Some considerations on the treatment of torsion and its interaction with other internal forces in EN 1993-1-1. A new approach

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Abstract

This paper presents and clarifies the treatments included in EN 1993-1-1 [1], relating to checking the resistance of the steel cross-sections under torsion and its interaction with other internal forces. Specifically, the origin of the formulations for shear-torsion interaction, which was not found in the literature, is presented. Furthermore, a very simple formulation based on the expressions used for shear-torsion interaction is developed, in order to take into account bending-warping torsion interaction for symmetrical double T cross-sections (IP and HE steel profiles). Such formulation

overcomes the overly conservative approach stated in EN 1993-1-1 [1], for Class 1 and Class 2 cross-sections (plastic and compact cross-sections). Finally, a rigorous method for the determination of the bending resistance of cross-sections is proposed, considering the interaction with shear and torsion. The proposal is well suited to the concept of cross-sectional class and it is perfectly consistent with the approaches set out to consider the bending-shear and bending-warping torsion interactions.

1. Introduction

After a thorough analysis of the specifications contained in EN 1993-1-1 [1] some doubts may arise about the conceptual basis of the specifications laid down in that document, related to the consideration of torsion effects for the design of steel structures (Bordallo, J. [2]). This paper develops some aspects aimed at understanding the treatment of torsion in EN 1993-1-1 [1] and improving such treatment, taking into account its interaction with bending and shear.

2. Origin of the formulations for shear-torsion interaction

The origin of the expressions stated in EN 1993-1-1 for determining the resistance of Class 1 and Class 2 cross-sections, under combined shear force and torsion is now presented.

2.1. Current content in EN 1993-1-1

Under the combined effects of shear force and torsion, in accordance with the specifications contained in EN 1993-1-1 [1], the plastic shear resistance accounting for torsional effects should be reduced and the following expression should be verified

$$V_{Ed} \leq V_{pl,T,Rd} \tag{1}$$

where

V_{Ed} is the design value of the shear force

$V_{pl,T,Rd}$ is the design plastic shear resistance accounting for torsional effects

Such resistance $V_{pl,T,Rd}$ is given by the following expressions, depending on the type of cross-section:

For I and H cross-sections:

$$V_{pl,T,Rd} = \sqrt{1 - \frac{\tau_{t,Ed}}{1{,}25 \cdot (f_y / \sqrt{3}) / \gamma_{M0}}} \cdot V_{pl,Rd} \tag{2}$$

For channel cross-sections:

$$V_{pl,T,Rd} = \left[\sqrt{1 - \frac{\tau_{t,Ed}}{1{,}25 \cdot (f_y / \sqrt{3}) / \gamma_{M0}}} - \frac{\tau_{w,Ed}}{(f_y / \sqrt{3}) / \gamma_{M0}} \right] \cdot V_{pl,Rd} \tag{3}$$

For hollow cross-sections:

$$V_{pl,T,Rd} = \left[1 - \frac{\tau_{t,Ed}}{(f_y / \sqrt{3}) / \gamma_{M0}} \right] \cdot V_{pl,Rd} \tag{4}$$

In these expressions, $\tau_{t,Ed}$ is the shear stress due to St. Venant torsion, $\tau_{w,Ed}$ is the shear stress due to warping torsion, f_y is the yield strength, γ_{M0} is the partial safety factor and $V_{pl,Rd}$ is the design plastic shear resistance in the absence of torsion.

2.2. Justification of the current expressions

2.2.1. Doubly symmetric I cross-sections

A symmetrical double T cross-section is subjected to torsional moment T_{Ed} and shear force V_{Ed}. Linear shear stresses $\tau_{t,Ed}$ due to St. Venant torsion (uniform torsion) and uniform shear stresses $\tau_{z,Ed}$ due to shear force are induced in the web. Shear stresses due to warping torsion are null in the web (see Fig. 1).

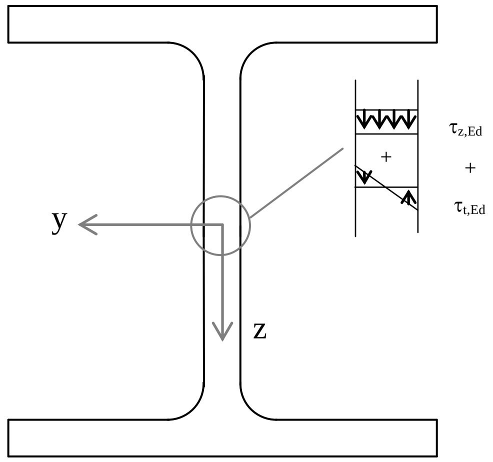

Figure 1. Representation of the shear stresses in the web due to shear force V_{Ed} and torsional moment T_{Ed} (St. Venant torsion).

It is possible to analyse the interaction of both shear stress distributions as an analogy with the normal stress distributions induced in a rectangular cross-section by axial force and bending moment. Then, for a rectangular cross-section of differential thickness *d* and width t_w - the web thickness of the double T cross-section-, the bending moment $M = \int_A \tau_{t,Ed} \cdot y \cdot dA$ produced by the shear stresses acting on the web due to uniform torsion and the axial force $N = \int_A \tau_{z,Ed} \cdot dA$ produced by the shear stress acting on the web due to shear force may be determined (see Fig. 2). Without diminishing generality to the analysis, a uniform shear stress distribution through the shear area may be assumed as $\tau_{z,Ed} = V_{Ed} / A_v$, being A_v the shear area.

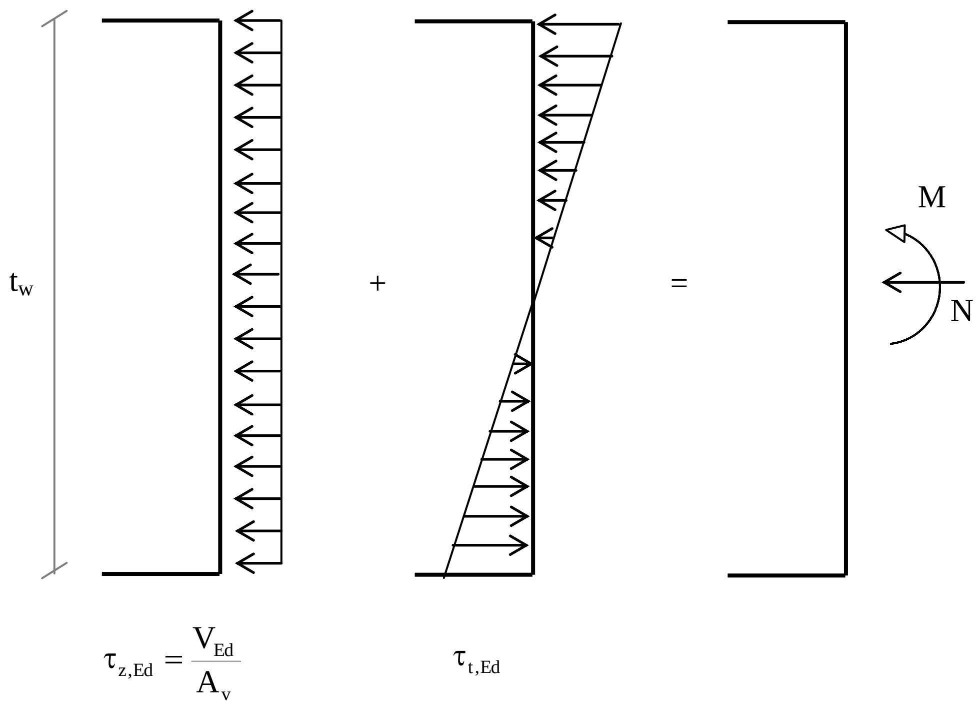

Figure 2. Representation of the shear stresses due to shear force and uniform torsion and the equivalent internal forces (bending and axial force) acting on a rectangular cross-section of differential thickness and width t_w

The current expressions in EN 1993-1-1 [1] for determining the reduced plastic shear resistance accounting for torsional effects can be obtained by checking a rectangular cross-section subjected to bending and axial force, considering shear stresses. Therefore, the von Mises stress should be less than the design value of yield strength of the steel, $\sigma_{von\,Mises} = \sqrt{3\tau^2} \leq \frac{f_y}{\gamma_{M0}}$, or what is the same, the cross-section will withstand a maximum shear stress equal to $\frac{f_y}{\sqrt{3}\gamma_{M0}}$.

