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Performance Analysis of NOMA in 5G Systems With HPA Nonlinearities

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ABSTRACT In this paper, we provide an analytical performance assessment of downlink non-orthogonal multiple access (NOMA) systems over Nakagami-*m* fading channels in the presence of nonlinear high-power amplifiers (HPAs). By modeling the distortion of the HPA by a nonlinear polynomial model, we evaluate the performance the NOMA scheme in terms of outage probability (OP) and ergodic sum rate. Hence, we derive a new closed-form expression for the exact OP, taking into account the undesirable effects of HPA. Furthermore, to characterize the diversity order of the considered system, the asymptotic OP in the high signal-to-noise (SNR) regime is derived. Moreover, the ergodic sum rate is investigated, resulting in new upper and lower bounds. Our numerical results demonstrate that the performance loss in presence of nonlinear distortions is very substantial at high data rates. In particular, it is proved that in presence of HPA distortion, the ergodic sum rate cannot exceed a determined threshold which limits its performance compared to the ideal hardware case. Monte-Carlo simulations are conducted and their results agree well with the analytical results.

INDEX TERMS Non-orthogonal multiple access (NOMA), high-power amplifiers (HPA), nonlinear polynomial model, outage probability (OP), ergodic sum rate.

I. INTRODUCTION

Non-orthogonal multiple access (NOMA) is considered as a promising multiple access technique in next-generation wireless communications. Unlike conventional orthogonal multiple access (OMA) scheme, NOMA can improve spectral efficiency and user fairness by allowing multiple users to be served in the same time via power domain [1] or code domain [2]. Motivated by the power domain NOMA multiplexing, all the users devices can decode their own information by considering the other messages as a perturbation noise. Successive interference cancellation (SIC) technique can be adopted at the receivers to separate the mixture signals in the power domain [3].

The performance of NOMA technique has been extensively investigated under various fading channels, including Rayleigh [4], Nakagami-*m* [5], and Rician [6] models. However, the previous literature have considered that all radio frequency (RF) components are perfect. Unfortunately, in practice, the system performance gets affected by the RF impairments such as high power amplifier (HPA) nonlinearity, in-phase and quadrature-phase (I/Q) imbalance and crosstalk [7], [8]. Balti and Guizani [9] have investigated the impact of the nonlinear distortion on multiple relay systems. They have proved that the system performance such as the outage probability, bits error rate (BER) and the ergodic capacity deteriorated compared to the linear HPA. The authors of [10] have evaluated the reliability and security of wireless-powered decode-and-forward (DF) multi-relay networks in presence I/Q imbalance and channel estimation errors (CEEs). They have carried out the asymptotic analysis and diversity orders for the outage probability (OP) in the high signal-to-noise ratio (SNR) regime under non-ideal and ideal conditions. Moreover, Li *et al.* [11] have investigated

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the impact of I/Q imbalance on the security of the ambient backscatte (AmBC) NOMA systems. They have derived the OP and the intercept probability (IP). They have proved that I/Q imbalance have reduced the reliability for the far user. In this work, we investigate the impact of the nonlinear distortions on the downlink NOMA system under Nakagami-m fading channels. It is well known that in the presence of HPA, the transmitted signal is severely nonlinearly distorted, which can lead to substantial performance degradation that should be carefully evaluated [12]. We note that the analytical performance assessment of NOMA systems with nonlinear HPA are relatively rare in the literature.

Recently, in [13] and [14] authors have investigated the impact of HPA on the NOMA-based relaying network. Closed-form analytical and high-SNR asymptotic expressions for the OP and system ergodic sum rate were obtained. The effects of residual hardware impairments and channels estimation errors on the cooperative NOMA system have been investigated in [15] over Nakagami-m channels, where the amplify-and-forward relay could harvest energy from the source.

An important fact that should be mentioned is that all the previous works [13]–[15], have considered a simple amplification factor gain to characterize the nonlinear distortions. However, such a model does not reflect the best behavior of the distorted signals. In our work, a memoryless nonlinear polynomial model will be considered to characterize the HPA nonlinearities [16], [17].

To the best of our knowledge, this is the first paper presenting a theoretical approach of downlink NOMA system with polynomial HPA nonlinearities. By considering the nonlinear HPA deleterious factors, we provide an accurate expression for the OP. Additionally, to highlight the diversity order, the asymptotic OP is derived. Moreover, we investigate the ergodic sum rate for the considered system in presence of HPA polynomial model and we derive explicit expressions for lower and upper bounds of the ergodic sum rate. Monte-Carlo simulation results are conducted to compare and validate our analytical approach. Numerical results manifested that the performance of power-domain NOMA network is notably affected by the hardware impairments, particularly when high achievable rates are required.

The rest part of this paper is structured as follows. Section II provides the downlink NOMA-HPA system model. In Section III, exact and asymptotic expressions of outage probability for users are derived. In Section IV, tight analytical expressions of the ergodic rate for each user are presented, where the expressions of lower and upper bound of the ergodic sum rate are obtained. Simulation results are provided in section V. Finally, Section VI concludes the paper.

