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Essays on Forestry Economics

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Contents

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Abstract

In this dissertation, optimal forest management and land use allocation are analyzed when biodiversity and carbon sequestration benefits are introduced into the forest agent problem. In particular, forest carbon sequestration is studied under two different scenarios: the first considers the problem of the small private owner, while the second focuses on the management of a public forest, when timber and land prices are endogenously determined.

In the first paper, based on a multiple rotation model *à la* Faustmann in which optimal land use conversion time is endogenous, we discuss the implementation of the optimal solution from the small private owner's perspective . Given the important role of the "permanence" issue in the context of the United Nations Framework Convention on Climate Change, two different accounting methods (the Carbon Flow and the Ton-Year Crediting) with constant and rising carbon prices are analyzed. It is shown that the use of different carbon methods strongly impacts optimal rotations and forest profitability, implying that short and long run timber supplies are also affected by the carbon accounting method choice. Moreover, the consideration of carbon stored in long-lived wood products affects the optimal land use conversion time when carbon prices are increasing. An application to the portuguese Eucalyptus forest confirms these results. In particular, as immediate land use conversion is optimal for most cases considered, the idea that forests may provide the economic incentives needed to change land-use decisions, buying time for the development and deployment of low carbon-based technological innovations, is reinforced.

The second paper adresses the question of optimal timber management when carbon benefits are introduced into a framework where both the price of timber and the price of land are endogenously determined. Building upon the multi vintage forest model developed by Salo and Tahvonen, the paper analyzes the problem of carbon sequestration under a forest sector scope. To compare forest carbon sequestration with avoided emissions, three different carbon accounting methods are considered: the carbon flow regime, the ton-year crediting and the average storage, where the carbon flow is the first-best solution. We compare the results obtained in each case with those without carbon sequestration, as well as the performances of the ton-year and the average storage with respect to the first-best solution on optimal land allocation between forestry and alternative uses, total carbon sequestered, timber production and social welfare, for different values of the most relevant parameters. In general, internalizing carbon sequestration benefits increases the optimal amount of land allocated to forest, and has implications to the optimal forest management. The induced impact in the timber market during the transition period depends upon the carbon accounting method generating interesting insights from the perspective of the implementation of the first-best solution. A full proof of long-run optimality of steady state forest is provided. The theoretical results are discussed based on numerical simulations that illustrate the setup's potential.

The recent recognition of the existence of possible conflicts between carbon sequestration policies and biodiversity has once more put biodiversitiy in the centre of the forestry literature debate While a complete assessment of the interactions between carbon sequestration policies and biodiversity conservation is still needed, there are previous questions in the biodivesity literature that remains to be addressed, namely, in what concerns the forest sector scope. To this end, in the third essay, biodiversity considerations are introduced into a multiple species, multi-vintage forest sector model with endogenously determined timber prices and land use allocation. Following recent ecological literature, biodiversity is modeled focusing on structural diversity, i.e, age classes and species distribution. We show that transition dynamics are strongly affected when biodiversity is introduced, contaminating both timber and land markets. Moreover, different ecological forest structures have distinct impacts on optimal land use distribution, therefore, affecting also timber prices. Finally, we observe major changes in optimal timber management. In fact, even after a long period of adjustment, optimal deviations from Faustmann's rotation combined with changes in land use allocation still occur.

The fourth essay extends the multi vintage forest model developed in the second by introducing net carbon sequestration benefits in a multiple species context. Based on the carbon flow accounting method, a full proof of long run optimality of steady state forest is provided. Based on sensitivity analysis with respect to the speed of growth, the carbon conversion factor and the amount of carbon that is stored in long-lived wood products among species, we conclude that they impact significantly on the optimal allocation of land to forest. In particular, when the fast growing species is also the one for which a lower fraction of wood is used in long-lived products, it may be optimal to allocate to the slow growing species a larger amount of land when compared to the case without carbon. Numerical simulations are performed, illustrating and confirming the results obtained.

Chapter 1

The Impact of Carbon Accounting on Optimal Forest Rotation

1.1 Introduction

The importance of the climate change debate in the current political agenda has put forestry in the centre of the environmental economics literature. In fact, Righelato and Spracklen [15] have recently shown that emissions avoided by the use of liquid biofuels over a 30-year period are much smaller (two to nine times) than the amount of carbon sequestered by forestation of an equivalent area of land. Moreover, several applied studies have examined the potential impact of forest carbon sink programs by estimating their cost-effectiveness and carbon sequestration capacity in a variety of settings, as Richards, Rosenthal, Edmonds and Wise [14], Sohngen and Mendelsohn [16] and more recently Tavoni, Bosetti and Sohngen [17], among others. Although the level of impact varies significantly depending upon the model used, in all cases introducing forests as carbon sinks reduce costs relative to policies that only consider fossil fuel emissions. Forestry seems, therefore, to be both an ecological and economically viable instrument to help mitigate climate change.[1]

However, in spite of the role that forests play in the global carbon cycle and the recent recognition of its potential contribution in an overall portfolio of greenhouse-gas mitigation strategies, the use of forest carbon sinks remains controversial in the context of the United Nations Framework Convention on Climate Change (UNFCCC). While many different reasons may lay behind this, the issue of ”permanence” in part explains the origin of the controversy. In fact, while managed forests assimilate carbon from the atmosphere, carbon is also gradually

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[1] According to the Third Assessment Report on the Intergovernmental Panel on Climate Change (IPCC), up to 20% of excessive emissions can be captured in forests and biological sinks over the next 50 years.

released after harvesting.

Therefore, "permanence" raises an important question about how to incorporate the services provided by carbon sequestration when modelling forest management: while forests can generate carbon offsets that may be used to compensate for GHG emissions, the net effect of sequestration has to be identical to that of avoided emissions. Hence, when carbon benefits are taken into account, not only the forested area is relevant, but also the amount of carbon released when the forest is harvested.

In the related literature, different accounting methods have been considered: the carbon flow regime, the lump-sum regime, and the carbon stock regime, among others. In what follows, we discuss the implementation of the optimal solution from the private owner's perspective based on a multiple rotation model *à la* Faustmann, focusing on the carbon flow regime and the ton-year crediting. Note that by considering these two accounting methods, we consider the main features addressed by the carbon accounting literature. The first method is essentially the Pigouvian tax/subsidy on the carbon externality, while the second represents the carbon accounting methods in which payments are made on the stock and redemption of carbon credits upon harvest is not required.[2] While we aim to analyze changes in the optimal timber management in the context of the small private forest owner,we consider that the price of timber and the price of land are exogenously determined.[3]

Most of the available studies addressing the question of carbon sequestration assume a constant carbon price. This hypothesis seems, however, to be rather inappropriate, as previous studies (Cline [3], [4], Maddison [10], Nordhaus [12], Peck and Teisberg [13], Sohngen and Mendelsohn [16]) that have computed economically efficient policies to mitigate climate change have shown that marginal damages of carbon should increase over time at rates varying from 1.5% to 4%. This issue is particularly relevant since it has important implications to the costs of sequestering carbon in forests, introducing new elements into the analysis. In fact, not only the profitability of forested land changes, but also the incentives to convert land from agriculture to forest is also affected, implying that the optimal timing of land conversion is also a decision variable of the landowner. Therefore, the optimal supply of carbon sequestration may shift relative to the case when carbon prices are assumed to be constant.

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[2] See Feng, Zhao and Kling [8] and Sohngen and Mendelsohn [16] for similar payment schemes.

[3] For the case with endogenous timber and land price see Costa Duarte, Cunha-e-Sá e Rosa [5].

When carbon prices increase over time, it is important to distinguish between the cases of land already forested (forested land) from those in which a potential incentive to convert agricultural land into forest may exist (agricultural land vs forest)).[4] In this last case, even if forestry becomes a more attractive option when carbon services are paid, it may be optimal not to convert land immediately.[5] This may be explained by the stylized fact reported in the literature according to which, following the pattern of trees' growth, the rate of carbon storage typically increases in young stands, and declines as the stand ages. Thus, an incentive is created to delay conversion as higher discounted carbon payments can be generated. However, as we will show later in the paper, the forest management policy (permanent versus rotative), as well as the chosen carbon accounting method are not without consequences in this context. These results are related to those derived in Velt and Plantinga [20], where the effect of rising carbon prices on the optimal portfolio of greenhouse-gas mitigation strategies is examined. In particular, carbon sequestration projects, where conversion of agricultural land into forest is considered, are compared to carbon abatement projects.

In this paper, we examine the problem of a small private forest manager under two different accounting methods with constant and rising carbon prices, in which optimal land use conversion time is endogenous. It is, therefore, our purpose to address the impacts on timber management's incentives resulting from the use of different carbon accounting schemes, allowing not only for changes in the optimal rotation period but also in the optimal timing of land use conversion. The impact of internalizing social benefits from carbon sequestration both on the optimal rotation age and land values is estimated for the Portuguese eucalyptus forest.

In general, we conclude that increasing carbon prices may not be enough to delay land conversion to forest. In fact, for very low opportunity costs of alternative use of land, it may be optimal to convert immediately. This result highlights the crucial role that forests may play in the context of a more global policy to combat climate change, giving rise to rapid reductions in CO_2 emissions, and, therefore, buying time for the development and deployment of low carbon based technological innovations.

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[4] Because more carbon is typically stored in forests than in lands used for agriculture, the conversion of agricultural land to forest achieves a net reduction in atmospheric CO_2 concentrations. Thus, in what follows, we only consider the eventual conversion of agricultural land to forest. Besides, when land is already forested, it is assumed that the choice in favour of forest use was optimal.

[5] The same may occur even when carbon prices remain constant over time. The effect, however, may become negligible.

In the case of the carbon flow, the future use of timber, reflected in the amount of carbon released at harvest plays a very important role in determining optimal rotations. While in the ton-year crediting regime the results reflect the way carbon benefits are accounted for, namely, based on the timber stock rather than the flow. In both cases, a sensitivity analysis to the initial carbon price is undertaken.

The remainder of the paper is organized as follows. Section 2 presents the theoretical model under the different methods of carbon accounting. Section 3 discusses the estimated results for the Portuguese eucalyptus forest. Section 4 concludes the paper. The tables and technical derivations are presented in the appendices.

1.2 The Model

The two following subsections present the theoretical model under constant and increasing carbon prices. The cases of already forested land and eventual optimal conversion of agricultural land to forest are discussed. In each case, both accounting methods, the carbon flow regime and the ton-year crediting, are considered.

1.2.1 Carbon Flow

Constant Prices

According to the carbon flow regime, as developed in Van Kooten, Binkley and G. Delcourt [19], the carbon credit cash flows are a function of the annual change in the forest carbon stock. Credit payments reflect the flow of carbon between land and the atmosphere through the carbon cycle, so a net increase in the carbon stock over a year means that carbon has been removed from the atmosphere and the owner is paid credits for it. Similarly, a fall in the carbon stock suggests carbon has been released into the atmosphere, and the owner surrenders the associated credits.

However, the amount of carbon released when the forest is harvested depends upon the use given to the timber harvested. Different uses will have different impacts on carbon release after harvest. To take this fact into account, and when there is not enough information, Van Kooten, Binkley and G. Delcourt [19] have introduced a parameter, β, which measures the fraction of timber that is harvested but goes into long-term storage in structures and landfills. Alternatively, decay functions to capture different uses can be considered, as in Alavalapati, Stainback and Carter [1]. This is especially relevant in the case of the Portuguese forest, since its two main species (pine tree and eucalyptus) have very distinct uses. Pine timber is mainly used to long-term carbon storage structures, while eucalyptus is used to produce pulpwood, releasing a larger amount of carbon.[6]

Under this accounting method, carbon benefits are a function of both the change in biomass and the amount of carbon per cubic meter, m^3. Thus, what is relevant to consider in carbon sequestration benefits' modelling is the change in the carbon uptake.[7]

The present value of benefits from carbon sequestration over a rotation of length T can be represented as follows:

$$\int_0^T P_c \alpha_i v_i'(t) e^{-rt} dt \tag{1.1}$$

where $v_i(t)$ represents the timber volume at age t, $v_i'(t)$ is the instantaneous growth in timber volume in period t, α_i converts timber volume in cubit feet to metric tons of carbon,[8] P_c is the social value of carbon sequestered,[9] r is the discount rate, and, finally, the subscript i accounts for the species.

Following Van Kooten et al. [19], the present value of the external cost of the carbon released at T is

$$P_c \alpha_i (1 - \beta_i) v_i(T) e^{-rT} \tag{1.2}$$

where β represents the fraction of the timber harvested and used to long-term storage structures. Depending on the use of the timber harvested, β varies in the unit interval. If $\beta = 0$, then all carbon is released at harvest time, while if $\beta = 1$ there are no social costs of carbon release.

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[6] When recycling is considered, the life cycle for carbon stored in pulpwood may increase significantly, determining a larger β for eucalyptus.

[7] Other sources of carbon as litter, branches, tops, stump and roots is simply recycled into the next stand of trees, due to the lack of information.

[8] The proportion of carbon in biomass varies with tree species, although it is generally in the range of $200kgs$ / m^3.

[9] P_c is the present value, for all time, of removing one unit of carbon from the atmosphere today. It is determined as the discounted value of the annual contribution to damage caused by one unit of carbon added over the expected number of years that the unit of carbon is present in the atmosphere.

The net present value of total benefits from timber production and sequestered carbon over multiple rotations of length T, is given by:

$$NPV_0 = \frac{(P_i v_i(T) - c)e^{-rT}}{(1-e^{-rT})} - \int_0^\infty a e^{-rt} dt + \\ + \frac{\int_0^T P_c \alpha_i v_i'(t) e^{-rt} dt - P_c \alpha_i (1-\beta_i) v_i(T) e^{-rT}}{(1-e^{-rT})} \tag{1.3}$$

where c represents the (constant) cost of replanting at t, P_i the price of a cubic feet of timber, and a the annual opportunity cost of forested land.

Maximizing (1.3) with respect to T and assuming that the second order conditions hold for a maximum, we obtain the first-order condition from which the optimal rotation period $T = T^C$ can be derived,

$$G_T' = \frac{r}{1-e^{-rT}} \left[G(T) - P_c \alpha_i (1-\beta_i) v_i(T) + \int_0^T P_c \alpha_i v_i'(t) e^{-rt} dt \right] - \\ - P_c \alpha_i \beta_i v_i'(T) \tag{1.4}$$

where $G(t) = P_i v_i(t) - c$.

This same expression can be restated as:

$$\frac{G_T'}{G(T)} = \frac{r}{1-e^{-rT}} + \\ + \frac{\frac{r}{1-e^{-rT}} \left[\int_0^T P_c \alpha_i v_i'(t) e^{-rt} dt - P_c \alpha_i (1-\beta_i) v_i(T) \right] - P_c \alpha_i \beta_i v_i'(T)}{G(T)} \tag{1.5}$$

Equation (1.5) clarifies the role of carbon benefits and timber use in deciding when to harvest. In Appendix A we show that when $\beta = 0$ we unambiguously conclude that the optimal rotation period increases relative to Faustmann's. In contrast, when $\beta = 1$, the optimal rotation period decreases. Moreover, for $0 < \beta < 1$, the final result is indeterminate.

From a private owner's perspective, the optimal solution could be implemented by assuming that public agencies provide payments for net CO_2 assimilation and tax net CO_2 emissions. Thus, an annual subsidy is paid to the forester equal to the total value of the carbon sequestered that year, while a tax is levied at harvest time that equals the external cost of the carbon released to the atmosphere, given the parameters of the model, in particular, the social value of carbon, P_c.

Increasing Carbon Prices

In this section, we consider the more general case where carbon prices increase over time according to:

$$P_{ct} = P_{c0} e^{\mu t} \tag{1.6}$$

where P_{c0} stands for the initial carbon price level, and μ for the rate of carbon price growth. Also, it is assumed that the carbon price growth rate is lower than the discount rate, $\mu < r$.

Forested Land When land is already forested, the net present value of profits per hectare when carbon prices increase is given by:

$$\begin{aligned} NPV_0 = {} & \frac{(P_i v_i(T) - c)e^{-rT}}{1 - e^{-rT}} - \int_0^{\infty} a e^{-rt} dt + \\ & + \frac{\int_0^T P_{c0} \alpha_i v_i'(t) e^{(\mu - r)t} dt}{1 - e^{(\mu - r)T}} - \frac{P_{c0} \alpha_i (1 - \beta_i) v_i(T) e^{(\mu - r)T}}{1 - e^{(\mu - r)T}} \end{aligned} \tag{1.7}$$

In this case, we assume that forest is the land use that maximizes the present value of profits per hectare.

The optimal T is obtained from the following first-order condition:

$$G_T' = \frac{r}{1 - e^{-rT}} G(T) +$$

$$+ \frac{(1 - e^{-rT}) e^{\mu T}}{(1 - e^{(\mu - r)} T)^2} P_c \alpha_i \left[\begin{array}{c} -\beta_i v_i'(T)(1 - e^{(\mu - r)} T) - (\mu - r) \int_0^T v_i'(t) e^{(\mu - r)t} dt + \\ +(\mu - r)(1 - \beta_i) v_i(T)(1 - e^{(\mu - r)T}) \end{array} \right] \tag{1.8}$$

It is not possible to unambiguously determine the impact of increasing carbon prices on the optimal rotation period as well as on the value of a hectare of land. It is possible, however, to observe that the impact of the cost of carbon release (third term inside the square brackets) relative to the permanent benefit change of carbon uptake at harvest (first term inside the square brackets) is now reduced. Hence, as β affects both terms in opposite directions, the ultimate impact of the different uses given to timber on the optimal rotation period may change in face of rising carbon prices. In fact, β has two distinct effects on the optimal rotation period: larger $\beta's$ determine lower costs of carbon released at harvest, and, consequently, lower rotations, but also originate higher changes of permanent carbon uptake at harvest time, contributing to larger rotation periods.

Agricultural Land versus Forest By improving forested land profitability, increasing carbon prices may create incentives to convert agricultural land into forest. Yet, even if conversion from agriculture to forest is optimal, it may be in the best interest of the forester to delay it. As trees sequester carbon at higher rates when they are younger, an incentive to delay conversion is created as higher future carbon payments are, thus, generated. This suggests that biology may play an important role in this context.

The net present value of forest investment per hectare is given by:

$$\begin{aligned} NPV_0 = {} & \frac{(P_i v_i(T) - c)e^{-rT}}{(1-e^{-rT})} e^{-rY} - \int_Y^\infty a e^{-rt} dt + \\ & + \frac{\int_0^T P_{c0}\alpha_i v_i'(t) e^{(\mu-r)t} dt e^{(\mu-r)Y}}{1-e^{(\mu-r)T}} - \frac{P_{c0}\alpha_i(1-\beta_i)v_i(T)e^{(\mu-r)(T+Y)}}{1-e^{(\mu-r)T}} \end{aligned} \tag{1.9}$$

where Y stands for the time of conversion of agricultural land to forest.

By maximizing (1.9) with respect to Y we obtain the first-order condition from which the optimal timing of conversion can be derived, as follows

$$\begin{aligned} \frac{\partial NPV_0}{\partial Y} = {} & -r\left[\frac{(P_i v_i(T) - c)e^{-rT}}{1-e^{-rT}} - \frac{a}{r}\right] e^{-rY} + \\ & + (\mu - r)\frac{\int_0^T P_{c0}\alpha_i v_i'(t) e^{(\mu-r)t} dt e^{(\mu-r)Y}}{1-e^{(\mu-r)T}} - \\ & - \frac{(\mu-r)}{1-e^{(\mu-r)T}}\left[P_{c0}\alpha_i(1-\beta_i)v_i(T)e^{(\mu-r)(T+Y)}\right] \end{aligned} \tag{1.10}$$

Rewriting this condition we obtain

$$\begin{aligned} \frac{\partial NPV_0}{\partial Y} = {} & \\ & -r\left[\begin{array}{c} \frac{(P_i v_i(T)-c)e^{-rT}}{1-e^{-rT}} e^{-rY} + \frac{\int_0^T P_{c0}\alpha_i v_i'(t) e^{(\mu-r)t} dt e^{(\mu-r)Y}}{1-e^{(\mu-r)T}} + \\ -\frac{\left[P_{c0}\alpha_i(1-\beta_i)v_i(T)e^{(\mu-r)(T+Y)}\right]}{1-e^{(\mu-r)T}} - \frac{a}{r}e^{-rY} \end{array}\right] + \\ & + \mu\left[\frac{\int_0^T P_{c0}\alpha_i v_i'(t) e^{(\mu-r)t} dt e^{(\mu-r)Y}}{1-e^{(\mu-r)T}} - \frac{\left[P_{c0}\alpha_i(1-\beta_i)v_i(T)e^{(\mu-r)(T+Y)}\right]}{1-e^{(\mu-r)T}}\right] \end{aligned} \tag{1.11}$$

where T should be evaluated at the optimal rotation period.

By inspection of (1.10), we conclude that the negative terms create incentives to immediate conversion, while the positive ones play for delay. The first term of the right hand side of (1.10) is unambiguously positive, the second is negative for a carbon price growth rate lower than the discount rate and $v' > 0$, while the last one, reflecting the impact of costs of carbon release at harvesting time, is positive for $\mu < r$. Therefore, for larger β, reflecting lower costs of carbon release, conversion occurs earlier than for lower β.

Velt and Plantinga [20] show that, for a permanent forest ($T = \infty$), implying no costs of carbon release, later conversion can be optimal for increasing carbon prices. However, when $\mu = 0$, that is, with constant carbon prices, these authors show that immediate conversion is optimal.[10]

In the case of this paper, for a rotative forest with $T < \infty$, when $\mu = 0$, once conversion is optimal (i.e. when the first term of the right hand-side in equation (1.11) is positive), delaying is never advantageous. However if $\mu > 0$, we cannot unambiguously show as in Velt and Plantinga [20] that immediate conversion is optimal when carbon prices are constant. So, our results extend theirs to the case of a multiple rotation forest management problem.

1.2.2 Ton-Year Crediting

Constant Prices

This approach attempts to determine the storing time of carbon sequestered in biomass for which the carbon stored is equivalent to a certain amount of avoided emissions. The calculations for this time period (T^e-equivalence time) are based on the residence time and decay pattern of atmospheric CO_2, its Absolute Global Warming Potential (AGWP) (see Fearnside, Lashof and Moura-Costa [7], and Moura-Costa and Wilson [11]). It was found that keeping a megagram (Mg) of CO_2 out of the atmosphere for a full 100 years is equivalent to 55 Mgyear (or ton-year) equivalents, rather than the full 100 Mg-years if the CO_2 entering the atmosphere had no movement to the ocean or other sinks. The number obtained, in this case 55, is denoted by the equivalent time, T^e. In addition, assuming a linear relationship between the residence of CO_2 in the atmosphere and its radiative forcing effect, the effect of storing 1 ton of CO_2 in forest biomass for 1 year was derived. Following Moura-Costa and Wilson [11], according to that rule, when the equivalence time considered is 55 years, it was found that

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[10] See Appendix B.

storing one ton of carbon for one year is equivalent to preventing the effect of 0.0182 tonnes CO_2 of emissions, which is denoted by the equivalence factor (E_f).

Based on the equivalence factor, the ton-year method consists of crediting a project with a fraction of its total yearly GHG benefit. This fraction is determined by the amount of carbon stored each year, which is then converted, using the E_f, to its equivalent amount of preventing effect. Notice that this method does not require redemption of carbon credits upon harvest.

Following this approach, and assuming that payments will occur on a yearly basis, the net present value of forest investment per hectare is given by:

$$NPV_0 = \frac{(P_i v_i(T) - c)e^{-rT}}{1 - e^{-rT}} - \int_0^\infty a e^{-rt} dt + \frac{P_c \alpha_i \int_0^T E_f v(t) e^{-rt} dt}{1 - e^{-rT}} \tag{1.12}$$

Maximizing (1.12) with respect to T yields the following first order condition:

$$G'(T) = \frac{r}{1 - e^{-rT}} \left[G(T) + P_c \alpha_i \int_0^T E_f v(t) e^{-rt} dt \right] - P_c \alpha_i E_f v(T) \tag{1.13}$$

Rewriting this condition we get:

$$\frac{G'(T)}{G(T)} = \frac{r}{1 - e^{-rT}} + \frac{\frac{r}{1-e^{-rT}} \left[P_c \alpha_i \int_0^T E_f v(t) e^{-rt} dt \right] - P_c \alpha_i E_f v(T)}{G(T)} \tag{1.14}$$

Notice that expression (1.14) is identical to that derived by Englin and Klan [6] denoted by “externalities balance”. In this case, it represents the “carbon balance”. Since v´>0, and v´´<0, this term is unambiguously negative, implying that harvesting will be postponed.[11]

The main difference between this method and the previous one is that payments here are based on the stock and not on the stock growth. This difference will impact significantly on the final results. In fact, it is possible to prove that in the absence of timber benefits it will be optimal to never harvest. (see Appendix C).

Increasing Carbon Prices

Forested Land

In this case, and for already forested land, the net present value of forest investment per hectare with increasing carbon prices is given by:

$$NPV_0 = \frac{(P_i v_i(T) - c)e^{-rT}}{1 - e^{-rT}} - \int_0^\infty a e^{-rt} dt + \frac{P_{c0} \alpha_i \int_0^T E_f v(t) e^{(\mu - r)t} dt}{1 - e^{(\mu - r)T}} \tag{1.15}$$

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[11] See Aronsson and Lofgen [2].

Once more, it is assumed that forest is the optimal land use for this piece of land. The corresponding first-order condition is given by:

$$\begin{aligned} G'(T) = {} & \frac{r}{1-e^{-rT}}G(T) - \\ & - \frac{1-e^{-rT}}{(1-e^{(\mu-r)T})^2}\left[(\mu-r)e^{\mu T}P_{c0}\alpha_i\int_0^T E_f v(t)e^{(\mu-r)t}dt\right] - \\ & - \frac{1-e^{-rT}}{(1-e^{(\mu-r)T})}e^{\mu T}P_{c0}\alpha_i E_f v(T) \end{aligned} \tag{1.16}$$

This can be rewritten as

$$\frac{G'(T)}{G(T)} = \frac{r}{1-e^{-rT}} +$$

$$+\frac{\frac{(1-e^{-rT})e^{\mu T}}{(1-e^{(\mu-r)T})}\left[-\frac{(\mu-r)}{1-e^{(\mu-r)T}}P_{c0}\alpha_i\int_0^T E_f v(t)e^{(\mu-r)t}dt - P_{c0}\alpha_i E_f v(T)\right]}{G(T)} \tag{1.17}$$

In (1.17), the term inside the square brackets is again the “externalities balance” or ”carbon balance”, except that, in this case, it is as if the discount rate has been reduced to $r-\mu < r$. Therefore, the same result applies, that is, for an increasing and concave $v(t)$, the balance is negative, implying that it will be optimal to cut later, relatively to the Faustmann solution.

Notice that with increasing carbon prices it is as if the exogenous discount rate that is applied to carbon net benefits is lower than the one used to calculate the present value of timber profits ($\mu < r$). Hence, the forest owner has an additional incentive to postpone cutting, compared to the case of constant carbon prices.

Agricultural Land vs Forest When carbon prices increase over time, by changing the incentives of optimal conversion, forestry may become more profitable when comparing to the previous case. When the optimal timing of conversion is endogenous, the net present value of forest investment per hectare is given by:

$$\begin{aligned} NPV_0 = {} & \frac{(P_i v_i(T)-c)e^{-rT}}{1-e^{-rT}}e^{-rY} - \int_Y^\infty a^{-rt}dt + \\ & + \frac{P_{c0}\alpha_i\int_0^T E_f v(t)e^{(\mu-r)t}dte^{(\mu-r)Y}}{1-e^{(\mu-r)T}} \end{aligned} \tag{1.18}$$

where Y represents the time of conversion.

Maximizing (1.18) with respect to Y, we obtain

$$\begin{aligned}\frac{\partial NPV_0}{\partial Y} &= -re^{-rY}\left[\frac{(P_i v_i(T)-c)e^{-rT}}{1-e^{-rT}}-\frac{a}{r}\right]+\\ &+(\mu-r)e^{(\mu-r)Y}\left[\frac{P_{c0}\alpha_i\int_0^T E_f v(t)e^{(\mu-r)t}dt}{1-e^{(\mu-r)T}}\right]\end{aligned} \tag{1.19}$$

the above expression can be rewritten as

$$\begin{aligned}\frac{\partial NPV_0}{\partial Y} &= -r\left[\frac{(P_i v_i(T)-c)e^{-rT}}{1-e^{-rT}}e^{-rY}+\frac{P_{c0}\alpha_i\int_0^T E_f v(t)e^{(\mu-r)t}dt}{1-e^{(\mu-r)T}}e^{(\mu-r)Y}-\frac{a}{r}e^{-rY}\right]+\\ &+\mu e^{(\mu-r)Y}\left[\frac{P_{c0}\alpha_i\int_0^T E_f v(t)e^{(\mu-r)t}dt}{1-e^{(\mu-r)T}}\right]\end{aligned} \tag{1.20}$$

Expression (1.19) identifies the forces playing for and against later optimal conversion. As in the carbon flow method, the negative terms create an incentive to immediate conversion, while positive terms have the opposite effect.

If $\mu = 0$, and assuming it is optimal to implement a forest project after introducing carbon benefits (i.e. if the expression inside parenthesis of the first term in (1.20) is positive), then conversion should take place immediately. However, if $\mu > 0$, the last term in (1.20) is positive and it is impossible to unambiguously determine the optimal policy. On the other hand, from the third term of (1.18), we conclude that carbon payments are based upon the carbon stock. Thus, delaying conversion for one more year implies that for every subsequent period the forester will receive credits calculated on a lower basis. Consequently, for a given (finite) optimal rotation period, the incentives to delay conversion are reduced when compared to the previous method, in which compensatory payments are based on the stock growth, that is, the flow, and where costs of carbon release may also postpone optimal conversion time.

1.3 Empirical Results

In this section the previous models are applied to the case of eucalyptus Portuguese forest. The yield function used in this section, based on Globulus 2.1 Model (Tomé, Ribeiro and

Soares, [18]) , is given by:[12]

$$V(t) = 877.0924(1 - e^{(-(-0.0724+0.0214t))}) \tag{1.21}$$

Notice that for this empirical growth function, $V(t)$ is negative for values of t below $3,383178$. To correct for this, a zero volume value is instead considered. As the function is no longer strictly concave in the relevant domain, the results derived in the theoretical section do not necessarily hold.

A long-term real discount rate, $r = 4\%$ is used. Plantation costs of $750/ha$, a maintenance cost of $25/ha/year$ and a replanting cost incurred after every 3 rotations given by $1250/ha$ are also considered.[13] The opportunity cost, a, considered as a forgone rent, is of $114.02/ha$. This rent corresponds to the Faustmann's present value of forest investment.

The comparison between the results obtained in the two different scenarios is made. As the results can be very sensitive to the initial values of P_c, a sensitivity analysis is undertaken for an interval ranging from 0 to $30/ton$.

For the empirical application it will only be considered the case in which the carbon price growth rate is lower than the discount rate ($\mu < r$). Hence, for the calculations, $\mu = 3\%$, which is within the range proposed by the majority of integrated assessment models.

The empirical results were obtained using the MatLab optimization toolbox.

1.3.1 Carbon Flow Regime

Constant Prices

Based on these assumptions, we calculate the optimal rotation age and the present value for different $P_c's$ and $\beta's$. The results obtained are presented in Tables 1 and 2, respectively, in Appendix D.

When only timber benefits are considered, the optimal rotation period is $T^F = 13.994$ years (Faustmann model). From Table 1 it is clear that introducing carbon sequestration increases the optimal rotation. In fact, the decision about the optimal rotation period is very sensitive

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[12] In this model a relationship between total volume and the age of pure and mixed eucalyptus forest tree was considered, based on two variables: density at the age of three years (938), and a quality index given by the value of the dominant height at the age of ten years (19 meters).

[13] Income taxes are not considered, as it is assumed that the majority of Portuguese private forest owners do not pay them (See GANEC [9]). For an explanation on costs see also GANEC [9]. Notice that 1 hectare is equivalent to 10 000 m^2.

to changes in P_c and β: when $P_c = 25$ and $\beta = 0.5$, the optimal rotation age is $T^C = 15.069$, which represents a significant change with respect to Faustmann's ($T^F = 13.994$).

In this case, the optimal rotation period decreases with β, that is, the lower the costs of carbon release at harvest the smaller the optimal rotation period. Therefore, the costs of carbon release at harvest are leading these results.

When carbon benefits are considered, the net present value of the forested land per hectare increases significantly and is always positive (i.e. the present value with carbon sequestration benefits is always above Faustmann). From Table 2, we observe that for $P_c = 25$ and $\beta = 0.5$, the net present value per hectare is given by $910.590/ha$, while for the considered limit case in which $P_c = 30$ and $\beta = 1$ this value is $1799.60/ha$. Therefore, the increase in the profitability of the forested land per hectare is rather substantial.

Increasing Prices

Forested Land Once more, we calculate the optimal rotation age and the present value for different initial carbon price levels and $\beta's$. The results for infinite carbon price growth can be found in Tables 3 and 4.

When carbon prices increase, optimal rotations increase in response to changes in the initial carbon prices, as well as to changes in β (Table 3). This translates into larger optimal rotations periods when compared to the Faustmann solution and the constant carbon prices case with carbon benefits. For $P_{c0} = 25$ and $\beta = 0.5$, the optimal rotation age is now $T^C = 16.09$, approximately one year larger than in the case of constant carbon prices. Notice also that the optimal rotation period now increases with β, in contrast to the constant price case. Therefore, the lower the costs of carbon release at harvest, the larger the optimal rotation period, to take advantage of the increase in carbon prices.

The value of the forest investment becomes not only considerably more sensitive (see Table 4) to changes in the values of those parameters but it also increases significantly. For $P_{c0} = 25$ and $\beta = 0.5$, the net present value per hectare is now given by 3360.5, in contrast to 910.59 with constant carbon prices. For the limit case of $P_{c0} = 25$ and $\beta = 1$, the corresponding value is $6351.3/ha$ against $1498.40/ha$ for constant carbon prices. Moreover, the larger the initial carbon prices, the larger the present value of the hectare of forest when prices increase permanently.

Agricultural vs Forested Land In contrast to the case of constant carbon prices, immediate conversion of agricultural land to forest can be no longer optimal when prices increase over time. In fact, two factors may contribute for optimal delay: increasing carbon prices, as mentioned, and increases in the opportunity cost of land, here considered as a forgone rent, a. We found that carbon price increase alone may be not enough to optimally delay the implementation of forest sequestration projects. Therefore, a combined increase in carbon prices and land rents was instead considered.[14] Notice, however, that as the model developed in this study only allows for partial equilibrium analysis, changes in timber prices, land rents and carbon prices must always be taken as exogenous. Thus, a small land rent increase was considered ($a = 116$). When increasing carbon prices are not attractive enough to postpone conversion to forest, the benefits of afforestation can be immediately felt. This will most probably occur in the case of land with relatively low opportunity cost for alternative uses. Land use value cannot be ignored.

For an initial $P_{c0} = 5$ it was found that delaying conversion is optimal for small values of β (see Table 5). In particular, for $\beta = 0$, the forester should wait 40 years to plant a new forest, while for $\beta = 0.2$, conversion of agricultural land should take place immediately. When the initial carbon price changes, those figures also change significantly. For instance, if the initial carbon price is 15, conversion should take place in 4 years for $\beta = 0$, contrasting sharply with the 40 years in the 5 case. Therefore, when the costs of carbon release at harvest are maximum, that is, for $\beta = 0$, the delay in optimal conversion is also maximum. Once again, optimal rotations are always above Faustmann's. In particular, when it is optimal to convert immediately, the obtained optimal rotations are equal to the ones in the forested land scenario. Otherwise, rotations are even larger.

Also, the increase in net present value is non-negligible, even after considering the exogenous induced increase in the land rent. Also, the optimal rotation period increases with β. This result is similar to that found in the case of forested land when prices increase permanently.

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[14] See Costa-Duarte, Cunha-e-Sá and Rosa [5].

1.3.2 Ton-Year Crediting

Constant Prices

The calculations were performed based on an equivalence factor of 0.018 ($T^e = 55\ years$). The results show that the optimal harvesting time is always postponed implying larger carbon supplies for already forested areas (see Tables 6 and 7). These changes are, however, much smaller than in the case of the previous carbon accounting method.

The net present value of forest investment is always larger when carbon benefits are considered. However, this increase is not substantial. Actually, when using this method, the value of forest investment for $P_{co} = 25$ is lower than the one obtained under the carbon flow method for a price of carbon of 5 and $\beta = 0.5$, meaning that carbon benefits are playing a minor role.

Thus, the accounting method used to estimate and to compensate for the carbon benefits is not without consequences, as it impacts both on the optimal rotation period and on the profitability of forested land. From a policy-maker's perspective, this fact cannot be disregarded given its implications on the supply of timber and carbon, both in the short-run and in the long run.

Increasing Prices

Forested Land The results are presented in Tables 8 and 9. Carbon benefits increase significantly and optimal rotations are now larger when compared to the constant carbon price case. For $P_{c0} = 25$ harvesting should only take place every 18.295 years (Table 8), while with constant carbon prices, optimal rotation is 14.617 years.

The present value increases considerably as the initial carbon price increases and is also higher when compared to the constant price case, making forest investment more attractive.

Agricultural Land versus Forest We find that conversion should take place immediately not only for the constant carbon price case, but also for most of the initial prices considered when prices are increasing; the results are presented in Table 10. Once again, a combined increase of carbon prices and land rent is necessary for optimal delay.

This is an expected result, as carbon payments are based upon the carbon stock and not on the stock growth. One of the main forces behind conversion delay, that is, the stylized fact according to which the rate of carbon storage typically increases in young stands and declines as the stand ages, is not valued by this carbon accounting method.

1.4 Conclusions

By sequestering and storing GHGs from the atmosphere, forests can generate carbon offsets, which may be used to compensate for GHG emissions. However, the net effect of sequestration has to be identical to that of avoiding emissions. This issue raises an important question about how to incorporate the benefits provided by carbon sequestration when modelling forest management.

In this paper, we focused on two carbon accounting methods: the carbon flow regime and the ton-year crediting method, with constant and increasing carbon prices. In this last case, not only the profitability of forested land changes but also the incentives to convert land from agriculture to forest, implying that the optimal timing of land conversion is also a decision variable of the landowner.

When net benefits from carbon sequestration are accounted for, the impact on the optimal rotation period depends upon a “carbon balance” representing a balance between the amount of carbon sequestered up to the harvest time (for each rotation) and carbon sequestered at harvest time. The sign of this carbon balance depends upon the carbon accounting method used.

In the case of the carbon flow regime, while payments for carbon benefits create an incentive to cut earlier, the cost of carbon emissions at harvest pushes for delaying. Under this accounting method, and for constant carbon prices, the optimal rotation period decreases with β, suggesting that the cost of carbon release is leading the results. In fact, for a strictly concave yield function, while for $\beta = 0$ optimal rotation period increases relative to Faustmann's, when $\beta = 1$ the optimal rotation period decreases. When carbon prices increase over time, the opposite may be true, as optimal rotation can actually increase with β. These results were found for the Portuguese eucalyptus forest.

When conversion is also a decision variable of the forester, the results are driven by the increase in carbon prices and the opportunity cost of alternative use of land. Immediate conversion can be optimal in cases where the opportunity cost of the alternative use of land is very low. In other words, if this is the case, independently of increasing carbon prices, it is optimal to convert to forest immediately. Moreover, delaying conversion is only optimal for low $\beta's$, and it occurs earlier the larger are initial carbon prices.

If the carbon accounting is based upon the ton-year crediting method, the carbon balance is always negative, implying that postponing harvest is always optimal relative to the Faustmann's solution. This effect is reinforced with increasing carbon prices. Moreover, incentives for optimal conversion are stronger under the carbon flow method, as the benefits from carbon sequestration are based on the timber stock rather than on the flow. These results are confirmed for the case of the Portuguese eucalyptus forest.

Finally, in both cases (carbon flow and ton-year crediting), the present value per hectare of forested land is lower in the constant case when compared to the increasing one.

In order to implement a sustainable forest management, from a policy maker's perspective, the results in this paper give interesting insights. In particular, using the international carbon market to value the carbon contained in standing forests may provide the economic incentives needed to change land-use decisions, buying time for the development and deployment of low carbon-based technological innovations.

1.5 Appendix A

Equation (1.5), after integration by parts and considering $\beta = 0$, can be rewritten as:

$$\frac{G'_T}{G(T)} = \frac{r}{1-e^{-rT}} +$$

$$+\frac{\frac{r}{1-e^{-rT}}\left[\int_0^T rP_c\alpha_i v_i(t)e^{-rt}dt\right] - rP_c\alpha_i v_i(T)}{G(T)}$$

Note that the second term on the right hand side, representing a balance between the amount of carbon sequestered up to the harvest time (for each rotation) and carbon sequestered at harvest time, can be interpreted as a “carbon balance”. For $v(t)$ strictly concave we have that $v(t)$ is increasing , implying that the optimal rotation increases relative to Faustmann as the balance is negative(see Aronsson and Lofgren[2]).

We now consider the case for $\beta = 1$. Using again (1.5), and substituting now $\beta = 1$, we obtain:

$$\frac{G'_T}{G(T)} = \frac{r}{1-e^{-rT}} +$$

$$+\frac{\frac{r}{1-e^{-rT}}\left[\int_0^T P_c\alpha_i v'_i(t)e^{-rt}dt\right] - P_c\alpha_i v'_i(T)}{G(T)}$$

Once again, the second term on the right hand side can de interpreted as representing a ”carbon balance”. For $v(t)$ strictly concave we have that $v'(t)$ is decreasing , implying now that the optimal rotation decreases relative to Faustmann, as the balance is positive(see Aronsson and Lofgren[2]).

1.6 Appendix B

Using our model we obtain the same result. For $T = \infty$ we have

$$NPV(Y,\mu) = \int_Y^\infty P_{co}\alpha_i v'_i(t-Y)e^{(\mu-r)t}dt - \int_Y^\infty ae^{-rt}dt$$

If immediate conversion is profitable with constant carbon prices, then it has to be the case that

$$NPV(Y,0) \geqslant 0 \Leftrightarrow \frac{a}{r} \leq \int_0^\infty P_{co}\alpha_i v_i'(t-Y)e^{-rt}dt$$

evaluated at $Y=0$. Moreover, if it is optimal to convert immediately, then

$$\frac{\partial NPV(Y,0)}{\partial Y} \leq 0 \Rightarrow a \leq \int_0^\infty P_{co}\alpha_i v_i''(t-Y)e^{-rt}dt$$

Integrating by parts the last integral we obtain

$$\frac{a}{r} \leq \int_0^\infty P_{co}\alpha_i v_i'(t-Y)e^{-rt}dt$$

Therefore, not only immediate conversion is profitable, but also optimal, as in Velt and Platinga [20].

1.7 Appendix C

In the absence of timber benefits the net present value of forest investment is given by

$$NPV_0 = \frac{P_c\alpha_i \int_0^T E_f v(t)e^{-rt}dt}{1-e^{-rT}} - \int_0^\infty ae^{-rt}dt$$

From the first order conditions we obtain:

$$\frac{\partial NPV_0}{\partial T} = -\frac{r}{1-e^{-rT}} \int_0^T E_f v(t)e^{-rt}dt + v(T) > 0$$

For $v\prime > 0$ and $v" < 0$ the sign of this expression is always positive (negative of the carbon balance). (see Aronsson and Lofgen [2]).

1.8 Appendix D

Carbon Flow

Table 1 - Optimal Rotation Period

- Carbon Flow Method - a=114,02

T^F =13,994

	Pc			
β	**5**	**10**	**25**	**30**
0	14.257	14.536	15.490	15.853
0.2	14.236	14.488	15.309	15.606
0.5	14.205	14.418	15.069	15.292
0.7	14.185	14.374	14.929	15.111
1	14.156	14.311	14.743	14.877

Table 2 - Net Present Value

- Carbon Flow Method - a=114,02

	Pc			
β	**5**	**10**	**25**	**30**
0	61.562	124.080	325.640	396.680
0.2	108.940	219.080	559.220	675.640
0.5	180.030	361.100	910.590	1095.80
0.7	227.430	455.710	1145.40	1376.80
1	298.550	597.700	1498.40	1799.60

Constant Carbon Prices

Table 3 - Optimal Rotation Period

- Carbon Flow Method - a=114,02 / μ=0,03

T^F =13,994

	Pc			
β	**5**	**10**	**15**	**25**
0	14.307	14.644	15.011	15.853
0.2	14.37	14.755	15.149	15.968
0.5	14.46	14.901	15.318	16.09
0.7	14.516	14.986	15.411	16.149
1	14.595	15.099	15.528	16.217

Table 4 - Net Present Value

- Carbon Flow Method - a=114,02 / μ=0,03

	Pc			
β	**5**	**10**	**15**	**25**
0	68.896	140.155	213.9884	370.29
0.2	306.6834	616.68	929.9683	1566.2
0.5	663.53	1331.9	2004.5	3360.5
0.7	901.5152	1809	2721.2	4556.8
1	1258.7	2524.8	3796.5	6351.3

Increasing Carbon Prices - Forested Land

Table 5

- Carbon Flow Method - a=116 / μ=0,03

	P_{c0}=5			P_{c0}=10			P_{c0}=15			P_{c0}=25		
β	**T***	**Y***	**NPV***	**T***	**Y***	**NPV***	**T***	**Y***	**NPV***	**T***	**Y***	**NPV***
0	15.162	40.659	38.1	15.162	17.556	96.007	15.162	4.0415	164.85	15.853	0	370.29
0.2	14.37	0	257.183	14.755	0	567.18	15.149	0	880.468	15.968	0	1516.7
0.5	14.46	0	614.03	14.901	0	1282.4	15.318	0	1955	16.09	0	3311
0.7	14.516	0	852.015	14.986	0	1759.5	15.411	0	2671.7	16.149	0	4507.3
1	14.595	0	1209.2	15.099	0	2475.3	15.528	0	3747	16.217	0	6301.8

Increasing Carbon Prices - Agricultural vs Forested land

Ton-Year

Table 6 - Optimal Rotation Period

- Ton-Year Crediting - a=114,02 / Ef=0,018

	P_c			
E_f	**5**	**10**	**25**	**30**
0,018	14.112	14.234	14.617	14.753

Table 7 - Net Present Value

- Ton-Year Crediting - a=114,02 / Ef=0,018

	P_c			
E_f	**5**	**10**	**25**	**30**
0,018	27.815	55.952	142.49	172.07

Constant Carbon Prices

Table 8 - Optimal Rotation Period				
- Ton-Year Crediting - a=114,02 / Ef=0,018				
	P_c			
E_f	**5**	**10**	**25**	**30**
0,018	14.581	15.263	18.295	19.959

Table 9 - Net Present Value				
- Ton-Year Crediting - a=114,02 / Ef=0,018				
	P_c			
E_f	**5**	**10**	**25**	**30**
0,018	127.1117	262.6381	736.271	925.8473

Increasing Carbon Prices - Forested Land

Table 10												
- Ton-Year Crediting - a=116 / μ=0.03 -												
	Pc0											
	5			**10**			**15**			**25**		
	T*	**Y***	**NPV***	**T***	**Y***	**NPV***	**T***	**Y***	**NPV***	**T***	**Y***	**NPV***
E_f= 0,018	15.162	20.71	84.64	15.263	0	213.14	16.071	0	358.47	18.295	0	686.77

Increasing Carbon Prices - Agricultural vs Forested Land

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Chapter 2

Forest Vintages and Carbon Sequestration

2.1 Introduction

Given the rising concern with CO_2 levels, and the recognition in the Kyoto Protocol of the important role that can be played by forests in the global carbon cycle to limit the impact of GHGs (greenhouse gases) emissions, the consideration of carbon sequestration benefits is in the centre of recent developments in forestry literature. Building in the multiple vintage forest model developed in Salo and Tahvonen [18], [19] and [20], the present paper´s contribution consists of introducing net carbon sequestration benefits' accounting in that setting.

