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Procedia Structural Integrity 5 (2017) 1318–1325

Structural Integrity Procedia

www.elsevier.com/locate/procedia

2nd International Conference on Structural Integrity, ICSI 2017, 4-7 September 2017, Funchal, Madeira, Portugal

Application of the Effective Impulse Approach to Stairs

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Abstract: One of the most commonly used simplified methods for predicting man induced vibrations in floors with high fundamental frequencies is the Effective Impulsive approach, first developed by the ARUP's company and later modified by the design guide SCI P354. Since the Effective Impulse approach was designed to be used in floors, its use in stairs can be arguable. To better understand the effectiveness of this method in stairs, in this paper are experimentally measured vibrations on a staircase with a poor dynamic behavior and then compared to the vibrations predicted using the Effective Impulse approach. The results indicate that this approach can be used, especially in the stair descends. The serviceability of the analyzed staircase was also verified by comparing the measured and predicted vibrations with the acceptable limits proposed by various authors and design guides.

Peer-review under responsibility of the Scientific Committee of ICSI 2017

Keywords: Vibration Serviceability; Impulsive Response; Effective Impulse; Man induced vibrations;

1. Introduction

Vibration serviceability is becoming an increasingly important research topic thanks to pedestrian structures such as footbridges and monumental stairs that have failed, in full public view, to perform adequately under human dynamic loading. Depending on the type of dynamic response, structures can be divided into two categories, low and high frequency structures. Low frequency structures (LFS), as the name implies, are those whose fundamental frequency is low, while high frequency structures (HFS) are those whose fundamental frequency is high. The main difference between these two kinds of structures is that LFS respond harmonically with a resonant response and HFS respond

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Peer-review under responsibility of the Scientific Committee of ICSI 2017
10.1016/j.prostr.2017.07.140

impulsively with a transient response (Brownjohn and Middleton (2007)). Currently staircases are usually designed with high fundamental frequencies in order to avoid the occurrence of resonant effects. However, due to the high stiffness and low mass normally seen in this type of construction, the resulting impulsive responses can also be significant. Staircases with fundamental frequencies lower than 16Hz should be treated as being LFS with the potential for resonant effects to occur. This cut-off frequency is higher than the one commonly used for floors (10Hz) (Middleton and Brownjohn (2009)). Based on Kerr's (1998) walking force data, ARUP's company (Wilford et al. (2006a; 2006b)) developed a simplified method for predicting vibrations in floors with high fundamental frequencies, based on an 'Effective Impulse'. Although they claim the Effective Impulse approach is superior to other methods because it is based on fundamental theory, the same was only developed to be applied in floors and is questionable his further use in other structures. The design guide SCI P354 (2009) also presents an Impulsive Effective approach, although with some modifications, based on the EN 1990 annex C (2009) (see Section 6). Since there isn't at the moment a method for adequately predict the response of high frequency stairs to walking forces, the objective of this paper is to assess if the Effective Impulse approach can properly predict the vibrations in this type of structures. In order to do that, the vibrations on a steel staircase, which had a well-known level of liveness, were measured experimentally and then compared with the vibrations calculated using the Effective Impulsive approach. The predicted and measured vibrations were also compared to the acceptability criteria proposed by the authors and design guides mentioned in Section 6 in order to demonstrate that impulsive responses can also lead to expressive vibration problems.

2. Effective Impulse

2.1. Arup's approach

The Effective Impulse approach was first developed by Wilford et al. (2006a; 2006b) with the objective of providing to the ARUP's structural design company a simplified method to calculate vibrations numerically in high frequency floors (HFF).

The method was obtained from the application of more than 800 footfall forces with different step frequencies, recorded by Kerr (1998) using a force plate, to SDOF (single degree of freedom) oscillators with unit mass and a range of natural frequencies. Accordingly with the first principles of dynamics, if an impulse is applied to a SDOF oscillator with unit mass, the resultant velocity is equal to the magnitude of the impulse. Consequently if in a SDOF oscillator with unit mass and a certain natural frequency is applied a load function with a given step frequency, its resultant velocity corresponds to an impulse which can be considered as the maximum force caused by a footfall for that particular natural frequency of the oscillator (Midleton and Brownjohn (2008)). Curve fitting was applied to the responses obtained for the 800 load functions and the Effective Impulse given by the Equation (1) was determine considering a weight of an average person of 700 N:

$$I_{eff} = C f_s^{1,43} / f_i^{1,30} \tag{1}$$

Where I_{eff} is the Effective Impulse (Ns), f_s is the step frequency (Hz) and f_i is the natural frequency of the mode i of the floor (Hz). C is a constant in which the mean value is set at 42 Ns, but for design purposes is set at 54 Ns with a 75% probability of not being exceeded.