II. SYSTEM MODEL

A. BASIC SIGNAL AND HPA MODEL

We consider a downlink NOMA system where the BS serving K single-antenna users, U_k. The BS sends a superposition of the individual messages, i.e, $\sum_{k=1}^{K}\sqrt{\xi_k P_s}x_k$, where P_s is the transmitted power at the source, ξ_k is is the power allocation coefficient, i.e. $\xi_1 \geq \cdots \geq \xi_K$ and x_k is the message for U_k. To model the HPA distortions, we consider the polar memoryless nonlinearity characterized by the amplitude-to-amplitude (AM/AM) and the amplitude-to-phase (AM/PM) conversion functions [18]. Furthermore, we assume that the HPA nonlinearity at the k-th user is instigated using a compensation approach such as the predistortion technique given in [18]. A popular choice for modeling these nonlinear PA characteristics in the BS, is the polynomial model providing analytical tractability [17]. Using this model, the signal at the output of the memoryless nonlinear HPA is given by

$$x_{out} = x_{in}\sum_{l=0}^{L} a_{2l+1}|x_{in}|^{2l}, \tag{1}$$

where x_{in} is the base-band equivalent input signal and L is the order of the polynomial function and a_{2l+1} represents a set of coefficients. In the strictly memoryless case with real value coefficients, only the AM/AM conversion is taken into account. We note that only odd-order product terms are considered in (1) due to the fact that the signals generated from the even order terms are outside the frequency band of interest. In this paper, the HPA coefficients are assumed to be known, to simplify the analysis.

B. NOMA TRANSMISSION

In the presence of HPA nonlinearity, the received signal at U_k using NOMA transmission is given by

$$y_k^{hpa} = h_k\left(\sqrt{\xi_k P_s}x_k\sum_{l=0}^{L} a_{2l+1}|\sqrt{\xi_k P_s}x_k|^{2l} + \sum_{i\neq k}^{K}\sqrt{\xi_i P_s}x_i\sum_{l=0}^{L} a_{2l+1}|\sqrt{\xi_i P_s}x_i|^{2l}\right) + n_k, \tag{2}$$

where h_k is the complex channel coefficients between the BS and U_k and n_k denotes zero-mean circularly symmetric additive white Gaussian noise at the k^{th} user with zero mean and variance σ_k^2. We assume that the all channels between the BS and the users are independent and identically distributed ($i.i.d$) Nakagami-m fading, with different fading parameters $m_k = m$ and different average fading powers $\Omega_i = \omega$. Without loss of generality, we assume that users are ordered on their channel quality i.e., $\left(\| h_1 \|^2 \leq \| h_2 \|^2 \leq \cdots \leq \| h_K \|^2\right)$. According to the power domain NOMA scheme, the SIC will be carried out at the users to decode the corresponding signals. In this case, the k^{th} user detects the n^{th} user's signals where $n < k$ and remove the message from its observation, in a successive iteration technique [4]. Therefore, from (2), the received signal at the U_k from the BS can be rewritten as

$$y_k^{hpa} = h_k\sqrt{P_s}z_k + n_k, \tag{3}$$

where z_k represents the following different terms

$$z_k = \underbrace{a_1\sqrt{\xi_k}\, x_k}_{\text{L-gain}} + \underbrace{\sqrt{\xi_k}x_k \sum_{l=1}^{L} a_{2l+1}|\sqrt{\xi_k}x_k|^{2l}}_{\text{NL-distortion noise}} + \underbrace{\sum_{u=k+1}^{K} \sqrt{\xi_u}x_u \sum_{l=0}^{L} a_{2l+1}|\sqrt{\xi_u}x_u|^{2l}}_{\text{IUI}} + \underbrace{\sum_{n=1}^{k-1} \sqrt{\xi_n}x_n \sum_{l=0}^{L} a_{2l+1}|\sqrt{\xi_n}x_n|^{2l}}_{\text{SIC}}. \quad (4)$$

For generality, the k^{th} user adopts only the purely linear instantaneous term as the useful signal term in the detection while treats everything else as interference (NL-distortion noise and he inter-user interference (IUI)). In order to quantify the effective distortion for U_k, we adopt and define the instantaneous signal-to-interference-plus-noise ratio (SINR) as the key performance metric. We note, that with large values of the parameter L, it is not straightforward to derive the SINR expression. Based on the Bussgang's theorem [19], when the transmitted symbol x_{in} is Gaussian distributed, the nonlinearly distorted signal can be expressed as the sum of two uncorrelated components, i.e.,

$$x_{out} = cx_{in} + d, \quad (5)$$

where c is the gain of the linear part, d is the nonlinear distortion noise uncorrelated with the input signal. As a good approximation, we focus on the case with 3-order polynomial model for HPA i.e. $L = 1$. Consequently, from equations (3) and (4), we can expressed the SINR at user k as follows

