Proceedings Paper:

Konstantopoulos, G. orcid.org/0000-0003-3339-6921 and Zhong, Q. (2016) Nonlinear control of dc/dc power converters with inherent current and power limitation. In: Proceedings of the 2016 24th Mediterranean Conference on Control and Automation (MED). 24th Mediterranean Conference on Control and Automation, 21-24 June 2016, Athens, Greece. IEEE , pp. 949-954. ISBN 978-1-4673-8345-5

https://doi.org/10.1109/MED.2016.7535991

Nonlinear control of dc/dc power converters with inherent current and power limitation

George C. Konstantopoulos and Qing-Chang Zhong

***Abstract*— A nonlinear controller with an inherent current-limiting capability is presented in this paper for different types of dc/dc power converters (boost, buck-boost). The proposed controller is based on the idea of applying a dynamic virtual resistance in series with the inductor of the converter, which varies according to a nonlinear dynamical system. It is shown that the proposed approach acts independently from the converter parameters (inductance, capacitance) or the load and has a generic structure that can be used to achieve different regulation scenarios, e.g. voltage, current or power regulation. Based on the nonlinear model of the boost and the buck-boost converter, it is analytically proven that the inductor current remains always bounded below a given maximum value using input-to-state stability theory under a suitable choice of the controller parameters. Hence, the proposed control strategy offers an inherent protection property since the power of the converter is limited below a given value during transients or unrealistic power demands. Simulation results for both types of dc/dc converters are presented to verify the desired controller performance.**

I. Introduction

DC/DC power converters are widely used to change a dc voltage level to a higher or lower value. In this framework, three main types of dc/dc converters are introduced: i) the boost converter, where the output voltage is controlled to a higher level that the input voltage, ii) the buck converter, where the output voltage is regulated to a lower level than the input and iii) the buck-boost converter, which offers the flexibility of controlling the output voltage to a higher or lower value than the input [1]. The desired operation is achieved by controlling the switching element of the converter, usually based on a pulse-width-modulation technique [2]. These types of power converters can be found in various applications including photovoltaic systems, wind power systems, energy storage, electric vehicles, dc microgrids, etc. [3], [4], [5], [6], [7].

Among these dc/dc converters, the boost and the buck-boost converter represent power electronic devices with increased interest for control and power engineers, since the output voltage and the inductor current can reach high values that can destabilize the system and damage the converter even when the device is connected to a strictly dissipative load. In this framework, various control methods have been designed to achieve a desired regulation scenario (usually output voltage regulation), including traditional Proportional-Integral (PI) controllers which are based on the small-signal model of the converter [8]. More advanced techniques have been developed using sliding control [9], [10], [11] or model predictive control [12] to guarantee precise output voltage regulation under a constraint for the control input, which is the duty ratio of the converter.

The nonlinear continuous-time dynamic description of the dc/dc converters, which provides the duty ratio control input and allows the investigation of different control algorithms, can be obtained using average analysis [2], [13], [14]. Based on this model, several nonlinear control methods have been designed to guarantee the stability of the closed-loop system [15], [16], [5], [17]. Based on the Hamiltonian structure of the converter model, passivity-based controllers have been effectively applied to achieve accurate output voltage regulation with a rigorous proof of stability [2], [18]. However, most of the existing control methods for dc/dc converters require accurate knowledge of the converter parameters (inductance, capacitance) or the load, which can change during the operation. Hence, a more robust method for controlling a dc/dc power converter has been presented in [19] using the interconnection and damping assignment passivity-based control. Since modern load types introduce complex dynamics (usually nonlinear) that can increased the nonlinearities and the number of states of the complete system, there is an obvious need for more advanced controllers that act independently from the system parameters and can guarantee the stable operation of the converter at all times. Particularly, a limitation of the inductor current below a given value is of major importance to protect the converter during transients or unrealistic power demands.