Considering plastic stress distribution in a rectangular cross-section subjected to bending and axial force, it can be understood that part of the cross-section is only resisting the axial force -the shear stresses due to shear force- while the other part of the cross-section is only resisting the bending moment -the shear stresses due to uniform torsion- (see Fig. 3).

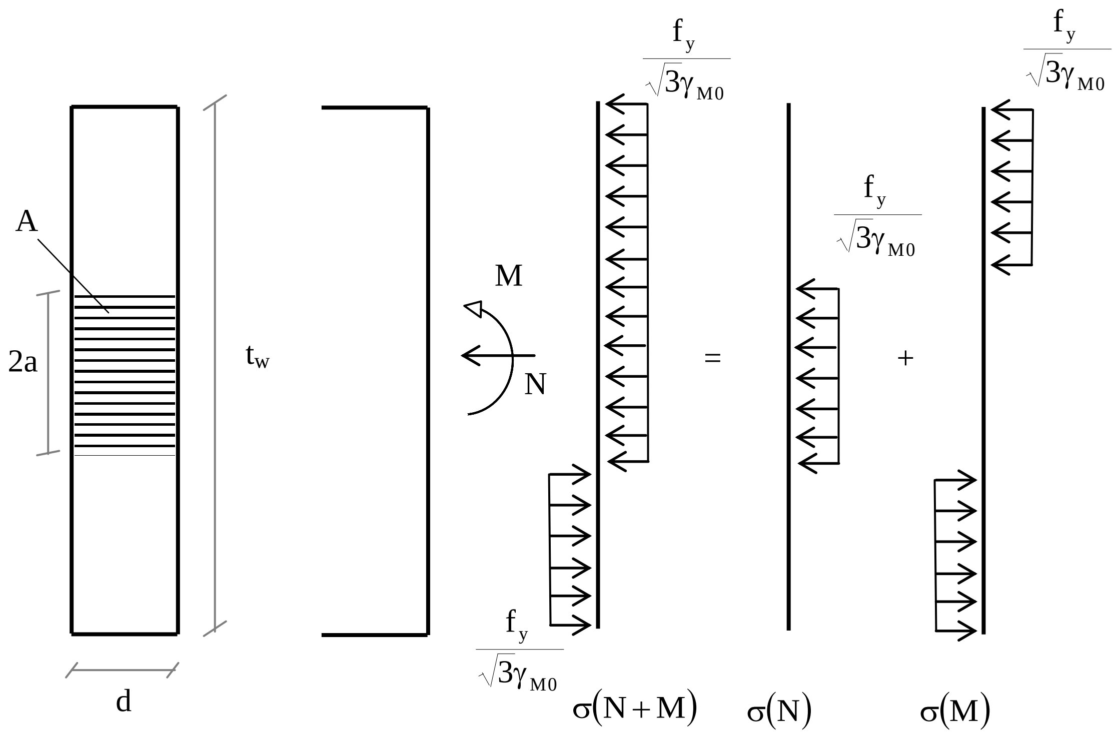

Figure 3. Plastic shear stress distribution in the web due to shear force and torsional moment (St. Venant torsion)

The value of the induced bending moment can be obtained by the integration of the shear stresses due to uniform torsion:

$$M = \int_A \tau_{t,Ed} \cdot y \cdot dA = W \cdot \tau_{t,Ed} = \frac{f_y}{\sqrt{3} \cdot \gamma_{M0}} \cdot \left(d \cdot \left(\frac{t_w}{2} - a \right) \right) \left(\frac{t_w}{2} + a \right) = \frac{f_y}{\sqrt{3} \cdot \gamma_{M0}} \cdot d \cdot \left(\frac{t_w{}^2}{4} - a^2 \right) \qquad (5)$$

From the above formula the resistant half-width *a* for axial force can be obtained as

$$a = \sqrt{\left(\frac{t_w{}^2}{4} - \frac{M}{\frac{f_y}{\sqrt{3} \cdot \gamma_{M0}} \cdot d} \right)} \qquad (6)$$

It can be seen that depending on the magnitude of the moment induced by the shear stresses due to uniform torsion, the resistant half-width *a* for axial force will be greater or less. Thus, the value of the reduced design plastic resistance to axial force, $N_{pl,T,Rd}$, i.e., the axial force resisted by the portion of the rectangular cross-section not exhausted by the effect of the bending caused by the shear stresses due to uniform torsion is:

$$N_{pl,T,Rd} = N_{pl,Rd} - 2\cdot\frac{f_y}{\sqrt{3}\cdot\gamma_{M0}}\cdot d\cdot\left(\frac{t_w}{2} - a\right) \tag{7}$$

Introducing in this expression the value of the resistant half-width *a* to axial force previously found (see Eq. (6)), it is obtained that

$$N_{pl,T,Rd} = N_{pl,Rd} - 2\cdot\frac{f_y}{\sqrt{3}\cdot\gamma_{M0}}\cdot d\cdot\left(\frac{t_w}{2} - \sqrt{\left(\frac{t_w^{\,2}}{4} - \frac{M}{\frac{f_y}{\sqrt{3}\cdot\gamma_{M0}}\cdot d}\right)}\right) \tag{8.a}$$

$$N_{pl,T,Rd} = N_{pl,Rd} - 2\cdot\frac{f_y}{\sqrt{3}\cdot\gamma_{M0}}\cdot d\cdot\frac{t_w}{2}\left(1 - \sqrt{\left(1 - \frac{M}{\frac{f_y}{\sqrt{3}\cdot\gamma_{M0}}\cdot d\cdot\frac{t_w^{\,2}}{4}}\right)}\right) \tag{8.b}$$

$$N_{pl,T,Rd} = N_{pl,Rd} - N_{pl,Rd}\left(1 - \sqrt{\left(1 - \frac{M}{\frac{f_y}{\sqrt{3}\cdot\gamma_{M0}}\cdot d\cdot\frac{t_w^{\,2}}{4}}\right)}\right) = N_{pl,Rd}\cdot\sqrt{\left(1 - \frac{M}{\frac{f_y}{\sqrt{3}\cdot\gamma_{M0}}\cdot d\cdot\frac{t_w^{\,2}}{4}}\right)} \tag{8.c}$$

