The Embeddedness of Organizational Performance: Multiple Membership Multiple Classification Models for the Analysis of Multilevel Networks♠

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Abstract

We develop a Multiple Membership Multiple Classification (MMMC) model for analysing variation in the performance of organizational units embedded in a multilevel network of dependence relations. The model postulates that the performance of organizational sub-units varies across network levels defined in terms of: (i) direct relations between organizational sub-units; (ii) relations between organizations containing the sub-units, and (iii) cross-level relations between sub-units and organizations. We demonstrate the empirical merits of the model in an analysis of inter-hospital patient mobility within a regional community of health care organizations. In the empirical case study we develop, organizational sub-units are departments of emergency medicine (EDs) located within hospitals. Networks within and across levels are delineated in term of patient transfer relations between EDs (lower-level, emergency transfers), hospitals (higher-level, elective transfers), and between EDs and hospitals (cross-level, non-emergency transfers). Our main analytical objective is to examine the effect of these interdependent and partially nested levels of action on the variation in waiting time among EDs – one of the most commonly adopted and accepted measures of ED performance. After controlling for institutional and organizational differences between hospitals, we find that the largest component of network variation in patient waiting-time is accounted for at the level of the hospital-to-hospital patient transfers. We take this result as clear evidence of the presence of multilevel network effects on organizational sub-unit performance. We also find that EDs connected by direct patient transfer relations tend to attain similar levels of performance. We interpret this performance spillover effect as evidence that collaborative network relations afford and sustain processes of interorganizational learning and knowledge transfer at the sub-unit level. We discuss further extensions to the model for more general analyses of multilevel network dependencies.

Keywords: *Health care organizations; Interorganizational fields; Interorganizational networks; Multilevel networks; Multiple Membership Multiple Classification Model; Interorganizational networks; Interorganizational fields; Health care organizations; Organizational performance.*

1. INTRODUCTION

Interest in the analysis of multilevel networks has been rapidly growing in recent years (Snijders et al., 2013; Wang et al., 2013). Despite such interest, the general view persists that: "This area of network modelling remains thoroughly underdeveloped." (Snijders, 2011: 137). This seems to be particularly the case in the study of formal organizations, whose nested hierarchical structure makes the analysis of multilevel networks unavoidable (Lomi et al., 2014). Nevertheless, the multiple levels spanned by networks within and between organizations are typically considered independent and analysed separately as independent levels of action (Borgatti and Foster, 2003; Kilduff and Tsai, 2003). As Moliterno and Mahony concluded in their extensive review of the literature (2011: 444): "[W]hile some recent network scholarship has begun considering multiple levels of analysis, the majority of scholarship in this area has examined single- and within-level network structures and relationships."

For this reason, extant studies of organizational networks are generally unable to deliver on their central promise to provide a bridge across multiple structural levels of action (Contractor, Wasserman and Faust, 2006). This is particularly the case in the study of interorganizational relations, where the nodes are individual organizations, characterized by an internal structure with multiple hierarchical levels (DiMaggio, 1986). In this paper, we present a new model for the analysis of multilevel networks - a Multiple Membership Multiple Classification (MMMC) model – which addresses this problem directly. An MMMC model was recently applied to the analysis of (single level) social network and group dependencies (Tranmer et al., 2014). We present here, for the first time, an extension of this model to the analysis of multilevel networks. To establish the empirical value of the MMMC model in this context, we present an illustrative application to the empirical analysis of interorganizational relations – a natural multilevel setting (DiMaggio, 1986; Laumann, Galaskiewicz, and Marsden, 1978).

Our work extends existing research in at least two ways. Firstly, like Hierarchical Linear Models (HLMs), our study spans multiple levels of analysis. Unlike HLMs, however, our model takes explicitly into account dependencies induced by network ties within and between structural levels through multiple affiliations. In this way, our study contributes to research in this field by extending available statistical models for multilevel systems to the analysis of multilevel social networks. Secondly, like recent studies based on Multilevel Exponential Random Graph Models

(MERGMs), we are analysing multilevel network data (Wang et al., 2013). However, unlike MERGMs, whose target of inference is multilevel network structure, as defined by ties within and across levels, the focus of the MMMC model is on variation in *outcomes* associated with attributes of the lower-level nodes across and between the levels of a multilevel network. More specifically, the MMMC model focuses on the way in which multilevel network dependencies are associated to variations in a *behavioural dependent variable* defined for nodes at the lowest level, rather than on the presence or absence of ties among such nodes. Also, unlike MERGMs, the models we propose are not restricted to binary networks, but are applicable to a broader range of weighted networks.

MMMC models have only recently been applied to network data (Tranmer et al, 2014). To our knowledge, however, they have not been applied to multilevel networks. Here, we specify the MMMC for a multilevel network and illustrate its application to data that we have collected on Emergency Department (ED) waiting times – a generally accepted measure of ED operational performance (Horwitz, Green and Bradley, 2009; Lambe et al., 2003). The network we examine is multilevel because it implicates multiple interdependent levels of action. The first level is defined in terms of transfers of emergency patients between EDs (lower-level nodes). The second level is defined in terms of transfers of elective patients between hospitals (higher-level nodes) containing the ED units. Finally, the third level involves transfer of non-emergency patients between EDs and hospitals (cross-level relations). The empirical example illustrates in practice how the MMMC that we have developed may be adopted to address recent calls to develop multilevel approaches to the analysis of intra and interorganizational networks (Baker and Faulkner, 2002; Brass et al., 2004). The multilevel data structure that we will be analysing in the empirical part of the paper is described, schematically, in **Figure 1.**

----- Insert Figure 1 about here -----

Our analysis focuses on variations in ED waiting times. Waiting times are an important measure of how well EDs respond to patient needs, and are commonly used as an indicator of the timeliness, efficiency, safety and patient-centeredness of emergency care (Horwitz, Green, and Bradley, 2009). The illustrative example we present as an application of the MMMC model for multilevel networks is almost ideal for at least two reasons: (i) detailed information on the various forms of patient transfers is publicly available, and (ii) the attention paid by health authorities in assessing health care outcomes makes the data particularly reliable.

We organize the article as follows. In Section two, we outline the motivation for developing statistical models for multilevel network dependencies. In the third section, we review methods and models for single level networks, including network autocorrelation models, and multilevel approaches via the Multiple Membership (MM) Model. We outline their conceptual similarities and differences. In the fourth section, we introduce the MMMC model for the analysis of multilevel networks, explaining how it is an extension of the single level network MM model, and what the parameter estimates from such a model indicate regarding multilevel network dependencies. In the fifth section, we describe the research design of our example, and the approach that we adopt for the estimation and evaluation of the model. In the sixth section we present the results of the analysis, and discuss their possible interpretation and implications. We conclude the article with a reflection on the limitations, general usefulness, and applicability of MMMC models to more general studies of multilevel network dependencies.

2. THE MULTILEVEL STRUCTURE OF INTERORGANISATIONAL NETWORKS

The recent interest in models for multilevel networks is driven in part by advances in statistical modelling (Wang et al., 2013) and in part by the rediscovery of the classic theoretical insight that social networks connect multiple levels of action (Boorman and White, 1976; White et al., 1976). Statistical models to investigate how structural levels of action may be (de)coupled have only become available recently.

Organizations provide an ideal setting for exploring the joint implications of these parallel trends (Rousseau, 1985). The networks in which they are embedded are multilevel social constructions (Simon, 1996; DiMaggio, 1986). This is the case because micro-level relations between individuals (or organizational units) create macro (or field-level) relations that eventually cumulate into community-level structure (Breiger and Pattison, 1978; Laumann, Galskiewicz and Marsden, 1978). There is a strong need for theory and analysis of multilevel networks in this context (Aguinis et al., 2011; Mathieu and Chen, 2010; Oh, Labianca, and Chung, 2006; Rousseau, 2011).

In his work on the interorganizational field of US Resident theatres, DiMaggio sets the stage for the development of models for multilevel networks that we present in this paper (1986: 363):

> "The insight that organizations consist of individuals and subunits with quite different agendas and objectives is exceptionally difficult to capture in an interorganizational framework, where the smallest units of analysis are organizational nodes in networks. Indeed, network imagery militates toward treating and talking about organizations as if they were unitary actors with constant and uncontested objective functions."

We explain how this "exceptional" difficulty may be addressed by considering organizations as the macro level in a multilevel network for which the internal structure of "organizational nodes" represents the micro level. This allows us to represent in the one model the way in which organizational behaviour is affected by: (i) network relations between organizations; (ii) networks of relations between sub-units contained within organizations, and finally (iii) networks connecting units to organizations across-levels.

3. MODELS FOR CROSS-SECTIONAL NETWORK DEPENDENCE

We begin by reviewing existing approaches for analysing single level and multilevel networks, focusing on the cross-sectional case, and identify the targets of inference for such analyses. We especially focus on models for network dependence. We explain the similarities and differences of these approaches, starting with the single-level network, before considering multilevel networks.