The response caused by the Effective Impulse (Equation (1)) can be seen as the response caused by a footfall on floors with high natural frequencies (Wilford et al. (2006)). The Arup's Effective Impulse approach is widely accepted as the most rational and the one that presents more realistic results (Brownjohn and Middleton (2007a; 2008b; 2009c)) and forms the basis of two of the most important design guides in the UK (SCI P354 (2009); CSTR43 (2005)).

2.2. SCI P354 approach

The design guide SCI P354 (2009) presents some alterations to the Arup's Effective Impulse (Wilford et al. (2006a; 2006b). Equation (2) demonstrates the Effective Impulse given by SCI P354 (2009).

$$I_{eff} = 60 f_s^{1,43} / f_i^{1,30} (\frac{P}{700}) \quad (2)$$

Where P is the static force exerted by an 'average person', normally taken as 746N. The constant C is increased to 60 Ns in accordance with EN 1990 annex C (2009). The main advantage of using the Effective Impulse from SCI P354 (2009) is because this design guide presents a simplified expression to calculate the accelerations caused by it, which is the most common way to quantify human induced vibrations. The accelerations generated by the Effective Impulse (Equation (2)) can be determined, according to SCI P354 (2009), by the Equation (3):

$$a(t) = \sum_{n=1}^{N} 2\pi f_i \sqrt{1-\xi^2} \mu_{e,i} \mu_{r,i} \frac{I_{eff}}{M_i} \sin\left(2\pi f_i \sqrt{1-\xi^2} t\right) e^{-\xi^2 \pi f_i t} W_i \quad (3)$$

Where $\mu_{e,i}$ is the mode shape amplitude, from the unity or mass normalized FE output, at point on the floor where the Effective Impulse I_{eff} is applied, $\mu_{r,i}$ is the mode shape amplitude, from the unity or mass normalized FE output, at the point where the response is to be calculated, M_i is the modal mass of mode i (equal to 1 if mode shapes are mass normalized) (Kg) and W_i is a weighting factor for human perception of vibrations, which depends on the direction of the vibrations on the human body using the basicentric coordinate system and the frequency of the mode under consideration f_i.

3. Experimental Program

3.1. Description of the staircase

The monumental staircase studied in this paper, which has a steel structure, is located in a public building in Funchal, Madeira. It connects the first and second levels of the building. Figure 1 is a photo showing a partial view of the stair. The public building staircase in addition to have a high vibration level, its dynamic behavior is different from what is normally verified in the generality of the stairs. In the steel staircase analyzed the vibrations are at the local level, i.e., the vibrations verified at the level of the steps are independent of those verified in the rest of the structure of the staircase.

Since the vibrations are local it is only of interest to describe the geometrical proprieties of the steps. The steps are composed by a metal plate with a thickness of 6 mm and a coating of synthetic rubber sheet. In Figure 1, are shown its dimensions, height, length and width measured on the site. The connection between the steps and the rest of the structure of the stair is made by means of an auxiliary plate that is welded to the steps and to the stringers.