Recalling that the value of the plastic resistance of a cross-section subjected to pure axial force is $N_{pl,Rd} = A_d\cdot\frac{f_y}{\sqrt{3}\cdot\gamma_{M0}}$, i.e. the area of the differential cross-section multiplied by the yield strength of steel, then the plastic resistance to axial force, reduced by uniform torsion, can be written as $N_{pl,T,Rd} = 2a\cdot d\cdot\frac{f_y}{\sqrt{3}\cdot\gamma_{M0}}$, that is, the product of the area of the resistant differential cross-section to axial force and the yield strength of the steel; or put another way, this resistance can be obtained as a product of the area of the differential cross-section by a reduced yield strength.

$$N_{pl,T,Rd} = A_d \cdot \frac{2a \cdot d}{A_d} \cdot \frac{f_y}{\sqrt{3} \cdot \gamma_{M0}} = A_d \cdot f_{y,T} \tag{9}$$

where A_d is the area of the differential section $(A_d = t_w \cdot d)$ and $f_{y,T}$ is the design yield strength, reduced by the effect of torsion. It can therefore be written that:

$$N_{pl,T,Rd} = N_{pl,Rd} \cdot \sqrt{\left(1 - \frac{M}{\frac{f_y}{\sqrt{3} \cdot \gamma_{M0}} \cdot d \cdot \frac{t_w^{\ 2}}{4}}\right)} \tag{10.a}$$

$$A_d \cdot f_{y,T} = A_d \cdot \frac{f_y}{\sqrt{3} \cdot \gamma_{M0}} \cdot \sqrt{\left(1 - \frac{M}{\frac{f_y}{\sqrt{3} \cdot \gamma_{M0}} \cdot d \cdot \frac{t_w^{\ 2}}{4}}\right)} \tag{10.b}$$

$$f_{y,T} = \frac{f_y}{\sqrt{3} \cdot \gamma_{M0}} \cdot \sqrt{\left(1 - \frac{M}{\frac{f_y}{\sqrt{3} \cdot \gamma_{M0}} \cdot d \cdot \frac{t_w^{\ 2}}{4}}\right)} \tag{10.c}$$

Thus, to determine the resistance of a rectangular cross-section subjected to bending and axial force, it can be assumed that the whole cross-section is resistant but the stress that can be achieved in it must be reduced by the ratio:

$$\rho = \sqrt{\left(1 - \frac{M}{\frac{f_y}{\sqrt{3} \cdot \gamma_{M0}} \cdot d \cdot \frac{t_w^{\ 2}}{4}}\right)} \tag{11}$$

Recall now the analogy made according to which the axial force is obtained from the integration of the shear stresses produced by the shear force at a differential cross-section. Then, the design shear resistance accounting for torsional effects is obtained by performing the integration on the shear area A_v.

$$V_{c,T,Rd} = \frac{f_y}{\sqrt{3}\cdot\gamma_{M0}}\cdot\rho\cdot A_v = V_{c,Rd}\cdot\sqrt{\left(1-\frac{M}{\frac{f_y}{\sqrt{3}\cdot\gamma_{M0}}\cdot d\cdot\frac{t_w^{\ 2}}{4}}\right)} \tag{12}$$

Moreover, by adopting linear elastic shear stress distribution due to uniform torsion through the thickness of the web ($W = W_{el} = \frac{1}{6}d\cdot t_w^{\ 2}$), it is obtained that:

$$M = W_{el}\cdot\tau_{t,Ed} = \tau_{t,Ed}\cdot\frac{1}{6}d\cdot t_w^{\ 2}.$$

So,

$$V_{c,T,Rd} = V_{c,Rd}\cdot\sqrt{\left(1-\frac{\tau_{t,Ed}\cdot\frac{1}{6}d\cdot t_w^{\ 2}}{\frac{f_y}{\sqrt{3}\cdot\gamma_{M0}}\cdot d\cdot\frac{t_w^{\ 2}}{4}}\right)} = V_{c,Rd}\cdot\sqrt{\left(1-\frac{\tau_{t,Ed}}{1{,}5\frac{f_y}{\sqrt{3}\cdot\gamma_{M0}}}\right)} \tag{13}$$

However, if a plastic shear stress distribution due to uniform torsion through the web thickness is adopted, ($W = W_{pl} = \frac{1}{4}d\cdot t_w^{\ 2}$), it holds:

$$M = W_{pl}\cdot\tau_{t,Ed} = \tau_{t,Ed}\cdot\frac{1}{4}d\cdot t_w^{\ 2}$$

Thereby,

$$V_{c,T,Rd} = V_{c,Rd}\cdot\sqrt{\left(1-\frac{\tau_{t,Ed}\cdot\frac{1}{4}d\cdot t_w^{\ 2}}{\frac{f_y}{\sqrt{3}\cdot\gamma_{M0}}\cdot d\cdot\frac{t_w^{\ 2}}{4}}\right)} = V_{c,Rd}\cdot\sqrt{\left(1-\frac{\tau_{t,Ed}}{\frac{f_y}{\sqrt{3}\cdot\gamma_{M0}}}\right)} \tag{14}$$

Now adopting an elastic-plastic shear stress distribution due to uniform torsion through the web thickness, a new resistant modulus would be obtained such that $W = \frac{1}{\alpha} \cdot d \cdot t_w^2$

$\frac{1}{\alpha} \in [\frac{1}{4}, \frac{1}{6}]$, so that

$$M = W_{epl} \cdot \tau_{t,Ed} = \tau_{t,Ed} \cdot \frac{1}{\alpha} \cdot d \cdot t_w^{\,2}$$

Therefore

$$V_{c,T,Rd} = V_{c,Rd} \cdot \sqrt{\left(1 - \frac{\tau_{t,Ed} \cdot \frac{1}{\alpha} \cdot d \cdot t_w^{\,2}}{\frac{f_y}{\sqrt{3} \cdot \gamma_{M0}} \cdot d \cdot \frac{t_w^{\,2}}{4}}\right)} = V_{c,Rd} \cdot \sqrt{\left(1 - \frac{\tau_{t,Ed}}{\frac{\alpha}{4} \frac{f_y}{\sqrt{3} \cdot \gamma_{M0}}}\right)} \tag{15}$$

When considering a stress distribution between the elastic and the plastic stress distributions, a value of coefficient α comprised between 4 and 6 may be adopted, or what is the same, a value in the ratio $\alpha / 4$ between 1 and 1,5. Taking the mean value of 1,25 for this ratio, the expression of EN 1993-1-1 [1] that allows for the design plastic shear resistance accounting for torsional effects for doubly symmetric I cross-sections is achieved.

$$V_{pl,T,Rd} = \sqrt{1 - \frac{\tau_{t,Ed}}{1{,}25 \cdot (f_y / \sqrt{3}) / \gamma_{M0}}} \cdot V_{pl,Rd} \tag{16}$$

2.2.2. Channel cross-sections

For channel cross-sections, the expression contained in EN 1993-1-1 to determine the design plastic shear resistance accounting for torsional effects is

$$V_{pl,T,Rd} = \left[\sqrt{1 - \frac{\tau_{t,Ed}}{1{,}25 \cdot (f_y / \sqrt{3}) / \gamma_{M0}}} - \frac{\tau_{w,Ed}}{(f_y / \sqrt{3}) / \gamma_{M0}}\right] \cdot V_{pl,Rd} \tag{17}$$

This expression can be similarly obtained to the above one, taking into account the existence of the uniform shear stress distribution across the web thickness, $\tau_{w,Ed}$, due to warping torsion.