In this paper, a nonlinear controller that acts independently from the converter parameters and the load is proposed to guarantee a current-limiting property for both a boost and a buck-boost power converter. The proposed control strategy is based on the idea of applying a dynamic virtual resistance in series with the converter inductor which varies according to a nonlinear dynamical system. Based on input-to-state stability (ISS) theory [20], it is shown that with a suitable choice of a controller parameter, the inductor current will never violate a maximum limit that can be defined by the control operator, independently from the regulation scenario (voltage, current or power regulation). In this way, the converter is protected at all times since the injected power is always limited, even if an undesired high power demand is requested. An

G. C. Konstantopoulos is with the Department of Automatic Control and Systems Engineering, The Univesity of Sheffield, Sheffield S1 3JD, U.K. g.konstantopoulos@sheffield.ac.uk

Q.-C. Zhong is with the Department of Electrical and Computer Engineering, Illinois Institute of Technology, Chicago, IL 60616, USA, and the Department of Automatic Control and Systems Engineering, The Univesity of Sheffield, Sheffield S1 3JD, U.K. zhongqc@ieee.org

The financial support from the EPSRC, U.K. under Grant No. EP/J01558X/1 is greatly appreciated.

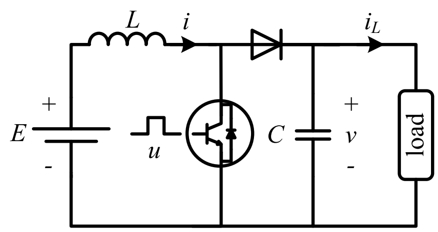


Fig. 1. Boost converter

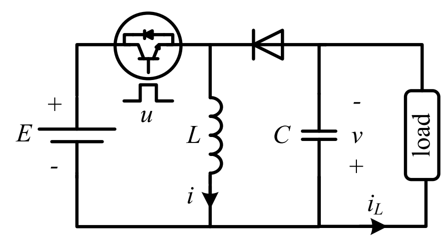


Fig. 2. Buck-boost converter

analytic framework for selecting all the controller parameters is also provided to complete the implementation procedure of the proposed controller. Extensive simulation results are presented for both a boost and a buck-boost converter to verify the desired operation of the proposed strategy and its current-limiting capability.

The rest of the paper is organised as follows. In Section II, the dynamic models of the boost and the buck-boost converters are presented. In Section III, the proposed nonlinear controller is presented and the current-limiting property is proven. A framework for choosing the controller parameters is also provided. In Section IV, simulation results are presented and finally in Section V, some conclusions are drawn.

II. Dynamic model of dc/dc power converters

Both the boost and the buck-boost power converters consist of an inductor L, a capacitor C, a diode and a switching element. Considering as E the dc input voltage and i_L the output load current, the schematic diagram of the boost and the buck-boost converters are shown in Fig. 1 and Fig. 2, respectively. Since in most applications the converter is operated using pulse-width-modulation with high switching frequency, it has been shown that using average analysis, the switching dynamic model of a dc/dc converter can be transformed into a continuous-time dynamic model where the control input is defined as the duty ratio $u \in [0, 1]$, which is a continuous-time signal [2].

Hence, using Kirchhoff laws, the dynamic representation of each dc/dc power converter becomes:

boost converter model:

$$L\frac{di}{dt} = -(1-u)v + E \quad (1)$$

$$C\frac{dv}{dt} = (1-u)i - i_L \quad (2)$$

buck-boost converter model:

$$L\frac{di}{dt} = -(1-u)v + uE \quad (3)$$

$$C\frac{dv}{dt} = (1-u)i - i_L. \quad (4)$$

Both power converters are nonlinear systems since the control input u is multiplied with the system states i and v. By considering a steady-state equilibrium (i_e, v_e) corresponding to a duty ratio u_e, it results from (1) and (3) that $u_e = 1 - \frac{E}{v_e}$ for the boost converter and $u_e = 1 - \frac{E}{v_e+E}$ for the buck-boost converter, which shows that the equilibrium point is unique in both converter cases but when $u = 1$ both systems become unstable (the inductor current continuously increases). Maintaining the inductor current limited and particularly below a given value is a crucial property that should be guaranteed at all times for the protection of the power converter.