$$\gamma_k^{hpa} = \frac{\bar{\gamma}_k |a_1|^2 |h_k|^2 \xi_k}{\bar{\gamma}_k |h_k|^2 \left[\sum_{u=k+1}^{K} \xi_u \left(|a_1|^2 + |a_3|^2\, \xi_u^2\right) + |a_3|^2\, \xi_k^3\right] + 1}, \quad (6)$$

where $\bar{\gamma}_k = \frac{P_s}{\sigma_k^2}$ is the average SNR at the k^{th} user. Hence and according to [13], the achievable data rate at U_k to detect U_j's signal ($j \leq k$) is

$$R_{j\to k} = \frac{1}{2}\log\left(1 + \gamma_{j\to k}^{hpa}\right), \quad (7)$$

where $\gamma_{j\to k}^{hpa}$ is the SINR of U_k's message decoded by U_j, given by

$$\gamma_{j\to k}^{hpa} = \frac{\bar{\gamma}_k |a_1|^2 |h_k|^2 \xi_j}{\bar{\gamma}_k |h_k|^2 (\alpha + \beta) + 1}, \quad (8)$$

where α and β are expressed as

$$\alpha = \sum_{u=j+1}^{K} \xi_u \left(|a_1|^2 + |a_3|^2\, \xi_u^2\right), \quad (9)$$

and

$$\beta = |a_3|^2\, \xi_k^3. \quad (10)$$

The K^{th} user need to decode all the other users data signals, in this case, the SNR for the K^{th} user to decode its own signal in presence of nonlinearities effects can be written as

$$\gamma_K^{hpa} = \frac{\bar{\gamma}_K |a_1|^2 |h_K|^2 \xi_j}{\gamma_K |h_K|^2\, |a_3|^2\, \xi_K^3 + 1}. \quad (11)$$

III. OUTAGE PROBABILITY OF NOMA

A. EXACT OP

In this section, we evaluate the performance of the considered system in term of OP. We consider the case that a preset target rate $\tilde{R}_k$ is determined by the user's quality of service (QoS) requirements [4]. In this case, it is interesting to examine the probability of the event that the user can cancel others user's data, i.e. $\tilde{R}_j \leq R_{j\to k} \leq \tilde{R}_k, j < k$. We define the OP at U_k as

$$P_k^o = 1 - \Pr\left(\gamma_{j\to k}^{hpa} > \theta_j, \forall j \in \{1 \leq j \leq k\}\right), \quad (12)$$

where $\theta_j = 2^{\tilde{R}_j} - 1$. The above probability is given by

$$\Pr\left(\gamma_{j\to k}^{hpa} > \theta_j, \forall j \in \{1 \leq j \leq k\}\right) \underset{\delta}{=} \Pr\left(|h_k|^2 > \frac{\theta_j}{\bar{\gamma}_k\left[|a_1|^2\xi_j - \theta_j(\alpha+\beta)\right]}\right), \quad (13)$$

where the step (δ) is the condition to obtain (13). i.e.

$$|a_1|^2\xi_j > \theta_j(\alpha + \beta). \quad (14)$$

We define $\phi_j = \frac{\theta_j}{\bar{\gamma}_k\left[|a_1|^2\xi_j - \theta_j(\alpha+\beta)\right]}$ for $j < K$, and $\phi_j^* = \max(\phi_1, \phi_2, \cdots, \phi_k)$. Consequently, the OP is

$$P_k^o = \Pr\left(|h_k|^2 < \phi_j^*\right). \quad (15)$$

When the condition (δ) is satisfied and thanks to the order statistics and the binomial theorem [20], the OP is [4]

$$P_k^o = \int_0^{\phi_j^*} K! f_{|\tilde{h}_k|^2}(x) \frac{\left(F_{|\tilde{h}_k|^2}(x)\right)^{k-1}\left(1 - F_{|\tilde{h}_k|^2}(x)\right)^{K-k}}{(k-1)!\,(K-k)!} dx, \quad (16)$$

where $f_X(.)$ and $F_X(.)$ are the PDF and the CDF of the unordered channel gain $|\tilde{h}_k|^2$, respectively, given by

$$f_X(x) = \frac{m^m x^{m-1}}{(\bar{\gamma}_k\Omega)^m\Gamma(m)} e^{-x\frac{m}{\bar{\gamma}_k\Omega}}, \quad (17)$$

and

$$F_X(x) = 1 - e^{-x\frac{m}{\bar{\gamma}_k\Omega}} \sum_{l=0}^{m-1} \frac{1}{l!}\left(\frac{mx}{\bar{\gamma}_k\Omega}\right)^l, \quad (18)$$

where $\Gamma(.)$ is the Gamma function.

