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PSEUDO REAL-TIME FORECASTING: A MODEL COMPARISON FOR PORTUGUESE GDP

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Abstract

GDP is one of the most important economic indicators, yet it presents a significant publishing delay. Many nowcasting models have proven to be successful and have outperformed standard forecasting regressions. This paper compares different nowcasting approaches for estimating quarterly Portuguese GDP, using estimated factors from mixed frequency real-time data.

We discuss the out-of-sample forecasting accuracy for each of the models. Furthermore, we investigate the contribution of current-quarter monthly data to the forecasting performance. The results point to an outperformance of the dynamic model averaging and using current-quarter monthly data only improves the forecasts of one of the models.

Keywords: Nowcasting, forecasting, dynamic factor model, dynamic model averaging, mixed data sampling.

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I. Introduction

"The state of the economy today is not as simple as assessing the state of the weather. It consists of millions of transactions and activities across a wide geographical area. Nevermind forecasting the future, forecasting the present is the real challenge in Economics." Jasper McMahon (at TEDxWarwick, 2013).

In order to conduct macroeconomic policy in real-time, one needs to evaluate the current state of the economy. The Gross Domestic Product (GDP) is known to be the most unified indicator of the economy's state. However, GDP is usually only published every quarter, and with a large release delay, therefore GDP figures are available with a large time lag. These features imply that one has to use nowcasting methods in order to forecast real-time GDP to be able to assess the economic state in real-time. Nowcasting is a framework for predicting the current and recent future, and the past. There has been much interest, especially from central banks, in developing nowcasting frameworks in order to conduct economic policy decisions based on real-time information (see Bánbura et al. 2010; Bánbura and Runstler, 2013; Bánbura and Modugno, 2014).

Nowcasting methods deal with two issues: timeliness and noisiness. Timeliness refers to the different frequency in the data, and the publication lags. Noisiness refers to volatility in the data that has nothing to do with the economy itself, due to survey results.

The main purpose of this research is to compare empirically the different approaches of (pseudo) [1]real-time forecasting, in the presence of mixed-frequency and unbalanced data. In particular, we apply different methods to a large Portuguese data set containing about 105 monthly indicators and 3 quarterly indicators for nowcasting and short-term forecasting of Portuguese GDP growth.

[1] Since we do not take revisions into account, we refer to such datasets as pseudo real-time (as opposed to vintages).

The literature has studied different methods to nowcast GDP. The use of monthly indicators, which are often available before GDP is a standard approach (see Luís C. Nunes, 2005; Kunovac and Spalat, 2014). On one hand, some research papers use bridge equations, using monthly information as explanatory variables in order to obtain nowcasts of quarterly GDP. On the other hand, a very popular method for macroeconomic nowcasting is using factor models (see Boivin and Ng, 2005; Forni et al., 2005; D'Agostino and Giannone, 2006; Giannone et al., 2004; Marcellino et al., 2003; Stock and Watson 2011).

This research focuses on comparing some of the most popular methods in nowcasting in terms of forecast accuracy, namely comparing the projections of an Expectation-Maximization (EM) algorithm, a 2-stage-aggregation method using a bridge equation, a factor dynamic model averaging, and a factor unrestricted mixed data sampling (Factor-U-MIDAS) regression. These methods are based on factor nowcasting, where factors are first estimated and later on they enter specific projection models. We are contributing to the nowcasting literature, more specifically to the literature using Portuguese data, by performing a more comprehensive model comparison.

The approach pursued here is to use mixed-frequency economic data, monthly and quarterly, taking into consideration the approximate delays of the data releases. We use factor models in order to estimate monthly factors, which will then be used to forecast current-quarter GDP according to each projection method. These are compared in terms of their forecasting accuracy by performing an out-of-sample evaluation.

We follow the approach of estimating the factors by applying the EM algorithm to account for missing observations in the variables, due to different release lags and different frequencies, similar to the approach of Schumacher and Breitung (2006), Mariano and Murasawa, (2003) and Bok et al. (2017). This algorithm can iteratively estimate monthly factors and use them to construct quarterly GDP estimates. We also estimate the factors using the two-stage aggregation method by following the approach in Giannone et al. (2008) and Mariano and Murasawa, (2003). As opposed to the EM algorithm, this method requires a previous transformation of the dataset, in order to make it a balanced panel, as it depends on principal components in the first stage. The aggregation option is relevant when one has a dependent variable - in this case GDP - of lower frequency than the explanatory variables. The monthly factors are converted in order to represent quarterly quantities, which will be used to forecast GDP through the bridge equation.

We also consider dynamic model averaging as a nowcasting approach. We use the factors previously estimated as the regressors and introduce time-varying parameters to the model. This approach will allow us to see if model averaging improves forecast accuracy. Finally, we regress the factors using a factor unrestricted mixed-data sampling approach (Factor-U-MIDAS). This is another popular approach for dealing with mixed-frequency data and hence will be included in the forecast comparison.

We employ the Clark-West and the Diebold-Mariano out-of-sample tests in order to compare the models' forecasts. Moreover, we present the root mean squared error and the mean absolute error of the forecasts for a more comprehensive forecast comparison.

Furthermore, we aim at investigating the importance of monthly current-quarter observations in nowcasting GDP growth, by constructing another dataset where current-quarter monthly observations are not available for the quarter we want to forecast, as done in Schumacher and Breitung (2006).

II. Literature Review

Although nowcasting is a recent field in time series econometrics, there has been an increasing interest in the subject, and different approaches to the problem have been developed. Most studies have addressed the comparison of more sophisticated models using standard univariate autoregressive models as benchmarks. This study aims at comparing a variety of factor nowcasting methods, to provide a more comprehensive model comparison. This contributes to the literature on the topic as this is a gap in the current research for the Portuguese nowcasting literature.

Several methods have been developed to nowcast GDP using related economic indicators: the use of bridge models as presented in Baffigi et al. (2004), obtaining predictions of quarterly GDP using monthly indicators; a related approach, based on mixed-data sampling (MIDAS), is considered by Kuzin et al. (2011) and the use of dynamic factor models, as previously mentioned.

Most of the nowcasting literature has focused on nowcasting the GDP of the United States or Germany. There is some nowcasting literature that uses Portuguese data, although it is not as extensive compared with other countries. Dias et al. (2006) show that large factor models have higher nowcasting and short-term forecasting performance than standard autoregressive models, although this finding is not applicable for longer forecast horizons. Since we are interested in forecasting GDP in the near future (one-quarter ahead), rather than long-term forecasts, it seems appropriate to choose a factor model. Morgado et al. (2007) compared current-quarter estimates of Portuguese GDP using a dynamic factor model with the alternative approach of combining the forecasts obtained from the dynamic factor models for each major cost component.

Central banks and other institutions have mainly used dynamic factor models for macroeconomic forecasting. As factor models can summarize the information contained in large data sets efficiently, by relying on only a few latent factors, they make forecasting with a large data set possible without running into dimensionality issues. Many authors (as the ones mentioned in the Introduction) have shown the successful performance of these model in forecasting, and other studies have used dynamic factor models specifically for the problem of nowcasting (see Schumacher and Breitung, 2006; Bok et al., 2017).

Schumacher and Breitung (2006) estimate the factors by principal components and use an EM algorithm to carry out the conversion of the different frequencies of the data. The EM algorithm is able to handle arbitrary patterns of missing values, thus being less restrictive than the two-stage method with the variable's frequency. This algorithm has the additional advantage of being able to directly provide monthly forecasts as it does not require a bridge equation.

In order to handle a mixed frequency data set, we follow the approach of Mariano and Murasawa (2003) to model monthly and quarterly data jointly. In this way, quarterly GDP can be explained by monthly variables. For the EM algorithm, we partition factors into three groups, following the approach done in Bok et al. (2017), who divided the factors into *global, real* and *labour* groups.

Giannone et al. (2008) construct a large bridge model using monthly information in order to update GDP nowcasts, accounting for the releases of monthly data throughout the quarter. The nowcast is obtained by projecting the quarterly GDP on the common factors estimated from the monthly data. For the two-stage aggregation method forecasts, we use a similar approach.

Eraslan and Schröder (2019) have proposed integrating a time-varying parameter mixed-frequency dynamic factor model in a dynamic model averaging framework for macroeconomic nowcasting. They extend the algorithm proposed by Koop and Korobilis

(2012), by using monthly GDP interpolations, to account for mixed-frequency data. Their findings point to a forecast performance improvement when accounting for time-varying parameters. We follow a similar approach, using the estimated factors as the explanatory variables and performing model averaging with time-varying parameters, except we do not use monthly interpolations of GDP, instead we work with quarterly GDP and quarterly factors. Koop and Korobilis (2012) also find that dynamic model averaging can bring substantial forecasting improvements compared to simple baseline regression.

Mixed-data sampling (MIDAS) are convenient regressions to estimate dynamic equations that can explain a low-frequency variable by its own lag and by other high-frequency variables and their lags. In this research, we derive U-MIDAS regressions, an unrestricted version of MIDAS based on linear lag polynomials, from the monthly factors in order to explain quarterly GDP growth. Foroni et al. (2011) found that U-MIDAS outperform the standard MIDAS when using quarterly and monthly data. Marcellino et al. (2010) also tested different nowcasting approaches, using factor models that can handle unbalanced data, by using different versions of a factor-based mixed data sampling (Factor-MIDAS), and comparing them with respect to their nowcasting performance. We use a similar approach for the factor U-MIDAS and also assess the informational content of current-quarter monthly indicators in a U-MIDAS regression.

Since these models have shown to outperform simple benchmark regressions, we will compare them side by side and address each model's advantages and disadvantages, as well as forecast accuracy and predictive power [2].

[2] In this research, the forecasting process and data preparation are done in R, with the help of the packages "nowcasting" (Valk, de Mattos, and Ferreira, n.d.),"fDMA" (Drachal 2020) and "midasr" (Kvedaras, 2019) packages.

III. Pseudo real-time mixed-frequency dataset

The quarterly dataset comprises 4 quarterly series available from the first quarter of 1995 until the second quarter of 2020. These include GDP, Gross Fixed Capital Formation, Imports, and Exports.

The monthly dataset compiled for the Portuguese economy comprises 105 series that can be categorized into hard and soft data. It covers consumers, manufacturing, services, construction, and retail trade surveys (68 series), employment, hours worked and wage indices in the industry, construction, commerce, retail trade and services (7 series), turnover in retail trade and services (15 series), consumer price index (7 series), industrial production (5 series), PSI-20, unemployment and IHPC.

The quarterly and monthly data was retrieved from *Instituto Nacional de Estatística* and *Banco de Portugal*. The exact series used are shown in section II of the Appendix.

Most of this data is only available from January 2005 until July 2020. We choose to start the dataset in March 2004, even though this will imply we still have some missing values at the beginning of the sample, so as to reduce the number of missing values and reduce the need of massive imputation when performing the factor estimation.

For the series that were not already seasonally adjusted, although the majority of the series are provided on a seasonally adjusted basis, a seasonal adjustment was conducted resorting on X-13-ARIMA-SEATS with the R package "seasonal".