From a theoretical point of view, when carbon benefits are considered, not only the forested area is relevant, but also the flow of carbon between land and the atmosphere through the carbon cycle, namely, the amount of carbon released when the forest is harvested. To account for all these impacts the typical analytical framework of the one stand forest or any other that does not take into account the internal age-structure of forests, are not appropriate.[1] In the single stand case, the decision on the optimal allocation between alternative uses can only be assessed in marginal terms, not allowing to address the global (or regional) impact of some policy incentive measures. Besides, since it typically represents the decision model

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[1] A one stand or single vintage forest is characterized by a plot of land with trees of the same species and of the same age, where the price of timber is an exogenous constant. In this context, we should mention, among others, Van Kooten, Binkley and Delcourt [26], who modeled a scheme to allocate carbon credits, under which the carbon credit cash flows are a function of the annual change in the forest carbon stock (carbon flow regime), Spring, Kennedy, and Nally [8] that study the effect of carbon sequestration, fire frequency and water scarcity in tree harvest decision, and Cunha-e-Sá and Rosa [3] where different accounting methods of carbon sequestration benefits in the model of the private forester are examined with constant and rising carbon prices. Also, Velt and Plantinga [27] explore the effect of rising carbon prices on the optimal portfolio of greenhouse-gas mitigation strategies based on the carbon flow accounting regime, and considers the optimal timing to convert agriculture land to forest.

of the private owner, both the price of land and the price of timber are exogenous constants. In contrast, in this paper, both the price of timber and the price of land are endogenously determined. Therefore, only in this context it is possible to study the transition path to the new steady-state, and, more generally, to perform comparative welfare analysis between the different carbon accounting methods. Hence, a multi-vintage forest setting with possible conversion to alternative land uses should be considered instead.[2]

By sequestering and storing GHG's from the atmosphere, forests can be used to compensate for GHG emissions. However, for this compensation to occur, the net effect of sequestration has to be comparable to that of avoided emissions. This issue raises an important question about how to incorporate the services provided by this activity when modeling forest management, which depends upon the choice of the carbon accounting method. Following the IPCC Special Report on Land Use, Land Use Change and Forestry, we consider three different carbon accounting methods - the carbon flow method, the ton-year crediting regime and the average storage. In contrast to the others, the carbon flow method is essentially a Pigouvian tax/subsidy on the carbon externality (first-best), as it fully internalizes at any point in time the carbon flows between forest and the atmosphere. Based on numerical simulations of the theoretical model, the results obtained in the carbon flow method on optimal land allocation between forestry and alternative uses, total carbon sequestered, timber production and social welfare, are compared to those obtained in the other two methods and to the case without carbon sequestration benefits. Sensitivity analysis with respect to the most relevant parameters is performed.

There is a vast empirical literature that attempts to estimate the costs of forest-based carbon sequestration, and compare them with those obtained in alternative energy-saving options such as reducing emissions from fossil fuel use. Despite their differences, in general, they conclude that the carbon sequestration option is surprisingly cost-effective relative to policies that only consider fossil fuel emissions.

A recent review of empirical studies for the US can be found in Stavins and Richards [24]. According to these authors, three general approaches have been used to estimate the economic costs of diverting land from other uses to forest carbon sinks: econometric studies of the revealed preferences of agricultural land owners, bottom-up engineering cost studies, and optimization models that account for behavioral response in the forest and agricultural sectors.

Recent econometric studies by Stavins [23], Newell and Stavins [15], Plantinga *et al.* [16],

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[2] Reinforcing the interest of this modeling framework for empirical studies, Getz and Haight [6] refer that biological populations are typically described by discrete time demographic models for reasons like seasonal cyclicality in reproduction or in concentration in harvesting, as in the case of forests.

Plantinga and Mauldin [17], Kerr, Pfaff and Sanchez [9] and Lubowski *et al.* [10], among others, have provided an alternative approach to modeling the potential costs of land for carbon sequestration in the US. These authors have analyzed how landowners have historically allocated land use between agriculture and forests in response to differences in prices. Rather than assuming maximization of profits by landowners a revealed-preference approach based on observed practices is used.

The majority of the studies fall in the second category, where land and timber prices are taken as exogenous constants. To obviate to this problem, studies by Alig *et al.* [2] and Adams *et al.* [1] have addressed this issue using the Forestry and Agricultural Sector Model (FASOM). This model is a multi-period, price endogenous, spatial equilibrium model that links the forest and agricultural sectors in the US, where the welfare of producers and consumers in the two sectors is maximized. Besides, it also estimates where and how much conversion of land between forest and agricultural uses would be induced by a carbon sequestration program. Also, Sedjo and Sohngen [21] developed a world timber supply model to examine and assess the interactions between carbon sequestration forestry, particularly, newly created carbon forests, and the markets for timber. However, in all these models, the numerical solutions are computed by imposing that the forest age class structure must reach the normal forest distribution in finite time.[3],[4] In contrast, in this paper, we do not impose a normal forest.

Despite that the concept of normal forest has been widely used in forest economics, only recently, in Salo and Tahvonen [20], the analytical conditions under which the normal forest is the long-run steady-state solution of the original forestry model with any number of age classes (Mitra and Wan, [12], [13]) and endogenous land allocation between forestry and alternative land uses were derived. This line of research addresses one of the most important theoretical issues that has been discussed in the context of forest economics literature, that is, the optimal evolution of an age-class structured forest over time, in particular, whether it converges to the normal or regulated forest. By extending the results in Salo and Tahvonen [20] to the presence of net carbon sequestration benefits' accounting, we prove that, in general, the long-run stationary state converges to the normal forest. In the case where all land is forested land, optimal forest management can lead to optimal cyclical harvesting. Alternatively, when it is optimal to allocate part of the forest land to other land uses, cycles optimally vanish, and the normal forest becomes a local saddle point equilibrium. While all the adjustments following the internalization of carbon sequestration benefits are undertaken through either changes in land allocation between forest and alternative uses, or the time to harvest, or both, in Salo

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[3] A normal, syncronized or regulated forest is a forest where total land is evenly allocated between existing age-classes and only the oldest age class (with, e.g., Faustmann financial maturity) is clearcut in each period.

[4] In contrast, Sohngen and Mendelsohn [22] determine endogenously the efficient shadow price of carbon.

and Tahvonen [20], the optimal rotation is always given by Faustmann's.

Based on numerical simulations of the theoretical model, we conclude that, for a given carbon price, the transition paths to the steady-state in the three accounting methods are rather similar in what concerns the dynamic behavior of forested area, as the optimal adjustment on land allocation is almost instantaneous, moving fast to cycles stabilization. In contrast, timber consumption performs rather distinctively among carbon accounting methods. While in the first-best case the timber market is characterized by major short-run adjustments, following the change in the optimal rotation period, in the other two accounting methods the adjustment is mainly driven by the allocation of land between forest and alternative uses. Therefore, we conclude that the first-best policy determines larger medium-term costs when compared to the less efficient solutions, thus making its implementation eventually less attractive in the short/medium run.

When different carbon prices are considered, in general, higher carbon prices increase both the optimal rotation period and land allocated to forest, increasing in all cases the amount of carbon sequestered relative to the case without carbon. The carbon flow method has the biggest impact in terms of carbon sequestration.

In general, welfare deviations of both the ton-year and the average storage with respect to the carbon flow solution increase as carbon prices increase. Whenever the carbon price only changes slightly the rotation period, that is, for low carbon prices, the ton-year performs worse than the average storage, while for high carbon prices, it performs better, as it allows for adjustments in the optimal rotation.

The use of forests as carbon sinks depends upon the species and, therefore, on the amount of carbon sequestered into long-term structures, that is, not released at harvest. Interestingly enough, we show that increases in the amount of carbon sequestered into long-term structures do not necessarily determine an increase in the amount of carbon sequestered in forest biomass. Ultimately, this is an empirical question.

The remainder of the paper is organized as follows. Section 2 presents the different accounting methods of carbon sequestration benefits, Section 3 extends the theoretical multiple vintage model to account for carbon sequestration benefits. Section 4 develops the model for the three carbon accounting methods considered: the carbon flow regime, the ton-year crediting and the average storage method. The results are compared to those obtained without carbon sequestration benefits. In Section 5 the results are discussed based on numerical simulations of the theoretical model. Section 6 concludes the paper. Technical details, figures and tables are presented in the Appendices.

2.2 Accounting Methods

By sequestering and storing GHG's from the atmosphere, forests can generate carbon offsets, which may be used to compensate for GHG emissions. However, for this compensation to occur, the net effect of sequestration has to be comparable to that of avoided emissions. This issue raises two important questions: first, how to compare forest carbon sequestration with avoided emissions, examined in this section, and second, how to incorporate the services provided by this activity when modeling forest management, considered below.

The IPCC Special Report on Land Use, Land-Use Change and Forestry [7] considers different accounting methods to apply to forest or land use change investment projects, namely, the stock change method, the average stock method and the ton-yearly crediting. In this paper, these methods are adjusted in order to account for the time dimension of carbon sequestration and storage.

According to the carbon flow regime, as developed in Van Kooten, Binkley and G. Delcourt [26], social benefits are a function of the annual change in the forest carbon stock, as well as of the amount of carbon permanently stored in timber products and landfills. A net increase in the forest carbon stock over a year means that carbon has been removed from the atmosphere. Similarly, a fall in the forest carbon stock suggests that carbon has been released into the atmosphere. In this context, while carbon released at harvest is taxed, depending upon the timber use, sequestered carbon is subsidized yearly. Therefore, the carbon flow method is essentially a Pigouvian tax/subsidy on the carbon externality, representing a first-best solution.

An alternative approach is the ton-year crediting regime. The ton-year method consists of crediting a forestry project with a fraction of its total yearly GHG benefit, based on what is called an equivalence factor E_f. This fraction is determined by the stock of carbon stored each year, which is then converted, using E_f, to its equivalent amount of preventing effect. In the context of this approach, two alternative calculations have been proposed by Moura-Costa and Wilson [14], and by Fearnside, Lashof and Moura-Costa [5], respectively. In both, they are based on the residence time and decay pattern of atmospheric $C0_2$, its Absolute Global Warming Potential (AGWP), taking explicitly into account the decay pattern of GHGs in the atmosphere. As a consequence, notice that this method does not require redemption of carbon credits upon harvest.

Moura Costa and Wilson [14] aim to determine the storing time of carbon sequestered in biomass for which the carbon stored is equivalent to an amount of avoided emissions (equivalence time). It was found that keeping a megagram (Mg) of CO_2 out of the atmosphere for a full 100 years is equivalent to 55 Mg-year (or ton-year) equivalents, rather than the full 100 Mg-years if the CO_2 entering the atmosphere had no movement to the ocean or other sinks. The number obtained, in this case 55, is denoted by the equivalent time, T_e. In addition, assuming a linear relationship between the residence of CO_2 in the atmosphere and its radiative forcing effect, the effect of storing 1 ton of CO_2 in forest biomass for 1 year was derived. According to this rule, storing one ton of carbon for one year is equivalent to preventing the effect of 0.0182 tons of CO_2 emissions, which is denoted by the equivalence factor, $E_f = 1/T_e$. Therefore, to store one ton of carbon for one year is equivalent to receiving a subsidy for preventing the effect of $1/T_e$ of CO_2 emissions.

Also based on a Absolute Global Warming Potential (AGWP) function, Fearnside et al.[5] estimate the incremental credit that can be awarded for each additional year that carbon stocks remain sequestered. For this purpose these authors assume as the benchmark "keeping a Mg of CO_2 out of the atmosphere for a full 100 years". If the stock remains intact for 100 years, the cumulative awarding of ton-year credits would equal the credits from a "permanent" emission reduction of the same magnitude. If the stock is released at any time prior to the 100-year time horizon, only the corresponding partial credit amount would be awarded.

The average carbon storage method consists of averaging the amount of carbon stored in a site over the long run, assuming an average cycle rotation period. As a result, the forest owner receives the corresponding subsidy. Finally, notice that in contrast to the carbon flow method, the ton-year and the average carbon storage are second-best solutions.

2.3 The Model

The model used in this paper follows closely the multiple vintage forest model developed in Salo and Tahvonen [20], which can be summarized as follows. The model assumes multi vintages forest land, where $s = 1, ..., n$ represents the age of trees, $x_{s,t}$ the area of forest land allocated to the age class s in period t, f_s the biomass content in timber per unit of land with trees of age class s, and $0 \leq f_1 \leq .... \leq f_n$. Land allocation must satisfy

$$0 \leq y_t = 1 - \sum_{s=1}^{n} x_{s,t} \tag{2.1}$$

that is, total land area equals 1, and y_t is the area of land allocated to an alternative use (agriculture or urban use).

Let us denote by $U(c_t) = \int d(c)dc$ the social utility from timber consumption, where $d(.)$ is the inverse demand for timber, and assume $U(.)$ is a continuous, twice differentiable, increasing and strictly concave function. Also, $W(y_t)$ represents the social utility of alternative land use, where $W(.)$ is a continuous, twice differentiable, increasing and concave function and $W'(y_t)$ is the opportunity cost of a unit of land. Finally, S_t represents the social value of carbon sequestration by forests [5] and depends on how the benefits from carbon sequestration are accounted for, as shown below.

Thus, the problem of optimal forest harvesting and allocation of land is obtained by maximizing the present value of social utility from the use of land, as follows:

$$v(x_{1,0},....x_{n,0})^i = \underset{\{x_{s,t+1},s=1,...n,t=0,...\}}{Max} \sum_{t=0}^{\infty} b^t \left[U\left(c_t\right) + S_t^i + W\left(y_t\right)\right] \tag{2.2}$$

subject to

$$c_t = \sum_{s=1}^{n-1} f_s \left(x_{s,t} - x_{s+1,t+1}\right) + f_n x_{n,t} \tag{2.3}$$

$$y_t = 1 - \sum_{s=1}^{n} x_{s,t} \tag{2.4}$$

$$x_{s+1,t+1} \leq x_{s,t}, s = 1, ...n-1 \tag{2.5}$$

$$\sum_{s=1}^{n} x_{s,t+1} \leq 1 \tag{2.6}$$

$$x_{s,t} \geq 0, s = 1, ..., n \tag{2.7}$$

for all $t = 0, 1....$ Moreover, the initial land distribution satisfies

$$x_{s,0} \geq 0, s = 1, ..., n, \sum_{s=1}^{n} x_{s,0} \leq 1 \tag{2.8}$$

Therefore, given the discount factor b, the problem consists of choosing the next period state, that is, the land allocation between different vintages and competing uses of land for all $t = 1, ...$[6]

The necessary conditions for optimal solutions can be obtained from the following Lagrangian problem. For (2.2-2.8) it can be stated as

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[5] S_t can alternatively be interpreted as the actual payment scheme given to forest owners to induce carbon sequestration. In this sense, the model can be used to evaluate and compare actual policy measures.

[6] In Salo and Tahvonen [20] no harvesting or plantation costs are considered nor any type of forest externalities. Under these conditions, m, as defined in (2.10), is the Faustmann rotation period in the one stand model. As consumption is constant in the steady-state, so is the marginal utility of consumption $U'(.) = p$, the long-run market equilibrium price of timber. In addition, this condition also corresponds to the maximum value in the steady-state of a marginal unit of bare forest land.

$$L^i = \sum_{t=0}^{\infty} b^t \left[U(c_t) + S_t^i + W(y_t)\right] + \lambda_t \left(1 - \sum_{s=1}^{n} x_{s,t+1}\right) + \sum_{s=1}^{n-1} \left[p_{s,t}\left(x_{s,t} - x_{s+1,t+1}\right)\right] \tag{2.9}$$

where $p_{s,t}$ and λ_t are the Lagrangian multipliers, and $i = c, t, a$. While $p_{s,t}$ can be interpreted as the value of marginal changes in forest land area of vintage s at the beginning of period $t+1$, λ_t represents the value of marginal changes in land allocation between forest and alternative uses.

Salo and Tahvonen [20] provide a full proof on the long-run optimality of the normal forest steady-state for the above problem, when $S_t = 0$. A forest is called an Optimal Faustmann Forest (OFF) if the age-class structure $x = (x_1, ..., x_n)$ has the property $x_s = 0$ for $s = m+1, ..., n$ and if harvesting only trees of age m is the optimal solution for the above problem when $x_0 = x$. An OFF is an interior OFF if $x_s > 0$ for $s = 1, ..., m$. In addition, an OFF with the normal forest structure is $x = (1/m, ..., 1/m, 0, ..., 0)$, and in each period it yields a constant consumption level of f_m/m. An OFF with consumption that is periodic with period length equal to m can be expressed as $x = (1/m + \phi_1, ..., 1/m + \phi_m, 0, ...0) \in S$, where ϕ^k represents the largest number ϕ that satisfies $x = (1/m + \phi_1, ..., 1/m + \phi_m, 0, ...0) \in K$ for all $|\phi_s| < \phi$, $s = 1, ..., m, \sum_{s=1}^{m} \phi_s = 0$. The Faustmann rotation period, denoted by m, $1 \leq m \leq n$, is assumed to be unique and satisfies the following condition:[7]

$$b^m f_m/(1 - b^m) \geq b^s f_s/(1 - b^s), \ s = 1, ..., n. \tag{2.10}$$

Salo and Tahvonen [20] show that, if all land is allocated to forestry, optimal forest management can lead to optimal cyclical harvesting because smoothening an age class structure that deviates from the normal forest is not optimal. On the contrary, if it is optimal to allocate part of the land to alternative land use then optimal stationary cycles cannot exist.[8]

Using similar notation, let m^i, for $i = c, t, a$, denote the optimal rotation period with net carbon sequestration benefits for each accounting method. Assume that m^i is unique, for $i = c, t, a$. A forest is called an Optimal Carbon Forest (OCF) if the age-class structure $x = (x_1, ..., x_n)$ is characterized by OFF for m^i, $i = c, t, a$, where m^i can be different from m.

In this paper, the full proof on the long-run optimality of the normal forest steady state is extended to the case of carbon sequestration benefits.

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[7] See Salo and Tahvonen [20], Proposition 1 and Corollary 1, pages 518-520.

[8] From now on, let i_∞ represent the stationary state level of variable i.

2.4 Introducing Carbon Sequestration Benefits

In this section, for the three methods, the age-class and land allocation forestry decision problem of the social planner is presented and the necessary and sufficient optimality conditions are derived. It is shown how the optimal rotation period, the long run equilibrium and the optimal land allocation are affected by introducing carbon sequestration benefits in the three different cases. For each method, the comparison with the case without carbon sequestration benefits is provided.

When formalizing net carbon benefits, we assume in all cases that the social value of one unit of carbon removed from the atmosphere is constant and given by p_c.[9] That is, the price of carbon is the value of the marginal damage of an additional unit of carbon added over to the atmosphere. Alternatively, if p_c is considered as a tax/subsidy to be payed to the forest owners, then the model can be used to estimate the cost of the policy in each case. Finally, S_t can be endogeneized by imposing quantitative targets in terms of carbon sequestration amounts or afforestation areas. In these cases, the shadow price of carbon implicit in the constraint can be estimated for each method.

In what follows, we consider that the amount of carbon per cubic feet of timber biomass growing in forest land is constant and equal to β.

2.4.1 Carbon flow regime

The carbon flow regime measures the change of the carbon stock in the standing trees, as well as the amount of carbon that is assumed to remain as permanently stored in timber products and landfills. This last amount depends upon the different uses of timber. We introduce a parameter θ which measures the fraction of timber that is harvested but goes into long-term storage in structures and landfills. Notice that once carbon has been sequestered, no further carbon benefits will be obtained, therefore what is relevant here is the change in the per period carbon uptake.[10]

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[9] Assuming a constant price means that forests have only a partial (marginal) impact on carbon sequestration markets. In addition, since the time horizon is infinite in this model, it is not realistic to assume that prices increase indefinitely. However, in the numerical simulations, there is the possibility of considering increasing carbon prices for finite periods.

[10] The carbon flow regime with $\theta = 0$ is similar to the rental approach that was proposed in the context of the Kyoto Protocol, namely, of the Clean Development Mechanism (CDM), as discussed in Marland, Fruit and Sedjo [11]. In the carbon flow regime presented in this paper, the forest owner is fully liable for the eventual carbon released and receives full credit for the amount of carbon sequestered for the whole duration period of the contract. This is similar to the rental approach, except that, in this case, the agent that receives full credit and the one that is liable are typically not the same, as well as the duration pf the contract. Besides, as we assume that there is perfect information, and the ton of carbon is payed at its shadow price value, the problem is simplified in this case, as there are no issues of property rights, credibility, asymmetry of information, uncertainty or any other market imperfections that are present, for instance, in the CDM context. We are

Since the carbon flow internalizes correctly the carbon externality, given that p_c is the social value of carbon, we may obtain an estimate of the efficient opportunity benefit of forests as carbon sinks.

The current net benefits from carbon sequestration at any period t, S_t^c, can be represented as follows:

$$S_t^c = \sum_{s=0}^{n-1} p_c \beta (f_{s+1} - f_s) x_{s+1,t+1} - p_c \beta (1-\theta) c_t \tag{2.11}$$

where the first term represents the value of the carbon stock increase in forest standing biomass, for all the area of forest land, and the last term represents the value of the carbon released due to harvest at t, that is, the amount that is not permanently stored in timber products or landfills.

By solving the problem (2.2-2.8) and taking S_t^c given by (2.11), as in Salo and Tahvonen [20], we first study the existence of optimal stationary cycles in a regime where the oldest age class is clear-cut and immediately regenerated at the end of each period.

Denote the optimal rotation period by m^c, that satisfies $1 \leq m^c \leq n$ [11] and for which

$$(p - \beta p_c(1-\theta)) \frac{b^{m^c} f_{m^c}}{1 - b^{m^c}} + \beta p_c \frac{\sum_{i=0}^{m^c-1} b^i (f_{i+1} - f_i)}{1 - b^{m^c}} \geq (p - \beta p_c(1-\theta)) \frac{b^s f_s}{1 - b^s} +$$

$$+\beta p_c \frac{\sum_{i=0}^{s-1} b^i (f_{i+1} - f_i)}{1 - b^s} \tag{2.12}$$

$s = 1, ..., n$, holds. Assume that m^c is unique. We show in Appendix A that $m^c \geq m$ when all carbon is released at harvest ($\theta = 0$). When $\theta = 1$ and $\{f_i - f_{i-1}\}$ is a decreasing sequence, $m^c \leq m$. Otherwise, for $\theta = 1$, $m^c \overset{\geq}{\underset{<}{=}} m$. [12]

Proposition 1: *Given* $g \equiv \frac{[U'(f_{m^c}/m^c) - \beta p_c(1-\theta)] b^{m^c} f_{m^c}}{1 - b^{m^c}} + \frac{\beta p_c}{1 - b^{m^c}} \sum_{i=0}^{m^c-1} b^i (f_{i+1} - f_i) - \frac{b}{1-b} W'(0) > 0$, $m^c \geq 2$, *and* $b < 1$, *there exists a set of interior Optimal Carbon Forests with* $\phi^k > 0$.

Proof. The proof is in Appendix A. ■

In Proposition 1 it is shown that optimal stationary cycles exist when it is optimal to allocate all land to forestry. From (2.34) in Appendix A, we may conclude that when carbon sequestration benefits are accounted for the maximum cycle radius may either increase or decrease.[13]

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grateful to an anonimous referee that called our attention to this point.

[11] Since there are no carbon intakes after n, it is never optimal to postpone harvest after n.

[12] In fact, in the case of the typical Faustmann model discussed in the forestry literature (continuous-time model), when the timber growth function is strictly concave we can show that the optimal rotation increases for lower values of θ, while it decreases for values of θ close to 1. However, this is not the case when the timber growth function is only increasing, as shown in Appendix 1 for the discrete-time setting. Notice that, if the timber growth sequence is increasing, but its increments are decreasing with the age of the tree, s, the same result as in the continuous-time case applies. See Cunha-e-Sá and Rosa [3].

[13] For a more detailed explanation see Salo and Tahvonen [18], pages 8-9 and 15.

Corollary 1: If $g \equiv \frac{[U'(f_{m^c}/m^c)-\beta p_c(1-\theta)]b^{m^c}f_{m^c}}{1-b^{m^c}} + \frac{\beta p_c}{1-b^{m^c}}\sum_{i=0}^{m^c-1} b^i(f_{i+1}-f_i) - \frac{b}{1-b}W'(0) \leq 0$, *optimal stationary cycles with* $y_\infty \geq 0$ *and* y_∞ *constant do not exist.*

Proof. The proof is in Appendix A. ■

When it is optimal to allocate land to alternative uses, Corollary 1 shows that optimal cycles are eliminated and the remaining equilibrium is the normal forest steady-state. Assuming that m^c is unique, for a stationary state, we have that $p_{s,t} = p_{s,\infty}$, $c_t = c_\infty$, $y_t = y_\infty$, $\lambda_t = 0$, and $x_{m,t} = x_\infty$, where c_∞, y_∞, x_∞, and $p_{s,\infty}$, for $s = 1, ..., n-1$, are constant. Direct substitution shows that in this case:

$$p_s = W'(y_\infty)\sum_{i=0}^{s-1} b^{-i} - f_s\left[U'(c_\infty) - \beta p_c(1-\theta)\right] - \beta p_c \sum_{i=0}^{s-1} b^{i-s}(f_{i+1}-f_i) \tag{2.13}$$

where $\sum_{i=0}^{s-1} b^{-i} = -b\frac{1-b^{-s}}{1-b}$, for $s = 1, ..., n$.

With some more algebra, we can write (2.13) for $s = m^c$ and as $p_{m^c,\infty} = 0$, obtaining:

$$W'(y_\infty)\frac{b}{1-b} - \frac{b^{m^c}f_{m^c}}{1-b^{m^c}}\left[U'(\frac{(1-y_\infty)f_{m^c}}{m^c}) - \beta p_c(1-\theta)\right] - \frac{\beta p_c}{1-b^{m^c}}\sum_{i=0}^{m^c-1} b^i(f_{i+1}-f_i) = 0 \tag{2.14}$$

Solving for y_∞, all the other steady-state variables are fully defined and, from (2.14), the allocation of land between forestry and the alternative use is optimal when the present value of output from a marginal unit of land equals the present value of a marginal unit of bare forest land, where both timber value and the net benefits from carbon sequestration are accounted for. From Appendix A, we conclude that the long-run optimal steady state will be characterized by an increase in the forest area and the opportunity cost of land when compared to the case without carbon benefits.

2.4.2 Ton-year crediting

The ton-year accounting method consists of crediting a forestry project with a fraction of its total yearly GHGs' benefit. This fraction is based on the stock of carbon stored each year, which is then converted, using E_f, to its equivalent amount of preventing effect.[14]

In this case, S_t^t can be defined as follows:

$$S_t^t = p_c(\beta E_f \sum_{s=1}^{n-1} f_s x_{s+1,t+1}) \tag{2.15}$$

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[14] Here, we consider E_f constant. This assumption is consistent with Moura-Costa and Wilson' [14] approach, and also with Fearnside et al. [5], if in this last case we assume that the equivalence factor measures only the benefit of storing carbon in the forest for one additional year. To be fully consistent with Fearnside et al. [5], the equivalence factor should be different for each age class s, that is, $E_f(s)$. However, all the main results also apply.

where the term in parenthesis represents the equivalent amount of emissions avoided in year t due to the amount of carbon stored during year t. By considering $f_s x_{s+1,t+1}$, this formalization excludes from benefits' accounting all possible harvesting of younger age classes, in period t. Notice also that there is no liability for carbon releases.

The necessary conditions for optimal solutions of problem (2.2-2.8) and S_t^t given by (2.15), are similar to the previous case and are presented in Appendix B.

Denote the optimal rotation period by m^t that satisfies $1 < m^t$ and for which[15]

$$p\frac{b^{m^t} f_{m^t}}{1-b^{m^t}} + \beta p_c E_f \frac{\sum_{i=1}^{m^t-1} b^i f_i}{1-b^{m^t}} \geq p\frac{b^s f_s}{1-b^s} + \beta p_c E_f \frac{\sum_{i=1}^{s-1} b^i f_i}{1-b^s}, \quad s = 1, ..., n. \tag{2.16}$$

holds. Assume that m^t is unique. We show in Appendix B that $m^t \geq m$.

Proposition 2: Given $g \equiv \frac{U'(f_{m^t}/m^t) b^{m^t} f_{m^t}}{1-b^{m^t}} + \frac{\beta p_c E_f}{1-b^{m^t}}(\sum_{i=1}^{m^t-1} b^i f_i) - \frac{b}{1-b}W'(0) > 0$, $m^t \geq 2$, *and* $b < 1$, *there exists a set of interior Optimal Carbon Forests with* $\phi^k > 0$.

Proof. The proof is in Appendix B. ■

From Proposition 2 if all land is forested land, cyclical harvesting with consumption that is periodic with period length equal to $m^t \geq m$ is optimal. By inspection, from (2.64) in Appendix B, we observe that the maximum radius cycle can either increase or decrease.

Corollary 2: If $g \equiv \frac{U'(f_{m^t}/m^t) b^{m^t} f_{m^t}}{1-b^{m^t}} + \frac{\beta p_c E_f}{1-b^{m^t}} \sum_{i=1}^{m^t-1} b^i f_i - \frac{b}{1-b}W'(0) \leq 0$, *optimal stationary cycles with* $y_\infty \geq 0$ *and* y_∞ *constant do not exist.*

Proof. The proof is in Appendix B. ■

From Corollary 2 we conclude that the cycles are eliminated, and it is optimal to allocate land both in forestry and in an alternative use. Assuming again that m^t is unique, for a stationary state, we have that $p_{s,t} = p_{s,\infty}$, $c_t = c_\infty$, $y_t = y_\infty$, $\lambda_t = 0$, and $x_{m,t} = x_\infty$, where c_∞, y_∞, x_∞, and $p_{s,\infty}$, for $s = 1, ..., n-1$, are constant. Direct substitution shows that, for $s = 1, ..., n$,:

$$p_s = W'(y_\infty)\sum_{j=0}^{s-1} b^{-j} - f_s U'(c_\infty) - \beta p_c E_f \sum_{i=1}^{s-1} b^{i-s} f_i \tag{2.17}$$

With some more algebra, we can write (2.17) for $s = m^t$ and as $p_{m^t,\infty} = 0$, obtaining:

$$W'(y_\infty)\frac{b}{1-b} - \frac{b^{m^t} f_{m^t}}{1-b^{m^t}} U'(\frac{(1-y_\infty) f_{m^t}}{m^t}) - \frac{\beta p_c E_f}{1-b^{m^t}} \sum_{i=1}^{m^t-1} b^i f_i = 0 \tag{2.18}$$

In this case, the net benefits from carbon sequestration (third term of (2.18)) are the present value of "emissions equivalence reduction" of a marginal unit of forest bare land with a rotation period of dimension m^t. Also, as carbon sequestration benefits have always a positive

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[15] Also, it may be optimal never to harvest the forest.

net value, the present value of forest land increases and consequently more land will be put to forest reducing the area in alternative uses. Moreover, as the optimal rotation period may change, the steady-state timber consumption level, $\frac{(1-y_\infty)f_{mt}}{m^t}$, will also change. In empirical terms it may increase or decrease compared to the case without carbon benefits.

Despite that at the one stand level it may be optimal to never harvest the forest, in a general equilibrium land allocation model this result is less likely and would require additional assumptions, namely, the existence of a choke price on timber.

2.4.3 Average Storage Method

The average storage accounting method consists of yearly crediting a forestry project with the amount of carbon benefits that the land allocated to forest generates, on average, during a rotation period. Hence, S_t^a can be defined as follows:

$$S_t^a = p_c \beta C \sum_{s=1}^{n} x_{s,t} \tag{2.19}$$

where the term C is taken as a constant,[16] representing the average carbon stored in each stand.

The necessary conditions for optimal solutions of problem (2.2-2.8) are similar to the previous case and are presented in Appendix C.

Let us denote the optimal rotation period by m^a, that satisfies $1 \leq m^a \leq n$, and for which:

$$p\frac{b^{m^a} f_{m^a}}{1-b^{m^a}} + \beta p_c b \frac{C}{1-b} \geq p\frac{b^s f_s}{1-b^s} + \beta p_c b \frac{C}{1-b}, \quad s = 1, ..., n. \tag{2.20}$$

holds. By comparing (2.20) with (2.10), we conclude that $m^a = m$, implying that the optimal rotation period is the same as Faustmann's.

Proposition 3: Given $g \equiv \frac{U'(f_{m^a}/m^a)b^{m^a} f_{m^a}}{1-b^{m^a}} + \frac{b}{1-b}D - \frac{b}{1-b}W'(0) > 0$, $m^a \geq 2$, *and* $b < 1$, *there exists a set of interior Optimal Carbon Forests with* $\phi^k > 0$, *where* $D = \beta p_c C$.

Proof. The proof is in Appendix C. ■

According to Proposition 3, if all land is forested land, optimal forest management can lead to optimal cyclical harvesting, but here the maximum radius cycle is the same as without carbon sequestration benefits.

Corollary 3: If $g \equiv \frac{U'(f_{m^a}/m^a)b^{m^a} f_{m^a}}{1-b^{m^a}} + \frac{b}{1-b}D - \frac{b}{1-b}W'(0) \leq 0$, *optimal stationary cycles with* $y_\infty \geq 0$ *and* y_∞ *constant do not exist.*

Proof. The proof is in Appendix C. ■

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[16] In particular, the average carbon stock stored can be given by $C = \frac{\sum_{s=1}^{m^a-1} f_s}{(m^a)^2}$.

Assuming again that m^a is unique, for a stationary state, we have that $p_{s,t} = p_{s,\infty}$, $c_t = c_\infty$, $y_t = y_\infty$, $\lambda_t = 0$, and $x_{m,t} = x_\infty$, where c_∞, y_∞, x_∞, and $p_{s,\infty}$, for $s = 1, ..., n-1$, are constant. Direct substitution shows that, for $s = 1, ..., n$:

$$p_s = W'(y_\infty) \sum_{j=0}^{s-1} b^{-j} - f_s U'(c_\infty) - \sum_{j=0}^{s-1} b^{-j} D \tag{2.21}$$

With some more algebra, we can write (2.21) for $s = m^a$, and $p_{m^a,\infty} = 0$, obtaining:

$$W'(y_\infty)\frac{b}{1-b} - \frac{b^{m^a} f_{m^a}}{1-b^{m^a}} U'(\frac{(1-y_\infty)f_{m^a}}{m^a}) - \frac{b}{1-b} D = 0 \tag{2.22}$$

Here, the net benefits from carbon sequestration (third term of (2.22)) are the present value of the yearly constant payment to a marginal unit of forest land, D. Since $m^a = m$ is unique, it is clear from (2.22) that y_∞ has to decrease, when compared to the case without carbon benefits. As the optimal rotation period is the same as Faustmann's, steady-state timber consumption increases and market equilibrium price decreases. As well, more land will be put to forest when compared to the case without carbon benefits, and, at the steady-state, the incremental forest land area will be evenly distributed among the different vintages.

2.5 Discussion of Numerical Results

In this section, we follow the example in Salo and Tahvonen [20] to simulate the theoretical models developed in the previous sections and to illustrate the potential use of this setting to applied empirical studies. The results obtained with the different accounting methods are compared with respect to the optimal land allocation between forestry and alternative uses, total carbon sequestered, timber production and social welfare to the case without carbon sequestration benefits. The ton-year and the average storage's performance with respect to the carbon flow is provided, both at the steady-state and in the transition to steady-state. Sensitivity analysis with respect to the most relevant parameters of the model is undertaken.

Henceforth, the following utility functions for consumption and non-forestry land are considered: $U(c) = \frac{c^{0.7}}{0.7}, W(y) = 0.5[\frac{y^{0.2}}{0.2}]$. The vector f_s containing the biomass content in timber per unit of land with age classes of trees, $s = 1, ...24$, is given by

$$f_s =$$

$$[0, 0, 0, 15, 22, 30, 39, 51, 65, 82, 101, 123, 148, 175, 204, 234, 263, 293, 321, 346, 370, 390, 408, 423]$$

and $b = 0.95$.[17]

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[17] Using this example, as in Salo and Tahvonen [20] without carbon, the solution reaches the saddle point path where only the oldest age class ($m = 19$) is harvested in period $t = 40$. After 120 periods the land allocation was approximately constant and the forest distribution was very close to the normal.

All the main results of internalizing carbon benefits are presented and illustrated in Tables 1, 2, 3, and Figures 1, 2, 3, 3A-3E, 4.[18] The simulations presented assume the same "price of carbon" for all accounting methods and are based on the following parameter values: $p_c = p_{ss} = 0.4368$ where p_{ss} is the steady-state timber price in the baseline,[19] that is, without carbon benefits, $\beta = 0.2$.[20] The equivalence factor for the ton-year is $E_f = 0.0182$.

We examine first, for a given carbon price, the time paths for the optimal land allocated to forest and timber consumption following the internalization of carbon sequestration (see Figures 1 and 2). We conclude that the optimal land allocation evolves towards a stationary state where both the area dedicated to forest land and timber production increase. In all cases, an adjusted normal forest is also the long-run equilibrium, confirming the theoretical results.

If we compare the three accounting methods, there are no major differences in the dynamic behaviour towards optimal forested area (see Figure 1). The optimal adjustment on land allocation is almost instantaneous, moving fast to cycle stabilization. Changes in the forested area take place through allocating more land to the area that is harvested each period. In contrast, the timber consumption paths perform rather distinctively among accounting methods, as a consequence of adjustments both on the optimal rotation period and on land allocation (see Figure 2). In the carbon flow method, adjustments both in the optimal rotation period and land allocation occur independently of the level of carbon prices (see Table 1). However, in the ton-year case, this is only observed for high carbon prices, while for low carbon prices, only the land allocation changes. Finally, in the average storage, the optimal rotation period never changes, implying that all the adjustments occur through land use changes. For $p_c = p_{ss}$ (see Figure 2), timber consumption decreases significantly in the short-run only for the carbon flow method, as this is the only method where rotation is adjusted. Therefore, it is optimal to preserve a fraction of the age class previously harvested, creating a shortage of timber in the market. For the other two methods, the impact on consumption is only due to changes in the forested area. Therefore, it is postponed, as changes in the distribution of land between age classes have impact only a cycle ahead.

Given each of the above time paths, it is possible to endogenously estimate the yearly impact on timber and land markets as well as on the amount of carbon sequestered (see Figure 4). The welfare gains of internalizing the social benefits of forests as carbon sinks

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[18] See tables and figures at the end of the paper.

[19] Here it is assumed $p_c = 0.468$ as the benchmark for comparing the three accounting methods; however, any other hypotheses can be easily implemented. In all simulations, initial land distribution is the steady-state of the model without carbon benefits.

[20] Following Salo and Tahvonen [28], in each iteration we use $t = 60$ as the period length. The number of iterations ranges from 200 to 500.

can then be obtained, either in terms of only steady-states' comparisons or also including the transition as in Table 3. They are positive in all cases when comparing only steady-state values, and are higher for the carbon flow method, as expected. The welfare gains reflect the increase in the social value of the timber market together with an increase in carbon benefits, net of the increase in the opportunity cost of land use. Despite that they are lower when the transition is included, they are still non-negative in all cases. However, note that the transition costs are higher in the carbon flow case (see Figure 3). This can be explained by the severe short run negative shock on timber markets due to the adjustment in the optimal rotation period (see Figures 3A-3C). In contrast, in the other two cases, the impacts on timber markets are smoother and postponed a rotation cycle, implying that the transition costs are lower (see Figure 3, 3D, and 3E). From a policy perspective this is a relevant result because the implementation of the first best solution presents the highest transition costs. Therefore, short run considerations may compromise the choice of the efficient solution.

Finally, we also conclude that, for the same price of carbon, the carbon flow accounting method has the larger impact on the additional amount of carbon sequestered when compared to the other methods (see Figure 4). This is due both to an increase in the optimal rotation period and in forested land. In contrast, the other two have smaller impacts in terms of additional carbon sequestered, because, for most carbon prices, the adjustment only occurs on the forest land (see Table 1).

In Table 1, the results of the sensitivity analysis to the price of carbon and to the value of θ are summarized. By inspection, we conclude that, in general, higher carbon prices increase both the optimal rotation period and land allocated to forest, increasing in all cases the amount of carbon sequestered relative to the case without carbon. The carbon flow method has the biggest effect in terms of carbon sequestration. Notice, in addition, that, by changing carbon prices, we can also obtain a carbon supply function for each carbon accounting method.

From a welfare point of view, an increase in carbon prices will always increase welfare gains. Given that the carbon flow is a first best solution, the welfare deviations of both the ton-year and the average storage from the optimal can be estimated, as presented in the Table 2. In general, welfare deviations of both second best methods increase as carbon prices increase. In the carbon flow case, changes in carbon prices induce adjustments both in the optimal rotation period and the optimal allocation of land. For low carbon prices, the average storage performs better than the ton-year, as the optimal rotation period is the same in both, while the average storage is closer to the carbon flow with respect to the optimal allocation of land (see Table 2, for $p_c = p_{ss}$ and $p_c = 2p_{ss}$). However, when carbon prices are high, the ton-year performs better, as it allows for adjustments in the optimal rotation (see Table 2, for $p_c = 6p_{ss}$).

An important issue that has also been the subject of discussion in the context of the use of forests as carbon sinks is related to the value of θ, that is, the amount of carbon stored in long-term structures. Considering now the impact of different values of θ, we observe that higher values of θ are always associated with larger amounts of land dedicated to forests, thus contributing to an increase in the total amount of carbon sequestered. However, since the optimal rotation period also adjusts, varying inversely with θ, the total amount of carbon sequestered on forest biomass is ambiguous (see Table 1, net cumulative biomass carbon), depending upon which effect dominates.

Finally, as the discount factor approaches one, for $\theta = 0$, that is, when no carbon is sequestered in long-term structures, the introduction of carbon benefits has no impact relative to the case without carbon, as in Tahvonen [25].

2.6 Conclusion

In this paper, the multiple vintage forest model developed by Salo and Tahvonen [20] is extended to internalize carbon sequestration benefits. All the adjustments occur through both optimal land allocation and the rotation period.

In order to compare the net effect of sequestration to that of avoided emissions three different carbon accounting methods are considered, namely, the carbon flow regime, the ton-year crediting and the average carbon storage. The carbon flow case is considered a first best solution because this accounting method fully internalizes at any point in time the carbon flows between forest and the atmosphere. In contrast to the one stand version of the model, typically representing the decision model of the private owner, both the price of timber and the price of land are endogenously determined. Therefore, only in this context, it is possible to study the transition path to the steady-state, and, more generally, comparative welfare analysis between the different methods can be performed. In addition, timber and carbon supply functions can also be estimated.

A full proof of the long-run optimality of steady-state forest is provided for all cases considered. Although the major theoretical results still apply, the extension to the presence of carbon sequestration benefits is not without consequences. First, in the corner solution situation where all land is forested land, optimal harvest is cyclical and the maximum radius cycle changes when compared to the case without carbon benefits, except in the average carbon storage case. Second, the optimal allocation area to forest will, in general, increase, as the net value from accounting carbon sequestration benefits is positive although the impacts differ with the accounting method used. Third, the optimal forest rotation period may or may not change depending on the value of carbon and on the accounting method considered. For all cases considered the changes in optimal rotation and in land allocation will determine the total amount of forest biomass carbon sequestration.

Formally, it is not possible to compare the impact of the different accounting methods both on the cycles dimension and on the optimal land allocation, because they are based on distinct parameters, θ, E_f, and C, respectively. However, numerically, depending on the values taken by the different parameters, comparisons can be undertaken.

Based on the numerical simulations, we conclude that the three accounting methods have distinct impacts on timber and land markets. Therefore, significant differences in social welfare paths are observed. One interesting result is that the carbon flow regime, a first best solution, is also the accounting method that generates the larger negative impact in the transition period, namely, in the short/medium run. Moreover, welfare deviations of both the ton-year and the average storage from the first-best increase as carbon prices increase. Whenever carbon prices induce minor changes in the optimal rotation period, the ton-year performs worse than the average storage; however, for high carbon prices, the ton-year performs better, as it allows for adjustments in the optimal rotation period.