Theorem 1: The exact OP of the k^{th} user with HPA distortions under Nakagami-m fading channels is given by

$$P_k^o = Q_k \sum_{i=0}^{K-k}\sum_{\lambda=0}^{i+K-k} \binom{i+k-1}{\lambda}\binom{K-k}{i}(-1)^{i+\lambda} \times \sum_{\eta=0}^{\lambda(m-1)} (1-\lambda)^{-(\eta+m)}\, a_{\lambda,\eta}\, \bar{\Gamma}\left(\eta+m, \frac{m}{\bar{\gamma}_k\Omega}\phi_j^*\right), \quad (19)$$

where $Q_k = \frac{K!}{(k-1)!(K-k)!\Gamma(m)}$ and $\bar{\Gamma}(.)$ is the normalized complementary incomplete Gamma function. $a_{\lambda,\eta}$ are the coefficients of x^l in the expansion $\left[\sum_{i=0}^{m} \frac{1}{i!}x^i\right]^\lambda$ defined as

$$a_{\lambda,\eta} = \begin{cases} 1, & \eta = 0 \\ \lambda, & \eta = 1 \\ [(m-1)!]^\lambda, & \eta = \lambda(m-1) \\ \sum_{n=1}^{\min(\eta,m-1)} \frac{n(\lambda+1)-\eta}{ln!} a_{\lambda,\eta-n}, & 2<\eta<\lambda(m-1). \end{cases} \quad (20)$$

Proof: Please see Appendix A for the proof.

It is worthy to point out that the OP is conditioned on $\tilde{R}_j \leq R_{j\to k} \leq \tilde{R}_k$. When such condition is not verified, the HPA will made the network in full outage state, i.e. $P_k^o = 1$. Furthermore, compared to the linear case studied in [4], NOMA system with nonlinear imperfections is more sensitive to the choice of HPA polynomial coefficients. From (δ) condition, we can see that HPA coefficients are not chosen arbitrarily. In particular, according to (16), we can easily obtain the OP in linear case by substituting $a_1 = 1, a_3 = 0$.

B. ASYMPTOTIC OP

In this Section, we consider that the OP performance at high SNR regime. When $\bar{\gamma}_k \to \infty$, we have $\phi_j^* \to 0$. Therefore a high SNR approximation for $F_X^\infty(\phi_j^*)$ is given by

$$F_X^\infty(\phi_j^*) \simeq \left(\frac{\phi_j^* m}{\Omega}\right)^m \left(\frac{1}{m!}\right). \quad (21)$$

Thus, the asymptotic OP can be expressed as

$$P_k^{out-hpa,\infty} = \frac{K!}{k!\,(K-k)!}\left(\frac{\phi_j^* m}{\Omega}\right)^{mk}\left(\frac{1}{m!}\right)^k o\left[\bar{\gamma}_k^{-mk}\right], \quad (22)$$

where $o(.)$ is the higher order terms. According to (22), the diversity order achieved at U_k is mk. It was shown in [20], that the outage diversity order for NOMA without HPA is mk. Hence, we conclude that the diversity order is not effected by the HPA nonlinearity degradation.

IV. ERGODIC SUM RATE

In this study, we consider that the target SINR of the mobile users are determined opportunistically by the users' channel condition, $\tilde{R}_k = R_k$. Therefore, the condition $R_{j\to k} \geq \tilde{R}_j$ is always verified since $\left(\| h_1 \|^2 \leq \| h_2 \|^2 \leq \cdots \leq \| h_K \|^2\right)$. In this case, the sum rate is given by

$$R_{sum}^{hpa} = \sum_{k=1}^{K-1} \frac{1}{2}\log_2\left(1+\gamma_k^{hpa}\right) + \frac{1}{2}\log_2\left(1+\bar{\gamma}_K |a_1|^2 |h_K|^2 \xi_K\right). \quad (23)$$

Alternatively, the ergodic sum capacity can be written as

$$R_{ave}^{hpa} = \int_0^\infty \frac{1}{2}\log_2\left(1+\frac{\bar{\gamma}_K x |a_1|^2 \xi_j}{\gamma_K x\,|a_3|^2\,\xi_K^3+1}\right) f_{|h_K|^2}(x)dx + \sum_{k=1}^{K-1}\int_0^\infty \frac{1}{2}\log_2\left(1+\frac{\bar{\gamma}_k x |a_1|^2 \xi_k}{\bar{\gamma}_k x(\alpha+\beta)+1}\right) \times f_{|h_k|^2}(x)dx, \quad (24)$$

where $f_{|h_k|^2}(x)$ is the PDF of the ordered variable $|h_k|^2$. Unfortunately, it is not easily to get an exact expression for the ergodic sum rate due to the high complexity of integrals. Hence, we will focus on the lower and upper bounds for R_{ave}^{hpa}. In the high SNR regime, R_{ave}^{hpa} can be expressed as

$$R_{ave}^{hpa} = \sum_{k=1}^{K-1}\frac{1}{2}\log_2\left(1+\frac{|a_1|^2\xi_k}{\alpha+\beta}\right) + \underbrace{\int_0^\infty \frac{1}{2}\log_2\left(1+\frac{\bar{\gamma}_K x |a_1|^2 \xi_K}{\gamma_K x\,|a_3|^2\,\xi_K^3+1}\right) f_{|h_K|^2}(x)dx}_{\Lambda} \quad (25)$$