To ensure stationarity, the quarterly series are converted into quarterly rate of change. The monthly series are transformed into monthly rates of change, except for the series already expressed in monthly or yearly growth rates. Then, we applied first differences to the monthly series that were non-stationary. We refer to section I and II in the Appendix for the data transformations used. We use the standard notation in the nowcasting literature in order to construct a monthly dataset with the quarterly and monthly series.[3] For each quarterly variable we construct a partially observed monthly corresponding variable. Quarterly observations are hence "assigned" to the third month of each quarter. In order to construct a (pseudo) real-time dataset, we take into account the publishing delays of the variables, so the dataset can reflect the real-time availability of these series. The delays, expressed in days, for each time series are shown in the Appendix. The GDP time series and the other quarterly series have the lowest degree of timeliness, followed by a group of indexes, which have a publication lag of 30 to 40 days, and finally survey results, which have the smallest publication lag. From one release to another, the availability of the data changes, which imply missing values at the end of the sample, creating the "ragged-edge" dataset.

IV. Methodology

I. Dynamic Factor Models

The dynamic factor model is formulated as follows:

$$X_t = \Lambda' f_t + e_t \tag{1}$$

$$f_t = \sum_{i=1}^{p} \psi_i \, f_{t-i} + \mu_t \tag{2}$$

where $X_t,\ t = 1, \dots, T,$ is an N-dimensional vector of time series for period t, f_t is a vector of r unobserved common factors, Λ is an (N x r) matrix of factor loadings, e_t is the N-dimensional vector of idiosyncratic terms and μ_t are the factor innovations, which are assumed to be uncorrelated with e_t.

[3] As is usual in the literature, the relationship between the observed quarterly GDP, y_t^q and the unobserved monthly observation y_t^m can be written as: $y_t^q = y_t^m + y_{t-1}^m + y_{t-2}^m$ for t = 3, 6, 9, …, T and unobserved otherwise.

The presuppose of a dynamic factor model is that the components of the large vector of time-series variables, X_t, can be explained by a few unobserved dynamic factors, f_t. The variables X_t are affected by a vector of mean-zero idiosyncratic disturbances, e_t, represented in equation (1). The variables from X_t will be loaded into the unobserved factors f_t through Λ. The unobserved factors can be estimated through several techniques, mainly relying on principal components analysis (PCA). The unobserved factors f_t follow a time series process, which is assumed to be a vector autoregression of order p, VAR(p), represented in equation (2), hence being referred as dynamic factors.

For the factor estimation we used the following two methods.

The Two-Stage Aggregation Method

In the two-stage method the factors are calculated based on the monthly variables, on which the dependent variable y (GDP in this case) will be regressed. In this application, X_t contains only the monthly series. As mentioned in the Introduction, we will use the aggregation option for this method, where monthly factors are transformed into quarterly quantities.

As mentioned above, before performing the Principal Component Analysis (PCA), X_t is transformed into a balanced panel. Then, in the first stage of the method, the parameters of the matrices Λ and f_t are estimated by PCA. The number of factors is chosen according to an information criterion similarly to Bai and Ng (2002).

In the second stage, following the routine provided by Giannone, Reichlin, and Small (2008), the factors are estimated using the monthly explanatory variables, after which the transformation from Mariano and Murasawa (2003) is applied in order to obtain factors representing quarterly quantities. The estimated factors are denoted by $\hat{f}_t$. These will be used to forecast the dependent variable y using the following bridge equation:

$$y_t = \beta_0 + \beta' \hat{f}_t + e_t. \tag{3}$$

The parameters of equation (3) are estimated through ordinary least squares (OLS), and the GDP forecast for h steps ahead, $\hat{y}_{t+h}$, is given by:

$$\hat{y}_{t+h} = \hat{\beta}_0 + \hat{\beta}' \hat{f}_{t+h}. \tag{4}$$

The Expectation Maximization Method

In this method, no bridge equation is needed, as opposed to the two-stage method. Here X_t is a joint vector containing both quarterly (monthly assigned) and monthly series.

Just as in Bok et al. (2017b) , we group the factor loading matrix into blocks of factors. The variables are grouped into global, soft and labour variables. We restrict the number of factors to one per block, which gives a total of 3 factors. The global factor affects all variables. As for the remaining blocks, these are included to account for common characteristics in particular subclasses of series. In order to control for the local correlations in survey data, we include variables representing economic agents' perceptions and sentiments in the *soft* block. We also include an additional local block for *labour* variables, such as hours worked and employment indexes. These blocks lead to the following factor and matrix loading structure,

$$f_t = \begin{pmatrix} f_t^G \\ f_t^S \\ f_t^L \end{pmatrix}, \qquad \Lambda = \begin{pmatrix} \Lambda_G \\ \Lambda_S \\ \Lambda_L \end{pmatrix}.$$

Then equation (1) becomes,

$$X_t = (\Lambda_G \;\; \Lambda_S \;\; \Lambda_L) \begin{pmatrix} f_t^G \\ f_t^S \\ f_t^L \end{pmatrix} + e_t.$$

The EM algorithm is an iterative approach that finds the maximum likelihood estimates of parameters, and alternates between two modes. The first mode known as the estimation-step (E-step) attempts to estimate the missing or latent variables. The second mode known as the maximization-step (M-step) attempts to optimize the parameters of the model by maximizing the conditional expectation of the likelihood.

First, we use an initial estimate of the missing data and initial monthly estimates of the quarterly data provided by the PCA estimates as in the first step of the above described two-stage aggregation method. Specifically,

1. E-step: Compute an updated estimate of the monthly or missing observation, $\widehat{X}_{\iota}^{\ j}$, with the conditional expectation of the likelihood, using the previous estimates of the parameters $\theta^{(j)} = (\Lambda^{(j)}, \psi_i^{(j)}, \Xi_{e_t}^{(j)}, \Xi_{\mu_t}^{(j)})$, where j is the previous iteration, with $\Xi_{e_t}^{(j)}$and $\Xi_{\mu_t}^{(j)}$ being the covariance matrix for e_t and μ_t, respectively.
2. M-step: Compute $\theta^{(j+1)}$ by maximization of the conditional expectation computed in the E-step. The estimate re-enters the E-step until some convergence criteria is reached. In this work the convergence is achieved when the log-likelihood is less than 10^{-4}.

The GDP monthly forecast is retrieved from the corresponding vector in $\widehat{X}_t$. The quarterly forecast is retrieved from the monthly assigned value for the corresponding quarter.

Although we do not discuss in-sample properties, in Section V of the Appendix we present both methods' fitted values for the entire sample.

II. Dynamic Model Averaging

Supposing we have a set of K models, with $K \leq 2^m$, and a set of m predictors, x_t. Denoting the latter by $x_t^{(k)}$, with k = 1,.., K, our set of models can be written as:

$$y_t = \left(x_t^k\right)^T \theta_t^k + \varepsilon_t^k \quad (5)$$

$$\theta_t^k = \theta_{t-1}^k + \mu_t^k \quad (6)$$

where, as before, y_t, is the time-series we want to forecast, GDP growth in this case, and θ_t^k is an $m \times 1$ vector of coefficients for each model. The general idea is that the observed y_t depends on an unobserved state θ_t, the latent factors. These latent factors are time-varying.

This approach considers a variety of models at each point in time, and then performs a model averaging, hence the terminology “dynamic model averaging”. This model allows for the combination of predictors for GDP forecasts to change over time, and also allows their marginal effect to change over time. Each variable can either be included or not included in the model, therefore two choices are possible for each predictor, constituting 2^m possibilities. The problem is that the number of models arises exponentially with the number of predictors, which can imply a time-consuming and infeasible estimation process if we use more than 20 variables, as the time to compute grows exponentially (see Drachal, 2020). Since the number of variables in our dataset would exceed the number of variables which would make the DMA computationally feasible, we take advantage of the factor model properties and use the previously estimated dynamic factors, which summarize the information contained in the data set, making computations a lot less burdensome. Since our model cannot deal directly with mixed-frequency datasets, the monthly factors are previously aggregated into quarterly factors.

We used the approximations proposed by Raftery et al. (2010), that involves two parameters, λ and α , called the “forgetting factors”, i.e., fixed numbers between 0 and 1. We refer to this paper for a discussion on the interpretation of these parameters. As discussed in this same paper, these approximations allow for fast real time forecasting.

In this work, the choice of the forgetting factors is done by choosing the ones that minimize the root mean squared error and the mean absolute error.

We refer to section VI in the Appendix for presentation of this choice.

In view of the previous considerations, the model-averaged point prediction of the system output is then computed from the following equation:

$$\widehat{y_t}^{DMA} = \sum_{k=1}^{K} \pi_{(t|t-1,k)} \hat{y}_t^{(k)} \sum_{k=1}^{K} \pi_{(t|t-1,k)} \left(\hat{x}_t^{(k)}\right)^T \hat{\theta}_{t-1}^{(k)} \tag{7}$$

where $\pi_{\left(t|(t-1),k\right)}$ are called the posterior probabilities, which depend on the forgetting factors. This expression and its derivation can be found in Koop and Korobilis (2012).

So as to compute the one-step-ahead GDP forecast, we take the first lag of the independent variables (the factors) to be the predictors.

For more details and plots of the dynamic model averaging performed in this research, consult section VI in the Appendix.

III. (Factor) Unrestricted MIDAS

Generally speaking, this class of models is used when the data frequency of the dependent variable is different from the data frequency of the independent one. In our case, y is observed quarterly, whereas the explanatory variables, x, are observed monthly. The unrestricted MIDAS can be expressed as in the following equation:

$$c(L^k)\omega(L)y_t = \delta_1(L)x_{1(t-1)} + \cdots + \delta_N(L)x_{Nt-1} + \epsilon_t \tag{8}$$

$$t = k, 2k, 3k, \ldots$$

where $\omega(L) = \omega_0 + \omega_1 L + \cdots + \omega_{k-1}L^{k-1}$, which characterizes the temporal aggregation scheme, $c(L^k) = (1 - c_1 L^k - \cdots - c_c L^{kc})$, and $\delta_j(L) = \left(\delta_{j,0} + \delta_{j,1}L + \cdots + \delta_{j,v}L^v\right), j = 1, \ldots, N$. The polynomials $\delta_i(L)$ refer to the high-frequencies variables, while $c(L^k)$ is the lag polynomial of the low-frequency variable. The low-frequency variable y , in this case GDP growth, is regressed on its own lag and on the lags of $x_{j,t}$, the j monthly variable, at time t. This specification is known as the approximate unrestricted MIDAS model. We refer to Foroni et al. (2011) for more details.

As an illustration, suppose that both current-quarter and previous-quarter monthly data have explanatory power. The structure of the model can be represented as:

$$\begin{bmatrix} y_2 \\ y_3 \\ \cdot \\ \cdot \\ y_n \end{bmatrix} = a \begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ y_{n-1} \end{bmatrix} + \begin{bmatrix} x_{1,6} & \cdots & x_{1,1} \\ \vdots & \ddots & \vdots \\ x_{1,3n} & \cdots & x_{1,3n-5} \end{bmatrix} \begin{bmatrix} \delta_{1,0} \\ \dots \\ \delta_{1,5} \end{bmatrix} \dots + \begin{bmatrix} x_{N,6} & \cdots & x_{N,1} \\ \vdots & \ddots & \vdots \\ x_{N,3n} & \cdots & x_{N,3n-5} \end{bmatrix} \begin{bmatrix} \delta_{N,0} \\ \dots \\ \delta_{N,5} \end{bmatrix} + \begin{bmatrix} \epsilon_2 \\ \epsilon_3 \\ \cdot \\ \cdot \\ \epsilon_n \end{bmatrix} \quad (9)$$

The coefficients a and δ_j can be estimated by OLS.