Finally, it is not always the case that higher amounts of carbon sequestered in long-term structures necessarily generate increases in the total carbon sequestered in forest biomass.

To conclude, the theoretical and the simulation model developed in this paper can be a useful tool to study the impact of using forests as carbon sinks.

2.7 Appendix A

The necessary conditions for optimal solutions of the problem (2.2-2.8) and S_t^c given by (2.11), which can be derived from the Karush-Kuhn-Tucker conditions for all $t = 0, ...,$ are as follows:

$$b^t \frac{\partial L^c}{\partial x_{1,t+1}} = bf_1 U'(c_{t+1}) + f_1 p_c \beta - bf_1 p_c \beta(1-\theta) -$$

$$-bW'(y_{t+1}) - \lambda_t + bp_{1,t+1} \leq 0 \tag{2.23}$$

$$b^t \frac{\partial L^c}{\partial x_{s+1,t+1}} = -f_s U'(c_t) + bf_{s+1} U'(c_{t+1}) + (f_{s+1} - f_s) p_c \beta + f_s p_c \beta(1-\theta) -$$

$$-bf_{s+1} p_c \beta(1-\theta) - bW'(y_{t+1}) - \lambda_t + bp_{s+1,t+1} - p_{s,t} \leq 0 \tag{2.24}$$

for $s = 1, ..., n-2$

$$b^t \frac{\partial L^c}{\partial x_{n,t+1}} = -f_{n-1} U'(c_t) + bf_n U'(c_{t+1}) + (f_n - f_{n-1}) p_c \beta + f_{n-1} p_c \beta(1-\theta) -$$

$$-bf_n p_c \beta(1-\theta) - bW'(y_{t+1}) - \lambda_t - p_{n-1,t} \leq 0 \tag{2.25}$$

$$x_{s,t+1} \geq 0, x_{s,t+1} \frac{\partial L^c}{\partial x_{s,t+1}} = 0, s = 1, ..., n \tag{2.26}$$

$$p_{s,t} \geq 0, p_{s,t}(x_{s,t} - x_{s+1,t+1}) = 0, s = 1, ..., n-1 \tag{2.27}$$

$$\lambda_t \geq 0, \lambda_t (1 - \sum_{s=1}^{n} x_{s,t+1}) = 0 \tag{2.28}$$

The existence of optimal solutions for bounded utility and $b < 1$ follows from Theorem 4.6 in Stokey and Lucas (p. 79).

Proof of Proposition 1:

Proof. Following Salo and Tahvonen [20], by convexity of problem (2.2)-(2.8), if there exist multipliers $p_{s,t}$ satisfying conditions (2.23)-(2.28) under harvesting at m^c, then the resulting age class structure is an interior OCF. The optimality follows since with harvesting at m^c, $\frac{\partial U}{\partial x_{s,t}}$ and $x_{s,t}$ remain bounded satisfying transversality conditions which, together with (2.23)-(2.28) are sufficient for optimality.

For $s = 1, ..., m^c - 1$ using (2.23) to eliminate λ_t from (2.24) and (2.25), and to satisfy (2.26) we obtain a system of m^c x $(m^c - 1)$ equality equations:

$$b(p_{s+1,t+1+k} - p_{1,t+1+k}) - p_{s,t+k} = -b\left[U'(c_{t+k+1}) - \beta p_c(1-\theta)\right](f_{s+1} - f_1) +$$

$$+\left[U'(c_{t+k})-\beta p_c(1-\theta)\right]f_s-\beta p_c(f_{s+1}-f_s)+\beta p_c f_1 \tag{2.29}$$

$$-bp_{1,t+1+k}-p_{m^c-1,t+k}=-b\left[U'(c_{t+k+1})-\beta p_c(1-\theta)\right](f_{m^c}-f_1)+$$

$$+\left[U'(c_{t+k})-\beta p_c(1-\theta)\right]f_{m^c-1}-\beta p_c(f_{m^c}-f_{m^c-1})+\beta p_c f_1 \tag{2.30}$$

where $s=1,...m^c-2$, $k=0,...,m^c-1$.

This system is linear in the Lagrangian multipliers $p_{s,t+k}$, $s=1,...,m^c-1$, $k=0,...,m^c-1$ and solving for any multiplier yields:

$$p_{s,t}=\frac{b^{m^c}f_{m^c}}{1-b^{m^c}}\left[b^{-s}(U'(c_{t+m^c-s})-\beta p_c(1-\theta))-(U'(c_t)-\beta p_c(1-\theta))\right]-$$

$$-f_s\left[U'(c_t)-\beta p_c(1-\theta)\right]+A_s^c \tag{2.31}$$

where A_s^c is given by

$$A_s^c=\frac{\beta p_c}{1-b^{m^c}}\left[(1-b^s)\sum_{i=0}^{m^c-1}b^{i-s}\left(f_{i+1}-f_i\right)-\left(1-b^{m^c}\right)\sum_{i=0}^{s-1}b^{i-s}\left(f_{i+1}-f_i\right)\right] \tag{2.32}$$

for $s=1,...,m^c-1$, $t=0,....,$ as can be verified by direct substitution into the two equations above. Moreover, from (2.32), we observe that $A_{m^c}^c=0$, and that A_s^c decreases to zero as s increases to m^c. Condition (2.27) requires, for the indefinitely repeated cycle, that $p_{s,t+k}\geq 0$ for $s=1,...,m^c-1$, $k=0,...,m^c-1$. Thus, the fact that $x\in K$ implies by (2.31) that

$$\frac{U'(c_{t+k})-\beta p_c(1-\theta)}{U'(c_{t+k+m^c-j})-\beta p_c(1-\theta)}\leq\frac{b^{m^c-j}f_{m^c}}{f_j+b^{m^c}(f_{m^c}-f_j)}+$$

$$+\frac{A_j^c(1-b^{m^c})}{\left[U'(c_{t+k+m^c-j})-\beta p_c(1-\theta)\right]\left[f_j+b^{m^c}(f_{m^c}-f_j)\right]} \tag{2.33}$$

for $k=0,...,m^c-1$, $j=1,...,m^c-1$. Using (2.3) and the definition of optimal harvesting, we can write $c_{t+k}=f_{m^c}x_s$ and $c_{t+k+m^c-j}=f_{m^c}x_{s-m^c+j}$ where $s-m^c+j$ is understood as $s-j$, if $s-m^c+j\leq 0$. Equation (2.33) takes the form

$$\frac{U'(f_{m^c}x_s)-\beta p_c(1-\theta)}{U'(f_{m^c}x_{s-m+j})-\beta p_c(1-\theta)}\leq\frac{b^{m^c-j}f_{m^c}}{f_j+b^{m^c}(f_{m^c}-f_j)}+$$

$$+\frac{A_j^c(1-b^{m^c})}{\left[U'(f_{m^c}x_{s-m+j})-\beta p_c(1-\theta)\right]\left[f_j+b^{m^c}(f_{m^c}-f_j)\right]} \tag{2.34}$$

for $s=1,...,m^c$, $j=1,...,m^c-1$.

We now show that the right-hand side of (2.34) larger than one is equivalent to (2.12) for any $j < m^c$. By rearranging the right-hand side of (2.34) we obtain

$$U'(f_{m^c}x_{s-m^c+j})b^{m^c}f_{m^c}(1-b^j) - U'(f_{m^c}x_{s-m^c+j})b^j f_j(1-b^{m^c}) +$$

$$+\beta p_c b^j\left[(1-b^j)\sum_{i=0}^{m^c-1} b^{i-j}\left(f_{i+1}-f_i\right) - (1-b^{m^c})\sum_{i=0}^{j-1} b^{i-j}\left(f_{i+1}-f_i\right)\right] > 0$$

or

$$U'(f_{m^c}x_{s-m^c+j})b^{m^c}f_{m^c}(1-b^j) - U'(f_{m^c}x_{s-m^c+j})b^j f_j(1-b^{m^c}) +$$

$$+\beta p_c \left[(1-b^j)\sum_{i=0}^{m^c-1} b^{i}\left(f_{i+1}-f_i\right) - (1-b^{m^c})\sum_{i=0}^{j-1} b^{i}\left(f_{i+1}-f_i\right)\right] > 0 \qquad (2.35)$$

On the other hand, by reducing to the same denominator, (2.12) can be restated as (2.35). Therefore, if there exists a $m^c \gtreqless m$ such that (2.12) holds, the right-hand side of (2.34) is larger than one. Then, by the strict concavity of U, there must exist a $\phi > 0$, such that (2.34) is satisfied if $x_s = 1/m^c + \phi_s$, $s = 1, ..., m^c$, for all $|\phi_s| < \phi$, $\sum_{s=1}^{m^c} \phi_s = 0$, proving that optimal harvesting is cyclical harvesting and that it is not optimal to cut before m^c.

Similarly, for $s = m^c + 1, ..., n$, and $k = 0, ..., m^c - 1$, the optimality of the harvesting at m^c requires that land is not allocated to age classes $s = m^c + 1, ..., n$. Since $x_{m^c,t} > 0$ and $x_{m^c+1,t+1} = 0$ in (2.27), we obtain $p_{m^c,t} = 0$, for $t = 0, ...,$ and $p_{st} \geq 0$, for $s = m^c + 1, ..., n-1$, $t = 0, ...,$as can also be checked in (2.31). Using this and conditions (2.24) and (2.25), yields

$$b(p_{m^c+1,t+1+k} - p_{1,t+1+k}) - p_{m^c,t+k} \leq -b\left[U'(c_{t+k+1}) - \beta p_c(1-\theta)\right](f_{m^c+1} - f_1) +$$

$$+\left[U'(c_{t+k}) - \beta p_c(1-\theta)\right] f_{m^c} - \beta p_c(f_{m^c+1} - f_{m^c}) + \beta p_c f_1$$

$$b(p_{s+1,t+1+k} - p_{1,t+1+k}) - p_{s,t+k} \leq -b\left[U'(c_{t+k+1}) - \beta p_c(1-\theta)\right](f_{s+1} - f_1) +$$

$$+\left[U'(c_{t+k}) - \beta p_c(1-\theta)\right] f_s - \beta p_c(f_{s+1} - f_s) + \beta p_c f_1$$

for $s = m^c + 1, ..., n-2$, and

$$-bp_{1,t+1+k} - p_{n-1,t+k} \leq -b\left[U'(c_{t+k+1}) - \beta p_c(1-\theta)\right](f_n - f_1) +$$

$$+\left[U'(c_{t+k})-\beta p_c(1-\theta)\right]f_{n-1}-\beta p_c(f_n-f_{n-1})+\beta p_c f_1$$

where $k=0,...,m^c-1$. Using (2.31), by direct substitution we can show that the first two inequalities are satisfied as equalities. By eliminating $p_{1,t+1+k}$ and $p_{n-1,t+k}$ from the last inequality, using (2.31), and the facts that $c_{t+k+1}=f_{m^c}x_s$ and $c_{t+k+m^c-n+1}=f_{m^c}x_{s-m^c+n}$, we can write the last inequality above as follows:

$$\frac{U'(f_{m^c}x_s)-\beta p_c(1-\theta)}{U'(f_{m^c}x_{s-m^c+n})-\beta p_c(1-\theta)}\leq\frac{b^{m^c-n}f_{m^c}}{f_n+b^{m^c}(f_{m^c}-f_n)}+$$

$$+\frac{A_n^c(1-b^{m^c})}{\left[U'(c_{t+k+m^c-n})-\beta p_c(1-\theta)\right]\left[f_n+b^{m^c}(f_{m^c}-f_n)\right]}$$

for $s=m^c+1,...,n$.

The conditions $p_{s,t+k}\geq 0$ for $s=m^c+1,...,n-1$, $k=0,...,m^c-1$ together with the last inequality yield

$$\frac{U'(f_{m^c}x_s)-\beta p_c(1-\theta)}{U'(f_{m^c}x_{s-m^c+j})-\beta p_c(1-\theta)}\leq\frac{b^{m^c-n}f_{m^c}}{f_n+b^{m^c}(f_{m^c}-f_n)}$$

$$+\frac{A_n^c(1-b^{m^c})}{\left[U'(f_{m^c}x_{s-m^c+j})-\beta p_c(1-\theta)\right]\left[f_n+b^{m^c}(f_{m^c}-f_n)\right]} \tag{2.36}$$

for $s=1,...,m^c$,and $j=m^c+1,...,n$. Similarly, it is easy to show that the right-hand side of (2.36) larger than one is equivalent to (2.12).

Consequently, there exists a $\phi>0$ such that (2.36) is satisfied if $x_s=1/m^c+\phi_s$, $s=1,...,m^c$, $x_s=0$ for $s=m^c+1,...,n$, for all $|\phi_s|<\phi$, $\sum_{s=1}^{m^c}\phi_s=0$, and simultaneously it is never optimal to postpone harvest after m^c.

In addition, a stationary cycle with all land allocated to forestry must satisfy $\lambda_t\geq 0$, for $t=0,....$ Solving (2.23) or (2.24) for λ_t, eliminating $p_{s,t}$, for $s=1,...,m^c-1$, $t=0,...,$ using (2.31), we obtain

$$\lambda_{t+k}=\frac{\left[U'(c_{t+k})-\beta p_c(1-\theta)\right]b^{m^c}f_{m^c}}{1-b^{m^c}}-\frac{\left[U'(c_{t+1+k})-\beta p_c(1-\theta)\right]b^{m^c+1}f_{m^c}}{1-b^{m^c}}+$$

$$+\frac{\beta p_c}{1-b^{m^c}}\sum_{i=0}^{m^c-1}b^i(1-b)\left(f_{i+1}-f_i\right)-bW'(0)\geq 0 \tag{2.37}$$

for $s=1,...,m^c$, where $c_{t+1+m^c}=c_{t+1}$. Writing $c_{t+k}=f_{m^c}x_s$ and $c_{t+1+k}=f_{m^c}x_{s-1}$, $s=1,...,m^c$, where $x_0=x_{m^c}$ yields

$$\lambda_s=\frac{\left[U'(f_{m^c}x_s)-\beta p_c(1-\theta)\right]b^{m^c}f_{m^c}}{1-b^{m^c}}-\frac{\left[U'(f_{m^c}x_{s-1})-\beta p_c(1-\theta)\right]b^{m^c+1}f_{m^c}}{1-b^{m^c}}+$$

$$+\frac{\beta p_c}{1-b^{m^c}}\sum_{i=0}^{m^c-1} b^i(1-b)\left(f_{i+1}-f_i\right)-bW'(0)\geq 0 \tag{2.38}$$

for $s=1,...,m^c$.

Given $g=\frac{[U'(f_{m^c}/m^c)-\beta p_c(1-\theta)]b^{m^c}f_{m^c}}{1-b^{m^c}}+\frac{\beta p_c}{1-b^{m^c}}\sum_{i=0}^{m^c-1} b^i\left(f_{i+1}-f_i\right)-\frac{b}{1-b}W'(0)>0$, there must exist a $\phi>0$ such that (2.38) is satisfied if $x_s=1/m^c+\phi_s$, $s=1,...,m^c$, for all $|\phi_s|<\phi$, $\sum_{s=1}^{m^c}\phi_s=0$. ■

Let i_∞ represent the stationary state level of variable i.

We next show that if $g\leq 0$ there exists a stationary state that satisfies all the necessary conditions for optimality.[21]

Proof of Corollary 1:

Proof. Given $g\leq 0$, no solutions for (2.38) exist. Thus, by letting $\lambda_t=0$ in (2.23) or (2.24), eliminating $p_{s,t}$, $s=1,...,m^c-1$, $t=0,...$, using (2.31), and writing (2.23) analogously to (2.38), we obtain for $s=1,...,m^c$:

$$\frac{\left[U'(f_{m^c}x_s)-\beta p_c(1-\theta)\right]b^{m^c}f_{m^c}}{1-b^{m^c}}-\frac{\left[U'(f_{m^c}x_{s-1})-\beta p_c(1-\theta)\right]b^{m^c+1}f_{m^c}}{1-b^{m^c}}+$$

$$+\frac{\beta p_c}{1-b^{m^c}}\sum_{i=0}^{m^c-1} b^i(1-b)\left(f_{i+1}-f_i\right)-bW'(y_\infty)\geq 0 \tag{2.39}$$

This system is linear in $\left[U'(f_{m^c}x_s)-\beta p_c(1-\theta)\right]$, $s=1,...,m^c$ and its solution is given by:

$$\left[U'(f_{m^c}x_s)-\beta p_c(1-\theta)\right]+\frac{\beta p_c}{b^{m^c}f_{m^c}}\sum_{i=0}^{m^c-1} b^i\left(f_{i+1}-f_i\right)=\frac{W'(y_\infty)\sum_{i=0}^{m^c-1}b^i}{b^{m^c-1}f_{m^c}}, s=1,...,m^c \tag{2.40}$$

as can be verified by direct substitution. Thus, $x_s=(1-y_\infty)/m^c$, $s=1,...,m^c$ and optimal stationary cycles cannot exist. ■

Impact on the optimal rotation period:

We now show that for $\theta=0$, $m^c\geq m$:

At the steady-state, if there exists a $m^c\neq m$, for which

$$\frac{(p-p_c\beta)b^{m^c}f_{m^c}}{1-b^{m^c}}+\frac{p_c\beta\sum_{i=0}^{m^c-1}b^i\left(f_{i+1}-f_i\right)}{1-b^{m^c}}\geq\frac{(p-p_c\beta)b^m f_m}{1-b^m}+\frac{p_c\beta\sum_{i=0}^{m-1}b^i\left(f_{i+1}-f_i\right)}{1-b^m} \tag{2.41}$$

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[21] The results obtained in Salo and Tahvonen [20] regarding convergence and stability of the stationary steady states (Lemma 1 and Lemma 2, pg. 523) still apply in the case of this paper, as the difference equation for $x_{m_t^i}$, for $i=c,t,a$, is similar to equation (34), pg. 522, in the paper. The additional terms that are present in our case are independent of $x_{m_t^i}$. Therefore, the marginal conditions yielding the corresponding characteristic polynomials turn out to be similar.

holds, then it is optimal to cut at m^c, where $m^c \gtreqless m$. If (2.41), which is the same as (2.12), holds in particular for $m^c = m+1$, then $m^c \geq m$, while if it holds in particular for $m^c = m-1$, $m^c \leq m$. If (2.41) holds as an equality, $m^c = m$.

By making $m^c = m-1$, we show below that (2.41) never holds, implying that $m^c \geq m$, that is, it is optimal to postpone harvest. Also, for some $m^c = m+1$, (2.41) may be satisfied.

Let $m^c = m-1$ in (2.41). By rearranging and collecting terms we obtain

$$pb^{m-1}f_{m-1}(1-b^m) - pb^m f_m(1-b^{m-1}) >$$

$$\begin{gathered} p_c\beta(b^m - b^{m-1})\sum_{i=0}^{m-2} b^i(f_{i+1}-f_i) - p_c\beta b^m f_m(1-b^{m-1}) + \\ +p_c\beta b^{m-1}(f_m - f_{m-1})(1-b^{m-1}) + p_c\beta b^{m-1}f_{m-1}(1-b^m) \end{gathered} \tag{2.42}$$

which can be restated as

$$p_c\beta(b^m - b^{m-1})\sum_{i=0}^{m-2} b^i(f_{i+1}-f_i) - p_c\beta f_m(b^m - b^{m-1})(1-b^{m-1}) - p_c\beta f_{m-1}b^{m-1}(b^m - b^{m-1}) \tag{2.43}$$

and, finally, as

$$p_c\beta(b^m - b^{m-1})\left[\sum_{i=0}^{m-1} b^i f_{i+1} - \sum_{i=0}^{m-1} b^i f_i - f_m\right] \tag{2.44}$$

In (2.42) the left-hand side is negative. Since

$$\sum_{i=0}^{m-1} b^i f_{i+1} = \sum_{i=0}^{m-1} b^i(f_{i+1} - f_m) + \sum_{i=0}^{m-1} b^i f_{m-1}$$

substituting above we obtain

$$p_c\beta(b^m - b^{m-1})\left[\sum_{i=0}^{m-1} b^i(f_{i+1} - f_m) - f_m + \sum_{i=0}^{m-1} b^i(f_m - f_i)\right]$$

Given that $\{f_i\}$, for $i = 1, ..., m-1$, is an increasing sequence, and $b < 1$, we may conclude this expression is positive, as the sign of the algebraic sum inside the square brackets is negative. Consequently, the right-hand side of (2.42) is positive, implying that (2.42) never holds for any $m^c < m$. Also, we can show that it may hold for some $m^c > m$. Therefore, when $\theta = 0$, it is never optimal to cut earlier, that is, $m^c \geq m$.

In the case $\theta = 1$, we will consider two cases. In case (i) we assume that the sequence $\{f_i - f_{i-1}\}$ is a decreasing sequence. In case (ii) we only assume that the sequence $\{f_i\}$ is an increasing sequence.

(i) Let $m^c = m + 1$ in (2.41). By rearranging and collecting terms we obtain

$$pb^{m+1} f_{m+1}(1 - b^m) - pb^m f_m(1 - b^{m+1}) >$$

$$p_c\beta \left[- (f_{m+1} - f_m)(1 - b^m)b^m + \left(b^m - b^{m+1}\right) \sum_{i=0}^{m-1} b^i (f_{i+1} - f_i) \right] \tag{2.45}$$

In (2.42) the left-hand side is negative. Dividing (2.42) by $(1 - b^m)b^m$, the right-hand side can be stated as follows:

$$p_c\beta \left[- (f_{m+1} - f_m) + \frac{b^m - b^{m+1}}{(1 - b^m)b^m} \sum_{i=0}^{m-1} b^i (f_{i+1} - f_i) \right]$$

Since

$$\sum_{i=0}^{m-1} b^i (f_{i+1} - f_i) = \sum_{i=0}^{m-1} b^i \left[(f_{i+1} - f_i) - (f_{m+1} - f_m) \right] + \sum_{i=0}^{m-1} b^i (f_{m+1} - f_m)$$

implying that

$$\sum_{i=0}^{m-1} b^i (f_{i+1} - f_i) = \sum_{i=0}^{m-1} b^i \left[(f_{i+1} - f_i) - (f_{m+1} - f_m) \right] + \frac{1 - b^m}{1 - b} (f_{m+1} - f_m)$$

Substituting above we obtain

$$p_c\beta \left[\frac{b^m - b^{m+1}}{(1 - b^m)b^m} \left(\sum_{i=0}^{m-1} b^i \left[(f_{i+1} - f_i) - (f_{m+1} - f_m) \right] + \frac{1 - b^m}{1 - b} (f_{m+1} - f_m) \right) - (f_{m+1} - f_m) \right]$$

which can be restated as

$$p_c\beta \left[\frac{b^m - b^{m+1}}{(1 - b^m)b^m} \left(\sum_{i=0}^{m-1} b^i \left[(f_{i+1} - f_i) - (f_{m+1} - f_m) \right] \right) + \left(\frac{b^m - b^{m+1}}{(1 - b^m)b^m} \frac{1 - b^m}{1 - b} - 1 \right) (f_{m+1} - f_m) \right]$$

Given that $\{f_i - f_{i-1}\}$, for $i = 1, ..., m$, is a decreasing sequence, we may conclude that this expression is positive, as the term that multiplies $(f_{m+1} - f_m)$ vanishes. Consequently, the right-hand side of (2.42) is positive, implying that (2.42) never holds for any $m^c > m$. By inspection, we observe that it may hold for some $m^c < m$. Therefore, for $\theta = 1$, it is never optimal to postpone harvest, that is, $m^c \leq m$.

(ii) Let $m^c = m + 1$ in (2.41). By rearranging and collecting terms we obtain

$$pb^{m+1} f_{m+1}(1 - b^m) - pb^m f_m(1 - b^{m+1}) >$$

$$p_c\beta \left[- \left(f_{m+1} - f_m\right)(1 - b^m)b^m + \left(b^m - b^{m+1}\right) \sum_{i=0}^{m-1} b^i \left(f_{i+1} - f_i\right) \right] \tag{2.46}$$

Since

$$\sum_{i=0}^{m-1} b^i f_i = \sum_{i=0}^{m-1} b^i (f_i - f_{m+1}) + \sum_{i=0}^{m-1} b^i f_{m+1}$$

which can be rewritten as

$$\sum_{i=0}^{m-1} b^i f_i = \sum_{i=0}^{m-1} b^i (f_i - f_{m+1}) + \frac{1 - b^m}{1 - b} f_{m+1}$$

By substituting above, we obtain

$$p_c\beta \left[- \left(f_{m+1} - f_m\right)(1 - b^m)b^m \right] +$$

$$+ p_c\beta \left(b^m - b^{m+1}\right) \left(\sum_{i=0}^{m-1} b^i f_i - \sum_{i=0}^{m-1} b^i \left(f_i - f_{m+1}\right) - \frac{1 - b^m}{1 - b} f_{m+1} \right)$$

Finally, by collecting terms, we get

$$p_c\beta \left(-(1 - b^m)b^m - \left(b^m - b^{m+1}\right) \frac{1 - b^m}{1 - b} \right) f_{m+1} + p_c\beta f_m (1 - b^m) b^m$$

$$+ p_c\beta \left(b^m - b^{m+1}\right) \sum_{i=0}^{m-1} b^i f_i - p_c\beta \left(b^m - b^{m+1}\right) \sum_{i=0}^{m-1} b^i \left(f_i - f_{m+1}\right)$$

where the first-term is negative and the other three are positive. In particular, the last term is positive as long as $\{f_i\}$ is an increasing sequence for $i = 1, ..., m + 1$. Therefore, the sign of this expression, that is, the right-hand side of (2.46) can be either positive or negative. Since the left-hand side of (2.46) is negative, it may be optimal to postpone harvest. This is in contrast to case (i), in which by imposing a more restrictive assumption, namely, that the sequence $\{f_i - f_{i-1}\}$ is decreasing, it is never optimal to postpone harvest.

Let now $m^c = m-1$ in (2.41). By rearranging and collecting terms we obtain

$$pb^{m-1}f_{m-1}(1-b^m) - pb^m f_m(1-b^{m-1}) >$$

$$p_c\beta\left[(f_m - f_{m-1})\,(1-b^{m-1})b^{m-1} + \left(b^m - b^{m-1}\right)\sum_{i=0}^{m-1} b^i\,(f_{i+1}-f_i)\right] \tag{2.47}$$

which can be rewritten as

$$p_c\beta\left[(f_m - f_{m-1})\,(1-b^{m-1})b^{m-1} + \left(b^m - b^{m-1}\right)\left(\sum_{i=0}^{m-2} b^i(f_{i+1}-f_i)\right)\right]$$

Using the same procedure as before, we may write

$$\sum_{i=0}^{m-2} b^i\left[(f_{i+1}-f_i)-(f_m-f_{m-1})\right] = \sum_{i=0}^{m-2} b^i + \sum_{i=0}^{m-2} b^i\,(f_m - f_{m-1})$$

that is,

$$\sum_{i=0}^{m-2} b^i\left[(f_{i+1}-f_i)-(f_m-f_{m-1})\right] = \sum_{i=0}^{m-2} b^i\left[(f_{i+1}-f_i)-(f_m-f_{m-1})\right] + \frac{1-b^{m-1}}{1-b}(f_m - f_{m-1})$$

Substituting above and collecting terms, we obtain

$$p_c\beta\left(b^m - b^{m-1}\right)\left(\sum_{i=0}^{m-2} b^i\left[(f_{i+1}-f_i)-(f_m-f_{m-1})\right]\right) +$$

$$+p_c\beta\left(\left(b^m - b^{m-1}\right)\frac{1-b^{m-1}}{1-b} + (1-b^{m-1})b^{m-1}\right)(f_m - f_{m-1})$$

Therefore, the right-hand side of (2.47) can be rewritten as the algebraic sum of the two above terms. The first term is negative, as long as $\{f_i - f_{i-1}\}$ is a decreasing sequence for $i = 1,...,m-1$, while the second one is positive, as the term that multiplies $(f_m - f_{m-1})$ is positive. Therefore, it may be optimal to cut earlier than m. The same result is obtained if, instead, we consider a less restrictive assumption such that $\{f_i\}$ is an increasing sequence for $i = 1,...,m+1$.

2.8 Appendix B

The Karush-Kuhn-Tucker conditions for optimal solutions of problem (2.2-2.8) and S_t^t given by (2.15) for all $t = 0, ...,$ as follows:

$$b^t \frac{\partial L^t}{\partial x_{1,t+1}} = bf_1 U'(c_{t+1}) - bW'(y_{t+1}) - \lambda_t + bp_{1,t+1} \leq 0 \tag{2.48}$$

$$b^t \frac{\partial L^t}{\partial x_{s+1,t+1}} = -f_s U'(c_t) + bf_{s+1} U'(c_{t+1}) + f_s p_c \beta E_f -$$

$$-bW'(y_{t+1}) - \lambda_t + bp_{s+1,t+1} - p_{s,t} \leq 0 \tag{2.49}$$

for $s = 1, ..., n-2,$

$$b^t \frac{\partial L^t}{\partial x_{n,t+1}} = -f_{n-1} U'(c_t) + bf_n U'(c_{t+1}) + f_{n-1} p_c \beta E_f - bW'(y_{t+1}) - \lambda_t - p_{n-1,t} \leq 0 \tag{2.50}$$

$$x_{s,t+1} \geq 0, x_{s,t+1} \frac{\partial L^t}{\partial x_{s,t+1}} = 0, s = 1, ..., n \tag{2.51}$$

$$p_{s,t} \geq 0, p_{s,t}(x_{s,t} - x_{s+1,t+1}) = 0, s = 1, ..., n-1 \tag{2.52}$$

$$\lambda_t \geq 0, \lambda_t (1 - \sum_{s=1}^{n} x_{s,t+1}) = 0 \tag{2.53}$$

The existence of optimal solutions for bounded utility and $b < 1$ follows from Theorem 4.6 in Stokey and Lucas (p.79).

Proof of Proposition 2:

Proof. Based on this new formulation, using a similar procedure as used to prove Proposition 1, for $s = 1, ..., m^t - 1$, using (2.48) to eliminate λ_t from (2.49) and (2.50), and to satisfy (2.51) we obtain a system of m^t x $(m^t - 1)$ equality equations:

$$b(p_{s+1,t+1+k} - p_{1,t+1+k}) - p_{s,t+k} = -bU'(c_{t+k+1})(f_{s+1} - f_1) + \left[U'(c_{t+k}) - \beta p_c E_f\right] f_s \tag{2.54}$$

$$-bp_{1,t+1+k} - p_{m^t-1,t+k} = -bU'(c_{t+k+1})(f_{m^t} - f_1) + \left[U'(c_{t+k}) - \beta p_c E_f\right] f_{m^t-1} \tag{2.55}$$

where $s = 1, ..., m^t - 2$, $k = 0, ..., m^t - 1$. This system is linear in the Lagrangian multipliers $p_{s,t+k}$, $s = 1, ..., m^t - 1$, $k = 0, ..., m^t - 1$. Solving for any multiplier yields

$$p_{s,t} = \frac{b^{m^t} f_{m^t}}{1 - b^{m^t}} \left[b^{-s} U'(c_{t+m^t-s}) - U'(c_t)\right] - f_s U'(c_t) + A_s^t \tag{2.56}$$

where A_s^t is given by

$$A_s^t = \frac{\beta p_c E_f}{1-b^{m^t}} \left[(1-b^s) \sum_{i=1}^{m^t-1} b^{i-s} f_i - (1-b^{m^t}) \sum_{i=1}^{s-1} b^{i-s} f_i \right] \tag{2.57}$$

for $s = 1, ..., m^t - 1$, $t = 0, ...$, as can be verified by direct substitution into the two equations above. Moreover, from (2.57), we observe that $A_{m^t}^t = 0$.

Condition (2.52) requires, for the indefinitely repeated cycle, that $p_{s,t+k} \geq 0$ for $s = 1, ..., m^t - 1$, $k = 0, ..., m^t - 1$. Thus, the fact that $x \in K$ implies by (2.56) that

$$\frac{U'(c_{t+k})}{U'(c_{t+k+m^t-j})} \leq \frac{b^{m^t-j} f_{m^t}}{f_j + b^{m^t}(f_{m^t} - f_j)} + \frac{A_j^t(1-b^{m^t})}{\left[f_j + b^{m^t}(f_{m^t} - f_j)\right] U'(c_{t+k+m^t-j})} \tag{2.58}$$

for $k = 0, ..., m^t - 1$, $j = 1, ..., m^t - 1$, where A_j^t is given by (2.57). Using (2.3) and the definition of carbon harvesting, we can write $c_{t+k} = f_{m^t} x_s$ and $c_{t+k+m^t-j} = f_{m^t} x_{s-m^t+j}$ where $s - m^t + j$ is understood as $s - j$, if $s - m^t + j \leq 0$. Equation (2.58) takes the form

$$\frac{U'(f_{m^t} x_s)}{U'(f_{m^t} x_{s-m^t+j})} \leq \frac{b^{m^t-j} f_{m^t}}{f_j + b^{m^t}(f_{m^t} - f_j)} + \frac{A_j^t(1-b^{m^t})}{\left[f_j + b^{m^t}(f_{m^t} - f_j)\right] U'(f_{m^t} x_{s-m^t+j})} \tag{2.59}$$

or, alternatively,

$$\frac{U'(f_{m^t} x_s)}{U'(f_{m^t} x_{s-m^t+j})} \leq \frac{b^{m^t-j} f_{m^t}}{f_j + b^{m^t}(f_{m^t} - f_j)} + \frac{A_j^t(1-b^{m^t})}{U'(f_{m^t} x_{s-m^t+j}) b^{m^t-j} f_{m^t}} \tag{2.60}$$

for $s = 1, ..., m^t$, $j = 1, ..., m^t - 1$.

We now show that the right-hand side of (2.60) larger than one is equivalent to (2.16) for any $j < m^t$. By rearranging the right-hand side of (2.60), we obtain

$$U'(f_{m^t} x_{s-m^t+j}) b^{m^t} f_{m^t}(1-b^j) - U'(f_{m^t} x_{s-m^t+j}) b^j f_j (1-b^{m^t}) +$$

$$+\beta p_c E_f b^j \left[(1-b^j) \sum_{i=1}^{m^t-1} b^{i-j} f_i - (1-b^{m^t}) \sum_{i=1}^{j-1} b^{i-j} f_i \right] > 0$$

or

$$U'(f_{m^t} x_{s-m^t+j}) b^{m^t} f_{m^t}(1-b^j) - U'(f_{m^t} x_{s-m^t+j}) b^j f_j (1-b^{m^t}) +$$

$$+\beta p_c E_f \left[(1-b^j) \sum_{i=1}^{m^t-1} b^i f_i - (1-b^{m^t}) \sum_{i=1}^{j-1} b^i f_i \right] > 0 \tag{2.61}$$

On the other hand, by reducing to the same denominator, (2.16) can be restated as (2.61). Therefore, using a similar reasoning as in the previous case, we conclude that not only it is not optimal to cut earlier than m^t, but also, by the strict concavity of U, there must exist a $\phi > 0$, such that (2.59) is satisfied if $x_s = 1/m^t + \phi_s$, $s = 1, ..., m^t$, for all $|\phi_s| < \phi$, $\sum_{s=1}^{m^t} \phi_s = 0$.

Similarly, we now derive the results for $s = m^t + 1, ..., n$, and $k = 0, ..., m^t - 1$. For the cases $m^t < n$, the optimality of the carbon harvesting requires that land is not allocated to age classes $s = m^t + 1, ..., n$. Since $x_{m^t t} > 0$ and $x_{m^t+1,t+1} = 0$ in (2.52), we obtain $p_{m^t t} = 0$, for $t = 0, ...$, as can be checked in (2.56). Using this and conditions (2.49), (2.50), and $p_{st} \geq 0$, for $s = m^t + 1, ..., n - 1$, $t = 0, ...$, yields

$$b(p_{m^t+1,t+1+k} - p_{1,t+1+k}) - p_{m^t,t+k} \leq -bU'(c_{t+k+1})(f_{m^t+1} - f_1) + [U'(c_{t+k}) - \beta p_c E_f]\, f_{m^t} \tag{2.62}$$

$$b(p_{s+1,t+1+k} - p_{1,t+1+k}) - p_{s,t+k} \leq -bU'(c_{t+k+1})(f_{s+1} - f_1) + [U'(c_{t+k}) - \beta p_c E_f]\, f_s$$

for $s = m^t + 1, ..., n - 2$, and

$$-bp_{1,t+1+k} - p_{n-1,t+k} \leq -bU'(c_{t+k+1})(f_n - f_1) + [U'(c_{t+k}) - \beta p_c E_f]\, f_{n-1} \tag{2.63}$$

where $k = 0, ..., m^t - 1$. Using (2.56), by direct substitution we can show that the first two inequalities are satisfied as equalities. By eliminating $p_{1,t+1+k}$ and $p_{n-1,t+k}$ from the last inequality, using (2.56), and the facts that $c_{t+k+1} = f_{m^t} x_s$ and $c_{t+k+m^t-n+1} = f_{m^t} x_{s-m^t+n}$, we can write the last inequality above as follows:

$$\frac{U'(f_{m^t} x_s)}{U'(f_{m^t} x_{s-m+n})} \leq \frac{b^{m^t-n} f_{m^t}}{f_n + b^{m^t}(f_{m^t} - f_n)} + \frac{(1 - b^{m^t})A_n^t}{[f_n + b^{m^t}(f_{m^t} - f_n)]\, U'(f_{m^t} x_{s-m^t+n})}$$

for $s = 1, ..., n$.

The conditions $p_{s,t+k} \geq 0$ for $s = m^t + 1, ..., n - 1$, $k = 0, ..., m^t - 1$ together with the last inequality yield

$$\frac{U'(f_{m^t} x_s)}{U'(f_{m^t} x_{s-m+j})} \leq \frac{b^{m^t-j} f_{m^t}}{f_j + b^{m^t}(f_{m^t} - f_j)} + \frac{(1 - b^{m^t})A_j^t}{[f_j + b^{m^t}(f_{m^t} - f_j)]\, U'(f_{m^t} x_{s-m^t+j})} \tag{2.64}$$

for $s = 1, ..., m^t$, and $j = m^t + 1, ..., n$. Similarly, it is easy to show that the right-hand side of (2.64) larger than one is equivalent to (2.16).

Consequently, there exists a $\phi > 0$ such that (2.64) is satisfied if $x_s = 1/m^t + \phi_s$, $s = 1, ..., m^t$, $x_s = 0$ for $s = m^t + 1, ..., n$, for all $|\phi_s| < \phi$, $\sum_{s=1}^{m^t} \phi_s = 0$, and simultaneously it is never optimal to postpone harvest after m^t.

In addition, a stationary cycle with all land allocated to forestry must satisfy $\lambda_t \geq 0$, for $t = 0, ...$. Solving (2.48) or (2.49) for λ_t, eliminating $p_{s,t}$, $s = 1, ..., m^t - 1$, $t = 0, ...$, using (2.56), we obtain

$$\lambda_{t+k} = \frac{U'(c_{t+k}) b^{m^t} f_{m^t}}{1 - b^{m^t}} - \frac{U'(c_{t+1+k}) b^{m^t+1} f_{m^t}}{1 - b^{m^t}} + \frac{\beta p_c E_f}{1 - b^{m^t}} \sum_{i=1}^{m^t-1} b^i (1 - b) f_i - bW'(0) \geq 0 \tag{2.65}$$

for $s = 1, ..., m^t$, where $c_{t+1+m^t} = c_{t+1.}$ Writing $c_{t+k} = f_{m^t}x_s$ and $c_{t+1+k} = f_{m^t}x_{s-1}$, $s = 1, ..., m^t$, where $x_0 = x_{m^t}$ yields

$$\lambda_s = \frac{U'(f_{m^t}x_s)b^{m^t}f_{m^t}}{1-b^{m^t}} - \frac{U'(f_{m^t}x_{s-1})b^{m^t+1}f_{m^t}}{1-b^{m^t}} + \frac{\beta p_c E_f}{1-b^{m^t}} \sum_{i=1}^{m^t-1} b^i(1-b)f_i - bW'(0) \geq 0 \tag{2.66}$$

for $s = 1, ..., m^t$.

Given $g = \frac{U'(f_{m^t}/m^t)b^{m^t}f_{m^t}}{1-b^{m^t}} + \frac{\beta p_c E_f}{1-b^{m^t}} \sum_{i=1}^{m^t-1} b^i f_i - \frac{b}{1-b}W'(0) > 0$, there must exist a $\phi > 0$ such that (2.66) is satisfied if $x_s = 1/m^t + \phi_s$, $s = 1, ..., m^t$, for all $|\phi_s| < \phi$, $\sum_{s=1}^{m^t} \phi_s = 0$. ■

Proof of Corollary 2:

Proof. Given $g \leq 0$, no solutions for (2.66) exist. Thus, by letting $\lambda_t = 0$ in (2.48) or (2.49), eliminating $p_{s,t}$, $s = 1, ..., m^t - 1$, $t = 0, ...$, using (2.56), and writing (2.48) analogously to (2.66), we obtain, for $s = 1, ..., m^t$.:

$$\frac{U'(f_{m^t}x_s)b^{m^t}f_{m^t}}{1-b^{m^t}} - \frac{U'(f_{m^t}x_{s-1})b^{m^t+1}f_m}{1-b^{m^t}} + \frac{\beta p_c E_f}{1-b^{m^t}} \sum_{i=1}^{m^t-1} b^i(1-b)f_i - bW'(y_\infty) \geq 0 \tag{2.67}$$

This system is linear in $U'(f_{m^t}x_s)$, $s = 1, ..., m^t$. Its solution is given by

$$U'(f_{m^t}x_s) + \frac{p_c\beta E_f \sum_{i=1}^{m^t-1} b^i f_i}{b^{m^t}f_{m^t}} = \frac{W'(y_\infty)\sum_{i=0}^{m^t-1} b^i}{b^{m^t-1}f_{m^t}}, \ s = 1, ..., m^t \tag{2.68}$$

as can be verified by direct substitution. Thus, $x_s = (1 - y_\infty)/m^t$, $s = 1, ..., m^t$ and optimal stationary cycles cannot exist. ■

Impact on the optimal rotation period:

We now show that $m^t \geq m$:

At the steady-state, if there exists a $m^t \neq m$, for which

$$\frac{pb^{m^t}f_{m^t}}{1-b^{m^t}} + \frac{p_c\beta E_f \sum_{i=1}^{m^t-1} b^i f_i}{1-b^{m^t}} \geq \frac{pb^m f_m}{1-b^m} + \frac{p_c\beta E_f \sum_{i=1}^{m-1} b^i f_i}{1-b^m} \tag{2.69}$$

holds, then it is optimal to cut at m^t, where $m^t \gtreqless m$. If (2.69), which is the same as (2.16), holds in particular for $m^t = m + 1$, then $m^t \geq m$, while if it holds in particular for $m^t = m - 1$, $m^t \leq m$. If (2.69) holds as an equality, $m^t = m$.

By making $m^t = m - 1$, we show below that (2.69) never holds, implying that $m^t \geq m$, that is, it is optimal to postpone harvest. Also, for $m^t = m + 1$, (2.69) can be satisfied.

Let $m^t = m - 1$ in (2.69). By rearranging and collecting terms we obtain

$$pb^{m-1}f_{m-1}(1-b^m) - pb^m f_m(1-b^{m-1}) >$$

$$p_c\beta E_f\left[(b^m - b^{m-1})\sum_{i=1}^{m-2} b^i f_i + (1-b^{m-1})b^{m-1}f_{m-1}\right] \tag{2.70}$$

In (2.70) the left-hand side is negative. Dividing (2.70) by $b^{m-1}(1-b^{m-1})$, and since $b = \frac{1}{1+r}$, we have that

$$\frac{1}{b^{m-1}}\frac{b^m - b^{m-1}}{1-b^{m-1}} = b\left[-\frac{r}{1-b^{m-1}}\right]$$

where the term in square brackets is the equivalent in discrete time to the discounting term $\frac{r}{1-e^{-rT}}$ in the continuous time, as long as $e^r \cong 1+r$, and $T = m-1$. Therefore, the right-hand side of (2.70) becomes

$$P_c\beta E_f\left[b\left[\frac{r}{1-b^{m-1}}\right]\sum_{i=1}^{m-2} -b^i f_i + f_{m-1}\right]$$

Since

$$\sum_{i=1}^{m-2} b^i f_i = -b\frac{1-b^{m-1}}{r}f_{m-2} + \sum_{i=1}^{m-2} b^i(f_i - f_{m-2})$$

we obtain

$$P_c\beta E_f\left[\frac{-br}{1-b^{m-1}}\sum_{i=1}^{m-2} b^i(f_i - f_{m-2}) + \left(f_{m-1} - \frac{b-b^{m-1}}{1-b^{m-1}}f_{m-2}\right)\right] \tag{2.71}$$

Since $\{f_i\}$ is an increasing sequence, we may conclude that $f_i - f_{m-2} \leq 0$ for $i = 1, ..., m-2$. Since $\frac{b-b^{m-1}}{1-b^{m-1}} < 1$, the second-term in the expression above is positive. This implies that the first term of the right-hand side of (2.70) is positive. Notice that if $\{f_i\}$ is strictly increasing and for $m \geq 2$, then it is strictly positive. Since the second term is also positive, the inequality never holds. Moreover, since (2.70) can be satisfied for $m^t = m+1$, this implies that $m^t \geq m$. This is also similar to the result obtained in the continuous version of the one stand model, as shown in Costa-Duarte, Cunha-e-Sá and Rosa [4].

