IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received February 8, 2020, accepted March 1, 2020, date of publication March 4, 2020, date of current version March 13, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.2978298

Exploiting Orientation Information to Improve Range-Based Localization Accuracy

SLAVISA TOMIC[1], MARKO BEKO[1,2], AND MILAN TUBA[3], (Member, IEEE)
[1]COPELABS, Universidade Lusófona de Humanidades e Tecnologias, 1749-024 Lisboa, Portugal
[2]UNINOVA, 2829-516 Caparica, Portugal
[3]Singidunum University, 11000 Belgrade, Serbia

Corresponding author: Slavisa Tomic (slavisa.tomic@ulusofona.pt)

This work supported in part by the Fundação para a Ciência e a Tecnologia under Project IF/00325/2015, Project foRESTER PCIF/SSI/0102/2017, and Project UIDB/04111/2020, and in part by the Universidade Lusófona/ILIND internal project TESLA.

ABSTRACT This work addresses target localization problem in precarious surroundings where possibly no links are line of sight. It exploits the known architecture of available reference points to act as an irregular antenna array in order to estimate the azimuth angle between a reference point and a target, based on distance estimates withdrawn from integrated received signal strength (RSS) and time of arrival (TOA) observations. These fictitious azimuth angle *observations* are then used to *linearize* the measurement models, which triggers effortless derivation of a new estimator in a closed-form. It is shown here that, by using fixed network geometry in which target orientation with respect to a line formed by a pair of anchors can be correctly estimated, the localization performance can be significantly enhanced. The new approach is validated through computer simulations, which corroborate our intuition of profiting from inherent information within a network.

INDEX TERMS Non-line-of-sight (NLOS), weighted least squares (WLS), received signal strength (RSS), time of arrival (TOA), azimuth angle.

I. INTRODUCTION

Achieving accurate location of people and objects will be a paramount task in many applications of the future Internet of things systems based on 5G networks. To this end, different properties of radio signals could be exploited, *e.g.* received signal strength (RSS) [1]–[4], time of arrival (TOA) [5]–[8] or angle of arrival [9]–[14].

The topic of target localization based on amalgamated RSS and TOA observations has evoked much attention in the research society recently [15]–[19]. In the works of [15] and [16], the authors considered the problem of range estimation founded on these two radio measurements. The authors in [17]–[19], addressed the target localization problem in mixed line-of-sight (LOS)/non-line-of-sight (NLOS) surroundings. In [17], the authors started by identifying the kind of path of each link by employing Nakagami distribution after which they derived an estimator in the form of weighted least squares (WLS). Based on the type of the identified path (LOS/NLOS), the WLS estimator than exploits TOA-only/RSS-only observations. In [18], the authors proposed an iterative estimator based on squared range WLS. They partially mitigated the resinous impact of NLOS biases by introducing a single balancing term, together with employing an intermittent scheme to determine an estimate of the target's location. The authors in [19], studied a worst-case setting of the problem in which it was assumed that none of the links are LOS. Building upon this, as well as the supposition that the knowledge about the magnitude of the NLOS bias is (imperfectly) available, the authors derived a robust estimator formulated in a generalized trust region sub-problem (GTRS) framework, by following a min-max criterion. The existing algorithms are limited in the sense that they either require perfect distinguishment between LOS/NLOS links [17] (which might not be feasible in practice) or their computational complexity is not completely satisfactory [18], [19].

In this work, we take a completely different approach from the existing ones: by taking advantage of the network topology of known reference points to play a role of an irregular antenna array and the available distance estimates (from RSS and TOA observations), we first estimate the azimuth angle between a reference point and a target. Note that this is in huge contrast with [9]–[12], [14], [20], where

The associate editor coordinating the review of this manuscript and approving it for publication was Ahmet M. Elbir.

azimuth angle was measured by using directional antennas or antenna arrays. These estimates are then used to *linearize* the measurement models based on Cartesian to polar coordinates transformation, which permits effortless development of a new estimator whose solution is given in closed-form. The biggest challenge in the proposed approach is to guarantee correct estimation of the target's orientation with respect to a line between two reference points. Owing to the elevated level of difficulty of the problem of interest, this is extremely hard to achieve always in ad-hoc networks. Nevertheless, in some fixed network architectures of high practical interest, where such guarantees are attainable, the new approach exhibits superior performance over the existing ones. Hence, the principal contributions of the present work are the following.

- Novel procedure for azimuth angle estimation between a reference point and a target. The procedure is based only on exploiting the known network architecture of the reference points and distance measurements obtained through RSS and TOA measurements, and requires no additional hardware. This is in sharp contrast with the existing methods in [9]–[12], [14], [20], where the azimuth angle was measured by using directional antennas or antenna arrays.
- New estimator for hybrid RSS-TOA localization in adverse indoor environments. Our proposed approach takes advantage of the estimated azimuth angle to facilitate derivation of an efficient estimator (both in terms of computational complexity and localization accuracy) in a closed-form. The new estimator shows superior performance over the existing ones, owing to the exploitation of the inherent information from the network architecture.

The remainder of this work is structured as follows. Section II introduces the considered measurement models and formulates the target localization problem. In Section III, derivation of the proposed procedure to estimate the azimuth angle between a reference point and a target is presented. Section IV describes how to exploit the estimated azimuth angle in order to efficiently solve the localization problem at hand. Sections V and VI validate the performance of the proposed localization algorithm in terms of computational complexity and localization accuracy, respectively. Lastly, Section VII summarizes the main findings of this work.

II. PROBLEM CONCEPTUALIZATION

Let us focus on a 2-dimensional network of sensors, where $\boldsymbol{a}_i$ and $\boldsymbol{x}$ respectively represent the known location of the i-th reference point (also called anchor) ($i = 1, \dots, N$) and the unknown location of the target. The target emits a radio signal, picked-up by anchors, which are capable of extracting the RSS and the TOA information from it. In NLOS environments, the two radio measurements (where TOA observations were converted to distance) can be modeled [15], [16] as

$$P_i = P_0 - b_i - 10\gamma \log_{10}\frac{\|\boldsymbol{x}-\boldsymbol{a}_i\|}{d_0} + n_i, \tag{1a}$$

$$d_i = \|\boldsymbol{x}-\boldsymbol{a}_i\| + \beta_i + m_i, \tag{1b}$$

where P_0 denotes the RSS (in decibel-milliwatts, dBm) measured at a short reference distance d_0 ($\|\boldsymbol{x}-\boldsymbol{a}_i\| \geq d_0$), b_i (in decibels, dB) and β_i (in meters, m) denote the (positive) NLOS biases, γ represents the path loss exponent between two sensors, indicating the rate at which the received strength decreases with distance, n_i denotes the log-normal shadowing term (dB) modeled as $n_i \sim \mathcal{N}(0, \sigma_{n_i}^2)$, and $m_i \sim \mathcal{N}(0, \sigma_{m_i}^2)$ is the measurement noise (m). Analogously to the previous works in [5]–[8], the magnitude of the NLOS biases is considered upper-bounded by a given parameter, *i.e.*, $0 \leq b_i \leq b_{\max}$ and $0 \leq \beta_i \leq \beta_{\max}$. Note that this assumption does not imply anything about the distributions of b_i or β_i, since they are not known in general.