3.1 Single level networks.

A single level network may be defined as a set of nodes for which connections exist that could be undirected or directed. Attributes often exist for each node, and one or more of these may be regarded as a dependent, y, variable. Other node attributes may comprise a set of explanatory variables, $\mathbf{X}$. When the target of inference is the network structure, in the context of social selection or social influence, we can investigate it with an Exponential Random Graph Model (ERGM), for which the application, software and literature are now all well established (Lusher, Koskinen and Robins, 2013). ERGMs can be fitted with or without attribute information, to assess whether such attributes are associated with the network tie structure.

Alternatively, network dependence might be the focus of the study, where the target of inference could be the extent to which the relationship between the dependent and explanatory variables is associated with the network connections, or where we wish to allow for network connections when estimating the relationship between the dependent and explanatory variables. Well established models for such cases are network autocorrelation models.

3.2 Network Autocorrelation Models.

Network autocorrelation models originally developed from spatial autocorrelation models - see, for example, Ord (1975), Doreian (1980). The model formulations are identical for the spatial and network cases, using spatial or network data respectively. The names of these models differ in the literature, but we will refer to two important cases below as the *network effects model* and the *network disturbances model*; these are the names used by Leenders (2002). We assume that the dependent variable is interval scale in all models defined below. For a single level network, the network effects (NE) model is defined as:

$$y = \rho \mathbf{W}\mathbf{y} + \mathbf{X}\mathbf{B} + \varepsilon$$

Where:

$$\varepsilon \sim N(0, \sigma_\varepsilon^2) \qquad (1)$$

In (1), y is an attribute of each node of the network, that we regard as a dependent variable, and **X** is a set of explanatory variables, also attributes of each node, which we wish to relate to the dependent variable. **W**, is a set of weights summarising network connections. These are usually standardised to sum to 1 across each row of **W** (Leenders, 2002). As an example, consider a network with 10 nodes, for which the first node of the network, corresponding to the first row of **W**, has three connected nodes. In the absence of other information, we can define their weights equally as 1/3, summing to 1 across row 1. The seven unconnected nodes to node 1 (including the self-connection), have weights of 0. The **W** can also be defined for unequal weights, such as where connections are based on valued ties. The diagonal of **W** is assumed to be zero to disallow self-connections (loops).

The parameters of model (1) are ρ, the coefficients **B** for any explanatory variables, and the variance of the errors, σ_ε^2. In this model, y appears on both sides of the equation. **W** provides information about the other nodes to which each node of the network is connected.

Substantively, this model is useful when we think that there is a direct connection of the values of y for connected nodes when predicting the value of y for each node. In other words, using an ego-net as an example, to determine whether the average value of the dependent variable of the alters is associated with the value of the dependent variable for the ego. The average strength of such a connection, conditional on the other explanatory variables in the model, is estimated by the (auto)correlation parameter ρ. The ε are differences in the predicted values of y from their true values, which are assumed to be Normally distributed with mean zero and variance σ_{ε}^{2}. Model (1) can be fitted before and after the inclusion of explanatory variables, **X.**

An alternative formulation to the network effects model, using the same data input as (1), is the network disturbances (ND) model:

$$
\begin{aligned}
y &= \mathbf{XB} + \varepsilon \\
\varepsilon &= \rho \mathbf{We} + \upsilon \\
\upsilon &\sim N(0, \sigma_{\upsilon}^{2}) \qquad (2)
\end{aligned}
$$

The first line of (2) looks like a standard Ordinary Least Squares (OLS) regression model, however the second line indicates how the network connections are taken into account in the error part of this model. The model parameters are the regression coefficients ρ, the coefficients **B,** and the variance of the error.

Model (2) takes into account the network through the error, **e**, which we assume is a vector of network random effects. Model (2) is useful for situations where we hypothesise there is an omitted variable, for which network structure exists, or network heterogeneity in y that cannot be explained by the explanatory variables alone. Model (2) is also useful when we want to estimate the regression for y on **X** given the fact the network nodes are connected; unlike OLS, the model coefficients and their standard errors will be estimated having taken the network structure into account. In model (2), ρ gives us an estimate of the extent to which the error, ε, of the focal node is correlated with values of the errors for connected nodes, conditional on the explanatory variables in the model, **X**. If there is no network structure to these errors, ρ is estimated as zero, and (2) reduces to a standard OLS model (Also true for model (1) when ρ is estimated as zero). Where network structure exists in the errors, ρ, will typically be positive. Single level NE and

ND models may be fitted in software such as `R` (R core team, 2013), using the `SNA` (Butts, 2008) or `SPDEP` (Bivand et al 2005) packages.

3.3 Multiple Membership Models.

Multilevel modelling is a common technique for investigating variations in response variables in structured populations (Snijders and Bosker, 2012). Tranmer, et al. (2014) explain that an MMMC model, a type of multilevel model, is useful for analysing network dependencies in a single level network, when other groups in the population such as areas, or schools, exist alongside the network. They show how an MMMC model may be used to compare network, school and area variations in educational performance.

When we have one network substructure, such as the ego-nets, in a single level network, the MMMC reduces to a Multiple Membership (MM) model (3) because there is only one classification.

$$\left.\begin{aligned} & y_i = \mathbf{x}_i'\beta + \sum_{j \in \mathrm{net1}(i)} w_{1i,j} u_j^{(1)} + e_i \\ & i = 1,\ldots,n; \qquad \mathrm{net1}(i) \subset (1,\ldots,J_{1\mathrm{net}}); \\ & u_j^{(1)} \sim N(0,\sigma^2_{u^{(1)}}); \quad e_i \sim N(0,\sigma^2_e); \quad \mathrm{Cov}(u_j^{(1)}, e_i) = 0. \end{aligned}\right\} \qquad (3)$$

In Model (3), y_i is a node level dependent variable for each of the n nodes, $\mathbf{x}_i$ is a vector of fixed explanatory variables, and β is the vector of their regression coefficients. Here, $\mathrm{net1}(i)$ is the set of network subgroups, which we will assume are ego-nets, but these could also be defined in other ways, such as cliques. The term $\sum_{j \in \mathrm{net1}(i)} w_{1i,j} u_j^{(1)}$ involves a set of $J_{1\mathrm{net}}$ random effects $u_j^{(1)}$, where $J_{1\mathrm{net}}$ is the total number of subgroups (here ego-nets) in the network. The weight that is given to each node for their ego-net membership is $w_{1i,j}$. These weights sum to 1 for each individual, as was the case for the network autocorrelation models defined earlier. As is typical in multilevel modelling, the random effects at the individual and network levels are assumed to be uncorrelated: $\mathrm{Cov}(u_j^{(1)}, e_i) = 0$. The between-network subgroup variance component, $\sigma^2_{u^{(1)}}$, allows us to assess the extent to which the dependent variable varies between the network subgroups, such as ego-nets, and by extension, how much variation in the dependent variable is within these subgroups.

3.4 Similarities and differences of ND and MM models.

Models (1), (2) and (3) all take into account network dependencies in cross-sectional network data. Tranmer et al. (2014; Table 4) compared ND models (2) with MM models (3) in an empirical analysis of academic performance, given information about networks and schools. They found similar estimated model coefficients, standard errors, and goodness of fit for both approaches. These authors compared the ND and MM models because both models include the network information in the random effects (error) part of the model.

As well as obtaining estimates of the model coefficients and their standard errors in the ND and MM models, some information about network dependency is estimated. In the ND model (2), $\hat{\rho}$ gives an estimate of the extent to which the errors are correlated for connected nodes given the explanatory variables. In the MM model (3) the variance component $\hat{\sigma}^2_{u^{(1)}}$ gives an estimate of the extent to which the errors vary on average between the different ego-nets in the network. If there is no correlation or variation of errors, ρ in model (2) and $\sigma^2_{u^{(1)}}$ in model (3) would both be estimated as zero.

As Tranmer et al. (2014) show, model (3) may be extended to a MMMC to include an additional level (classification) for school, allowing the relative share of variation in academic performance of students due to networks and due to schools to be assessed. Moreover, as the authors discuss in their 2014 paper, model (3) could be extended to include random coefficients – allowing, for example, network variation to be different for girls and boys with respect to academic performance.

Model (1) uses the average value of y for connected nodes in the prediction of y for the focal node, scaled by the (auto)correlation parameter estimate, ρ. Similar approaches can be made in the multilevel framework by first creating a “peer effect” variable, using:

$$\overline{y}_i = \sum_{j \in \text{net1}(i)} w_{1i,j} y_{ij} \qquad (4)$$

For ego-nets, this gives the average of *y* for alters (i.e. their peers) not including the ego's value of *y*. Similar calculations can be made for the explanatory variables. The choice of weights will affect the values of these peer effect variables.

If we fit a single level model:

$$y_i = \beta_0 \overline{y}_i + \varepsilon_i,$$

Where:

$$\varepsilon_i \sim N(0, \sigma_\varepsilon^2) \qquad (5)$$

we obtain an estimate of the average peer effect, $\hat{\beta}_0$, which is similar to the estimated average auto-correlation, $\hat{\rho}$, in (1) when *y* has a mean of zero. ε_i is then the difference in the predicted value, $\hat{y}_i$, based on $\overline{y}_i$, the average values of y_i for the alters of *i*, and we obtain an estimate of its variance, $\hat{\sigma}_\varepsilon^2$. As a quick illustration – and to foreshadow some of our empirical results - we fitted models (1) and (5) to our empirical data, using the row-standardised single-level network of ED connections, where the *y* variable was standardised to have a mean of zero and a standard deviation of 1. We obtained the following estimates: $\hat{\rho}$=0.091, $\hat{\beta}_0$=0.104, respectively. These similar values indicate the presence of network autocorrelation; there is some similarity in the values of *y* for connected individuals.