However, in order to assess the importance of current-quarter monthly observations, another model is constructed that depends only on previous-quarter information. That model has the following structure:

$$\begin{bmatrix} y_2 \\ y_3 \\ \cdot \\ \cdot \\ y_n \end{bmatrix} = a \begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ y_{n-1} \end{bmatrix} + \begin{bmatrix} x_{1,3} & \cdots & x_{1,1} \\ \vdots & \ddots & \vdots \\ x_{1,3n-3} & \cdots & x_{1,3n-5} \end{bmatrix} \begin{bmatrix} \delta_{1,0} \\ \dots \\ \delta_{1,5} \end{bmatrix} \dots + \begin{bmatrix} x_{N,3} & \cdots & x_{N,1} \\ \vdots & \ddots & \vdots \\ x_{N,3n-3} & \cdots & x_{N,3n-5} \end{bmatrix} \begin{bmatrix} \delta_{N,0} \\ \dots \\ \delta_{N,5} \end{bmatrix} + \begin{bmatrix} \epsilon_2 \\ \epsilon_3 \\ \cdot \\ \cdot \\ \epsilon_n \end{bmatrix} \quad (10)$$

The difference between the standard MIDAS approach and our Factor-U-MIDAS is that in the latter the explanatory variables are the estimated factors. To see the models' fitted values throughout the sample, consult section VII of the Appendix.

For each of the models, we use the factors estimated by both factor estimation methods (2-stage-aggregation and EM) in order to assess how these two factor estimation methods integrated into forecasting models compare in terms of the out-of-sample forecasting performance.

V. Real-time forecasting of Portuguese GDP: Results

In this section, we compare the accuracy of the forecasts computed in each quarter based on increasing availability of the indicators. The forecasts are direct one-quarter-ahead[4] out-of-sample forecasts, which means the models are re-estimated every time a new data release is available. We use quarterly forecasts in order to compare the observations we estimated with the actual data. For each estimated forecast we use current-quarter available monthly data while the last GDP available figure is only available for the previous quarter, since we are working with a real-time dataset. For example, for the data release March 2018, GDP is available only for the last quarter of 2017, although our monthly data is available for February 2018. Our out-of-sample period is from March 2018 until December 2019.[5] This period corresponds to 8 one-quarter-ahead forecasts. The models' notation can be seen in section III of the Appendix.

The number of factors selected for the 2-stage aggregation method and for the EM algorithm was four[6] and three, respectively. We use p = 1 for the EM and p = 2 for the 2-stage aggregation method. The choice of the factors is explained in section IV of the Appendix.

In Table 1, the out-of-sample forecasting results for Portuguese GDP growth are shown.

Table 1: *Out-of-sample forecast results*

Out-of-sample	**2018 Q1**	**2018 Q2**	**2018 Q3**	**2018 Q4**	**2019 Q1**	**2019 Q2**	**2019 Q3**	**2019 Q4**
Actual values	0.0106	0.0106	0.0101	0.0066	0.0182	0.0007	0.0108	0.0083
EM	0.0092	0.0053	0.0062	0.0054	0.007	0.0008	0.0037	0.0061
2-stg-agg	0.0136	0.0062	0.0089	0.0049	0.0076	0.0055	0.0084	0.0033
DMA-EM	0.0091	0.0081	0.0087	0.0070	0.0103	0.0074	0.0101	0.0059
DMA-2stg	0.0074	0.0083	0.0631	0.0020	0.0081	0.004	0.0102	0.0059

[4] In the sense that the last available GDP was published one quarter before our forecast

[5] We exclude 2020 from our analysis due to the major economic downturn caused by the Covid-19 pandemic.

[6] For the UMIDAS model using the 2-stage factors, we use the three first factors (instead of four), since we cannot use this many independent predictors in a multiple regression with the given sample size of the low frequency observations. We assume this does not change much since the first factors are typically more important in explaining the data.

UMIDAS-EM-1	0.0238	0.0068	0.0167	-0.0125	0.0203	-0.0057	0.0267	0.0058
UMIDAS-2stg-1	-0.01047	0.0047	0.0007	0.0108	0.0124	0.0049	0.0078	0.0074
UMIDAS-EM-2	0.0131	0.0129	0.013	0.0087	0.0086	0.0096	0.0102	0.0084
UMIDAS-2stg-2	0.0070	0.0063	0.0001	0.0050	0.0031	0.0068	0.0037	0.0069

The plot of these out-of-sample forecasts are presented below. The exact point forecasts are indicated with a circle. The red plot represents the model using the EM factors and the green plot the 2-stage factors.

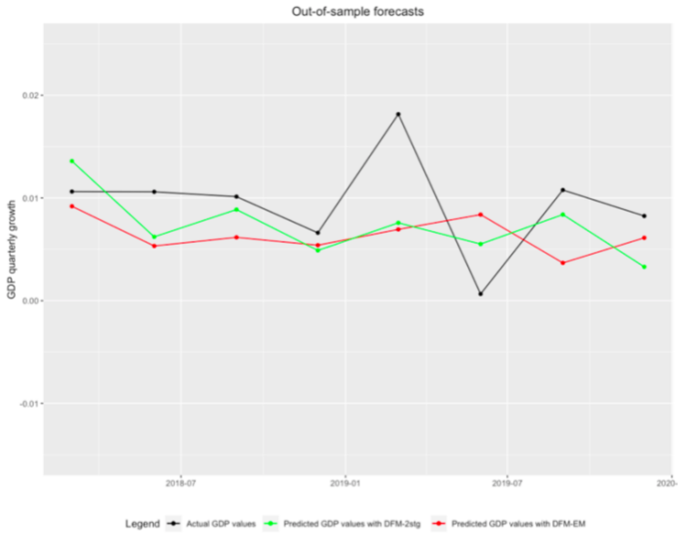

Figure 1: *Out-of-sample one-step forecast comparisons between models DFM-EM, represented by the red plot, and DFM-2stg, represented by the green plot*

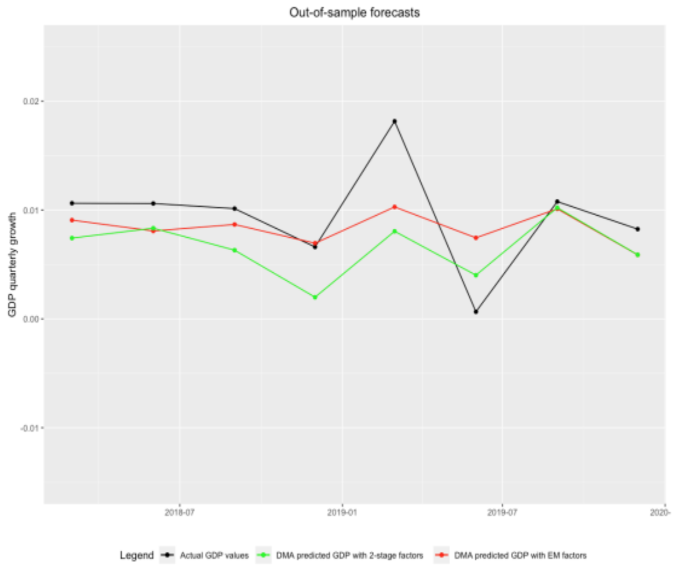

Figure 2: *Out-of-sample one-step forecast comparisons between models DMA-EM, represented by the red plot, and DMA-2stg, represented by the green plot*

Figure 3: *Out-of-sample one-step forecast comparisons between models UMIDAS-EM-1, represented by the red plot, and UMIDAS-2stg-1, represented by the green plot*

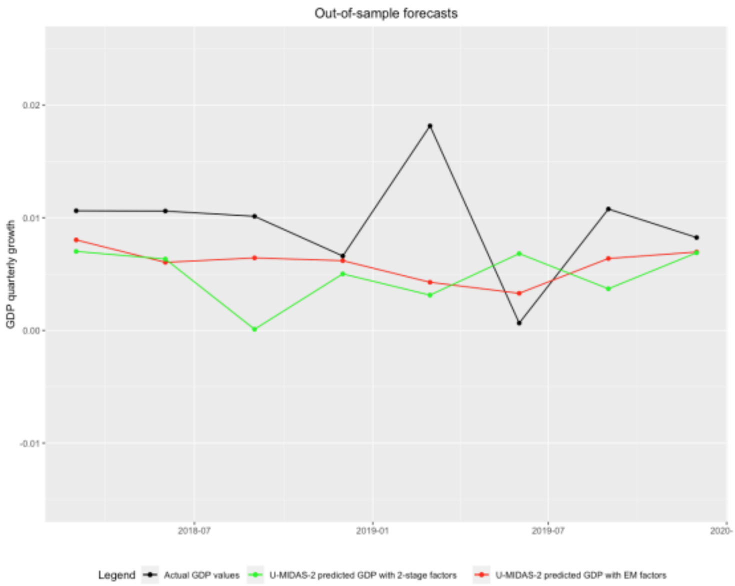

Figure 4: *Out-of-sample one-step forecast comparisons between models UMIDAS-EM-2, represented by the red plot, and UMIDAS-2stg-2, represented by the green plot*

Visually, we can already see that the DMA forecasts present a more accurate representation of GDP growth, compared to the other models. The UMIDAS-1 models, particularly the UMIDAS-EM-1, seem to have a poor forecasting performance, although it was the best model in capturing a peek in GDP growth in March 2019. The UMIDAS-

2 models did not show much variation in GDP growth, which suggest these models might not do so well in periods of significant economic changes.

Some descriptive statistical measures of these forecasts are presented below, specifically the Root Mean Squared Error and the Mean Absolute error.

Table 2: *Out-of-sample forecast measures* [7]

	RMSE	**MAE**
DFM-EM	6.00	5.01
DFM-2stg	4.97	4.14
DMA-EM	4.08	3.17
DMA-2stg	4.37	3.47
UMIDAS-EM-1	10.60	8.68
UMIDAS- 2stg-1	5.04	4.20
UMIDAS-EM-2	5.72	4.18
UMIDAS- 2stg-2	7.50	6.14

The models with smaller RMSE and MAE are the DMA models. The models UMIDAS-EM-1 and UMIDAS-2stg-2 seem to have the worst performance. Since these are descriptive statistics, we use the Clark-West test and the Diebold-Mariano tests for a more formal out-of-sample forecast comparison.

We use the Clark-West Test (CW) to compare if the competing out-of-sample forecasts from nested models are equally accurate. The CW statistic compares the Mean Squared Prediction Errors (MSPE) of two competing models, while accounting for a bias in the MSPE that arises from comparing nested models. The null hypothesis is that the two models have equivalent forecast performance, while under the alternative hypothesis the second (alternative) model's forecasts are better. The null model should be the most parsimonious model while the alternative is an extended form of the null model. The CW test statistic specificities can be found in section VIII in the Appendix.

[7] The numbers are expressed in 10^{-3}.