A tight approximation to $\log_2\left(1+\frac{\bar{\gamma}_K |a_1|^2 x \xi_K}{\gamma_K x |a_3|^2 \xi_K^3+1}\right)$ is given by [13, eq.22]

$$\log_2\left(1+\frac{\bar{\gamma}_K x |a_1|^2 \xi_K}{\gamma_K x\,|a_3|^2\,\xi_K^3+1}\right) \triangleq \log_2\left(1+\bar{\gamma}_K x \alpha_K\right) - \log_2\left(1+\bar{\gamma}_K x \beta_K\right), \quad (26)$$

where $\alpha_K = \xi_K\left(|a_1|^2+|a_3|^2\xi_K^2\right)$ and $\beta_K = a_3^2\xi_K^3$. Therefore, the upper and lower bounds for Λ can be expressed as

$$\Lambda_{UB} = \frac{\bar{\gamma}_K \alpha_K}{2\ln(2)}\int_0^\infty \frac{1-F_{|h_K|^2}(x)}{1+x\bar{\gamma}_K\alpha_K}dx - \frac{\bar{\gamma}_K \beta_K}{2\ln(2)}\int_0^\infty \frac{1-F_{|h_K|^2}(x)}{1+x\bar{\gamma}_K\beta_K}dx \quad (27)$$

and

$$\Lambda_{LB} = \frac{\bar{\gamma}_K \alpha_K/2}{2\ln(2)}\int_0^\infty \frac{1-F_{|h_K|^2}(x)}{1+x\bar{\gamma}_K\alpha_K/2}dx - \frac{\bar{\gamma}_K \beta_K/2}{2\ln(2)}\int_0^\infty \frac{1-F_{|h_K|^2}(x)}{1+x\bar{\gamma}_K\beta_K/2}dx \quad (28)$$

Theorem 2: The lower and upper bounds for R_{ave}^{hpa} in presence of HPA nonlinear distortions can be derived as

$$R_{ave,UB}^{hpa} = \frac{\bar{\gamma}_K}{2\ln(2)}\sum_{\lambda=0}^{K}\binom{K}{\lambda}(-1)^\lambda e^{\frac{\lambda m}{\bar{\gamma}_k\Omega}}\sum_{\eta=0}^{\lambda(m-1)}\left(\frac{m}{\bar{\gamma}_k\Omega}\right)^\eta a_{\lambda,\eta} \times \left(\alpha_K\Psi\left(\bar{\gamma}_K\alpha_K\right)-\beta_K\Psi\left(\bar{\gamma}_K\beta_K\right)\right) + \frac{1}{2}\sum_{k=1}^{K-1}\log\left(1+\frac{|a_1|^2\xi_k}{\alpha+\beta}\right), \quad (29)$$

and

$$R_{ave,LB}^{hpa} = \frac{\bar{\gamma}_K}{2\ln(2)}\sum_{\lambda=0}^{K}\binom{K}{\lambda}(-1)^\lambda e^{\frac{\lambda m}{\bar{\gamma}_k\Omega}}\sum_{\eta=0}^{\lambda(m-1)}\left(\frac{m}{\bar{\gamma}_k\Omega}\right)^\eta a_{\lambda,\eta} \times \left(\frac{\alpha_K}{2}\Psi\left(\bar{\gamma}_K\alpha_K/2\right)-\frac{\beta_K}{2}\Psi\left(\bar{\gamma}_K\beta_K/2\right)\right) + \frac{1}{2}\sum_{k=1}^{K-1}\log\left(1+\frac{|a_1|^2\xi_k}{\alpha+\beta}\right), \quad (30)$$

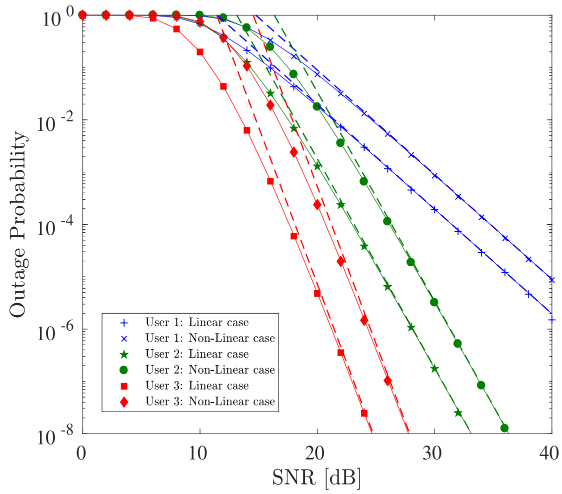

FIGURE 1. OP for $K = 3$, $\xi_1 = 1/2$, $\xi_2 = 1/3$, $\xi_3 = 1/6$, $a_1 = 1$, $a_2 = 0.15$, $\tilde{R}_1 = 0.8$ BPCU, $\tilde{R}_2 = 1.2$ BPCU, $\tilde{R}_3 = 1.9$ BPCU, $m = 2$.

where

$$\Psi(\tau) = \sum_{\varsigma=0}^{\eta} \frac{\eta!}{(\eta-\varsigma)!} \left(\frac{(-1)^{\eta-\varsigma-1}}{(\tau)^{\eta-\varsigma}} e^{-(\tau)} \mathrm{Ei}\left(-\frac{1}{\tau}\right) + \sum_{\kappa=0}^{\eta-\varsigma} \frac{(\kappa-1)!}{-(\tau)^{\eta-\varsigma-\kappa}} \right) \quad (31)$$

Proof: Please see Appendix B for the proof.