Whilst (5) is a single level model, it could be extended to include an ego-net level as in Model (3). The resulting model would allow us to estimate the remaining variation in the error between ego-nets in the network that has not been explained by the peer effect of *y*, allowing substantive hypotheses regarding variation in the network to be tested.

3.5 Multilevel Networks.

In general, a multilevel network can be defined for two levels as a series of level 1 nodes and their connections, a series of level 2 nodes and their connections, and often also the cross-level network between level 1 nodes and level 2 nodes. Attribute information is often available for the level 1 and level 2 nodes.

An example is Lazega et al's (2008) "linked design" approach. Here, the level 1 nodes are individual cancer researchers, connected by advice seeking; the level 2 nodes are the laboratories in which they work, connected by resource sharing. There is also a cross-level network connecting individual researchers to laboratories that are not their usual place of work. Level 1 node attributes include age, gender, research impact score, level 2 node attributes include size, expenditure, speciality. In Lazega et al's data there is usually one researcher per laboratory. If there was exactly one researcher in every laboratory, the data could at first appear to be a multiplex network. However this network is multilevel because of the nature of the connections: the first relates to level 1 nodes (researchers) the second to level 2 nodes (laboratories), and the third to cross-level connections between level 1 and level 2 nodes.

Wang et al. (2013) developed and applied their MERGM approach to the Lazega et al's (2008) data. Their targets of inference were various aspects of the multilevel network structure. When the targets of inference are multilevel network dependencies for the level 1 network, the level two network and the cross-level network, we can extend the MM model (3) to include three sets of classifications for these networks, making it a Multiple Membership Multiple Classification (MMMC) Model.

4. THE MMMC MODEL FOR MULTILEVEL NETWORKS.

We provide a general description of the MMMC for investigating variations in a dependent variable for the level 1 nodes across the various components of the multilevel network. We focus on the case of a two-level network.

MMMC models may be fitted when the targets of inference are the sources of variation in a level 1 node dependent variable $y_{i,}$ across the three components of the multilevel network, and at the individual level, before and after the inclusion of explanatory variables.

We can assess these sources of variation from the estimated variances of the random effects in the MMMC model. Moreover, when an MMMC model is fitted with a set of explanatory variables, we estimate their coefficients and standard errors taking into account the complex multilevel network structure of the data in the analysis.

The structure of a multilevel network with two levels can be defined as a set of m_1 nodes at level 1. For each of these level 1 nodes, we have a response variable, y_i . We can represent this network by an m_1 by m_1 adjacency matrix. At level 2 we have a network of m_2 nodes. Sometimes there will be more than one level 1 node contained in each level 2 node. However, an important special case is where there is exactly one level 1 node contained in each level 2 node; either through data availability, or in the population. When this is the case, $m_1 = m_2 = m$, and, the cross-level network will also be of dimension m by m. We will define and discuss the MMMC model for this special case.

The level 1 network, level 2 network, and cross-level network can each be represented as a non-symmetric valued m by m matrix. We define these three matrices as $\mathbf{W}_1$, $\mathbf{W}_2$ and $\mathbf{W}_C$, respectively. Each row of $\mathbf{W}_1$, indexed by $i=1,\ldots,m$, of these matrices is the ego-net of each level 1 network node, where the row corresponds to the ego, and each non-zero element in that particular row represents an alter in that ego-net. Similarly, the rows of $\mathbf{W}_2$ and $\mathbf{W}_C$ are also ego-nets. For each row, the total number of non-zero elements is the ego-net size (that is, the number of alters in the ego-net). If $\mathbf{W}_1$, $\mathbf{W}_2$ and $\mathbf{W}_C$ are binary matrices, the sum of each row i of each of these matrices, n_{1i}, n_{2i} and n_{Ci} is the total number of alters for the ego-net of each level 1 node, level 2 node, and for the cross-level nodes, respectively. If $\mathbf{W}_1$, $\mathbf{W}_2$ and $\mathbf{W}_C$ are valued matrices, the sum of each row of these matrices gives the sum of the tie values in that ego-net, and we may find that there are much greater tie values for some of the alters in a particular ego-net than others, which we should take into account in the model specification.

We can think of the alters of an ego-net as the members of ego's group, and can assign weights for ego-net membership. Usually these will be row-standardised as discussed above (in 3.3). To illustrate how network weights can be assigned for a multilevel network in the context of our empirical example, **Table 1** shows an example row of patient transfer data.

- Insert Table 1 about here -

In this example, ED 1 transfers patients to seven other EDs. Hospital 1, in which ED1 is contained, transfers patients to six other hospitals. There are two hospitals, other than hospital 1, to which patients are transferred back from ED1. In total, 73 patients are transferred from (ego) ED 1 to seven other (alter) EDs as emergency transfers, with the number transferred ranging from 1 to 51. ED weights can hence be calculated as $w_{m1net1,j} = n_{11j} / \sum_j n_{11j}$, summing to 1 across the row. Hospital 1 makes a total of 24 elective transfers of patients to six other hospitals in its hospital ego net, and weights $W_{m2net1,j}$ can be calculated as for the EDs, and these sum to 1 across the row. For the cross-level network, ED 1 transfers 130 non-emergency patients to two hospitals, the first of which, to where the majority (129) of patients are transferred, is the hospital in which the ED is located. The weights $W_{cross1,j}$ can hence be calculated, again summing to 1 across the row.

We adapt the MMMC model approach of Tranmer et al (2014) to a multilevel network, as defined in Equation (6):

$$y_i = \beta_0 + \mathbf{x}'_i \beta + \sum_{j \in m1net(i)} w_{m1neti,j} u_j^{(1net)} + \sum_{j \in m2net(i)} w_{m2neti,j} u_j^{(2net)} + \sum_{j \in mCross(i)} w_{Crossi,j} u_j^{(Cross)} + e_i$$

$$i = 1,...,n \quad m1net(i) \subset (1,...,J_{1net}), \quad m2net(i) \subset (1,...,J_{2net}), \quad mCross(i) \subset (1,...,J_{Cross})$$

$$u_j^{(1net)} \sim N(0, \sigma^2_{u^{(1net)}}), \quad u_j^{(2net)} \sim N(0, \sigma^2_{u^{(2net)}}), \quad u_j^{(Cross)} \sim N(0, \sigma^2_{u^{(Cross)}}), \quad e_i \sim N(0, \sigma^2_e)$$
$$\mathrm{Cov}(u_j^{(1net)}, u_j^{(2net)}) = \mathrm{Cov}(u_j^{(1net)}, u_j^{(Cross)}) = \mathrm{Cov}(u_j^{(2net)}, u_j^{(Cross)}) = 0 \qquad (6)$$

The fixed part of the model is given by $y_i = \beta_0 + \mathbf{x}'_i \beta$, where y_i is a dependent variable for each level 1 node, *i*, β_0 is a constant term and $\mathbf{x}'_i \beta$ is a set of explanatory variables and their respective coefficients. The three terms, $\sum_{j \in m1net(i)} w_{m1neti,j} u_j^{(1net)}$, $\sum_{j \in m2net(i)} w_{m2neti,j} u_j^{(2net)}$ and $\sum_{j \in mCross(i)} w_{Crossi,j} u_j^{(Cross)}$ are sums of the random effects for the level 1 network, $u_j^{(1net)}$, level 2 network, $u_j^{(2net)}$, and cross-level network, $u_j^{(Cross)}$, each multiplied by the appropriate weights: $w_{m1neti,j}$, $w_{m2neti,j}$ and $w_{Crossi,j}$, respectively. The terms $m1net(i)$, $m2net(i)$ and $mCross(i)$ indicate the alters, *j,* of ego *i* in the level 1 network, level 2 network and cross-level network, respectively. The maximum number of network subgroups (e.g. ego-nets) in these three components of the multilevel network is J_{1net} , J_{2net}, and J_{Cross} , respectively. Here, the weights are based on ego-net membership, but could be defined for other network substructures such as cliques. The weights could also be row standardised on a different function to the inverse of total patient transfers, such as the inverse square root.

The random effects, $u_j^{(1net)}, u_j^{(2net)}$ and $u_j^{(Cross)}$ and e_i, are assumed to be uncorrelated with one another, and each is assumed to be normally distributed with mean zero, and variances $\sigma^2_{u^{(1net)}}$, $\sigma^2_{u^{(2net)}}$, $\sigma^2_{u^{(Cross)}}$, and σ^2_e, respectively. The total variation in y_i, therefore has four components: level 1 network, the level 2 network, cross-level network, and the individual level. The estimated parameters from this model allow us to estimate the extent and relative share of variation in y_i across the multilevel network with respect to these four components.