Since our dynamic model averaging models are an extended time-varying parameter form of the dynamic factor model we use the Clark-West test in order to compare the models' forecast accuracy. The UMIDAS-1 are also an extended version of the UMIDAS-2, since it only uses more information, hence these models are treated as nested models, with UMIDAS-1 being treated as the null model and UMIDAS-2 as the alternative. Below, we present the Clark-West test results for the one-quarter-ahead forecasts:

Table 3: *Clark West out-of-sample comparison tests*

Model 1 (null)	Model 2 (altern.)	p-value
DFM-EM	DMA-EM	0.010**
DFM-2stg	DMA-2stg	0.013**
UMIDAS-EM-2	UMIDAS-EM-1	0.029**
UMIDAS-2stg-2	UMIDAS-2stg-1	0.066*

*Note: *, ** and *** indicate significance at the 10%, 5% and 1% significance level, respectively.*

For a fair comparison, in the Clark West tests we compare different models' forecasts using the same set of factors. For both factor estimation methods the FDMA model seems to outperform the DFM model.

The Clark-West test shows that UMIDAS-2stg-1 and UMIDAS-2stg-2 have equivalent forecast performance. It also shows the UMIDAS-EM-1 outperforms the UMIDAS-EM-2, which confirms that using current quarter monthly observations improves forecast accuracy. Even so, we need more tests to confirm these results, since we suspect that the UMIDAS-EM-1 has worse forecasting performance than the UMIDAS-EM-2.

We also use the Diebold-Mariano test, which can be used with non-nested models, to determine whether forecasts are significantly different. The Diebold-Mariano test statistic's specificities can be found in section VIII of the Appendix.

The test hypothesis differ, so we clearly define each hypothesis in the table below. We stick to comparing different model's forecasts using the same set of factors. We also compare the forecast performance of the same model but with different factors, in order to check which factor estimation method produces the best outcomes.

Table 4: Diebold Mariano out-of-sample comparison tests

Model 1	Model 2	H1	H0	p-value
DMA-2stg	UMIDAS-2stg-2	Forecasts of 1 are more accurate than 2	Forecasts of 1 are not more accurate than 2	0.0152**
DMA-2tsg	UMIDAS-2stg-1	1 and 2 have different forecast accuracy	1 and 2 have same forecast accuracy	0.7597
DMA-EM	UMIDAS-EM-1	Forecasts of 1 are more accurate than 2	Forecasts of 1 are not more accurate than 2	0.0252**
DMA-EM	UMIDAS-EM-2	1 and 2 have different forecast accuracy	1 and 2 have same forecast accuracy	0.2945
UMIDAS-2stg-1	UMIDAS-2stg-2	1 and 2 have different forecast accuracy	1 and 2 have same forecast accuracy	0.1692
UMIDAS-EM-1	UMIDAS-EM-2	1 and 2 have different forecast accuracy	1 and 2 have same forecast accuracy	0.1565
DFM-2stg	UMIDAS-2stg-2	Forecasts of 1 are more accurate than 2	Forecasts of 1 are not more accurate than 2	0.0287**
DFM-2stg	UMIDAS-2tg-1	1 and 2 have different forecast accuracy	1 and 2 have same forecast accuracy	0.9682
DFM-EM	UMIDAS-EM-1	Forecasts of 1 are more accurate than 2	Forecasts of 1 are not more accurate than 2	0.0701*
DFM-EM	UMIDAS-EM-2	1 and 2 have different forecast accuracy	1 and 2 have same forecast accuracy	0.7776
DMA-EM	DFM-EM	Forecasts of 1 are more accurate than 2	Forecasts of 1 are not more accurate than 2	0.0056***
DMA-2stg	DFM-2stg	1 and 2 have different forecast accuracy	1 and 2 have same forecast accuracy	0.4342
UMIDAS-2stg-1	UMIDAS-EM-1	Forecasts of 1 are more accurate than 2	Forecasts of 1 are not more accurate than 2	0.0379**
UMIDAS-EM-2	UMIDAS-2stg-2	Forecasts of 1 are more accurate than 2	Forecasts of 1 are not more accurate than 2	0.0084***

DMA-EM	DMA-2stg	1 and 2 have different forecast accuracy	1 and 2 have same forecast accuracy	0.4239
DFM-EM	DFM-2stg	1 and 2 have different forecast accuracy	1 and 2 have same forecast accuracy	0.1161

*Note: *, ** and *** indicate significance at the 10%, 5% and 1% significance level, respectively.*

In contrast with the Clark-West test, the Diebold-Mariano test indicates that the forecasts produced by the DMA-2stg model and the DFM-2stg model have equivalent performance. Nonetheless, it confirms an outperformance of the DMA-EM in comparison to the DFM-EM, and the measures described in Table 2 indicate that the DMA's forecasts are more accurate in comparison to its competing model DFM. This tests also indicate that the DMA-2stg and DMA-EM models' forecasts seem to outperform, respectively, the UMIDAS-2stg-2 and UMIDAS-EM-1. The latter are also outperformed by the competing DFM models, confirming their poor forecasting performance compared with the other models, which is corroborated by the measures in Table 2.

When it comes to comparing the results from the two different factor estimation methods, we find that the factors estimated from the 2-stage-aggregation method seem to produce the same accuracy forecasts than the ones estimated by the EM algorithm, except for the UMIDAS models.

VI. Investigating the importance of monthly observations

In order to investigate the importance of monthly observations that are available before GDP, another dataset is constructed, where the last release date of the monthly data is the same as the GDP. The plots below show how the forecasts of both models differ. The red plot represents the forecasts of the model with previous and current quarter (referring to the quarter we want to forecast) monthly observations, while the blue plot represents the forecast of the model with only previous quarters monthly observations. The plots for the DFM-EM and the DFM-2stg models are represented below.

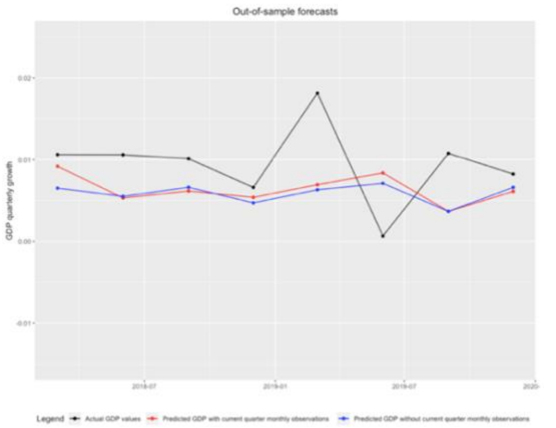

Figure 5: *DFM-EM one-step forecasts with current-quarter monthly observations (red plot) and without (blue)*

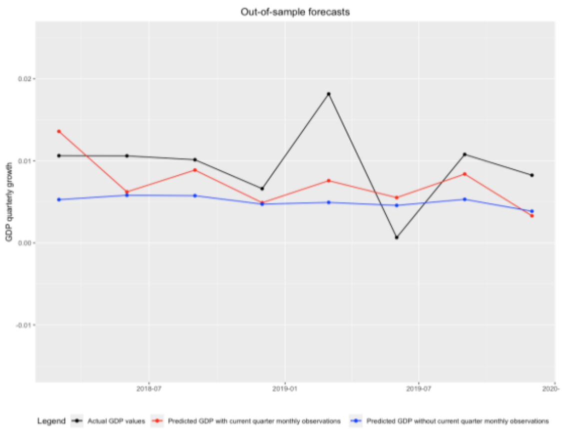

Figure 6: *DFM-2stg one-step forecasts with current-quarter monthly observations (red plot) and without (blue plot)*

The DFM-EM plots are very similar, and the current-quarter monthly observations model does not seem to produce significantly better forecasts. However, the DFM-2stg model without current quarter monthly observations seems to deliver a worse forecasting performance and it is not capable of predicting significant economic changes.

Our models are nested since they only differ in the number of monthly observations. We hence perform a Clark-West test to compare their forecast accuracy.

Table 5: *Clark West out-of-sample tests for evaluating importance of current quarter monthly observations*

Model 1	Model 2	p-value
DFM-2stg without current monthly observations	DFM-2stg with current monthly observations	0.0189**
DFM-EM without current monthly observations	DFM-EM with current monthly observations	0.3135

*Note: *, ** and *** indicate significance at the 10%, 5% and 1% significance level, respectively.*

The Clark-West shows there is an advantage of using the monthly observations in the current quarter for the DFM-2stg model, but not for the DFM-EM.

We also perform the Diebold-Mariano test, presented in Table 6 below.

Table 6: *Diebold-Mariano out-of-sample tests for evaluating importance of current quarter monthly observations*

Model 1	Model 2	H1	H0	p-value
DFM-EM without current monthly observations	DFM-EM with current monthly observations	1 and 2 have different forecast accuracy	1 and 2 have same forecast accuracy	0.8292
DFM-2stg without current monthly observations	DFM-2stg with current monthly observations	Forecasts of 1 are less accurate than 2	Forecasts of 1 are not less accurate than 2	0.0287**

Note: *, ** and *** indicate significance at the 10%, 5% and 1% significance level, respectively.

The test confirms that in the DFM-EM model, using current and previous quarter observations as opposed to only using previous quarter observations has no significant impact on forecast accuracy. However, for the DFM-2stg model, we confirm that the use of timely monthly observations can greatly improve the forecast accuracy.

In the previous section, we also compared the U-MIDAS model with previous and current quarter monthly factors (UMIDAS-1) and the model using only previous quarter monthly factors (UMIDAS-2). The Diebold-Mariano tests show that using current quarter monthly observations has no significant impact in the forecast performance, although the measures in Table 2 point to different forecast accuracy of the UMIDAS-1 and UMIDAS-2 models. This finding is not consistent with the literature. Schumacher and Breitung (2006) have found that including timely monthly observations in dynamic factor models leads to substantial improvements in the forecast performance when using the EM algorithm for factor estimation, although in this research this is the case when using the 2-stage-aggregation method.

This experiment was not performed for the DMA models since these models use quarterly factors that are available at the same time as quarterly GDP.

VII. Concluding remarks

The real-time forecasting perspective adopted in this research accounts for the publication delays of statistical economic data that policy makers are confronted with in order to assess the current state of the economy. It is thus required to have specific forecasting solutions that can use information from relevant economic indicators, that are not only subject to different publication lags but also come in different frequencies, and so cause ragged-edge data. Although not all models put to test in this research are able to deal directly with ragged-edge data issue, they can be used for nowcasting purposes.

As for the performance differences between the forecasting methods, the results suggest an outperformance of the dynamic model averaging method integrated into a dynamic factor model, as opposed to the standard dynamic factor model. These findings are consistent with the literature (Eraslan and Schröder, 2019), although they use monthly interpolations of GDP. However, compared to the other models this model is not able to deal directly with mixed frequencies, although providing the best forecasts.

The unrestricted factor MIDAS model has not shown to outperform the standard dynamic factor model or the dynamic model averaging, although it is one of the easiest models, that is able to deal with mixed-frequency data, to implement.

The choice of the factor estimation techniques has not much impact on the nowcasting performance, except for the factor U-MIDAS models. However, the EM algorithm has the advantage of being able to tackle a higher proportion of missing values.

When investigating the impact of current quarter monthly observations in the forecasting performance, we find that, contrary to the literature, there is no clear advantage in using these versus using only previous quarter monthly observations. A possible direction of future work may be expanding on the scope of this investigation and use a final vintage dataset, considering data revisions.

