THE EFFECTS OF BIG BANG ON THE GILT-EDGED MARKET:

Term Structure Movements and Market Efficiency

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ABSTRACT

This study is concerned with the impact of the 1986 Stock Market deregulation, or Big Bang, on the efficiency of the United Kingdom government securities market. The main theoretical finding is that the change to dual capacity dealing with negotiated commissions cannot be justified economically without the inclusion of a best execution rule for broker/dealers.

The empirical section of the study has three parts. The first part uses established and new auto-correlation techniques to test market efficiency in the traditional weak-form efficient market hypothesis paradigm. The second part tests market efficiency through an analysis of pricing residuals from fitting term structure curves. A new method to fit these curves is developed. The third section tests market efficiency by examining evidence of anomalies in the shape and movements of the term structure. From all three sources, there is strong evidence that the changes introduced by Big Bang improved efficiency in the gilt-edged market.

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CHAPTER ONE

Introduction

1.1. Objectives of the Study

The deregulation of the United Kingdom Stock Exchange in October 1986, known colloquially as "Big Bang", saw the creation of a very different market for government securities, "gilt-edged" stock. The market makers, whose number increased in excess of three hundred per cent, were to be permitted direct access to customers rather than being bound to conduct business through broking agents; and commissions on all transactions were to be negotiable rather than subject to a sliding scale of charges, with a fixed minimum level. At the same time, the state of technology permitted a transfer of the "market-place" from the floor of the Stock Exchange to the video screens of market makers' dealing rooms.

This study seeks to examine the impact of these changes on the efficiency of the market. There are three kinds of market efficiency: allocative efficiency, a measure of optimal resource allocation; operational efficiency, a measure of the cost of transacting; and informational efficiency, a measure of the extent to which prices reflect relevant information. This study is concerned primarily with examining the impact of deregulation on informational efficiency.

The weak-form of the efficient markets model, which states that prices fully and instantaneously reflect all past price information and reward rational investors for accepting risk (Fama (1970), Jensen (1978)), is usually tested in the following manner. The statistical representation of the model can be written as:

$$x_t = \mu_t + e_t$$

$$\mu_t = RF_t + RP_t \geq 0. \qquad (1.1.1)$$

$$E(e_t) = 0, \qquad E(e_t e_{t+i}) = 0 \quad (i \neq 0), \qquad \text{cov}(\mu_s, e_t) = 0 \quad (\text{ all } s, t).$$

where the return from holding the asset from day $t-1$ to day t is defined by $x_t = \log_e(P_t / P_{t-1})$, P_t being the price at the end of day t, where RF_t is the return from risk-free investments and where RP_t is a risk premium. This model, and the constant mean (random walk) model, imply that daily returns in an efficient market are uncorrelated, and autocorrelation based studies have become the dominant testing paradigm. However, such tests have, on many occasions, been shown to have low power, and there is yet to be developed a uniformly most powerful alternative.[1]

However, for the gilt-edged market (and other bond markets) it is possible to build more powerful efficiency tests using knowledge of the term structure of interest rates. In a perfect market, the relationship between the term structure of interest rates and the prices of coupon bonds is given by

$$P = \frac{C_1}{(1+R_1)} + \frac{C_2}{(1+R_2)^2} + \cdots + \frac{C_n}{(1+R_n)^n} \tag{1.1.2}$$

where, the price of the bond is P, the cash flow payments to be made at the ends of periods $1,2,...,n$ are $C_1, C_2, \ldots, C_n$, and the spot interest rates applicable to these payments are the series $R_1, R_2, R_3, \ldots, R_n$. The set of these spot rates may be regarded as the term structure of interest rates.

There are two elements of this decomposition that may be used to develop efficiency tests. Firstly, observations on the term structure can be used to create the present value of the coupon stream for a bond, and the residual between the present value and the market price can be used to discover evidence of market inefficiency.[2] The second approach comes from an examination of the term structure itself. By studying the dynamics of the term structure and those factors contributing to its shape and movements over time, it will be possible to determine whether there are anomalies in the shape of the term structure which persist over time, indicating market inefficiency, and the presence of possible arbitrage trading profits.

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1 See Summers (1983) and Chapter 5.

2 As a theoretical model, equation (1.1.2) is a no arbitrage condition (see Schaefer 1980a), and thus any residual is evidence of inefficiency. Practical considerations such as observation error, short selling constraints, tax effects and the term structure estimation procedure permit residuals to take on qualities from which efficiency may be assessed (see chapter 6).

1.2. Organisation of the Study

The fact that this study brings together many hitherto unacquainted and extensive literatures means that a literature survey in the usual sense is neither appropriate nor reasonably feasible. Instead literature surveys are included within particular chapters at a location designed to indicate how this work progresses from the current state of knowledge or existing technology, and why it is useful to do so.

Thus, the following chapter (usually reserved for a literature survey) describes the development of the gilt-edged market up to the point of deregulation, and considers the theoretical aspects of proposed changes to the structure of the gilt-edged market. Chapter three considers the impact of these changes on the operational efficiency of the gilt-edged market, both from a theoretical and empirical viewpoint, and examines how well this new market structure absorbed the shocks of October 1987.

These extensively theoretical and institutional issues balance an otherwise wholly empirical and technical thesis. Chapters 5 to 8 contain the empirical and technical studies implied by the objective outlined above: testing the impact of Big Bang on informational-efficiency. Chapter 4 is in some sense a literature review for this part of the thesis, particularly so for chapter 5, but also contains an important theoretical result. We draw on a recent reinterpretation of the traditional theories of the term structure of interest rates to provide an interpretation of the efficient markets hypothesis in terms of these traditional theories.

Tests of market efficiency using traditional weak-form autocorrelation techniques are conducted in chapter 5. The low power of the standard tests leads to the use of more powerful tests which have been recently applied to commodity futures prices.[3] Observations on the term structure are required for the construction of efficiency tests based upon the term structure, or the pricing residuals from it. In chapter 6, a new technique for measuring the term structure is

[3] See Taylor (1985) and chapter 5.

developed, and tests on pricing residuals conducted. The dynamics of the term structure are examined in chapters 7 and 8. The dynamics of key interest rates in the term structure are examined in chapter 7 and the term structure as a whole examined in chapter 8. The efficiency tests based upon the shape of the term structure are included in chapter 8. Chapter 9 provides a summary of the main results and technical developments, and contains suggestions for further work.

CHAPTER TWO

The Gilt-Edged Market and the Economics of Big Bang

2.1. Introduction

Gilt-edged securities are redeemable fixed interest securities issued by, or guaranteed by, H.M.Government, on which is paid semi-annual interest. In their simplest form, they are characterized by name, coupon (a nominal interest rate), and redemption date. For example, 10% Treasury Stock 2003, makes two equal semi-annual payments of 5% and will be repaid at face value (usually £100) at a predetermined date in the year 2003. There are currently over 100 stocks with a variety of names, with coupons ranging from 2% to 15.5% and maturities of between only a few months and approaching 30 years. Several of these stocks have a redemption period rather than a single date: the government has the option to repay the stock at any time during this period. Additionally, there are a group of stocks having index-linked payments, and another group of undated stocks. This study is concerned with those gilts that are dated and have fixed payments.

This chapter is structured as follows. Section 2.2 considers the operations of the market before Big Bang. It provides a self-contained history of the development of the market up to the point of deregulation. This section discusses such topics as the operations and identity of the market participants; the size of turnover, commissions and spreads; the relationship between gilts and monetary policy; and the problems that the market had encountered during its development and the ways in which these problems were tackled. Thus the discussion concerns issues of operational and allocative efficiency.

The next section of this chapter, section 2.3, contains a theoretical analysis of the causes of Big Bang and discusses whether there is an economic justification for the changes introduced. Popular opinion has identified the start of the events that led to deregulation with the infamous deal between the Stock Exchange chairman and the Secretary of State for Industry in

1983, in the wake of the challenge by the Director General of Fair Trading to the rule book of the Stock Exchange. However, we consider the event in the context of longer term economic events and find a rather different conclusion, based upon economic principles and technological development rather than legal expediency and compromise. Conclusions are summarized in section 2.4.

2.2. The Gilt-Edged Market Before Big Bang

Along side National Savings, Treasury bills, short term borrowing from the Bank of England, loans on overseas markets and direct loans such as those from the IMF, gilts make up the "National Debt". The first systematic issues of national debt appear to have been to raise £1*m* to finance the wars against Louis XIV of France. In fact the Bank of England was founded at the same time, 1694, as the quid pro quo to the financiers who raised the initial sum was their being granted a charter to form a bank.

One of the major factors subsequently contributing to the growth of both the debt and the gilt market has been the financing of war. By 1873, the nominal value of gilts stood at £800*m*. By 1913, it had increased to £1,000*m* while the national debt stood £650*m*. By the end of the first world war, the national debt was some £7,500*m* and the gilt market had a nominal value of £5,400*m*. During the inter-war years the market grew a further £500*m*, but the second world war had pushed up the national debt to £21,000*m* and the size of the market to £12,800*m* by 1945. Following the post war programme of Nationalisation, the Radcliffe committee (1959) reported that the gilt market now stood at £14,700*m*. In order to finance the corporate expansion programmes of these new public corporations and also local authorities, the gilt market had increased to £16,200*m* in 1970. In the seventies, for reasons which will be discussed later, gilts market management became a major tool of government monetary policy and in 1986, just prior to Big Bang, market size had reached £122,000*m*. As such, it was substantially larger than the PSBR and represented 80% of the total UK sterling debt.

Although debt grew roughly six fold in the forty years since 1945 (see figure 2.1), its growth was slower than that of GDP. Over this period the ratio of debt to GDP fell from 2.37 to 0.45. However, the prolonged period of debt financing incurred a substantial growth in debt service costs. This could have run the potentially explosive problem of issuing more debt to simply pay the interest on existing debt. Furthermore the recent sales of nationalised industries had artificially lowered the PSBR. Political response concentrated on the fact that interest payments are taxable transfers and hence some of the outflow was retrieved by the government. The psychology of whether investors take this tax effect into their decision making process, and the theory concerning the situation in which debt financing becomes cumulatively unstable, are beyond the scope of this study.[1]

The main form of issuing gilt edged securities had been through an offer for sale to the general public. Until the infamous "Battle of Watling Street" in February 1979, the price was usually fixed at the time of announcement, normally three working days before the application deadline. This meant that if market prices rose in the intervening period, the new issue was available at a substantial discount. Between the announcement and issue date of Treasury 13 3/4% 2000/2003, the long end of the market rose dramatically, causing the issue department of the Bank of England (in Watling Street) to be swamped with applications, and to experience "disorderly scenes". Subsequently, issue was primarily through public tender. The government generally set a minimum price and stock was issued to those tendering the highest price and continuing with the next highest and so on until the issue was fully allotted. The lowest price at which applications were successful was the price actually paid by all allottees. If the issue was undersubscribed when all allottees paid the minimum tender price or if, where no minimum was specified, tender prices were unacceptable, the balance of the issue became the

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[1] The seminal works on this topic are Barro (1974) and Blinder and Solow (1973). A comprehensive recent survey of the issues involved is Buiter (1986).

current "tap stock". This meant that it was taken in by the issue department for later sale to the market in response to demand. The tap process required a considerable degree of finesse from the issue department. Lowering the price of tap stock excessively caused immediate losses to existing holders and could deter them from tendering for future issues. On the other hand, unless the market was moving upwards, a price higher than the issue price was unlikely to attract further investors. However, if it was feared that the tap price might gradually increase, new investors might not have preferred to defer purchase. In any case, the issue department had the option to wait until the climate was right before turning on the tap.

2.2.1. The Secondary Market

From at least 1970 until Big Bang, gilts continually accounted for about 11% of the total value of the secondary market in all quoted securities. Even in nominal terms, with the effects of share prices and interest rates removed, the proportion remained fairly constant, though at the rather higher 44.6% in 1970 to 48% in 1983. However, we do not see the true size of the gilt market unless we consider turnover. In a survey conducted by the Stock Exchange just prior to Big Bang, it was found that while gilts accounted for only 10.3% of total bargains, they accounted for 65.4% of the total value of trades. Not only this, the proportion had fallen from around 72% in 1975. This reflected the reversal of the trend of the seventies when, as will be explained below, there was a strong emphasis on gilts market management as a tool of monetary policy. Despite the falling proportion, actual daily turnover had increased from £370*m* in 1975 to £2,215*m* in 1986.

The above figures suggest that the average bargain size for gilts was large relative to other securities. The average gilts bargain was just under £0.5*m* whereas the average equity bargain was just under £10,000. These figures reflected the characteristics of trading in the gilts market. It was dominated by institutional investors conducting primarily switching operations in response to changes in interest rate expectations.[2] The Exchange reported that the percentage of turnover in medium and long gilts carried out by individuals and agents was only 6.6% in 1986, having fallen from only 19.8% in 1983. However, this sector still accounted for around 60% of trades as more than half of the trades were for sums of £10,000 or less. The percentage of turnover carried out by institutions had also fallen during the same period due to the rise of in-house and overseas dealings, but remained at well over 67%. This had not always been the case. Table 2.1 shows the rise of the institutional share of the market and the decline of the banking and individuals' share of the market. Interestingly, the proportion of gilts in the overall financial assets of the institutions had declined noticeably since 1979 due to the relaxation of exchange controls. In other words, though the institutions dominated the market, gilts were less important to them than when they did not dominate that market. The proportion in personal financial assets had also declined, from 36% in 1966 to 4% in 1986. This was due to: firstly, the tax incentives of investing via the institutions which is clearly borne out in the above figures showing the changes in the respective shares of the market; secondly, the volatility of interest rates channelling savings to safer environments; and thirdly, the MIRAS scheme redirecting savings into house purchases.

As with the equity market before Big Bang, trading in the secondary market in gilts was conducted under a single capacity system. This meant that the process of trading was split between jobbers and brokers. A broker received orders to buy and sell from his clients. He did not normally take a position in a security. Then he would visit a jobber's stand on the floor of the exchange and ask the jobber - the wholesalers of securities - for a price, without declaring whether he wished to buy or sell. The jobber, who was only permitted to deal with registered brokers or other jobbers, would respond by quoting two prices, a higher one for buying and a lower one for selling. Jobbers' remuneration was derived from the "spread" between the two prices. The broker would then execute the deal with the jobber who offered the most

[2] Switching operations are transactions involving switching from one or more gilts (selling) to one or more other gilts (buying). There are a number of motivations for such transactions such as the change in interest rate expectations described above.

favourable price. The broker made his money by charging his client a commission for carrying out the deal. With the exception of short-dated gilts, these commissions were determined by the Exchange's rules and were a sliding scale of charges, with a fixed minimum level, related to deal size.

By 1985 there were eight jobbing firms, Wedd Durlacher Mordaunt, Akroyd & Smithers PLC, Pinchin Denny, Wilson & Watford, Charlesworth, Giles and Cresswell, Moulsdale (Liverpool), and Aitken Campbell (Glasgow). The first two accounted for 80% of the business and with the medium sized firm Pinchin Denny virtually the whole of it. The five others comprised 3 small London firms and 2 regional firms. Prior to Big Bang spreads on short maturity gilts (under five years to redemption) were roughly 2-4 ticks (£2/32-£4/32 per cent) and on medium and long term gilts were 4-8 ticks. Brokers' commission charges prior to Big Bang were about 0.05% on long gilts and 0.008% on short gilts. The turnover profile described above, was reflected in the profile of brokers' commission income, the share from individuals and agents having fallen, while gilts commission accounted for only 10% of total commission income, due to their predominantly low volume, high value nature. Also, there were probably about 12 brokers carrying out 90% of the business by value, and with a fairly restricted client base.

It is not surprising to learn that in such a tight knit community, views were generally shared and consequently the market had the potential to be highly sensitive to any moves in opinion. In order to reduce the possibility of embarrassment to jobbing firms and patchiness of liquidity, two avenues of assistance were available to the jobbers: the Government Broker and the Money Brokers.

The office of Government Broker, colloquially "GB", had been established in 1786 and since 1829 had been the senior partner in the broking firm Mullen's & Co. The Government Broker and his department, entirely divorced from the rest of Mullen's & Co., dealt in the market on behalf of the Issue department of the Bank and the Commissioners for the

Reduction of the National Debt. The Government Broker acted as a "Jobber of Last Resort". Though there was no formal undertaking, he was prepared, at prices of his own choosing, to buy and sell stock from the jobbers in response to particular market conditions.

In the fifties and sixties, there had been three Stock Exchange Money Brokers (SEMBs). By 1986 there were six, all specialist departments of the following stockbroking firms, Cazenove, Sheppards & Chase, Laurie Millbank, Rowe and Pitman, James Capel & Co., and Hoare Govett. Essentially, they acted as a market for loans of cash and stock. If a jobber bought stock, he could borrow money to settle the following day, against security of the stock. Prior to 1914, there had been account dealing. Since 1918, the market operated a system of cash settlement the following day. If a jobber sold stock, he could borrow the stock against the security of cash received. Money Brokers were permitted to use other financial institutions to balance their own accounts.

2.2.2. Monetary Policy and Gilt Market Management

In the Bank of England Quarterly Bulletin for 1966, the objectives for those who managed the gilt-edged market were disclosed. Most importantly, there was the task of maximising the long run desire of investors at home and abroad to hold British government debt; preserving the "attraction, health and capacity" of the market. The reason simply was the Government's continuing need at that time for large quantities of long-term finance both for current borrowing requirements and to replace maturing debt. Subsidiary to the above, and of a shorter term, was the task of assisting economic policy. There were two parts to this objective, firstly the promotion or maintenance of an "appropriate pattern of interest rates", and secondly, the provision of assistance in reaching the targets of credit policy, by limiting government borrowing from the banking sector.

In 1966, the quantitative objective of monetary policy was the limiting of the growth in bank lending to the domestic sector. The government sought short term finance from the banking sector through the tender for Treasury bills and the Bank of England's financing operations in the money markets. This allowed the government the necessary flexibility to pursue the aims of long term gilt-edged management. This management was conducted in the secondary market by the Government Broker. By being prepared to deal in the way described earlier, the government could ensure the liquidity and hence the long term appeal of the market to the investors. Furthermore, the Government Broker could equally well supply or absorb stock without influencing the price, as do so to influence it. Thus market observation of the Government Broker was essential to detect any changes in policy which could directly influence the gilts market.

In 1968, in agreement with the IMF, domestic credit expansion (DCE) replaced bank lending as a monetary target. This had differing effects in the primary and secondary market. In the primary market there was no obvious effect as the government's requirement for finance at that time was small. Furthermore, it was not seen as a lasting change in monetary policy. However, in the secondary market, the direction of attention to targeting the quantity of a monetary aggregate, as measured by the DCE, meant the freeing of its price. Fluctuating interest rates causes fluctuating gilt prices. This was evidenced by the Bank's tendency to alter prices at which it was prepared to deal more quickly to avoid disruptive volatility. In September 1971, the publication of the "Competition and Credit Control Document" marked a major change in monetary policy operations. The quantitative limits on bank advances ended and DCE control was modified in line with the broader aim of regulating the growth of the money supply by varying interest rates.

Predating this by 4 months, the Bank of England had decided to modify its operations in the secondary market. As this remained the system until Big Bang, it is useful to describe the changes. The principal change was that the Government Broker was no longer prepared to respond to requests to buy stock outright, except for those with a maturity of one year or less; and he would only buy longer stocks at his discretion and initiative. He would undertake exchanges of stock at prices of his own choosing only if it did not shorten the maturity of debt in public hands, and he would continue to respond to bids for tap stocks and other stocks which he wanted to sell. These changes led to increased short term fluctuations in prices and a reduction in liquidity. As one channel of assistance had closed, jobbers responded by increasing spreads and changing prices more abruptly. The Bank did not believe however that "the longer-term health of the market need suffer in consequence" (BEQB (1979), p.138).

Unfortunately, the authorities began to find that the more volatile interest rates required for monetary control clashed with the desire to maintain the long term health of the market. Ceteris paribus, such a situation could be resolved (see the discussion of variable rate stocks below). However, the seventies saw a period of rising and more variable inflation and an increased budget deficit (PSBR). During the sixties inflation had ranged from 1% to 8%. Between 1970 and 1978, it varied from 7% to 27%. In 1970, the PSBR was £1*bn*. In 1975, it was £10.5*bn*, and it averaged £6*bn* during the seventies. Volatile inflation causes volatile nominal interest rates compounding gilt management problems, and although an increased PSBR required increased borrowing of some form, gilts might not be the most preferred candidate. So there has to be a significant other reason for the Government's unprecedented resort to the gilt-edged market during the seventies to finance the PSBR.

The key point is that if Government spending is financed by making sales of securities to the non-bank public, the money supply in aggregate remains unchanged, since the amount placed in public hands by virtue of the Government's spending is exactly counterbalanced by the reduction in the total stock of money caused by these purchases. In contrast when the Government borrows from the banking system its spending increases the amount of money in circulation, as before, but there is no counterbalancing reduction in bank deposits, and as a consequence the money supply is expanded.

2.2.3. Developments in the Primary Market

Against such an uncertain background, it is not surprising to discover that the scale of gilt edged sales to the non-bank public referred to above was not achieved without some degree of market innovation. The new developments were the introduction of partly-paid stocks, convertible stocks, variable rate stocks and index-linked stocks, and the use of "taplets". Other innovations were suggested, but they were either never implemented or swept aside by the wave of Big Bang changes. These were: firstly, permitting sharper changes in the prices at which the Bank sold stock to the market; secondly, the development of a "special relationship" between the Bank and the main gilt-edged investors; and thirdly, introducing a further short term asset.

Following the sterling crisis in 1976 when the Government found it necessary to make an application to the IMF for stand-by credit due to the huge growth of the PSBR relative to GDP, monetary control was accorded a high economic priority. From a post-war low of -0.10%, the PSBR as a percentage of GDP reached a peak of 9.58% in 1975. In March of 1977, the Bank of England resorted to a device which had not been used since 1940. It announced the issue of stock on a partly-paid basis, the calls being timed to match expected financing requirements and to reduce the monthly fluctuations in monetary aggregates.

Also, new forms of market instrument were introduced which were designed to be attractive under the prevailing conditions of uncertainty, especially to the non-bank public, and "The Grand Old Duke of York" made his market debut. At times of funding crises (1976), the authorities adopted the tactic of forcing up interest rates, causing gilts prices to fall, and allowing tap stocks to be re-established at a much lower level. It was then hoped that demand for the high yielding tap stock would be forthcoming and market prices would begin to rise once more. The market thus adopted the oscillatory progress of the well-known nursery rhyme character. Since 1981, the "Duke" has not been recalled into active service.

The first of the new market instruments was convertible stocks introduced in March 1973. They are shorter stocks (with yields close to the market yield for that maturity) which carry the option to convert the holding into predefined amounts of a longer stock (with a yield close to that for longer dated stocks at the time the shorter dated stock was issued), at some time or times in the future. Conversion terms cause this to be a more expensive way of acquiring the longer stock at a conversion date than purchasing in the market, ceteris paribus. This difference is the premium investors pay for the option to convert. Expectations of long-term rates essentially determine whether a particular investor will prefer to hold long-term bonds or short term bonds. The convertible bond provides the opportunity to exchange a short term bond for a long term bond and the attractiveness of this opportunity depends on the variability of the price ratio of the long term bond and the short term bond. Whether or not conversion occurs, that is whether or not expectations were fulfilled, is of relatively little importance to the Government compared to whether the stock was attractive in the first place. For if interest rates make conversion unfavourable, the longer bond is presumably more attractive as a market purchase, so it does not seem unreasonable, for portfolio balance reasons, that it should pick up a substantial quantity of the cash from the redeemed short stock. Although convertible issues are more expensive to manage than ordinary issues, they have significantly aided the operation of the primary market.

It was mentioned earlier how the increase in the volatility of interest rates clashed with the desire to maintain the "health" of the gilt market. Finding a solution to this conflict was made more difficult in that it contained "Catch-22" characteristics. To invest in gilts, the non-bank financial institutions who were already considerably saturated with gilts needed the prospect of falling interest rates and confidence that the money supply was in control; and, whilst the opposite conditions prevailed, these investors were noticeably absent. The required solution was a capital protective instrument marketable during times of high interest rates that were not expected to fall. The preferred solution was the introduction of variable rate stocks. There were three issues of Treasury Variable Rate Stock, with maturities of 1981,1982, and 1983; the first issue being made in 1977. These stocks provided a degree of insurance against rising short term rates in the following way. The half-yearly coupon was set equal to the average Treasury bill rate for the six months prior to the coupon date, plus a fixed margin of 1/2%. Apart from being more complex to analyse than fixed coupon gilts, their success was limited. As they were short term stocks, they tended to be more attractive to the banking sector rather than the non-bank sector at which they were primarily aimed. Consequently, they were fairly thinly traded and perhaps did not enjoy their deserved price stability; but, in comparison to fixed coupon gilts of the same maturity, their prices were considerably less variable. During 1977-79, variable rate prices ranged about 3 points (£3 per £100 nominal face value), while fixed coupon prices fluctuated by 12 points.

To directly challenge the inflationary environment, index-linked stocks were introduced in the budget speech of 10 March 1981. The idea had been around for a number of years (see e.g. BEQB 1979 p147), and although inflation was falling by 1981, their attraction is that the "real" return to maturity is fairly clear. Institutional investors welcomed them as their liabilities rose with inflation. In fact, all investors concerned by the effect of rising inflation on nominal interest rates and hence on prices, can gain substantial compensation against the inflation component of capital losses. For the first three issues between March 1981 and March 1982 (£1000*m* 2% Index-Linked Stock 1996,£1000*m* 2% Index-Linked Stock 2006 and £750*m* 2 1/2% Index-Linked Stock 2011), ownership was restricted to pension funds or institutions writing pension business. This resulted in poor marketability and frustrated other willing purchasers. The Treasury and Bank of England responded by freeing ownership of these and subsequent issues beginning with £750*m* 2% Index-Linked 1988 in March 1982. Coupon and redemption payments are weighted by the RPI for the month eight months before the payment date. This time lag allows for: firstly, the coupon payments to be known six months in advance as with ordinary gilts; secondly, the approximate 37 day ex-dividend period; and thirdly, the timing of RPI announcements. It is the minimum period practical. However, it does mean that the redemption value on the date of issue is not 100, and that there is no protection against inflation during the last eight months to maturity. By 1985, there were 11 index-linked stocks, of varying maturities, accounting for about 8% of the market. They have proved popular with investors.

After 1980, the authorities modified the issue procedure to assist their sales of debt. They created small additional tranches of existing stocks ("tranchettes" or"taplets") and sold them to the Bank, which in turn "tapped" them out to the market as demand dictated. Initially regarded as a temporary measure, their convenience made them a regular market feature. Stock could be quickly issued, with no prospectus or difficult pricing problems, to meet unexpected demand when a new issue might have been less easily digested. Also, any need to reduce the price involved less embarrassment then with a new issue.

For completeness, I will discuss those ideas that were not implemented in the gilt market. The first suggestion, a modification of pricing practices, consisted of two distinct aspects: a modification of the pricing of tap stocks, and the adoption of tendering as the general sale practice for new issues. The latter suggestion was introduced to some extent, but the former, which was based upon the proposition that "a sufficient fall in the price at which the stock is obtainable will ...(ceteris paribus)... produce the required demand", was not thought appropriate. Although more flexible pricing, of which three forms were considered by the Bank (BEQB,1979), would assist tap sales which were dependent on the state of the market, the Bank believed that the additional element of uncertainty would "seriously impede the making of a market, in any size, in gilt-edged stocks whether by jobbers, as at present, or under some different institutional arrangement." The view expressed in this comment also demonstrated that the Bank did not think that the pre-Big Bang system was sufficiently robust to handle such an innovation. It believed that the potential threat to liquidity could damage the long term attractiveness of the market, which could not easily be balanced against any advantage of flexible pricing. What happened instead was the introduction of taplets, as described earlier.

A second idea was to establish a more direct relationship between the Government and the major investors. Suggestions included the negotiation of underwriting by the institutions rather than the Bank, and the negotiation of direct placings with the institutions. The problem was that it was unclear that in a bear market the institutions would be any more likely to buy by way of placement than in the open market, unless offered a significant yield inducement. Neither would it be prudent to underwrite the issues, unless given the freedom to move the underwriting price. Conversely, in a bull market, the present system was quite satisfactory. Furthermore, and without entering arguments concerning the efficacy of government influence over major investors, it was likely that such an influence would hold yields artificially low and hence reduce the long term attractiveness of gilts to other investors. While the flow of funds from institutions may be stabilised there was no reason to expect this to be the case for the market overall.

Finally, there was the suggestion of introducing a new form of short term debt. Though primarily targeted at institutional funds awaiting investment and company bank deposits (which form part of the money supply), it could also appeal to private investors to compete with non-marketable assets (e.g. National Savings). The two marketable short term instruments already available, Treasury bills and gilts approaching maturity, are eligible reserve assets to the banking system. So, compared to other short term assets with which they compete, these have a particular value to the banks and their yield is on occasion bid down to levels unattractive to other investors. Therefore it was suggested that a short term asset that was not an eligible liability might be introduced. However, such an area was already heavily populated by local authority loans, and to compete with this may have reduced the flow of funds into the public sector as a whole. As for prospective demand, the proportion of short term funds which were held in bank deposits suggested that a close substitute would be necessary to attract funds away. Unfortunately, this would not have produced any meaningful reduction in liquidity in the economy. Furthermore, a higher yield would need to have been offered, and as non-marketable assets were offering high yields, the cost was likely to be unacceptable.

2.2.4. Conclusion

Despite the innovations in the primary market, none had worked as successfully as hoped, and were in any case borne out of an atmosphere of concern over the ability to secure sufficient public sector funding in the economic climate of the late sixties to early eighties. Furthermore, those alternatives that had been suggested seemed to need a more robust dealing system in order to produce the desired allocative result.

The secondary market was also quite aware of its limitations. Jobbers believed that the assistance of the Government Broker and money brokers often failed to meet their needs, and that there were not a sufficient number of firms to provide the market with the level of liquidity it really required.

The new gilt-edged market and the impacts of Big Bang on operational efficiency are described in chapter three. The next section, 2.3, considers the causes of Big Bang and considers whether there is an economic justification for the changes introduced (in particular in the dealing system), beyond a simple response to the pressures upon liquidity examined above.

2.3. The Economics of Big Bang: Causes and Justification

On the 27 October 1986, the UK Stock Market underwent substantial changes. The official phrase to describe the events was "Deregulation". However, the use of a single day to introduce the great majority of these changes caused the event to be known, colloquially, as the "Big Bang". Deregulation not only changed the way in which agents operated in the market but also the structure of the market itself. It is well documented that there were three major components to the changes. They were: permitting member firms to act in dual capacity, that is, they can act as both jobber and broker; the abolition of the system of fixed minimum commissions to be replaced by negotiated commissions; and the freeing of membership restrictions to the Exchange.

2.3.1. The Causes of Big Bang

The developments which led up to deregulation fall into three broad categories; legal reasons, economic reasons and technological reasons. Most of the focus of attention on the causes of Big Bang has tended to be on the legal happenings and, hence as we shall see, its roots are deemed to extend back only to 1983. However, there were significant economic and technological reasons and more deep rooted legal reasons why the road to deregulation began even earlier. One implication of this line of argument is that while Big Bang fitted the "pro-competitive" ideology of the Government at the time, this was coincidental as deregulation was an inevitable conclusion of legal and, more importantly, economic and technological events.

In 1976, the powers of the Office of Fair Trading (OFT) were extended to include service industries. Two years later, the Director General of Fair Trading gave notice that he wished to challenge the Stock Exchange's rule book in the Restrictive Practices court. The DGFT informed the Exchange that he had found a number of restrictions as defined by the 1973 Restrictive Practices Act. These were: limits to the freedom of brokers to supply their services (i.e. minimum commissions were in force), limits to the operation of the market system (i.e. single capacity), and limits to the use of agents (i.e. controls on membership of the exchange). At first the Exchange mounted an expensive defence knowing it would take years for the case to come to court.

By 1981, the case was well underway and the prospect of a long and expensive legal battle faced both sides. The Exchange had hoped that the change of government in 1979 would provide some relief. But when the Chairman, Sir Nicholas Goodison, asked for a Royal Commission to look into the matter of the restrictive practices of the Exchange, he was refused this.

He had argued that a court was not the right forum since it could only decide innocence or guilt. What was needed was a body able to recommend reform. The then Secretary of State for Trade and Industry, John Nott, felt that to make the Exchange an exception to the whole "derestrictive" nature of government policy was inappropriate. However, his successor, in 1983, Cecil Parkinson, agreed, in July of the same year, with the Chairman of the Exchange to exempt the Exchange from the restrictive practices legislation on certain conditions, namely the abandonment of fixed minimum commissions and allowing freedom of entry for outside firms. Hence, several commentators have dated the beginnings of the City deregulation from this point.

However, the change was not as clear cut as this may suggest. In 1979, the new Conservative government had abolished Exchange controls. Though this decision fitted their ideological preference for free markets, it had become an economic necessity. North Sea Oil was causing a large current account deficit as the rising price of the pound was making imports more attractive. A capital outflow was required if payments were to balance. But more seriously the controls were no longer performing their intended function - controlling the value of currencies and the direction of investment. Despite them western countries could not hold fixed parities, and with them, multinationals who earned profits abroad were encouraged to leave them there. Since the ending of these controls, there have been several foreign banks operating in the euro-markets admirably placed to channel funds overseas. To respond to this, the Exchange began to allow dealings between its own members and "Designated Dealers".[3] As foreign brokers were prepared to pay reciprocal commissions to British brokers who supplied them with business, the flow of funds overseas was profitable on both sides. In response, member firms of the London Stock Exchange established subsidiaries to trade on overseas exchanges, under their house rules. The implications are clear. If a client wished buy or sell some securi-

[3] Members of another stock exchange who were prepared to perform the same function for foreign shares as jobbers did in London, that is, maintaining a continuous two-way price.

ties and his broker operated in the UK he would pay minimum commissions. But if he traded in Wall Street, a lower negotiated commission was possible.[4] Thus, there was already great pressure on the exchange to remove the fixed commissions system. By 1984, the London Stock Exchange had formalised the procedure by defining an "international dealer" as a subsidiary of a member firm, but not subject to the minimum commission rules.[5] Even before this, American Depository Receipts (ADR's) and Automated Real Time Investment Exchange Limited (ARIEL) had been used to circumvent the commission system.[6]

Contrary to popular belief, the issue of dual capacity was not part of the Goodison-Parkinson pact: it did not need to be. Stockbrokers and jobbers only have an incentive to maintain single capacity dealing under a system of fixed minimum brokerage commissions. If commissions are negotiable, the operation of dual capacity dealing will naturally follow. The reasons for the link between these two aspects of market structure will now be analysed.

Single capacity dealing had been introduced by law in 1908 to formalize the informal specialisation of roles that had evolved among participating firms during the nineteenth century, and to reestablish the lapsed regulatory control of the City authorities. However, it soon became apparent that it was impossible to prevent a jobber from dealing directly with a customer, as he could put the deal through a "friendly" broker or his own broking division as a formality. At the same time, brokers competing on commission levels have a strong incentive to make markets in shares to capture the jobbers' spread income in place of commission income. A fixed level of brokerage commissions would remove the usefulness to the jobber of the dummy broker because this meant that the dummy broker received payment for no service and

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[4] The UK securities internationally traded tended to be the large "blue chip" companies, and the clients, big UK institutions.

[5] The foreign arms of these international could act in dual capacity in the foreign market. They dealt in UK equities in ADR form with foreign investors in the foreign market. ADR's are described below.

[6] American Depository Receipts were huge blocks of UK blue chip securities bought by US institutions in the UK. Then all subsequent dealings were done in the US where there was no duty. Discontent among the prominent Merchant banks (Accepting Houses) over the commission levels led to their setting up Automated Real Time Investment Exchange Limited through which they dealt with each other, by-passing the market and its charges. To avoid a fragmented market, the system was bought out by the Exchange.

the jobber was not paid for his broking service. Fixed commission levels would also reduce the incentive for brokers to take positions in the market. So a system of fixed minimum commissions was introduced in 1912. Thus it is clear that these two aspects of market structure are inextricably linked, and so when the Goodison-Parkinson pact signalled the end of minimum commissions, it also marked the end of single capacity dealing.

While fixed commissions are a necessary condition for the maintenance of single capacity dealing, they are certainly not a sufficient condition. Brokers still have an incentive, albeit reduced under fixed commissions to take positions in stocks to capture the jobbers spread income. The actions pursued by both brokers and jobbers in response to this incentive demonstrate an interesting irony concerning the causes of Big Bang. In response to the potential for capturing spread income and facilitated by the rise of the institutional investor (see table 2.1), broking firms began to transact "put-throughs". A firm finding itself with a matching buy and sell order would make the market itself, checking the price with a jobbing firm. Indeed, the Wilson committee (1980) reported that 10% of broking business was conducted this way. The jobbers' counter-measure was to adopt short run collusive agreements, to maximize their joint income. For example, a group of jobbers might make a joint book (market) in a security, or set an agreed spread on a security. The irony of this state of affairs is that these practices were highlighted by the 1978 Monopolies Commission report, and thus in some sense, the system of fixed minimum commissions contributed to the removal of single capacity when it was designed to maintain it. Furthermore, the fact that the minimum commission system was under investigation anyway meant that commission levels certainly could not be raised in order to discourage the anti-competitive practices described above.

The third component of Big Bang was the relaxation of the Stock Exchanges membership restrictions. In fact the rule that outside firms must have less than a thirty per cent share in a member firms ended on the 1st of March 1986. There followed a large number of mergers and takeovers among merchant banks, brokers and jobbers.[7] The synergy in ability to monitor a

[7] For example, merchant bank S G Warburg teamed up with broker Rowe & Pitman Mullen and jobber

share from inception in the primary market, through the secondary market, until a possible redesign following a merger or takeover, belies some of the underlying economic reasons for these mergers among different financial institutions. Once again, the deregulation of the foreign exchange market had a key impact. A merchant banks' traditional role had been to design suitable issues of securities for its client companies, arrange for the issue to be underwritten and then distribute the securities. The abolition of exchange controls meant that client firms could often obtain finance cheaper from abroad by taking advantage of short run opportunities of low interest rates and appreciating currencies. Simultaneously, these same companies had begun to employ staff of equal calibre to the merchant banks, and could obtain similar information. Some companies even began to organise the design and underwriting of the issue, leaving the bank with only the distribution function. This led to smaller profits for the banks, who sought to make them up by taking a position in the market. The cheapest way to obtain the necessary expertise seemed to be the acquisition of broking and jobbing firms. The City was also host to a growing number - about 120 - of foreign security firms and banks, who also used existing firms as their route to membership.[8]

Although the focus of discussion has been on demonstrating that Big Bang was more the product of economic rather than legal forces, there is one further factor to consider; the role of technology. The recent technological advances in computers and telecommunications laid beneath many of the pressures for a change in the market structure. The ARIEL system, the institutional dealer arrangements and the opportunities for firms to obtain cheap short run finance from overseas all relied on such developments. Recently, it has been argued that technological factors are the root cause of the recent growth in new forms of securities (e.g. Swaps, Options and Futures) rather than the more visible short run exploitation of taxation or regulatory anomalies (see e.g. Cooper (1986) and Miller (1986)). Indeed Cooper further argues

Akroyd and Smithers to form "Mercury Securities".

[8] For example, the Hong Kong and Shanghai Bank acquired James Capel and Co.

and provides examples which show that market structure is an endogenous response to these developments. Whether or not this was the case for the changes proposed to the equity and gilt market, the American market deregulation in 1975 had shown that it would be impossible to run an efficient and liquid market post deregulation without these technological aids.

2.3.2. The Economic Justification for the Big Bang Changes

Any threat to the liquidity of the market - the risk of having non-matching buyers and sellers - could hinder the nation's ability to raise finance. Therefore any change in the system could be expected to be contentious to say the least. It is often said that stability provides a consistent environment which, in turn, instils confidence in those operating in that environment. But to believe this means a faultless system is illusory.

The single capacity / fixed commissions market system had provided stability, but had not been able to (or wished to) react to recent threats to its continuity. Because of the distorting influence of Britain's tax structure, investment in shares had largely been channelled through the big institutions, and they tended to buy only "safe" shares in well-established companies. Jobbers who had specialised in other sectors had therefore done badly and the overall number of jobbers was falling.[9] As a result, gaps had developed in the market; that is, there were some sectors in which no jobber who would quote a price.

First let us consider the economic justifications for maintaining the fixed minimum commissions system. It has already been seen that minimum commissions can be justified as a necessary condition for the maintenance of single capacity. But, this implies that we have already justified single capacity: we have not.[10]

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[9] Taking the equity and gilt-edged market as a whole, there were over 100 jobbing firms in 1970, but only 17 in 1984.

[10] The arguments developed in this section are similar to those used by Brealey and Hodges (1978) in their paper prepared for use by the Stock Exchange as part of its defense against the restrictive practices case. At that time the details of the Big Bang changes were not known, and their arguments could not be conclusive. With the full details available, a strong conclusion concerning the economic justification for these changes can now be made.

Economic theorists are widely agreed that perfectly competitive markets will allocate resources efficiently, in the sense that no one may be made better off without making someone else worse off. A firm in such a market faces a given market price and hence will produce until price equals marginal cost. In contrast, a profit maximising monopolist or cartel will restrict production to the level at which marginal revenue equals marginal cost. Output is lower and prices higher than under competition. This suggests that there is a prima facie case for supposing that any restrictive practice which limits output or imposes a minimum price above the competitive outcome is not in the public interest. Principal defences for such practices are either that there is some structural instability facing the industry or that there are social benefits or costs to production which are not reflected in the price.

The first plausible defence is that fixed commissions may not imply abnormal profits, but rather induce competition in service rather than in price. Unlike New York, the London Stock Exchange did not limit the number of seats and there were few costs or barriers to entry. If there were large monopoly profits to broking, would-be brokers should have been clamouring to enter the market. The actual picture was quite the reverse - there was a general decline in the number of firms and exchange membership during the seventies and early eighties. Even under free-entry, price fixing does not guarantee monopoly profits. Rather it reduces overall demand for broking services and encourages competition through non-price means, such as research services. Though successful non-price strategies increase firms business, any increase in revenue is most likely to be used in paying the salaries of a superior analysis team, rather than boosting profits. Finally, except for gilts, there were few restrictions to setting up alternative dealing systems. Though ADR's and ARIEL were born out of dissatisfaction with fixed commissions, the absence of any other competition implied that either fixed commissions were fiction rather than fantasy (as explained earlier) or their level was little higher than would exist in a competitive market.

Secondly, and furthermore, it will be argued below that any margin between a negotiated rate and a competitive rate does not represent a social cost but rather is the inevitable cost of transferring information from private to public hands. If the additional income did reflect a social cost, we cannot conclude that it would be better to abolish minimum commissions without taking into account the costs of introducing negotiated rates. The experience of the USA and, with the benefit of hindsight, the UK suggest that these adjustment costs are not negligible. Thus there is choice between repeated small and uncertain gains on the one hand and once-and-for-all substantial and certain losses on the other.

It has been argued that brokerage houses do not make excess returns. Instead of competing in price they do so in terms of research. Hence the case for minimum commissions stands on the argument that it must be socially desirable to have competition in service rather than in price. This means that there must be something special about the fact that brokerage firms are dealing in information rather than tangible goods. Salient features of information are that it can lead to both public and private gains and that it is impossible to determine property rights to it. Let us see how these two features can justify a system in which the collection and dissemination of information is subsidized by the community at large.

Under negotiated commissions brokers would undertake less research and would sell it for cash on a confidential basis. This research would not be in the public domain. There would also be an incentive for investing institutions to undertake research for their own private benefit. Such research is socially wasteful. One investor's gain is another's loss. It would pay institutions instead to agree not to undertake private research. This could never be monitored. But the way to ensure it is not undertaken is if they consent to subsidise research which is made public. Hence investors are prepared to pay effectively higher commissions to generate publically available information; a not unrealistic description of the system of minimum commissions. Then these payments guarantee that institutions will not undertake socially wasteful research that only results in private wealth transfers between investors.

Now let us consider the issue of single capacity. If there were no costs to investors' continuous participation in the market, there would be no need for a jobber to take positions. Sale and purchase could be by public auction. In practice such a continuous public auction would be costly: few investors leave a continuous set of limit orders and , therefore, orders arrive in a discontinuous ("lumpy") and haphazard way. In the absence of a market-maker, there would be large short-term price variations dependent on the precise sequence of orders. Hence it is worth paying a market maker to make the market. By observing the sequence of orders or conducting his own research the market maker can form an estimate of the equilibrium price of the share. By offering to buy below and sell above this price, he reduces the short term variability. His price spread is the price of intermediacy.

While it has been argued that jobbers perform an important market-making function, it is far from easy to show why they must be isolated from direct contact with investors. Three defences have been suggested.

Firstly, it has been argued that the separation of jobber and broker is necessary for the maintenance of a single market place. This seems implausible. There are strong economic pressures for a central market system and these would not be eliminated by allowing a jobber access to investors.

Secondly, not only is minimum commissions necessary for maintaining single capacity, but, the research advantages of minimum commissions are upheld by single capacity. It has been shown how single capacity is linked to minimum commissions. The abandonment of fixed commissions leads to a broker seeking to take positions in the market, and a jobber to seek contact with investors, using a broker only as a formality. It was argued above that there are welfare gains to having research undertaken by independent firms who then disseminate it widely. If brokers were permitted to deal on their own account, this system would no longer be viable. Institutions would strongly object to paying for research that is then used against them. In other words, if we want to have brokers undertake independent research then the separation of jobbers and brokers is essential.

Thirdly, the combining of principal and agent could lead to a conflict of interests. If a broker holds stock on his own account, his independence will become in doubt. Moreover, if he knows the identity of his customer he may know whether he is a buyer or a seller and will quote accordingly. Not only is this bad news for the customer but also it reduces the value of the broker's services. As far as a dealer is concerned, the above could equally apply if he can deal directly with an investor. But, by interposing an agent between the dealer and the customer (i.e. single capacity), the customer is able to protect his anonymity. Though this is an important motivation for single capacity (and other agency relations), it is difficult to show that in the absence of rules investors would not out of self-interest go to an independent broker anyway. In other words, the argument can be used to defend a rule that brokers may not deal on their own account, but it cannot itself justify a rule that jobbers may not deal directly with the public. If we wish to appeal to a conflicts of interest argument to justify the latter rule, it seems that we are forced to contend that the public does not know what is in its best interest.

If there was no need to prevent brokers from dealing with the public as principals (for the reasons described above), the strongest argument for maintaining single capacity is to uphold the "independent research" advantages of minimum commissions. However, the strongest defences for minimum commissions are, firstly, precisely this research advantage and, secondly, to maintain single capacity. Therefore we could not justify one without the presence of the other to back it up. So, the maintenance of both rests on the conflict of interest argument regarding brokers. This may account for the introduction of the "best execution" rule for post-Big Bang brokers. This forbids them acting as principals unless they can better the price on offer by the market makers. Thus the potential conflict of interests should not occur. Hence there is no economic justification for a return to the old system of single capacity and fixed minimum commissions.

Furthermore, that same old system was not free of a potential conflict of interests situation arising, albeit of a different form. Since under single capacity most of the broker's revenue comes from transacting business which follows from advice given to their clients; they may be tempted to persuade their clients to buy and sell more often than is strictly necessary or even desirable. This is not, though, a problem confined to stockbroking.[11]

The last element of the changes was the freeing of membership restrictions, which led to numerous mergers in the years preceding Big Bang. In many cases motives were ill-defined and purchasers appear to have paid substantial sums for what at best is an insurance policy. However one of the main consequences of this wider ownership is that the Exchange will embrace a much wider variety of interests than hitherto and as a result may find it difficult to maintain agreement among its members. Heterogeneity of interests is perhaps the best guarantee of competition. With the mergers came an extensive transfer of employees across, into and out of these new partnerships. In every case substantial remunerations were paid.

2.4. Conclusions

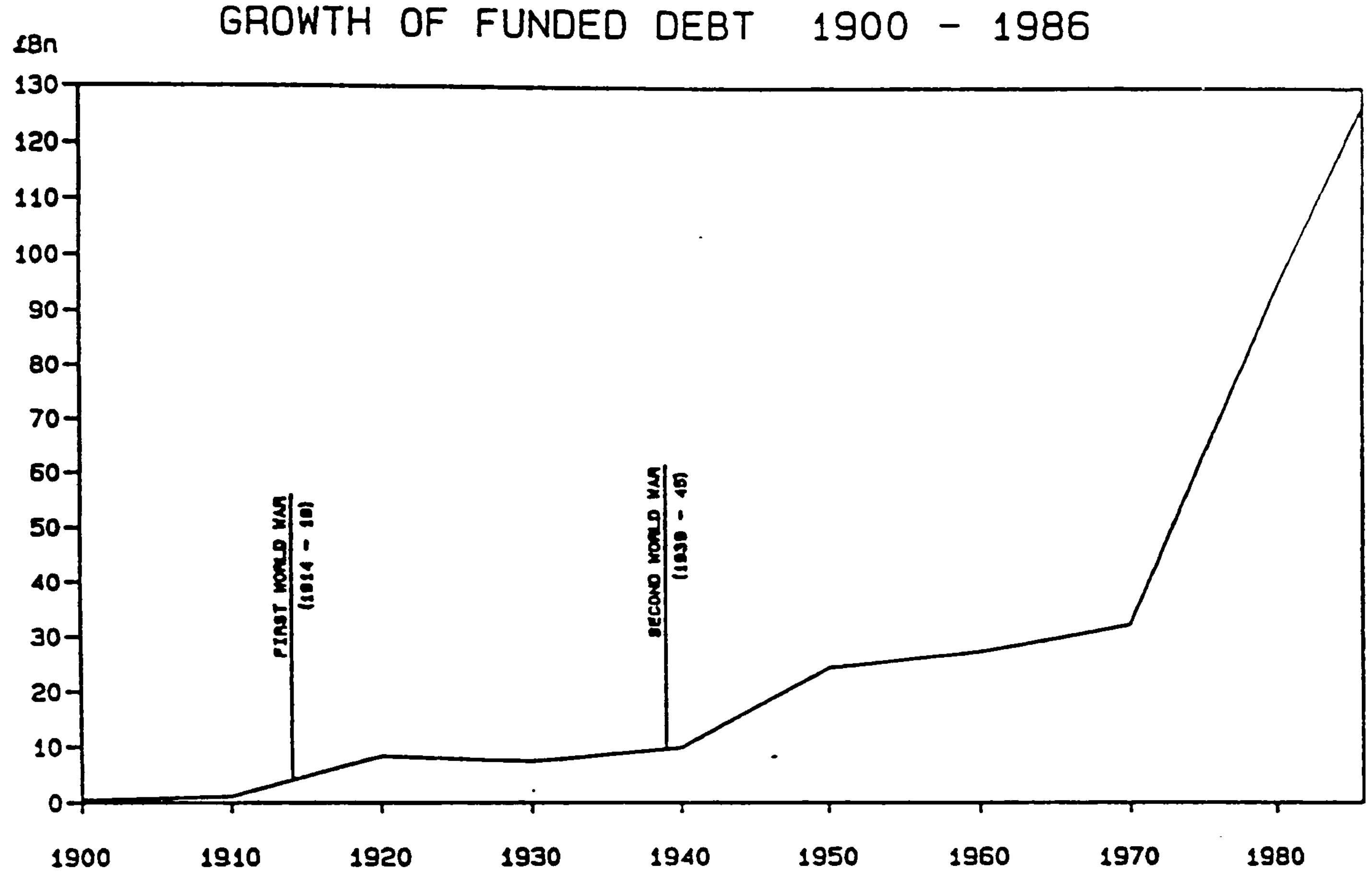
Following a description of the gilt-edged market as it stood prior to Big Bang, the causes of deregulation are found. Rather than it being a direct result of legal expediency and compromise, as represented by the Goodison-Parkinson pact, deregulation was much more the natural conclusion of a process of economic events and technological developments that had been in motion for many years before 1983. These factors facilitated low cost circumventions of the old system, and it was inevitable and essential that change would take place to maintain a liquid market place.

[11] It could occur if a surgeon prescribed operations that were not essential but brought in large fees.

Anecdotal evidence of the use of a system other than that in place, that is, dual capacity and negotiated commissions rather than single capacity and fixed commissions, does not mean that a change to such a system is economically justified. This issue analyzed and it is found that there is no economic justification for maintaining the old system if the new system operates a "best-execution" rule. Such a rule was introduced as part of the Big Bang changes.