2.9 Appendix C

The Karush-Kuhn-Tucker conditions for optimal solutions of problem (2.2-2.8), S_t^a given by (2.19), for all $t = 0, ...,$ as follows:

$$b^t\frac{\partial L^a}{\partial x_{1,t+1}} = bf_1U\acute{}(c_{t+1}) - bW'(y_{t+1}) - \lambda_t + bp_{1,t+1} + bD \leq 0 \tag{2.72}$$

$$b^t \frac{\partial L^a}{\partial x_{s+1,t+1}} = -f_s U'(c_t) + b f_{s+1} U'(c_{t+1}) + bD -$$

$$-bW'(y_{t+1}) - \lambda_t + b p_{s+1,t+1} - p_{s,t} \leq 0 \tag{2.73}$$

for $s = 1, ..., n-2$,

$$b^t \frac{\partial L^a}{\partial x_{n,t+1}} = -f_{n-1} U'(c_t) + b f_n U'(c_{t+1}) + bD - bW'(y_{t+1}) - \lambda_t - p_{n-1,t} \leq 0 \tag{2.74}$$

$$x_{s,t+1} \geq 0, x_{s,t+1} \frac{\partial L^a}{\partial x_{s,t+1}} = 0, s = 1, ..., n \tag{2.75}$$

$$p_{s,t} \geq 0, p_{s,t}(x_{s,t} - x_{s+1,t+1}) = 0, s = 1, ..., n-1 \tag{2.76}$$

$$\lambda_t \geq 0, \lambda_t (1 - \sum_{s=1}^{n} x_{s,t+1}) = 0 \tag{2.77}$$

The existence of optimal solutions for bounded utility and $b < 1$ follows from Theorem 4.6 in Stokey and Lucas (p.79).

Proof of Proposition 3:

Proof. Following Salo and Tahvonen [20], for $s = 1, ..., m^a - 1$ using (2.72) to eliminate λ_t from (2.73) and (2.74), and to satisfy (2.75) we obtain a system of m^a x $(m^a - 1)$ equality equations

$$b(p_{s+1,t+1+k} - p_{1,t+1+k}) - p_{s,t+k} = -bU'(c_{t+k+1})(f_{s+1} - f_1) + f_s U'(c_{t+k}) \tag{2.78}$$

$$-b p_{1,t+1+k} - p_{m^a-1,t+k} = -bU'(c_{t+k+1})(f_{m^a} - f_1) + f_{m^a-1} U'(c_{t+k}) \tag{2.79}$$

where $s = 1, ..., m^a - 2$, $k = 0, ..., m^a - 1$,.

This system is linear in the Lagrangian multipliers $p_{s,t+k}$, $s = 1, ..., m^a - 1$, $k = 0, ..., m^a - 1$. Solving for any multiplier yields

$$p_{s,t} = \frac{b^{m^a} f_{m^a}}{1 - b^{m^a}} \left[b^{-s} U'(c_{t+m^a-s}) - U'(c_t) \right] - f_s U'(c_t) \tag{2.80}$$

for $s = 1, ..., m^a - 1$, $t = 0, ...,$ as can be verified by direct substitution into the two equations above. condition (2.76) requires, for the indefinitely repeated cycle, that $p_{s,t+k} \geq 0$ for $s = 1, ..., m^a - 1$, $k = 0, ..., m^a - 1$. Thus, the fact that $x \in K$ implies by (2.80) that

$$\frac{U'(c_{t+k})}{U'(c_{t+k+m^a-j})} \leq \frac{b^{m^a-j} f_m}{f_j + b^{m^a}(f_{m^a} - f_j)} \tag{2.81}$$

for $k = 0, ..., m^a - 1$, $j = 1, ..., m^a - 1$. Using (2.3) and the definition of carbon harvesting, we can write $c_{t+k} = f_{m^a} x_s$ and $c_{t+k+m^a-j} = f_{m^a} x_{s-m^a+j}$ where $s - m^a + j$ is understood as $s - j$, if $s - m^a + j \leq 0$. Equation (2.81) takes the form

$$\frac{U'(f_{m^a} x_s)}{U'(f_{m^a} x_{s-1})} \leq \eta_j^a \equiv \frac{b^{m^a - j} f_{m^a}}{f_j + b^{m^a}(f_{m^a} - f_j)} \tag{2.82}$$

for $s = 1, ..., m^a$, $j = 1, ..., m^a - 1$. Since $\eta_j^a > 1$, the right-hand side of (2.82) is larger than one. Moreover, this is equivalent to (2.20), as can be easily checked. Then, by the strict concavity of U, there must exist a $\phi > 0$, such that (2.82) is satisfied if $x_s = 1/m^a + \phi_s$, $s = 1, ..., m^a$, for all $|\phi_s| < \phi$, $\sum_{s=1}^{m^a} \phi_s = 0$.

Similarly, results can be derived for $s = m^a + 1, ..., n$, and $k = 0, ..., m^a - 1$. Following the previous cases, we can show that a similar condition to (2.82) can be obtained for $s = m^a, ..., n$, $j = m^a + 1, ..., n$,

$$\frac{U'(f_{m^a} x_s)}{U'(f_{m^a} x_{s-m^a+j})} \leq \eta_j^a \equiv \frac{b^{m^a - j} f_{m^a}}{f_j + b^{m^a}(f_{m^a} - f_j)} \tag{2.83}$$

to which all we have shown above for $s = 1, ..., m^a$, $j = 1, ..., m^a - 1$, still applies. Consequently, there exists a $\phi > 0$ such that (2.83) is satisfied if $x_s = 1/m^a + \phi_s$, $s = 1, ..., m^a$, $x_s = 0$ for $s = m^a + 1, ..., n$, for all $|\phi_s| < \phi$, $\sum_{s=1}^{m^a} \phi_s = 0$. Moreover, $m^a = m$, as it can be easily observed by comparing (2.20) with (2.10).

In addition, a stationary cycle with all land allocated to forestry must satisfy $\lambda_t \geq 0$, for $t = 0, ...$. Solving (2.72) or (2.73) for λ_t, eliminating $p_{s,t}$, $s = 1, ..., m^a - 1$, $t = 0, ...$, using (2.80), we obtain

$$\lambda_{t+k} = \frac{U'(c_{t+k}) b^{m^a} f_{m^a}}{1 - b^{m^a}} - \frac{U'(c_{t+1+k}) b^{m^a+1} f_{m^a}}{1 - b^{m^a}} + bD - bW'(0) \geq 0 \tag{2.84}$$

for $s = 1, ..., m^a$, where $c_{t+1+m^a} = c_{t+1}$. Writing $c_{t+k} = f_{m^a} x_s$ and $c_{t+1+k} = f_{m^a} x_{s-1}$, $s = 1, ..., m^a$, where $x_0 = x_{m^a}$ yields

$$\lambda_s = \frac{U'(f_{m^a} x_s) b^{m^a} f_{m^a}}{1 - b^{m^a}} - \frac{U'(f_{m^a} x_{s-1}) b^{m^a+1} f_m}{1 - b^{m^a}} + bD - bW'(0) \geq 0 \tag{2.85}$$

for $s = 1, ..., m^a$.

Given $g = \frac{U'(f_{m^a}/m^a) b^{m^a} f_{m^a}}{1 - b^{m^a}} + \frac{b}{1-b} D - \frac{b}{1-b} W'(0) > 0$, there must exist a $\phi > 0$ such that (2.85) is satisfied if $x_s = 1/m^a + \phi_s$, $s = 1, ..., m^a$, for all $|\phi_s| < \phi$, $\sum_{s=1}^{m^a} \phi_s = 0$. ■

Proof of Corollary 3:

Proof. Given $g \leq 0$, no solutions for (2.85) exist. Thus, by letting $\lambda_t = 0$ in (2.72) or (2.73), eliminating $p_{s,t}$, $s = 1, ..., m^a - 1$, $t = 0, ...$, using (2.80), and writing (2.72) analogously to (2.85), obtaining:

$$\frac{U'(f_{m^a} x_s) b^{m^a} f_{m^a}}{1 - b^{m^a}} - \frac{U'(f_{m^a} x_{s-1}) b^{m^a+1} f_{m^a}}{1 - b^{m^a}} + bD - bW'(y_\infty) \geq 0 \tag{2.86}$$

for $s = 1, ..., m^a$.

This system is linear in $U'(f_{m^a} x_s)$, $s = 1, ..., m^a$. Its solution is given by

$$U'(f_{m^a} x_s) + \frac{\sum_{i=0}^{m^a-1} b^i}{b^{m^a-1} f_{m^a}} D = \frac{W'(y_\infty) \sum_{i=0}^{m^a-1} b^i}{b^{m^a-1} f_{m^a}}, s = 1, ..., m^a \tag{2.87}$$

as can be verified by direct substitution. Thus, $x_s = (1 - y_\infty)/m^a$, $s = 1, ..., m^a$ and optimal stationary cycles cannot exist. ■

2.10 Appendix D

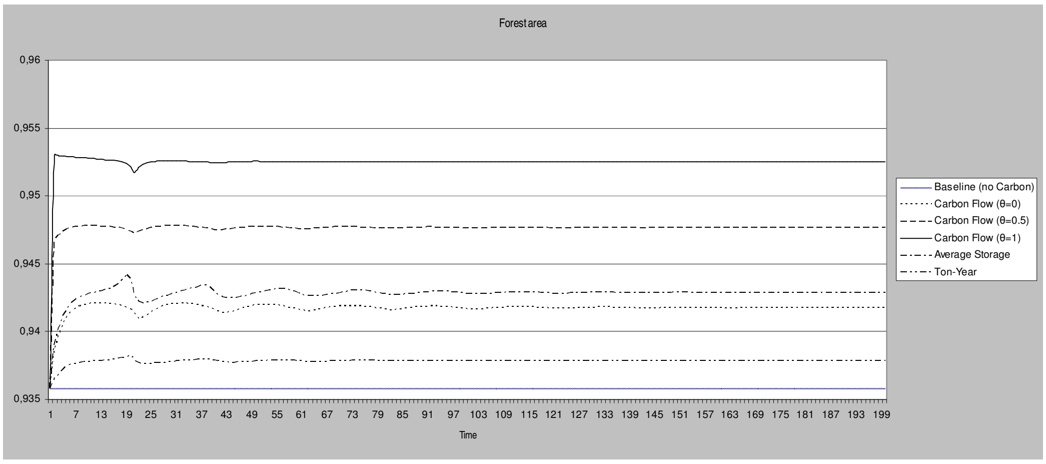

Figure 1. Forest Area

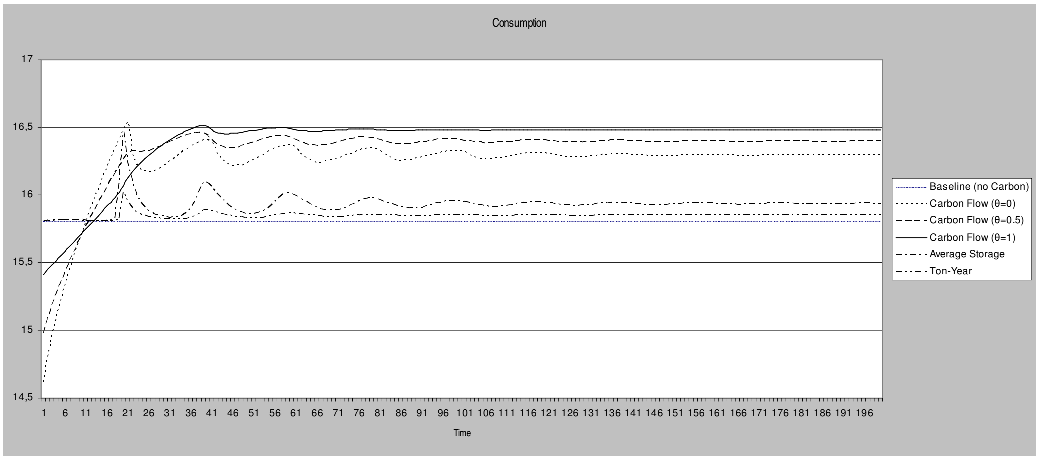

Figure 2. Timber Consumption

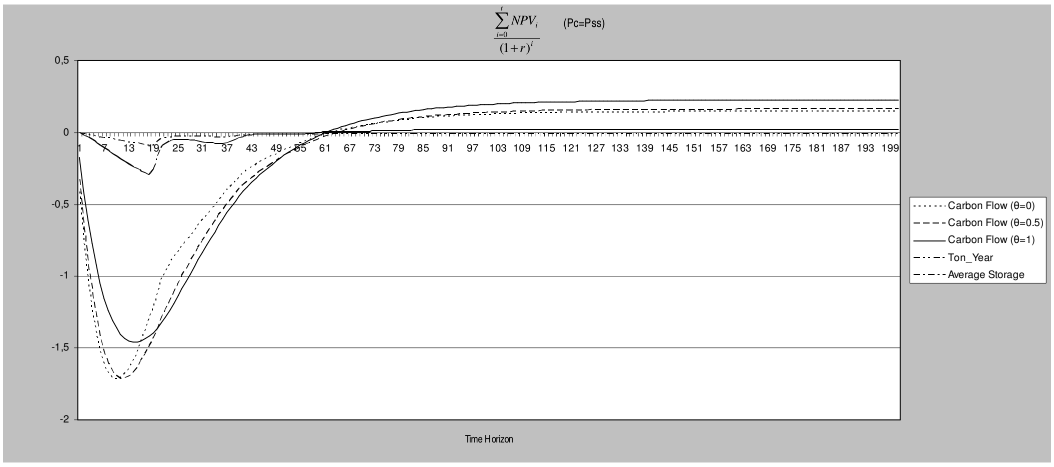

Figure 3. Cumulative Net Present Value

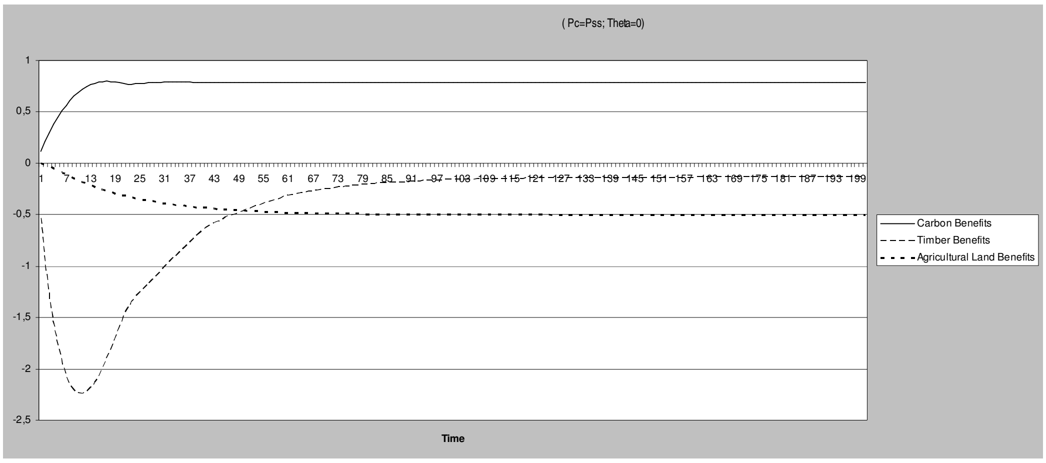

Figure 3a. Cumulative Net Present Value (components) – Carbon Flow Method

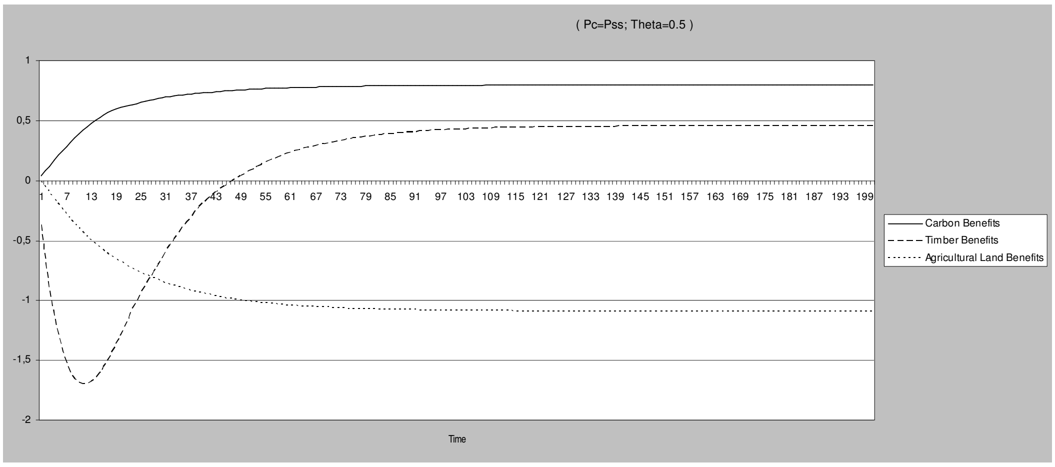

Figure 3b. Cumulative Net Present Value (components) – Carbon Flow Method

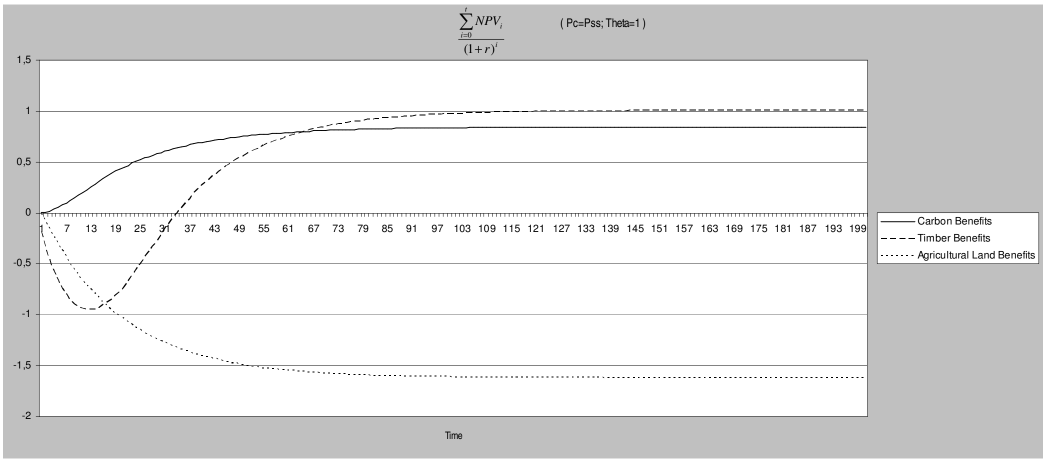

Figure 3c. Cumulative Net Present Value (components) – Carbon Flow Method

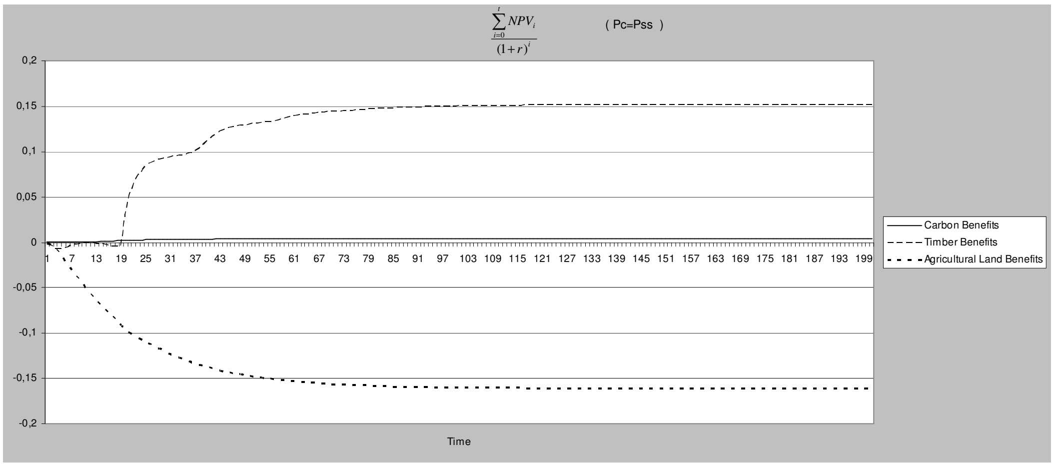

Figure 3d. Cumulative Net Present Value (components) – Ton-Year Method

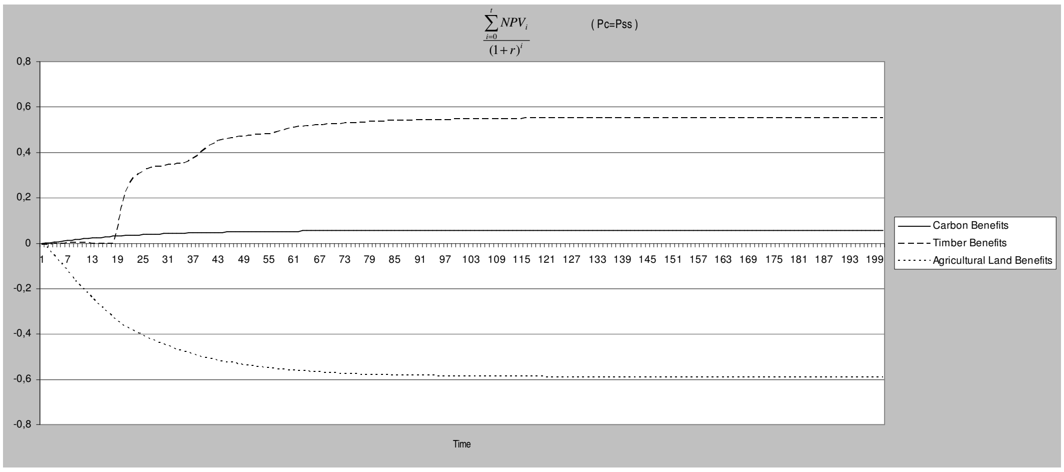

Figure 3e. Cumulative Net Present Value (components) – Average Storage Method

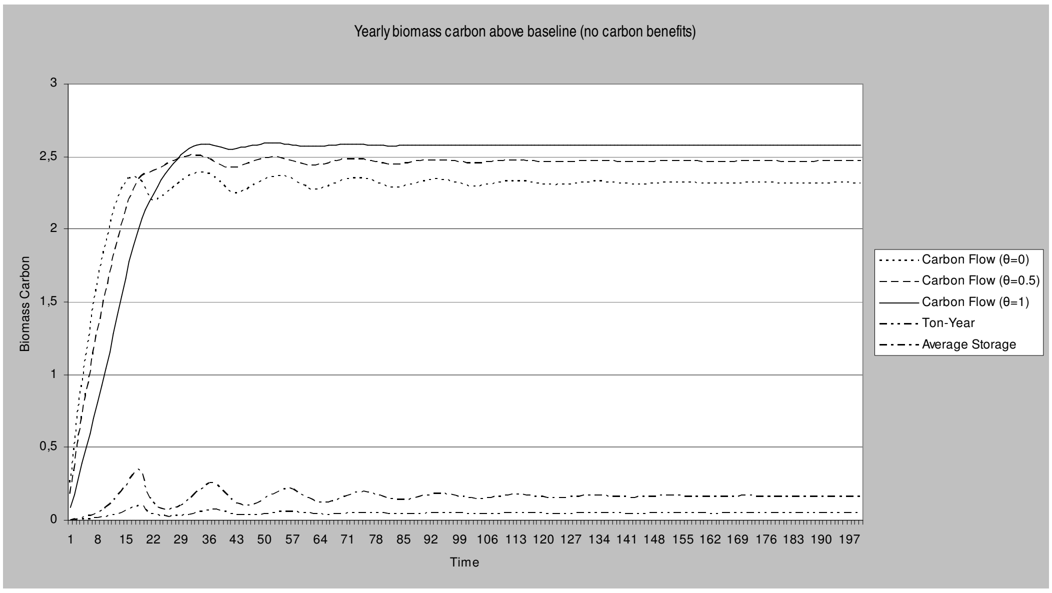

Figure 4. Yearly biomass carbon

Ton-Year

	No carbon benefits	Pc_Pss	Pc_2Pss	Pc_6Pss
Forested area (1 - y8)	0,9358	0,93783	0,93945	0,94616
Optimal rotation	19	19	19	21
Area by age xs,8	0,04925	0,04936	0,04944	0,0451
Price of timber	0,4368	0,4365	0,4363	0,43003
Unit cost of land W'(y8)	4,4971	4,6143	4,713	5,18
Timber Consumption	15,814	15,848	15,875	16,67
Net Cumulative Biomass Carbon	0	9,279	16,932	909,93

Average

	No carbon benefits	Pc_Pss	Pc_2Pss	Pc_6Pss
Forested area (1 - y8)	0,9358	0,94286	0,94864	0,96395
Optimal rotation	19	19	19	19
Area by age xs,8	0,04925	0,049623	0,049928	0,050736
Price of timber	0,4368	0,43582	0,43504	0,43336
Unit cost of land W'(y8)	4,4971	4,9363	5,3763	7,136
Timber Consumption	15,814	15,933	16,029	16,237
Net Cumulative Biomass Carbon	0	31,308	56,899	124,42

Carbon Flow

Pc=Pss

	No carbon benefits	θ		
		0	0,5	1
Forested area (1 - y8)	0,9358	0,94177	0,94767	0,95252
Optimal rotation	19	20	20	20
Area by age xs,8	0,04925	0,04709	0,04738	0,04762
Price of timber	0,4368	0,43288	0,43207	0,43143
Unit cost of land W'(y8)	4,4971	4,8631	5,2964	5,7252
Timber Consumption	15,814	16,297	16,399	16,481
Net Cumulative Biomass Carbon	0	451,19	473,85	484,41

Pc=2Pss

	No carbon benefits	θ		
		0	0,5	1
Forested area (1 - y8)	0,9358	0,94748	0,95666	0,96333
Optimal rotation	19	21	21	20
Area by age xs,8	0,04925	0,045119	0,045554	0,048156
Price of timber	0,4368	0,42985	0,42853	0,42984
Unit cost of land W'(y8)	4,4971	5,2831	6,1578	7,0389
Timber Consumption	15,814	16,683	16,854	16,684
Net Cumulative Biomass Carbon	0	916,74	953,72	575,47

Pc=4Pss*

	No carbon benefits	θ		
		0	0,5	1
Forested area (1 - y8)	0,9358	0,95694	0,96851	0,97582
Optimal rotation	19	23	21	21
Area by age xs,8	0,04925	0,04161	0,04612	0,04647
Price of timber	0,4368	0,42758	0,42742	0,42726
Unit cost of land W'(y8)	4,4971	6,1898	7,9521	9,8215
Timber Consumption	15,814	16,98	17,002	17,022
Net Cumulative Biomass Carbon	0	1807,9	1054,3	1082,7

*These values are obtained after 300 iterations

Pc=6Pss*

	No carbon benefits	θ		
		0	0,5	1
Forested area (1 - y8)	0,9358	0,96439	0,97573	0,98218
Optimal rotation	19	24	22	21
Area by age xs,8	0,04925	0,04	0,044348	0,046766
Price of timber	0,4368	0,4273	0,42524	0,42344
Unit cost of land W'(y8)	4,4971	7,2058	9,7921	12,541
Timber Consumption	15,814	17,031	17,294	17,54
Net Cumulative Biomass Carbon	0	2293,3	1540,3	1127,1

*These values are obtained after 500 iterations

The Net Cumulative Biomass Carbon is calculated as the sum of the yearly sequestered biomass carbon above the one resulting from the baseline scenario(no carbon benefits). The considered time horizon is of 200 years.

Table 1. Simulation Results

Pc=Pss

θ=0

	v^i	forest area	rotation
Carbon Flow	226,3744	0,94177	20
Ton-year	226,2331	0,93783	19
Average	226,2407	0,94286	19

Deviations from optimal solution

	Ton-year	Average
Welfare Loss $[(v^i - v^{cf})/v^{cf}]$	-0,06%	-0,06%
$(\text{forest area}^i - \text{forest area}^{cf})/\text{forest area}^{cf}$	-0,39%	0,11%

θ=0,5

	v^i	forest area	rotation
Carbon Flow	240,2064	0,94767	20
Ton-year	240,0632	0,93783	19
Average	240,111	0,94286	19

Deviations from optimal solution

	Ton-year	Average
Welfare Loss $[(v^i - v^{cf})/v^{cf}]$	-0,06%	-0,04%
$(\text{forest area}^i - \text{forest area}^{cf})/\text{forest area}^{cf}$	-0,98%	-0,48%

θ=1

	v^i	forest area	rotation
Carbon Flow	254,0965	0,95252	20
Ton-year	253,8933	0,93783	19
Average	253,9813	0,94286	19

Deviations from optimal solution

	Ton-year	Average
Welfare Loss $[(v^i - v^{cf})/v^{cf}]$	-0,08%	-0,05%
$(\text{forest area}^i - \text{forest area}^{cf})/\text{forest area}^{cf}$	-1,47%	-0,97%

Pc=2Pss

θ=0

	v^i	forest area	rotation
Carbon Flow	227,6339	0,94748	21
Ton-year	226,2678	0,93945	19
Average	226,2992	0,94864	19

Deviations from optimal solution

	Ton-year	Average
Welfare Loss $[(v^i - v^{cf})/v^{cf}]$	-0,60%	-0,59%
$(\text{forest area}^i - \text{forest area}^{cf})/\text{forest area}^{cf}$	-0,80%	0,12%

θ=0,5

	v^i	forest area	rotation
Carbon Flow	255,2039	0,95666	21
Ton-year	253,9539	0,93945	19
Average	254,1353	0,94864	19

Deviations from optimal solution

	Ton-year	Average
Welfare Loss $[(v^i - v^{cf})/v^{cf}]$	-0,49%	-0,42%
$(\text{forest area}^i - \text{forest area}^{cf})/\text{forest area}^{cf}$	-1,72%	-0,80%

θ=1

	v^i	forest area	rotation
Carbon Flow	282,998	0,96333	20
Ton-year	281,64	0,93945	19
Average	281,9713	0,94864	19

Deviations from optimal solution

	Ton-year	Average
Welfare Loss $[(v^i - v^{cf})/v^{cf}]$	-0,48%	-0,36%
$(\text{forest area}^i - \text{forest area}^{cf})/\text{forest area}^{cf}$	-2,39%	-1,47%

Pc=6Pss

θ=0

	v^i	forest area	rotation
Carbon Flow	240,1809	0,96439	24
Ton-year	234,4237	0,94616	21
Average	226,8387	0,96395	19

Deviations from optimal solution

	Ton-year	Average
Welfare Loss $[(v^i - v^{cf})/v^{cf}]$	-2,40%	-5,56%
$(\text{forest area}^i - \text{forest area}^{cf})/\text{forest area}^{cf}$	-1,82%	-0,04%

θ=0,5

	v^i	forest area	rotation
Carbon Flow	319,9694	0,97573	22
Ton-year	316,7785	0,94616	21
Average	311,1256	0,96395	19

Deviations from optimal solution

	Ton-year	Average
Welfare Loss $[(v^i - v^{cf})/v^{cf}]$	-1,00%	-2,76%
$(\text{forest area}^i - \text{forest area}^{cf})/\text{forest area}^{cf}$	-2,96%	-1,18%

θ=1

	v^i	forest area	rotation
Carbon Flow	402,9598	0,98218	21
Ton-year	399,1327	0,94616	21
Average	395,4119	0,96395	19

Deviations from optimal solution

	Ton-year	Average
Welfare Loss $[(v^i - v^{cf})/v^{cf}]$	-0,95%	-1,87%
$(\text{forest area}^i - \text{forest area}^{cf})/\text{forest area}^{cf}$	-3,60%	-1,82%

** The welfare values obtained for the ton-year and average storage method are calculated using the carbon flow accounting to obtain the benefits of the internalization of the carbon externality (S).*

Table 2. Welfare deviations: Ton-year vs Carbon Flow / Average Storage vs Carbon Flow

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Chapter 3

Biodiversity and the Forest Sector

3.1 Introduction

Climate change and the loss of biodiversity are amongst the most serious challenges facing the international community. Given their global character, a combined action of the different parties involved in the process is required. As a response to this problem two international agreements were designed: the Kyoto Protocol (KP) and the Convention on Biodiversity (CBD). However, given the prominence of the climate change debate on the international policy arena, while a several studies on the use of forests as carbon sinks have been undertaken, only a much smaller part on the economics literature has been devoted to biodiversity.

Recently, though, as the existence of possible conflicts between carbon sequestration policies and biodiversity has been recognized (UNCBD [27]), biodiversity is once more at the centre of the forestry literature debate (Englin and Callaway [12], Creedy *et al* [11], Caparrós and Jacquemont [8], Matthews *et al.*[20]).

Building upon the pioneer work by Hartmann [14], who extended the one stand Faustmann model to incorporate forest's provision of amenity services, a vast literature on this subject is already available. However, while this framework has been proved to be a powerful analytical tool to analyze stand level forests' questions, it only allows for exogenous timber and land prices, leaving many questions unanswered. In fact, this is a major drawback of these models, as biodiversity policies by increasing/decreasing rotation periods and introducing different incentives on land and species use, may create considerable pressure both on timber and land markets. Actually, land use change is a crucial element on the design of an optimal biodiversity policy: according to the Intergovernmental Panel on Climate Change (IPPC), land-use change is one of the major factors affecting not only climate change but also biodiversity loss (IPCC [16]). Therefore, the possibility of endogenous land allocation should be considered in a biodiversity model. In addition, the one stand forest analytical framework only allows for static comparisons, implying that transition dynamics are left aside, a central aspect when considering welfare costs. Some important exceptions are the models by Alig *et al.* [1] , Sedjo and Sohngen [24], Tahvonen [26], and more recently by Costa-Duarte, Cunha-Sá and Rosa [10]. However, except for Tahvonen [26], these papers focus their analysis on timber and carbon sequestration aspects.

While a complete assessment of the interactions between carbon sequestration policies and biodiversity conservation is still needed, there are previous questions in the biodivesity literature that remains to be addressed, namely, in what concerns the forest sector scope. To this end, biodiversity considerations are introduced into a multiple species, multi-vintage forest sector model with endogenously determined timber prices and land use allocation. That is the purpose of this paper.

When biodiversity is introduced into an economic model, two additional difficulties arise: the first relates to the measurement of economic values provided by biodiversity, while the second results from the modelling of the complex nature of biological relationships that constitutes an ecosystem. Therefore, most studies focus on a single (or few) amenity service. However, as forests species and canopy variety are closely related to biological diversity (Biesterfeldt and Boyce [3], Burton *et al.* [7], Hunter [15]), management of its ecological services can be achieved by focusing only on structural diversity, i.e., age classes and species distribution (Buongiorno *et al.* [5], Buongiorno *et al.* [6], Önal [21]). In what follows, we shall measure biodiversity based on this literature, as it not only saves us from the problem of enumerating all possible amenities forests may provide, but it also identifies an ecologically meaningful framework, where environmental benefits are a result of the forest structure condition (Bowes and Krutilla [4]). In addition, we also avoid the problem of biodiversity valuation and incorporate it, instead, as an ecological constraint into the problem of the public forest manager that maximizes timber and alternative uses benefits. Note, however, that it is also possible in this context to calculate the resulting losses in timber consumption value from introducing biodiversity considerations, and, therefore, to find a threshold value to be satisfied by policy implementation.

While an obvious ecologically valuable forest structure to consider is the old growth, to focus only on these type of forests can be an oversimplification, as different stages of a stand provide different habitats that are favorable for some plants and animals but not to others (e.g .seedlings are favorable to dears while a higher proportion of saplings improve conditions for the existence of songbirds) (Bisterfeldt and Boyce [3], Lin and Buongiorno [19]). Therefore, a target distribution between species and age classes is instead considered (Buongiorno *et al.* [6], Krcmar *et al.* [18], Önal [21]).

Despite the fact that biodiversity tends to increase when agricultural lands are converted into forests, the species used for afforestation or reforestation are not without consequences. In fact, the use of exotic over native species is one of the most contentious aspects concerning biodiversity management in plantation forests. While a consensus is not achieved within the scientific community, it is usually recommended that native species should be favored over exotics (Hartley [13]). According to this literature, we will consider that biodiversity's concerns are focused on the area devoted to the native species forest(s). To this end, three different biodiversity scenarios are analyzed.

In the first scenario, the forest structure to be considered only accounts for species 1 area distribution without distinguishing age class structure. The public forest managers are thus only interested in guaranteeing that a sufficiently large amount of forest area is devoted to native species. Under the other two alternative scenarios age class is also considered. The first focus on the more obvious biodiversity forest structure, the old growth. When this constraint is imposed, the public forest manager focuses only on the total amount of land devoted to species 1 old growth area. In the second case, constraints are imposed both on old growth and younger stand areas. Therefore, the already mentioned literature claiming that stands with younger trees may also contribute to biodiversity is also taken into account (Biesterfeldt and Boyce [3], Lin and Buongiorno [19]). While a full proof on the long-run optimality of the normal forest steady-state is provided for the first scenario, the other two will be treated only by the simulation of numerical examples. In fact, the main insights of this paper are brought by the simulation results and, consequently, a large part of the analysis is dedicated to them. In addition, as the solution to the unconstrained biodiversity problem, as well as its main properties, are already well known, all major comparisons with the baseline scenario can be easily undertaken.

Based on the numerical simulations, we observe that the optimal transition path towards the stationary state is characterized by major disturbances concentrated in a short period of time. Consequently, timber and land markets are also highly affected. Moreover, timber management along the transition path is very different from the one in the baseline case (in which biodiversity is not taken into account), as both deviations from Faustmann rotations and changes in land use allocation are present for a long period. Hence, consumption cycles also tend to persist. Finally, we observe that timber price behaviour is also highly affected by the type of forest structure considered.

The remainder of the paper is organized as follows. Section 2 extends the theoretical multiple vintage model to account for biodiversity in a multi-species context and provides a full proof of the optimal steady state for the first biodiversity scenario. In Section 3 numerical simulations illustrate the main results for the three above mentioned biodiversity forests structures. Section 4 concludes the paper. Technical details and figures are presented in the Appendices.

3.2 The Model

In this section we provide a full proof on the long-run optimality of the normal forest steady-state under the first biodiversity scenario, i.e., when the targeted forest structure only accounts for species 1 area distribution and no constraint is imposed on the age class structure.

The model used in this paper follows closely the multiple vintage forest model developed in Salo and Tahvonen [23], which can be summarized as follows. The model assumes multi species multi vintages forest land, where $s = 1, ..., n$ represents the age of trees for species $l = 1, ..., L$, $x^l_{s,t}$ the area of forest land allocated to the age class s of species l in period t, f^l_s the biomass content in timber per unit of land with trees of age class s and species l, and $0 \leq f^l_1 \leq .... \leq f^l_n$, for each l. Land allocation must satisfy

$$0 \leq y_t = 1 - \sum_{l=1}^{L} \sum_{s=1}^{n} x^l_{s,t} \tag{3.1}$$

that is, total land area equals 1, and y_t is the area of land allocated to an alternative use (agriculture or urban use).

Let us denote by $U^l(c_t) = \int d^l(c)dc$ the social utility from timber consumption for species l, where $d^l(.)$ is the inverse demand for timber for species l, and assume each $U^l(.)$ is a continuous, twice differentiable, increasing and strictly concave function. Also, $W(y_t) = \int q(y)dy$, where $W(.)$ is a continuous, twice differentiable, increasing and concave function.

Thus, the problem of optimal forest harvesting and allocation of land is obtained by maximizing the present value of social utility from the use of land, as follows:

$$v(x^l_{1,0}, ...x^l_{n,0}) = \underset{\{x^l_{s,t+1}, s=1,...n, l=1,...,L, t=0,...\}}{Max} \sum_{t=0}^{\infty} b^t \left[\sum_{l=1}^{L} U^l \left(c^l_t \right) + W \left(y_t \right) \right] \tag{3.2}$$

subject to

$$c^l_t = \sum_{s=1}^{n-1} f^l_s \left(x^l_{s,t} - x^l_{s+1,t+1} \right) + f^l_n x^l_{n,t},\ l = 1, ..., L \tag{3.3}$$

$$y_t = 1 - \sum_{l=1}^{L} \sum_{s=1}^{n} x^l_{s,t} \tag{3.4}$$

$$x^l_{s+1,t+1} \leq x^l_{s,t}, s = 1, ...n-1,\ l = 1, ..., L \tag{3.5}$$

$$\sum_{l=1}^{L}\sum_{s=1}^{n} x^l_{s,t+1} \leq 1 \tag{3.6}$$

$$\alpha \sum_{l=1}^{L}\sum_{s=1}^{n} x^l_{s,t+1} \leq \sum_{s=1}^{n} x^1_{s,t+1}, \qquad 0 < \alpha < 1 \tag{3.7}$$

$$x^l_{s,t} \geq 0, s = 1, ..., n,\ l = 1, ..., L \tag{3.8}$$

for all $t = 0, 1....$ Moreover, the initial land distribution satisfies

$$x^l_{s,0} \geq 0, s = 1, ..., n, \sum_{l=1}^{L}\sum_{s=1}^{n} x^l_{s,0} \leq 1 \tag{3.9}$$

Therefore, given the discount factor b, the problem consists of choosing the next period state, that is, the land allocation between different vintages and competing uses of land for all $t = 1, ...$

Equation (3.7) introduces biodiversity by imposing a particular forest structure with ecological value. Without loss of generality we chose, for the case here considered, species 1 total area to be at least a proportion, α, of the total forested area. [1]

The necessary conditions for optimal solutions can be obtained from the following Lagrangian problem. For (3.2-3.9) it can be stated as

$$L = \sum_{t=0}^{\infty} b^t \left[\sum_{l=1}^{L} U^l\left(c^l_t\right) + W\left(y_t\right)\right] + \lambda_t \left(1 - \sum_{l=1}^{L}\sum_{s=1}^{n} x^l_{s,t+1}\right) +$$

$$+ \sum_{l=1}^{L}\sum_{s=1}^{n-1} \left[p^l_{s,t}\left(x^l_{s,t} - x^l_{s+1,t+1}\right)\right] + \mu_t \left[\sum_{s=1}^{n} x^1_{s,t+1} - \alpha \sum_{l=1}^{L}\sum_{s=1}^{n} x^l_{s,t+1}\right] \tag{3.10}$$

where $p^l_{s,t}$, λ_t and μ_t are the Lagrangian multipliers. While $p^l_{s,t}$ can be interpreted as the value of marginal changes in forest land area of vintage s for species l at the beginning of period $t+1$, λ_t represents the value of marginal changes in land allocation between forest and alternative uses. μ_t can be interpreted as the value of a marginal change in the total forested land proportion area's constraint.

Salo and Tahvonen [23] provide a full proof on the long-run optimality of the normal forest steady-state for the above problem when equation (7) is not considered or binding (or $\mu_t = 0$) and there is only one species.

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[1] The same proof can be easily extented to the case of a restricted set of species

A forest is called an Optimal Biodiversity Faustmann Forest (OBFF) if the age-class structure $x^l = (x^l_1, ..., x^l_n)$ has the property $x^l_s = 0$ for $s = m^l + 1, ..., n$ and if harvesting only trees of age m^l is the optimal solution for the above problem when $x^l_0 = x^l$. An OBFF is an interior OBFF if $x^l_s > 0$ for $s = 1, ..., m^l$. In addition, an OBFF with the normal forest structure is $x^l = (1/m^l, ..., 1/m^l, 0, ..., 0)$, and in each period it yields a constant consumption level of f_{m^l}/m^l. An OBFF with consumption that is periodic with period length equal to m^l can be expressed as $x^l = (1/m^l + \phi^l_1, ..., 1/m^l + \phi^l_{m^l}, 0, ...0) \in S$, where ϕ^{kl} represents the largest number ϕ^l that satisfies $x^l = (1/m^l + \phi_1, ..., 1/m^l + \phi_{m^l}, 0, ...0) \in K$ for all $\left|\phi^l_s\right| < \phi^l$, $l = 1, ..., L$, $s = 1, ..., m^l, \sum_{s=1}^{m^l} \phi^l_s = 0$, The Faustmann rotation period for species l, denoted by m^l, $1 \leq m^l \leq n^l$, is assumed to be unique and satisfies the following condition: [2]

$$b^{m^l} f^l_{m^l}/(1 - b^{m^l}) \geq b^s f^l_s/(1 - b^s),\ s = 1, ..., n. \tag{3.11}$$

Salo and Tahvonen [23] show that, if all land is allocated to forestry, optimal forest management can lead to optimal cyclical harvesting because smoothening an age class structure that deviates from the normal forest is not optimal. On the contrary, if it is optimal to allocate part of the land to alternative land use then optimal stationary cycles cannot exist.[3]

By solving the problem (3.2-3.9) for each species l, we first study the existence of optimal stationary cycles in a regime where the oldest age class is clear-cut and immediately regenerated at the end of each period.

Proposition 1:Given $g^l \equiv \frac{U^{l\prime}(f^l_{m^l}/m^l) b^{m^l} f^l_{m^l}}{1 - b^{m^{cl}}} - \frac{b}{1-b} W'(0) - \frac{b}{1-b} \mu\alpha > 0$, $m^{cl} \geq 2$, *and* $b < 1$, *for* $l = 1, ..., L$, *there exists a set of interior Optimal Biodiversity Faustmann Forests with* $\phi^{kl} > 0$.

Proof. The proof is in Appendix A. ■

In Proposition 1 it is shown that optimal stationary cycles exist when it is optimal to allocate all land to forestry. From (3.34) and (3.35) in Appendix A it is still possible to conclude that for more stringent biodiversity restrictions, that is, the higher is μ, the lower will be the cycles for the unconstrained species $(l = 2, ...L)$ and the higher for the constrained one $(l = 1)$.

Corollary1: If $g^l \equiv \frac{U^{l\prime}(f^l_{m^l}/m^l) b^m f^l_{m^l}}{1 - b^{m^l}} - \frac{b}{1-b} W'(0) - \frac{b}{1-b} \mu\alpha \leq 0$, *for* $l = 1, ..., L$, *optimal stationary cycles with* $y_\infty \geq 0$ *and* y_∞ *constant do not exist.*

Proof. The proof is in Appendix A. ■

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[2] In Salo and Tahvonen [23] no harvesting or plantation costs are considered nor any type of forest externalities. Under these conditions, m, as defined in (3.11), is the Faustmann rotation period in the one stand model. As consumption is constant in the steady-state, so is the marginal utility of consumption $U'(.) = p$, the long-run market equilibrium price of timber. In addition, this condition also corresponds to the maximum value in the steady-state of a marginal unit of bare forest land.

[3] See Salo and Tahvonen [23], Proposition 1 and Corollary 1, pages 518-520.