2.2.3. Hollow cross-sections

For hollow cross-sections the design plastic resistance for shear and torsion is

$$V_{pl,T,Rd} = \left[1 - \frac{\tau_{t,Ed}}{(f_y/\sqrt{3})/\gamma_{M0}}\right] \cdot V_{pl,Rd} \tag{18}$$

It is easily obtained if one takes into account that warping torsion in hollow sections is negligible and noting that the shear stress distribution due to uniform torsion is constant through the thickness of the hollow section.

3. Bending-warping torsion interaction

The application rules stated in EN 1993-1-1 [1] to check the resistance of a cross-section subjected to bending moment and warping torsion are presented in the next section. As will be discussed later, these rules are correct but excessively conservative in the presence of cross-sections capable of developing plastic resistance (Class 1 and Class 2 cross-sections).

3.1 The current approach in EN 1993-1-1

According to EN 1993-1-1 [1], for the elastic verification the following well-known yield criterion may be applied

$$\left(\frac{\sigma_{x,Ed}}{f_y/\gamma_{M0}}\right)^2 + \left(\frac{\sigma_{z,Ed}}{f_y/\gamma_{M0}}\right)^2 - \left(\frac{\sigma_{x,Ed}}{f_y/\gamma_{M0}}\right)\left(\frac{\sigma_{z,Ed}}{f_y/\gamma_{M0}}\right) + 3\left(\frac{\tau_{Ed}}{f_y/\gamma_{M0}}\right)^2 \leq 1 \tag{19}$$

In addition, EN 1993-1-1 [1] states in the same section that for determining the plastic moment resistance of a cross-section due to bending and torsion, only torsion effects B_{Ed} should be derived from elastic analysis. EN 1993-1-1 [1] informs about how the structural analysis should be performed to determine the internal warping torsion and consequently the bimoment B_{Ed}. However, EN 1993-1-1 [1] does not offer any information about possible formulations to consider bending-warping torsion interaction in plastic design.

As a first approximation, the interaction between bending and warping torsion could be made from the following equation

$$M_{c,B,Rd} = \left[1 - \frac{\sigma_{w,Ed,\max}}{f_y / \gamma_{M0}} \right] \cdot M_{c,Rd} \qquad (20)$$

where the maximum longitudinal normal warping stress $\sigma_{w,Ed,\max}$ can be obtained by the following equation (Kollbrunner and Basler, [3])

$$\sigma_{w,Ed,\max} = \frac{B_{Ed} \cdot \omega_{\max}}{I_A} \qquad (21)$$

being $\omega_{\max}$ the maximum value of the normalized sectorial coordinate and I_A the warping section constant (linear elastic warping normal stress distribution is assumed).

This expression is perfectly consistent for Class 3 and Class 4 cross-sections in which the stress in the extreme compressed fibre of the steel member assuming an elastic distribution of stresses can reach the yield strength. Obviously in Class 4 cross-sections effective widths should be used to make necessary allowances for reductions in resistance due to the effects of local buckling. Then, adding the normal stresses due to bending and warping torsion at the cross-section relevant point, the expression proposed in EN 1993-1-1 [1], although not explicitly, is achieved.

$$\sigma_{w,Ed,\max} + \sigma_{M,Ed} \leq \frac{f_y}{\gamma_{M0}} \tag{22}$$

The expression is clearly conservative for Class 1 or Class 2 cross-sections because those sections can develop their plastic moment resistance. In order to calculate the bending resistance, taking into account interaction with warping torsion, EN 1993-1-1 [1] implicitly suggests the above expression by which the design yield strength of the steel is reduced for the whole cross-section by an amount equal to $\left[1 - \frac{\sigma_{w,Ed,\max}}{f_y / \gamma_{M0}}\right]$. Thus, it is assuming that at every point of the cross-section the normal warping stress is equal to the maximum normal warping stress $\sigma_{w,Ed,\max}$, which is not true; in addition, it is assumed that the sign of the stress (compressive or tensile) induced by bending and warping torsion is the same at any point of the cross-section, which is not true either.

3.2 Proposal for the determination of the plastic bending resistance for doubly symmetric I Class 1 and Class 2 cross-sections subjected to bending and warping torsion

Due to the overly conservative formulation of EN 1993-1-1 [1], in this section an alternative, less conservative and more accurate and realistic, formulation for the determination of the plastic bending resistance of doubly symmetric I cross-sections subjected to bending about its major principal axis of inertia and warping torsion is proposed.

Figure 4 shows the direct stress distribution $\sigma_{w,Ed}$ due to the bimoment moment B_{Ed} and the direct stress distribution $\sigma_{x,Ed}$ due to the bending moment for a doubly symmetric I cross-section. Now, the bending resistance accounting for torsional effects will be determined.

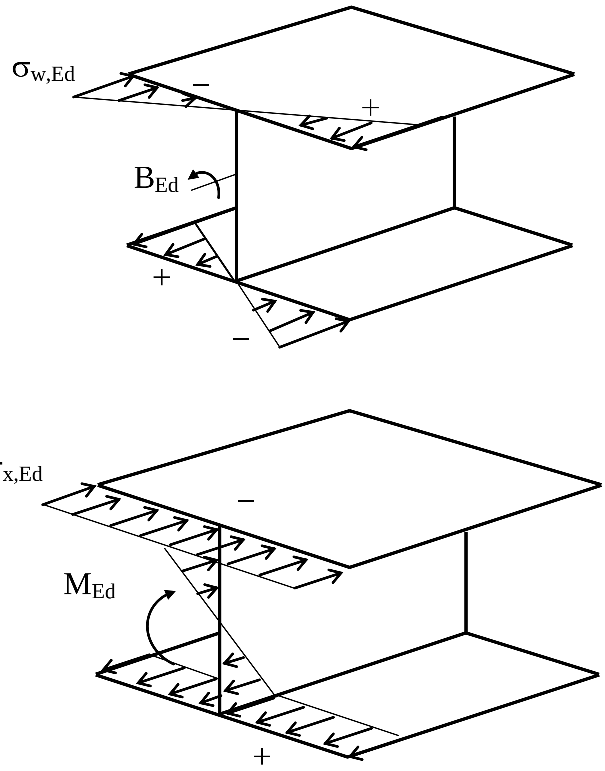

Figure 4. Normal stress distributions due to warping torsion B_{Ed} and bending moment M_{Ed}

The flanges of such doubly symmetric I cross-sections can be understood as rectangular cross-sections subjected to a vertical axis bending moment $M_{z,Ed} = \int_{A_f} \sigma_{w,Ed} \cdot y \cdot dA$ produced by the normal warping stresses ($\sigma_{w,Ed} = \dfrac{B_{Ed} \cdot \omega}{I_A}$) and an axial force $N_{Ed} = \int_{A_f} \sigma_{x,Ed} \cdot dA$ produced by the normal bending stresses ($\sigma_{x,Ed} = \dfrac{M_{Ed}}{W}$) (W is the section bending resistant modulus) (see Fig. 5).