Figure 2.1

GROWTH OF FUNDED DEBT 1900 - 1986



GROWTH OF FUNDED DEBT 1694 - 1899

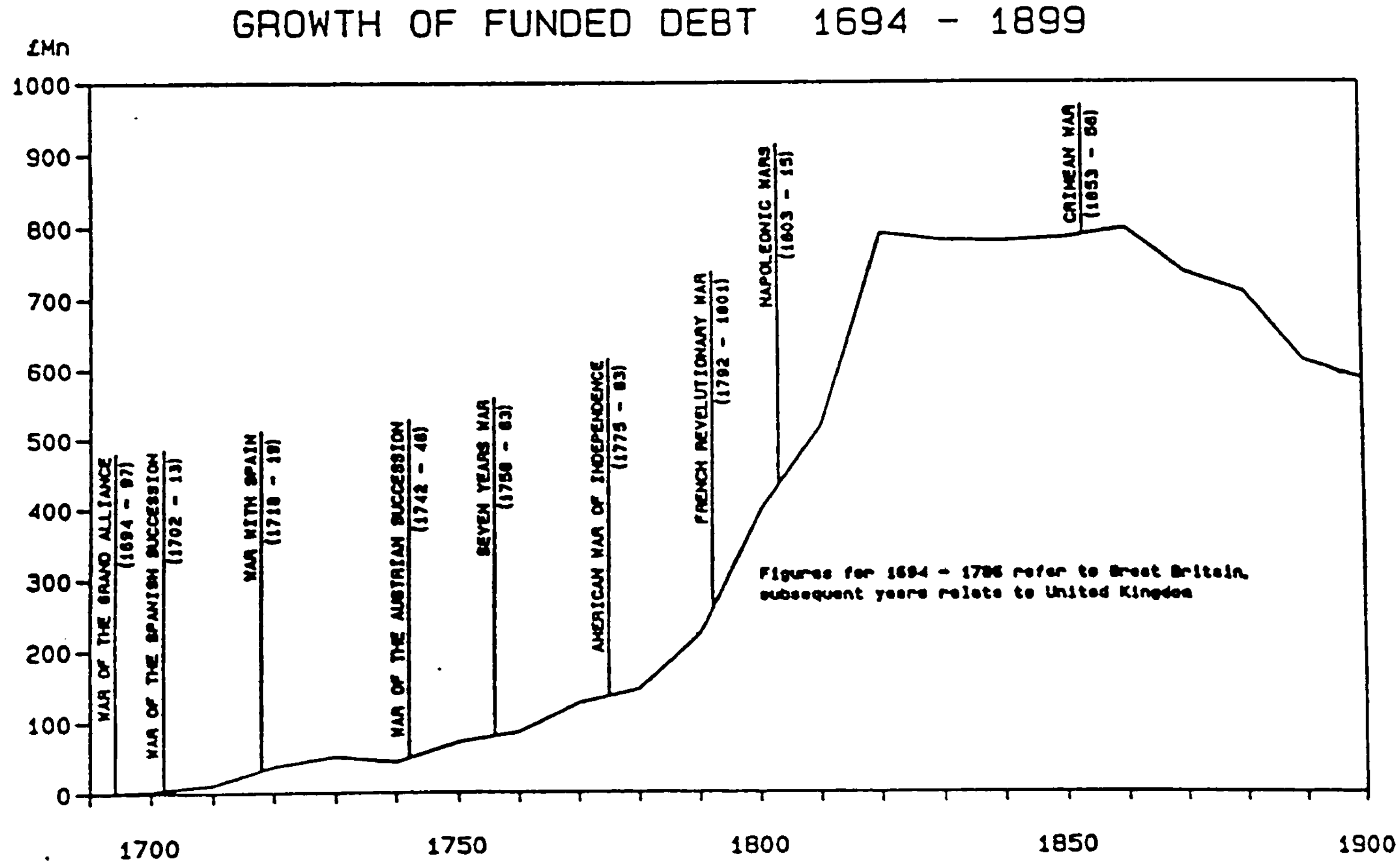


TABLE 2.1

OWNERSHIP OF GILT-EDGED SECURITIES (in billions of pounds)				
SECTOR	1959	1970	1984	1987
Banking	3.2 (22%)	2.1 (13%)	7.6 (7%)	8.8 -
Individuals	3.4 (23%)	3.6 (22%)	14.4 (12.5%)	14.5 -
Overseas	- -	2.3 (14%)	10.4 (9%)	14.4 -
Non-bank financial institutions	- (18.5%) *	5.9 (36%)	63.4 (62.2%)	76.3 -
-of which-				
Pension funds and insurance companies	- -	- -	42.2 (42%)	66.5 -

* : 1957 figure.
Source: Thomas (1987) and BEQB various issues.

CHAPTER THREE

The Effects of Big Bang on Operational Efficiency

3.1. Introduction

Dual capacity dealing in the presence of negotiated commissions was introduced in the United Kingdom Stock Exchange on Monday 26th October 1986. However, these were not the only changes to the structure of the gilt-edged market that were introduced at that time. These additional measures or modifications to existing services were designed to assist the promotion of liquid and operationally-efficient trading within the new market structure. This chapter describes these changes and seeks to determine the impact on liquidity and operational efficiency.

Thus, section 3.2 provides a complete description of the operational changes introduced in the gilt-edged market. In the next section, 3.3, relevant theories of market microstructure are examined, and the predictions that these theories are able to make concerning the likely impact of the changes on operational efficiency are drawn out. Then, the extent to which these theoretical predictions are borne out by the experience of the post-Big Bang market is considered in section 3.4, where also the effects of the Stock Market Crash are discussed. Section 3.5 concludes.

3.2. The Gilt-Edged Market after Big Bang.

Under the new market system, brokers and jobbers will be replaced by a new form of Stock Exchange member firms known as "broker/dealers". They are able to conduct business with their clients either as principals or agents. That is they may operate in a "dual capacity". As principals, they are able to transact their customers' business with their own book; while as agents, they put together deals for negotiated commission on their clients' behalf. When presented with a customer order, they may choose in which capacity they wish to deal.

In addition to the broker/dealers, there is a group of gilt-edged market makers (GEMMs) who may also act in a dual capacity, but who do have certain dealing commitments, undertaken in return for certain dealing privileges. On 12 April 1984, the Bank invited applications to become registered market makers. Their function was clearly identified. "The essential liquidity of the new gilt-edged market will be provided by a number of market makers who undertake to make, on demand and in any trading conditions, continuous and effective two-way prices at which they stand committed to deal, in an appropriate size as discussed in advance with the Bank." (BEQB June 1985). They are expected to make such markets across the whole spread of gilts, and are restricted in which other instruments they may make a market.[1] There were 29 successful applications to become market makers, or "Primary Dealers" as the Bank preferred them to be known. Initially, over 150 firms had expressed an interest, but there were 31 serious contenders. Some major institutions decided not to apply at all and two withdrew after being initially accepted, leaving 27 GEMMs registered market makers at Big Bang.[2]

To assist the market makers, who demonstrate the capacity - in terms of capital and of management and operational resources - to do so, and who are prepared to accept the Bank's prudential oversight, the following facilities (privileges) are available. Firstly, the market makers have a direct and exclusive relationship with the bank. The following undertakings by the Bank, replaced the office of the Government Broker, who dealt on the Stock Exchange floor, by a team of staff within the Bank's gilt division, who deal by telephone. The nature of the relationship is not significantly altered. The Bank is prepared to receive, on a discretionary basis, from market makers - just before or at any time during business hours - outright bids for stock, including particularly tap stocks, which it may have in its portfolio. Also on a discre-

[1] For example they can deal in related instruments such as gilt futures and options, but not equities.

[2] The established firms of Schroders and Smith New Court decided not to apply, while Bank of America and Union Discount Securities pulled out of the market. The remaining 27 GEMMs are listed in the appendix to this chapter. Further details can be found in Phillips (1987, p.14-15)

tionary basis, it will undertake switches of stock proposed by market makers on such terms as it may agree. It is prepared to bid, at a price of its own choosing, for index-linked and stock with less than three months to maturity offered by market makers. It is prepared to purchase outright, at its discretion and chosen price, other stock offered by market makers.

Furthermore, the market makers have borrowing facilities at the Bank against approved security up to maximum amounts related to their capital and reserves. They also enjoy the two tax concessions previously available to gilt market jobbers. Firstly, they can claim relief against tax for the full trading loss made by buying stock cum-dividend and selling it ex-dividend in the course of ordinary business regardless of the time interval between purchase and sale. Secondly, they can use the "bull and bear" dividend arrangement, thereby offsetting, for tax purposes, dividends on stock they have sold against dividends received on stock purchased.

The market makers have continued, and now exclusive, access to the money brokers. With the commitment to make markets, the possibility of zero inventory in a stock, necessitates the borrowing of stock to meet a buying order. When next the market maker has a selling order, he can unwind his borrowed position. Borrowing is done against security of the cash received from the customer. As this may be invested at money market rates, it nearly always pays a market maker to borrow stock for which the charge is about 3/4% per annum. Prior to Big Bang, the Bank was concerned with ensuring an adequate supply of borrowable stock and invited applications, shortly after those to become market makers, to be money brokers. The result saw Lazard Brothers (a merchant bank), King and Shaxson (a discount house), and Prudential Bache and Clive Discount in unison added to those assisting the jobbers before Big Bang. These firms are obliged to keep money broking entirely separate from any other business.

Although market makers are quite at liberty to deal with each other, they also have the facility to unwind positions with each other anonymously. This business is handled by new Inter-dealer Brokers (IDBs), as already existed, but in a slightly less regulated framework, in

New York. Each market maker may input any number of bids and offers. These are displayed to market makers through a screen based system. Those wishing to conduct one of the deals available contacts the IDB who acts as principal, concealing the identities of the market makers involved, and matching the deal. The IDBs are remunerated by charging a small fee to the originator of the business, of 1/128%. This facility is not available to broker/dealers or investors. To confine the IDB network to market makers would ensure creditworthiness when dealing 'blind', and prevent business from being taken away from market makers when business was good and so hindering their operations should business become less favourable. Again, applications were invited by the Bank, and firms had to demonstrate a potential demand for their services from the market makers. This proved slightly awkward in so far as the market makers felt that three or four firms were sufficient, yet there were six equally suited candidates. The Bank decided to authorise all six, namely: Charles Fulton (IDB), Garban Gilts, Mabon Nugent International, Fundamental & Marshall Brokers, Tullett & Tokyo (Gilts), and Williams Cooke Lott and Kissack.

The market officially opens at 9am. when the IDB network opens. However, market makers may operate before this. There is no closing time. The market makers are permitted to display mid-prices on the Stock Market's screen based automated quotation system, SEAQ. However, unlike equity market making, they do not have to display two-way prices and deal size quotes whether firm or indicative. There are two reasons for this particular visibility decision that concern the problems in maintaining an up-to-the-minute set of prices for strongly interrelated securities. Firstly, suppose interest rates change, this affects all gilts. If a market maker is not quick enough to change all his prices he could easily be embarrassed. Equities reflect fundamentals which generally are specific to one security or at most an industry. An interest rate change affects them too, but in a much more differentiated manner. Secondly, displaying prices contains much information about their positions in the stocks which again could leave them vulnerable. Market makers do have the option of displaying two-way quotes within a closed access group on the TOPIC system.[3] Of course, market makers are committed to making a market on application by telephone. However, they are at liberty to alter prices between an enquiry call and a call to proceed with the deal. But, they are often prepared to "take out" a bid at a certain price to protect a client. Details of trades are published the following day in the Stock Exchange Daily Official List (SEDOL). Broker/dealers are clearly at a disadvantage in terms of facilities and scope for action. They can still view market makers quotations on SEAQ and TOPIC, but are unable to post prices, and, as explained in chapter two, they are subject to the "best execution" rule.

Settlement procedures have been automated to assist with the conduct of business. A Central Gilts Office (CGO) project produced the new system in two stages. Phase one devised a rapid transfer of stock by an electronic system, and was brought in before Big Bang. Phase two, devised an assured payment arrangement, which would provide for irrevocable instructions for payment for stock to be simultaneously generated with the movement of stock between accounts in the system. Phase two was introduced on Big Bang day itself.

Supervision of the new market is shared between the Stock Exchange in its role, following merger with the International Securities Regulatory Organisation (ISRO), as a "Self-regulating Organisation" ("SRO"), and the Bank of England. The former monitors the actual bargains using various reporting and direct observation techniques. The Bank using the most extensive and complex guidelines (see BEQB (December 1984) (June 1985)), monitors the capital structure and risk exposure of the market makers, the facilities offered to them by firms such as IDB's and money brokers, and also has certain reporting requirements.

The Bank also decided to add to the existing forms of issue procedure for gilts, to maintain its continued long run management aims. On May 13th 1987, the first auction of a new gilt issue occurred, when £1*bn* of Treasury 8% 1992 was offered to the public by this method

[3] The Exchange's view data system.

in £50 partly-paid form. In this procedure, which loosely resembles US Treasury Bond auctions, there is neither a minimum price nor any formal underwriting agreement, yet the Bank expected to fully allot the issue and actively encouraged all GEMMs, as part of their duty to the market, to take part in the auction. The Bank reserved the right not to allot all the stock if the auction was covered only at a deep discount, in which case the Bank would absorb the balance into its own portfolio. This particular issue method transfers the pricing risk, that is, the risk that an issue price will become seriously out of line with the market between announcement and issue date, from the Bank to the market. To ease the risk facing the market, the Bank sanctioned a "when issued" or "grey" market for trading in the new security before the issue date. In the event, the issue was well covered (2.3 times) and this prompted a second auction in September 1987 which, finding itself in a pocket of favourable market sentiment, was 1.5 times covered. In contrast to the first auction, the stock issued was long dated, 9% Treasury 2008. Finally, the only form of instrument not in issue since Big Bang is the variable rate stock.

3.3. Market Microstructure Theory and Operational Efficiency

This section considers how these changes just described have been modelled in economic theory, and what predictions such theories might imply for operational efficiency, as measured in particular by the costs of dealing.

Much economic theory, in the tradition of Walras, still proceeds as if prices were set in a gigantic "market-place" auction in which all potential buyers and sellers participate directly. Market microstructure theory expands the cast of participants to include market makers as intermediaries who close gaps arising from the imperfect synchronization between the arrivals of buyers and sellers.

The main prediction of this theory is that in such market structures, non-trivial bid-ask spreads are an equilibrium property of the market. That is, at any given time, a customer wishing to buy faces a higher price than a customer wishing to sell. Demsetz (1968) provided the first formal model of the bid-offer spread as a "mark-up that is paid for predictable intermediacy of exchange in organized markets" (1968, p.36), i.e. a return for providing (passive) intermediacy. He argued that the return will just equal the cost of the service, given sufficient competition among market makers.

Since that time, the literature has divided into two groups, differentiated by the assumptions concerning the behaviour of market makers and the behaviour of the customers. One group of models considers the setting of bid and ask prices by risk averse market makers so as to maximize their expected utility of terminal wealth, subject to uncertainty concerning the flow of orders and the returns on their inventory of stock and cash. Key studies in this group are Garman (1976), Amihud and Mendelson (1980), Mildensteen and Schleef (1983), Ho and Stoll (1980, 1981 and 1983) and Grossman and Miller (1988). These models divide participants into two groups; market makers, and customers. Customers trade when they perceive a differential, at current prices, between their desired holdings of an asset and their actual holdings of that asset. By responding to this trade, market makers are supplying immediacy to the customers allowing them to shed the price risk involved in waiting to close trade (the search for equal and opposite buy and sell orders). The market makers charge for bearing this risk by offering the sellers (buyers) a price that is not uncertain, but that is lower (higher), on average, than they could expect from delaying to close the trade, thus generating a bid-ask spread.

The other group of models consider the determination of bid ask spreads by one or more risk-neutral market markers who are maximizing expected profits subject to uncertainty concerning the class of investor with which they are dealing. In particular, the investor may be either a liquidity trader (as described above) whose trading decisions are governed by prices, or an information trader who trades when he has an informational advantage over the market maker. Studies developing this form of model include: Bagehot (pseud. Treynor) (1971), Copeland and Galai (1983), Glosten and Milgrom (1985), Roell (1988) and Dennert (1989). The main feature of these models are that spreads are set to recoup losses to well-informed traders (who know better than the market maker the value of the asset), with profits from liquidity traders. The knowledge that some investors are better informed than the market maker means that a buy (sell) order could be (with some probability) a signal that the true value of a security is higher (lower) than the market maker had expected. To guard against this risk, a bid-ask spread is set about the market makers ex-ante expectation of the value of the security.

There are two features of the deregulation of the gilt-edged market that could usefully be modelled using techniques from the microstructure literature: firstly, the impact on spreads of the change in the dealing and the price quotation system; and secondly, the impact on spreads of the increase in market maker numbers. However, the majority of existing studies have limited applicability by assumption or construction. Before considering those models most likely to predict the outcome of a thus deregulated gilt-edged market, it is useful to consider the shortcomings of the remaining studies as useful models for our purposes.

The principal weakness of the majority of microstructure models is that while they recognize that market makers are in competition (rather than being monopoly suppliers of intermediacy), they nevertheless analyze only one representative market maker with these interrelationships remaining exogenous. Consequently, they do not address the second question of the impact on spreads of a change in the number of market makers; that number is fixed. Although changes in dealing system are still of interest in the context of representative market maker models, such considerations have only been discussed in the relatively few multi-dealer models.

The four studies which explicitly recognize dealer competition, albeit in different ways, are: Ho and Stoll (1980,1983), Grossman and Miller (1988), and Dennert (1989). The immediate disadvantage is that each of these models fall into one of the two groups of studies mentioned earlier. It is clearly desirable for a model to combine elements of both the risk aversion and the asymmetric information stories. This would not be a problem were it not for the fact that the predictions from each of these groups relating to our two questions are quite different, and seem to depend upon the assumptions concerning the degree of risk aversion on the part of the market maker.

The model of Dennert (1989) provides a game theoretic treatment of the effect of adverse selection in dealer markets, which explicitly recognizes the price competition between dealers. He argues that as the number of market makers increases, so the chance of having to deal with an informed participant increases. The reason is that the uniformed will go to the "cheapest" market maker, whereas the informed traders will exploit every instance of "cheapness", gradually dominating the market. By arguments expressed earlier, an increased probability of dealing with an informed trader will, ceteris paribus, increase individual spreads. This growing individual risk exposure is shown to also increase the market spread (the touch). This result relies on two assumptions: firstly, that dealers precommit themselves, in the sense that prices are firm rather than indicative; and secondly, that market makers are risk neutral. The former assumption means that the model is applicable only to the alpha equity market and not the gilt-edged market. The second assumption causes the model not to reflect an important consideration in market making; inventory risk.

Indeed, the models that derive results from the assumption of risk averse market makers (i.e. Ho and Stoll (1983) and Grossman and Miller (1988)) suggest that the market spread should fall as the number of market makers increases. In these models, a higher number of market makers will lead to a better allocation of the inventory risk (which plays no role in a risk neutral setting) of being a market maker, and thus to lower spreads. But, these models do not incorporate the risk exposure effect or model the interaction between market makers as a strategic game.

The opposing results of these studies perhaps suggests that market making is a natural oligopoly, since spreads increase whether the number of market makers increases or declines. However, the risk aversion studies also do not consider the information content in order flow, the resource over which market makers are competing. An increased number of market makers increases the scarcity of this type of information, which would tend to lead to increased rather than decreased spreads. Clearly, there are many competing forces at work, and a definitive prediction concerning the impact of the increased number of market makers is difficult to establish.

The impact of screen based dealing is perhaps more predictable. The increased visibility in the market should reduce the amount of informational advantages to be had. This will both reduce the chances of having to trade with an informed customer, and reduce the overall uncertainty facing a market maker, for he will be better able to infer the likely price risk of a security from the range of quotes of other dealers. Thus there should be pressure from this source to reduce spreads.

Finally, and furthermore, the Ho and Stoll (1983) model predicts that an increase in inter-dealer trading (as is expected in the gilt-edged market with the introduction of the IDBs) will lead to reduced market spreads.

3.4. Big Bang and Operational Efficiency

Prior to Big Bang, expectations among existing participants and many commentators of the consequences in the gilts market were somewhat pessimistic. As with the rest of the Stock Market, emotions ran high, for example, "If there is to be blood on the floor anywhere...it is most likely to be on the floor of the UK government gilt-edged market." (Financial Times 27/10/86), was typical of sentiment at that time. It was expected that turnover would increase, but concern that it would not increase in the same proportion as the number of market makers. The forecast of increased turnover had two contributory factors. Firstly, US experience had shown that the introduction of IDBs increased trading volume substantially. Secondly, it was expected that increased competition among market makers for business would lead to a reduction in spreads that would encourage further trading activity. We now know that such a definite expectation concerning the impact of increased dealer numbers on spreads was somewhat naive, but the model from which this kind of expectation was drawn ("increased competition reduces prices") was also rather naive.

Table 3.1 shows that turnover after Big Bang did indeed increase substantially. This is both in terms of customer business where the level of business per month is running on average 50% higher than the annual figure before Big Bang. Then each month intra-market business, openly between market makers or anonymously via IDBs, roughly matches the customer business figure. There was a significant decline in turnover in the summer months of 1987, but this was short-lived. Rapidly falling equity prices in the crash of October 1987 made the relative security of gilts attractive to investors and, in November 1987, both customer and intra-market turnover reached record levels. The latest available figures (to March 1989) indicate that turnover is maintaining the high levels experienced immediately after Big Bang. However, there was a noticeable reduction in intra-market business in March 1989 due to the withdrawal of two IDBs, Mabon Nugent Gilts and Tullet & Tokyo (Gilts) Limited.

Before Big Bang, when market making was almost in duopoly supply, there was negligible intra-market business. The two jobbers saw so much of the market that it was unlikely that their portfolios could be out of line with the trading pattern. Hence, initiating such business would reveal their position exactly to the other jobber. With many market makers the likelihood of encountering a more mixed order flow is significantly increased, and the need to unwind positions is greater. The desire for anonymity in carrying out such transactions is evidenced by the fact that the majority of intra-market business is conducted through IDBs. But, if the size of IDB trading reflects the dispersion of market making and the need to unwind mixed orders, then the size of IDB trading must also reflect the increased customer trading.

Customer turnover was expected to increase as dealing costs were expected to fall, both in terms of dealing spreads as explained above and from reduced brokerage commissions. Before examining these in detail, it is interesting to consider three sources of downward pressure on customer business in the post-Big Bang market. Firstly, a significant number of the major customers in the pre-Big Bang market entered the post-Big bang market as GEMMs, in particular Discount Houses and overseas security firms. Secondly, in February 1986, accrued interest on gilts with over five years to maturity became an element of income rather than capital gains for tax purposes. It had been advantageous for high rate income tax payers to switch out of a stock immediately before the dividend date and reinvest in a similar stock with a longer time to the next dividend date. Investors not subject to tax on income would, conversely, hold the stock for the short period around the dividend date before reversing the deal to restore their original portfolio. This tax switching was substantial, and was eliminated. Thirdly, on 2 July 1986 capital gains tax on gilts was abolished. Previously, gilts held over one year were exempt. This meant that investors could use gilt capital losses (effected by a "bed and breakfast" deal - selling and immediately repurchasing, so leaving holdings unaltered) over a year as a credit against capital gains elsewhere. Trading from this source was also eliminated. That customer turnover did increase indicates the attractiveness of the market to new investors which, in turn, must reflect the relative trading costs in that market. Indeed, in the gilt-edged market the link between dealing costs and trading volume is very strong due to the nature of gilt-edged trading. Most trading is driven by price anomalies between equivalent securities; lower dealing costs mean that smaller anomalies are now tradeable.

Both commissions and spreads in the market are now substantially lower than pre-Big Bang. Virtually all large value bargains (over £250,000, and on average £2.1m) are dealt net of commissions, direct with market maker. Only 20% of trading occurs through an agent (average size £200,000). For private clients, commission rates varied widely, but are slightly lower on average, at 1.0% for bargains of £1,000, and 0.1% on bargains of around £10,000 to £50,000. On the most actively traded stocks spreads are now about 1-2 ticks on short dated gilts and 2-4 ticks on long dated gilts. Comparing the total dealing costs before and after Big Bang on a deal of normal market size (around £1*m* nominal), the Bank of England found that there was a 60% to 70% reduction in dealing costs following Big Bang. Such a comparative study was facilitated by the fact that the size distribution of trades does not appear to have changed significantly in either short or long gilts. About 90% of turnover value is for deals of over £1*m*. This accounts for about 20% of the medium and long gilt bargains and 12% of the short gilt bargains. The majority of transactions are for £10000 or less, and about 60% of transactions account for less than 0.5% of turnover value.

There are two other issues to examine regarding dealing costs: how spreads change as deal size changes; and the impact of the equity market crash. For deals larger than normal market size, spreads will depend upon the extent to which such a transaction leaves the market maker unbalanced, and his expectations of the price to balance or hedge the position in another market. The Stock Exchange found that for deals up to £10*m*, there was no discernible spread premium. A market with little spread variation over deal size is said to be a "deep" market. It also reported that there had been a reduction in the volatility of spreads which it attributed to the force of competition not to widen spreads in adverse trading conditions. For smaller deals, the fixed costs of dealing are more significant, and less advantageous terms are to be found. Although, spreads increased at the time of the crash, they have narrowed once again. However, dealing costs are on average higher than they were immediately after Big Bang. This does not signify a reduction in liquidity, but rather reflects the fact that the deal size for the quoted spreads has increased from £1m to £5m for shorts and from £2.5m to £5m for longs.

At the time of Big Bang, the view was widely expressed that twenty seven GEMMs was probably a larger number than the market could support over a prolonged period; the Bank admitted prior to Big Bang that their combined market share expectations totalled 175%. However, the Bank accepted this number as it was satisfied that the applicants had adequate capacity to perform the desired functions. Since Big Bang, but especially since the crash, competitive pressures have proved to be at least as great as foreseen; with many GEMMs experiencing operating losses and seven withdrawing from dealing. The Bank in a review in early 1989 stated that, after most GEMMs had suffered losses during the summers of 1987 and 1988, by the end of 1988 one third of the GEMMs were achieving positive returns, one third were containing losses to less than £1*m*, and the remaining third were making significant operating losses. It believed that there would be a further reduction in the number of GEMMs, although, significantly, it has granted GEMM status to two further firms, Daiwa Europe (Gilts) Limited and Nomura Gilts Limited.[4] The Bank also reported that throughout the post-Big Bang era the GEMMs performed their market making function well.

In the early summer of 1987, financial journalists were suggesting that gilt-edged market making was a highly concentrated service industry (see e.g. Financial Times 26th May p.9). The Stock Exchange Quality of Markets group confirmed this view in their summer issue. The Exchange found that on one day, five firms undertook 40% of trading and, with a further five, over two thirds of the business. However, a more recent survey by the Bank (1989) interprets this finding differently. It finds that throughout 1987, there appeared to be three groupings: about six firms with relatively large market shares of at least 5%, accounting for around 45% of turnover; another six with a relatively small market share 1% - 2½% each accounting for around 10% of turnover; and the remainder in between with market shares in the range 2½% - 5% each, accounting for the remaining 45% of turnover. The Bank does not consider this a high degree of concentration for two reasons: firstly, although the shares above remained constant, firms have been continually changing category; and secondly, there has been no tendency

[4] A listing of Gilt-Edged Market Makers registered between March 1986 and March 1989 can be found in the appendix to this chapter. The source of this list is the Bank of England Quarterly Bulletin (1989, February).

for this level of concentration to increase, even temporarily, or for any one firm to become progressively more dominant.

The Bank also reported that the new market structure is providing an efficient service for small deals. Five of the GEMMs provide a regular market making service in small business. One, Aitken-Campbell (Glasgow), has operated as a specialist in small deals, continuing its pre Big Bang jobbing specialisation in that type of business; the other four, Baring Wilson and Watford, James Capel Gilts, NatWest Gilts, and Phillips and Drew Moulsdale, have provided a small deals service as an adjunct to their wholesale market making activity. The Bank viewed this development as ensuring the maintenance of the attractiveness of gilts with the investing public. Agency broking has also continued in gilts and accounts for around 10% of customer business.

At the time of Big Bang, three further firms were permitted to act as money brokers; making a total of nine firms. Although there was some concern that there may not be sufficient stock available to meet the needs of an expanded market making population, and that the existing firms would not be able to fully respond to the new active trading environment, neither of these fears were realized. The SEMBs have expanded the stock available for lending and successfully adapted their operations. Furthermore an additional money broking licence has been granted, to SLH Gilts Money Brokers Ltd.

In December 1987, membership of the gilt-edged settlement procedure (phase 2) (see chapter 2) was made available to any participant in the gilt-edged market, either through direct membership on the same terms as existing members (i.e. all the major participants: GEMMs, IDBs, SEMBs, etc), or via a nominee existing member. This was planned to further reduce the overall paperwork involved in stock transfers, and has worked well with capacities never exceeded.

Although, as explained earlier, it was deemed desirable not to force a centralised price display on the market, there was a fear that inconsistencies in prices may occur between

GEMM trades with customers and GEMM trades with IDBs, and also between deals of normal market size and larger than normal market size. Reporting in the immediate aftermath Big Bang, the Stock Exchange found no evidence of such inconsistencies.

The developments in the primary market since the crash have been arguably more significant than those in the secondary market. The immediate post crash period followed much as before, with another auction of stock planned for January 1988. However, the general market background for the third auction was much less comfortable than it had been for the two previous auctions. In the wake of the stock market crash, interest rates were reduced to stave off recession and correspondingly yields on gilts fell. Uncertainties surrounding the December G7 meeting were also undermining market confidence. Against this background, the auction stock possessed features designed to attract purchasers: it was partly paid; free of tax to residents abroad (FOTRA); could be hedged in the new medium gilt futures contract; and, to enhance its liquidity, comprised part of the largest gilt issue (8 3/4% Treasury 1997 - medium dated). However, bidders held back ahead of the U.S. trade figures announced two days later, and the issue was only just covered (1.06 times). However, the resulting spread of accepted bids and good U.S. trade figures stimulated retail interest thereafter. At that point two further auctions were planned for August 1988 and February 1989.

However, during 1988 government policy towards debt and debt financed budget deficits changed. It was clear that the government intended to achieve a PSBR surplus (or at least avoid at deficit), and that this was going to be achieved, not by raising taxes or cutting expenditure but by using the revenue raised from a series of privatizations in 1987. As a result the PSBR moved into surplus and the government took the opportunity of reducing the national debt by buying back, through the Government Broker, certain issues of stock. The ultimate reversal of plans occurred in January 1989, when the Bank conducted a reverse auction of stock, purchasing £500m of two short dated stocks. There have been no new issues of gilt-edged stock since the end of 1988, and significant buying back (particularly of long dated stock) since the middle of 1988.

A further feature of the post Big Bang era is the important role played by derivative products - notably the gilt-edged futures contracts operated by LIFFE. Although activity in the short and medium contracts is subdued, the long gilt contract has contributed substantially to the market's liquidity, by enabling participants greater scope for risk management. LIFFE's market in options on the long gilt future has served a similar purpose. The Stock Exchange has also developed a framework for negotiated options on individual gilts and some GEMMs have marketed gilt warrants. Subject to appropriate regulation, derivative products seem likely to make an increasing contribution to the liquidity of this market.

In contrast to the successes, and the markets able handling of the aftermath of the equity market crash, there have been a few problems. Firstly, the increased volume of transactions has put heavy pressure on the paper settlements procedures. The Stock Exchange is however, accelerating the introduction of the new automated settlement system (TAURUS). A surprising problem was created as a by product of the new telephone based dealing system. Though committed to deal, if the market is loosing confidence, then a market maker need not pick up the telephone. The wish to do so must however be matched against the loss of any "profitable" business. There has also been some evidence that some of the new merchant bank / broker / jobber conglomerates are suffering from diseconomies of scale. The integration of different activities and personnel have proved less straightforward than was perhaps anticipated.

3.5. Conclusions

This chapter has provided a description of the new gilt-edged market structure introduced at Big Bang, and considered the impact of the changes on operational efficiency. A simple transformation from single capacity and fixed commissions to dual capacity and negotiated commissions bely some extensive changes which took place in the market, such as the introduction of inter-dealer brokers and the substantial increase in market making capacity.

The impact on operational efficiency was analyzed from both a theoretical and empirical perspective. However, it was found difficult to draw predictions from the microstructure theory for two reasons. Firstly, the predictions seemed heavily dependent upon the assumptions of the particular model studied and there was no consensus of opinion among models. Secondly, as they have been so far constructed, providing predictions of relevant outcomes to Big Bang requires them to perform tasks for which they were not designed. Consequently, the impact on operational efficiency is best appreciated from an empirical point of view. It was found that the costs of dealing were around 65% lower after Big Bang than beforehand across the market in deals of average size, and that there was no significant spread premium for deals of sizes well in excess of this figure. Some market makers have also established specialist departments for small deals. The provision of liquidity by the market makers has been highly satisfactory, and robust to the Stock Market Crash. Thus the Big Bang changes have resulted in a significant improvement in the operational efficiency of the gilt-edged market.

TABLE 3.1

GILT-EDGED MARKET DAILY AVERAGE TURNOVER (in millions of pounds)		
YEAR OR MONTH	CUSTOMER BUSINESS	INTRA-MARKET BUSINESS
1981	580	-
1982	804	-
1983	836	-
1984	1066	-
1985	1038	-
1986	1372 *	-
Nov 1986	1400	1844
Dec 1986	1480	1565
Jan 1987	1913	2057
Feb 1987	1907	2155
Mar 1987	2992	2770
Apr 1987	2338	2384
May 1987	2586	2334
Jun 1987	2579	2240.
Jul 1987	1971	1836
Aug 1987	1683	1665
Sep 1987	1746	1723

* : First nine months of 1986 only.

TABLE 3.1 (cont.)

GILT-EDGED MARKET DAILY AVERAGE TURNOVER (in millions of pounds)		
YEAR OR MONTH	CUSTOMER BUSINESS	INTRA-MARKET BUSINESS
Oct 1987	2388	2194
Nov 1987	3114	2735
Dec 1987	2017	1787
Jan 1988	2241	2290
Feb 1988	2219	2185
Mar 1988	2694	2350
Apr 1987	2481	2290
May 1988	1963	1823
Jun 1988	2303	2196
Jul 1988	1991	1933
Aug 1988	1802	1890
Sep 1988	1911	2013
Oct 1988	2343	2135
Nov 1988	2045	2067
Dec 1988	1958	1773
Jan 1989	2632	1737
Feb 1989	2351	2084
Mar 1989	2246	1333

APPENDIX TO CHAPTER THREE

Gilt-Edged Market Makers (March 1986 - March 1989)

Aitken Campbell (Gilts) Limited	*
Bank of America	--
Barclays de Zoete Wedd Gilts Limited	*
Baring Wilson & Watford	*
BT Gilts Limited	
Cater Allen Securities Limited	
Chase Manhattan Gilts Limited	
Citicorp Scrimgeour Vickers Limited	-
CL-Alexanders Laing & Cruickshank Gilts Limited	
CSFB (Gilts) Limited	
Daiwa Europe (Gilts) Limited	+
Gerrard and National Securities Limited	
Goldman Sachs Government Securities (U.K.) Limited	
Greenwell Montagu Gilt-Edged	
Hill Samuel Wood Mackenzie (Sterling Debt) Limited	-
Hoare Govett Sterling Bonds Limited	-
James Capel Gilts Limited	
Kleinwort Benson Gilts Limited	*
Lloyds Merchant Bank (Government Bonds) Limited	-
Merrill Lynch Government Securities Company	*
Morgan Grenfell Government Securities Limited	* -
J P Morgan Sterling Securities Ltd	
NatWest Gilts Limited	
Nomura Gilts Limited	+
Phillips & Drew Moulsdale	*
Prudential-Bache Capital Funding (Gilts) Limited	-
RBC Gilts Limited	-
Salomon Brothers UK Limited	
Shearson Lehman Hutton Gilts Limited	
S G Warburg, Akroyd, Rowe & Pitman, Mullens (Gilt-Edged) Ltd	*
Union Discount Securities	--

Key

* = acquired or were a gilts market jobber pre- Big Bang

-- = withdrew pre-Big Bang

- = withdrew post-Big Bang

+ = joined since Big Bang

CHAPTER FOUR

An Efficient Markets Model of Gilt-Edged Price Movements

4.1. Introduction

In 1970, in an extensive study of U.S. Treasury Bills, Roll developed and tested an efficient markets model of that market. He invoked the established term structure theories to derive models of bond price movements in an efficient market. He found that the pure expectations model was inconsistent with market efficiency, while hypotheses recognizing a premium between forward rates and expected future spot rates performed well. In 1981, these established theories were subjected to a rigorous theoretical examination by Cox, Ingersoll and Ross. They demonstrated that the "expectations hypothesis" of the term structure has several interpretations. Hence the pure expectations model used by Roll is one member of a set of quite distinct alternative models. Furthermore, it is shown that only one of these models, characterized by the equality of expected holding returns for one specific interval, is sustainable in a continuous time rational expectations equilibrium. This model is not the pure expectations model used by Roll. In this chapter, we use the formal term structure analysis of Cox, Ingersoll and Ross, together with the approach of Roll, to develop a model of gilt price movements that can then be used to test the efficient markets hypothesis, in the traditional manner.

This chapter has three main sections. The next section, 4.2, provides a review of the traditional theories of the term structure within the formal framework developed by Cox, Ingersoll and Ross (1981). Then, section 4.3 discusses the relationship between the term structure of interest rates and yield to maturity, to provide further useful insights into the linkages between bond prices and interest rates. In section 4.4, the preceding ideas are used to interpret the efficient markets model in terms of bond prices and the term structure of interest rates. Section 4.5 provides a summary.

4.2. A Review of the Theory of the Term Structure of Interest Rates

In a perfect capital market, the price, P at which any individual would be willing to hold marginal amounts of a security promising a default-free stream of payments is given by

$$P=\frac{C_1}{(1+R_1)}+\frac{C_2}{(1+R_2)^2}+\cdots+\frac{C_n}{(1+R_n)^n} \tag{4.1}$$

where $C_1,C_2,\ldots,C_n$ are the payments in sterling to be made at the ends of periods $1,2,\ldots,n$ and where $R_1,R_2,R_3,\ldots,R_n$ are the market rates of interest applicable to each payment. The relationship between the rates $R_1,R_2,\ldots,R_n$ is called the term structure of interest rates.

It has become conventional to analyse the term structure along lines suggested by Hicks (1946,pp.146-7) who showed that the market for both long and short term loans implies a forward market for loans analogous to a commodities futures market. For example, an individual wishing to make a one period loan commencing in $(n-1)$ periods time may achieve this by simultaneously making a spot loan for n periods and borrowing for $(n-1)$ periods. If the rate of interest on the spot loan is R_n and on the spot borrowing is R_{n-1}, then the rate of interest on the one period forward loan between periods $(n-1)$ and n is

$$\frac{(1+R_n)^n}{(1+R_{n-1})^{n-1}}-1. \tag{4.2}$$

This is called the forward interest rate and is denoted f_n.

4.2.1. Notation

A discussion of the term structure requires that we consider interest rates at various points in time rather than just at a single date, the notation is thus modified to include a further subscript denoting the market date at which the interest rate applies. So,

$$R_{j,t},R_{j,t+1},R_{j,t+2},\ldots \tag{4.3}$$

denotes a time series of "j-period" spot rates for successive market dates $t,t+1,t+2,\ldots$ (the time series of "j-period" "long rates"). Similarly,

$$R_{1,t}, R_{2,t}, R_{3,t}, \ldots, R_{j,t}, \ldots, R_{n,t} \tag{4.4}$$

denotes a set of contemporaneous spot rates applicable to payments to be made at $t+1, t+2, \ldots$ (the set of "long rates" at time t), i.e. the term structure at market date t.

All forward rates are expressed as one period rates, thus $f_{j,t}$ is the forward rate for the period beginning $t+j-1$ determined at market date t. Hence

$$f_{j+2,t-2}, f_{j+1,t-1}, f_{j,t}, f_{j-1,t+1}, f_{j-2,t+2} \tag{4.5}$$

denotes a time series of forward rates all referring to the same period in calendar time, i.e. $t+j-1$ to $t+j$, while

$$f_{1,t}, f_{2,t}, f_{3,t}, \ldots, f_{j,t}, \ldots, f_{n,t} \tag{4.6}$$

is the set of implicit forward rates determined at market date t, for periods $t, t+1, t+2, \ldots$

Since coupon bonds may be regarded as portfolios of pure discount bonds we may for the present assume without loss of generality that all bonds have zero coupons. We also assume that bond markets are perfect.

4.2.2. The Term Structure with Certainty

The way in which the theory is most easily introduced is by considering a simple example. Suppose there is an individual who, at time t, wishes to invest for two periods. One way to do this is to purchase a one period bond, and reinvest the proceeds into another one period bond which matures at time $t+2$. The return over the two periods is

$$(1+R_{1,t})\,(1+R_{1,t+1}). \tag{4.7}$$

An alternative way is to purchase a two period bond in which case the proceeds over the two year period would be

$$(1+R_{2,t})^2 = (1+R_{1,t})\,(1+f_{2,t}). \tag{4.8}$$

The difference between these two returns is

$$(1+R_{1,t})\,(1+R_{1,t+1}) - (1+R_{1,t})\,(1+f_{2,t}) = (1+R_{1,t})(f_{2,t} - R_{1,t+1}) \tag{4.9}$$

and it is the relationship between the forward rate and the future spot rate which determines whether this difference is positive, negative or zero. But, suppose that the individual wished to invest for only one period. In that case he could buy a one period bond with a return $(1+R_{1,t})$. Alternatively, he could purchase a two period bond and sell it after one period. In that case the return would be

$$\frac{(1+R_{1,t})\,(1+f_{2,t})}{(1+R_{1,t+1})}\ . \tag{4.10}$$

The difference between the returns on the two strategies is

$$\frac{(1+R_{1,t})\,(1+f_{2,t})}{(1+R_{1,t+1})} - (1+R_{1,t}) = \frac{(1+R_{1,t})}{(1+R_{1,t+1})}\,(f_{2,t}-R_{1,t+1}) \tag{4.11}$$

which is also entirely dependent on the difference between future spot rate and forward rate for its sign.

Term Structure theory tries, inter alia, to answer the question "What determines the difference between forward rates and future spot rates?". If the future spot rate was known with certainty, then individuals would choose the investment with the highest return. Thus for more than one bond to be sustainable, equilibrium must be characterized by indifference between between investment alternatives, that is, the forward rate and the future spot rate must be equal.

4.2.3. The Term Structure under Uncertainty

In reality, the future spot rates are unknown. Now the individual with the two period horizon can obtain a certain return by investing in the two period bond or an uncertain return by investing in two one period bonds. Similarly, the individual with the one period horizon can obtain a certain return by investing in the one period bond or an uncertain return by purchasing the two period bond and selling it after one year. The unknown future spot rate is the cause of the uncertainty in each case. Term structure hypotheses under uncertainty seek to explain the relationship between the forward rate implicit in the known return and the expectation of the value of the unknown future spot rate, that is, the relationship between short and long rates of interest.[1]

The Expectations Hypothesis

Most of the theory underlying the Expectations Hypothesis was not developed until the late 1930's by notably, Hicks (1939) and Lutz (1940). These and other authors have a set out a number of economic propositions concerning the relationship between long and short rates. For example, Lutz (1940,pp.37,49) states "An owner of funds will go into the long market if he thinks the return he can make there over the time for which he has funds available will be above the return he can make in the short market over the same time, and vice versa," and "a lender who wants to invest for only one year is in principle prepared to buy (...) a bond of any maturity and sell it again after the first year."

Probably the broadest interpretation of these propositions is the conclusion that bond market equilibrium is characterized by an equality among expected returns on all possible investment strategies over all holding periods. Thus for the holding period t to $t+n$, the return on any feasible series of investment must have the same expectation:

$$E\left[\frac{\tilde{P}_{t+1,t+1}}{P_{t,t+1}}.\frac{\tilde{P}_{t+2,t+2}}{\tilde{P}_{t+1,t+2}}.\quad\cdots\quad.\frac{\tilde{P}_{t+n,t+n}}{\tilde{P}_{t+n-1,t+n}}\right]=\phi_{t+n,t} \tag{4.12}$$

where the price of a bond quoted at $t+i$ which matures at time $t+j$ is given by $P_{t+i,t+j}$.[2] The expected return ϕ must be independent of the arbitrary reinvestment times $t+i$ $(i=0,1,...,n)$ and the bonds selected, as denoted by their maturity dates $t+j$ $(t+j \geq t+i)$.

However, this relationship can be simply invalidated (see, for example, Cox, Ingersoll and Ross, (CIR) (1981), p.775). We note that equation (4.6) requires that the expected return

[1] This review section will not consider the recent arbitrage theories of, inter alia, Vasicek (1977), Brennan and Schwartz (1979) and Cox, Ingersoll and Ross (1985). These are discussed in chapter seven, prior to empirical work set within that particular framework. A review of empirical tests of the theories discussed below can be found in Dobson, Sutch and Vanderford (1976).

[2] The reader should note that where there are two subscripts on price notation, the first subscript relates to the date of quotation. For interest rates, where two subscripts are used, the date of quotation is the second subscript (see section 4.2.1).

on a bond maturing at t+2 over the period $(t,t+1)$ must be equal to the certain return on a bond maturing at t+1

$$\frac{1}{P_{t,t+1}} = \frac{E(P_{t+1,t+2})}{P_{t,t+2}} \tag{4.13}$$

where $P_{t+n,t+n}$, the price at maturity, equals unity. Furthermore, the return expected over $t,t+2$ on a bond maturing at t+1 rolled over at maturity into a bond maturing at t+2 must equal the guaranteed return on the bond maturing at t+2,

$$\frac{1}{P_{t,t+2}} = \frac{1}{P_{t,t+1}} E\left[\frac{1}{P_{t+1,t+2}}\right]. \tag{4.14}$$

Together these equations imply that

$$E\left[\frac{1}{P_{t+1,t+2}}\right] = \frac{P_{t,t+1}}{P_{t,t+2}} = \frac{1}{E(P_{t+1,t+2})}. \tag{4.15}$$

But by Jensen's inequality, (4.9) cannot be true except in the case of certainty. So all expected returns for all holding periods can never be the same in equilibrium.

This contradiction can be avoided if it is postulated that expected holding period returns are equal for one specific holding period. The natural choice is the next basic (i.e. "shortest") interval. Under this assumption the Expectations Hypothesis is characterized by

$$\frac{E_t(P_{t+1,t+n})}{P_{t,t+n}} = 1+R_{1,t} \tag{4.16}$$

which when evaluated recursively leads to the statement of equilibrium

$$P_{t,t+n} = E_t([(1+R_{1,t})(1+R_{1,t+1})...(1+R_{1,t+n-1})]^{-1}). \tag{4.17}$$

Equation (4.16) has been labelled the *Local Expectations Hypothesis* , (L-EH), by Cox, Ingersoll and Ross (1981).

Traditionally, the equilibrium relation of the expectations hypothesis has been stated in a different fashion. It is often assumed, as implied by Lutz's first quote above that the guaranteed return from holding any discount bond to maturity is equal to the return expected from rolling over a series of single period bonds. This is a special case of (4.6) where the arbitrary reinvestment times are restricted to be at single period intervals, and can be written as

$$\frac{1}{P_{t,t+n}} = E_t[(1+R_{1,t})(1+R_{1,t+1})...(1+R_{1,t+n-1})]. \tag{4.18}$$

Cox, Ingersoll and Ross (1981) label this model the *Returns to Maturity Expectations Hypothesis* , (RTM-EH).

Alternatively, some authors make a similar assumption concerning the equality of expected yields. For example, Malkiel (1966,p.20) states that "...all differentials in anticipated holding period yields [are] completely eliminated..."; i.e.,

$$[P_{t,t+n}]^{-1/n} = E_t([(1+R_{1,t})(1+R_{1,t+1})...(1+R_{1,t+n-1})]^{-1/n}). \tag{4.19}$$

This model is called the *Yield to Maturity Expectations Hypothesis* , (YTM-EH).

In other cases it is assumed that forward rates are unbiased estimates of future spot rates. This form of the hypothesis, associated primarily with Lutz (1940) and Meiselman (1962), is known as the *Pure* or *Unbiased Expectations Hypothesis* (U-EH), and can be written as

$$(1+f_{n,t}) = E_t(1+R_{1,t+n-1}). \tag{4.20}$$

This was the form of the Expectations Hypothesis examined by Roll (1970). In this form, the implications of the Expectations Hypothesis for the shape of the term structure are easily seen. If investors expect the future spot rate to be higher than the current spot rate, then according to (4.14), the forward rate will also be higher than the current spot rate. Thus the two period rate must be greater than the one period rate, and hence the term structure is upward sloping. Conversely, this hypothesis says that the term structure will only slope downwards if investors expect future spot rates to be lower than current spot rates.

Equation (4.20) may be equivalently written (for zero coupon bonds only) as

$$\frac{P_{t,t+n-1}}{P_{t,t+n}} = E_t(1+R_{1,t+n-1}). \tag{4.21}$$

A recursive evaluation of (4.21) gives the equilibrium condition

$$\frac{1}{P_{t,t+n}} = (1+R_{1,t})E_t(1+R_{1,t+1})...E_t(1+R_{1,t+n-1}). \tag{4.22}$$

This is equivalent to the (RTM-EH) hypothesis if the levels of future interest rates are not autocorrelated. However, empirical evidence suggests that interest rate levels are autocorrelated.

Many researchers have not always been careful in distinguishing among the hypotheses, perhaps (also) not realizing that the first three are (pairwise) incompatible. Now if we define the random variable $Z = [(1+R_{1,t})(1+R_{1,t+1})...(1+R_{1,t+n-1})]^{-1}$, then equations (4.17) through (4.19) can be rewritten as

$$P = E[Z] \qquad (4.17')$$

$$P^{-1} = E[Z^{-1}] \qquad (4.18')$$

$$P^{-1/n} = E[Z^{-1/n}]. \qquad (4.19')$$

Jensen's inequality assures us that at most only one of these expressions is valid. If equation (4.19) describes equilibrium, then the yield on an n period bond will be greater than the value given in (4.17) and less than the value given in (4.18). On the other hand, if equilibrium condition (4.17) [condition (4.18)] is valid, the equations (4.18) and (4.19) [equations (4.17) and (4.19)] give long yields which are too large [small].

The Expectations Hypothesis in Continuous Time

In a continuous time formulation, the U-EH is equivalent to the YTM-EH (see CIR,1981,p.776) and not the RTM-EH as was possible (but unlikely) in a discrete time framework. Furthermore, CIR (1981) show that the L-EH is the only version of the Expectations Hypothesis which is sustainable in a continuous-time rational expectations equilibrium. In this case, if r_t is the instantaneous rate of interest at time t, then the L-EH can be expressed as

$$\frac{E_t(dP_{t,t+n})}{P_{t,t+n}} = r_t dt \qquad (4.23)$$

which by integration becomes

$$P_{t,t+n} = E\left[\exp\left[-\int_t^{t+n} r_s ds\right]\right]. \qquad (4.24)$$

This equation says that the bond's current price is the expected discounted value of the promised unit payment.

However, Ahn and Thompson (1988) have shown that if the path of the short rate is assumed to follow a jump-diffusion process (i.e. displays discontinuities), then not even the L-

EH is obtained in the CIR equilibrium, as the premium associated with the jump process is still present in the L-EH. Before considering alternative hypotheses, we consider the alleged role of risk neutrality in the Expectations Hypothesis.

The Expectations Hypothesis and Risk Neutrality

Originally, it was thought that the Expectations Hypothesis was generally obtained under conditions of risk neutrality. That is when the only parameter governing investment decisions is expected returns. However, Meiselman (1962, p.10, discussing in particular the U-EH) argued that it was not necessary to assume that risk aversion was entirely absent since, "individual transactors may still speculate or hedge on the basis of risk aversion, but the speculators who are indifferent to uncertainty will bulk sufficiently large to determine market rates on the basis of their mathematical expectations alone." On the other hand, Bierwag and Grove (1967, p.50) pointed out that it was difficult to imagine an equilibrium in a frictionless market dominated by 'plungers' of this kind unless they had identical expectations.

CIR (1981), for their preferred version of the hypothesis, the L-EH, show that it is also not the natural consequence of universal risk neutrality, except when interest rates are non stochastic or other special circumstances obtain. They show firstly that in a risk neutral pure exchange economy interest rates must be non stochastic. Thus all forms of the EH are sustainable but only in the singular sense of a certainty model. Secondly, they show that in a risk-neutral production-exchange economy interest rates may be certain or stochastic. In the former case, the Expectations Hypotheses hold (again in the singular sense); in the latter case the L-EH does not generally obtain. Finally, they show that for each type of economy non trivial conditions exist under which the L-EH is sustained for risk averse agents.

The Liquidity Preference Hypothesis

Building on the Keynesian idea of "normal backwardation," it was argued (first by Hicks,1939), that the forward rate will normally exceed the expected spot rate. In other words, the expected rate of return on a long bond must exceed the expected rate on a short bond by a premium which compensates the lender for assuming the increased risks of price fluctuations. Hence, this model contains the strong behavioural assumption that risk premia are uniformly positive.

Hicks (1946,p.146-7) made his argument in three parts. First, "these persons [borrowers] will want to hedge their future supplies of loan capital, just as they will want to hedge their future supplies of raw materials. They will have a strong propensity to borrow long." Second, "if no extra return is offered for long lending, most people (and institutions) would prefer to lend short, at least in the sense that they prefer to hold their money on deposit on some way or other." Finally, he argued that to offset this "constitutional weakness" in the supply of long funds, speculators will borrow short and lend long only in return for a premium as "compensation for the risk they are incurring."

Market Segmentation Hypotheses

In his Hedging Pressure Hypotheses, Culbertson (1957) asserts that the market is dominated by hedging rather than speculative behaviour. He argued that individuals have strong maturity preferences and that bonds of different maturities trade in separate and distinct markets. The demand and supply of bonds of a particular maturity are supposedly little affected by the prices of bonds of neighbouring maturities. There is now no reason for the term premiums to be positive and an increasing function of maturity. However, as Copeland and Weston (1983,p.70) point out that "While the market segmentation hypothesis can explain why implied forward rates and expected rates may differ, the direction and magnitudes are not systematic". Furthermore, the argument that bonds of close maturity will not be close substitutes, is difficult to sustain.

The Preferred Habitat Theory of Modigliani and Sutch (1966) began with the same approach. They defined an investor to have n-period habitat if "he has funds which he will not need for n periods and which, therefore, he intends to keep in bonds for n periods." However, recognizing the implied inefficiency, they did not go so far as to assume that this investor considers only n-period bonds. Investors can be "tempted out of their natural habitats by the lure of higher expected returns." They intended their approach as a plausible rationale for term premiums that does not restrict them in sign or monotonicity, rather than as a necessary causal explanation.[3] In fact, the hypothesis implies that at least some term premia will be non-zero and the relationship between them and maturity will be a smooth one.

Roll (1970) distinguishes two possible general forms of the Market Segmentation Hypothesis. Firstly, where the premium relating to a particular period in calendar time remains constant through time; secondly, where the premium relating to a particular maturity remains constant.

4.2.4. Remarks on the Term Structure Theory

It should be noticed that the above hypotheses are not mutually exclusive. In terms of its implications, the Liquidity Premium Hypothesis can be regarded as a special case of the Preferred Habitat Hypothesis, namely that case in which all investors have a habitat equal to the shortest holding period. Furthermore, the pure expectations hypothesis which asserts that all term premia are zero can be regarded as a form of the preferred habitat (where the preferred habitat of all market participants encompasses the whole maturity spectrum).

A theoretical attempt to validate the various models has been undertaken by CIR (1981). They show, using a simple production economy, that unlike Hicks, term premiums may be either positive or negative. Furthermore, they give a different interpretation to this result than that of Modigliani and Sutch. They show that it is not preference for consumption at different times which creates "habitats," but rather the degree of risk aversion; that is, the tendency to hedge against changes in interest rates, as suggested by Merton (1973).

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[3] See CIR (1985) footnote 2.