When it is optimal to allocate land to alternative uses, Corollary 2 shows that optimal cycles are eliminated and the remaining equilibrium is the normal forest steady-state. Assuming that m^l is unique, for a stationary state with binding biodiversity constraint, we have that $p^l_{s,t} = p^l_{s,\infty}$, $c^l_t = c^l_\infty$, $y_t = y_\infty$, $\lambda_t = 0$, $\mu_t > 0$ and $x^l_{m^l,t} = x^l_\infty$, where c^l_∞, y_∞, x^{cl}_∞, and $p^l_{s,\infty}$, for $s = 1, ..., n-1$, and $l = 1, ..., L$, are constant. Following Salo and Tahvonen ([23]) page 521 is easy to show that the following equations for the p^l_{st} Lagrangian multipliers guarantee that there exists a unique stationary state for the allocation of land between alternative use and forestry and between the forest age classes.[4]

$$p^1_s = W'(y_\infty)\sum_{i=0}^{s-1} b^{-i} - f^1_s U^{1\prime}(c^1_\infty) - \mu_\infty(1-\alpha)\sum_{i=0}^{s-1} b^{-i} \tag{3.12}$$

and

$$p^l_s = W'(y_\infty)\sum_{i=0}^{s-1} b^{-i} - f^l_s U^{l\prime}(c^l_\infty) + \mu_\infty \alpha \sum_{i=0}^{s-1} b^{-i} \tag{3.13}$$

$l = 2, .., L$, $s = 1, ..., n$,

After some algebra, (3.12) and (3.13) can be rewritten for $s = m^l$ and $p_{m^l} = 0$ as:

$$\frac{W'(y_\infty)b}{1-b} - \frac{\mu_\infty(1-\alpha)b}{1-b} - \frac{b^{m^1} f^1_m}{1-b^{m^1}} U^{1\prime}\left[(1 - y_\infty - \sum_{l=2}^{L}\sum_{s=1}^{n} x^l_{s,t+1}) f^1_m / m^1\right] = 0 \tag{3.14}$$

and

$$\frac{W'(y_\infty)b}{1-b} + \frac{\mu_\infty \alpha b}{1-b} - \frac{b^{m^1} f^1_m}{1-b^{m^1}} U^{1\prime}\left[(1 - y_\infty - \sum_{v=1}^{L}\sum_{s=1}^{n} x^{v\neq l}_s) f^l m^l / m^l\right] = 0 \tag{3.15}$$

Combining these two equations and solving for y_∞, all the other state variables are fully defined. Consequently, the allocation between all species age stands and the alternative use can be obtained.

3.3 Numerical Simulations

We now proceed with simulations for the theoretical model developed in the previous section under three different types of biodiversity forest structure. Our analysis is based on the following example: $U(c^l_t) = \frac{(c^l_t)^{0.7}}{0.7}$ is the utility function from consumption for both species[5] and $W(y_t) = 0.5[\frac{y_t^{0.2}}{0.2}]$ is the utility from non-forestry land, that is, from the alternative use of

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[4] The full proof can be provided by request.

[5] It is our aim to focus on factors such as species speed of growth and forest structure. Therefore we consider the same utility function for both types of timber.

land[6]. The vectors f_s^l of the biomass content in timber per unit of land for species $l = 1, 2$ and for age classes $s = 1, ..., 12$, are given by

$$f_s^1 = [0, 0, 22, 39, 65, 101, 148, 204, 263, 321, 370, 408]$$

$$f_s^2 = [5, 35, 90, 160, 227, 269, 296, 315, 331, 346, 357, 367]$$

Note that species 2 initially increases at a higher rate than species 1. For both species the initial land distribution is given by $x_{s0}^l = 0.1,\ s = 1, .., 5$ and $x_{s0}^l = 0,\ s = 6, ..., 12$. The used discount factor is $b = 0.9025$.

Species' type is one of the main concerns regarding biodiversity conservation in forestry. Despite that a consensus is not achieved within the scientific community, it is usually recommended that native species should be favored over exotics (Hartley [13], Krcmar *et al.* [18]). Moreover, the widespread use of non-native fast growing species for timber production (Caparrós *et al.* [9]), and more recently suggested for carbon sequestration activities (Krcmar *et al.* [18], Van Kooten [28], IPCC [17]), is pointed out as one of the major threats to biodiversity. Thus, we assume that biodiversity's concerns will be focused on the area devoted to the native species' forest. In particular, species 1 (the slow growing species) represents the native forest, while the second one represents the non-native type. Bearing this in mind, we focus on three different kinds of target biodiversity forest structures.

The simplest forest structure to be considered only accounts for species area distribution while no distinction is made on age class structure. In this case, the public forest managers are only interested in guaranteeing that a sufficiently large amount of forest area is devoted to the native species. When age class is also considered, two cases are addressed. The first focus on the more obvious biodiversity forest structure, the old growth. When this constraint is imposed, the public forest manager is concerned about the total amount of land devoted to species 1 old growth area. For the second case, requirements are imposed both on old growth and younger stand areas.

Note that the initial land distributions may not instantaneously satisfy the imposed forest structure conditions. As a result, an adjustment time interval, T^b, is necessary. Here we will consider two values for this variable, $T^b = 40$ and $T^b = 10$ time units.

In what follows, we focus on transition dynamics, disturbances and adjustments both on timber and land markets and, finally, on the cyclical approach to the forest steady state distribution.

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[6] Here we are using the same example as in Salo and Tahvonen [23]. The same algorithm is also applied (see Andersond and Ye [2])

3.3.1 Baseline - The Unconstrained Problem

If only timber benefits are considered and no constraint is imposed, the steady state normal forest structure is reached approximately after 50 periods (cycles completely vanish). Timber consumption is, consequently, fully smoothened. While different initial distributions of land strongly impact the transition paths, the new steady state is independent of the initial conditions. When the steady state is reached, harvesting patterns follows Faustmann rotation: only trees of age class 10 are harvested for species 1, and of age class 5 for species 2. Most area is allocated to forest use, while a significantly higher fraction is devoted to the fast growing species (approximately 85% of total forest area).

The approach to the steady state consumption path, where the normal forest structure is reached, is cyclical. For the initial periods, when the stand's area's distribution is still far from the optimal, adjustments are made by both deviations from the Faustmann rotation and adjustments on the distribution between species forest area and the alternative use. However, when age class structure is already close to the normal forest, marginal changes in utility approach zero as timber consumption is smoothened, while costly deviations from the Faustmann rotation are strictly positive (note that time is a discrete variable). Therefore, marginal adjustments are no longer obtained by deviating from Faustmann rotation but exclusively through land distribution allocation (see Salo and Tahvonen [23]). These adjustments, then, proceed as follows: when the stand area to be harvested is above the normal forest structure distribution, next period allocation for stand 1 is decreased and the inverse process occurs if the harvested stand area is below the optimal distribution allocation.

Along the optimal transition path, consumption cycles are positively correlated between species. Periods of higher consumption for species 2 match with higher consumption for species 1. The same pattern is observed for periods of low consumption. As most area is devoted to species 2, stand area distribution for species 1 is changed to match the consumption cycle path for species 2, and cycles of five periods are, therefore, created.[7].

3.3.2 Species' Area

We now consider the first and less stringent constrained problem, where forest structure is not imposed over age classes but only on species' area distribution. If timber benefits are maximized without any forest structure constraint, species 1 forest area accounts only for 14.5% of total forest area. Therefore, we shall consider three cases. In the first species 1 forest area equals 20% of total forest area, while for the second and third that area increases to 60%

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[7] Note that if both biomass vector were equal to f_s^1, these cycles would be in periods of 10 time units.

and 90%, respectively.

When $T^b = 40$, the transition to the new optimal steady state is not smooth when compared to the unconstrained solution. During the initial periods, when the constraint on species 1 area is still far in the future, the new transition dynamics is the same as in the unconstrained problem. Therefore, severe changes are concentrated in a short period of time, just before and after T^b, the time period at which the constraint has to be met (see Figures 1a and 1b). Adjustments are tighter the larger is the restriction on species 1 total area, implying, in some cases, almost discrete jumps on timber consumption, prices and land distribution among species (see Figures 2 -3). Consequently, major disturbances are reflected on both timber and land markets.

Total forest area decreases as more demanding constraints are imposed on species 1 area. However, land use distribution between forest and the alternative use is rather stable: no major changes take place even for very high percentage values of land dedicated to species 1 forest area (90%) (see Figure 2c).

If the $T^b = 10$, transition dynamics start soon to differ from that obtained in the baseline scenario. More important, however, is the impact of this parameter on Timber Consumption Present Value, as differences in the former seem to be the most important factor driving decreases in the latter. This is due to the fact that major adjustments are now taking place during the initial periods, considerably reducing consumption for species 2 when the effect of discounting is still small (see Figures 4a and 4b).

In contrast to the unconstrained problem, timber consumption cycles are no longer always positively correlated. In fact, the period in which consumption is the highest for species 1 coincides with that in which consumption for species 2 is the lowest. Moreover, species 1 cycles no longer change to perfectly match species 2 cycles as in the previous section (figure 5). To understand what lies behind this result, let us assume that consumption for both species is positively correlated. If both timber consumptions (and stand areas) were above the normal forest steady state structure, optimal adjustments would require that in the following periods forested area had to diminish. To satisfy the constraint, however, area adjustments would have to be coordinated or, in alternative, it would be possible to increase stand areas for age classes above Faustmann and then adjust (reduce) stand areas to approach the normal forest steady state. However, Faustmann rotation deviations are costly when compared to changes in forest area. Therefore, periods of decreasing timber consumption for species 1 will tend to coincide with periods where consumption is increasing for species 2. By doing so, when it is optimal to reduce one species forest area, it will be optimal to increase or maintain the other, and only minor changes in Faustmann's rotation periods are necessary.

3.3.3 Old Growth

We now analyze the case in which optimal biodiversity forest structure is given by old growth. When old growth is considered, a slight modification to the problem presented in (3.2 - 3.9) is necessary. In fact, an additional age class $(n+1)$ has to be introduced, representing the land area for the age classes older than n.[8] Moreover, (3.7) should be rewritten as:

$$\alpha \sum_{l=1}^{L} \sum_{s=1}^{n+1} x_{s,t+1}^{l} = x_{n+1,t+1}^{1}, \qquad 0 < \alpha < 1 \tag{3.16}$$

Once again, the transition to the new optimal steady state solution is not smooth and a similar pattern to the previous constrained problem is obtained. Drastic changes are, therefore, concentrated in a short period of time, approximately 15/20 periods, when several disturbances contaminate both land and timber markets (see Figures 6a, 6b and 8c).

The steady state total forest area decreases as a higher forest area is imposed to be old growth. However, if the targeted area is small (0.05 and 0.1), changes in the steady state land use distribution between forest and the alternative use are also small when compared to the one in the baseline scenario (see Figure 7c). On the other hand, changes on species' area composition are more substantial, as the slow growing species is favored over the fast growing one. In fact, species 1 total forest area can be even higher than the one in the baseline scenario (see figures 7a and 7b) . While species' 1 total forest area increases, its timber consumption actually decreases. This is due to the fact that the increase in total area also includes the area devoted to old growth. If only considered the area used for timber production, this number is actually smaller. Timber species 2 forest area also declines. As a consequence of these two facts, both timber prices are higher under the old growth constrained problem (see Figures 8a and 8b).

As in the previous subsection, when $T^b = 10$, transition dynamics start soon to differ from that obtained under the baseline scenario. More important, however, is the fact that if this period is too short, forest area may suffer a temporarily large reduction to meet the constraint, while the optimal steady state area distribution is again approached in the future (see figure 9). Consequently, major disturbances are felt both in timber and land markets. Once more, changes in T^b seem to be the most important factors driving reductions on total present value. Moreover, under this type of restriction, the present value of timber consumption decrease by more when compared to the previous subsection. In fact, the present value for a 10% target old growth area is lower than the one obtained for the extreme case of a 90% species 1 total forest area restriction for the previous type of forest structure.

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[8] For a complete presentation of this modified problem see Tahvonen [26]

Also, there seems to be an upper limit to the old growth target (30%) above which the total forest area steadily declines towards zero (see Figures 7a and 7b),[9] implying that as timber consumption declines, prices rapidly increase. Therefore, tightening the constraint on old growth area may have a huge impact on timber and land markets. Moreover, as the forest structure is defined over the proportion of total forest area, and not over the total available amount of land, this optimal decrease in total forested area may even result in allocations without biodiversity meaning.

As in the unconstrained problem, stand area distribution for species 1 is changed to match the consumption cycle path for species 2. Consumption cycles are, therefore, positively correlated, unlike the optimal solution under the previous forest structure constraint. After 200 time periods, the normal forest structure is still not reached and approximations are cyclical. In fact, adjustments towards the normal forest are now more difficult as changes in stands areas have to be met by changes in the old growth area. As a consequence, stand ages above Faustmann rotation are now used to alleviate the costs of adjustment. For instance, when the stands to be harvested, that is, Faustmann's, are above the optimal normal forest structure, it is optimal to leave a fraction of this area to age class 11 for species 1 (remember that Faustmann rotation is given by age class 10). By doing so, the area for these stands can be optimally reduced while total forest area is not, resulting that no adjustments on the old growth area are necessary. Note that this is a major change in forest management relative to the unconstrained problem, where adjustments took place early on by changing areas' distribution and not by deviating from Faustmann's rotation.

3.3.4 Old growth and younger stands

Finally, we analyze the more complex forest structure type where restrictions are imposed both on old growth and younger age classes. To account for this type of forest structure we consider restrictions in the total area of the three younger age stands and, as in the previous subsection, in the oldest age class. For illustrative purposes we will focus our analysis on the following cases: old growth area constraint: 10% and 20%; younger age stands: 5%, 10% and 20%.

Under this type of restriction, (3.7) should be rewritten as:

$$\alpha_1 \sum_{l=1}^{L} \sum_{s=1}^{n+1} x_{s,t+1}^{l} = \sum_{s=1}^{3} x_{s,t+1}^{1}, \qquad 0 < \alpha_1 < 1 \tag{3.17}$$

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[9] Note that, for the functions used in this simulations, the steady state allocation will always have a positive fraction of the land devoted to both forest species, as timber marginal utility of consumption tends to inifinity when consumption goes to zero.

and

$$\alpha_2 \sum_{l=1}^{L} \sum_{s=1}^{n+1} x_{s,t+1}^{l} = x_{n+1,t+1}^{1}, \qquad 0 < \alpha_2 < 1 \tag{3.18}$$

The transition dynamics follow the same pattern as in the two previous constrained problems: major changes in relation to the unconstrained problem are concentrated in a short period of time. Thus, similar effects are felt on both timber and land markets (see Figures 11a, 11b and 13c).

As for the previous type of forest structure constraint, total forest area decreases for higher constraints on species 1 area (see Figure 12c). However, forest area actually increases for this species (implying that species 2 area decreases). Also, for the same percentage constraint in old growth, the absolute area obtained for this type of imposed structure is higher than the one in the previous section. Timber consumption for species 1 is also higher, driving price reductions. The opposite is observed for species 2 (see Figures 13a and 13b).

Minor increases (5% to 10%) in the total younger stands' area requirement imply major changes on species mix composition: species 1 area may surpass species 2, while in the unconstrained case most of the land was dedicated to the latter type of tree (see Figures 12a and 12b). As for the previous type of restriction, species composition seem to be a more important factor of adjustment rather than changes between forest and the alternative use of land.

Land allocation adjustment to satisfy the restriction on the three younger stands are not undertaken by equally increasing the area for these stands. Instead, in each period, to one (or two) stand(s) is devoted an amount of land above the unconstrained solution, guaranteeing that the constraint is satisfied. Note that by doing so, the alternative of having three stands permanently above the optimal normal forest distribution is avoided. Moreover, Faustmann's deviations will also take place, as the higher stand can be partially harvested at age 9, or even 3. These adjustments will obviously impact on the optimal consumption path, and for substantially higher thresholds on the younger stands areas (10% or 20%), both species will present consumption cycles' lenghts of 3 time units (see Figure 14).

3.4 Concluding Remarks

In this paper, we analyze the impact of introducing biodiversity into a multiple species, multi-vintage forest sector model with endogenously determined timber prices and land use allocation. Biodiversity was incorporated as an additional ecological constraint in the problem of the public forest manager, avoiding the additional problem of economic valuation.

Following recent ecological literature, we model biodiversity focusing on structural diversity, i.e, age classes and species distribution. To account for the most representative structures, three different constraints were studied. In the first, the structure is only imposed on species' area distribution, without distinguishing age class structure, while in the second the constraint is imposed on the total area of species 1 old growth age class. Finally, in the last forest structure considered, both young and old growth stand areas are targeted.

For all the considered forest structures, the transition to the new optimal steady state is not smooth when compared to the unconstrained solution. In fact, for the initial periods, when we are far away from the time period at which the constraint has to be satisfied, the new transition dynamics is similar to the ones obtained in the unconstrained problem. Severe adjustments are, therefore, concentrated in a short period of time, when major additional disturbances imposed by biodiversity considerations are felt both on timber and land markets. In addition, as more stringent constraints are imposed, total forest area diminishes for all forest structures considered. It is, however, in the case of old growth that this reduction is the highest, leading to a shrinking total forest area that may even result in allocations without biodiversity meaning. Land markets are, therefore, more affected when the imposed forest structure accounts for old growth stands.

As timber prices are endogenous in this model, it is also possible to analyze the impacts on timber markets from imposing different forest structures. While in the first case, timber prices for species 1 tend do diminish and to increase for species 2, in the old growth case both prices increase. In fact, while in the former the area used for timber production for species 1 increases and for species 2 decreases, in the latter both timber production areas decrease. If, however, the targeted forest structure is the third case considered, stringent constraints imposed in the younger stands area tend to increase total native forest area and, consequently, reduce timber prices, while those on the old growth area tend to decrease them. In this latter type of restriction species 2 timber prices always increase.

While biodiversity is introduced into the problem as a constraint it is still possible to calculate the resulting losses on timber consumption present value. As expected, introducing biodiversity always results in losses in the timber consumption present value. These losses, however, are greater when the imposed forest structure includes old growth. In fact, the present value for a 10% target old growth area is lower than the one obtained for the extreme case of a 90% species 1 total forest area requirement, in the first type of forest structure.

Cycles persist for a very long period, even for the first constraint type, for which a proof on the optimality of the normal forest steady state structure was provided in section 2. More important, however, is to observe that cycles properties differ widely, depending on the considered targeted forest structure. In the first case, cycles between species are not positively correlated: the period in which consumption is the highest for species 1 coincides with that in which consumption for species 2 is the lowest. Moreover, species 1 cycles no longer match species 2 cycles as in the baseline case. In the second constrained problem, when old growth is the imposed forest structure, cycles perfectly match. Finally, when analyzing the last forest structure type, we observe that for stringent requirements in the younger stands (10% or 20%), both species presented consumption cycles of 3 time units, suggesting that the number of age classes included in this constraint determines consumption cycles.

Finally, we observe major changes in optimal timber management. In fact, even after a long period of adjustment, optimal deviations from Faustmann's rotation combined with changes in land use allocation still occur. This is in contrast to the baseline case, where Faustmann's rotation deviations only occur in the initial periods. Thus, introducing biodiversity into the multispecies, multi-vintage model gives interesting insights about the adjustment dynamics in timber and land markets. From a policy perspective analysis this is of great relevance and, as mentioned in the introduction, should be taken into account before carbon sequestration benefits are considered into the analysis.

3.5 Appendix A

The necessary conditions for optimal solutions of the problem (3.2-3.9) for species 1, which can be derived from the Karush-Kuhn-Tucker conditions for all $t = 0, ...,$ are as follows:

$$b^{-t}\frac{\partial L}{\partial x^1_{1,t+1}} = bf^1_1 U^{1\prime}(c^1_{t+1}) - bW'(y_{t+1}) - \lambda_t + bp^1_{1,t+1} + b\mu_{t+1}(1-\alpha) \leq 0, \tag{3.19}$$

$$b^{-t}\frac{\partial L}{\partial x^1_{s+1,t+1}} = -f^1_s U^{1\prime}(c^1_t) + bf^1_{s+1} U^{1\prime}(c^1_{t+1}) +$$

$$-bW'(y_{t+1}) - \lambda_t + bp^1_{s+1,t+1} - p^l_{s,t} + b\mu_{t+1}(1-\alpha) \leq 0 \tag{3.20}$$

for $s = 1, ..., n-2$

$$b^{-t}\frac{\partial L}{\partial x^1_{n,t+1}} = -f^1_{n-1} U^{1\prime}(c^1_t) + bf^1_n U^{1\prime}(c^1_{t+1}) +$$

$$-bW'(y_{t+1}) - \lambda_t - p^1_{n-1,t} + b\mu_{t+1}(1-\alpha) \leq 0 \tag{3.21}$$

for the remaining species ($l = 2, ..., L$), these conditions are given by:

$$b^{-t}\frac{\partial L}{\partial x^l_{1,t+1}} = bf^l_1 U^{l\prime}(c^l_{t+1}) - bW'(y_{t+1}) - \lambda_t + bp^l_{1,t+1} - b\alpha\mu_{t+1} \leq 0, \tag{3.22}$$

$$b^{-t}\frac{\partial L}{\partial x^l_{s+1,t+1}} = -f^l_s U^{l\prime}(c^l_t) + bf^l_{s+1} U^{l\prime}(c^l_{t+1}) +$$

$$-bW'(y_{t+1}) - \lambda_t + bp^l_{s+1,t+1} - p^l_{s,t} - b\alpha\mu_{t+1} \leq 0 \tag{3.23}$$

for $s = 1, ..., n-2$ and $l = 2, ..., L$

$$b^{-t}\frac{\partial L}{\partial x^l_{n,t+1}} = -f^l_{n-1} U^{l\prime}(c^l_t) + bf^l_n U^{l\prime}(c^l_{t+1}) +$$

$$-bW'(y_{t+1}) - \lambda_t - p^l_{n-1,t} - b\alpha\mu_{t+1} \leq 0 \tag{3.24}$$

for $l = 2, ..., L$

$$x^l_{s,t+1} \geq 0, x^l_{s,t+1}\frac{\partial L}{\partial x^l_{s,t+1}} = 0, s = 1, ..., n \textit{ and } l = 1, ..., L, \tag{3.25}$$

$$p^l_{s,t} \geq 0, p^l_{s,t}(x^l_{s,t} - x^l_{s+1,t+1}) = 0, s = 1, ..., n-1, \textit{ and } l = 1, ..., L, \tag{3.26}$$

$$\lambda_t \geq 0, \lambda_t(1 - \sum_{l=1}^{L}\sum_{s=1}^{n} x^l_{s,t+1}) = 0 \tag{3.27}$$

$$\mu_t \geq 0, \mu_t \left(\sum_{s=1}^{n} x^1_{s,t+1} - \alpha \sum_{l=1}^{L}\sum_{s=1}^{n} x^l_{s,t+1} \right) \tag{3.28}$$

The existence of optimal solutions for bounded utility and $b < 1$ follows from Theorem 4.6 in Stokey and Lucas (p. 79).

Proof of Proposition 1:

Proof. Following Salo and Tahvonen [23], by convexity of problem (3.2)-(3.9), if there exist multipliers $p^l_{s,t}$ satisfying conditions (3.19)-(3.28) under harvesting at m^l, then the resulting age class structure is an interior OFF. The optimality follows since with harvesting at m^l, $\frac{\partial U^l}{\partial x^l_{s,t}}$ and $x^l_{s,t}$ remain bounded satisfying transversality conditions which, together with (3.19)-(3.28) are sufficient for optimality.

For $s = 1, ..., m-1$ using (3.19) for species 1 and (3.22) for $l = 2, ..., L$ to eliminate λ_t from (3.20), (3.21),(3.23), (3.24), and to satisfy (3.25) we obtain a system of m^l x $(m^l - 1)$ equality equations:

$$b(p^l_{s+1,t+1+k} - p^l_{1,t+1+k}) - p^l_{s,t+k} = -bU^{l\prime}(c^l_{t+k+1})(f^l_{s+1} - f^l_1) + U^{l\prime}(c_{t+k})f^l_s \tag{3.29}$$

$$-bp^l_{1,t+1+k} - p^l_{m^l-1,t+k} = -bU^{l\prime}(c^l_{t+k+1})(f^l_{m^l} - f^l_1) + U^{l\prime}(c^l_{t+k})f^l_{m^l-1} \tag{3.30}$$

where $s = 1, ...m^l - 2$, $k = 0, ..., m^l - 1$ and $l = 1, ..., L$.

Note that equations (3.29) and (3.30) are equal to the ones obtained in Salo and Tahvonen [23] for the case of a single species. Therefore, the result obtained in Salo and Tahvonen [23] follows directly.

Solving the above system for any multiplier p^l_{st} yields:

$$p^l_{s,t} = \frac{b^{m^l} f_{m^l}}{1 - b^{m^l}} \left[b^{-s}(U^{l\prime}(c^l_{t+m^l-s}) - U^{l\prime}(c^l_t) \right] - f^l_s(U^{l\prime}(c^l_t) \tag{3.31}$$

The same argument applies for the case where $m < n$.

In addition, a stationary cycle with all land allocated to forestry must satisfy $\lambda_t \geq 0$, for $t = 0, ...$ Solving (3.19) or (3.20) for λ_t, eliminating $p^1_{s,t}$, for $s = 1, ..., m^1 - 1$, $t = 0, ...$, using (3.31), we obtain

$$\lambda_{t+k} = \frac{U^{1\prime}(c^1_{t+k})b^{m^1}f^1_{m^1}}{1-b^{m^1}} - \frac{U^{1\prime}(c^1_{t+1+k})b^{m+1}f^1_{m^1}}{1-b^{m^1}} - bW'(0) + b\mu_{t+1+k}(1-\alpha) \geq 0 \tag{3.32}$$

using the same procedure with (3.22) and (3.23) yields:

$$\lambda_{t+k} = \frac{U^{l\prime}(c^l_{t+k})b^{m^l}f_{m^l}}{1-b^{m^l}} - \frac{U^{l\prime}(c^l_{t+1+k})b^{m+1}f^l_{m^l}}{1-b^{m^l}} - bW'(0) - b\mu_{t+1+k}\alpha \geq 0 \tag{3.33}$$

for $l = 2, ..., L$ and $k = 0, ..., m$, where $c^l_{t+1+m^l} = c^l_{t+1.}$ Writing $c^l_{t+k} = f^l_{m^l}x^l_s$ and $c^l_{t+1+k} = f^l_{m^l}x^l_{s-1}$, $s = 1, ..., m^l$, where $x^l_0 = x^l_{m^l}$ yields

$$\lambda_s = \frac{U^{1\prime}(f^1_{m^1}x^1_s)b^{m^1}f^1_{m^1}}{1-b^{m^1}} - \frac{U^{1\prime}(f^1_{m^1}x^1_{s-1})b^{m^1+1}f^1_m}{1-b^{m^1}} - bW'(0) + b\mu_{t+1+k}(1-\alpha) \geq 0 \tag{3.34}$$

$$\lambda_s = \frac{U^{l\prime}(f^l_{m^l}x^l_s)b^{m^l}f^l_{m^l}}{1-b^{m^l}} - \frac{U^{l\prime}(f^l_{m^l}x^l_{s-1})b^{m^l+1}f^l_m}{1-b^{m^l}} - bW'(0) - b\mu_{t+1+k}\alpha \geq 0 \tag{3.35}$$

for $s = 1, ..., m^l$ and $l = 2, ..., L$.

Note that, as expected, if $\mu_{t+1} = 0$ the above equations are the same as in Salo and Tahvonen [23].

Given $g^l = \frac{U^{l\prime}(f^l_{m^l}/m^l)b^{m^l}f^l_{m^l}}{1-b^{m^{cl}}} - \frac{b}{1-b}W'(0) - \frac{b}{1-b}\mu\alpha > 0$, there must exist a $\phi^l > 0$ such that (3.34) and (3.35) are satisfied if $x^l_s = 1/m^l + \phi^l_s$, $s = 1, ..., m^l$, for all $\left|\phi^l_s\right| < \phi^{cl}$, $\sum_{s=1}^{m^l}\phi^l_s = 0$, and $l = 1, ..., L$.

Let i^l_∞ represent the stationary state level of variable i^l.

We now show that if $g^l \leq 0$ there exists a stationary state that satisfies all the necessary conditions for optimality.[10] ■

Proof of Corollary 1

Proof. Given $g^l \leq 0$, no solutions for (3.35) exist. Thus, by letting $\lambda_t = 0$ in (3.19) or (3.20), eliminating $p^1_{s,t}$, $s = 1, ..., m^1 - 1$, $t = 0, ...$, using (3.31), and writing (3.19) analogously to (3.34), we obtain for $s = 1, ..., m^1$

$$\frac{U^{1\prime}(f^1_{m^1}x^1_s)b^{m^1}f^1_{m^1}}{1-b^{m^1}} - \frac{U^{1\prime}(f^1_{m^1}x^1_{s-1})b^{m^1+1}f_{m^1}}{1-b^{m^1}} - bW'(y_\infty) + b\mu_\infty(1-\alpha) \geq 0 \tag{3.36}$$

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[10] The results obtained in Salo and Tahvonen [23] regarding convergence and stability of the stationary steady states (Lemma 1 and Lemma 2, pg. 523) still apply in the case of this paper, as the difference equation for x_{m_t}, for is similar to equation (34), pg. 522, in the paper. The additional terms that are present in our case are independent of $x_{m^i_t}$. Therefore, the marginal conditions yielding the corresponding characteristic polynomials turn out to be similar.

This system is linear in $U^{1\prime}(f^1_{m^1}x^1_s)$, $s = 1, ..., m^1$ and its solution is given by:

$$U^{1\prime}(f^1_{m^1}x^1_s) = \frac{(W'(y_\infty) - \mu_\infty(1-\alpha))\sum_{i=0}^{m^1-1} b^i}{b^{m^1-1}f^1_{m^1}} \tag{3.37}$$

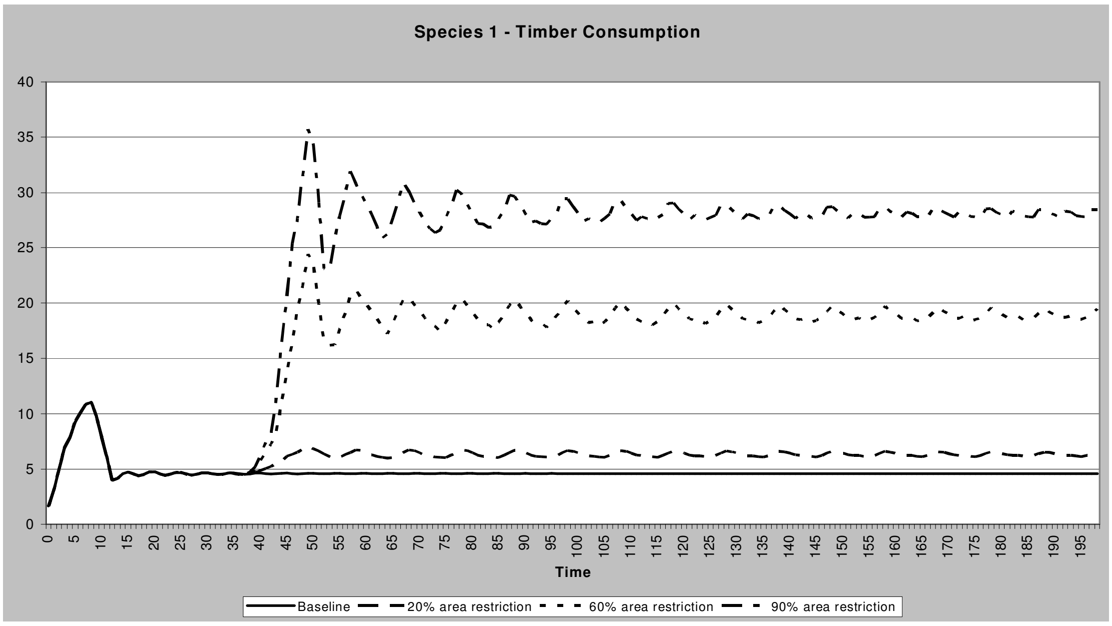
$s = 1, .., m^1$. Thus, $x^1_s = (1 - y_\infty - \sum_{l=2}^{L}\sum_{s=1}^{n} x^l_s)/m^1$, $s = 1, ..., m^l$ and optimal stationary cycles cannot exist.

Following the same steps for species $l = 2, ..., L$. using (3.22) or (3.23), (3.35). The obtained solution is given by:

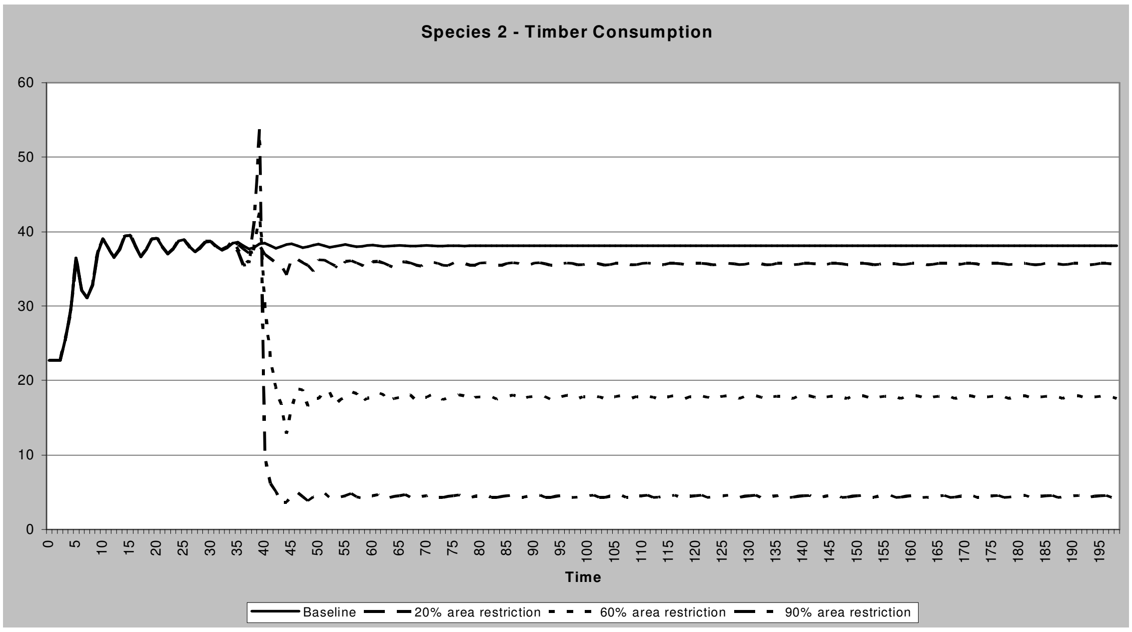
$$U^{l\prime}(f^l_{m^l}x^l_s) = \frac{(W'(y_\infty) + \mu_\infty\alpha)\sum_{i=0}^{m^l-1} b^i}{b^{m^l-1}f^l_{m^l}} \tag{3.38}$$

for $s = 1, ..., m^l$, $l = 2, ..., L$, as can be verified by direct substitution. Thus, $x^l_s = (1 - y_\infty - \sum_{v=1}^{L}\sum_{s=1}^{n} x^{v\neq l}_s)/m^l$, $s = 1, ..., m^l$ and optimal stationary cycles cannot exist. ■

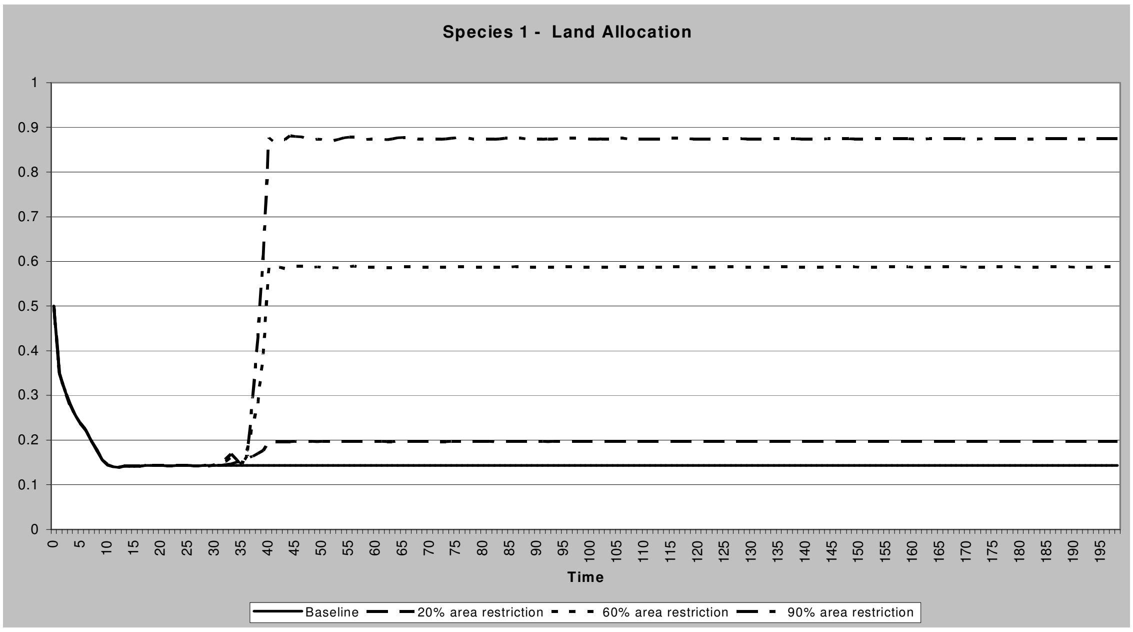
3.6 Appendix B



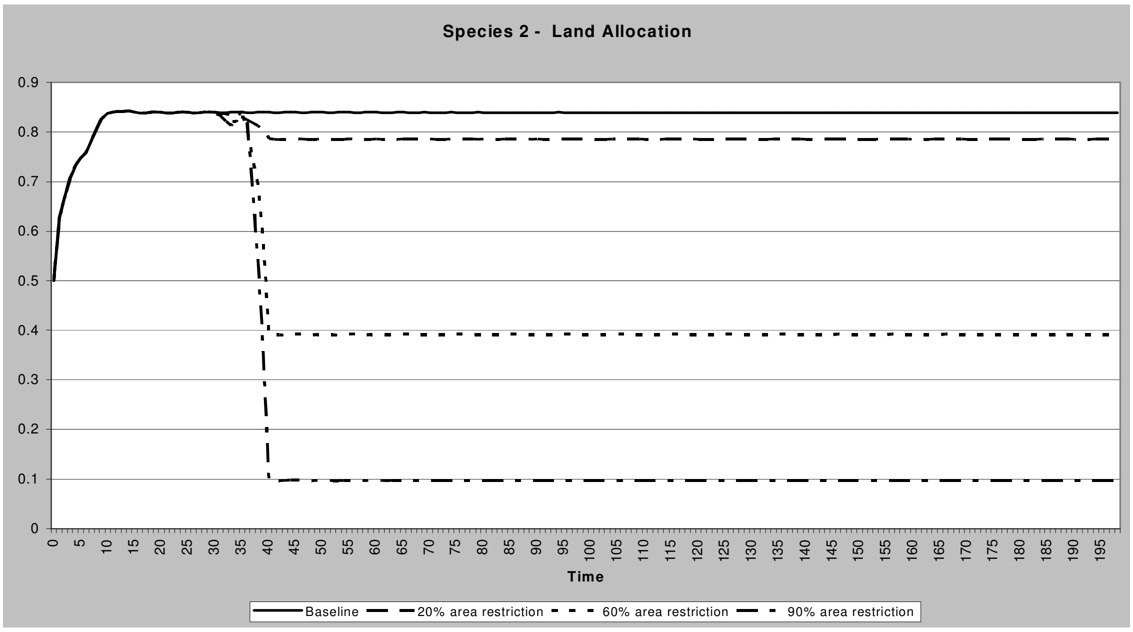
SPECIES AREA - Figure 1a



SPECIES AREA - Figure 1b



SPECIES AREA - Figure 2a



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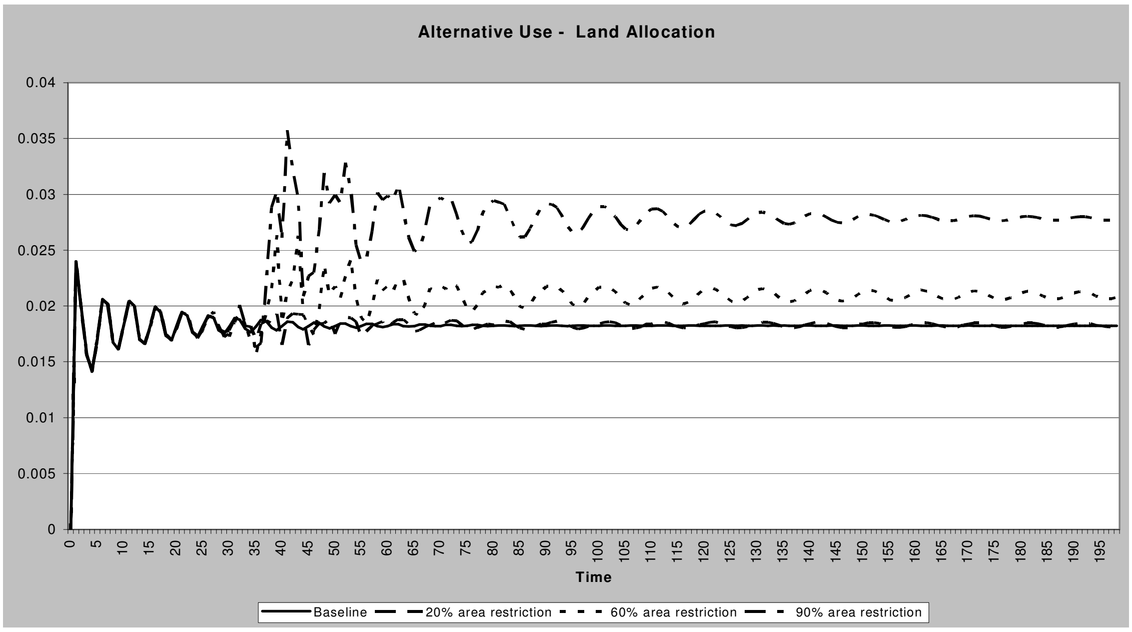

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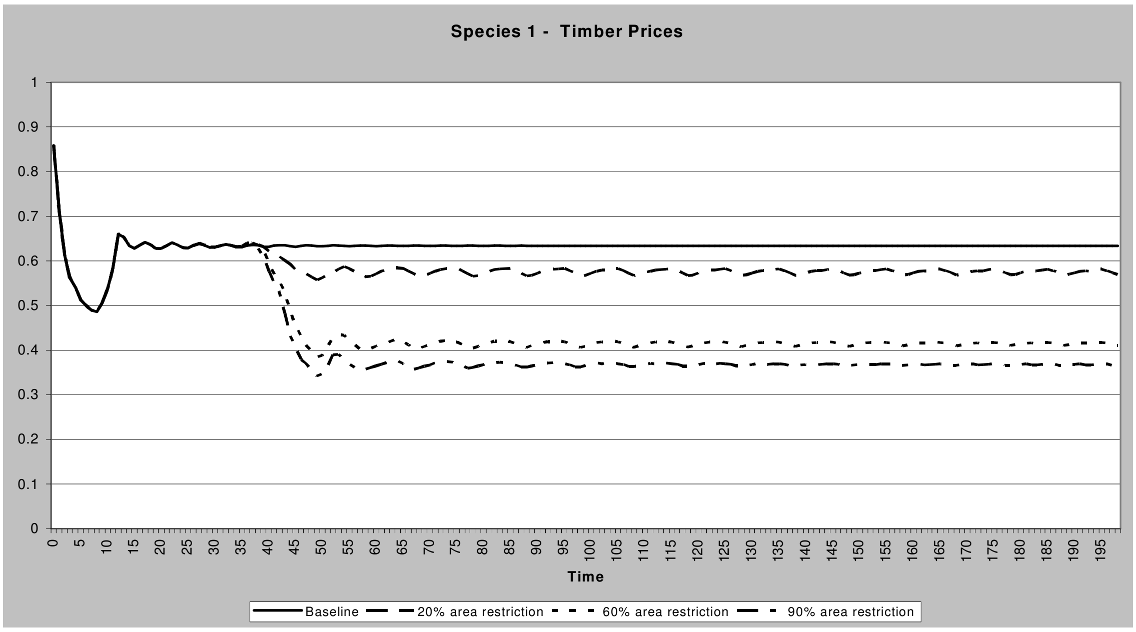
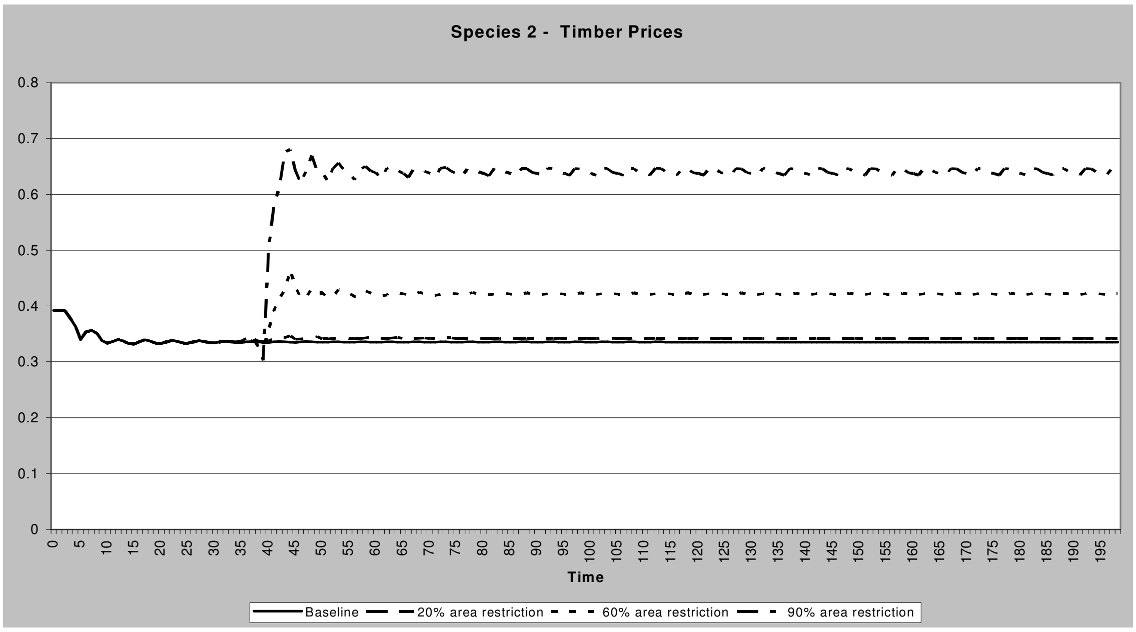
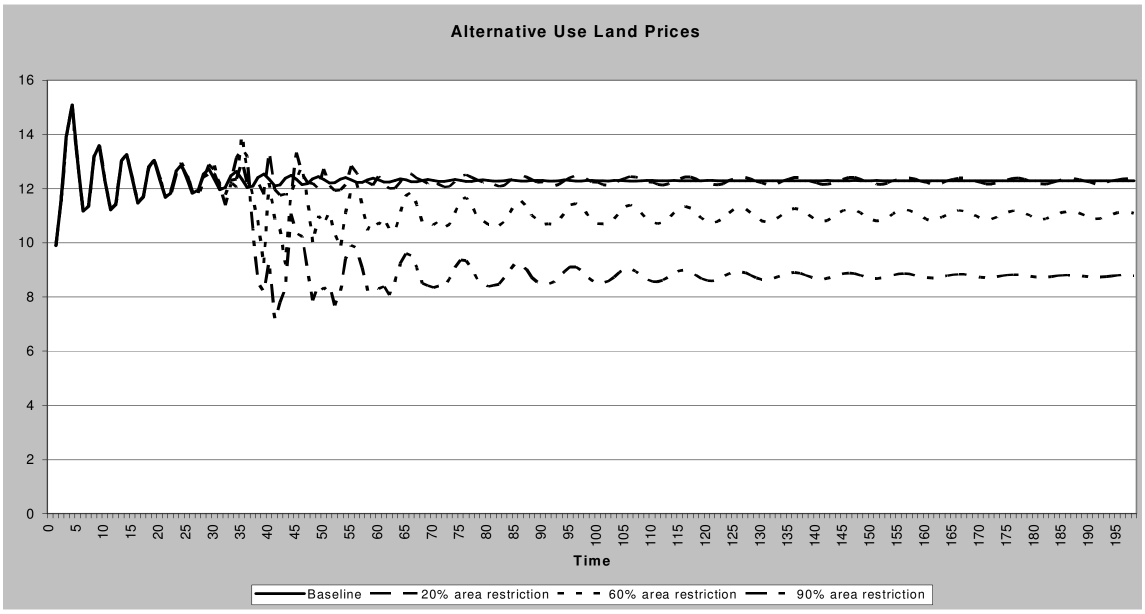