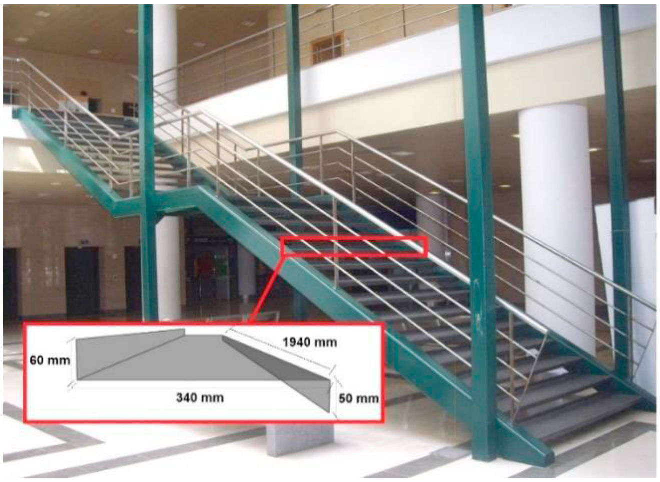

Figure 1 – Steel staircase studied and geometrical properties of the steps

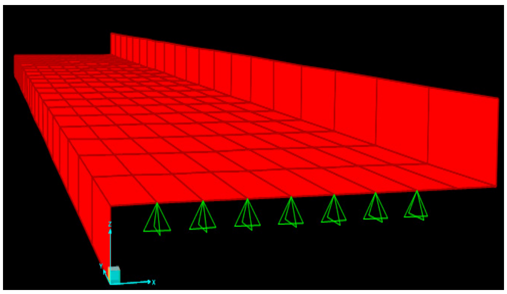

Figure 2 – Numerical Model of the Step

3.2. Dynamic properties

Modal tests were conducted to estimate the natural frequencies and damping of the stair. The frequencies of the first two local modes are respectively 24.0Hz and 45.6Hz. These frequencies are different from the frequencies of the global modes obtained experimentally, clearly demonstrating that the dynamic behavior of the steps are independent from the rest of the staircase. This can be explained by the fact that the welding that joins the steps to the stringers, due to the wear and tear suffered over the years, has a practically null rotational stiffness (low degree of fixation). The frequencies obtained at the local level have high values, evidencing that the response of the stair should be impulsive and not in resonance.

In addition to define the frequencies of the modes, it is important to characterize experimentally their modal shapes. The shapes of the first two vibration modes obtained experimentally, in their respective modal coordinates, are represented in Figures 3 and 4. It is possible to observe that the first mode (24.0Hz) is vertical with some torsion and the second mode (45.6Hz) is exclusively of torsion.

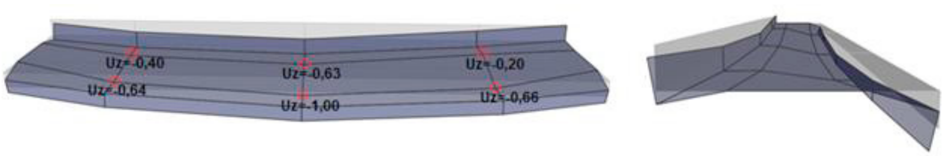

Figure 3 – Experimental shape of the first mode (24.0Hz)

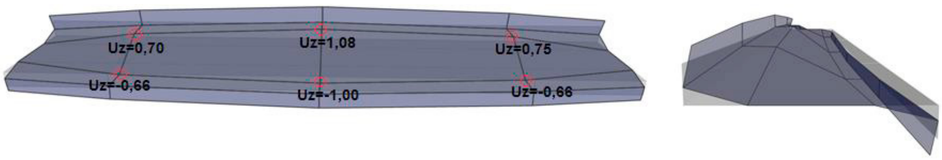

Figure 4 – Experimental shape of the second mode (45.6Hz)

The damping was estimated to be 0.82% of the critical, using the half-power bandwidth method. This value is in agreement with the authors González (2013), Bishop et al. (1995) and Davis et al. (2009) who obtained in their measurements a damping of approximately 1%.

3.3. Walking tests

As can be seen in the previous subsection the frequencies of the local modes have high values, this means that the step frequencies usually used in stairs (2.0 to 4.5Hz) will hardly excite it so that a resonant response is obtained. In addition, in each stair step only one footfall is applied, so there isn't possible a resonant build-up. For this reason, were selected in the walking tests step frequencies that appear to be more plausible when ascending and descending a staircase and not step frequencies in order to obtain a resonant response. For an isolated pawn it was decided to use the following step frequencies: 2.0Hz for a normal ascend, 3.0Hz for a fast ascent, 2.5Hz for a normal descent and 3.5Hz for a fast descent.

In the determination of the accelerations due to the footfall forces it were used two accelerometers. Both were placed at midspan of a random step because individuals when walking a staircase (if not accompanied) tend to place the foot approximately in that location and also for the fact that is the point where larger responses are generated. In Table 1 is the description of the experimental individual tests carried out on the public building steel staircase.

In order to verify the group effect, experimental tests with a group of individuals were also performed. As the stair steps have a relatively long length (1.94 m), it is possible for individuals to walk them in two distinct ways: one behind the other or side-by-side. With this in mind two types of experimental tests were performed, using a group of 4 individuals, designated 1+1+1+1 and 2+2. In the group tests (1+1+1+1) the 4 individuals walked the staircase sequentially, one behind the other, with a separation step between each individual and in the group tests (2+2) the 4 subjects walked the staircase side-by-side, two elements at a time. It was also chosen in this type of group test to leave a spacing of one step between each pair of individuals. In both types of group test there was an attempt for the individuals to cross the stair steps with the same pacing rate and phase shift between them, according to Kerr (1998) this is the case that gives rise to a larger group enhancement effect. For a group of walkers the following step frequencies were chosen: 2.0Hz for normal ascent, 2.5Hz for normal descent and 3.5Hz for rapid descent. In Table 2 is the description of the experimental tests performed for a group of walkers.