The Term Structure and Uncertain Inflation

Term structure theory has generally assumed that future consumption prices are known, i.e. that the inflation rate is deterministic. Thus uncertainty about future rates of interest was assumed to reflect only uncertainty in real rates of return. However, most bonds are denominated in nominal terms. Indeed inflation uncertainty comprises the predominant portion of interest rate risk at least for short term bonds (e.g. Treasury Bills).

A paper by Brealey and Schaefer (1977) took the opposite approach. They assume that the ex-ante real rate of interest is deterministic, but that the rate of inflation is uncertain. By assuming that investors make rational forecasts of the inflation rate, they show that real returns on bonds are, under their assumptions, serially independent, and, by further assuming suitable utility functions, were able to derive an equilibrium solution to the term structure. The relationship between forward rates and expected spot rates which they derive is superficially similar to the liquidity preference hypothesis in that term premia are universally non-negative. However, these premia must be interpreted as risk premia rather than liquidity premia. This is because, under their assumptions, the term structure is independent of the maturity preferences of borrowers and lenders. In their analysis long term bonds are riskier than short term bonds for all market participants.

CIR (1981) use a model economy, in which the term structure of real interest rates is non stochastic, to examine the Expectations Hypothesis under uncertain inflation. The no arbitrage condition under deterministic real rates is that those bonds with payoffs denominated in the real numeraire must be equal (e.g. inflation indexed bonds). For these bonds, then, all forms of the EH will hold. However, with uncertain inflation nominal interest rates will be stochastic, and nominal bonds of different maturities may have different expected rates of return. CIR show that if inflation is stochastic, only the L-EH can be sustained for nominal rates if risk premia are zero. Furthermore, they show that where investors are risk neutral, a sufficient condition to generate this result is that inflation must have no real consequences within the economy. Hence they conclude, "This, or similar cases, appear to be the only situation in which risk-neutrality is compatible with both the EH and stochastic interest rates, and it obtains only because inflation has no real consequences in the economy." (CIR, 1981,pp.788-789). More recently, CIR have incorporated uncertain inflation into a general model with a stochastic term structure of real interest rates, (1985, p.401-405).

Implications of Market Efficiency

The traditional term structure theories are essentially only hypotheses which say little more than that forward rates should or need not equal expected spot rates. Furthermore they are all theories couched in ex-ante terms and they must be linked with ex-post realizations to be testable.

Attempts to deal with these two elements constitute a further area of work on the term structure. An important result concerning the first element is given in Roll (1970) who shows that the sequence of forward rates , adjusted by corresponding term premia, e.g. $f_{j,t}-\pi_{j,t}$, applicable to a given period in calendar time follows a pure martingale sequence. That is:

$$E_{t-1}(f_{j,t} - \pi_{j,t}) = f_{j+1,t-1} - \pi_{j+1,t-1} \tag{4.25}$$

Roll calls this the "fundamental dynamic equation for an efficient loan market" (1970, pp.22-33). He derives it from firstly the futures market implicit in a loan market and secondly Samuelson's (1965) results concerning the fluctuations of futures prices.[4]

With regard to the second element, Roll (1970), for example, has built and tested a mean-variance model and treated bonds symmetrically with other assets and used a condition of market efficiency to relate ex ante and ex post concepts. If rationality requires that ex post realizations do not differ systematically from ex ante views, then statistical tests can be made on ex ante propositions by using ex post data.

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4 It should be noted that the risk premia in this equation refer to the same period in calendar time, and hence have a different interpretation from those in equation (4.35).

Taxation

According to the market segmentation hypotheses, investors have strict preferences for certain types of bonds, and these trade in distinct markets. This expression of choice can be explained where, for example, investors face different rates of income tax. The differential effect of taxes means that, if at some tax rate two bonds with different coupons offer the same return, they must offer different returns at all other tax rates. Taxes therefore produce segmentation in the market. Non-tax payers will favour the highest coupon bonds and high tax payers will favour the lowest coupon bonds. Since payments on short bonds consist primarily of payments of principal, short bonds will also be relatively more attractive to high rate tax payers. Long bonds, which consist primarily of payments of coupon, will be relatively attractive to low rate tax payers. Bonds therefore have a "clientele" of investors who rationally hold that bond, and spot rates will be different across tax brackets.

It has also been shown (Schaefer, 1980) that the differential effect of taxes causes the basic bond pricing equation to no longer be a no-arbitrage equilibrium relationship. To remove arbitrage possibilities, a friction in the trading process has to be introduced, and a sufficient (but not necessary) condition is to restrict short selling. This restores the no arbitrage condition, but redefines the equilibrium as an inequality with price greater than or equal to present value.

Work on tackling the tax effects present in the market has occupied the designers of complex mathematical models of the gilt-edged market. The approach is essentially one of finding a suitable three dimensional model (equation) linking price, coupon and maturity. Feldman (1977) suggested that at any particular time, there exist two discounting functions, one for income items (which are taxable) and one for capital payments. Furthermore, the coefficients of these functions can be found by applying least squares to the model and from these coefficients model prices can be found. The primary drawback with this model is that it is linear in coupon, when in fact investors tend to place price premia on stocks at the extremities of the coupon range - the "high rate tax payer / low coupon" effect already described. In response, Clarkson (1979) eschews the aforementioned compound interest functions and establishes a relationship instead between the flat yield g/P (coupon divided by price), and the proportionate capital gain to redemption, $1/P$, for all stocks. Yip (1986) provides evidence in favour of the Schaefer treatment of taxes, as against the mathematical models, and the agnostic view is presented by Leung (1980) who combines insights from both Schaefer and Feldman/Clarkson in a gilt market model.

4.3. The Term Structure and Yield to Maturity

The measure used to describe bond market returns is known as yield to maturity (or, redemption yield). For a gilt-edged security with coupon rate $g\%$ and redemption value F, yield to maturity y solves the equation

$$P = \frac{gF}{(1+y)} + \frac{gF}{(1+y)^2} + \cdots + \frac{gF}{(1+y)^n} + \frac{F}{(1+y)^n} \qquad (4.26)$$

In the term structure discounting equation (4.1), payments to different bonds at the same point in time were discounted at the same rate, and payments to the same bond at different points in time were discounted at different rates. In the above yield to maturity equation, payments to the same bond at different points in time are discounted at the same rate, and payments to different bonds at the same point in time are discounted by different rates. Yield to maturity is, therefore, the rate of interest that would produce the same present value or price, as was obtained by discounting by the different spot rates, that is, it is an internal rate of return. It is clear from this expression that yield is a complex average of spot rates. In fact its exact form as the root of a polynomial of order n cannot be written down.

4.3.1. The Problems with Redemption Yields[5]

Redemption yields continue to be used by investment managers as a basic measure of return and hence the relative attractiveness (implicitly, therefore, value) of particular bonds. So it is worth considering the limitations of this concept both as a measure of return and as a measure of value.

As a measure of return it can describe only two facts. Firstly, given the current price, the yield that solves equation (4.26) will only be the return over the next period in one of two cases; a situation of certainty for all time, and a situation of certainty over the next period. Secondly, if the bond is held until redemption, yield is the equivalent rate of return per period. Yield clearly has strong limitations as a measure of return.

The cash flows from holding a coupon bond to maturity are equivalent to a package containing an n-year annuity of gF, plus a single payment of F in year n. The yield to maturity is discounting all the cash payments at the same rate of interest, and measures the average annual cost of the gF annuity and the final payment. Typically, different bonds will represent different combinations of annuity and final payment, and different packages will have different yields. Hence a comparison of yields, is not a comparison of like with like. Unless bonds offer the same package of cash flows over time they will have different yields.

The dependence of yield to maturity on the coupon level has several implications. It means that a conventional yield curve - a smooth curve drawn through yields to maturity plotted against maturity - is to some extent arbitrary because the points do not even in theory lie on a smooth curve. In order to obtain smooth curves, it is necessary to plot a separate curve for each coupon level. However, even supposing that this could be done, it is unclear what information it would convey. Yields are derived from bond prices and hence contain no additional information concerning value than that represented by the bond's price.

[5] The arguments in this subsection were originally published by Schaefer (1977) and proofs are given in Schaefer (1979).

It is more useful to estimate the sequence of spot rates. These correspond to the yields on bonds which pay no coupons. They are harder to estimate but once known, can be used to evaluate any bond. The spot rates may well lie on a smooth curve. Once this curve is known a further useful form of yield curve can be derived from it. This is called the annuity yield curve and it shows the correct yield for a constant stream of annuity payments of any length. As annuity yield is itself an average of spot rates, it will tend to lie below a rising spot rate curve and above a falling curve.

The spot rate curve depicts the yield to maturity on a zero coupon bond. The annuity yield curve depicts the yield to maturity on a zero principal bond. Since most bonds lie between these two extremes: they offer positive coupons and positive principal, so the yield to maturity on coupon bonds must lie between these two curves. Clearly, the lower the coupon level, the closer will be the yield curve to the spot rate curve, and the higher the coupon level, the closer will be the yield curve to the annuity yield curve. If the bond is very short, the present value of principal repayment is likely to be much larger than the present value of the coupon payments. So, the yield to maturity will be fairly close to the spot rate curve. If the bond is very long, the present value of principal is relatively small and the yield curve tends to the annuity curve. For a monotonic spot rate curve, we arrive at a situation depicted in figure 4.1. Notice that the 1% coupon curve displays a pronounced hump and that all yield curves except the spot rate curve tend towards the annuity curve. In fact these same curves are all asymptotically horizontal no matter what shape the spot rate curve adopts.

The problems with yield to maturity also extend to measures based upon it, of which volatility and duration are cases.

4.3.2. Duration and Volatility

We have seen how, in a certain world, price change is related to the level of the yield to maturity. From the algebra of the yield formula, it is also possible to establish the following relationships. Ceteris paribus, the percentage price change for any given percentage change in yield is larger (a) the smaller is the initial yield level, (b) the larger is the term to maturity, and (c) the smaller is the coupon rate. These relationships are shown in figures 4.2-4.4.

The above price/yield relationships are summarized in a measure of the average life of a security which is known as the "duration" of the security. Duration takes its name from its interpretation as a weighted average of the dates on which cash flows are promised. The weight used at any date is the ratio of the present value of the cash flow at that date to the price. Thus if $C=gF$ is the coupon payment, duration D is given by

$$D=1\frac{C}{P(1+y)}+2\frac{C}{P(1+y)^2}+...+n\frac{C+F}{P(1+y)^n}. \tag{4.27}$$

It is argued that duration is a preferable measure of the life of a security than maturity as it is a function of all the cash flows. It is straightforward to show that duration is equivalent to the measure of the responsiveness of price to a change in yield. Differentiating equation (4.26) term by term with respect to yield we obtain

$$\frac{dP}{dy}\approx\frac{-C}{(1+y)^2}+\frac{-2C}{(1+y)^3}+... \tag{4.28}$$

and so

$$\frac{-dP}{dy}.\frac{(1+y)}{P}\approx\frac{C}{P(1+y)}+2\frac{C}{P(1+y)^2}+...+n\frac{C+F}{P(1+y)^n}. \tag{4.29}$$

which is just the duration formula.[6] A related formula is known as volatility, that is, the proportional change in price per unit change in yield, i.e.,

$$V=\frac{dp}{dy}.\frac{1}{p} \tag{4.30}$$

or

$$V=-(1+y)^{-1}\,.\,D$$

Volatility is used more frequently in the U.K. by bond market practitioners than duration. It is

[6] Note that this formula is only a close approximation to duration when changes in yield are small.

used as both a measure of risk and to select bonds given an assessment of future interest rates. Thus if bond A has a higher volatility than bond B it would be argued that a given fall in yield would produce a larger proportional price increment in A than B, and, if yields were predicted to decline, A would be chosen rather than B.

However, we saw earlier that the yield to maturity on a bond with a given maturity depends on the size of the coupon and, as a result, yields to maturity on bonds with different coupon levels cannot usefully be compared. This problem applies not just to yields to maturity, but also volatility and duration which are based upon it.

This means that a given change in spot rates will usually produce different yield to maturity changes on bonds with different coupons. So volatility cannot be used to assist prediction of relative returns on bonds in the manner described above. The relationship between price change, volatility and change in yield to maturity is a tautology for a particular bond but does not help us to predict returns on other bonds as the term structure changes. A solution has been provided however within the field of bond portfolio immunization.

4.3.3. Immunization and Generalized Duration

An immunization strategy is one in which a portfolio of (component) bonds is managed so that its value is always as close as possible to the value of another asset: the target. The idea, and the term immunization, were introduced by Redington (1952), an actuary who proposed it as a means for life assurance companies to mitigate the effects of interest rate changes on their net worth. The essence of Redington's strategy is to set the duration of the assets and liabilities equal.

Such a measure of duration must fully capture the effects of changes in factors affecting the term structure, and is therefore better served by a formula which relates bond price to spot rates rather than yield to maturity. We may construct a generalized model of duration in the following manner.[7]

Given a continuous stream of cash flows $C(t)$, $t=0,1,...,n$ for a bond with price P, we rewrite the basic discounting equation (4.1) in continuous time as

$$P = \int_0^n C(t)\exp^{-tR(t)}\, dt \tag{4.1'}$$

Using equation (4.2), also in continuous time

$$R = \frac{1}{n}\int_0^n f(s)\, ds, \tag{4.2'}$$

we may replace the spot rates by the implied forward rates since, and so

$$P = \int_0^n C(t)\exp^{-\int_0^t f(s)\, ds}\, dt \tag{4.31}$$

If we assume that the forward rate to moves as a function of one factor, namely the short rate r, we may rewrite equation (4.31) as

$$P = \int_0^n C(t)\exp^{-\int_0^t f(s,r)\, ds}\, dt \tag{4.32}$$

Differentiating with respect to the short rate r and dividing by P to obtain the proportional change gives

$$\frac{dP}{dr}.\frac{1}{P} = \frac{\int_0^n \left[\int_0^t \frac{\partial f}{\partial r}(s,r)\, ds\right] C(t)\exp^{-\int_0^t f(s,r)\, ds}\, dt}{P} \tag{4.33}$$

This is the generalization of the usual duration formula. Its principal difference from the yield to maturity special case is that the cash flow date t in the numerator integral in the yield case is itself replaced by an integral of forward rate elasticities in the general case. For parallel shifts in the yield curve, the integral of forward rate elasticities equals t and we have the conventional duration measure.

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[7] This generalization of duration follows Hodges (1975).

The limitation with this duration formula is that it is only a one factor model (the forward rate curve just depends on the short rate). By specifying a joint stochastic process for k influencing factors and applying results from stochastic calculus, Nelson and Schaefer (1983 pp.63-4) have fully generalized this result.[8]

4.4. An Efficient Markets Model of Gilt-Edged Prices

The movement of prices in speculative markets has traditionally been couched in terms of the random walk hypothesis. For a return from holding the asset from day $t-1$ to day t defined by $x_t=\log_e(P_t/P_{t-1})$, where P_t is the price at the end of day t, the random walk hypothesis says that daily returns are uncorrelated and have a constant mean, i.e.

$$x_t = \mu_x + e_t \qquad (4.34)$$
$$E(e_t) = 0, \qquad E(e_t e_{t+i}) = 0 \quad (i \neq 0).$$

This can be distinguished from the weak form of the efficient markets model, which states that prices accurately reflect all past price information and reward rational investors for accepting risk (Fama, 1970; Jensen, 1978). Let RF_t be the return from risk free investments and let $RP_{x,t}$ be the risk premium. The statistical representation of the model can be written as:

$$x_t = \mu_{x,t} + e_t$$
$$\mu_{x,t} = RF_t + RP_{x,t} \geq 0. \qquad (4.35)$$
$$E(e_t) = 0, \qquad E(e_t e_{t+i}) = 0 \quad (i \neq 0), \qquad \text{cov}(\mu_s, e_t) = 0 \quad (\text{all } s,t).$$

Furthermore, both the random walk model for $\mu_x \geq 0$ and the efficient markets model are special cases of the sub-martingale process (Fama, 1970):

$$E[x_t \mid \text{all } x_{t-i},\ i \geq 1] \geq 0. \qquad (4.36)$$

Both this and the special cases can be assessed by evaluating trading rules, since the sub-martingale property implies that a rule based on past price information cannot outperform a

[8] Further details of this study and the stochastic calculus, arbitrage models are given in chapter seven.

buy and hold strategy.

Using results presented in the previous sections, the efficient markets model can be interpreted in terms of bond prices and the term structure of interest rates. This will provide an efficient markets model of bond price movements.

Let us consider the CIR preferred model of the Expectations Hypothesis, the L-EH, as described by equation (4.16). This can be alternatively written as

$$\frac{E_t(P_{t+1,t+n})-P_{t,t+n}}{P_{t,t+n}} = R_{1,t} \tag{4.16'}$$

which simply says that the one period expected return from time t to time $t+1$ is the one period rate for loans beginning at time t. In other words the expected return for bonds of different maturities over the next holding interval is the same. If we define the observed relative change in price from $t-1$ to t as x_t, we obtain the model

$$\begin{gathered} x_t = \mu_{x,t} + e_t \\ \mu_{x,t} = R_{1,t-1} \\ E(e_t) = 0, \qquad E(e_t e_{t+i}) = 0 \quad (i \neq 0), \qquad \text{cov}(R_{1,s}, e_t) = 0 \quad (\text{all } s,t). \end{gathered} \tag{4.37}$$

This is exactly, what would obtain in the efficient markets model (4.35) where there were no risk differentials between securities, that is, the expected rate of return is equal to the risk free rate of return. Furthermore, if the risk free rate were constant through time, the random walk model can be interpreted as the L-EH. If risk preferences influence investment decisions or taxes segment the market, the efficient markets model can capture fully such effects with the term $RP_{x,t}$, in (4.35).[9] Finally, we note once more that although term structure theory assumes bonds pay no coupons, that coupon bonds are simply portfolios of pure discount bonds allows us to carry across the above relationships to model coupon bonds.

[9] It should be noted that the risk premium term here is different from that used by Roll in his "fundamental dynamic equation". For example, Roll compares the premium on a five year bond next now with that on a four year bond next year. In this case, we are concerned with the movements of "the five year risk premium" through time.

4.5. Summary

This chapter has developed an efficient markets model of gilt price movements. It is shown how such a model follows naturally from the recent reinterpretation of the traditional theories of the term structure by Cox, Ingersoll and Ross (1981).

In the next chapter, this model will be used to examine the impact of the Big Bang deregulation on the efficiency of the gilt-edged market, within a traditional efficient markets testing framework.

Figure 4.1. The Relationship between Spot Rates, Annuity Yields and Yields on Bonds with Constant Coupon. Source: Schaefer 1977

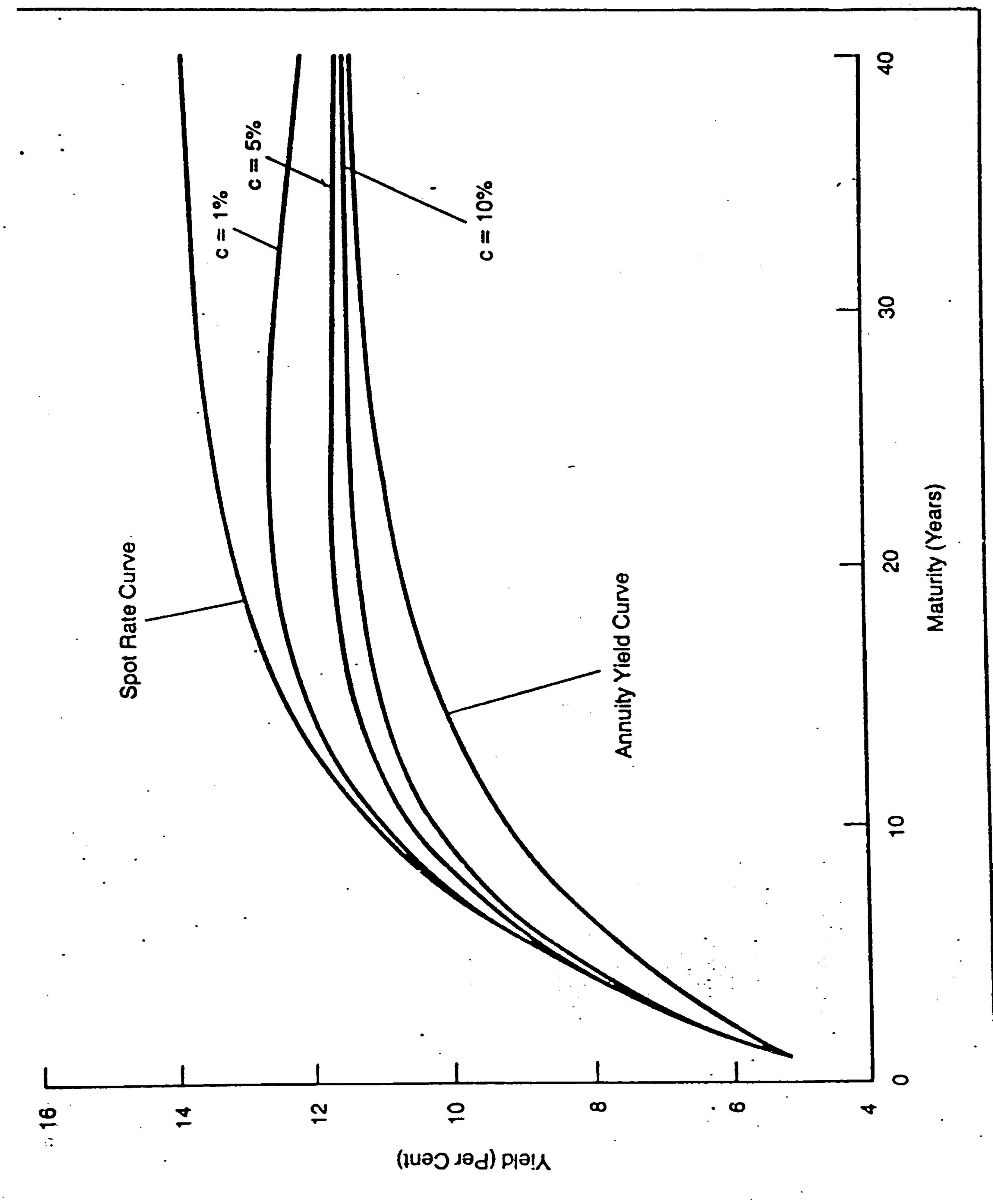


Figure 4.2, Bond Price / Yield Relationship

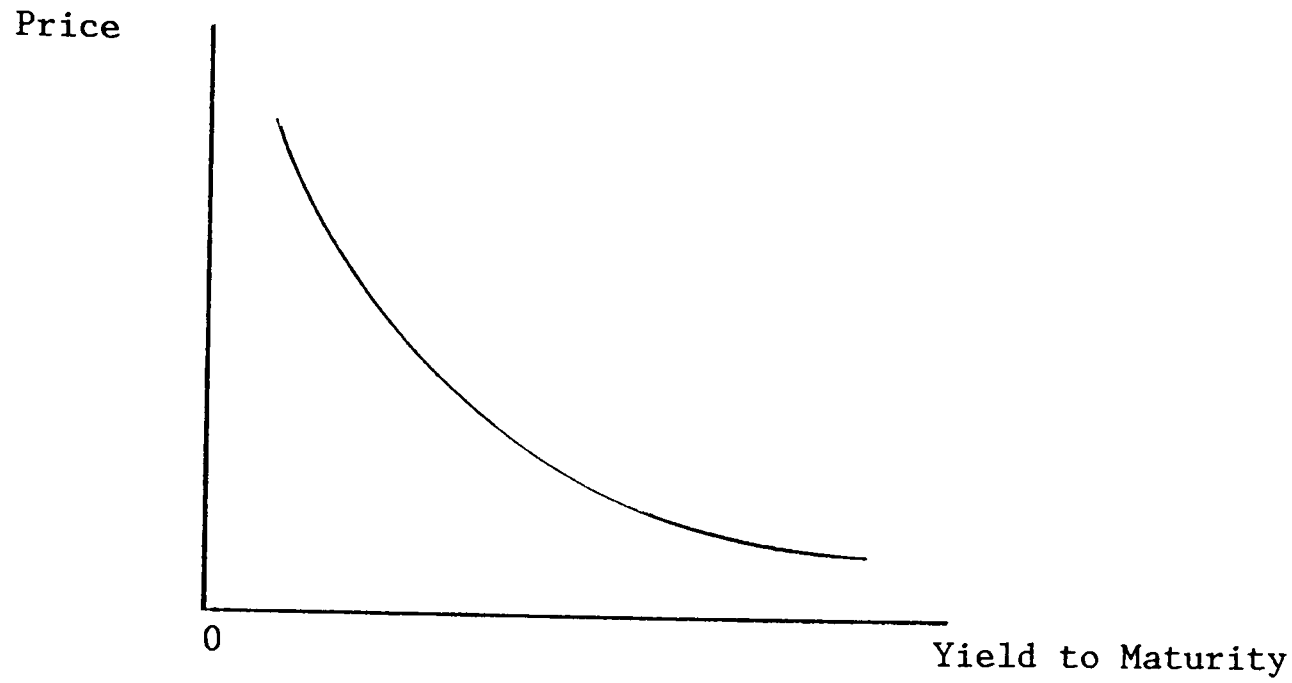


Figure 4.3, Bond Price / Yield Relationship:
Coupon Rate = 10%; Bond 1 Maturity greater than
Bond 2 Maturity

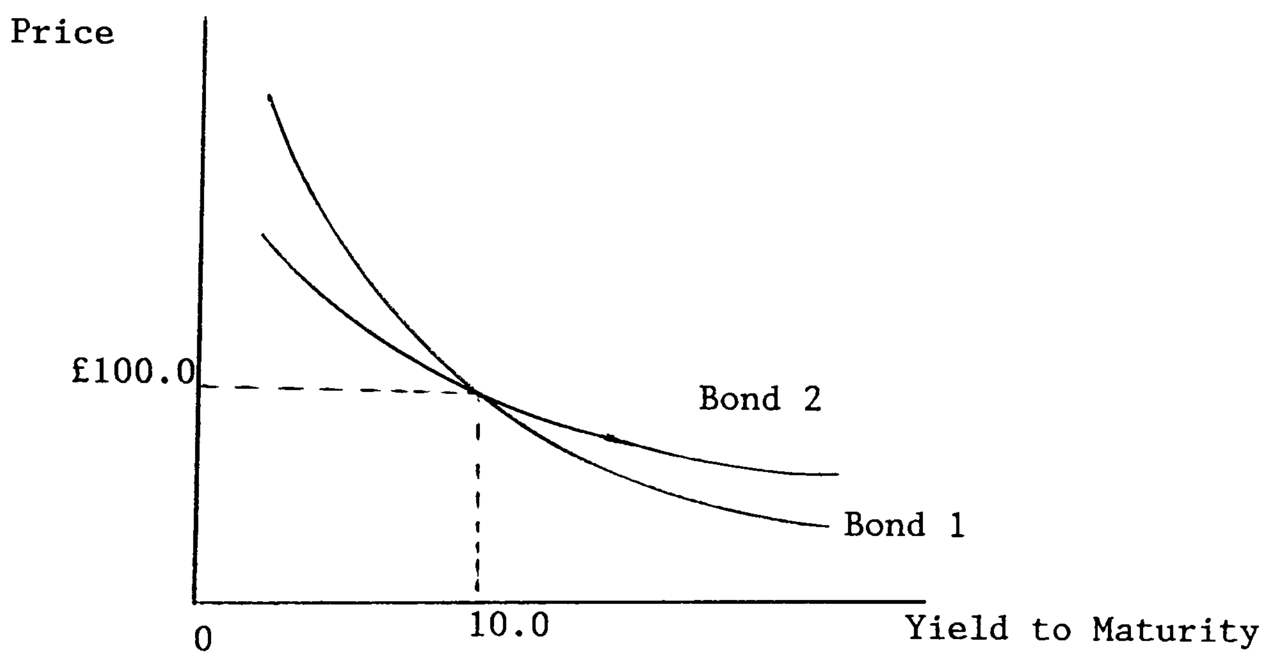
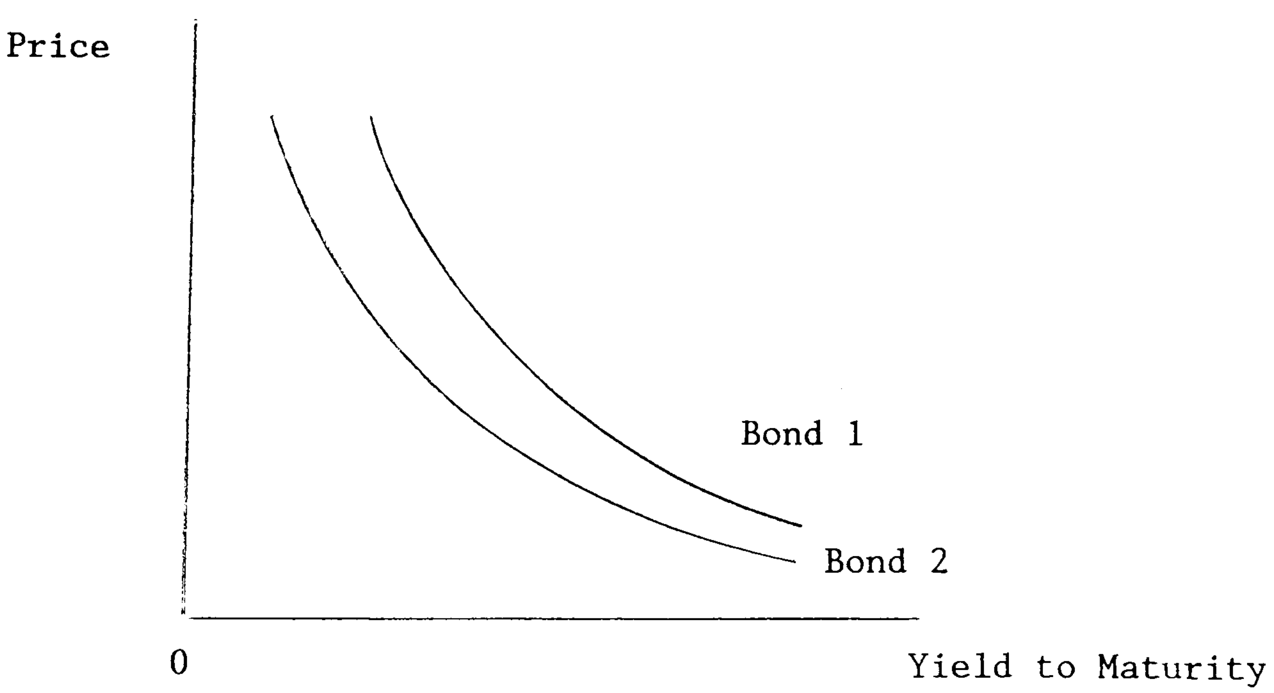


Figure 4.4, Bond Price / Yield Relationship:
Maturity = 10 years; Bond 1 Coupon greater than
Bond 2 Coupon



CHAPTER FIVE

The Effects of Big Bang on Market Efficiency

5.1. Introduction

Many years and many volumes of empirical work had preceded the theoretical underpinning of the efficient markets hypothesis by Samuelson in 1965. More recently the rational expectations hypothesis in economic theory, that individuals make no systematic mistakes in forecasting the future, has provided the efficient markets hypothesis, with an even stronger theoretical foundation. Yet still, the question of market efficiency remains an empirical one. The evidence, predominantly on equities and on overseas securities, suggests that although there are some anomalies (for example, January and weekend effects, small firm effects and low price to earnings ratio effects), markets are efficient.[1]

The weak form of the efficient markets hypothesis says that prices fully reflect all past price information and reward rational investors for accepting risk (Fama, 1970; Jensen, 1978). In chapter four, the statistical representation of this model, and the special case of the random walk hypothesis, were interpreted in terms of traditional term structure theories to provide a model to test market efficiency in the gilt-edged market. The efficient markets model implies that returns are serially uncorrelated, and autocorrelation based studies have become the dominant testing paradigm. However, over the last decade, several authors have criticised the statistical power of these tests and have suggested alternative procedures (for example, Shiller (1981)). Recently, motivated by the deficiencies with the traditional autocorrelation tests, Taylor (1986) has developed techniques and related efficiency tests that are free from the majority of these problems, but that remain within traditional paradigm.

Hence this chapter is composed as follows. Section 5.2 reviews the historical develop-

[1] A strongly dissenting view is that of Shiller (1981). The controversy surrounding his volatility tests is discussed in section 5.2 below.

ment of efficient market and random walk tests, and the results of previous British studies are noted. The volatility test controversy is discussed also. Section 5.3 examines the distribution of daily returns. While any change in distributional characteristics at Big Bang is interesting in itself, such a study will determine whether the returns series have the appropriate sample properties necessary for powerful efficiency tests. Section 5.4 reports on the methodology and results of the efficiency tests, which adopt the framework of Taylor (1986). Conclusions regarding the effect of Big Bang on market efficiency are drawn in section 5.5.

5.2. The History of the Random Walk and the Efficient Market Hypotheses[2]

Historically, market efficiency was tested using a random walk model. This is more restrictive than the efficient markets model and says that returns are uncorrelated and have a constant mean. Though not using the term "random walk", Bachelier (1900) had concluded that prices in a competitive market followed a random walk. Though there was a small number of studies in the first half of this century, Bachelier's work remained largely unappreciated and certainly unmatched. In 1953, Kendall published a paper which attempted to discover patterns in speculative prices of a similar nature to the alleged trade cycles in economic data. He was extremely negative about his result that the "data behave almost like a wandering series" saying further that

> "To the statistician there is some pleasure in the thought that the symmetrical distribution reared its graceful head undisturbed amid the uproar of the Chicago wheat-pit. The economist, I suspect, or at any rate the trade cyclist, will look for statistical snags before he is convinced of the absence of systematic movements." (1953 p.13),

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2 This review is primarily concerned with the major developments in empirical methodology and their application to British security markets. A good review of the theoretical refinements of the concept of an "efficient market" can be found in Strong and Walker (1988, ch.6).

Once again the significance of the result in support of the random walk hypothesis was not fully appreciated at the time. However part of the discussion after the presentation of the paper (see Bartlett, 1953) foreshadowed later work (e.g. Alexander (1961) and Cootner (1962)) by suggesting that price changes might be dependent but not correlated, from distributions being either non-linear or non-normal.

The modern interest in this subject began in 1959 following papers by Roberts and Osborne. Roberts presented the heuristic reasoning behind the random walk hypothesis of stock prices. His basic proposition was that a competitive market eliminates excess profits: only normal profits are earned. Hence, only a monopolist earns supernormal profits. So any rules, technical analysis, designed to earn excess returns cannot expect to be useful. His explanation of the "faith" in Chartism was that graphed market levels seemed to give clear signals. However, when weekly changes in the Dow-Jones index were examined statistically their behaviour was as if they had been generated by a pure chance model. So market levels looked like the cumulation of chance results. In other words, a series of cumulated random numbers is likely to give as clear (and as false) signals as the series of stock price levels. The implication was that price changes are independent of past history.

Osborne (1959) demonstrated the fit of share price data to a model of Brownian Motion. He further argued, by reference to psychological considerations, that it is not absolute changes in price, but rather changes in the log of price that are independent. This transformation has since been justified on both theoretical and empirical, econometric grounds. Granger and Morgenstern (1970, p107-8) provides a full account: a summary of the main arguments follows. Theoretically, the distribution of prices is bounded from below by zero but is unbounded from above. The logarithmic transformation results in a distribution which is symmetrically unbounded, and hence may be more symmetric about its mean. The transformation is consistent with investor utility maximising objectives. The scale for measuring prices should be such that a given change in the transformed variable yields a utility function that is independent of the price level. If investor's utility is based approximately on rate of return then the logarithmic transformation has the effect of making changes independent of position. The transformation therefore allows the hypothesis to be tested that investors want to maximize expected gain as a well as maximize expected rate of return. Empirically, transformed data have more symmetric and more nearly normal histograms. They appear to have more time invariant first and second sample moments, and appear to be much closer to being independent observations from a random process. More recently, this transformation has been shown (Taylor and Kingsman, 1979) to be the best choice from the Box-Cox set of transformations.

In 1962, Moore published empirical results supporting the random walk / Brownian Motion model. Both autocorrelation tests, first used by Kendall (1953), and the runs tests were used. The runs test, introduced by Cowles and Jones (1937) into the field of finance, uses the fact that if returns are independent, the number of sequences of returns of the same sign e.g. (++) or (--) should equal the number of reversals (+-) or (-+). Unlike the autocorrelation test it is unaffected by distributional considerations : it is non-parametric. However, the null hypothesis, that returns are independent, is a stronger condition than a null hypothesis of no autocorrelation among returns.

In 1963, Granger and Morgenstern introduced the techniques of spectral analysis into the study of share prices. The spectrum of a time series is a representation (through a Fourier transformation) of the autocorrelation function of that series. The spectrum gives a complete picture of autocorrelation in any stochastic process with finite variance. But, if the series is non-stationary or if the variance is infinite, results of this and the time domain autocorrelations could be ambiguous or incorrect. It was clear, even then, that distributional considerations were important for obtaining robust test results.

So it is perhaps not surprising that, simultaneously, studies of the distribution along with more sophisticated tests were being undertaken. Work by Mandelbrot (1963) and Fama (1963) argued that the distributions were non-Gaussian and have infinite variance, and they believed that the "Pareto-Levy" distribution was more appropriate. This distribution captured the symmetric, fat-tailed nature of empirical distributions. However, empirical evidence has conclusively rejected this hypothesis (Blattberg and Gonedes,1974; Hagerman,1978; Perry,1983). Furthermore a decade later Fama preferred to use the normal distribution (Fama,1976,Ch.1.). The currently preferred explanation of fat-tails is that it occurs in distributions with non-constant variance (see section 5.4.2 below). However, at that time, it was appreciated that standard serial correlation tests were not sufficient to fully capture the complex interrelationships of price changes. The first test to recognize non-linear dependence was Alexander (1961). He said that if stock price increments were independent, no trading rule could produce profits. He tested filter rules and found evidence of profits. Such a rule can commence by purchasing an asset at time i. It should then be sold on the first day j after time i for which the price is x percent lower than the highest price between time i and $j-1$ inclusive. When the price is x percent more than the least price on or after day j then the asset should be bought again. The parameter x is generally assessed within the range 0.5 to 25. However, the success of such a strategy could imply either non-stationarity or non-linearity. Cootner (1962) provided further evidence supporting either dependence or non-stationarity, and demonstrated the leptokurkic shape of the distribution. However, in a most extensive study of filter rules by Fama and Blume (1966), they concluded that "for measuring the direction and degree of dependence in price changes, the standard statistical tools are probably as powerful as the Alexandrian filter rules".

The application of any trading rule requires the payment of transaction costs on each trade. Consequently, small price dependencies may not in fact produce profits in excess of costs, and the market should not be regarded as inefficient. Keane (1983) classifies a market with no price dependencies as "perfectly efficient" and one with small, but unexploitable price dependencies as "near efficient". Trading costs comprise brokerage commissions and taxes, market-making spreads and the costs of searching the lowest cost method of carrying out the transaction. The conclusion of Fama and Blume (1966), based upon empirical study, was that "when commissions were taken into account the largest profits under the filter technique are those of the broker...when commissions are omitted, the returns from the filter technique were, of course, greatly improved, but are still not as large as the returns from simply buying and holding".

The City deregulation has altered the structure of trading costs as explained in chapter three: on the whole they were lower afterwards. This implies that small profits from exploiting price dependencies, which would not cover transactions costs before Big Bang, may well do so afterwards. At least at an intuitive level, we can think of the efficiency losses for the market as consisting of two kinds: (1) transactions costs (representing operational efficiency), (2) losses to exploiting price dependencies (representing an aspect of informational efficiency). As the former are reduced, the latter will increase unless the price dependencies also decrease.

5.2.1. The Results of some Past U.K Studies

Brealey (1970) examined correlations in the FT all-share index and found only minor evidence of dependence. Also, evidence of a leptokurkic distribution was found. Dryden (1970), using serial correlation tests, runs tests and filter rules produced further support for the random walk hypothesis. Kemp and Reid (1971) used the runs test on share prices, but their results ran into difficulties when attempting to account for no change runs and cannot be seen as conclusive. Griffiths (1970) found small negative correlations, but Guy (1976) explained that this was probably due to measurement error. Cunningham (1973) and Girmes and Damant (1975) provide evidence of trading rules generating profits. However the former was not compared to a buy and hold strategy and neither looked capable of covering trading costs. Taylor (1986) does not reject the random walk model for share prices when tested against a model reflecting the slow interpretation of information.

5.2.2. The Volatility Test Controversy

Relatively recently, it was argued that even though the standard statistical tests were generally unable to reject the null hypothesis of market efficiency this could not be taken as evidence in favour of the hypothesis. This principle applies to all scientific theories. Experiments can falsify a theory by contradicting one of its implications. But the verification of one of its predictions cannot be taken to prove or establish the theory. The positive outcome of this negative criticism, was an alternative testing procedure, due to Shiller (1981), which examines whether the efficient markets model is able to account for the historic volatility of prices. Shiller's argument runs as follows. Given perfect foresight, the price of an asset would be the present value of the known future dividends. Given a constant discount factor, the efficient markets hypothesis implies that the actual prices are participants' optimal forecasts of the perfect foresight price. Since optimal forecasts should vary less than the variable they are trying to predict, the actual variance of the stock price should be less than that of the perfect foresight price. This condition is known as the "variance bounds restriction". Shiller found the opposite situation in practice, the actual variance of stock prices was much greater than the variance of the perfect foresight path. He concluded that this "excess volatility" was evidence of inefficiency.

However, numerous authors (e.g. Kleidon (1986)) have argued that the tests are flawed. Firstly, the actual "efficient markets" price variance is underestimated by Shiller. The perfect foresight price is based on information of all future dividends, whereas the actual price is based on current information. It is quite likely that as dividends change over time, participants revise their forecasts of future dividends. Since the movements of the perfect foresight price are restricted so as to produce a return equal to the discount rate, it is quite likely that actual prices will appear more volatile. Secondly, and related to this objection, is work suggesting that dividends should not be treated as exogenous but rather as a choice made by managers of firms (Marsh and Merton, 1986,1987).[3] Models of aggregate dividends that assume managers try to

[3] The discussion centres on the question of whether the series of aggregate dividends is non-stationary

keep smooth dividends closely fit historical data. If dividends are smoothed and do not vary proportionately with profits, the observed dividend will vary little, and prices will appear much more volatile than the present value of observed dividends. Thirdly, the tests assume a constant discount factor (expected rate of return), a fluctuating discount factor can explain much of the variance in stock prices. This is essentially the same non-stationarity weakness of the traditional tests. The recent work seems to suggest that an efficient market can account for excess volatility.

A stronger criticism of efficient market tests has come from Summers (1983), who argued that even with negligible excess returns, it is still possible for there to be disequilibrium in prices. Hence for any model based on excess return arguments, "Do we really know that financial markets are efficient?" However, though there may be pricing errors, it is of course an empirical issue whether such miss-pricing exists. Brennan and Schwartz (1982) report that the differences between actual and equilibrium bond prices are not persistent. Furthermore, to establish miss-pricing requires the specification of an equilibrium price. While some analysts may estimate an equilibrium price, many others may accept the market price as an equilibrium price conditional on the market's information set and focus on predicting the marginal effect on price of new information. A study of the nature of price anomalies in the gilt-edged market is contained in chapter six.

Summers also demonstrates the effects on test power of assuming stationarity and linearity in the returns generating process. However, Taylor (1986) has provided a framework which overcomes these difficulties. This framework is adopted in the final section of this chapter. It was Summers also who argued that the failure to refute the hypothesis of market efficiency cannot be seen as evidence in favour of it. The solution is to ensure that the power of tests used is maximized and the tests' design made as robust as possible. The next two sections

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or stationary about a trend as assumed by Shiller.

adopt an overall approach which is believed to meet these criteria.

5.3. The Distribution of Daily Returns

Observed returns, x_t, are calculated as the change in the logarithm of successive days prices

$$x_t = \log_e(P_t) - \log_e(P_{t-1}) \tag{5.1}$$

where P_t is the price or price index at time t. The data are a set of thirty nine individual gilt-edged securities plus four market indexes.[4] The individual price series studied are daily observations on those gilts that were quoted fully paid in the secondary market throughout the period of fifty two weeks either side of Big Bang. Thus all stocks with a maturity of less than two years in October 1985 are excluded. We also exclude all irredeemable, variable rate and conversion stocks. In chapters six through eight, which involve cross-sectional analysis, tax effects are an important consideration (see chapter six, section 6.4 on tax effects). So to maintain a consistent data set throughout this study, we remove those bonds liable to such tax effects at this stage, that is, low coupon bonds. If the remaining thirty nine bonds are not representative of the whole market from a time series point of view, this will be picked up by the results for the index series.[5]

The summary statistics for the returns series are contained in table 5.1. The figures for the period before Big Bang sit directly above those for the period after Big Bang. The first column of figures, headed n, gives the sample sizes. On a daily basis the sample means, $\bar{x}$, would be expected to be close to zero as, for example, an annual rate of 10% is equivalent to a daily return of 0.038%. Standard deviations, s, measure the extent to which prices are chang-

[4] The results for the index series provide a useful, albeit unsophisticated, check upon whether the reactions to the Big Bang in our sample bonds are untypical of those experienced by the market (considered as a whole), and the well-established maturity sub-sections of it.

[5] Precise details of the data and their source are contained in the appendix to this chapter.

ing. A low value indicates that the probability of a large price change is relatively low. Thus the riskiness of alternative securities may be contrasted by a comparison of their standard deviations. Though the estimated standard deviations are small, the gilt market result is quite consistent with the traditional measure of gilt market volatility. The shorter the security, the greater the proportional reduction in time to maturity, when comparing the period after and the period before Big Bang. The smaller the term to maturity, the less volatile the security. Consequently, the greater the reduction in maturity, the bigger the impact on volatility. Hence the short securities should show the greatest reduction in volatility. This is observed to be the case; the short securities do show the greatest reduction in volatility, (table 5.1, column s).

Tests for changes in standard deviation (or variance) usually specify normal distributions in the null hypothesis. We have no reason to impose these restrictions and so we must use a test which is robust to departures from normality. Though non normality robust tests for changing standard deviations exist (e.g. Layard, 1973 and Brown and Forsythe, 1974), such testing here will be encompassed by the use of a non parametric test for changes in all the characteristics of the distribution (see below).

The skewness statistics (column headed "Skew." in table 5.1) assess the symmetry of the distribution. The symmetric normal distribution has zero skewness. All the sample series exhibit non-zero skewness. Medium and long maturity gilts generally show a positive value prior to Big Bang and a negative value after Big Bang. Gilt indexes show a stronger skew to the left after Big Bang, and short maturity gilts show a weaker skew to the right. The gilt market is therefore united in its response to Big Bang.[6] Furthermore, it can be seen that the sign of the skewness for individual securities cannot be inferred from the market indexes.

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[6] The index of short maturity gilts in fact shows a reduction in negative skewness. This is probably due to the exclusion from the data set of any securities redeemed during the data period. Thus all securities with a maturity of less than two years at the start of the period are excluded. Since an index of shorts includes stocks of a maturity up to 5 years, it is likely to have constantly changing constituents and to be a biased aggregate of its constituents. So, contrasting results are not surprising.

Outliers can often cause high values of skewness and so the presence of outliers provides an additional source of evidence concerning normality. For the gilts considered individually, the maximum number of observations beyond two standard deviations for any bond is seventeen and this declines with increasing bond maturity to about twelve. The maximum number of observations beyond three standard deviations is six and this also declines with increasing maturity to just one. Since outliers are smoothed on aggregation, the results for gilt indexes show a fewer number of outliers. On the null hypothesis that the samples had been drawn from a standard normal distribution, we would expect (for our sample sizes) no more than eleven observations beyond two standard deviations and not one beyond three standard deviations. None of the securities or indexes meet this standard, however the margin by which they overshoot is not that large given the sample size. It would therefore be unwise to make a judgment on normality solely on these criteria. The number of outliers beyond 2,3 and 4 standard deviations from the sample mean are given in the columns headed >2, >3 and >4 respectively in table 5.1.

Normal distributions have a kurtosis equal to 3. All but the short maturity gilts and their index show kurtosis less than 5, but all are greater than three (see column headed "Kurt." in table 5.1). The sample distributions thus possess a greater peakedness, or "leptokurtosis", than a normal distribution. We also observe that as maturity increases so there is an increasing tendency for kurtosis to increase after the Big Bang. This is borne out by a similar tendency among the outlying observations. As maturity increases, the number of outliers overall declines after the Big Bang, however, there are relatively more extreme outliers. This accounts for the observed increase, post Big Bang, in "fat tails" to the distributions as bond maturity increases. A non-parametric test will be used to test the significance of the departures from normality.

The normality test determines the goodness of fit of a normal distribution based upon the sample moments to the particular sample distribution itself. The particular test used here, which is suitable for continuous distributions, is the Kolmogorov-Smirnov Test. The test involves specifying the cumulative frequency distribution that would occur under the hypothesised theoretical distribution and comparing it with the observed cumulative frequency distribution. The point of maximum divergence between these distributions is determined, and reference to the known sampling distribution indicates whether such a divergence would probably occur if the observations were really a random sample from, in this case, the normal distribution.

The probability value associated with each series is given in the column labelled "KS1" in table 5.1. At a significance level of 5 percent, which defines the region of rejection as those values less than 0.05, the following results emerge. The short maturity securities tended to reject the normality hypothesis, but less so after Big Bang and with increased maturity. There is some evidence of non-normality among the medium maturity gilts in the period after Big Bang, but this is small and not systematic. This probably reflects the gradual rise in kurtosis after Big Bang in these bonds. The long maturity bonds are unable to reject a hypothesis of normality, and none of the index series display departures from normality.

The small evidence of non-normality observed here is in sharp contrast to results for individual equity securities. In a study by Taylor (1986), for example, the value of the kurtosis figure is well in excess of six for most series. A normality test such as the above was not carried out: the conclusion of non-normality was taken as directly evidenced by the size of the kurtosis statistics. The figures here do not provide such a strong message, and hence normality tests are essential.

The strength of the above result is dependent on the power of the test. While it is true that the weaker the assumptions that constitute a statistical model, the more general are the conclusions, such a benefit is obtained at the expense of lower power. A relatively small sample for any given significance level, will have a relatively low power by definition. The power / generality trade-off is true for any given sample size, but for the comparison of two tests on samples of unequal size it may not be true. For example, test A may be more powerful than test B using samples of size 30. But test B may be more powerful in a sample of 30 than A is in a sample of 20. In other words we can avoid the problem of choosing between power and generality by selecting a test of broad generality and then increasing its power to that of the most powerful available by enlarging the size of the sample. The concept of "power-efficiency" is concerned with the amount of increase in sample size which is necessary to make test B as powerful as test A. For example if test B requires a sample of $N_A = 25$ cases to have the same power as test A with $N_B = 20$ cases, then test B has a power efficiency of 80% given by the following formula.

$$\text{Power-efficiency of test B} = (100)\,\frac{N_A}{N_B}\ \text{per cent}$$

The power-efficiency of the Kolmogorov one sample test can be shown to be superior to any alternative non parametric tests in this situation (see Seigel (1956)). Though a comparison to a parametric test is not performed, the two sample version of this test which will be used below has a known and highly satisfactory power-efficiency (see Massey (1951)).

So far, tests of the normality of each period have been carried out and a description of their comparative summary statistics been reported. A test will now be undertaken to determine the significance, if any, of the difference in the distributions between the two samples; the periods before and after Big Bang. The test determines whether the two samples come from the same population, though we do not explicitly say what that population might be. Single sample tests have arrived at a conclusion on this question above. There are two forms of difference which may be tested. Firstly, we may test whether the samples differ in central tendency, independently of their higher order moments. Secondly, we may test whether the samples are from populations which differ in every respect. The most powerful tests in each category are, respectively, the Mann-Whitney U and Kolmogorov Smirnov Two Sample tests.

The null hypothesis for the Mann Whitney test is that the two samples are from the same population. The alternative hypothesis is that one sample is stochastically larger than the other.

This is a directional hypothesis which means that we reject the null hypothesis if the probability that an observation from one sample is larger than an observation from the other is greater than one half. If the evidence supports the alternative hypothesis, then the "bulk" of the population for one sample is higher than the "bulk" of the population for the other sample. A two-tailed test, where we do not impose the direction of the difference, accepts the alternative hypothesis if the above probability does not equal one half.

To apply the U test, we first combine the observations from both groups and rank these in order of increasing size retaining each observation's identity as either sample 1 or sample 2. The largest negative number has the lowest rank and the largest positive number has the highest rank. Then choosing either the observations from group 1 or group 2, we count how many times an observation in that sample is preceded by an observation in the other sample. The statistic U is the cumulative total of these occurrences and its sampling distribution under the null hypothesis is known.

For large samples we may use the following formula instead of the then tedious counting procedure. Identical results are obtained. We assign the rank of 1 to the lowest score in the combined samples, assign the rank 2 to the next lowest and so on. Then

$$U = n_1 n_2 + n_1 \frac{(n_1 + 1)}{2} - R_1 \tag{5.2}$$

where n_1 and n_2 are the sizes of samples 1 and 2 respectively and R_1 is the sum of the ranks assigned to sample 1. A similar formula is available which considers the sum of the ranks in sample 2. As it is the smaller of these two U values which is required, we apply the following transformation to ensure we obtain it.

$$U = n_1 n_2 - U' \tag{5.3}$$

Also in the case of large samples, it has been shown (Mann and Whitney (1947)) that as the sample sizes increase, the sampling distribution of U rapidly approaches the normal distribution, with mean = $n_1 n_2/2$ and standard error = $\sqrt{n_1 n_2(n_1+n_2+1)/12}$. So we may standardize the U statistic by subtracting its mean and dividing by the standard error, and this is then distributed as standard normal. Furthermore this normal approximation means that it does not matter whether U or U' is used. The sign of the standardized statistic will change, but not its value.

The power-efficiency of this test approaches $3/\pi$ = 95.5 per cent as the size of the combined sample increases (Mood, 1954), and is close to 95 per cent for even moderate sized samples. It is therefore an excellent alternative to the t test and is distribution free. The probability values of this test are given in the "MW" column of table 5.1. With a 5% significance level, no figure is statistically significant and so we conclude that there has been no significant change to the mean of the distribution of returns of either individual gilts or indexes following Big Bang.