SPECIES AREA - Figure 3a



SPECIES AREA - Figure 3b



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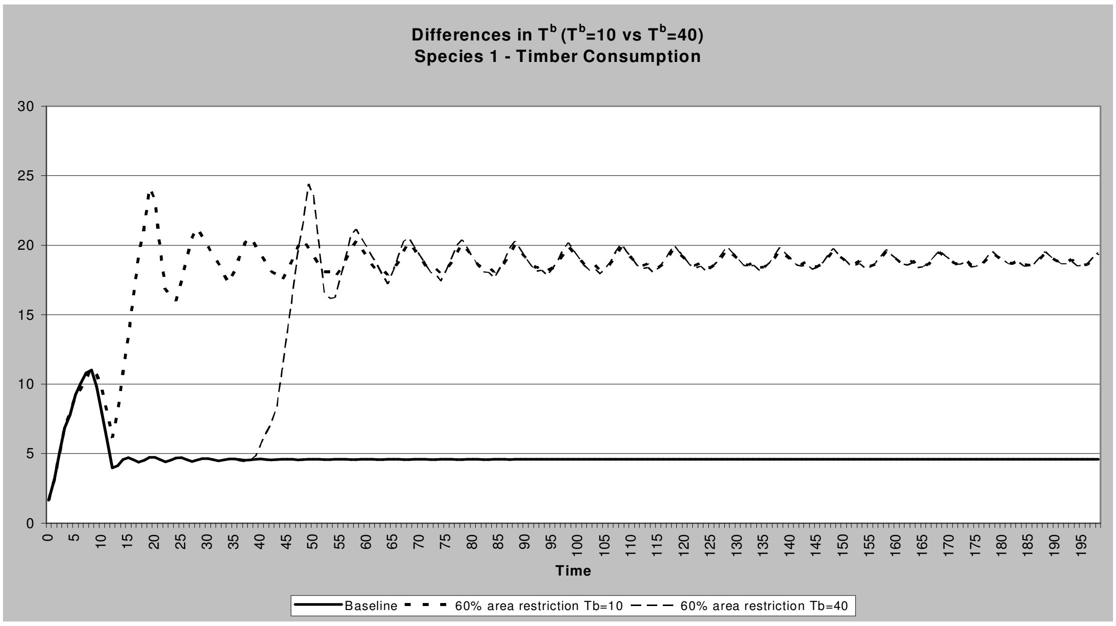
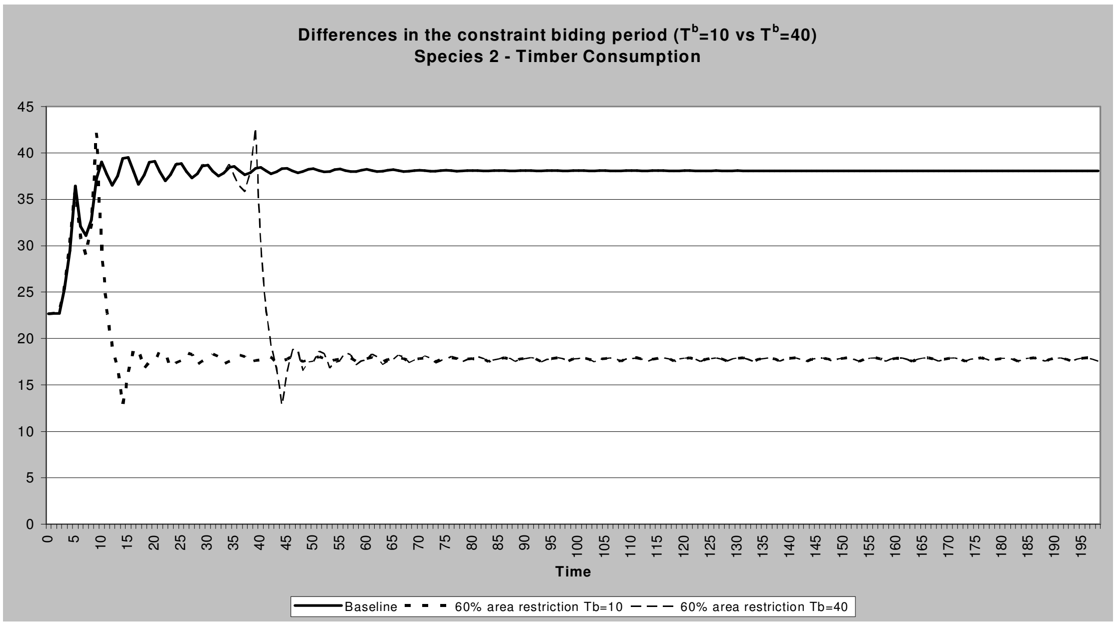


SPECIES AREA - Figure 4a



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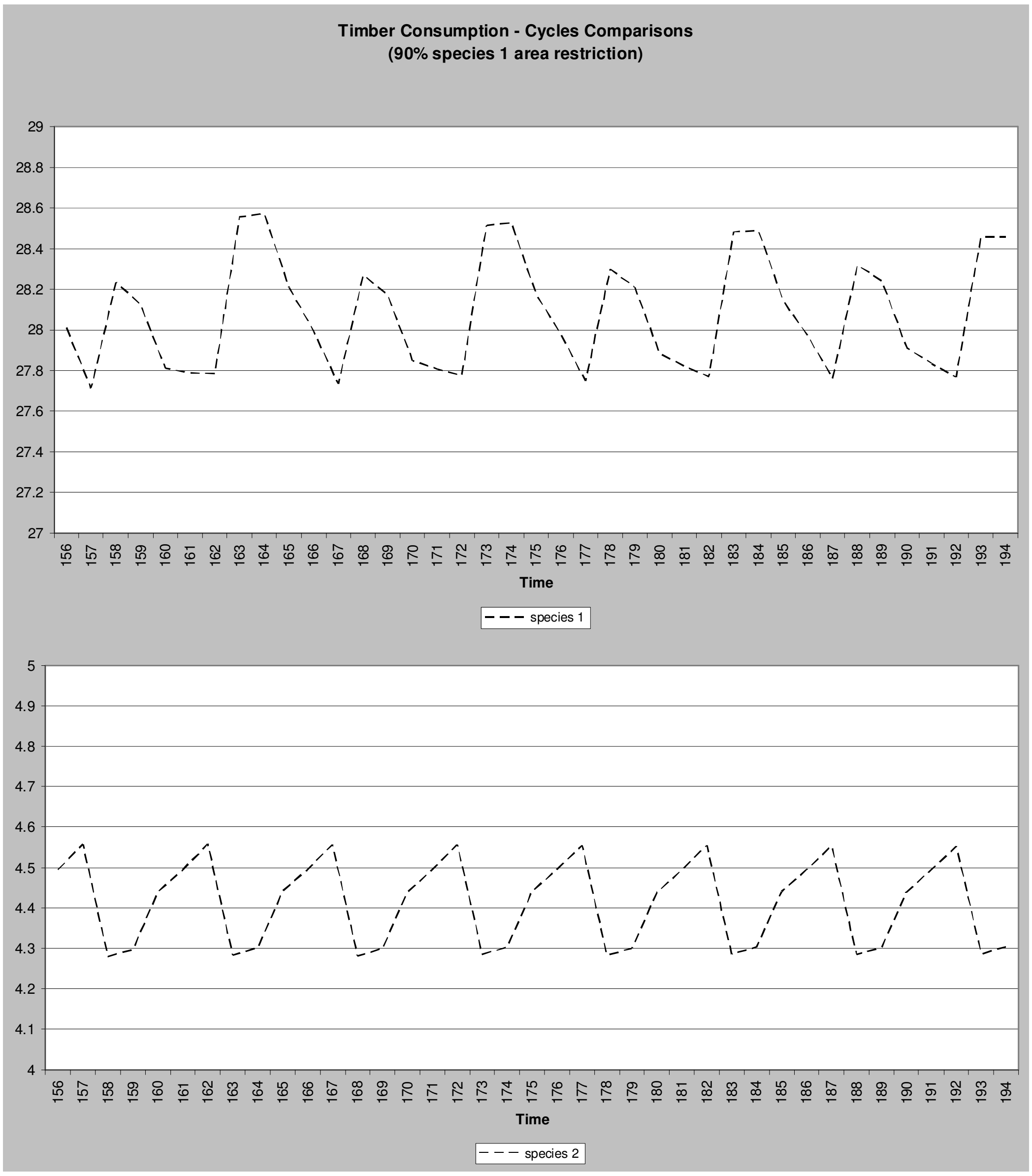
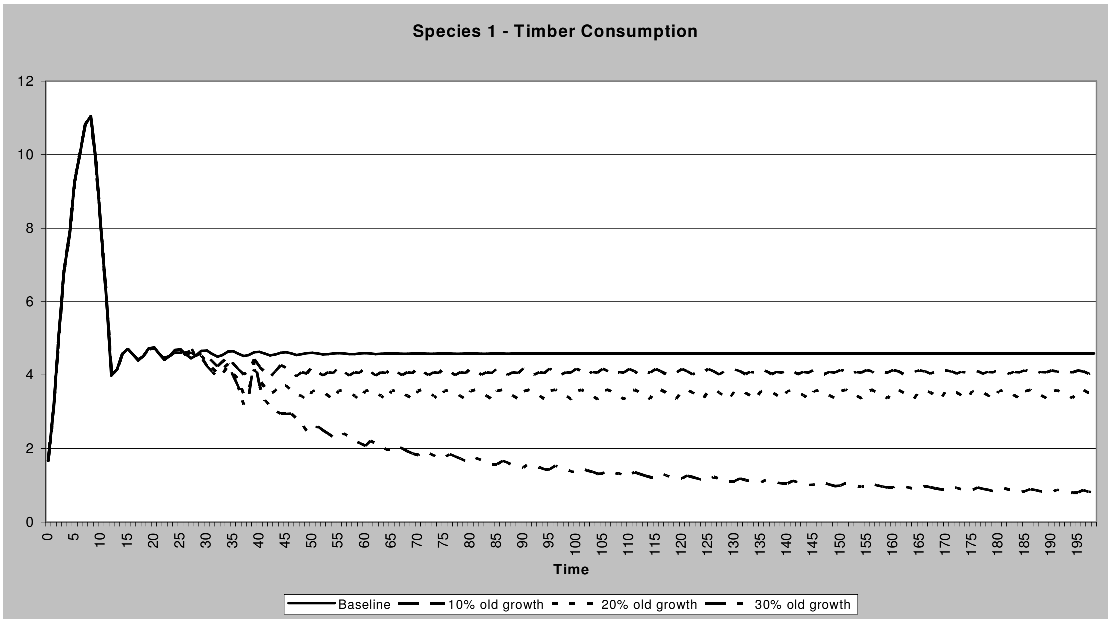
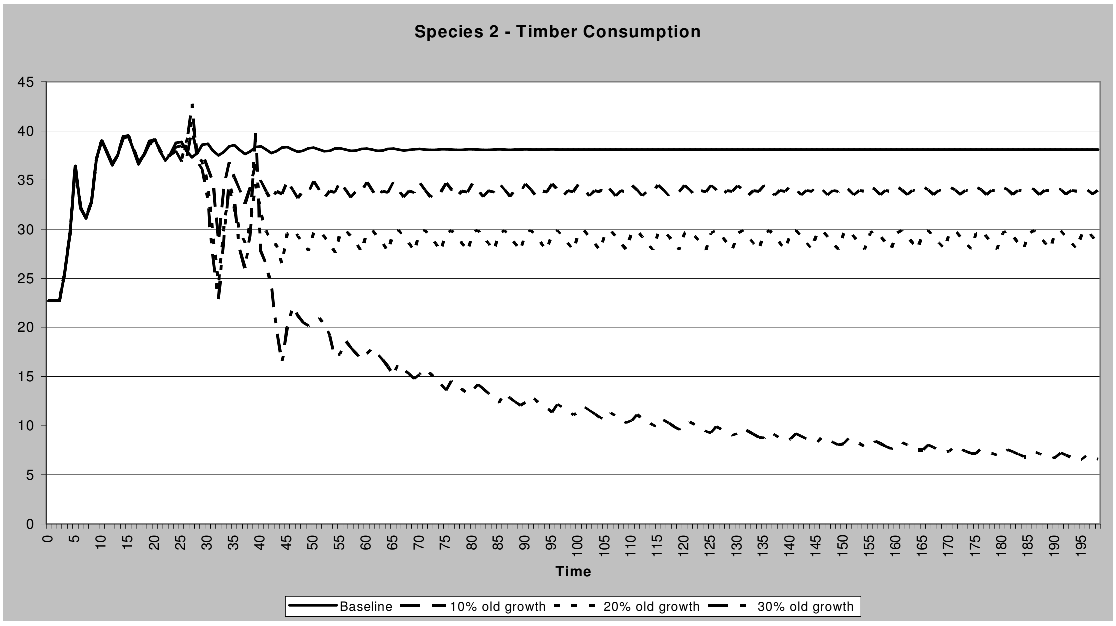


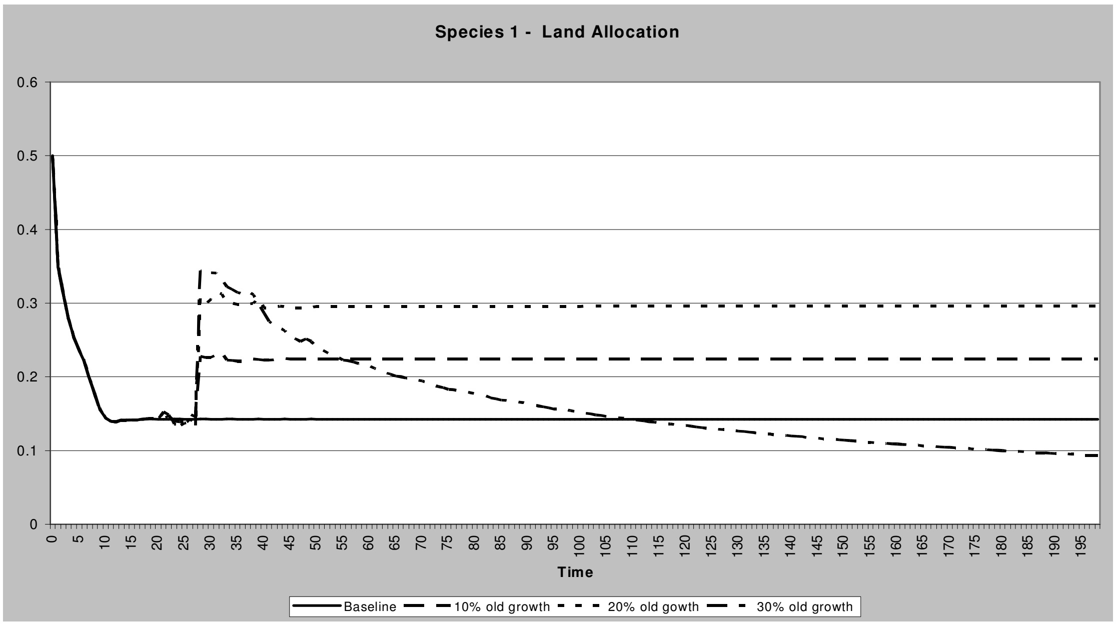
SPECIES AREA - Figure 5



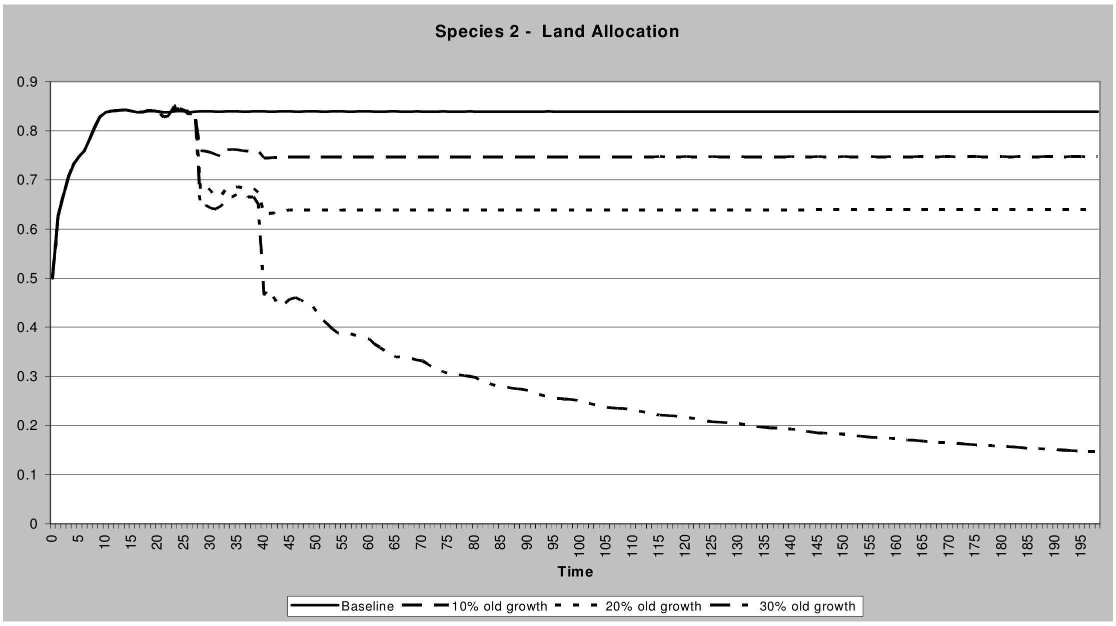
OLD GROWTH - Figure 6a



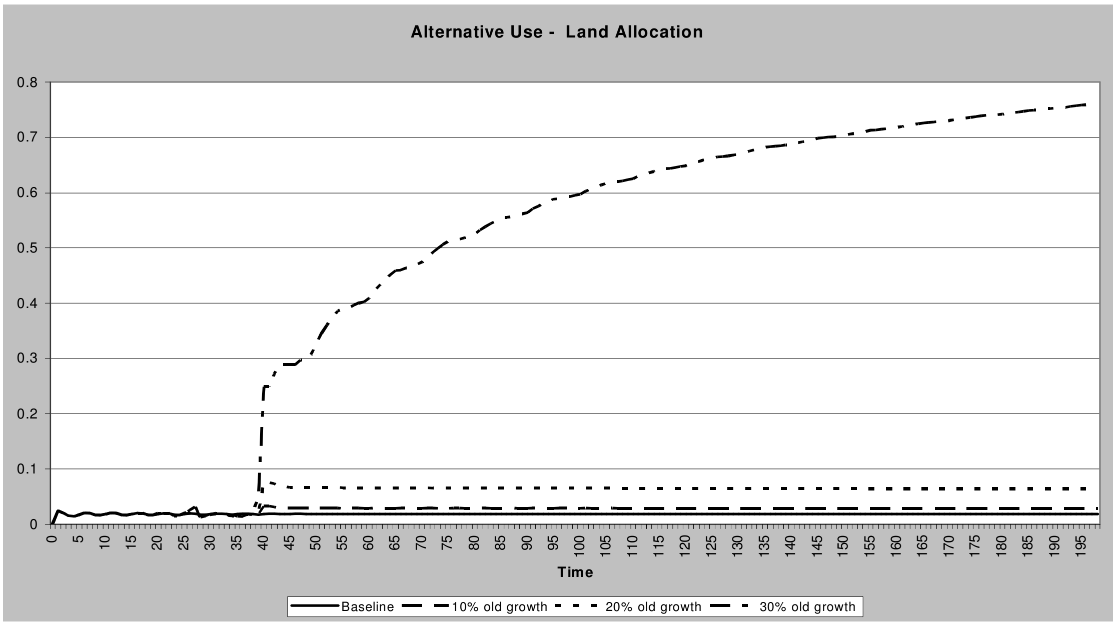
OLD GROWTH - Figure 6b



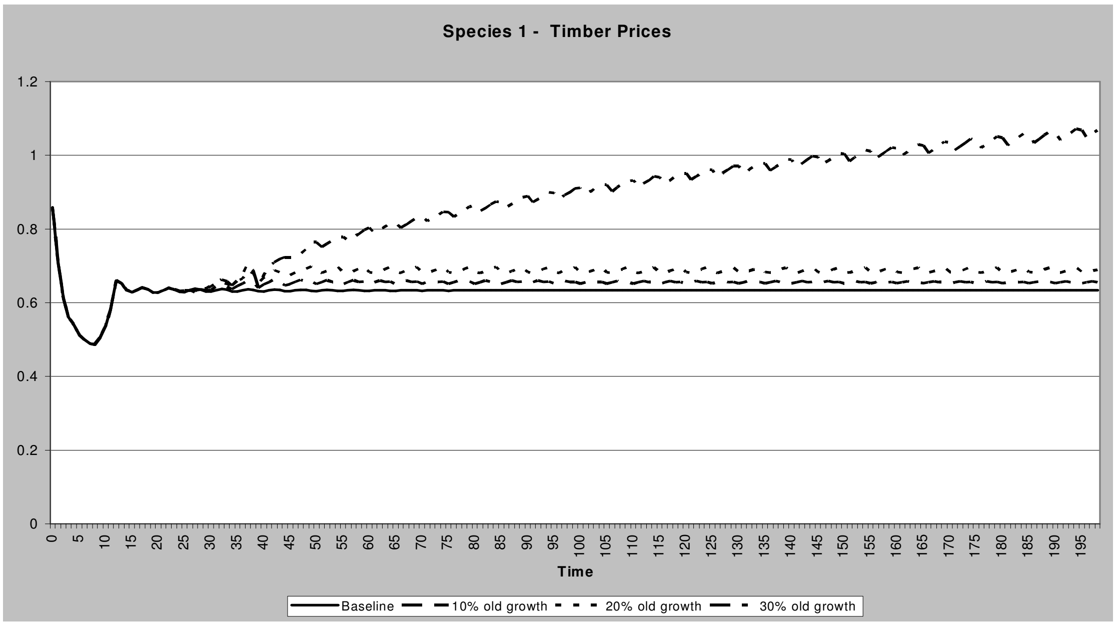
OLD GROWTH - Figure 7a



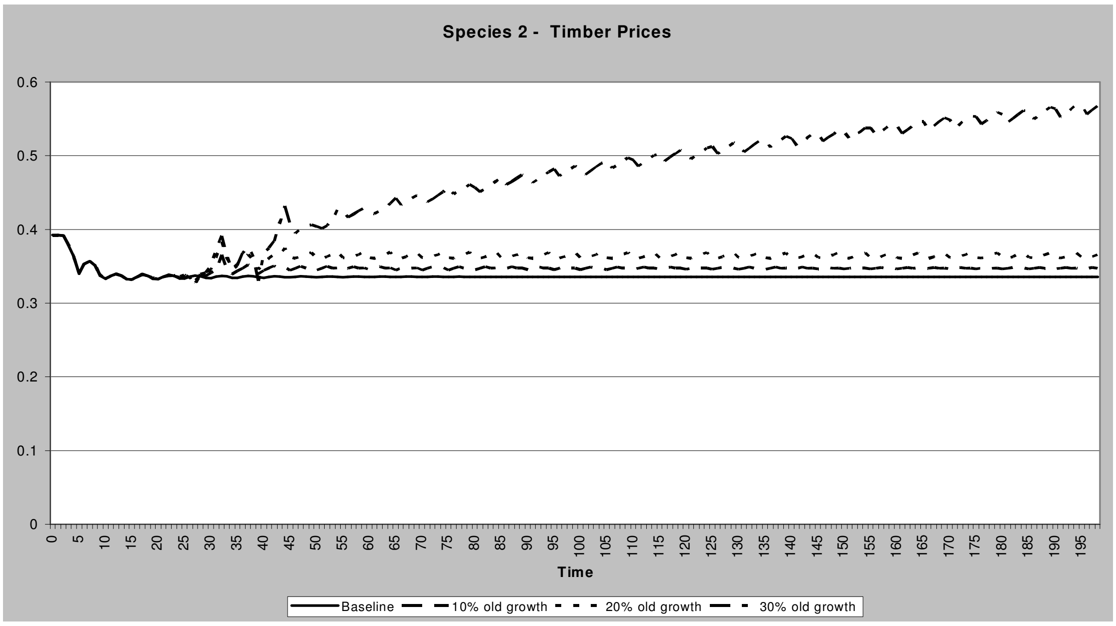
OLD GROWTH - Figure 7b



OLD GROWTH - Figure 7c



OLD GROWTH - Figure 8a



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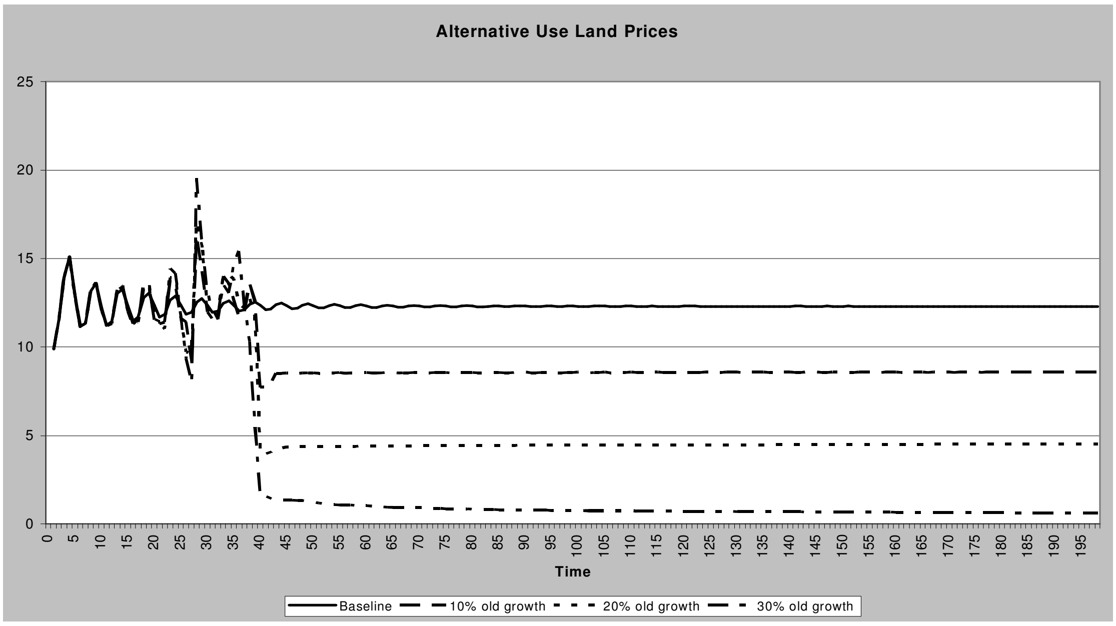

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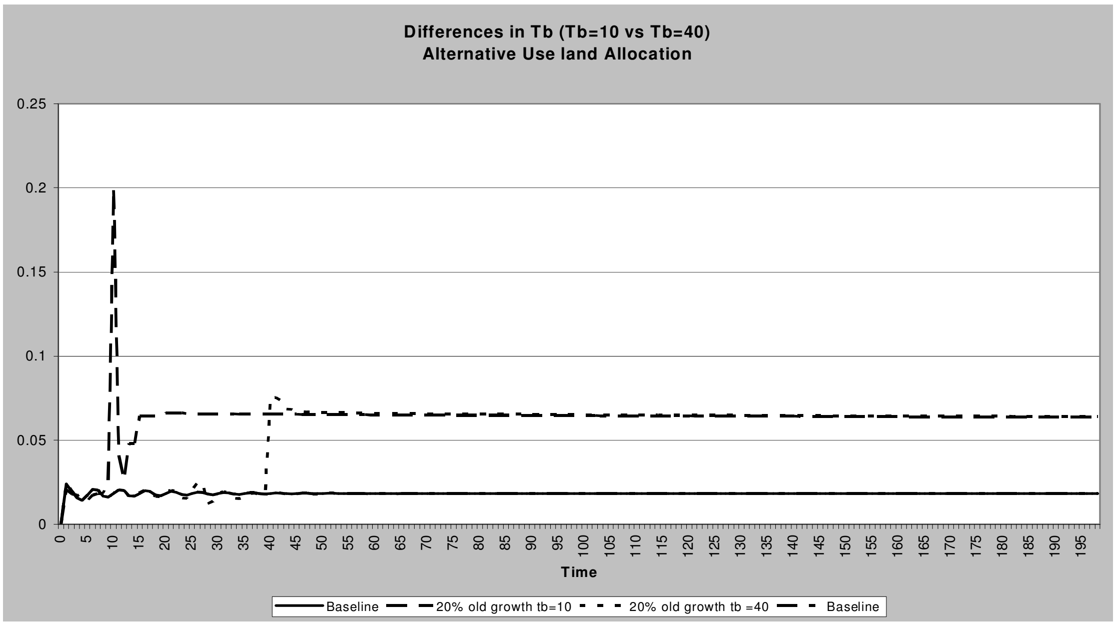
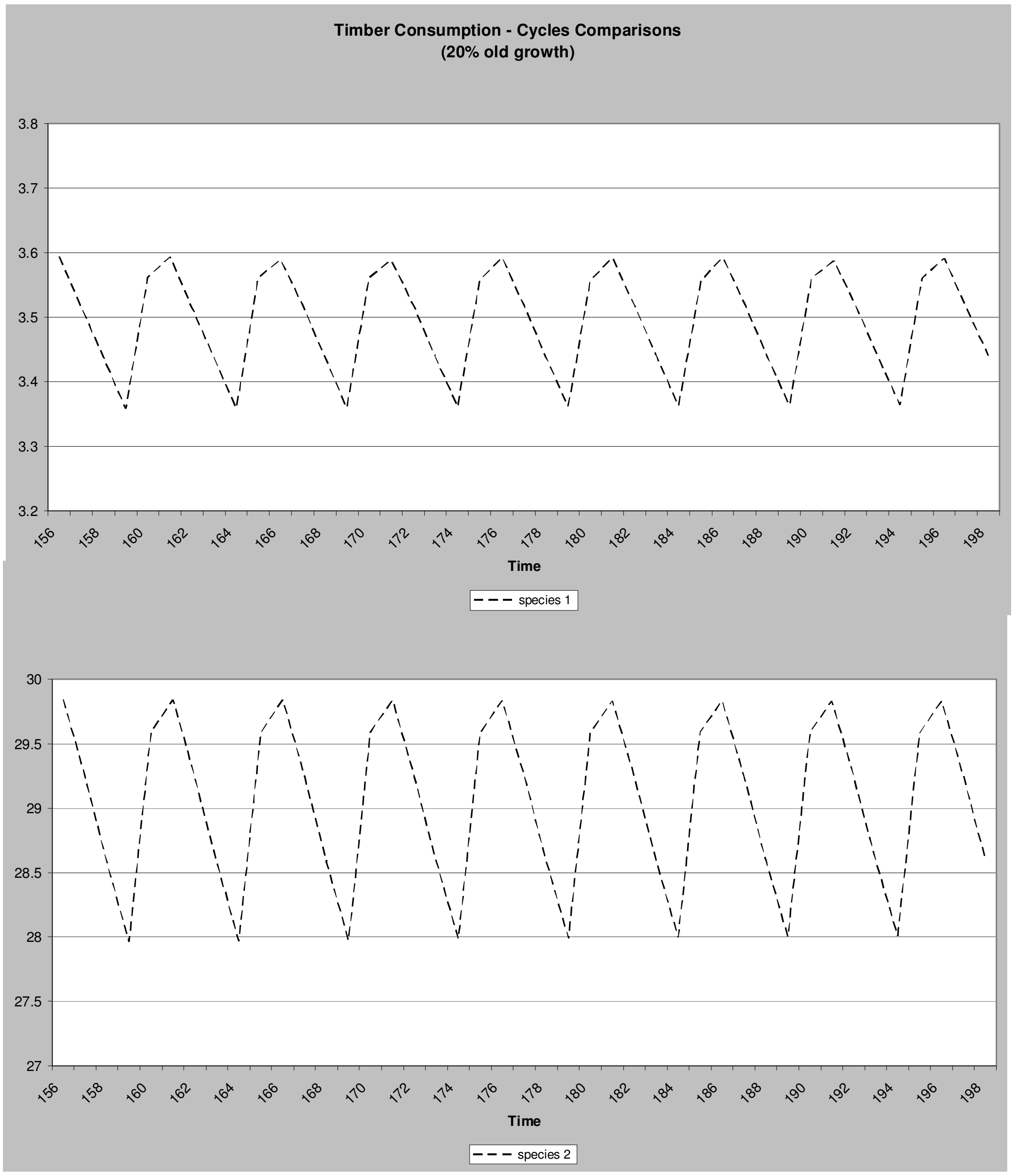


OLD GROWTH - Figure 9



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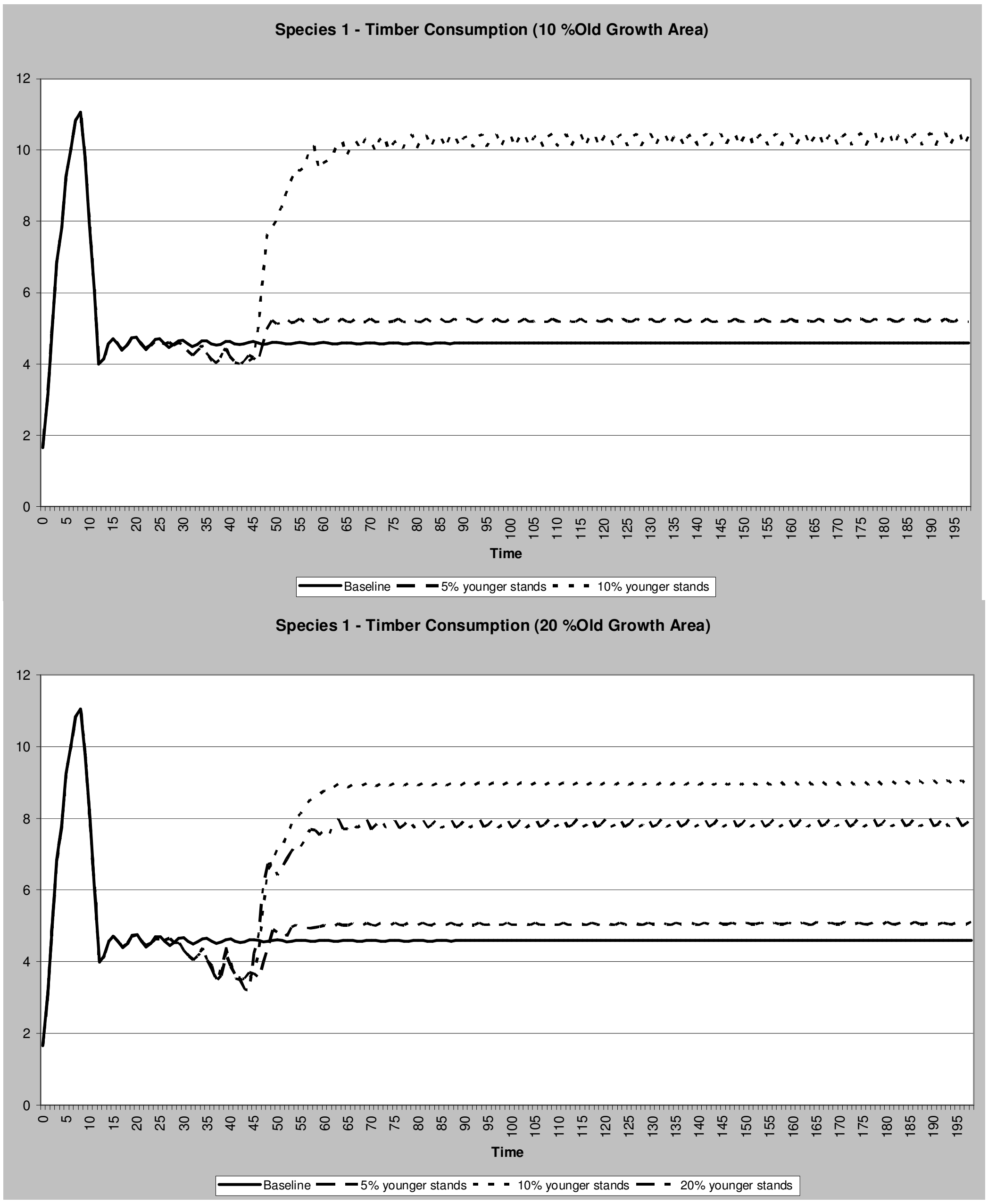
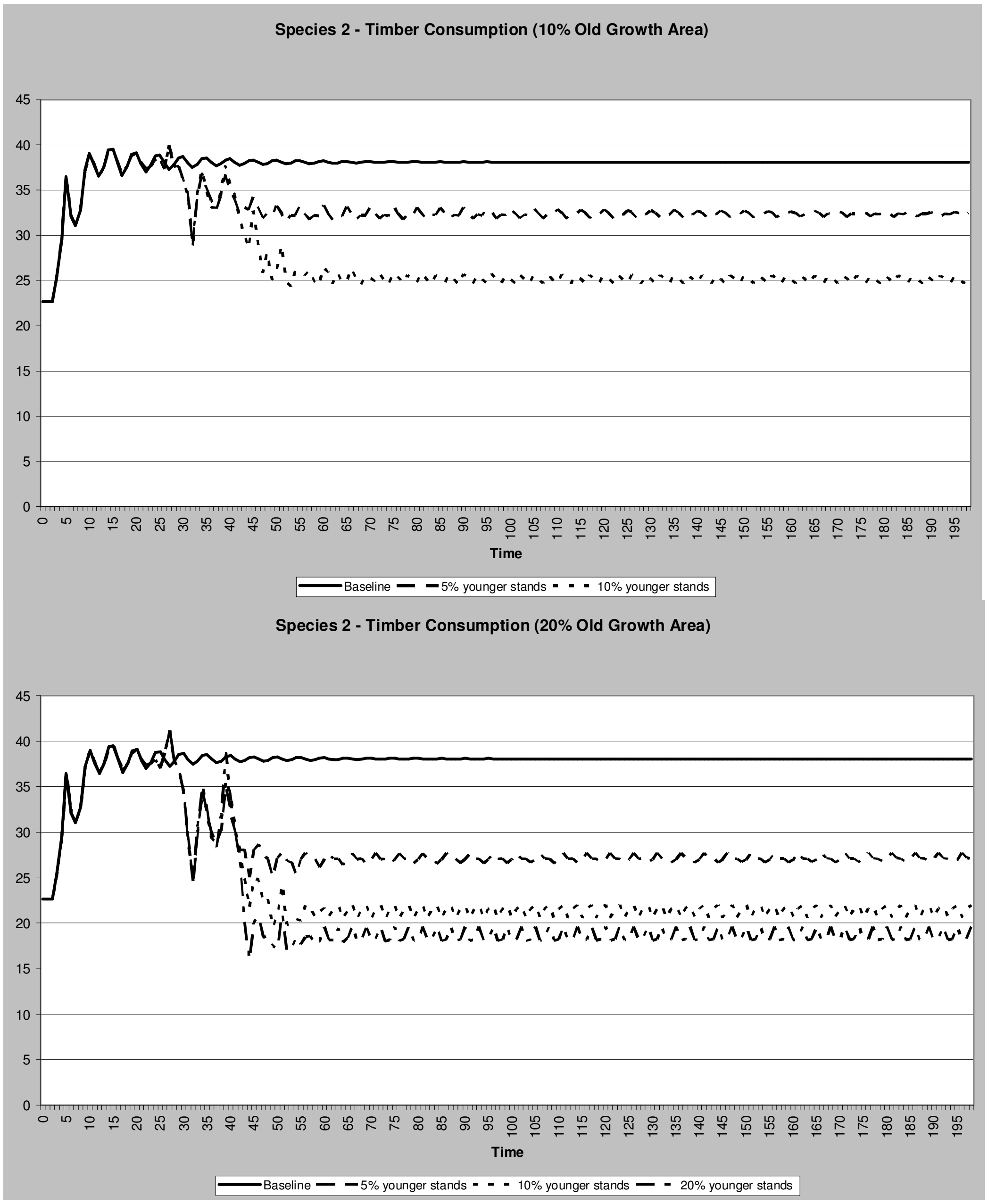
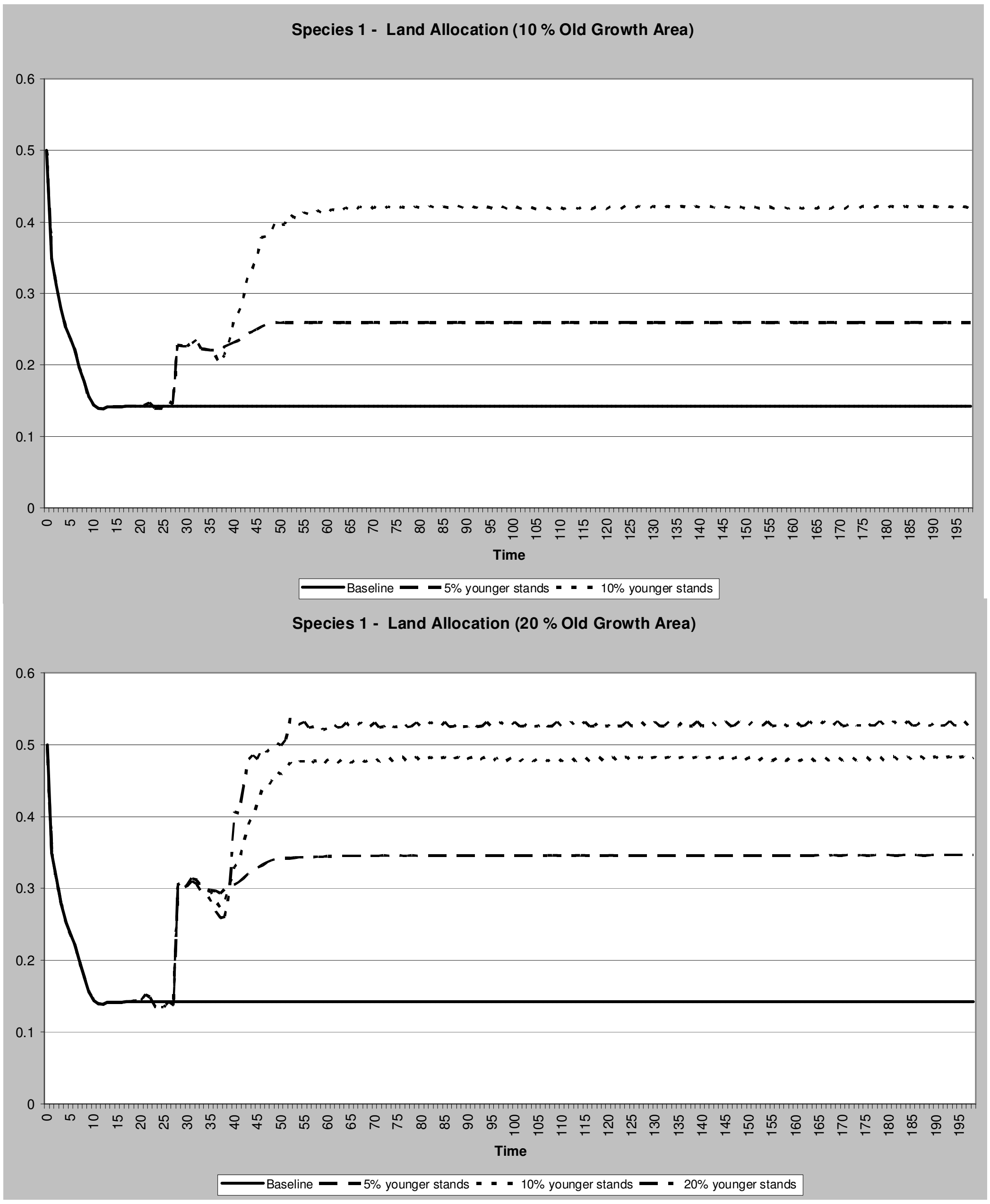