Based on (1a) and the fact that n_i and m_i presumably follow a Normal distribution, the combined RSS-TOA maximum likelihood (ML) estimator of $\boldsymbol{x}$, b_i and β_i can be formulated as If RSS and TOA measurements were taken from individual origins, the problem in (2), shown at the bottom of this page would represent the exact joint ML estimator [16]. Nevertheless, the experimental trials in [15] and [21] corroborate that these observations withdrawn simultaneously from a single radio signal are weakly correlated; hence, the supposition that the measurements are uncorrelated is not unacceptable. Howsoever, (2) is far from being convex and is not determined, since the set of unknowns ($2N+2$) is greater than the set of observations ($2N$). Hence, instead of tackling it directly, we first estimate the azimuth angle between an anchor and the target, which then allows us to *linearize* the measurement models and derive a different estimator in a closed-form.[1]

III. AZIMUTH ANGLE ESTIMATION

In the *mean* ML sense,[2] the best estimators of $\|\boldsymbol{x}-\boldsymbol{a}_i\|$ from (1a), are

$$\widehat{d}_i^{\,\text{RSS}} = d_0 10^{\frac{P_0 - P_i - \frac{b_{\max}}{2}}{10\gamma}}, \tag{3a}$$

$$\widehat{d}_i^{\,\text{TOA}} = d_i - \frac{\beta_{\max}}{2}. \tag{3b}$$

[1] An alternative (iterative) approach for achieving (local) *linearization* might be through gradient search. However, these methods highly rely on initialization and might lead to imprecise solutions due to local minima.

[2] *E.g.*, the best estimate of $\|\boldsymbol{x}-\boldsymbol{a}_i\|$ in (1b) in the *mean* ML perspective is $d_i - \beta_{\max}/2 = \widehat{d}_i^{\,\text{TOA}}$, where $\beta_{\max}/2$ denotes the mean value of the interval $[0, \beta_{\max}]$, from which β_i is stipulated. Analogous reasoning was applied for defining $\widehat{d}_i^{\,\text{RSS}}$.

$$\left\{\widehat{\boldsymbol{x}}, \widehat{b}_i, \widehat{\beta}_i\right\} = \arg\min_{\boldsymbol{x}, b_i, \beta_i} \sum_{i=1}^{N} \frac{\left(P_i - P_0 + b_i + 10\gamma\log_{10}\frac{\|\boldsymbol{x}-\boldsymbol{a}_i\|}{d_0}\right)^2 \sigma_{m_i}^2 + \left(d_i - \|\boldsymbol{x}-\boldsymbol{a}_i\| - \beta_i\right)^2 \sigma_{n_i}^2}{\sigma_{n_i}^2 \sigma_{m_i}^2} \tag{2}$$

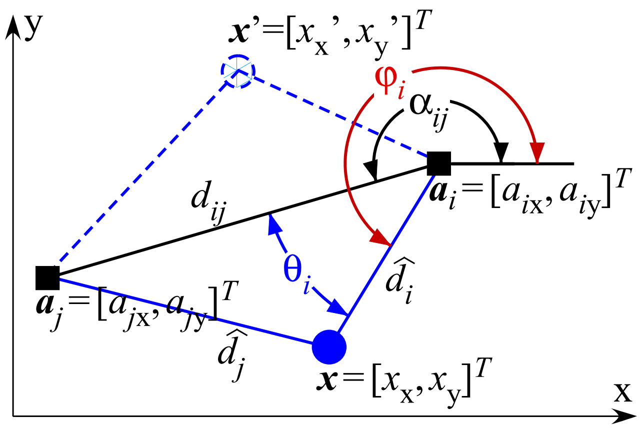


FIGURE 1. Geometrical interpretation of the azimuth angle estimation process.

As shown in Fig. 1 where $\widehat{d}_i$ instead of $\widehat{d}_i^{\text{RSS}}$ and $\widehat{d}_i^{\text{TOA}}$ is used for notation simplicity, by having these distance estimates at hand, together with the known locations of anchors, one could form triangles between a couple of anchors and the target. Note that in any of these triangles, all three side lengths are (possibly imperfectly) known. Hence, by using the law of cosines, the angle θ_i (see Fig. 1) can be easily estimated as follows.

$$\hat{\theta}_i = \arccos\left(\frac{d_{ij}^2 + \widehat{d}_i^2 - \widehat{d}_j^2}{2 d_{ij} \widehat{d}_i}\right), \tag{4}$$

for both types of measurements (RSS and TOA). In addition, the azimuth angle between the i-th anchor and the target at the i-th anchor (from both RSS and TOA) is estimated as

$$\hat{\varphi}_i = \alpha_{ij} \pm \hat{\theta}_i, \tag{5}$$

where α_{ij} stands for the (known) azimuth angle among $\boldsymbol{a}_i$ and $\boldsymbol{a}_j$, and $\pm$ is used to capture all potential scenarios relative to the location of the target (depending on whether the target is located above or below the line formed by the anchors i and j). In general, the correct sign in (5), *i.e.*, the target's orientation with respect to a line formed by a pair of anchors can be estimated by finding a rough estimate of $\boldsymbol{x}$. For instance, one could achieve this by resorting to an ordinary trilateration method, like the one in [9], or by any of the existing RSS-TOA localization algorithms [17]–[19]. Still, because of the elevated level of difficulty of the considered problem (non-convex and under-determined) and perhaps unfavorable geometry in ad hoc networks (*e.g.*, when sensors are approximately deployed on a line), it is not realistic to attain perfect orientation estimates in all cases. Naturally, this would reflect negatively on the localization performance. Nonetheless, there are many fixed network deployments of practical interest (as the one considered in Section VI), where one can simply rely on known network topology to guarantee the correct orientation estimation in all situations. Hence, we will show that in such cases, the artificially fabricated information about the estimated azimuth angle can considerably enhance localization performance. To this end, the following section gives details about how this additional information could be exploited.

