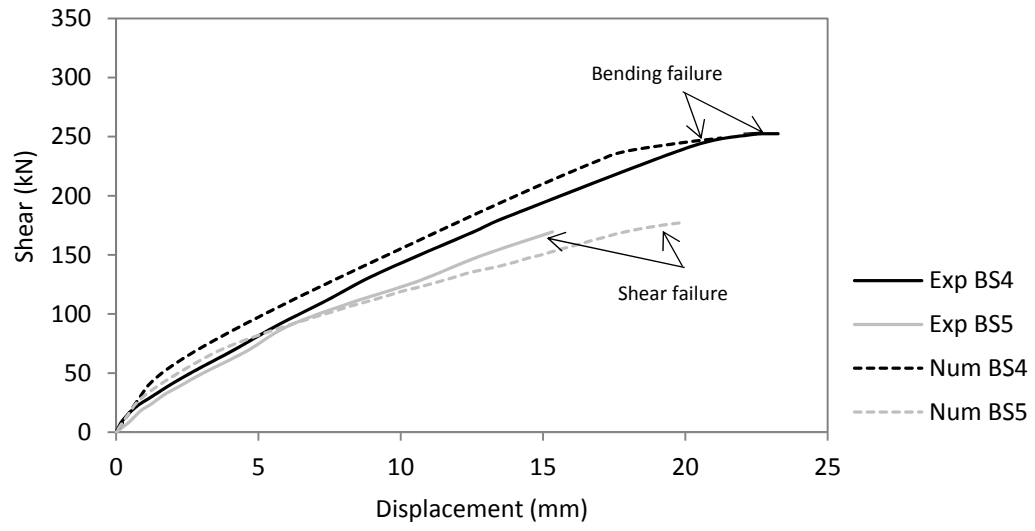
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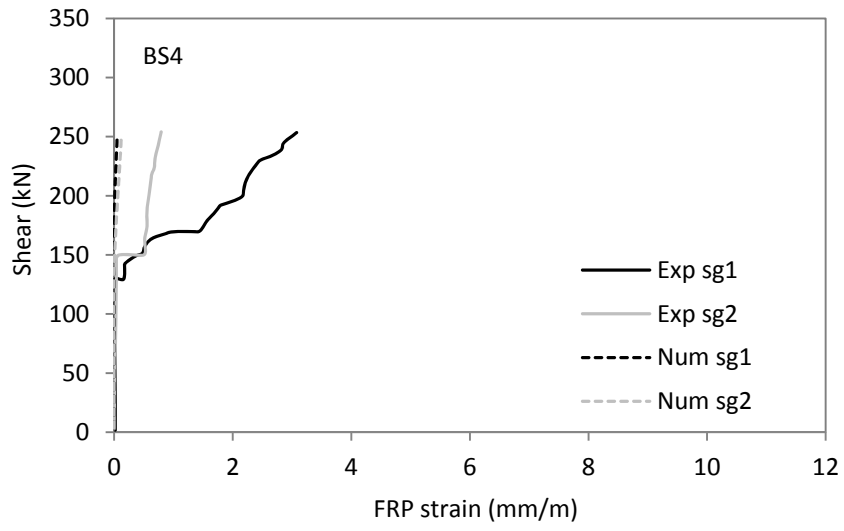


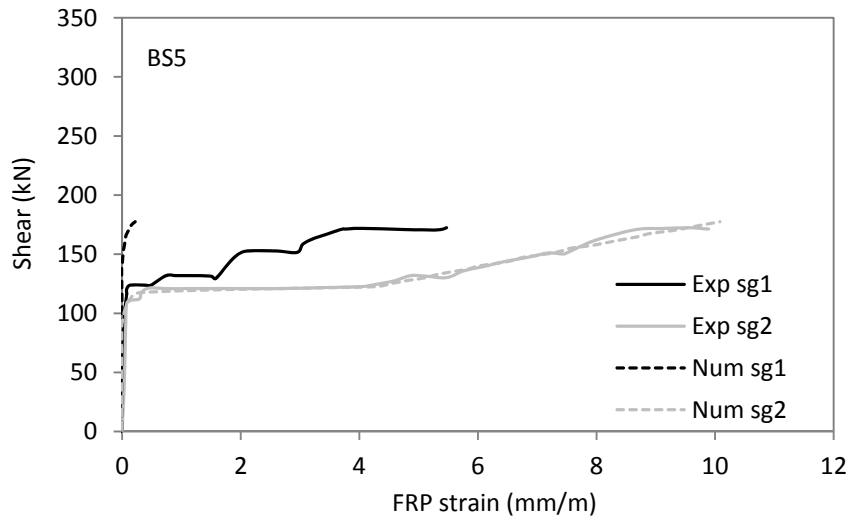
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- Nodes
  - Elements
  - ▬ Shear resistant fibers
  - ▬ Non shear resistant fibers