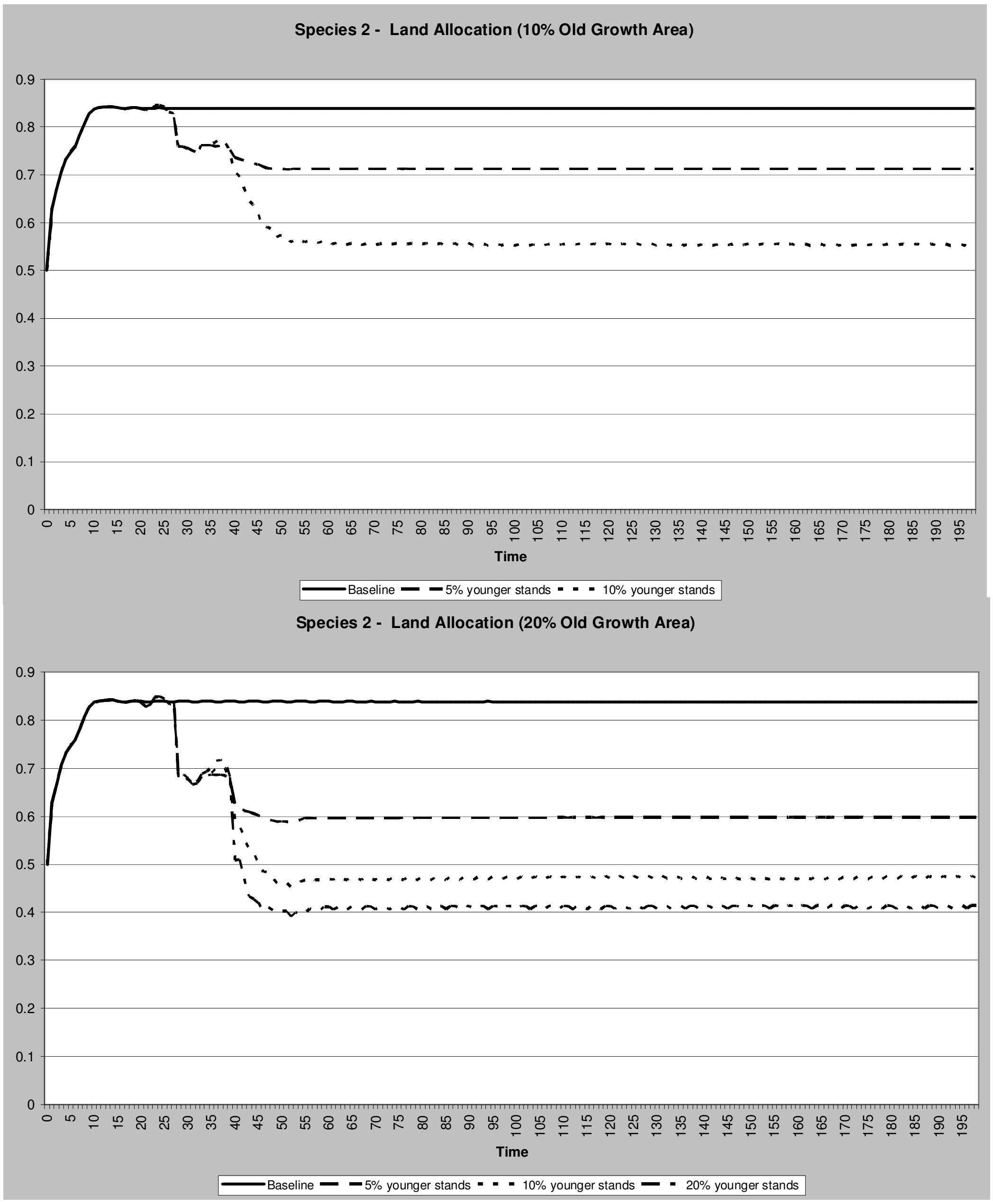
OLD GROWTH AND YOUNGER STANDS - Figure 11a



OLD GROWTH AND YOUNGER STANDS - Figure 11b



OLD GROWTH AND YOUNGER STANDS - Figure 12a



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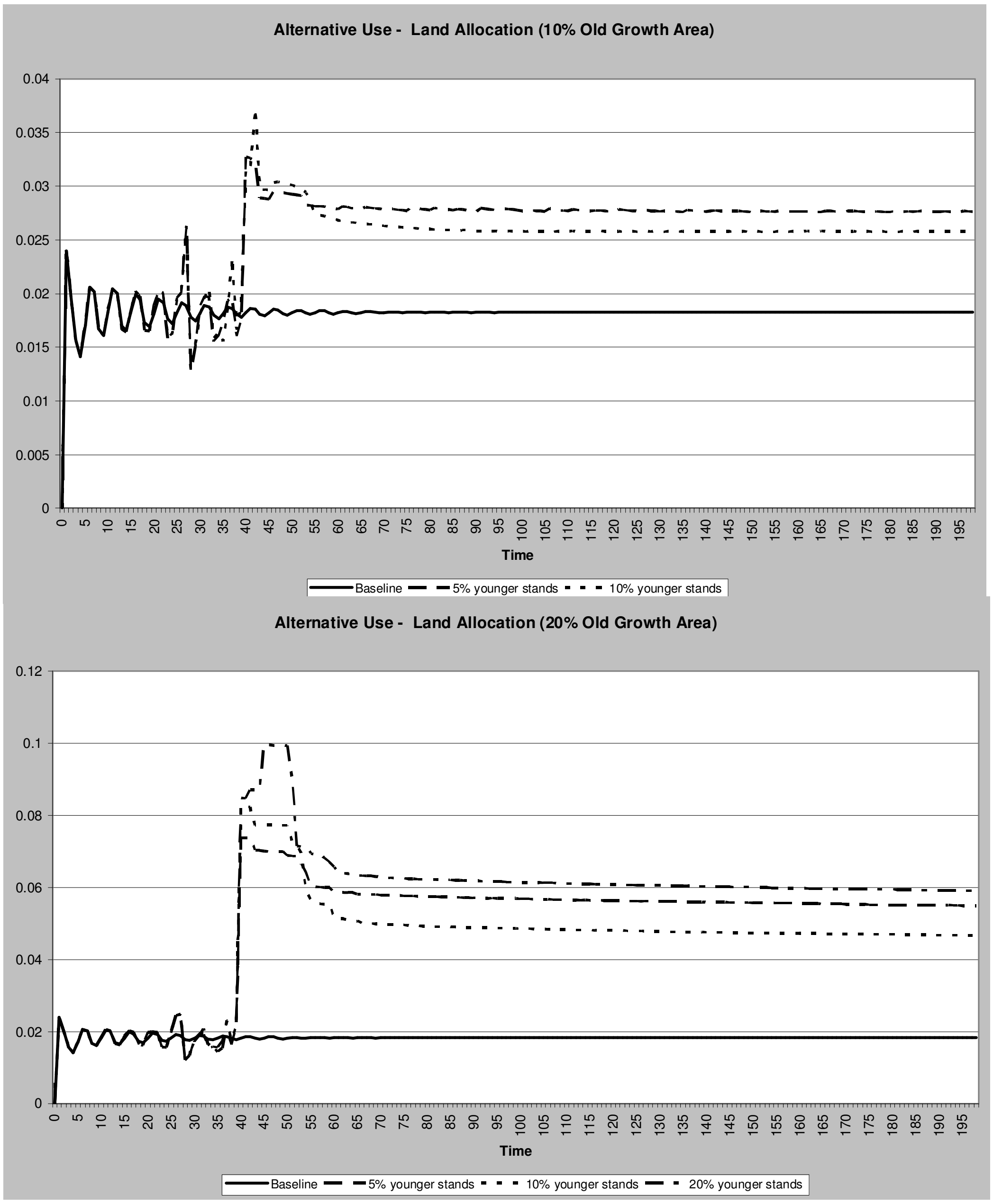

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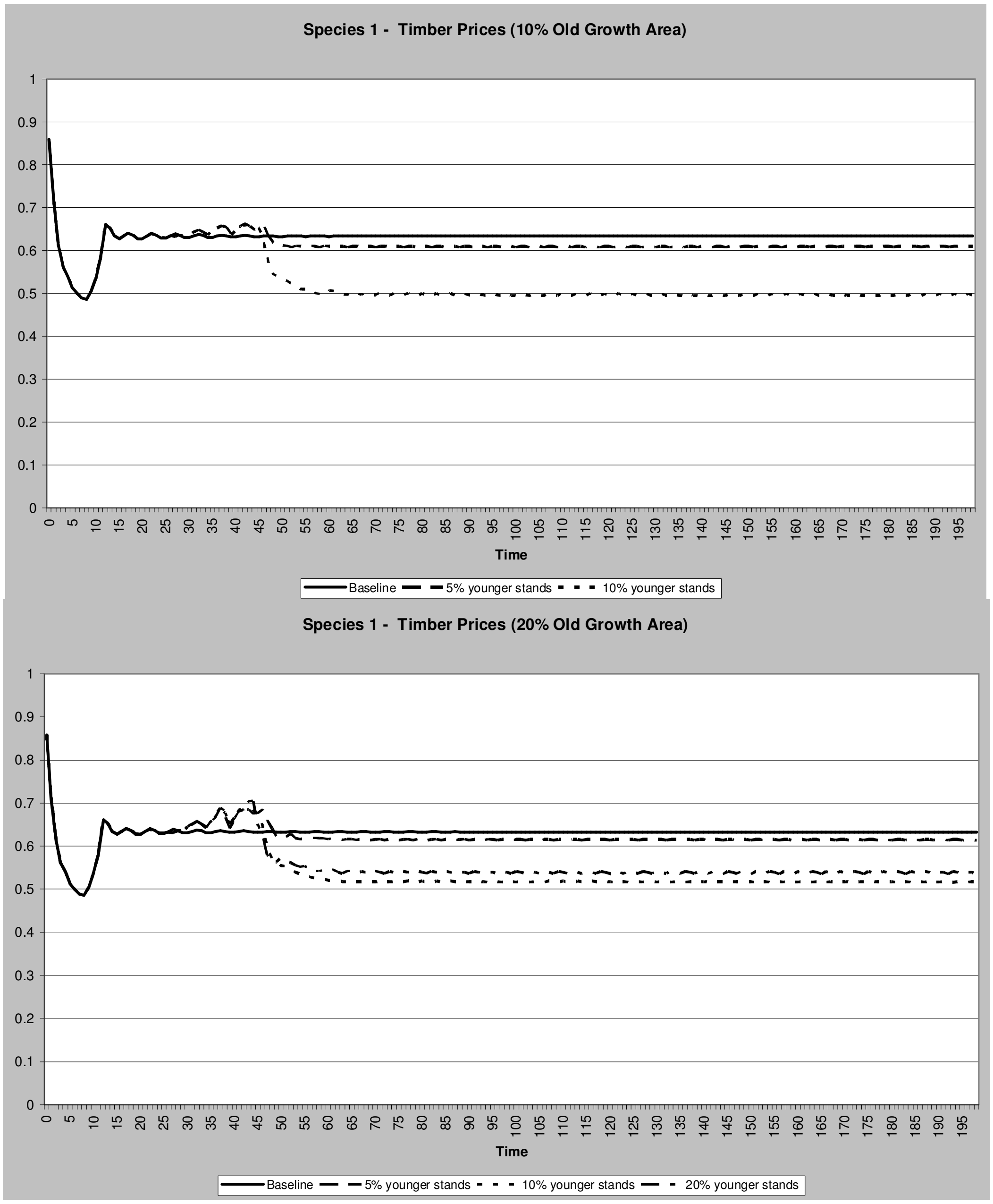

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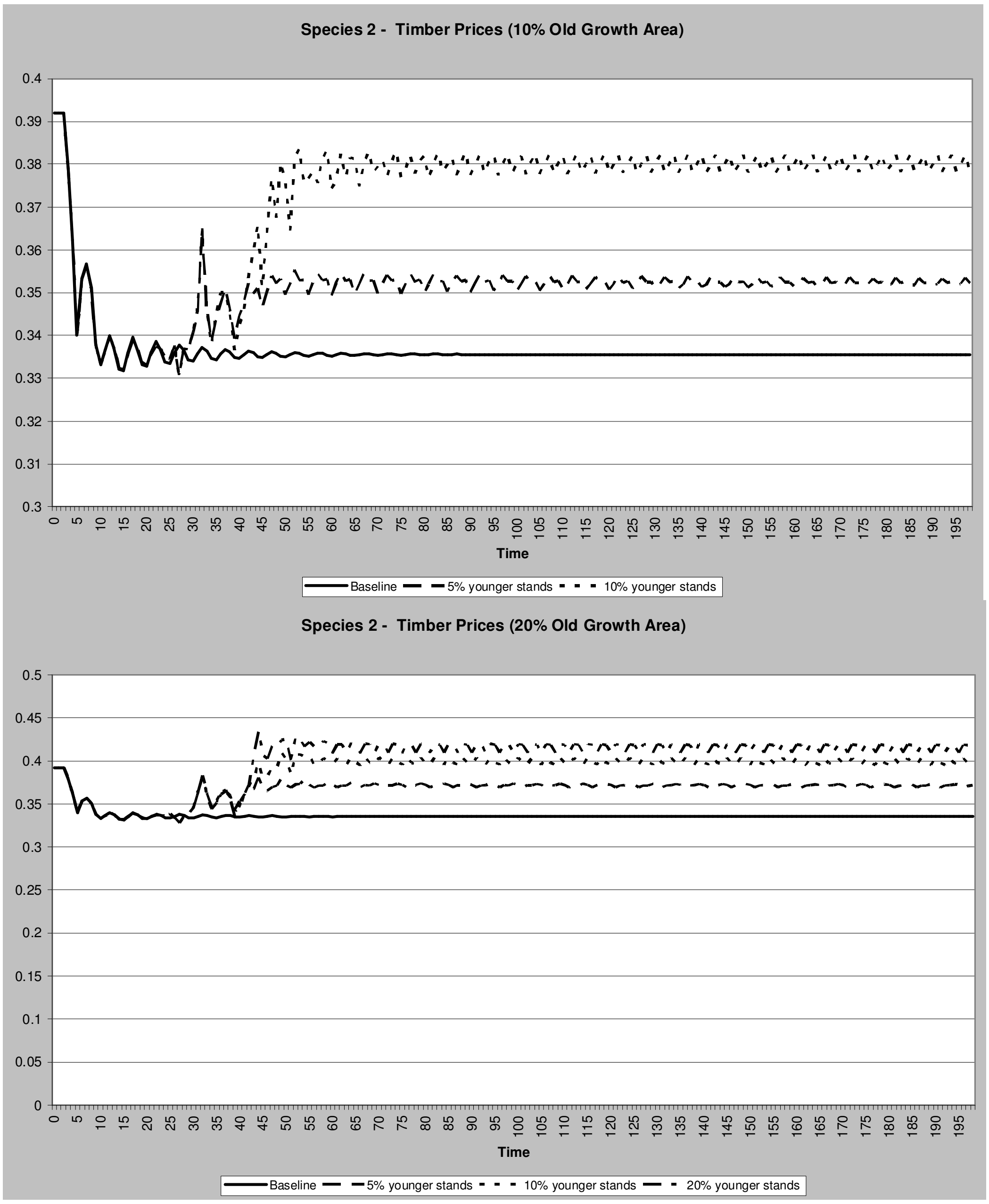

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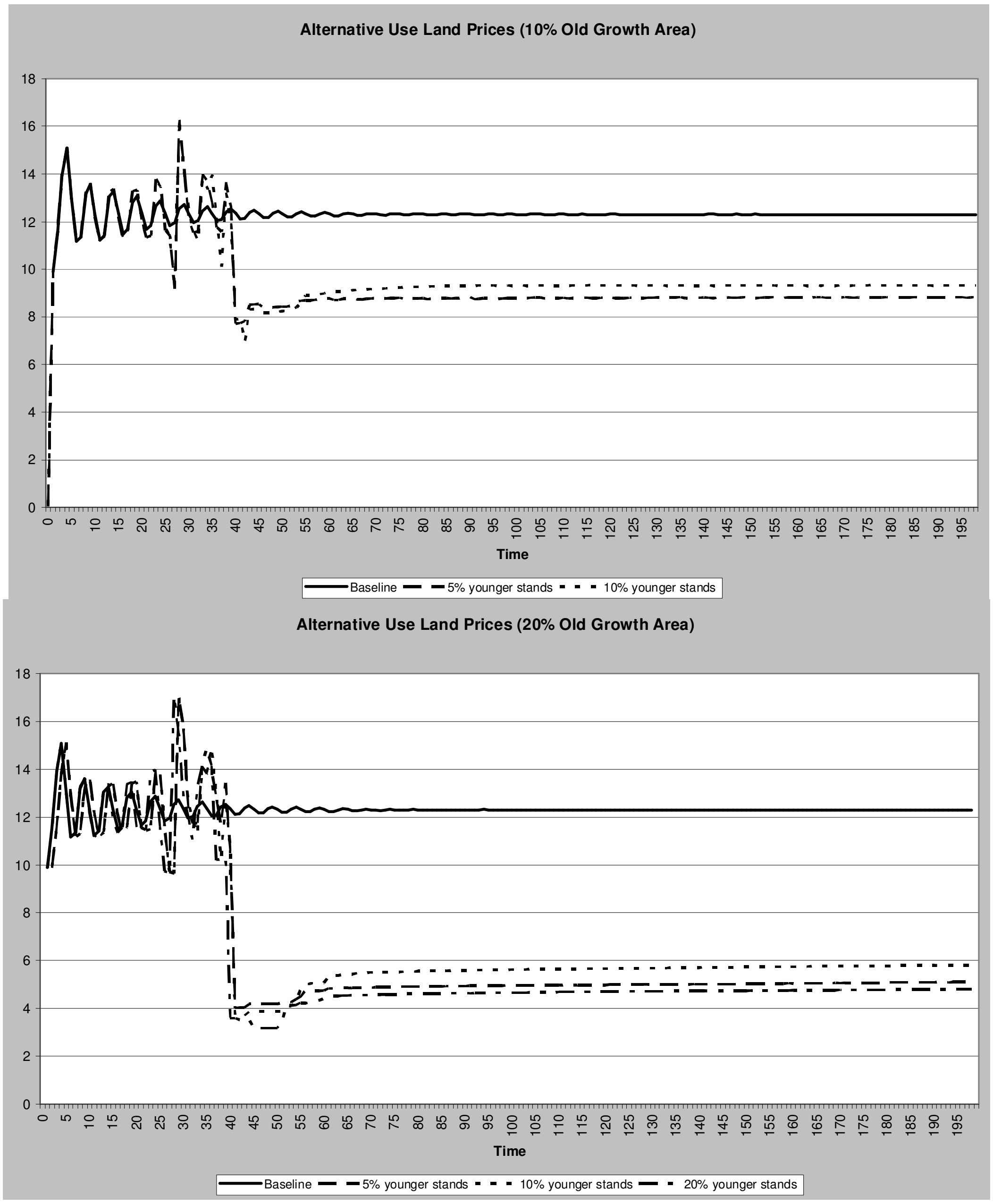

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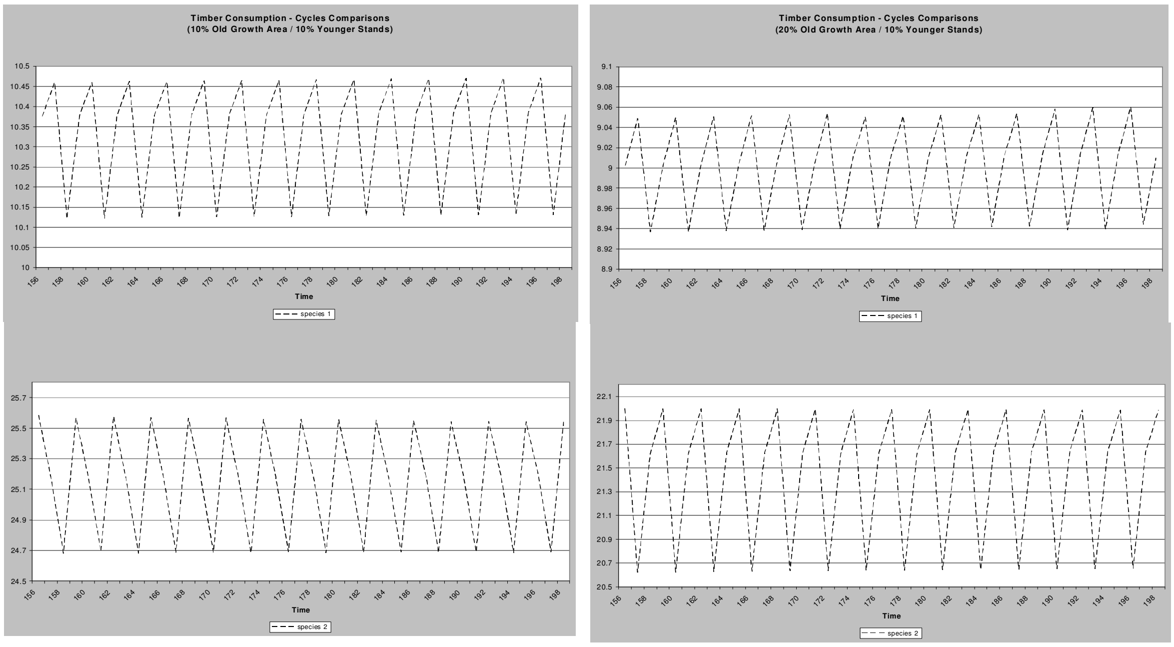

OLD GROWTH AND YOUNGER STANDS - Figure 14

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Chapter 4

Multi-Species Forest Vintages and Carbon Sequestration

4.1 Introduction

Given the rising concern with CO_2 levels, and the recognition in the Kyoto Protocol of the important role that can be played by forests in the global carbon cycle to limit the impact of GHGs (greenhouse gases) emissions, the consideration of carbon sequestration benefits is in the centre of recent developments in forestry literature. Recently, Righelato and Spracklen [25] have shown that the emissions avoided by the use of the liquid biofuels over a 30-year period is much smaller (two to nine times) than the amount of carbon sequestered by forestation of an equivalent area of land.

Several studies have shown that the carbon sequestration option was surprisingly cost-effective in the context of greenhouse gas emissions stabilization plan. Different applied studies have examined the potential impact of forest carbon sink programs by estimating their cost-effectiveness and carbon sequestration capacity in a variety of settings, as Richards, Rosenthal *et al.* [23], Sohngen and Mendelsohn [30], and more recently Tavoni *et al.* [33], among others. Although the level of impact varies significantly depending upon the model used, in all cases introducing forests as carbon sinks reduce costs relative to policies that only consider fossil fuel emissions.

Moreover, forest conservation can be implemented almost immediately at a global scale. As a matter of fact, know-how for forest conservation is available today and may be used as a means to buy needed time until the low-carbon technologies become available. Using the growing international carbon market to value the carbon contained in standing forests will also provide the incentives to change land-use decisions, making a standing forest more valuable than alternative uses of land.

From a theoretical point of view, when carbon benefits are considered, not only the forested area is relevant, but also the flow of carbon between land and the atmosphere through the carbon cycle, namely, the amount of carbon released when the forest is harvested. To account for all these impacts the typical analytical framework of the one stand forest or any other that does not take into account the internal age-structure of forests, are not appropriate.[1] In the single stand case, the decision on the optimal allocation between alternative uses can only be assessed in marginal terms, not allowing to address the global (or regional) impact of some policy incentive measures. Besides, since it typically represents the decision model of the private owner, both the price of land and the price of timber are exogenous constants. In contrast, in this paper, both the price of timber and the price of land are endogenously determined. Therefore, only in this context it is possible to study the transition path to the new steady-state, and, more generally, to perform comparative welfare analysis when carbon benefits are considered. Hence, a multi-vintage forest setting with possible conversion to alternative land uses should be considered instead. [2]

Despite that forest management activities play a key role through mitigation of climate change, forests are also affected by climate change which may under some circumstances reduce the net impact of those mitigation activities, as recent literature indicates, as a consequence of fire, pests, drought and heat waves, affecting forestry production including timber.[3] Forest mitigation options include reducing emissions from deforestation and forest degradation, enhancing the sequestration rate in existing and new forests, providing wood fuels as a substitute for fossil fuels, and providing wood products for more energy-intensive materials. Therefore, if properly designed and implemented, forestry mitigation options may generate benefits in terms of employment and income generation opportunities, biodiversity and watershed conservation, provision of timber and fibre, as well as aesthetic and recreational services.

In this context, the mitigation and adaptation trade-offs and synergies in the forestry

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[1] A one stand or single vintage forest is characterized by a plot of land with trees of the same species and of the same age, where the price of timber is an exogenous constant. In this context, we should mention, among others, Van Kooten, Binkley and Delcourt [35], who modeled a scheme to allocate carbon credits, under which the carbon credit cash flows are a function of the annual change in the forest carbon stock (carbon flow regime), Spring, Kennedy, and Nally [12] that study the effect of carbon sequestration, fire frequency and water scarcity in tree harvest decision, and Cunha-e-Sá and Rosa [7] where different accounting methods of carbon sequestration benefits in the model of the private forester are examined with constant and rising carbon prices. Also, Velt and Plantinga [36] explore the effect of rising carbon prices on the optimal portfolio of greenhouse-gas mitigation strategies based on the carbon flow accounting regime, and considers the optimal timing to convert agriculture land to forest.

[2] Reinforcing the interest of this modeling framework for empirical studies, Getz and Haight [10] refer that biological populations are typically described by discrete time demographic models for reasons like seasonal cyclicality in reproduction or in concentration in harvesting, as in the case of forests.

[3] See Nabuurs, Masera, Andrasko, Benitez-Ponce, Boer, Dutschke, Elsiddig, Ford-Robertson, Frumhoff, Karjalainen, Krankina, Kurz, Matsumoto, Oyhantcabal, Ravindranath, SanzSanchez, and Zhang [19].

sector should be carefully addressed. Several adaptation practices can be used in the forest sector, including changes in land use choice, management intensity, hardwood/softwood species mix, timber growth and harvesting patterns within and between regions, changes in rotation periods, shifting to species more productive under the new climatic conditions, landscape planning to minimize fire and insect damage, among others. In some specific regions, namely, in Europe, afforestation and reforestation are the dominant mitigation options. Currently, these activities are included under Article 3.3 and in Articles 6 and 12 (CDM) of the Kyoto Protocol. Plantations consisting of multiple species may be an attractive adaptation option as they are less vulnerable to climate change, because of larger tolerance of some species to climate change, different migration abilities and different resilience to invading species.

This is related to a more general argument according to which diversity raises productivity and robustness of natural ecosystems, and therefore of the Earth's life-support systems. Diversity helps natural ecosystems to make the best adjustments to changes in environmental conditions. There is no single subset of species that by itself would serve to operate all ecosystems services all over the planet. So diversity in a given location may increase productivity and ecosystem functions in that location, while diversity at the regional or global level is necessary for the operation of important ecosystems in all geographic regions.

The purpose of this paper is to develop a framework where carbon sequestration benefits are internalized in a multi species and multi vintages context. By allowing to endogenously determine the price of timber and the unit price of land, the implications to the forest sector of introducing changes in management practices, different species mix, alternative uses of land, externalities, taxation, among other policy relevant questions, can be studied. The model follows closely the multiple vintage forest model developed in Salo and Tahvonen [26], [27] and [28], extending it by introducing net carbon sequestration benefits on optimal land allocation and optimal forest management in a multi-species discrete-time setting.

Based on the IPCC Special Report on Land Use, Land Use Change and Forestry, we consider the carbon flow accounting method, which is essentially a Pigouvian tax/subsidy on the carbon externality. The proofs on the existence of optimal stationary steady-states are extended to this more general context. Besides, we show how the optimal rotation period, the long run equilibrium and the optimal land allocation are affected by introducing carbon sequestration benefits as well as by the species mix.

In general, we conclude that with or without benefits from carbon sequestration, and in the absence of externalities among species, the long-run stationary state is the normal forest for each species. In the case where all land is forested land, optimal forest management can lead to optimal cyclical harvesting. Alternatively, when it is optimal to allocate part of the forest land to other land uses, the remaining equilibrium is the normal forest steady state. When cycles optimally vanish, in general, the normal forest becomes a local saddle point equilibrium. Also, we conclude that biology plays an important role, as fast growing species' plantations increase when carbon benefits are introduced. Carbon in long-lived products is a critical aspect, as its consideration may actually reverse optimal allocations of land in favor of slow growing species when compared to the case without carbon. Moreover, the net social benefits of a carbon sequestration policy are considerably higher in a framework where this carbon pool is also accounted for.

The remainder of the paper is organized as follows. Section 2 extends the theoretical multiple vintage model to account for carbon sequestration benefits in a multi-species context. Section 3 develops the model for the carbon flow accounting regime. In Section 4 numerical simulations illustrate the main results. Section 5 concludes the paper. Technical details and figures are presented in the Appendices.

4.2 The Model

The model used in this paper follows closely the multiple vintage forest model developed in Salo and Tahvonen [28], which can be summarized as follows. The model assumes multi species multi vintages forest land, where $s = 1, ..., n$ represents the age of trees for species $l = 1, ..., L$, $x^l_{s,t}$ the area of forest land allocated to the age class s of species l in period t, f^l_s the biomass content in timber per unit of land with trees of age class s and species l, and $0 \leq f^l_1 \leq .... \leq f^l_n$, for each l. Land allocation must satisfy

$$0 \leq y_t = 1 - \sum_{l=1}^{L} \sum_{s=1}^{n} x^l_{s,t} \tag{4.1}$$

that is, total land area equals 1, and y_t is the area of land allocated to an alternative use (agriculture or urban use).

Let us denote by $U^l(c_t) = \int d^l(c)dc$ the social utility from timber consumption for species l, where $d^l(.)$ is the inverse demand for timber for species l, and assume each $U^l(.)$ is a continuous, twice differentiable, increasing and strictly concave function. Also, $W(y_t) = \int q(y)dy$, where $W(.)$ is a continuous, twice differentiable, increasing and concave function. Finally, S^{jl}_t depends on how the benefits from carbon sequestration are accounted for.

Thus, the problem of optimal forest harvesting and allocation of land is obtained by maximizing the present value of social utility from the use of land, as follows:

$$v(x_{1,0},....x_{n,0})^j = \underset{\{x^l_{s,t+1}, s=1,...n, l=1,...,L, t=0,...\}}{Max} \sum_{t=0}^{\infty} b^t \left[\sum_{l=1}^{L} (U^l\left(c^l_t\right) + S^{jl}_t) + W\left(y_t\right) \right] \tag{4.2}$$

subject to

$$c^l_t = \sum_{s=1}^{n-1} f^l_s \left(x^l_{s^l,t} - x^l_{s^l+1,t+1}\right) + f^l_n x^l_{n,t},\ l = 1,...,L \tag{4.3}$$

$$y_t = 1 - \sum_{l=1}^{L} \sum_{s=1}^{n} x^l_{s,t} \tag{4.4}$$

$$x^l_{s+1,t+1} \leq x^l_{s,t}, s = 1,.......n-1,\ l = 1,...,L \tag{4.5}$$

$$\sum_{l=1}^{L} \sum_{s^l=1}^{n^l} x^l_{s,t+1} \leq 1 \tag{4.6}$$

$$x^l_{s,t} \geq 0, s = 1,....,n,\ l = 1,...,L \tag{4.7}$$

for all $t = 0, 1...$, where S^{jl}_t represents the net benefits from carbon sequestration for the chosen carbon accounting method, indexed by j. Moreover, the initial land distribution satisfies

$$x^l_{s,0} \geq 0, s = 1,....,n, \sum_{l=1}^{L} \sum_{s=1}^{n} x^l_{s,0} \leq 1 \tag{4.8}$$

Therefore, given the discount factor b, the problem consists of choosing the next period state, that is, the land allocation between different vintages and competing uses of land for all $t = 1, ...$

The necessary conditions for optimal solutions can be obtained from the following Lagrangian problem. For (4.2-4.8) it can be stated as

$$L^j = \sum_{t=0}^{\infty} b^t \left[\sum_{l=1}^{L} (U^l\left(c^l_t\right) + S^{il}_t) + W\left(y_t\right) \right] + \lambda^j_t \left(1 - \sum_{l=1}^{L} \sum_{s=1}^{n} x^l_{s,t+1}\right) +$$

$$+ \sum_{l=1}^{L} \sum_{s=1}^{n-1} \left[p^{jl}_{s,t} \left(x^l_{s^l,t} - x^l_{s^l+1,t+1}\right) \right] \tag{4.9}$$

where $p^{jl}_{s,t}$ and λ^j_t are the Lagrangian multipliers. While $p^{jl}_{s,t}$ can be interpreted as the value of marginal changes in forest land area of vintage s for species l at the beginning of period $t+1$,

λ_t^j represents the value of marginal changes in land allocation between forest and alternative uses for each accounting method i.

Salo and Tahvonen [28] provide a full proof on the long-run optimality of the normal forest steady-state for the above problem, when $S_t = 0$ and there is only one species. A forest is called an Optimal Faustmann Forest (OFF) if the age-class structure $x^l = (x_1^l, ..., x_n^l)$ has the property $x_s^l = 0$ for $s = m^l + 1, ..., n$ and if harvesting only trees of age m^l is the optimal solution for the above problem when $x_0^l = x^l$. An OFF is an interior OFF if $x_s^l > 0$ for $s = 1, ..., m^l$. In addition, an OFF with the normal forest structure is $x^l = (1/m^l, ..., 1/m^l, 0, ..., 0)$, and in each period it yields a constant consumption level of f_{m^l}/m^l. An OFF with consumption that is periodic with period length equal to m^l can be expressed as $x^l = (1/m^l + \phi_1^l, ..., 1/m^l + \phi_{m^l}^l, 0, ...0) \in S$, where ϕ^{kl} represents the largest number ϕ^l that satisfies $x^l = (1/m^l + \phi_1, ..., 1/m^l + \phi_{m^l}, 0, ...0) \in K$ for all $\left|\phi_s^l\right| < \phi^l$, $l = 1, ..., L$, $s = 1, ..., m^l, \sum_{s=1}^{m^l} \phi_s^l = 0$, The Faustmann rotation period for species l, denoted by m^l, $1 \leq m^l \leq n^l$, is assumed to be unique and satisfies the following condition: [4]

$$b^{m^l} f_{m^l}^l/(1 - b^{m^l}) \geq b^s f_s^l/(1 - b^s),\ s = 1, ..., n. \tag{4.10}$$

Salo and Tahvonen [28] show that, if all land is allocated to forestry, optimal forest management can lead to optimal cyclical harvesting because smoothening an age class structure that deviates from the normal forest is not optimal. On the contrary, if it is optimal to allocate part of the land to alternative land use then optimal stationary cycles cannot exist.[5]

In this paper, the full proof on the long-run optimality of the normal forest steady state is extended to the case of carbon sequestration benefits with multi-species, building upon the results previously obtained in the case of only one species.

Using similar notation, let m^{jl}, for $l = 1, ..., L$, denote the optimal rotation period with net carbon sequestration benefits for each accounting method j and for species l. Assume that m^{jl} is unique. A forest is called an Optimal Carbon Multi-Species Forest (OCMSF) if the age-class structure for each species l, $x^l = (x_1^l, ..., x_n^l)$, is characterized by OFF for m^{il} and for each $l = 1, ..., L$, where m^{jl} can be different from m^l, for the carbon accounting method j.

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[4] In Salo and Tahvonen [28] no harvesting or plantation costs are considered nor any type of forest externalities. Under these conditions, m, as defined in (4.10), is the Faustmann rotation period in the one stand model. As consumption is constant in the steady-state, so is the marginal utility of consumption $U'(.) = p$, the long-run market equilibrium price of timber. In addition, this condition also corresponds to the maximum value in the steady-state of a marginal unit of bare forest land.

[5] See Salo and Tahvonen [28], Proposition 1 and Corollary 1, pages 518-520.

4.3 Introducing Carbon Sequestration Benefits: Carbon Flow Regime

By sequestering and storing GHG's from the atmosphere, forests can generate carbon offsets, which may be used to compensate for GHG emissions. However, for this compensation to occur, the net effect of sequestration has to be comparable to that of avoided emissions. This issue raises two important questions: first, how to compare forest carbon sequestration with avoided emissions, examined in this section, and second, how to incorporate the services provided by this activity when modeling forest management.

The IPCC Special Report on Land Use, Land-Use Change and Forestry [11] considers different accounting methods to apply to forest or land use change investment projects, namely, the stock change method, the average stock method and the ton yearly crediting. In the economic forestry literature, similar accounting methods have also been considered: the carbon flow regime, the lump-sum regime, and the carbon stock regime, among others, as referred in Locatelli and Pedroni [14]. In what follows, we consider the carbon flow regime.

According to the carbon flow regime, as developed in Van Kooten, Binkley and G. Delcourt [35], social benefits are a function of the annual change in the forest carbon stock, as well as of the amount of carbon permanently stored in timber products and landfills. A net increase in the forest carbon stock over a year means that carbon has been removed from the atmosphere. Similarly, a fall in the forest carbon stock suggests that carbon has been released into the atmosphere. However, the amount of carbon released when the forest is harvested depends upon the use given to the timber harvested. Different uses will have different impacts on the amount of carbon released after harvest, as some uses are able to provide long term carbon storage in structures like furniture or houses. In this context, while carbon released at harvest is taxed, depending upon the timber use, sequestered carbon is subsidized yearly. Therefore, the carbon flow method is essentially a Pigouvian tax/subsidy on the carbon externality.

In this context, the age-class and land allocation forestry decision problem of the social planner is presented and the necessary and sufficient optimality conditions are derived. It is shown how the optimal rotation period, the equilibrium cycles and the optimal land allocation are affected by carbon sequestration benefits in a multi-species context. Sensivity analysis with respect to the speed of growth, the carbon conversion factor and the amount of carbon that is stored in long-lived wood products among species, is undertaken.

When formalizing net carbon benefits, we assume in all cases that the social value of one unit of carbon removed from the atmosphere is constant and equal to p_c.[6] Also, we consider

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[6] p_c is the present value, for all time, of removing one unit of carbon from the atmosphere today. It is

that the amount of carbon per cubic feet of timber biomass growing in forest land is constant and equal to β^l for species l for $l = 1, ..., L$. Notice that once carbon has been sequestered, no further carbon benefits will be obtained. Thus, in this case, what is relevant when modeling carbon sequestration benefits in a standing forest is the change in the per period carbon uptake. Finally, to take into account different uses of timber we introduce a parameter θ^l which measures the fraction of timber that is harvested but goes into long-term storage in structures and landfills for species l.

Under these assumptions, the current net benefits from carbon sequestration at any period t for each species l, S_t^{cl}, can be represented as follows:

$$S_t^{cl} = \sum_{s=0}^{n-1} p_c \beta^l (f_{s+1}^l - f_s^l) x_{s+1^l,t+1}^l - p_c \beta^l (1 - \theta^l) c_t^l \tag{4.11}$$

where the first term represents the value of the carbon stock increase in forest standing biomass, for all the area of forest land, and the last term represents the value of the carbon released due to harvest at t.

By solving the problem (4.2-4.8) and taking S_t^{cl} given by (4.11) for each species l, as in Salo and Tahvonen [28], we first study the existence of optimal stationary cycles in a regime where the oldest age class is clear-cut and immediately regenerated at the end of each period.

Denote the optimal rotation period by m^{cl}, that satisfies $1 \leq m^{cl} \leq n$ and for which

$$\sum_{l=1}^{L} \left(p^l - \beta^l p_c (1 - \theta^l)\right) \frac{b^{m^{cl}} f_{m^{cl}}}{1 - b^{m^{cl}}} + \sum_{l=1}^{L} \beta^l p_c \sum_{i=0}^{m^{cl}-1} b^i \frac{(f_{i+1}^l - f_i^l)}{1 - b^{m^{cl}}} \geq$$

$$\geq \sum_{l=1}^{L} \left(p^l - \beta^l p_c (1 - \theta^l)\right) \frac{b^s f_s^l}{1 - b^s} + \sum_{l=1}^{L} \beta^l p_c \sum_{i=0}^{s-1} b^i \frac{(f_{i+1}^l - f_i^l)}{1 - b^s} \tag{4.12}$$

for $s = 1, ..., n$, holds.[7] Assume that m^{cl} is unique, for each l. We show in Appendix B that $m^{cl} \geq m^l$, for each l, when all carbon is released at harvest ($\theta^l = 0$). When $\theta = 1$ and $\{f_i - f_{i-1}\}$ is a decreasing sequence, $m^{cl} \leq m^l$. Otherwise, $m^{cl} \gtreqqless m^l$. In fact, in the absence of external effects between species, there is no $m^{cl} \geq m^c$ common to all species that improves upon m^{cl} for each l, as any common m^c is always dominated by the optimal m^{cl} for each l.

Proposition 1: Given $g^{cl} \equiv \frac{\left[U^{l\prime}(f_{m^{cl}}^l/m^{cl}) - \beta^l p_{c(1-\theta^l)}\right] b^{m^{cl}} f_{m^{cl}}^l}{1-b^{m^{cl}}} + \sum_{l=1}^{L} \beta^l p_c \sum_{i=0}^{m^{cl}-1} b^i \frac{(f_{i+1}^l - f_i^l)}{1-b^{m^{cl}}} - \frac{b}{1-b} W'(0) > 0$, $m^{cl} \geq 2$, *and* $b < 1$, *for* $l = 1, ..., L$, *there exists a set of interior Optimal Multi Species Carbon Forests with* $\phi^{kl} > 0$.

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determined as the discounted value of the annual contribution to damage caused by one unit of carbon added over the expected number of years that the unit of carbon is present in the atmosphere.

[7] These conditions correspond to the optimal conditions of the discrete version of the one stand model as proved in Costa-Duarte, Cunha-e-Sá and Rosa [8] in the case of only one species.

Proof. The proof is in Appendix A. ■

In Proposition 1 it is shown that optimal stationary cycles exist when it is optimal to allocate all land to forestry. From (4.34) in Appendix A, we may conclude that when carbon sequestration benefits are accounted for the maximum cycle radius may either increase or decrease.

Corollary 1: If $g^{cl} \equiv \frac{\left[U^{l\prime}(f^l_{m^{cl}}/m^{cl})-\beta^l p_c(1-\theta^l)\right]b^{m^{cl}}f^l_{m^{cl}}}{1-b^{m^{cl}}}\sum_{l=1}^{L}\beta^l p_c\sum_{i=0}^{m^{cl}-1}b^i\frac{(f^l_{i+1}-f^l_i)}{1-b^{m^{cl}}}-\frac{b}{1-b}W'(0)\leq 0$, *for* $l=1,...,L$, *optimal stationary cycles with* $y_\infty \geq 0$ *and* y_∞ *constant do not exist.*

Proof. The proof is in Appendix A. ■

When it is optimal to allocate land to alternative uses, Corollary 2 shows that optimal cycles are eliminated and the remaining equilibrium is the normal forest steady-state. Assuming that m^{cl} is unique, for a stationary state, we have that $p^{cl}_{s,t}=p^{cl}_{s,\infty}$, $c^{cl}_t=c^{cl}_\infty$, $y_t=y_\infty$, $\lambda^c_t=0$, and $x^{cl}_{m^{cl},t}=x^{cl}_\infty$, where c^{cl}_∞, y_∞, x^{cl}_∞, and $p^{cl}_{s,\infty}$, for $s=1,...,n-1$, and $l=1,...,L$, are constant. Direct substitution shows that in this case:

$$p^{cl}_s = W'(y_\infty)\sum_{i=0}^{s-1}b^{-i} - f^l_s\left[U^{l\prime}(c^l_\infty)-\beta^l p_c(1-\theta^l)\right] - \sum_{l=1}^{L}\beta^l p_c\sum_{i=0}^{s-1}b^{i-s}(f^l_{i+1}-f^l_i) \tag{4.13}$$

for $s=1,...,n$, where $\sum_{i=0}^{s-1}b^{-i}=-b\frac{1-b^{-s}}{1-b}$.