In an MMMC model, the variances of random effects $u_j^{(1net)}, u_j^{(2net)}$ and $u_j^{(Cross)}$, with associated weights $w_{m1neti,j}$, $w_{m2neti,j}$ and $w_{Crossi,j}$, respectively, should be scaled to reflect ego-net membership, as Tranmer et al (2014) showed. To estimate a typical share of variation in the response, y_i , for the different network components, we can use the average group membership weights for each set of multilevel network components. This can be achieved by multiplying each variance estimate $\hat{\sigma}^2_{u^{(1net)}}, \hat{\sigma}^2_{u^{(2net)}}$ and $\hat{\sigma}^2_{u^{(Cross)}}$ by the average of the non-zero elements of its associated group membership weight matrix. With these scaled estimates, it is then possible to estimate the percentage variation in y_i for each of the four components. It is typical in multilevel modelling to first obtain these estimates with a null model without explanatory variables, to measure the overall extent of variation for each component, before adding explanatory variables to the full model. Usually, the covariates will explain some of the variation in the response variable, but the sizes of variance components from the full model will also indicate where most of the unexplained variation remains. Again, we can estimate the relative share of this in the four estimated variance components, conditional on the inclusion of covariates in the model.

In MMMC models it is often appropriate to define explanatory variables with respect to the alters (peers) of each focal (ego) node: for example, the average size of alter hospitals to which the ego hospital transfers patients. As for dependent variable in Equation (4) above, we can calculate peer explanatory variables, by multiplying each of these by its appropriate ego-net membership weights.

In the next section, we illustrate an application of the MMMC model to Italian health data.

5. RESEARCH DESIGN

5.1 Empirical Setting and Data

We collected data on a community of hospital organizations in Lazio. Similarly to other Italian regions, Lazio is partitioned into Local Health Units (LHUs), designed to ensure availability of, and access to, homogenous service throughout the region. 110 hospitals provide care services for the region, 56% of which are publicly owned. Fifty-seven EDs, located within hospitals, provide emergency care services in the region. Not all hospitals have EDs; for those that do, there is exactly one ED per hospital.

Patient transfer is one of the main forms of inter-hospital collaboration (Lomi and Pallotti, 2012). The specialized health care literature on interorganizational networks has long recognized the relevance of patient transfer relations for inter-hospital collaboration both in emergency and non-emergency settings (Lee et al., 2011; Iwashyna and Courey, 2011; Veinot et al., 2012). For the case of elective inter-hospital patient transfer, for example, Lomi et al. (2014) found that inter-hospital collaboration allows patients to access better care because patients systematically flow from lower to higher quality hospitals. In a study of inter-hospital transfer of Acute Myocardial Infarction (AMI) patients, Veinot et al (2012) found that partner selection is more likely to be based on institutional routines than on consideration of quality or performance.

We focus on three different types of relations as shown in **Figure 1.** The first type involves emergency transfers of patients between pairs of EDs in our sample. For patients admitted to an ED, highly formalized care routines are in place to determine as rapidly as possible whether these patients need to be transferred, and if so, to which ED destination. Transfers between EDs typically occur because capacity, capabilities, and expertise are unevenly distributed between hospitals. Emergency patients may be transferred because the sender ED does not have the capacity to either treat the condition itself or because of complications that might arise from treatment (Bosk et al., 2011). Emergency transfers occur within 24 hours from admission to an ED. Other important aspects related to emergency transfers include the identification of an accepting physician in the receiving ED, the timely transmission of information accompanying the patient, and the set-up of a well-functioning infrastructure to support emergency patient transfers. Data on emergency transfers were collected among all 57 EDs.

The second type of relation involves the elective transfer of patients between all the hospitals. This is a second level of action. Elective patients (also referred to as in-patients) are individuals who have already acquired the status of "admitted patient" and, therefore, who have agreed to follow treatments administered by professional medical staff who are clinically responsible and legally liable for their conditions. This is an important qualification, because elective transfers are the outcome of individual organizational decisions over which patients have surrendered control at admission. Transfers of elective patients are driven overwhelmingly by clinical and practical constraints. For example, insufficient expertise or available capacity in the sender hospital. However, a sender hospital may choose from any number of recipient hospitals for the same patient. Elective transfers follow an organizational model of coordination that is mainly based on informal arrangements and routines established between partner hospitals (Lomi et al., 2014; Veinot et al., 2012). Unlike the transfers of emergency patients, decisions to transfer hospitalized patients can take several days and are usually discussed during periodic administrative meetings.

Finally, the third type of relation shown in **Figure 1** involves patients transferred from EDs to hospitals. We call them "non-emergency transfers" because they involve patients with stabilized conditions being transferred to a "regular" hospital wards to undergo further non-emergency treatment. These cross-level transfers are meaningful and important because they signal unexpected acts of cooperation between EDs and hospitals. They are acts of cooperation because the sender ED needs to rely on and trust the receiver hospital, which needs to accept the patient. They are unexpected because – by default – patients admitted into an ED are typically retained by the hospital to which that ED belongs. For these reasons, transfers of non-emergency patients are driven mainly by lack of available capacity (i.e., staffed beds) in the sending hospital. In our sample, ED to hospital transfers generate cross-level networks. Because each ED (lower level unit) may transfer patients to multiple hospitals (higher level units), these cross-level relations define a situation consistent with the MMMC model.

The three types of patient transfer relations give rise to a multilevel network of relations between hospitals, between EDs located within hospitals, and between EDs and hospitals. As our dependent variable is a measure of ED performance, we only considered those 57 hospitals containing an ED in our analysis, and excluded the 53 hospitals that do not have an ED. Based on these 57 cases, we constructed three 57 by 57 non-symmetric valued square matrices. The first matrix contains in the rows (columns) the EDs sending (receiving) patients, and the matrix element the number of patients transferred from the row ED *i* to the column ED *j* (the level 1 network**)**. The second matrix contains in the rows (columns), the hospitals sending (receiving) patients, and in the matrix elements, the number of patients transferred from the row hospital *i* to the column hospital *j* (level 2 network). The third matrix is defined by the number of patients being transferred from EDs back to hospitals, and is the cross-level network.

5.2 Variables and measures

The dependent variable of interest in this study is a measure of ED waiting time based on the time elapsed between admission and treatment of patients (Horwitz, Green, and Bradley, 2009). In our analysis, for each ED we used the (standardised) percentage of patients who received treatment within 4 hours after their admission to the hospital ED. According to the Italian NHS standards, this is the maximum waiting time for emergency patients admitted to an ED (Agenzia di Sanita' Pubblica, 2006)[1]. ED waiting time is one of the most commonly used measures of ED effectiveness and performance (Horwitz, Green, and Bradley, 2009, Guttmann et al., 2011; Dunn, 2003). This is the case because emergency services in hospitals do not typically supply care services. Their main task is to stabilize the conditions of incoming patients so that they may be transferred as soon as possible to an appropriate unit within the hospital, or – should this not be possible - to a unit in a different hospital. Previous studies have shown that prolonged waiting time in EDs reduces the quality of care and increases the chance of adverse events for patients (Lew, Lew and Kennedy, 2003; Hoot and Aronsky, 2008; Vieth and Rodhes, 2006). Also, prolonged waiting time decreases patient satisfaction (Taylor and Benger, 2004), and increases the number of patients who leave an ED before being seen (Fernandes Price, and Christenson, 1997). For these reasons, ED waiting time has always received attention from both administrators and policy makers as it represents an important measure of effectiveness, efficiency and safety in emergency care (Horwitz, Green, and Bradley, 2009).

Details of the various control covariates included in our empirical models are given in **Table 2.**

- Insert Table 2 about here -

[1] Waiting time has been adjusted to not include any time spent in EDs due to inability to find a bed for patients requiring hospitalization.

We control for the effects of two broad categories of factors that may account for the variability in performance of EDs. The first category captures salient features of hospitals – and hence are attributes of the hospitals, the level 2 network nodes. To control for the effect of the size of hospital on the performance of EDs we use the total number of staffed beds (**Size**). We also include the average percentage of occupied beds (**Capacity**) to control for the possible effect of availability of in-patient beds into which to move emergency patients. We control for the ability of hospitals to treat, on average, more complex cases by including the Case Mix Index[2] as a measure of hospital **Complexity**. Because the ability of hospitals to accommodate emergency admissions may affect the ED waiting time, we use **Retention rate** as measured by the percentage of in-patients admissions from EDs over the total number of patients admitted to EDs. Given that the geographical location of hospitals may affect the volume of patients coming to an ED, we include the variable **Capital city**. This is a binary indicator variable taking the value of 1 if the hospital is located in Rome, the capital city, and 0 otherwise.

The second category of factors refers to specific features of EDs – i.e., to the level 1 network nodes. Because higher-volume EDs may be more crowded, thus increasing the percentage of patients in the ED with long waiting times, we include the total number of admissions (**Volume**) to control for the effect of the volume of activity. We include **Uncertainty** as measured by the proportion of red-triage admissions over the total number of admissions, which gives an indication of the medical condition of patients arriving at EDs[3]. Finally, we use the variable **ED code** to control for the complexity of the care processes performed by EDs. **ED code** is a three category variable based on a classification of EDs according to their resources (e.g., human resources, technologies) and care processes, for which we created two indicator variables. The baseline category comprises the least specialized EDs (with regard to internal staff and care processes), typically providing immediate assistance to non-urgent cases, ED code = 2 is used to

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[2] The Case Mix Index is a composite index frequently used in the specialized literature to measure the intensity of resource consumption for patients admitted to a particular hospital during a specific time frame. It measures the average severity of illness for discharged acute care inpatients. It can be used for comparing patient sets across hospitals, specialties, and departments.