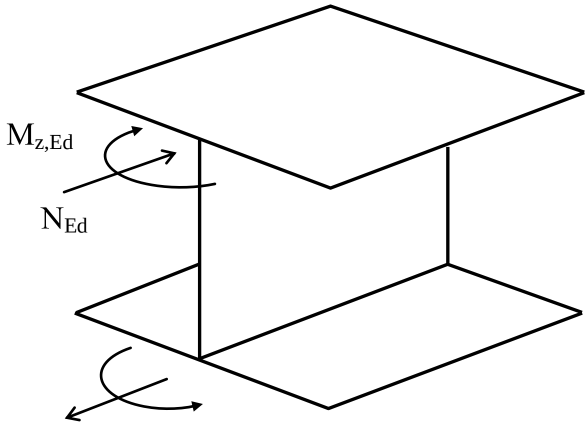

Figure 5. Representation of bending moments and axial forces on the flanges induced by normal warping stresses and normal bending stresses

The plastic bending moment resistance for doubly symmetric I sections Class 1 and 2 cross-sections subjected to bending and warping torsion can be obtained by checking a rectangular cross-section of thickness t_f and width b subjected to bending and axial force. The dimensions t_f and b are the thickness and the width of the flange, respectively. For Class 1 and Class 2 cross-sections, where an axial force is present, allowance should be made for its effect on the plastic moment resistance. In the case of rectangular cross-sections (the flanges of doubly symmetric I cross-sections) subjected to combined loading (bending and axial force) only a part of the cross-section is resisting the axial force (the normal stresses due to bending) whilst the other part only resists the bending moment (the normal warping stresses) (see Fig. 6).

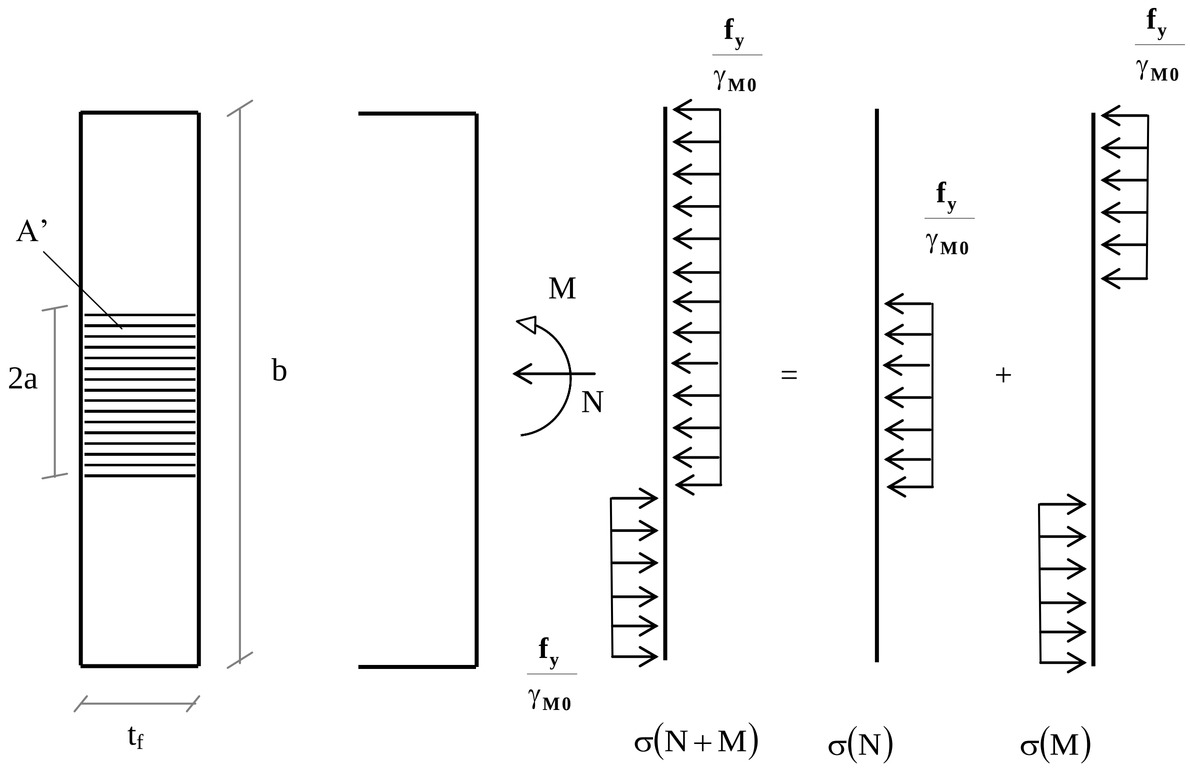

Figure 6. Plastic distribution of the normal stresses on the flanges due to axial force and bending

The value of the moment induced by the normal warping stresses is obtained by integrating the warping stress distribution over the area of the flange:

$$M_{z,Ed} = \int_{A_f} \sigma_{w,Ed}(y) \cdot y \cdot dA = W \cdot \frac{f_y}{\gamma_{M0}} = \frac{f_y}{\gamma_{M0}} \cdot \left(t_f \cdot \left(\frac{b}{2} - a \right) \right) \left(\frac{b}{2} + a \right) = \frac{f_y}{\gamma_{M0}} \cdot t_f \cdot \left(\frac{b^2}{4} - a^2 \right) \qquad (23)$$

From the above formula the resistant half-width *a* for axial force can be obtained as

$$a = \sqrt{\left(\frac{b^2}{4} - \frac{M_{z,Ed}}{\frac{f_y}{\gamma_{M0}} \cdot t_f} \right)} \qquad (24)$$

Depending on the magnitude of the moment induced by the normal warping stresses, the resistant half-width *a* to axial force will be greater or less. Thus, the value of the reduced design plastic resistance to axial force, $N_{pl,T,Rd}$, i.e., the axial force resisted by the portion of the rectangular cross-section not exhausted by the effect of the bending caused by the normal warping stresses is

$$N_{pl,T,Rd} = N_{pl,Rd} - 2\cdot\frac{f_y}{\gamma_{M0}}\cdot t_f\cdot\left(\frac{b}{2} - a\right) \tag{25}$$

Introducing in this expression the value of the resistant half-width *a* to axial force previously found (see Eq. (24)), it is obtained that

$$N_{pl,T,Rd} = N_{pl,Rd} - 2\cdot\frac{f_y}{\gamma_{M0}}\cdot t_f\cdot\left(\frac{b}{2} - \sqrt{\left(\frac{b^2}{4} - \frac{M_{z,Ed}}{\frac{f_y}{\gamma_{M0}}\cdot t_f}\right)}\right) \tag{26.a}$$

$$N_{pl,T,Rd} = N_{pl,Rd} - 2\cdot\frac{f_y}{\gamma_{M0}}\cdot t_f\cdot\frac{b}{2}\left(1 - \sqrt{\left(1 - \frac{M_{z,Ed}}{\frac{f_y}{\gamma_{M0}}\cdot t_f\cdot\frac{b^2}{4}}\right)}\right) \tag{26.b}$$

$$N_{pl,T,Rd} = N_{pl,Rd} - N_{pl,Rd}\left(1 - \sqrt{\left(1 - \frac{M_{z,Ed}}{\frac{f_y}{\gamma_{M0}}\cdot t_f\cdot\frac{b^2}{4}}\right)}\right) = N_{pl,Rd}\cdot\sqrt{\left(1 - \frac{M_{z,Ed}}{\frac{f_y}{\gamma_{M0}}\cdot t_f\cdot\frac{b^2}{4}}\right)} \tag{26.c}$$