Table 1 – Description of the experimental tests performed for an isolated pawn

Isolated pawn	Number of trials
Ascent 2.0Hz	4
Ascent 3.0Hz	4
Descent 2.5Hz	4
Descent 3.5Hz	4

Table 2 – Description of the experimental tests performed for a group of walkers

Group of walkers (1+1+1+1)	Group of walkers (2+2)	Number of trials
Ascent 2.0Hz	Ascent 2.0Hz	4
Descent 2.5Hz	Descent 2.5Hz	4
Descent 3.5Hz	Descent 3.5Hz	4

4. Numerical model

To implement the effective impulse approach some dynamic properties (mode shapes) of the structure are required (see Section 2). In order to accurately determine the dynamic properties of the public building staircase, a finite element (FE) model was created using the structural analysis program SAP2000 (2013). Since the vibrations are local (see Subsection 3.2) it was only necessary to create a numerical model of one of the stair steps. The metal plate that constitutes the step was modelled by shell elements with a thickness of 6 mm. The synthetic rubber sheet coating has a much reduced thickness, and its contribution to the stiffness of the step can be negligible, therefore in the modelling only its mass (6 kg/m^2) was taken into account. The numerical model was created considering the dimensions of the stair step seen in Figure 1. The connection (through welding) between the step and the stringers was simulated using pinned supports. Figure 2 shows the numerical model of the stair step.

After the construction of the FE model standard eigenvalue analysis was used to predict its vibration modes and respective natural frequencies. In Table 3 are compared the local vibration modes obtained numerically with those measured experimentally (see Subsection 3.2). It can be observed from Table 3 that the numerical model created was able to predict approximately the vibration modes. The use of pinned supports adequately simulated the lack of rotational stiffness in the connection between the steps and the stringers verified in the actual stair.

Table 3 – Local vibration modes obtained numerically and measured experimentally

Nº	Shape	Numerical Frequency (Hz)	Experimental Frequency (Hz)
1	Vertical w/ torsion	24,1	24,0
2	Torsion	42,6	45,6

5. Numerical Analysis

The accelerations were calculated numerically using the SCI's (2009) Effective Impulse approach (Equation (2) and (3)) described in Subsection 2.3, for the reasons mentioned in there. The parameters utilized to define Equation (3) and generate the accelerations due to Effective Impulse (Equation (2)) are presented on Table 4. These parameters were obtained from the local numerical model described in Section 4. To determine the accelerations were used step frequencies of 2.0Hz, 2.2Hz, 3.0Hz and 3.3Hz to be coherent with those verified after the evaluating of the experimental results. In total four simulations were done, one for each step frequency.

The modes shapes $\mu_{e,i}$ and $\mu_{r,i}$ have equivalent amplitude because the accelerations were analyzed on the same node where the Effective Impulse was applied.

The Effective Impulse is applied at the middle of the step and the second mode of vibration is torsional, however the mode shape amplitude $\mu_{e,2}$ is not exactly equal to zero (see Table 4) because the second vibration mode doesn't has a pure torsion, in other words the rotation is not made exactly around the middle axis of the step.

The weighting factor W_i was obtained by Equation (4) present in SCI P354 (2009). In accordance with SCI P354 (2009). Equation (4) should be employed when there are structures whose natural frequencies are higher than 16Hz and the direction of incidence of vibrations is the vertical axis.

$$W_i = \frac{16}{f_i} \tag{4}$$

Where f_i corresponds to the natural frequency of the mode under consideration.

Table 4 – Parameter values utilized to define Equation (3)

1º mode	Step frequency				2º mode	Step frequency			
Parameters	2.0Hz	2.2Hz	3.0Hz	3.3Hz	Parameters	2.0Hz	2.2Hz	3.0Hz	3.3Hz
$I_{eff,1}$ $[N.s]$	2,5	2,9	4,5	5,1	$I_{eff,2}$ $[N.s]$	1,2	1,4	2,1	2,5
M_1 $[Kg]$	1,0				M_2 $[Kg]$	1,0			
f_1 $[Hz]$	24,1				f_2 $[Hz]$	42,6			
W_1	0,67				W_2	0,38			
$\mu_{e,1} = \mu_{r,1}$	0,1883				$\mu_{e,2} = \mu_{r,2}$	0,0635			
ξ [%]	0,82				ξ [%]	0,82			