Different control tasks can be considered for these types of converters depending on the application, with most common being the regulation of the output voltage v, the inductor current i or the power. To this end, in the sequel, a nonlinear controller that can achieve all different regulation tasks and inherits a current-limiting property is investigated.

III. Nonlinear control design and analysis

A. The proposed controller

In order to achieve a desired regulation scenario (voltage, current or power regulation) together with a current limitation for a dc/dc power converter, a dynamic virtual resistance w is applied in series with the inductor of the converter which decouples the dynamics of the input current. To this end, the following nonlinear controller is applied for each type of dc/dc converter:

boost converter:

$$u = 1 - \frac{w}{v}i, \quad (5)$$

buck-boost converter:

$$u = 1 - \frac{w}{v+E}i, \quad (6)$$

where the virtual resistance changes according to the nonlinear dynamics

$$\dot{w} = -cw_q^2 g(E, i, v, i_L) \quad (7)$$

$$\dot{w}_q = \frac{c(w-w_m)w_q g(E, i, v, i_L)}{\Delta w_m} - k\left(\frac{(w-w_m)^2}{\Delta w_m^2} + w_q^2 - 1\right) w_q,$$

with c, k, w_m, Δw_m being positive constants and $g(E, i, v, i_L)$ being a smooth function that describes the desired regulation scenario, i.e. $g(E, i_e, v_e, i_{Le}) = 0$ at the desired equilibrium point. For example, when the control task is the output voltage regulation to a reference value v_{ref}, then $g(E, i, v, i_L) = v_{ref} - v$. Equivalently, this function can take the form $g(E, i, v, i_L) = i_{ref} - i$ (current regulation), $g(E, i, v, i_L) = P_{ref} - vi_L$ (power regulation), etc.

To investigate the nonlinear controller dynamics of w and w_q, consider the following Lyapunov function candidate

$$W = \frac{(w - w_m)^2}{\Delta w_m^2} + w_q^2. \quad (8)$$

By calculating the time derivative of W and substituting the controller dynamics (7), it yields

$$\dot{W} = -2k \left(\frac{(w - w_m)^2}{\Delta w_m^2} + w_q^2 - 1 \right) w_q^2, \quad (9)$$

which is zero on the ellipse

$$W_0 = \left\{ w, w_q \in R : \frac{(w - w_m)^2}{\Delta w_m^2} + w_q^2 = 1 \right\} \quad (10)$$

and on the horizontal axis $w_q = 0$. Additionally, $\dot{W} < 0$ outside the ellipse and $\dot{W} > 0$ inside the ellipse except from $w_q = 0$. By choosing the initial conditions w_0, w_{q0} on the ellipse W_0:

$$\dot{W} = 0, \Rightarrow W(t) = W(0) = 1, \forall t \geq 0,$$

which means that the controller states w and w_q will start and travel at all times on the ellipse W_0 (see Fig. 3). For simplicity, the initial conditions can be chosen as

$$w_0 = w_m, \, w_{q0} = 1. \quad (11)$$

Since the controller states are restricted on W_0, then $w \in [w_{min}, w_{max}] = [w_m - \Delta w_m, w_m + \Delta w_m], \forall t \geq 0$. By considering the mathematical transformation

$$\begin{aligned} w &= w_m + \Delta w_m \sin\phi \\ w_q &= \cos\phi, \end{aligned}$$

from (7), there is

$$\dot{\phi} = \frac{c w_q g(E, i, v, i_L)}{\Delta w_m}, \quad (12)$$

which is the angular velocity of the controller states w and w_q while moving on the ellipse W_0 (Fig. 3). As a result, when the desired regulation scenario is achieved, i.e. $g(E, i, v, i_L) = 0$, the angular velocity tends to zero and the controller states stop and converge to two constant values w_e and w_{qe}. By selecting

$$w_m > \Delta w_m > 0,$$

the ellipse W_0 stays on the right-half plane and $w \in [w_{min}, w_{max}] > 0, \forall t \geq 0$, resulting in a positive dynamic virtual resistance.