There is no prior reason to expect that the effects of Big Bang on the distribution of daily gilt-edged market returns should be confined to changes in the central tendency of the distribution. To determine whether the two samples are from populations with the same or different distributions we may use the Kolmogorov-Smirnov Two-Sample Test. The test is sensitive to any kind of differences in the distributions from which the two samples were drawn.

Like the one-sample version, the Kolmogorov-Smirnov test is concerned with the agreement between two cumulative distributions. If the two samples have in fact been drawn from the same population distribution, then the cumulative distributions of both samples can be expected to be fairly close to each other, inasmuch as they both only show random deviations from the population distribution. The sampling distribution of the largest deviation between each cumulative distribution function is known. The test has a power efficiency of 96 percent in small samples and this decreases slightly for larger samples (Dixon, 1954). The probability values of this test are given in the column labelled "KS2" in table 5.1. Using a 5% significance level, the test indicates the we cannot reject the hypothesis that the samples before and after Big Bang are drawn from the same or identical distributions, except for the four shortest maturity gilts.

In summary, the evidence points to there having been little impact following Big Bang on the parameters of the distribution of daily returns of both medium and long gilts. This is seen as evidence in favour of the distributions of returns closely approximating a stationary process. However, the short maturity gilts do show a significant change in distribution. Again with the exception of the short maturity gilts, we are unable to reject the hypothesis that the distributions of daily returns examined are drawn from normal populations with their sample mean and variance as parameters.

5.4. Testing the Efficient Market Hypothesis

This section of the chapter is comprised of five elements. The first three sub-sections conduct an analysis of autocorrelations within the context of three different stochastic models for price movements. These models are: firstly, the random walk model; secondly, a conditional variance model, which can explain not only the high kurtosis of observed returns but also high autocorrelation variance that gives rise to further difficulties when using autocorrelation tests; and thirdly, a price trend model, which captures features suggestive of market inefficiency. The fourth sub-section conducts tests of the random walk hypothesis using information obtained from the autocorrelation analysis. The fifth sub-section considers the implications of these results for market efficiency.

5.4.1. The Random Walk Hypothesis and Autocorrelation Analysis

Let us suppose that prices follow a random walk which says that daily returns are uncorrelated and have a constant mean, i.e.

$$H_0 : \; x_t = \mu + e_t \tag{5.4}$$

$$E(e_t) = 0, \qquad E(e_t e_{t+i}) = 0 \quad (i \neq 0).$$

The market may be said to make efficient use of past price information as prices will adjust fully and instantaneously when new information becomes available.

Evidence against the random walk hypothesis will be provided by evidence of non-zero autocorrelation in the daily returns series. The autocorrelations of returns are usually estimated from the sample autocorrelation function, that is,

$$r_{\tau,x}=\frac{\sum_{t=1}^{n-\tau}(x_t-\bar{x})(x_{t+\tau}-\bar{x})}{\sum_{t=1}^{n}(x_t-\bar{x})^2}, \qquad \tau>0. \tag{5.5}$$

where n is the sample size, τ is the lag in days and $\bar{x}$ is the sample arithmetic mean.

Figures for these autocorrelations for the periods either side of the market deregulation are given in the left-hand portion of table 5.2 under the heading "Autocorrelations of Returns". The coefficients for lags 1 to 30 are assigned to one of six classes according to their size: (1) $r<-0.1$, (2) $-0.1\leq r<-0.05$, (3) $-0.05\leq r<0$, (4) $0\leq r\leq 0.05$, (5) $0.05<r\leq 0.1$, (6) $0.1<r$. The coefficient for lag one is also reported independently as $r_{1,x}$. For the short and long gilts the coefficient at lag one is generally smaller after Big Bang, while for gilts of a medium maturity the converse is largely true. All of these coefficients are positive. When the coefficients to lag thirty are examined, the nature of the change is more apparent. Around 23% of the coefficients exceeded 0.01 before Big Bang while less than 5% were in the same range (class (6)) after the event. This move was not countered by a significant shift in all coefficients towards negative values, but rather an overall reduction in the absolute size of autocorrelations.

Tests based on these statistics are generally derived from a theorem about the asymptotic properties of sample autocorrelations, proved by Anderson and Walker (1964). This theorem implies that the variance of the sample autocorrelation for n observations drawn from a finite variance, strict white noise process is approximately $1/n$. By strict white noise, we mean an identically and independently distributed process. Since the null hypothesis of zero autocorrelation does not imply a strict white noise process, $1/n$ may not be an appropriate sampling variance for our autocorrelation coefficients. If, however, returns were generated by strict white noise, our coefficients would yield the following conclusions. Mostly all coefficients at lag one are insignificantly different from zero. Considering coefficients to lag thirty, where the number of significant negative [positive] coefficients is given in the column of table 5.2 marked $\#_1$ [$\#_6$], over 12% of the coefficients were significant before Big Bang, while less than one percent were significant afterwards.

Estimating Autocorrelation Variances

It is possible to determine whether the variance approximation $1/n$ is valid, by calculating the sample autocorrelation variances in a manner described by Taylor (1984). This method uses the null hypothesis that $\{X_t\}$ is an uncorrelated process and, in addition, assumes only that the multivariate density of the process is symmetric and, initially, that the expected value of every X_t is known and can be assumed to be zero.[7] An assumption of stationarity is not necessary to achieve the results. Multivariate symmetry defines the "special null hypothesis" (1984,p.301)

$$H_s : f(x) = f(|x|) \quad \text{for all vectors} \quad x \ , \tag{5.6}$$

under which the process will also be uncorrelated. If H_s is true then we may deduce the following about the density function: every sequence x_t^*, $1 \le t \le n$, for which each x_t^* is either x_t or $-x_t$ has equal likelihood, namely the likelihood of the observed data. If all the returns are non-zero, there are 2^n such sequences: zero returns reduce the number of sequences. Each sequence could give a realization of $R_{\tau,X}$, and these equally likely sequences thus provide a discrete conditional distribution for $R_{\tau,X}$. The variance of this conditional distribution can be shown to be

$$a_\tau = \frac{\sum_{t=1}^{n-\tau} x_t^2 x_{t+\tau}^2}{(\sum_{t=1}^{n} x_t^2)^2} \tag{5.7}$$

and is an unbiased estimate. There can be some bias if H_s is false, but a_τ is still an appropriate

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[7] Capital letters are used to denote population quantities when a distinction between sample quantities is desirable.

estimate if the null hypothesis is true (Taylor (1986), p.123-4).

The above estimates a_τ apply the assumption that the means of the X_t are all zero. For a known constant mean μ, all terms x_t in (5.7) are replaced by $x_t-\mu$. For observed returns, μ is close to zero but unknown, so x_t is replaced by $x_t-\bar{x}$. The estimate b_τ of $n[\text{var}(R_{\tau,X})]$ then becomes

$$b_\tau=\frac{\sum_{t=1}^{n-\tau}(x_t-\bar{x})^2(x_{t+\tau}-\bar{x})^2}{(\sum_{t=1}^{n}(x_t-\bar{x})^2)^2} \tag{5.8}$$

and if the process is strict white noise, $b_\tau \approx 1$. When b_τ is greater than 1, the significance level of a test involving $R_{\tau,X}$ will be underestimated. For example, if $\{X_t\}$ is uncorrelated and the autocorrelation coefficients are normally distributed with mean zero and variance $2/n$, but we use a 5% significance level and falsely assume the coefficients variance is $1/n$, then chance of rejecting the null hypothesis (of zero autocorrelation) is 17 per cent.

Estimates of b_τ for lags from one to five days for all series are given in the five columns which form the right-hand portion of table 5.2. The most noticeable feature is that the coefficients for the period before Big Bang are generally greater than unity and larger than those for the following period, which are very infrequently greater than unity. We would therefore be unwilling to trust the results of tests based upon the standard $1/n$ rule for the period before Big Bang, particularly among the short maturity gilts. In order to conduct a comparative exercise, we need to be confident of test results for both periods. By considering a possible cause of large autocorrelation variances, we motivate a solution to the problem.

5.4.2. Stochastic Volatility and Conditional Variance Models

High autocorrelation variances can be explained by changes in the variance of the process, or changes in the conditional variance $\text{var}(X_t|x_{t-j},j>0)$ given a constant variance. Simulations

(Taylor, 1984) based upon these propositions indicate that the order of magnitudes observed here among the autocorrelation variance coefficients are consistent with the second view. Several studies (e.g. Granger and Morgenstern (1970), Clark (1973) and Tauchen and Pitts (1983)) have linked conditional standard deviations to trading volume. These models may be written in the form

$$X_t = \mu + e_t \qquad (5.9)$$
$$e_t = V_t \; U_t$$

with $\{U_t\}$ a standardized process having zero mean and unit variance for all t, and $\{V_t\}$, representing volatility, a process of positive random variables usually having $\mathrm{var}(X_t | v_t) = v_t^2$. Stationary models for the standard deviation process have been developed by, for example Taylor (1982a) and Engle (1982). In the former model changes in conditional variance are driven by economic forces independent of the market. One innovation drives the level of trading activity, determining v_t which are assumed to be autocorrelated while, independently, the innovation $\{U_t\}$ determines the price response. In the second model, conditional variances are driven by the past history of returns, and hence are called autoregressive conditional heteroscedasticity (ARCH) models.[8]

It can be shown, (Taylor (1984), p.303), that the autocorrelation variances for both these processes can be arbitrarily large and as τ increases decay towards zero. If the problem is a changing conditional variance, then the solution is to construct a series with a reasonably homogeneous conditional variance.

Rescaled Returns and Autocorrelation Analysis

Such a series could be given by

$$y_t = (x_t - \bar{x}) / \hat{v}_t \qquad (5.10)$$

which will be called the rescaled returns series. It will be similar to the unobservable

[8] ARCH models are considered in greater detail in chapter seven as plausible models for spot interest rates processes.

standardized return $u_t = (x_t - \mu)/v_t$ whenever the estimates of mean and variance are good.

Forecasts of v_t can be obtained from the non-linear models described above. However, there may be occasions when the assumption of stationarity in these models is not appropriate. Any change in unconditional variances that could cause non-stationarity implies that it would be preferable to have a variance forecasting method that will reflect this change quickly and accurately. An exponentially-weighted moving average (EWMA) forecast achieves this by assigning most weight to recent observations. We now summarize the principal arguments of this method (Taylor and Kingsman, 1979).

The main idea is to use past observed returns to estimate the current level of the V-process, working from the results that $E(V_t)$ equals $|x_t - \mu|/\delta$, with $\delta = E|U_t|$. A past return x_{t-s}, $s>0$, gives the crude estimate $|x_t - \bar{x}|/\delta$ for v_{t-s}. Exponentially weighted averages of these estimates provide improved estimates of the v_t if the V-process is changing slowly.[9] Thus consider the estimates

$$\begin{aligned} \hat{v}_t &= \gamma \sum_{s=0}^{\infty} (1-\gamma)^s |x_t - \bar{x}|/\delta \\ &= (1-\gamma)\hat{v}_{t-1} + \gamma |x_t - \bar{x}|/\delta \end{aligned} \tag{5.11}$$

To implement (5.11), we use γ=0.04 based on the results of extensive back-optimizing (see Taylor (1986), p.112, table 4.2).[10] Making the assumption that the returns have conditional normal distributions (see section 5.3), δ is approximately equal to 0.798. An initial estimate $\hat{v}_{21}$ is calculated from the first twenty returns using $1.253 \sum_{t=1}^{20} |x_t - \bar{x}|/20$.

The benefits from rescaling can be appreciated when the variances of the autocorrelations coefficients for the rescaled returns are calculated. The estimated variances of the autocorrela-

[9] Evidence in Taylor (1986) based on modelling the two conditional variance processes mentioned above for series of share prices, commodity prices and commodity and currency futures prices suggested that conditional variances change slowly.

[10] I have experimented with the range of optimal γ values for the different series studied in Taylor (1986). The effects are negligible.

tion coefficients for returns $\{x_t\}$ were obtained by assuming the multivariate symmetry hypothesis H_s. When H_s is true for $\{X_t\}$ it will also be true for the process $\{Y_t\}$ generating the rescaled returns $\{y_t\}$, if we ignore the difference between $\bar{x}$ and μ. This follows directly from the definition of $\hat{v}_t$ as a function of past returns which does not depend on the sign of $x_{t-s}-\bar{x}, s>0$. Hence, the estimate b_τ^* of n^* [var$(R_{\tau,Y})$] is

$$b_\tau^* = n^* \frac{\sum_{t=21}^{n-\tau} (y_t-\bar{y})^2 (y_{t+\tau}-\bar{y})^2}{(\sum_{t=1}^{n} (y_t-\bar{y})^2)^2} \tag{5.12}$$

where n^* is $(n-20)$ since the first twenty observations were used to initialize $\hat{v}$. The right hand side of table 5.3 gives figures for b_τ^* for lags up to five, in the same manner as table 5.2. For all but the short maturity gilts, the value at lag one is less than unity, both in the periods before and after Big Bang. Thus conclusions of tests based upon the sampling properties of autocorrelation coefficients should be more reliable. Furthermore, we note that the kurtosis of the rescaled variables is generally less than that for the unscaled variables. Thus the maintained assumption of normality for the medium and long maturity gilts is not adversely affected by this transformation.

Autocorrelation coefficients for the rescaled returns are given are given in the left-hand portion of table 5.3.[11] In general, the coefficient at lag one is greater in the period after Big Bang. This compares with the mixed result among unadjusted returns. Once more, all of the coefficients are positive but most are insignificantly different from zero. To summarize the coefficients to lag thirty, each is assigned to six classes in the same manner, and taking the same values, as the unadjusted returns. Before Big Bang nearly 15% of the coefficients were significant (columns headed with a # as with the unadjusted returns). After Big Bang, less than three percent were significant. There has been a considerable change to the number of

[11] Substantial autocorrelation cannot be induced by the rescaling transformation. The likely maximum autocorrelation so induced is less than 0.004 (see Taylor, 1980).

coefficients exceeding 0.01. Before Big Bang, 24% were in class (6) (see table 5.3) but around 11% were in the same range after the event. The values of the autocorrelation coefficients of the rescaled returns demonstrate that the Big Bang had a significant effect on the pattern and level of autocorrelation in the gilt-edged market, but that the magnitude of the impact is exaggerated if unadjusted returns are used.

5.4.3. Price Trend Models

A sensible alternative hypothesis to the random walk, which implies that information is rapidly reflected in prices, is one which captures the idea that prices do not adjust fully and instantaneously when new information becomes available. Instead some new information is incorporated slowly into prices. Slow interpretation of a particular item of information will cause several returns to be partially determined by the same information. Thus we have a price-trend model (Taylor, 1980) given by

$$x_t = \mu_t + e_t$$
$$\mu_t - \bar{\mu} = \alpha_t(\mu_{t-1} - \bar{\mu}) + \eta_t \qquad (5.13)$$
$$E(e_t) = 0, \qquad E(e_t e_{t+i}) = 0 \quad (i \neq 0), \qquad \text{cov}(\mu_s, e_t) = 0 \quad (\text{all } s,t).$$

We assume initially that the returns process has a constant variance, and denote var(e_t) by σ_e^2, var(μ_t) by σ_μ^2, and $E(\mu_t)$ by $\bar{\mu}$. The trend model is interpreted as follows. We assume that trend values are determined by the current information about demand and supply, (that is, some process which is not interpreted as an inflation term or risk adjusted expected return, and which does not restrict μ_t to be non-negative). We also assume that information arrives randomly at the market. In the trend equation, η_t measures the impact of information on day t, and is assumed to be a series of identically distributed random variables having zero mean, with each η_t independent of the past trend values $\{\mu_t,$ all $s<t\}$. α_t measures the importance of the past trend information in the determination of the current trend value. α_t are independent random variables, and α_t and η_s are independent for all $s \neq t$. α_t is not independent of η_t. Also,

$E(\eta_t | \alpha_t)$=0 and, in general, var($\eta_t | \alpha_t$) depends on α_t.

The random walk hypothesis says that ρ_τ, the theoretical autocorrelation function, is zero for all τ. The above returns equation has a theoretical autocorrelation function

$$\rho_\tau = \frac{cov(\mu_t, \mu_{t+\tau})}{var(\mu_t) + var(e_t)} \tag{5.14}$$

For the trend model we have var(e_t) = σ_e^2, var(μ_t) = σ_μ^2, and cov($\mu_t, \mu_{t+\tau}$) = $\{E(\alpha_t)\}^\tau \sigma_\mu^2$. Thus denoting $E(\alpha_t)$ by p, and substituting these into (4.6) we obtain,

$$\rho_\tau = p_\tau \sigma_\mu^2 / (\sigma_\mu^2 + \sigma_e^2) \tag{5.15}$$

Thus the coefficients depend on p, the speed at which imperfectly reflected information is incorporated into prices, and the ratio $\sigma_\mu^2/\sigma_e^2 = R$ alone. They are all positive and small if R<<1. The values decline slowly if p is close to unity. Thus trends cause positive autocorrelations, and the effect of current information which is not fully reflected in the current price, upon future returns, diminishes as time goes on. Thus we define the "price-trend hypothesis" as

$$H_a : \rho_\tau = A\ p^\tau, \tag{5.16}$$

for some A>0, and 0<p<1, for all τ>0. These autocorrelations can be shown to be those of an ARMA(1,1) process where the moving average coefficient is chosen to be the solution of a quadratic equation involving p and A (see e.g. Taylor (1980), p.343-344)

The trend model can incorporate a stochastic volatility term, as shown by Taylor (1986, p.79-83). In one model, the stochastic volatility effects are confined to the innovations term U_t, while the other supposes that $\mu_t - \bar{\mu}$ and U_t are multiples of a common volatility V_t. However, in either case, if all processes are stationary then the autocorrelations of X_t are still $A\ p^\tau$ or very similar, which does not affect the results of the test to be described below.

5.4.4. Testing the Random Walk Hypothesis

Many random walk tests have been based upon autocorrelation coefficients calculated from returns (see e.g. the U.K. studies mentioned earlier) and can be criticised for being incorrect as they assume sampling properties about the autocorrelation coefficients which are likely to be unrealistic. However, if we use instead the autocorrelation coefficients calculated from rescaled returns, we cannot be similarly at fault. Unfortunately, these tests have also been criticised for having low power even, more recently, against plausible alternative hypotheses (see e.g. Summers, 1983). However, in this paper we outline a test developed by Taylor (1982b) which is powerful against the alternative hypothesis of the price trend model described above. These tests are described below and the results are given in table 5.4. As with the earlier tables, the figures for the period before Big Bang sit directly above those for the period after Big Bang. Critical values for each statistic are given at the top of each table: the null hypothesis is rejected for figures in excess of the critical values.

A simple test uses the first autocorrelation coefficient. It applies the result $\sqrt{n}R_1 \sim N(0,1)$, approximately, when H_0 is true, and rejects the null hypothesis at the 5% level of significance if $\sqrt{n^*}|r_{1,y}|>1.96$. This test is both logical and powerful if dependence is expected to be between consecutive returns. An alternative method to examine the coefficients collectively is the Box-Pierce (1970) statistic

$$Q_k = n \sum_{\tau=1}^{k} r_\tau^2 \tag{5.17}$$

which is approximately χ_k^2 when H_0 is true and given the assumed independence of the $R's$. The statistic is calculated for $k=10$, 30 , and 50.

By examining the figures in the four appropriate columns of table 5.4, we can see that, firstly, in most of the short maturity gilts, there has been a substantial drop in the size of the first order autocorrelation, enough to alter the significance of the coefficient on the shortest maturity security. Secondly, although there has appeared to be a rise in nearly all coefficients on longer maturity gilts, it has not been of a size to send all but a handful of coefficients from their insignificant state before Big Bang into a state of being statistically different from zero afterwards. So in general, the magnitude of first order autocorrelation seems to have remained on balance insignificant. This pattern is not however repeated in the consideration of coefficients to higher lags. Virtually all the gilts have experienced a fall in the level of autocorrelation when considering lags up to 10, with 90% transferring from the significant to insignificant regions. Up to lag 30, over 90% have experienced a fall in the level of autocorrelations, with over 70% moving from the region of significance to insignificance. To lag 50, still some 80% saw a decline in the level of autocorrelation, but now only a quarter moved from significance to insignificance. This latter statistic is not contradictory, for the remainder have stayed as they were prior to Big Bang, insignificant. So, in summary, there has been a dramatic fall in the level and reduction in the significance of autocorrelation as a result of Big Bang.

An alternative to studying autocorrelations is spectral analysis. This is particularly appropriate when cycles in returns are the alternative hypothesis to random behaviour. Spectral theory relevant to economic studies is described by Granger and Newbold (1977, Ch.2). The spectral density function for a stationary process can be defined as

$$s(\omega)=\sigma^2/(2\pi)\left[1+2\sum_{\tau=1}^{\infty}\rho_\tau\cos(\tau\omega)\right] \qquad 0 \le \omega \le 2\pi \tag{5.18}$$

with $\sigma^2 = \text{var}(X_t)$. The integral of $s(\omega)$ from 0 to 2π equals σ^2 and $s(\omega)=s(2\pi-\omega)$, so it is only necessary to consider the frequency range 0 to π. If the random walk hypothesis is true, $s(\omega)$ will be constant for all ω. So to test H_0 we need to estimate $s(\omega)$ and test for a constant spectral density.

Estimates of $f(\omega)=2\pi s(\omega)/\sigma^2$ have the general form

$$\hat{f}(\omega) = 1 + 2\sum_{\tau=1}^{M-1} \psi_\tau r_\tau cos(\tau\omega) \tag{5.19}$$

with positive and monotonically decreasing ψ_τ ensuring consistent estimates. The ψ_τ 'lag window generators' are defined, for fixed M by (Parzen,1961)

$$\begin{aligned}\psi_\tau &= 1 - 6\tau^2(M-\tau)/M^3 && 0 < \tau \le M/2 \\ &= \frac{2(M-\tau)^3}{M^3} && M/2 \le \tau < M.\end{aligned} \tag{5.20}$$

Spectral estimates are calculated using $M=100$. Praetz(1979) has shown that $\hat{f}(\omega_1)$ and $\hat{f}(\omega_2)$ are correlated estimates only if $|\omega_1-\omega_2| \leq 3\pi/M$. Hence tests are based on $\hat{f}(\omega)$ evaluated for $\omega=0,4\pi/M,8\pi/M,...,pi$, giving $1+M/4$ potential statistics. These statistics can be standardized using the asymptotic theory for sample autocorrelations to give

$$f_j = [\hat{f}(4j\pi/M)-1]/\sqrt{\left\{4\sum_{\tau=1}^{M-1}[\lambda_\tau \cos(4j\tau\pi/M)]^2/n\right\}} \tag{5.21}$$

for $j=0,1,...,M/4$. The f_j are effectively independent observations from $N(0,1)$ for large sample sizes n when H_0 is true. The most plausible cycle is one week, $\omega=2\pi/5$ and the standardized spectral statistic is f_j, $j=M/10$. This test statistic is denoted f_w in table 5.4 and is a one-tailed test.

The results of this test indicate that there was no evidence of any weekly cycles in the gilt-edged market. Big Bang did not need to rectify an anomaly in this instance, though for about a quarter of the series, the statistic moved further into the region of insignificance.

As returns may have a non-normal and perhaps non-stationary distribution, non-parametric tests could be more appropriate.[12] The only such test used to date is the runs test. A positive [no change] [negative] run is a sequence of positive [zero] [negative] returns. Let x_t^* be 1,0,−1 for positive, zero, or negative x_t. Also let h_t be 0 if $x_t^*=x_{t+1}^*$ and 1 otherwise. Then $h_t=1$ signifies that x_{t+1} begins a new run and so the total number of runs of all types is

$$H=1+\sum_{t=1}^{n-1}h_t \tag{5.22}$$

Suppose there are n_1 positive returns, n_2 zero returns, and n_3 negative returns in the series. Then the mean and variance of the variable $\tilde{H}$ generating H, conditional upon n_1,n_2, and n_3, are

$$E[\tilde{H}] = n+1-\left[\sum n_j^2/n\right]$$

---

12 This test is particularly appropriate for short maturity gilts where the hypothesis of normality was rejected (see section 5.3).

and

$$var(\tilde{H}) = \left\{ \sum n_j^2 (\sum n_j^2 + n + n^2) - 2n \sum n_j^3 - n^3 \right\} / (n^3 - n) \quad (5.23)$$

when H_0^* is true, summing over $j=1,2,3$ (Mood,1940). This null hypothesis H_0^* is that the x_t^* are generated by a strict white noise process $\{X_t^*\}$. It is usually assumed that there is no practical difference between H_0 and H_0^*. For large n, $\tilde{H}$ is approximately normal so tests can use

$$K = (H - E[\tilde{H}]) / \sqrt{var(\tilde{H})} \quad (5.24)$$

rejecting H_0^* (and H_0) at the 5% level if $|K|>1.96$. The advantage of this form of test is the avoidance of problems arising from variance changes. However, it has low power and thin trading can cause several no change runs. The latter may be responsible for less total runs than expected thereby refuting independence (and H_0^*) but not the RWH (H_0).

The runs test (column headed K in table 5.4) in general shows no significant evidence of runs in the market either before or after Big Bang. A few short maturity gilts have seen a transfer into the region of significance, but not very far into it, while for the rest of the market, the evidence is is less weak after Big Bang. However, runs tests are subject to the thin trading problems described earlier and these could account for the results. However, applying the tests to market indexes does not change the result in support of the random walk hypothesis. It is more likely, therefore, that the runs test is picking up the change in the distribution of the short rates after Big Bang, as described in section 5.3.

An alternative non-parametric test, due to Durbin (1967) uses the cumulated periodgram (proportionate to the sample spectral density).If the theoretical spectral density function for strict white noise is horizontal, then the cumulated periodgram is the line with coordinates $(j, j/M)$, with j and M being defined earlier. A Kolmogorov - Smirnov type test may then be carried out upon the maximum deviation of the sample cumulated periodgram and the line.[13] The results of this test unanimously enforce those of the parametric spectral tests, and are not

[13] Other Kolmogorov-Smirnov type tests are discussed in greater detail in section 5.3, earlier.

reported in the results table 5.4.

By considering the likelihood ratio statistic for tests of H_0 (the random walk hypothesis of zero autocorrelation) against H_a (the price trend hypothesis), using as "data" the series of sample autocorrelation coefficients (r_τ), Taylor (1982b) has shown that a suitable set of statistics for accu.ately recorded prices is given by

$$T_{k,\phi} = \sum_{\tau=1}^{k} \phi^\tau r_\tau \qquad (0<\phi<1). \tag{5.25}$$

These statistics use the fact that in H_a the sample autocorrelation coefficients have monotonically decreasing positive expectations. Errors in price series decrease r_1, since an error in price at time t causes errors in the returns at both time t and time $t+1$, one positive and one negative. As test power can be lost because of errors in the data, a better set of test statistics is, in practice,

$$U_{k,\phi} = \sum_{\tau=2}^{k} \phi^\tau r_\tau \qquad (0<\phi<1). \tag{5.26}$$

It is necessary to chose k,ϕ and the significance level prior to performing the test. Taylor's opinions derived from work on commodity prices (1978), were used in a selection procedure which culminated in the recommendations k=30 and ϕ=0.92 (Taylor, 1982b). These figures are still recommended (Taylor, 1986) for all speculative prices.

For a constant variance random walk, $\sqrt{n}\, r_\tau$ has asymptotic distribution $N(0,1)$ and, for all $\tau \neq s$, r_τ and r_s are asymptotically independent (Anderson and Walker, 1964). Therefore when H_0 is true,

$$U^* = U_{30,0.92} / (\sum_{\tau=2}^{30} 0.92^{2,\tau} n^{-1})^{1/2}$$

$$U^* = 0.4649 \sqrt{n} \sum_{\tau=2}^{30} 0.92^\tau r_\tau \tag{5.27}$$

is asymptotically distributed as $N(0,1)$. The test is one tailed rejecting H_0 when U^* exceeds a critical value determined by the significance level. The incorporation of asymptotic sample properties of the autocorrelation coefficients into the construction of these tests means that it is essential that these tests are carried out using the rescaled returns.

The tests against the price trend alternative are not only the most powerful but also provide the clearest picture of the impact of Big Bang. The two autocorrelation tests T^* and U^* (labelled as such in table 5.4) in every case show a significant movement against rejecting the random walk hypothesis after Big Bang. In one third of the cases the series had significant U^* statistics before Big Bang and insignificant ones afterwards.

There will be a single thin peak at ω=0 in the spectral density function if $\alpha \approx 1$ and the price trend hypothesis is true. This fact has been used to develop a powerful spectral tests against the alternative of a price trend (see Taylor, 1986 p.143-5). Except in a few stocks of either extremely long or extremely short maturity, this spectral statistic f_0 shows that the evidence in favour of the price trend hypothesis is reduced after Big Bang, though in all cases, at the 5% level the null hypothesis of a random walk is rejected.

5.4.5. The Implications for Market Efficiency

As Taylor (1986, p.161-163) has shown, the U^* statistic can be modified to test the null hypothesis of market efficiency against the alternative hypothesis of a price trend. This recognizes that the trend term μ_t, which is the non-random component of the daily returns in equation (5.13) - the price trend model, could be interpreted as a time-varying risk premium. It can be shown that if the term μ_t is a time varying risk premium then an upper bound on the autocorrelations of returns in an efficient market can be determined. This bound is given by the ratio of the variance of the risk premium to the variance of the returns. By adjusting each autocorrelation coefficient by this upper bound value, ρ^*, i.e.

$$U^*_{EMH} = 0.4649\sqrt{n}\sum_{\tau=2}^{30} 0.92^{\tau}(r_\tau - \rho^*) \ , \qquad (5.28)$$

the U^* test has, as the null hypothesis, the efficient markets hypothesis rather than the random walk hypothesis.

Now, let us suppose that the absolute premium over the sample is less than 20%, which is intended to generous. Hence, daily, $|\mu_t| < 0.20/252 = 0.0007$. Assuming μ_t has a uniform distribution over the range -0.0007 to 0.0007 gives $\text{var}(\mu_t) = (0.0014)^2/12 = 1.63 \times 10^{-7}$. This is very small with respect to the variance of the returns (a typical value from table 5.1 is 0.005), and hence the upper bound is likely to take a very small value relative to the autocorrelations, (approximately 3.26×10^{-5}).

So, given the strength of the results for tests of the effects of Big Bang assuming a null hypothesis of the random walk, it is very unlikely that they could be called into question if the null hypothesis is changed to the efficient markets hypothesis. Thus the Big Bang is deemed to have created a greater level of informational-efficiency in the gilt-edged market than had prevailed beforehand.

5.5. Summary and Conclusion

This chapter sought to establish the impact of market deregulation on informational efficiency in the gilt-edged market, using a traditional framework, yet adopting recent developments in the available methodology. The development of the traditional framework for testing market efficiency was analyzed and the strengths and weaknesses drawn out. One principal weakness was that certain tests made assumptions about the distribution of sample returns that were not confirmed or simply not true. An analysis of the distribution of returns was conducted to prevent the use of inappropriate test statistics. It was found that the distribution of returns was insignificantly different from a normal distribution, excepting very short maturity gilts, and that the returns distribution had not changed significantly after Big Bang. These results formed the basis for the maintained hypotheses for the subsequent work; that is, returns closely approximate a normally distributed stationary process. This reasonable assumption increases the power of subsequent tests of the random walk and market efficiency hypotheses. However, steps are still taken to ensure that these tests are as powerful as possible; this involved analysing the distributional properties of the sample autocorrelations. Using this technique and other powerful tests, including using an explicit alternative hypothesis which reflected market inefficiencies, it was found that the gilt-edged market experienced a significant fall in the level of informational inefficiencies. For all long dated and most medium dated stocks, a hypothesis of market efficiency could not be rejected after Big Bang.

However, the alternative hypothesis used in this chapter is just one of many alternative hypotheses. Thus despite all the steps taken to make the power of the tests in this chapter as strong as possible, it is still possible that there may exist stronger tests. The development of further tests forms the basis of the next three chapters. The ultimate aim being to verify the preliminary result from this chapter concerning market efficiency and the impact upon it of the Big Bang in the gilt-edged market.

TABLE 5.1

SUMMARY STATISTICS "Shorts"											
SERIES	n	$10^4\bar{x}$	$10^2 s$	>2	>3	>4	Skew.	Kurt.	KS1	MW	KS2
EX88	251	-0.20	0.20	14	6	1	0.41	6.16	0.000	0.207	0.000
	246	0.45	0.09	16	5	1	0.10	6.36	0.000		
TR89	251	-0.64	0.26	18	5	-	0.34	5.47	0.029	0.233	0.003
	246	0.40	0.16	12	4	1	0.02	5.97	0.026		
T10H	251	-0.36	0.31	17	6	-	0.20	5.29	0.012	0.196	0.010
	246	0.72	0.18	14	4	-	-0.23	5.50	0.006		
EX10	251	-0.28	0.30	15	5	-	0.46	5.24	0.003	0.152	0.018
	246	0.91	0.19	13	4	-	-0.14	4.93	0.032		
EX11	251	-0.52	0.30	15	5	-	0.26	4.86	0.001	0.148	0.008
	246	0.67	0.19	13	4	-	-0.02	4.59	0.024		
T13	251	-1.19	0.32	16	1	-	0.11	5.00	0.010	0.181	0.062
	246	0.24	0.24	14	2	-	-0.27	4.34	0.047		
EX90	251	-0.55	0.33	17	6	-	0.32	5.41	0.002	0.157	0.095
	246	0.67	0.26	12	2	-	-0.22	4.30	0.045		
E12H	251	-0.97	0.33	17	3	-	0.24	4.57	0.009	0.223	0.154
	246	0.34	0.25	13	2	-	-0.20	4.17	0.078		
"Short/Mediums"											
T91	251	-0.76	0.35	15	3	-	0.20	4.57	0.045	0.237	0.166
	246	0.68	0.29	12	3	-	-0.21	4.87	0.031		
EX91	251	-0.66	0.42	16	-	-	-0.02	3.77	0.152	0.536	0.101
	246	1.01	0.32	13	-	-	-0.15	4.49	0.074		

SUMMARY STATISTICS (cont.)
"Short/Mediums" (cont.)

SERIES	n	$10^4\bar{x}$	10^2s	>2	>3	>4	Skew.	Kurt.	KS1	MW	KS2
T12T	251	-0.98	0.44	14	2	-	0.01	4.35	0.193	0.474	0.106
	246	0.71	0.37	14	3	1	0.07	5.66	0.030		
T10	251	-0.71	0.47	17	2	-	0.00	3.60	0.201	0.427	0.193
	246	1.47	0.39	10	4	-	-0.24	4.68	0.091		
EX92	251	-0.89	0.46	16	3	-	-0.25	4.11	0.162	0.456	0.078
	246	0.99	0.39	13	4	1	-0.17	5.05	0.120		
E92	251	-0.98	0.45	14	3	-	-0.10	4.18	0.237	0.394	0.058
	246	0.81	0.38	15	5	1	-0.17	5.11	0.068		
"Mediums"											
T93	251	-0.88	0.48	14	1	-	-0.01	3.73	0.613	0.515	0.045
	246	0.96	0.39	11	5	1	-0.30	5.33	0.030		
T13T	251	-1.13	0.46	13	4	-	-0.02	4.44	0.257	0.352	0.116
	246	0.75	0.40	13	3	-	-0.28	4.70	0.032		
T14H	251	-1.08	0.44	15	4	-	-0.04	4.19	0.255	0.447	0.118
	246	0.60	0.39	12	4	-	-0.28	4.42	0.110		
E94	251	-1.02	0.47	14	3	-	-0.01	3.87	0.376	0.366	0.175
	246	0.86	0.42	14	4	-	-0.37	4.26	0.046		
EX94	251	-0.93	0.49	14	4	-	0.00	4.08	0.174	0.421	0.396
	246	1.04	0.44	13	4	-	-0.43	4.93	0.129		
TR12	251	-0.73	0.50	14	2	-	0.00	3.77	0.508	0.572	0.118
	246	0.94	0.47	14	4	-	-0.35	4.78	0.048		

SUMMARY STATISTICS (cont.)
"Mediums" (cont.)

SERIES	n	$10^4\bar{x}$	10^2s	>2	>3	>4	Skew.	Kurt.	KS1	MW	KS2
T95	251	-0.81	0.49	14	3	-	-0.16	4.22	0.309	0.548	0.177
	246	0.84	0.46	15	4	-	-0.40	4.98	0.016		
TR96	251	-1.01	0.47	16	3	-	-0.09	4.07	0.114	0.442	0.152
	246	0.79	0.44	15	3	-	-0.33	4.65	0.112		
T15Q	251	-1.19	0.47	14	1	-	0.05	3.70	0.315	0.498	0.343
	246	0.85	0.44	15	3	-	-0.31	4.39	0.149		
EX96	251	-1.08	0.51	14	1	-	0.07	3.78	0.160	0.462	0.260
	246	0.93	0.47	15	3	-	-0.43	4.51	0.042		
T97	251	-0.10	0.52	17	1	-	0.16	3.76	0.311	0.444	0.204
	246	0.96	0.48	15	3	-	-0.39	4.58	0.008		
EX15	251	-1.14	0.50	12	2	-	-0.02	3.65	0.377	0.487	0.372
	246	0.92	0.47	16	4	-	-0.35	4.59	0.090		
T15H	251	-1.04	0.49	14	2	-	0.08	3.75	0.324	0.350	0.336
	246	0.84	0.47	14	3	-	-0.49	4.62	0.063		
EX98	251	-0.95	0.58	14	2	-	0.11	3.62	0.345	0.501	0.594
	246	0.90	0.56	16	4	-	-0.28	4.93	0.121		
EX99	251	-0.95	0.57	14	1	-	0.09	3.85	0.412	0.515	0.545
	246	0.96	0.54	16	1	-	-0.44	4.72	0.000		
TR13	251	-2.36	0.66	9	2	4	-1.83	18.33	0.021	0.576	0.398
	246	0.93	0.54	14	3	-	-0.36	4.57	0.024		

SUMMARY STATISTICS (cont.) "Medium/Longs"											
SERIES	n	$10^4\bar{x}$	10^2s	>2	>3	>4	Skew.	Kurt.	KS1	MW	KS2
T01	251	-0.88	0.54	11	3	-	0.16	4.13	0.088	0.493	0.385
	246	0.83	0.51	14	3	-	-0.50	4.65	0.129		
EX02	251	-2.30	0.67	9	2	3	-1.76	17.38	0.045	0.483	0.461
	246	1.03	0.55	13	3	-	-0.45	4.30	0.090		
"Longs"											
T03	251	-2.39	0.65	11	2	5	-2.48	24.94	0.011	0.574	0.753
	246	1.00	0.56	13	3	1	-0.02	5.87	0.087		
T11H	251	-1.49	0.72	12	2	4	-1.48	14.48	0.062	0.539	0.735
	246	1.11	0.62	12	4		-0.52	4.66	0.123		
TR10	251	-0.62	0.62	10	1	-	0.23	3.92	0.174	0.391	0.184
	246	1.46	0.63	12	3	-	-0.36	4.01	0.202		
EX05	251	-0.88	0.61	15	1	1	0.37	4.11	0.116	0.333	0.466
	246	3.16	0.68	13	3	-	0.32	6.22	0.261		
T12H	251	-0.97	0.61	13	2	-	0.24	3.88	0.213	0.383	0.266
	246	1.10	0.62	12	3	-	-0.37	4.48	0.123		
T11T	251	-0.90	0.65	14	2	-	0.15	3.78	0.194	0.389	0.0376
	246	1.19	0.64	13	3	-	-0.36	4.56	0.136		
T13H	251	-0.24	0.60	13	2	-	0.17	3.81	0.260	0.380	0.510
	246	1.19	0.61	15	3	-	-0.32	4.41	0.100		

SUMMARY STATISTICS (cont.) "Indexes"											
INDEX	n	$10^4\bar{x}$	10^2s	>2	>3	>4	Skew.	Kurt.	KS1	MW	KS2
FTAGOV "All stocks"	251	-0.79	0.43	13	2	1	-0.449	4.752	0.421	0.553	0.800
	244	0.86	0.44	12	2	1	-0.631	4.769	0.133		
FTAGOV "0-5 years"	251	-0.13	0.22	12	2	2	-0.705	6.345	0.059	0.620	0.823
	244	0.38	0.20	14	3	1	-0.629	5.316	0.138		
FTAGOV "5-15 years"	251	-0.98	0.52	14	2	1	-0.438	4.585	0.389	0.554	0.742
	244	1.10	0.51	13	2	-	-0.588	4.612	0.151		
FTAGOV ">15 years"	251	-0.88	0.65	16	1	-	-0.219	3.865	0.399	0.619	0.794
	244	1.16	0.69	10	3	1	-0.494	4.225	0.350		

TABLE 5.2

RETURNS "Shorts"															
SERIES	AUTOCORRELATIONS OF RETURNS										AUTOCORRELATION VARIANCES				
	Lag 1	Lags 1-30, frequency by class*									Estimates b_τ from returns x_t at lag				
	$r_{1,x}$	$\#_1$	1	2	3	4	5	6	$\#_6$		1	2	3	4	5
EX88	0.155 0.127	- -	0 0	1 3	7 4	8 12	8 8	6 3	(4) (2)		1.72 0.71	1.74 1.12	0.96 0.91	1.27 0.69	1.25 1.04
TR89	0.145 0.114	- -	0 0	2 2	5 7	8 13	7 6	8 2	(4) -		1.62 1.12	1.60 0.95	1.05 0.99	1.36 0.62	1.57 0.92
T10H	0.040 0.089	- -	0 1	2 2	4 6	12 13	10 7	2 1	(1) -		1.72 1.24	1.25 0.95	1.50 1.04	1.33 0.71	1.51 1.02
EX10	0.103 0.124	- -	1 1	2 1	4 9	6 11	11 6	6 2	(3) -		1.41 1.15	1.07 1.15	1.07 1.01	1.55 0.67	1.14 1.05
EX11	0.103 0.096	- -	0 2	2 1	4 7	8 9	8 10	8 1	(4) -		1.49 1.00	1.23 1.02	1.24 1.08	1.56 0.65	1.20 0.95
T13C	0.123 0.002	(1) -	1 0	1 2	4 8	9 11	6 9	9 0	(4) -		1.56 1.12	1.07 1.14	1.14 1.03	1.38 0.70	1.19 1.05
EX90	0.084 0.007	- -	1 0	2 3	2 9	8 10	11 8	6 0	(4) -		1.50 0.99	1.28 1.17	1.16 1.09	1.25 0.69	1.05 0.95
E12H	0.102 0.037	- -	0 0	2 2	3 8	8 10	10 10	7 0	(4) -		1.42 0.98	1.15 1.06	1.11 1.56	1.28 0.70	1.16 0.97
"Short/Mediums"															
T91C	0.150 0.082	- -	0 1	2 1	4 4	4 17	11 6	9 1	(5) -		1.37 0.83	1.18 0.92	1.47 1.14	1.26 0.84	1.35 0.92
EX91	0.068 0.105	- -	0 2	1 0	6 8	7 10	9 6	7 4	(3) -		1.32 0.85	1.24 0.85	1.70 1.17	1.29 0.88	1.60 0.89
T12T	0.042 -0.029	- -	0 1	2 3	6 7	7 12	11 6	4 1	(3) (1)		1.16 1.13	1.21 0.78	1.79 1.02	1.20 0.67	1.52 0.71
T10C	0.012 0.014	- -	0 2	0 0	6 9	10 14	8 3	6 2	(2) (1)		1.32 0.92	1.31 0.93	1.53 1.39	1.36 0.97	1.50 0.98

RETURNS
"Short/Mediums" (cont.)

SERIES	AUTOCORRELATIONS OF RETURNS									AUTOCORRELATION VARIANCES				
	Lag 1	Lags 1-30, frequency by class*								Estimates b_τ from returns x_t at lag				
	$r_{1,x}$	$\#_1$	1	2	3	4	5	6	$\#_6$	1	2	3	4	5
EX92	0.036	-	0	2	5	8	9	6	(3)	1.32	1.30	1.66	1.61	1.59
	0.060	-	1	1	8	10	9	1	-	0.76	0.69	1.35	0.93	0.87
E92C	0.026	-	0	0	7	9	9	5	(2)	1.19	1.18	1.73	1.53	1.54
	0.046	-	1	1	9	12	6	1	(1)	0.69	0.78	1.22	0.95	0.91
"Mediums"														
T93C	0.038	-	0	0	8	6	9	7	(3)	1.10	1.12	1.55	1.11	1.47
	0.095	(1)	1	1	8	9	10	1	-	0.85	0.80	1.35	1.04	1.01
T13T	0.075	-	0	2	3	9	5	11	(4)	1.09	1.17	1.71	1.28	1.32
	0.071	-	0	2	7	11	10	0	-	0.87	0.78	1.28	0.92	0.94
T14H	0.073	-	0	0	4	10	5	11	(5)	1.23	1.08	1.79	1.23	1.37
	0.113	-	0	2	8	11	8	1	-	0.84	0.81	1.14	1.07	0.93
E94C	0.061	-	0	1	4	7	10	8	(6)	1.21	1.12	1.72	1.19	1.37
	0.083	-	1	2	6	10	8	3	-	0.91	0.89	1.13	1.14	0.86
EX94	0.057	-	0	2	4	8	8	8	(5)	1.16	1.11	1.58	1.15	1.36
	0.100	-	1	1	7	12	7	2	-	0.76	0.75	1.33	1.06	0.82
TR12	0.084	-	0	1	4	11	8	6	(3)	1.14	1.16	1.64	1.11	1.43
	0.084	-	1	2	8	9	9	1	-	0.69	0.77	1.22	1.07	0.85
T95C	0.044	-	0	0	7	8	8	7	(6)	1.34	1.16	1.85	1.23	1.40
	0.087	-	1	2	6	10	9	2	-	0.83	0.70	1.46	1.07	0.92
TR96	0.040	-	0	0	4	10	7	9	(6)	1.32	1.05	1.74	1.19	1.23
	0.124	-	1	1	5	11	11	1	-	0.83	0.76	1.13	1.15	0.96
T15Q	0.091	-	0	0	7	6	8	9	(8)	1.15	1.26	1.50	1.13	1.59
	0.129	-	0	2	7	10	10	1	(1)	0.86	0.77	1.09	0.95	1.01
EX96	0.044	-	0	0	7	9	5	9	(6)	1.07	1.24	1.33	1.14	1.49
	0.090	-	0	2	8	13	6	1	-	0.71	0.77	1.20	1.52	0.76

RETURNS
"Mediums" (cont.)

SERIES	AUTOCORRELATIONS OF RETURNS									AUTOCORRELATION VARIANCES				
	Lag 1	Lags 1-30, frequency by class*								Estimates b_τ from returns x_t at lag				
	r_{1x}	$\#_1$	1	2	3	4	5	6	$\#_6$	1	2	3	4	5
T97C	0.041	-	0	1	4	11	3	11	(3)	1.00	1.07	1.41	1.34	1.24
	0.090	-	0	2	7	10	10	1	-	0.79	0.73	1.25	0.94	0.88
EX15	0.059	-	0	0	5	10	4	11	(7)	1.09	1.25	1.73	1.21	1.43
	0.119	-	1	2	5	12	9	1	-	0.83	0.77	1.20	1.12	0.98
T15H	0.082	-	0	1	3	9	6	11	(5)	1.19	1.15	1.42	1.20	1.35
	0.126	-	1	1	5	14	8	1	(1)	0.80	0.79	1.26	1.08	0.93
EX98	0.045	-	0	2	5	9	9	5	(2)	1.10	1.04	1.20	1.19	1.43
	0.083	-	1	2	7	14	5	1	-	0.68	0.64	1.39	1.13	0.91
EX99	0.043	-	1	0	7	7	7	8	(3)	1.09	1.14	1.23	1.26	1.50
	0.095	-	0	3	9	8	9	1	-	0.76	0.77	1.29	1.10	0.97
TR13	0.073	-	0	2	5	11	7	5	(2)	0.79	0.81	0.90	1.10	0.87
	0.093	(1)	1	2	7	10	8	2	-	0.74	0.71	1.24	1.09	1.01
"Mediums/Longs"														
T01C	0.092	-	1	1	7	8	5	8	(6)	1.04	1.04	1.20	1.06	1.48
	0.116	-	1	1	8	10	9	1	-	0.70	0.70	1.35	1.01	0.99
EX02	0.066	-	1	0	8	8	8	5	(2)	0.99	0.69	0.90	0.70	0.93
	0.083	-	1	2	7	14	5	1	-	0.73	0.76	1.32	1.06	0.98
"Longs"														
T03C	0.049	-	0	0	8	9	10	3	(2)	0.68	0.70	0.75	0.74	0.92
	0.050	(1)	1	1	10	7	11	0	-	0.74	0.64	1.16	1.05	0.90
T11H	-0.083	-	0	5	6	11	5	3	-	2.51	1.01	0.73	0.79	1.21
	-0.002	-	2	2	8	11	5	2	-	0.91	0.68	1.43	1.69	0.96
TR10	0.047	-	0	2	6	12	5	5	(3)	1.19	1.06	1.27	1.17	1.66
	0.031	(1)	1	3	6	10	9	0	-	0.81	0.80	1.24	1.04	0.87
EX05	0.054	-	0	1	5	13	5	6	(3)	1.24	1.10	1.08	1.17	1.73
	0.038	-	1	3	7	10	9	0	-	0.76	0.72	1.02	1.13	0.88

RETURNS "Longs" (cont.)															
SERIES	AUTOCORRELATIONS OF RETURNS									AUTOCORRELATION VARIANCES					
	Lag 1	Lags 1-30, frequency by class*								Estimates b_τ from returns x_t					
	$r_{1,x}$	$\#_1$	1	2	3	4	5	6	$\#_6$	1	2	3	4	5
T12H	0.077 0.064	- (1)	0 1	2 3	6 5	7 12	10 9	5 0	(2) -	1.24 0.74	1.24 0.68	1.28 1.18	1.16 1.02	1.62 0.86
T11T	0.049 0.038	- (1)	0 1	2 1	8 10	7 11	6 5	7 2	(2) -	1.14 0.75	1.16 0.68	1.13 1.12	1.16 1.06	1.54 0.81
T13H	0.080 0.074	- (1)	0 1	4 3	2 7	11 12	5 7	8 0	(5) -	1.19 0.73	1.30 0.73	1.20 1.16	1.21 0.93	1.59 0.91
"Indexes"															
FTGOVT	0.038 0.125	- -	0 1	0 4	11 4	6 8	6 12	7 1	(5) -	1.65 0.81	0.95 0.67	1.28 0.81	1.20 1.35	1.18 0.95
FTAGOV "All Stocks"	0.041 0.057	- (1)	0 1	2 4	5 6	10 6	8 12	5 1	(5) -	1.54 0.82	1.13 0.69	1.70 1.18	1.72 1.21	1.29 1.06
FTAGOV "0-5 years"	0.059 -0.007	- -	0 1	3 3	2 10	11 5	10 8	4 3	(2) (1)	1.48 1.34	1.45 0.79	1.35 1.25	1.37 1.05	1.10 1.06
FTAGOV "5-15 years"	0.038 0.076	- (1)	0 1	2 3	5 6	7 8	9 10	7 2	(5) -	1.48 0.80	1.11 0.70	1.70 1.16	1.66 1.21	1.31 1.06
FTAGOV ">15 years"	0.042 0.013	- -	0 1	4 5	6 6	8 8	5 10	7 0	(4) -	1.47 0.82	1.05 0.73	1.54 0.93	1.68 1.12	1.29 0.99

* The six classes are (1) $r<-0.1$, (2) $-0.1\le r<-0.05$, (3) $-0.05\le r<0$, (4) $0\le r\le 0.05$, (5) $0.05<r\le 0.1$, (6) $0.1<r$.

TABLE 5.3

SERIES	AUTOCORRELATIONS OF RETURNS Lag 1 r_{1y}	Lags 1-30, frequency by class* $\#_1$	1	2	3	4	5	6	$\#_6$	AUTOCORRELATION VARIANCES Estimates b_τ^* from returns y_t at lag 1	2	3	4	5
RESCALED RESTURNS "Shorts"														
EX88	0.175 0.127	- -	0 0	2 3	5 1	9 12	8 8	6 6	(4) (2)	1.65 0.49	1.95 1.04	0.94 0.95	1.49 0.61	0.97 1.16
TR89	0.137 0.107	- -	0 0	0 2	8 6	8 11	7 9	7 2	(4) -	1.49 0.88	1.78 0.87	0.99 1.04	1.27 0.54	2.49 0.82
T10H	0.043 0.118	- -	0 1	1 2	7 7	10 8	8 9	4 3	(3) -	1.57 0.84	1.22 0.86	1.17 1.07	1.12 0.61	2.45 1.08
EX10	0.114 0.134	- -	0 0	2 2	4 4	8 13	9 9	7 2	(4) (2)	1.52 0.80	1.18 1.01	0.85 1.00	1.39 0.62	1.04 1.02
EX11	0.114 0.098	- -	0 1	0 2	8 5	10 9	3 11	9 2	(4) -	1.56 0.70	1.23 0.89	1.03 0.96	1.41 0.61	1.00 0.84
T13C	0.135 0.012	- -	0 0	1 2	6 8	9 11	6 7	8 2	(5) -	1.65 0.86	1.07 1.15	0.99 1.01	1.19 0.66	1.11 1.04
EX90	0.095 0.030	- -	0 0	3 5	5 4	7 11	8 8	7 2	(4) -	1.60 0.70	1.26 1.10	0.99 1.00	1.20 0.76	1.06 1.00
E12H	0.121 0.049	- -	1 0	1 2	4 7	12 10	4 10	8 1	(5) -	1.56 0.78	1.24 1.04	1.00 1.12	1.25 0.67	0.99 1.09
"Short/Mediums"														
T91C	0.175 0.101	- -	0 0	1 2	6 7	10 9	6 9	7 3	(6) -	1.24 0.64	1.20 0.83	1.61 1.02	1.23 0.77	1.07 1.03
EX91	0.105 0.120	- -	0 1	1 1	3 6	12 12	8 5	6 5	(5) (2)	1.08 0.72	1.05 0.80	1.62 1.05	1.17 0.84	1.20 0.94
T12T	0.049 0.057	- -	0 1	0 3	7 5	10 10	8 8	5 3	(5) (1)	0.97 0.69	1.06 0.84	1.61 1.10	1.07 0.74	1.12 0.84
T10C	0.035 0.040	- -	0 1	0 3	4 5	13 14	7 5	6 2	(5) (2)	0.99 0.70	1.09 0.92	1.42 1.18	1.22 1.00	1.23 1.13

RESCALED RESTURNS
"Short/Mediums" (cont.)