V. NUMERICAL RESULTS

In this section, we present some numerical results to demonstrate the performance of the downlink NOMA-HPA in terms of OP and ergodic sum rate under Nakagami-m fading channels. Here, the power allocation coefficients are $\xi_k = \frac{K-k+1}{\epsilon}$, where ϵ is to ensure $\sum_{k=1}^{K} \xi_k = 1$. Moreover, we consider the case of three users by setting $K = 3$. The values for the parameters of the nonlinear polynomial model are assumed to be: $a_1 = 1$ and $a_3 = 0.15$. In all the plots, the marker lines are obtained from Monte Carlo simulations of linear (without HPA) and nonlinear (with HPA) NOMA systems, respectively. The solid lines correspond to the theoretical close-form expressions. The dashed lines are obtained by the asymptotic expression.

Fig. 1, depicts the OP with or without HPA where $\tilde{R}_k$ is the upper targeted rate for the conventional scheme (bit per channel use (BPCU)). We can observe that exact analytical and simulation results match well over the entire SINR range, and the asymptotic results are very tight at high SINR. The nonlinear effects of HPA degrades the user's OP. In fact, the OP is only sightly degraded by the HPA at the low SINR, but the performance loss induced by HPA increases substantially as the SINR gets higher. Moreover, as the level user k increase, the performance loss compared to the linear case, increases. Fig. 2 illustrates the impact of target rate R_k on the OP for ideal and non-ideal condition. It can be seen that the diversity is reached with or without HPA when $R_1 = 0.5$, $R_2 = 1$ and $R_3 = 1.7$. However, when $R_1 = 0.7$, $R_2 = 2$

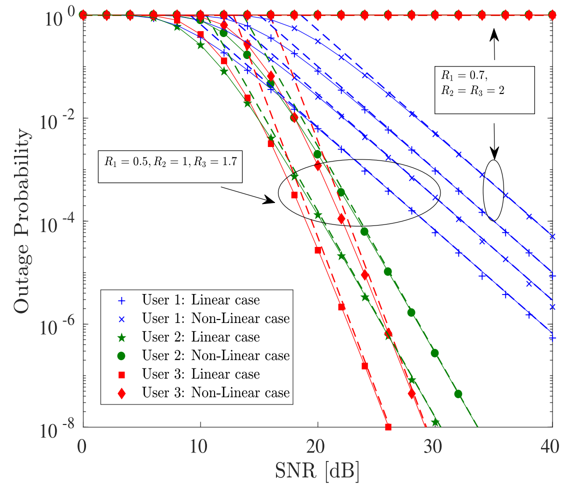

FIGURE 2. OP diversity for $K = 3$, $m = 2$, $\tilde{R}_1 = 0.8$ BPCU, $\tilde{R}_2 = 1.2$ BPCU, $\tilde{R}_3 = 1.9$ BPCU.

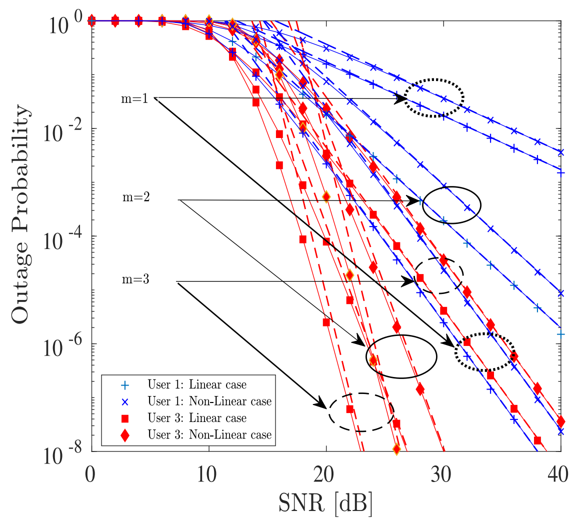

FIGURE 3. OP for different users $K = 3$, $\tilde{R}_1 = 0.8$ BPCU, $\tilde{R}_2 = 1.2$ BPCU, $\tilde{R}_3 = 1.9$ BPCU and different fading parameters $m = \{1, 2, 3\}$.

and $R_3 = 2$, we observe that the system performance for user 2 and user 3 are in full outage in linear and nonlinear NOMA cases, due to the fact that $\tilde{R}_2 < R_2$ and $\tilde{R}_3 < R_3$. Consequently, it is impossible for user 2 and user 3 to detect there own message. Moreover, the degradation of the OP performance in both linear and nonlinear cases, is inversely proportional with the target rate. This interpretation can be justified for user 1 when $R_1 = 0.5$ and $R_1 = 0.7$. Moreover, the performance loss by the HPA distortion increases as the target rate increases.