IV. THE PROPOSED WLS ESTIMATOR

This section shows how to *linearize* the measurement model (1a) by exploiting the estimated azimuth angles in (5), and how to solve the localization problem in a closed-form. Notice that from (3) one can write

$$\lambda_i \|\boldsymbol{x} - \boldsymbol{a}_i\| \approx \eta d_0, \tag{6a}$$
$$\|\boldsymbol{x} - \boldsymbol{a}_i\| \approx d_i - \frac{\beta_{\max}}{2}, \tag{6b}$$

where $\lambda_i = 10^{\frac{P_i + \frac{b_{\max}}{2}}{10\gamma}}$ and $\eta = 10^{\frac{P_0}{10\gamma}}$. Moreover, by exploiting $\hat{\varphi}_i$ in (5) and applying simple geometry, one can approximate

$$\hat{\varphi}_i \approx \arctan\left(\frac{x_y - a_{iy}}{x_x - a_{ix}}\right),$$

where x_y and a_{iy} (x_x and a_{ix}) denote the y-coordinates (the x-coordinates) of the target and the i-th anchor respectively. After performing certain elementary algebraic operations, the previous expression can be written in the following vector form.

$$\boldsymbol{c}_i^T (\boldsymbol{x} - \boldsymbol{a}_i) \approx 0, \tag{7a}$$
$$\boldsymbol{k}_i^T (\boldsymbol{x} - \boldsymbol{a}_i) \approx 0, \tag{7b}$$

where $\boldsymbol{c}_i = [-\sin(\hat{\varphi}_i^{\text{RSS}}), \cos(\hat{\varphi}_i^{\text{RSS}})]^T$ and $\boldsymbol{k}_i = [-\sin(\hat{\varphi}_i^{\text{TOA}}), \cos(\hat{\varphi}_i^{\text{TOA}})]^T$, for the estimates obtained according to RSS and TOA observations.

Then, according to the least squares criterion based on (6a) and (7a), an estimate of $\boldsymbol{x}$ can be obtained by solving

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \sum_{i=1}^{N} (\lambda_i \|\boldsymbol{x} - \boldsymbol{a}_i\| - \eta d_0)^2 + \sum_{i=1}^{N} \left(\boldsymbol{c}_i^T (\boldsymbol{x} - \boldsymbol{a}_i)\right)^2$$
$$+ \sum_{i=1}^{N} \left(\|\boldsymbol{x} - \boldsymbol{a}_i\| - \widehat{d}_i^{\text{TOA}}\right)^2 + \sum_{i=1}^{N} \left(\boldsymbol{k}_i^T (\boldsymbol{x} - \boldsymbol{a}_i)\right)^2. \tag{8}$$

The estimator in (8) is not convex owing to the norm terms in the first two summations. Nevertheless, since we have (fabricated) azimuth angle information at our disposal, and it is well known that polar space is more suitable when dealing with directional data [22], we shortly switch from Cartesian to polar space and (indirectly) tackle (8) in the latter space. To do so, we express $\boldsymbol{x} - \boldsymbol{a}_i = r_i \boldsymbol{u}_i$ in (6a), with $r_i \geq 0$ and $\|\boldsymbol{u}_i\| = 1$, which results in $\|\boldsymbol{x} - \boldsymbol{a}_i\| = r_i$. Moreover, we define the unit vector as $\boldsymbol{u}_i = [\cos(\hat{\varphi}_i^{\text{RSS}}), \sin(\hat{\varphi}_i^{\text{RSS}})]^T$. Similar procedure is applied in (6b), where $\boldsymbol{x} - \boldsymbol{a}_i = \rho_i \boldsymbol{v}_i$, with the unit vector defined as $\boldsymbol{v}_i = [\cos(\hat{\varphi}_i^{\text{TOA}}), \sin(\hat{\varphi}_i^{\text{TOA}})]^T$. To invert the treatment and return to the original space, it is enough to multiply the two equations by $\boldsymbol{u}_i^T \boldsymbol{u}_i = 1$, *i.e.*, $\boldsymbol{v}_i^T \boldsymbol{v}_i = 1$. Hence, by applying the described steps to (6a), we get

$$\lambda_i \boldsymbol{u}_i^T r_i \boldsymbol{u}_i = \eta d_0 \Leftrightarrow \lambda_i \boldsymbol{u}_i^T (\boldsymbol{x} - \boldsymbol{a}_i) = \eta d_0, \tag{9a}$$
$$\boldsymbol{v}_i^T \rho_i \boldsymbol{v}_i = \widehat{d}_i^{\text{TOA}} \Leftrightarrow \boldsymbol{v}_i^T (\boldsymbol{x} - \boldsymbol{a}_i) = \widehat{d}_i^{\text{TOA}}. \tag{9b}$$

With the goal to attribute more trust to *nearby* connections (to both RSS and TOA links), weights, $\boldsymbol{w} = [\sqrt{w_i}]^T$ and $\boldsymbol{\omega} = [\sqrt{\omega_i}]^T$, are introduced, with $w_i = 1 - \widehat{d}_i^{RSS} / \sum_{i=1}^N \widehat{d}_i^{RSS}$ and $\omega_i = 1 - \widehat{d}_i^{TOA} / \sum_{i=1}^N \widehat{d}_i^{TOA}$. Hence, following the WLS criterion, the following estimator can be derived.