Note that from (12) the angular velocity $\dot{\phi}$ becomes zero on the horizontal axis, i.e. when $w_q = 0$. This is desirable to avoid a possible oscillating behavior of the controller dynamics around the ellipse W_0 on the $w - w_q$ plane. To further explain this, assume that the controller states pass the desired equilibrium point during a transient and try to reach the horizontal axis. Then $w_q \to 0$ which means that $\dot{\phi} \to 0$ independently from the function $g(E, i, v, i_L)$. Thus, the controller states slow down until the angular velocity changes

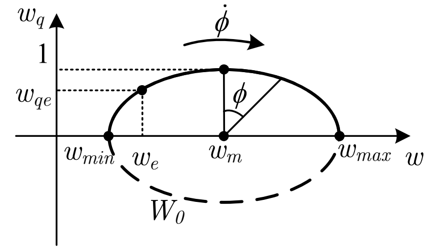


Fig. 3. Phase portrait of the controller dynamics

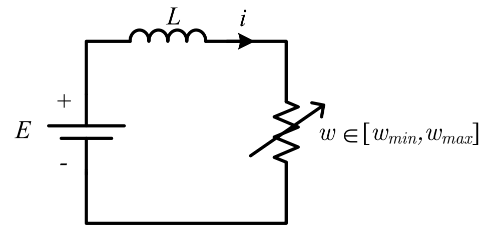


Fig. 4. Equivalent circuit of closed-loop current dynamics

sign and forces them to return to the desired equilibrium. As a result, w and w_q cannot travel around the whole ellipse W_0 and, based on the given initial conditions (11), they are restricted on the upper semi-ellipse of W_0 as shown in Fig. 3. Hence, $w_q \in [0, 1]$.

Now, by substituting (5) into (1) for the boost converter and (6) into (3) for the buck-boost converter, the closed-loop dynamics of the inductor current become in both cases

$$L \frac{di}{dt} = -wi + E, \quad (13)$$

from which it is clear that the proposed controller introduces a dynamic virtual resistance w in series with the inductor L, as shown in Fig. 4.

By considering the Lyapunov function candidate

$$V = \frac{1}{2} L i^2$$

for system (13), then the time derivative of V is calculated as

$$\dot{V} = L i \frac{di}{dt} = -wi^2 + Ei \leq -w_{min} i^2 + E |i| .$$

Hence

$$\dot{V} < 0, \forall |i| > \frac{E}{w_{min}}$$

which means that system (13) is ISS, where E is the dc input voltage [20]. As a result, if initially $|i(0)| \leq \frac{E}{w_{min}}$, then

$$|i(t)| \leq \frac{E}{w_{min}}, \forall t \geq 0. \quad (14)$$

Therefore, by selecting w_{min} according to

$$w_{min} = \frac{E}{i_{max}}, \quad (15)$$

where i_{max} denotes the maximum allowed current of the converter, then by substituting (15) into (14) it results in

$$|i(t)| \leq i_{max}, \forall t \geq 0,$$

which guarantees the desired current-limiting capability of the converter.

Assuming a constant (or bounded) input voltage E, the current limitation results in a power limitation of both converter types. For the boost converter $P = Ei \leq Ei_{max}$ and for the buck-boost converter $P = Eui \leq Ei_{max}$ for a given maximum value i_{max}. Hence, both dc/dc converters operating under the proposed controller are always protected during transients or unrealistic power demands. To further clarify this, consider the case of output voltage regulation for a boost converter, i.e. $g(E, i, v, i_L) = v_{ref} - v$ and assume that v_{ref} is chosen as a high value such that $P_{ref} = v_{ref} i_L > Ei_{max}$. Then $v_{ref} - v > 0$ and from (12) the controller states w and w_q will travel counter-clockwise and eventually $w_{min} \rightarrow 0$ and $w_q \rightarrow 0$ which is also an equilibrium point of the system according to (7). As a result, $i \rightarrow i_{max}$ and the power of the converter will be $P \rightarrow Ei_{max} < P_{ref}$ showing that the converter will be protected at all times.