SERIES	AUTOCORRELATIONS OF RETURNS										AUTOCORRELATION VARIANCES				
	Lag 1	Lags 1-30, frequency by class*									Estimates b_τ^* from returns y_t at lag				
	$r_{1,y}$	$\#_1$	1	2	3	4	5	6	$\#_6$		1	2	3	4	5
EX92	0.054 0.067	- -	0 1	2 3	6 5	8 10	8 9	6 2	(3) -		1.09 0.55	1.22 0.64	1.49 1.23	1.56 0.91	1.28 0.99
E92C	0.030 0.049	- -	0 1	1 3	4 5	9 13	10 6	6 2	(4) -		0.91 0.49	1.12 0.73	1.49 1.08	1.44 0.95	1.19 1.02
"Mediums"															
T93C	0.059 0.103	- (1)	0 1	0 1	6 5	7 11	8 7	9 5	(7) -		0.98 0.63	1.04 0.74	1.38 1.24	1.03 0.99	1.26 1.06
T13T	0.091 0.088	- -	0 0	1 2	4 6	8 9	9 10	8 3	(5) -		0.86 0.75	1.05 0.75	1.55 1.18	1.23 0.89	1.03 0.98
T14H	0.104 0.131	- -	0 0	0 2	3 6	11 9	8 8	8 5	(7) (2)		0.95 0.75	1.00 0.79	1.63 1.08	1.06 1.02	1.17 1.04
E94C	0.072 0.106	- -	0 1	2 1	2 6	11 10	8 5	7 7	(5) (1)		0.99 0.83	1.06 0.86	1.58 1.20	1.09 1.05	1.15 0.83
EX94	0.089 0.118	- -	0 0	1 4	5 6	8 10	7 5	9 5	(5) (1)		0.95 0.71	1.03 0.75	1.48 1.36	1.19 0.95	1.12 0.78
TR12	0.096 0.094	- -	0 0	2 2	1 7	13 10	7 8	7 3	(7) (1)		0.92 0.64	1.08 0.75	1.59 1.26	1.08 0.95	1.26 0.84
T95C	0.073 0.107	- -	0 0	0 3	5 6	8 7	8 8	9 6	(6) -		0.95 0.74	0.98 0.76	1.72 1.99	1.08 0.92	1.20 0.95
TR96	0.073 0.145	- -	0 1	1 1	4 7	10 8	7 9	8 4	(6) (2)		0.96 0.78	0.95 0.74	1.90 1.21	1.06 0.98	1.00 1.00
T15Q	0.119 0.151	- -	0 1	2 1	2 5	8 12	8 6	10 5	(8) (2)		1.04 0.82	1.13 0.69	1.37 1.26	1.10 0.86	1.38 1.11
EX96	0.080 0.116	- -	0 0	1 1	3 6	11 13	6 6	9 4	(8) (1)		0.88 0.62	1.13 0.75	1.22 1.34	1.07 1.42	1.26 0.78

RESCALED RESTURNS
"Mediums" (cont.)

SERIES	AUTOCORRELATIONS OF RETURNS									AUTOCORRELATION VARIANCES				
	Lag 1	Lags 1-30, frequency by class*								Estimates b_τ^* from returns y_t at lag				
	$r_{1,y}$	$\#_1$	1	2	3	4	5	6	$\#_6$	1	2	3	4	5
T97C	0.055	-	0	1	4	9	8	8	(5)	0.82	1.08	1.32	1.29	0.96
	0.105	-	0	2	6	8	11	3	(1)	0.74	0.76	1.38	0.75	0.87
EX15	0.083	-	0	1	2	11	5	11	(8)	0.89	1.11	1.55	1.06	1.24
	0.154	-	0	2	6	8	10	4	(1)	0.78	0.75	1.41	1.02	1.09
T15H	0.105	-	0	1	4	7	7	11	(6)	0.93	1.09	1.24	1.10	1.20
	0.157	-	1	1	4	11	9	4	(1)	0.73	0.73	1.55	0.95	0.95
EX98	0.051	-	0	2	3	9	12	4	(4)	0.92	1.02	1.14	1.04	1.21
	0.100	-	0	2	6	10	9	3	(1)	0.62	0.62	1.60	1.12	0.93
EX99	0.055	-	0	2	6	6	9	7	(4)	0.94	1.12	1.16	1.10	1.22
	0.125	-	0	2	8	8	8	4	(2)	0.67	0.69	1.50	1.03	0.96
TR13	0.075	-	0	1	4	13	9	3	-	0.60	0.61	0.92	0.99	0.39
	0.114	-	0	2	8	7	8	5	(1)	0.66	0.69	1.43	1.01	0.99
"Medium/Longs"														
T01C	0.081	-	0	3	7	5	9	6	(3)	0.87	1.02	1.16	0.95	1.20
	0.135	-	1	1	5	11	9	3	(1)	0.64	0.68	1.54	0.96	1.01
EX02	0.018	-	0	3	6	9	8	4	(1)	1.09	0.39	0.82	0.49	0.47
	0.093	-	1	1	7	11	7	3	-	0.65	0.80	1.56	0.97	1.06
"Longs"														
T03C	0.016	-	0	1	7	13	5	4	-	0.38	0.66	0.47	0.54	0.60
	0.067	-	1	2	8	5	9	5	(1)	0.64	0.61	1.28	0.99	0.94
T11H	-0.100	-	1	5	7	11	5	1	(1)	2.17	1.24	0.63	0.50	0.86
	0.020	-	1	3	4	9	9	4	-	0.83	0.69	1.66	1.79	0.94
TR10	0.036	-	0	1	7	10	6	6	(3)	1.04	0.94	1.13	1.05	1.45
	0.049	-	0	4	6	8	10	2	-	0.75	0.85	1.34	0.99	0.86
EX05	0.051	-	0	2	3	11	10	4	(3)	0.99	1.04	1.00	0.99	1.40
	0.062	-	0	3	6	8	9	4	(1)	0.70	0.74	1.03	1.05	0.86

RESCALED RESTURNS
"Longs" (cont.)

SERIES	AUTOCORRELATIONS OF RETURNS									AUTOCORRELATION VARIANCES				
	Lag 1	Lags 1-30, frequency by class*								Estimates b_τ^* from returns y_t at lag				
	$r_{1,y}$	$\#_1$	1	2	3	4	5	6	$\#_6$	1	2	3	4	5
T12H	0.073	-	0	1	5	8	9	7	(3)	1.00	1.17	1.19	0.99	1.29
	0.082	-	2	2	7	7	9	3	-	0.66	0.72	1.22	1.00	0.84
T11T	0.046	-	0	2	7	7	7	7	(2)	0.99	1.09	1.06	1.04	1.29
	0.060	-	1	3	7	7	10	2	(1)	0.66	0.74	1.18	1.03	0.79
T13H	0.076	-	0	2	4	6	9	9	(4)	0.99	1.10	1.08	0.95	1.54
	0.091	-	2	2	7	6	12	1	(1)	0.64	0.77	1.23	0.89	0.93
"Indexes"														
FTGOVT	0.054	-	0	1	10	4	6	9	(6)	1.18	0.99	1.14	0.98	1.13
	0.153	-	0	4	5	8	11	2	(2)	0.79	0.66	1.90	1.32	0.97
FTAGOV "All Stocks"	0.063	-	0	3	2	9	10	6	(5)	0.95	1.23	1.30	1.40	1.29
	0.080	-	1	3	5	8	10	3	(1)	0.81	0.77	1.15	1.07	1.07
FTAGOV "0-5 years"	0.062	-	1	3	5	5	11	5	(2)	0.99	1.38	0.97	1.26	1.28
	0.016	-	0	2	7	11	7	3	(2)	1.34	0.77	1.07	0.93	1.00
FTAGOV "5-15 years"	0.061	-	0	1	3	9	10	7	(5)	0.96	1.26	1.33	1.37	1.33
	0.096	(1)	1	4	4	8	10	3	-	0.80	0.79	1.23	1.04	1.09
FTAGOV ">15 years"	0.046	-	0	3	6	10	6	5	(5)	0.93	1.16	1.27	1.32	1.21
	0.019	-	1	2	10	7	8	2	-	0.77	0.73	0.95	1.03	1.02

* The six classes are (1) $r<-0.1$, (2) $-0.1\le r<-0.05$, (3) $-0.05\le r<0$, (4) $0\le r\le 0.05$, (5) $0.05<r\le 0.1$, (6) $0.1<r$.

TABLE 5.4

RANDOM WALK STATISTICS "Shorts"									
SERIES	T^*	U^*	$\sqrt{n^*} r_{1,y}$	Q_{10}	Q_{30}	Q_{50}	f_0	f_ω	K
C.Val.	1.65	1.65	±1.96	18.31	43.77	67.50	1.65	1.65	±1.96
EX88	4.73 3.95	4.01 3.48	2.65 1.92	30.08 15.65	44.40 36.44	51.50 54.75	3.40 4.58	-0.35 -0.56	-4.03 -1.84
TR89	5.40 2.75	4.98 2.30	2.08 1.60	36.62 9.70	50.14 19.88	58.10 41.18	3.84 3.42	-0.78 -1.22	-0.67 -1.72
T10H	3.80 2.77	3.85 2.26	0.65 1.77	24.93 12.28	37.00 27.49	47.67 48.87	2.91 3.41	-1.22 -1.01	0.34 -3.33
EX10	4.81 3.02	4.48 2.43	1.74 2.01	31.68 12.15	43.94 24.69	55.11 45.53	3.54 3.74	-0.42 -0.99	0.11 -2.63
EX11	5.11 3.12	4.82 2.77	1.73 1.48	33.84 12.22	42.76 27.58	51.47 50.19	3.63 3.87	-0.40 -0.88	-0.73 -2.46
T13C	5.21 1.51	4.79 1.57	2.05 0.18	40.33 5.43	52.12 19.91	63.29 39.14	3.71 2.46	-0.03 -0.94	0.04 -0.37
EX90	4.66 1.29	4.45 1.21	1.45 0.46	34.33 4.65	46.47 20.91	56.30 39.19	3.25 2.20	-0.56 -1.11	-0.85 0.04
E12H	5.32 1.56	4.99 1.38	1.85 0.73	35.37 5.76	48.19 18.48	58.87 36.45	3.62 2.46	0.04 -0.86	-0.71 -1.08
"Short/Mediums"									
T91C	6.25 2.16	5.66 1.70	2.65 1.51	48.15 6.38	60.12 22.83	68.47 42.59	4.58 2.99	-1.23 -0.60	-1.10 -1.24
EX91	5.97 2.64	5.81 2.09	1.59 1.81	41.34 10.18	55.04 32.16	66.37 54.34	4.28 3.39	-1.03 -0.83	-0.73 -0.49
T12T	4.99 1.66	5.10 1.43	0.75 0.86	35.06 7.24	48.07 28.43	62.07 50.16	3.63 2.45	-1.13 -0.79	-0.57 -0.40
T10C	5.53 1.52	5.79 1.40	0.53 0.60	35.81 5.11	47.94 22.41	55.14 44.10	4.07 2.35	-1.10 -0.54	0.58 0.13

RANDOM WALK STATISTICS
"Short/Mediums" (cont.)

SERIES	T^*	U^*	$\sqrt{n^*}r_{1y}$	Q_{10}	Q_{30}	Q_{50}	f_0	f_ω	K
C.Val.	1.65	1.65	±1.96	18.31	43.77	67.50	1.65	1.65	±1.96
EX92	4.88 1.86	4.95 1.59	0.83 1.01	34.00 6.91	46.14 24.94	56.68 44.05	3.50 2.68	-1.15 -0.74	-0.25 -0.72
E92C	5.17 1.96	5.44 1.82	0.45 0.73	30.93 6.86	45.44 26.59	54.64 45.27	3.65 2.79	-1.04 -0.68	0.49 -0.34
"Mediums"									
T93C	5.97 2.71	6.11 2.29	0.89 1.54	36.17 10.63	53.93 30.65	64.68 52.57	4.21 3.52	-0.98 -0.80	0.12 -1.64
T13T	6.39 2.58	6.36 2.24	1.39 1.33	43.98 9.42	60.48 24.61	69.53 47.77	4.63 3.45	-0.84 -0.64	-1.21 -0.97
T14H	6.83 3.01	6.76 2.43	1.57 1.97	49.01 13.20	62.56 31.89	70.80 55.73	4.94 3.86	-0.68 -0.52	-0.48 -1.59
E94C	6.15 2.90	6.22 2.47	1.09 1.59	44.42 11.36	59.59 36.21	69.16 53.77	4.55 3.88	-1.02 -0.67	-0.64 -0.53
EX94	6.22 2.61	6.18 2.08	1.35 1.77	43.11 10.40	59.40 31.87	66.88 53.42	4.60 3.47	-0.88 -0.78	-0.22 -0.34
TR12	5.96 2.43	5.86 2.04	1.46 1.42	38.58 7.79	49.46 27.87	55.31 49.83	4.25 3.29	-0.87 -0.60	0.44 0.16
T95C	7.09 2.79	7.24 2.34	1.11 1.61	52.32 11.33	69.13 33.64	77.14 56.23	5.10 3.65	-1.20 -0.96	0.36 -0.33
TR96	7.04 3.44	7.18 2.80	1.11 2.19	55.56 11.94	73.36 34.93	82.33 58.24	5.10 4.25	-1.00 -0.59	-0.47 -1.67
T15Q	7.69 3.26	7.59 2.58	1.87 2.27	58.28 11.46	82.49 30.61	89.06 56.38	5.50 4.16	-1.26 -0.41	-0.54 -0.36
EX96	6.72 2.78	6.79 2.28	1.21 1.75	47.27 7.79	65.48 25.66	77.28 50.04	4.71 3.59	-1.01 -0.85	0.44 -0.31

RANDOM WALK STATISTICS
"Mediums" (cont.)

SERIES	T^*	U^*	$\sqrt{n^*}r_{1,y}$	Q_{10}	Q_{30}	Q_{50}	f_0	f_ω	K
C.Val.	1.65	1.65	±1.96	18.31	43.77	67.50	1.65	1.65	±1.96
T97C	5.95 2.93	6.12 2.51	0.83 1.58	41.58 10.22	58.28 28.62	70.79 51.70	4.34 3.77	-0.96 -0.48	0.11 -1.35
EX15	6.94 3.41	7.01 2.72	1.26 2.32	49.76 14.71	68.95 33.56	77.53 60.56	5.01 4.12	-1.08 -0.69	0.07 -1.45
T15H	7.11 3.61	7.05 2.92	1.59 2.36	53.63 14.47	75.67 32.54	87.94 61.42	5.24 4.35	-0.85 -0.77	0.21 -1.42
EX98	4.81 2.16	4.91 1.71	0.77 1.51	31.50 7.25	46.33 24.62	56.05 45.57	3.54 2.99	-1.13 -0.51	-0.28 -1.06
T99C	5.20 2.14	5.36 1.62	0.68 1.66	36.59 7.96	56.42 25.86	62.39 50.81	3.94 3.01	-0.93 -0.72	-0.02 -1.70
EX99	4.89 2.58	4.97 2.00	0.83 1.88	36.69 8.66	51.50 27.51	61.83 51.53	3.53 3.36	-1.02 -0.70	-0.38 -1.25
TR13	3.06 2.72	2.84 2.22	1.14 1.72	11.65 8.13	21.19 30.39	27.68 55.18	2.24 3.57	-1.03 -0.69	-0.25 -0.68
"Medium/Longs"									
T01C	3.75 2.88	3.55 2.26	1.24 1.97	20.08 13.20	42.39 31.89	48.18 55.73	3.04 3.86	-0.48 -0.52	-0.33 -1.59
EX02	2.86 2.37	2.99 1.98	0.28 1.39	12.74 7.44	27.84 25.82	34.52 46.76	1.97 3.20	-1.07 -0.80	0.39 -0.57
"Longs"									
T03C	2.72 2.33	2.85 2.10	0.24 1.01	11.10 12.28	21.87 32.22	26.60 52.92	1.79 3.30	-0.82 -0.60	-0.25 0.10
T11H	0.33 1.88	1.00 1.92	-1.51 0.30	11.67 8.87	22.87 29.68	31.48 45.95	0.273 2.93	-0.27 -0.71	0.39 0.12
TR10	3.60 1.72	3.69 1.55	0.54 0.74	18.85 7.96	35.48 27.16	45.79 46.67	2.86 2.68	-1.14 -0.52	0.13 0.12
EX05	4.74 1.93	4.83 1.70	0.77 0.94	28.93 8.47	42.23 27.89	52.10 48.32	3.37 2.85	-1.13 -0.60	0.04 0.67

RANDOM WALK STATISTICS
"Longs" (cont.)

SERIES	T^*	U^*	$\sqrt{n^*} r_{1,y}$	Q_{10}	Q_{30}	Q_{50}	f_0	f_ω	K
C.Val.	1.65	1.65	±1.96	18.31	43.77	67.50	1.65	1.65	±1.96
T12H	4.88 1.78	4.83 1.40	1.10 1.24	29.66 9.89	44.87 29.44	53.80 50.88	3.49 2.69	-1.06 -0.43	0.03 -1.10
T11T	4.10 1.61	4.17 1.37	0.70 0.91	24.36 6.91	39.41 27.90	50.72 47.50	3.06 2.54	-0.99 -0.56	0.06 0.00
T13H	5.29 1.96	5.26 1.55	1.15 1.36	32.70 10.88	53.10 31.79	63.82 53.31	3.86 2.85	-1.22 -0.60	-0.68 -0.51
"Indexes"									
FTGOVT	5.04 2.91	5.13 2.18	0.82 2.29	29.96 12.43	52.86 31.88	59.40 65.06	3.64 3.76	-0.99 -0.00	-0.25 -1.03
FTAGOV "All Stocks"	5.25 2.35	5.31 2.05	0.95 1.19	33.76 13.58	59.55 31.88	70.28 61.20	3.81 3.19	-1.12 -0.62	0.45 -0.12
FTAGOV "0-5 years"	4.44 2.34	4.42 2.44	0.95 0.24	35.27 16.21	53.95 30.22	65.56 52.22	2.79 3.31	-0.60 -0.62	-0.27 -1.68
FTAGOV "5-15 years"	5.64 2.38	5.74 1.97	0.92 1.43	35.55 13.44	62.73 33.00	74.72 62.76	4.15 3.03	-1.19 -0.75	-0.06 -0.54
FTAGOV ">15 years"	3.71 1.07	3.74 1.04	0.70 0.29	23.20 7.09	47.12 22.73	59.52 45.12	2.76 2.05	-1.17 -0.41	-0.05 0.03

APPENDIX TO CHAPTER FIVE

The Quality, Source and Nature of the Data Set

The Nature of the Data Set

This study uses a set of daily closing price observations on a group of representative gilt-edged securities. The data set runs from 27 October 1985 to 16 October 1987, providing a comparison of the year before and the year after Big Bang. To qualify for inclusion in the data set, the bonds had to be quoted fully paid in the market, not be undated, index-linked, variable rate or conversion stocks, and have high coupons. These forty-five bonds are listed in table 5.A.1., where their codes, used in the results tables of chapters five and six, are given.

The high coupon requirement allows this set of gilts to be used for all parts of the study. In later chapters, the presence of tax effects would change maintained theoretical relationships and could distort empirical results. For example, in chapter six, where we seek to measure the underlying term structure of interest rates in the market, the presence of taxes would define distinct no-arbitrage term structures for each tax bracket. What we are seeking in chapter six, in fact, is the zero-tax rate term structure.[1] Using the knowledge that, as explained in chapter 4 (section 4.2.4), relatively high coupon bonds tend to be preferred by relatively low rate tax paying investors, it is likely that high coupon bonds will be predominantly held by low (zero) tax rate paying investors. However, there is a spectrum of coupon levels, a range of maturities and a set of tax brackets, and it is thus possible that some high coupon bonds can be efficiently held by non-zero rate (albeit low) investors.[2] The choice of a simple high-low division does not imply that gilts can be partitioned into two homogeneous groups, but is a simple device to facilitate the development and testing of subsequent procedures and hypotheses. Clearly, such a division will not necessarily eliminate the effects of taxes from the sample entirely, but is

[1] Even this term structure is not generally unique in the presence of buy/sell spreads or the restrictions on short selling necessary to prevent tax arbitrage. The term structure is conditional on what cash flows are being sought. However, in practice, this distinction is not important for our current purposes.

[2] See Schaefer (1980b).

believed to be justifiable given the tasks ahead. Checks on the magnitude of any remaining tax effects are made as necessary later in the study.[3]

In the time series analysis of chapter five and chapter six, those bonds that were either redeemed or issued during the two years of the sample are not included, as one half of the data set would have none or very little data. Thus there were thirty nine gilts studied in chapter five, between forty two and forty five used for the cross-sectional work in chapter six, and the base thirty nine used for the time series analysis in chapter six.

The Source of the Data

The source of the data is the Datastream historical gilt price series. Datastream extract this data from TOPIC, the view data system of the Stock Exchange.[4] The prices are the daily closing values of the SEAQ mid-prices for the individual gilts.[5] These closing values (4.30 p.m.) are calculated by the Stock Exchange Price Reporting department from prices contained in the GEMMs Closed User Group pages of TOPIC. Each market maker has twenty such pages on TOPIC allowing him to input price data and other information in any format and make it available to a selected (closed) client base. The Stock Exchange mid-price for a stock is "the average of two market makers prices selected daily on a most suitable basis". The market makers are unaware of whether their prices are being used on any particular day.

To prevent spurious results when comparing the periods before and after Big Bang, it is essential to determine whether this current price dissemination arrangement was materially different before Big Bang. Firstly, Datastream report that they have used this same data source right throughout the two year sample period. However, the Stock Exchange, must have changed its price reporting system, due to the introduction of new technology at Big Bang, and

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[3] See e.g. chapter 6, section 6.8.

[4] I am grateful to Martin Wheatley of the Stock Exchange and the Customer Services department of Datastream for assistance in verifying data sources.

[5] The SEAQ mid-price service is described in chapter three, section 3.2.

the movement of the market place from the floor of the Stock Exchange to the dealing room desks.

Before Big Bang, price reporters wandered around the Stock Exchange floor and collected prices from the Jobbers' blackboards, throughout the day. These were then manually input to the TOPIC system and displayed on the screens. The closing prices were taken from the screens at about 3.00 p.m. each day. Thus both the reporting system and the timing changed at Big Bang. However, the Stock Exchange believe that this is unlikely to cause problems of inconsistency for the following reason. The gilt-edged market has a very low relative volume of trade (and hence likely price changes) in the last ninety minutes of the day. This is primarily due to the closure of LIFFE before the closure of the gilt-edged market. Thus prices taken at different times during this period are unlikely to be at all different.

The Quality of the Data

The quality of the price data currently, is entirely dependent on the quality of the underlying GEMMs CUGS prices. A Stock Exchange survey of the system noted the following points. The quality of prices and speed of update varies considerably between market makers and also appears to depend on the time of day. The service is noticeably slower at lunch time. In several cases the updates were seen to be not as regular as promised by the market makers and not all prices had a time tag. The most frequently updated stocks were generally the most heavily traded ones. Medium and long stocks were updated twice as quickly as short dated stocks, thus the runners pages provide the most reliable indication of market movements.

Despite an understanding that under heavy trading conditions the market makers have insufficient time to input prices into the trading system, the surveys showed that, as indicated above, the most accurate prices are those for which there is much activity.

The degree to which closing prices are dependent on the quality of prices during the day is probably quite small, and certainly as a source of closing prices, the SEAQ mid-price cannot be outperformed by any other publically available source. The Reuters service is similar but also depends entirely on market maker input from either the Topic screens or from specific telephone enquiry, though it appeared to be more quickly updated during the day. Again this is not critical for closing prices. Other sources are the individual market makers, who would not necessarily provide a consensus "market" price. Transaction prices are available in the SEDOL, but large transactions are not recorded unless both parties agree. This potentially useful series is therefore full of invisible holes.

The quality of data before Big Bang, was dependent on the quotes given to the price reporters. It is not clear whether these quotes would be any more or less likely to be up-to-date than those obtained after Big Bang. On average, there has probably been little change in this area.

TABLE 5.A.1

Identity of Gilt-Edged Securities

Name	Coupon	Redemption	Code in Results Tables	FOTRA=#
"Shorts"				
Exchequer	11.75%	1986	*	
Treasury	12.00%	1986	*	
Exchequer	14.00%	1986	*	
Exchequer	13.25%	1987	*	
Treasury	10.00%	1987	*	
Treasury	12.00%	1987	*	
Exchequer	10.50%	1988	EX88	
Treasury	11.50%	1989	TR89	
Treasury	10.50%	1989	T10H	
Exchequer	10.00%	1989	EX10	
Exchequer	11.00%	1989	EX11	
Treasury	13.00%	1990	T13	#
Exchequer	11.00%	1990	EX90	#
Exchequer	12.50%	1990	E12H	
"Short/Mediums"				
Treasury	11.75%	1991	T91	
Exchequer	11.00%	1991	EX91	
Treasury	12.75%	1992	T12T	#
Treasury	10.00%	1992	T10	
Exchequer	12.25%	1992	EX92	
Exchequer	13.50%	1992	E92	
"Mediums"				
Treasury	12.50%	1993	T93	#
Treasury	13.75%	1993	T13T	#
Treasury	14.50%	1994	T14H	#
Exchequer	13.50%	1994	E94	
Exchequer	12.50%	1994	EX94	
Treasury	12.00%	1995	TR12	
Treasury	12.75%	1995	T95	#
Treasury	14.00%	1996	TR96	
Treasury	15.25%	1996	T15Q	#
Exchequer	13.25%	1996	EX96	#
Treasury	13.25%	1997	T97	#
Exchequer	15.00%	1997	EX15	
Treasury	15.50%	1998	T15H	#
Exchequer	12.00%	1998	EX98	
Exchequer	12.25%	1999	EX99	
Treasury	13.00%	2000	TR13	

TABLE 5.A.1 (cont.)

Identity of Gilt-Edged Securities

Name	Coupon	Redemption	Code in Results Tables	FOTRA=#
"Medium/Longs"				
Treasury	14.00%	1998-2001	T01	
Exchequer	12.00%	1999-2002	EX02	
"Longs"				
Treasury	13.75%	2000-2003	T03	
Treasury	11.50%	2001-2004	T11H	
Treasury	10.00%	2004	TR10	
Exchequer	10.50%	2005	EX05	
Treasury	12.50%	2003-2005	T12H	
Treasury	8.50%	2007	+	
Treasury	11.75%	3003-2007	T11T	
Treasury	13.50%	2004-2008	T13H	
Treasury	9.00%	2008	+	
Treasury	8.00%	2009	+	
Indexes				
Financial Times Government Securities Index			FTGOVT	
FT Actuaries Government Securities Indexes			FTAGOV(.)	

Key

*	=	Redeemed before 16 October 1987
+	=	Issued after 27 October 1985
Short	=	Maturity of less than 5 years at 27 October 1985
Short / Medium	=	Maturity of less than 5 years at 16 October 1987
Medium	=	Maturity of less than 15 years at 27 October 1987 and greater than 5 years at 16 October 1987
Medium / Long	=	Maturity of less than 15 years and greater than 5 years at 16 October 1987
Long	=	Maturity of greater than 15 years at 16 October 1987

CHAPTER SIX

Price Anomalies and Market Efficiency[1]

6.1. Introduction

In a perfect market, the market price of a bond should be equal to the present discounted value of the future cash flows to be received by the bond. This can be expressed in terms of the familiar discounting equation

$$P=\frac{C_1}{(1+R_1)}+\frac{C_2}{(1+R_2)^2}+\cdots+\frac{C_n}{(1+R_n)^n} \tag{6.1}$$

The market price of the bond is P, and the payments to be made at the ends of periods $1,2,\ldots,n$ are $C_1, C_2, \ldots, C_n$. The spot interest rates applicable to these payments are the series $R_1, R_2, R_3, \ldots, R_n$, and we may regard the term structure as being the series of spot rates.

Studies to estimate the term structure have used various methods of fitting the above discounting equation. The particular method chosen is largely determined by the intended use of the interest rate estimates, however, two techniques seem to prevail. Both estimate a linear approximation to the discount function, but differ in their choice of approximation functions. McCulloch (1971) used polynomial spline functions, whereas Schaefer (1973,1981) used a set of Bernstein polynomials. The residuals between the market price and the fitted price (the price calculated by discounting the stream of coupons by the estimated spot rates) can be used to test market efficiency.

From a theoretical viewpoint, in a frictionless market, equation (6.1) represents a 'no arbitrage' condition. Consider a set of m riskless bonds each paying cash flows over periods 1,...,n. Then, as Schaefer (1980a) has shown, by applying a variant of Farkas' lemma[2] we may show that either (a) it is possible to find a set of spot rates $R_1, R_2, R_3, \ldots, R_n$ that satisfy equa-

[1] A paper based upon this chapter has been accepted for publication by the *Journal of Business Finance and Accounting*.

[2] See e.g. Gale (1960) p.48.

tion (1.1.2) for each bond, or (b) it is possible to construct a zero net investment portfolio that provides a strictly positive cash flow in some period (1,...,n or the current period 0). Alternatively, (b) is pure arbitrage. The implication of this result is clear. If we find any residuals at all, then the market is inefficient as pure arbitrage is possible.

From a practical point of view, the interpretation of pricing residuals is not as clear cut. Firstly, as discussed in chapter 4 (section 4.2.4) and the appendix to chapter 5, the presence of taxes would cause equation (6.1) to no longer be a no-arbitrage relationship for all bonds, and a friction in trading (such as restrictions on short selling) is necessary to restore a no-arbitrage relationship (with distinct term structures defined for each tax bracket).[3] For the moment we shall side step this issue and consider the implications of the procedure for fitting the term structure. As will be seen, a key consideration in fitting term structure curves is that of ensuring sufficient degrees of freedom for reliable estimation.[4] While, it is possible to fit an approximating function to all the kinks in the term structure, this is, typically, neither necessary nor desirable. Sufficiently reliable measurements of the term structure can be obtained from relatively low degree functions, and thus the residuals from fitting the term structure are proxying the finer points of the curve. Furthermore, other sources of residuals such non-synchronous price data, differences in liquidity, and measurement error, do permit the residuals to be interpreted as indicators of the pricing efficiency of the market. Finally,it is also not clear that observed residuals could be regarded as pure arbitrage if the practical costs (e.g. expensive or restricted short selling) are prohibitive.

In this chapter, we measure the term structure, calculate the fitted prices and examine the properties of the pricing residuals. In measuring the term structure, the opportunity is taken to develop a new method for fitting the term structure: the use of "B-Splines". In addition to the purpose of analyzing residuals, there are a number of other reasons why measurements of the

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[3] See also Schaefer (1980a,b) and later in this chapter.

[4] See sections 6.4 and 6.5

term structure are useful. They can be used to value other certain cash flows (such as new bond issues), or test the assumptions of various term structure theories. Indeed, this technique is used to provide a database for the next two chapters, which examine the dynamics of the term structure and seek to determine whether the dynamics of the term structure are efficient.

6.2. Measuring the Term Structure of Interest Rates

Conceptually, term structure estimation is reasonably straightforward. Let us assume that we have a set of m default free bonds, where the i^{th} bond has a price P_i, pays an amount $C_{i,j}$ at time t_j, and where there are n different periods. If the number of bonds with linearly independent vectors of cash flows exceeds the number of payment dates, then because future cash flows are known and prices are observable, the discount function can be estimated by ordinary least squares from the following equation

$$P_i = \sum_{j=1}^{n} C_{i,j}\, d_{t_j} \tag{6.2}$$

where d_{t_j} is the discount factor for date t_j.[5] This discount factor is equivalently the price of a pure discount bond paying £1 at time t_j in every state of nature. Each discount factor defines a spot rate, the rate at which the discount factor must be continuously compounded until t_j to reach £1, and the set of these rates may be regarded as the term structure of interest rates. Carleton and Cooper (1976) successfully employed this method by selecting a sample of bonds which only incurred four different payment dates per year. However, to ensure the cash flow matrix had sufficient rank, it was also necessary to restrict the maximum maturity under consideration to seven years. Consequently spot rates for gilts of medium and long maturities

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[5] The reader is asked to note that there are some differences between the notation used in this chapter and that used in other chapters. This makes the presentation of the complex formulae described in this chapter far less cluttered. The notation used in this chapter is fully explained in the course of the text.

could not be obtained in this manner. Further difficulties are caused by only obtaining discrete point estimates. Discrete spot rate estimates will generate forward rates that do not lie on a smooth curve.[6]

6.2.1. Approximation Functions

An alternative approach that avoids these difficulties consists of estimating a linear approximation to the (continuous) discount function. In other words, instead of estimating each d_{t_j} value directly we substitute an approximation of the form

$$d(t) = \sum_{l=1}^{L} \alpha_l f_l(t) \tag{6.3}$$

and estimate the α_l coefficients which are applied to the L approximating functions chosen. On substitution of this function into our price equation (6.2), we obtain

$$P_i = \sum_{l=1}^{L} \alpha_l \sum_{j=1}^{n} C_{i,j} \, f_l(t). \tag{6.4}$$

We still have a linear regression equation but now we can choose how many coefficients we wish to estimate.

The Weierstrass Theorem has been used to justify polynomial approximation. This theorem says that we can approximate arbitrarily closely over a given interval any continuously differentiable function by a polynomial. Depending on the required degree of accuracy, a higher order polynomial may be necessary. There are dangers, however, in using high order polynomials. Although these may provide a greater degree of accuracy, if they are fitted through limited data, then it is possible for the approximation to fluctuate wildly over its range.

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[6] However, this methodology more closely reflects economic reality than the approximation methodology in the sense that no clear economic meaning can be attached to a spot rate estimated for a date at which no bond actually makes a payment.

6.2.2. Spline Functions

Sections of high degree polynomials can be closely approximated, however, by several low degree polynomials. When joined together, they are able to approximate a continuous but complex shape. These piecewise polynomial functions are called spline functions, and, unlike polynomials have uniform convergence properties. Furthermore, they provide a high order of derivative continuity, which has the added advantage of fixing some of the freedom of the approximating function, and reducing the number of parameters to be estimated. The borders are defined at abscissa values and are called knots.

It can be shown (e.g. Powell, 1981 ch.3) that a k order polynomial spline function over the interval $a=t_0<t_1< \cdots <t_n=b$ (i.e. fitted in n segments over $[a,b]$) is of the form

$$d(t) = \sum_{l=0}^{k}\gamma_l t^l + \frac{1}{k!}\sum_{l=1}^{n-1}\delta_l (t-t_l)_+^k \qquad (6.5)$$

where the subscript $_+$ has the meaning $(t-t_l)_+ = \max[0,(t-t_l)]$. The function has $k+n$ coefficients $(\gamma_l : l=0,1,...,k)$ and $(\delta_l : l=0,1,...,n-1)$, and third degree $(k=3)$ functions are most usually encountered. This is the minimum necessary to ensure a smooth forward rate curve.

Spline functions were first used to estimate the term structure by McCulloch (1971). He used a piecewise quadratic function to approximate the discount function for U.S. Treasury and a number of corporate issues. As this function is only once continuously differentiable, the estimated forward rate curves have discontinuous first derivatives - or 'knuckles' , as McCulloch terms it. In later work (1975), he used a cubic spline to approximate a family of tax adjusted discount functions and yield curves. McCulloch's spline functions are not of the form of the polynomial spline function given above, but they have been shown to be equivalent (Shea, 1982), as estimators from both models are equivalent linear transformations of the discount factors.

Alternative approximation functions were used by Schaefer (1973) in the form of a set of Bernstein polynomials. The important characteristic of these functions is that it is relatively easy, through the use of simple non-negativity constraints on the approximation function coefficients α_l, to constrain the discount function to ensure non-negative forward rates.[7] The result was an estimate of the term structure that was reasonably well fitted over its entire range. Computationally, however, the speed of convergence of Bernstein polynomials is relatively slow compared to spline functions.

6.3. Appropriate Spline Functions: B-Splines

Extreme care is required when choosing the form of the spline functions that are to be used. Not all bases are equally capable of defining spline regressors useful for reliable estimation. Powell (1981, p. 227-8) shows that it is extremely bad practice to work with the coefficients γ_j and δ_j as inaccuracies arise from the subtraction of large numbers. The inaccuracies arise because this and certain other bases generate a regressors' matrix which is nearly perfectly collinear.[8] Instead, it is recommended that a basis of "B-splines", which are identically zero over a large portion of the approximation space, is used. These prevent the loss of accuracy due to cancellation and they also have good convergence properties.

The function

$$B_p^k(t) = \sum_{l=p}^{p+k+1} \left[\prod_{h=p,h \neq l}^{p+k+1} \frac{1}{(t_h - t_l)} \right] (t - t_l)_+^k \qquad -\infty < t < \infty \tag{6.6}$$

is known as a k-order B-spline. The subscript p denotes that $B_p^k(t)$ is only non-zero if t is in the interval $[t_p, t_{p+k+1}]$. Example graphs of first, second and third order B-splines are given in figure 6.1. At this point, for clarity of exposition, we shall introduce a simple example. A linear B-spline function ($k=1$) would be given by

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7 The reasons for and desirability of constraining the discount function are described in section 6.4.1, below.

8 Shea (1982) has shown that these problems arise in the McCulloch formulation.

$$B_p(t) = \sum_{l=p}^{p+2}\left[\prod_{h=p,h\neq l}^{p+2}\frac{1}{(t_h-t_l)}\right](t-t_l)_+ \qquad -\infty<t<\infty. \tag{6.7}$$

It is non-zero over the interval $[t_p, t_{p+2}]$ and takes the following values

$$B_p(t) = \begin{cases} 0 & \text{for } t \le t_p \\ (t - t_p)/(t_{p+1} - t_p)(t_{p+2} - t_p) & \text{for } t_p \le t \le t_{p+1} \\ (t_{p+2} - t)/(t_{p+2} - t_p)(t_{p+2} - t_{p+1}) & \text{for } t_{p+1} \le t \le t_{p+2} \\ 0 & \text{for } t_p \le t \end{cases}.$$

The first natural step to construct our basis of B-Splines is to include the $n-k$ functions $\{B_p : p=0,1,...,n-k-1\}$ because they are linearly independent. We shall continue with the simple example and assume our approximation space has two segments (n=2 and, say, p=0). Hence, we first include the function $B_0^1(t)$ which can be evaluated in the manner described above, and appears as the solid line in figure 6.2. It is zero until t_0, after which point it has a positive linear slope until t_1, whereupon it takes a negative linear slope until becoming zero once more at t_2. In fact, we require a total of $(n+k)$ basis functions, and so another $2k$ basis functions are needed. A convenient way of choosing them so that they are also B-Splines is to introduce some extra knots outside the interval $[a,b]$.[9] Then we construct a total basis of B-Splines, that is, $\{B_p^k(t) : p=-k,-k+1,...,n-1\}$. In our example, we add the knots t_{-1} and t_3 and span the approximation interval with the $n+k$=3 functions $B_{-1}^1(t)$, $B_0^1(t)$ and $B_1^1(t)$ (figure 6.2). For spline functions of higher degree the extra $2k$ functions have the effect of including the right-hand "tails" of those functions that were first non-zero in regions to the left of t_0. In our linear example, over the interval t_0 to t_1, the right hand portion of $B_{-1}^1(t)$ is added to the existing left hand portion of $B_0^1(t)$. Hence we note the general result that each segment has present within it non zero portions of $k+1$ functions, in our example this numbers two.

[9] Specifically, we let $\{t_j : j=-k,-k+1,...,-1\}$ and $\{t_j : j=n+1,n+2,...,n+k\}$ be any points on the real line that satisfy the conditions

$$t_{-k}<t_{-k+1}<...<t_{-1}<t_0=a$$

$$b=t_n<t_{n+1}<t_{n+2}<...<t_{n+k}$$

6.3.1. Calculation of Higher Degree B-Splines

In order to obtain a smooth forward rate curve, we must have a spline function of at least order three. The calculation, by the earlier formula (6.6), is inconveniently complicated; manually or computer assisted. However, Powell (1981, p.234-5) has shown that the following recurrence relation holds for k-order B-splines for all real values of t

$$B_p^k(t) = \frac{(t-t_p)B_p^{k-1}(t) + (t_{p+k+1}-t)B_{p+1}^{k-1}(t)}{(t_{p+k+1}-t_p)} \tag{6.8a}$$

and it is recommended that they are calculated from the tableau, figure 6.3, computing columns in sequence from the left. If t is in the interval $[t_p, t_{p+1}]$, then the numbers in the first column have the values

$$\begin{aligned} B_q^1(t) &= 0 \qquad q \neq p-1, \qquad q \neq p \\ B_{p-1}^1(t) &= (t_{p+1} - t)/[(t_{p+1} - t_{p-1})(t_{p+1} - t_p)] \\ B_p^1(t) &= (t - t_p)/[(t_{p+1} - t_p)(t_{p+2} - t_p)] \end{aligned} \tag{6.8b}$$

We already know that we need $k+1$ functions for each portion of the approximation interval, and for the segment $t_p \le t \le t_{p+1}$ these begin with $B_{p-k}^k(t)$ and end with $B_p^k(t)$. It is straightforward to see that the recurrence relation will deliver the first of the $k+1$ required functions, i.e. $B_{p-k}^k(t)$, from initial calculation of the k functions $B_{p-k}^1(t)$ to $B_{p-1}^1(t)$.[10] To get the first two of the required functions i.e. $B_{p-k}^k(t)$ and $B_{p-k+1}^k(t)$, the $k+1$ functions $B_{p-k}^1(t)$ to $B_p^1(t)$ are needed. Similarly, and in general, to get the $k+1$ functions $B_{p-k}^k(t)$ to $B_p^k(t)$, the calculation of the $2k$ initial functions $B_{p-k}^1(t)$ to $B_{p+k-1}^1(t)$ is required.[11]

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10 To see this, exchange the subscript p in the tableau for $p-k$ (Figure 6.3).

11 The papers by Shea (1984,1985) recommend a form of basis spline derived by DeBoor (1978) which also avoid the ill-conditioning problem. It is simple to show that these functions are non-trivially different from those presented here. A comparison of the implied zero-order splines, which are special cases of those considered here, and the results of DeBoor's recurrence program are adequate proof. Furthermore, the Powell presentation is more accessible and consequently the potential of the B-spline technology is more easily appreciated and certainly more simple to use.

6.4. Estimation Procedure

In matrix notation, the linear regression problem described by equation (6.2) may be written as

$$P = D\,\alpha + u \tag{6.9}$$

where P is a vector of gross price observations on m different bonds, α is a vector of $L=(n+k)$ approximation coefficients, D is the $(m \times L)$ matrix of the summed products of the cash flows and evaluated basis splines, and u is a vector of residuals.

As a theoretical model, equation (6.9) leaves no room for residuals (apart from observation error). However, as explained in the introduction, we can interpret the residuals as estimation error. We assume that they possess the classical properties, in order to support the statistical procedures in this chapter. The presence of tax effects (see sections 4.2.4 and the appendix to chapter 5) would invalidate the necessary properties of the residuals. Hence, the evidence on whether damaging tax effects do exist in this sample is also examined, later, in section 6.8.

Mathematically, the regressors matrix D is cosntructed as follows. If t_{ij} is the point in time when bond i receives cash flow j, then the general element of the matrix D is given by

$$D_{il} = \sum_{j=1}^{n} C_{ij}\, f_l(t_{ij}) \qquad i=1,\ldots,M., \qquad l=1,2,\ldots,L. \tag{6.10}$$

where

$$f_l(t_{ij}) = B_p^k(t_{ij}) \quad \text{and} \quad t_p \le t_{ij} < t_{p+1}.$$

The least squares estimate of the vector of coefficients on the basis functions, α, is given by

$$\hat{\alpha} = (D'D)^{-1}D'P \quad , \tag{6.11}$$

and may be substituted into equation (6.3) to obtain an estimate of the discount function.

6.4.1. Constraining the Discount Function

Several authors such as Rose and Schworm (1980) and Shea (1984) have noted that term structures estimated using polynomial spline functions would often generate forward rates that were unstable and fluctuated widely, often drifting off to negative values. Vasicek and Fong (1982) have pointed out that discount functions are principally exponential decays and since polynomials have different curvature, a polynomial spline function will tend to oscillate around an exponential discount function, and that this explained the earlier results. However, Shea (1985) argued that the difficulties of modelling exponential decays with polynomial functions do not extend to local polynomial approximations to exponential functions. He further demonstrated that in practice the Vasicek and Fong functions are equally likely to generate unstable forward rates. Chambers, Carleton and Waldman (1984) have incorporated the exponential characteristic in a different manner. They suggested that the spot rate curve rather than the discount function should be approximated, using an exponential function. Unfortunately, they clouded the potential of this method by considering a highly selected sample of bonds. Nevertheless, the need for non-linear estimation makes it computationally more difficult, particularly if the confidence intervals on the estimated interest rates are to be calculated.

If it is desirable, we may introduce constraints on the forward rates as done by Schaefer (1973,1981) and examined by Shea (1984). Schaefer constrained the slope of the discount function to be everywhere negative, facilitated by his choice of approximation function. To impose that particular constraint within this framework is considerably more awkward. Shea, using a spline function approach, reported that simple restrictions of fixed proportions between first derivatives were often sufficient. For example, he found that constraining the slope at the last knot to be one half of the slope near to the next-to-last knot was sufficient to remove negative forward rates at the long maturity end of the curve. Within B-Splines technology, these derivatives may be obtained by differentiating, with respect to time t, both the recurrence relation relation and the associated first order B-spline formulae (6.8), and then constructing a tableau similar to Figure 6.3 to guide the calculation of the derivatives in the same manner as the levels.

However, the results of Shea (1984) also demonstrate that the imposition of ad hoc constraints can be counter-productive. He shows that non-negativity constraints can dramatically alter the structure of the forward rate curve in places other than where negative rates are constrained away. Furthermore, there may be some explanation which makes it undesirable to impose constraints upon estimation. In fact, the only natural constraint to impose on the function is that it should take a value of unity at time zero, that is

$$d(0) = \sum_{l=p-k}^{l=p+n} \alpha_l \, B_l^k(0) = 1.0 \tag{6.12}$$

where $t=0$ is in the interval $(t_p \le t < t_{p+1})$, and, by construction, only the first $k+1$ of the above summed functions will be non-zero, that is, those from $p-k$ to p. By using only this one constraint, we both minimise the potential loss of degrees of freedom and see whether the basis functions described above are sufficiently robust and flexible to require no measures to prevent negative forward rates. The effectiveness of just this one constraint will be discussed later.

The linear restriction on the constraints may be written as

$$W \alpha = w \tag{6.13}$$

where W is an L element row vector of basis functions evaluated at $t=0$, and w is the scalar, unity. The restricted least squares estimator of α is defined as (see e.g. Johnston (1984) p. 205)

$$a = \hat{\alpha} + (D'D)^{-1} W' [W (D'D)^{-1} W']^{-1}(w - W \hat{\alpha}) \tag{6.14}$$

where $\hat{\alpha}$ is the least squares estimator of α.

Given the restricted least squares estimates of the coefficients of the basis functions, we may solve for the estimated discount function using equation (6.3), in matrix notation

$$\hat{d} = F a \tag{6.15}$$

where $\hat{d}$ is a T element column vector of discount factors at particular time points, and F is a $(T \times L)$ matrix of basis splines evaluated at the corresponding time points. The general element of F is given by

$$F_{tl} = f_l(t) = B_p^k(t) \qquad t_p \le t < t_{p+1}. \tag{6.16}$$

For each discount factor d_t, there is a spot rate of interest R_t, and the set of these is the term structure of interest rates. The formula to calculate the spot rates is simply

$$R_t = \left[\frac{1}{d_t}\right]^{1/t} - 1 \tag{6.17}$$

and the implied forward rates are given by

$$r_t = \left[\frac{d_{t-1}}{d_t} - 1\right]. \tag{6.18}$$

6.4.2. Confidence Intervals

A feature conspicuous by its absence in the majority of papers on term structure estimation is a report upon the accuracy of the estimated discount function and interest rates. The formulae suggested by McCulloch (1971) and also used in his later paper (1975) have been largely ignored in all subsequent papers. However, Schaefer (1981) does indicate those interest rates which are "unreliable" due to being estimated beyond the maturity of the longest available bond.

That the standard errors of estimated interest rates have for so long gone unreported is perhaps not surprising when it is remembered that the errors in this model should (theoretically) be identically zero. However, as explained earlier, the residuals are being interpreted as estimation error, possessing the classical properties of statistical regression models.[12] A further reason for their absence is the fact that the procedure to retrieve the relevant variance-covariance matrices is, relative to the rest of the procedure, statistically complex. In this thesis, we apply some established statistical results to generate sufficiently robust standard errors for all the products derived from the least squares procedure.

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[12] Again, we note that these properties are dependent upon there being no tax effects present. The evidence on this issue is discussed in section 6.8.

The standard variance-covariance estimator for an unrestricted least squares model is given by

$$\mathrm{var}(\hat{\alpha}) = \hat{\sigma}^2(D'D)^{-1}. \tag{6.19}$$

When the regression is run subject to the restriction in (6.12), the variance-covariance matrix can be shown to be (see e.g. Goldberger (1964 p.257))

$$\mathrm{var}(a) = \sigma^2 \,\{(D'D)^{-1} - (D'D)^{-1}\, W'\, [W\, (D'D)^{-1}\, W']^{-1} W\, (D'D)^{-1}\,\} \tag{6.20}$$

if the restriction is accepted as true. We shall denote this $(L \times L)$ matrix A. The variance-covariance matrix of the discount function may be obtained by applying the result that the variance of a linear combination $x'y$ is given by the quadratic form $x'\mathrm{var}(y)x$, and hence

$$\mathrm{var}(\hat{d}) = FAF'. \tag{6.21}$$

Since forward rates are essentially a ratio of discount factors, the variance of the forward rate may be estimated by $\mathrm{var}(d_{t-1}/d_t)$. On applying the formula for the variance of a ratio (see e.g. Bulmer 1979, p.79), we obtain the following approximation for the variance of the forward rate

$$\mathrm{var}(\hat{r_t}) \approx \frac{d_{t-1}^2}{d_t^2}\left\{\frac{\mathrm{var}(d_{t-1})}{d_{t-1}^2} + \frac{\mathrm{var}(d_t)}{d_t^2} - \frac{2\,\mathrm{cov}\,(d_{t-1},d_t)}{d_{t-1}d_t}\right\} \tag{6.22}$$

This formula can be viewed as an alternative to that of McCulloch (1971) which is derived from the above formula for use with instantaneous rates. The McCulloch formula pre and post multiplies the matrix A by a matrix representing the difference between the ratio of matrix F and vector $\hat{d}$, and the derivatives of the same. The formula suggested above is computationally less difficult. It is a satisfactory approximation provided that the variances of the discount factors used are substantially smaller than the square of their expected values (the square of discount factors themselves, for unbiased estimates), and provided that the distribution of the denominator discount function is positive.

The McCulloch (1971) formula for spot rates relies on an approximation to a logarithmic function which may not be very accurate given the range of variables involved. When estimated, it was systematically smaller and hence less robust than the alternative formula suggested below. We already have standard errors and hence confidence bounds upon the discount factors which can be used to define, through application of equation (6.17), upper and lower bounds upon the spot rates. These may be simply reinterpreted as standard errors by appropriately differencing the confidence bounds. In particular, the following formula is suggested

$$\text{s.e.}(\hat{R}_t) \approx \left[\frac{\left[\dfrac{1}{d_t - k.\ \text{s.e.}(d_t)} \right]^{1/t} - \left[\dfrac{1}{d_t + k.\ \text{s.e.}(d_t)} \right]^{1/t}}{2k} \right] \tag{6.23}$$

Because of the non-linear relationship between the discount factor and spot rate, there must be an asymmetric confidence region about the spot rate. Although, the arithmetic averaging in the above formula suppresses this, the discrepancy from linearizing is generally small. Furthermore, this formula is intuitively consistent with the forward rate formula, and empirically $\text{s.e.}(\hat{r}_1) = \text{s.e.}(\hat{R}_1)$ for all k in $0.01<k<30$. McCulloch's (1971) formulae do not possess this property.

6.5. Data and Parameter Definitions

The data used for this exercise consisted of closing mid-market prices on forty nine high coupon, fixed interest, redeemable gilts.[13] Observations were drawn at weekly intervals before and after Big Bang. During that period six gilts in the sample matured and three, with the required characteristics, were issued and subsequently quoted fully-paid. Sample sizes thus range between forty two and forty five. To illustrate the properties of B-splines and the estimated confidence intervals, it is more convenient to work with a sub-set of this data set

[13] High coupon stocks are used in an attempt to estimate the term structure without distortions due to the low coupon / short maturity preference of high rate tax payers. Details of the data used can be found in the appendix to chapter 5.

(monthly intervals between March 1986 and October 1987). However, the full weekly data set is used for the efficiency testing in section 6.7 later.

There are several parameters which have to be chosen. To minimise the losses in degrees of freedom yet still imply smooth forward rate curves, cubic (k=3) B-splines were used. Due to the relatively limited supply of bonds with a maturity of greater than eighteen years, this was the maximum term (T=18) over which discount factors were estimated. In the sample chosen, the maximum bond maturity encountered was twenty four years and so the edges of the approximation space can be set at say a=0 and b=25. The flexible nature of the approximation process means that the only constraint on the setting of the b knot is that it is greater than the maximum maturity of any bond present in the sample. In fact the knot was set at b=40. The further out the knot the further out can the discount function be estimated, however once past the maximum maturity bond (and often before, with a limited supply of data), the estimates become very unreliable. This phenomenon is clearly shown in Schaefer (1981 table 3. p.430-1), though he employs a different technique of estimation.