With some more algebra, we can write (4.13) for $s=m^{cl}$ and as $p^{cl}_{m^{cl},\infty}=0$, obtaining

$$W'(y_\infty)\frac{b}{1-b}-\frac{b^{m^{cl}}f^l_{m^{cl}}}{1-b^{m^{cl}}}\left[U^{l\prime}(c^l_\infty)-\beta^l p_c(1-\theta^l)\right]-\sum_{l=1}^{L}\beta^l p_c\sum_{i=0}^{m^{cl}-1}\frac{b^i}{1-b^{m^{cl}}}(f^l_{i+1}-f^l_i)=0 \tag{4.14}$$

for $l=1,...,L$, where

$$c^l_\infty=\frac{(1-y_\infty-\sum_{v=1}^{L}x^{v\neq l}_\infty)f^l_{m^{cl}}}{m^{cl}},\ \text{and}\ c^{v\neq l}_\infty = x^{v\neq l}_\infty f^l_{m^{cl}}\ \text{for}\ v\neq l=1,...,L \tag{4.15}$$

Therefore, we obtain a system of L equations, one for each species, in L unknowns (y_∞, and c^l_∞, for $l=1,...,L-1$). The allocation of land between the different types of forest and the alternative use that results is optimal when the present value of output from a marginal unit of land in the alternative use equals the present value of a marginal unit of bare forest land for each species, where both timber value and the net benefits from carbon sequestration are accounted for, as follows:

$$W'(y_\infty)\frac{b}{1-b}=$$

$$=\frac{b^{m^{c1}}f^1_{m^{c1}}}{1-b^{m^{c1}}}\left[U^{1\prime}\left(\frac{(1-y_\infty-\sum_{l=2}^{L}x^{cl}_\infty)f^1_{m^{cl}}}{m^{c1}}\right)-\beta^1 p_c(1-\theta^1)\right]+\sum_{l=1}^{L}\beta^l p_c\sum_{i=0}^{m^{cl}-1}\frac{b^i}{1-b^{m^{cl}}}(f^l_{i+1}-f^l_i)=...$$

$$\dots = \frac{b^{m^{cL}} f^L_{m^{cL}}}{1-b^{m^{cL}}}\left[U^{L\prime}(x^L_\infty f^L_{m^{cL}}) - \beta^L p_c(1-\theta^L)\right] + \sum_{l=1}^{L}\beta^l p_c \sum_{i=0}^{m^{cl}-1}\frac{b^i}{1-b^{m^{cl}}}(f^l_{i+1}-f^l_i) \tag{4.16}$$

Let us assume that we only have two species, that is, $l = 1, 2$. In this case, the above conditions become

$$W'(y_\infty)\frac{b}{1-b} =$$

$$= \frac{b^{m^{c1}} f^1_{m^{c1}}}{1-b^{m^{c1}}}\left[U^{1\prime}\left(\frac{(1-y_\infty-x^{c2}_\infty)f^1_{m^{c1}}}{m^{c1}}\right) - \beta^1 p_c(1-\theta^1)\right] + \sum_{l=1}^{2}\beta^l p_c \sum_{i=0}^{m^{cl}-1}\frac{b^i}{1-b^{m^{cl}}}(f^l_{i+1}-f^l_i) =$$

$$= \frac{b^{m^{c2}} f^2_{m^{c2}}}{1-b^{m^{c2}}}\left[U^{2\prime}(x^2_\infty f^2_{m^{c2}}) - \beta^2 p_c(1-\theta^2)\right] + \sum_{l=1}^{2}\beta^l p_c \sum_{i=0}^{m^{cl}-1}\frac{b^i}{1-b^{m^{cl}}}(f^l_{i+1}-f^l_i) \tag{4.17}$$

which also imply, for an interior solution, that is, with land optimally allocated to both species, that

$$\frac{\frac{b^{m^{c1}} f^1_{m^{c1}}}{1-b^{m^{c1}}}\left[U^{1\prime}\left(\frac{(1-y_\infty-x^{c2}_\infty)f^1_{m^{c1}}}{m^{c1}}\right) - \beta^1 p_c(1-\theta^1)\right] + \sum_{l=1}^{2}\beta^l p_c \sum_{i=0}^{m^{cl}-1}\frac{b^i}{1-b^{m^{cl}}}(f^l_{i+1}-f^l_i)}{\frac{b^{m^{c2}} f^2_{m^{c2}}}{1-b^{m^{c2}}}\left[U^{2\prime}(x^2_\infty f^2_{m^{c2}}) - \beta^2 p_c(1-\theta^2)\right] + \sum_{l=1}^{2}\beta^l p_c \sum_{i=0}^{m^{cl}-1}\frac{b^i}{1-b^{m^{cl}}}(f^l_{i+1}-f^l_i)} = 1 \tag{4.18}$$

Since the second term both in the numerator and the denominator is the same, a larger first term in the numerator implies a lower one in the denominator and vice-versa. The equilibrium can be recovered by increasing the amount of land dedicated to forest and relatively more the part corresponding to the most valuable species.

4.4 Numerical Simulations

In this section, we simulate the theoretical model developed in the previous sections. We start by considering a case with two different species but without carbon benefits. For this scenario, we base our analysis on the following example: $U(c^l) = \frac{(c^i)^{0.7}}{0.7}$ is the utility function from consumption for both species and $W(y) = 0.5[\frac{y^{0.2}}{0.2}]$ is the utility from non-forestry land, that is, from the alternative use of land. The vectors f^l_s of the biomass content in timber per unit of land for species $l = 1, 2$ and for age classes $s = 1, \dots 24$, are given by

$$f^1_s = [0, 0, 0, 15, 22, 30, 39, 51, 65, 82, 101, 123, 148, 175, 204, 234, 263,$$

$$293, 321, 346, 370, 390, 408, 423]$$

$$f_s^2 = [5, 15, 35, 60, 90, 125, 160, 194, 227, 249, 269, 284, 296, 306, 315, 323,$$

$$331, 339, 346, 352, 357, 362, 367, 372]$$

For both species the initial land distribution is given by $x_{s0} = 0.1$, $s = 1, .., 5$ and $x_{s0} = 0$, $s = 6, ..., 24$. The discount factor is $b = 0.95$

From Figure 1 we conclude that differences on species' growth may cause wide disparities on the optimal distribution of tree species. Actually, if we consider, instead, that all land is initially allocated to species 1 (see Figure 2), we obtain a path that illustrates what has been observed in Portugal for the last decades, where pinus forest has been replaced by eucalyptus plantations.

Departing from this baseline scenario, we now introduce carbon benefits and analyze some of the most important trade-offs regarding carbon sequestration in a multispecies framework.

i) speed of growth across species: $f_s^l s$

We begin by analyzing the effects of growth differences between species. Except for f_s^1 and f_s^2, all parameters are equal among species, namely, $p_c = 0.4368$, $\theta^1 = \theta^2 = 0.5$ and $\beta^1 = \beta^2 = 0.2$. The results obtained show that differences on species' growth do matter when carbon sequestration is introduced, as more land is dedicated to the fast growing species. However, this increase does not seem to be very sensitive to increases in the value of p_c (see Figure 3).

ii) carbon conversion factor: β

Different tree species sequester distinct amounts of carbon due, not only to growth, but also to other biological features, e.g., types of leaves. To capture this effect, we consider a different β for each species, namely $\beta^1 = 0.2$ and $\beta^2 = 0.5$, and the same growth vector for both species, given by f_s^1. From Figure 4, we conclude that the carbon conversion factor plays a very important role, as a large gap is now optimally generated between the species with the larger β and the other. Note that β may be interpreted as each species' ability to either sequester carbon in branches or to retain it in the soil.

iii) long-lived wood products: θ

Bearing in mind a post-Kyoto world, a very important discussion about the role of carbon sequestration is now taking place. In fact, the current Protocol assumes that all carbon is released once a tree is harvested ($\theta = 0$ in our model). However, as a substantial part of the harvested wood goes into long-lived wood products, carbon actually remains sequestered for decades and, in some cases, even centuries, implying that such an assumption may actually end up being too restrictive. To account for that, a sensitivity analysis for θ is performed, assuming that growth and all other parameters are the same across species. In Figure 5, we observe that the value of θ affects significantly the optimal species' distribution, as a higher fraction of land is allocated to trees with higher $\theta's$. Furthermore, when carbon sequestered in long-lived wood products is taken into account, the net social benefit per ton of carbon removed by forest sinks increases considerably (from to 0.1809 to 0.3957).

We now additionally allow for differences in species' growth. Species 1 is represented by vector f_s^1 and species 2 by vector f_s^2. Given that the majority of fast growth species is used for paper production, a lower θ is associated to species 2. From Figure 6, we conclude that differences in θ are actually enough to counteract differences in growth when carbon benefits are introduced (see Figure 2). In fact, if compared with a scenario where $\theta = 0$, a higher fraction of land is allocated to the slow growing forest (associated with a higher θ), in contrast to the fast growing species.

We can conclude that the optimal land allocation evolves towards a stationary state and that the area dedicated to forested land and timber production increases. However, the increase in timber production associated to an expansion of forested land is not very significant, as the increase occurs in all classes and only the oldest one is harvested. Nonetheless, increases on carbon retained in forests are rather substantial.

Biology plays a important role, as fast growing species' plantations increase when carbon benefits are introduced. Carbon in long-lived products is a critical aspect, as its consideration may actually reverse optimal allocations of land in favor of slow growing species. Moreover, the net social benefits of a carbon sequestration policy are considerably higher in a framework where this carbon pool is also accounted for.

4.5 Concluding Remarks

The introduction of carbon sequestration benefits in the multiple vintage forest model developed by Salo and Tahvonen [28] is undertaken by considering the carbon flow regime in a multi-species context. However, in contrast, the partial equilibrium setting of this model with endogenously determined timber and land prices allows us to endogenously determine the optimal impact on timber and land markets from internalizing carbon sequestration benefits in a multi species context.

A full proof of the long-run optimality of steady-state forest is provided. Although the major theoretical results still apply, the extension to the presence of carbon sequestration benefits with multi-species is not without consequences. The results obtained, based on numerical simulations, may suggest how to increase efficiency on forest management from a social point of view.

Depending on the values taken by the different parameters, namely, the speed of growth, the carbon conversion factor and the amount of carbon that is stored in long-lived wood products among species, we observe that either fast growing species or species with larger carbon conversion factors are optimally allocated to a larger fraction of land. Moreover, a similar result applies to species whose timber is used in long-lived wood products. In particular, this effect may even dominate the impact of the different speeds of growth among species. We may observe that the slow growing species may be the one that benefits the most when carbon sequestration has value

To conclude, the theoretical and simulation models developed in this paper can be a useful tool to empirical studies on forestry policy in general, or, in particular, to examine the impact of policy measures on forest management to mitigate GHGs emissions in the post-Kyoto context.

4.6 Appendix A

The necessary conditions for optimal solutions of the problem (4.2-4.8) and S^{cl}_t given by (4.11) for each species l, which can be derived from the Karush-Kuhn-Tucker conditions for all $t = 0, ...$, are as follows:

$$b^t \frac{\partial L^{cl}}{\partial x^l_{1,t+1}} = bf^l_1 U^{l\prime}(c^l_{t+1}) + bf^l_1 p_c \beta^l - bf^l_1 p_c \beta^l (1-\theta^l) -$$

$$-bW'(y_{t+1}) - \lambda^c_t + bp^{cl}_{1,t+1} \leq 0,\ l = 1, ..., L, \tag{4.19}$$

$$b^t \frac{\partial L^{cl}}{\partial x^l_{s^l+1,t+1}} = -f^l_{s^l} U^{l\prime}(c^l_t) + bf^l_{s+1} U^{l\prime}(c^l_{t+1}) + (f^l_{s+1} - f^l_s) p_c \beta^l + f^l_s p_c \beta^l (1-\theta^l) -$$

$$-bf^l_{s+1} p_c \beta^l (1-\theta^l) - bW'(y_{t+1}) - \lambda^c_t + bp^l_{s+1,t+1} - p^l_{s,t} \leq 0 \tag{4.20}$$

for $s = 1, ..., n-2$, and $l = 1, ..., L$,

$$b^t \frac{\partial L^{cl}}{\partial x^l_{n,t+1}} = -f^l_{n-1} U^{l\prime}(c_t) + bf^l_n U^{l\prime}(c_{t+1}) + (f^l_n - f^l_{n-1}) p_c \beta^l + f^l_{n-1} p_c \beta^l (1-\theta^l) -$$

$$-bf^l_n p_c \beta^l (1-\theta^l) - bW'(y_{t+1}) - \lambda^c_t - p^l_{n-1,t} \leq 0 \tag{4.21}$$

for $l = 1, ..., L$,

$$x^l_{s,t+1} \geq 0, x^l_{s,t+1} \frac{\partial L^{cl}}{\partial x^l_{s,t+1}} = 0, s = 1, ..., n \ and\ l = 1, ..., L, \tag{4.22}$$

$$p^{cl}_{s,t} \geq 0, p^{cl}_{s,t}(x^l_{s,t} - x^l_{s+1,t+1}) = 0, s = 1, ..., n-1,\ and\ l = 1, ..., L, \tag{4.23}$$

$$\lambda^c_t \geq 0, \lambda^c_t (1 - \sum_{l=1}^{L} \sum_{s=1}^{n} x^l_{s,t+1}) = 0 \tag{4.24}$$

The existence of optimal solutions for bounded utility and $b < 1$ follows from Theorem 4.6 in Stokey and Lucas (p. 79).

Proof of Proposition 1:

Proposition:Following Salo and Tahvonen [28], by convexity of problem (4.2)-(4.8), if there exist multipliers $p^{cl}_{s,t}$ satisfying conditions (4.19)-(4.24) under harvesting at m^{cl}, then the resulting age class structure is an interior OCMSF. The optimality follows since with harvesting *at* m^{cl}, $\frac{\partial U^l}{\partial x^l_{s,t}}$ *and* $x^{cl}_{s,t}$ *remain bounded satisfying transversality conditions which, together with (4.19)-(4.24) are sufficient for optimality.*

Proof. For $s = 1, ..., m^{cl} - 1$ using (4.19) to eliminate λ^c_t from (4.20) and (4.21), and to satisfy (4.22) we obtain a system of m^{cl} x $(m^{cl} - 1)$ equality equations:

$$b(p^{cl}_{s+1,t+1+k} - p^{cl}_{1,t+1+k}) - p^{cl}_{s,t+k} = -b\left[U^{l\prime}(c^l_{t+k+1}) - \beta^l p_c(1-\theta^l)\right](f^l_{s+1} - f^l_1) +$$

$$+\left[U^{l\prime}(c_{t+k}) - \beta^l p_c(1-\theta^l)\right] f^l_s + \beta^l p_c f^l_1 - \beta^l p_c(f^l_{s+1} - f^l_1) \tag{4.25}$$

$$-bp^{cl}_{1,t+1+k} - p^{cl}_{m^c-1,t+k} = -b\left[U^{l\prime}(c^l_{t+k+1}) - \beta^l p_c(1-\theta^l)\right](f^l_{m^c} - f^l_1) +$$

$$+\left[U^{l\prime}(c^l_{t+k}) - \beta^l p_c(1-\theta^l)\right] f^l_{m^c-1} - \beta^l p_c(f^l_{m^{cl}} - f^l_{m^{cl}-1}) + \beta^l p_c f^l_1 \tag{4.26}$$

where $s = 1, ...m^{cl} - 2$, $k = 0, ..., m^{cl} - 1$.

This system is linear in the Lagrangian multipliers $p^{cl}_{s,t+k}$, $s = 1, ..., m^{cl} - 1$, $k = 0, ..., m^{cl} - 1$ and solving for any multiplier yields:

$$p^{cl}_{s,t} = \frac{b^{m^{cl}} f_{m^{cl}}}{1 - b^{m^{cl}}}\left[b^{-s}(U^{l\prime}(c^l_{t+m^{cl}-s}) - \beta^l p_c(1-\theta^l)) - (U^{l\prime}(c^l_t) - \beta^l p_c(1-\theta^l))\right] -$$

$$-f^l_s(U^{l\prime}(c^l_t) - \beta^l p_c(1-\theta^l)) + A^{cl}_s \tag{4.27}$$

where A^{cl}_s is given by

$$A^{cl}_s = \frac{\beta p_c}{1 - b^{m^{cl}}}\left[(1 - b^s)\sum_{i=0}^{m^{cl}-1} b^{i-s}\left(f^l_{i+1} - f^l_i\right) - (1 - b^{m^{cl}})\sum_{i=0}^{s-1} b^{i-s}\left(f^l_{i+1} - f^l_i\right)\right]$$

for $s = 1, ..., m^{cl} - 1$, $t = 0, ...,$ as can be verified by direct substitution into the two equations above. Condition (4.23) requires, for the indefinitely repeated cycle, that $p^{cl}_{s,t+k} \geq 0$ for $s = 1, ..., m^{cl} - 1$, $k = 0, ..., m^{cl} - 1$. Thus, the fact that $x^{cl} \in K^{cl}$ implies by (4.27) that

$$\frac{U^{l\prime}(c^l_{t+k}) - \beta^l p_c(1-\theta^l)}{U^{l\prime}(c^l_{t+k+m^c-j}) - \beta^l p_c(1-\theta^l)} \leq \frac{b^{m^{cl}-j} f^l_{m^{cl}}}{f^l_j + b^{m^{cl}}(f^l_{m^{cl}} - f^l_j)} + \tag{4.28}$$

$$+\frac{A^{cl}_j(1 - b^{m^{cl}})}{\left[U^{l\prime}(c_{t+k+m^{cl}-j}) - \beta^l p_c(1-\theta^l)\right]\left[f^l_j + b^{m^{cl}}(f^l_{m^{cl}} - f^l_j)\right]}$$

for $k = 0, ..., m^{cl} - 1$, $j = 1, ..., m^{cl} - 1$. Using (4.3) and the definition of optimal harvesting, we can write $c^l_{t+k} = f_{m^c} x^l_s$ and $c^l_{t+k+m^{cl}-j} = f^l_{m^{cl}} x^{cl}_{s-m^{cl}+j}$ where $s - m^{cl} + j$ is understood as $s - j$, if $s - m^{cl} + j \leq 0$. Equation (4.28) takes the form

$$\frac{U^{l\prime}(f^l_{m^{cl}} x^{cl}_s) + \beta^l p_c \theta^l}{U^{l\prime}(f^l_{m^{cl}} x^{cl}_{s-m+j}) + \beta^l p_c \theta^l} \leq \frac{b^{m^{cl}-j} f^l_m}{f^l_j + b^{m^{cl}}(f^l_{m^c} - f^l_j)} + \tag{4.29}$$

$$+ \frac{A^{cl}_j (1 - b^{m^{cl}})}{\left[U^{l\prime}(f^l_{m^{cl}} x^{cl}_{s-m^{cl}+j}) - \beta^l p_c (1 - \theta^l)\right] \left[f^l_j + b^{m^{cl}}(f^l_{m^{cl}} - f^l_j)\right]} \tag{4.30}$$

for $s = 1, ..., m^{cl}$, $j = 1, ..., m^{cl} - 1$.

///By (4.12) $\eta^{cl}_j > 1$, implying that the right-hand side of (4.30) is larger than one and larger than η^{cl}_j. Then, by the strict concavity of U^l, there must exist a $\phi^{cl} > 0$, such that (4.30) is satisfied if $x^{cl}_s = 1/m^{cl} + \phi^{cl}_s$, $s = 1, ..., m^{cl}$, for all $\left|\phi^{cl}_s\right| < \phi^{cl}$, $\sum_{s=1}^{m^{cl}} \phi^{cl}_s = 0$, proving that optimal harvesting is cyclical harvesting and that it is not optimal to cut before m^{cl}.

Similarly, for $s = m^{cl} + 1, ..., n$, and $k = 0, ..., m^{cl} - 1$, the optimality of the harvesting at m^{cl} requires that land is not allocated to age classes $s = m^{cl} + 1, ..., n$. Since $x^{cl}_{m^{cl},t} > 0$ and $x^l_{m^{cl}+1,t+1} = 0$ in (4.23), we obtain $p^{cl}_{m^{cl},t} = 0$, for $t = 0, ...$, and $p^{cl}_{st} \geq 0$, for $s = m^{cl} + 1, ..., n - 1$, $t = 0, ...$,as can also be checked in (4.27). Using this and conditions (4.20) and (4.21), yields

$$b(p^{cl}_{m^{cl}+1,t+1+k} - p^{cl}_{1,t+1+k}) - p_{m^{cl},t+k} \leq -b\left[U^{l\prime}(c^l_{t+k+1}) + \beta^l p_c \theta^l\right](f^l_{m^{cl}+1} - f^l_1) +$$

$$+ \left[U^{l\prime}(c^l_{t+k}) + \beta^l p_c \theta^l\right] f_{m^{cl}} \tag{4.31}$$

$$b(p^{cl}_{s+1,t+1+k} - p^{cl}_{1,t+1+k}) - p^{cl}_{s,t+k} \leq -b\left[U^{l\prime}(c^l_{t+k+1}) + \beta^l p_c \theta^l\right](f^l_{s+1} - f^l_1) +$$

$$+ \left[U^{l\prime}(c^l_{t+k}) + \beta^l p_c \theta^l\right] f^l_s \tag{4.32}$$

for $s = m^{cl} + 1, ..., n - 2$, and

$$-bp^{cl}_{1,t+1+k} - p^{cl}_{n-1,t+k} \leq -b\left[U^{l\prime}(c^l_{t+k+1}) + \beta^l p_c \theta^l\right](f^l_n - f^l_1) +$$

$$+ \left[U^{l\prime}(c^l_{t+k}) + \beta^l p_c \theta^l\right] f^l_{n-1} \tag{4.33}$$

where $k = 0, ..., m^{cl} - 1$. Using (4.27), by direct substitution we can show that the first two inequalities are satisfied as equalities. By eliminating $p^{cl}_{1,t+1+k}$ and $p^{cl}_{n-1,t+k}$ from the last inequality, using (4.27), and the facts that $c^l_{t+k+1} = f^l_{m^{cl}} x^{cl}_s$ and $c^l_{t+k+m^c-n+1} = f^l_{m^c} x^{cl}_{s-m^{cl}+n}$, we can write the last inequality above as follows:

$$\frac{U^{l\prime}(f^l_{m^c} x^{cl}_s) + \beta^l p_c \theta^l}{U^{l\prime}(f^l_{m^{cl}} x^{cl}_{s-m^{cl}+n}) + \beta^l p_c \theta^l} \leq \frac{b^{m^{cl}-n} f^l_{m^{cl}}}{f^l_n + b^{m^{cl}}(f^l_{m^{cl}} - f^l_n)}$$

for $s = m^{cl} + 1, ..., n$.

The conditions $p^{cl}_{s,t+k} \geq 0$ for $s = m^{cl} + 1, ..., n-1$, $k = 0, ..., m^{cl} - 1$ together with the last inequality yield

$$\frac{U^{l\prime}(f^l_{m^{cl}} x^{cl}_s) + \beta^l p_c \theta^l}{U^{l\prime}(f^l_{m^c} x^{cl}_{s-m^{cl}+j}) + \beta^l p_c \theta^l} \leq \eta^{cl}_j \equiv \frac{b^{m^{cl}-j} f^l_{m^c}}{f^l_j + b^{m^{cl}}(f^l_{m^{cl}} - f^l_j)} \tag{4.34}$$

for $s = 1, ..., m^{cl}$,and $j = m^{cl}+1, ..., n$. A similar condition to (4.30) can be obtained, concluding that it is not optimal to postpone harvest at m^{cl}.

In addition, a stationary cycle with all land allocated to forestry must satisfy $\lambda^c_t \geq 0$, for $t = 0, ...$ Solving (4.19) or (4.20) for λ^c_t, eliminating $p^{cl}_{s,t}$, for $s = 1, ..., m^{cl} - 1$, $t = 0, ...,$ using (4.27), we obtain

$$\lambda^c_{t+k} = \frac{\left[U'(c^l_{t+k}) + \beta^l p_c \theta^l\right] b^{m^{cl}} f_{m^{cl}}}{1 - b^{m^{cl}}} - \frac{\left[U^{l\prime}(c^l_{t+1+k}) + \beta^l p_c \theta^l\right] b^{m^{cl}+1} f^l_{m^{cl}}}{1 - b^{m^{cl}}} - bW'(0) \geq 0 \tag{4.35}$$

for $s = 1, ..., m^{cl}$, where $c^l_{t+1+m^{cl}} = c^l_{t+1.}$ Writing $c^l_{t+k} = f^l_{m^c} x^{cl}_s$ and $c^l_{t+1+k} = f^l_{m^{cl}} x^{cl}_{s-1}$, $s = 1, ..., m^{cl}$, where $x^l_0 = x_{m^{cl}}$ yields

$$\lambda^c_s = \frac{\left[U^{l\prime}(f^l_{m^{cl}} x^{cl}_s) + \beta^l p_c \theta^l\right] b^{m^{cl}} f^l_{m^{cl}}}{1 - b^{m^{cl}}} - \frac{\left[U^{l\prime}(f^l_{m^{cl}} x^{cl}_{s-1}) + \beta^l p_c \theta^l\right] b^{m^{cl}+1} f^l_m}{1 - b^{m^{cl}}} - bW'(0) \geq 0 \tag{4.36}$$

for $s = 1, ..., m^{cl}$.

Given $g^{cl} = \frac{\left[U^{l\prime}(f^l_{m^{cl}}/m^{cl}) - \beta^l p_c(1-\theta^l)\right] b^{m^{cl}} f^l_{m^{cl}}}{1-b^{m^{cl}}} + \sum_{l=1}^{L} \beta^l p_c \sum_{i=0}^{m^{cl}-1} b^i \frac{(f^l_{i+1} - f^l_i)}{1-b^{m^{cl}}} - \frac{b}{1-b} W'(0) > 0$, there must exist a $\phi^l > 0$ such that (4.36) is satisfied if $x^l_s = 1/m^{cl} + \phi^{cl}_s$, $s = 1, ..., m^{cl}$, for all $\left|\phi^{cl}_s\right| < \phi^{cl}$, $\sum_{s=1}^{m^{cl}} \phi^{cl}_s = 0$, and $l = 1, ..., L$. ■

Let i^l_∞ represent the stationary state level of variable i^l.

We now show that if $g^{cl} \leq 0$ there exists a stationary state that satisfies all the necessary conditions for optimality.[8]

Proof of Corollary 1:

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[8] The results obtained in Salo and Tahvonen [28] regarding convergence and stability of the stationary steady states (Lemma 1 and Lemma 2, pg. 523) still apply in the case of this paper, as the difference equation for $x_{m^i_t}$, for $i = c, t, a$, is similar to equation (34), pg. 522, in the paper. The additional terms that are present in our case are independent of $x_{m^i_t}$. Therefore, the marginal conditions yielding the corresponding characteristic polynomials turn out to be similar.

Proof. Given $g^{cl} \leq 0$, no solutions for (4.36) exist. Thus, by letting $\lambda_t^c = 0$ in (4.19) or (4.20), eliminating $p_{s,t}^{cl}$, $s = 1, ..., m^{cl} - 1$, $t = 0, ...$, using (4.27), and writing (4.19) analogously to (4.36), we obtain for $s = 1, ..., m^{cl}$ and $l = 1, ..., L$,

$$\frac{\left[U^{l\prime}(f_{m^{cl}}^{l} x_s^{cl}) - \beta^l p_c(1-\theta^l)\right] b^{m^{cl}} f_{m^{cl}}^{l}}{1 - b^{m^{cl}}} + \sum_{l=1}^{L} \beta^l p_c (1-b) \sum_{i=0}^{m^{cl}-1} b^i \frac{(f_{i+1}^l - f_i^l)}{1 - b^{m^{cl}}} -$$

$$-\frac{\left[U^{l\prime}(f_{m^{cl}} x_{s-1}^{cl}) - \beta^l p_c(1-\theta^l)\right] b^{m^{cl}+1} f_{m^{cl}}}{1 - b^{m^{cl}}} - bW'(y_\infty) \geq 0 \tag{4.37}$$

This system is linear in $\left[U^{l\prime}(f_{m^{cl}}^{l} x_s^{cl}) - \beta^l p_c(1-\theta^l)\right]$, $s = 1, ..., m^{cl}$ and its solution is given by:

$$U^{l\prime}(f_{m^{cl}}^{l} x_s^{cl}) - \beta^l p_c(1-\theta^l) + \sum_{l=1}^{L} \beta^l p_c \sum_{i=0}^{m^{cl}-1} b^i \frac{(f_{i+1}^l - f_i^l)}{1 - b^{m^{cl}}} = \frac{W'(y_\infty) \sum_{i=0}^{m^{cl}-1} b^i}{b^{m^{cl}-1} f_{m^{cl}}^{l}} \tag{4.38}$$

for $s = 1, ..., m^{cl}$, $l = 1, ..., L$, as can be verified by direct substitution. Thus, $x_s^{cl} = (1 - y_\infty - \sum_{v=1}^{L} \sum_{s=1}^{n} x_s^{cv \neq l})/m^{cl}$, $s = 1, ..., m^{cl}$ and optimal stationary cycles cannot exist. ■

4.7 Appendix B

The value of θ and the optimal rotation period:

Since there is no optimal rotation period for the whole forest, in what follows, it is enough to consider only one species. Therefore, to simplify notation, we eliminate the superscript l.

We now show that for $\theta = 0$, $m^c \geq m$:

At the steady-state, if there exists a $m^c \neq m$, for which

$$\frac{(p - p_c\beta) b^{m^c} f_{m^c}}{1 - b^{m^c}} + \frac{p_c\beta \sum_{i=0}^{m^c-1} b^i (f_{i+1} - f_i)}{1 - b^{m^c}} \geq \frac{(p - p_c\beta) b^m f_m}{1 - b^m} + \frac{p_c\beta \sum_{i=0}^{m-1} b^i (f_{i+1} - f_i)}{1 - b^m} \tag{4.39}$$

holds, then it is optimal to cut at m^c, where $m^c \gtreqless m$. If (4.39), which is the same as (4.12), holds in particular for $m^c = m+1$, then $m^c \geq m$, while if it holds in particular for $m^c = m-1$, $m^c \leq m$. If (4.39) holds as an equality, $m^c = m$.

By making $m^c = m - 1$, we show below that (4.39) never holds, implying that $m^c \geq m$, that is, it is optimal to postpone harvest. Also, for some $m^c = m+1$, (4.39) may be satisfied.

Let $m^c = m - 1$ in (4.39). By rearranging and collecting terms we obtain

$$pb^{m-1} f_{m-1}(1 - b^m) - pb^m f_m(1 - b^{m-1}) >$$

$$p_c\beta(b^m - b^{m-1}) \sum_{i=0}^{m-2} b^i(f_{i+1} - f_i) - p_c\beta b^m f_m(1 - b^{m-1}) + p_c\beta b^{m-1}(f_m - f_{m-1})(1 - b^{m-1}) + p_c\beta b^{m-1} f_{m-1}(1 \tag{4.40}$$

which can be restated as

$$p_c\beta(b^m - b^{m-1}) \sum_{i=0}^{m-2} b^i(f_{i+1} - f_i) - p_c\beta f_m(b^m - b^{m-1})(1 - b^{m-1}) - p_c\beta f_{m-1} b^{m-1}(b^m - b^{m-1}) \tag{4.41}$$

and, finally, as

$$p_c\beta(b^m - b^{m-1}) \left[\sum_{i=0}^{m-1} b^i f_{i+1} - \sum_{i=0}^{m-1} b^i f_i - f_m \right] \tag{4.42}$$

In (4.40) the left-hand side is negative. Since

$$\sum_{i=0}^{m-1} b^i f_{i+1} = \sum_{i=0}^{m-1} b^i(f_{i+1} - f_m) + \sum_{i=0}^{m-1} b^i f_{m-1}$$

substituting above we obtain

$$p_c\beta(b^m - b^{m-1}) \left[\sum_{i=0}^{m-1} b^i(f_{i+1} - f_m) - f_m + \sum_{i=0}^{m-1} b^i(f_m - f_i) \right]$$

Given that $\{f_i\}$, for $i = 1, ..., m-1$, is an increasing sequence, and $b < 1$, we may conclude this expression is positive, as the sign of the algebraic sum inside the square brackets is negative. Consequently, the right-hand side of (4.40) is positive, implying that (4.40) never holds for any $m^c < m$. Also, we can show that it may hold for some $m^c > m$. Therefore, when $\theta = 0$, it is never optimal to cut earlier, that is, $m^c \geq m$.

In the case $\theta = 1$, we will consider two cases. In case (i) we assume that the sequence $\{f_i - f_{i-1}\}$ is a decreasing sequence. In case (ii) we only assume that the sequence $\{f_i\}$ is an increasing sequence.

(i) Let $m^c = m + 1$ in (4.39). By rearranging and collecting terms we obtain

$$pb^{m+1} f_{m+1}(1 - b^m) - pb^m f_m(1 - b^{m+1}) >$$

$$p_c\beta\left[-\left(f_{m+1}-f_m\right)(1-b^m)b^m+\left(b^m-b^{m+1}\right)\sum_{i=0}^{m-1}b^i\left(f_{i+1}-f_i\right)\right] \qquad (4.43)$$

In (4.40) the left-hand side is negative. Dividing (4.40) by $(1-b^m)b^m$, the right-hand side can be stated as follows:

$$p_c\beta\left[-\left(f_{m+1}-f_m\right)+\frac{b^m-b^{m+1}}{(1-b^m)b^m}\sum_{i=0}^{m-1}b^i\left(f_{i+1}-f_i\right)\right]$$

Since

$$\sum_{i=0}^{m-1}b^i\left(f_{i+1}-f_i\right)=\sum_{i=0}^{m-1}b^i\left[\left(f_{i+1}-f_i\right)-\left(f_{m+1}-f_m\right)\right]+\sum_{i=0}^{m-1}b^i(f_{m+1}-f_m)$$

implying that

$$\sum_{i=0}^{m-1}b^i\left(f_{i+1}-f_i\right)=\sum_{i=0}^{m-1}b^i\left[\left(f_{i+1}-f_i\right)-\left(f_{m+1}-f_m\right)\right]+\frac{1-b^m}{1-b}(f_{m+1}-f_m)$$

Substituting above we obtain

$$p_c\beta\left[\frac{b^m-b^{m+1}}{(1-b^m)b^m}\left(\sum_{i=0}^{m-1}b^i\left[\left(f_{i+1}-f_i\right)-\left(f_{m+1}-f_m\right)\right]+\frac{1-b^m}{1-b}(f_{m+1}-f_m)\right)-\left(f_{m+1}-f_m\right)\right]$$

which can be restated as

$$p_c\beta\left[\frac{b^m-b^{m+1}}{(1-b^m)b^m}\left(\sum_{i=0}^{m-1}b^i\left[\left(f_{i+1}-f_i\right)-\left(f_{m+1}-f_m\right)\right]\right)+\left(\frac{b^m-b^{m+1}}{(1-b^m)b^m}\frac{1-b^m}{1-b}-1\right)\left(f_{m+1}-f_m\right)\right]$$

Given that $\{f_i-f_{i-1}\}$, for $i=1,...,m$, is a decreasing sequence, we may conclude that this expression is positive, as the term that multiplies $(f_{m+1}-f_m)$ vanishes. Consequently, the right-hand side of (4.40) is positive, implying that (4.40) never holds for any $m^c>m$. By inspection, we observe that it may hold for some $m^c<m$. Therefore, for $\theta=1$, it is never optimal to postpone harvest, that is, $m^c\leq m$.

(ii) Let $m^c=m+1$ in (4.39). By rearranging and collecting terms we obtain

$$pb^{m+1}f_{m+1}(1-b^m)-pb^mf_m(1-b^{m+1})>$$

$$p_c\beta\left[-\left(f_{m+1}-f_m\right)(1-b^m)b^m+\left(b^m-b^{m+1}\right)\sum_{i=0}^{m-1}b^i\left(f_{i+1}-f_i\right)\right] \tag{4.44}$$

Since

$$\sum_{i=0}^{m-1}b^if_i=\sum_{i=0}^{m-1}b^i(f_i-f_{m+1})+\sum_{i=0}^{m-1}b^if_{m+1}$$

which can be rewritten as

$$\sum_{i=0}^{m-1}b^if_i=\sum_{i=0}^{m-1}b^i(f_i-f_{m+1})+\frac{1-b^m}{1-b}f_{m+1}$$

By substituting above, we obtain

$$p_c\beta\left[-\left(f_{m+1}-f_m\right)(1-b^m)b^m\right]+$$

$$+p_c\beta\left(b^m-b^{m+1}\right)\left(\sum_{i=0}^{m-1}b^if_i-\sum_{i=0}^{m-1}b^i\left(f_i-f_{m+1}\right)-\frac{1-b^m}{1-b}f_{m+1}\right)$$

Finally, by collecting terms, we get

$$p_c\beta\left(-(1-b^m)b^m-\left(b^m-b^{m+1}\right)\frac{1-b^m}{1-b}\right)f_{m+1}+p_c\beta f_m(1-b^m)b^m$$

$$+p_c\beta\left(b^m-b^{m+1}\right)\sum_{i=0}^{m-1}b^if_i-p_c\beta\left(b^m-b^{m+1}\right)\sum_{i=0}^{m-1}b^i\left(f_i-f_{m+1}\right)$$

where the first-term is negative and the other three are positive. In particular, the last term is positive as long as $\{f_i\}$ is an increasing sequence for $i=1,...,m+1$. Therefore, the sign of this expression, that is, the right-hand side of (4.44) can be either positive or negative. Since the left-hand side of (4.44) is negative, it may be optimal to postpone harvest. This is in contrast to case (i), in which by imposing a more restrictive assumption, namely, that the sequence $\{f_i-f_{i-1}\}$ is decreasing, it is never optimal to postpone harvest.

Let now $m^c=m-1$ in (4.39). By rearranging and collecting terms we obtain

$$pb^{m-1}f_{m-1}(1-b^m)-pb^mf_m(1-b^{m-1})>$$

$$p_c\beta\left[\left(f_m-f_{m-1}\right)(1-b^{m-1})b^{m-1}+\left(b^m-b^{m-1}\right)\sum_{i=0}^{m-1}b^i\left(f_{i+1}-f_i\right)\right] \tag{4.45}$$

which can be rewritten as

$$p_c\beta\left[(f_m - f_{m-1})\,(1-b^{m-1})b^{m-1} + \left(b^m - b^{m-1}\right)\left(\sum_{i=0}^{m-2} b^i(f_{i+1}-f_i)\right)\right]$$

Using the same procedure as before, we may write

$$\sum_{i=0}^{m-2} b^i\left[(f_{i+1}-f_i)-(f_m-f_{m-1})\right] = \sum_{i=0}^{m-2} b^i + \sum_{i=0}^{m-2} b^i\,(f_m-f_{m-1})$$

that is,

$$\sum_{i=0}^{m-2} b^i\left[(f_{i+1}-f_i)-(f_m-f_{m-1})\right] = \sum_{i=0}^{m-2} b^i\left[(f_{i+1}-f_i)-(f_m-f_{m-1})\right] + \frac{1-b^{m-1}}{1-b}(f_m-f_{m-1})$$

Substituting above and collecting terms, we obtain

$$p_c\beta\left(b^m-b^{m-1}\right)\left(\sum_{i=0}^{m-2} b^i\left[(f_{i+1}-f_i)-(f_m-f_{m-1})\right]\right)+$$

$$+p_c\beta\left(\left(b^m-b^{m-1}\right)\frac{1-b^{m-1}}{1-b}+(1-b^{m-1})b^{m-1}\right)(f_m-f_{m-1})$$

Therefore, the right-hand side of (4.45) can be rewritten as the algebraic sum of the two above terms. The first term is negative, as long as $\{f_i - f_{i-1}\}$ is a decreasing sequence for $i = 1, ..., m-1$, while the second one is positive, as the term that multiplies $(f_m - f_{m-1})$ is positive. Therefore, it may be optimal to cut earlier than m. The same result is obtained if, instead, we consider a less restrictive assumption such that $\{f_i\}$ is an increasing sequence for $i = 1, ..., m+1$.

4.8 Appendix C

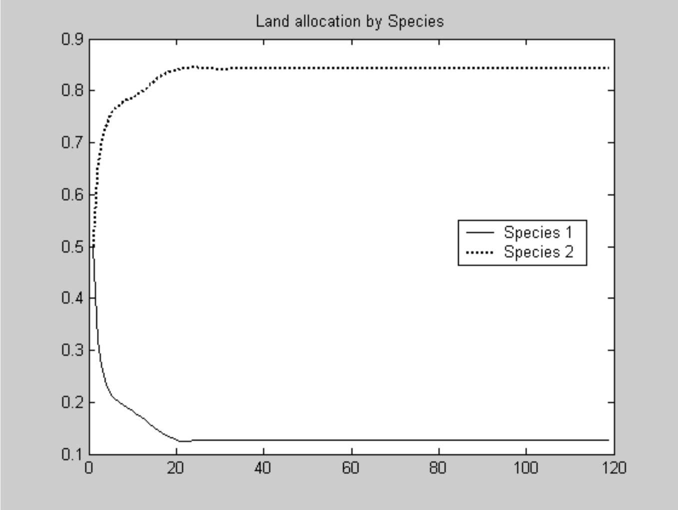

Figure 1

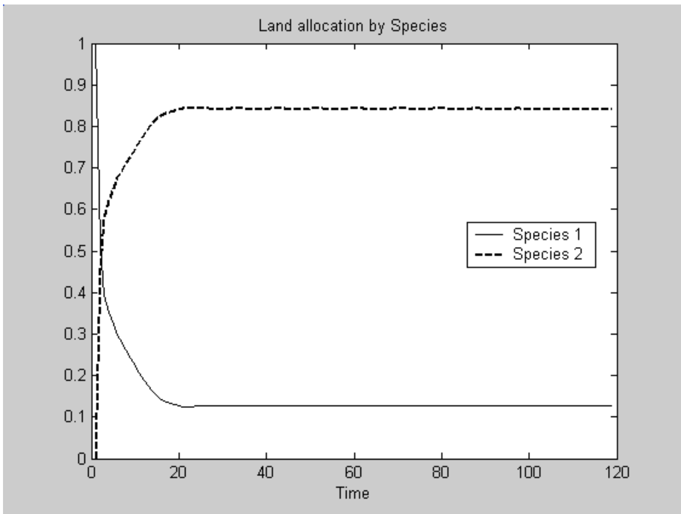

Figure 2

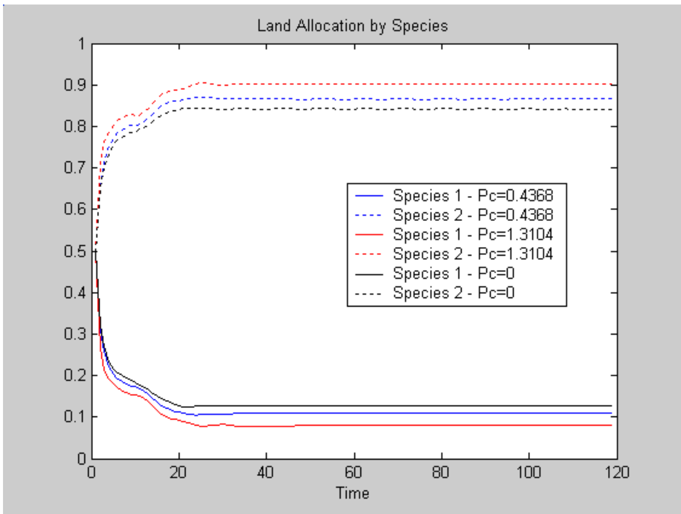

Figure 3

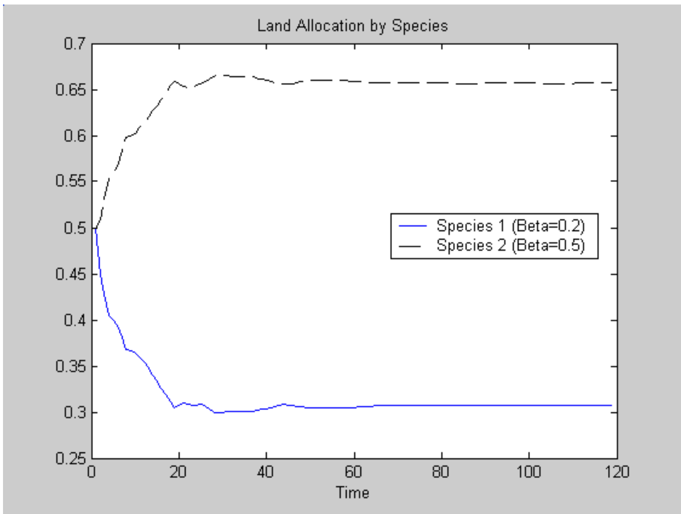

Figure 4

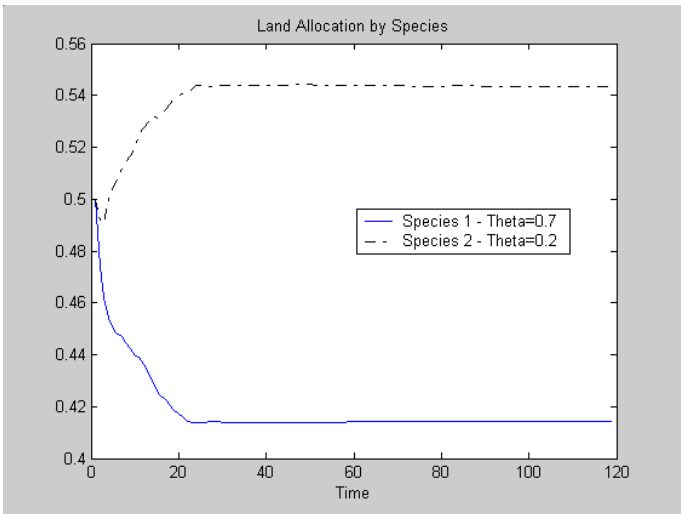

Figure 5

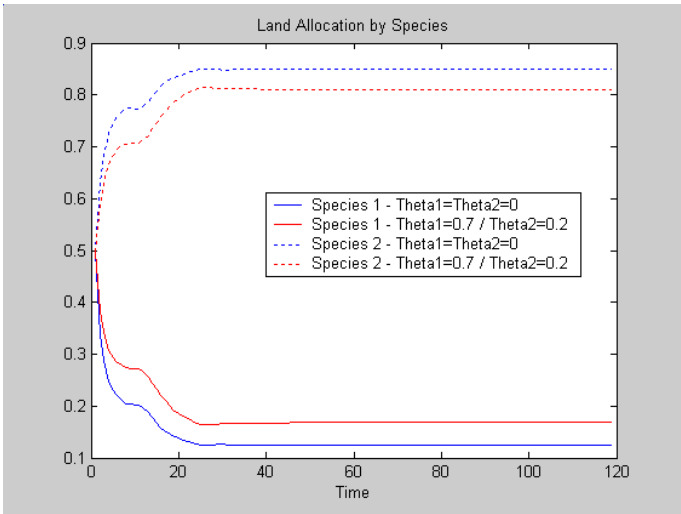
Land Allocation by Species
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
20
40
60
80
100
120
Time
Species 1 - Theta1=Theta2=0
Species 1 - Theta1=0.7 / Theta2=0.2
Species 2 - Theta1=Theta2=0
Species 2 - Theta1=0.7 / Theta2=0.2

Figure 6

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