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Appendix

I. Data transformation codes

In order to make our series stationary, we resorted to the following data transformations:

trans = 0: the observed series is preserved

trans = 1: monthly rate of change $\frac{x_{i,t}-x_{i,t-1}}{x_{i,t-1}}$

trans = 2: monthly difference $x_{i,t} - x_{i,t-1}$

trans = 7: quarterly rate of change $\frac{x_{i,t}-x_{i,t-3}}{x_{i,t-3}}$

II. List of variables used

Variable	Frequency	Release delay	Transformation code
GDP (CP) (NA)	Q	60	7
Gross Fixed Capital formation (CP) (NA)	Q	60	7
Exports (CP) (NA)	Q	60	7
Imports (CP) (NA)	Q	60	7
Services turnover index – Total	M	40	2
Index of turnover in retail trade – retail trade except motor vehicles and motorcycles	M	30	2
Index of turnover in retail trade – retail trade except motor vehicles, motorcycles and fuel	M	30	2
Index of turnover in retail trade – retail sale of food, beverages and tobacco	M	30	2

Index of turnover in retail trade – retail sale in non-specialized stores with food, beverages or tobacco predomination	M	30	0
Index of turnover in retail trade – retail sale of food, beverages and tobacco in specialized stores	M	30	2
Index of turnover in retail trade – retail trade of non-food products	M	30	2
Index of turnover in retail trade – retail trade of non-food products (except fuel)	M	30	2
Index of turnover in retail trade – retail sale in non-specialized stores other than food, beverages or tobacco predominating	M	30	2
Index of turnover in retail trade – retail sale of automotive fuel in specialized stores	M	30	2
Index of turnover in retail trade – retail sale of audio and video equipment, hardware, paints and glass, electrical household appliances in specialized stores	M	30	2
Index of turnover in retail trade – retail sale of textiles, clothing, footwear and leather goods in specialized stores	M	30	2
Index of turnover in retail trade – retail sale of computer, peripheral units and software, telecommunications equipment, books and other products in specialized stores	M	30	0
Index of turnover in retail trade – retail sale of dispensing chemist, medical and orthopedical goods, cosmetic and toilet articles in specialized stores	M	30	2

Index of turnover in retail trade – other retail sale not in stores, stalls or markets	M	30	0
Index of gross wages and salaries in retail trade	M	30	2
Index of employment in retail trade (NA)	M	30	2
Index of hours worked in retail trade	M	30	2
Index of hours worked in construction	M	40	1
Index of hours worked in industry	M	38	1
Index of hours worked in services	M	40	0
Economic sentiment indicator-Total	M	5	1
Index of consumer prices-Total	M	30	0
Index of consumer prices-Total except housing	M	30	0
Index of consumer prices-Total except non-transformed food products and energy products	M	30	0
Index of consumer prices-Total except non-transformed products	M	30	0
Index of consumer prices-Total except energy products	M	30	0
Index of consumer prices-non-transformed food products	M	30	0
Index of consumer prices-energy products	M	30	0
PSI-20 (BP)	M	10	1
Index of harmonized consumer prices	M	18	0
Industrial production index-Total	M	30	0
Industrial production index-Consumer goods	M	30	0
Industrial production index-Intermediate goods	M	30	0
Industrial production index-Investment goods	M	30	0
Industrial production index-Energy	M	30	0
Evaluation of employment in the last 3 months-Total	M	5	0

Evaluation of the activity over the last 3 months for services-Total	M	5	2
Confidence indicator in services	M	5	2
Evaluation of the activity over the last 3 months for construction-Total	M	5	2
Evaluation of the activity over the last 3 months for construction-development of building projects and construction of buildings	M	5	2
Evaluation of the activity over the last 3 months for construction-civil engineering	M	5	2
Evaluation of the activity over the last 3 months for construction-specialized construction activities	M	5	2
Evaluation of current overall order books for construction-Total	M	5	2
Evaluation of current overall order books for construction-development of building projects and construction of buildings	M	5	2
Evaluation of current overall order books for construction-civil engineering	M	5	2
Evaluation of current overall order books for construction-specialized construction activities	M	5	2
Expected evolution of employment over the next 3 months for construction-Total	M	5	2
Expected evolution of employment over the next 3 months for construction-Development of building projects, construction of buildings	M	5	2
Expected evolution of employment over the next 3 months for construction-Civil engineering	M	5	2

Expected evolution of employment over the next 3 months for construction-Specialized construction activities	M	5	2
Expected changes in prices charged over the next 3 months for construction-Total	M	5	2
Expected changes in prices charged over the next 3 months for construction- Development of building projects, construction of buildings	M	5	2
Expected changes in prices charged over the next 3 months for construction- Civil engineering	M	5	2
Expected changes in prices charged over the next 3 months for construction- Specialized construction activities	M	5	2
Expected evolution of employment over the next 3 months for manufacturing industry-Total	M	5	2
Expected evolution of employment over the next 3 months for manufacturing industry-Final consumption goods	M	5	2
Expected evolution of employment over the next 3 months for manufacturing industry-Investment goods	M	5	0
Expected evolution of employment over the next 3 months for manufacturing industry-Motor vehicles	M	5	0
Expected evolution of employment over the next 3 months for manufacturing industry-Other investment goods	M	5	0
Expected evolution of employment over the next 3 months for manufacturing industry-Intermediate consumption goods	M	5	2
Expected change in prices charged over the next 3 months-Total	M	5	0

Expected change in prices charged over the next 3 months- Final consumption goods	M	5	0
Expected change in prices charged over the next 3 months-Investment goods	M	5	0
Expected change in prices charged over the next 3 months-Motor vehicles	M	5	0
Expected change in prices charged over the next 3 months-Other investment goods	M	5	0
Expected change in prices charged over the next 3 months-Intermediate consumption goods	M	5	0
Expected evolution of the activity over the next 3 months for manufacturing industry-Total	M	5	0
Expected evolution of the activity over the next 3 months for manufacturing industry- Final consumption goods	M	5	0
Expected evolution of the activity over the next 3 months for manufacturing industry-Investment goods	M	5	0
Expected evolution of the activity over the next 3 months for manufacturing industry-Motor vehicles	M	5	0
Expected evolution of the activity over the next 3 months for manufacturing industry- Other investment goods	M	5	2
Expected evolution of the activity over the next 3 months for manufacturing industry-Intermediate consumption goods	M	5	2
Evaluation of domestic demand for manufacturing industry-Total	M	5	2
Evaluation of domestic demand for manufacturing industry-Final consumption goods	M	5	2
Evaluation of domestic demand for manufacturing industry-Investment goods	M	5	2

Evaluation of domestic demand for manufacturing industry-Motor vehicles	M	5	2
Evaluation of domestic demand for manufacturing industry-Other investment goods	M	5	2
Evaluation of domestic demand for manufacturing industry-Intermediate consumption goods	M	5	2
Evaluation of the activity over the last 3 months for manufacturing industry-Total	M	5	2
Evaluation of the activity over the last 3 months for manufacturing industry-Final consumption goods	M	5	2
Evaluation of the activity over the last 3 months for manufacturing industry-Investment goods	M	5	2
Evaluation of the activity over the last 3 months for manufacturing industry-Motor vehicles	M	5	0
Evaluation of the activity over the last 3 months for manufacturing industry-Other investment goods	M	5	2
Evaluation of the activity over the last 3 months for manufacturing industry-Intermediate consumption goods	M	5	2
Confidence indicator for manufacturing industry	M	5	2
Expected evolution of the activity in the next 3 months for commerce	M	5	2
Expected evolution of prices in the next 3 months for services	M	5	2
Economic climate indicator	M	5	2
Expected changes in prices over the next 3 months for trade	M	5	2
Expected evolution of employment over the next 3 months for trade	M	5	2
Expected evolution of the activity over the next 3 months for trade	M	5	2

Evaluation of turnover over the last 3 months for trade	M	5	2
Confidence indicator for trade	M	5	2
Expected change in prices over the next 12 months	M	5	2
Evaluation of change in prices over the last 12 months	M	5	2
Expected evolution of the economic situation in the country over the next 12 months	M	5	2
Evaluation of the economic situation in the country over the last 12 months	M	5	2
Expected likelihood to spend money on major purchases over the next 12 months	M	5	2
Expected likelihood to save over the next 12 months	M	5	2
Expected evolution of unemployment over the next 12 months	M	5	2
Household indebtedness level	M	5	2
Expected evolution of the financial situation of households over the next 12 months	M	5	2
Evaluation of the financial situation of households over the last 12 months	M	5	2
Unemployment rate- Total	M	30	

Note: The release delays for variables from *Instituto Nacional de Estatística* were retrieved from the *Boletim Metodológico* for each statistics category. The publishing delays for variables retrieved from Banco de Portugal were taken as the average annual release delays in the *BPstat Calendário de difusão estatística.*

(CP) Current prices

(NA) Non-adjusted for calendar effects

(BP) Retrieved from *Banco de Portugal* statistical database (*BPst*at). All other series were retrieved from Instituto Nacional de Estatística.

III. Model notation

Model	Notation
Expectation Maximization algorithm forecasts	**DFM-EM**
2-stage-aggregation factor estimation method with bridge equation forecasts	**DFM-2stg**
Dynamic model averaging forecasts estimated with the EM factors	**DMA-EM**
Dynamic model averaging forecasts done with the 2-stage aggregation factors	**DMA-2stg**
Factor-U-MIDAS forecasts estimated with the EM factors using the model described in (9)	**UMIDAS-EM-1**
Factor-U-MIDAS forecasts estimated with the 2-stage-aggregation factors using the model described in (9)	**UMIDAS-2stg-1**
Factor-U-MIDAS forecasts estimated with the EM factors using the model described in (10)	**UMIDAS-EM-2**
Factor-U-MIDAS forecasts estimated with the 2-stage-aggregation factors using the model described in (10)	**UMIDAS-2stg-2**

IV. Choice of the number of factors

Our choice of factors is based on the minimization of an information criteria, as shown in the graphs. The first graph represents the number of factors according to information criteria 1 and the second according to information criteria 2. More details on the used information criteria can be found in Bai and Ng, S. (2002).

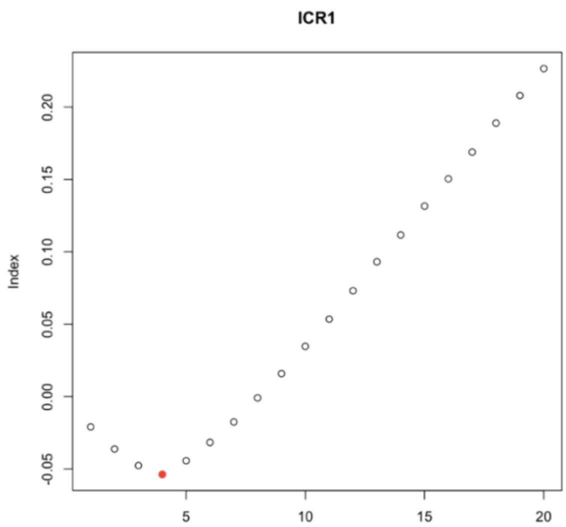

Figure 7: *Number of factors according to information criteria 1*

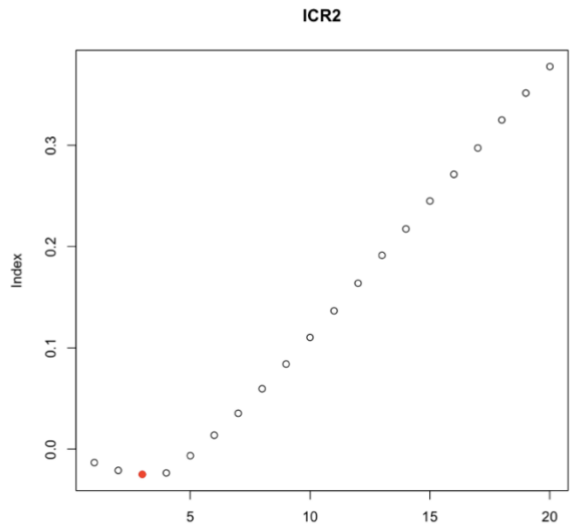

Figure 8: *Number of factors according to information criteria 2*

The first information criteria points to four factors, while the second points to three factors. For the 2-stage-aggregation method we chose to work with four factors instead of three since they provided the best forecast accuracy.