There is essentially only one a priori guideline for setting the within sample knots, that is, dividing the bonds into short, medium and long maturity as classified by the market. There are two difficulties with this approach. Firstly, the market definition of "short" and some participants' definitions are different: under five years and under seven years respectively. Secondly, such a definition causes a strong clustering of bonds, leaving the long end of the market poorly represented. Conversely, the clustering at the short end might imply a further division is useful. In order to avoid biases and inefficiencies from accepting an under parameterized function while still preserving consistency, we commence with a likely over-parameterized model and change the position and reduce the number of knots until the standard errors of the interest rates are minimized.[14] This involved changing the position of the knot near the position of

[14] This "general to specific" modelling approach was popularized by Hendry (1979), and has become standard econometric practice.

worst interest rate definition, as determined by the estimated standard errors. If the knot moved so close to a neighbour to no longer be informative, it was eliminated. As the process was repeated, it was found that the interest rate estimates tended to stay within a relatively narrow set of values long before the standard errors were finally minimized. This is a strength of this methodology, as it allows reasonable confidence about the interest rate estimates even if standard errors are not completely minimized. The outcome of this process for the samples studied here suggested the following division into n=3 portions; up to five years, up to ten years and over ten years. This division placed approximately equal quantities of bonds in each segment. Such a starting point is recommended for future studies, although the number of segments has still to be chosen. This strategy maximizes degrees of freedom over individual portions of the discount function.[15]

6.6. Estimated Coefficients and Interest Rates

There were n=3 plus k=3, that is, L=6 approximation function coefficients to estimate for each date. The vectors of estimates a for the illustrative sub-set of data are provided in (Table 6.1). It can be used as a databank for other researchers who need only begin from equation (6.15) to generate interest rates of varying terms as required. However, it should be noted that until July 1986, the maximum bond maturity in the samples used was 18 years, and hence interest rates calculated beyond this maturity are likely to be unreliable. Although the maximum maturity thereafter increased to twenty four years, the paucity of data means that estimation much beyond eighteen years is also likely to give relatively less reliable estimates.

Figures 6.4 and 6.5 show the evolution of the estimated spot and forward rate curves for maturities of between one and eighteen years. The three dimensional plots were drawn from

[15] Including those outside the sample, the knot settings used were −3,−2,−1,0,5,10,40,45,50,60.

the set of interest rate estimates for whole numbers of years, calculated for the illustrative data sub-set. The spot rate curve (figure 6.4) appears humped in shape and to have moved extensively in a parallel fashion. The forward rate curve (figure 6.5) exaggerates the shape of spot rate curve as would be expected. The short rate $R_1 = r_1$ is in the foreground in both diagrams and fluctuates more than the long rate. We note also that the forward rate curve is never negative, and confirm that even for unreliable estimates beyond eighteen years there is no tendency for rates to become negative. We are therefore confident in our procedure, and maintain our belief that even if there were negative forward rates, there may be some explanation that makes it undesirable to impose constraints upon estimation.

We may further gauge this degree of confidence by examining the standard errors for the interest rate estimates. Figures 6.6 and 6.7 show two examples of spot rate curves and forward rate curves respectively with bands at one standard error drawn either side.[16] For the spot rate curves, the standard error is typically 0.04 percent and this rises with maturity to around 0.07 percent at eighteen years. Similarly, the standard error of the forward rate curves rises with maturity to a value of 0.35 percent at year eighteen from a typical value of 0.20 percent. The size of the standard errors for the two chosen examples are echoed among the figures for the intervening periods and suggest a high degree of reliability in our estimates. This may be compared to the situation prior to July 1986, when there were no bonds with a maturity of greater than eighteen years in the chosen samples. The typical standard errors for spot rates at a maturity of eighteen years are little changed at around 0.08 percent, but the forward rate standard errors are around 0.95 percent. For maturities beyond eighteen years, the difference is

[16] Specifically, these dates are chosen to be at the end of the sample and at the first date which has bonds with more than eighteen years until maturity. Consequently, we can be relatively confident about the estimates yet still appreciate the enlargement of standard errors as data becomes sparse with ever increasing maturity. The spot rate confidence bounds depicted have not been linearized by equation (6.23). They are simply

$$\left[\frac{1}{d_t \pm \text{s.e.}(d_t)} \right]^{1/t} .$$

more marked. At dates after July 1986, the twenty and eighteen year figures are little different, whereas before July 1986, the twenty year standard error is typically 0.25 percent for spot rates and 2.50 percent for forward rates. Clearly it is unwise to estimate interest rates for maturities beyond the highest bond maturity in the sample.

Figures 6.6 an 6.7 also show that the confidence bands tend to expand at very short maturities. This means that constraining the discount function to pass through unity at time zero may not identify the short maturity end of the term structure with the required level of accuracy. This suggests that in markets where an appropriate short rate of interest exists, it may be preferable to constrain the short end of the term structure to take this particular value rather than constrain the discount function.

6.7. Price Anomalies and Market Efficiency

As outlined in the introduction, the residuals between the market price and the fitted price (the price calculated by discounting the stream of coupons by the estimated spot rates) can be used to test market efficiency. This comes from the interpretation of residuals as reflecting factors such as differences in liquidity, and the quality of prices. Although tax effects and estimation error could also give rise to errors in practice, there is no reason to expect these effects would be significantly different either side of Big Bang; certainly the estimation technique was identical.[17]

For those dates depicted in figures 6.4 and 6.5, most recreated prices are less than fifty pence away from the market price (quoted in pounds), that is, most prices are less than one half of one percent away from the market. Significantly, this is less than the average bid-offer spread. Furthermore, no estimated price is greater than one percent different from the market price. What are primarily interested in here, is whether the size of this error (as a

[17] Tax effects are discussed in the next section. It is noted again that, theoretically, in a perfect market, there should be no residuals present at all, and that any residuals imply pure arbitrage. It has been argued earlier (see the introduction) that this might not necessarily be the case, in practice.

representation of price inefficiency) is significantly altered after Big Bang, and we will use some simple descriptive statistics and time series tests to examine this issue.

Table 6.2 provides descriptive and autocorrelation statistics for the pricing errors based upon weekly term structure observations. The columns give figures for the simple mean pricing error $\bar{e}$, the mean absolute pricing error $|\bar{e}|$, the skewness of the distribution of absolute errors, and three autocorrelation statistics, which were used in chapter five. The top figure in each row refers to the year before Big Bang and the lower figure to the year after Big Bang. In thirty out of the thirty nine bonds common to each sample date, the mean absolute error is smaller after Big Bang than beforehand. In twenty five cases the absolute value of the simple mean error is smaller after Big Bang.[18] The errors are denominated in pounds and are of similar magnitude to the monthly observations derived from figures 6.4 and 6.5. The conclusion from that small monthly observed sample that the errors are numerically insignificant cannot be refuted for this much larger weekly observed sample. The desirable characteristic of a distribution of absolute errors is that it should strongly positively skewed, that is, have most of the distribution near zero. Unfortunately this means that the benefits of a general decrease in the mean absolute size of error could be offset by a significant reduction in the skewness of the distribution after Big Bang. This has not been the case. The skewness of the distributions have not lowered to a significant extent.

If changes in the discrepancy between the market and model prices were predictable then it would suggest market inefficiencies exploitable by particular trading rules. Using three of the autocorrelation tests described in chapter five, the predictability of pricing errors before and after Big Bang is examined. The first test uses the first autocorrelation coefficient of the series of pricing errors. It applies the result $\sqrt{n}R_1 \sim N(0,1)$, approximately, under the null hypothesis of no autocorrelation. The T^* and U^* statistic are Taylor's (1982b) maximum likelihood

[18] The simple mean errors are the focus of the next section on tax effects.

statistics for testing the null hypothesis of no autocorrelation against an alternative hypothesis based upon a trend model (see also chapter five). In thirty of the thirty nine bonds examined, there was significantly less evidence of systematic effects after Big Bang than beforehand as measured by all three statistics.

Taken as a whole, the message from table 6.2 is that pricing errors have been signficantly reduced after Big Bang. To the extent that this represents an improvement in the quality of price reporting in the market, this may be taken as support for the findings of chapter five. If this quality improvement is indicative of changes in operational as well as informational efficiency, these results support the findings of chapter three.

6.8. The Effects of Taxes

We have deliberately chosen our sample to avoid the type of tax effects examined by Schaefer (1981).[19] As explained in the appendix to chapter 5, it is possible nevertheless that the choice rule applied has not completely removed them. Any remaining effects will be manifested by a strong negative relationship between coupon size and residual. This is because the market value of a relatively low coupon stock which is predominantly held by high rate tax payers is likely to be under-valued by model assuming a zero tax rate (i.e. using the zero-rate discount factors). To assess the extent of remaining effects, the correlations between coupon and residual were calculated, both on an individual observation basis (i.e. for all bonds at each data observation date) and collectively (i.e. for the mean residuals of all bonds across the observation sets each side of Big Bang).

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[19] Details of the sample are given in the appendix to chapter five. Discussion of the effects of taxes can be found in chapter 4, section 4.2.4 as well as earlier in this chapter, in particular in the introduction and sections 6.4.2, 6.5 and 6.7.

On a date by date basis, they were generally small (rarely over thirty percent), with the vast majority between five and fifteen percent. Collectively, the mean residuals for the periods either side of Big Bang together are given in the first column of table 6.2. The correlation coefficients between the residuals and coupon level for either side of Big Bang are -0.38 and -0.41 respectively. This is believed to be insufficient to seriously bias the term structure estimates. However, these figures are probably sufficiently large to necessitate further support being offered for their unimportance. This comes from the fact that the term structure estimates produced residuals that were small and therefore generated model prices that fitted well.[20] Consequently, the correlations between residual and coupon (or, if squared, equal to the coefficient of variation from a regression of residual on coupon) are based on a variable (i.e. residual) that provides a relatively insignificant proportion of the variation of the actual price compared to that provided by the fitted price. Thus although it was correct to be concerned about the relationship between coupon and residual, when considered relative to the role played by the fitted prices in determining actual prices, it is not significant. Finally, we note the finding that the term structure estimates are robust to minor changes in constituent securities.

6.9. Summary and Conclusions

This chapter sought to use the price residuals from measuring the term structure to test market efficiency. Studies to estimate the term structure often use spline functions. However, unless spline functions are carefully chosen, certain matrices formulated in the estimation are likely to be ill-conditioned. Hence, this chapter presents a method of estimating the term structure using spline functions which will not incur this problem. This involves using a form of spline function defined by Powell (1981) and known as B-splines. Comprehensive details of the form of B-Splines and the estimation processes involved are given in order to establish a definitive pro-

[20] The correlation between the time series of actual and fitted prices for each of the bonds in table 6.3 are all over 96% with two exceptions at 81% and 74%.

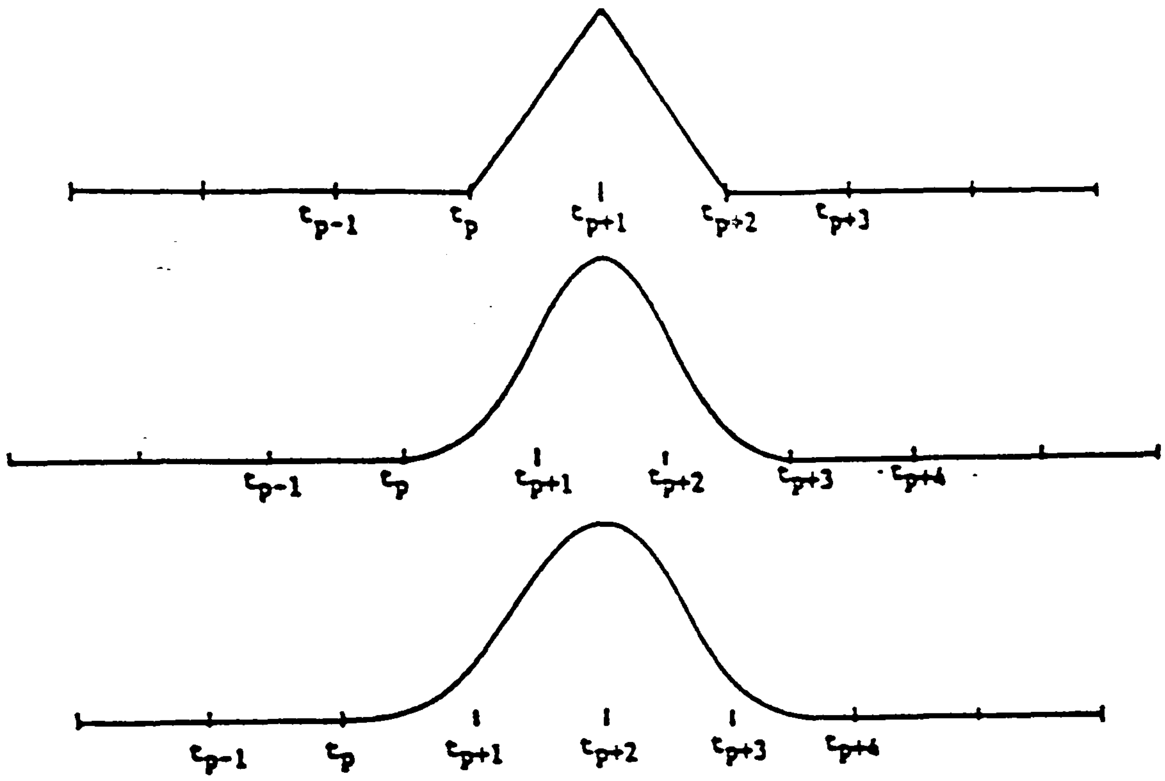
cedure for this form of term structure estimation.

It is argued that the calculation of appropriate standard errors provides both useful information during the estimation stages and a necessary guide to performance at the end. The path of the term structure estimates for several dates is provided, and the estimated approximation function coefficients are provided as a database for other researchers. By criteria discussed above, the estimated interest rates fit well and thus the spline procedure is re-established as a robust alternative to the Bernstein polynomial approach.

In the analysis of the properties of pricing errors, the market was found to have experienced a significant reduction in the presence of systematic errors after the Big Bang together with a reduction in the average size of pricing error. This result complements both the findings of chapter five on the impact of Big Bang on market informational efficiency, and those of chapter three on operational efficiency. Finally the correlations between error and coupon were calculated to assess whether there were any remaining tax effects in the data that could bias the interest rate measurements. It was found that the effects of taxes had been adequately taken into account prior to estimation.

Figure 6.1

B-splines of degrees one, two and three.



Source: Powell (1981)

Figure 6.2

B-splines of degree one

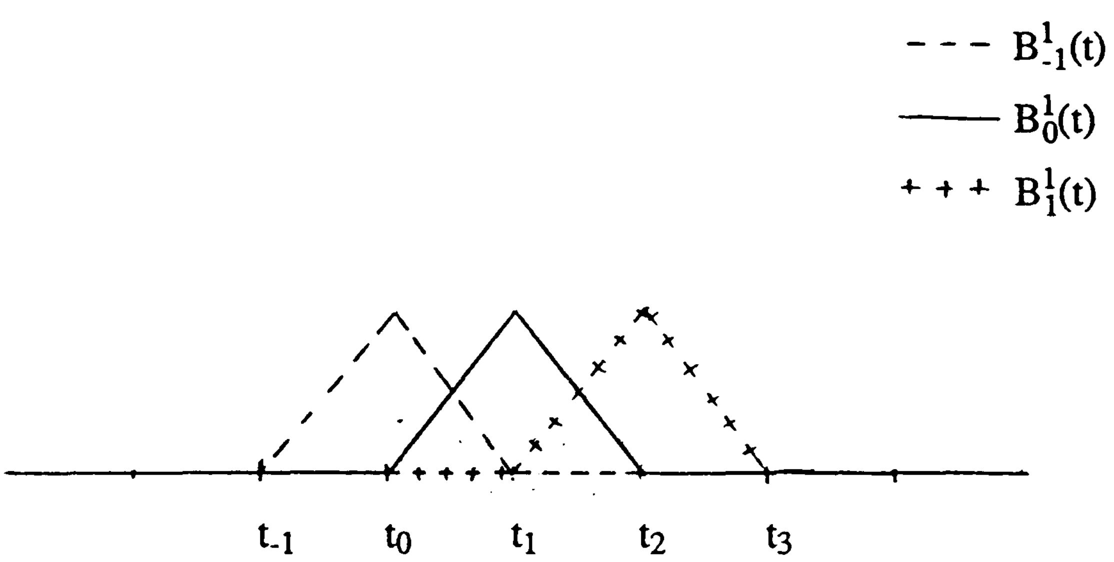
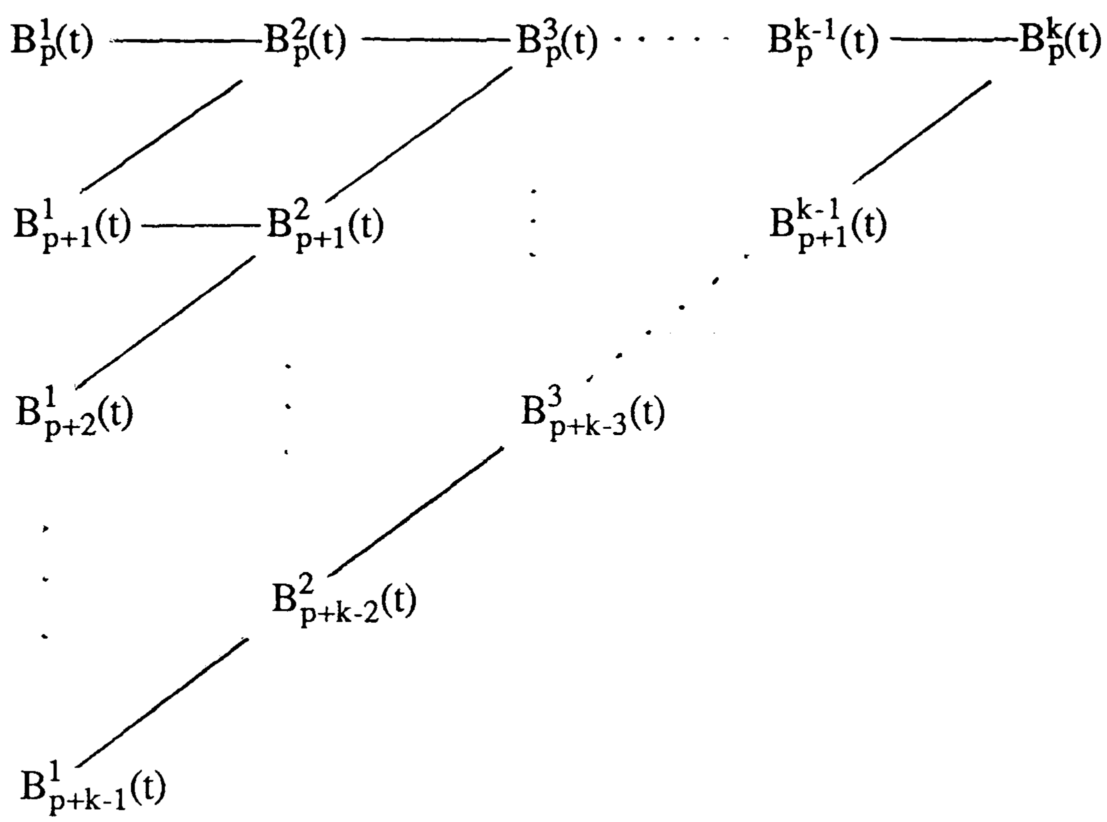


Figure 6.3

Tableau for calculating higher order B-splines



Source: Powell (1981)

TABLE 6.1

APPROXIMATION FUNCTION COEFFICIENTS							
Date	Sample Size	a_1	a_2	a_3	a_4	a_5	a_6
06-03-86	44	8.9494	10.0127	24.4558	-3.5955	26.4838	-135.8266
03-04-86	44	8.8771	10.1582	26.2281	-1.7565	22.1166	-95.6332*
01-05-86	44	8.7397	10.4710	26.4010	-1.4472*	18.2164	-62.4611*
29-05-86	43	8.7496	10.4473	26.4933	-3.3381	23.7268	-102.8525*
26-06-86	43	8.7595	10.4316	25.8893	-3.5968	24.4475	-104.4071*
24-07-86	44	8.8014	10.3438	25.1638	-2.9285	20.0201	-59.2118
21-08-86	44	8.7700	10.4124	25.4618	-2.9927	22.0531	-75.2774
18-09-86	44	8.8211	10.3130	23.8958	-3.6029	21.1760	-78.0470
16-10-86	44	8.9050	10.1334	22.7766	-3.8561	22.8458	-85.9986
13-11-86	44	8.8981	10.1554	22.2320	-4.2376	23.0039	-92.2683
11-12-86	44	8.9173	10.1061	22.7165	-4.8246	24.9539	-98.3064
08-01-87	43	8.8736	10.1947	23.7305	-2.9116	22.1254	-89.1255
05-02-87	43	8.8411	10.2651	24.0780	-3.4023	22.6534	-86.6559
05-03-87	43	8.7945	10.3569	25.4065	-2.4153	20.2634	-71.6676
02-04-87	44	8.7470	10.4647	25.4821	-2.3971	22.5476	-93.5764
30-04-87	44	8.7093	10.5456	25.9762	-1.9789	23.5021	-98.6961
28-05-87	43	8.7157	10.5329	25.8075	-1.5656	23.4407	-96.3109
25-06-87	43	8.7237	10.5168	25.6018	-2.5597	26.0113	-112.3366
23-07-87	43	8.7626	10.4329	25.1455	-2.9624	25.6546	-110.3432
20-08-87	43	8.8653	10.2140	23.6945	-3.6420	24.5855	-103.7808
17-09-87	43	8.8087	10.3412	23.9178	-3.5880	24.7398	-102.5023
15-10-87	42	8.8314	10.2908	23.7782	-4.7753	27.1726	-119.9067

* : not significant at a 5% level.

Time path of the Spot Rate Curve

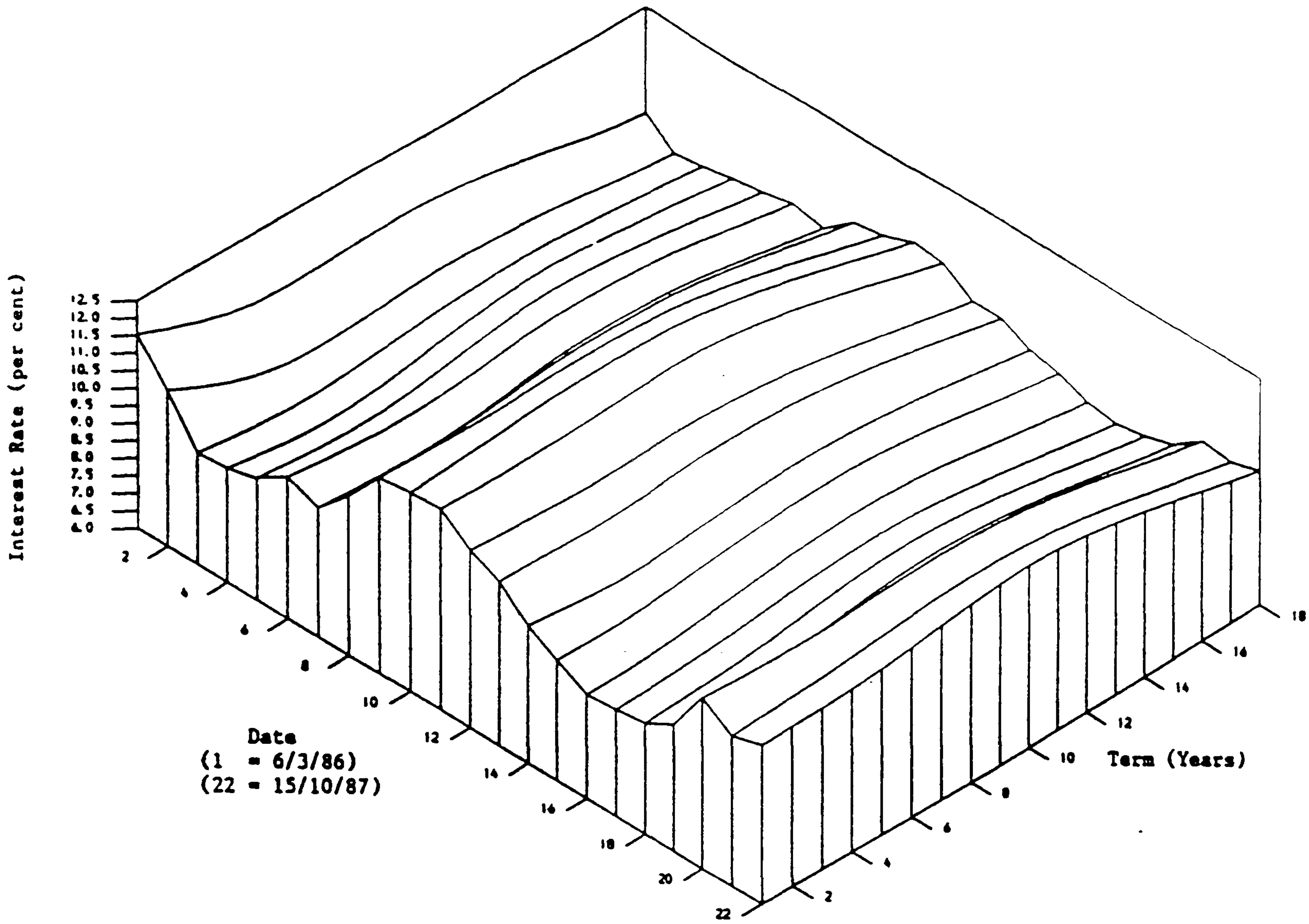


Figure 6.5

Time path of the Forward Rate Curve

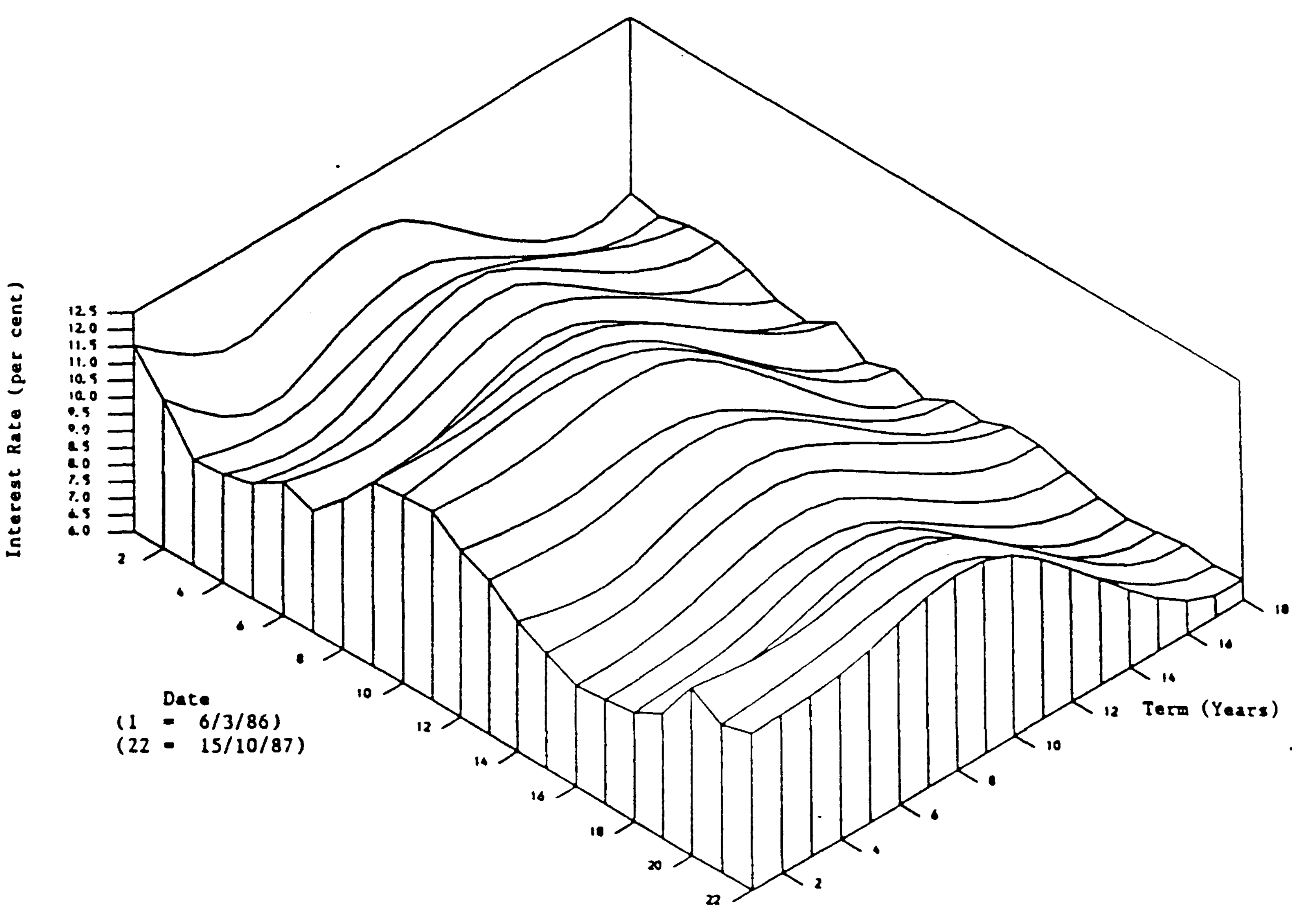
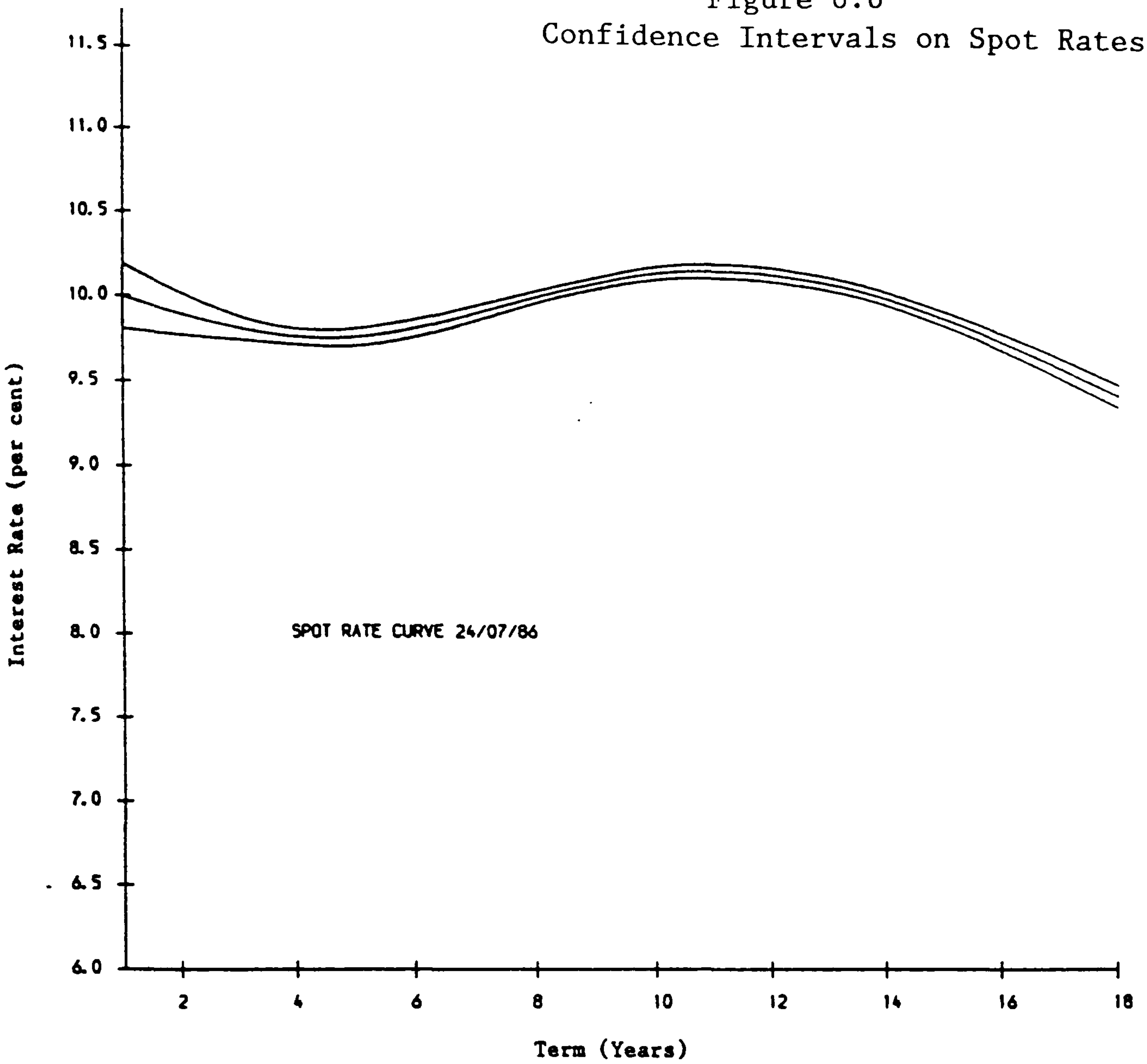


Figure 6.6
Confidence Intervals on Spot Rates



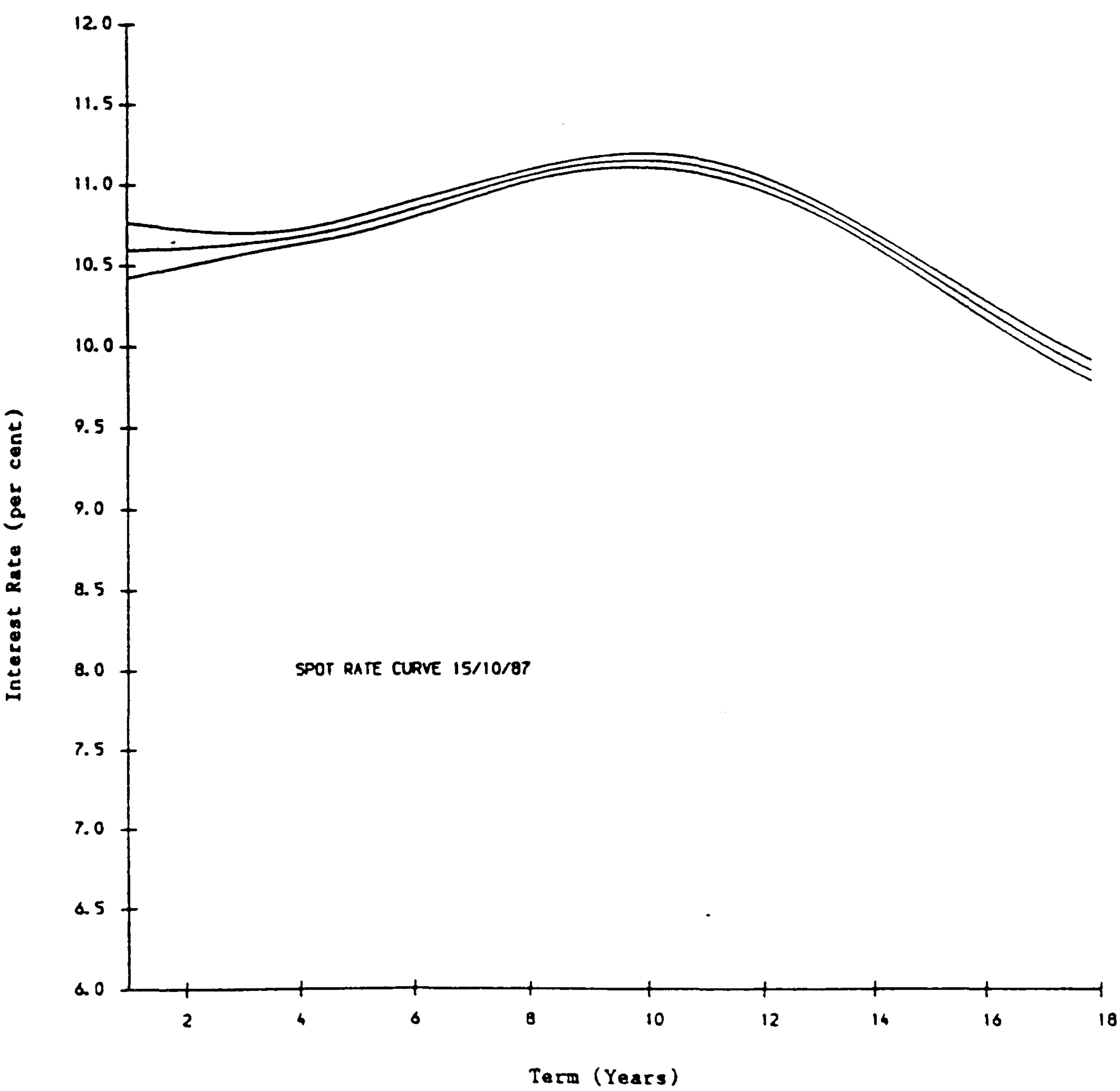


Figure 6.7

Confidence Intervals on Forward Rates

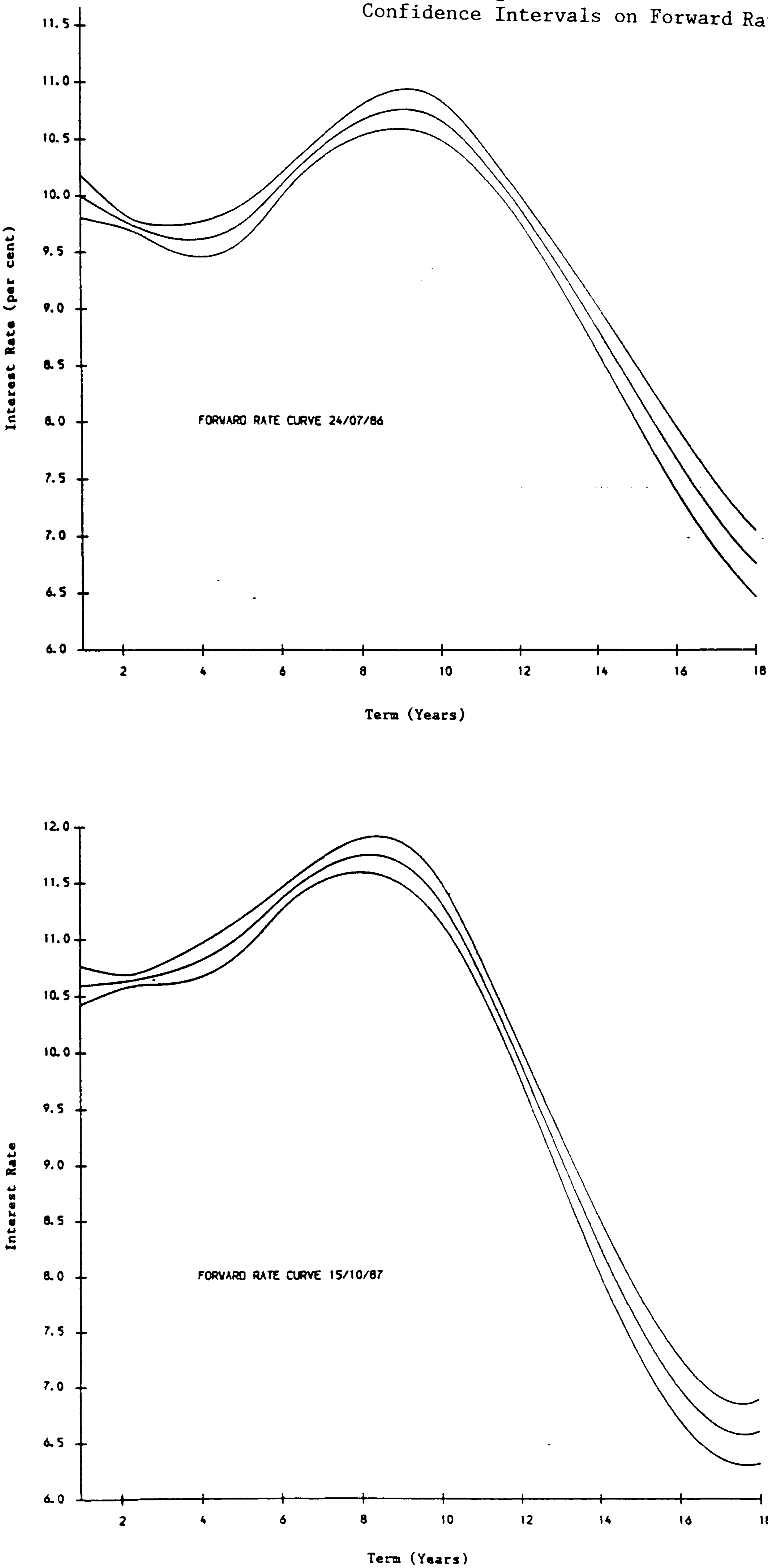


TABLE 6.2

Descriptive and Autocorrelation Statistics on Pricing Errors
"Shorts"

SERIES	$\bar{e}$	$\lvert\bar{e}\rvert$	Skew	$\sqrt{n}r_{1,e}$	T^*	U^*
C.Val.	-	-	-	±1.96	1.65	1.65
EX88	0.179 0.055	0.205 0.089	0.935 0.959	4.85 2.69	10.33 1.92	38.79 3.38
TR89	0.000 -0.035	0.074 0.073	1.217 1.378	2.44 3.56	0.82 0.41	5.14 0.44
T10H	-0.083 -0.044	0.137 0.077	5.829 1.149	-0.08 4.26	-1.81 1.86	-6.77 0.98
EX10	-0.110 -0.084	0.164 0.095	4.176 1.235	0.43 2.12	-0.92 0.26	-2.48 0.50
EX11	-0.063 -0.074	0.104 0.089	2.076 2.076	3.91 1.17	-0.92 -0.52	-1.08 0.57
T13C	0.198 0.116	0.221 0.141	0.365 0.582	5.11 3.98	3.02 3.01	14.50 11.48
EX90	0.098 0.028	0.113 0.068	0.912 1.317	4.06 1.42	-0.64 -0.70	-0.11 -3.91
E12H	0.081 -0.012	0.113 0.077	0.428 1.621	3.47 1.59	0.60 -0.34	0.84 0.33
"Short/Mediums"						
T91C	-0.034 -0.048	0.181 0.095	1.033 3.102	4.12 1.97	7.90 -0.28	29.62 -0.26
EX91	0.130 0.122	0.193 0.195	0.815 1.440	5.34 4.43	5.94 2.81	24.46 6.88
T12T	-0.021 0.117	0.195 0.155	0.762 2.068	5.52 0.84	3.88 0.10	20.06 0.55
T10C	0.467 0.441	0.468 0.442	1.257 7.025	5.45 -0.09	5.44 -1.13	20.49 -3.40

Descriptive and Autocorrelation Statistics on Pricing Errors "Short/Mediums"						
SERIES	$\bar{e}$	$\lvert\bar{e}\rvert$	Skew	$\sqrt{n}\, r_{1,e}$	T^*	U^*
C.Val.	-	-	-	±1.96	1.65	1.65
E92C	-0.257 -0.104	0.267 0.152	0.524 1.674	4.92 1.77	3.28 1.83	16.64 5.48
E92C	-0.458 -0.283	0.472 0.291	1.773 2.003	2.64 1.14	2.86 2.98	11.96 12.50
"Mediums"						
T93C	0.219 0.102	0.236 0.157	0.339 1.828	4.29 1.62	3.91 0.68	12.79 5.33
T13T	-0.141 -0.089	0.156 0.154	0.274 3.131	4.13 3.28	1.91 3.44	9.05 10.58
T14H	0.538 0.260	0.538 0.283	-0.423 1.268	6.24 1.68	16.16 2.86	62.09 14.80
E94C	-0.336 -0.306	0.337 0.306	2.056 2.279	2.68 2.10	1.14 -0.32	4.09 -2.33
EX94	-0.038 -0.131	0.101 0.156	0.628 1.178	4.26 -0.17	5.96 -1.48	20.12 -7.33
TR12	0.089 0.088	0.142 0.159	1.561 0.438	3.11 1.73	1.66 0.22	6.60 1.21
T95C	0.485 0.483	0.485 0.498	-0.330 0.101	2.88 3.62	0.19 2.02	0.27 4.35
TR96	-0.335 -0.365	0.350 0.365	0.076 2.783	4.66 1.55	2.18 -0.63	2.47 -4.32
T15Q	-0.294 -0.054	0.305 0.245	1.080 6.012	3.83 0.67	5.80 -0.12	25.51 2.22
EX96	0.706 0.610	0.706 0.610	-0.260 0.000	6.41 3.47	17.08 6.73	64.03 22.88

Descriptive and Autocorrelation Statistics on Pricing Errors "Mediums"						
SERIES	$\bar{e}$	$\lvert\bar{e}\rvert$	Skew	$\sqrt{n}r_{1,e}$	T^*	U^*
C.Val.	-	-	-	±1.96	1.65	1.65
T97C	0.488 0.466	0.490 0.508	-0.008 0.961	3.78 1.04	5.93 -0.12	23.15 2.98
EX15	-1.220 -1.025	1.220 1.025	1.000 3.673	5.77 -0.54	14.74 -1.51	50.99 -4.53
T15H	-1.007 -0.772	1.007 0.772	1.003 3.236	6.23 0.58	18.36 -0.51	66.28 1.11
EX98	0.149 0.384	0.374 0.438	1.003 2.549	5.73 0.65	13.18 -1.00	44.78 -2.57
EX99	0.241 0.381	0.427 0.439	0.221 2.773	5.03 0.86	10.29 -0.49	34.61 -1.29
TR13	0.014 -0.210	0.601 0.215	1.950 5.004	4.60 0.64	3.55 -0.58	13.07 -1.82
"Medium/Longs"						
T01C	-0.309 -0.585	0.708 0.585	1.414 5.291	6.10 0.22	3.66 0.31	12.30 0.16
EX02	0.917 0.727	0.933 0.727	2.124 7.113	5.57 -0.27	7.92 28.22	-0.98 -3.09
"Longs"						
T03C	-0.188 -0.530	0.846 0.530	1.629 4.039	5.35 -0.54	4.90 -0.46	17.35 -0.81
T11H	1.485 0.749	1.485 0.785	1.830 -0.095	6.38 1.51	12.04 5.26	43.38 17.72
TR10	0.332 0.094	0.413 0.114	1.470 0.762	6.04 2.51	4.12 0.60	12.42 2.56
EX05	0.257 -0.356	0.457 0.356	1.971 0.094	6.17 -0.46	15.47 -0.04	55.21 1.64

Descriptive and Autocorrelation Statistics on Pricing Errors "Longs"						
SERIES	$\bar{e}$	$\lvert\bar{e}\rvert$	Skew	$\sqrt{n}\, r_{1,e}$	T^*	U^*
C.Val.	-	-	-	±1.96	1.65	1.65
T12H	-0.601 -0.060	0.601 0.108	1.225 0.701	6.46 3.19	18.61 5.57	66.27 23.16
T11T	-0.091 0.621	0.497 0.622	1.501 -0.586	6.34 1.86	13.92 3.89	48.68 17.10
T13H	-0.526 -0.090	0.562 0.121	1.029 1.039	4.05 3.47	12.34 8.77	43.16 35.21

CHAPTER SEVEN

Modelling the Dynamics of Interest Rates

7.1. Introduction

The purpose of this chapter is to use a series of observations on the term structure of interest rates, obtained using the techniques developed in chapter six, to examine whether the arbitrage theories of the term structure (e.g. Cox,Ingersoll and Ross (CIR) (1985) and Brennan and Schwartz (1979)) are able to provide a full explanation of the dynamics of the term structure. Previous authors, for example Brown and Dybvig (1986), have suggested that this might not be the case. As we wish to examine the efficiency of the dynamics of the term structure in the next chapter and since efficiency tests usually subsume a particular equilibrium model, it is desirable to examine these theories before constructing something richer. These arbitrage theories are tested through an examination of certain assumptions that they make, in particular, concerning the dynamics of interest rate processes.

The chapter begins with a review of the arbitrage term structure theories in order to establish a class of stochastic processes for comparison, section 7.2. These theories assume that the term structure may be fully characterized by a limited number of state variables, and the literature can be regarded as divided between single factor models (e.g. Vasicek (1977)) and two factor models (e.g. Brennan and Schwartz (1979)). This categorization provides a useful means of dividing the empirical work in this chapter. In section 7.3, the stochastic processes for the short and long rate of interest are identified and estimated, and compared to the processes used in single factor models. In section 7.4, in the spirit of two factor models, the long rate is permitted to enter the model of the short rate and, similarly, the short rate is allowed to influence the dynamics of the long rate. With this modification, new stochastic processes are identified and estimated. The "orthogonality proposition" (Ayres and Barry (1979,1980)) is examined. This property of interest rates provides a key simplification in the solution of two factor models, and prompts the study of the dynamics of the process generating the spread (the difference) between the long rate and the short rate of interest. Finally, an indication of explanatory power of the different theories is obtained. The conclusion, section 7.5, summarizes the main results.

7.2. A Review of the Arbitrage Term Structure Models

The problem with the traditional term structure hypotheses, discussed in chapter four, is that those that incorporate a term premium tend not to specify the exact relationship between maturity and premium. However, if certain assumptions are made about the stochastic evolution of interest rates in a continuous time model, much richer theories can be derived that constrain the relationship between premium and maturity. This was an important motivation for the arbitrage theories of the term structure.

This group of models adopts the arbitrage, or hedged portfolio, pricing techniques first used in the options pricing model of Black and Scholes (1973). Black and Scholes proposed that the value of an option is dependent upon the price of the underlying asset and the time to expiry only. The asset price is assumed to be lognormally distributed and to be generated by a diffusion process. It is taken to be the sole source of uncertainty and its dynamics are assumed to be fully described by one state variable. It is therefore possible to combine the option, the underlying asset and the riskless asset in a fully hedged (zero-variance) portfolio yielding the risk-free rate. This hedging condition defines a partial differential equation which may be solved to find the price of the option.

In terms of bond pricing, it is assumed that the dynamics of the term structure can similarly be described by a limited number of state variables, and the literature can be divided between those models that include one state variable (e.g. Vasicek (1977) and CIR (1985)) and those that use two state variables (e.g. Brennan and Schwartz (1979) and Schaefer and

Schwartz (1984)). Let us begin by examining the single factor models.

7.2.1. Single Factor Term Structure Models

The short rate is assumed to be the sole source of uncertainty and to follow a diffusion process, described by the stochastic differential equation

$$dr = \mu(r,t)dt + \sigma(r,t)dz \tag{7.1}$$

where $r(t)$ is the short rate, having drift $\mu(r,t)$ and variance $\sigma^2(r,t)$, and where $z(t)$ is a Wiener process with zero mean and variance dt. The price of a bond is assumed to be a function of time maturity and the path of the short rate only, that is

$$P(t,n)=P(t,n,r(t)) \tag{7.2}$$

where $P(t,n)$ is the price of a bond at time t which matures at time n. The market is assumed to be informationally-efficient (as defined in chapter four) and investors are assumed to make rational decisions. In the presence of a single state variable, the instantaneous returns on all bonds are locally perfectly correlated, to the extent that they are all correlated with the short rate, the only source of uncertainty in this model.

If this were a deterministic model, then to measure the response of price to changes in the interest rate and calendar time, we would apply the 'chain rule' of differentiation to equation (7.2). Where $r(t)$ is stochastic, we may use 'Ito's Lemma' to differentiate P, provided that $P(t,n,r(t))$ is at least twice continuously differentiable in $r(t)$ and once continuously differentiable in t. Hence

$$dP = \frac{\partial P}{\partial r}dr + \frac{\partial P}{\partial t}dt + \tfrac{1}{2}\frac{\partial^2 P}{\partial r^2}(dr)^2. \tag{7.3}$$

Now we can substitute for dr from (7.1) in the first term of the right-hand side. Since dz is of order $\sqrt{dt}$, $(dr)^2=\sigma^2 dt+$ higher order terms in dt. Thus (7.3) becomes

$$dP = \left[\mu\frac{\partial P}{\partial r} + \frac{\partial P}{\partial t} + \tfrac{1}{2}\sigma^2\frac{\partial^2 P}{\partial r^2}\right]dt + \sigma\frac{\partial P}{\partial r}dz. \tag{7.4}$$

$$= P(t,n,r(t))\,\mu'(t,n,r(t))\,dt \;+\; P(t,n,r(t))\,\sigma'(t,n,r(t))\,dz \tag{7.5}$$

where

$$\mu' = \frac{1}{P}\left[\mu\frac{\partial P}{\partial r} + \frac{\partial P}{\partial t} + \tfrac{1}{2}\sigma^2\frac{\partial^2 P}{\partial r^2}\right] \tag{7.6}$$

and

$$\sigma' = \frac{1}{P}\sigma\frac{\partial P}{\partial r} \tag{7.7}$$

Equation (7.5) holds for bonds of all maturities.

The models are completed by invoking the third assumption to prevent riskless arbitrage, in the spirit of the Black-Scholes option pricing model. By constructing a portfolio of two bonds with different maturities such that it is instantaneously riskless, arbitrage is prevented if this portfolio realizes the same return as a loan at the short rate. This condition implies that the ratio $(\mu'(t,n,r(t))-r(t))/\sigma'(t,n,r(t))$ must be the same for both bonds. Since the initial choice of maturities was arbitrary, this ratio, which is denoted by $q(t,r(t))$ and called the "price of risk", is constant for all maturities.

By substituting (7.7) into the "price of risk" we obtain

$$\mu' = r\frac{1}{P}q\sigma\frac{\partial P}{\partial r}. \tag{7.8}$$

which if used to replace μ' in (7.6) gives after rearrangement

$$\frac{\partial P}{\partial t} + (\mu - \sigma q)\frac{\partial P}{\partial r} + \tfrac{1}{2}\sigma^2\frac{\partial^2 P}{\partial r^2} - rP = 0. \tag{7.9}$$

This is the basic bond pricing (or "term structure") equation for which the solution, subject to the boundary condition $P(n,n,r(n))=1$ has been shown to be (see e.g. Vasicek (1977,p.182-3))

$$P(t,n) = E_t[\exp A(n)] \tag{7.10}$$

where

$$A(n) = -\int_t^n r(s)ds \;-\; \tfrac{1}{2}\int_t^n q^2(s,r(s))ds \;+\; \int_t^n q(s,r(s))dz(s)$$

The solution in (7.10) can be successfully employed only in special situations, for if we do not know the probability density of the exponent, $A(t)$, then the 'expectation' in (7.10) cannot be evaluated. However, in this case, numerical methods could be applied directly to equation (7.9). Even when we do know the probability density function of $A(t)$, evaluation of the 'expectation' will generally require numerical techniques. Equation (7.10) has been shown to have analytical closed form solutions for some specific distributions, that is, where specific choices concerning the process $r(t)$ and risk parameter q are made.