In Fig. 3, we plot the OP for different fading parameters m. Firstly we can observe that exact analytical results and simulations match well. It can be seen that the best behavior is achieved when $m = 3$. This fact can be explained from the diversity order. However, it can be observed that, increasing diversity order, the gap between the ideal and non-ideal case has become significant.Indeed, from this plot, we can see

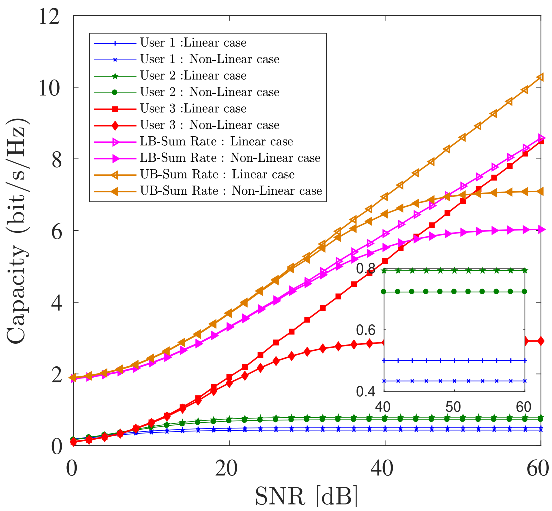

FIGURE 4. Capacity rate for $K = 3$, $\xi_1 = 1/2$, $\xi_2 = 1/3$, $\xi_3 = 1/6$, $m = 2$.

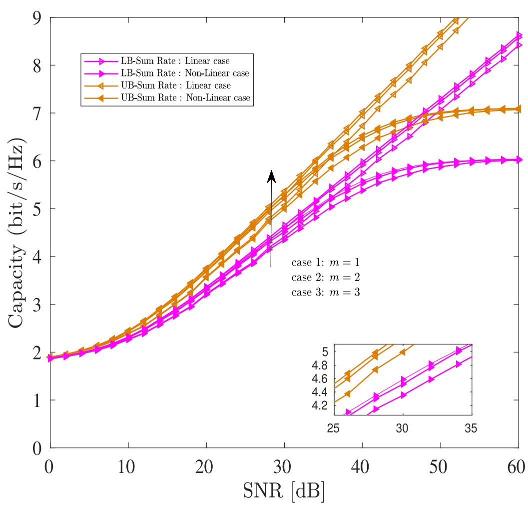

FIGURE 5. Ergodic sum rate for $K = 3$, $\xi_1 = 1/2$, $\xi_2 = 1/3$, $\xi_3 = 1/6$ and different fading parameters $m = \{1, 2, 3\}$.

that the gap in term of OP, when $m = 3$, is more greater compared to the special case of Rayleigh fading, i.e., $m = 1$. Consequently, we can deduce that the HPA is very sensitive to the channel parameters.

In Fig. 4, we present the impact of nonlinear effects on the ergodic rates for different users versus system SINR. Firstly, we can confirm that the analytic results of the ergodic rates for all users matches very well with the simulation results in linear and nonlinear case respectively. Furthermore, the figure shows a perfect agreement between numerical analysis and the Monte-Carlo simulations of the upper and lower bounds for the ergodic sum rate in ideal and non-ideal cases, respectively. From this figure, we can observe that user 1 and user 2 are sightly degraded by the HPA effects. Moreover, the achievable ergodic rate for these users are constants in medium-and high-SNR for linear and nonlinear cases. However user 3 is widely affected by HPA nonlinearity especially at high-SNR compared to the linear case. Fig. 4 also shows the lower and upper bound on the system capacity obtained according to (29) and (30). We can deduce from this figure that HPA nonlinearities have an undesirable impact at high SINR. Precisely, both lower and upper capacity behaviors saturate and approach to a deterministic value limiting its performance.

In Fig. 5, we evaluate the ergodic sum rate of NOMA-HPA under different channel configurations. From the figure, it can be shown that the ergodic sum rate of ideal-hardware system monotonically increases as the fading parameter m increases. Similar interpretation are also observed for the ergodic sum rate of the nonlinear system specially in the low-to-medium SNR regions. However, the advantage offered to the ergodic sum rate by increasing the factor m is limited at the high SNR due to HPA distortions.

VI. CONCLUSION

In this paper, we have studied the impact of nonlinear HPA on the performance of downlink NOMA system. The main difference between our approach and the previous works is that the HPA polynomial model has been adopted to highlight the effects of nonlinearities distortions. We provided new closed form expressions for the OP of the ordered users over Nakagami-m fading channels. Based on the theoretical results, the diversity orders achieved by the users were been obtained. A tractable upper and lower bounds for the ergodic sum rate of all users were derived. Our analytical manifested that the OP performance of downlink NOMA system is notably affected by the HPA distortions, particularly when high achievable rates are required. Then, obtained results show that the scale parameter of the Nakagami-m distribution degraded the OP. The capacity behavior characterize the impact of HPA nonlinearities and demonstrate the existence of a deterministic value that cannot be crossed by increasing the signal powers. It depend only on the HPA parameters.