Knowing the value of the plastic resistance of a cross-section subjected to pure axial force is $N_{pl,Rd} = A_f\cdot\frac{f_y}{\gamma_{M0}}$, being in this case A_f the flange area multiplied by the yield strength of steel, then the plastic resistance to axial force, reduced by warping torsion, can be written as $N_{pl,T,Rd} = 2\cdot a\cdot t_f\cdot\frac{f_y}{\gamma_{M0}}$, that is, the product of the resistant area of the flange to axial force and the yield strength of the steel. Or put another way, this resistance could be obtained as the product of the flange area and a reduced yield strength $f_{y,T}$ due to warping torsion; then, the following equation is reached

$$N_{pl,T,Rd} = A_f \cdot \frac{2a \cdot t_f}{A_f} \cdot \frac{f_y}{\gamma_{M0}} = A_f \cdot f_{y,T} \tag{27}$$

where A_f is the flange area $(A_f = b \cdot t_f)$ and $f_{y,T}$ is the design yield strength, reduced by the warping torsion. It can therefore be written that

$$N_{pl,T,Rd} = N_{pl,Rd} \cdot \sqrt{\left(1 - \frac{M_{z,Ed}}{\frac{f_y}{\gamma_{M0}} \cdot t_f \cdot \frac{b^2}{4}}\right)} \tag{28.a}$$

$$A_f \cdot f_{y,T} = A_f \cdot \frac{f_y}{\gamma_{M0}} \cdot \sqrt{\left(1 - \frac{M_{z,Ed}}{\frac{f_y}{\gamma_{M0}} \cdot t_f \cdot \frac{b^2}{4}}\right)} \tag{28.b}$$

$$f_{y,T} = \frac{f_y}{\gamma_{M0}} \cdot \sqrt{\left(1 - \frac{M_{z,Ed}}{\frac{f_y}{\gamma_{M0}} \cdot t_f \cdot \frac{b^2}{4}}\right)} \tag{28.c}$$

Thus, to determine the resistance of a rectangular cross-section subjected to bending and axial force, it can be assumed that the whole cross-section is resistant but the stress that can be achieved in it must be reduced by the ratio:

$$\rho = \sqrt{\left(1 - \frac{M_{z,Ed}}{\frac{f_y}{\gamma_{M0}} \cdot t_f \cdot \frac{b^2}{4}}\right)} \tag{29}$$

Therefore, for combined bending moment and warping torsion the plastic bending moment resistance accounting for torsional effects (the bimoment B_{Ed}) should be reduced from $M_{c,Rd}$ to $M_{c,B,Rd} = \frac{f_y}{\gamma_{M0}} \cdot \rho \cdot W$

$$M_{c,B,Rd} = M_{c,Rd} \cdot \sqrt{\left(1 - \frac{M_{z,Ed}}{\frac{f_y}{\gamma_{M0}} \cdot t_f \cdot \frac{b^2}{4}}\right)} \tag{30}$$

Adopting a linear elastic normal warping stress distribution, the resistant modulus of the flange is $W = W_{el} = \frac{1}{6} t_f \cdot b^2$, therefore

$$M_{z,Ed} = W_{el} \cdot \sigma_{w,Ed} = \sigma_{w,Ed} \cdot \frac{1}{6} t_f \cdot b^2$$

So,

$$M_{c,B,Rd} = M_{c,Rd} \cdot \sqrt{\left(1 - \frac{\sigma_{w,Ed} \cdot \frac{1}{6} t_f \cdot b^2}{\frac{f_y}{\gamma_{M0}} \cdot t_f \cdot \frac{b^2}{4}}\right)} = M_{c,Rd} \cdot \sqrt{\left(1 - \frac{\sigma_{w,Ed}}{1{,}5 \frac{f_y}{\gamma_{M0}}}\right)} \tag{31}$$

However, if a plastic normal warping stress distribution is adopted on the flange, the resistant modulus is $W = W_{pl} = \frac{1}{4} t_f \cdot b^2$, therefore

$$M_{z,Ed} = W_{pl} \cdot \sigma_{w,Ed} = \sigma_{w,Ed} \cdot \frac{1}{4} t_f \cdot b^2$$

Thereby,

$$M_{c,B,Rd} = M_{c,Rd} \cdot \sqrt{\left(1 - \frac{\sigma_{w,Ed} \cdot \frac{1}{4} t_f \cdot b^2}{\frac{f_y}{\gamma_{M0}} \cdot t_f \cdot \frac{b^2}{4}}\right)} = M_{c,Rd} \cdot \sqrt{\left(1 - \frac{\sigma_{w,Ed}}{\frac{f_y}{\gamma_{M0}}}\right)} \tag{32}$$

Adopting now an elastic-plastic normal warping stress distribution on the flange, a new resistant modulus would be obtained such that $W = \frac{1}{\alpha} \cdot t_f \cdot b^2$ with $\frac{1}{\alpha} \in [\frac{1}{4}, \frac{1}{6}]$, so that

$$M_{z,Ed} = W \cdot \sigma_{w,Ed} = \sigma_{w,Ed} \cdot \frac{1}{\alpha} \cdot t_f \cdot b^2$$

Therefore,

$$M_{c,B,Rd} = M_{c,Rd} \cdot \sqrt{\left(1 - \frac{\sigma_{w,Ed} \cdot \frac{1}{\alpha} \cdot t_f \cdot b^2}{\frac{f_y}{\gamma_{M0}} \cdot t_f \cdot \frac{b^2}{4}}\right)} = M_{c,Rd} \cdot \sqrt{\left(1 - \frac{\sigma_{w,Ed}}{\frac{\alpha}{4} \frac{f_y}{\gamma_{M0}}}\right)} \qquad (33)$$

When considering a normal stress distribution between the elastic and the plastic stress distributions, a value of coefficient α comprised between 4 and 6 may be adopted, or what is the same, a value in the ratio $\alpha / 4$ between 1 and 1,5. Taking the mean value of 1,25 for this ratio, the proposed expression that allows for the design plastic bending moment resistance accounting for torsional effects for doubly symmetric I cross-sections is achieved.

$$M_{c,B,Rd} = \left[\sqrt{1 - \frac{\sigma_{w,Ed,\max}}{1{,}25 \cdot \frac{f_y}{\gamma_{M0}}}} \right] \cdot M_{c,Rd} \qquad \text{for Class 1 and Class 2 cross-sections} \qquad (34)$$

3.3 Comparison between the implicit elastic current approach of EN 1993-1-1 and the proposed formula

A comparative analysis between the reduction coefficient assumed in the implicit elastic approach of EN 1993-1-1 [1] (see Eq. (20)) and the reduction coefficient proposed in this work (see Eq. (34)) is carried out. Both reduction coefficients are used to take into account bending moment-torsion interaction. This is done through the graphical representation of the reduction coefficient in function of the maximum normal warping stress. Warping stresses are obtained according to the classic formula of the warping torsion (Kollbrunner and Basler [3]). The value of the maximum normal warping stress is shown as a percentage of the design yield strength of the steel.