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \sum_{i=1}^N w_i \left(\lambda_i \boldsymbol{u}_i^T (\boldsymbol{x} - \boldsymbol{a}_i) - \eta d_0\right)^2 + \sum_{i=1}^N w_i \left(\boldsymbol{c}_i^T (\boldsymbol{x} - \boldsymbol{a}_i)\right)^2 + \sum_{i=1}^N \omega_i \left(\boldsymbol{v}_i^T (\boldsymbol{x} - \boldsymbol{a}_i) - \widehat{d}_i^{TOA}\right)^2 + \sum_{i=1}^N \omega_i \left(\boldsymbol{k}_i^T (\boldsymbol{x} - \boldsymbol{a}_i)\right)^2. \quad (10)$$

The problem in (10) can be rewritten in a vector form as

$$\underset{\boldsymbol{x}}{\text{minimize}} \ \|\boldsymbol{W}(\boldsymbol{A}\boldsymbol{x} - \boldsymbol{b})\|^2, \quad (11)$$

where $\boldsymbol{W} = \text{diag}\left(\left[\boldsymbol{w}^T, \boldsymbol{w}^T, \boldsymbol{\omega}^T, \boldsymbol{\omega}^T\right]\right)$, and diag($\bullet$) denotes a diagonal matrix whose elements on the main diagonal are defined by the vector in the argument, and

$$\boldsymbol{A} = \begin{bmatrix} \lambda_1 \boldsymbol{u}_1^T \\ \vdots \\ \lambda_N \boldsymbol{u}_N^T \\ \boldsymbol{c}_1^T \\ \vdots \\ \boldsymbol{c}_N^T \\ \boldsymbol{v}_1^T \\ \vdots \\ \boldsymbol{v}_N^T \\ \boldsymbol{k}_1^T \\ \vdots \\ \boldsymbol{k}_N^T \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} \lambda_1 \boldsymbol{u}_1^T \boldsymbol{a}_1 + \eta d_0 \\ \vdots \\ \lambda_N \boldsymbol{u}_N^T \boldsymbol{a}_N + \eta d_0 \\ \boldsymbol{c}_1^T \boldsymbol{a}_1 \\ \vdots \\ \boldsymbol{c}_N^T \boldsymbol{a}_N \\ \boldsymbol{v}_1^T \boldsymbol{a}_1 + \widehat{d}_1^{\text{TOA}} \\ \vdots \\ \boldsymbol{v}_N^T \boldsymbol{a}_N + \widehat{d}_N^{\text{TOA}} \\ \boldsymbol{k}_1^T \boldsymbol{a}_1 \\ \vdots \\ \boldsymbol{k}_N^T \boldsymbol{a}_N \end{bmatrix}.$$

The closed-form solution[3] to (11) is thus

$$\hat{\boldsymbol{x}} = \left(\boldsymbol{A}^T \boldsymbol{W}^T \boldsymbol{W} \boldsymbol{A}\right)^{-1} (\boldsymbol{A}^T \boldsymbol{W}^T \boldsymbol{b}).$$

V. COMPLEXITY ANALYSIS

In this section, besides the proposed WLS algorithm in (11), three existing algorithms are considered: the HWLS algorithm which identifies the type of path by relying on Nakagami distribution [17], the SR-WLS algorithm which approximates the NLOS biases by balancing parameters [18], and the R-GTRS algorithm which uses a robust approach to turn the original problem into a min-max one [19].

An overview of the computational complexity of the relevant algorithms is given in Table 1. Note that S_{max} in Table 1 denotes the maximum allowed number of iterations in the bisection scheme. The table exhibits that the considered algorithms have linear complexities in N. Nevertheless, the complexities of SR-WLS and R-GTRS are somewhat higher due to the iterative nature of the bisection procedure.

[3] Note that after solving (11), one could apply an alternating approach in which $\hat{\boldsymbol{x}}$ could be used to obtain an estimate of the mean NLOS bias (both RSS and TOA) and vice versa. Nevertheless, in our simulations, this iterative approach showed only marginal improvements in terms of localization accuracy, which does not justify its use.

TABLE 1. Summary of the considered algorithms.

Algorithm	Description	Complexity
WLS	The proposed method in (11)	$\mathcal{O}(N)$
HWLS	The HWLS method in [17]	$\mathcal{O}(N)$
SR-WLS	The SR-WLS method in [18]	$2 \times \mathcal{O}(S_{\max} N)$
R-GTRS	The R-GTRS method in [19]	$\mathcal{O}(S_{\max} N)$

TABLE 2. Anchors' deployment within the simulation surrounding.

Index	1	2	3	4	5	6	7	8
$\boldsymbol{a}_i$ (m)	0	0	B	B	0	$B/2$	$B/2$	B
	0	B	0	B	$B/2$	0	B	$B/2$

VI. PERFORMANCE ASSESSMENT

In this section, we present a set of computer simulation results in order to assess the performance of the proposed algorithm in terms of localization accuracy. All presented results were obtained by implementing the considered algorithms in MATLAB. All observations were simulated by following (1a), whereas the known locations of the reference points are given in Table 2. The true unknown location of the target was generated randomly within a region of $B \times B$ m^2 in every Monte Carlo, M_c, run. It is worth mentioning that the first N (where N might vary in each setting) anchors from Table 2 were employed invariably. The rest of the fixed simulation parameters were set as: $P_0 = 20$ dBm, $d_0 = 1$ m, $\gamma = 3$, and $M_c = 50000$. Furthermore, for each link and in each M_c run, the NLOS biases (for both RSS and TOA measurements) were retrieved arbitrary from a uniform distribution on the interval $[0, \text{bias}_{\max}]$ (dB, m), *i.e.*, $\text{bias}_i \sim \mathcal{U}[0, \text{bias}_{\max}]$, $i = 1, \ldots, N$. The main metric for performance assessment is the root mean squared error (RMSE), defined as RMSE $= \sqrt{\sum_{i=1}^{M_c} \frac{\|\boldsymbol{x}_i - \widehat{\boldsymbol{x}}_i\|^2}{M_c}}$, where $\widehat{\boldsymbol{x}}_i$ represents the estimate of the true target location, $\boldsymbol{x}_i$, in the i-th M_c run.

The considered network geometry is chosen intentionally, since it allows for perfect estimation of the correct sign in (5). For instance, to determine $\hat{\varphi}_1$ one could use $\boldsymbol{a}_2$ and $\boldsymbol{a}_3$ to form pairs with $\boldsymbol{a}_1$, since the only feasible target orientations would be to the right and above the lines formed by these pairs of anchors, respectively. To determine $\hat{\varphi}_5$ one could use $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$ to form pairs with $\boldsymbol{a}_5$, since the only feasible target orientation would be to the right of the lines formed by these pairs of anchors, and so on. Whenever feasible, in all simulations presented here, each anchor used two additional anchors to assist it in estimating its azimuth angle to the target.

The new algorithm is compared with the existing ones (see Table 1). In all simulations, the first N anchors in Table 2 are considered, and $K = 30$ was used for SR-WLS and R-GTRS.