The number of factors chosen for the EM algorithm was restricted to one per block, which accounts to three, following the literature on factor blocks. The number of lags (p) for the VAR of the dynamic factors was chosen accordingly to the forecast accuracy.

V. Dynamic Factor Model

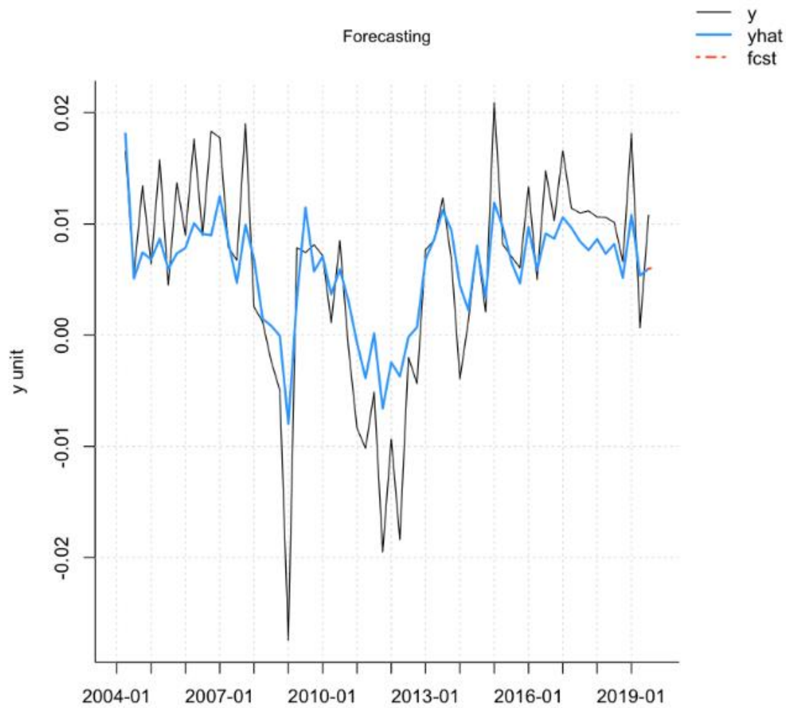

Figure 9: *Nowcasting plot for the EM algorithm. The blue plot represents the fitted values of GDP growth and the red dashed line the out-of-sample forecasts for 2020*

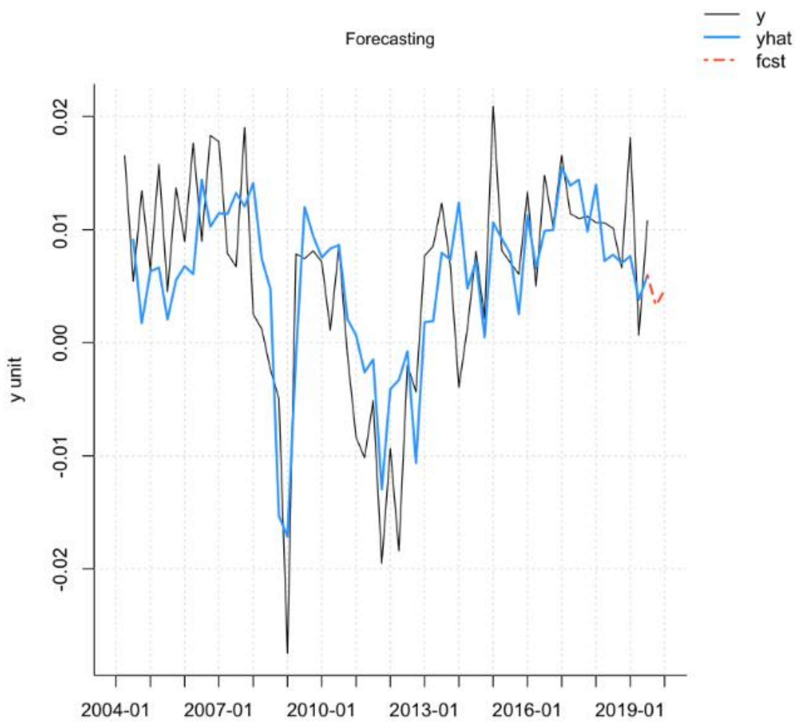

Figure 10: *Nowcasting plot for the 2-stage aggregation with bridge equation method. The blue plot represents the fitted values of GDP growth and the red dashed line the out-of-sample forecasts for 2020*

These two figures show the fitted values for GDP growth throughout the entire sample as well as out-of-sample forecasts for 2020 (which we are not interested in for this paper).

VI. Dynamic model averaging

Model	**Number of averaged models (K= 2^m)**
DMA-EM	8
DMA-2st	16

As stated in the Methodology section, the choice of the forgetting factors was based on the minimization of the root mean squared error and the mean absolute error of the models. Below we present the tables that support our choice of forgetting factors.

Choice of the forgetting factors

For DMA-EM

RMSE

λ / a	0.99	0.98	0.97
0.99	0.0101	0.0102	0.0102
0.95	0.0104	0.0104	0.0104
0.9	0.0095	0.0095	0.0095

MAE

λ / a	0.99	0.98	0.97
0.99	0.0075	0.0075	0.0076
0.95	0.0078	0.0078	0.0078
0.9	0.0072	0.0072	0.0072

Forgetting factors chosen: $a = 0.98, \lambda = 0.9$.

For DMA-2stg

RMSE

λ / a	0.99	0.98	0.97
0.99	0.0103	0.0103	0.0103
0.95	0.0095	0.0095	0.0095
0.9	0.0089	0.0088	0.0088

MAE

λ / a	0.99	0.98	0.97
0.99	0.0079	0.0079	0.0079
0.95	0.0072	0.0072	0.0072
0.9	0.0069	0.0069	0.0069

Forgetting factors chosen: $a = 0.98, \lambda = 0.9$.

Below we present some relevant plots for the dynamic model averaging performed.

These two figures show the fitted values for GDP growth throughout the entire sample.