B. Parameter selection

Since the minimum virtual resistance w_{min} is related to the maximum current i_{max}, in the same framework the maximum value w_{max} of the virtual resistance will correspond to the minimum inductor current i_{min}. Although the minimum current of both the boost and the buck-boost converter is theoretically zero, in practice a very small current flows through the parasitic elements of the converter. Hence, w_{max} can be selected as

$$w_{max} = \frac{E}{i_{min}}, \tag{16}$$

where i_{min} can be sufficiently small (mA or μA). Having defined the maximum and minimum values of the virtual resistance, then the parameters w_m and Δw_m that define the ellipse W_0 are given as

$$w_m = \frac{w_{max} + w_{min}}{2} = \frac{E}{2}\left(\frac{1}{i_{min}} + \frac{1}{i_{max}}\right), \tag{17}$$

$$\Delta w_m = \frac{w_{max} - w_{min}}{2} = \frac{E}{2}\left(\frac{1}{i_{min}} - \frac{1}{i_{max}}\right). \tag{18}$$

The controller gain k is multiplied by the term $\frac{(w-w_m)^2}{\Delta w_m^2} + w_q^2 - 1$ in (7), which is zero on the ellipse W_0. The role of this gain is to make the controller dynamics of w_q robust to external disturbances or calculation errors in the sense that if the controller states are disturbed from W_0 they will quickly converge to the desired ellipse. Therefore, k can be chosen as a sufficiently high positive constant.

Finally, the choice of parameter c has a direct impact on the dynamic performance of the controller since it affects the angular velocity $\dot{\phi}$ in (12). To define a framework for choosing c, consider a worse case scenario where w and w_q start from point $(w_{max}, 0)$ and reach point $(w_{min}, 0)$ at the steady state by traveling on the upper semi-ellipse of W_0, i.e. they travel on an arc with central angle π rad. Assuming a settling time t_s for this operation, then in the worst case

TABLE I

SYSTEM AND CONTROLLER PARAMETERS

Parameters	Values	Parameters	Values
L	4 mH	switching frequency	20 kHz
C	100 μF	k	100
E	100 V	i_{max}	2 A
c	4×10^5	i_{min}	1 mA

where the angular velocity $\dot{\phi}$ is constant and equal to its maximum value, there is

$$\dot{\phi}_{max} = \frac{\pi}{t_s} = \frac{c \max\{|g(E, i, v, i_L)|\}}{\Delta w_m}$$