Fig. 2 illustrates the performance comparison in terms of the number of anchors, N, in the setting where the number of NLOS links, $|\mathcal{L}_{\text{NLOS}}|$, is equal to N. The figure shows that WLS outperforms significantly the existing approaches,

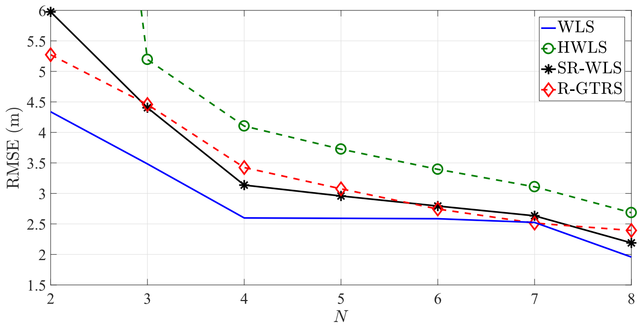


FIGURE 2. RMSE versus N comparison, when $\text{bias}_{\text{max}} = 5$ (dB, m), $\text{bias}_i \sim \mathcal{U}[0, \text{bias}_{\text{max}}]$, $\sigma_i = 3$ (dB, m), $|\mathcal{L}_{\text{NLOS}}| = N$, $B = 20$ m.

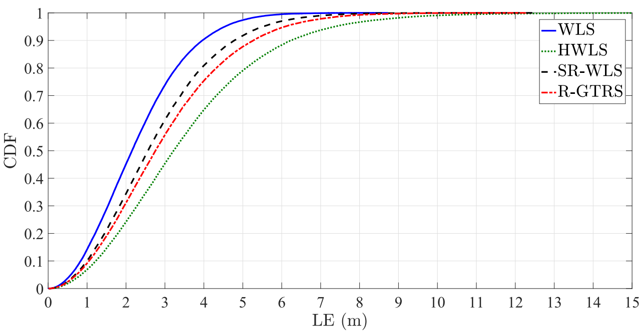


FIGURE 3. CDF versus LE (m) comparison, when $N = 4$, $\text{bias}_{\text{max}} = 5$ (dB, m), $\text{bias}_i \sim \mathcal{U}[0, \text{bias}_{\text{max}}]$, $\sigma_i = 3$ (dB, m), $|\mathcal{L}_{\text{NLOS}}| = 4$, $B = 20$ m.

especially for low N. This result was anticipated, since the new approach benefits from additional information from the estimated azimuth angles. Naturally, as N grows, the amount of information acquired in the network becomes sufficiently large to permit reasonably good performance to all methods.

Fig. 3 illustrates the cumulative distribution function (CDF) of localization error (LE), when $N = 4$. We define the LE according to $\text{LE} = \|\boldsymbol{x}_i - \widehat{\boldsymbol{x}}_i\|$ (m), for $i = 1, \ldots, M_c$. It can be seen from the figure that the proposed estimator accomplishes $\text{LE} \leq 4$ in roughly 90% of the cases, which is fairly better in comparison with the existing methods.

Fig. 4 illustrates the performance comparison in terms of noise powers, σ_i (dB, m). In order to give a deeper insight on the impact of noise powers on localization performance, the magnitude of the NLOS biases, bias_{max} (dB, m), was fixed to a reasonably low value, *i.e.*, $\text{bias}_{\text{max}} = 1$ (dB, m) was considered. Fig. 4 shows that WLS performs well in noisy environments, offering a considerable error reduction in comparison with the state of the art approaches.

Fig. 5 illustrates the performance comparison in terms of the magnitude of the NLOS biases, bias_{max} (dB, m). Analogously to the previous figure, here, we set $\sigma_i = 1$ (dB, m). From Fig. 5, one can notice a margin in the performance between the new and the existing approaches for every examined value of bias_{max} (dB, m). This result suggests that WLS handles NLOS bias more efficiently than other approaches.

Fig. 6 illustrates the performance comparison in terms of the number of NLOS links, $|\mathcal{L}_{\text{NLOS}}|$. Note that $|\mathcal{L}_{\text{NLOS}}| = 0$ corresponds to a completely LOS setting, while $|\mathcal{L}_{\text{NLOS}}| = N$

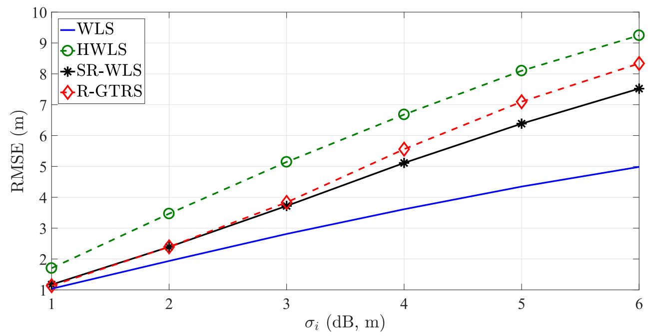


FIGURE 4. RMSE versus σ_i (dB, m) comparison, when $N = 3$, $\text{bias}_{\text{max}} = 1$ (dB, m), $\text{bias}_i \sim \mathcal{U}[0, \text{bias}_{\text{max}}]$, $|\mathcal{L}_{\text{NLOS}}| = 3$, $B = 15$ m.

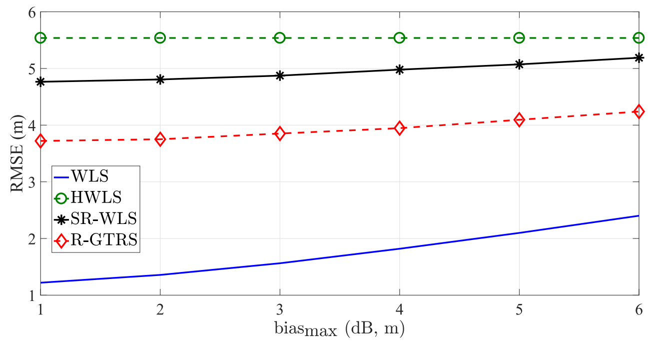


FIGURE 5. RMSE versus bias_{max} (dB, m) comparison, when $N = 2$, $\sigma_i = 1$ (dB, m), $\text{bias}_i \sim \mathcal{U}[0, \text{bias}_{\text{max}}]$, $|\mathcal{L}_{\text{NLOS}}| = 2$, $B = 15$ m.