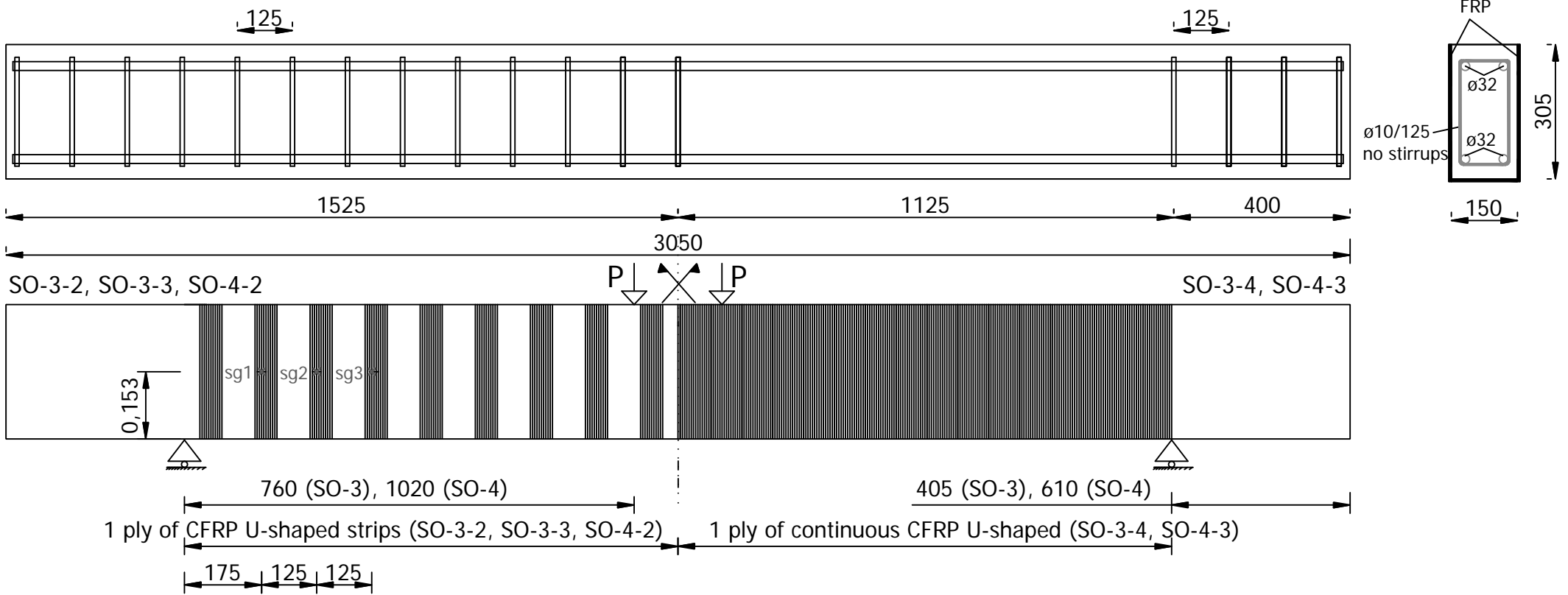


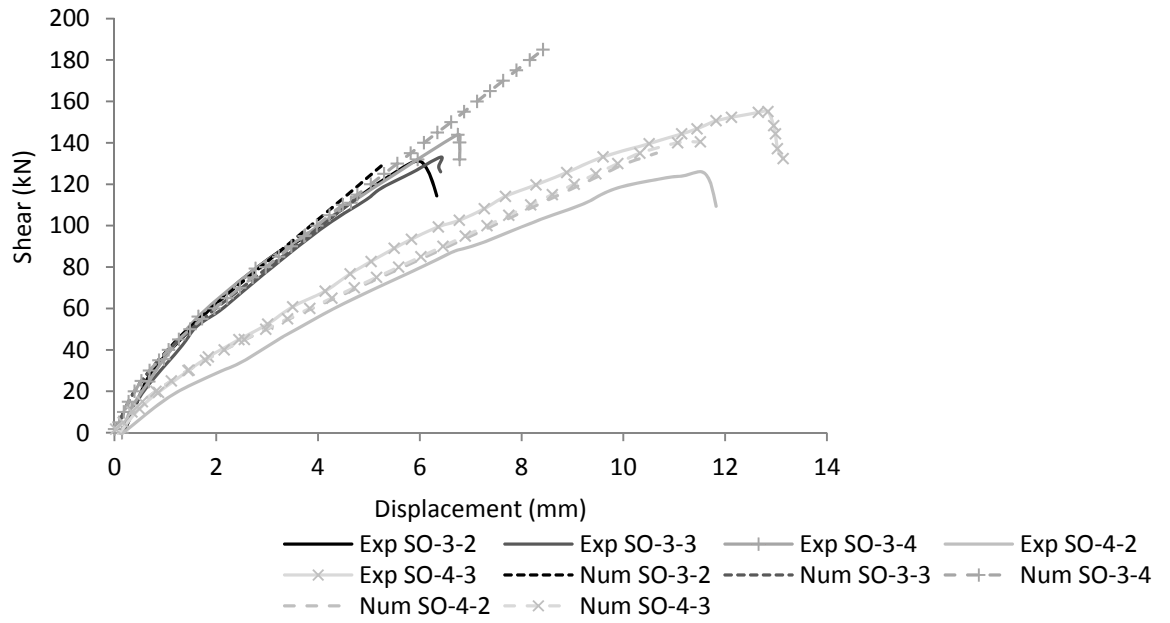


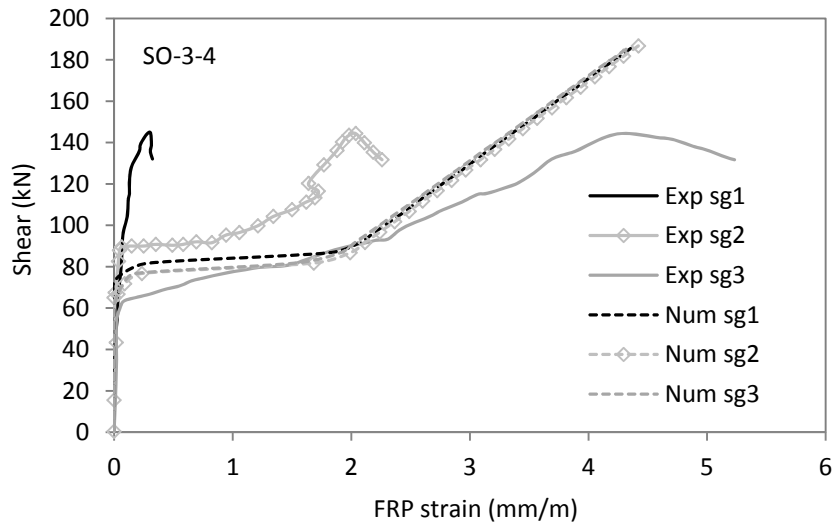


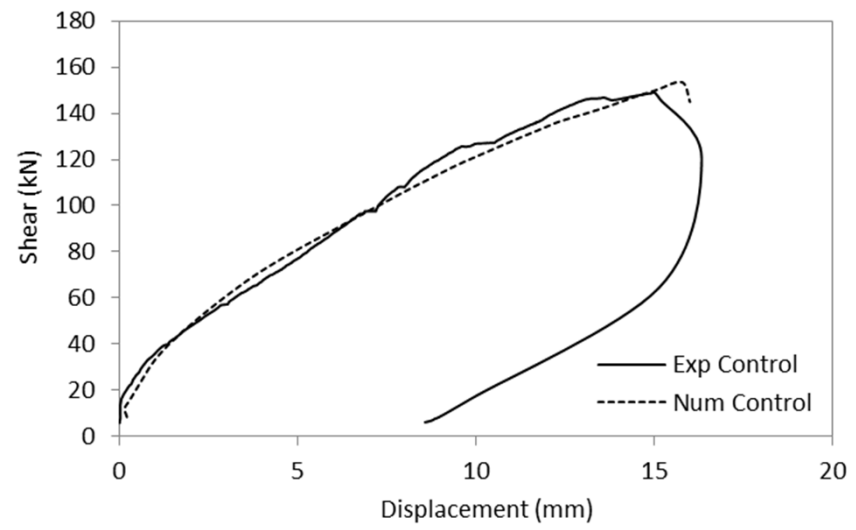


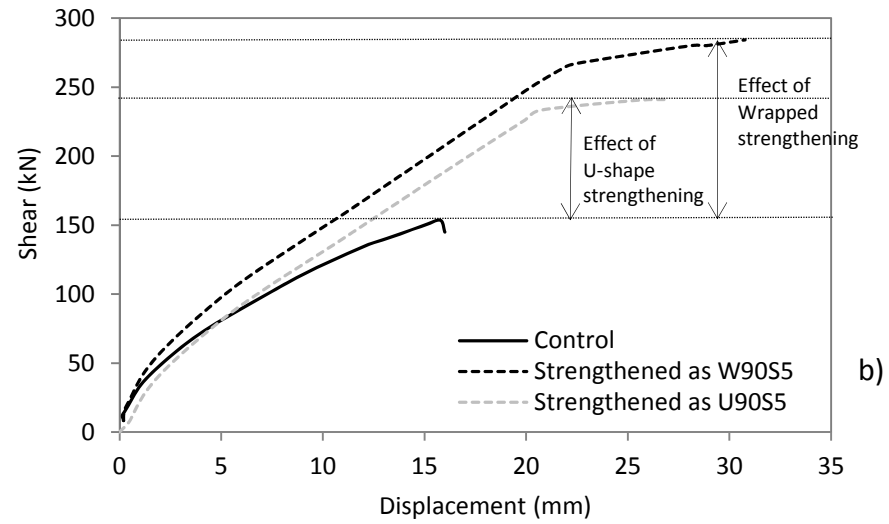
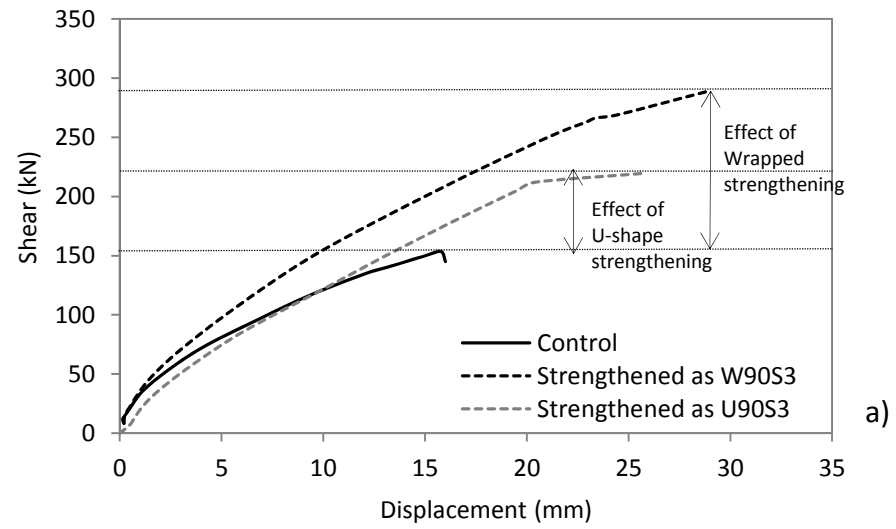












**Fig. 1.** Bonded length of FRP shear strengthening systems in a side-bonded or U-shaped configuration

**Fig. 2.** Fundamentals of the shear-sensitive fibre beam model for FRP shear strengthened elements

**Fig. 3.** Sectional model and FRP bond failure checking procedure

**Fig. 4.** Tensional scheme in the concrete fibre and FRP for bond failure checking procedure

**Fig. 5.** Geometry, reinforcement, strengthening configurations and instrumentation of the beams tested by Alzate (2012). Dimensions in mm.

**Fig. 6.** Debonding failure of the U-shaped beam from Alzate (2012)

**Fig. 7.** Test set-up and mesh of the numerical model for the experimental program of Alzate (2012). Dimensions in mm.