since $0 \leq w_q \leq 1$, which yields

$$c = \frac{\pi \Delta w_m}{t_s \max\{|g(E, i, v, i_L)|\}}, \tag{19}$$

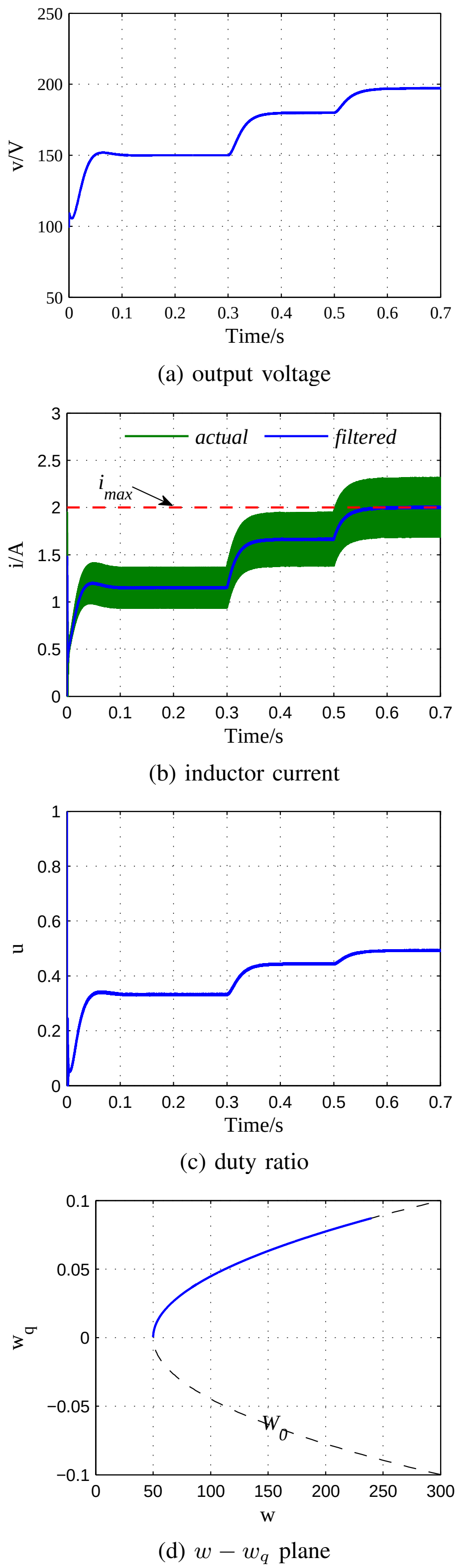
where $\max\{|g(E, i, v, i_L)|\}$ denotes the maximum possible absolute value of function g. For example, for a voltage regulation scenario where $g = v_{ref} - v$ and a boost converter application, $\max\{|g(E, i, v, i_L)|\} = v_{max} - E = \frac{Ei_{max}}{i_L} - E$. Note that (19) provides a framework for a starting value of c. Since function g will decrease as soon as the system approaches the equilibrium point and w_q will be less than 1, then c can be chosen as a higher value, i.e. it can be increased until a satisfactory response is achieved.

IV. SIMULATION RESULTS

To verify the proposed control strategy, both a boost and a buck-boost converter connected to a resistive load of $200\,\Omega$ are simulated using the Simpower Systems toolbox of Matlab/Simulink. The parameters of the system and the controller are shown in Table I and are the same for both converters. The control task is to regulate the output voltage to v_{ref}, i.e. $g(v) = v_{ref} - v$. It should be underlined that since the actual switching model of each converter is simulated and not the average model, a low-pass filter is required at the measurement of the inductor current to remove the switching ripples.

Case 1: boost converter

Since the dc input voltage is $E = 100\,$V and the boost converter is investigated, the output reference voltage v_{ref} is set initially to $150\,$V, at the time instant $t = 0.3\,$s it changes to $180\,$V and finally at $t = 0.5\,$s it increases to $250\,$V which will require a large inductor current in order to test the current-limiting property of the proposed strategy. As it is shown in Fig. 5(a), during the first $0.5\,$s the output voltage is regulated at the desired level after a short transient. However, when the reference voltage v_{ref} is set to $250\,$V, the output voltage is regulated near $200\,$V because the inductor current tries to violate the maximum value i_{max}. This is clearly shown in Fig. 5(b), where the average value of the inductor current (used in the control implementation) stays always below i_{max} to protect the converter from the unrealistic power demand. The duty ratio response is given in Fig. 5(c),