[3] Triage is a way for emergency departments to prioritize patients by acuity level into categories indicating how quickly they should be seen by a health provider (National Center for Health Statistics, 2013; Schrader and Lewis, 2013). The acuity of visits is classified internationally into four categories: emergent, urgent, semi-urgent, and non-urgent. In the Italian health care system, red-triage refers to emergent cases, yellow-triage to urgent cases, green triage to semi-urgent cases, and white triage to non-urgent cases.

indicate moderately specialized EDs, typically providing assistance to semi-urgent and urgent cases. ED code = 3 is used to indicate the most specialized EDs providing assistance to emergent cases.

5.3 Empirical model specification

We fitted a series of MMMC models to these data, and single-level models for baseline comparison. The results appear in Section 6. In every case, the dependent variable was the percentage of ED waiting time < 4 hours, standardized to have a mean of zero and a standard deviation of 1. Because this dependent variable has an approximately Normal distribution for the 57 EDs, we fitted linear MMMC models. Some models include explanatory variables in the fixed part of the model. Rather than fitting these covariates directly as the values for each ED or hospital node, we adjusted them for peer effects by taking into account the alters of each ego-net or hospitals or ED. In each case, the explanatory variables we fitted are, therefore, ego's value of the measure minus the average values for all of their alters. These measures thus indicate the difference between ego's value of the measure and the average of this measure for alters it transfers patients to, either at the ED or hospital level as appropriate. All interval scale variables were standardised prior to adjusting them for peer effects.

5.4 Model estimation and interpretation

We began by fitting a series of null models to estimate the extent of variation in ED waiting time across the multilevel network, via the estimated variance components. We evaluated the models statistically using the Deviance Information Criterion (DIC) - the smallest value indicates the best model fit, having accounted for model complexity (Browne, 2009) - and substantively by calculating the estimated share of variation in ED waiting time for the three multilevel network components and for the individual ED level, as described in Section 4.

After adding explanatory variables, we also evaluated the model fit statistically, and calculated the relative share of remaining variation in waiting time not explained by the explanatory variables. We tested the statistical significance of the estimated regression coefficients in the fixed part of the model using approximate t-ratios. By fitting a single level model in addition to the MMMC models, and comparing the estimated regression coefficients from this model with the MMMC models, we were able to assess whether we would have reached the same conclusions in each case.

MMMC models can be fitted with specialist software for modelling, such as MLwiN (Rasbash et al, 2012). The model results presented here were all estimated via an MCMC algorithm (Browne, 2009) using default flat priors for the fixed effects and a chain of 100,000 samples, implemented in MLwiN. In all models, standard diffuse (gamma) priors were assumed for the variance parameters.

6. RESULTS

6.1 Orienting question

We examine the components of variation in individual ED performance associated with: (i) the patient transfer network between EDs (emergency transfers); (ii) the patient transfer network between hospitals in which EDs are contained (elective transfers), and with (iii) the cross-level patient transfer network between EDs and hospitals (non-emergency transfers). Our orienting question is thus: which one of these three networks is associated with the largest relative share of variation in ED performance? This is the question around which we organize the empirical analysis.

Our main orienting question is theoretically valuable because extant research argues that patient transfer involves a collaborative relation that is highly conductive of mutual learning between partners – and even competing hospitals (Lomi and Pallotti, 2012). In organizational research, mutual learning is typically associated with diffusion of practices, routines and experiences (Levitt and March, 1998) – which in turn then leads to correlation or co-movements in organizational behaviour and performance. The analytical objective of the models we estimate is to identify the source of variation in ED performance across the multiple network levels in which these organizational units are embedded.

Our orienting question is not only theoretically valuable, but also empirically relevant. If a significant component of variation in ED performance can be associated with network relations among EDs, then the data would provide some indication that processes of organizational learning happen at the sub-organizational level – i.e., at the level of organizational units (Ingram, 2002). Conversely, if variation in ED performance is explained by (higher level) relations among hospitals, then we might conclude that the network ties between ED units do not produce autonomous learning effects. In this case, the appropriate level at which performance differentials between sub-units should be understood would be the more aggregate organizational level. A similar reasoning holds for cross-level relations. In this case, the main cause of differences in sub-unit performance would be the ability of ED sub-units to collaborate across the boundaries of the hospitals within which they are located. Finally, if variation in ED performance is explained mainly by differences in individual attributes rather than the networks, then we would conclude that network-based processes play only a limited role in supporting learning in this organizational community. The MMMC model is uniquely suited to adjudicate between these various sources of variation in organizational sub-unit performance.

6.2 Multilevel network structure

The smallest number of EDs to which any focal ED transferred patients to was 2, with a median of 24 and a maximum of 41. The smallest non-zero number of patients transferred between any two EDs was 1, the largest 1688. The smallest number of hospitals to which any of the 57 hospitals transferred patients to was 1, the maximum 36, with a median of 23. The smallest number of patients transferred between any two hospitals was 1, and the largest 236. For the cross-level networks, the smallest number of hospitals from which patients were transferred from EDs to hospitals was 2, the maximum 35, and the median 15. For transferred patients, the minimum was 14, with a maximum of 11,000. As expected, for these cross-level transfers, the majority of patients were transferred from the ED of a hospital to a non-emergency ward in the same hospital as the ED.

6.3 Null and Peer-Effect MMMC Models.

We fitted a series of null models to estimate the overall variation in ED waiting time for the four components of the population. We additionally fitted a peer-effect model, in which the average value of the dependent variables for alters (peers) in the ED ego-net is included as a fixed covariate, to assess whether the average value of the alter ED's standardised waiting times is a good predictor of the ego ED standardised waiting time. The results are shown in **Table 3.**

---- Insert Table 3 about here ----

In Table 3, M1 is a single level null model used as a baseline for comparison with the more realistically complex models. M2 includes a single-level network structure, by including random effects for the single level network of ED inter-connections. M3 includes random effects for single-level network hospital (H) inter-connections. The DIC drops for these models to values of around 160.5 compared with M1 with a DIC of about 164.7, indicating the better goodness of fit for the single-level network models compared with the single-level model, M1. M4 includes both the level 1 and level 2 networks, and is thus a model for multilevel network dependencies. The DIC of M4 of 159.3 does not reduce compared with the single level network models, but it does allow us to get an estimate of the share of variation of ED waiting times at the ED network and hospital network levels. Model M5 includes level 1, level 2 and cross-level network components, and the goodness of fit is improved (DIC=144.9), compared with the models that do not include a cross-level network component. This suggests that the cross-level network is associated with variations in waiting times, and allows us to estimate the share of variation in waiting times between the ED networks, hospital networks, and cross-level networks.

The peer-effects model, M6, includes a fixed effect for alters' average value of ED waiting time; that is, the average of ego's peers, in terms of ED patient transfer. Its estimated coefficient of 0.181 is not statistically significant, and the DIC is slightly worse than the model that does not include this effect 160.6 (compared with M4 of 159.3). This suggests that knowledge of waiting times for the alter EDs is not on its own a good predictor of ego ED's waiting time. We also tried peer-effect variables for ED waiting time based on hospital and cross-level ED to hospital connections, and obtained broadly the same results as those for M6.

The results in **Table 3** suggest that the model that includes all four components in the multilevel network (M5) has the best fit, statistically. This is a rather complex model for 57 observations but we note that alongside these 57 rows of data, the three differing 57 x 57 matrices of weights are also used in the identification of the model parameters; QAP correlations indicate that these three matrices are not highly correlated with one another.

We now consider the substantive interpretation of the null model results. **Table 4** shows the mean and median non-zero weights for the ED inter connections, hospital inter-connections and ED to hospital cross-connections.

---- Insert Table 4 about here ----

The results in **Table 5** first give the total estimated residual variance. Where networks are included in the model components, the mean weights are used to give typical values of the total variance. Based on these totals we can estimate the share of variation in waiting time for the different classifications in each model. The results suggest that of the three networks, hospital networks have the largest relative share of variation in waiting time above the individual level (4-6%), but that all three levels above the individual level have a non-zero share of the variation in waiting times (ED network M2: 2-4%; cross-level network M3: 1%).

---- Insert Table 5 about here ---

6.4 Full Models

We now add explanatory variables for the two nodes to the null models, as described in Section 5. Again, we fitted a single level model (M7) [an OLS regression], a model with random effects for the multilevel network of ED and hospital inter-connections (M8), and the model that also includes random effects for ED to hospital cross-connections (M9). Finally, we fitted a model that includes a fixed effect for alters' average value of waiting time (a peer-effect), as described for M6 above, to the model that includes ED and hospital network levels. The results are shown in **Table 6**.