7.2.2. Simple One Factor Models

The simplest model possible is the random walk model with $\mu(t,r)=\mu$ and $\sigma(t,r)=\sigma$, both constant through time. However, under a random walk the variance of the interest rate increases without limit; hence very large negative and positive rates become more and more likely. The Ornstein-Uhlenbeck process, or 'elastic random walk' model used by Vasicek, i.e.

$$dr = \alpha(r^* - r)dt + \sigma dz \qquad \alpha>0 , \tag{7.11}$$

does not have this problem, since the short rate is always drifting towards its long term equilibrium value, r^*. However, the elastic random walk does permit transient occurrences of negative rates. The probability of reaching negative values depends upon the parameters α, r^*, and σ and is, therefore, an empirical issue.

Cox, Ingersoll and Ross suggest the process

$$dr = \alpha(r^* - r)dt + \sigma\sqrt{r}dz. \tag{7.12}$$

which precludes the possibility of the short rate being negative. Since the sample path of the diffusion process is continuous, the interest rate cannot become negative without passing through zero. For this process, $\sigma(0,t)=0$ and $\mu(0,t)>0$. Thus, whenever the interest rate reaches zero, it is certain to become positive immediately. Dothan (1978) suggested the geometric random walk model

$$dr = \alpha(r^* - r)dt + \sigma r dz. \tag{7.13}$$

which has the same property.

The choice of functional form for the risk premium $q(t,r(t))$ has also differed among authors. Boyle (1979), who used the elastic random walk model for the spot rate within a study of portfolio immunization, assumed that $q(t,r)=0$. This is equivalent to assuming that the Local Expectations Hypothesis holds (see chapter four). Vasicek assumed that the risk premium was constant through time and independent of the spot rate, that is $q(t,r)=q$ a constant. The assumptions made by CIR on preferences, the dynamics of the investment opportunity set, and the dynamics of the state variable, restrict the form of $q(r,t)$ to be proportional to $\sqrt{r}$.

The shape of the term structures permitted by these models is restrictive, by virtue of the fact that apart from deterministic shifts in the utility dependent parameter $q(t,r)$, the whole of the term structure can be inferred from the current value of the instantaneous rate. Cross sectionally all the possible spot rate curves approach a common asymptote as $n \to \infty$, and so this "long rate" must be constant over time as well.

The graph of the spot rate curve over time (chapter six, figure 6.4) suggests that a constant long rate is not likely to be a reasonable representation of the long rate in the gilt-edged market. Brown and Dybvig (1986) made the same observation for U.S. government bonds. However, for real rather than nominal rates, recent U.K. evidence (Brown (1988)) from index-linked gilts suggests that this may be a more realistic view of the long real rate of interest.

7.2.3. Multi-Factor Models

The models thus far have assumed that the short interest rate was the only state variable affecting bond prices, and its movements were taken as exogenous. However, there are clearly underlying economic factors that influence the interest rate. Aggregate wealth, the distribution of wealth among investors, the expected rate of return on physical investment, taxes, government policy and inflation probably all influence the behaviour of the interest rate. Both Richard (1978) and CIR (1985) have explicitly considered non-interest rate variables. For example, in their two factor model, CIR have the expected inflation rate as one variable and the real rate of interest as the other variable. However, in practical applications, it has been usual to model bond prices using interest rates as instrumental variables for "economic" factors.

Thus Brennan and Schwartz (1979,1982) develop a two factor model where the state variables are the long rate and the instantaneous rate. This is based upon the assumption that the long rate contains information about the future values of the instantaneous rate. The model can be described by

$$dr = \mu_1(r,l,t)dt + \sigma_1(r,l,t)dz_1 \qquad (7.14)$$
$$dl = \mu_2(r,l,t)dt + \sigma_2(r,l,t)dz_2$$

where $r(t)$ is the short rate, $l(t)$ is the long rate, and $z_1(t)$ and $z_2(t)$ are instantaneously correlated Wiener processes both with zero mean and variance dt.

By Ito's Lemma and the no arbitrage condition, the resulting bond pricing equation is a function of r,l and the market price of short interest rate risk, say $q_1(r,l,t)$, which is again independent of maturity. The solution is independent of the market price of long interest rate risk, which is analogous to the Black-Scholes result that the price of an option is independent of the expected rate of return on the underlying asset (see also Brennan and Schwartz (1979)). As with the one factor models, choices for the form of the stochastic processes and risk parameter have to be made. For example, Brennan and Schwartz (1979) used the following process for the short rate within their two factor model

$$dr = \alpha(l-r)dt + \sigma r dz. \qquad (7.15)$$

Thus the short rate drifts towards the long rate, which follows a stochastic process that precludes negative values,i.e.

$$dl = l(\beta_1+\beta_2 r+\beta_3 l)dt + \sigma l dz. \qquad (7.16)$$

By the same argument that prevented the short rate in the CIR model equation (7.12) from taking negative values, the long rate in equation (7.16) is prevented from taking negative values. However, if the long rate were to take the value zero in equation (7.16), it would be certain to stay at this level. As with the elastic random walk model, this is only likely to be a serious problem if the interest rate is already close to zero.

A general difficulty with these models is that they do not permit a closed form analytical solution. However, Schaefer and Schwartz (1984) have shown that if the two stochastic processes are orthogonal (uncorrelated) an approximate closed form solution can be derived. The difficulty arises in determining orthogonal state variables. There is no reason for the long rate and the short rate used by Brennan and Schwartz to be uncorrelated. For example, the short rate could depend upon expectations of short term inflation, while the long rate could depend upon the long run rate of inflation. Indeed, Schaefer and Schwartz provide evidence that the short and long rate are far from orthogonal. However, these authors also found that the long rate and the spread (the difference between the long rate and short rate) were orthogonal, an idea that was first proposed and tested by Ayres and Barry (1979,1980). This empirical phenomenon has been consistently observed in a number of separate studies (e.g. Schaefer (1980c) and Nelson and Schaefer (1983)). Hence, Schaefer and Schwartz suggest a two factor model in which the long rate follows a CIR-type process and the spread follows an elastic random walk process, which is more reasonable than the same process for the short rate due to the transitory occurrence of negative values. The approximate closed form solution was found to give results close to those obtained from full numerical solution.

7.2.4. "Term Variable" Models

For the one factor models described above, the permissible shapes for the implied forward rate curve are too highly constrained and do not match the observed curves (see chapter 6, figure 6.4). Furthermore, the two factor models, though more flexible, still incorporate a risk premium factor which is ultimately dependent upon individual investor preferences. This makes the resulting formula difficult to estimate and difficult to implement (see e.g. Brennan and Schwartz (1982)). Motivated by these problems, Ho and Lee (1986), in the framework of a discrete trading economy, take the initial forward rate curve as (exogenously) given. Next, the entire curve is assumed to fluctuate according to a binomial process.[1] In this model, the no arbitrage condition places a restriction upon the parameters of the binomial process and produces a unique bond pricing formula. This formula is free of preference dependent parameters, and fully captures the initial forward rate curve.

Recent (unpublished) studies extending this work have highlighted the limitations of both the original Ho-Lee presentation and of this type of model generally, i.e. Heath, Jarrow and Morton (HJM) (1987), Carverhill (1988) and Dybvig (1989). The main problem with the Ho-Lee class of models is that a naive choice of the innovations process can lead to problematic model properties. For example, the innovations process used by Ho and Lee leads to the positive probability, albeit small, of negative interest rates. This has been remedied by HJM and Carverhill who have developed continuous time versions that do not permit negative interest rates. Dybvig (1989) has developed a model in the spirit of Ho and Lee, in that it uses the initial term structure, that is also free of the possibility of negative interest rates.

7.2.5. Conclusions

The main conclusion from the review of the previous studies is that the most fruitful future path is an empirical one. Indeed Dybvig has argued that all the theory has been done, and that the remaining questions are of an engineering nature.[2] It is certainly likely that further advances in theory will not precede a much greater understanding of the dynamics of the term structure of interest rates observed in markets around the world.

[1] The binomial approach was first used to model contingent claims (options) by Cox, Ross and Rubinstein (1979) and Rendleman and Bartter (1979).

[2] This view was expressed during discussions following a preview of his paper (1989).

Thus the empirical work in this chapter focuses on the time series properties of the interest rates that comprise the term structure in the market for British Government Securities. Particular attention is paid to modelling the variance of these processes, and assessing the explanatory power of the alternative models.

7.3. Modelling the Dynamics of the Short and Long Interest Rates

In this section, univariate time series models are identified and estimated for the short and long rate of interest. The identification stage commences with the specification of a general dynamic model (with mean reversion) which has the models denoted by equations (7.11-7.13) as special cases. This permits a comparison of the models determined by the subsequent data based simplification, and those used in the single factor arbitrage theories (i.e. Vasicek and CIR).

These single factor theories of the term structure assume that the state variable x follows a specific form of the diffusion process

$$dx = \mu(x,t)dt + \sigma(x,t)dz \ , \tag{7.17}$$

which has instantaneous drift μ, instantaneous variance σ^2, and where $z(t)$ is a Wiener process with zero mean and variance dt. In particular, they are all special cases of the following process

$$dx = \alpha \ (x^* - x) \ dt + \sigma(x) \ dz \ : \tag{7.18}$$

the Vasicek model (7.11) has $\sigma(x)=\sigma$, a constant; the CIR model (7.12) has $\sigma(x)=\sigma\sqrt{x}$; and the Dothan model (7.13) has $\sigma(x)=\sigma x$. The discrete time equivalent of (7.18), for use in empirical work, is

$$\Delta x_t = \alpha \ (x^* - x_{t-1}) + u_t. \tag{7.19}$$

where $\Delta x_t = x_t - x_{t-1}$, and where the disturbance term u_t is assumed not to be autocorrelated. However, equations (7.12) and (7.13) propose that u_t is not homoscedastic (as in equation

(7.11)), which will invalidate inferences concerning coefficients estimated under ordinary least squares. Specifically, heteroscedasticity will bias the sampling variance and cause inefficient estimation of the coefficient of mean reversion, α. However, correct inferences can be made by using White's (1980) heteroscedasticity-consistent covariance matrix estimator which generates adjusted standard errors for the estimated coefficients.

In order to distinguish between the single factor models, the form of the heteroscedasticity must be determined. The Breusch-Pagan (1979) test for heteroscedasticity may be used for this purpose. Under the null hypothesis of homoscedasticity in the residuals $\hat{u}_t$, $n.R^2$ (where n is the number of observations and R^2 is the coefficient of variation) from the regression of $\hat{u}_t^2$ on the k variables thought likely to influence the variance, is distributed as $\chi^2(k-1)$. Choosing, in addition to the constant term, the right hand side variables as x, and x^2, allows us to distinguish further between equations (7.12) and (7.13). For if the Breusch Pagan statistic detects heteroscedasticity, refuting the Vasicek model (7.11), then the influencing variable is likely to be relatively most significant in the Breusch Pagan equation. However, it is possible neither variable appears more significant than the other. In this case, a solution is to divide equation (7.19) by x and re-estimate the residuals. If equation (7.13) is the correct model, then the Breusch Pagan test should not detect any heteroscedasticity. If (7.12) is the correct model, the squared residuals will still be related to $(1/x)$, which may be used as the right hand side variable in the Breusch Pagan test equation.

The above procedure ignores the possibility that the variance term may be influenced by other variables and in more complex ways. In particular, it is possible that the variance changes through time, independently of the state variable. A class of models designed to handle this form of heteroscedasticity is the autoregressive conditional heteroscedasticity (ARCH) models. These models were first proposed by Engle (1982) for modelling inflation, but have become increasingly successful at modelling the variance of speculative price series (see e.g. Taylor (1986), Bollerslev (1987) and Engle, Lilien and Robins (1987)). Following Engle

(1982), the ARCH(q) model in the disturbance terms (applied to equation (7.19)) is given by

$$\Delta x_t = \alpha (x^* - x_{t-1}) + u_t , \qquad u_t \mid \Omega_{t-1} \sim \mathrm{IN}(0,h_t) \tag{7.20}$$
$$h_t = \theta_0 + \sum_{j=1}^{q} \theta_j u_{t-j}^2 ,$$

where Ω_{t-1} is the information set at time $t-1$ and h_t is the conditional variance which is a linear function of the last q squared innovations. An LM test for the presence of an ARCH(q) model is obtained by taking $ARCH(q)=n.R^2$ from regressing $\hat{u}_t^2$ on $\hat{u}_{t-1}^2 , \ldots , \hat{u}_{t-q}^2$. Under the null hypothesis of no ARCH effects, $ARCH(q)$ will have an asymptotic chi-squared distribution with q degrees of freedom.

An alternative more parsimonious model than the ARCH(q) model is the Generalized Autoregressive Conditional Heteroscedasticity GARCH model of Bollerslev (1986). The GARCH(p,q) model (applied to equation (7.19) is

$$\Delta x_t = \alpha (x^* - x_{t-1}) + u_t , \qquad u_t \mid \Omega_{t-1} \sim \mathrm{IN}(0,h_t) \tag{7.21}$$
$$h_t = \theta_0 + \sum_{j=1}^{q} \theta_j u_{t-j}^2 + \sum_{j=1}^{p} \phi_j h_{t-j}^2 .$$

Pantulla (1986) has shown that the variance equation can be expressed as

$$u_t^2 = \omega + \sum_{j=1}^{m} (\theta_j + \phi_j) u_{t-j}^2 - \sum_{j=1}^{p} \phi_j v_{t-j} + v_t ,$$

where $m = \max(p,q)$ and v_t is serially uncorrelated. Thus, u_t^2 will have the usual properties of an ARMA(m,p) process, so that identification tests for the orders of p and m can be carried out on the u_t^2 series. The GARCH model can be shown to be an infinite order ARCH model with exponentially declining weights and, therefore, allows the estimation of high order ARCH models in a parsimonious manner.

Not only is it possible that the form of the variance used by Vasicek and CIR is too simple, but also it is possible that the drift term fails to account for all the short run dynamics of the interest rate. In other words, the implicit AR(1) structure does not fully represent the dynamics of interest rates. In the spirit of Hendry (1979), and to preserve the desirable characteristics of a mean reverting series, we estimate the model

$$\Delta x_t = \alpha \left(x^* - x_{t-1} \right) + \sum_{j=1}^{j=n} \beta_j \Delta x_{t-j} + u_t \qquad (7.22)$$

with terms as defined in (7.19), for each of the interest rate series. The model (7.16) is the special case $\beta_j = 0$ for all $j = 1,2,\ldots,n$.

7.3.1. The Data and the Results

The data are weekly observations on the one year and eighteen year spot interest rates in the gilt-edged market for a year either side of the Big Bang.[3] This data was obtained using the approximation and estimation techniques developed and described in chapter six. Table 7.1 provides some descriptive statistics on the two series, and the two series in first differences. The short rate at time t will be denoted as r_t and the long rate at time t as l_t. The differenced series display some evidence of leptokurtosis, more especially in the short rate. A time-varying variance can account for an observed distribution with "fatter tails" than the normal distribution, providing additional reason to test for heteroscedastic residuals. However, the Kolmogorov-Smirnov test for departures from normality is unable to reject the null hypothesis of normality in both cases.[4]

Table 7.2 provides the estimated coefficients and t statistics for the autoregressions (equation 7.22) with lags up to four periods for both the short and the long rate. The autoregression technique is preferred to an examination of the sample autocorrelation and partial autocorrelation functions, in the manner proposed by Box and Jenkins (1976), as the standard errors of the autocorrelation coefficients can be biased in the presence of a time-varying variance. Lags up to four periods are assumed to be sufficient to capture adequately the short run dynamics of interest rates. Indeed, table 7.2 shows that the long rate may be modelled without inclusion of

[3] Although, we are not entirely satisfied with such a short data set for this particular time series analysis (for fear of not fully capturing mean reversion), the data set is adequate for the purpose of determining the effects of Big Bang, which is the overall aim of this study.

[4] This test is described in chapter 5, section 5.3

any lagged dependent variables while the current change in the short rate depends only on the previous change in the short rate. However, this immediately informs us that the short rate process which best describes the data does not possess the Markov characteristic required by the single factor term structure models. However, the long rate does possess this characteristic and thus could be used in these models.

After removing the insignificant lagged variables in the two interest rate models (see table 7.2), the coefficients were re-estimated and the results are given below. The figures under the estimated coefficients are the t statistics. Standard errors were calculated using the White (1980) heteroscedasticity-consistent covariance matrix estimator.

The Estimated Short Rate Process

$$\Delta \hat{r}_t = \underset{(2.425)}{0.064} \left(\underset{(22.45)}{10.05} - r_{t-1} \right) + \underset{(2.291)}{0.270} \, \Delta r_{t-1} \tag{7.23}$$

$\bar{R}^2$=0.08 $\hat{\sigma}$=0.268 $LM(4)$=3.51 $BP(2)$=4.49 $ARCH(1)$=0.91 $ARCH(4)$=4.34

The Estimated Long Rate Process

$$\Delta \hat{l}_t = \underset{(1.834)}{0.065} \left(\underset{(31.03)}{9.472} - l_{t-1} \right) \tag{7.24}$$

$\bar{R}^2$=0.02 $\hat{\sigma}$=0.182 $LM(4)$=3.18 $BP(2)$=1.50 $ARCH(1)$=1.07 $ARCH(4)$=5.61

The $\bar{R}^2$ figure is the R^2 coefficient of variation adjusted for degrees of freedom. The $BP(k)$ and the $ARCH(q)$ statistics are the Breusch Pagan and ARCH tests for heteroscedasticity explained earlier. The $LM(p)$ statistic is a Lagrange Multiplier statistic, distributed as $\chi^2(p)$, that tests the null hypothesis of zero autocorrelation in the residuals up to lag p. Autocorrelation has the same consequences for inference as heteroscedasticity, and can also be used to detect miss-specification of the model. For reference, critical values for the χ^2 distribution are

4 Note that $BP(k)$ is distributed as $\chi^2(k-1)$, whereas $ARCH(q)$ is distributed as $\chi^2(q)$.

given in table 7.3.

The short rate process has an estimated coefficient of mean reversion of 0.064 which is significantly different from zero. Furthermore, the estimated equilibrium rate $r^* = 10.05\%$ is not significantly different from the sample mean. There is no significant autocorrelation in the residuals which augers well for this model. There is no evidence of ARCH effects and, on examining the sample autocorrelation and partial autocorrelation coefficients of the squared residuals, also no evidence of GARCH effects. However, the Breusch Pagan test equations do indicate that the variance is not constant, contrary to the assumption made by Vasicek. However, these equations are unable to distinguish between a model with the variance related to the level of the interest rate (CIR) and a model with the variance related to the square of the level (Dothan). Furthermore, the tactic of scaling also provides inclusive results. What is clearly most important concerning this model is the significant influence of the previous change in the short rate on the current change in the short rate. This makes this particular series inappropriate for use as the short rate of interest in the CIR/Vasicek term structure theory.

However, it would be interesting to determine whether this result for the whole of the two year sample is repeated for both the periods before and after Bang Bang, modelled separately.[5] While the model (7.23) could not be refuted for the period before Big Bang, the previous change in the short rate was not important in determining the current change in the short rate after Big Bang. Thus, the short rate studied could be modelled as Markov process after Big Bang and, therefore, could be used in applications of the CIR/Vasicek theory.

The estimated equilibrium long rate l^* is 9.472%, and is not significantly different from the sample mean. The coefficient of mean reversion is 0.065, and although just not statistically significant at the five percent level, is sufficiently numerically significant to retain. To interpret

[5] Primarily however, we are working with the full two year weekly data sample in this chapter as no explicit comparative exercise is being undertaken. This maximizes the degrees of freedom and thus the power of the tests in this chapter. This means that we can begin the comparative exercises in the next chapter with the greatest degree of confidence in the results from this chapter, which is essential as they have a direct bearing on the courses of action taken in the next chapter.

further this mean reversion coefficient, equation (7.24) may be solved as a first order difference equation, and the median time of decay found. It is found that a shock to the long rate will take just over ten periods (weeks) to be half absorbed. As explained earlier, the elastic random walk model is preferred to the simple random walk model which can cause explosive time paths in both positive and negative directions. There is no significant evidence of ARCH effects or autocorrelation in the regression residuals given by the appropriate statistics. However, within the ARCH equations, and from the GARCH analysis of sample autocorrelation functions, there is some evidence of a significant relation between the current variance and that four periods (weeks) previously, which could indicate a regularity in the trading patterns by market participants. The Breusch Pagan statistic does not indicate the presence of heteroscedasticity of the CIR/Dothan type, but the individual testing equations indicate that there is sufficient cause to cast doubt the constant variance assumption. The evidence is less clear than in the case of the short rate and, like the short rate, does not enable a distinction to be made between the CIR or Dothan model.

7.4. Modelling Interest Rates in Two Factor Models

In two factor models, equations (7.14)-(7.16), each state variable is permitted to enter the mean and variance process of the other state variable. In section 7.3, current changes in the short rate were found to depend upon past changes in the short rate. It is possible that the lagged dependent variable was acting as a proxy variable for the long rate, which could be included in a two factor model. Equally, it is possible that the short run dynamics of the short rate will not be affected by the inclusion,of the long rate. Thus, with the inclusion of a lagged dependent variable in the short rate process, the Brennan and Schwartz processes (7.15) and (7.16) were estimated.

The results for the two processes, after removing insignificant lagged dependent variables, were as follows. Again the figures in parentheses are the heteroscedasticity-consistent t statistics.

$$\Delta \hat{r}_t = \underset{(2.288)}{0.072} \left(l_{t-1} - r_{t-1} \right) + \underset{(1.973)}{0.236} \, \Delta r_{t-1} \tag{7.25}$$

$\bar{R}^2$=0.10 $\hat{\sigma}$=0.265 $LM(4)$=5.90 $BP(4)$=1.91 $ARCH(1)$=0.43 $ARCH(4)$=3.91.

$$\Delta l_t = l_{t-1} \left(\underset{(1.269)}{0.054} - \underset{(-0.457)}{0.002} \, r_{t-1} - \underset{(-0.392)}{0.003} \, l_{t-1} \right) . \tag{7.26}$$

$\bar{R}^2$=0.00 $\hat{\sigma}$=0.182 $LM(4)$=5.94 $BP(4)$=3.91 $ARCH(1)$=1.51 $ARCH(4)$=6.73.

Immediately we see that the lagged dependent variable retains a significant statistical presence in the short rate process, though its influence is reduced by the inclusion of the long rate, almost to the critical value of statistical significance using the heteroscedasticity-consistent standard errors. There is no significant evidence of autocorrelation or ARCH effects in the residuals, and the Breusch Pagan test statistic and equations do not detect any significant heteroscedasticity. Thus a constant variance cannot be rejected despite the consequences concerning negativity.

We must reject the Brennan and Schwartz specification for the long rate of interest. On comparing this specification to the elastic random walk specification (7.24), it is found that none of the extra variables makes any significant additional contribution to explaining the long rate of interest. The residuals in (7.26) do pick up the same weak ARCH effect detected in the earlier model (7.24).

7.4.1. The Orthogonality Proposition and the "Spread" Process

The empirical evidence so far has highlighted the potential pitfalls of using certain term structure theories without ensuring that the dynamics of the state variables are fully represented in the model. In particular, the dynamics of the short rate may not be consistent with the models of CIR, Vasicek and Brennan and Schwartz.

Schaefer and Schwartz (1984) suggested that there may be good reason, ex ante, not to use the short rate in a two factor model. They demonstrated that by assuming the two state variables were orthogonal, a key simplification to generating an approximate closed form solution in such models could be obtained. Then they provided evidence (and cited other evidence) that the short rate and the long rate tended to be highly correlated. Instead, they recommended the use of the "spread" process (the long rate minus the short rate) in two factor models, after Ayres and Barry (1979,1980) who had noticed the regularity with which the spread was orthogonal to the long rate. Schaefer(1980c) and Nelson and Schaefer (1983) have also found the spread to be orthogonal to the long rate.

The correlations for the three interest rate processes are given in table 7.4, where $s_t = l_t - r_t$ denotes the spread process. The figures cannot reject the orthogonality proposition. Given the results for the short rate process, it would be useful to know whether the spread process could be used as a state variable in the two factor model. Descriptive statistics for the spread and the differenced spread series are given in table 7.1. Although the kurtosis for Δs is large relative to the normal distribution value of three, it is not significantly large to refute a hypothesis of normality by the Kolmogorov-Smirnov test. The general dynamic specification, equation (7.22), is used to model the spread process and, after discarding insignificant lagged dependent variables and re-estimating the final model, the result was as follows.

The Estimated Spread Process

$$\Delta \hat{s}_t = \underset{(3.095)}{0.132} \, (\underset{(-3.171)}{-0.605} - s_{t-1}) \tag{7.27}$$

$\bar{R}^2$=0.08 $\hat{\sigma}$=0.229 $LM(4)$=3.54 $BP(2)$=0.40 $ARCH(1)$=8.03 $ARCH(4)$=11.98

Equation (7.27) says that the spread process follows an elastic random walk with an equilibrium value of –0.605 and a coefficient of mean reversion of 0.132. This coefficient implies that a shock to the spread process would have a "half-life" of just less than five weeks. The observation of a negative equilibrium rate (and a negative sample mean) cannot be regarded as typical, as it implies that, on average, the term structure was downward sloping across the sample period. While the autocorrelation statistic and BP statistic are not significant, there is clear evidence of an ARCH(1) effect in the spread series. This is certainly the most likely cause of the leptokurtosis in the data distribution. The ARCH effect also shows up clearly when examining the sample autocorrelation and partial autocorrelation functions of the series of squared residuals from equation (7.27). However, ARCH effects do not prevent the spread process being used as a state variable in a two factor model of the gilt-edged market, presuming that the hypothesis of normality can still be sustained. But, their presence contrasts strongly with the assumption made by Schaefer and Schwartz (1984) that the spread follows an Ornstein-Uhlenbeck (constant variance, elastic random walk) process.[6]

The results for the spread and the long rate of interest indicate that these would be suitable interest rate processes for use in modelling the term structure of interest rates in the gilt-edged market using a two factor (Brennan and Schwartz - type) model. However, one may question whether the degree of explanation provided by this two factor model is as substantially different from the explanatory power of a comparable one factor model (using, for example, the long rate) as the increased flexibility of term structure shape gained from a two factor model might suggest. To gain a first insight into this question, and to motivate the empirical work undertaken in chapter eight, equations of the the following form were estimated, and the coefficients of variation (adjusted for degrees of freedom) compared,

[6] Such an assumption was not, of course, necessary for the solution procedure developed in that paper.

$$\Delta R_{i,t} = c_i + d_i \Delta l_t + \upsilon_{i,t} \ , \qquad i=2,3,...,17. \tag{7.28}$$

$$\Delta R_{i,t} = e_i + f_i \Delta l_t + g_i(\Delta l_t - \Delta r_t) + \nu_{i,t} \ , \qquad i=2,3,...,17. , \tag{7.29}$$

where $\Delta R_{i,t}$ is the change over time $t-1$ to t in the i^{th} year spot rate. The results are given in table 7.6 below. As expected two factors out perform one factor. But what is more significant is that except for low maturities, the added increment in $\bar{R}^2$ is less than the remaining proportion of unexplained variation.[7]

7.5. Conclusion

This chapter sought to describe the stochastic processes underlying key rates of interest in the term structure of interest rates in the market for British government securities. This permitted tests of certain assumptions made by the arbitrage theories of the term structure. In particular, it was found that the short interest rate possessed more complex short run dynamics than had been assumed, and that the variance of process defined as the spread (the difference) between the long and the short rate of interest possessed features that had not been previously modelled in this context, namely characteristics suggestive of an ARCH process.

The implication of these results is that if we wish to use these arbitrage models, we must verify the assumptions that they make to ensure that the explanatory power of these models is maximized. However, further tests are conducted that suggest that even if such considerations are taken into account a two factor arbitrage model may still not provide the desired level of explanatory power to fully capture the dynamics of the term structure. Indeed, the degree of variation in portions of the terms structure left unexplained was on occasions in excess of

[7] These tests are not, and should not be interpreted as, full and complete tests of the CIR and Brennan and Schwartz models. The actual models consider the price of risk, deal with infinitesimally small changes and combine state variables differently. However, these tests and earlier results are certainly suggestive of their maximum potential.

thirty percent. The suggestion is that a better model is likely to include more than two factors. This issue, and the implications for testing the efficiency of the dynamics of the term structure, are examined in the next chapter.

TABLE 7.1

SUMMARY STATISTICS

RATE	Mean	Min	Max	Std Dev	Variance	Skew	Kurt	K-S
r	10.12	8.59	11.70	0.903	0.815	0.276	1.853	0.216
Δr	-0.01	-0.74	0.87	0.278	0.077	-0.020	3.911	0.687
l	9.40	8.51	10.54	0.496	0.246	0.310	2.244	0.714
Δl	0.01	-0.40	0.53	0.166	0.027	0.369	3.609	0.965
s	-0.72	-1.95	0.42	0.514	0.264	-0.402	2.140	0.192
Δs	0.01	-0.72	0.70	0.221	0.049	0.047	5.880	0.326

TABLE 7.2

AUTOREGRESSIONS ON INTEREST RATE SERIES

SERIES	α	β_1	β_2	β_3	β_4
Δr_t	0.066 (2.333)	0.221 (1.845)	0.114 (0.981)	-0.064 (-0.769)	0.149 (1.365)
Δl_t	0.081 (2.065)	-0.079 (-0.633)	-0.041 (-0.434)	0.071 (0.681)	0.150 (1.644)

TABLE 7.3

Critical Values for the Chi-Square Distribution

Degrees of Freedom	90%	95%	99%
1	2.71	3.84	6.63
2	4.61	5.99	9.21
3	6.25	7.81	11.3
4	7.78	9.49	13.3

TABLE 7.4

ORTHOGONALITY TESTS		
$\Delta r , \Delta l$	$\Delta s , \Delta l$	$\Delta r , \Delta s$
0.612	-0.02	-0.802

TABLE 7.5

$\bar{R}_i^2 \times 100$ %																
Model	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1 Factor Δl	49	58	62	64	63	61	56	50	46	47	51	60	72	84	93	98
2 Factor $\Delta l , \Delta s$	98	92	86	81	78	76	72	68	63	62	64	70	78	87	94	99
Diff'nce	49	34	24	17	15	15	16	18	17	15	13	10	6	3	1	1
Unexpl'd	2	8	14	19	22	24	28	32	37	38	36	30	22	13	6	1

CHAPTER EIGHT

Term Structure Movements and Market Efficiency

8.1. Introduction

The lesson of chapter seven, which used a time-series approach to modelling the term structure of interest rates, was that the arbitrage theories of, for example, Cox, Ingersoll and Ross (1985) and Brennan and Schwartz (1979), were unable to provide a full explanation of the dynamics of the gilt-edged term structure. One reason was that, on occasions, the dynamics of the spot interest rates were not consistent with the dynamics assumed in the models. The other reason is that the typical empirical forms of these models may not include a sufficient number of state variables (factors) to capture fully the movements in the term structure.

The focus of this chapter is on the common factors affecting bond returns. Empirical techniques will be used to determine the number, and possibly the identity, of the common factors affecting bond returns. These factors will then be used to develop further efficiency tests. Firstly, as the extraction of common factors forms one half of the empirical procedure to test the arbitrage pricing model of Ross (1976), this is a natural first step. Since this theory is founded upon market efficiency, failure to refute the theory can also be interpreted as evidence that the market is efficient. Furthermore, since the term structure models in chapter seven can be regarded as arbitrage pricing models, a test of APT can be seen as a test of a multi-factor version of these theories. However, the main use of underlying factors is in determining whether there are anomalies in the shape of the term structure, upon which trading profits may be made. For example, if the term structure was characterized by a steep upward linear slope, then, ceteris paribus, a rise in the whole curve would be expected next period. However, if it was too steep given current expectations of future rates, the long end would fall and just the short end rise next period. The tests conducted in this chapter will be designed to test whether it is possible to trade profitably, and risklessly, on these inefficiencies.

The chapter begins with a description of the extraction of the common factors affecting movements in the term structure, and interprets these factors in terms of their geometrical influence on the spot rate curve.Section, 8.3, is concerned with testing the arbitrage pricing theory for the gilt-edged market, along the lines suggested by Roll and Ross (1980). While, all tests of APT are fraught with difficulty, it is felt that in a bond market they are considerably reduced relative to the equity market, so generating reasonably powerful tests. Section 8.4 considers the relationship between the common factors and bond returns with the purpose of testing market efficiency by seeking evidence that suggests the existence of profitable trading rules, as described above. That is, it considers "term structure movements and market efficiency".

8.2. The Extraction of Common Factors

To establish the number of common factors affecting bond returns, the technique of principal components is used.[1] In principal components, a set of m variables, say $x_1, \ldots, x_m$ are transformed linearly and orthogonally into an equal number of new variables $y_1, \ldots, y_m$. They are chosen such that y_1 has maximum variance, y_2 has maximum variance subject to being uncorrelated with y_1 and so on. The transformation is obtained by finding the latent roots (eigenvalues) and vectors (eigenvectors) of the covariance matrix. The eigenvalues, arranged in descending order of magnitude, are equal to the variances of the corresponding y-variables, which are the unstandardized principal components. Often the first few components account for a large proportion of the total variance of the x-variables and may then, for certain purposes, be used to summarize the original data.

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[1] Principal components formally resembles the technique of factor analysis. See Lawley and Maxwell (1971) for further details of factor analysis and the precise distinction between principal components and factors analysis.

In chapter seven, the ability of one and two factor models to explain the movements in the term structure was assessed by setting up simple regression equations linking changes in various points along the term structure and changes in either one or two common factors, suggested by the theory (see equations (7.28 & 7.29). The results suggested either that a two factor model may not provide a full explanation of movements in the term structure, or that the two factors suggested from the theoretical work are not the two factors affecting movements in the term structure, or are bad proxies for them. For now, we shall ignore the latter possibility, and embark upon a search for the number of factors affecting the movements in the term structure.

Thus, the principal components analysis was performed on the time series of first differences on the set of $j=1,2,...,18$ annual maturity spaced spot interest rates, i.e.,

$$R_{j,t} - R_{j,t-1} \tag{8.1}$$

where

$$R_{j,t} = \left[\frac{1}{d_{j,t}}\right]^{1/j} - 1$$

where $d_{j,t}$ is the discount factor at time t (with t incrementing in weeks) for payments receivable j whole years ahead from t. The sub-set of this weekly data set that was studied in chapter seven was the two series given by $j=1,18$.[2]

[2] Before describing the results of the principal components analysis, it is essential to remind the reader of the properties incumbent in the spot rate data as a direct result of the estimation method employed in chapter 6. In chapter 6, a series of term structures were generated using a linear approximating function comprised of six cubic B-spline basis functions. There was good reason for choosing this number of functions, but the consequence of this decision is that all the fitted values (e.g. interest rates and bond prices) have dimensionality essentially of six. Hence, the principal components analysis that is about to be conducted has a pre-determined maximum number of components, i.e. six. It is necessary to ensure that this data provides a good approximation to the actual values (i.e. as if the approximation methodology fitted every point on the term structure curve exactly) to prevent the restriction to six components (as a maximum) eliminating vital explanatory information. We have seen in chapter 6 how well the fitted prices track the actual market prices and consequently how little information is contained in the residuals from fitting the term structures. A similar comparison of the correlations between movements in actual and fitted prices would provide similarly useful information here. The correlations so calculated are all in excess of 96% with two exceptions at 71% and 79%. Clearly, little explanation remains in the residuals. Furthermore, a principal components analysis upon rates of return in the market prices of all the underlying bonds indicates that the first six components account for 95% of the total variation. So restricting the maximum number of components to six will not significantly affect the principal components analysis of spot rate movements.

The results of the principal components decomposition of the covariance matrix of the spot rate movements are given in tables 8.1, 8.2, and 8.3. The three tables give the eigenvalues and corresponding eigenvectors for the periods across, before and after Big Bang respectively. The particular software chosen for the principal components, standardizes the variables (here the rates of return), subtracts their means and divides by their standard deviations, before computing the principal components. The resulting components have the property that they are mean zero, have unit variance and are orthogonal. The correlation coefficients between a principal component vector and the set of original variables are identical to that components loading vector. The sum of squared factor loadings equals the characteristic root (eigenvalue). The fraction of the variance of the original variables explained by a principal component is its characteristic root divided by the number of variables.

The results of the principal components are striking. For each data set, there is one component which dominates the co-movements in the spot rates. This one component accounts for 87% of the variance of the original variables. Almost 100% explanation is provided by a further three components, explaining 7%, 4% and 2% respectively.[3] The dominant component corresponds to roughly parallel shifts in the spot rate curve. This strongly accords with the manner in which the spot rate curve moved through this data set (see figure 6.4). It appeared to have moved in a largely parallel fashion. The impact of this factor on the average term structure for the period may be seen by calculating and graphing the following relationship

$$\Delta\bar{R}_j \pm c.\,\mathrm{S.D.}(\Delta F_{t,k}).\beta_{j,k} \tag{8.2}$$

where $\Delta\bar{R}_j$ is the sample mean of $\Delta R_{j,t}$, $\beta_{j,k}$ is sensitivity of the jth spot rate to the k component (i.e. the element in the jth row of the kth eigenvector), and c is some constant number. This may be interpreted as adding (and subtracting) c standard deviations of factor k's influence on variable ΔR_j to the mean of that variable, and is analogous to the method of

[3] The remaining fourteen components are of negligible size and are not reported in the tables of eigenvalues and eigenvectors

constructing prediction intervals about simple regression lines. The graphs of these intervals for $c=2$, are given in figures 8.1-8.3 corresponding to tables 8.1-8.3. Considering figure 8.1, the the parallel shift effect of the first factor can be clearly seen. The second factor appears to correspond to changes in the overall slope of the spot rate curve. The third factor corresponds to changes in the curvature of the spot rate curve. The fourth factor is difficult to interpret and, anyway, has a very small effect on the spot rate curve. In the sub-sample groups, figures 8.2 and 8.3., the dominant component is still a parallel shift in the spot rate curve. However, while the period before Big Bang (table and figures 8.2) maintains the interpretation of the factors given to the whole data set, the same cannot be said of the period after Big Bang. Figures 8.3 show that the second most important factor refers to a change in curvature and the the third most important factors corresponds to a change in slope (the reverse of the other samples). Furthermore, the fourth factor can be interpreted as a change in curvature also.

Since commencing this study, I became aware of an unpublished paper by Dybvig (1989) which also uses principal components to study the factors moving bond interest rates. His study considered two data sets, a monthly time series of U.S. Treasury Bills with a monthly maturity up to nine months, and an annual time series of U.S. annually dated short maturity bonds (less than five years to maturity). He found that one factor completely dominated the market, similar to this study, but that its effect was even more significant accounting for over 95% of the total explanation in both data sets. The variables analysed were slightly different to those studies here, but not fundamentally so. Rather than work with the differences used here, he chose to analyse the innovations in the current term structure. These were obtained from first order vector autoregressions on the set of spot rates.[4]

[4] Although it is theoretically cleaner to work with innovations rather than first differences, it is in practice unlikely to give very different results.

8.3. Testing the Arbitrage Pricing Theory

An important body of research in financial economics is concerned with the forces that determine the prices of risky securities. The desire to understand these forces has led to the development of a number of competing models of asset pricing. These include the original Capital Asset Pricing models (CAPM) of Treynor (1961), Sharpe (1964), Lintner (1965) and Mossin (1966), the intertemporal models of Merton (1973), Long (1974), Rubinstein (1976), and Breeden (1979), and the Arbitrage Pricing Theory (APT) of Ross (1976). Each model derives a relationship between the expected return on an asset and one or more measures of exposure to systematic risk.

For many years, empirical work on asset pricing was dominated by the CAPM: its appeal being the postulation of a simple, measurable relationship between risk and expected return. The main implication of the theory is that expected return should be linearly related to an asset's covariance with the return on the market portfolio:

$$E(R_i) = \gamma_0 + \gamma_1 \beta_i \qquad (8.3)$$

where

$$\beta_i = \sigma_{im} / \sigma_m^2$$

is the "beta coefficient" of asset i, $E(R_i)$ its expected return, and γ_0 and γ_1 are constants that do not depend on i. In particular, γ_0 is the risk free rate of return R_f and γ_1 is the difference between the return on the market portfolio R_m and the risk free rate of return, i.e. the "market price of risk". Thus expected returns are given by the risk free rate plus a risk premium calculated as the price of risk times the quantity of risk (beta). The CAPM relationship relies upon a number of restrictive assumptions, for example, homogeneous expectations among investors, the absence of taxes, and the equality of borrowing and lending rates. Models with these assumptions relaxed have been derived by Lintner (1969), Brennan (1970) and Black (1972). Authors responsible for extending the model into a continuous time framework have been referenced above.

Black, Jensen and Scholes (1972) and Fama and MacBeth (1973) are important early examples of empirical tests of the CAPM. However, these and other works have been criticized in an influential article by Roll (1977). He argued that since the true market portfolio is empirically unobservable, the results of investigations which use a proxy for the market portfolio must be ambiguous. In particular, empirical rejection of the model may indicate a violation of the theory; alternatively, it may simply reflect the miss-specification of the proxy variable. He concludes that "the theory is not testable unless the exact composition of the true market portfolio is known and used in the tests".

There have been two forms of response to the "Roll Critique": the first, associated primarily with Shanken (see, Shanken (1986)) is concerned with refining the statistical techniques involved; which he terms, "Living with the Roll Critique". His work develops an empirical framework in which prior beliefs about the correlation between a proxy and the true market portfolio can be explicitly incorporated. The usual notation of a proxy is expanded to accommodate a vector of variables which, together, account for much of the variation in the market portfolio return. In this context, the focus is on the 'multiple' correlation between the proxy and the market portfolio. He finds that if the statistical evidence of the proxy's inefficiency is sufficiently strong, then the inefficiency of the true market may be correctly inferred and CAPM rejected. The strength of this method increases with the presumed correlation and is conditional upon the accuracy of this prior belief.

The second development that coincided with, though not necessarily in response to, the Roll critique, was the Arbitrage Pricing Theory produced by Ross (1976). Unlike, the CAPM, which has its basis in mean variance analysis and requires an optimizing investor to choose assets on the basis of expected returns and risk (variance), the APT is a mechanism which, given the process that generates security returns, derives asset prices from arbitrage arguments (similar to those used in the arbitrage term structure models in chapter seven). Although the strong assumptions concerning individuals preferences that are required in CAPM are not necessary in the APT, an assumption of homogeneous expectations is necessary, as is a greater understanding of the return generating process.

APT requires that returns on a security be linearly related to a set of K factors as shown in equation (8.4)

$$R_{it} = E(R_{it}) + \sum_{k=1}^{K} F_{kt}\beta_{ik} + u_{it} \tag{8.4}$$

where

$$E(F_{kt}) = 0\ , \quad E(u_{it} \mid F_{kt}) = 0\ , \quad E(u_{it}) = 0\ , \quad \text{all k}\ .$$

R_{it} is the return on the ith asset over the period $t-1$ to t, $E(.)$ is the expectations operator, F_{kt} is the value at time t of the kth factor that impacts on security i, β_{ik} is the sensitivity of security i to factor k, and u_i is a term representing factors specific only to security i. If investors expect equation (8.4) to hold, Ross (1976) demonstrates that APT implies that

$$E(R_i) = R_f + \sum_{k=1}^{K} \beta_{ik}\lambda_k \tag{8.5}$$

where

$$\lambda_k = E(R_k) - R_f\ .$$

R_k is the return on a portfolio that depends only upon the kth type of risk, and hence λ_k is the risk premium on such a portfolio. Thus, equation (8.5) says that the risk premium on an individual security depends upon the sensitivity of the security to the systematic risk factors multiplied by the risk premia attached to these risks.

In order to test the APT, it is necessary to test equation (8.5), which means that estimates of β_{ik}'s are needed. However, to estimate the these, the relevant factors must be defined. The most widely used approach to this problem is to estimate the factors and the sensitivities simultaneously. Given estimates of the sensitivities, under the assumption that returns are generated by equation (8.4), the basic hypothesis to test is

H_0 : There exists non-zero constants, $(R_f, \lambda_1, \ldots, \lambda_K)$

in the cross sectional regression (8.5).

A complete specification of equation (8.4) would requires all the factors and sensitivities to be defined so that the covariance between any residual return (the u_i's not explained by the equation) was zero. While it is not possible to produce this exact result, a close approximation can be achieved using principal components analysis. As explained earlier, principal components will extract a set of factors equal to the number of securities, but such that a limited number of them can explain all but a negligible proportion of the covariance between the returns on the securities.

The first step is to define a returns series for application of the principal components analysis. If $d_{i,t}$ is the price of a pure discount bond purchased at time t (with t incrementing in weeks) and having i whole years to maturity (with i incrementing in whole years), then the weekly rate of return on this bond would be given by

$$\frac{(d_{i-1/52,t} - d_{i,t-1})}{d_{i,t-1}} \tag{8.6}$$

This kind of rate of return implies the following pattern of trading. An investor purchases at time $t-1$, say, a bond with i whole years until maturity. At time t, a week later, she sells this bond and buys a new bond having the same i length of time until maturity, this time i beyond t, and so on. Thus the original purchase price at time $t-1$ is $d_{i,t-1}$, and the selling price at time t is $d_{i-1/52,t}$. The pruchase price for the new bond at time t is $d_{i,t}$, and it will sell at $d_{i-1/52,t+1}$.

Although the focus directed by the previous chapter was on the factors underlying the movemtns in the spot rate curve (hence part one of this chapter), a comparison of equations (8.1) and (8.6) shows that the rates of return being used in the test of APT are not the same as the first differenced spot rates used earlier. While it would be preferable to maintain a consistent choice of variables for analysis, the movements in the spot rate curve do not have an interpretable trading pattern, and so cannot be regarded as the rate of return on a security. Thus a further principal components analysis is carried out upon the returns series described by equation (8.6).[5]

The results for this principal analysis are given in tables 8.4 to 8.6. The eigenvectors are the sensitivities and the eigenvalues measure the proprotion of the variance of the original variables exlained by the factors. The factors themselves can be reconstructed from the eigenvectors and the set of returns. In the tables, the eigenvectors are scaled such that the squares of the elements equal the eigenvalue (as with the tables 8.1-8.3). Dealing firstly with the whole two year sample, it can be seen that the principal eigenvector and eigenvalueindicate that the first factor completely dominates the other factors. It corresponds to roughly parallel shifts in the set of returns on pure discount bonds. The second factor seems to correspond to a change in slope of the set of returns, and the third factor to twists, that is the set of retunrs becoming more concave or convex with respect to maturity. The fourth and subsequent factors are of negligible size, and we conclude that there are three factors influencing the rate of return on pure discount bonds.[6] In the pre-Big Bang sample, this result is echoed, whereas in the post Big Bang sample, the interpretation of the second and third most important factors may be interchanged. In all samples, the number of important factors and thier total explanatory power remains constant.

However, there have been some important studies which suggest that the number of factors may be arbitrary. This criticism is in part due to the manner in which the equity market has been studied: a large number of securities makes factor analysis a cumbersome procedure, and so the securities involved have usually been packaged into portfolios which are then analysed.[7] Shanken (1982) has proved that the number of factors depends upon the way in

[5] Since, the rates of return are non-linear combinations of the spot rate movements, it would be surprising if this analysis gives different results in terms of number and magnitude of components. This is also the case due to the dimensionality considerations and the very close relationship between the underlying price data on coupon bonds and the fitted prices and derivatives (discount factors, zero coupon bond prices, spot rates) explained earlier.

[6] The tables of eigenvalues and eigenvectors refer only to the first four components.

[7] Chen (1983) has described a method that allows APT to be tested across large numbers of securities. However, his procedure, which involves forming a small number of portfolios of securities based upon an initial factor solution for use in further tests, has been questioned by Dhrymes, Friend and Gultekin (1984).

which the securities are packaged into portfolios. Empirical evidence of this phenomenon has been provided by Diacogiannis (1986) for the U.K. market. Furthermore, Kryzanowski and To (1982) for Canada and Dhrymes, Friend and Gultekin (1984) for the U.S. have shown that the number of factors is an increasing function of the number of securities in each portfolio. That it is linear rather than convergent must cast doubt upon the robustness of factor analytic techniques. However, Roll and Ross (1984) point out that it is always possible to induce factors that are idiosyncratic rather than systematic when the sample size of securities increases. For example, factors specific to a given industry sector may begin to emerge as the number of securities in each portfolio become large, but such factors have nothing to do with APT because by definition they are diversifiable. Therefore, they argue that this phenomenon does not necessarily deprive APT of its empirical content.

The implications for this study are reasonably clear and relatively straightforward since it is concerned with bonds rather than equities. Firstly, there is no need with the current sample size of eighteen bonds to form portfolios. This immediately removes any arbitrariness from that source. As explained in the note to the first section, the dimensionality of the data is predetermined, and so including any further bonds would not have any significant impact.

The next step in the analysis is to test the cross-sectional model, equation (8.6). Roll and Ross (1980) showed that it was possible to use the method that Fama and MacBeth (1973) had used to test CAPM, in the case of the APT model. Fama and MacBeth estimated security "betas" from time series data and then performed the cross sectional regression (8.3) at monthly intervals over time. This provided estimates of the price of risk over time and the average value and standard error could be computed. In the context of the APT, this means estimating the factor sensitivities from time series data, and then running a set of cross-sectional regressions of returns upon these sensitivities, in order to estimate the price of risk, λ, for each factor. Roll and Ross also show that this methodology is analogous to a generalized least squares (GLS) procedure (1980,p.1090).

The disadvantage with the GLS procedure is that it will only take account of one known source of bias in the standard errors of the estimated λs; that is, the downward bias induced if one were to use the mean return vector for a single cross-sectional regression. It is possible that there are other sources of potential bias, for example, heteroscedasticity. Thus, the following approach is adopted: firstly, run an ordinary least squares regression on the mean return vector, but use adjusted standard errors which account so far as is possible for known biases; secondly, run a set of ordinary least squares regressions over time, as Fama and MacBeth, to act as a check on the first procedure.

The least squares regression results based upon the mean return vector are reported in table 8.7. The Fama-MacBeth time series of cross-sectional results did not contradict the findings using the mean return vector with adjusted standard errors. The first column in table 8.7 refers to the particular data set used to measure the mean returns and estimate the factor sensitivities. The first two rows correspond to the two year sample running across Big Bang, and the factor sensitivities used are those given in table 8.4. The third and fourth rows use the factor sensitivities given in table 8.5 which corresponds to the year before Big Bang; while the fifth and sixth rows correspond to the year after Big Bang, with factor sensitivities taken from table 8.6.

The difference between the two rows for each sample is in the choice of the risk free rate R_f in equation (8.5). A value for this term must be assumed rather than estimated. While, a value for R_f could be obtained as the constant term in the regression, Roll and Ross have shown (1980, p.1091), that this causes the estimates of the risk prices to be no longer independent, which invalidates inferences based upon t-statistics upon the individual coefficients. Instead it is recommended that a rate is assumed, taken over to the left hand side of equation 8.5, and subtracted from the expected returns $E(R_i)$ prior to estimation. Two interest rates are assumed: one that is thought to be close to the actual risk free rate over the sample periods, and one which is thought not be close. The two rates chosen, were 10% and 5% respectively.[8]

[8] It is possible that the risk free rates in the sub-samples are not equal to each other or that for the

The next four columns in table 8.7 give the estimated λ_k coefficients together with their t-statistics.[9] It is known that there is likely to be heteroscedasticity in the residuals of the regression and other sources of bias in the sampling variance, which causes inefficient estimation of the coefficients. To correct for this so far as is possible, the t-statistics are calculated using White's (1980) heteroscedasticity-consistent covariance matrix estimator, which adjusts the standard errors of the estimated coefficients. The R^2 figure is the coefficient of variation. The subsequent four columns report the results of diagnostic tests for autocorrelated and/or heteroscedastic residuals which will invalidate inferences as explained above. We will only achieve a powerful test of the APT if the diagnostic tests produce figures that are not statistically significant. The $LM(p)$ figure is a Lagrange Multiplier statistic, distributed as $\chi^2(p)$, which tests the null hypothesis of zero autocorrelation in residuals up to lag four. The $BP(g)$ figure is the Breusch-Pagan (1979) test for heteroscedasticity. Under the null hypothesis of no heteroscedasticity in the residuals, $\hat{u}_t$, $n.R^2$ (where n is the number of observations) from the regression of $\hat{u}_t^2$ on the g variables thought likely to influence the variance is distributed as $\chi^2(g-1)$. Here we include the factor loadings and, in the spirit of the later tests for the influence of quadratic terms, the squares of the factor loadings. The $ARCH(1)$ and $ARCH(4)$ figures are tests for autoregressive conditional heteroscedasticity in the variance, as proposed by Engle (1982) and described in detail in chapter seven. The $ARCH$ model has the conditional variance depending upon the past q squared innovations.

Considering firstly the two year sample, it can be seen that under the assumption of a ten percent risk free rate, the coefficients on the sensitivities of returns to the three important factors are all significantly different from zero, and the coefficient on the beta of the fourth factor is not significant. Furthermore, all the diagnostic tests are not significant at the five per cent

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whole sample. However, trying to account for this by choosing a different "close to the actual" rate for each sample introduces a further variable to be accounted for in the interpretation of the results. Hence the fixed rate assumption.

[9] These coefficients are therefore inclusive of the assumed risk free rate.

level and the $\bar{R}^2$ is 67%. The effect of assuming a wildly inaccurate risk-free rate is dramatic; three out of four of the diagnostic tests are significant, the $\bar{R}^2$ has fallen to 7% and the biases in the t-statistics are clear. The results for the sub-samples are less easy to interpret, since the diagnostic tests statistics take values close to or in excess of the critical values. This means that it is not possible to make a strong statement concerning the impact of Big Bang on market efficiency from this approach. However, when the data set is considered as a whole there is no reason to doubt market efficiency nor to reject the APT.

Several authors have pointed out that tests against specific alternative hypotheses might be more powerful than ones with no alternative hypothesis assumed. In the context of an equity market, it has been suggested that variables such as firm size or measures of idiosyncratic risk could be included in an alternative to the APT hypothesis.[10] Specifically, this involves estimating the coefficients of the usual cross-sectional regression, but augmented with variables representing these other hypothesized influences. Thus we have

$$E(R_i) = R_f + \sum_{k=1}^{K} \beta_{ik} \lambda_k + \gamma s_i \tag{8.7}$$

where s_i is the value of the additional variable for security i.