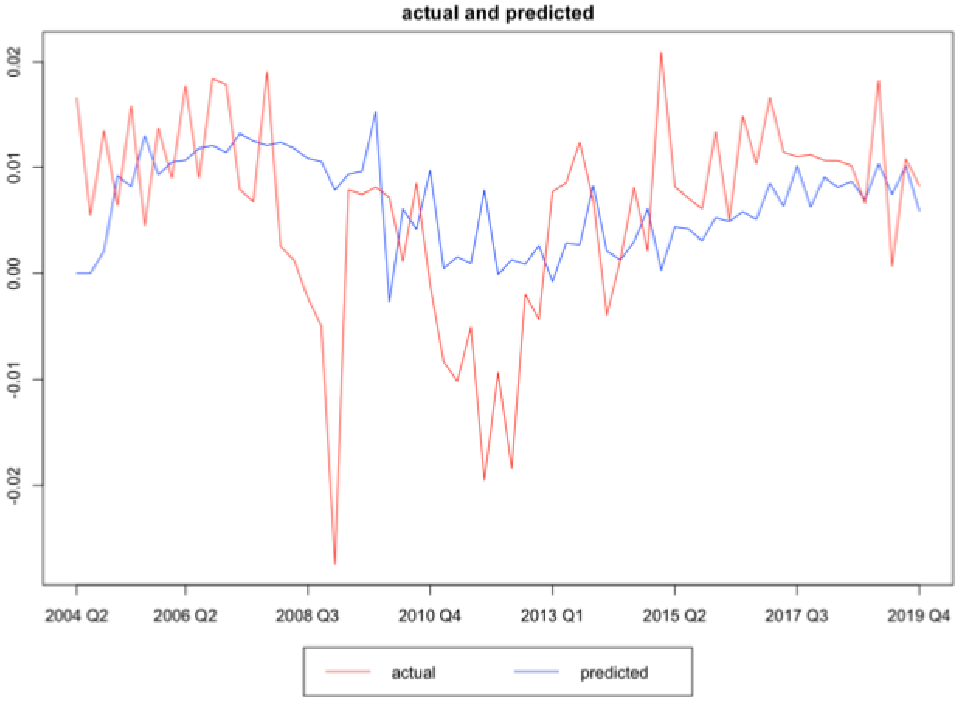

Figure 11: *Actual values for GDP growth and fitted values with DMA-EM*

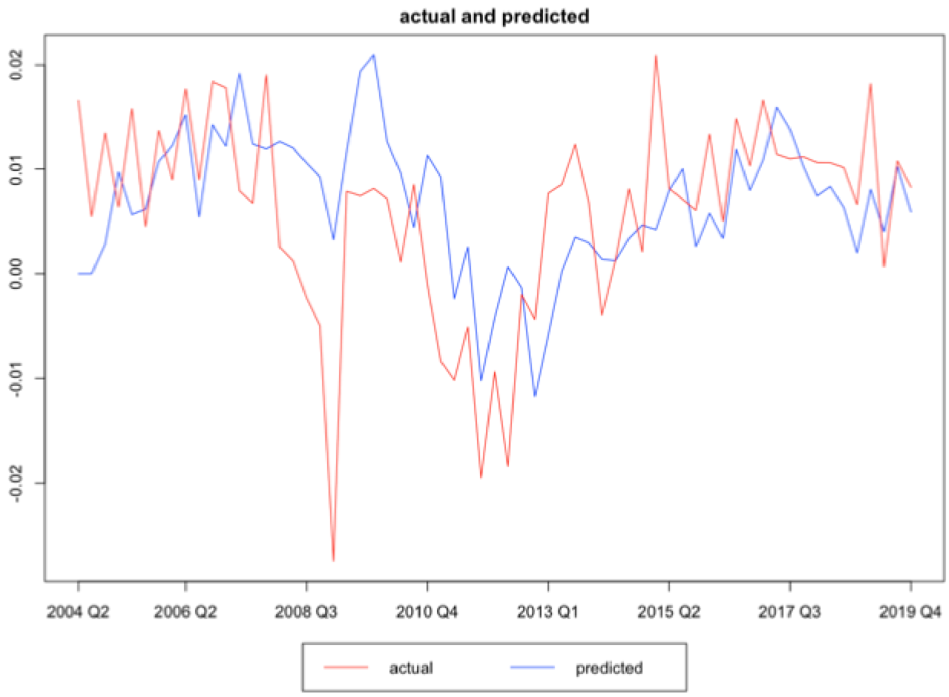

Figure 12*: Actual values for GDP growth and fitted values with DMA-2stg*

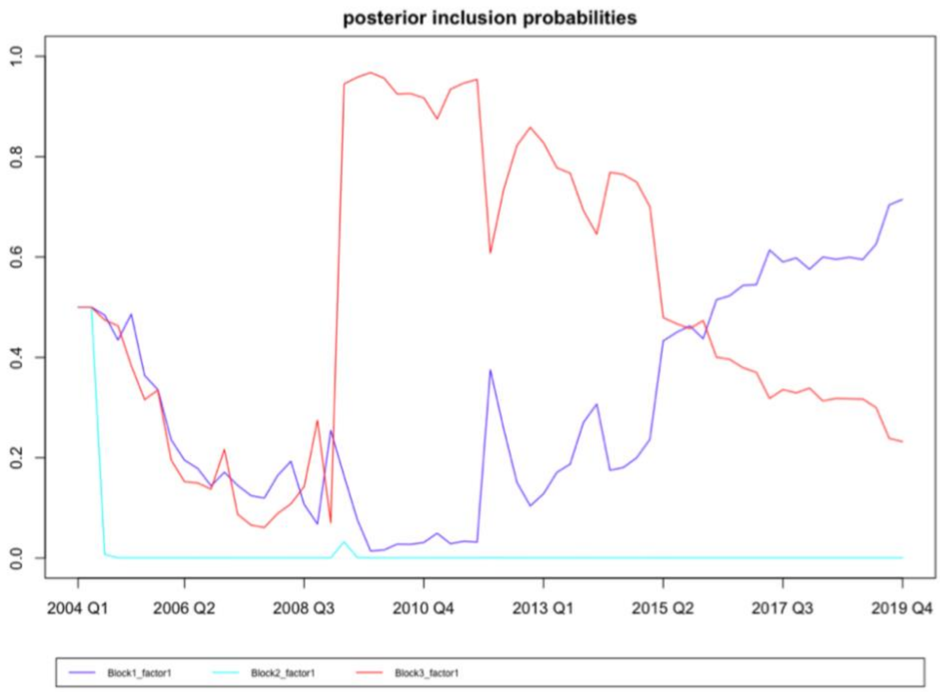

Figure 13*: Relative variable importance for DMA-EM*

Note: Block1 = Global, Block2 = Survey, Block 3 = Labor

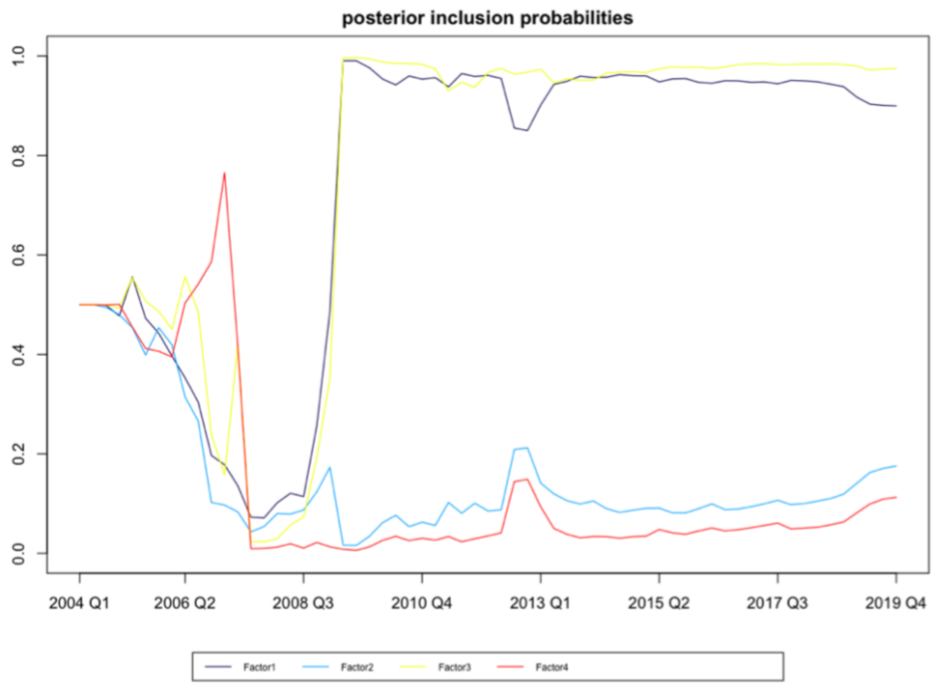

Figure 14*: Relative variable importance for DMA-2stg*

The relative variable importance of a given variable, also called the posterior inclusion probability) is the sum of the posterior probabilities of the models that include this variable. This measures how each specific variable impacts the set of considered models.

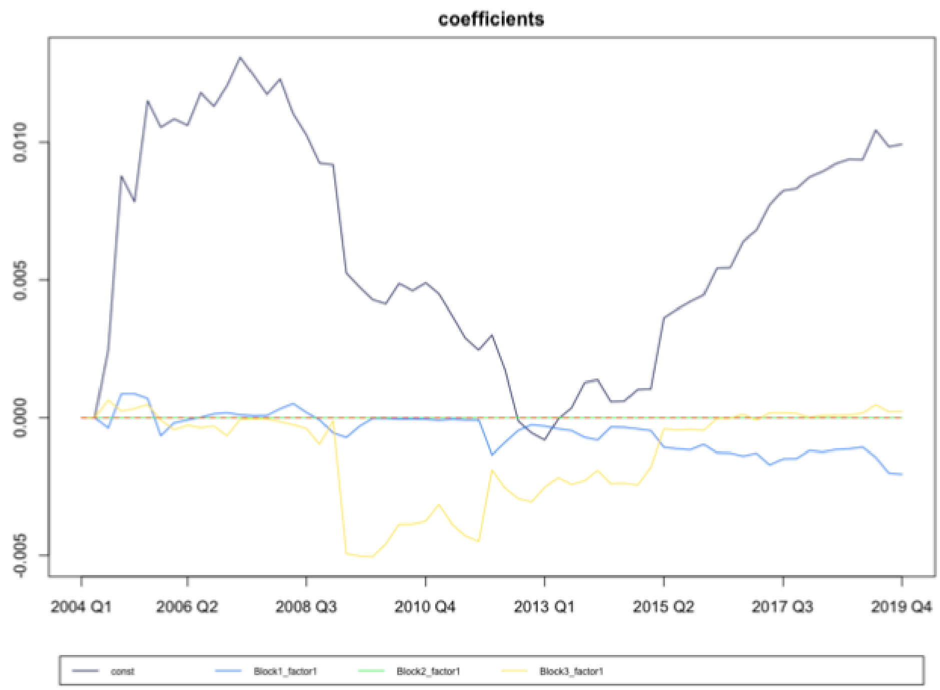

***Figure 15**: Expected values of regression coefficients for DMA-EM*

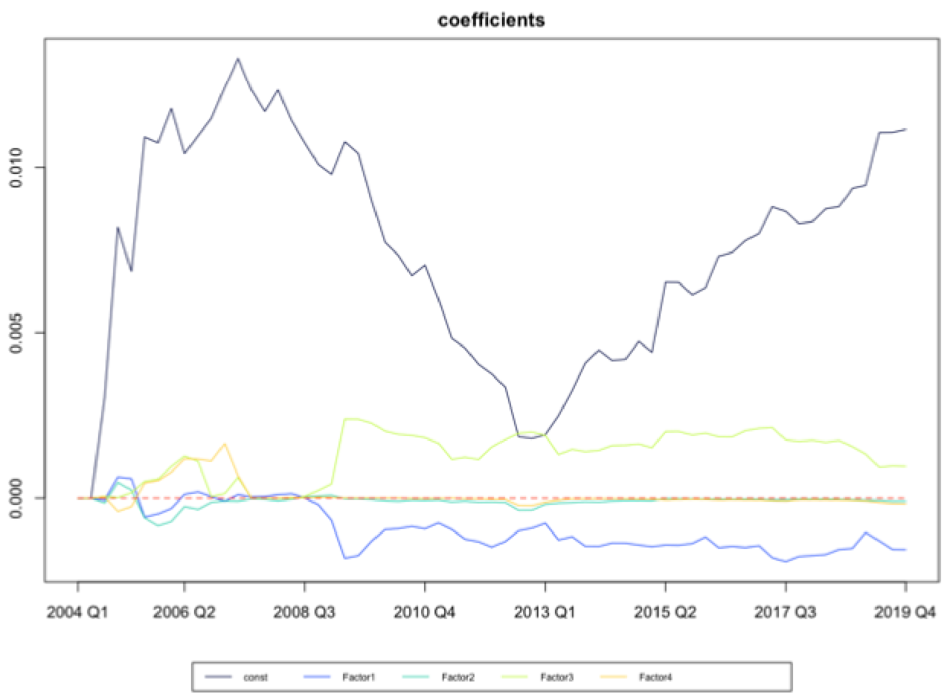

***Figure 16:** Expected values of regression coefficients for DMA-2stg*

These graphs show how the coefficients associated with each factor vary throughout the sample, as well as the constant term.

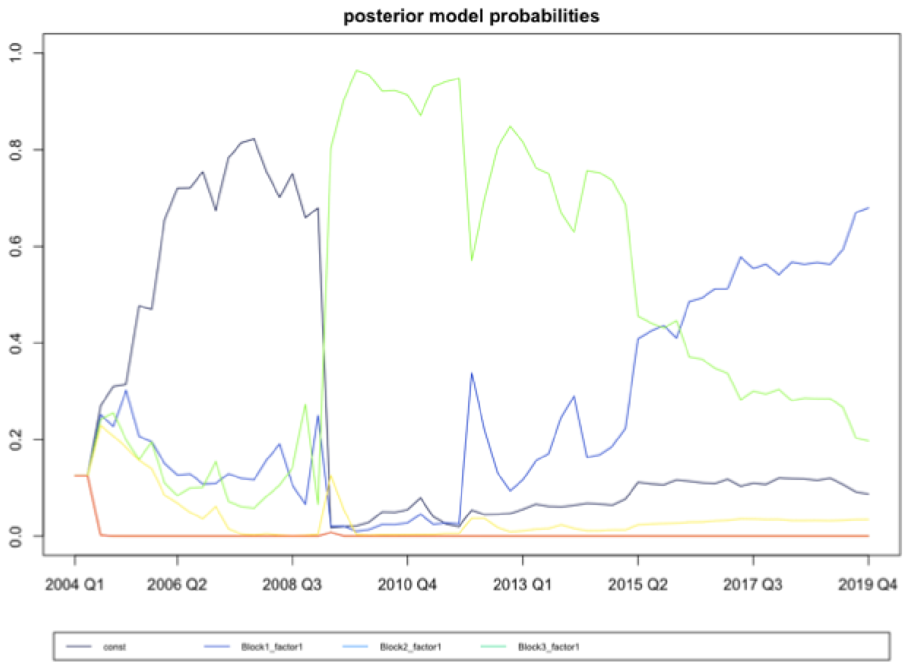

Figure 17: *Posterior model probabilities for DMA-EM*

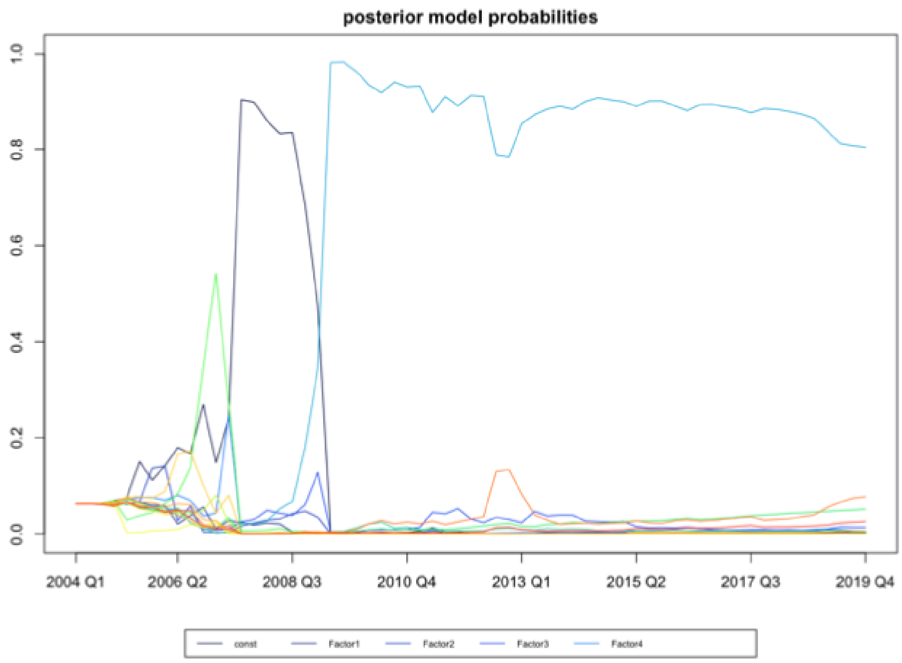

Figure 18: *Posterior model probabilities for DMA-2stg*

The posterior model probabilities act as weights attributed to the averaged models in dynamic model averaging.