---- Insert Table 6 about here ----

We begin by focusing on the coefficients in the fixed part of the model, and their respective standard errors. The covariates included in the full models are not highly correlated with one-another. Of the covariates included, we found that in M7, M8, and M9 in **Table 6,** capacity (a hospital measure) and uncertainty (an ED measure) were the only statistically significant covariates, both having a negative association with ED waiting time. In this example we would have come to the same overall conclusion about the association of the covariates with waiting time in terms of statistical significance in all three models, but we would not be able to investigate the nature of any multilevel network variation in waiting time unexplained by these variables using the single level model. Finally, for M10 we also include a fixed peer-effect for alters' average value of waiting time (mean Y of ED alters) to the model with a hospital and ED network. We see that this coefficient is statistically significant, but the DIC of 131.9 is slightly higher than the corresponding model that does not include this fixed effect (DIC of M8 = 129.6).

---- Insert Table 7 about here ----

In Table 7, as expected, the total residual variation in waiting time is reduced by the inclusion of covariates when compared with the total variation for the corresponding null models. The relative share of this residual variation for the different network classifications used in the models is estimated in **Table 7.** The remaining variation is predominantly at the individual level for these MMMC models. For the relative share at all other levels above the individual, hospital networks are most strongly associated with variations in waiting time (M8: 13%, M9: 20%, M10 4%). ED networks have a smaller share (M8: 1%, M9: 3%, M10: 2%), as do cross-level networks (M10: 3%). This is consistent both with previous literature on inter-hospital patient transfer relations (Lomi et al., 2014), and with our prior qualitative understanding of the nature and meaning of the different kinds of network ties that we have observed. Patient transfer relations at the hospital level signal the presence of deliberate co-ordination arrangements.

Inter-connections between EDs are more closely regulated in the context of emergency care networks. Finally, cross-level networks (EDs to Hospitals) are only occasional connections to hospitals other than the same one in which the ED is located, and probably determined by contingent factors related to the availability of capacity in the sender hospital. It is interesting that the analysis revealed these qualitative features of the multilevel network that we have observed. Consistently with the results for the null models, we see that the model including the multilevel network inter-connections and the cross-connections (M9) is statistically the best model in **Table 6** in terms of goodness of fit, as indicated by the values of the DIC (46.165). The large drop in this value compared with models that do not include a cross-level network component suggests we should treat this result with some caution given that the estimated percentage of cross-level network variation is small. However, inclusion of multilevel network components either with or without the cross-level network included improves the model fit compared with the single level model.

7. **CONCLUSIONS**

We have presented an MMMC model for the analysis of multilevel network dependencies. We have illustrated the empirical value of the model with an analysis of variations in ED waiting times in the context of multilevel patient transfer networks between hospitals (Iwashyna et al., 2009).

As summarized in Tables 5 and 7, we found that most of the variation in waiting time was between individual EDs, but a reasonable share of this variation was between networks of hospitals that transferred patients to one another. This conclusion seems to suggest that the hospitals in which EDs are located actually matter – despite the apparent autonomy of ED activities from the general hospital activities - because variations in the performance of EDs depend, at least in part, on hospital-level effects. This result is consistent with the findings of extant research in health care management. Horwitz et al. (2009), for instance, have provided evidence that hospital-level effects, such as inpatient occupancy rate, are important determinants of variability in ED performance, as measured by waiting time and visiting time. Similarly, Dunn (2003) showed that modest decreases in hospital occupancy rates resulted in highly significant reductions in ED waiting times, thus providing evidence for the importance of hospital-level characteristics on ED performance. Finally, in multiple hospital settings, Lucas et al. (2009) found a strong association between hospital census variables and ED length of stay. These results are also important in a more general sense because they suggest that the performance of individual organizational sub-units can only be assessed with reference to the overall composition of activities that organizations hold.

We also found that variation in waiting time between networks of EDs that transferred patients to one-another was relatively small – a result that may be taken as evidence of performance spill-over effects. One way to frame this result is in terms of assimilation of clinical practices – or interorganizational learning facilitated by network ties. This result is consistent with extensive research showing that connected organizations are more likely to become similar by adopting similar practices and behavioural orientations (DiMaggio and Powell, 1983; Strang and Soule, 1998). In the specific case we have examined this isomorphic tendency is sustained by the extensive exchange of clinical information that inter-hospital patient transfer relations entail.

Once the MMMC models are estimated, the shrunken residuals (Snijders and Bosker 2012) from levels above the individual could be used as a basis for comparing hospitals or EDs in a fair way in terms of waiting time using a realistically complex method. Examples of using multilevel analysis to make fair comparisons of schools can be found in the literature, for example, Leckie and Goldstein (2009). Similar approaches to those used in Education could be made using MMMC models for the health data.

Extensions to the MMMC models presented and applied here are clearly possible. Firstly, we could add random coefficients to the explanatory variables in the models. Substantively, this would mean that, for example, ED covariates could have different associations with waiting time in different ego-nets of EDs; similarly for hospitals. For example, the association of ED waiting time with hospital capacity could be stronger in some networks of hospitals than in others. Secondly, where data are collected over time, it would be possible to add time as a level in the MMMC model to assess the stability of multilevel network variations. Singer and Willett (2003), and Steele (2008) give a range of examples of multilevel models for longitudinal data analysis. Thirdly, the MMMC model may be used for multilevel networks with more than two levels, if such data are available. The extent to which it is possible to identify parameter estimates in these complex models will depend on the quality of the available data, and is the subject of on-going research. More empirical analysis is needed in this area, using a variety of multilevel network datasets.

The choice of weights as well as the choice of relations between nodes for the multilevel network will affect the results of the MMMC model analysis, as well as allowing different substantive theories to be tested. The weights we used in the analyses presented were based on the inverse of the total number of patients transferred in each ego-net. Alternative weighting schemes could be used, such as weights proportional to the inverse square root of the total, to take into account the variation in the numbers of patients transferred. We tried this alternative for the null models, and in comparison with the original analyses found that hospital networks still had the biggest share of network variation, and that the estimated proportion of variation in ED waiting time due to individual EDs reduced slightly. The weights could also be based on different relations to patient transfers, such as the geographical distance between pairs of hospitals (and hence pairs of EDs), or the Jaccard distance, which summarises the similarity, or difference, of each pair of hospitals in terms of whether they are, say, speciality eye hospitals, or general hospitals.

When modelling variations in waiting times for the hospital networks, we could use the model described in Equation (1) with three different relations on a single level of network nodes, rather than having to choose one relation. This enables the MMMC model to be used for a multiplex analysis; for example, to see which of the three relations is most strongly associated with variation in waiting times in networks of hospitals. However, a multiplex network is not a multilevel network, but is instead several different relations for a single level of network nodes. Moreover, a multilevel analysis of network data does not automatically make that network multilevel. For example, de Miguel Luken and Tranmer (2010) investigated single-level ego-networks using a multilevel model.

More research is needed on the association of network structure with network dependence, both for single level networks and for multilevel networks. Firstly, to assess the performance of the MMMC model for networks of differing density: from fairly sparse to very dense. Secondly, for a given network density, certain patterns of network variation may tend to be most often associated with certain network substructures, suggesting that the MERGM and MMMC model approaches might complement one another for multilevel network analysis.

References

Agenzia di Sanita' Pubblica della Regione Lazio, 2006. Descrizione dell'offerta sanitaria degli istituti di ricovero e cura per acuti nel Lazio. Available at www.asplazio.it/home/eventi/rapp_attivita_ospedaliera_2011/convegno_18dic2012.php.

Aguinis, H., Boyd, B.K., Pierce, C.A., Short, J.C., 2011. Walking new avenues in management research methods and theories: Bridging micro and macro domains. Journal of Management 37, 395-403.

Baker, W.E., Faulkner, R.F., 2002. Interorganizational networks. In: Baum J.A.C. (Ed.), The Blackwell Companion to Organizations. Oxford: Blackwell Publishers Ltd.

Bivand, R., Bernat, A., Carvalho, M., Chun, Y., Dormann, C., Dray, S., Sparse, M.S., 2005. The spdep package. Comprehensive R Archive Network, Version 0.3-13.

Boorman, S.A., White, H.C., 1976. Social structure from multiple networks: II. Role structures. American Journal of Sociology 81, 1384-1446.

Borgatti, S.P., Foster, P.C., 2003. The network paradigm in organizational research: A review and typology. Journal of Management 29, 991-1013.

Bosk, E.A., Veinot, T., Iwashyna, T.J., 2011. Which patients, and where: A qualitative study of patient transfers from community hospitals. Medical Care 49, 592-598.

Brass, D.J., Galaskiewicz, J., Greve, H.R., Tsai, W., 2004. Taking stock of networks and organizations: A multilevel perspective. Academy of Management Journal 47, 795-817.

Breiger, R., Pattison, P., 1978. The joint role structure of two communities' elites. Sociological Methods and Research 7, 213-226.

Browne, W.J., 2009. MCMC Estimation in MLwiN v2.1. Centre for Multilevel Modelling, University of Bristol.

Butts, C.T., 2008. Social network analysis with SNA. Journal of Statistical Software 24, 1-51.