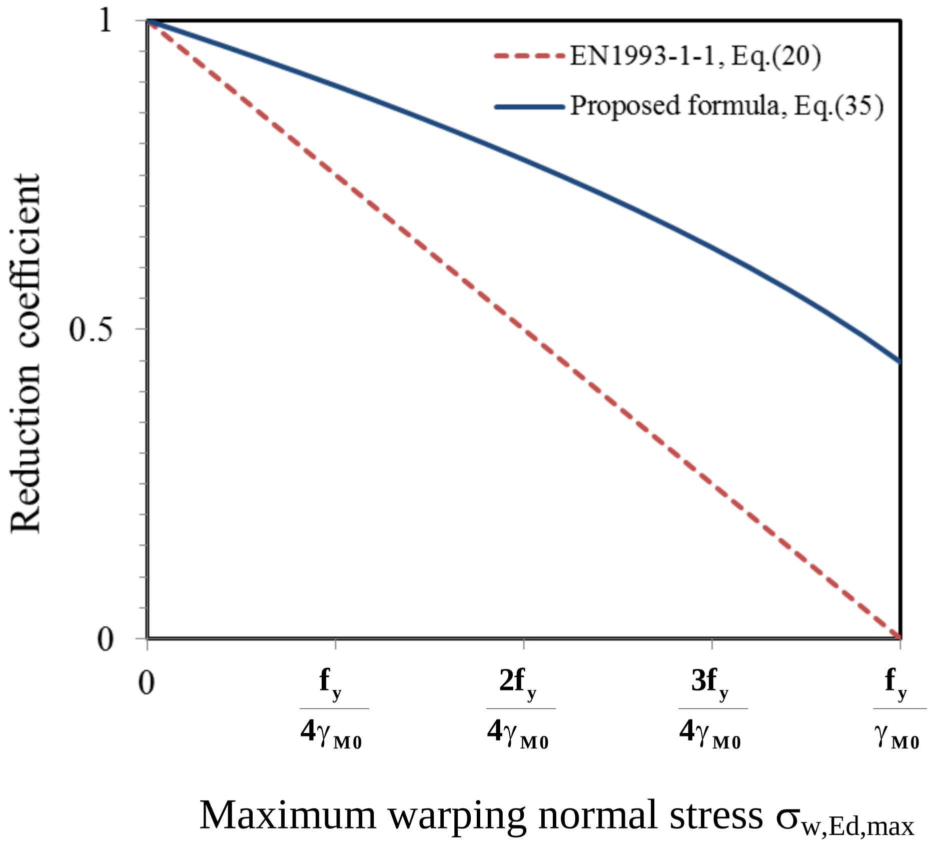

Figure 7. Comparative analysis of the reduction coefficient for the design bending moment resistance accounting for torsional effects, considering the EN 1993-1-1 [1] implicit approach and the proposed formula

Observing the curves shown in Figure 7 it is concluded that the formulation contained in EN 1993-1-1 [1] is very conservative. The curves apply to the case of doubly symmetric I cross-sections in bending around the major principal axis and warping torsion. It can be observed that the bending moment resistance considering its interaction with warping torsion for Class 1 and Class 2 cross-sections by means of the proposed formula in this work can be 40% higher than that obtained using the implicit current EN 1993-1-1 approach [1]. Moreover, the limitation that the resistance to bending moment considering interaction with torsion becomes zero for values of warping stress very close to the design yield strength is solved.

In summary, in this work a new formula for determining the resistance to bending moment, taking into account the interaction with torsion, is presented. This new approach is realistic and absolutely consistent with the concept of cross-sectional class, clearly improving the current approach of EN 1993-1-1 [1]. The presented formulation applies to the case of doubly symmetric I Class 1 and Class 2 cross-sections subjected to torsion and bending around the major principal axis. Following a similar methodology to the one presented herein for bending-torsion interaction or shear-torsion interaction, specific formulations for other types of cross-section may be developed.

4. Bending moment-shear-torsion interaction

The specifications stated in EN 1993-1-1 [1] for determining the resistance of a cross-section in bending when subjected to bending moment, torsion and shear force are not consistent with the formulation presented for bending moment-torsion interaction in this work.

4.1 Current specifications in EN 1993-1-1

For bending moment-torsion-shear force interaction, EN 1993-1-1 [1] states that when the design shear force V_{Ed} exceeds half the plastic shear resistance considering torsion effects $V_{pl,T,Rd}$, the reduced moment resistance should be taken as the design resistance of the cross-section, calculated using a reduced yield strength $(1-\rho)\cdot f_y$ for the shear area where

$$\rho = \left(\frac{2 \cdot V_{Ed}}{V_{pl,T,Rd}} - 1 \right)^2 \tag{35}$$

Considering exclusively the reduction in yield strength, associated with the shear area of the cross-section, it is only taken into account the interaction due to shear stresses caused by torsion. But the interaction due to normal warping stresses $\sigma_{w,Ed}$ is not taken into account to determine the reduced moment resistance. According to this approach, when the design shear force, V_{Ed}, exceeds half the plastic shear resistance considering torsion effects, $V_{pl,T,Rd}$, as EN 1993-1-1 proposes [1], the reduced yield strength caused by bending-torsion interaction is not taken into account. A discontinuity in the interaction diagram next to shear force values equal to half the plastic shear resistance to be produced, being on the unsafe side when the design shear force exceeds 50% the plastic shear resistance. Based on this interpretation the bending-shear-torsion interaction diagram would be

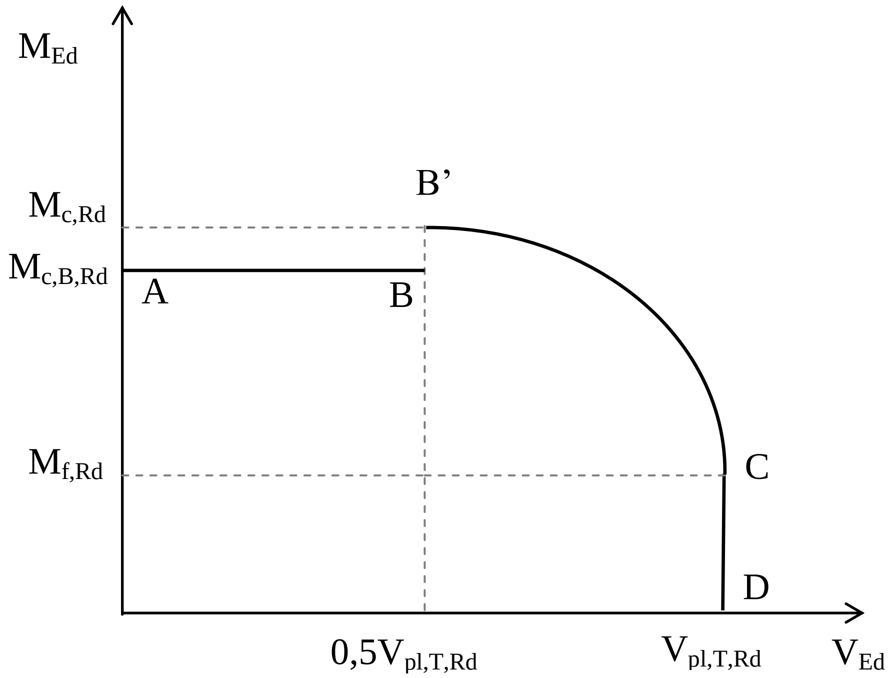

Figure 8. Bending-shear-torsion interaction diagram. Discontinuity at $V_{Ed} = 0{,}5V_{pl,T,Rd}$

4.2 New proposed bending-shear-torsion interaction diagram

To solve this inconsistency, the following method for the determination of the plastic resistance to bending is proposed, considering its interaction with torsion and shear. In the case of being in the presence of a cross-section subjected to bending, shear and torsion, when the design shear force V_{Ed} is less than 50% the reduced plastic shear resistance of the cross-section $V_{pl,T,Rd}$, the reduced yield strength $\frac{M_{c,B,Rd}}{M_{c,Rd}} \cdot f_y$ due to bending-torsion interaction will be assigned to the whole cross-section. The influence of the shear stresses due to shear force and torsion is not considered, but the influence of the normal warping stresses is considered. The reduced design plastic resistant moment is then $M_{c,B,Rd}$.