VII. Factor Unrestricted MIDAS

Similarly to the other models, we present the fitted values throughout the entire sample for the UMIDAS models.

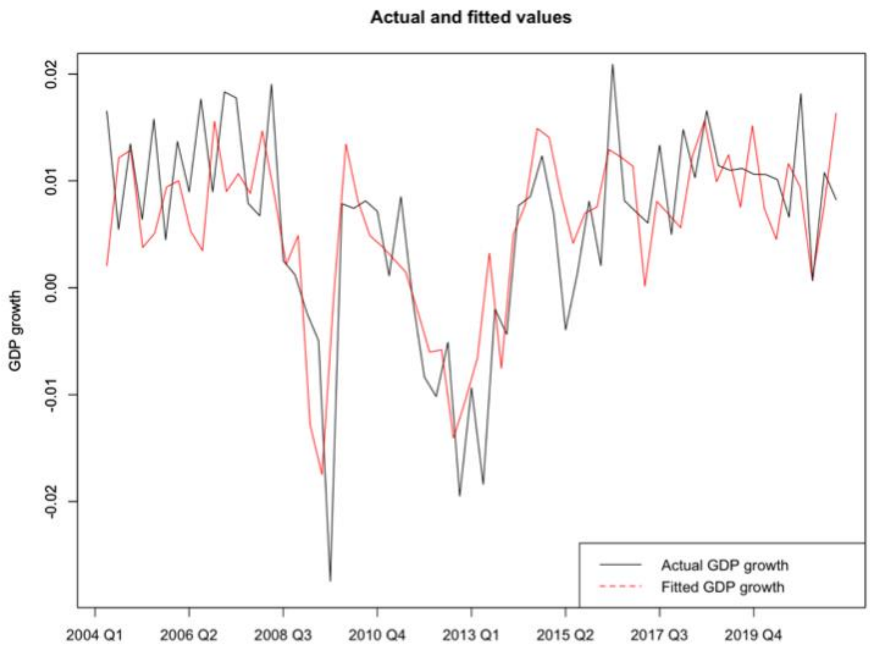

Figure 19: *Actual and fitted GDP growth values for UMIDAS-EM-1*

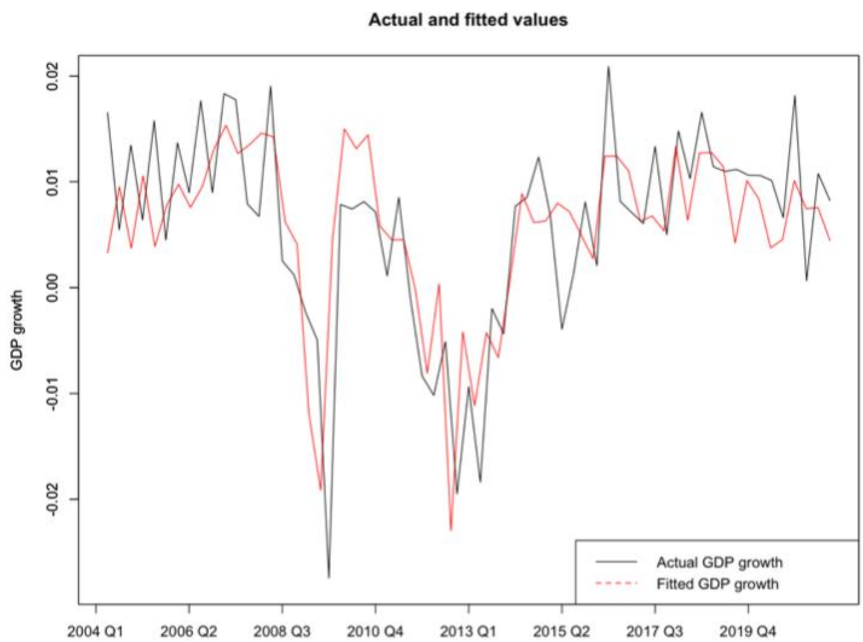

Figure 20: *Actual and fitted GDP growth values for UMIDAS-2stg-1*

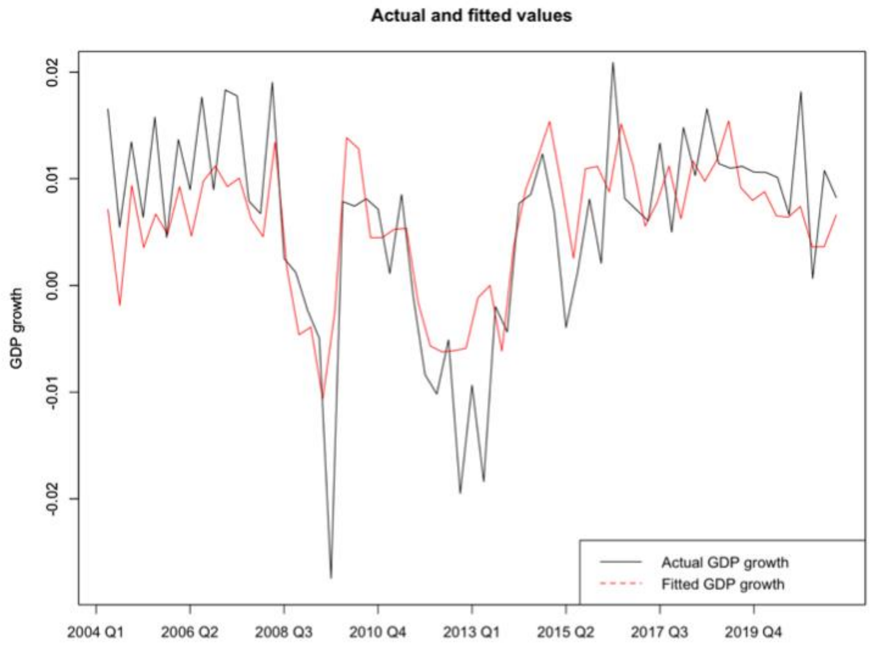

Figure 21: *Actual and fitted GDP growth values for UMIDAS-EM-2*

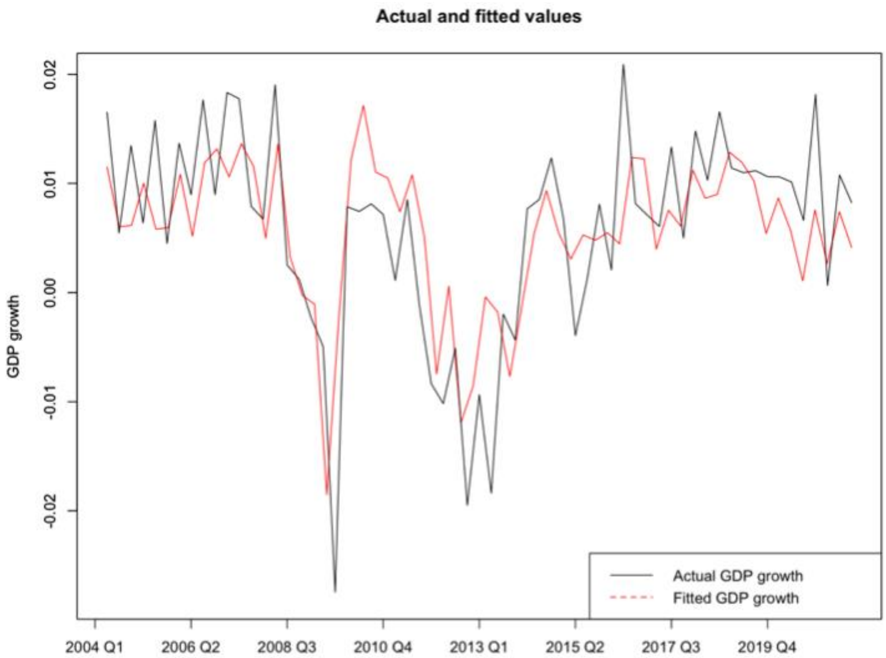

Figure 22: *Actual and fitted GDP growth values for UMIDAS-2stg-2*

VIII. Out-of-sample test specifications

The Clark-West test

$$\hat{s} = (y_{t+1} - \hat{y}_{m1,t+1})^2 - (y_{t+1} - \hat{y}_{m1,t+1})^2 + (\hat{y}_{m1,t+1} - \hat{y}_{m2,t+1})^2$$

$$CW = \frac{\bar{\hat{s}}\,\sqrt{P}}{\sqrt{Avar(\hat{s})}}$$

where $\hat{s}$ is the adjusted loss differential function, P is the number of forecasts used in $\hat{s}$, $\bar{\hat{s}}$ is the mean of $\hat{s}$, and $\sqrt{\text{Avar}(\hat{s})}$ is the asymptotic variance of the adjusted loss differential function.

We define our test hypothesis as:

H_0: Both models have equivalent forecast performance

H_1: The alternative model has better forecast performance

The Diebold-Mariano test

The forecast errors are calculated based on the squared errors "loss type". For h (steps ahead) ≥ 1, the Diebold-Mariano statistic is defined as follows:

$$DM = \frac{\sum_{i=1}^{n} (y_i - \hat{y}_{m1,i})^2 - (y_i - \hat{y}_{m2,i})^2}{\sqrt{\gamma_0 + 2\sum_{k=1}^{h-1} \gamma_k}}$$

where

$$\gamma_k = \frac{1}{n} \sum_{i=k+1}^{n} (\,(y_i - \hat{y}_{m1,i})^2 - (y_i - \hat{y}_{m2,i})^2)$$
$$- \frac{\sum_{i=1}^{n} (y_i - \hat{y}_{m1,i})^2 - (y_i - \hat{y}_{m2,i})^2}{n})(\,(y_{i-k} - \hat{y}_{m1,i-k})^2$$
$$- (y_i - \hat{y}_{m2,i-k})^2) - \frac{\sum_{i=1}^{n} (y_i - \hat{y}_{m1,i})^2 - (y_i - \hat{y}_{m2,i})^2}{n})$$