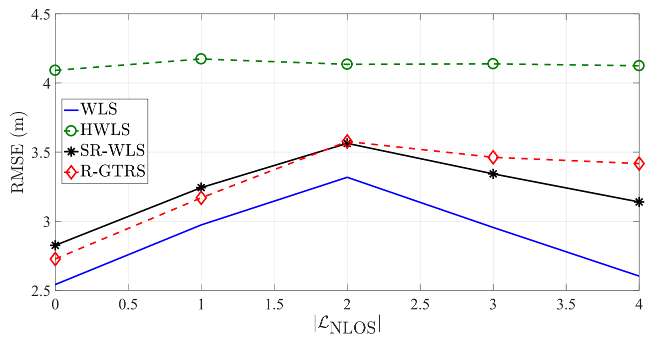


FIGURE 6. RMSE versus $|\mathcal{L}_{\text{NLOS}}|$ comparison, when $N = 4$, $\text{bias}_{\text{max}} = 5$ (dB, m), $\text{bias}_i \sim \mathcal{U}[0, \text{bias}_{\text{max}}]$, $\sigma_i = 3$ (dB, m), $B = 20$ m.

corresponds to a fully NLOS setting. The figure shows that, besides HWLS algorithm[4] all approaches perform better in the two radical scenarios (all links LOS/NLOS) than in the mixed environment. This might be justified to some proportion by the fact that all of them tend to cancel out the influence of the NLOS bias, which gets more miscellaneous as the number of LOS and NLOS links balance out, allowing only partial NLOS bias mitigation. Nevertheless, the figure shows superior performance of WLS in all considered proportion of LOS/NLOS links.

Next, we compare the proposed hybrid algorithm against its complements utilizing RSS-only and TOA-only

[4]Note that the authors in [17] assume that their estimator is able to *distinguish* among LOS/NLOS links. Hence, in Figs. 2-6, and 8, HWLS was implemented with perfect knowledge about bias_i (dB, m).

TABLE 3. Stability test results for the proposed estimator.

N	2	3	4	5	6	7	8
MEEC_{500}	$\begin{bmatrix} 7.25 & 0.34 \\ 0.34 & 12.21 \end{bmatrix}$	$\begin{bmatrix} 5.84 & 1.94 \\ 1.94 & 6.63 \end{bmatrix}$	$\begin{bmatrix} 3.51 & -0.23 \\ -0.23 & 3.37 \end{bmatrix}$	$\begin{bmatrix} 2.78 & -0.20 \\ -0.20 & 3.94 \end{bmatrix}$	$\begin{bmatrix} 3.31 & 0.46 \\ 0.46 & 3.33 \end{bmatrix}$	$\begin{bmatrix} 4.37 & 0.27 \\ 0.27 & 1.85 \end{bmatrix}$	$\begin{bmatrix} 1.94 & 0.12 \\ 0.12 & 2.03 \end{bmatrix}$
MEEC_{5000}	$\begin{bmatrix} 7.66 & -0.05 \\ -0.05 & 11.62 \end{bmatrix}$	$\begin{bmatrix} 5.86 & 1.72 \\ 1.72 & 6.03 \end{bmatrix}$	$\begin{bmatrix} 3.37 & -0.00 \\ -0.00 & 3.47 \end{bmatrix}$	$\begin{bmatrix} 2.80 & -0.04 \\ -0.04 & 4.10 \end{bmatrix}$	$\begin{bmatrix} 3.41 & 0.37 \\ 0.37 & 3.43 \end{bmatrix}$	$\begin{bmatrix} 4.49 & 0.08 \\ 0.08 & 1.83 \end{bmatrix}$	$\begin{bmatrix} 2.00 & 0.01 \\ 0.01 & 1.91 \end{bmatrix}$
MEEC_{50000}	$\begin{bmatrix} 7.69 & -0.03 \\ -0.03 & 11.26 \end{bmatrix}$	$\begin{bmatrix} 6.11 & 1.75 \\ 1.75 & 6.11 \end{bmatrix}$	$\begin{bmatrix} 3.41 & 0.01 \\ 0.01 & 3.41 \end{bmatrix}$	$\begin{bmatrix} 2.81 & 0.01 \\ 0.01 & 3.91 \end{bmatrix}$	$\begin{bmatrix} 3.36 & 0.37 \\ 0.37 & 3.37 \end{bmatrix}$	$\begin{bmatrix} 4.44 & 0.01 \\ 0.01 & 1.83 \end{bmatrix}$	$\begin{bmatrix} 1.93 & -0.01 \\ -0.01 & 1.90 \end{bmatrix}$
$\epsilon_i^{(500)}$	7.29 12.23	4.26 8.22	3.20 3.68	2.74 3.97	2.86 3.78	1.82 4.39	1.85 2.11
$\epsilon_i^{(5000)}$	7.66 11.63	4.23 7.67	3.36 3.47	2.80 4.11	3.05 3.78	1.82 4.49	1.91 2.01
$\epsilon_i^{(50000)}$	7.68 11.26	4.35 7.86	3.40 3.42	2.81 3.91	2.99 3.74	1.83 4.44	1.90 1.93

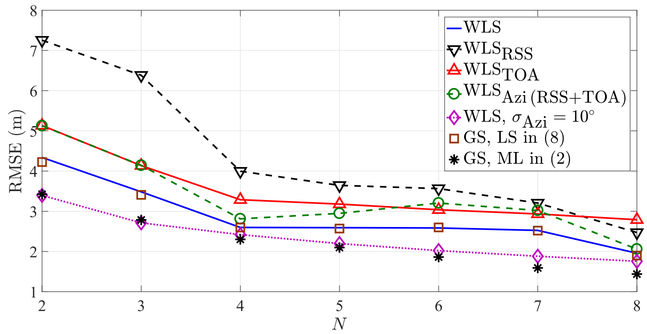


FIGURE 7. RMSE versus N comparison, when $\text{bias}_{\max} = 5$ (dB, m), $\text{bias}_i \sim \mathcal{U}[0, \text{bias}_{\max}]$, $\sigma_i = 3$ (dB, m), $|\mathcal{L}_{\text{NLOS}}| = N$, $B = 20$ m.