**Fig. 8.** Shear force vs. displacement at mid-span: a) test series S530 and b) beam series S300

**Fig. 9.** Strains in transversal reinforcement (stirrups and FRP) for beams with S530 FRP

**Fig. 10.** Strains in transversal reinforcement (stirrups and FRP) for beam with S300 FRP

**Fig. 11.** Stresses in transversal reinforcement (stirrups and FRP)

**Fig. 12.** Strains in transversal reinforcement for load phase correspondent to yielding of steel

**Fig. 13.** Stresses in transversal reinforcement along the height of the cross section at mid shear span for load phase correspondent to yielding of steel: a) steel and b) FRP

**Fig. 14.** Strains in transversal reinforcement for the debonding load phase

**Fig. 15.** Stresses in transversal reinforcement along the height of the cross section at mid shear span for debonding load phase: a) steel and b) FRP

**Fig. 16.** Geometry, reinforcement, strengthening configurations and instrumentation of the beams BS4 and BS5 tested by Matthys (2000). Dimensions in mm.

**Fig. 17.** Test set-up and mesh of the numerical model for the experimental program of Matthys (2000). Dimensions in mm.

**Fig. 18.** Shear force vs. displacement at mid-span for beams BS4 and BS5.

**Fig. 19.** Experimental to numerical strains in the FRP reinforcement for : a)BS4 and b)BS5

**Fig. 20.** Geometry, reinforcement, strengthening configurations and test set-up of beams SO-3-2, SO-3-3, SO-3-4, SO-4-2, SO-4-3, tested by Khalifa and Nanni (2002). Dimensions in mm

**Fig. 21.** Shear force vs. displacement at mid-span for beams SO-3-2, SO-3-3, SO-3-4, SO-4-2, SO-4-3

**Fig. 22.** Experimental to numerical strains in the FRP reinforcement for beam SO-3-4



**Fig. 23.** Response of the shear critical RC beam (Control specimen)

**Fig. 24.** Gain of shear resistance with FRP strengthening with different solutions: a) FRP S530 and b) FRP S300