6. Analytical Predictions and Comparison to Measurements

In order to facilitate the comparison between the numerical and experimental results relatively to an isolated pawn, two graphs were made, one for the ascents and one for the descents, including all the accelerations obtained. In both graphs, peak accelerations were used, since these are the conditioning values. The acceptability criteria proposed by the author's Bishop et al. (1995) and Davis et al. (2009) and by the design guides SCI P354 (2009) and AISC 11 (1997) were also inserted in the graphs with the intention to verify if the measured and predicted peak accelerations satisfy the proposed criteria. The limit proposed in AISC 11 (1997) is for indoor footbridges, however it is also commonly used for stairs. For a group of walkers, the numerical accelerations were quantified applying amplification factors, as described in subsection 6.3.

6.1. Stair ascents

The graph with the comparison between the predicted and measured peak accelerations obtained to one person ascending the stairs is represented in Figure 5. As can be observed with the Effective Impulse approach, both for the ascent at 2.0Hz and for the ascent at 3.0Hz, were obtained accelerations overestimated in comparison with those measured experimentally. The measured and predicted accelerations far exceed the proposed limits. Since accelerations increase with the step frequency this difference is even higher for ascents at 3.0 Hz.

Figure 6 shows a comparison between the accelerations measured experimentally due to a footfall obtained by a pedestrian ascending the stair at a pacing rate close to 3.0Hz and the ones calculated using the Effective Impulse approach for a pacing rate of 3.0Hz. The difference between the peaks predicted and measured is approximately 5 m/s^2. It also can be seen that the responses are at or near the natural frequency (24.0Hz), clearly demonstrating that the vibrations in the staircase studied are local and not global. As was expected the responses also have an impulsive character.

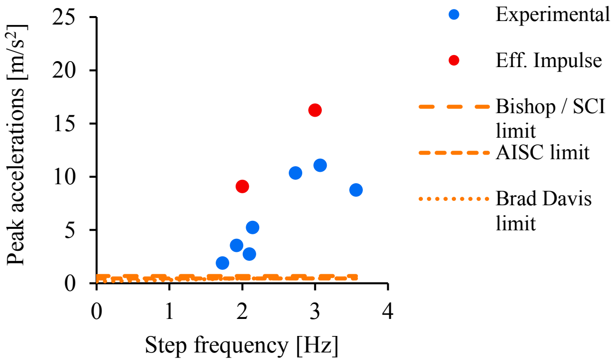

Figure 5 – Predicted and measured peak accelerations for ascents

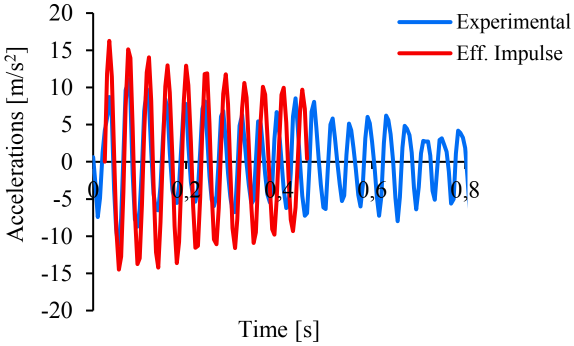

Figure 6 – Accelerations due to an ascent at 3.0Hz

6.2. Stair descents

The graph with the comparison between the predicted and measured peak accelerations obtained to one person descending the stairs is represented in Figure 7. The peak accelerations obtained by the Effective Impulse approach are close to the experimental measurements, both for the descent at 2.2Hz and for the descent at 3.3Hz. The fact that accelerations in stairs are higher in descents and the fact that the accelerations obtained with the Effective Impulse approach only increase with the increase in the step frequency can help to explain why the predicted and measured responses were closer in the descents. The accelerations in the descents, for some cases, reach values about twice the gravitational acceleration (≈ 9,8 m/s^2), clearly being much higher than the acceptable limits, even more than in ascents.

Figure 8 shows a comparison between the accelerations measured experimentally due to a footfall obtained by a pedestrian descending the stair at a pacing rate close to 3.30Hz and the ones calculated using the Effective Impulse approach for a pacing rate of 3.3Hz.