Contractor, N., Wasserman, S., and Faust, K., 2006. Testing multitheoretical, multilevel hypotheses about organizational networks. Academy of Management Review 31, 681-703.

de Miguel Luken, V., Tranmer, M., 2010. Personal support networks of immigrants to Spain: A multilevel analysis. Social Networks 32, 253-262.

DiMaggio, P., 1986. Structural analysis of organizational fields: A blockmodeling approach. Research in Organizational Behavior 8, 335–370.

DiMaggio, P.J., Powell. W.W., 1983. The iron cage revisited: Institutional isomorphism and collective rationality in organizational fields. American Sociological Review 48, 147–160.

Doreian, P., 1980. Linear models with spatially distributed data, spatial disturbances or spatial effects?. Sociological Methods & Research 9, 29-60.

Dunn, R., 2003. Reduced access block causes shorter emergency department waiting times: An historical control observational study. Emergency Medicine 15, 232-238.

Fernandes C.M., Price A., Christenson, J.M., 1997. Does reduced length of stay decrease the number of emergency department patients who leave without seeing a physician?. Journal of Emergency Medicine 15, 397-399.

Guttmann,A., Schull M.J., Vermeulen, M.J., Stukel, T.A., 2011. Association between waiting times and short term mortality and hospital admission after departure from emergency department: population based cohort study from Ontario, Canada. *British Medical Journal* 342:d2983.

Hoot, N.R., Aronsky, D., 2008. Systematic review of emergency department crowding: Causes, effects, and solutions. Annals of Emergency Medicine 52, 126-136.

Horwitz, L.I., Green, J., Bradley, E.H., 2009. US emergency department performance on wait times and length of visit. Annals of Emergency Medicine 55, 133-141.

Ingram, P. 2002. Interorganizational Learning. In: Baum, J.A.C. (Ed), The Blackwell Companion to Organizations. Blackwell, Malden, MA, pp. 642–663.

Iwashyna, T.J., Courey, A.J., 2011. Guided transfer of critically ill patients: Where patients are transferred can be an informed choice. Current Opinion in Critical Care 17, 641-647.

Iwashyna, T.J., Christie, J.D., Moody, J., Kahn, J.M., Asch, D.A., 2009. The structure of critical care transfer networks. Medical Care 47, 787-793.

Kilduff, M., Tsai, W., 2003. Social networks and organizations. Sage.

Lambe, S., Washington, D.L., Fink, A., Laouri, M., Liu, H., Fosse, J.S., Brook. R.H., Asch, S.M., 2003. Waiting times in California's emergency departments. Annals of Emergency Medicine 41, 35–44.

Laumann, E.O., Galaskiewicz, J., Marsden, P.V., 1978. Community structure as interorganizational linkages. Annual Review of Sociology 4, 455–484.

Lazega, E., Jourda, M.T., Mounier, L., Stofer, R., 2008. Catching up with big fish in the big pond? Multi-level network analysis through linked design. Social Networks 30, 159-176.

Leckie, G., Goldstein, H., 2009. The limitations of using school league tables to inform school choice. Journal of the Royal Statistical Society: Series A (Statistics in Society) 172, 835-851.

Lee, B.Y., McGlone, S.M., Song, Y., Avery, T.R., Eubank, S., Chang, C.-C., Bailey, R.R., Wagener, D.K., Burke, D.S., Platt, R., Huang, S.S., 2011. Social network analysis of patient sharing among hospitals in Orange County, California. American Journal of Public Health 4, 707-713.

Leenders, R.T.A., 2002. Modeling social influence through network autocorrelation: constructing the weight matrix. Social Networks 24, 21-47.

Levitt, B., March, J.G., 1998. Organizational learning. Annual Review of Sociology 14, 319-40.

Liew, D., Liew, D., Kennedy, M.P., 2003. Emergency department length of stay independently predicts excess inpatient length of stay. Medical Journal of Australia 179, 524-526.

Lomi, A., and Pallotti, F., 2012. Relational collaboration among spatial multipoint competitors. Social Networks 34, 101-111.

Lomi, A., Mascia, D., Vu, D., Pallotti, F., Conaldi, G., Iwashyna, T.J., 2014. Quality of care and interhospital collaboration: A study of patient transfers in Italy. Medical Care 52, 407-414.

Lucas, R., Farley, H., Twanmoh, J., Urumov, A., Olsen, N., Evans, B., Kabiri, H., 2009. Emergency department patient flow: The influence of hospital census variables on emergency department length of stay. Academy of Emergency Medicine 16, 597-602.

Lusher, D., Koskinen, J., Robins, G. (Eds). 2013. Exponential Random Graph Models for Social Networks. Theory, Methods, and Applications. NY: Cambridge University Press.

Mathieu, J.E., Chen, G., 2011. The etiology of the multilevel paradigm in management research. Journal of Management 37, 610-641.

Moliterno, T.P., Mahony, D.M., 2011. Network theory of organization: A multilevel approach. Journal of Management 37, 443-467.

National Center for Health Statistics, 2013. Health, United States, 2012: With Special Feature on Emergency Care. Hyattsville, MD.

Oh, H., Labianca, G., Chung, M.-H., 2006. A multilevel model of group social capital. Academy of Management Review 31, 569-582.

Ord, K., 1975. Estimation methods for models of spatial interaction. Journal of the American Statistical Association 70, 120-126.

R Core Team, 2013. R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. URL http://www.R-project.org/.

Rasbash, J., Steele, F., Browne, W.J. and Goldstein, H., 2012. A User's Guide to MLwiN, v2.26. Centre for Multilevel Modelling, University of Bristol.

Rousseau, D.M., 1985. Issues of level in organizational research: Multi-level and cross-level perspectives. In: Cummings, L.L., Staw B.M. (Eds.), Research in organizational behavior. Greenwich, CT: JAI Press, 7, 1-37.

Rousseau, D.M., 2011. Reinforcing the micro/macro bridge: Organizational thinking and pluralistic vehicles. Journal of Management 37, 429-442.

Schrader, C.D., Lewis, L.M., 2013. Racial disparity in emergency department triage. Journal of Emergency Medicine 44, 511–518.

Simon, H.A., 1996. The sciences of the artificial. Cambridge, Massachusetts: MIT Press.

Singer, J.D., Willett, J.B., 2003. Applied longitudinal data analysis: Modeling change and event occurrence. Oxford University Press.

Snijders, T.A., Lomi, A., Torló, V.J., 2013. A model for the multiplex dynamics of two-mode and one-mode networks, with an application to employment preference, friendship, and advice. Social Networks 35, 265-276.

Snijders, T.A.B., 2011. Statistical models for social networks. Annual Review of Sociology 37, 131-153.

Snijders, T.A.B., Bosker, R.J., 2012. Multilevel analysis: An introduction to basic and advanced multilevel modeling. Sage Publications. Thousand Oaks, CA.

Steele, F., 2008. Multilevel models for longitudinal data. Journal of the Royal Statistical Society, Series A (Statistics in Society) 171, 5-19.

Strang, D., Soule, S.A., 1998. Diffusion in organizations and social movements: From hybrid Corn to poison pills. Annual Review of Sociology 24, 265–290.

Taylor, C., Benger, J.R., 2004. Patient satisfaction in emergency medicine. Emergency Medicine Journal 21, 528-532.

Tranmer, M., Steel, D., Browne, W., 2014. Multiple Membership Multiple Classification Models for Social Network and Group Dependencies. Journal of the Royal Statistical Society, Series (A), 177, Part 2: 1-17.

Veinot, T.C., Bosk, E.A., Unnikrishnan, K.P., Iwashyna, T.J., 2012. Revenue, relationships and routines: The social organization of acute myocardial infarction patient transfers in the United States. Social Science & Medicine 75, 1800-1810.

Vieth, T.L., Rhodes, K.V., 2006. The effect of crowding on access and quality in an academic ED. American Journal of Emergency Medicine 24, 787-794.

Wang, P., Robins, G., Pattison, P., & Lazega, E., 2013,. Exponential random graph models for multilevel networks. *Social Networks*, *35*(1), 96-115.

White, H.C., Boorman, S.A., Breiger, R., 1976. Social structure from multiple networks: I. Blockmodels of roles and positions. American Journal of Sociology 81, 730-780.