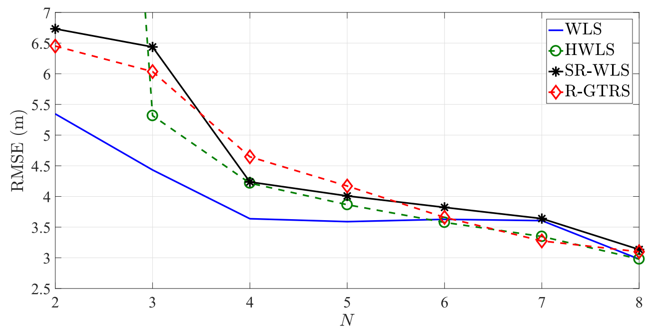


FIGURE 8. RMSE versus N comparison, when $\text{bias}_{\max} = 5$ (dB, m), $\text{bias}_i \sim \text{Exp}(\mathcal{U}[0, \text{bias}_{\max}])$, $\sigma_i = 3$ (dB, m), $|\mathcal{L}_{\text{NLOS}}| = N$, $B = 20$ m.

measurements, and azimuth angle-only estimates from (5), and present the results in Fig. 7. As a kind of a lower bound, we also include the results of WLS when, instead of estimating the azimuth angles, one is able to measure them (which requires additional hardware at anchors), denoted as "WLS, $\sigma_{\text{Azi}} = 10°$". For such a case, azimuth angle measurement error was retrieved from a von Mises distribution with zero-mean value [22], where the concentration parameter chosen to coincide with a Normal distribution with zero-mean whose noise power was set as noted explicitly in Fig. 7. Finally, the theoretical results of (2) and (8) obtained through grid search (GS) with a step of 0.1 m are also presented.[5] The figure exhibits that the hybrid approach indeed brings benefits in terms of localization accuracy in comparison with *traditional* ones. Also, it shows that the idea exploited within the proposed approach is a valid one, since the performance of WLS is competitive with its counterpart that is able to measure the azimuth angle with a relatively high precision of 10°. Lastly, it shows that the estimator in (8) is a good approximation of (2), *i.e.*, that the proposed derivation steps in Section IV make sense.

Note that the supposition that the upper bound of the NLOS biases is perfectly available, might be an oversimplified perspective of the reality. Therefore, with the objective to assess the performance of the new estimator in a more realistic setting, where the magnitude of the NLOS biases is not perfectly known, we randomly drawn the NLOS biases from an exponential distribution, where the rate parameter is generated according to a uniform distribution on the interval $[0, \text{bias}_{\max}]$ (dB, m), *i.e.*, $\text{bias}_i \sim \text{Exp}(\mathcal{U}[0, \text{bias}_{\max}])$, $i = 1, \ldots, N$, and we present the results in Fig. 8. Naturally, the figure exhibits slight performance deterioration of all considered algorithms, but the main conclusions drawn in previous settings remain unaltered, *i.e.*, the use of additional information can enhance localization performance, especially when N is low.

Lastly, we also performed a test in order to check the stability of the proposed estimator. Hence, we calculated the mean estimation error covariance (MEEC) matrix [23] as $\text{MEEC} = \sum_{i=1}^{M_c} \frac{(\boldsymbol{x}_i - \widehat{\boldsymbol{x}}_i)(\boldsymbol{x}_i - \widehat{\boldsymbol{x}}_i)^T}{M_c}$, after $M_c = 500, 5000$, and 50000, in order to to get a notation about the behaviour of the matrix. The results of this test are presented in Table 3, together with the results of the eigenvalues of MEEC, ϵ_i, for the corresponding M_c run. The table shows that MEEC is positive definite for all M_c runs; thus, the stability of MEEC is verified, *i.e.*, we can conclude that the estimation process is mean square error stable [23].

VII. CONCLUSION

This work addressed the RSS-TOA target localization problem in adverse NLOS surroundings. We took advantage of the network architecture to estimate the azimuth angle through available range observations. By making use of the fabricated azimuth information, the originally non-linear measurement models (RSS and TOA) were *linearized* by shortly switching from Cartesian to (a more natural space for exploiting

[5]In order to solve (2), true NLOS bias realizations and noise powers were used; thus, one can see the results obtained by solving (2) and (8) as theoretical lower bounds on the performance of WLS. Also, note that grid search algorithms are impractical for real-time applications due to their extensive execution time.

directional data) polar space after which the new estimator was effortlessly derived in closed-form. Hence, not only that we showed how to estimate the azimuth angle without using any auxiliary hardware (for instance antenna arrays or directional antennas), but we also showed how to amalgamate it together with RSS and TOA observations to improve localization accuracy. The new estimator exhibited superior performance over the existing ones in all considered scenarios, which is owed to fixed favorable deployment of the reference points that allowed us perfect estimation of the target's orientation in all cases. It is still an open question if perfect estimation of the target's orientation can be achieved always in ad-hoc networks, since it is extremely difficult to resolve the ambiguity problem in such a challenging setting (adverse NLOS environment with potentially inauspicious network deployments), but it is a part of our ongoing work. Nevertheless, here we showed that there is indeed a great potential in the inherent information *hidden* in the network topology, which, if availed in the right manner, can enhance significantly the localization performance.

REFERENCES

[1] R. W. Ouyang, A. K.-S. Wong, and C.-T. Lea, "Received signal strength-based wireless localization via semidefinite programming: Noncooperative and cooperative schemes," *IEEE Trans. Veh. Technol.*, vol. 59, no. 3, pp. 1307–1318, Mar. 2010.

[2] G. Wang and K. Yang, "A new approach to sensor node localization using RSS measurements in wireless sensor networks," *IEEE Trans. Wireless Commun.*, vol. 10, no. 5, pp. 1389–1395, May 2011.

[3] R. M. Vaghefi, M. R. Gholami, R. M. Buehrer, and E. G. Strom, "Cooperative received signal strength-based sensor localization with unknown transmit powers," *IEEE Trans. Signal Process.*, vol. 61, no. 6, pp. 1389–1403, Mar. 2013.

[4] S. Tomic, M. Beko, and R. Dinis, "RSS-based localization in wireless sensor networks using convex relaxation: Noncooperative and cooperative schemes," *IEEE Trans. Veh. Technol.*, vol. 64, no. 5, pp. 2037–2050, May 2015.

[5] G. Wang, H. Chen, Y. Li, and N. Ansari, "NLOS error mitigation for TOA-based localization via convex relaxation," *IEEE Trans. Wireless Commun.*, vol. 13, no. 8, pp. 4119–4131, Aug. 2014.

[6] S. Tomic and M. Beko, "A bisection-based approach for exact target localization in NLOS environments," *Signal Process.*, vol. 143, pp. 328–335, Feb. 2018.