On the other hand, when the design shear force V_{Ed} is greater than 50% the reduced plastic shear resistance of the cross-section $V_{pl,T,Rd}$, the reduced yield strength due to shear stresses should be considered for the shear area of the cross-section to determine the reduced design plastic resistant moment. Therefore, the reduced yield strength equal to $(1-\rho) \cdot \frac{M_{c,B,Rd}}{M_{c,Rd}} \cdot f_y$ will be taken for the shear area and, other than the above, the reduced yield strength equal to $\frac{M_{c,B,Rd}}{M_{c,Rd}} \cdot f_y$ will be taken for the rest of the cross-section

where $$\rho = \left(\frac{2 \cdot V_{Ed}}{V_{pl,T,Rd}} - 1 \right)^2 \qquad (36)$$

$V_{pl,T,Rd}$ is obtained in accordance with EN 1993-1-1 [1] and $M_{c,B,Rd}$ is obtained in accordance with the proposed expression in this work (see Eq. (34)). Thus, a bending-shear-torsion interaction diagram without any discontinuity and perfectly consistent with the concept of cross-sectional class is obtained.

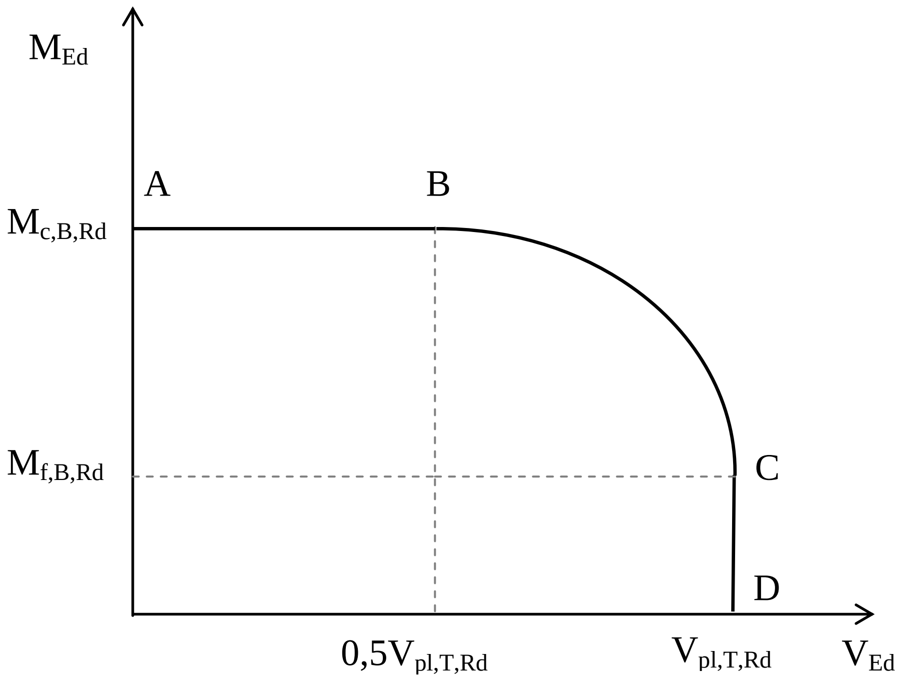

Figure 9. Proposed bending-shear-torsion interaction diagram

In this diagram $M_{f,B,Rd}$ represents the moment resisted by the cross section, excluding the shear area (it may be assumed that the web of the cross section is exhausted). As a good and easy approximation, $M_{f,B,Rd}$ is the moment resisted by the flanges of the cross-section when subjected to bending about the major axis, but considering the interaction with the normal warping stresses. Normal stress distributions in the cross-section, as a function of the different branches of the proposed interaction diagram (see Fig. 9), are shown in Figure 10.

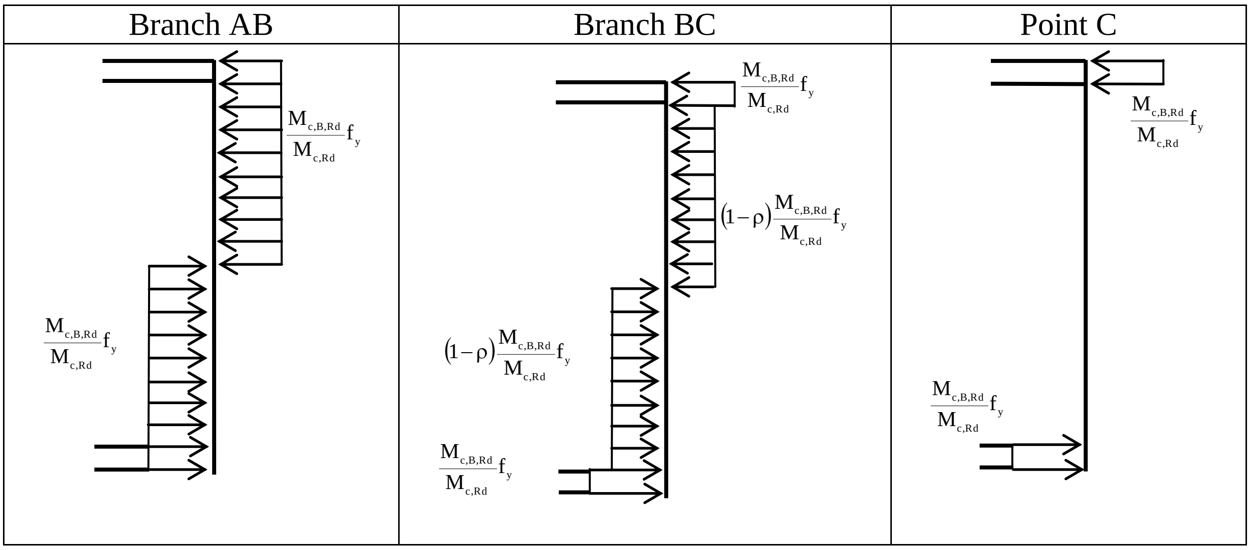

Figure 10. Normal stress distributions in the branches of the proposed bending-shear-torsion interaction diagram

5. Conclusions

In summary, in this paper a general formulation is provided to determine the design plastic resistant bending moment for plastic and compact cross-sections (Class 1 and Class 2), taking into account the interaction with shear force and torsion. Furthermore, the paper provides new information about the ambiguous specifications of EN 1993-1-1[1] on how to consider the effects of torsion and its interaction with bending and shear force for the verification of the ultimate limit state of the resistance of the cross-sections. The formulations proposed in this paper to consider the bending-shear-torsion interaction are consistent with the concept of cross-sectional class. In addition, the new formulations solve rigorously, efficiently and easily the verification of the resistance of the cross-sections subjected to combined internal forces.

6. References

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