In the context of discount bonds, an appropriate alternative hypothesis can be constructed by considering the linearity of the APT model. The APT hypothesis says that expected return is linearly related to the factor betas. It has been suggested (see Beenstock and Chan (1986)) that by forming an augmented regression with squares of betas as additional variables, the linearity hypothesis and hence the APT model may be tested.[11] As there is no reason why the square of one beta should have more influence than the square of another, a priori, all single and joint combinations of squared betas were added as explanatory variables and their significance tested.[12] In the full sample model, under the assumption of a ten per cent risk free

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10 See for example, Roll and Ross (1980), Reinganum (1981) and Chen (1983).

11 While the inclusion of quadratic terms is suggestive of non-linear effects, this model is still, of course, "linear in the parameters".

12 This also controls, in a rather crude fashion, for omitted variable bias.

rate, there is no statistically significant evidence of any non-linearity in the cross-sectional model. Thus the linear factor APT model cannot be rejected. However, in the sub-samples there is some evidence of non-linearity in the period prior to Big Bang. This could indicate a rejection of APT or evidence of market efficiency. Given the results for the sample overall, it seems reasonable to interpret this as evidence of an improvement in efficiency after Big Bang. Using a five per cent risk-free rate induces some of the coefficients on non-linear terms in all the samples to appear significant, again indicating the effects of assuming an unreasonable value for this rate.

8.4. Term Structure Movements and Market Efficiency

There are two elements to this section of the chapter. Firstly, the predictability of the individual factors is examined, to determine whether those factors underlying the movements in the market display inefficiencies. Secondly, the relationship between these factors and bond market returns are analyzed in a manner which will identify anomalies in the shape of the term structure upon which profitable, riskless, trades may be made. Thus, we conduct tests in the spirit of the traditional dichotomy of efficiency tests, autocorrelation tests and trading rule tests, albeit in a manner very different from the traditional equity market orientated tests.[13]

8.4.1. The Predictability of Common Factors

For this analysis, we use techniques explained in chapter seven to test for the Markov property in a time series, that is, we estimate

$$\Delta f_{k,t} = a \ (f_k^* - f_{k,t-1}) + \sum_{j=1}^{j=n} b_j \Delta f_{k,t-j} + u_{k,t}, \tag{8.8}$$

[13] Autocorrelation tests of the traditional form were conducted in chapter 5.

for each of the $k=1,...,4$ factors. The coefficient a is the coefficient of mean reversion, and if the factor is unpredictable the b coefficients representing short run dynamic adjustment should be insignificantly different from zero.

In general, there is little evidence that the b coefficients are significant, table 8.8. There is certainly no evidence of systematic effects among the first, and dominant, factor. In the full sample, only the third factor shows some persistence, which cannot be regarded as serious as this factor only accounts for some 1.3% of the total variance of returns. The pre-Big Bang sub-sample does not provide any evidence of non-Markovian properties, while the post-Big Bang data set produces non-Markovian second, third and fourth factors. This can be seen as evidence against market efficiency after Big Bang, but as with the autocorrelation tests described in chapter five, tests of this form generally have low power. Thus, it is desirable to conduct tests that can detect the possibility of profitable trading.[14]

8.4.2. Anomalies in the Shape of the Term Structure

An anomaly in the shape of the term structure is taken to mean that the yield curve takes a particular shape at time $t-1$, which does not fully reflect the shape it takes at time t, in the absence of news. For example, if the term structure were observed to be a positive linear function of maturity, it would be expected to rise above this level at all maturities next period.[15] If it was found that, in such situations, the short maturity end rose but the long end fell below current levels, we could say that the term structure was, on average, too steep. This is an anomaly, since a portfolio which took a long position in a long dated bond and short position in a short dated bond could have made a riskless profit. If the yield curve had a persistent tendency to be too steep or, alternatively, not steep enough, then this can be viewed as strong evidence of market inefficiency.

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[14] Trading rule tests of the traditional type, such as the Alexander (1961) filter rule (see chapter four), were not conducted in chapter five, as the implied trading strategy is not representative of gilt market trading activity.

[15] Though not necessarily by the same amount due to considerations of risk.

Unfortunately, the complex shape of the term structure and its subsequent movements means that such anomalies are not easy to detect from the yield curve itself.[16] However, the underlying principal components can be used to study the separate elements of the shape of the term structure. Hence, tests in the spirit of the introductory example may be conducted. Since, the principal components are derived from returns, that is, price movements, they measure movements in the elements of shape rather than the levels of these elements of shape. It is however straightforward to calculate levels from a series of differences.

Thus two series of factor levels were constructed; one measuring the current (linear) slope, and the other measuring the current curvature (concavity with respect to maturity). If there is high correlation between these levels and the subsequent return on appropriately constructed portfolios, then this is evidence of an anomaly in the shape of the term structure.

For the case of the slope component, an appropriate portfolio is one of the form described in the introductory example, that is, it has a long position in a long bond, a short position in a short bond. The holdings of each bond are determined, using the immunization idea of setting the duration of assets equal to the duration of liabilities, by matching the duration of the long and short bond holdings. Hence if q_l is the holding in the long bond and q_s is the holding in the short bond, then since the duration of a pure discount bond is simple its maturity, we choose q_s and q_l to solve

$$s \, . \, q_s \;=\; l \, . \, q_l \qquad (8.9)$$

where s and l are the maturities (durations) of the two bonds. Normalizing $q_l = 1$, gives solutions $q_s = l/s$.[17] For the purpose of constructing such portfolios, short bonds are defined as bonds with a maturity of no more than five years, medium bonds are those with a maturity of between six and fourteen years, and long bonds have maturities of fifteen years and over. This corresponds approximately with the turning points in curvature factors (tables 8.4-8.6). Given

[16] See figure 6.4., and the discussion of modelling difficulties in chapter seven.

[17] We note that since the holdings do not sum to unity, there will be a net cash position with these portfolios.

this definition, there are twenty portfolios that can be constructed, and the correlations between the returns series on the portfolio and slope level are given in table 8.9. The upper figure in each case refers to the period before Big Bang and the lower figure to the period after Big Bang. None of the correlations is large, they are all less than 0.45, however those above 0.28 are statistically significantly different from zero. A significant positive [negative] correlation indicates that the term structure was on average too steep [flat] over the period in question. Before Big Bang, all coefficients are significantly different from zero. Their positive sign indicates that the term structure was on average too steep before Big Bang. After Big Bang all the coefficients are not significantly different from zero, and so this shape anomaly has been removed. This is strong evidence of an improvement in efficiency after Big Bang.

To measure the impact of changes in concavity on a portfolio requires that it should have positions in a short, a medium and a long bond, such that if the medium bond has a long [short] position, then the short and long bonds have short [long] positions. Once more, use of the immunization idea, assists in the determination of the holdings in the portfolios. If the holding of the medium term bond is given by q_m and its duration by m, then we would choose the holdings to solve

$$m \, . \, q_m = s \, . \, q_s + l \, . \, q_l \, . \tag{8.10}$$

However, this equation alone does not define the holdings. By adding a zero cash requirement, i.e. $q_m = q_s + q_l$, and normalizing one of the holdings to unity, e.g. $q_m = 1$, we can determine the remaining holdings as

$$\begin{aligned} q_m &= 1 \\ q_l &= 1 - q_s \\ q_s &= \frac{m - l}{s - l} \, . \end{aligned} \tag{8.11}$$

Given the definition by maturity (duration) of short, medium and long bonds, there are one hundred and eighty possible portfolios that can be constructed. However, those portfolios with medium maturity bonds having maturities near the borders between the classes of bonds are less likely to give reliable conclusions regarding concavity anomalies, unless the concavity factor has turning points unsympathetic with the division of bonds used. The evidence (tables 8.4-8.6) suggests that some of the extreme medium maturity portfolios should be interpreted with caution.

Nevertheless the correlations between the return on all the three bond portfolios and the curvature factor were calculated, and comparative figures for the periods before and after Big Bang are given in table 8.10. Overall, just over half of the correlations are reduced in magnitude after Big Bang, providing further evidence of the improved efficiency after Big Bang. A significant positive [negative] correlation means that the term structure was on average too pointed / concave [flat] over the period in question. The results are enhanced by considering just those results for portfolios with medium maturity bonds having the most central maturities, and those positioned in the upper right hand portion of the tables. These two criteria maximize the ability of the portfolio to reflect concavity, by centralizing the middle maturity and spreading the extreme maturities of the portfolio. Within this sub-set of portfolios there is a reduction in correlation following Big Bang in almost seventy per cent of the portfolios. Although overall, most of the correlations are significantly different from zero, only a few are in excess of fifty percent, and then only by a small margin, and the number of significant correlations was reduced after Big Bang. Furthermore when the two periods are taken together, the maximum correlation is 0.39.

The two portfolio analyses in this section, based upon term structure movements seem to support earlier results concerning the impact of Big Bang on market efficiency. Although these tests are not trading rule tests in the strictest sense, the use of portfolios in the correlation analysis means that it is reasonable to infer that rules using these portfolios would have generated profits before Big Bang, but will not do so after Big Bang. So although there is a small reservation on the power of the tests, there is no reservation concerning the qualitative result with regards to Big Bang and market efficiency.

8.5. Conclusions

This chapter has examined the components underlying movements in the term structure to detect market inefficiencies evidenced by anomalies in the shape of the term structure curve. A principal components analysis of the covariance matrix of underlying returns produced evidence that three factors are important in determining movements in the terms structure. These were shown to influence the level of the curve, the slope of the curve and the concavity of the curve, with the most important factor being that influencing the level of the curve. This result is not surprising given that the term structure had been shown in chapter six to have moved in an extensively parallel fashion throughout the two year sample period.

These factors are used first used to test the APT model, using the procedures developed by Roll and Ross (1980). This achieves two aims: firstly, it determines whether linear factor models, of which the theoretical models in chapter seven are but one form, are appropriate models for this market; and, secondly, it is a joint test of the APT model and market efficiency. The tests could not reject the APT model, and also provided some weak evidence of the inefficiencies present in the market before Big Bang. It is also argued that many of the hazards involved in testing the APT are removed by using a bond market rather than an equity market, lending additional power to the tests.

The main efficiency testing involved examining the factors themselves. Firstly, the random walk hypothesis was tested, and it was found that, in general, the changes in the factors were unpredictable. The evidence regarding the impact of Big Bang was however fairly balanced. Hence, secondly, more powerful tests were constructed by examining the levels of the factors, precisely the level of the slope and the curvature factors. The current state of the shape of the term structure has certain implications for its future path, and hence bond returns. If the current shape does not fully reflect this movement, abnormal returns may be made by holding appropriately constructed bond portfolios. By examining the correlations between bond portfolio returns and the levels of the factors, the extent of any anomalies in the shape could be identified. The advantage of principal components is that the separate components of shape could be examined individually, preventing contamination of the results by other influences of shape. It was found that there were no anomalies in the slope after Big Bang but evidence that the slope was, on average, too steep before Big Bang. In terms of curvature, the effect of Big Bang is not so great, and the correlations larger. However, the balance of the evidence is favourable to Big Bang and market efficiency. It must be remembered that even though many of the correlations are significantly different from zero, to design profitable trades based upon a factor which accounts for less than two per cent of the variation in the term structure, probably requires much higher correlations than those produced here.

TABLE 8.1

Principal Components of $\Delta R_{1,t},\ldots,\Delta R_{18,t}$ (28/10/85 - 16/10/87)			
eigenvalues of $\mathrm{cov}_t(\Delta R_{1,t},\ldots,\Delta R_{18,t})$			
15.62406	1.14662	0.79642	0.37998
corresponding eigenvectors (columns)			
0.83300	0.40057	0.02697	0.37867
0.89622	0.38517	0.07387	0.20720
0.92943	0.35071	0.10557	0.03928
0.94017	0.30340	0.11279	-0.09874
0.94528	0.25072	0.08761	-0.18470
0.94709	0.19463	0.02477	-0.21120
0.97006	0.13199	-0.06521	-0.19305
0.97367	0.06125	-0.16381	-0.14444
0.96356	-0.01426	-0.25109	-0.07877
0.94786	-0.08620	-0.30056	-0.01451
0.94063	-0.15140	-0.29861	0.03945
0.94347	-0.20768	-0.24689	0.07540
0.95004	-0.25292	-0.15359	0.09473
0.95143	-0.28382	-0.02866	0.09686
0.94175	-0.29638	0.11220	0.08112
0.92103	-0.28883	0.25028	0.04708
0.89352	-0.26114	0.36508	-0.00202
0.85873	-0.20781	0.43653	-0.06856

TABLE 8.2

Principal Components of $\Delta R_{1,t},\ldots,\Delta R_{18,t}$ (28/10/85 - 24/10/86)			
eigenvalues of $\mathrm{cov}_t(\Delta R_{1,t},\ldots,\Delta R_{18,t})$			
15.71960	1.43908	0.54224	0.20302
corresponding eigenvectors (columns)			
0.81448	0.39754	0.39300	0.14314
0.88256	0.37947	0.27190	0.05401
0.92606	0.34738	0.14162	-0.03172
0.94499	0.30682	0.01814	-0.09753
0.95014	0.26464	-0.07999	-0.12924
0.95455	0.22564	-0.14586	-0.11872
0.96131	0.18478	-0.18808	-0.07706
0.96781	0.13562	-0.21067	-0.01648
0.97134	0.07463	-0.21342	0.05243
0.97066	0.00451	-0.19642	0.11865
0.97361	-0.08557	-0.14809	0.14190
0.97043	-0.17639	-0.08352	0.14144
0.95822	-0.25941	-0.01044	0.11435
0.93734	-0.32738	0.05906	0.06779
0.91564	-0.37248	0.11596	0.00988
0.90236	-0.39301	0.15069	-0.05553
0.90312	-0.38054	0.15365	-0.12547
0.90089	-0.30374	0.09825	-0.19628

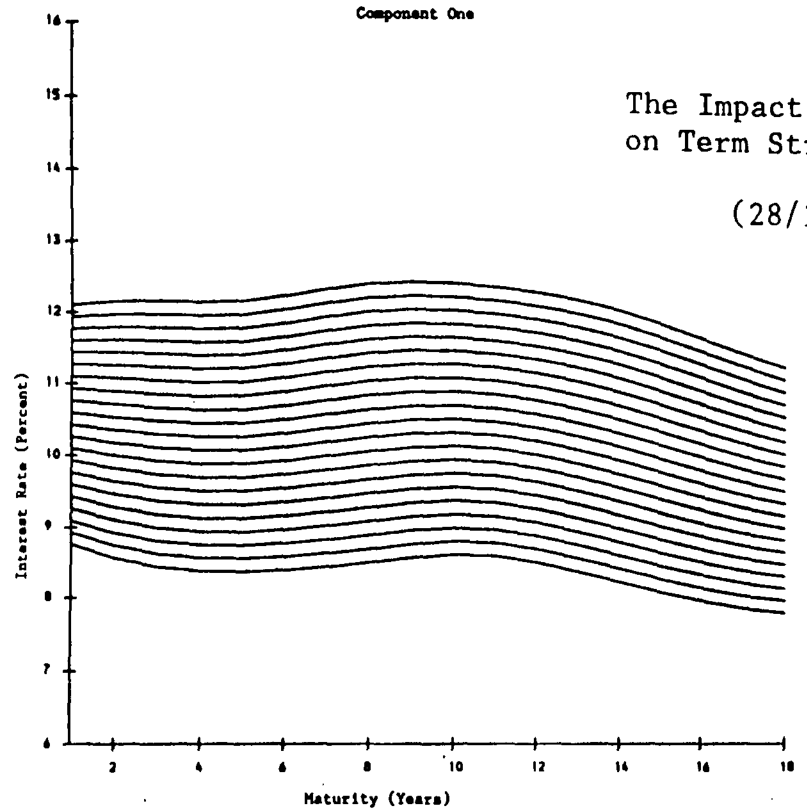
TABLE 8.3

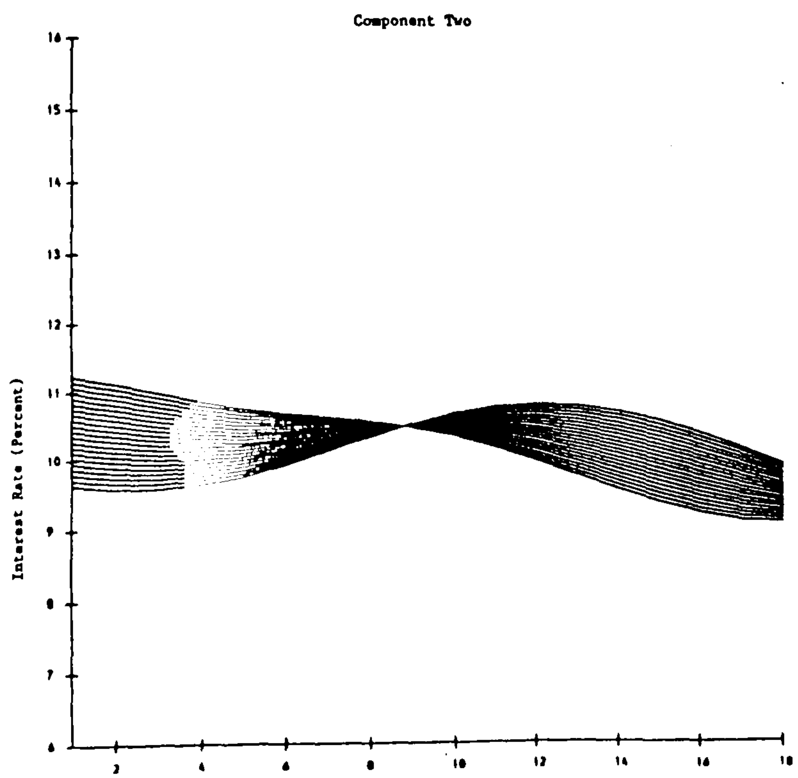
Principal Components of $\Delta R_{1,t},\ldots,\Delta R_{18,t}$ (27/10/86 - 16/10/87)			
eigenvalues of $\mathrm{cov}_t(\Delta R_{1,t},\ldots,\Delta R_{18,t})$			
15.65267	1.22760	0.78691	0.33036
corresponding eigenvectors (columns)			
0.85917	0.09322	0.36092	0.35046
0.91543	0.19942	0.29826	0.18233
0.93503	0.27070	0.22777	0.02303
0.93583	0.29485	0.16362	-0.10225
0.94047	0.26360	0.11912	-0.17833
0.95954	0.17053	0.09563	-0.20256
0.97819	0.03048	0.08549	-0.18682
0.97820	-0.12848	0.07772	-0.14327
0.95566	-0.27359	0.06553	-0.08627
0.92886	-0.36622	0.04013	-0.03257
0.91488	-0.40333	0.01063	0.01218
0.92265	-0.38163	-0.03615	0.04191
0.94409	-0.30914	-0.09628	0.06164
0.96534	-0.18755	-0.16551	0.07303
0.96767	-0.02875	-0.23860	0.07461
0.94018	0.14162	-0.30217	0.06728
0.89070	0.28723	-0.34768	0.05705
0.84038	0.38924	-0.37315	0.04646

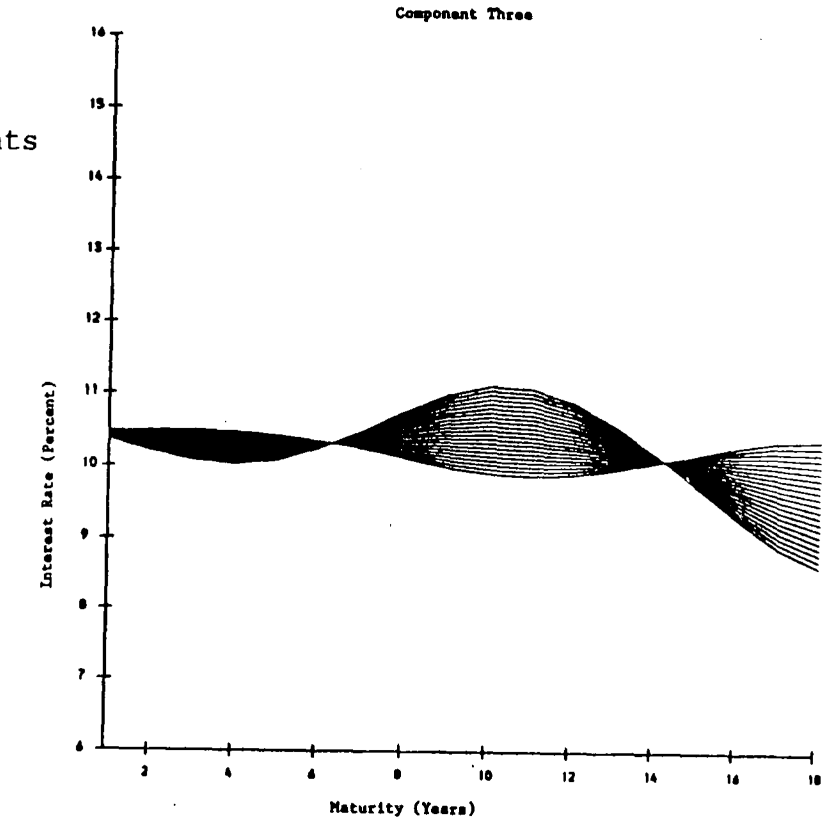
Figure 8.1

The Impact of Principal Components on Term Structure Curves

(28/10/85 - 16/10/87)







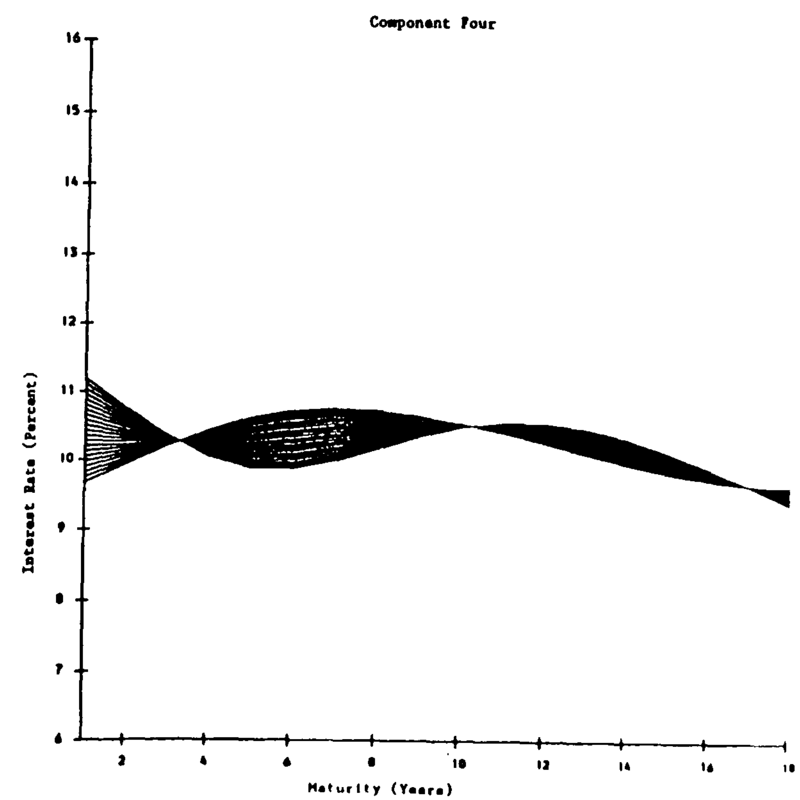
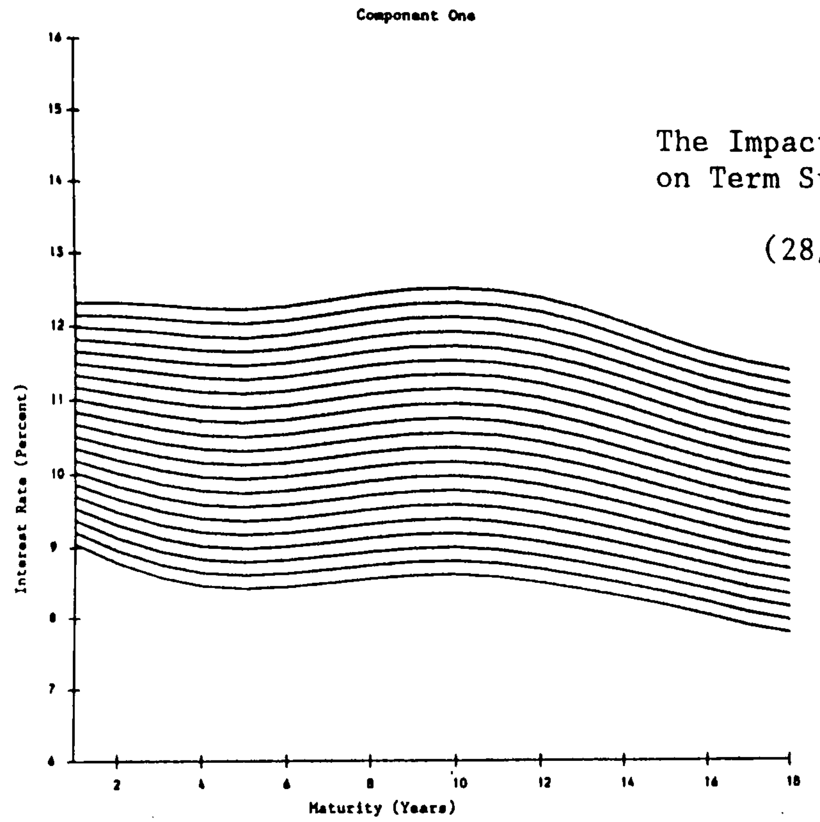
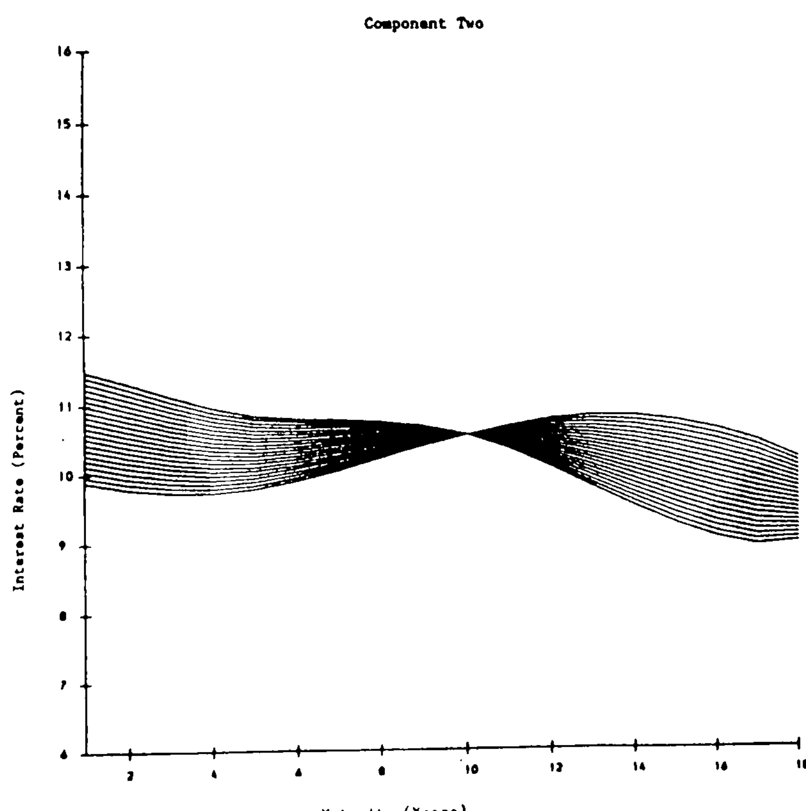


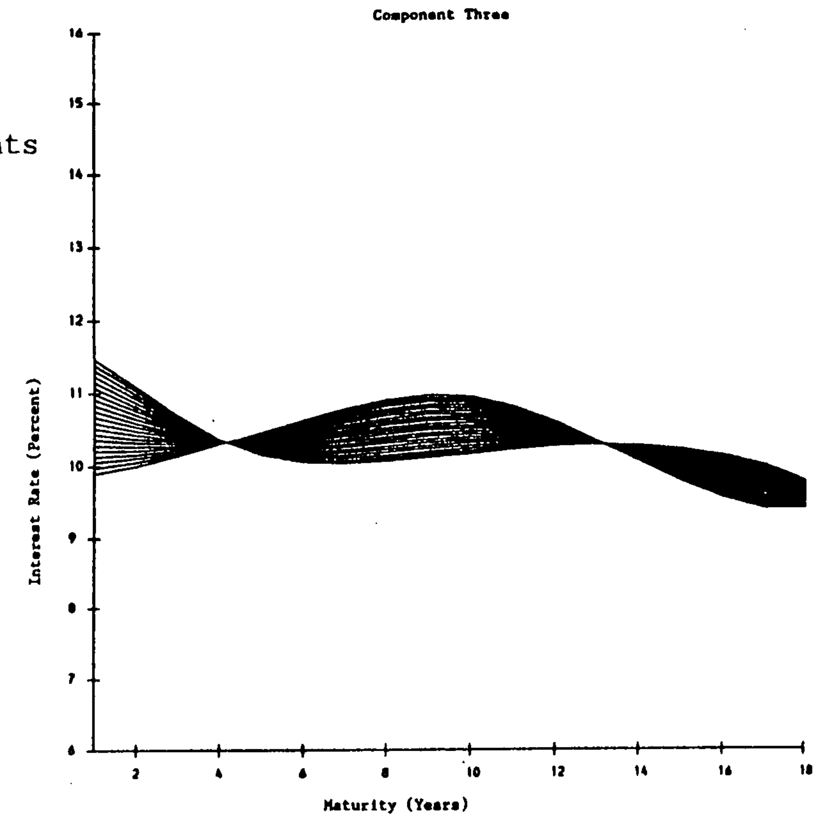
Figure 8.2

The Impact of Principal Components
on Term Structure Curves

(28/10/85 - 24/10/86)







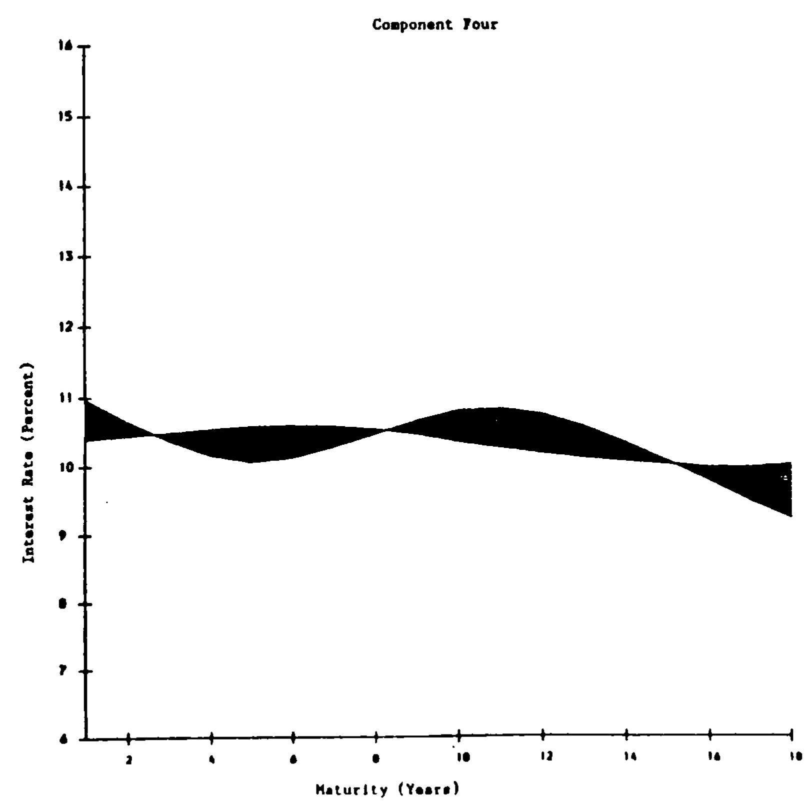


Figure 8.3

The Impact of Principal Components on Term Structure Curves

(27/10/86 - 16/10/87)

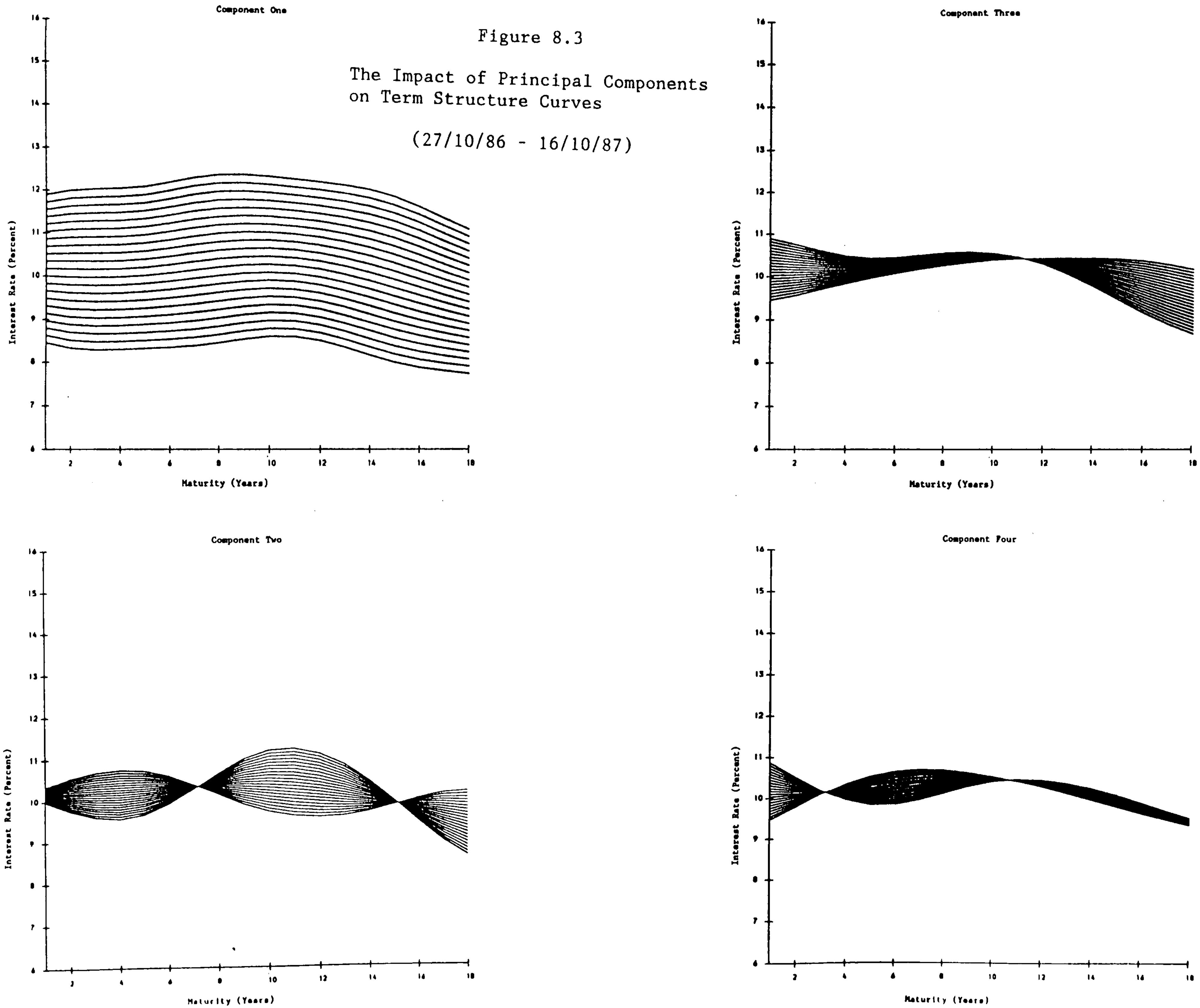


TABLE 8.4

Principal Components Analysis on Returns (28/10/85 - 16/10/87)			
eigenvalues of covariance matrix (%)			
0.00526	0.00039	0.00018	0.00003
(91.10)	(6.35)	(1.30)	(0.72)
corresponding eigenvectors (columns)			
0.00198	0.00036	0.00067	0.00092
0.00401	0.00050	0.00173	0.00133
0.00603	0.00065	0.00301	0.00152
0.00819	0.00058	0.00421	0.00139
0.01016	0.00084	0.00501	0.00091
0.01189	0.00145	0.00518	0.00061
0.01352	0.00262	0.00456	0.00012
0.01496	0.00393	0.00354	-0.00066
0.01618	0.00536	0.00185	-0.00098
0.01755	0.00625	-0.00015	-0.00129
0.01906	0.00635	-0.00175	-0.00136
0.01993	0.00525	-0.00317	-0.00026
0.02116	0.00310	-0.00381	0.00518
0.02218	0.00047	-0.00374	0.00115
0.02336	-0.00236	-0.00326	0.00135
0.02372	-0.00552	-0.00179	0.00136
0.02388	-0.00861	0.00008	-0.00001
0.02315	-0.01029	0.00268	-0.00288

TABLE 8.5

Principal Components Analysis on Returns (28/10/85 - 24/10/86)			
eigenvalues of covariance matrix (%)			
0.00499	0.00035	0.00007	0.00003
(89.38)	(6.59)	(3.09)	(0.45)
corresponding eigenvectors (columns)			
0.00216	0.00077	0.00044	-0.00106
0.00416	0.00180	0.00095	-0.00157
0.00615	0.00298	0.00142	-0.00182
0.00851	0.00405	0.00178	-0.00180
0.01036	0.00492	0.00194	-0.00134
0.01201	0.00554	0.00166	-0.00096
0.01356	0.00577	0.00119	-0.00036
0.01470	0.00609	0.00043	0.00057
0.01593	0.00560	-0.00067	0.00109
0.01713	0.00465	-0.00169	0.00149
0.01842	0.00315	-0.00257	0.00174
0.01918	0.00091	-0.00305	0.00037
0.02063	-0.00173	-0.00249	-0.00033
0.02209	-0.00355	-0.00169	-0.00128
0.02306	-0.00526	-0.00100	-0.00149
0.02284	-0.00605	0.00064	-0.00153
0.02303	-0.00595	0.00217	0.00011
0.02132	-0.00378	0.00476	0.00362

TABLE 8.6

Principal Components Analysis on Returns (27/10/86 - 16/10/87)			
eigenvalues of covariance matrix (%)			
0.00572	0.00061	0.00014	0.00001
(88.04)	(9.37)	(2.10)	(0.17)
corresponding eigenvectors (columns)			
0.00185	-0.00017	0.00062	0.00124
0.00393	-0.00020	0.00157	0.00165
0.00601	-0.00069	0.00280	0.00172
0.00806	-0.00149	0.00382	0.00093
0.01018	-0.00161	0.00462	0.00011
0.01201	-0.00101	0.00491	-0.00036
0.01374	0.00078	0.00456	-0.00070
0.01550	0.00279	0.00350	-0.00076
0.01672	0.00541	0.00222	-0.00070
0.01828	0.00751	0.00029	-0.00040
0.01998	0.00848	-0.00081	-0.00034
0.02097	0.00795	-0.00195	0.00050
0.02199	0.00598	-0.00226	0.00069
0.02265	0.00277	-0.00266	0.00024
0.02405	-0.00088	-0.00238	0.00004
0.02490	-0.00542	-0.00174	0.00028
0.02512	-0.01038	-0.00132	-0.00013
0.02529	-0.01384	-0.00077	-0.00047

TABLE 8.7

Estimated Risk Premia Coefficients of APT Model									
Data Set (R_f)	$\beta_1 \times 10^{-3}$	$\beta_2 \times 10^{-3}$	$\beta_3 \times 10^{-3}$	$\beta_4 \times 10^{-3}$	$\bar{R}^2$	*LM* (4)	*BP* (8)	*ARCH* (1)	*ARCH* (4)
C Val.	±1.96	±1.96	±1.96	±1.96		9.49	14.06	3.84	9.49
85-87 (10%)	0.463 (2.567)	1.026 (7.567)	0.671 (4.446)	-0.026 (-0.177)	0.67	6.85	7.32	0.18	1.59
85-87 (5%)	3.929 (19.293)	1.525 (9.502)	1.654 (8.700)	0.62 (2.428)	0.07	10.14	15.87	7.46	3.18
85-86 (10%)	-2.807 (9.730)	1.549 (5.935)	-0.083 (-0.380)	0.421 (1.883)	0.80	7.76	9.72	0.05	4.91
85-86 (5%)	0.680 (2.349)	2.504 (8.547)	0.364 (1.859)	-0.227 (-0.783)	0.64	5.84	12.84	1.577	0.758
86-87 (10%)	3.963 (16.803)	0.681 (3.507)	0.832 (5.066)	0.255 (2.032)	0.61	8.79	14.01	0.57	2.05
86-87 (5%)	7.411 (33.484)	0.909 (5.678)	1.983 (10.815)	1.202 (5.497)	0.54	10.285	12.957	0.01	0.98

TABLE 8.8

AUTOREGRESSIONS ON PRINCIPAL COMPONENTS					
SERIES	a	b_1	b_2	b_3	b_4
$\Delta f_{1,t}$ 85-87	0.515 (3.316)	-0.296 (-1.901)	-0.100 (-0.670)	0.012 (0.089)	0.038 (0.355)
$\Delta f_{2,t}$ 85-87	1.378 (5.400)	0.409 (1.864)	0.292 (1.626)	0.145 (1.016)	0.117 (1.126)
$\Delta f_{3,t}$ 85-87	2.304 (6.269)	0.732 (2.244)	0.429 (1.620)	0.289 (1.540)	0.207 (2.049)
$\Delta f_{4,t}$ 85-87	1.562 (5.306)	0.266 (1.056)	0.212 (1.025)	0.088 (0.542)	-0.073 (-0.728)
$\Delta f_{1,t}$ 85-86	0.419 (2.271)	-0.156 (-0.787)	-0.084 (-0.430)	0.187 (0.975)	0.078 (0.449)
$\Delta f_{2,t}$ 85-86	1.317 (3.557)	0.412 (1.275)	0.373 (1.465)	0.129 (0.578)	0.101 (0.572)
$\Delta f_{3,t}$ 85-86	1.036 (2.837)	-0.299 (-0.898)	-0.284 (-0.953)	-0.175 (-0.721)	-0.082 (-0.541)
$\Delta f_{4,t}$ 85-86	1.310 (2.883)	0.250 (0.671)	0.170 (0.559)	-0.103 (-0.419)	-0.232 (-1.384)
$\Delta f_{1,t}$ 86-87	0.559 (2.078)	-0.329 (-1.285)	-0.047 (-0.205)	-0.136 (-0.6725	-0.113 (-0.784)
$\Delta f_{2,t}$ 86-87	2.780 (4.943)	1.126 (2.277)	0.746 (1.902)	0.517 (1.882)	0.332 (2.283)
$\Delta f_{3,t}$ 86-87	1.923 (4.432)	0.838 (2.332)	0.488 (1.603)	0.307 (1.392)	0.176 (1.157)
$\Delta f_{4,t}$ 86-87	2.381 (4.691)	0.885 (2.105)	0.727 (2.240)	0.498 (2.056)	0.132 (0.894)

TABLE 8.9

Correlations between the return on a two bond portfolio and the "slope" factor

Short bond maturity	Long bond maturity			
	15 years	16 years	17 years	18 years
1 year	0.32 0.19	0.33 0.22	0.32 0.22	0.30 0.21
2 years	0.34 0.10	0.35 0.13	0.35 0.14	0.32 0.14
3 years	0.38 0.10	0.40 0.14	0.40 0.16	0.37 0.15
4 years	0.41 0.07	0.41 0.11	0.42 0.13	0.39 0.13
5 years	0.43 0.05	0.44 0.09	0.44 0.11	0.41 0.11

TABLE 8.10

Correlations between the return on a three bond portfolio and the "curvature" factor

Medium bond maturity = 6 years				
Short bond maturity	Long bond maturity			
	15 years	16 years	17 years	18 years
1 year	-0.38 0.35	-0.44 0.22	-0.44 0.13	-0.46 0.05
2 years	-0.38 0.34	-0.42 0.21	-0.43 0.11	-0.45 0.04
3 years	-0.39 0.31	-0.44 0.18	-0.44 0.09	-0.46 0.01
4 years	-0.39 0.20	-0.45 0.05	-0.46 -0.04	-0.48 -0.10
5 years	-0.38 0.14	-0.44 -0.03	-0.44 -0.08	-0.46 -0.13

Medium bond maturity = 7 years				
Short bond maturity	Long bond maturity			
	15 years	16 years	17 years	18 years
1 year	-0.40 0.17	-0.45 0.02	-0.46 0.07	-0.49 -0.14
2 years	-0.39 0.13	-0.45 -0.02	-0.46 -0.10	-0.49 -0.16
3 years	-0.40 0.08	-0.46 -0.08	-0.46 -0.15	-0.49 -0.21
4 years	-0.41 -0.41	-0.47 -0.47	-0.47 -0.48	-0.51 -0.51
5 years	-0.41 -0.21	-0.47 -0.30	-0.48 -0.32	-0.51 -0.35

Medium bond maturity = 8 years				
Short bond maturity	Long bond maturity			
	15 years	16 years	17 years	18 years
1 year	-0.38 -0.04	-0.44 -0.18	-0.45 -0.23	-0.48 -0.28
2 years	-0.37 -0.08	-0.43 -0.22	-0.44 -0.26	-0.47 -0.30
3 years	-0.37 -0.16	0.43 -0.27	-0.45 -0.30	-0.48 -0.34
4 years	-0.36 -0.28	-0.42 -0.36	-0.44 -0.37	-0.48 -0.40
5 years	-0.34 -0.36	-0.41 -0.42	-0.42 -0.41	-0.45 -0.41

Medium bond maturity = 9 years				
Short bond maturity	Long bond maturity			
	15 years	16 years	17 years	18 years
1 year	-0.42 -0.24	-0.49 -0.33	-0.50 -0.33	-0.53 -0.36
2 years	-0.42 -0.28	-0.48 -0.35	-0.49 -0.36	-0.52 -0.39
3 years	-0.42 -0.33	-0.49 -0.39	-0.50 -0.39	-0.53 -0.40
4 years	-0.41 -0.39	-0.48 -0.43	-0.49 -0.43	-0.53 -0.44
5 years	-0.40 -0.43	-0.47 -0.46	-0.47 -0.45	-0.51 -0.46

Medium bond maturity = 10 years				
Short bond maturity	Long bond maturity			
	15 years	16 years	17 years	18 years
1 year	-0.40 -0.41	-0.47 -0.44	-0.48 -0.42	-0.51 -0.44
2 years	-0.39 -0.43	-0.46 -0.46	-0.47 -0.44	-0.50 -0.45
3 years	-0.39 -0.46	-0.46 -0.48	-0.47 -0.46	-0.49 -0.47
4 years	-0.37 -0.49	-0.45 -0.50	-0.45 -0.48	-0.48 -0.49
5 years	-0.35 -0.51	-0.43 -0.52	-0.43 -0.49	-0.45 -0.51

Medium bond maturity = 11 years				
Short bond maturity	Long bond maturity			
	15 years	16 years	17 years	18 years
1 year	-0.40 -0.41	-0.49 -0.44	-0.49 -0.43	-0.51 -0.44
2 years	-0.40 -0.43	-0.48 -0.46	-0.48 -0.44	-0.50 -0.45
3 years	-0.39 -0.45	-0.47 -0.47	-0.47 -0.45	-0.49 -0.46
4 years	-0.37 -0.47	-0.45 -0.49	-0.45 -0.47	-0.46 -0.48
5 years	-0.35 -0.48	-0.43 -0.50	-0.42 -0.48	-0.42 -0.49

Medium bond maturity = 12 years				
Short bond maturity	Long bond maturity			
	15 years	16 years	17 years	18 years
1 year	-0.38 -0.51	-0.49 -0.52	-0.47 -0.49	-0.44 -0.49
2 years	-0.37 -0.52	-0.47 -0.53	-0.45 -0.49	-0.42 -0.50
3 years	-0.38 -0.53	-0.46 -0.54	-0.44 -0.50	-0.40 -0.51
4 years	-0.33 -0.55	-0.44 -0.55	-0.41 -0.52	-0.37 -0.52
5 years	-0.30 -0.56	-0.41 -0.56	-0.36 -0.52	-0.32 -0.52

Medium bond maturity = 13 years				
Short bond maturity	Long bond maturity			
	15 years	16 years	17 years	18 years
1 year	-0.25 -0.51	-0.44 -0.53	-0.38 -0.48	-0.31 -0.49
2 years	-0.23 -0.52	-0.42 -0.53	-0.36 -0.48	-0.29 -0.49
3 years	-0.22 -0.53	-0.40 -0.54	-0.33 -0.49	-0.26 -0.50
4 years	-0.18 -0.54	-0.37 -0.55	-0.29 -0.50	-0.23 -0.51
5 years	-0.14 -0.54	-0.33 -0.55	-0.24 -0.51	-0.19 -0.52

Medium bond maturity = 14 years				
Short bond maturity	Long bond maturity			
	15 years	16 years	17 years	18 years
1 year	-0.20 -0.51	-0.48 -0.55	-0.36 -0.49	-0.25 -0.50
2 years	-0.18 -0.52	-0.46 -0.55	-0.34 -0.49	-0.23 -0.51
3 years	-0.17 -0.53	-0.45 -0.56	-0.32 -0.50	-0.19 -0.52
4 years	-0.11 -0.53	-0.38 -0.56	-0.23 -0.50	-0.15 -0.52
5 years	-0.11 -0.54	-0.38 -0.56	-0.23 -0.51	-0.15 -0.52

CHAPTER NINE

Summary and Conclusions

9.1. Summary and Implications

The main theoretical findings of this study are contained in chapters 2, 3 and 4. In chapter 2, it was shown that the changes introduced by Big Bang could only be justified on economic grounds if there existed a "best-execution" rule. It was found that the only defence for maintaining the separation of stock-brokers and stock-jobbers was that of ensuring that there was no conflict of interests when the role of principal and agent were combined. Chapter 3 contained a theoretical analysis that provided predictions of the consequences of Big Bang for operational efficiency, together with an examination of the market to determine the extent to which such predictions were realized. In chapter 4, it was demonstrated that an interpretation of the efficient markets hypothesis in terms of the traditional theories of the term structure follows very naturally from a recent reinterpretation of these theories.

Chapters 2 and 3 also contain sections of interest to economic historians. As well as describing in detail the structure and operations of the market before and after Big Bang, the legal, technological and economic causes of Big Bang are analysed in order to set the event within the appropriate historical perspective. Contrary to popular opinion which has identified and dated the trigger mechanism for Big Bang as the deal between the Chairman of the Stock Exchange and the Government in 1983, it is argued that the path to stock exchange deregulation was the inevitable consequence of earlier, longer term and mostly economic events, facilitated by technological developments. Big Bang is thus the result of economic and principal and logic rather than legal expediency and compromise.

Chapters 6 and 7 contain sections of methodological interest. First, a type of approximation function, a basis of B-Splines, was adapted for use in approximating the discount function for coupon bonds. While the use of spline functions for this purpose is not new, these particular functions avoid the problems involving ill-conditioned matrices that affect the performance the standard spline functions. Other procedures to aid application to term structure measurement are also developed, including formulae for calculating appropriate confidence intervals. The method described in chapter 7 for analysing the the dynamics of interest rates is more general than that typically used when modelling the term structure. It is standard practice to posit some form of diffusion process to represent the dynamics of these rates. Application of the general-to-specific approach, used in econometric modelling, demonstrates that on occasions the dynamics of interest rates take on more complex forms than those usually assumed.

The main empirical results concern the impact of Big Bang on informational efficiency, and these are contained in chapters 5, 6, and 8. Traditional autocorrelation tests are conducted in chapter five, though use is made of some recently developed maximum likelihood autocorrelation tests which have greater power than the standard battery of tests used in this kind of study. The implication of the results of these tests and an extensive analysis of the autocorrelations themselves is that market efficiency improved substantially after Big Bang. In chapter 6, the present value of the coupon stream of each bond (discounted by the measured discount factors) is compared to its market price. Having controlled for tax effects, the residual between these figures should be identically zero, if the market is efficient. In the period before Big Bang, there was evidence of residuals in excess of a margin permitted for measurement error, and this was much less the case in the period after Big Bang. Chapter 8 tested efficiency by searching for anomalies in the shape of the term structure. This was done by examining the correlation between the returns on portfolios of pure discount bonds, which could risklessly profit from particular anomalies in shape (e.g. too steep or too flat), and principal components representing such components of shape. These tests also indicated that efficiency had been improved by the Big Bang.

The empirical results point conclusively to the fact that the informational efficiency of the gilt-edged market was substantially improved by the changes brought in by Big Bang. This is valuable information for both the Bank of England and the Stock Exchange who will wish to know the impacts of the changes that were introduced.

The methodology for estimating the term structure will have direct relevance for gilt-market investors, those wishing to price contingent claims on gilt-market securities and the Bank of England who seek to price new issues. Chapter 6 presents a method which is more accurate than the standard spline fitting technique, and that will certainly rival the alternative Bernstein Polynomial approach. This procedure is regarded as the major contribution of this thesis.

In chapter 8, it was found that there were three factors influencing the movements in the term structure. Current modelling exercises tend to use a two factor model, which is consistent with the first two of these factors. The presence of an important third factor will be of significance to all concerned with modelling the term structure and related securities. One future course of action is explored in the next section.

9.2. Suggestions for Further Research

In the area of theoretical research, the topic of modelling structural changes in securities markets could be fruitfully pursued. The majority of existing microstructure models do not consider the impact on spreads or volatility of prices of once and for changes in market structure, such as a changes in the number of participants or the manner in which prices are reported and trades actioned. Some of the comments in chapter three highlight the relevant issues, but further development would clearly be desirable.

There are several possible extensions to the empirical work in this study. Firstly, the measurement of term structures can be applied to other bond markets, such as the U.S. bond market. Secondly, the relative performance of the B-Spline and Bernstein Polynomial techniques could be analyzed. Thirdly, and most importantly, must be that of fitting term structures and fitting a term structure model, such as the Schaefer and Schwartz (1984) model, forming portfolios based upon the residuals between these two pricing models to search out arbitrage profits and thus test efficiency. The reason why this cannot be done at present is that, from the work in chapter eight, a three factor model would be needed. Currently, the most complex models use two factors, which can be viewed as representing the level and the slope of the term structure (the short rate and the spread process). The curvature probably reflects expectations of future volatility, and until our understanding of the process generating volatility improves, such a three factor model and extensions to the pricing of contingent claims will have to remain a future goal. An extension to that work would then be an examination of the real versus nominal (inflation inclusive) term structure of interest rates dichotomy, as recently examined by Cox, Ingersoll and Ross (1985, p.401-405) in the context of this form of model.

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