[7] S. Zhang, S. Gao, G. Wang, and Y. Li, "Robust NLOS error mitigation method for TOA-based localization via second-order cone relaxation," *IEEE Commun. Lett.*, vol. 19, no. 12, pp. 2210–2213, Dec. 2015.

[8] S. Tomic, M. Beko, R. Dinis, and P. Montezuma, "A robust bisection-based estimator for TOA-based target localization in NLOS environments," *IEEE Commun. Lett.*, vol. 21, no. 11, pp. 2488–2491, Nov. 2017.

[9] D. Niculescu and B. Nath, "Ad hoc positioning system (APS) using AOA," in *Proc. IEEE 22nd Annu. Joint Conf. IEEE Comput. Commun. Societies (INFOCOM)*, San Francisco, CA, USA, Apr. 2003, pp. 1734–1743.

[10] S. Tomic, M. Beko, and R. Dinis, "Distributed RSS-AoA based localization with unknown transmit powers," *IEEE Wireless Commun. Lett.*, vol. 5, no. 4, pp. 392–395, Aug. 2016.

[11] M. Khan, N. Salman, A. Kemp, and L. Mihaylova, "Localisation of sensor nodes with hybrid measurements in wireless sensor networks," *Sensors*, vol. 16, no. 7, p. 1143, Jul. 2016.

[12] S. Tomic, M. Beko, R. Dinis, and P. Montezuma, "A closed-form solution for RSS/AoA target localization by spherical coordinates conversion," *IEEE Wireless Commun. Lett.*, vol. 5, no. 6, pp. 680–683, Dec. 2016.

[13] Y. Wang and K. C. Ho, "An asymptotically efficient estimator in closed-form for 3-D AOA localization using a sensor network," *IEEE Trans. Wireless Commun.*, vol. 14, no. 12, pp. 6524–6535, Dec. 2015.

[14] S. Tomic, M. Beko, and R. Dinis, "3-D target localization in wireless sensor networks using RSS and AoA measurements," *IEEE Trans. Veh. Technol.*, vol. 66, no. 4, pp. 3197–3210, Apr. 2017.

[15] J. Zhang, Y. Wang, L. Hu, and L. Ding, "Measurement-based indoor NLoS ToA/RSS range error modelling," *Electron. Lett.*, vol. 52, no. 2, pp. 165–167, Jan. 2016.

[16] A. Coluccia and A. Fascista, "On the hybrid TOA/RSS range estimation in wireless sensor networks," *IEEE Trans. Wireless Commun.*, vol. 17, no. 1, pp. 361–371, Jan. 2018.

[17] S. Tiwari, D. Wang, M. Fattouche, and F. Ghannouchi, "A hybrid RSS/TOA method for 3D positioning in an indoor environment," *ISRN Signal Process.*, vol. 2012, pp. 1–9, Jan. 2012.

[18] S. Tomic, M. Beko, M. Tuba, and V. M. F. Correia, "Target localization in NLOS environments using RSS and TOA measurements," *IEEE Wireless Commun. Lett.*, vol. 7, no. 6, pp. 1062–1065, Dec. 2018.

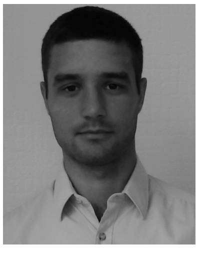
[19] S. Tomic and M. Beko, "A robust NLOS bias mitigation technique for RSS-TOA-Based target localization," *IEEE Signal Process. Lett.*, vol. 26, no. 1, pp. 64–68, Jan. 2019.

[20] S. Tomic, M. Beko, and M. Tuba, "A linear estimator for network localization using integrated RSS and AOA measurements," *IEEE Signal Process. Lett.*, vol. 26, no. 3, pp. 405–409, Mar. 2019.

[21] D. Macii, A. Colombo, P. Pivato, and D. Fontanelli, "A data fusion technique for wireless ranging performance improvement," *IEEE Trans. Instrum. Meas.*, vol. 62, no. 1, pp. 27–37, Jan. 2013.

[22] K. V. Mardia, *Statistics of Directional Data*. London, U.K.: Academic, 1972.

[23] B. Sinopoli, L. Schenato, M. Franceschetti, K. Poolla, M. I. Jordan, and S. S. Sastry, "Kalman filtering with intermittent observations," *IEEE Trans. Autom. Control*, vol. 49, no. 9, pp. 1453–1464, Sep. 2004.



SLAVISA TOMIC received the M.S. degree in traffic engineering according to the postal traffic and telecommunications study program from the University of Novi Sad, Serbia, in 2010, and the Ph.D. degree in electrical and computer engineering from the University Nova of Lisbon, Portugal, in 2017. He is currently an Assistant Professor with the Universidade Lusófona de Humanidades e Tecnologias, Lisbon, Portugal. His research interests include target localization in wireless sensor networks and non-linear and convex optimization.

MARKO BEKO was born in Belgrade, Serbia, in November 1977. He received the Ph.D. degree in electrical and computer engineering from the Instituto Superior Tecnico, Lisbon, Portugal, in 2008. He received the title of the Professor com Agregação of electrical and computer engineering from the Universidade Nova de Lisboa, Lisbon, in 2018. He is currently a Full Professor (a Professor Catedrático) with the Universidade Lusófona de Humanidades e Tecnologias, Lisbon. He is the winner of the 2008 IBM Portugal Scientific Award. He serves as an Associate Editor for the IEEE Open Journal of the Communications Society and *Journal on Physical Communication* (Elsevier).

MILAN TUBA (Member, IEEE) received the B.S. and M.S. degrees in mathematics, and the M.S., M.Ph., and Ph.D. degrees in computer science from the University of Belgrade and New York University. He is currently the Vice Rector of International Relations, Singidunum University, Belgrade, and the Head of the Department of Mathematical Sciences, State University of Novi Pazar. From 1983 to 1994, he was with Vanderbilt University, Nashville, TN, USA, and the Courant Institute of Mathematical Sciences, New York University, and later as an Assistant Professor of electrical engineering with the Cooper Union School of Engineering, New York. His research interests include heuristic optimizations applied to computer networks, image processing, and combinatorial problems. He is the author or coauthor of more than 150 scientific articles and coeditor or a member of the editorial board or scientific committee of number of scientific journals and conferences. He is a member of the ACM, AMS, SIAM, and IFNA.