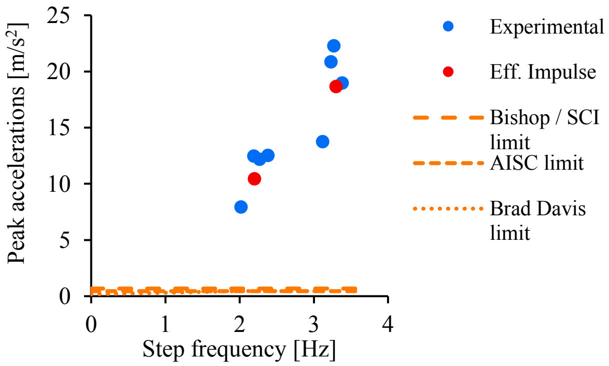

Figure 7 – Predicted and measured peak accelerations for descents

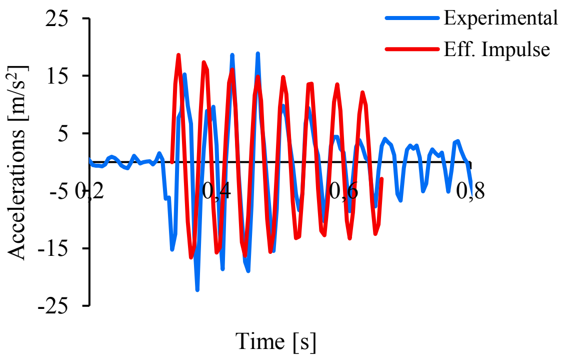

Figure 8 – Accelerations due to an descent at 3,30Hz

6.3. Results for a group of pedestrians

Experimentally the group enhancement effect was verified by obtaining amplification factors between the accelerations relative to a group of walkers and the accelerations relative to an isolated pawn. In the group tests (1+1+1+1), were obtained mean amplification factors of approximately 3.0, 1.5 and 1.0 respectively for ascents at 2.0 Hz and for descents at 2.5 Hz and 3.5 Hz. In the group tests (2 + 2), were obtained mean amplification factors of approximately 2.0, 1.5 and 1.0 respectively for ascents at 2.0 Hz and for descents at 2.5 Hz and 3.5 Hz. The group enhancement effect is practically negligible for descents at 3.5Hz, as can be seen from the amplification factors obtained for this step frequency. It is not possible to obtain accelerations relative to a group of walkers directly through the application of the Effective Impulse approach, therefore the group accelerations were obtained by multiplying the individual accelerations obtained with the Effective Impulse by the amplification factors mentioned previously. For the descents at 2.5Hz and at 3.5Hz, through the use of amplification factors, were obtained numerical accelerations close to those measured experimentally, for the two types of group tests (1+1+1+1 e 2+2) analyzed. For the ascents at 2.0Hz, were obtained numerical accelerations overestimated for both types of group tests (1+1+1+1 e 2+2) because the accelerations obtained for an isolated pawn using the effective impulse approach were also overestimated (see Figure 5).

7. Summary and Conclusions

Applying the Effective Impulse approach, for the ascent at 2.0Hz and 3.0Hz were obtained overestimated accelerations in comparison with those measured experimentally. For the descents at 2.2Hz and 3.3Hz, the Effective Impulse approach proved to be efficient, since the accelerations measured experimentally were approximated with those predicted. The SCI's and ARUP's Effective Impulse approach present satisfactory results for both descents, and these are the most conditioning cases. So, this method can be applied to stairs.

Regardless of being possible to obtain satisfactory results in the stair descends it is suggested that an impulsive effective approach should be conceived to be directly applied to stairs, to take in account the distinct dynamic forces and footfall rate employed when descending and ascending the stairs from that verified when walking across floors.

The effective impulse was not conceived to determine accelerations due to a group of walkers, this being one of the shortcomings of this methodology, so the numerical group accelerations were obtained through the use of amplification factors. Satisfactory results were obtained for the descents at 2.2Hz and 3.3Hz and overestimated results for the ascents at 2.0Hz.

The accelerations predicted and measured reach values close to the double of the gravitational acceleration ($\approx$ 9,8 m/s^2) for pacing rates close to 3.3 Hz, for an isolated walker and a group of walkers, far exceeding the acceptable limits proposed by the author's Bishop et al. (1995) and Davis et al. (2009) and by the design guides SCI P354 (2009) and AISC 11 (1997).

Acknowledgements

This work was financially supported by: Project POCI-01-0145-FEDER-007457 - CONSTRUCT - Institute of R&D In Structures and Construction, funded by FEDER funds through COMPETE2020 - Programa Operacional Competitividade e Internacionalização (POCI) and by national funds through FCT - Fundação para a Ciência e a Tecnologia.

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