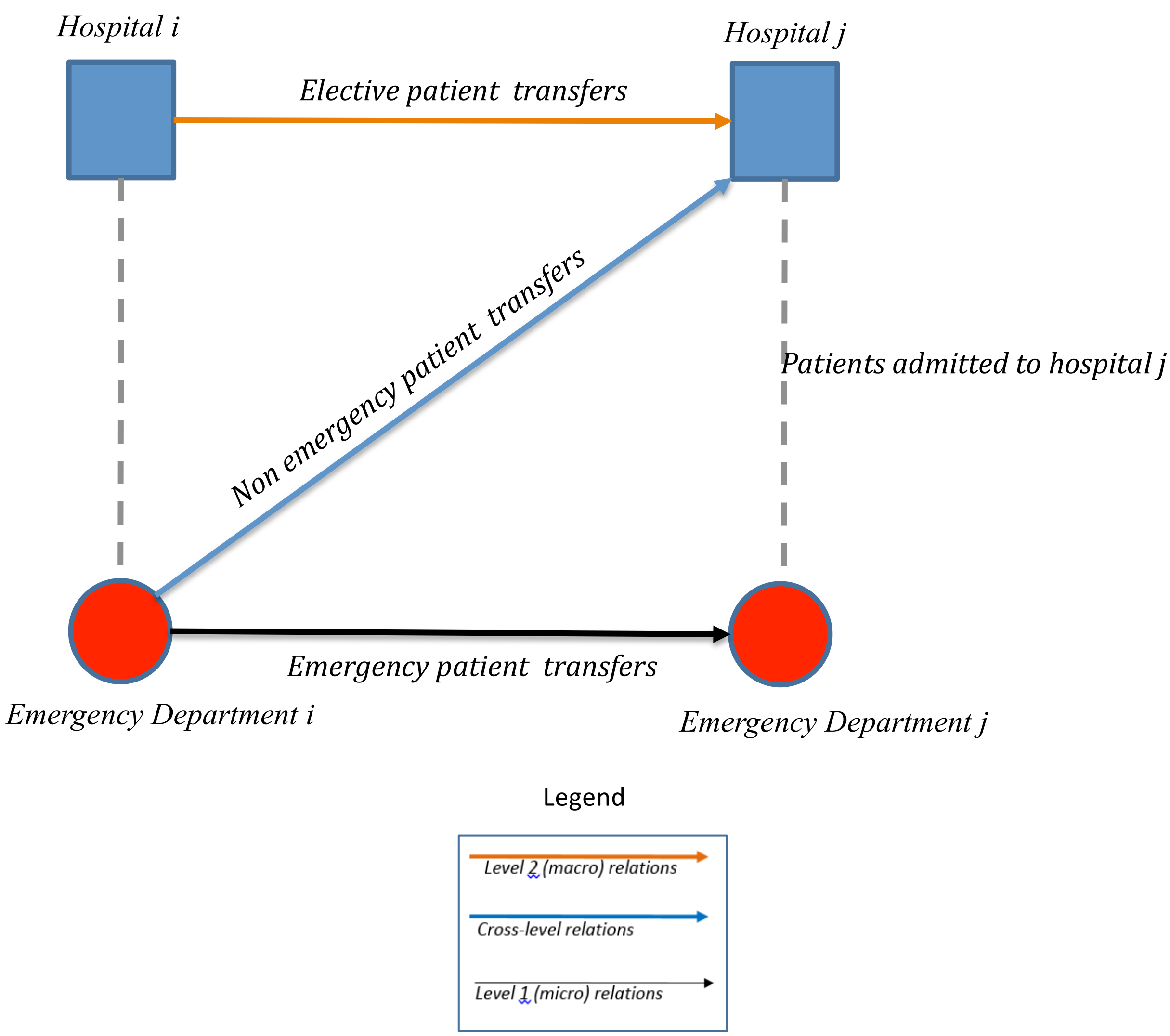

Figure 1. The multilevel network of hospitals and Emergency Departments (EDs)

Table 1: example row of patient transfer data and subsequent weights.

	ALTERS							
EGO: ED 1	**1ST ED**	**2nd ED**	**3rd ED**	**4th ED**	**5th ED**	**6th ED**	**7th ED**	**ROW TOTAL**
n_{11j}	5	1	8	1	51	6	1	73
$W_{m1net1,j}$	.07	.01	.11	.01	.70	.08	.01	1
EGO: H1	**1st H**	**2nd H**	**3rd H**	**4th H**	**5th H**	**6th H**		
n_{21j}	1	2	4	3	8	6		24
$W_{m3net1,j}$	.04	.08	.17	.13	.33	.25		1
EGO: cross (X)	**1st X**	**2nd X**						
n_{c1j}	129	1						130
$W_{Cross1,j}$	.99	.01						1

Table 2. Summary table for node level covariates.

Variable Name	Definition	Measure	Network Level	Included to control for:
Hospital Network:				
SIZE	Size – mean size of all alter hospitals.	Number of staffed beds	2	Effect of ego's organizational size relative to alters
CAPACITY	Capacity – mean capacity of all alter hospitals	Average percentage of beds occupied	2	Effect of ego's capacity constraint relative to alters
COMPLEXITY	Complexity – mean complexity of all alter hospitals	Case mix index	2	Effect of ego's complexity of cases Relative to alters
CAPITAL CITY	Capital City of ego – prop alter hospitals in Rome	1 = hospital in Rome; otherwise 0	2	Effect of ego's geographical location relative to alters
ED Network:				
VOLUME	Volume – mean alter ED volume	Total number of admissions	1	Effect of ego's volume of activity relative to alters.
UNCERTAINTY	Uncertainty - mean alter ED uncertainty	Proportion of red triage	1	Effect of ego's complexity of cases relative to alters
RETENTION RATE	Retention rate – mean ED	Percentage of in-patient admissions from emergency rooms	1	Effect of ego's capacity of EDs to retain emergency patients relative to alters.
ED CODES 2 & 3	ED code	1 = PS; 2 = ED I; 3= ED II	1	Effect of ego's complexity of activity relative to alters.
MEAN Y ED CONNECTIONS	Mean of dependent variable for all connected EDs	Mean of dependent variable for connected EDs	1	Does mean of alters' waiting time predict value for ego?

Table 3. Null and Peer-Effects MMMC results. Response variable is ED waiting time (Standardised).

Models:	**M1: null Individual only**	**M2: null ED**	**M3: null H**	**M4: null ED+H**	**M5: null ED+H+Cross**	**M6 Peer-Effects ED+H**
Coefficient:	$\hat{\beta}$	$\hat{\beta}$	$\hat{\beta}$	$\hat{\beta}$	$\hat{\beta}$	$\hat{\beta}$
CONSTANT	.00	-.132	-.09	-.138	-.15	-.08
Mean Y of ED alters						.181 (.478)
Variances:						
Level: Cross						
$\hat{\sigma}2$					.117	
Level: H						
$\hat{\sigma}2$			1.231	.886	.830	.833
Level: ED						
$\hat{\sigma}2$		.731		.482	.574	.496
Level: individual						
$\widehat{\sigma 2}$	1.037	.874	.875	.822	.723	.910
DIC:	164.74	160.57	160.54	159.25	**146.89**	**160.58**

Table 4. Summary statistics for patient transfer weights.

Patient transfer weights:	**Median**	**Mean**
ED	.0138	.0447
Hospital	.0206	.0488
Cross level	.0006	.0644

Table 5. Null and Peer-Effects Models: Estimated percentage variation at each level, based on mean patient transfer weights.

Model:	Tot. resid. Variance*	Individual	ED network	Hospital network	Cross-level network
M1: Single	1.037	100			
M2: ED	.907	96	4		
M3: H	.935	94		6	
M4: ED + H	.887	93	2	5	
M5: ED + H + cross	.800	91	3	5	1
M6: ED + H + peer	.972	94	2	4	

*Based on mean transfer weights for all models except M1, which does not include network levels.

Table 6. Full MMMCM results. Response variable is ED waiting time (Standardised). Figures in Bold are statistically significant at the 5% level.

Full Models	M7: Individual only		M8: ED+H		M9: ED+H+Cross		M10: ED + H	
Coefficients:	$\hat{\beta}$	s.e.	$\hat{\beta}$	s.e.	$\hat{\beta}$	s.e.	$\hat{\beta}$	s.e.
CONSTANT	-.426		-.568		-0.588		-.145	
Hospital NETWORK VARIABLES								
SIZE	-.109	.151	-.155	.149	-.152	.150	-.087	.143
COMPLEXITY	.116	.156	.08	.147	.086	.147	.170	.147
CAPACITY*	**-.400**	**.137**	**-.382**	**.126**	**-.399**	**.126**	**-.393**	**.124**
CAPITAL CITY	-.564	.388	-.693	.395	-.691	.403	-.335	.401
ED NETWORK VARIABLES								
VOLUME	.074	.162	.127	.157	.121	.160	.043	.156
UNCERTAINTY*	**-.243**	**.118**	**-.331**	**.110**	**-.332**	**.111**	**-.406**	**.115**
RETENTION RATE	.075	.115	-.028	.114	-.051	.112	-.044	.110
ED code=2	-.495	.286	-.408	.274	-.383	.282	-.304	.269
ED code=3	-.448	.432	-.284	.410	-.269	.408	-.393	.396
Mean Y of ED alters							**.950**	**.412**
Variances:								
Level: Cross								
$\hat{\sigma}2$						.177		
Level: H								
$\hat{\sigma}2$				1.359		1.406		.462
Level: ED								
$\hat{\sigma}2$				.21		.20		.191
Level: individual								
$\hat{\sigma}2$		.624		.416		.260		.462
DIC:		144.38		129.62		**46.165**		131.91

Table 7. Full Models: Estimated percentage residual variation at each level, based on mean patient transfer weights.

Model:	Tot. resid. Variance*	Individual	ED network	Hospital network	Cross-level network
M7: Single	.624	100			
M8: ED + H	.491	86	1	13	
M9: ED + H + cross	.349	74	3	20	3
M10: ED + H + Peer	.493	94	2	4	

*Based on mean transfer weights for all models except M7 which does not include network levels.