APPENDIX A
PROOF OF THEOREM 1

In order to proceed, P_k^o can be rewritten as

$$P_k^o = Q_k \sum_{i=0}^{K-k} \binom{K-k}{i} \int_0^{\phi_j^*} [F_X(x)]^{i+K-k} f_X(x)dx. \quad (32)$$

where $Q_k = \frac{K!}{(k-1)!(K-k)!}$. Applying the binomial expansion in [21, eq. (1.110-pp:25)] to $[F_X(x)]^{i+K-k}$ yields

$$\left[1 - e^{-x\frac{m}{\bar{\gamma}_k \Omega}} \sum_{l=0}^{m-1} \frac{1}{l!} \left(\frac{mx}{\bar{\gamma}_k \Omega}\right)\right]^i = \sum_{\lambda=0}^{i+K-k} \binom{i+K-k}{\lambda} e^{-\lambda x \frac{m}{\bar{\gamma}_k \Omega}} \left[\sum_{l=0}^{m-1} \frac{1}{l!} \left(\frac{mx}{\bar{\gamma}_k \Omega}\right)^l\right]^\lambda. \quad (33)$$

Then, we expand (33) according [22] to as follows

$$\left[\sum_{l=0}^{m-1}\frac{1}{l!}\left(\frac{mx}{\bar{\gamma}_k\Omega}\right)\right]^{\lambda}=\sum_{\eta=0}^{\lambda(m-1)}\left(\frac{mx}{\bar{\gamma}_k\Omega}\right)^{\eta}a_{\lambda,\eta}. \quad (34)$$

By applying (33) and (34) to P_k^o in (32), we obtain

$$P_k^o=\frac{Q_m m^m}{(\bar{\gamma}_k\Omega)^m}\left(\sum_{i=0}^{K-k}\sum_{\lambda=0}^{i+K-k}\binom{i+k-1}{\lambda}\binom{K-k}{i}(-1)^{i+\lambda}\right.$$
$$\left.\times\sum_{\eta=0}^{\lambda(m-1)}\left(\frac{m}{\bar{\gamma}_k\Omega}\right)^{\eta}a_{\lambda,\eta}\right)\int_0^{\phi_j^*}x^{\eta}x^{m-1}e^{-x\frac{(1+\lambda)m}{\bar{\gamma}_k\Omega}}dx. \quad (35)$$

So that, we apply the following identity [21, eq. (3.381.1)] to resolve the integral in equation (35)

$$\int_0^u\Psi^{\nu-1}e^{-\mu\Psi}d\Psi=\mu^{\nu}\bar{\Gamma}(\nu,\mu u), \quad (36)$$

where $\bar{\Gamma}(.)$ denotes the normalized complementary incomplete Gamma function. Therefore, we obtain the expression of (19).

APPENDIX B
PROOF OF THEOREM 2

By applying the largest order statistics and using the help of Lemma 1, $F_{|h_K|^2}(x)$ can be rewritten as

$$F_{|h_K|^2}(x)=\left[F_{|\tilde{h}_K|^2}(x)\right]^K$$
$$=\sum_{\lambda=0}^{K}\binom{K}{\lambda}(-1)^{\lambda}\sum_{\eta=0}^{\lambda(m-1)}\left(\frac{m}{\bar{\gamma}_K\Omega}\right)^{\eta}a_{\lambda,\eta}x^{\eta}e^{-x\frac{\lambda m}{\bar{\gamma}_K\Omega}}. \quad (37)$$

Substituting equations (37) in the integrals Λ_{UP}, we obtain

$$\Lambda_{UB}=\frac{\bar{\gamma}_K}{2\ln(2)}\sum_{\lambda=0}^{K}\binom{K}{\lambda}(-1)^{\lambda}e^{\frac{\lambda m}{\bar{\gamma}_K\Omega}}\sum_{\eta=0}^{\lambda(m-1)}\left(\frac{m}{\bar{\gamma}_K\Omega}\right)^{\eta}a_{\lambda,\eta}$$
$$\left(\int_0^{\infty}\alpha_K\log\left(1+x\bar{\gamma}_K\alpha_K\right)x^{\eta}e^{-x}dx\right.$$
$$\left.-\int_0^{\infty}\beta_K\log\left(1+x\bar{\gamma}_K\beta_K\right)x^{\eta}e^{-x}dx\right). \quad (38)$$

With the aid of [21, eq. (4.222.8)], the integral $\Psi(\tau)=\int_0^{\infty}\log\left(1+x\tau\right)\tau^{\eta}e^{-x}dx$ can be calculated as in equation (31). Finally, using some algebraic manipulations, the upper bounds for the ergodic sum rate can be obtained as in (29). Noting, the lower bound for the ergodic sum rate capacity can be obtained by substituting α_K and β_K by $\alpha_K/2$ and $\beta_K/2$, respectively.

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