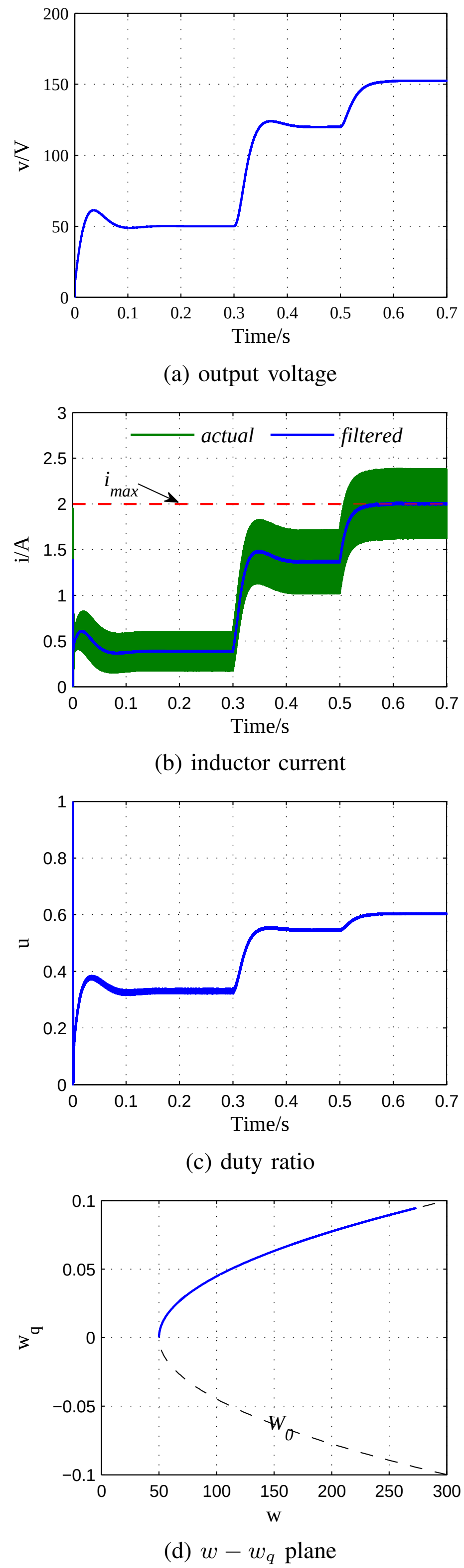
(a) output voltage

(b) inductor current

(c) duty ratio

(d) $w - w_q$ plane

Fig. 5. Simulation results of the boost converter under the proposed controller



(a) output voltage

(b) inductor current

(c) duty ratio

(d) $w - w_q$ plane

Fig. 6. Simulation results of the buck-boost converter under the proposed controller

while in Fig. 5(d) the controller states w and w_q are plotted on the $w-w_q$ plane to verify the Lyapunov theory, since they are restricted on the upper semi-ellipse of W_0 as explained in Subsection III-A.

Case 2: buck-boost converter

Similarly, a buck-boost converter is investigated and since it allows the output voltage to be regulated at a lower or higher level than the input, the output reference voltage v_{ref} is set initially to $50\,\text{V}$, at the time instant $t = 0.3\,\text{s}$ it changes to $120\,\text{V}$ and finally at $t = 0.5\,\text{s}$ it increases to $200\,\text{V}$. Once again, when the inductor current is below i_{max}, the output voltage reaches the desired level as shown in Fig. 6(a) for the first $0.5\,\text{s}$. When the reference voltage changes to $200\,\text{V}$, then the output voltage is regulated to a lower value since the current increases and reaches the limit (Fig. 6(b)). The control input (duty ratio) is shown in Fig. 6(c) and as in the case of the boost converter, the controller states remain on W_0 during the whole operation (Fig. 6(d)). Hence, it is verified that the proposed controller can protect both the boost and the buck-boost converter from high currents at all times, i.e. during transients or unrealistic requests of power.

V. Conclusions

A current-limiting controller with nonlinear dynamics was developed for both the boost and the buck-boost power converter. With an appropriate choice of the controller parameters, it is proven that the inductor current remains always limited below a given value, resulting in a limit of the converter power without requiring any knowledge of the converter inductance, capacitance or the load. Extensive simulation results for both types of dc/dc power converters suitably verified the proposed approach.

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